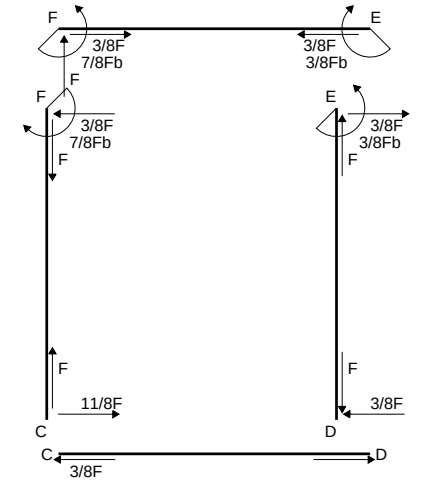
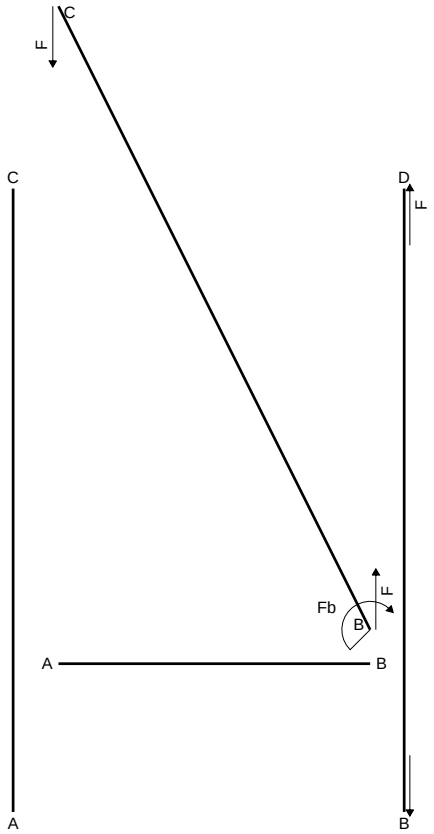
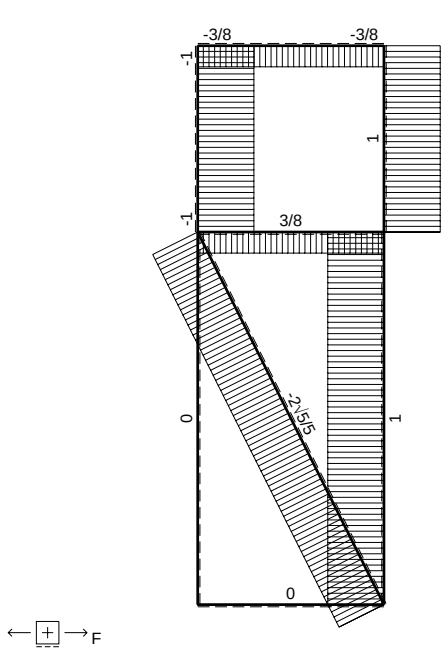
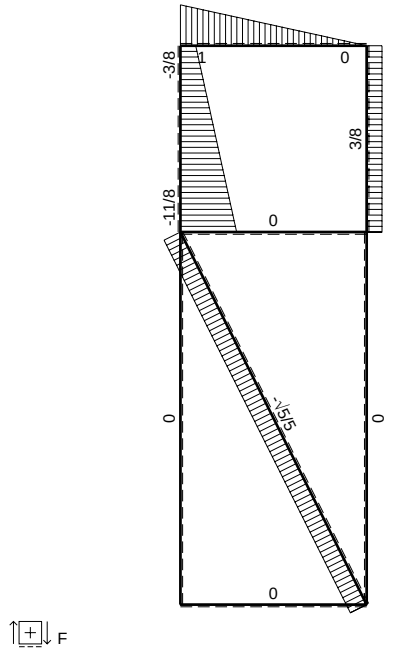
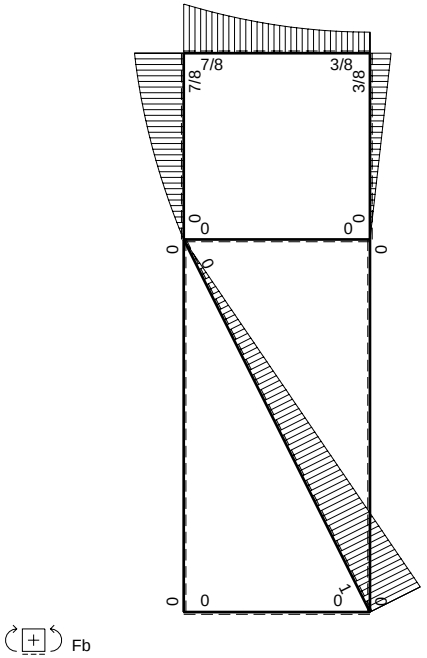

F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

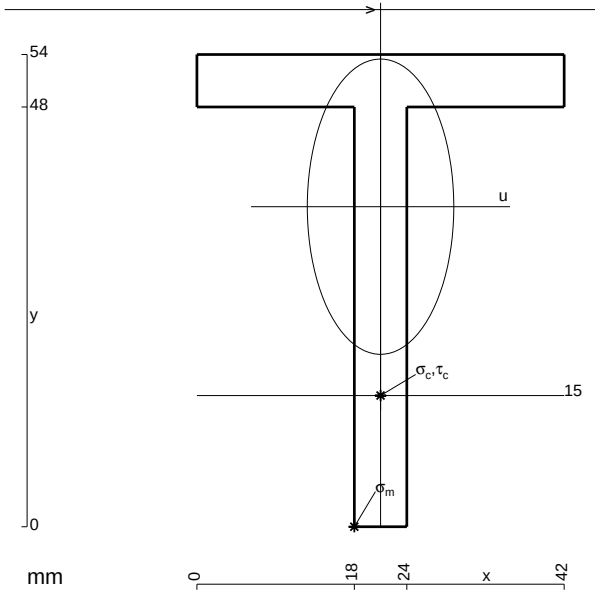
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211009 ESΣ09.xxxx.001



A = 540. mm^2
J_u = 154030. mm^4
J_v = 37908. mm^4
y_g = 36.6 mm
N = -1959. N
T_y = -979.4 N
M_x = 897900. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -36.6 mm
σ_m = N/A-Mv/J_u = 209.7 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -21.6 mm
σ_c = N/A-Mv/J_u = 122.3 N/mm^2
τ_c = 2.775 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.4 N/mm^2
S^* = 2619. mm^3

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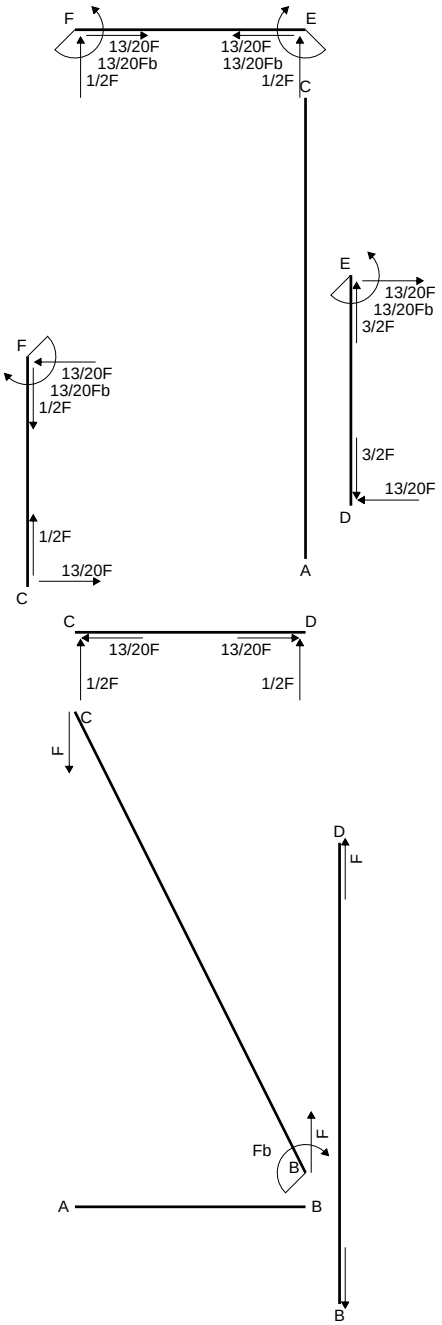
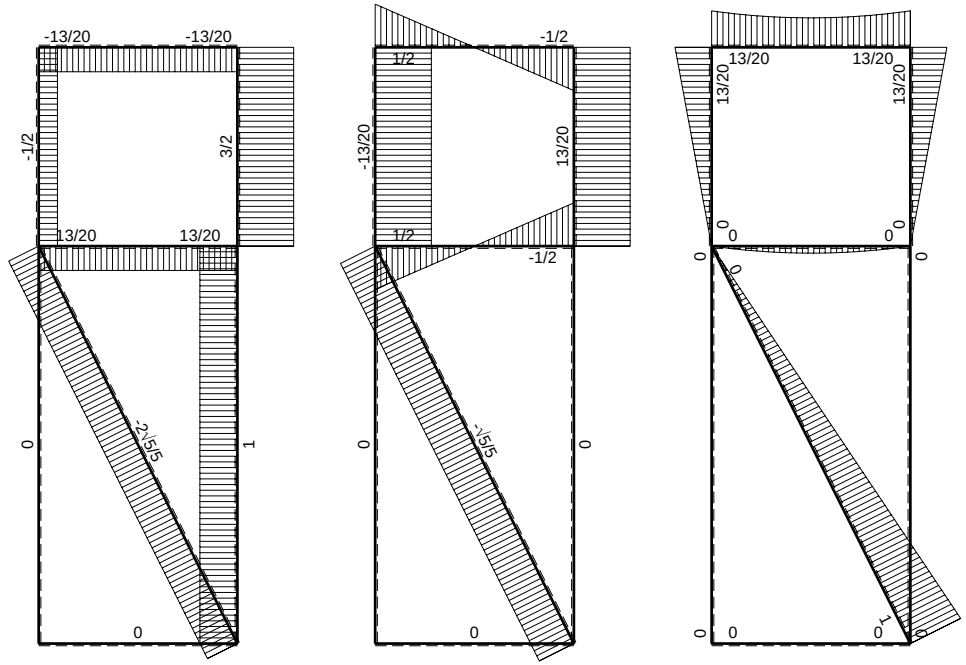
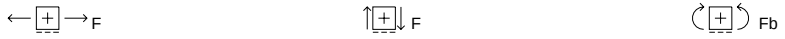
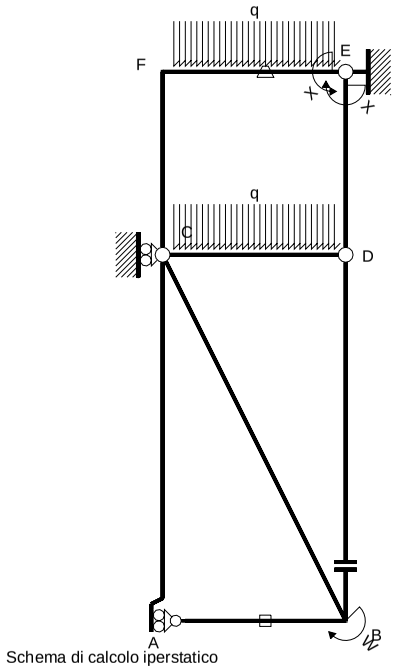
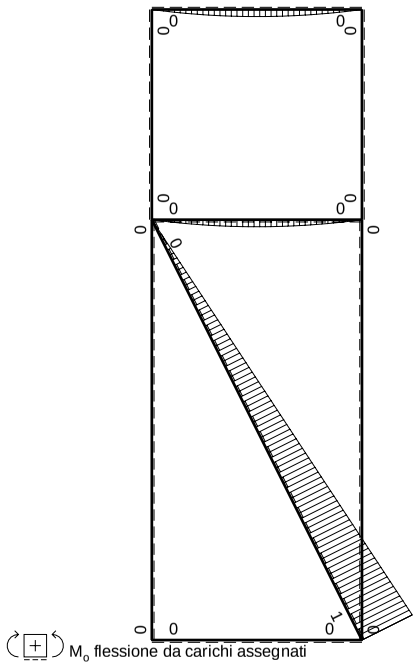

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

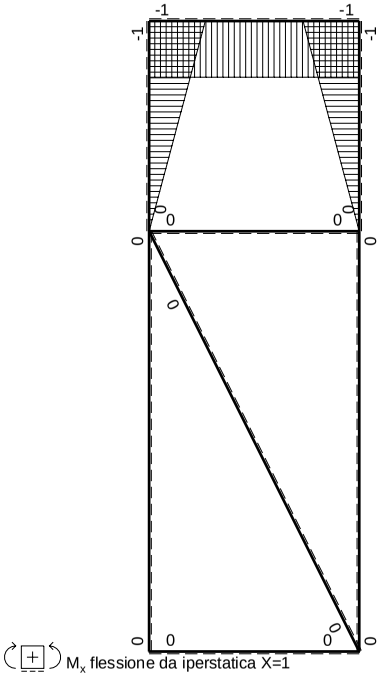

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 214234 ESΣ09.xxxx.002

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

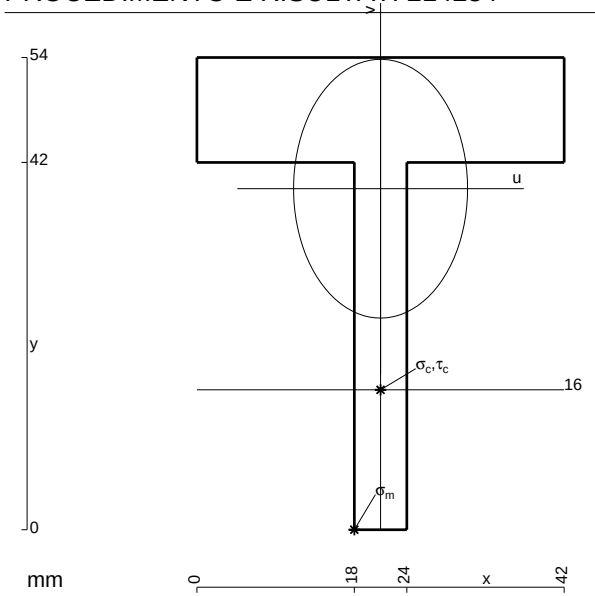
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 756. mm^2
J_u = 165564. mm^4
J_v = 74844. mm^4
y_g = 39. mm
N = -1869. N
T_y = -934.7 N
M_x = 940500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39. mm
$\sigma_m = N/A - Mv/J_u$ = 219.1 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -23. mm
$\sigma_c = N/A - Mv/J_u$ = 128.2 N/mm^2
τ_c = 2.8 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.3 N/mm^2
S^* = 2976. mm^3

ESΣ09.xxxx.002

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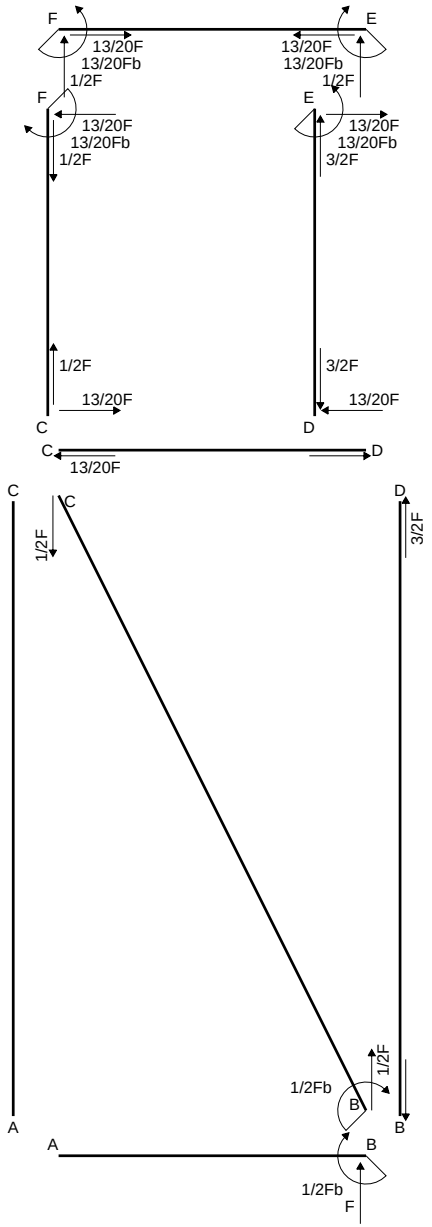
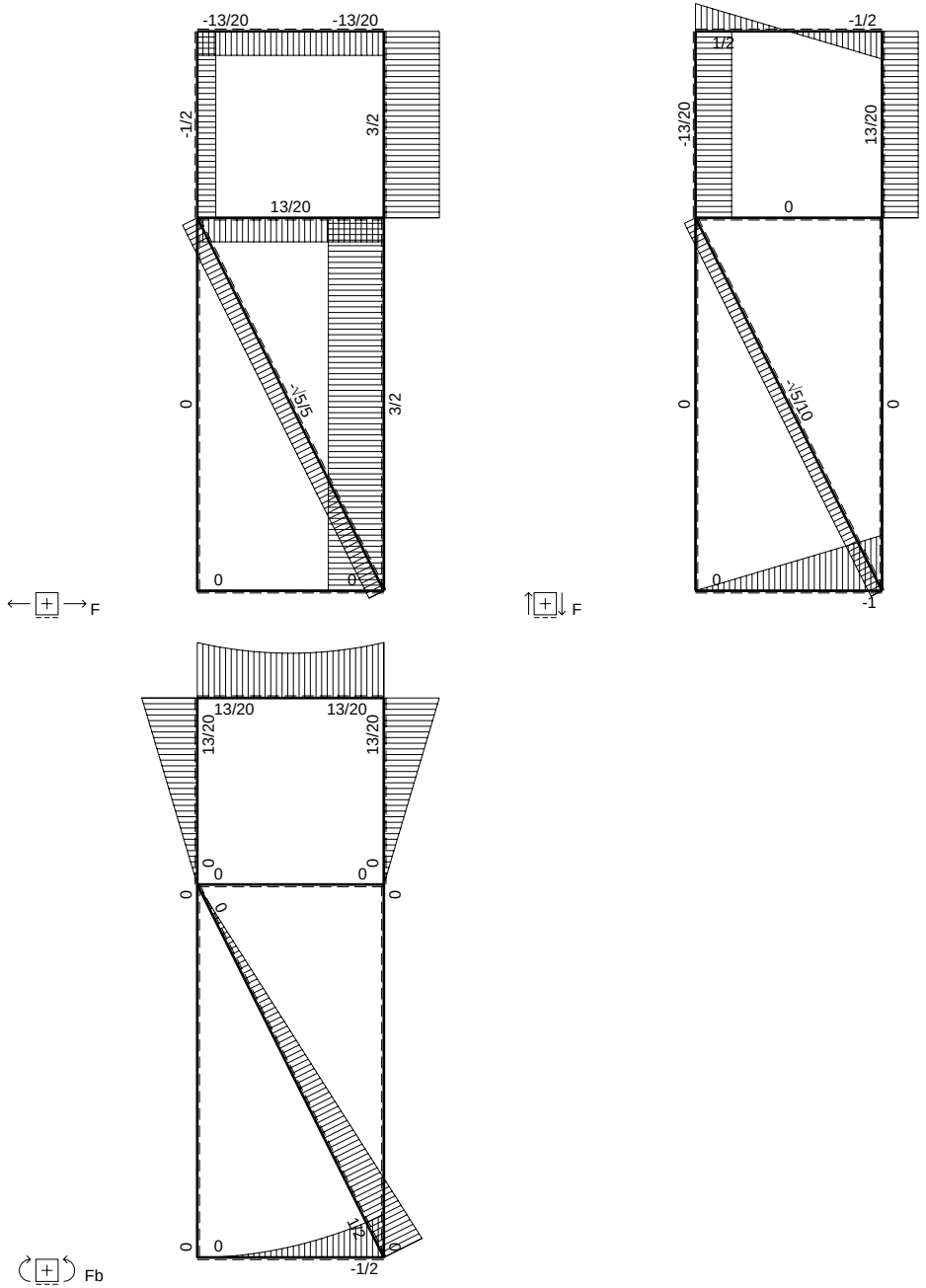
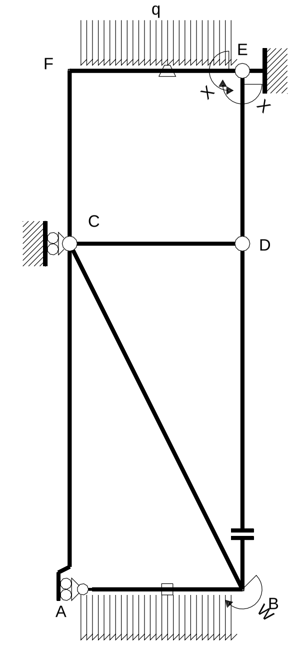
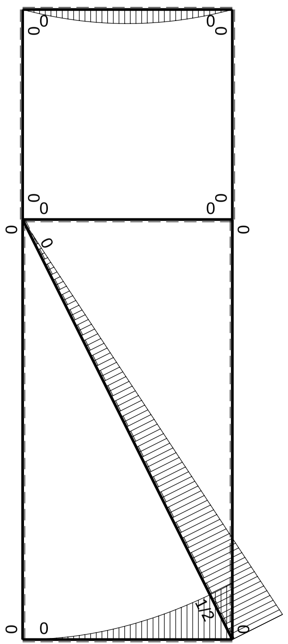

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

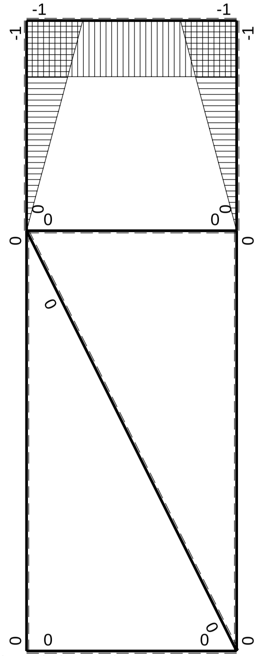

-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

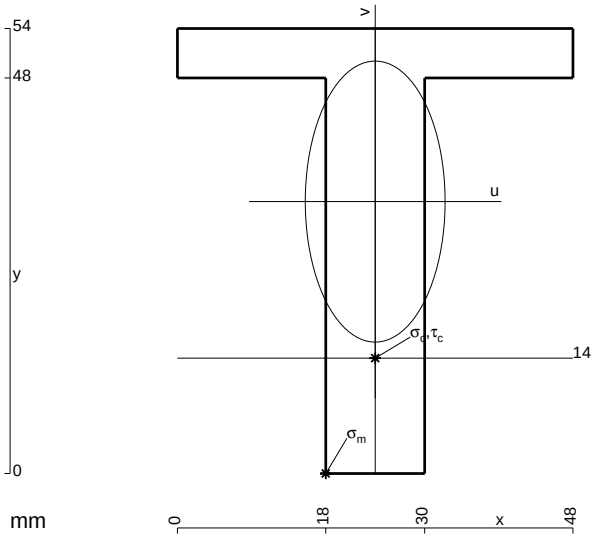
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 981883 ESΣ09.xxxx.003



A = 864. mm^2
J_u = 251424. mm^4
J_v = 62208. mm^4
y_g = 33. mm
T_y = -7150. N
M_x = -1751750. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33. mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19. mm
$\sigma_c = -Mv/J_u$ = -132.4 N/mm^2
τ_c = 10.35 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.6 N/mm^2
S^* = 4368. mm^3

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ESΣ09.xxxx.003

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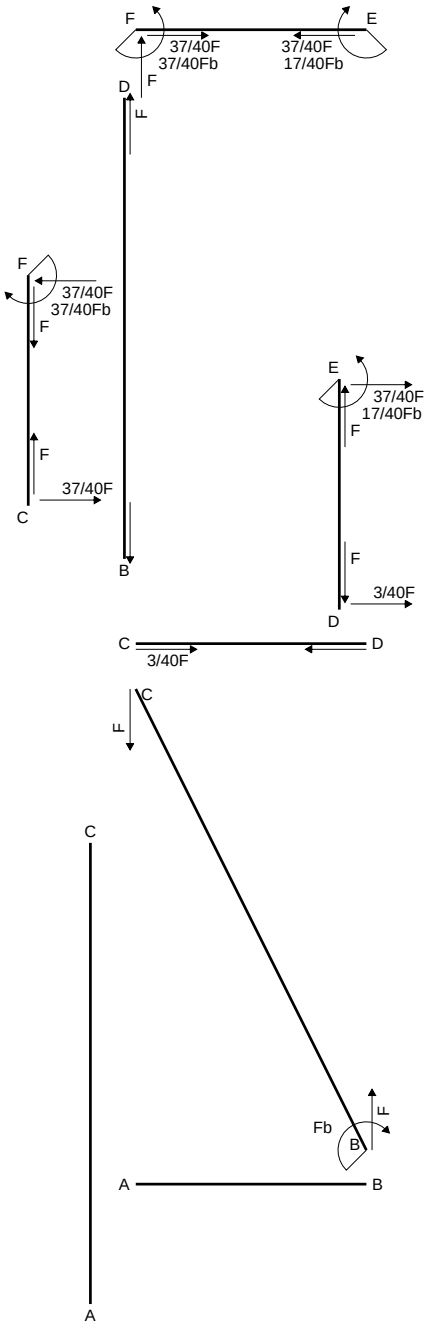
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E
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37/40Fb
37/40F
17/40Fb
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F
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37/40Fb
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C
B
E
37/40F
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3/40F
D
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C
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A
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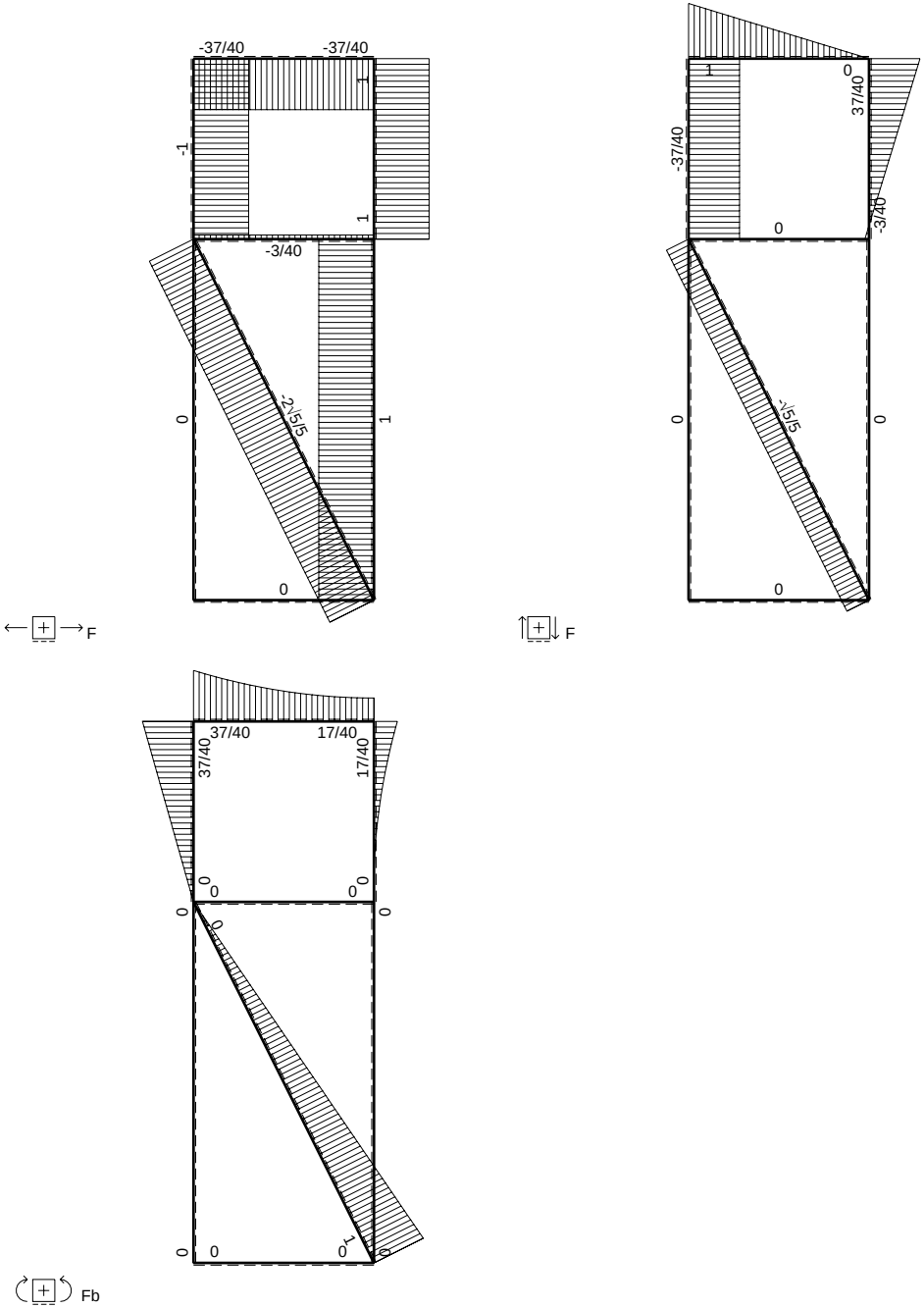
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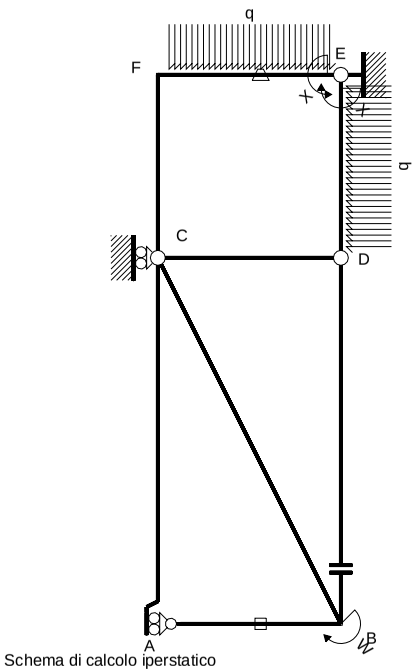
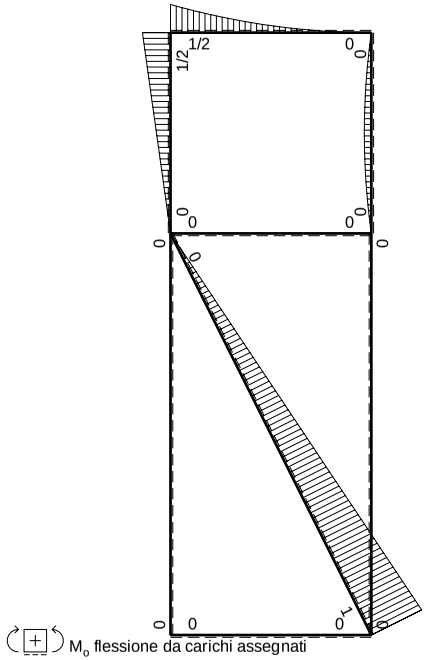
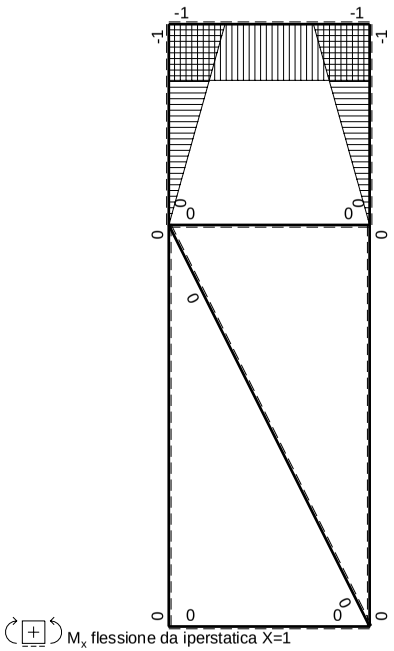
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982951 ESΣ09.xxxx.004

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

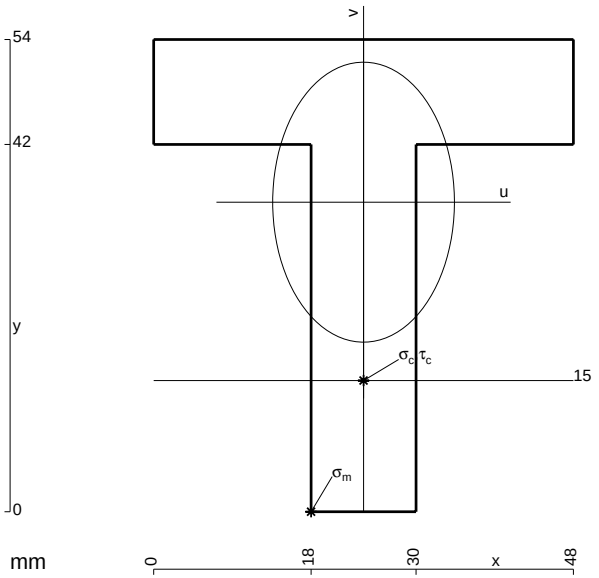
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 1080. mm^2
J_u = 276955. mm^4
J_v = 116640. mm^4
y_g = 35.4 mm
N = -3202. N
T_y = -1601. N
M_x = 1897400. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.4 mm
$\sigma_m = N/A - Mv/J_u$ = 239.6 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.4 mm
$\sigma_c = N/A - Mv/J_u$ = 136.8 N/mm^2
τ_c = 2.419 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 136.9 N/mm^2
S^* = 5022. mm^3

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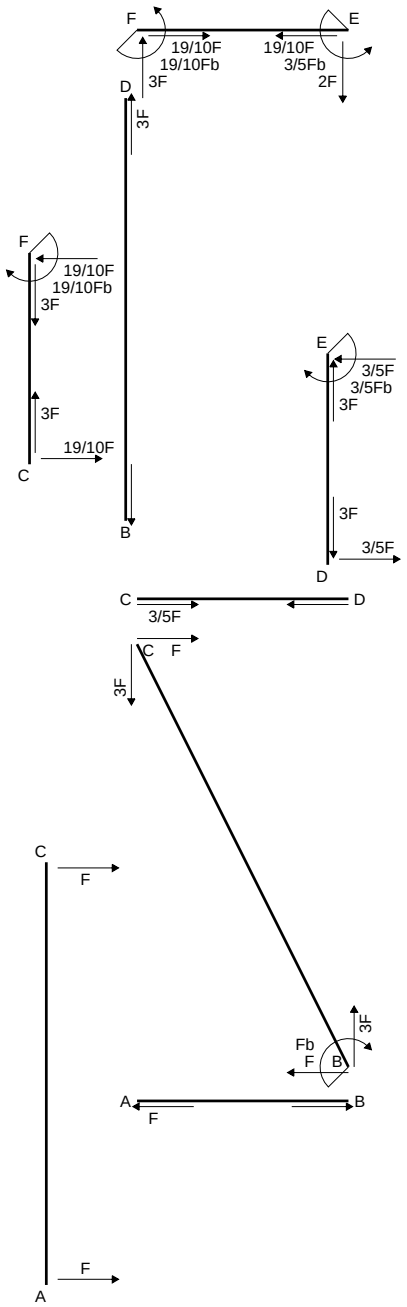
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19/10Fb
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3F
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19/10F
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3/5Fb
3F
3F
19/10F
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B
3F
3/5F
D
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D
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Fb
F
B
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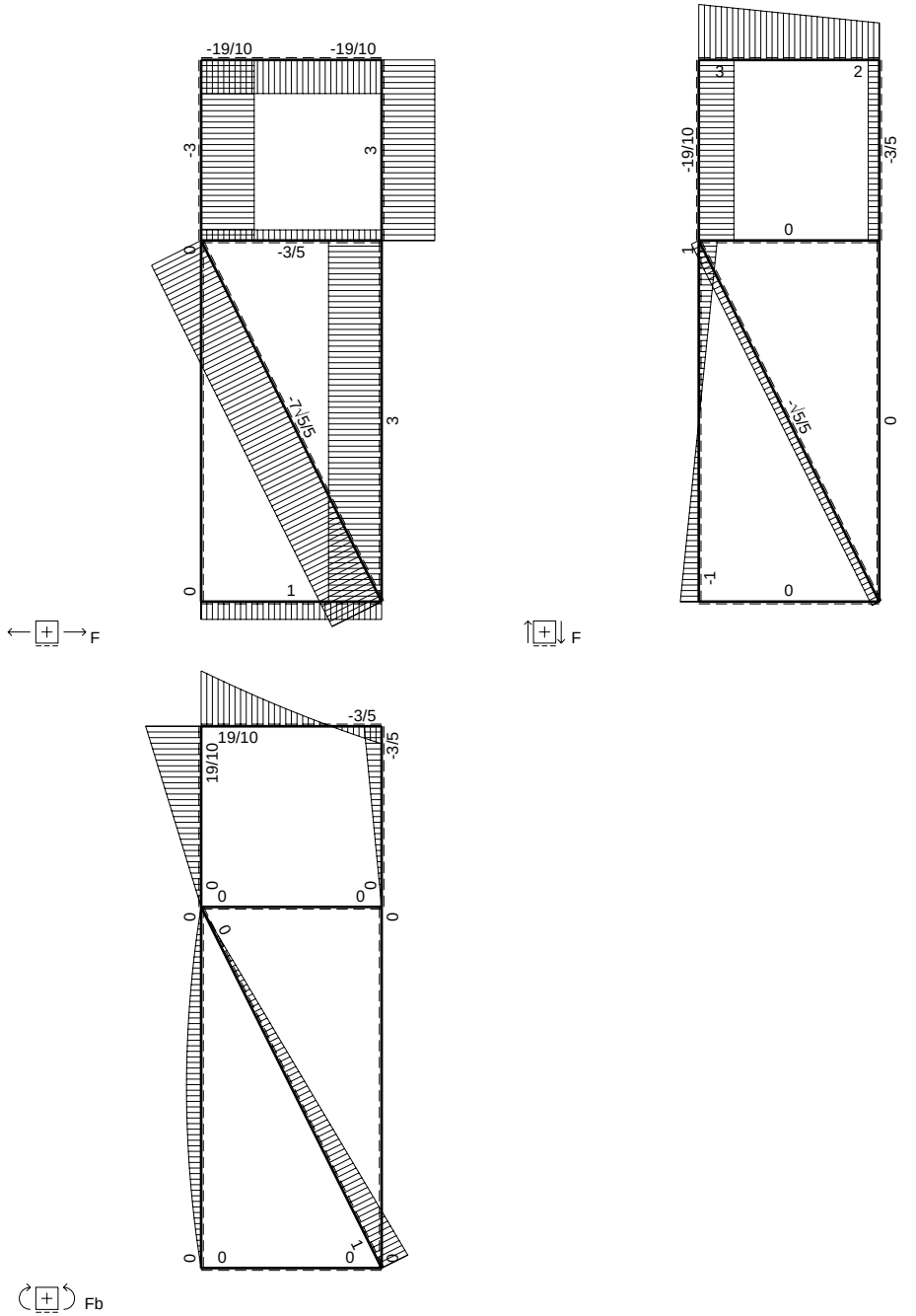
A
B
F
F
A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F
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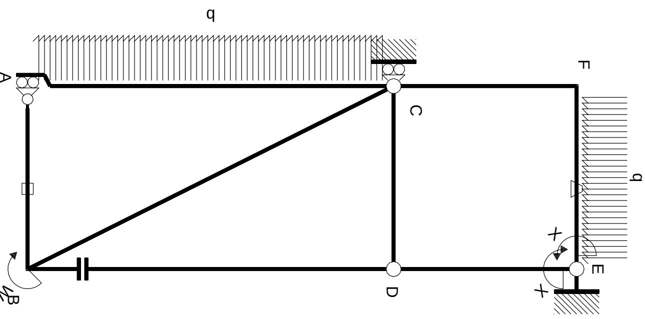
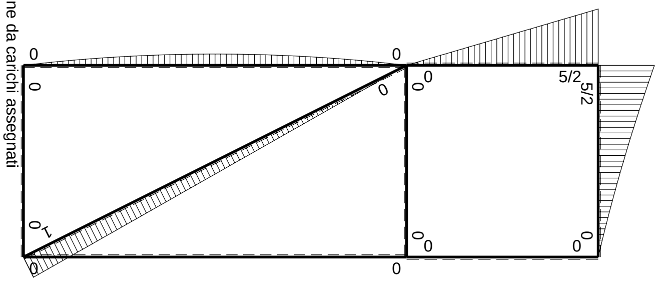
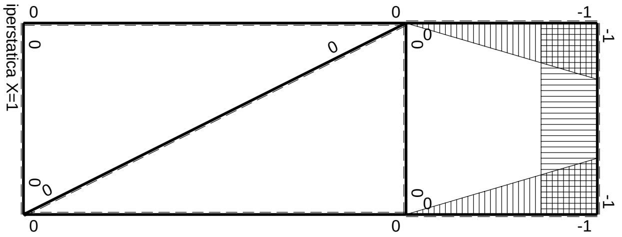
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

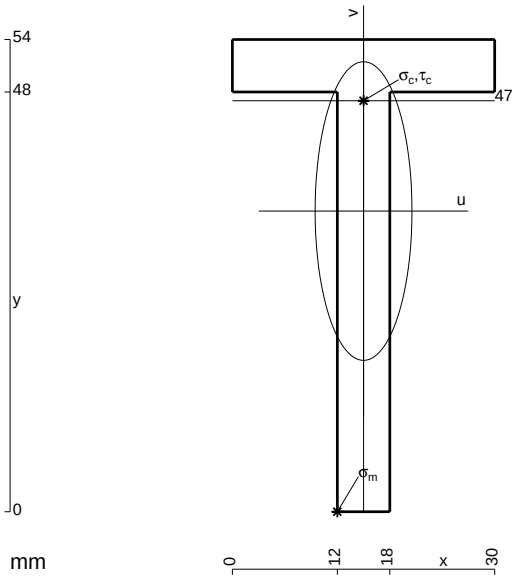
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 468. mm^2
J_u = 136587. mm^4
J_v = 14364. mm^4
y_g = 34.38 mm
N = -4571. N
T_y = -652.9 N
M_x = 832200. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -34.38 mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 15. mm
y_c = 47. mm
v_c = 12.62 mm
$\sigma_c = N/A - Mv/J_u$ = -86.63 N/mm^2
τ_c = 2.446 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 86.73 N/mm^2
S^* = 3069. mm^3

ESΣ09.xxxx.005

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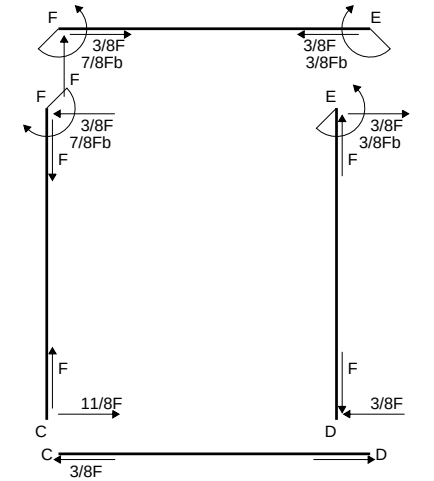
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E
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7/8Fb
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3/8Fb
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F
E
3/8F
7/8Fb
3/8F
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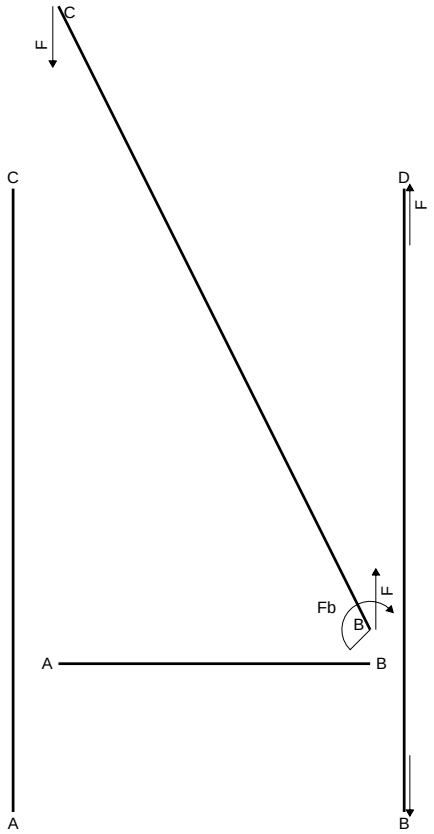
C
D
C
D
3/8F


C
F
C
D
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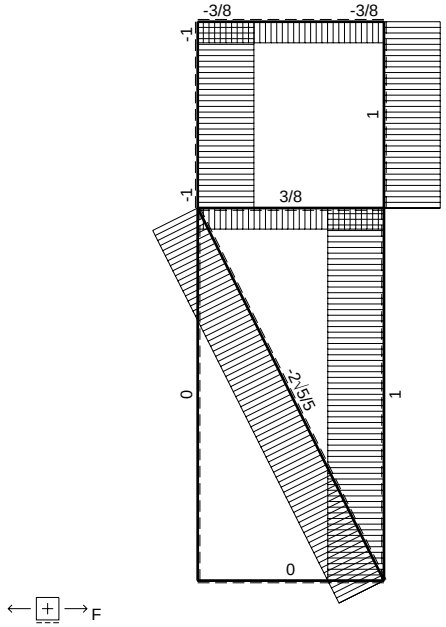
Fb
B
F
A
B
A
B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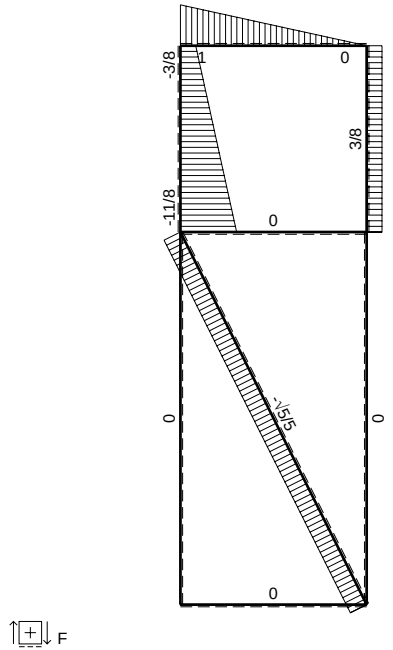
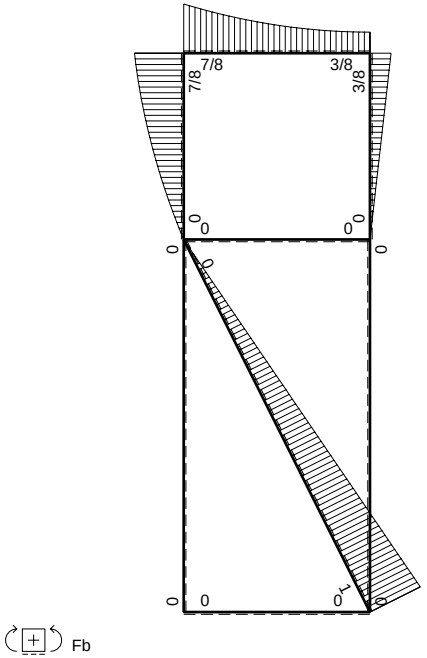
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
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0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

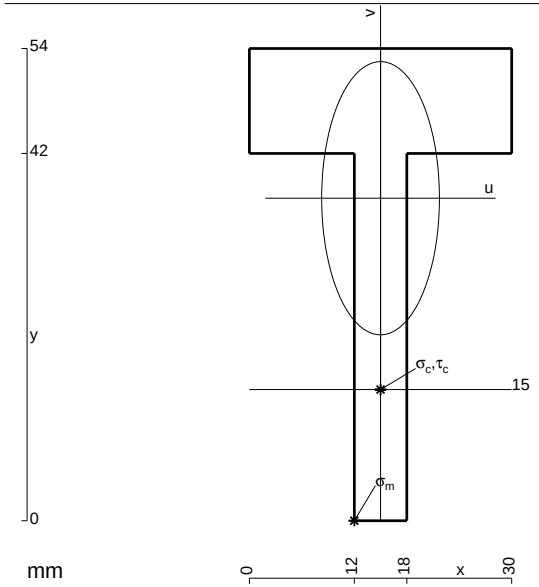
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 612. mm^2
J_u = 149427. mm^4
J_v = 27756. mm^4
y_g = 36.88 mm
N = -1279. N
T_y = -639.5 N
M_x = 858000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.88 mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.88 mm
$\sigma_c = N/A - Mv/J_u$ = 123.6 N/mm^2
τ_c = 1.886 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 123.6 N/mm^2
S^* = 2644. mm^3

ESΣ09.xxxx.006

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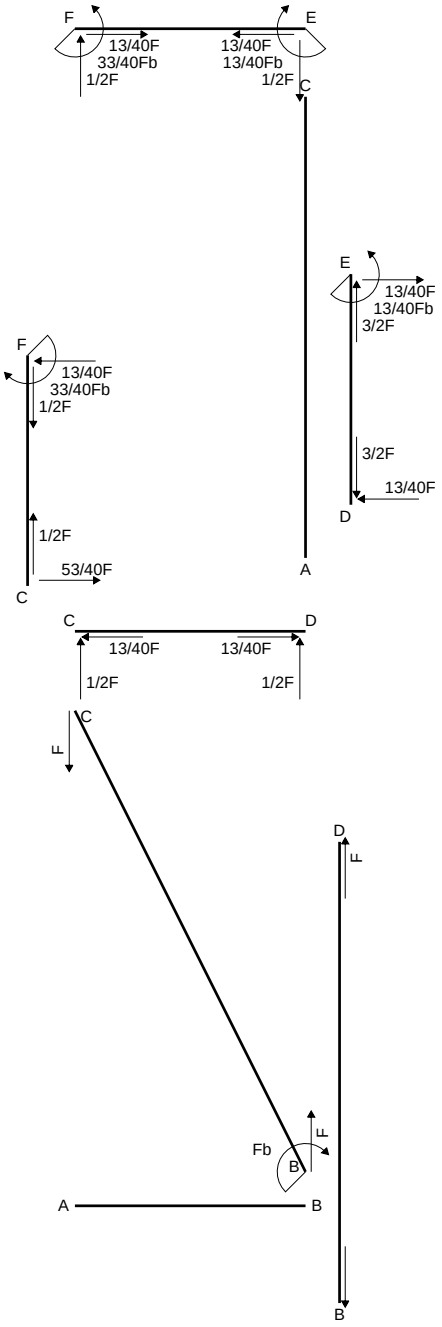
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E
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33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
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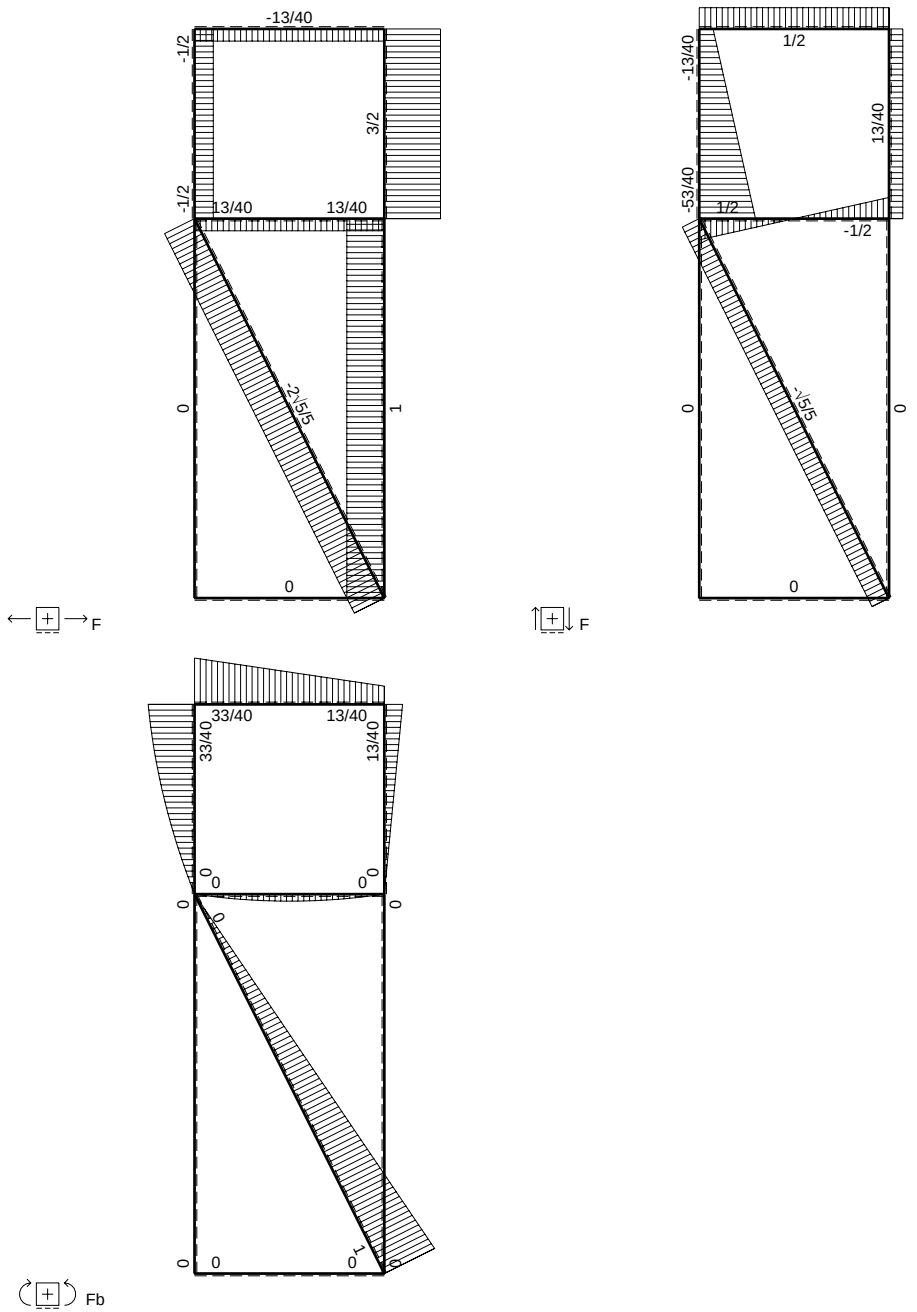
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
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3/2
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-√5/5
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0
0
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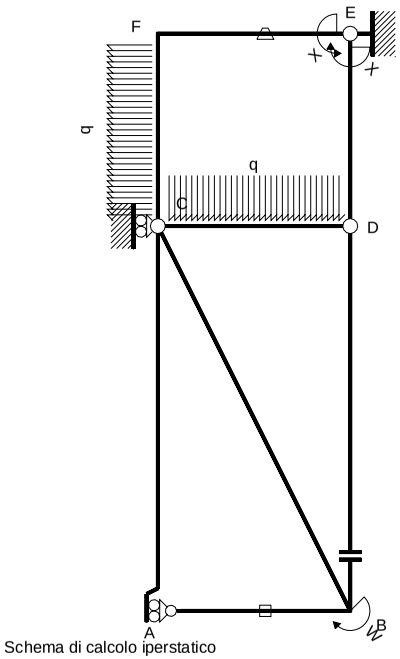
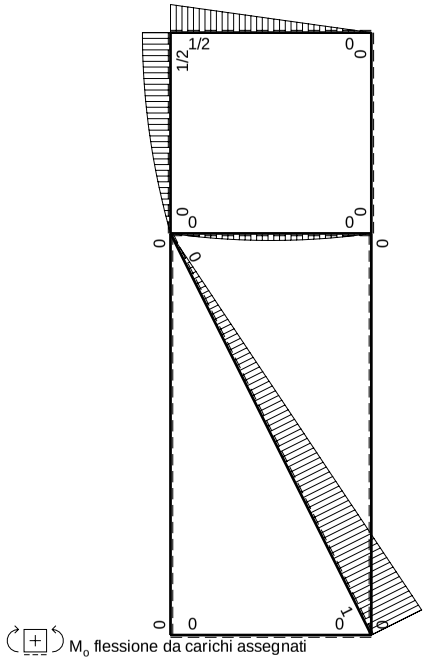
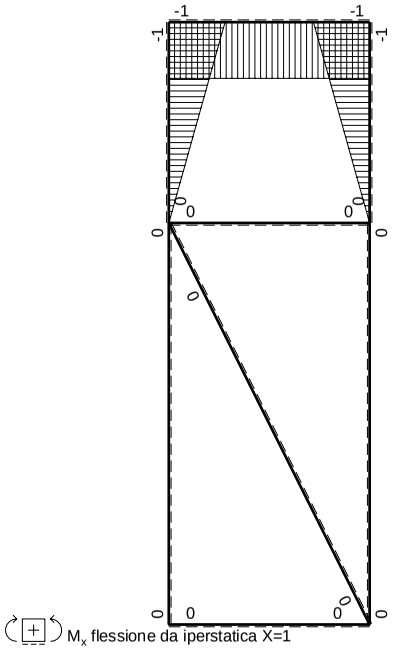
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 938810 ESΣ09.xxxx.007

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

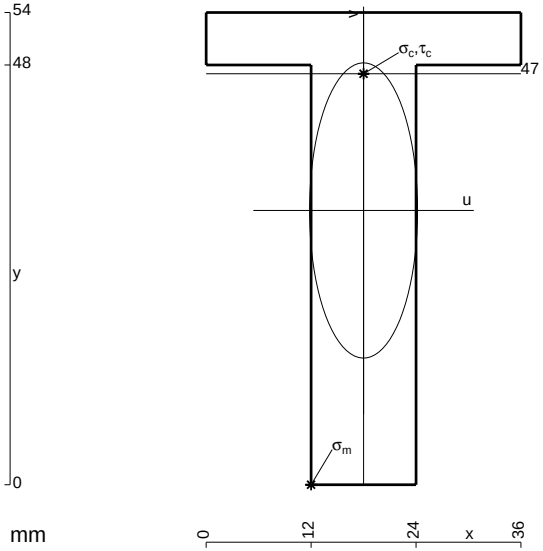
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 938810 ESΣ09.xxxx.007



A = 792. mm^2
J_u = 225759. mm^4
J_v = 30240. mm^4
y_g = 31.36 mm
N = -2236. N
T_y = -1118. N
M_x = 1600000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.36 mm
$\sigma_m = N/A - Mv/J_u$ = 219.5 N/mm^2
x_c = 18. mm
y_c = 47. mm
v_c = 15.64 mm
$\sigma_c = N/A - Mv/J_u$ = -113.6 N/mm^2
τ_c = 1.83 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.7 N/mm^2
S^* = 4435. mm^3

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ESΣ09.xxxx.007

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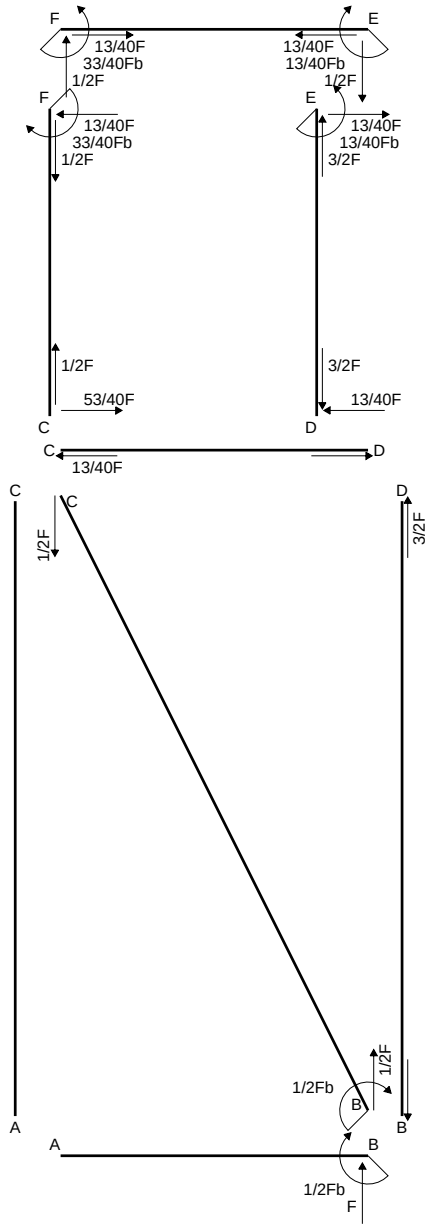
F
E
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33/40Fb
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1/2F
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E
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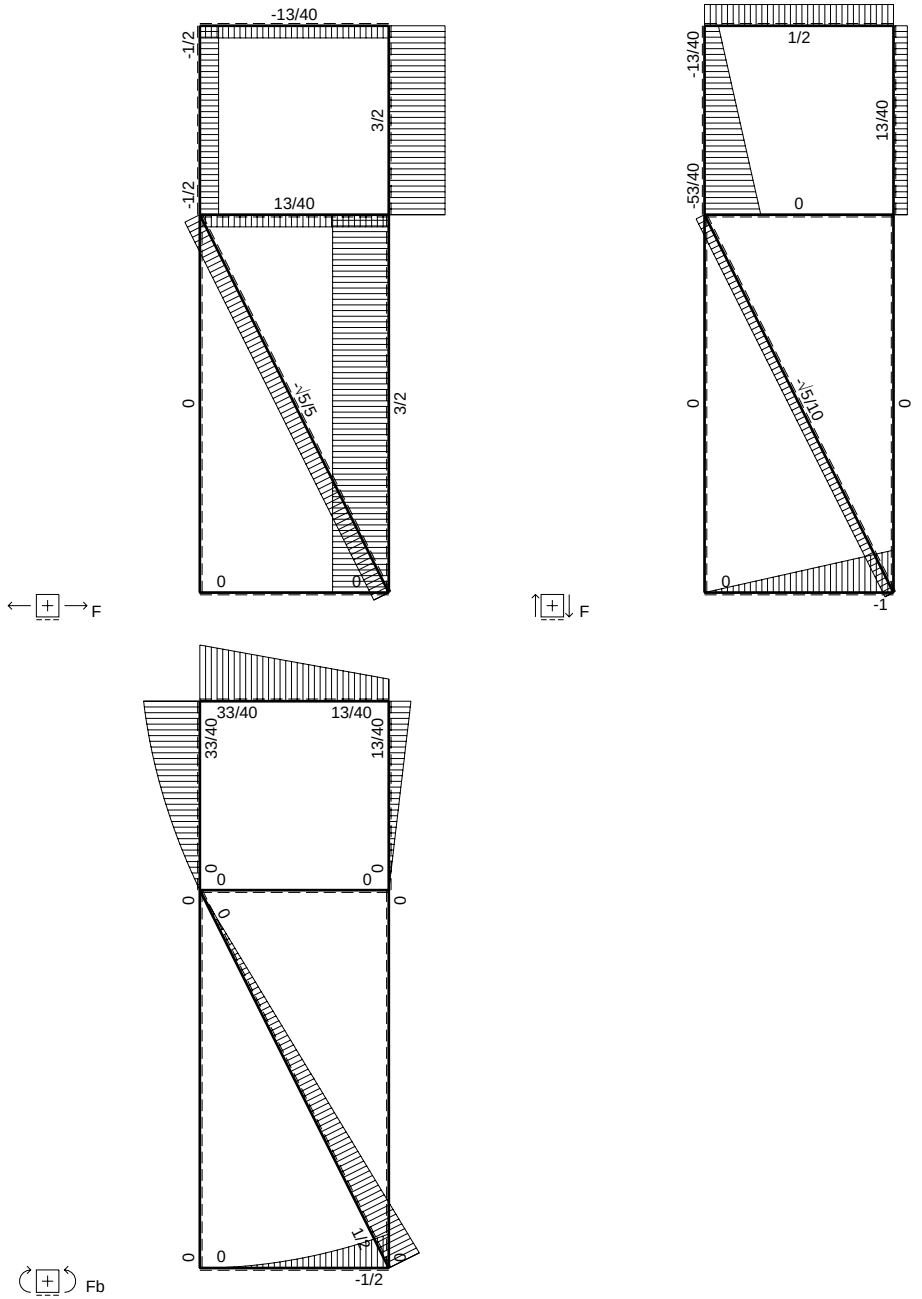
A
B
1/2Fb
F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13/40
-53/40
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0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210705 ESΣ09.xxxx.008

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

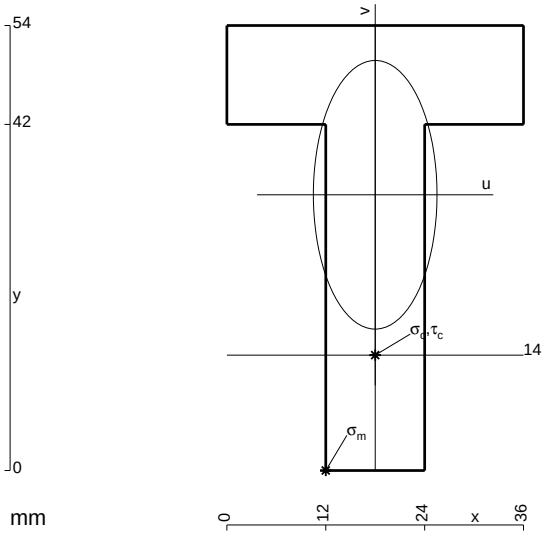
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 936. mm^2
J_u = 248849. mm^4
J_v = 52704. mm^4
y_g = 33.46 mm
T_y = -5030. N
M_x = -1710200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.46 mm
$\sigma_m = -Mv/J_u$ = -230. N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.46 mm
$\sigma_c = -Mv/J_u$ = -133.7 N/mm^2
τ_c = 7.488 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.4 N/mm^2
S^* = 4446. mm^3

ESΣ09.xxxx.008

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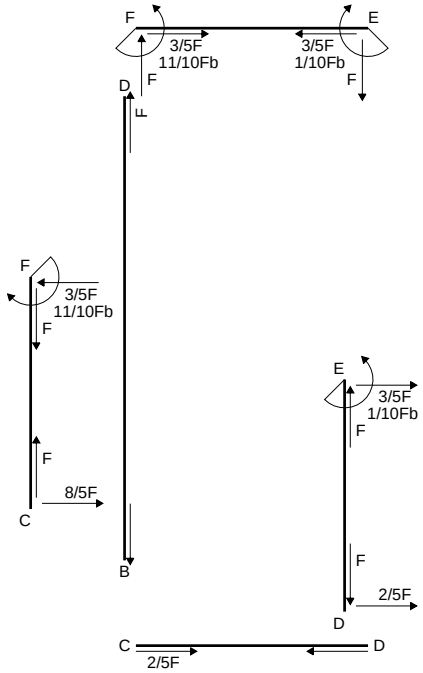
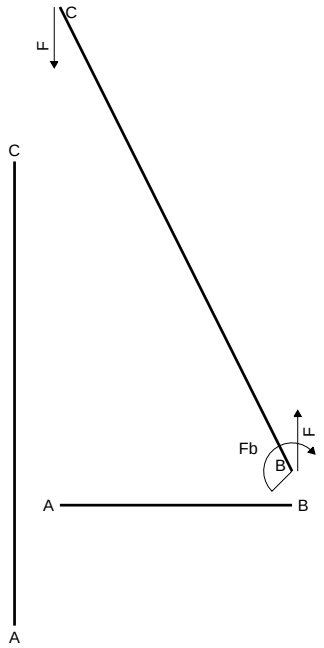
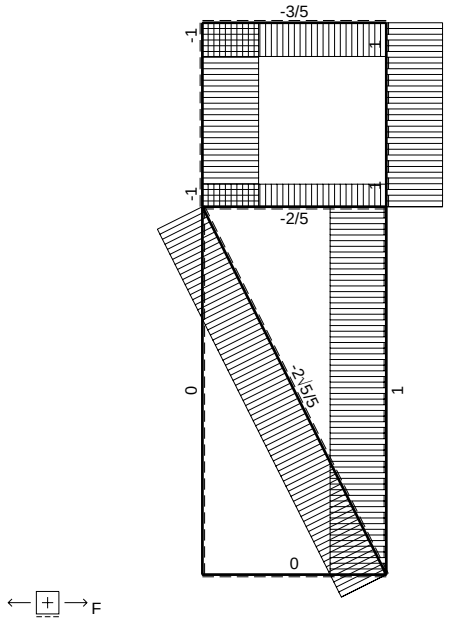
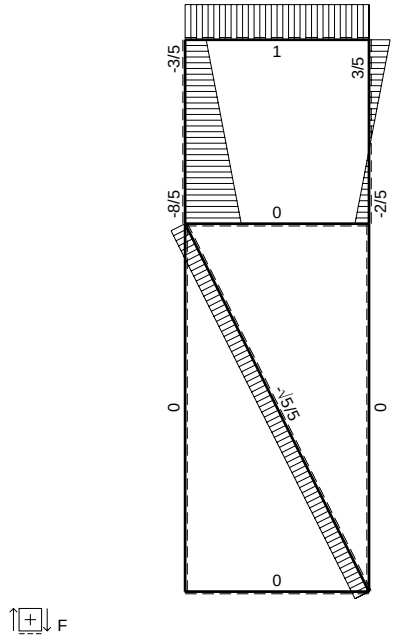
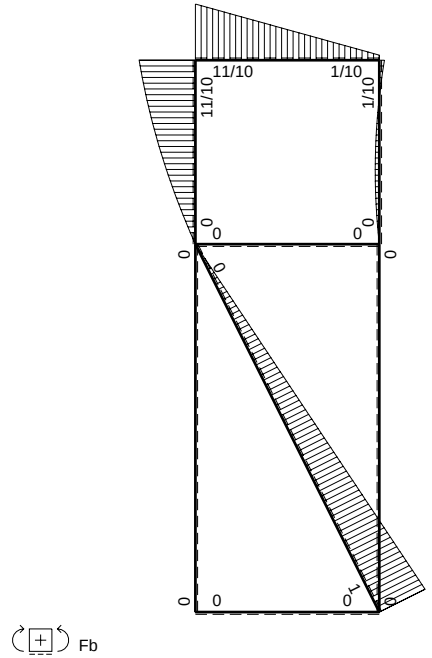

F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3XLb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb²/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx²	0	-Fb+3/2Fx-1/2qx³/b	0	1-2x/b+x²/b²	(-3/8+0)Fb²/EJ	1/3XLb/EJ
CF b	x/b	-3/2Fx+1/2qx²	0	-3/2Fx²/b+1/2qx³/b	0	x²/b²		
	totali						1/6Fb²/EJ	5/3XLb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209897 ESΣ09.xxxx.009

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

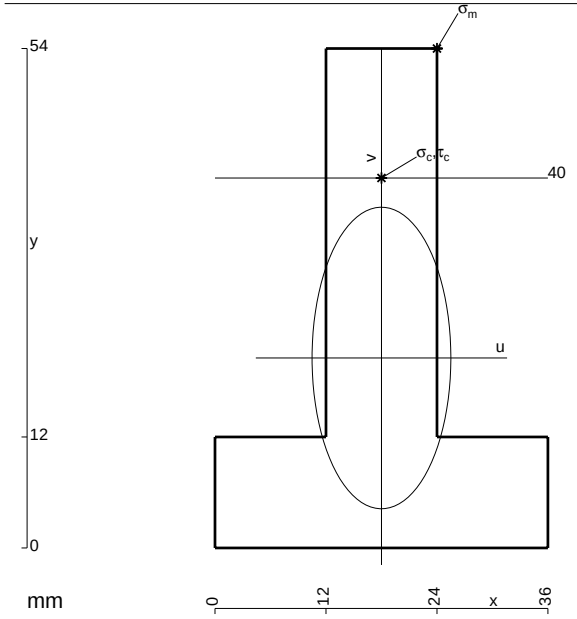
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 936. mm^2
J_u = 248849. mm^4
J_v = 52704. mm^4
y_g = 20.54 mm
N = -2191. N
T_y = -1096. N
M_x = 1764000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.46 mm
$\sigma_m = N/A - Mv/J_u$ = -239.5 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.46 mm
$\sigma_c = N/A - Mv/J_u$ = -140.3 N/mm^2
τ_c = 1.631 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.3 N/mm^2
S^* = 4446. mm^3

ESΣ09.xxxx.009

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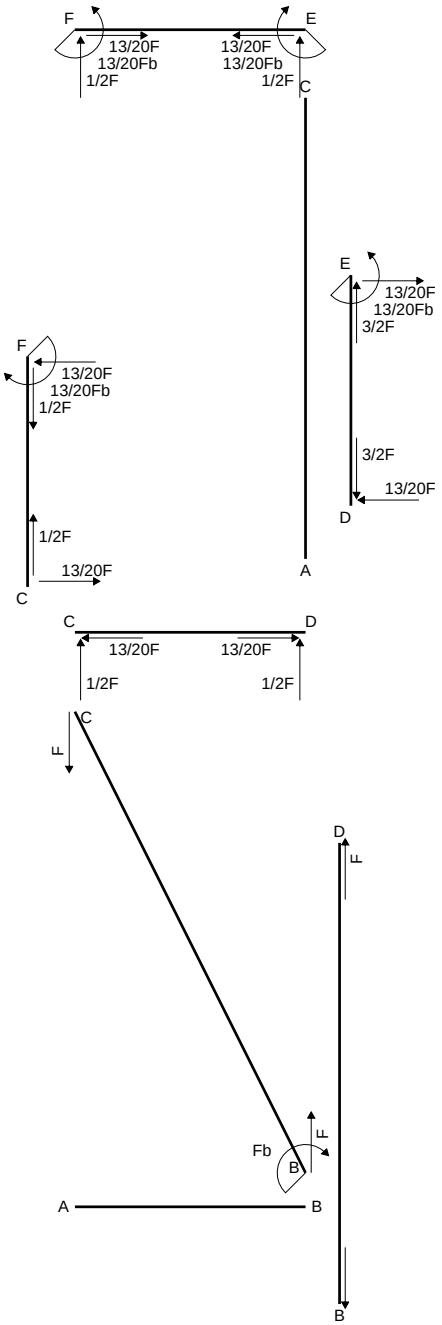
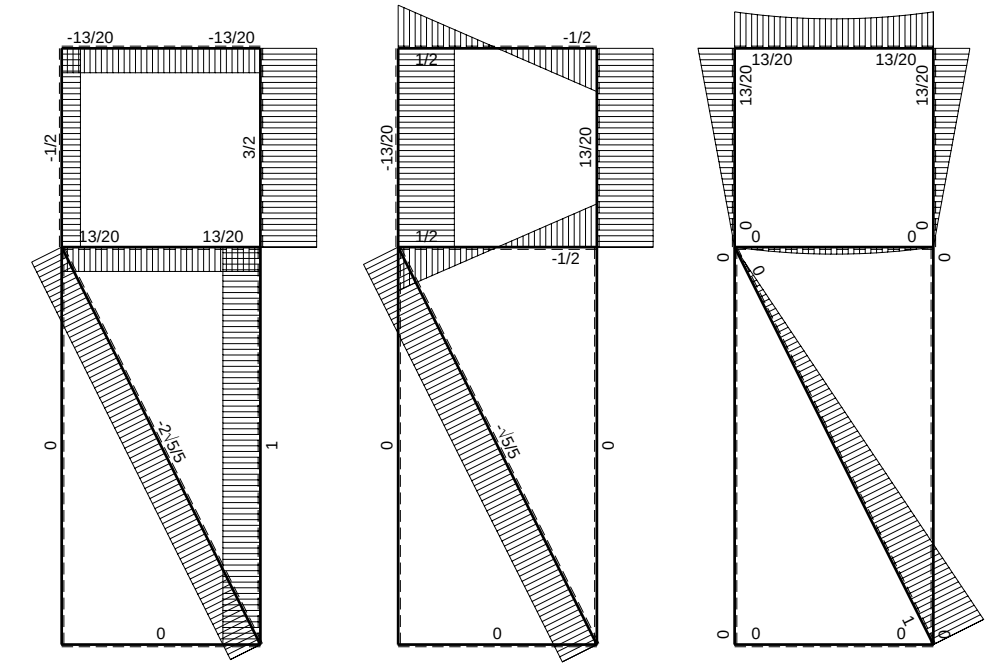
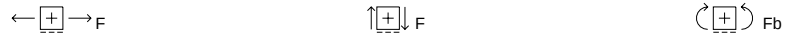
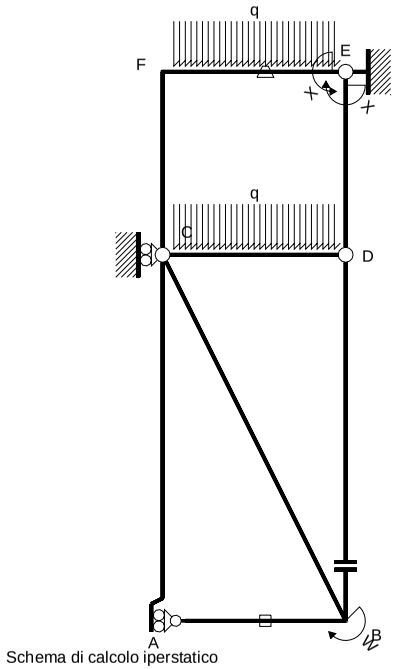
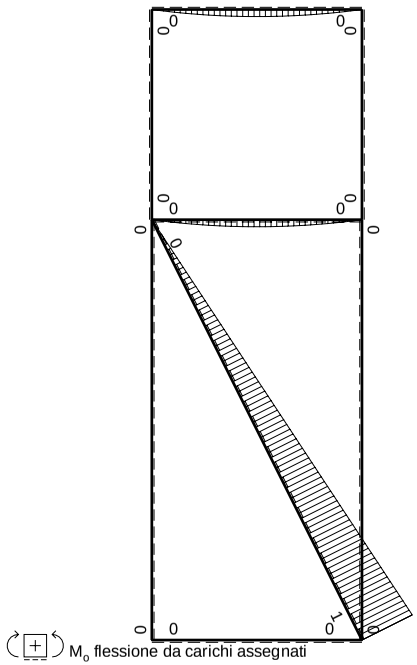

F
E
13/20F
13/20Fb
1/2F
C
E
13/20F
13/20Fb
3/2F
F
13/20F
13/20Fb
1/2F
3/2F
13/20F
D
1/2F
13/20F
C
A
C
D
13/20F
13/20F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/20
-13/20
-1/2
3/2
13/20
13/20
0
1
-2√5/5
0
-1/2
1/2
-13/20
13/20
-1/2
1/2
-√5/5
0
0
13/20
13/20
13/20
13/20
0
0
0
0
0
0
0
1
0

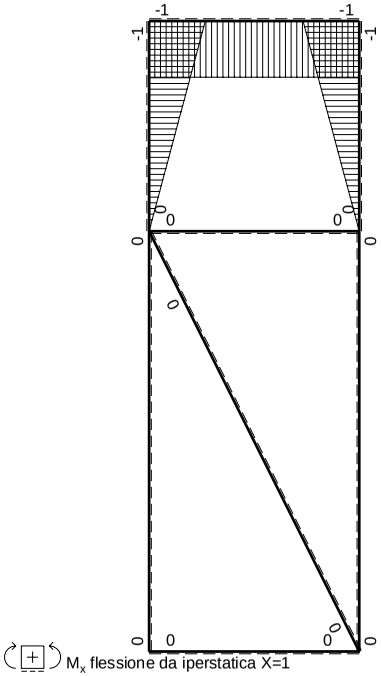

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957901 ESΣ09.xxxx.010

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

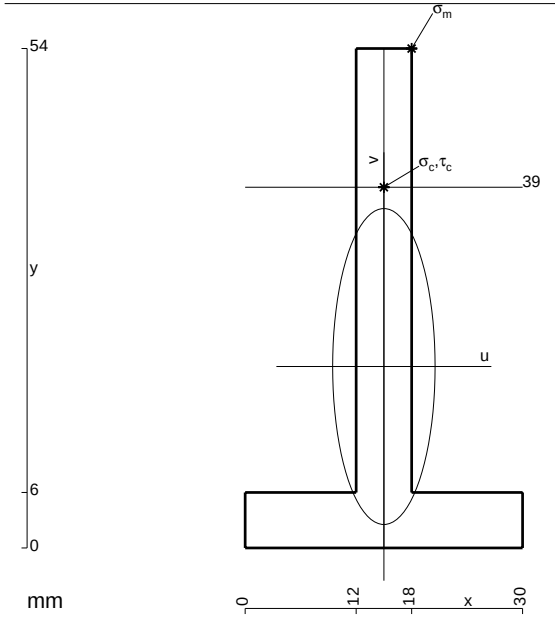
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 468. mm^2
J_u = 136587. mm^4
J_v = 14364. mm^4
y_g = 19.62 mm
N = -1825. N
T_y = -912.3 N
M_x = 775200. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 34.38 mm
$\sigma_m = N/A - Mv/J_u$ = -199. N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 19.38 mm
$\sigma_c = N/A - Mv/J_u$ = -113.9 N/mm^2
τ_c = 2.694 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114. N/mm^2
S^* = 2420. mm^3

ESΣ09.xxxx.010

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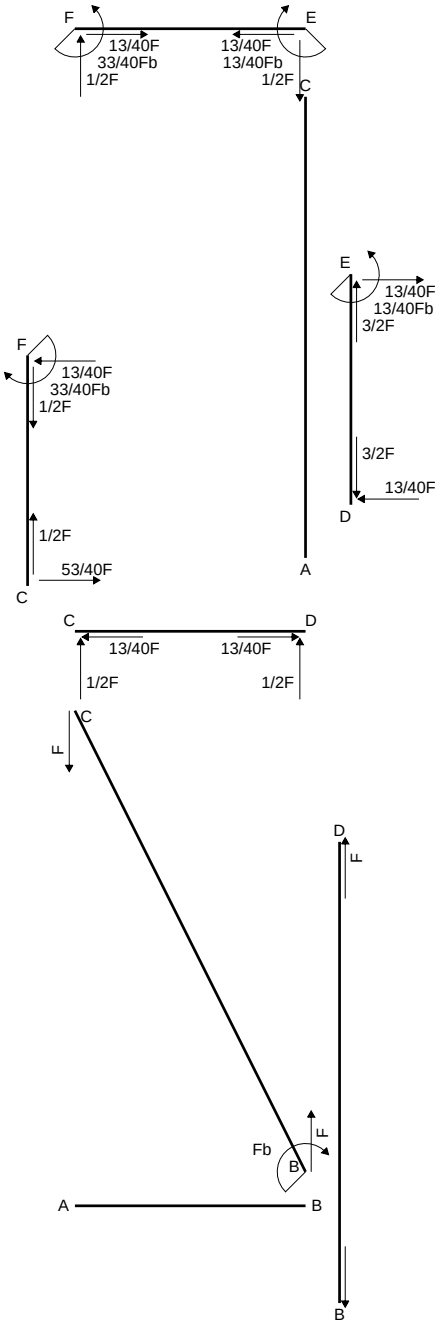
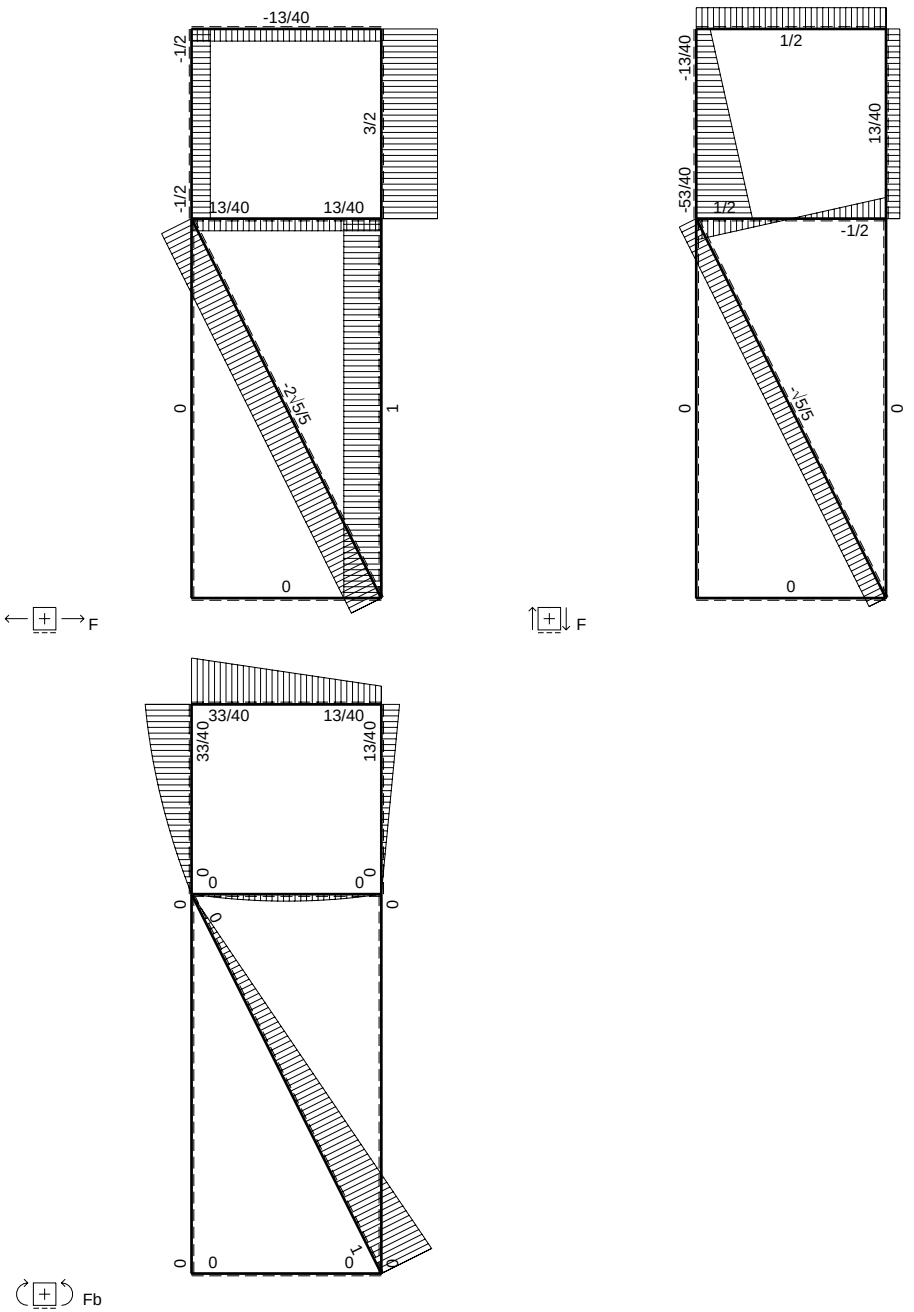

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F
-13/40
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13/40
-53/40
1/2
-1/2
-√5/5
0
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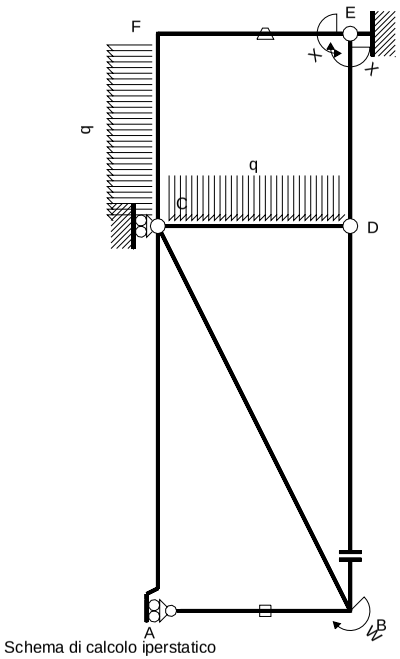
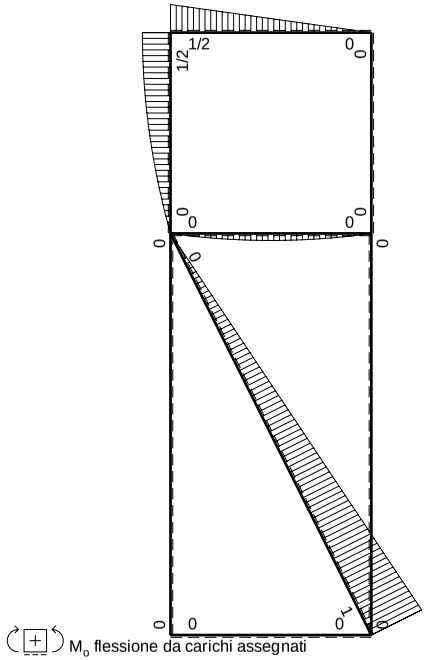
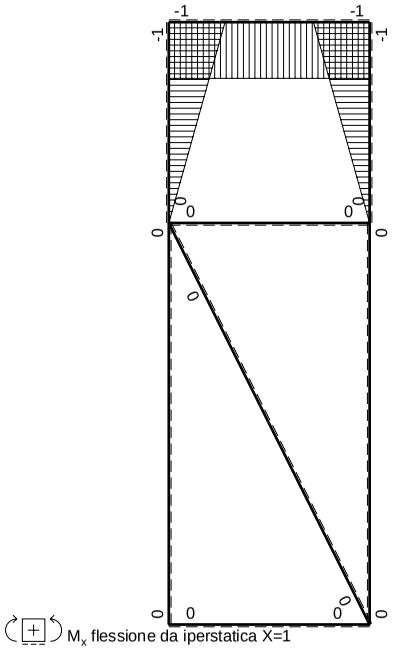
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981830 ESΣ09.xxxx.011

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

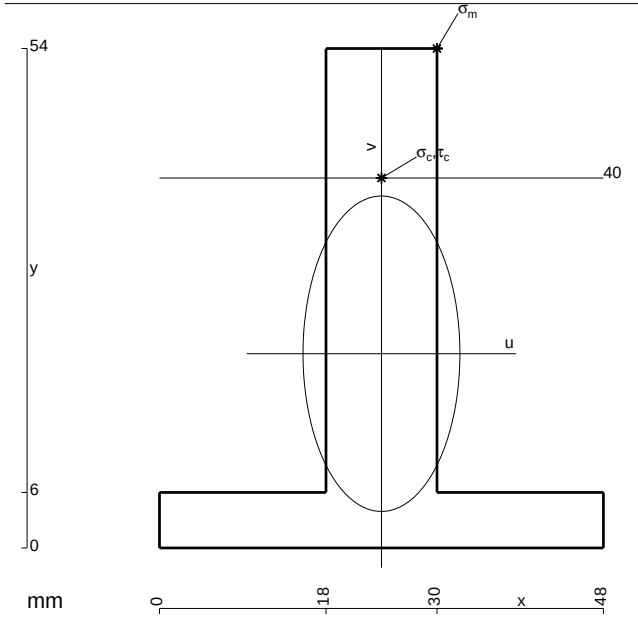
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 864. mm^2
J_u = 251424. mm^4
J_v = 62208. mm^4
y_g = 21. mm
N = -3417. N
T_y = -1708. N
M_x = 1566200. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33. mm
$\sigma_m = N/A - Mv/J_u$ = -209.5 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 19. mm
$\sigma_c = N/A - Mv/J_u$ = -122.3 N/mm^2
τ_c = 2.473 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.4 N/mm^2
S^* = 4368. mm^3

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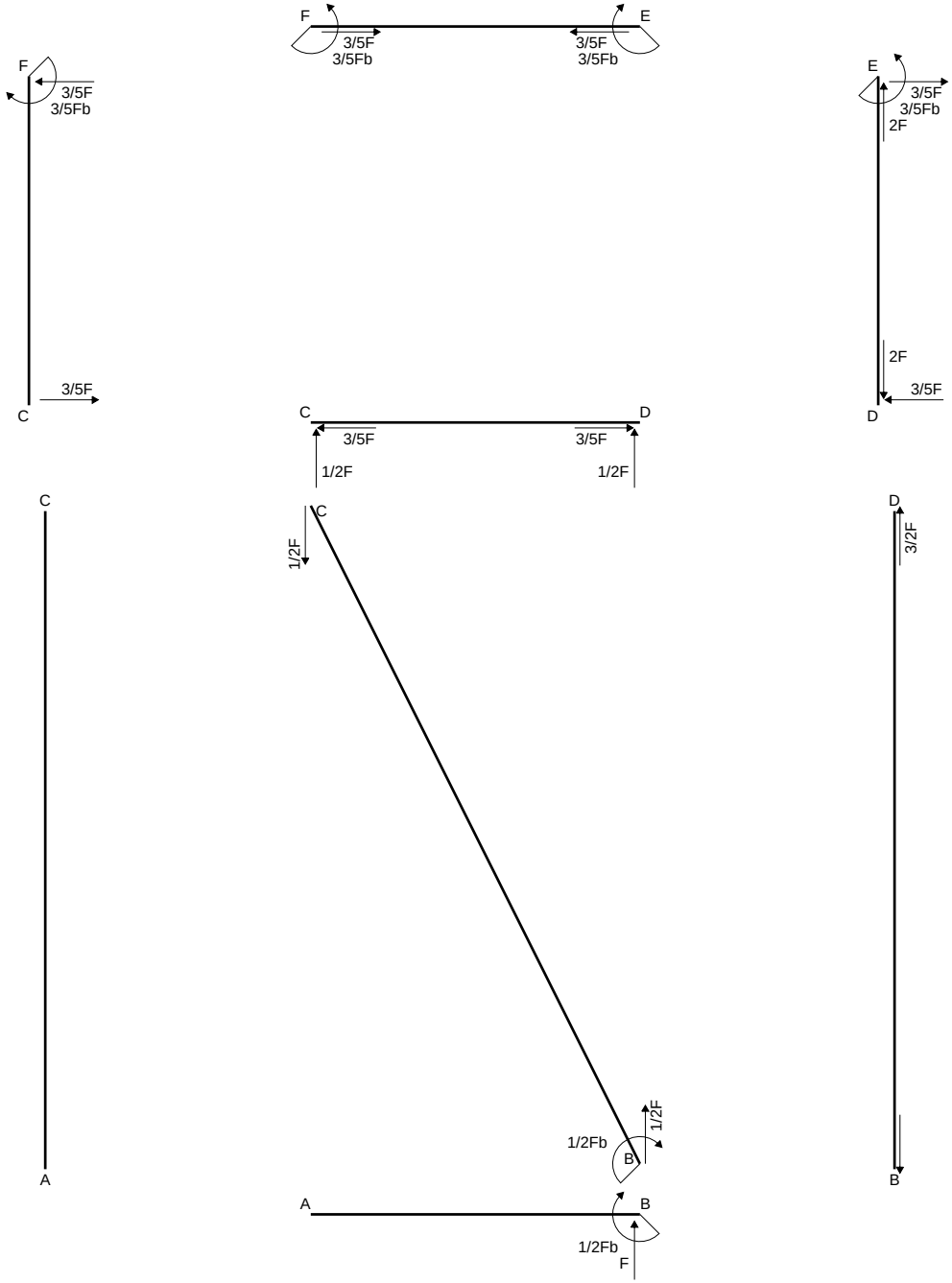
F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
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2F
C
3/5F
2F
D
3/5F
C
D
3/5F
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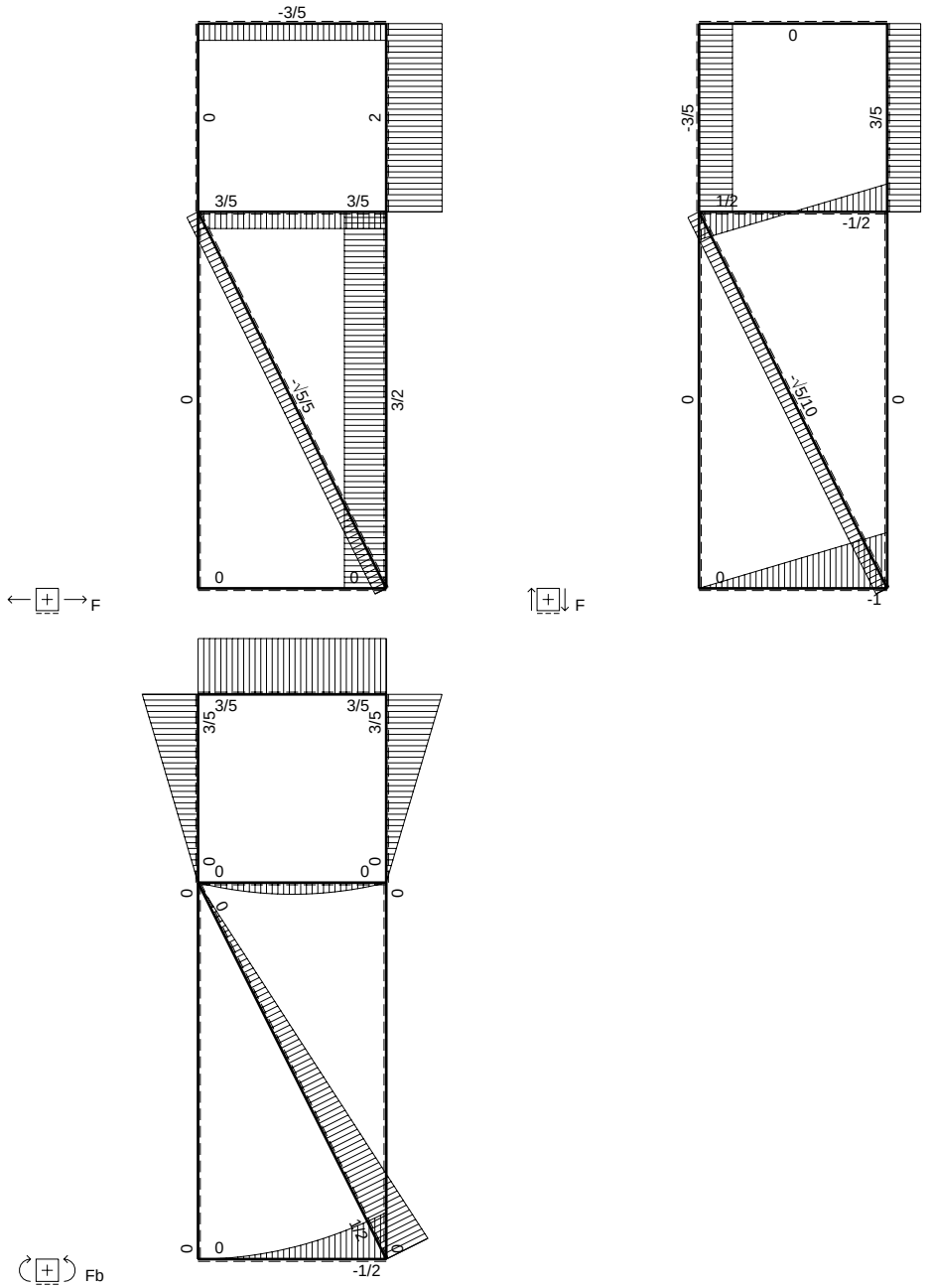
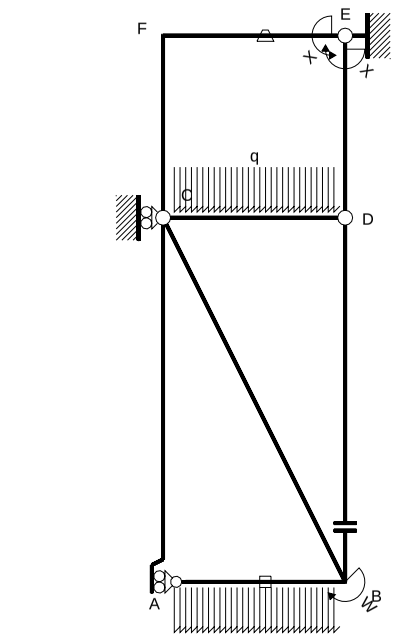
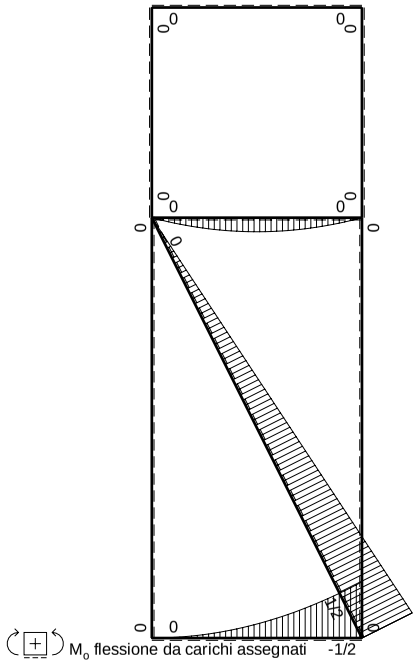
A
B
1/2Fb
F

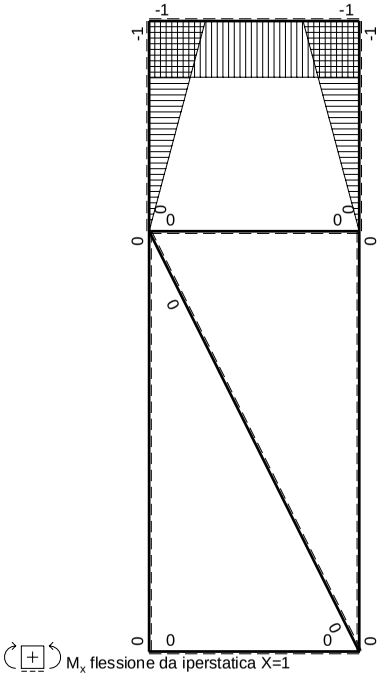

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}b$	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

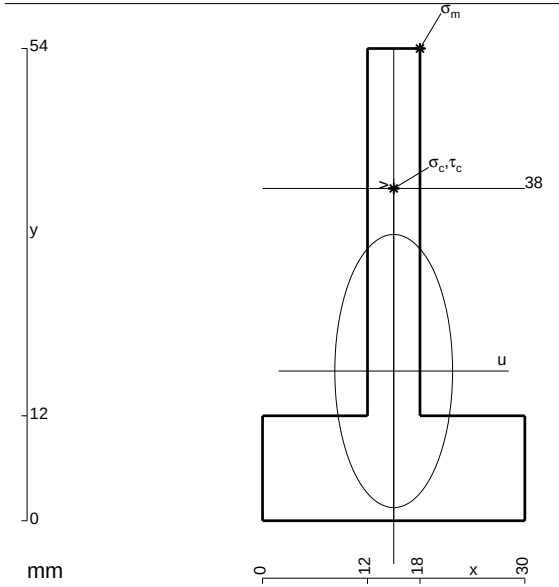
$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 209941 ESΣ09.xxxx.012



A = 612. mm^2
J_u = 149428. mm^4
J_v = 27756. mm^4
y_g = 17.12 mm
T_y = -3960. N
M_x = -891000. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 36.88 mm
$\sigma_m = -Mv/J_u$ = 219.9 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 20.88 mm
$\sigma_c = -Mv/J_u$ = 124.5 N/mm^2
τ_c = 12.25 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.3 N/mm^2
S^* = 2773. mm^3

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ESΣ09.xxxx.012

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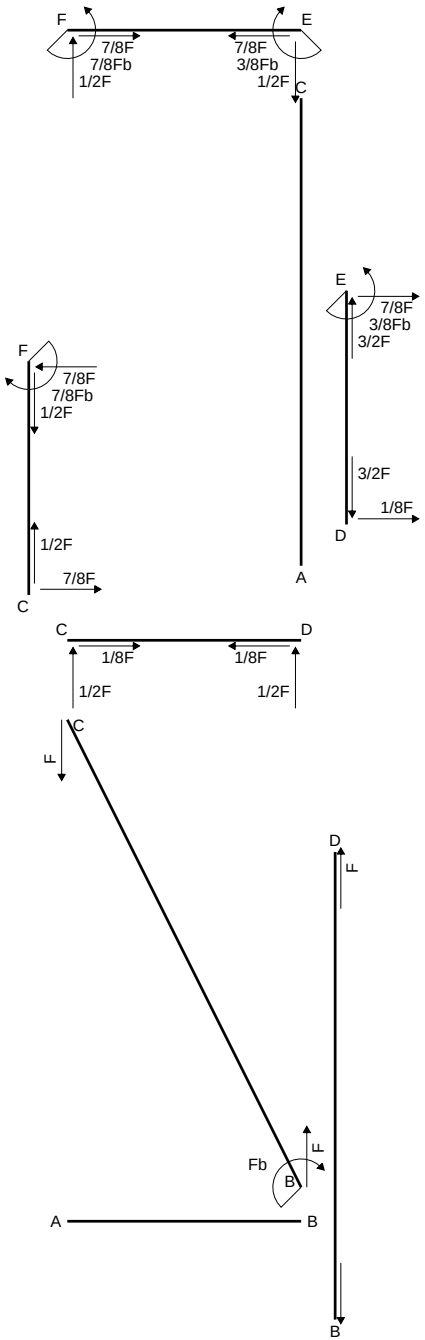
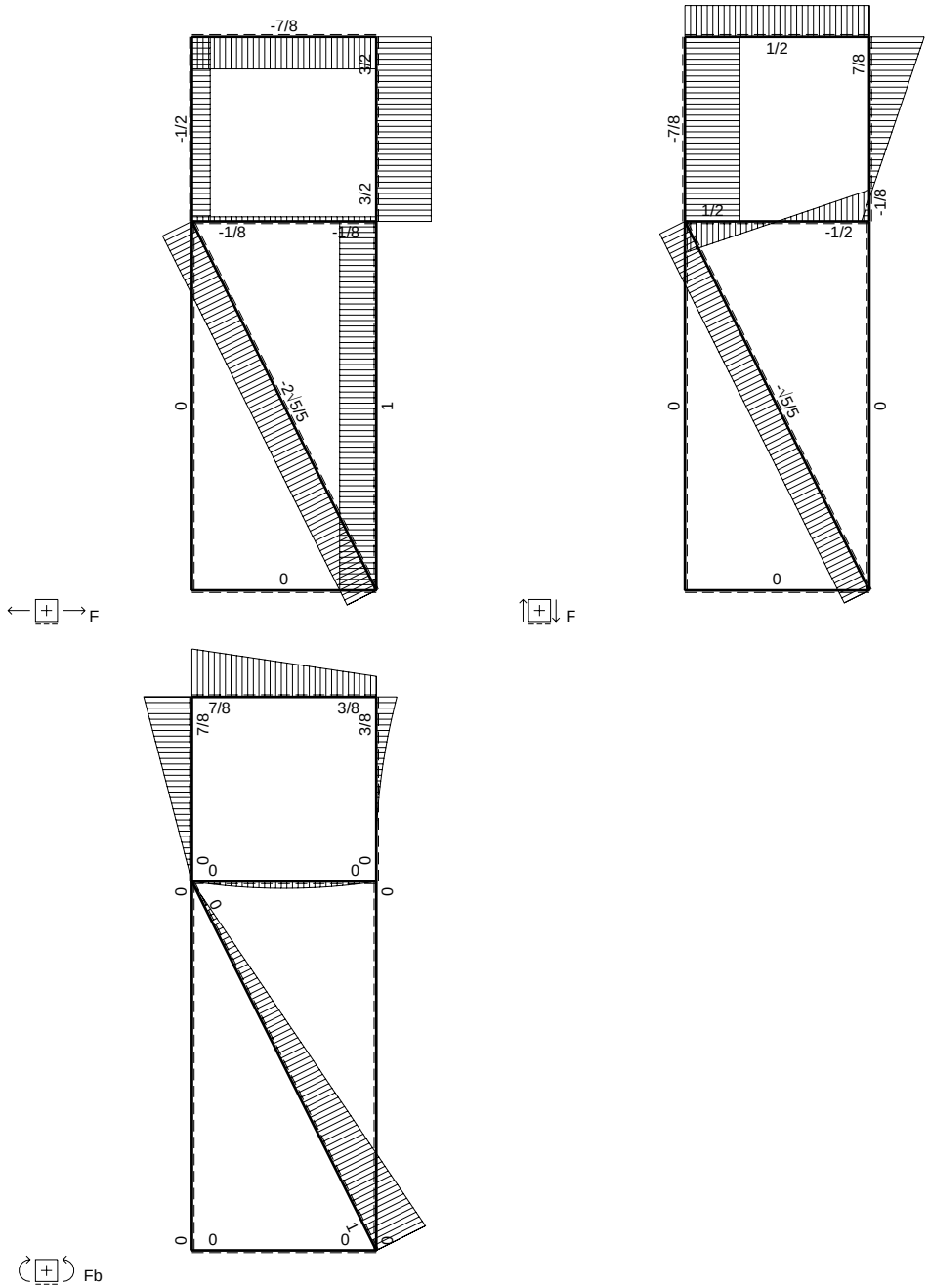
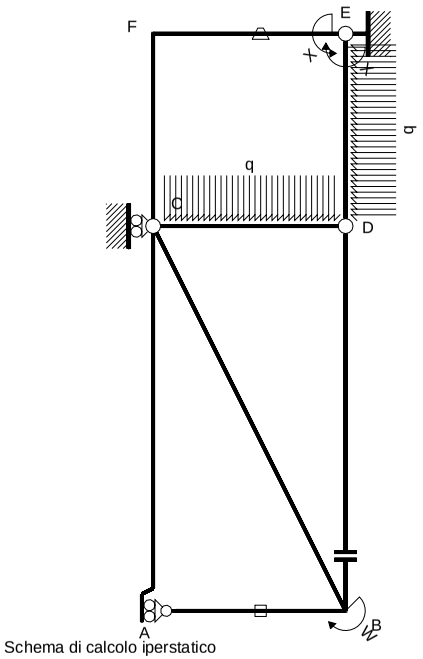
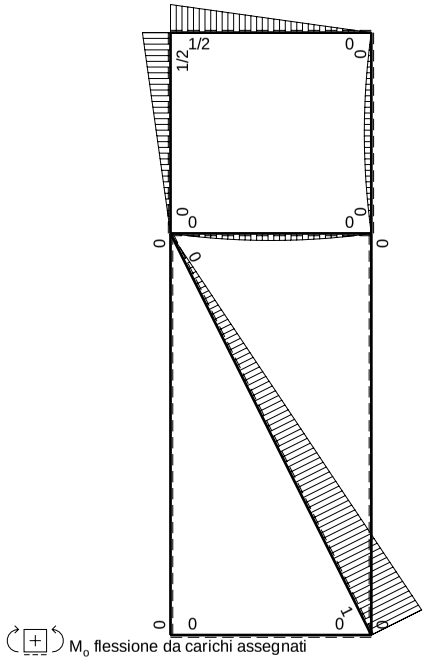

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B

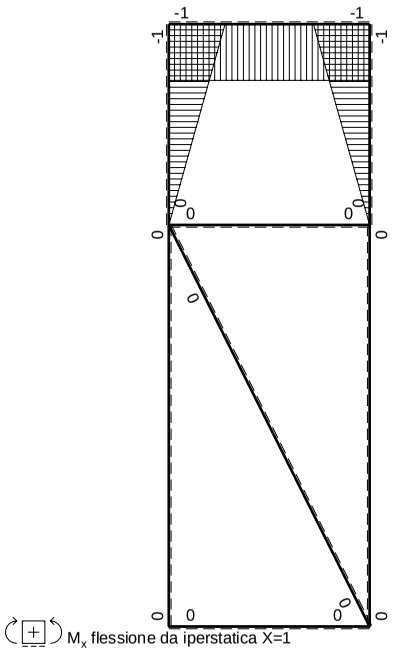

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x\theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
				totali			$5/8Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209770 ESΣ09.xxxx.013

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

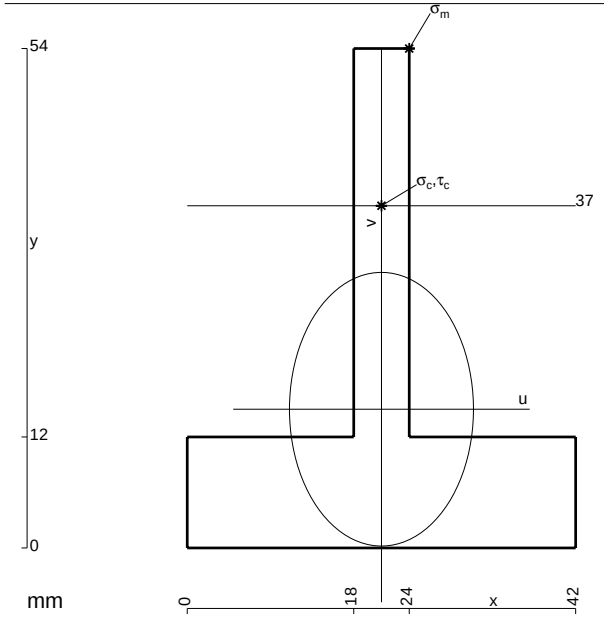
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 756. mm^2
J_u = 165564. mm^4
J_v = 74844. mm^4
y_g = 15. mm
N = -1762. N
T_y = -881. N
M_x = 965300. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 39. mm
$\sigma_m = N/A - Mv/J_u$ = -229.7 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22. mm
$\sigma_c = N/A - Mv/J_u$ = -130.6 N/mm^2
τ_c = 2.759 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.7 N/mm^2
S^* = 3111. mm^3

ESΣ09.xxxx.013

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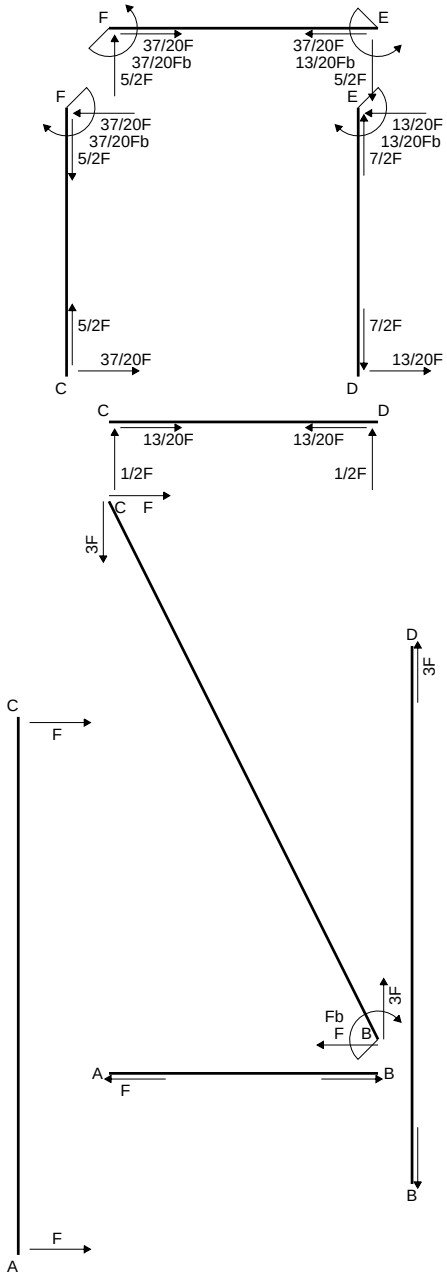
F
E
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37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
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3F
Fb
F B
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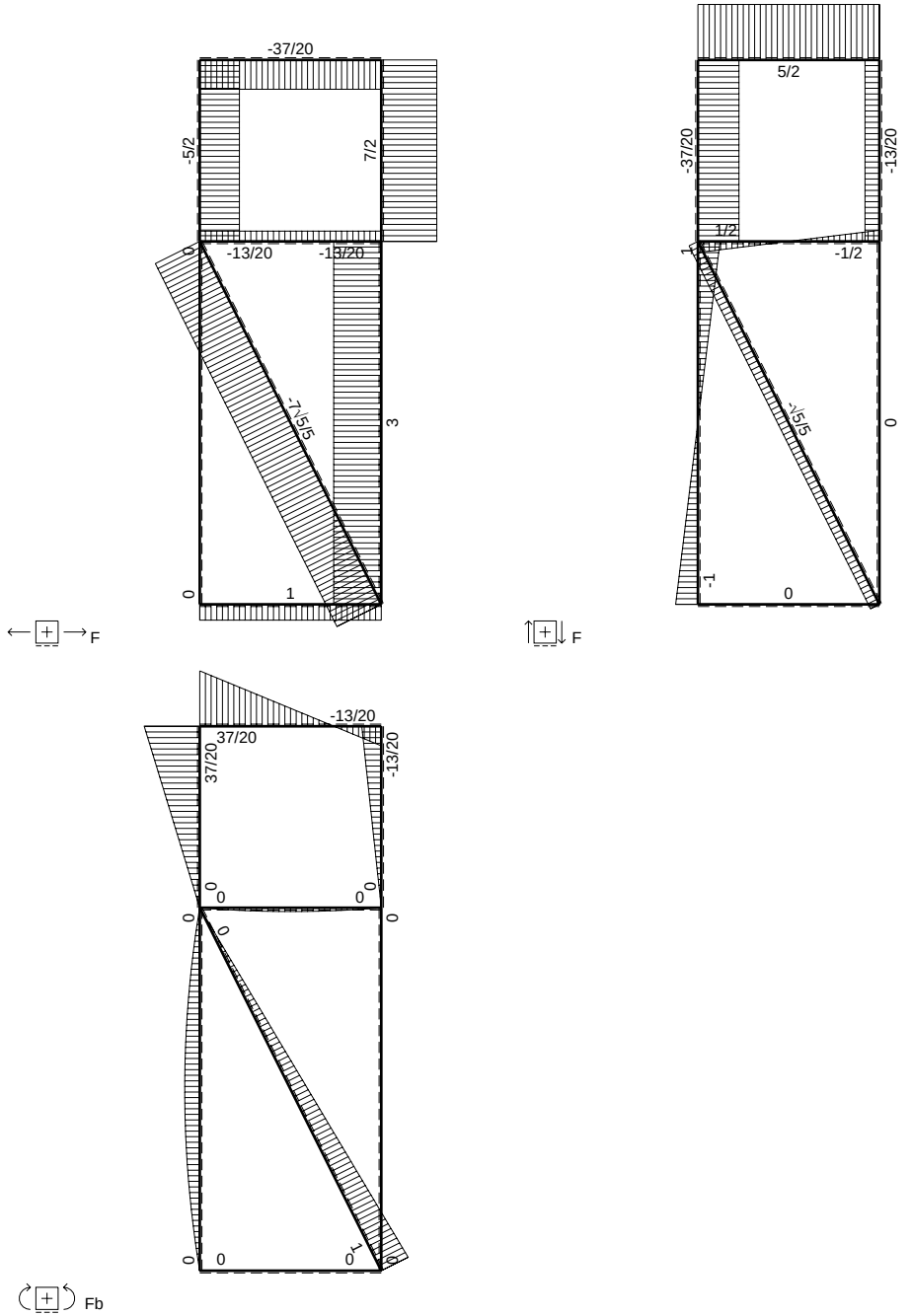
F
A
B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F
5/2
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-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
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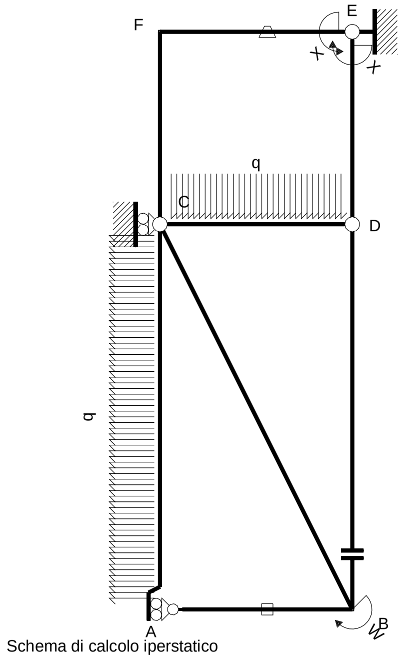
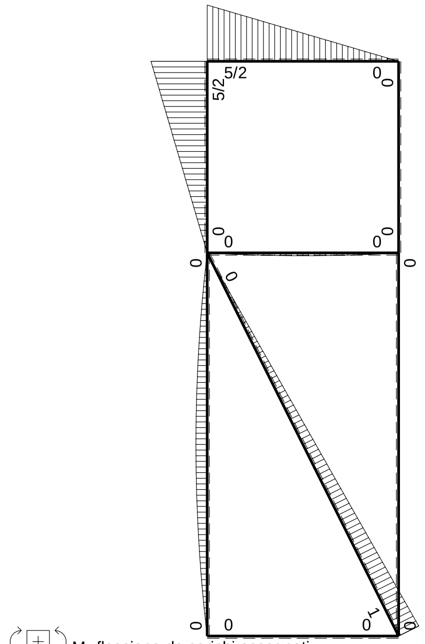
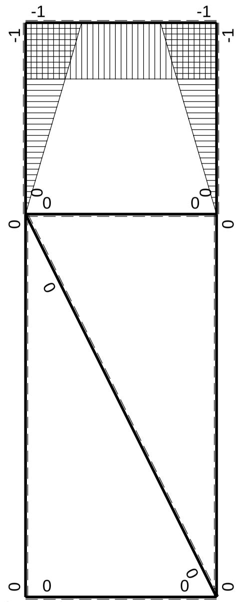
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

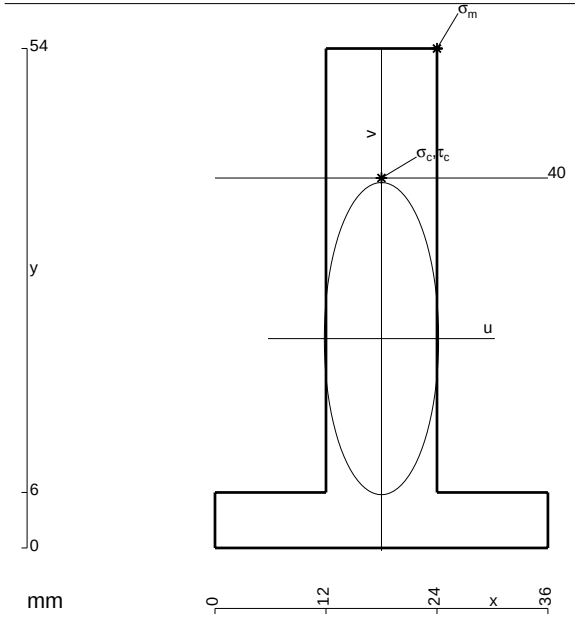
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 792. mm^2
J_u = 225759. mm^4
J_v = 30240. mm^4
y_g = 22.64 mm
N = -9673. N
T_y = -1382. N
M_x = 1637700. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 31.36 mm
$\sigma_m = N/A - Mv/J_u$ = -239.7 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 17.36 mm
$\sigma_c = N/A - Mv/J_u$ = -138.2 N/mm^2
τ_c = 2.088 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.2 N/mm^2
S^* = 4093. mm^3

ESΣ09.xxxx.014

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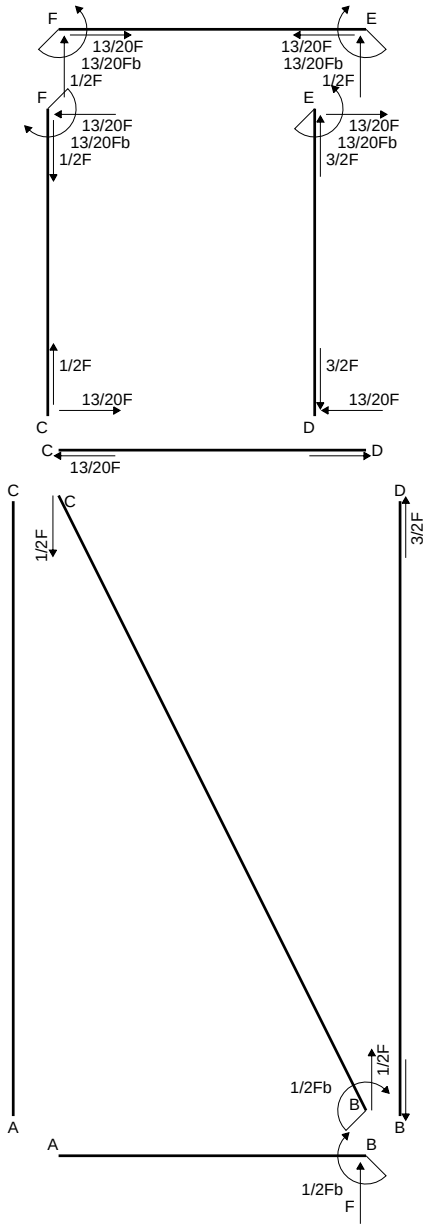
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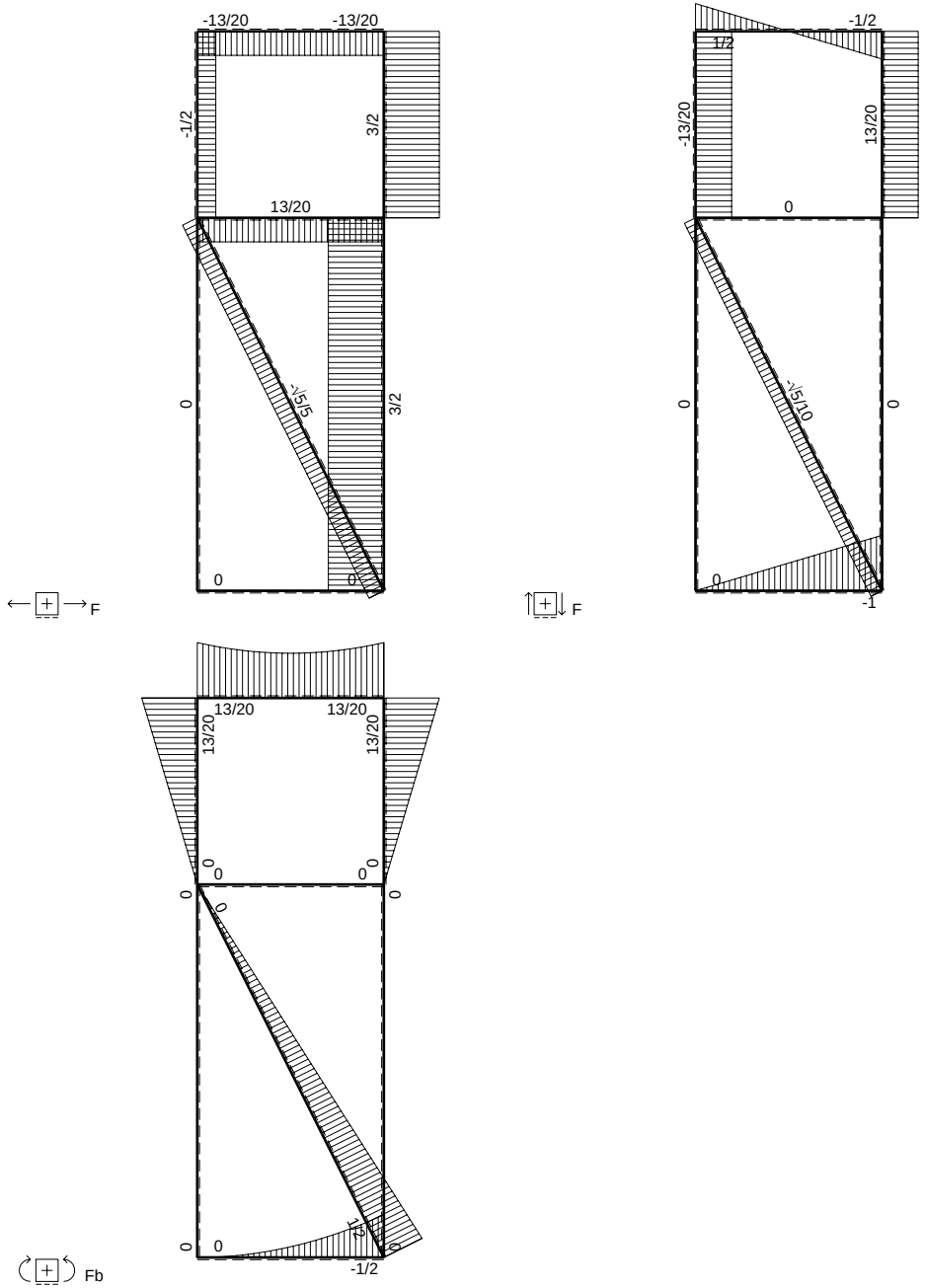
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
13/20
-1/2
3/2
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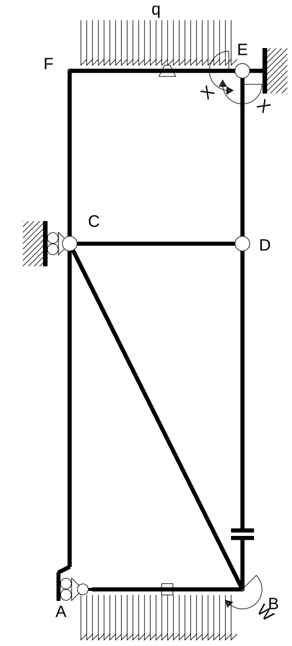
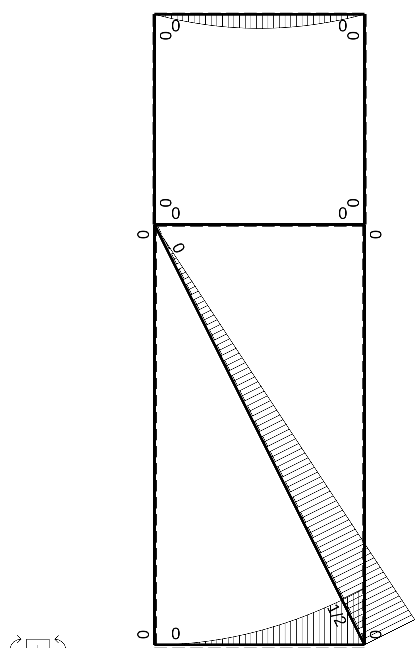
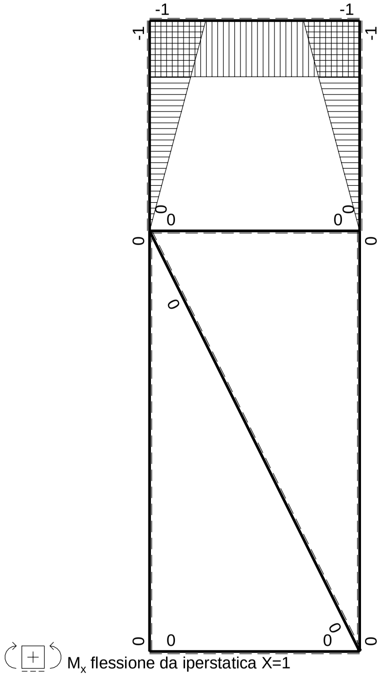
F
1/2
-√5/10
-1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

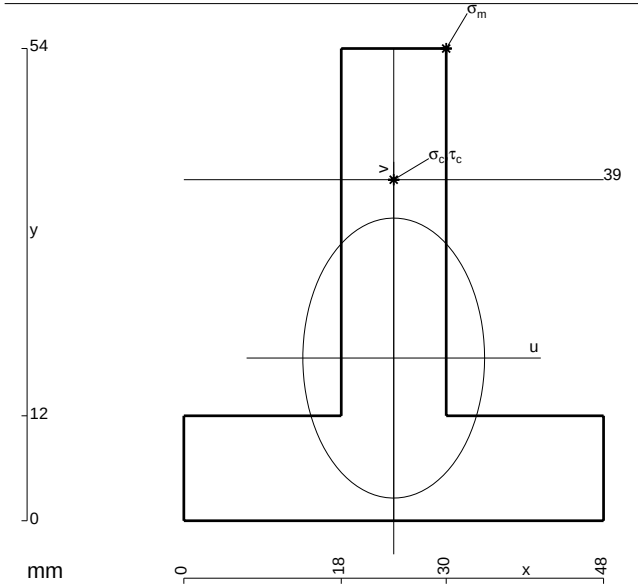
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 910789 ESΣ09.xxxx.015



A = 1080. mm^2
J_u = 276955. mm^4
J_v = 116640. mm^4
y_g = 18.6 mm
T_y = -5490. N
M_x = -1564650. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 35.4 mm
$\sigma_m = -Mv/J_u$ = 200. N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.4 mm
$\sigma_c = -Mv/J_u$ = 115.2 N/mm^2
τ_c = 8.296 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.1 N/mm^2
S^* = 5022. mm^3

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ESΣ09.xxxx.015

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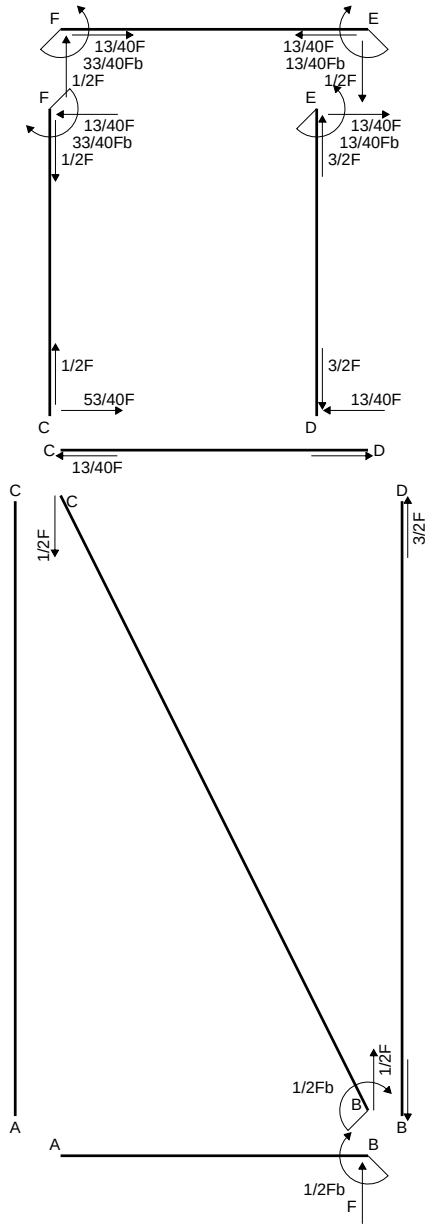
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13/40Fb
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3/2F
13/40F
C
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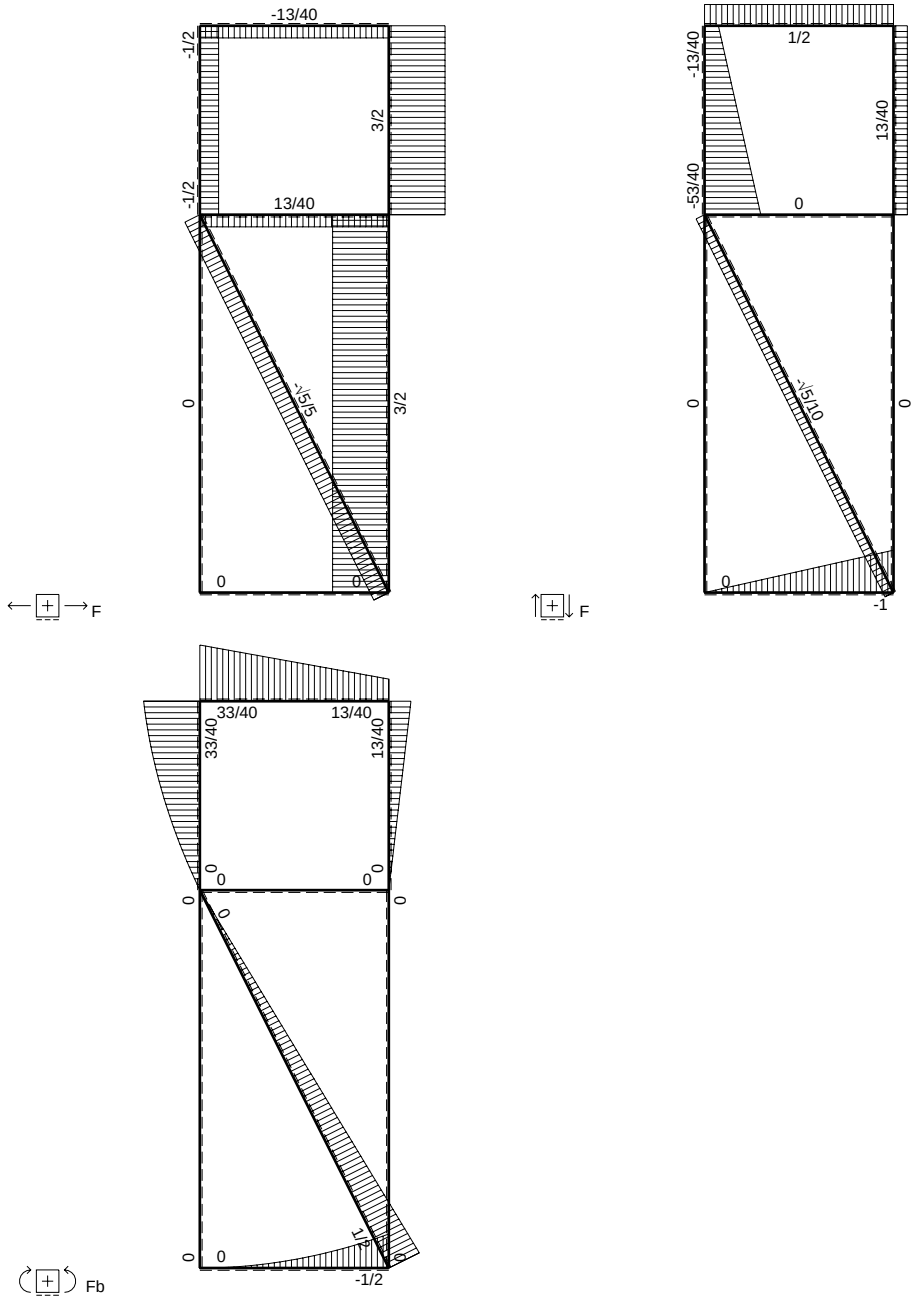
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3/2
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3/2
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0
F
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1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
0
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212858 ESΣ09.xxxx.016

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

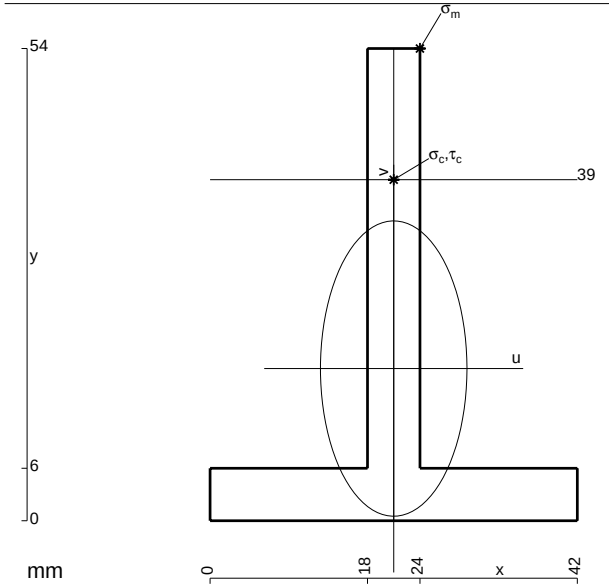
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 212858 ESΣ09.xxxx.016



A = 540. mm^2
J_u = 154030. mm^4
J_v = 37908. mm^4
y_g = 17.4 mm
T_y = -2940. N
M_x = -882000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 36.6 mm
$\sigma_m = -Mv/J_u$ = 209.6 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 21.6 mm
$\sigma_c = -Mv/J_u$ = 123.7 N/mm^2
τ_c = 8.332 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.5 N/mm^2
S^* = 2619. mm^3

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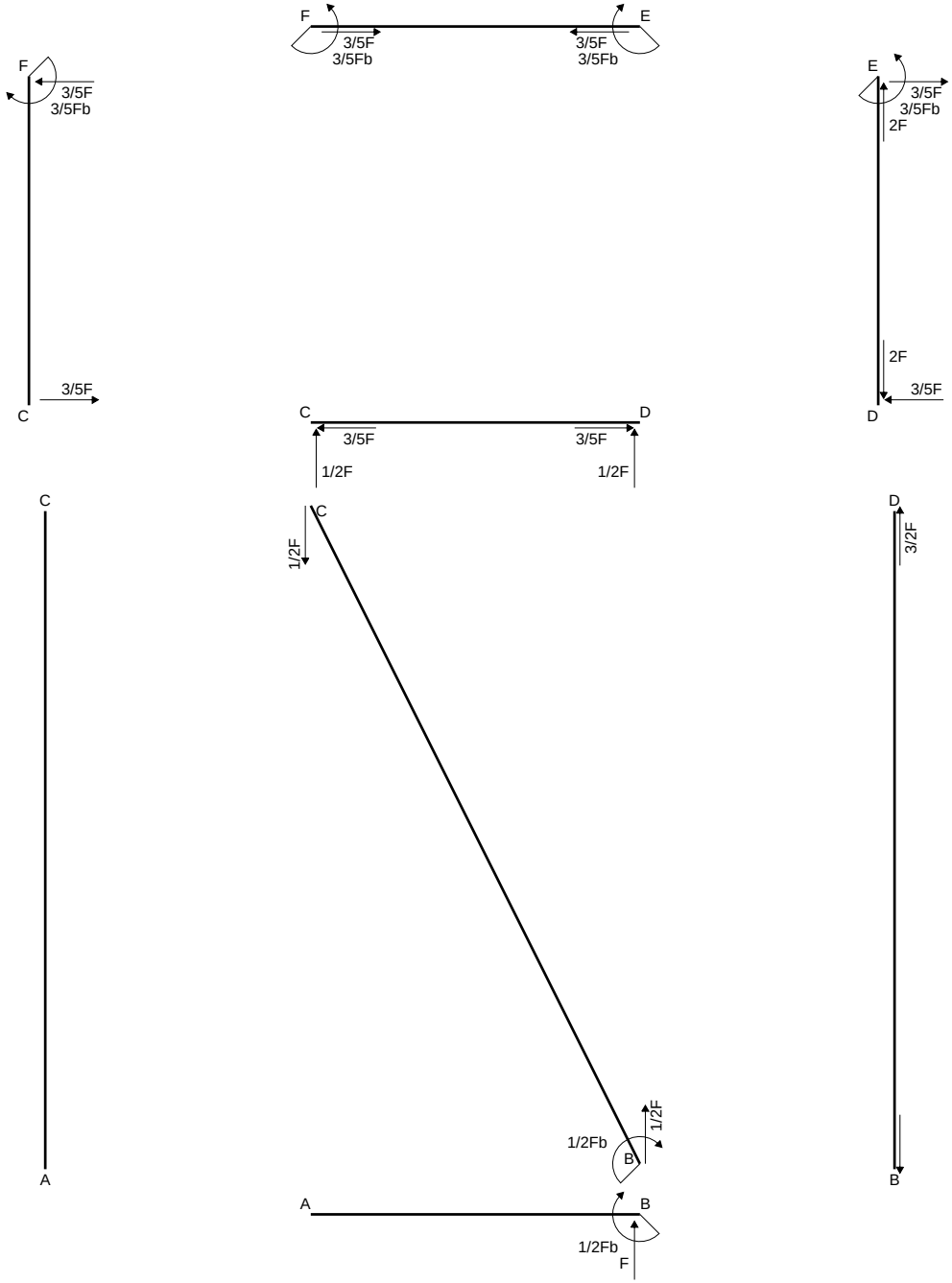
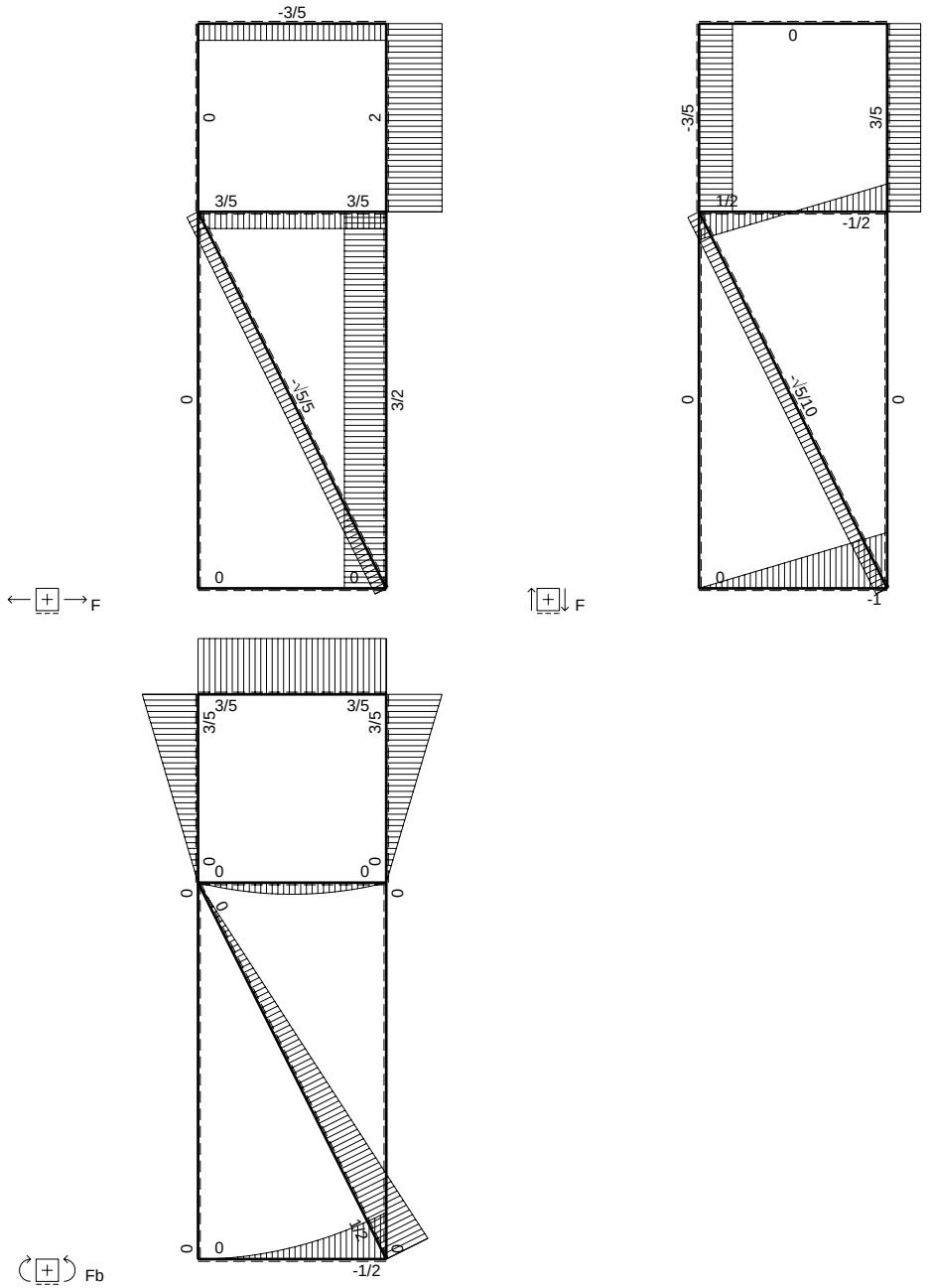

F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F
A
B


-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
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-3/5
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-1/2
0
-√5/10
0
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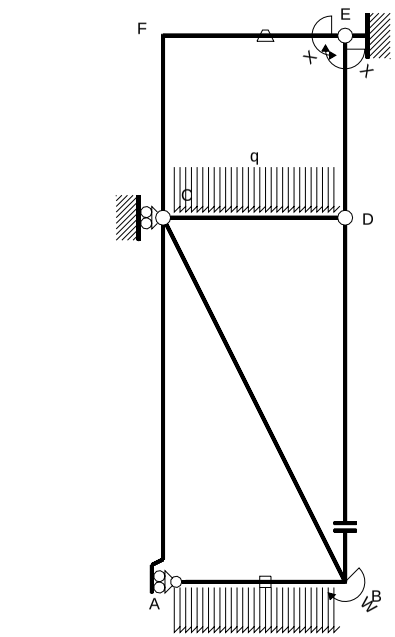
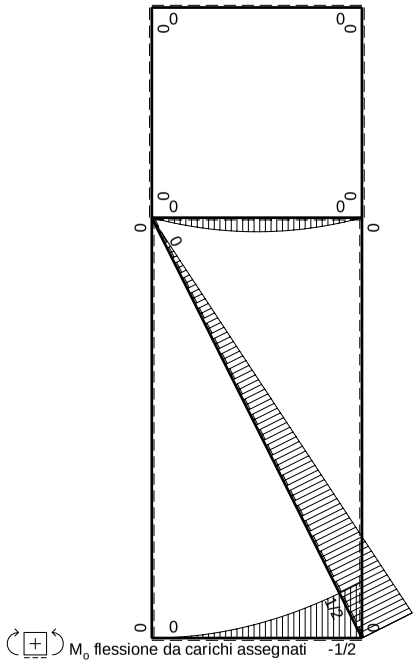
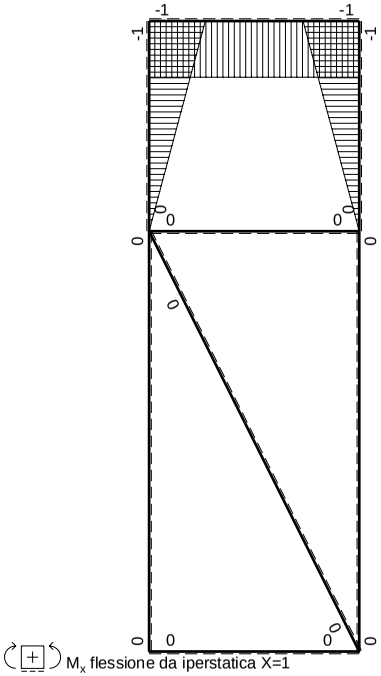
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

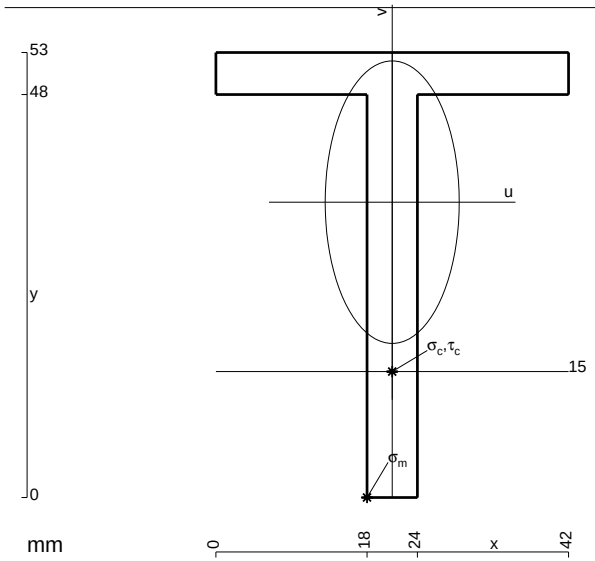
$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1\ )\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1\ )\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 498. mm^2
J_u = 141019. mm^4
J_v = 31734. mm^4
y_g = 35.17 mm
T_y = -2840. N
M_x = -880400. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -35.17 mm
$\sigma_m = -Mv/J_u$ = -219.6 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -20.17 mm
$\sigma_c = -Mv/J_u$ = -126. N/mm^2
τ_c = 8.36 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.8 N/mm^2
S^* = 2491. mm^3

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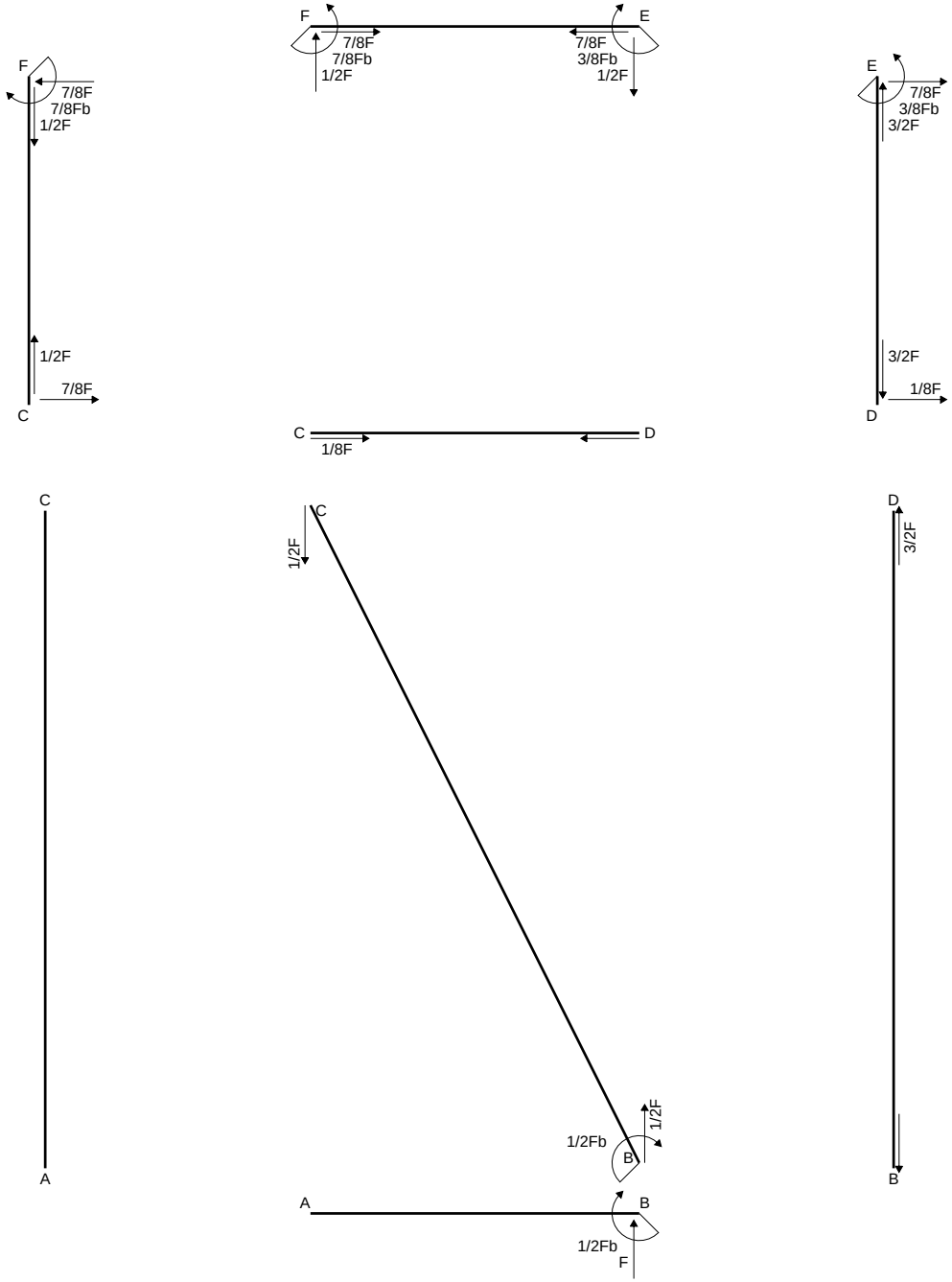
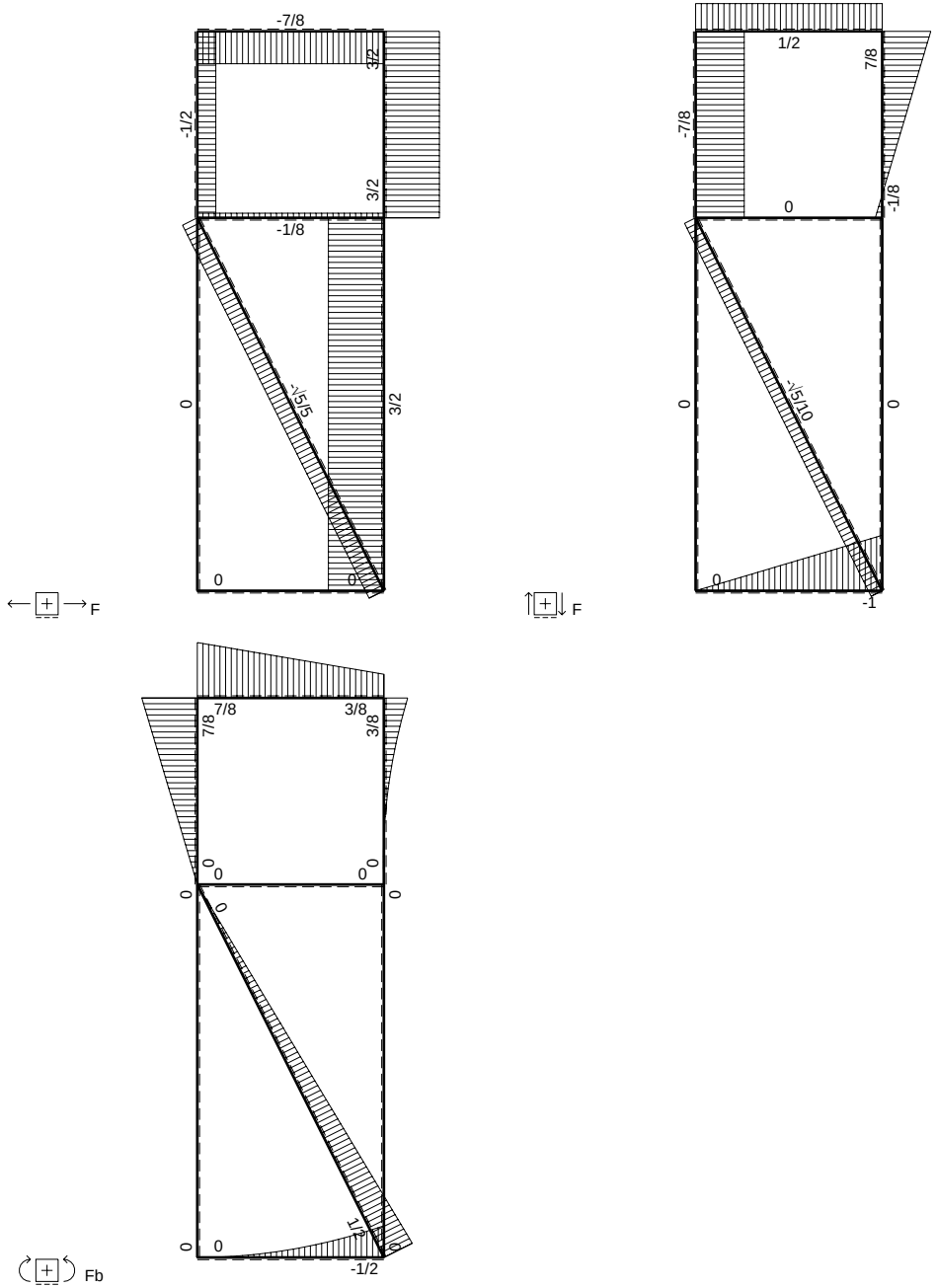

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
1/2
7/8
-1/8
-√5/10
-1
F
7/8
3/8
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

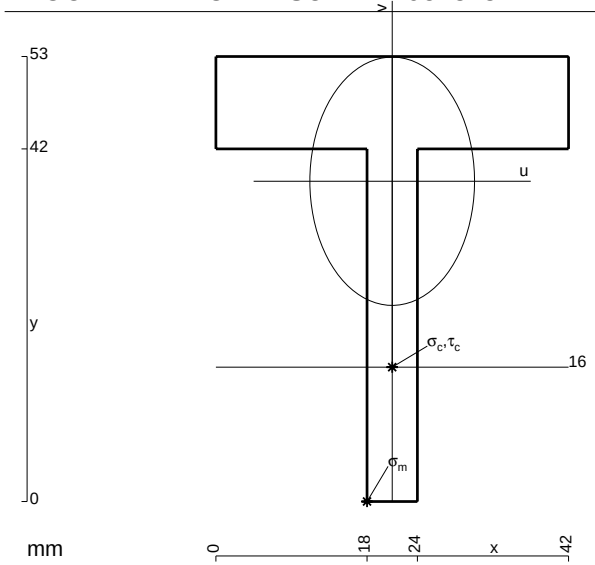
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 714. mm^2
J_u = 156210. mm^4
J_v = 68670. mm^4
y_g = 38.15 mm
T_y = -2850. N
M_x = -940500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.15 mm
$\sigma_m = -Mv/J_u$ = -229.7 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.15 mm
$\sigma_c = -Mv/J_u$ = -133.3 N/mm^2
τ_c = 8.8 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.2 N/mm^2
S^* = 2894. mm^3

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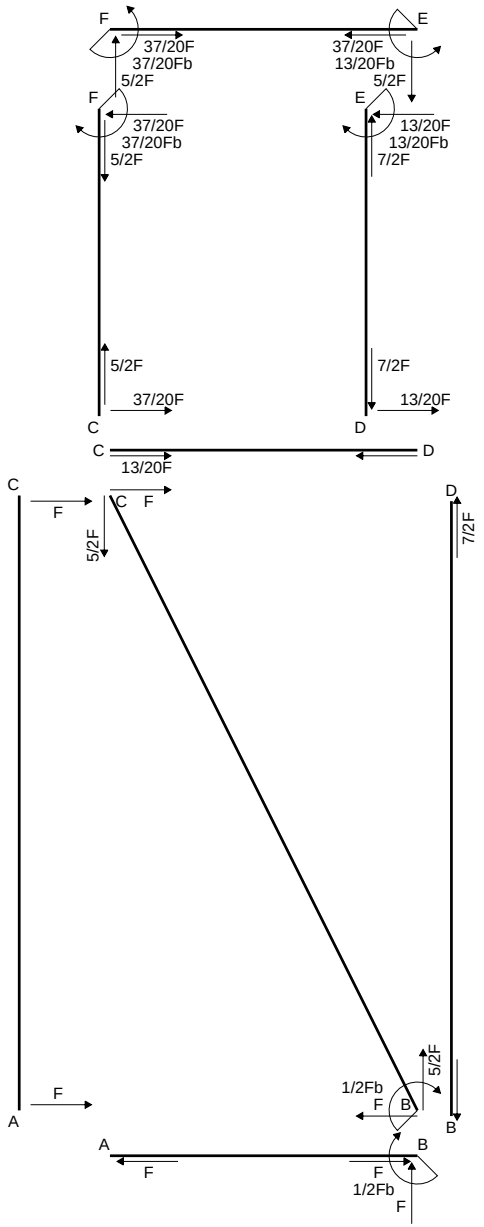
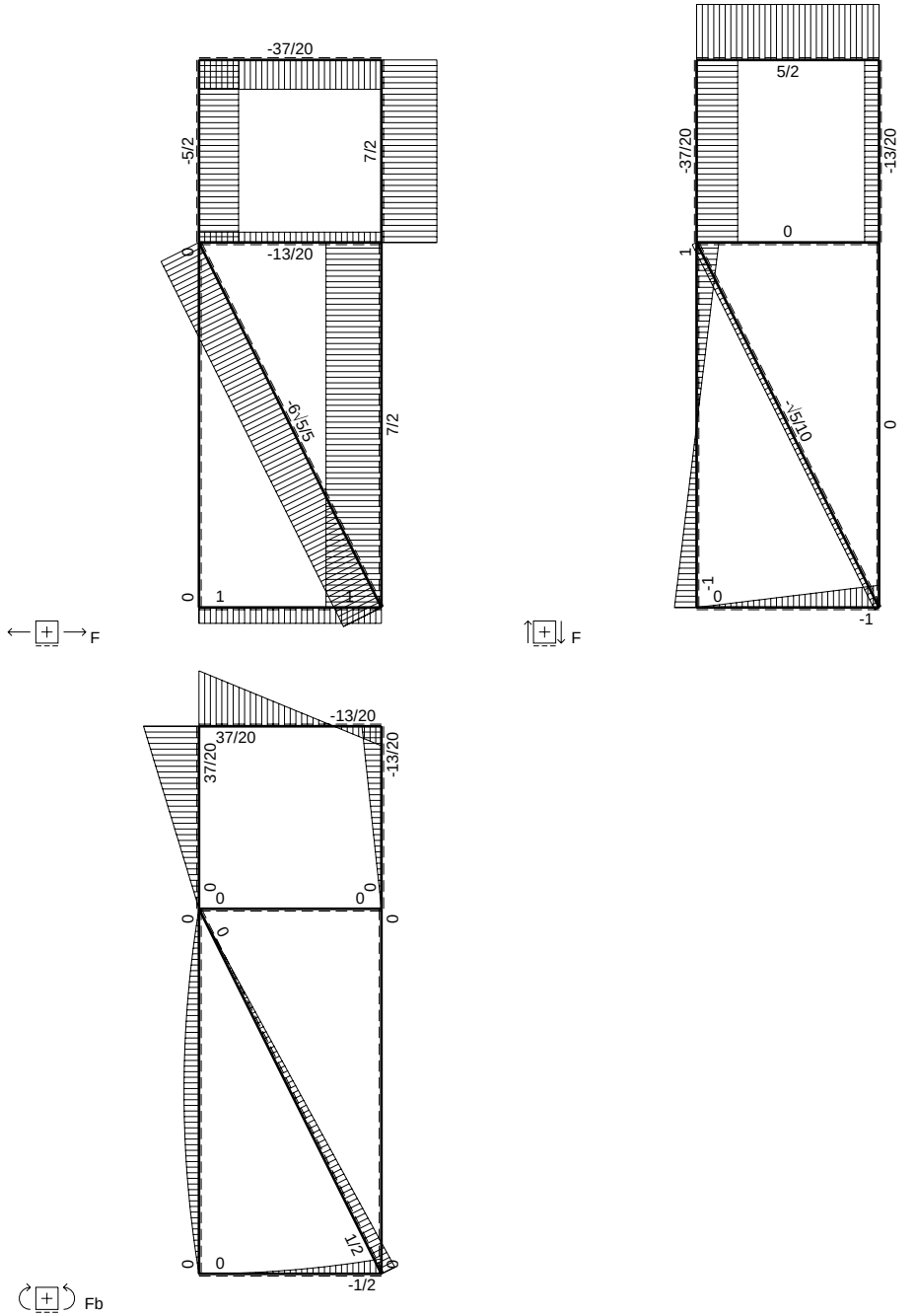

F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb


-37/20
-5/2
7/2
-13/20
-5\5/5
1
0
F
5/2
-37/20
-13/20
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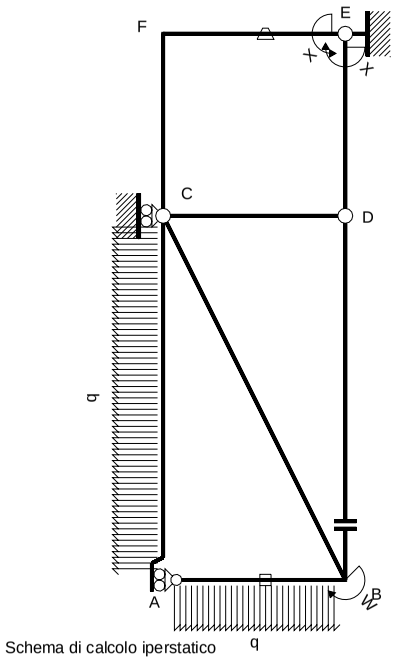
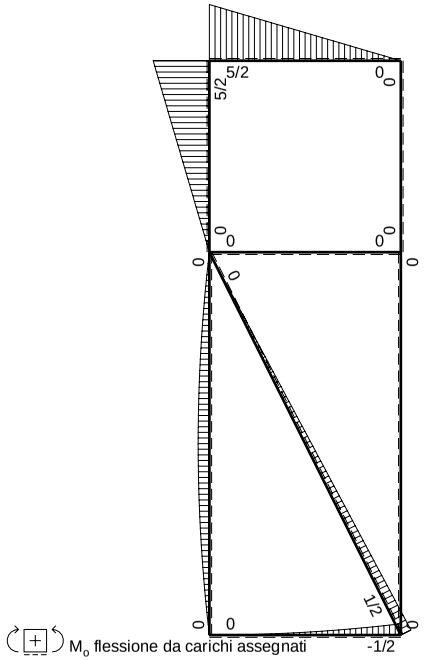
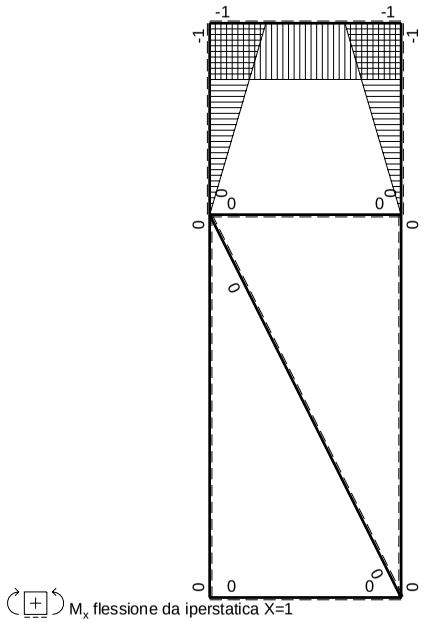
-1
37/20
Fb
1/2
-1/2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
				totali			$-13/12Fb^2$/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211691 ESΣ09.xxxx.019

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

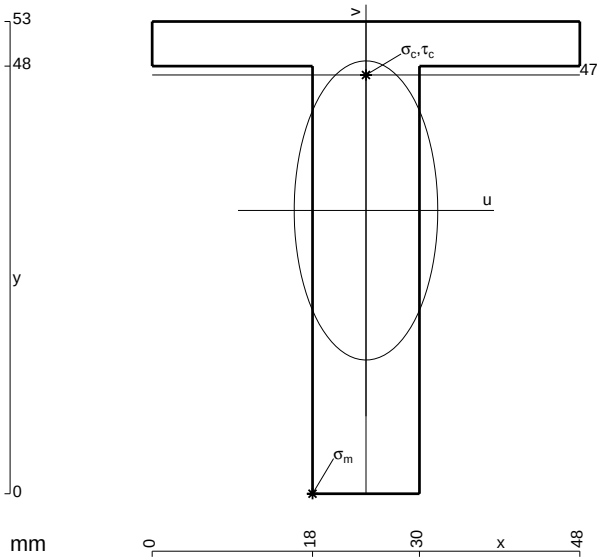
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 816. mm^2
J_u = 230061. mm^4
J_v = 52992. mm^4
y_g = 31.79 mm
N = 5090. N
T_y = -5090. N
M_x = -1781500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -31.79 mm
$\sigma_m = N/A - Mv/J_u$ = -240. N/mm^2
x_c = 24. mm
y_c = 47. mm
v_c = 15.21 mm
$\sigma_c = N/A - Mv/J_u$ = 124. N/mm^2
τ_c = 8.625 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.9 N/mm^2
S^* = 4678. mm^3

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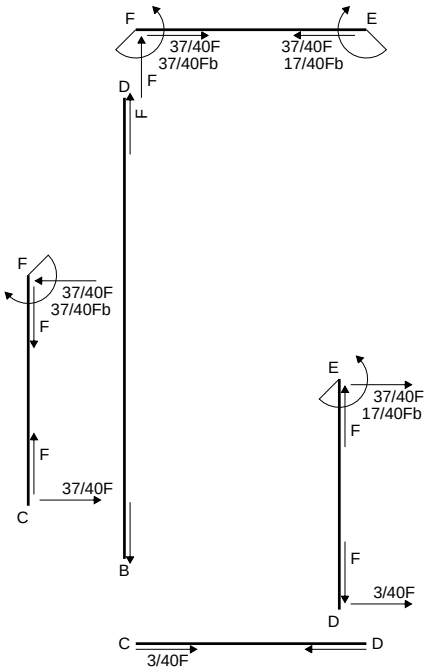
F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
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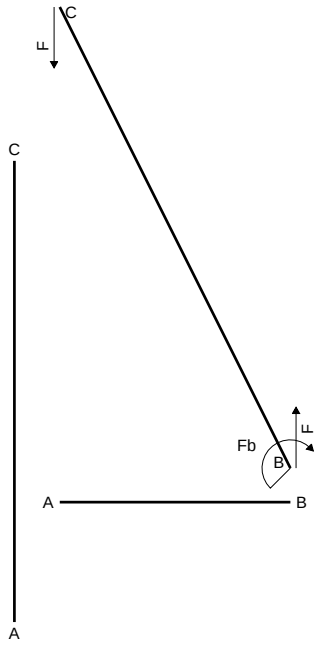
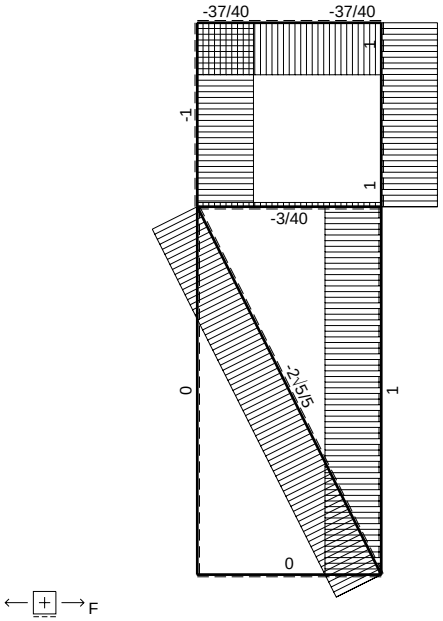
D
C
3/40F
D


C
F
C
Fb
F
B
A
B
A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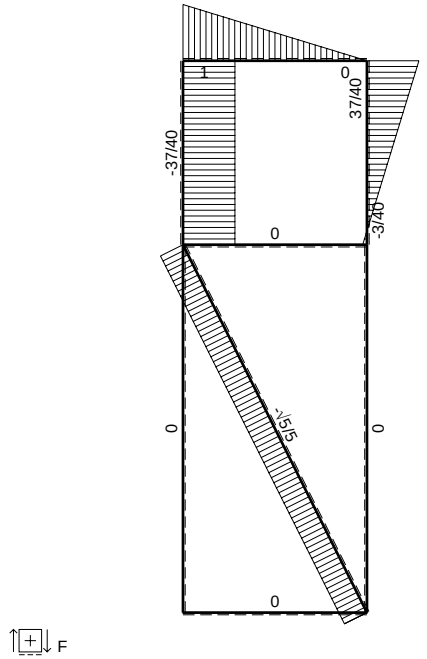
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
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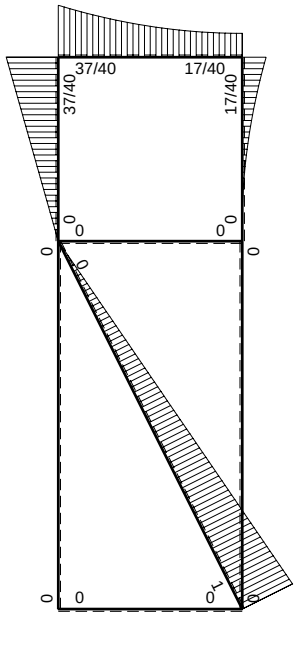
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F


37/40
17/40
37/40
17/40
0
0
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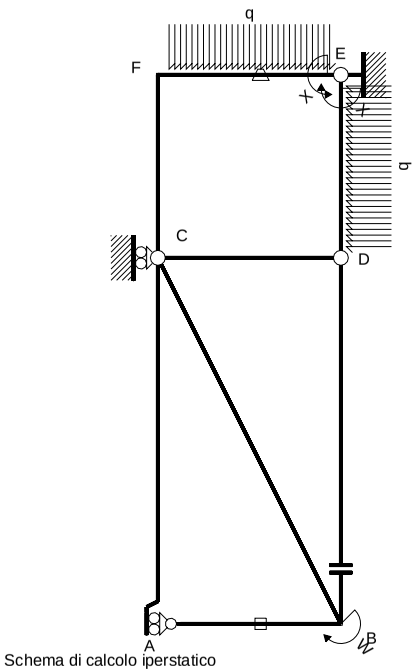
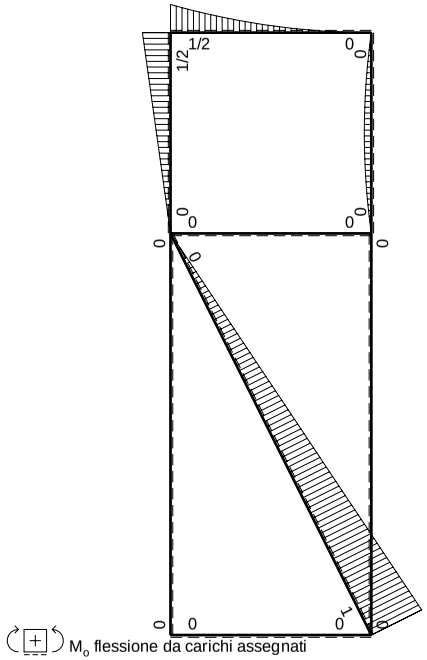
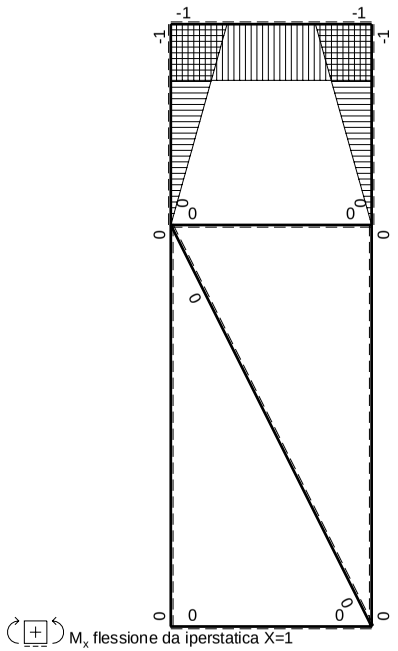
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 956563 ESΣ09.xxxx.020

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

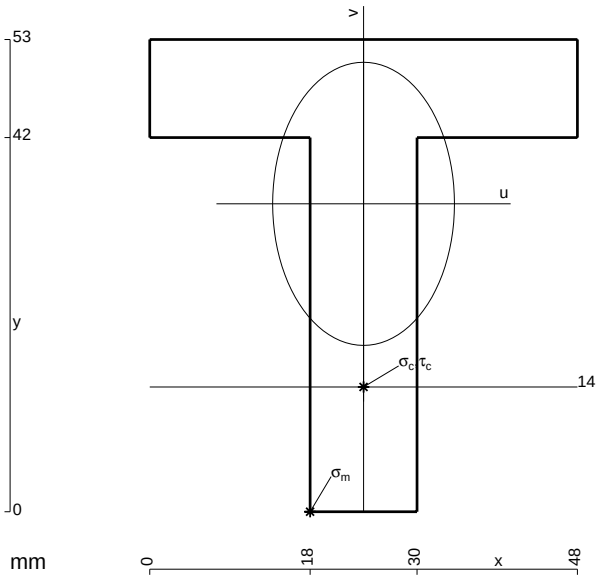
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 1032. mm^2
J_u = 260495. mm^4
J_v = 107424. mm^4
y_g = 34.56 mm
N = -3703. N
T_y = -1851. N
M_x = 1531800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.56 mm
σ_m = N/A-Mv/J_u = 199.6 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -20.56 mm
σ_c = N/A-Mv/J_u = 117.3 N/mm^2
τ_c = 2.742 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.4 N/mm^2
S^* = 4630. mm^3

ESΣ09.xxxx.020

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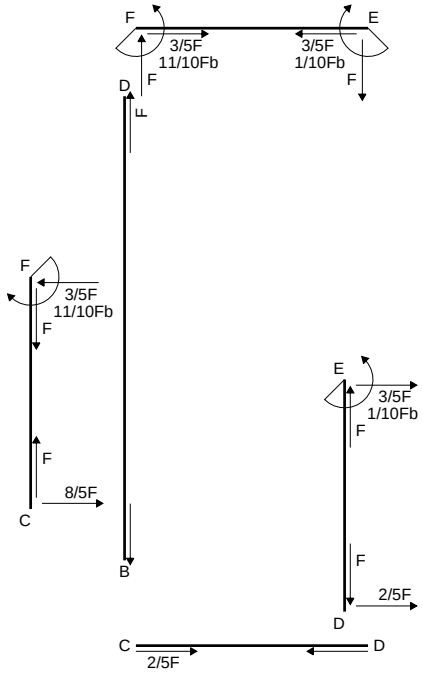
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
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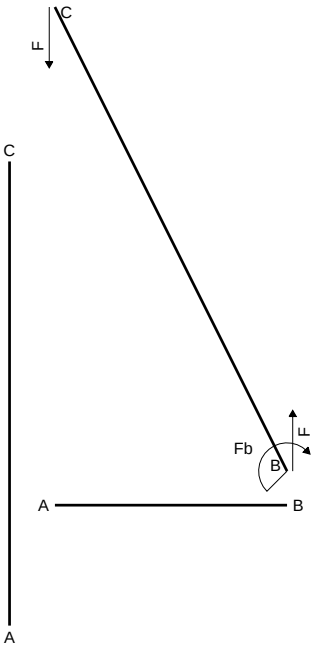
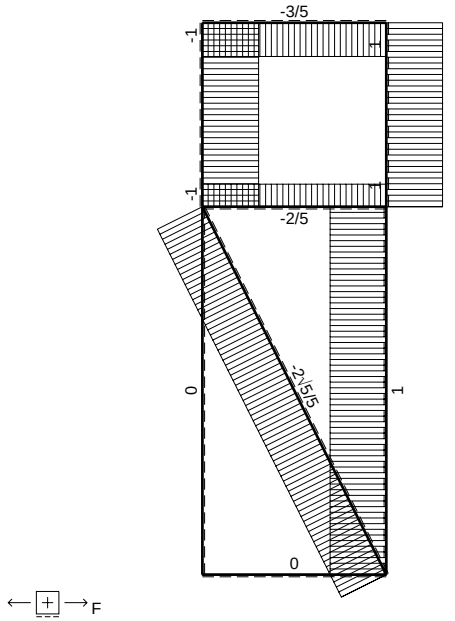
1/10Fb
F
F
8/5F
C
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
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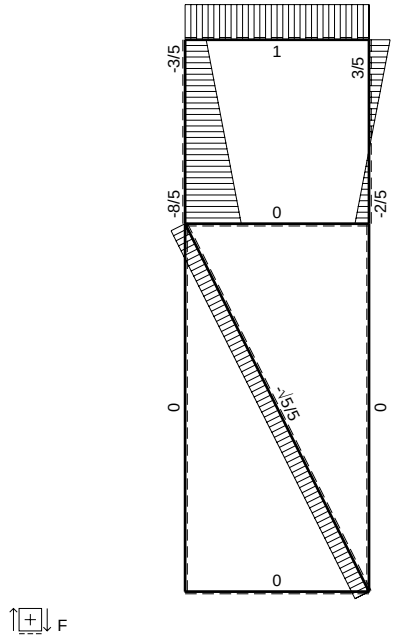
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
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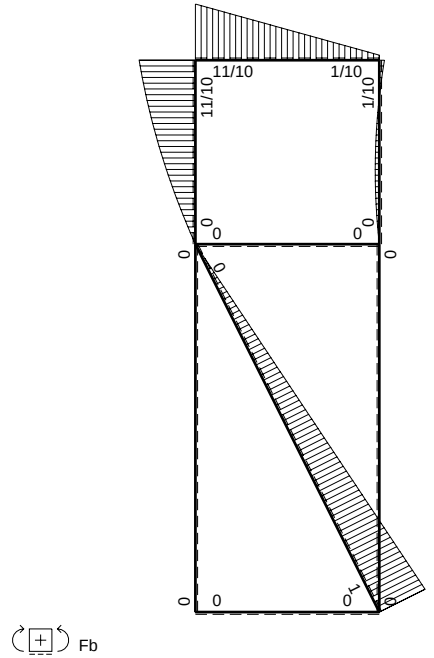
-2/5
0
-2/5/5
0
F


11/10
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11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3XMb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb²/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx²	0	-Fb+3/2Fx-1/2qx³/b	0	1-2x/b+x²/b²	(-3/8+0)Fb²/EJ	1/3XMb/EJ
CF b	x/b	-3/2Fx+1/2qx²	0	-3/2Fx²/b+1/2qx³/b	0	x²/b²		
	totali						1/6Fb²/EJ	5/3XMb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 212811 ESΣ09.xxxx.021

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

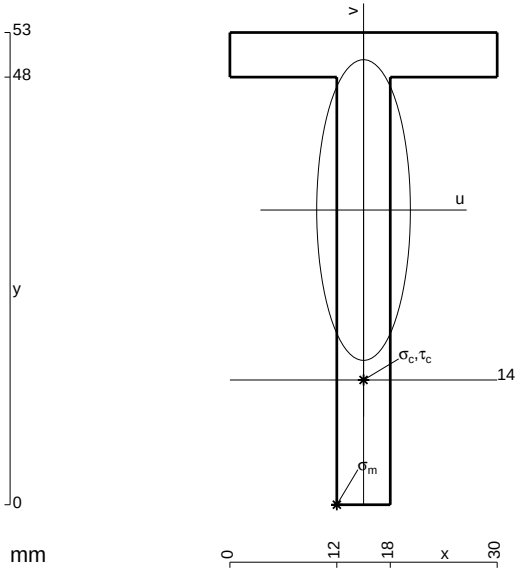
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 438. mm^2
J_u = 124871. mm^4
J_v = 12114. mm^4
y_g = 33.08 mm
N = -1807. N
T_y = -903.4 N
M_x = 808000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -33.08 mm
$\sigma_m = N/A - Mv/J_u$ = 209.9 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -19.08 mm
$\sigma_c = N/A - Mv/J_u$ = 119.3 N/mm^2
τ_c = 2.641 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.4 N/mm^2
S^* = 2190. mm^3

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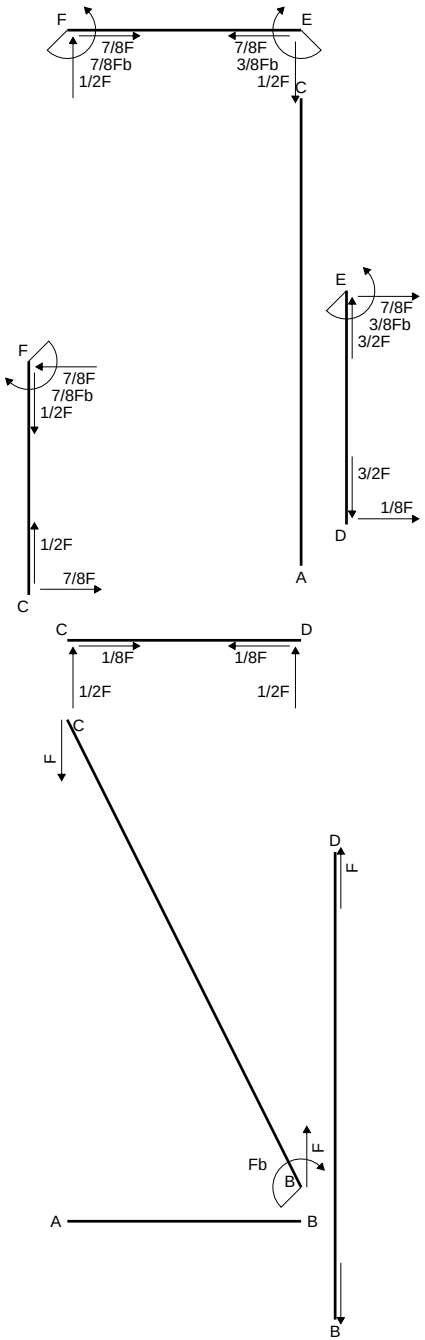
F
E
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7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
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3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
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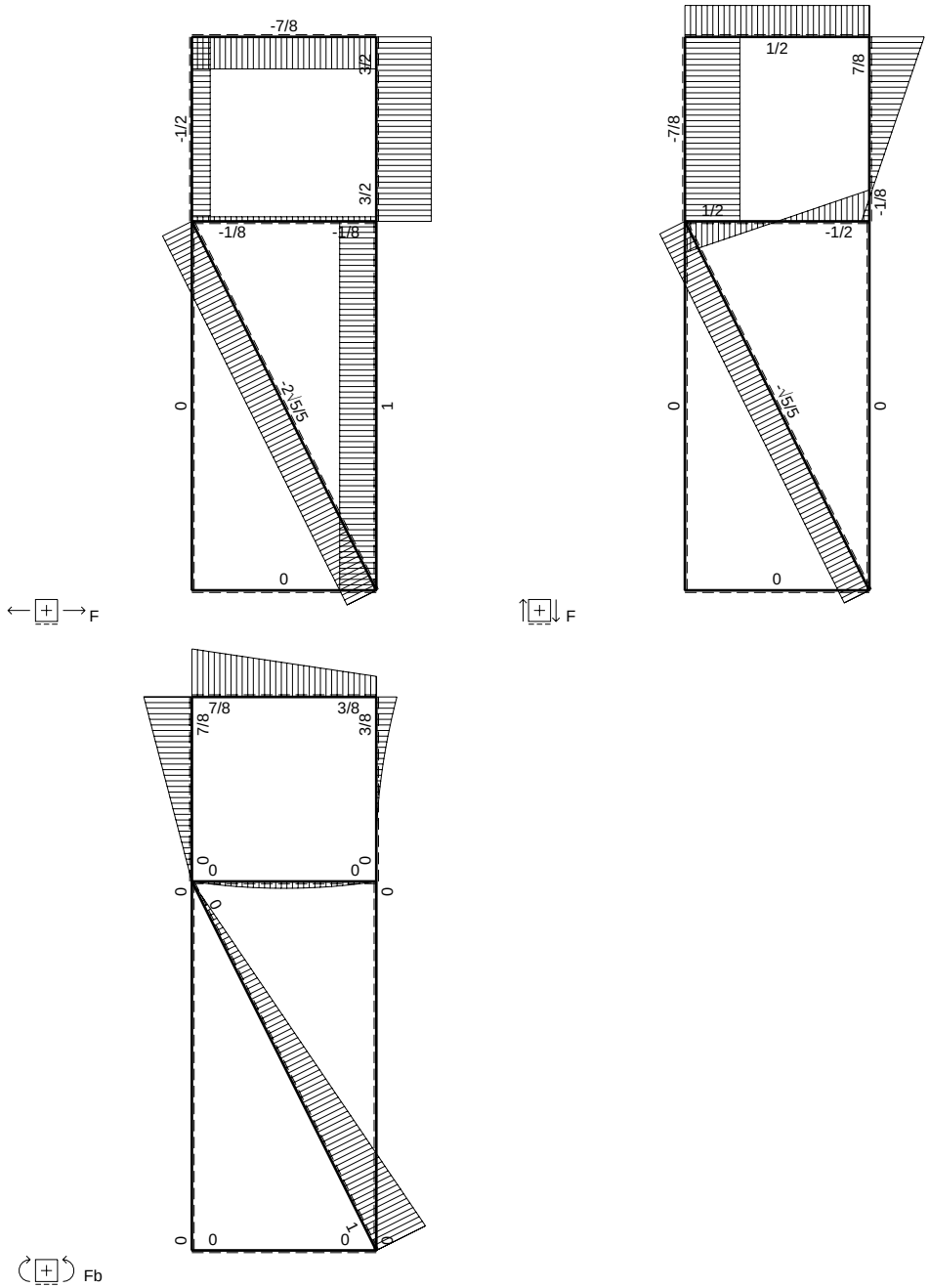
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
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-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
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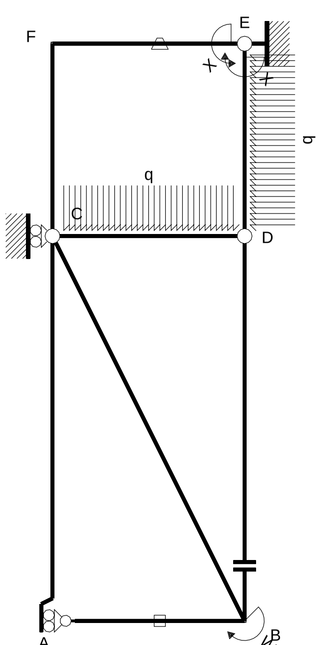
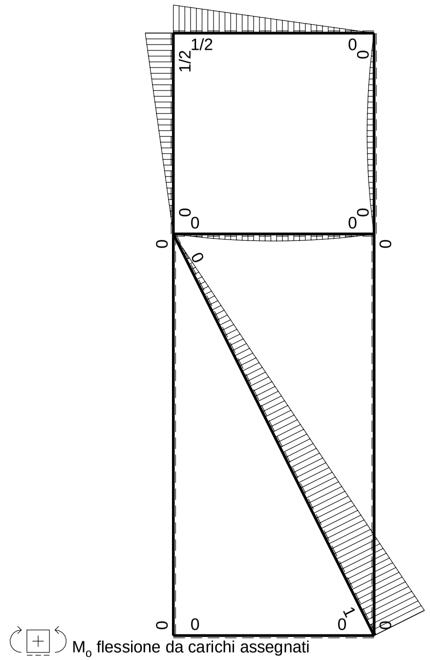
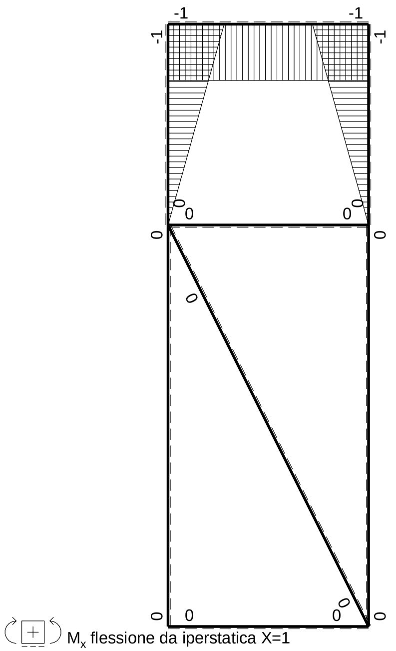
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

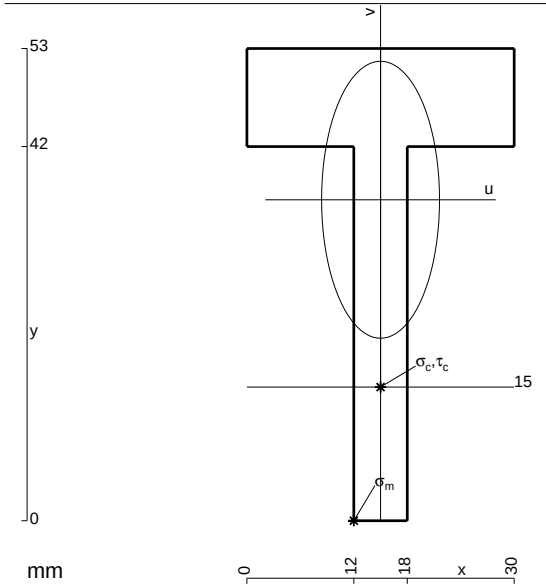
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 582. mm^2
J_u = 140714. mm^4
J_v = 25506. mm^4
y_g = 36.03 mm
N = -1762. N
T_y = -881. N
M_x = 866800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.03 mm
$\sigma_m = N/A - Mv/J_u$ = 218.9 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.03 mm
$\sigma_c = N/A - Mv/J_u$ = 126.5 N/mm^2
τ_c = 2.679 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.6 N/mm^2
S^* = 2567. mm^3

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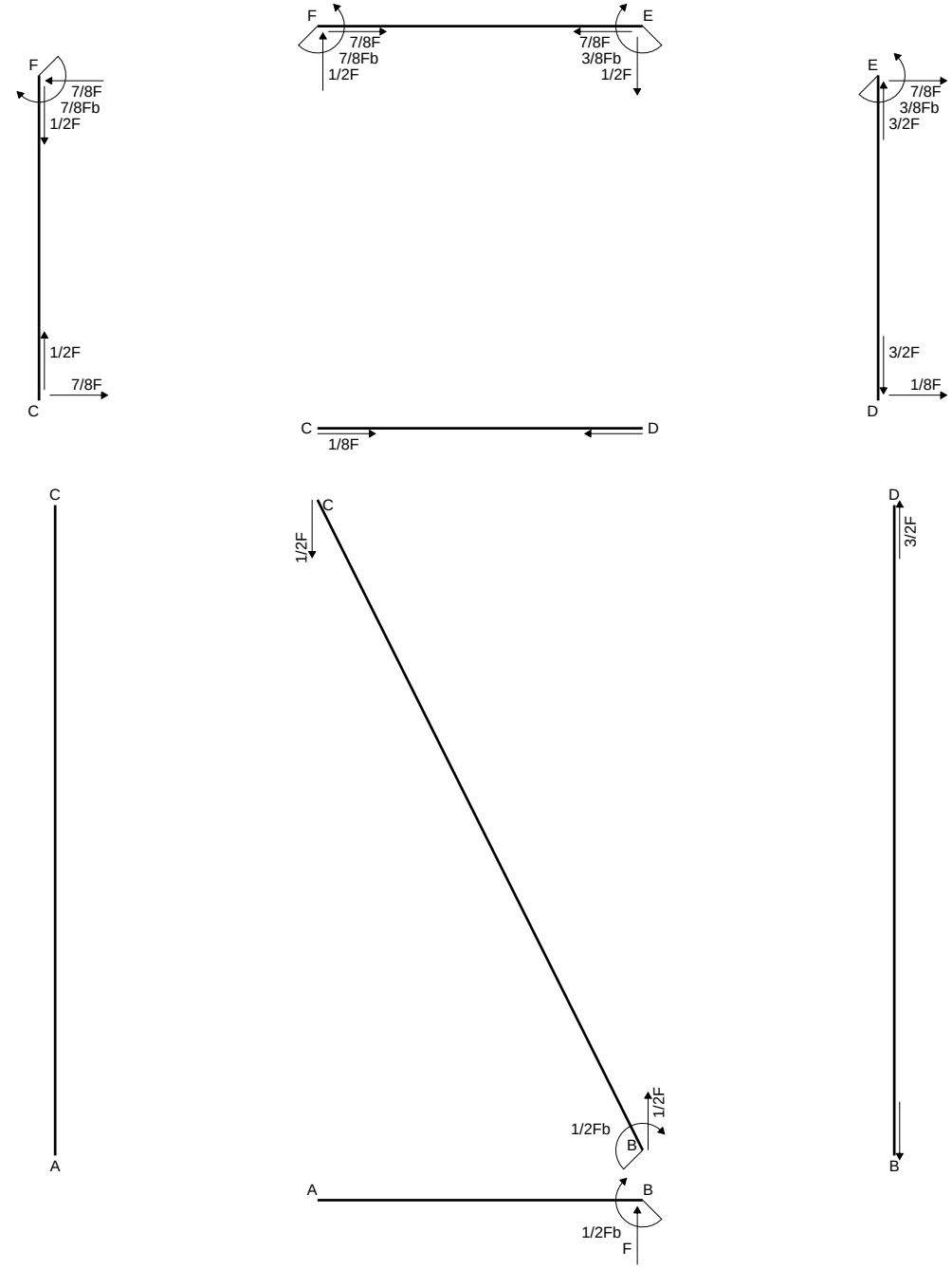
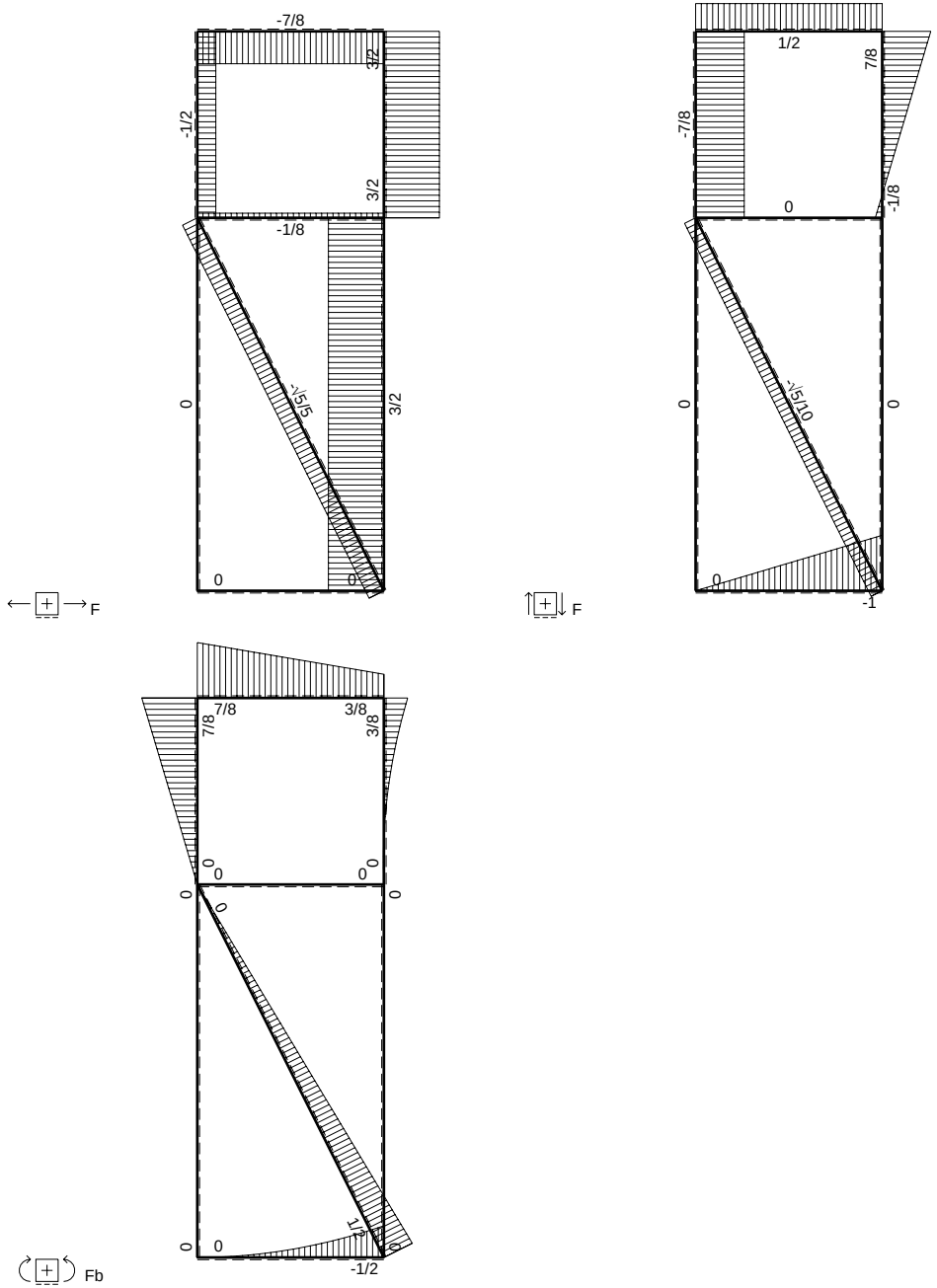

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
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1/2
7/8
-1/8
-√5/10
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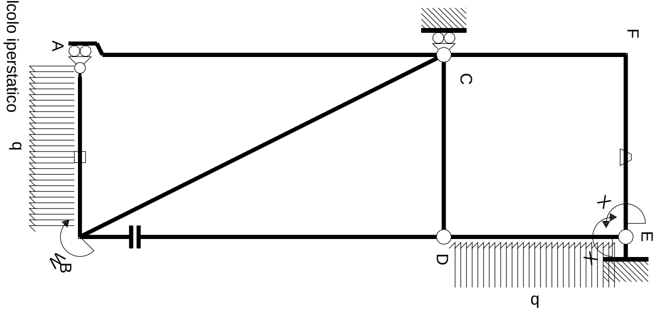
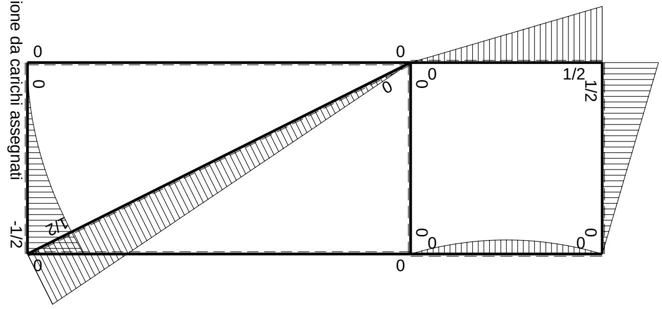
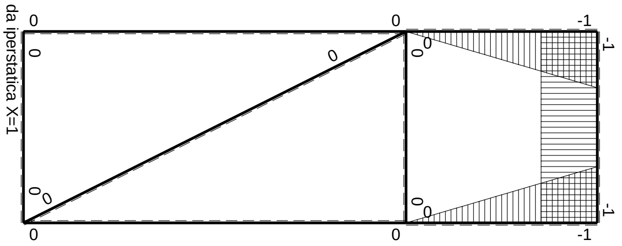
3/8
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

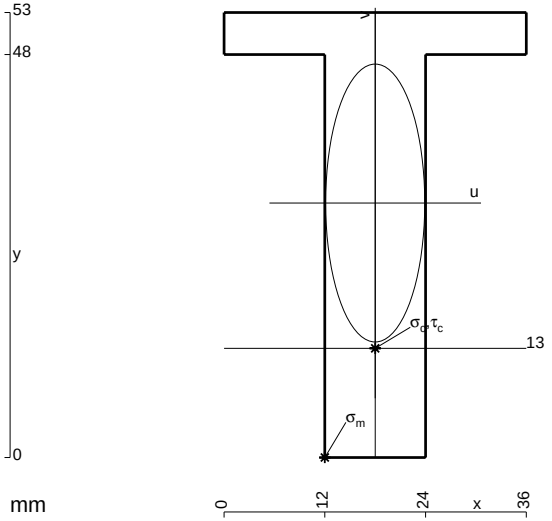
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 756. mm^2
J_u = 207276. mm^4
J_v = 26352. mm^4
y_g = 30.31 mm
T_y = -6550. N
M_x = -1572000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -30.31 mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -17.31 mm
$\sigma_c = -Mv/J_u$ = -131.3 N/mm^2
τ_c = 9.781 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 132.4 N/mm^2
S^* = 3714. mm^3

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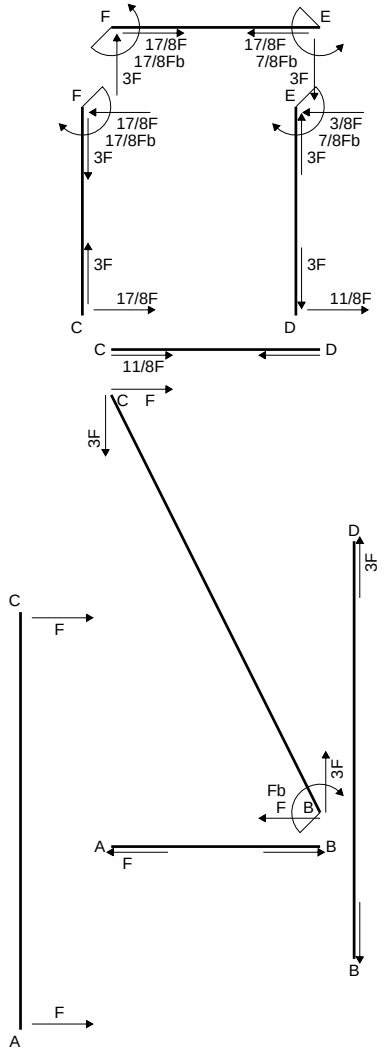
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17/8Fb
3F
17/8F
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3F
F
E
17/8F
17/8Fb
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7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
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3F
D
3F
C
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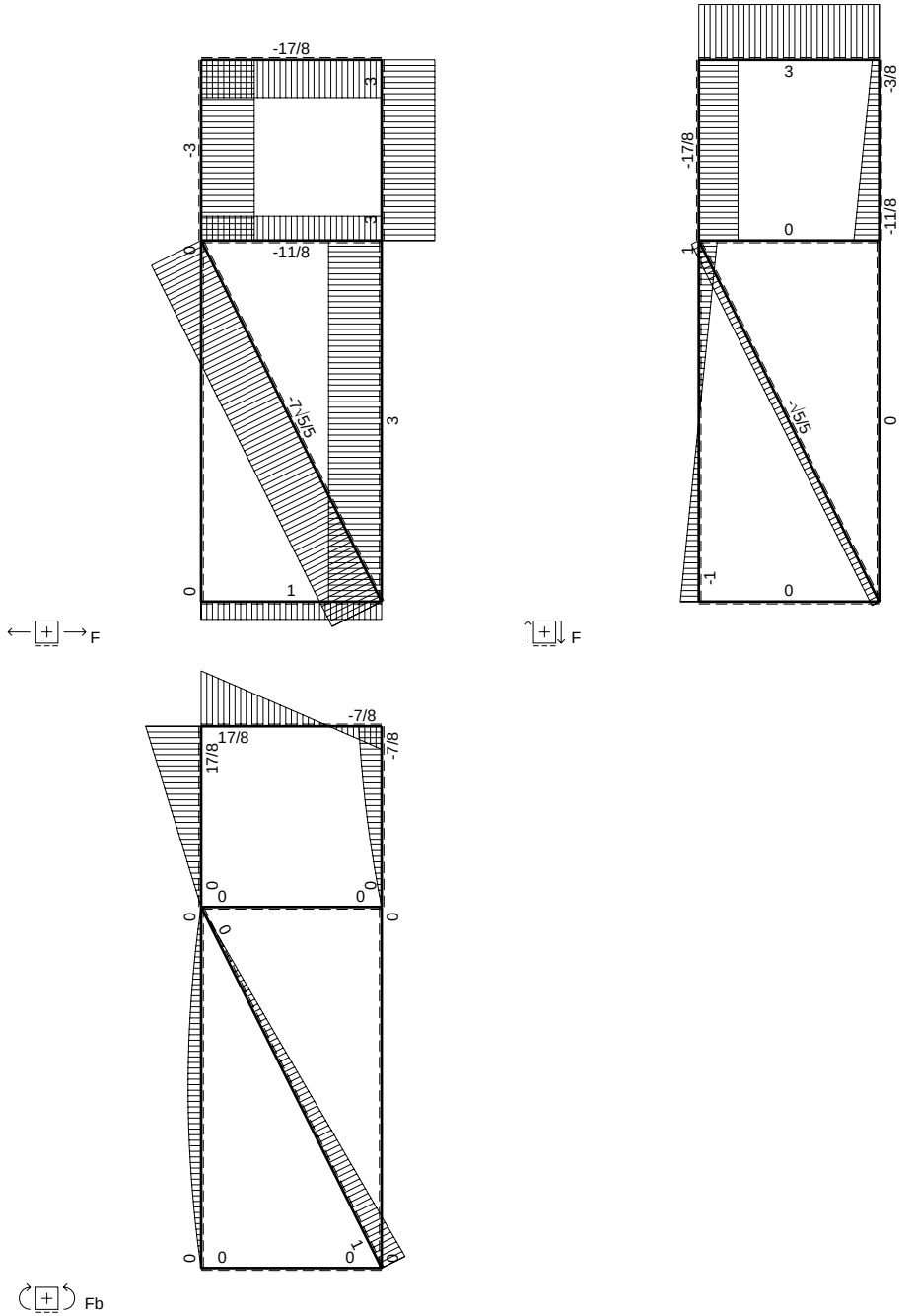
Fb
F
B
A
B
F
B
F
A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-11/8
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1
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F
3
-17/8
-3/8
-11/8
0
-√5/5
-1
F
-7/8
17/8
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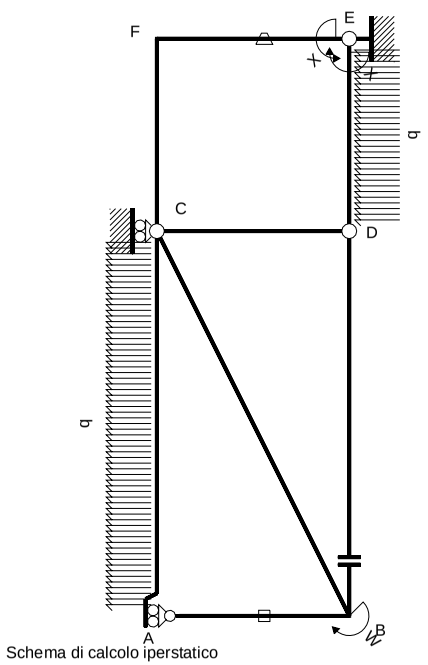
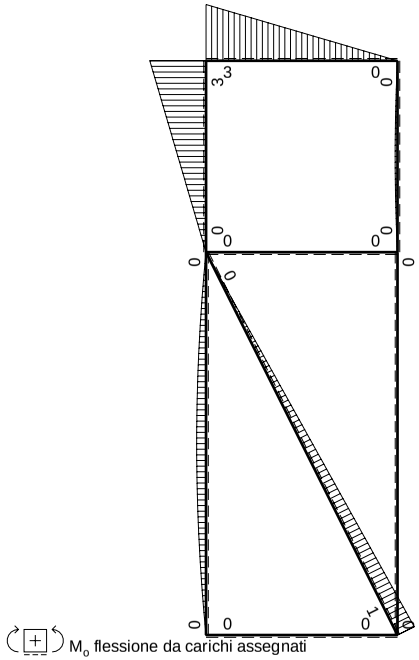
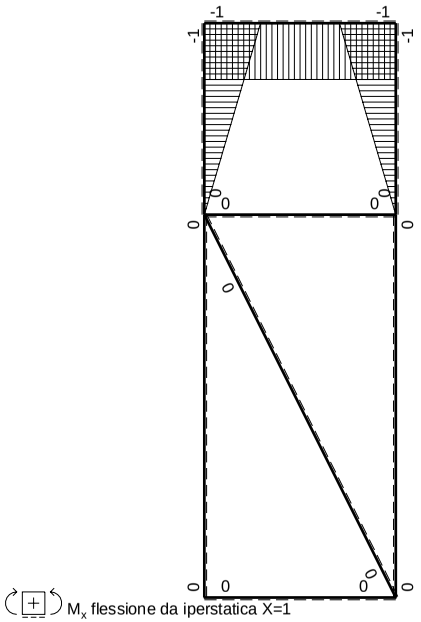
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
		totali					$-35/24Fb^2/EJ$	5/3Xb/EJ
		iperstatica $X=W_{EF}$					7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211309 ESΣ09.xxxx.024

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

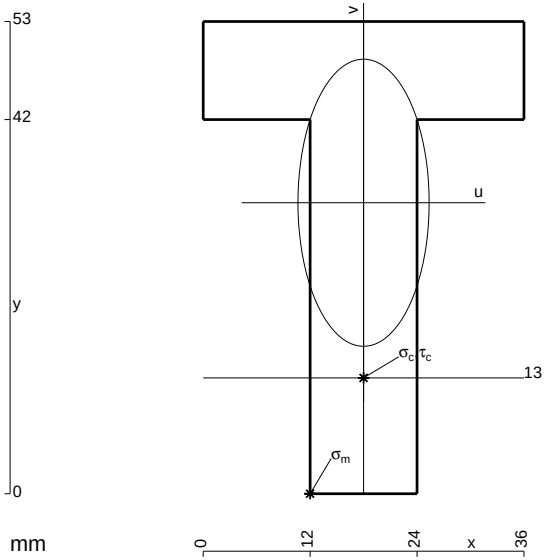
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 900. mm^2
J_u = 233812. mm^4
J_v = 48816. mm^4
y_g = 32.66 mm
N = -11082. N
T_y = -1583. N
M_x = 1805400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.66 mm
$\sigma_m = N/A - Mv/J_u$ = 239.9 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -19.66 mm
$\sigma_c = N/A - Mv/J_u$ = 139.5 N/mm^2
τ_c = 2.303 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 139.6 N/mm^2
S^* = 4081. mm^3

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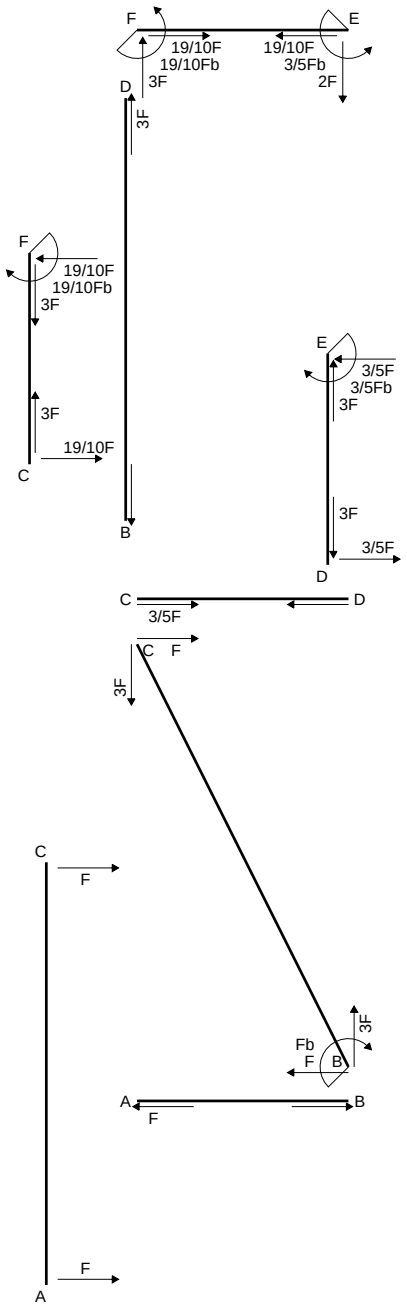
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19/10Fb
19/10F
3/5Fb
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3F
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3F
F
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E
3/5F
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3F
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B
3/5F
D
C
D
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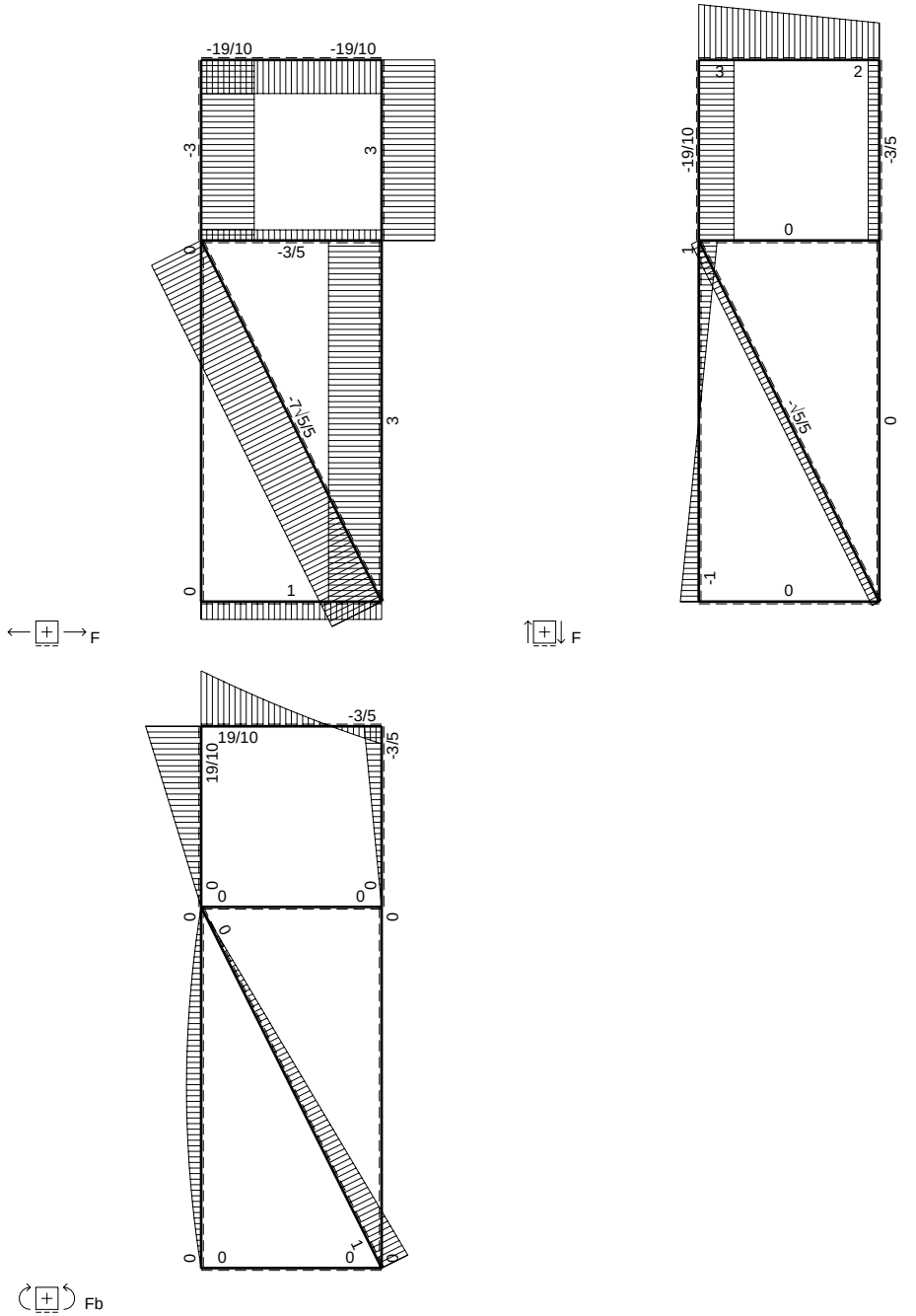
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A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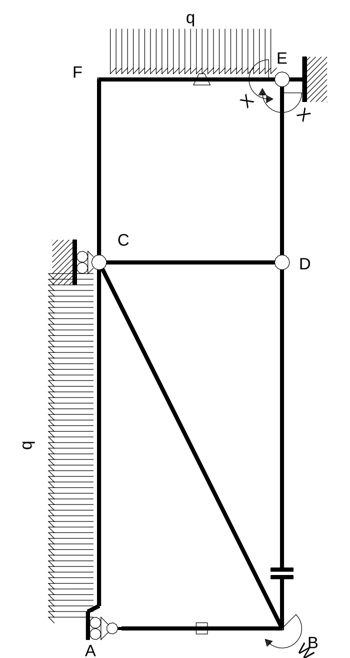
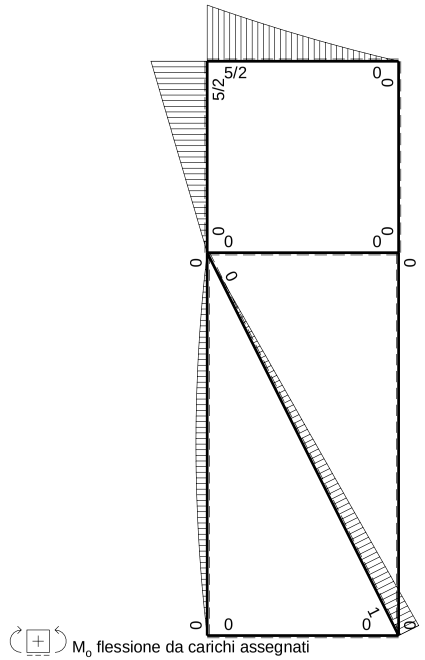
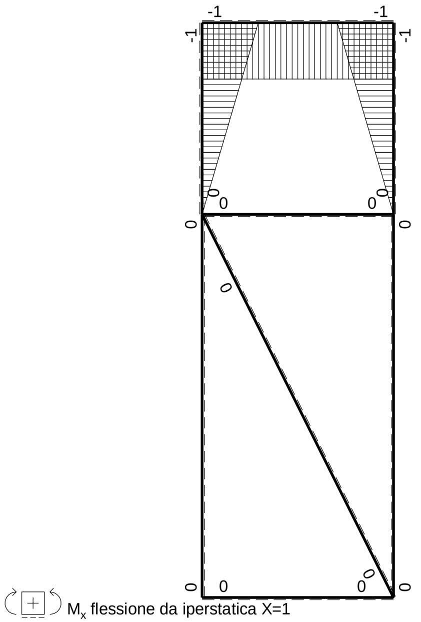
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

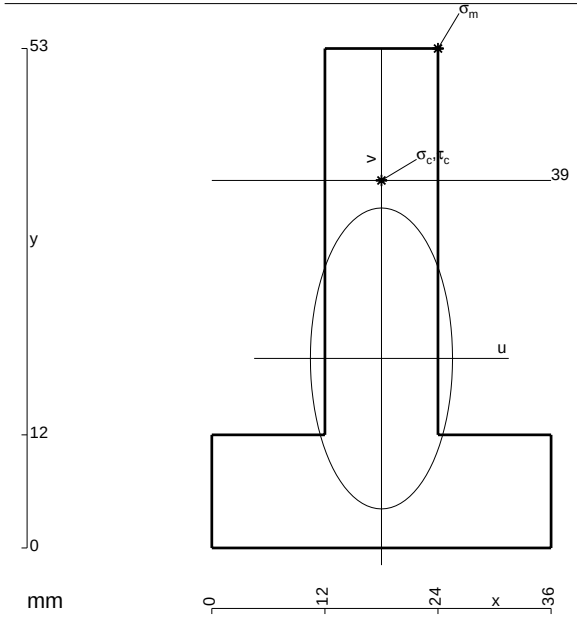
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 924. mm^2
J_u = 235641. mm^4
J_v = 52560. mm^4
y_g = 20.11 mm
N = -7795. N
T_y = -1114. N
M_x = 1369500. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 32.89 mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 18.89 mm
$\sigma_c = N/A - Mv/J_u$ = -118.2 N/mm^2
τ_c = 1.713 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.3 N/mm^2
S^* = 4349. mm^3

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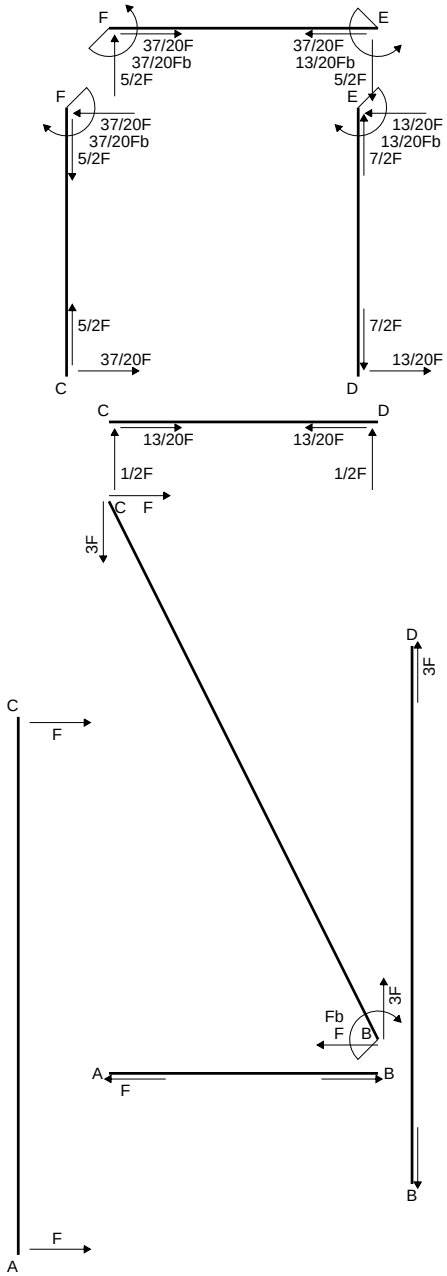
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37/20F
13/20Fb
5/2F
F
E
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13/20Fb
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37/20F
C
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13/20F
D
C
D
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13/20F
1/2F
1/2F
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D
3F
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3F
Fb
F B
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B
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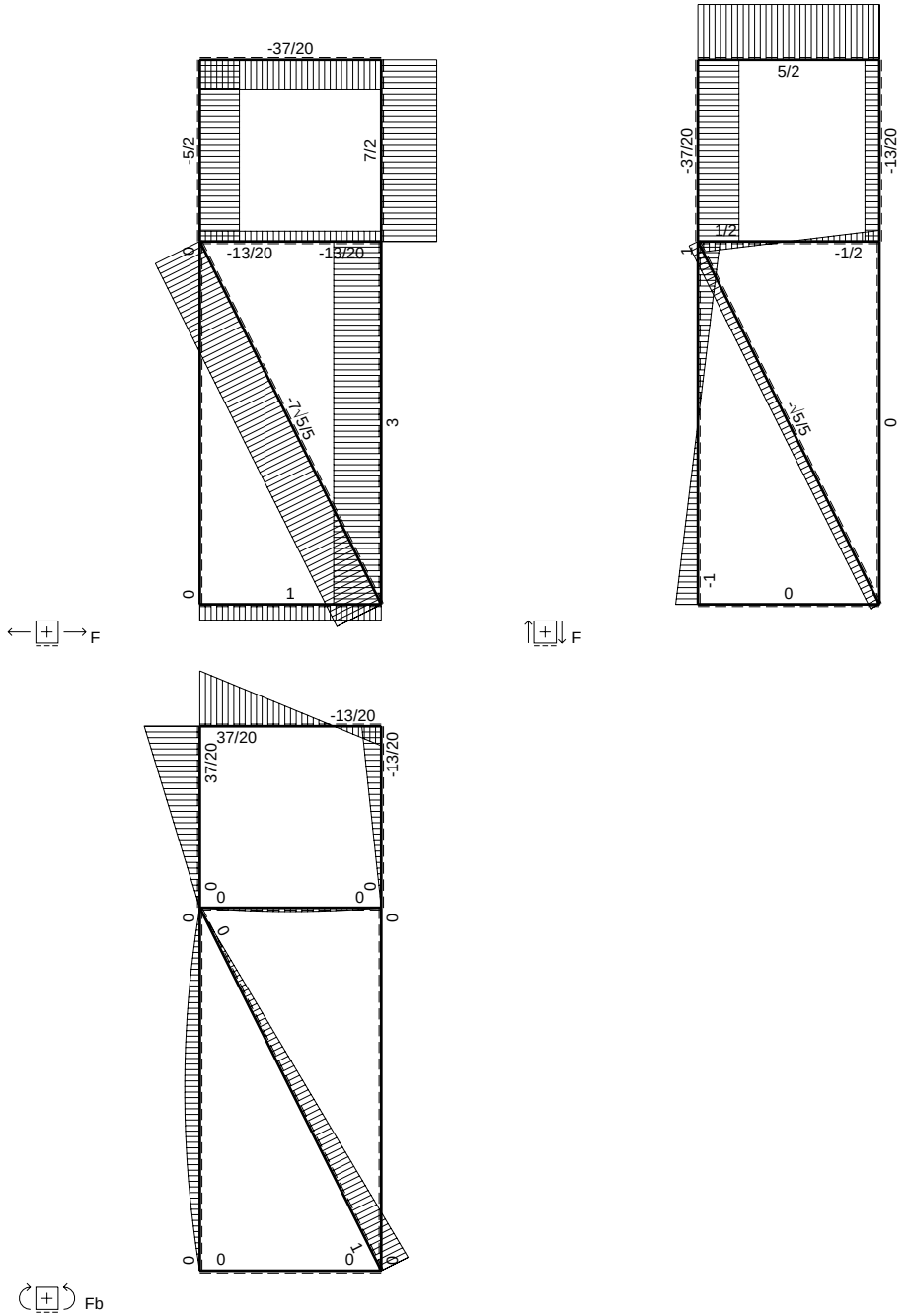
F
A
B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-13/20
1/2
-1/2
-7/5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

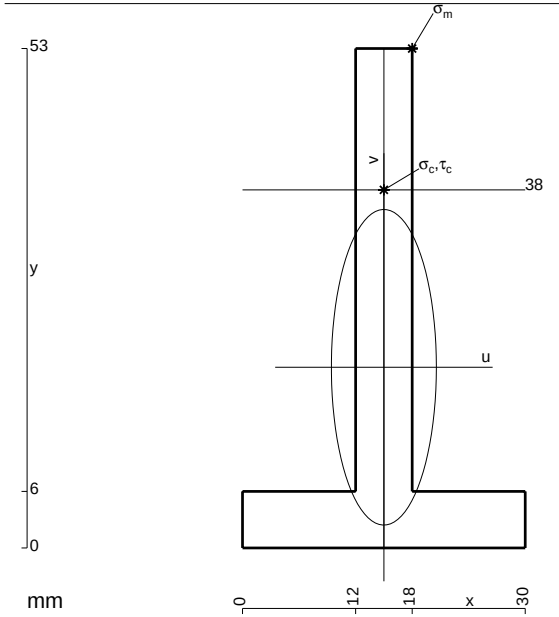
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 462. mm^2
J_u = 129608. mm^4
J_v = 14346. mm^4
y_g = 19.18 mm
N = -4070. N
T_y = -581.4 N
M_x = 767000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 33.82 mm
$\sigma_m = N/A - Mv/J_u$ = -209. N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 18.82 mm
$\sigma_c = N/A - Mv/J_u$ = -120.2 N/mm^2
τ_c = 1.771 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.3 N/mm^2
S^* = 2369. mm^3

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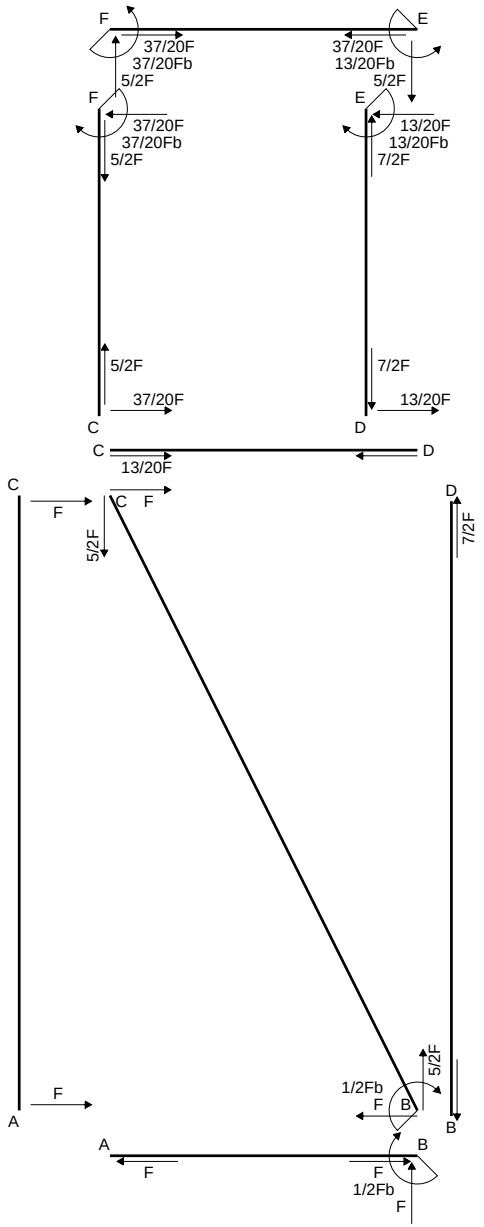
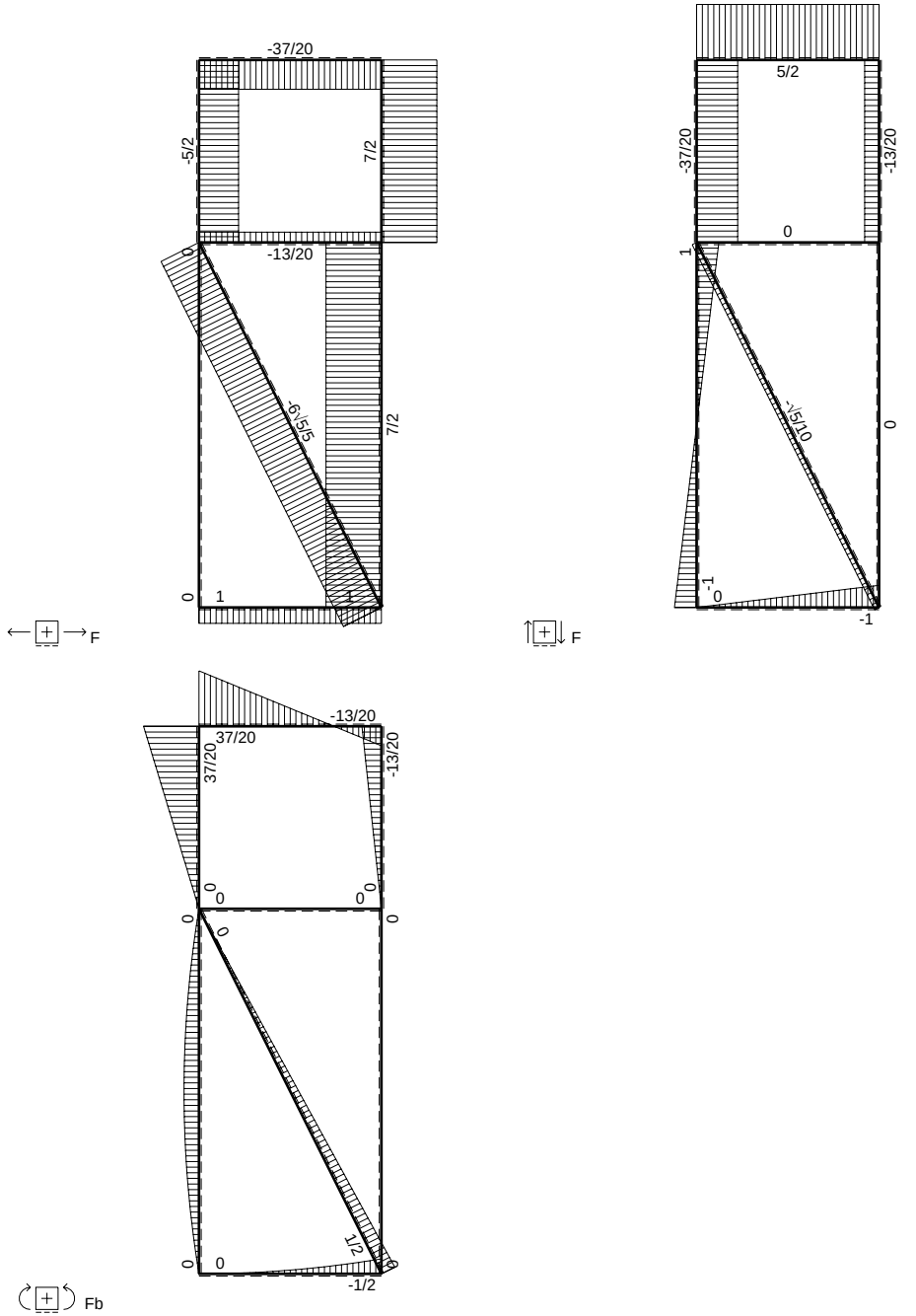

F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb


-37/20
-5/2
7/2
-13/20
-5\/5/5
1
0
F
5/2
-37/20
-13/20
-\5/10
-1
Fb
1/2
-1/2

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-FbEJ	-5/2Fx	FbEJ	1	$(-5/4+1)Fb^2/EJ$	XbEJ
FE b	1	-5/2Fb+5/2Fx	FbEJ	-5/2Fb+5/2Fx	FbEJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

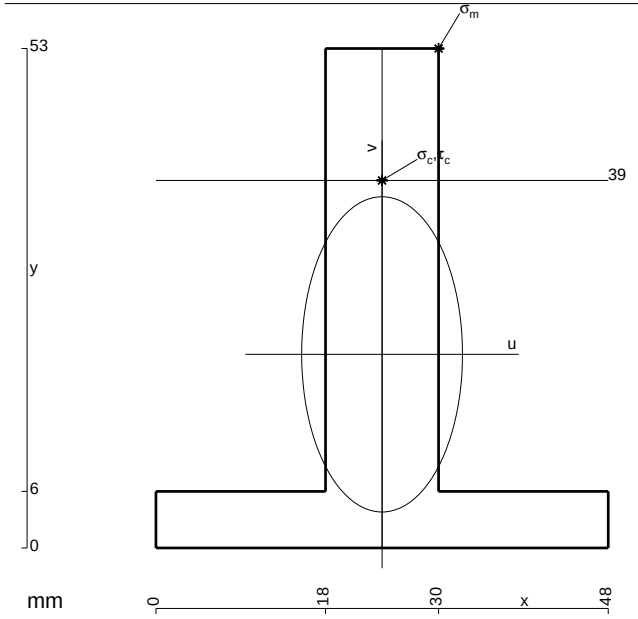
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 852. mm^2
J_u = 238569. mm^4
J_v = 62064. mm^4
y_g = 20.54 mm
N = 5070. N
T_y = -5070. N
M_x = -1571700. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 32.46 mm
$\sigma_m = N/A - Mv/J_u$ = 219.8 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 18.46 mm
$\sigma_c = N/A - Mv/J_u$ = 127.6 N/mm^2
τ_c = 7.574 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.2 N/mm^2
S^* = 4277. mm^3

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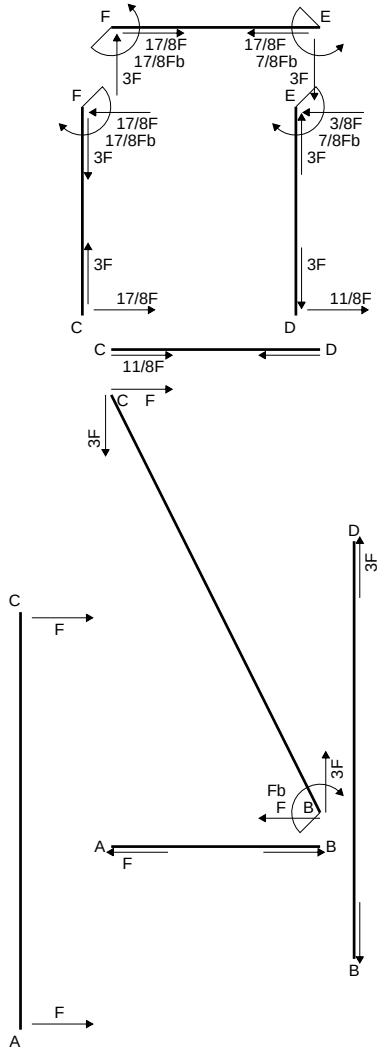
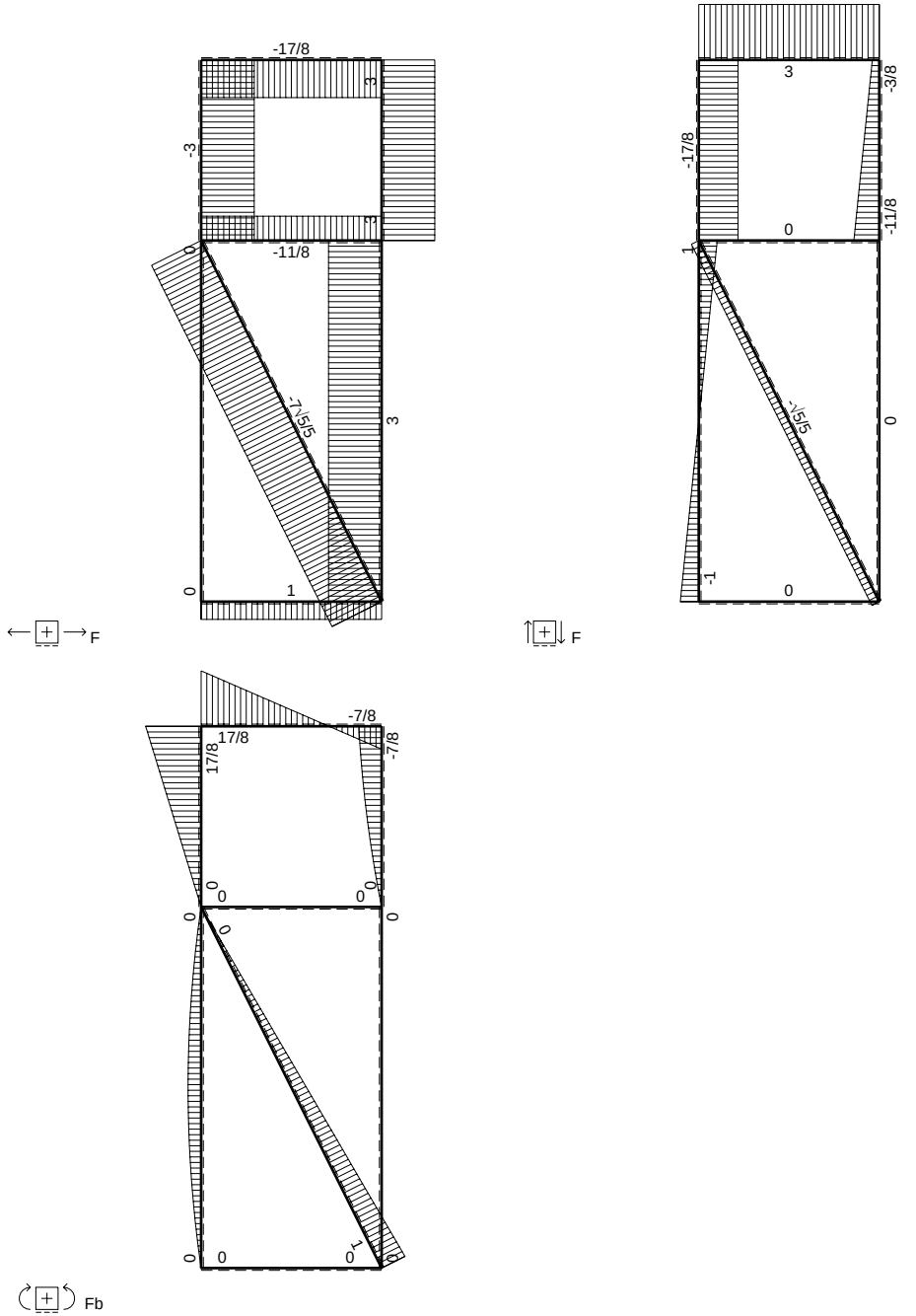

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A


-17/8
-3
-11/8
3
0
1
-7√5/5
F
3
-17/8
-3/8
-11/8
3
0
-1
-√5/5
F
-7/8
17/8
-7/8
17/8
0
0
0
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x²/b²	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x²/b²		
	totali						-35/24Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

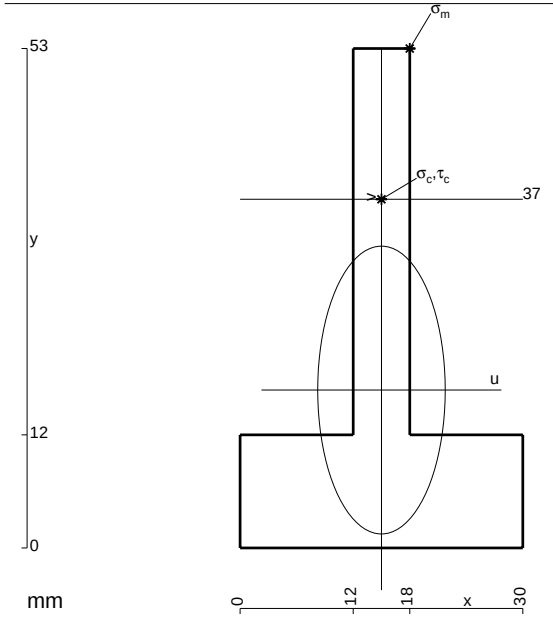
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 606. mm^2
J_u = 141406. mm^4
J_v = 27738. mm^4
y_g = 16.76 mm
N = -4101. N
T_y = -585.8 N
M_x = 864600. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.24 mm
$\sigma_m = N/A - Mv/J_u$ = -228.4 N/mm^2
x_c = 15. mm
y_c = 37. mm
v_c = 20.24 mm
$\sigma_c = N/A - Mv/J_u$ = -130.5 N/mm^2
τ_c = 1.872 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.6 N/mm^2
S^* = 2711. mm^3

ESΣ09.xxxx.028

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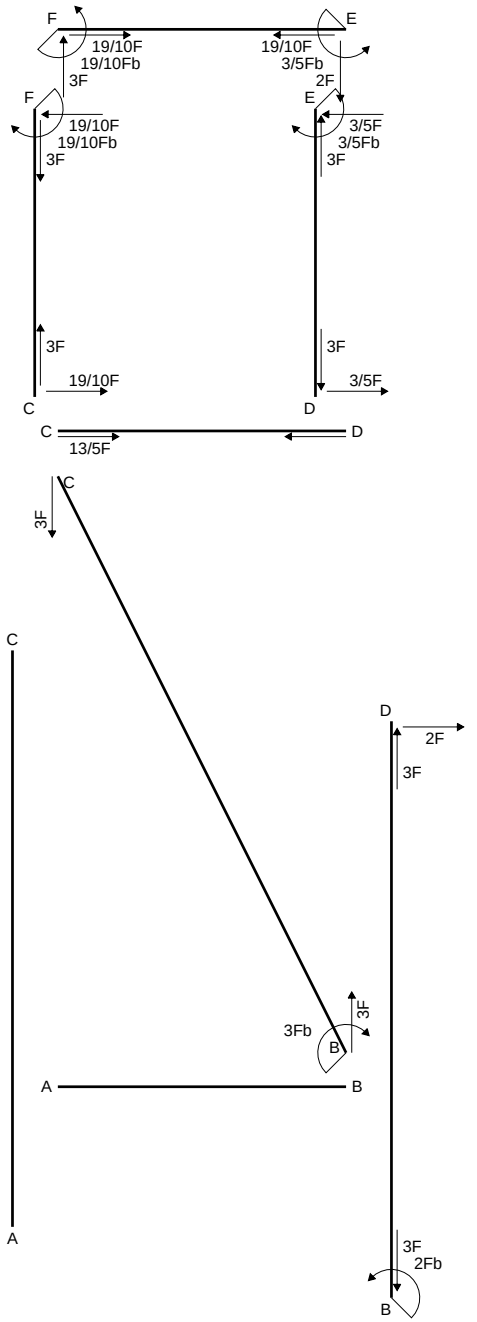
F
E
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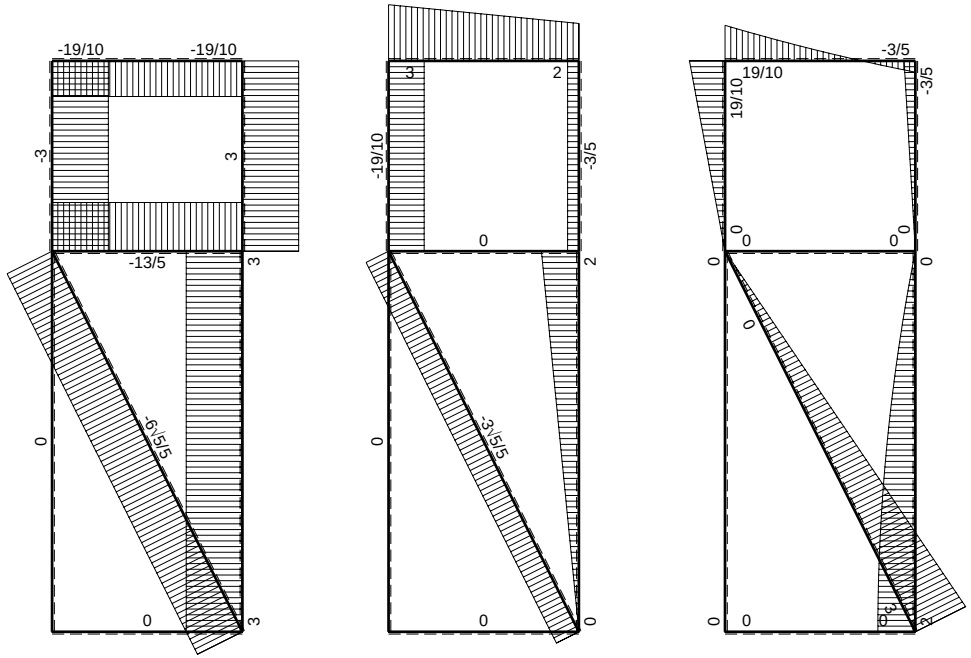
19/10Fb
19/10F
3/5Fb
3F
2F
F
E
19/10F
19/10Fb
3/5F
3/5Fb
3F
3F
3F
19/10F
3/5F
C
D
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
A
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
-19/10
3
2
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
0
0
0
0
0

F F Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
totali							-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210099 ESΣ09.xxxx.029

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

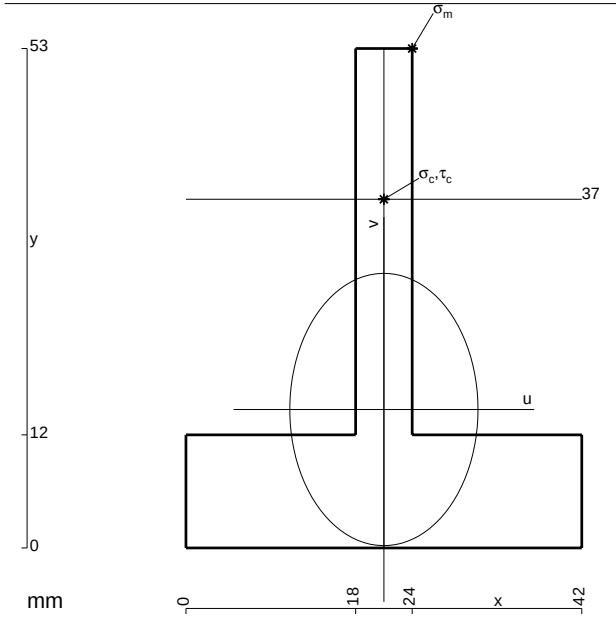
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 750. mm^2
J_u = 156599. mm^4
J_v = 74826. mm^4
y_g = 14.69 mm
N = -1234. N
T_y = -617.2 N
M_x = 966000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.31 mm
$\sigma_m = N/A - Mv/J_u$ = -238. N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.31 mm
$\sigma_c = N/A - Mv/J_u$ = -139.3 N/mm^2
τ_c = 1.911 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.3 N/mm^2
S^* = 2910. mm^3

ESΣ09.xxxx.029

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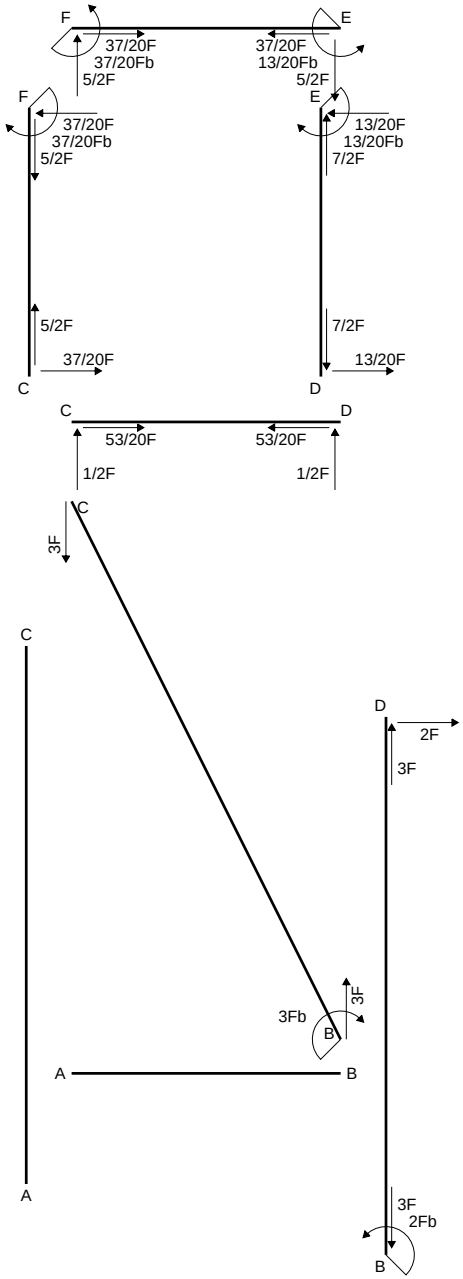
F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
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3F
C
D
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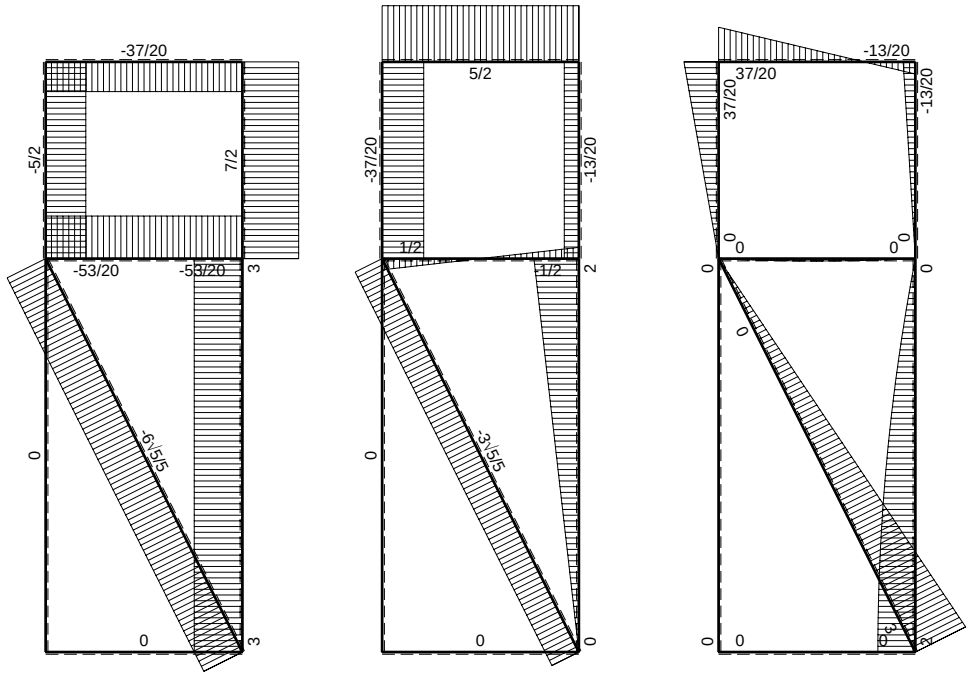
B
3F
A
B
A
3F
2Fb
B


-37/20
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7/2
-53/20
-53/20
3
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-6√5/5
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3
5/2
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-13/20
1/2
1/2
2
-3√5/5
0
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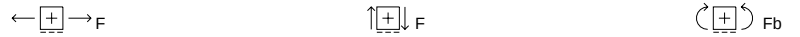
37/20
37/20
-13/20
0
0
0
0
0
0
0
-3√5/5
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi\theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212663 ESΣ09.xxxx.030

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

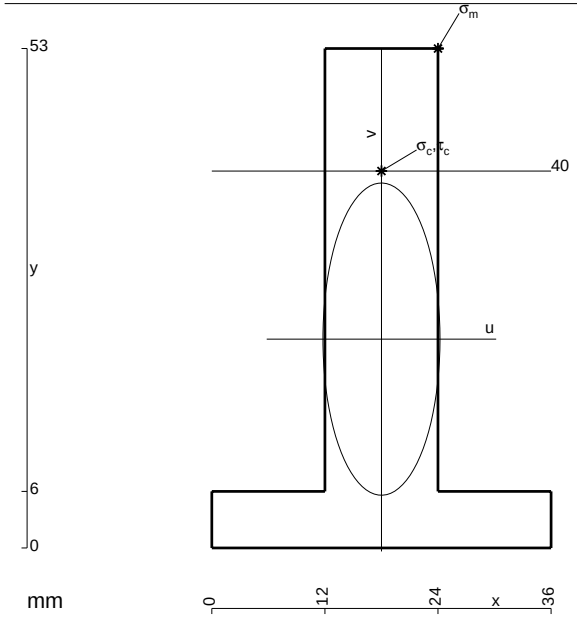
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 780. mm^2
J_u = 214152. mm^4
J_v = 30096. mm^4
y_g = 22.16 mm
N = -3274. N
T_y = -1637. N
M_x = 1354200. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 30.84 mm
$\sigma_m = N/A - Mv/J_u$ = -199.2 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 17.84 mm
$\sigma_c = N/A - Mv/J_u$ = -117. N/mm^2
τ_c = 2.418 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.1 N/mm^2
S^* = 3797. mm^3

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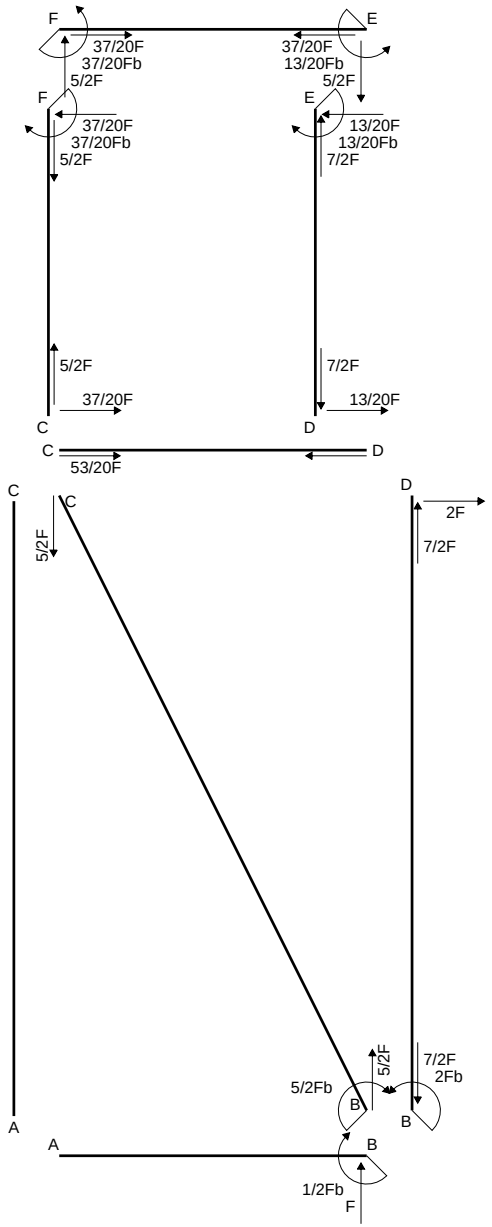
F
E
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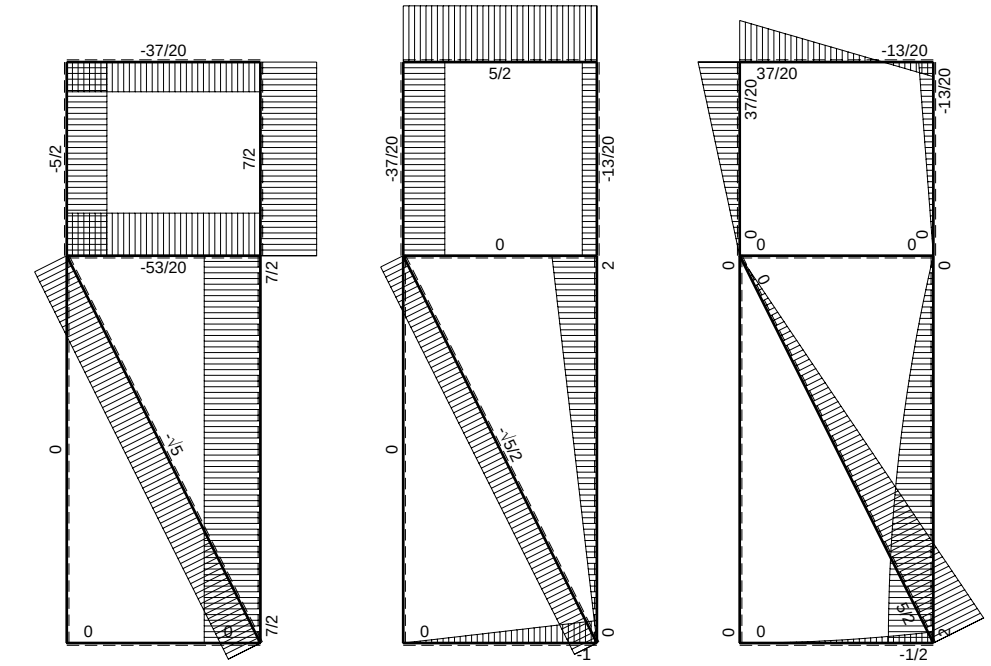
37/20Fb
5/2F
13/20Fb
37/20F
13/20F
13/20Fb
7/2F
5/2F
37/20F
13/20F
C
D
53/20F
2F
5/2Fb
2Fb
1/2Fb
A
B


-37/20
-5/2
7/2
-53/20
0
-13/20
5/2
2
-1
37/20
-1/2

F F Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						-13/12Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211730 ESΣ09.xxxx.031

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

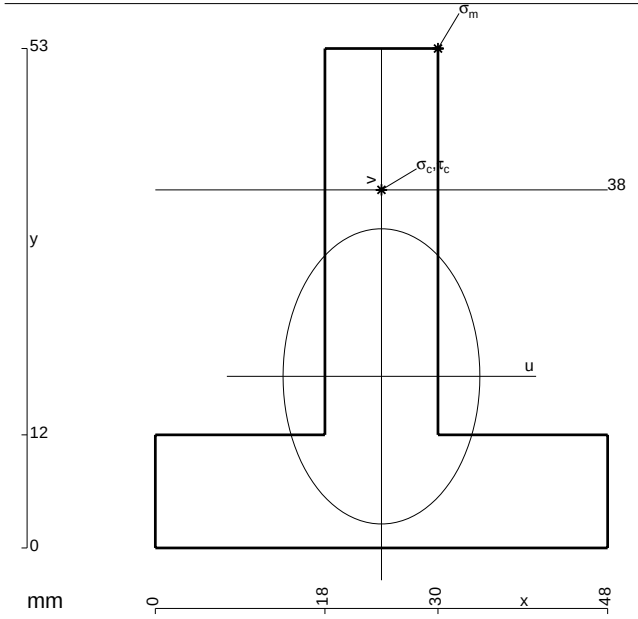
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1068. mm^2
J_u = 262174. mm^4
J_v = 116496. mm^4
y_g = 18.21 mm
N = -3466. N
T_y = -1733. N
M_x = 1550000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.79 mm
$\sigma_m = N/A - Mv/J_u$ = -208.9 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.79 mm
$\sigma_c = N/A - Mv/J_u$ = -120.3 N/mm^2
τ_c = 2.706 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.3 N/mm^2
S^* = 4913. mm^3

ESΣ09.xxxx.031

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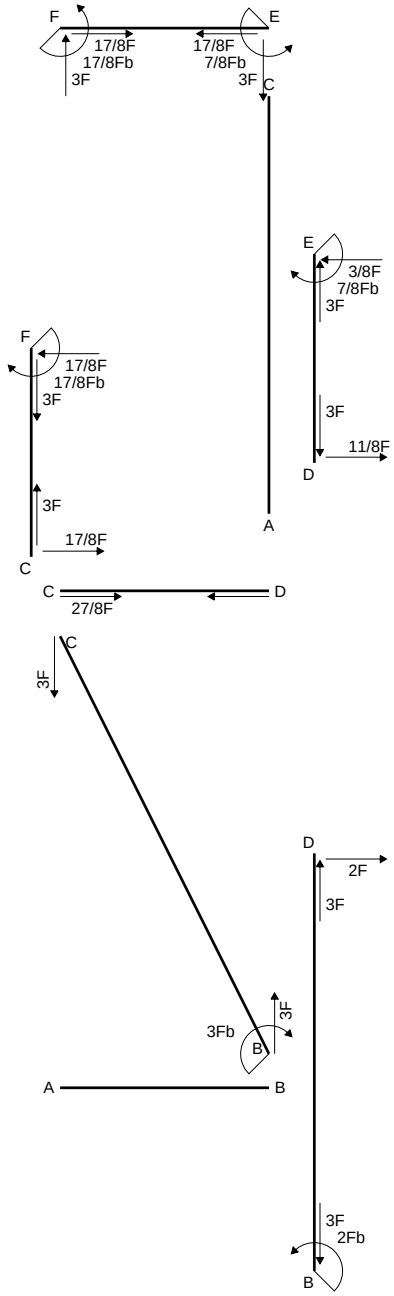
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17/8F
7/8Fb
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E
3/8F
7/8Fb
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17/8F
17/8Fb
3F
3F
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3F
A
17/8F
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3F
B
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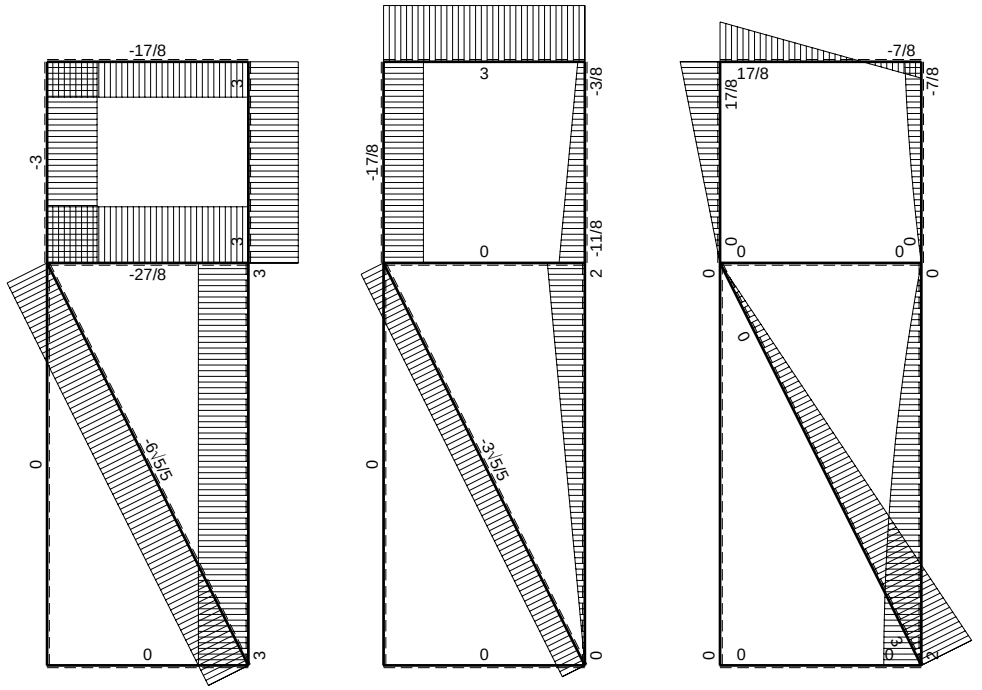
B
3F
2Fb
B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-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0

F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^{2}b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XkJ/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	XkJ/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3XkJ/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
	totali						-35/24Fb^2/EJ	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

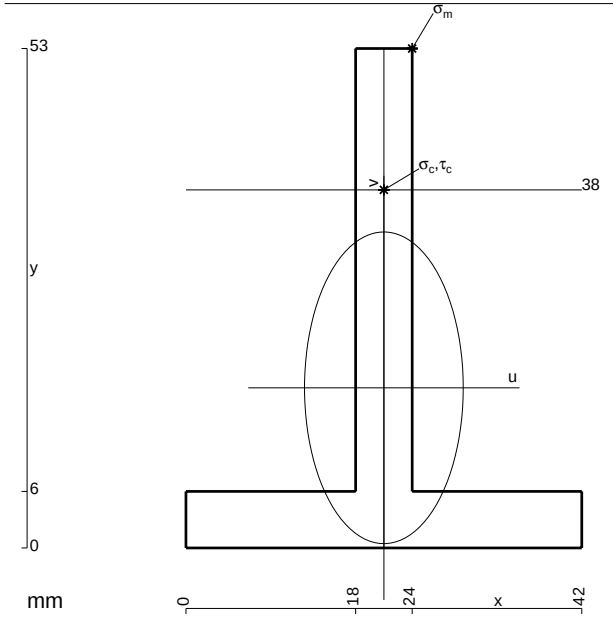
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 534. mm^2
J_u = 146122. mm^4
J_v = 37890. mm^4
y_g = 16.99 mm
N = -1771. N
T_y = -885.5 N
M_x = 871200. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.01 mm
$\sigma_m = N/A - Mv/J_u$ = -218. N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 21.01 mm
$\sigma_c = N/A - Mv/J_u$ = -128.6 N/mm^2
τ_c = 2.591 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.6 N/mm^2
S^* = 2566. mm^3

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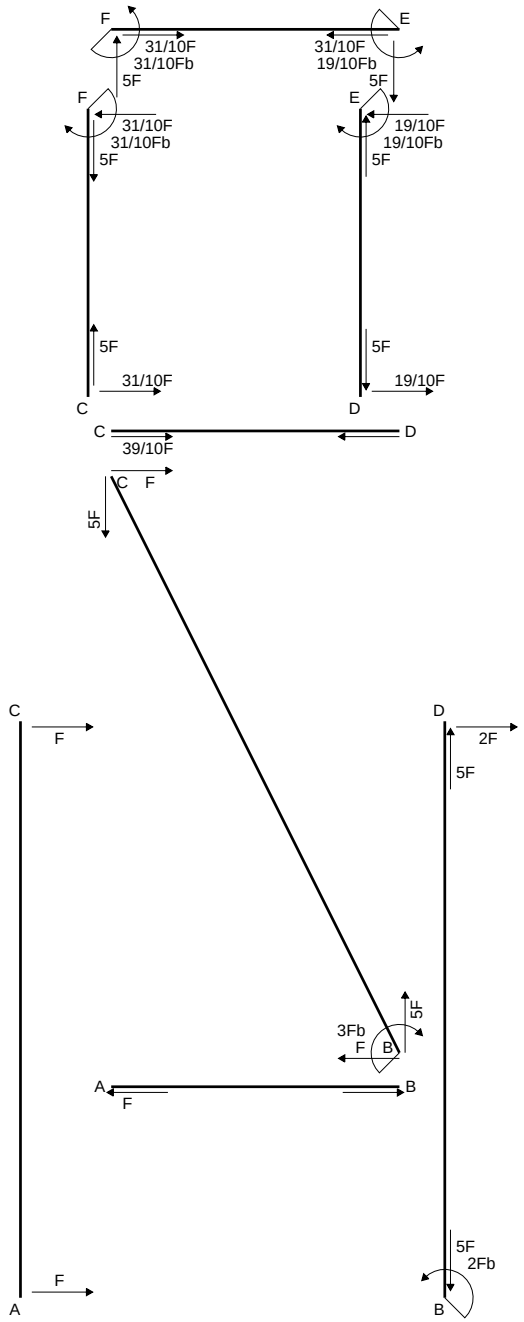
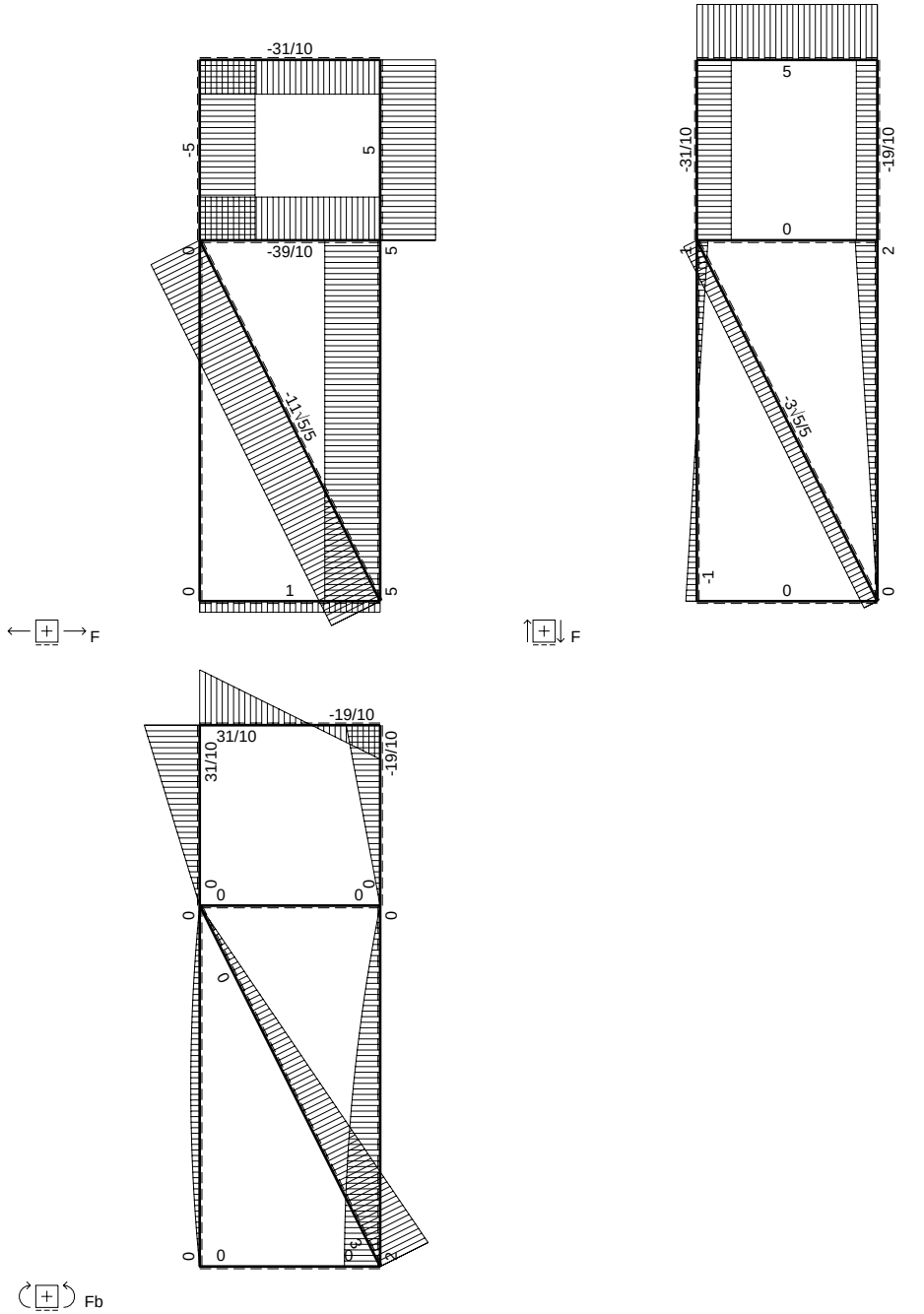

F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
	totali						-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212504 ESΣ09.xxxx.033

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

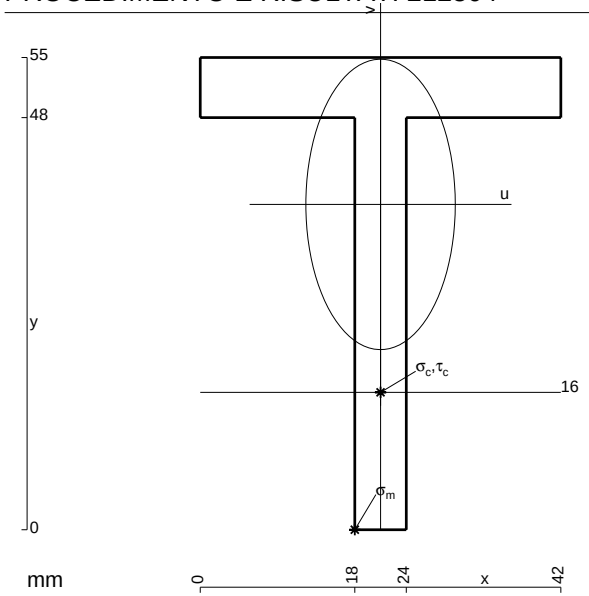
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$



A = 582. mm^2
J_u = 166519. mm^4
J_v = 44082. mm^4
y_g = 37.89 mm
N = -3394. N
T_y = -925.7 N
M_x = 1035000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.89 mm
$\sigma_m = N/A - Mv/J_u$ = 229.7 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.89 mm
$\sigma_c = N/A - Mv/J_u$ = 130.2 N/mm^2
τ_c = 2.659 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.3 N/mm^2
S^* = 2870. mm^3

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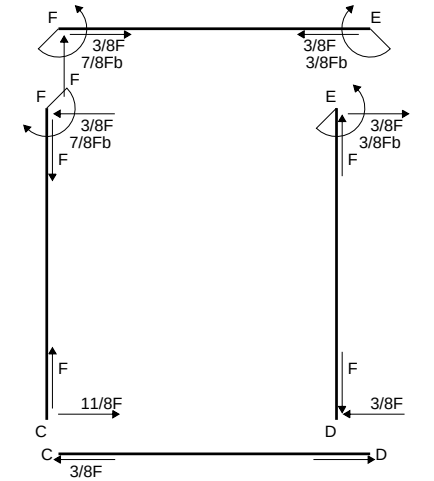
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
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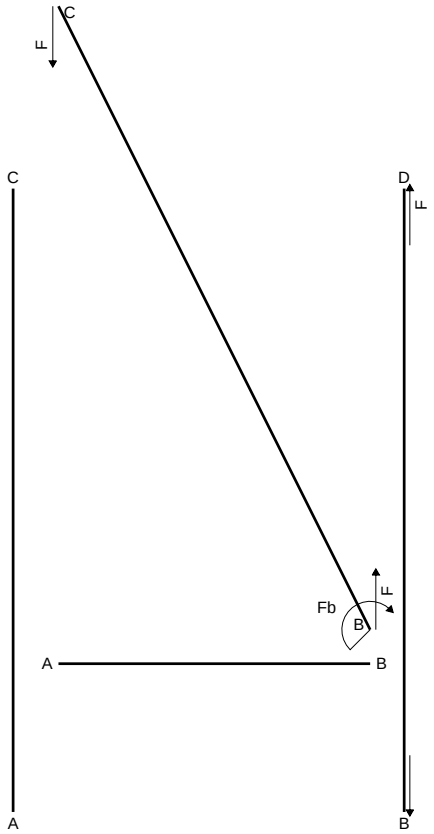
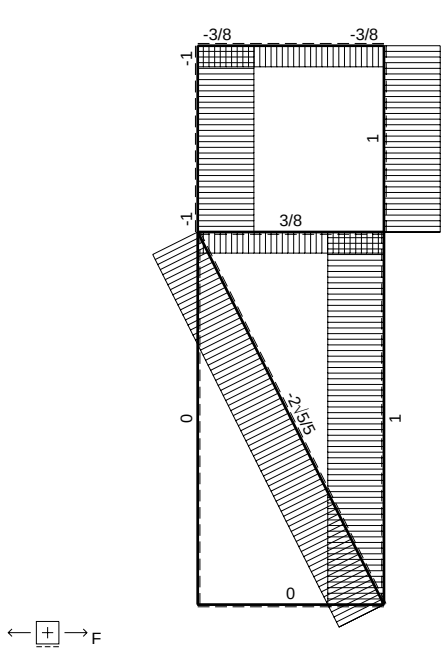
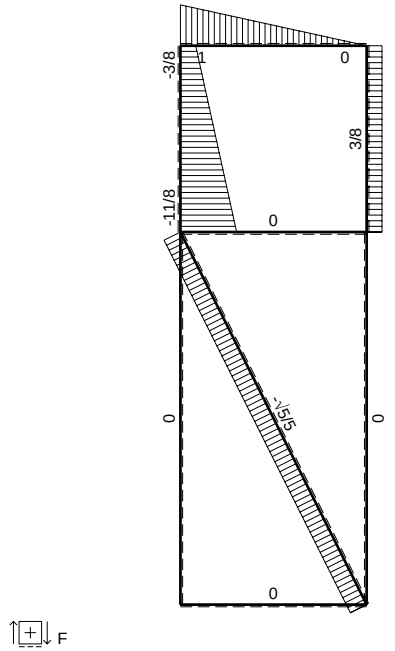
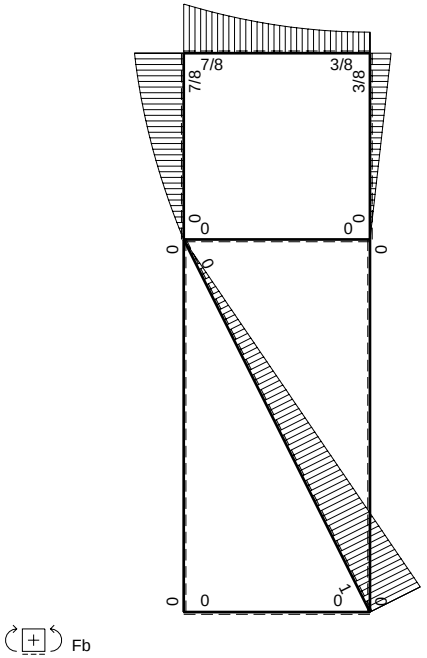
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

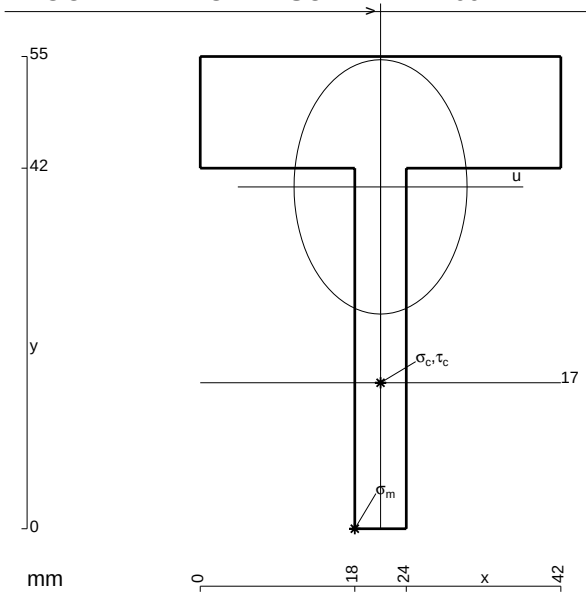
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 798. mm^2
J_u = 175127. mm^4
J_v = 81018. mm^4
y_g = 39.82 mm
N = -1762. N
T_y = -881. N
M_x = 1063800. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.82 mm
$\sigma_m = N/A - Mv/J_u$ = 239.7 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.82 mm
$\sigma_c = N/A - Mv/J_u$ = 136.4 N/mm^2
τ_c = 2.678 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.5 N/mm^2
S^* = 3194. mm^3

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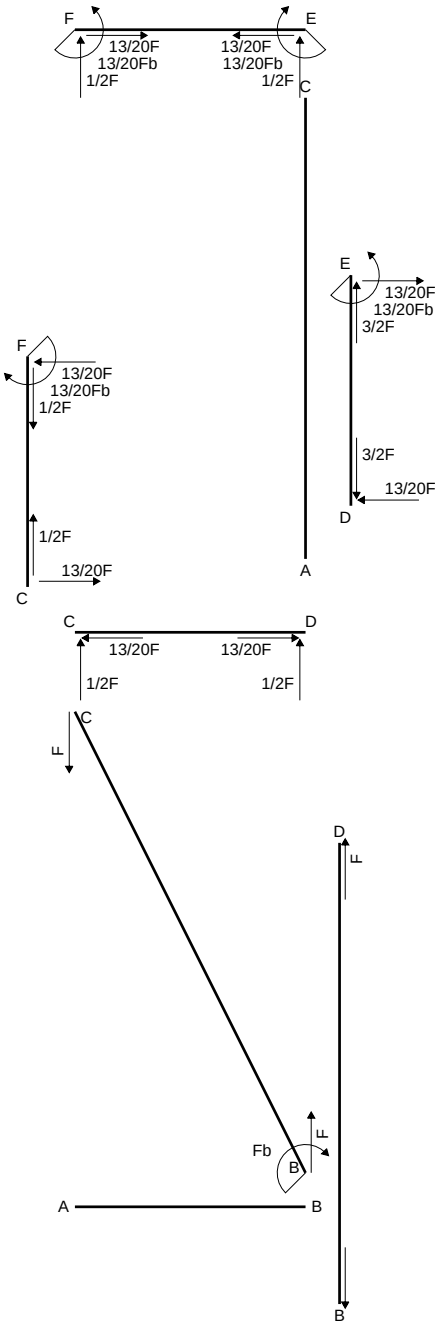
F
E
13/20F
13/20Fb
1/2F
C
E
13/20F
13/20Fb
3/2F
F
13/20F
13/20Fb
1/2F
3/2F
13/20F
D
1/2F
13/20F
C
A
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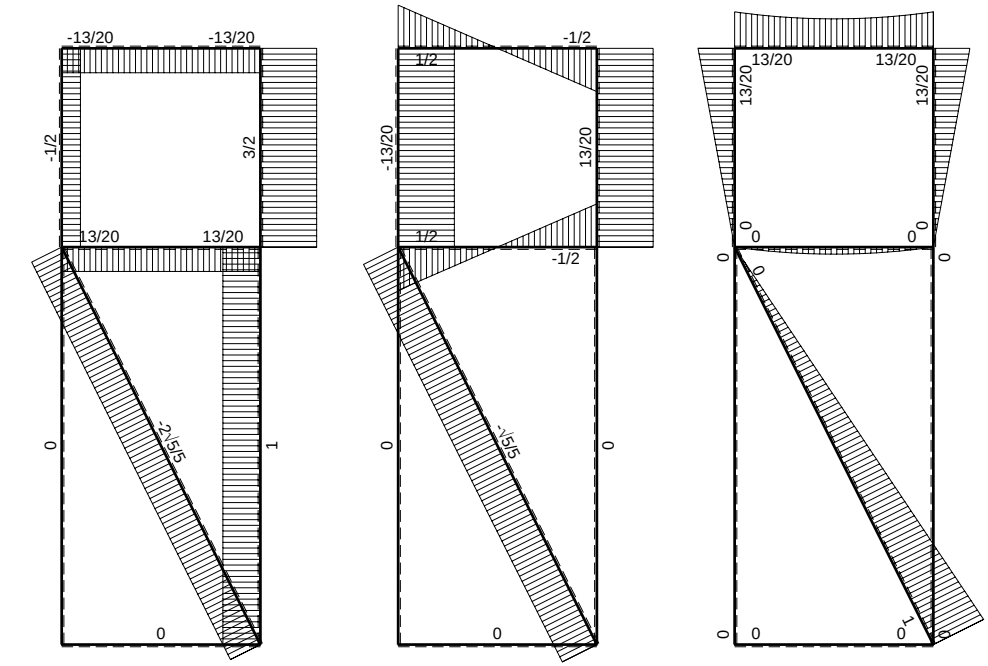
B
F
A
B
B


-13/20
-13/20
-1/2
3/2
13/20
13/20
0
1
-2√5/5
0
-1/2
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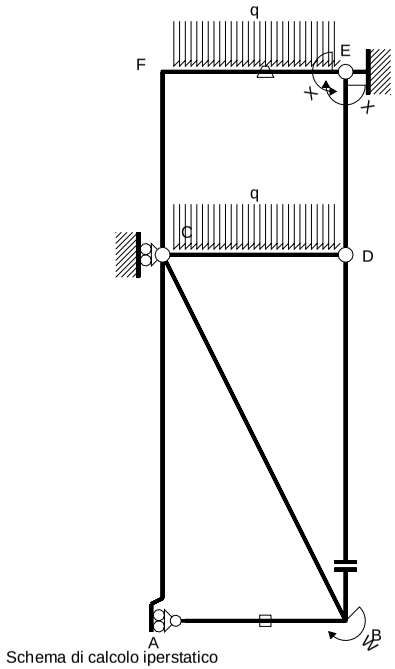
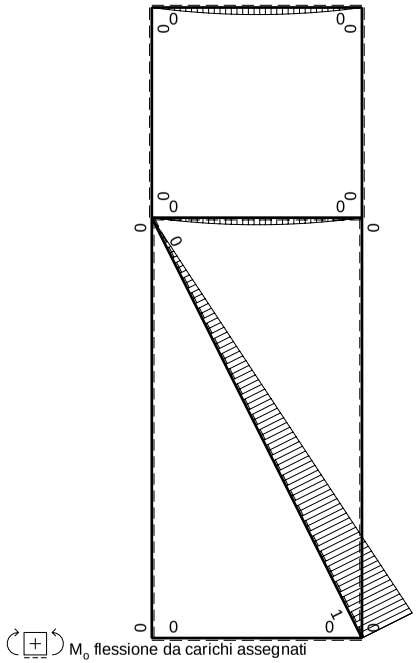
0
-1

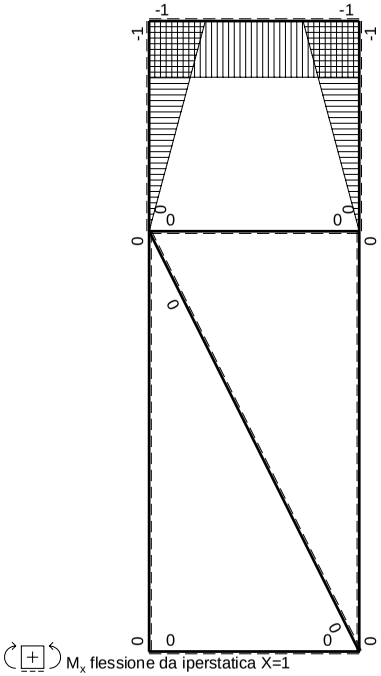
F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212877 ESΣ09.xxxx.035

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

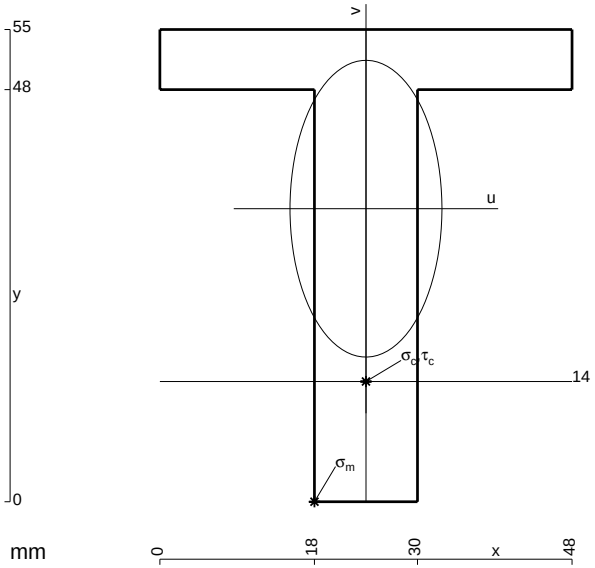
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 912. mm^2
J_u = 272448. mm^4
J_v = 71424. mm^4
y_g = 34.13 mm
N = -2495. N
T_y = -1248. N
M_x = 1618200. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.13 mm
σ_m = N/A-Mv/J_u = 200. N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -20.13 mm
σ_c = N/A-Mv/J_u = 116.8 N/mm^2
τ_c = 1.74 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 116.9 N/mm^2
S^* = 4558. mm^3

ESΣ09.xxxx.035

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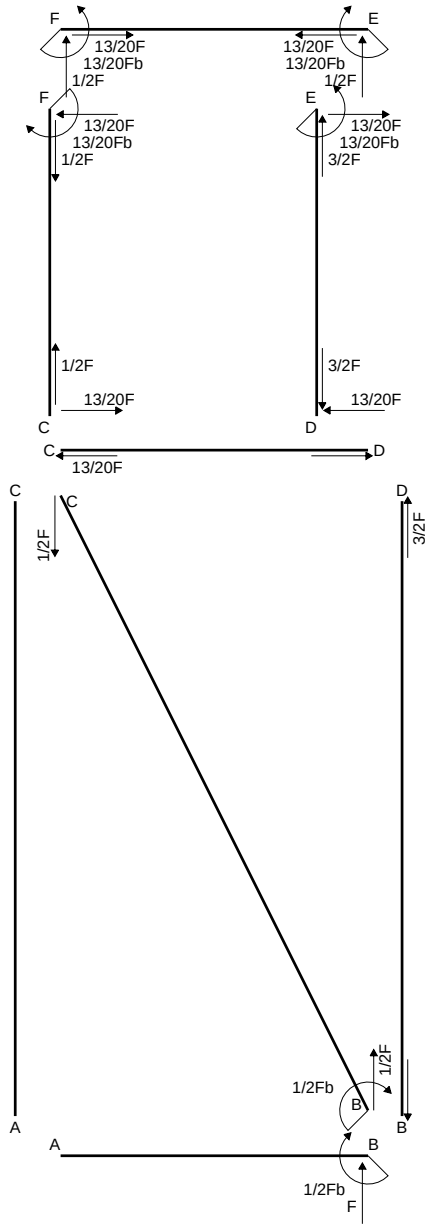
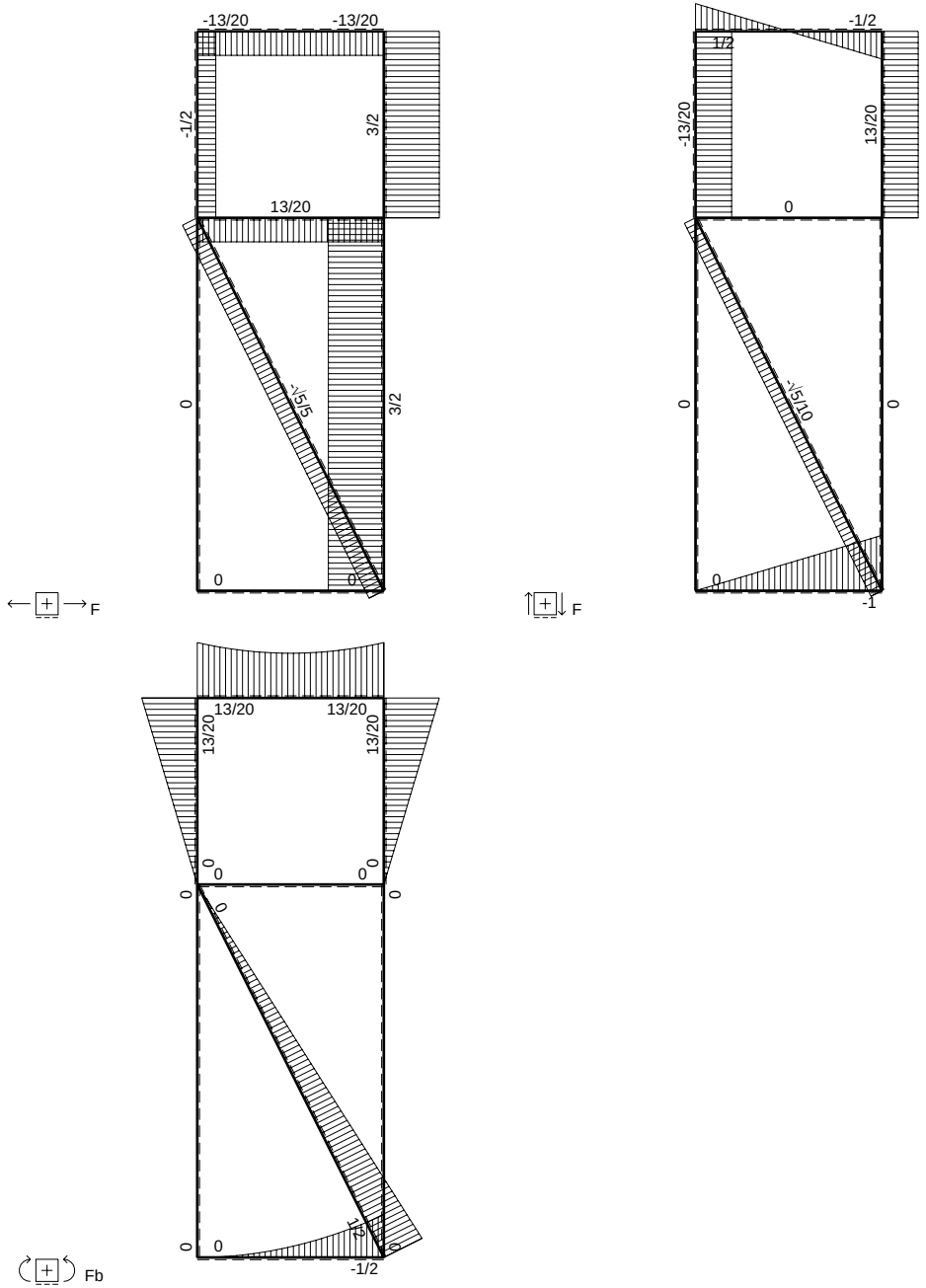
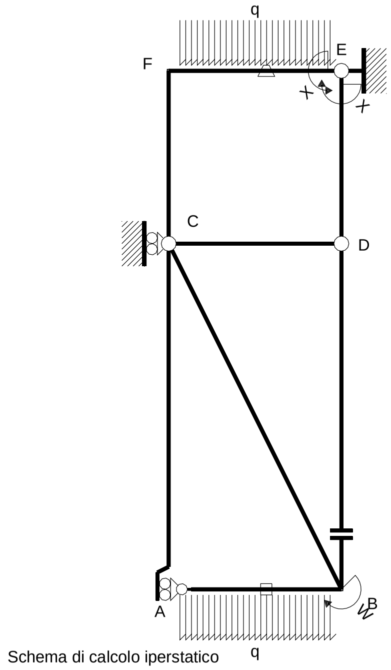
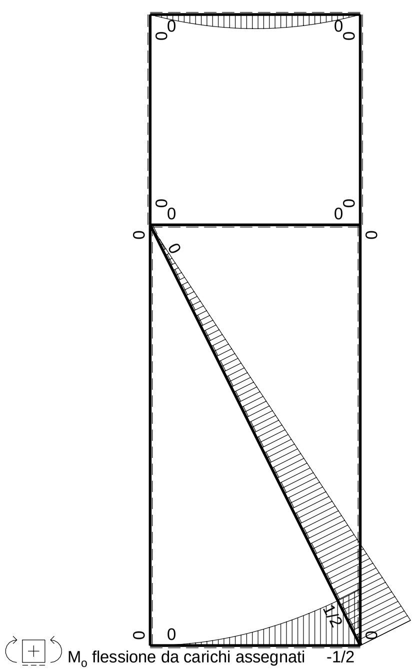

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

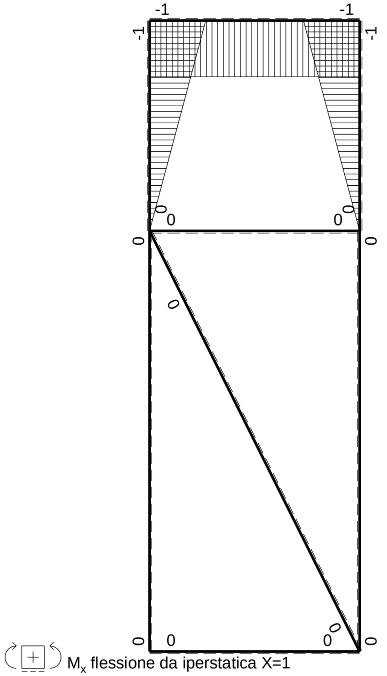

-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati -1/2



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

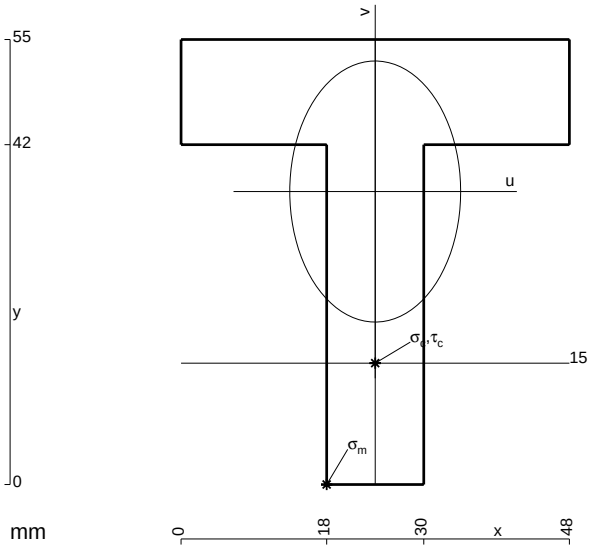
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

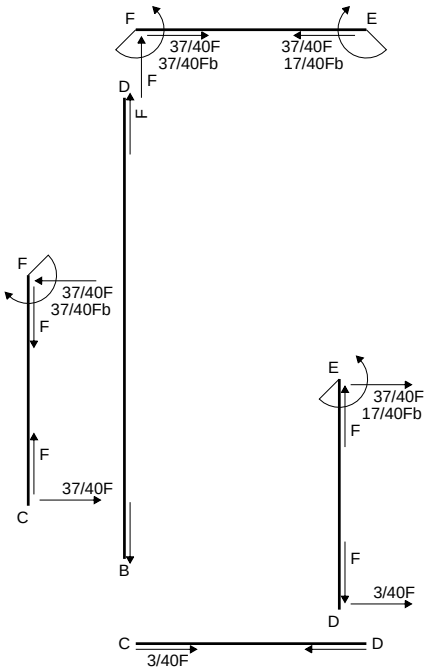
PROCEDIMENTO E RISULTATI 995596 ESΣ09.xxxx.036



A = 1128. mm^2
J_u = 293725. mm^4
J_v = 125856. mm^4
y_g = 36.21 mm
T_y = -5490. N
M_x = -1701900. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -36.21 mm
$\sigma_m = -Mv/J_u$ = -209.8 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -21.21 mm
$\sigma_c = -Mv/J_u$ = -122.9 N/mm^2
τ_c = 8.05 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.7 N/mm^2
S^* = 5168. mm^3

ESΣ09.xxxx.036

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24


F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
F
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C
B
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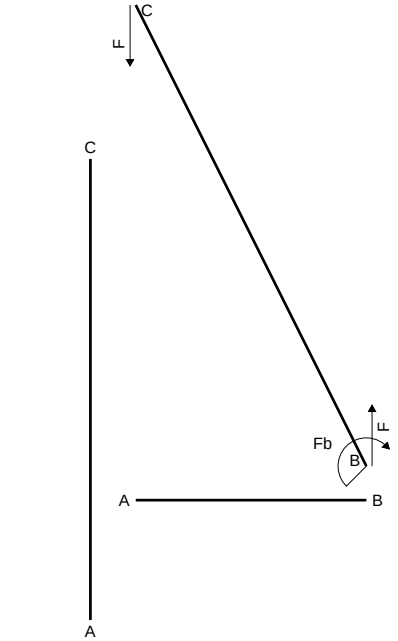
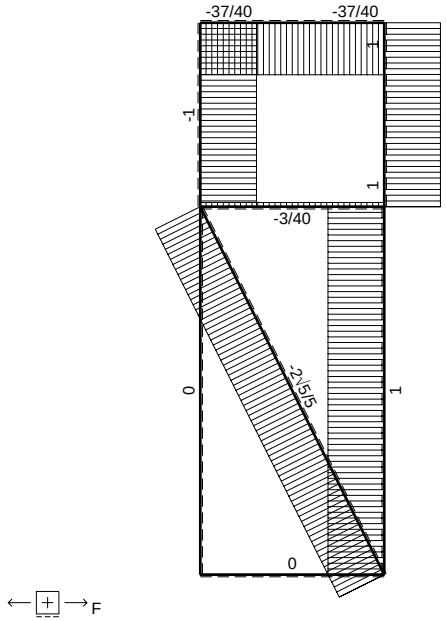
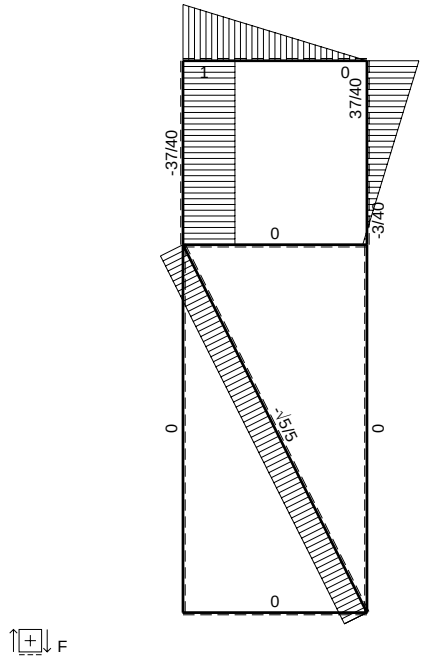
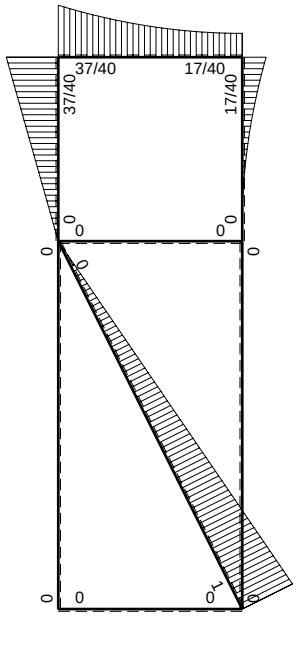
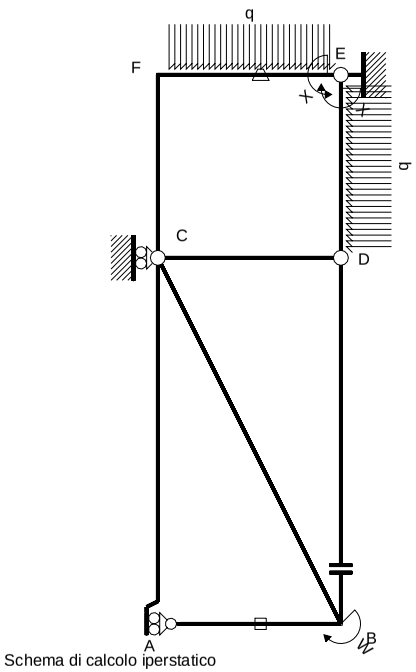
17/40Fb
F
F
3/40F
D
C
D
3/40F


C
F
C
Fb
F
B
A
B
A

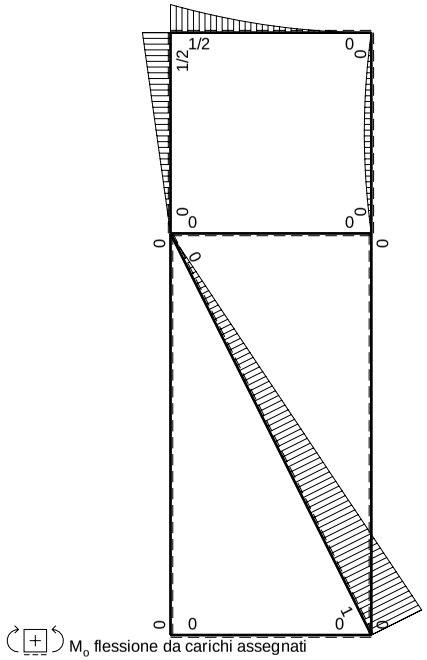

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
F

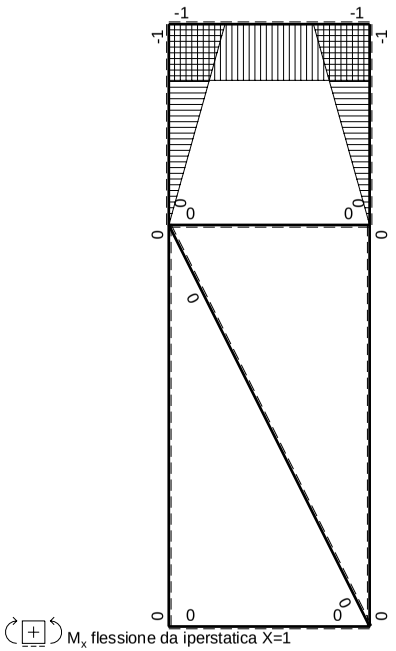

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209784 ESΣ09.xxxx.037

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

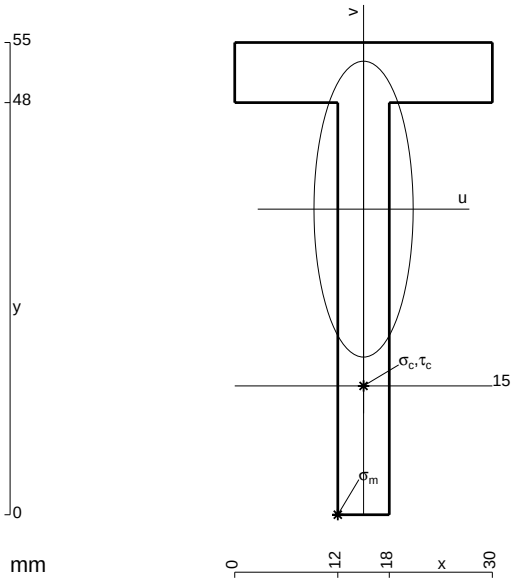
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 498. mm^2
J_u = 147997. mm^4
J_v = 16614. mm^4
y_g = 35.6 mm
N = -1252. N
T_y = -626.1 N
M_x = 924000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.6 mm
$\sigma_m = N/A - Mv/J_u$ = 219.7 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.6 mm
$\sigma_c = N/A - Mv/J_u$ = 126.1 N/mm^2
τ_c = 1.783 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.1 N/mm^2
S^* = 2529. mm^3

ESΣ09.xxxx.037

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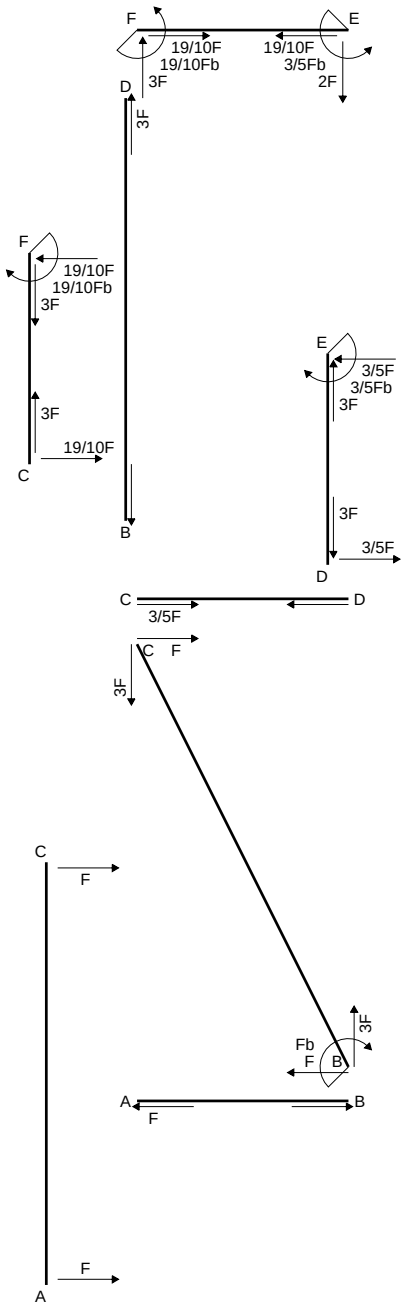
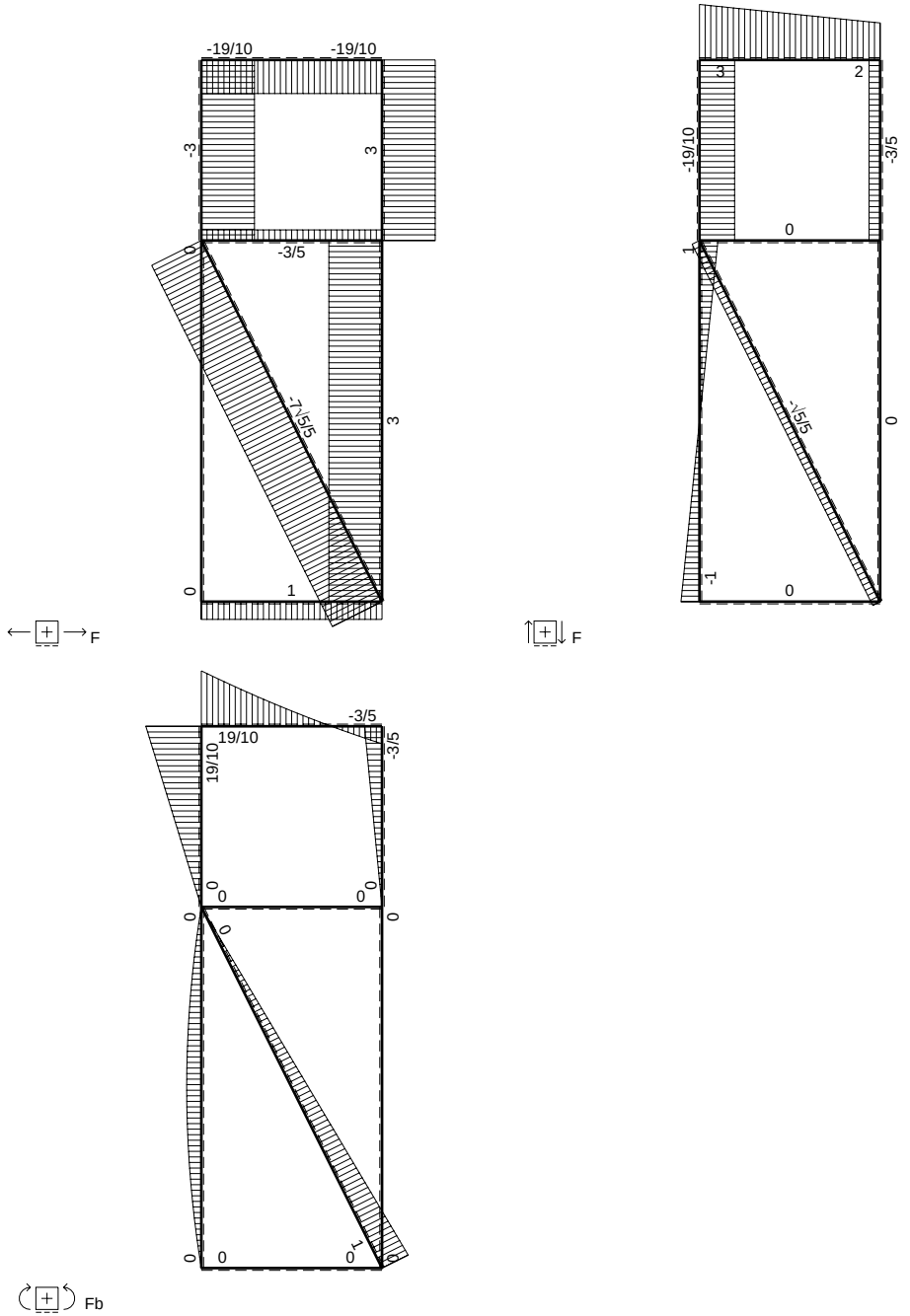
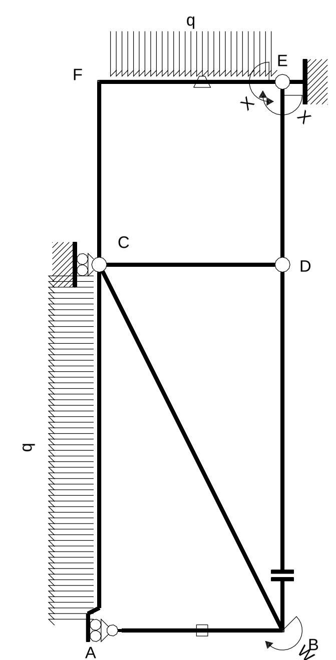
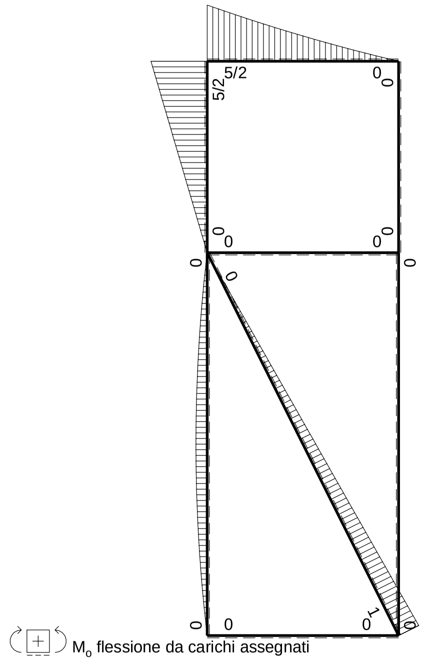

F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C F
3F
C
F
Fb
F
B
3F
A
B
F
F
A

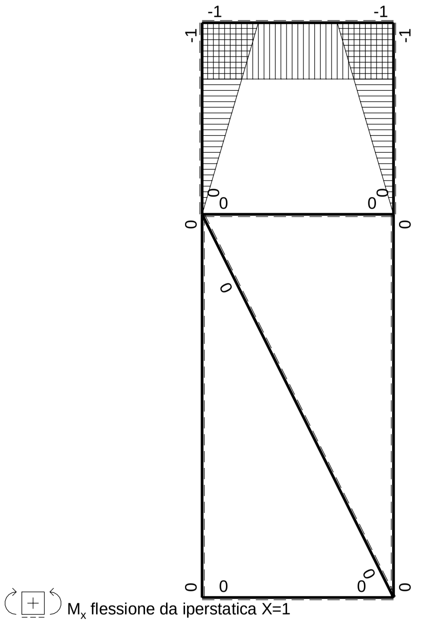

-19/10
-19/10
-3
3
-3/5
0
0
-7\5/5
3
1
F
-19/10
3
2
-3/5
0
1
-1\5/5
0
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	$Fb-\sqrt{5}/5Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

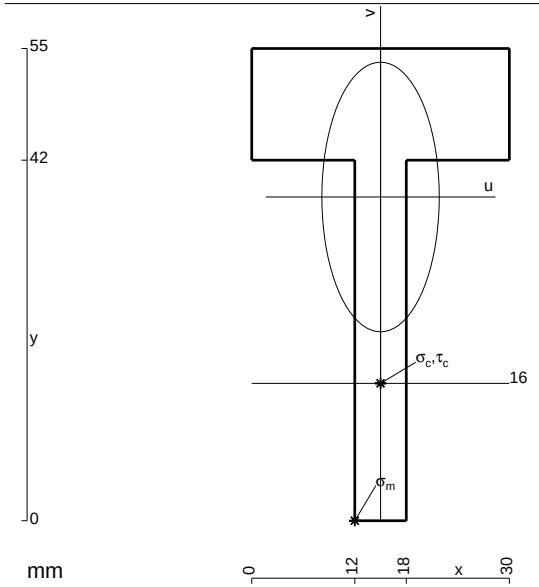
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 642. mm^2
J_u = 158306. mm^4
J_v = 30006. mm^4
y_g = 37.71 mm
N = -4445. N
T_y = -635. N
M_x = 994000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.71 mm
$\sigma_m = N/A - Mv/J_u$ = 229.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.71 mm
$\sigma_c = N/A - Mv/J_u$ = 129.4 N/mm^2
τ_c = 1.907 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.4 N/mm^2
S^* = 2852. mm^3

ESΣ09.xxxx.038

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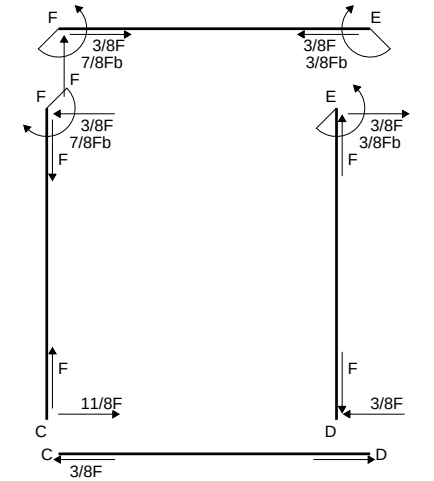
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
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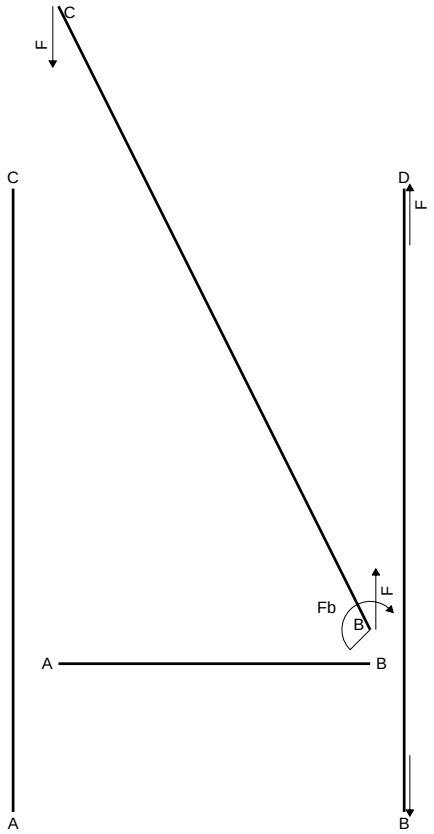
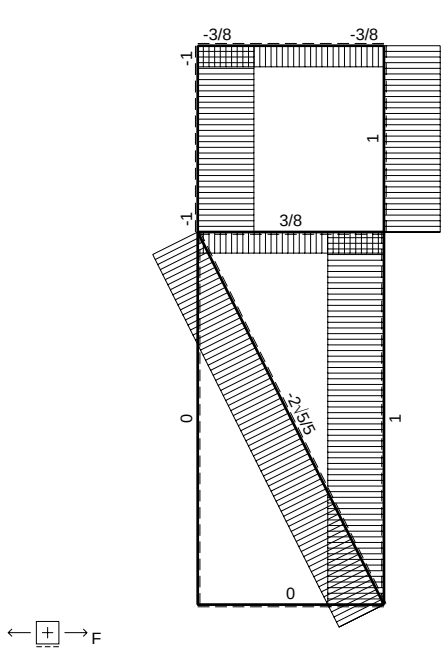
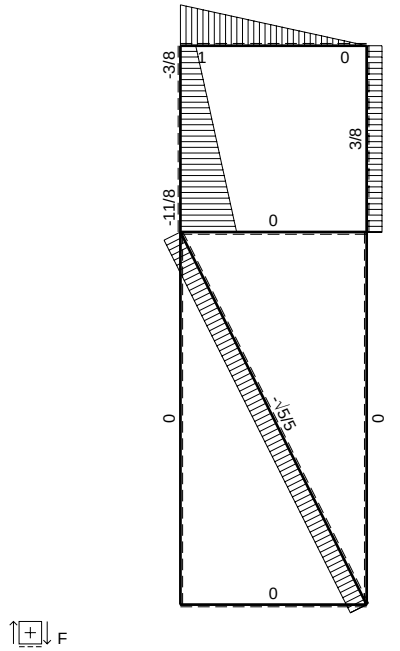
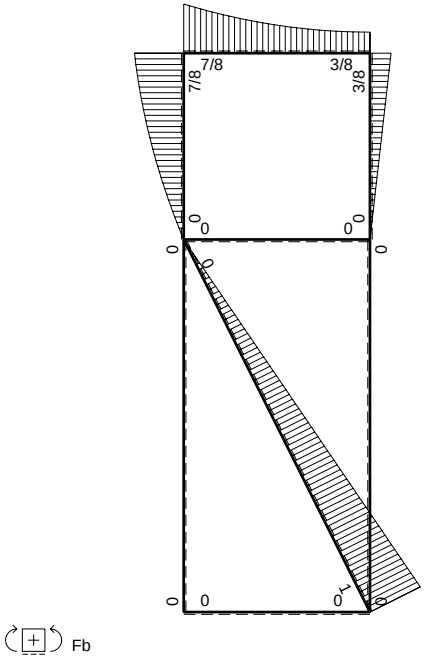
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
1
-1
3/8
1
0
-2√5/5
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

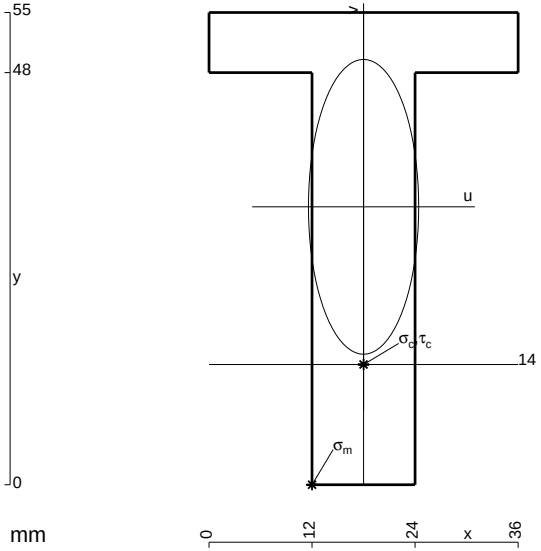
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 828. mm^2
J_u = 244195. mm^4
J_v = 34128. mm^4
y_g = 32.37 mm
N = -2209. N
T_y = -1105. N
M_x = 1827800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.37 mm
σ_m = N/A-Mv/J_u = 239.6 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.37 mm
σ_c = N/A-Mv/J_u = 134.8 N/mm^2
τ_c = 1.607 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.9 N/mm^2
S^* = 4262. mm^3

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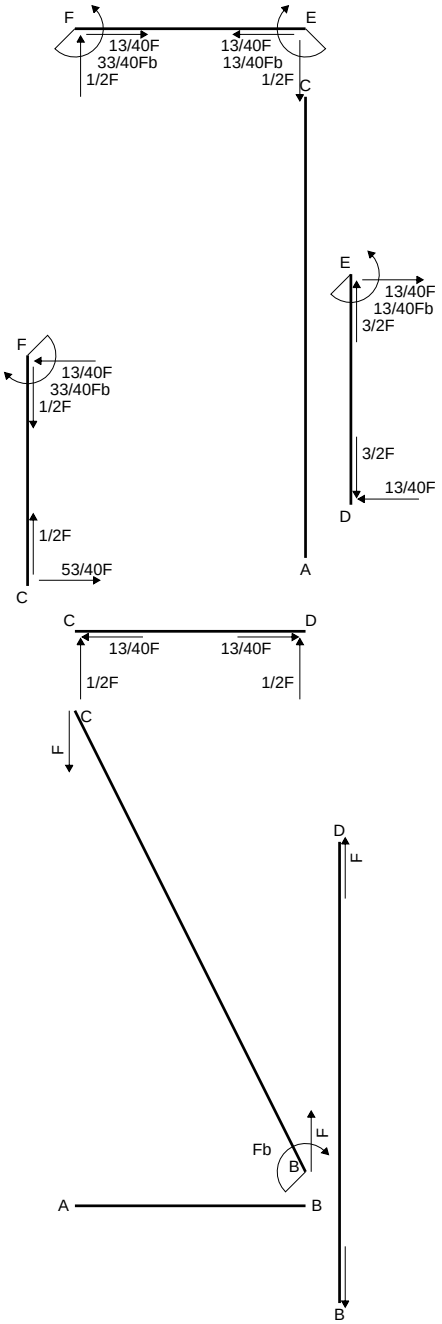
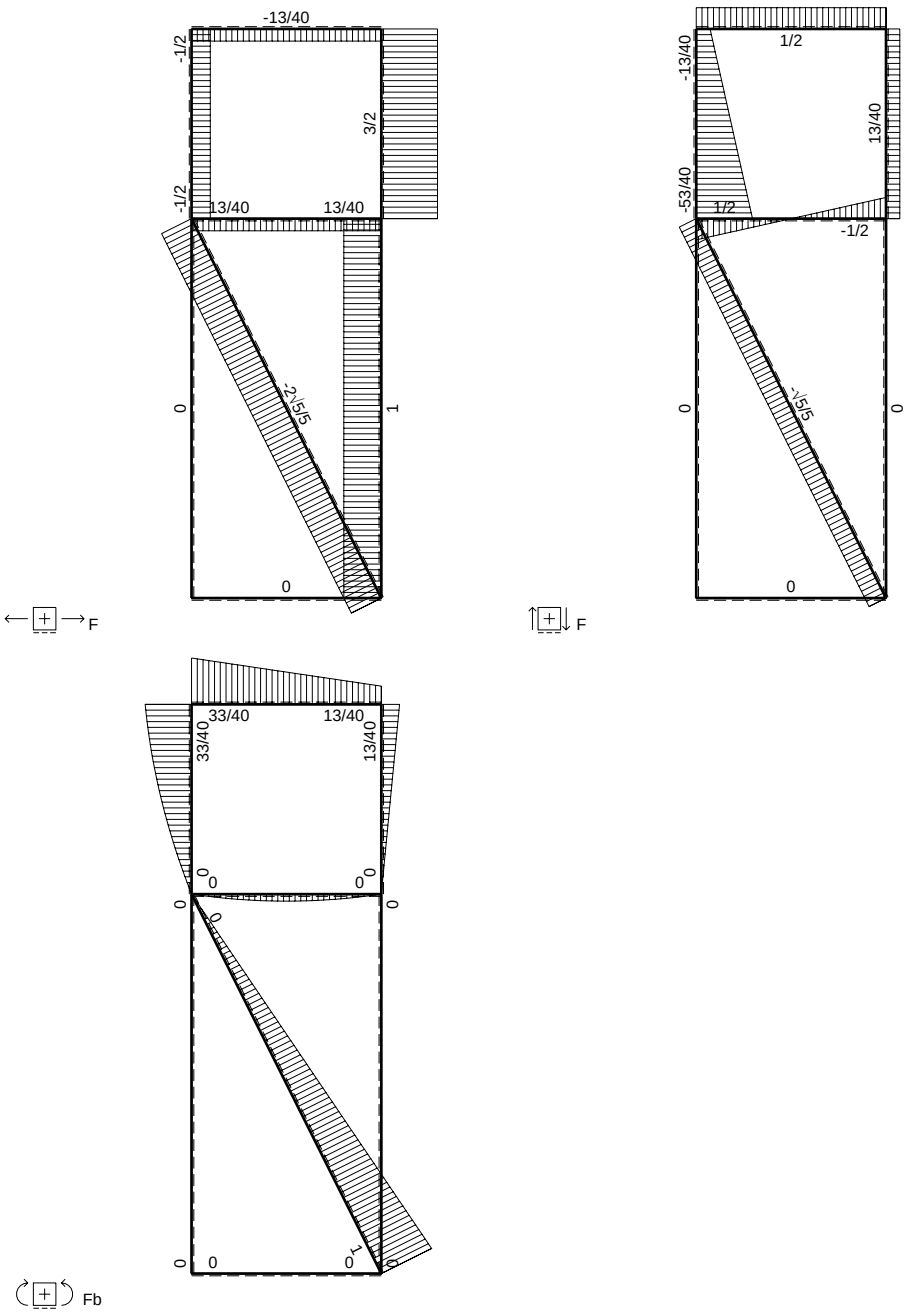

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
-2√5/5
0
1
0
F
-13/40
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13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

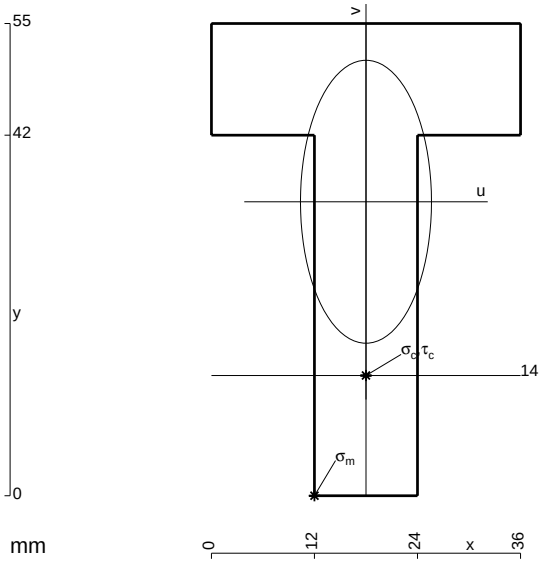
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 972. mm^2
J_u = 264196. mm^4
J_v = 56592. mm^4
y_g = 34.24 mm
N = -3605. N
T_y = -1802. N
M_x = 1571700. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.24 mm
$\sigma_m = N/A - Mv/J_u$ = 200. N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -20.24 mm
$\sigma_c = N/A - Mv/J_u$ = 116.7 N/mm^2
τ_c = 2.602 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.8 N/mm^2
S^* = 4576. mm^3

ESΣ09.xxxx.040

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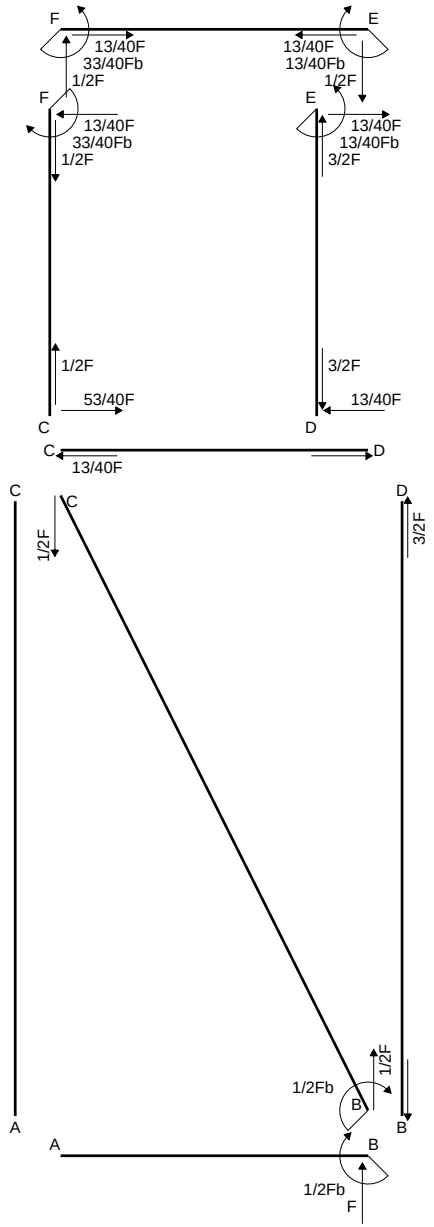
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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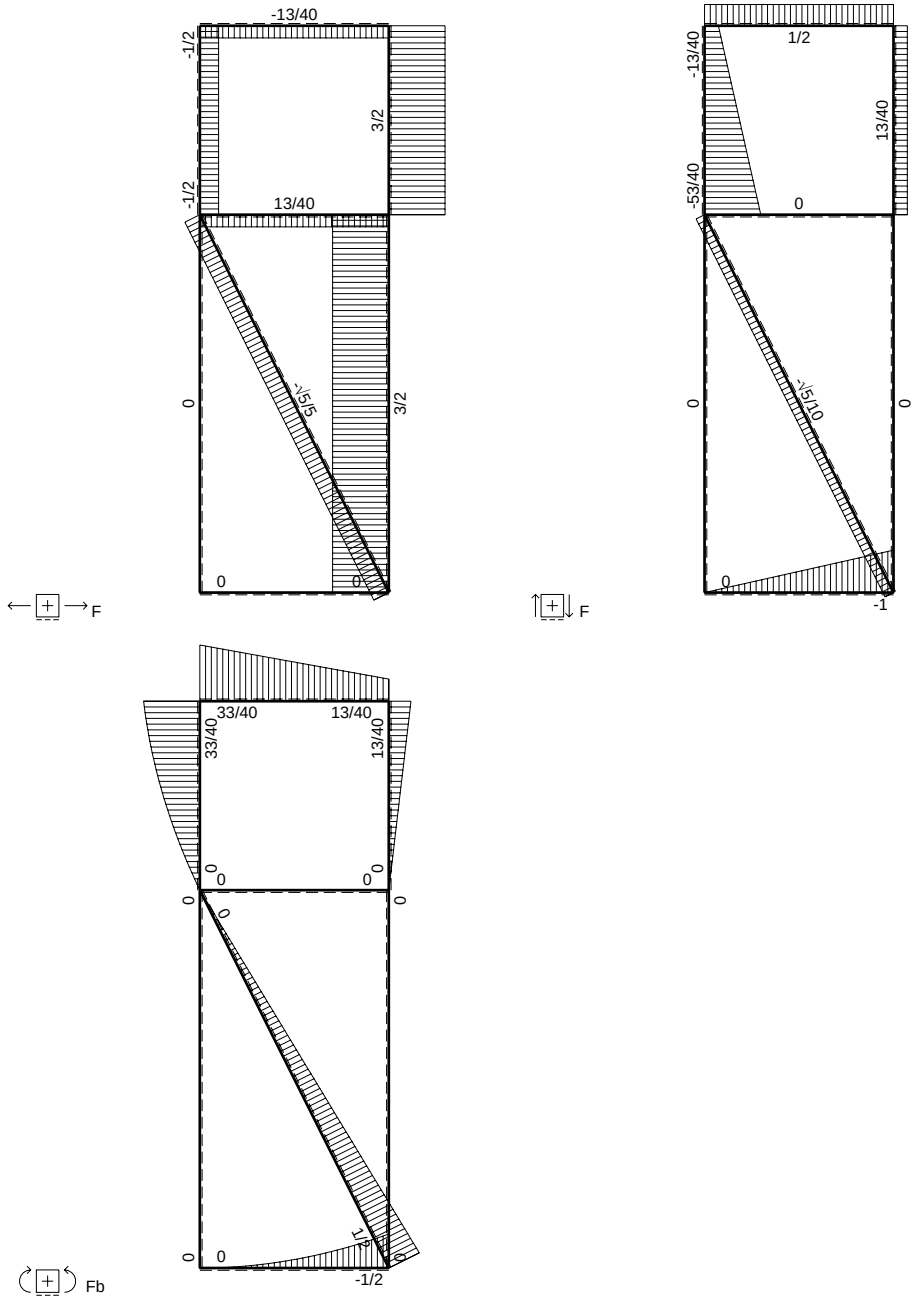
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
1/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XLb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XLb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3XLb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XLb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209727 ESΣ09.xxxx.041

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

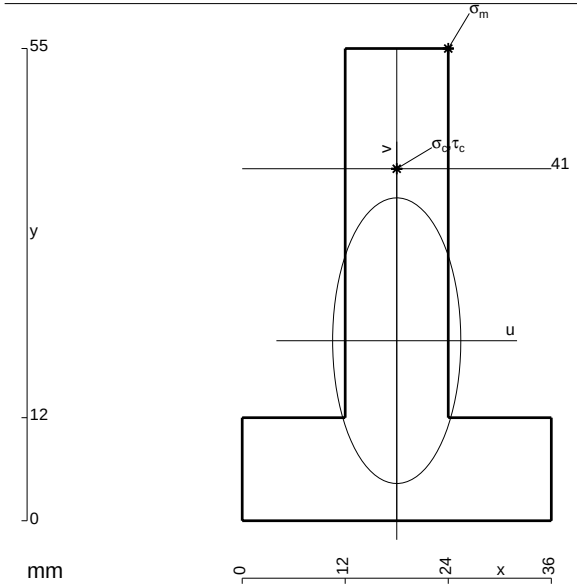
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 209727 ESΣ09.xxxx.041



A = 948. mm^2
J_u = 262515. mm^4
J_v = 52848. mm^4
y_g = 20.97 mm
T_y = -7710. N
M_x = -1619100. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 34.03 mm
$\sigma_m = -Mv/J_u$ = 209.9 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 20.03 mm
$\sigma_c = -Mv/J_u$ = 123.5 N/mm^2
τ_c = 11.11 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125. N/mm^2
S^* = 4541. mm^3

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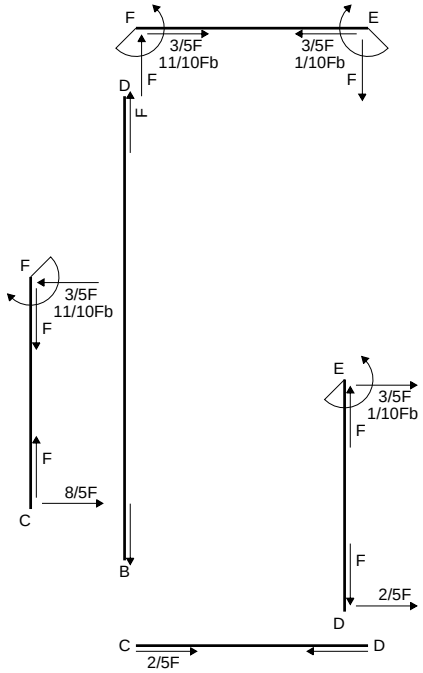
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
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1/10Fb
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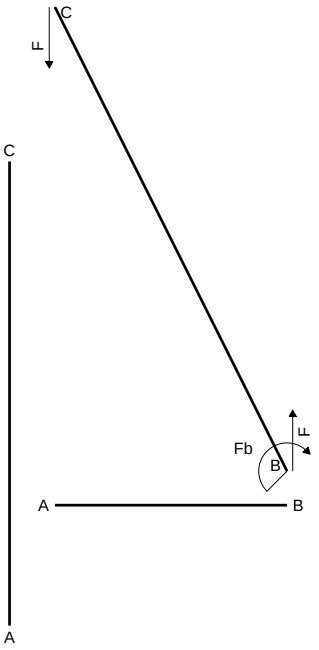
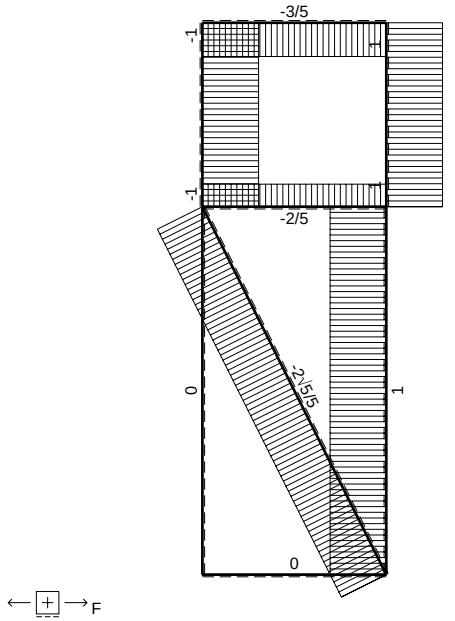
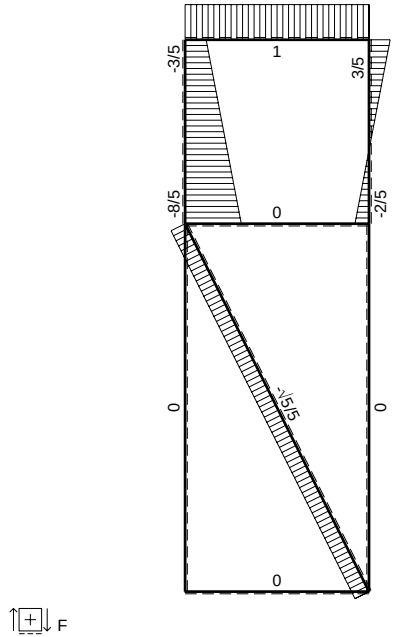
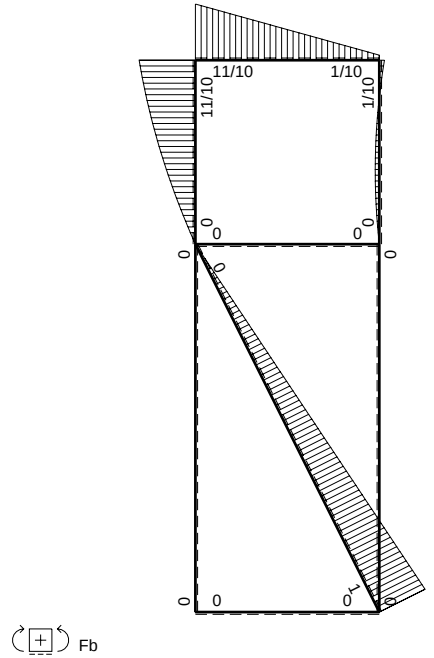
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XLb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	XLb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3XLb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
totali							1/6Fb^2/EJ	5/3XLb/EJ
iperstatica $X=W_{EF}$							-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211410 ESΣ09.xxxx.042

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ \ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ \ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

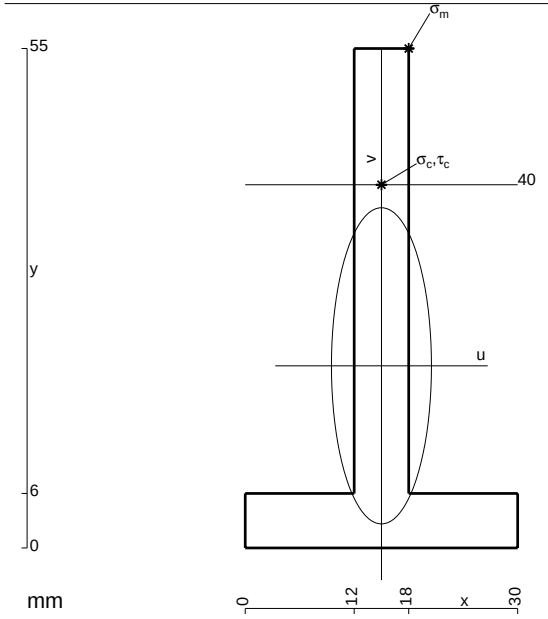
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = 1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ \ Fb^2/EJ$



A = 474. mm^2
J_u = 143796. mm^4
J_v = 14382. mm^4
y_g = 20.06 mm
N = -1726. N
T_y = -863.1 N
M_x = 887800. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 34.94 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 19.94 mm
$\sigma_c = N/A - Mv/J_u$ = -126.8 N/mm^2
τ_c = 2.471 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.8 N/mm^2
S^* = 2470. mm^3

ESΣ09.xxxx.042

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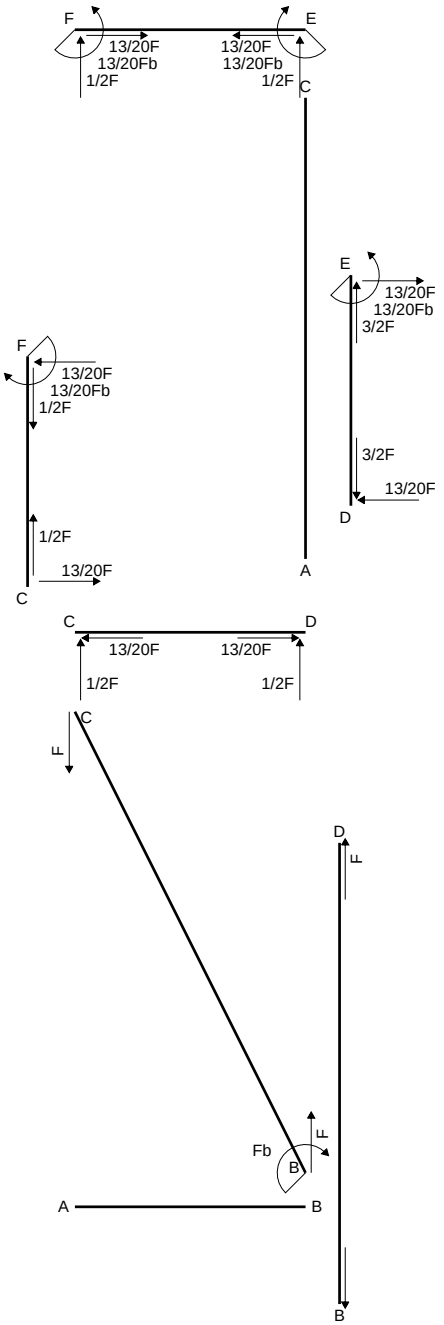
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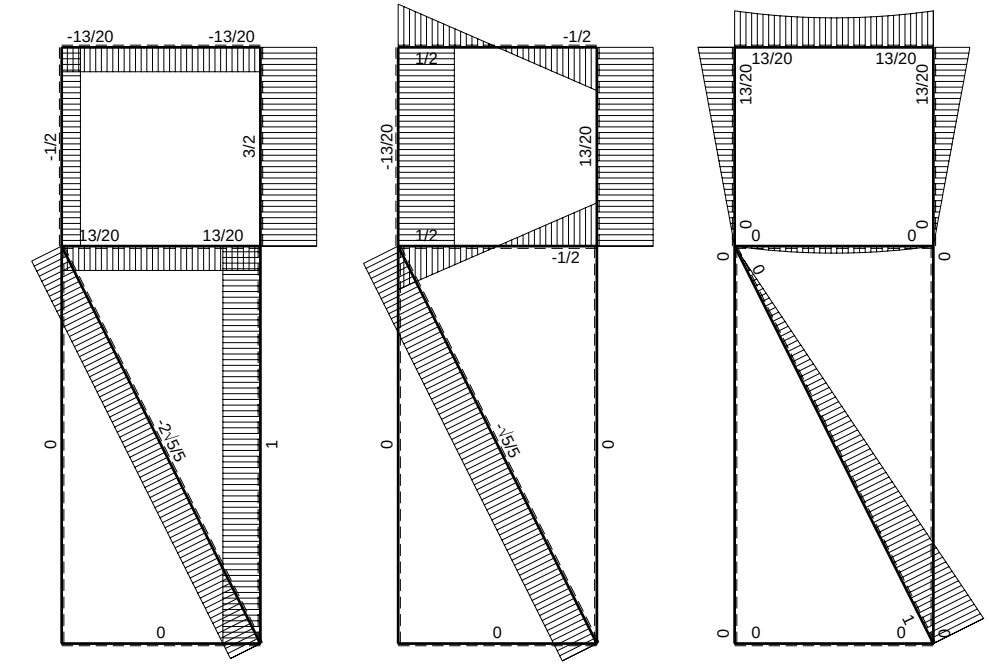
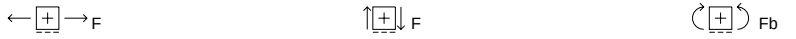
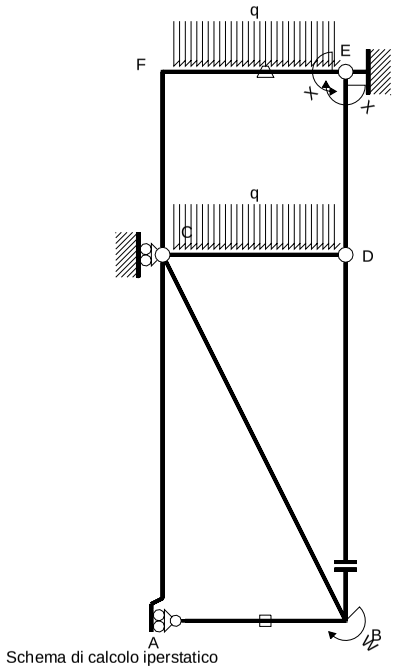
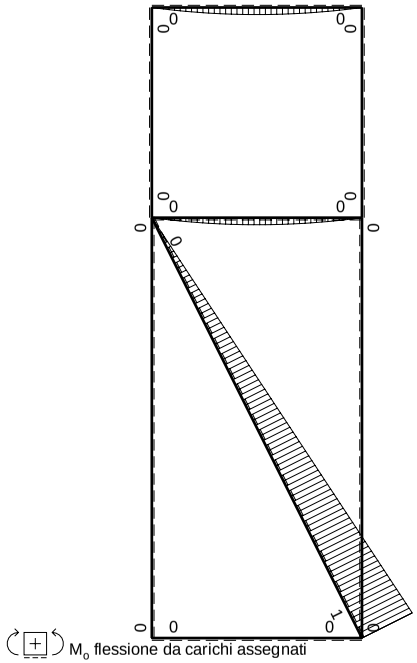
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

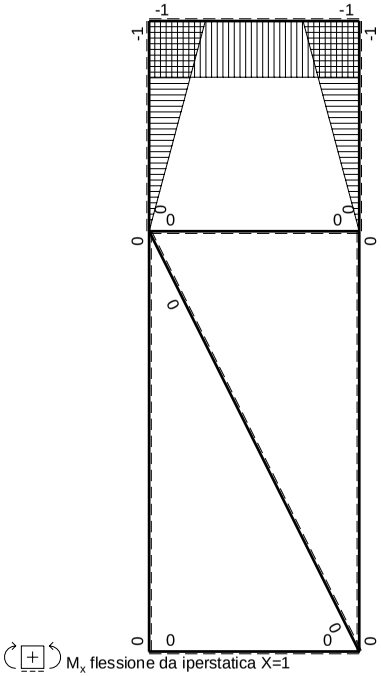

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984860 ESΣ09.xxxx.043

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

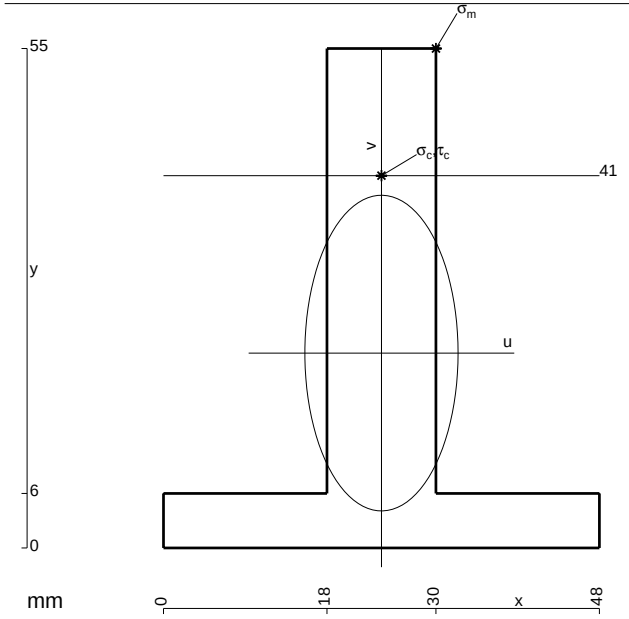
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 984860 ESΣ09.xxxx.043



A = 876. mm^2
J_u = 264708. mm^4
J_v = 62352. mm^4
y_g = 21.46 mm
N = -3193. N
T_y = -1597. N
M_x = 1785000. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 33.54 mm
$\sigma_m = N/A - Mv/J_u$ = -229.8 N/mm^2
x_c = 24. mm
y_c = 41. mm
v_c = 19.54 mm
$\sigma_c = N/A - Mv/J_u$ = -135.4 N/mm^2
τ_c = 2.241 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135.5 N/mm^2
S^* = 4459. mm^3

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ESΣ09.xxxx.043

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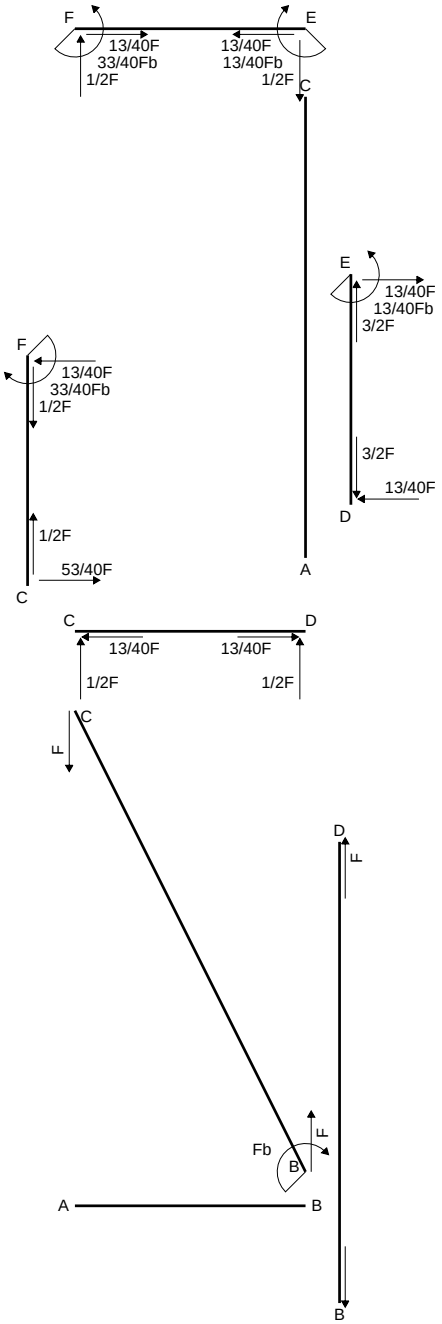
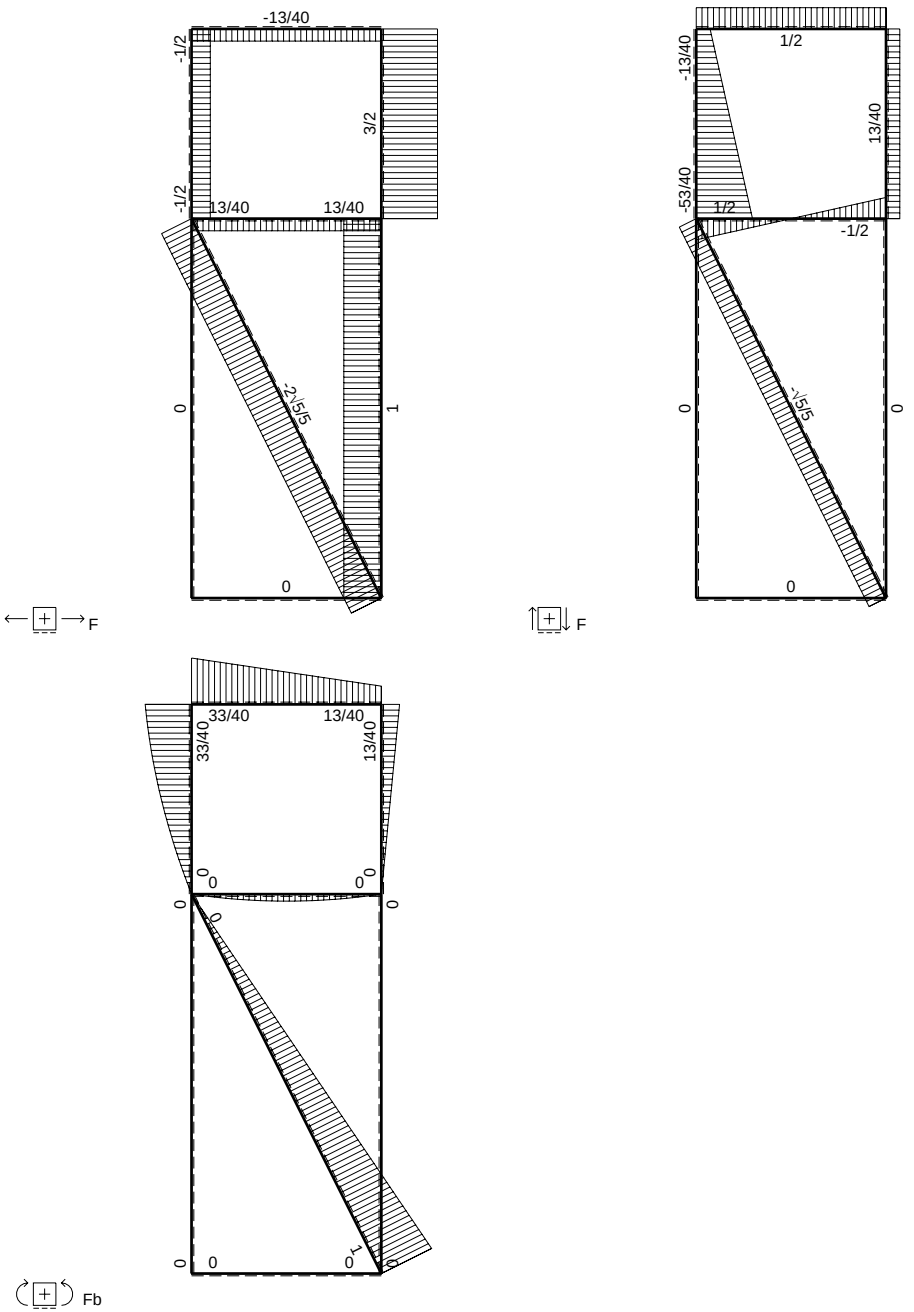

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
F
13/40F
33/40Fb
1/2F
3/2F
13/40F
D
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XkJ/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XbJ/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3XkJ/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

PROCEDIMENTO E RISULTATI 210329 ESΣ09.xxxx.044

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

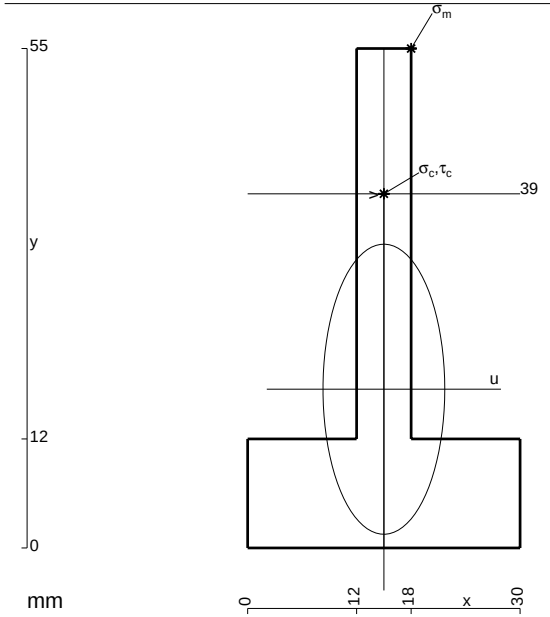
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 618. mm^2
J_u = 157731. mm^4
J_v = 27774. mm^4
y_g = 17.48 mm
N = -1646. N
T_y = -822.9 N
M_x = 993600. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.52 mm
$\sigma_m = N/A - Mv/J_u$ = -239. N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 21.52 mm
$\sigma_c = N/A - Mv/J_u$ = -138.2 N/mm^2
τ_c = 2.464 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.3 N/mm^2
S^* = 2834. mm^3

ESΣ09.xxxx.044

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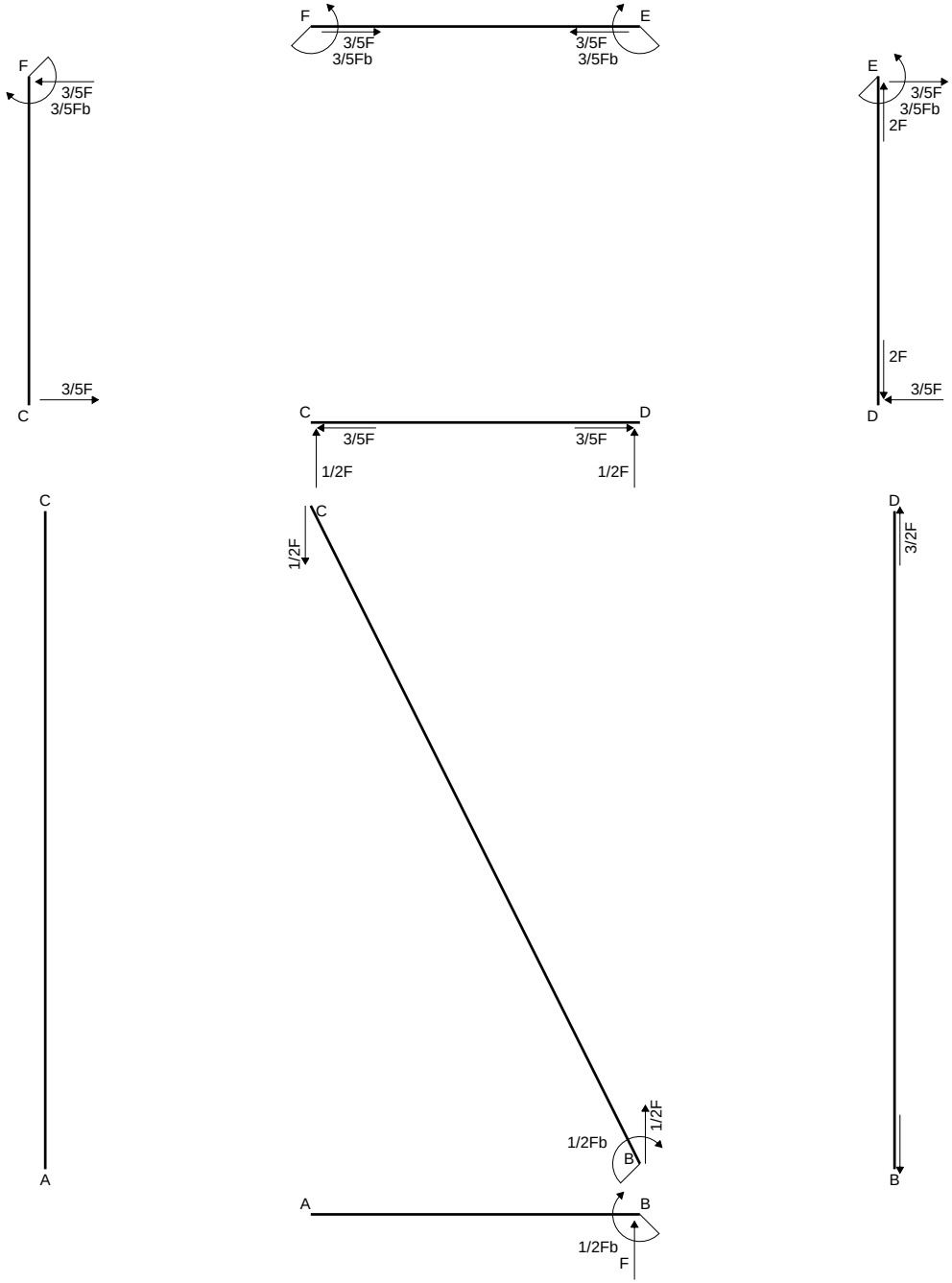
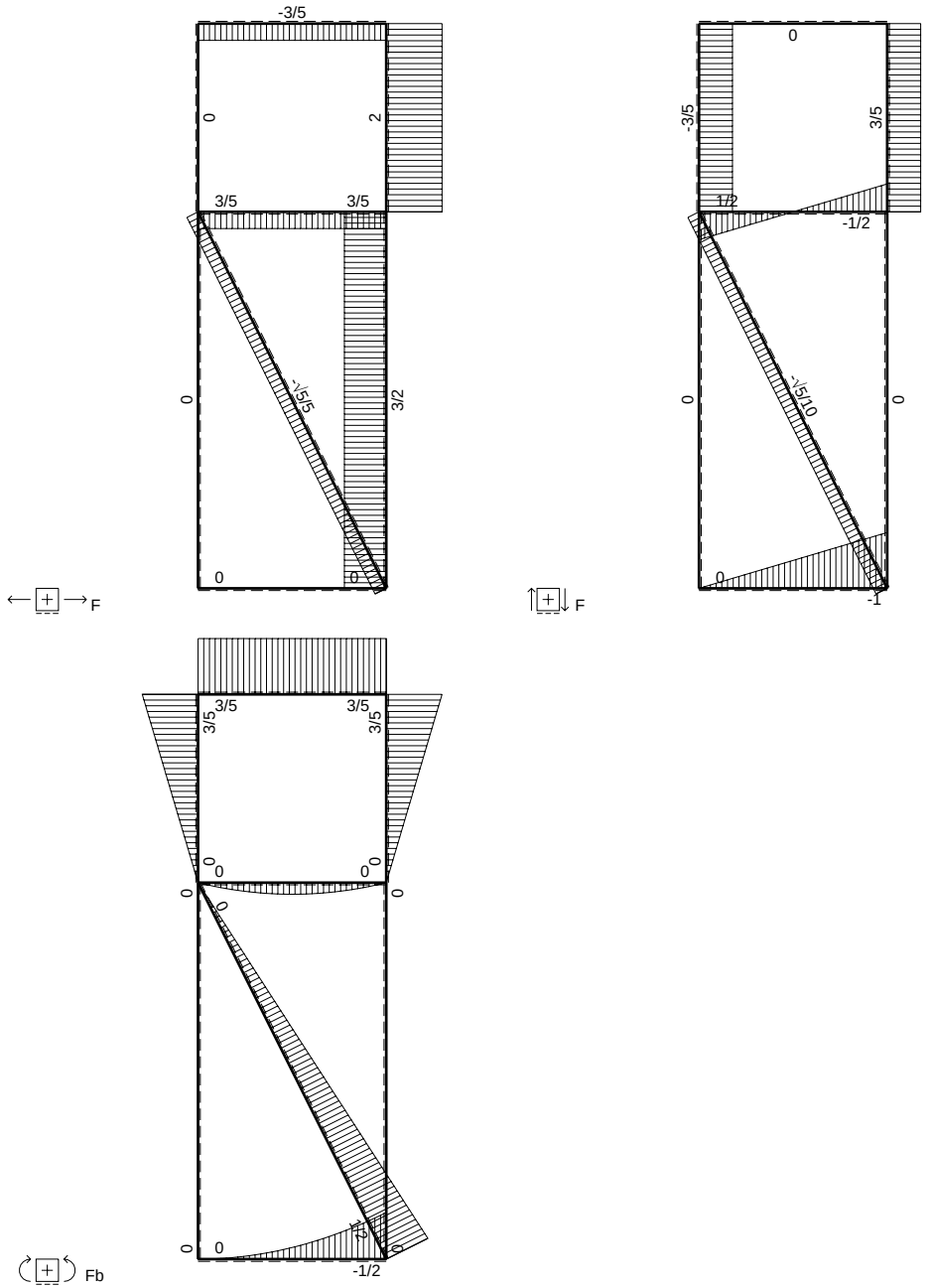

F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F


-3/5
0
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3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
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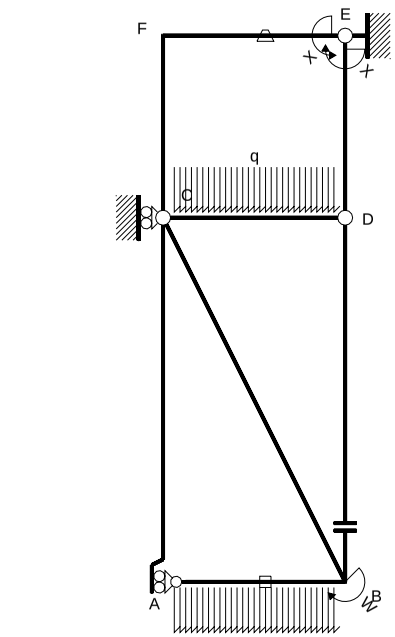
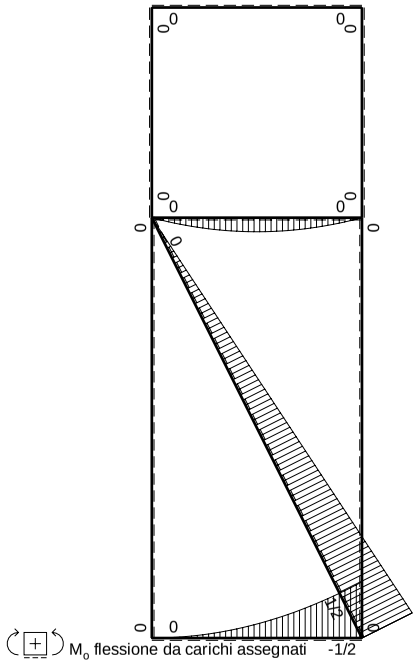
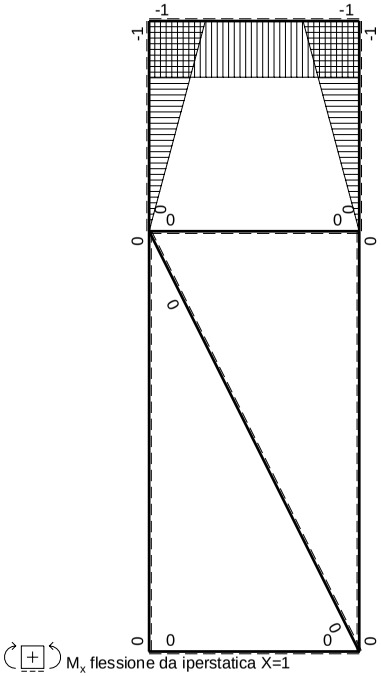
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

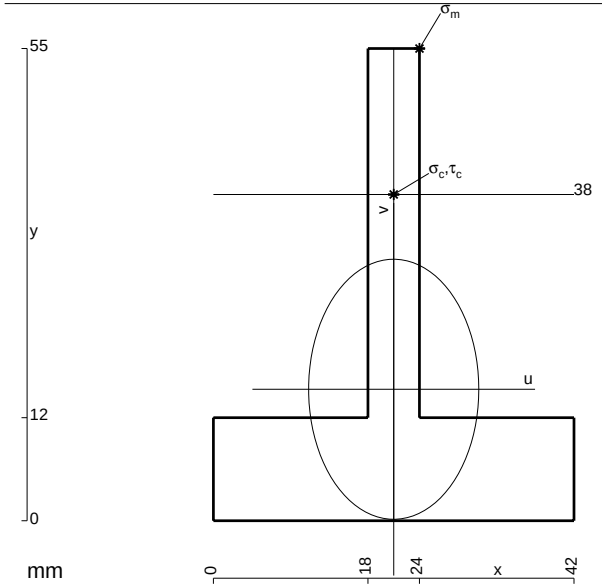
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 762. mm^2
J_u = 174852. mm^4
J_v = 74862. mm^4
y_g = 15.31 mm
T_y = -3030. N
M_x = -878700. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 39.69 mm
$\sigma_m = -Mv/J_u$ = 199.5 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 22.69 mm
$\sigma_c = -Mv/J_u$ = 114. N/mm^2
τ_c = 9.188 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.1 N/mm^2
S^* = 3181. mm^3

ESΣ09.xxxx.045

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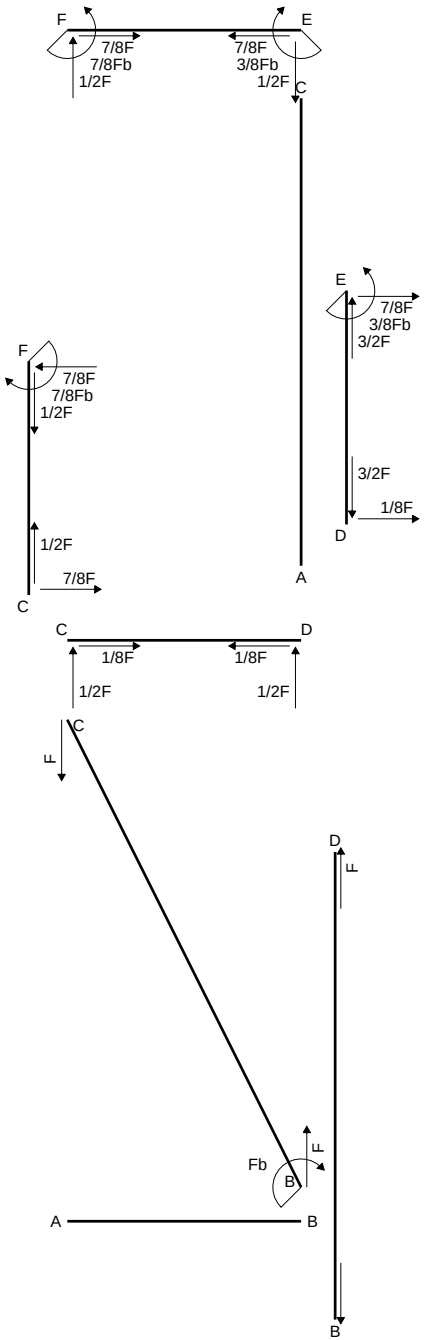
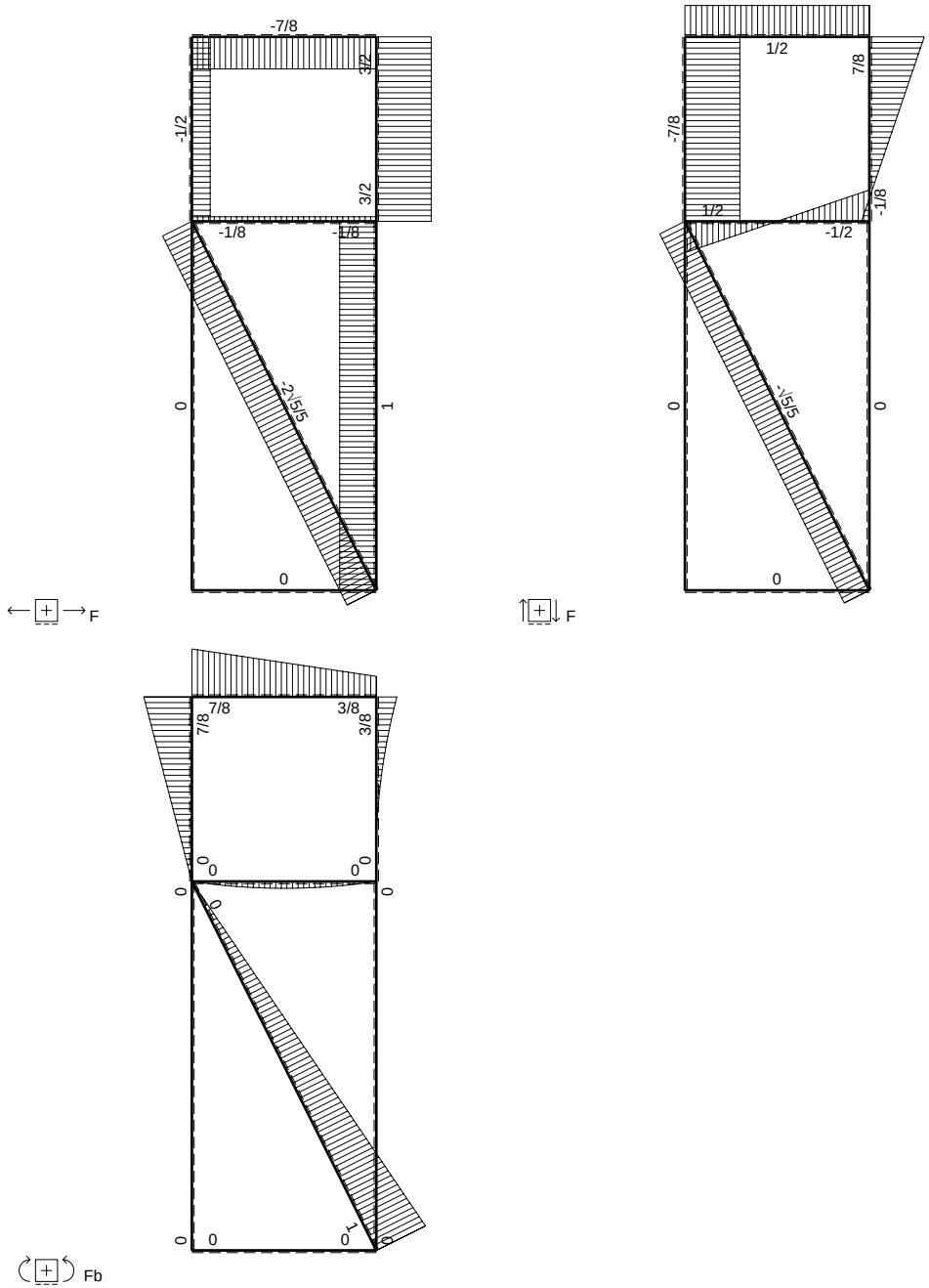

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
A
C
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
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-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XbEJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XbEJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3XbEJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3XbEJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

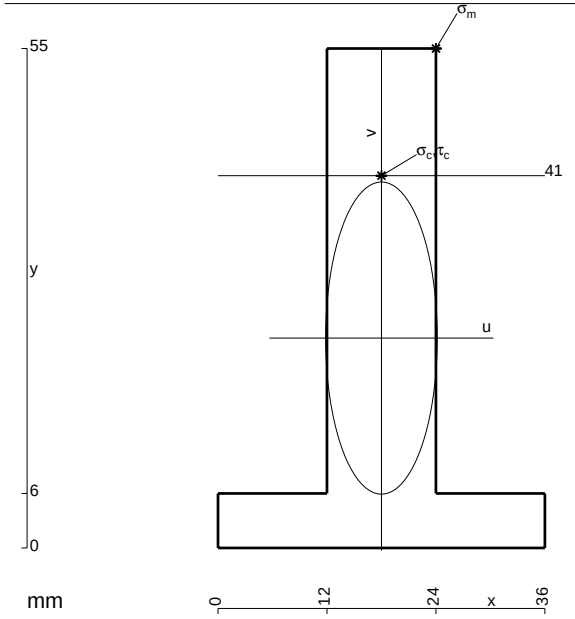
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 804. mm^2
J_u = 237762. mm^4
J_v = 30384. mm^4
y_g = 23.11 mm
N = -2227. N
T_y = -1114. N
M_x = 1543800. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 31.89 mm
$\sigma_m = N/A - Mv/J_u$ = -209.8 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 17.89 mm
$\sigma_c = N/A - Mv/J_u$ = -118.9 N/mm^2
τ_c = 1.632 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119. N/mm^2
S^* = 4181. mm^3

ESΣ09.xxxx.046

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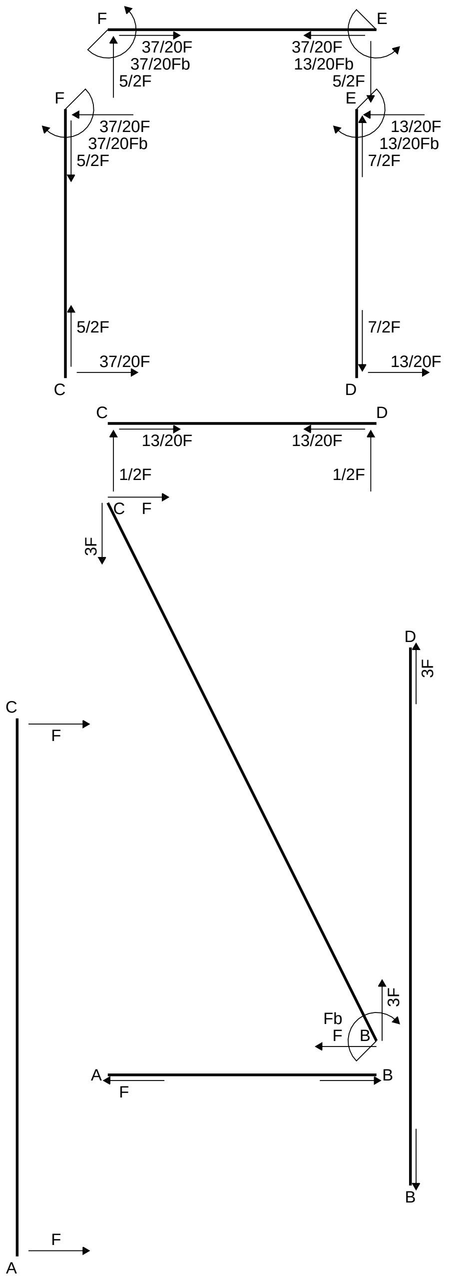
F
E
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37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
7/2F
37/20F
13/20F
C
D
C
D
13/20F
13/20F
1/2F
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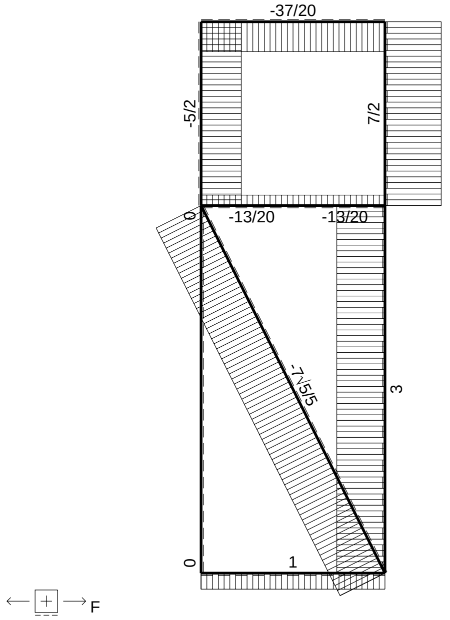
A
B
F
B
F
A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-13/20
-7√5/5
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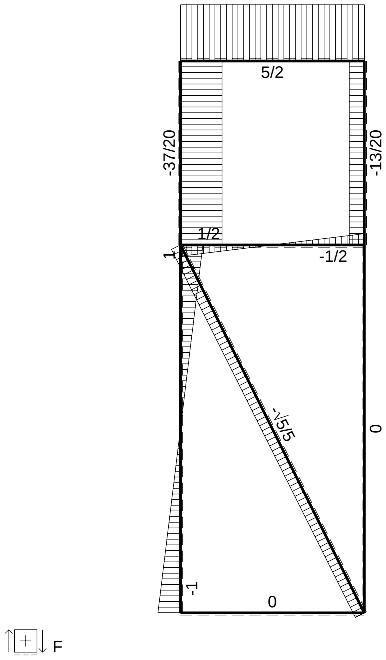
0
1
F


5/2
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-13/20
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-1/2
-√5/5
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-1
0
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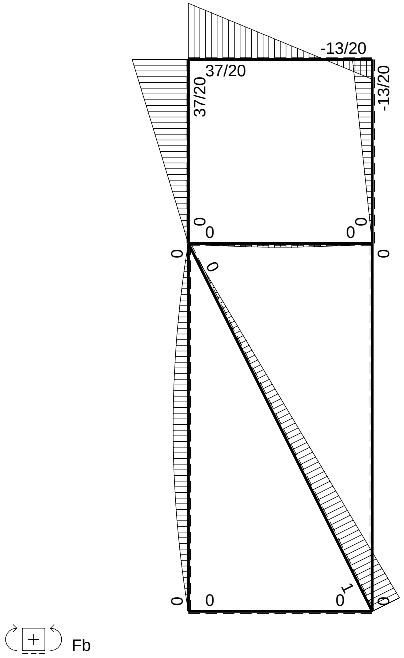
-13/20
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0
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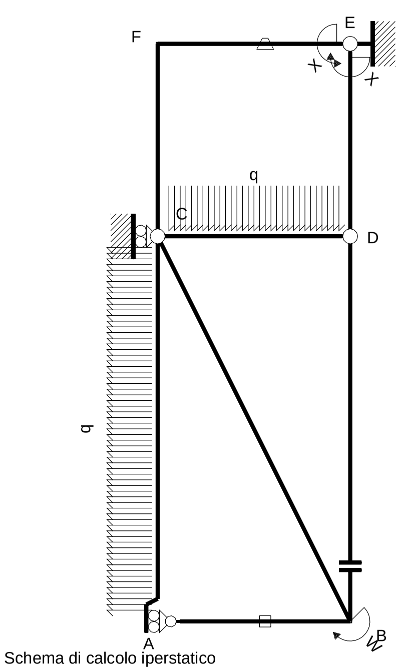
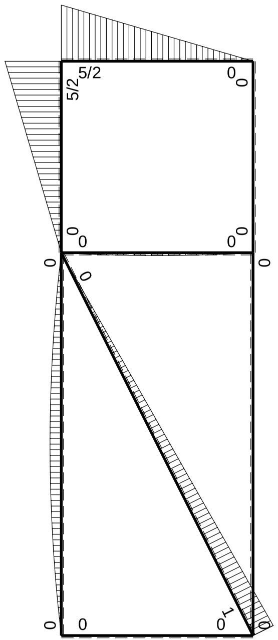
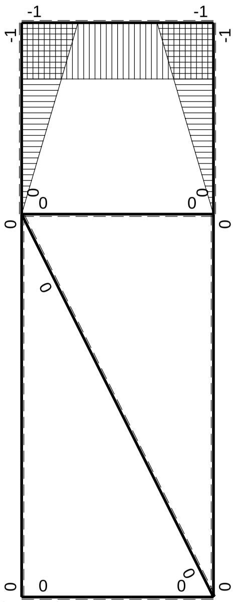
0
0
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

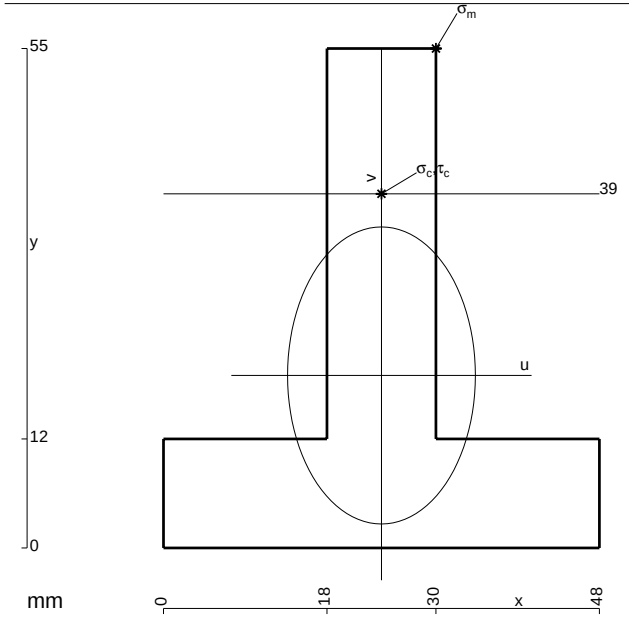
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1092. mm^2
J_u = 292252. mm^4
J_v = 116784. mm^4
y_g = 18.99 mm
N = -8171. N
T_y = -1167. N
M_x = 1722600. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 36.01 mm
$\sigma_m = N/A - Mv/J_u$ = -219.7 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.01 mm
$\sigma_c = N/A - Mv/J_u$ = -125.4 N/mm^2
τ_c = 1.79 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.4 N/mm^2
S^* = 5377. mm^3

ESΣ09.xxxx.047

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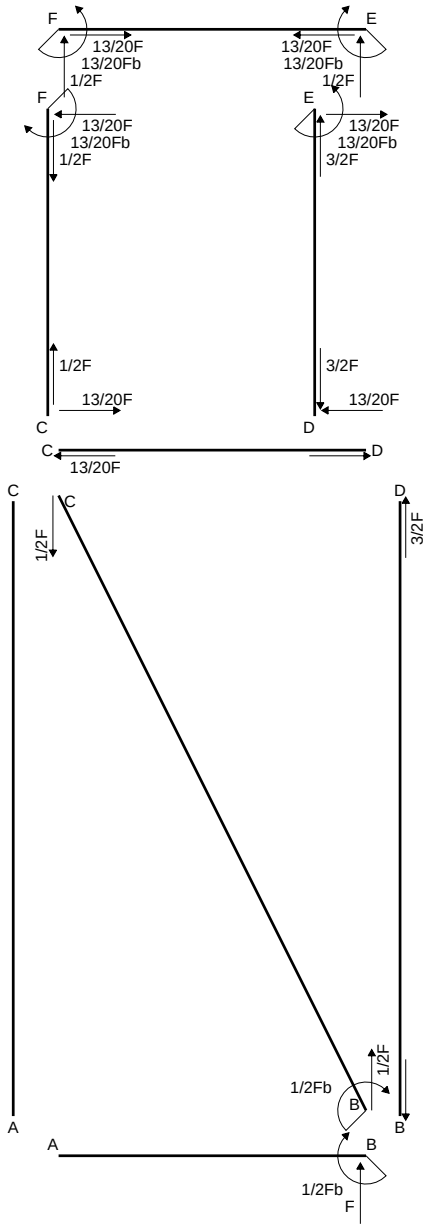
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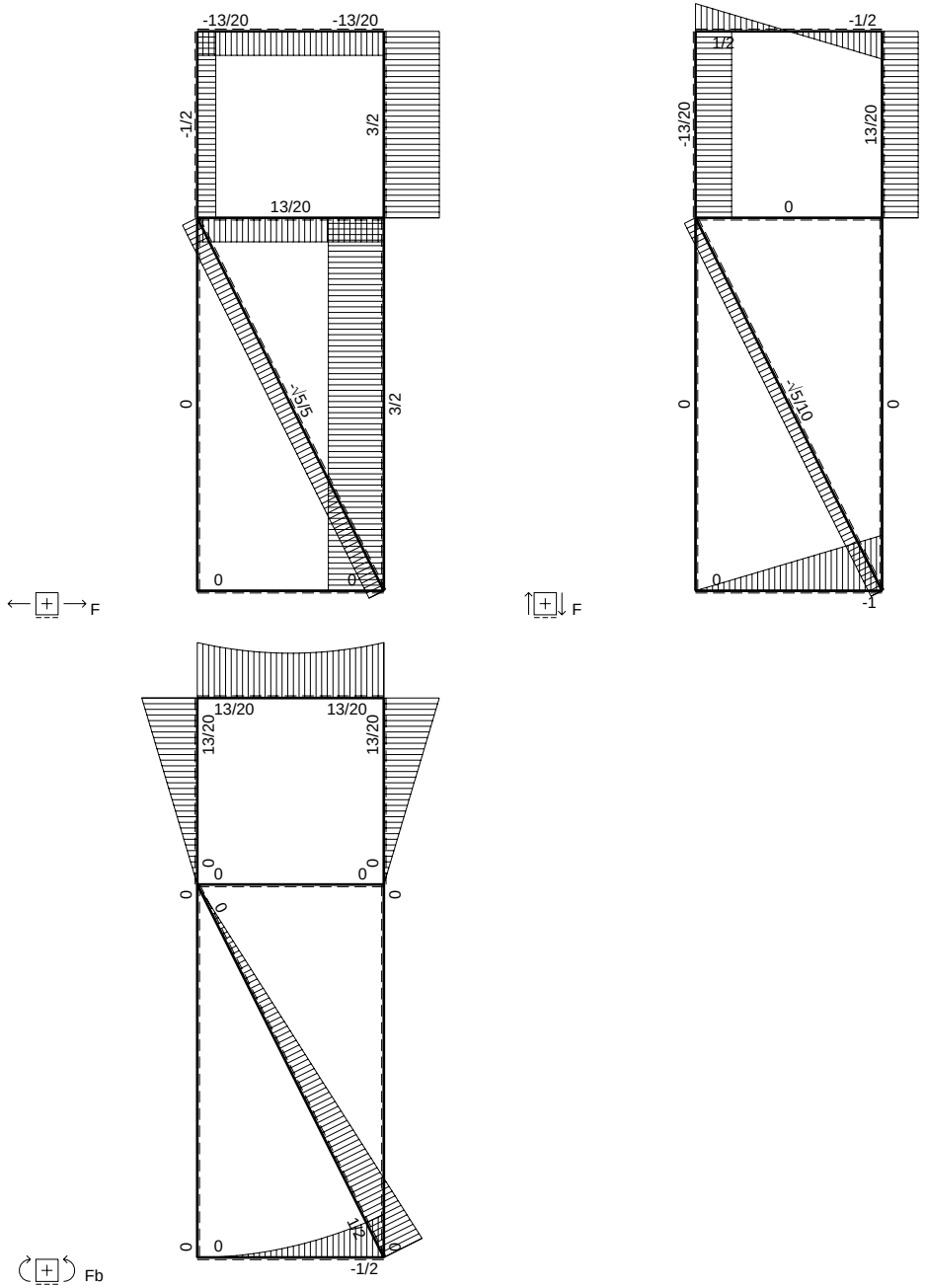
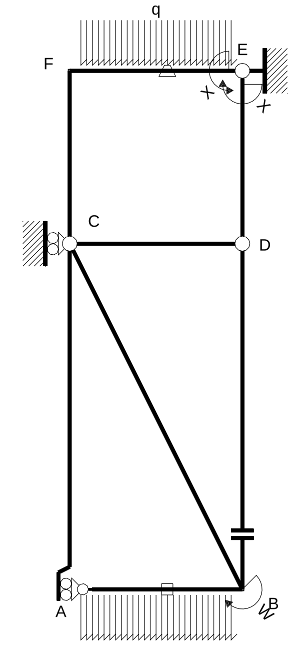
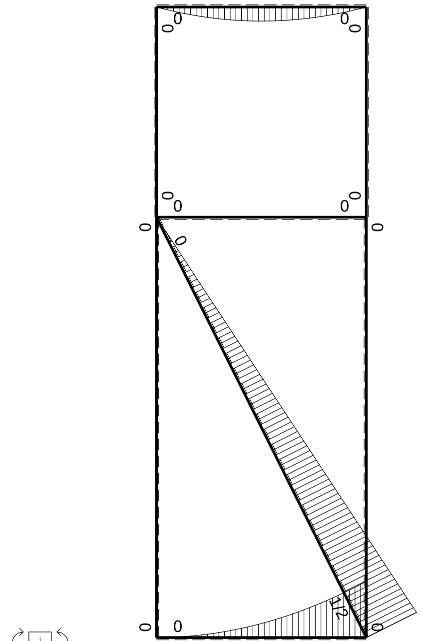
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

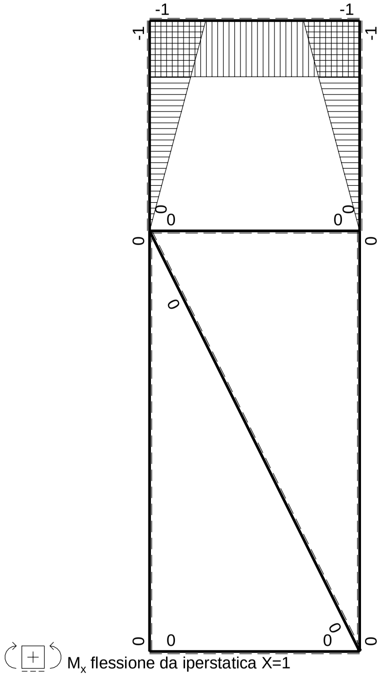

-13/20
-1/2
3/2
13/20
0
-√5/5
-√5/10
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati -1/2



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

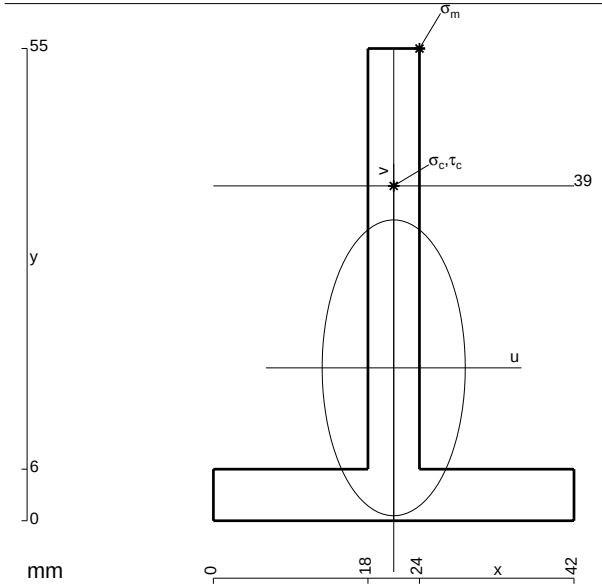
$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 546. mm^2
J_u = 162198. mm^4
J_v = 37926. mm^4
y_g = 17.81 mm
T_y = -2860. N
M_x = -1001000. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.19 mm
$\sigma_m = -Mv/J_u$ = 229.5 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 21.19 mm
$\sigma_c = -Mv/J_u$ = 130.8 N/mm^2
τ_c = 8.236 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.6 N/mm^2
S^* = 2802. mm^3

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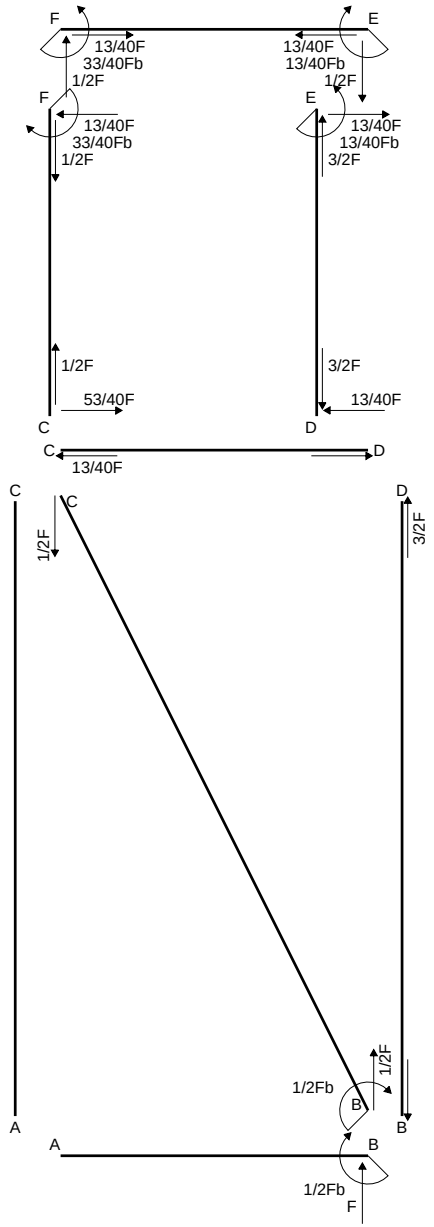
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
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1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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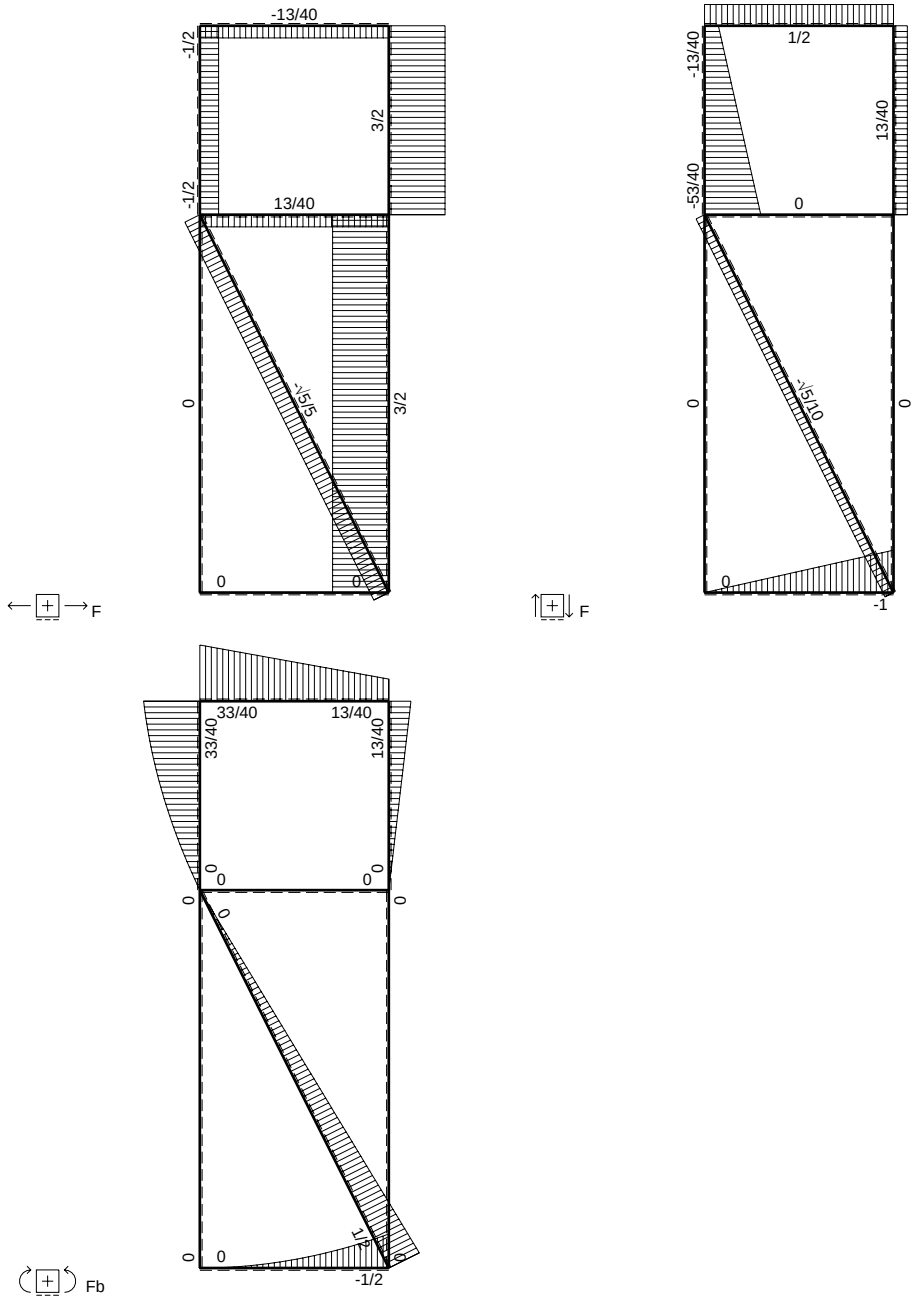
A
B
1/2Fb
F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3/2
-1/2
13/40
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-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
1/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	$5/3Xb/EJ$
iperstatica $X=W_{EF}$							-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982043 ESΣ09.xxxx.049

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14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

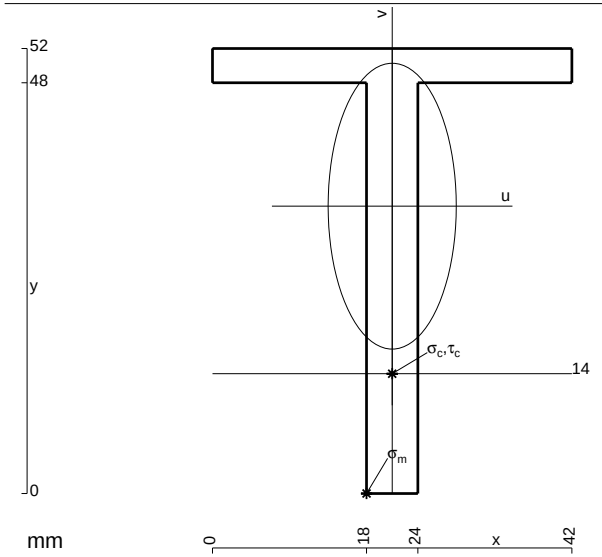
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 456. mm^2
J_u = 127247. mm^4
J_v = 25560. mm^4
y_g = 33.58 mm
T_y = -2590. N
M_x = -906500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -33.58 mm
$\sigma_m = -Mv/J_u$ = -239.2 N/mm^2
x_c = 21. mm
y_c = 14. mm
v_c = -19.58 mm
$\sigma_c = -Mv/J_u$ = -139.5 N/mm^2
τ_c = 7.574 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.1 N/mm^2
S^* = 2233. mm^3

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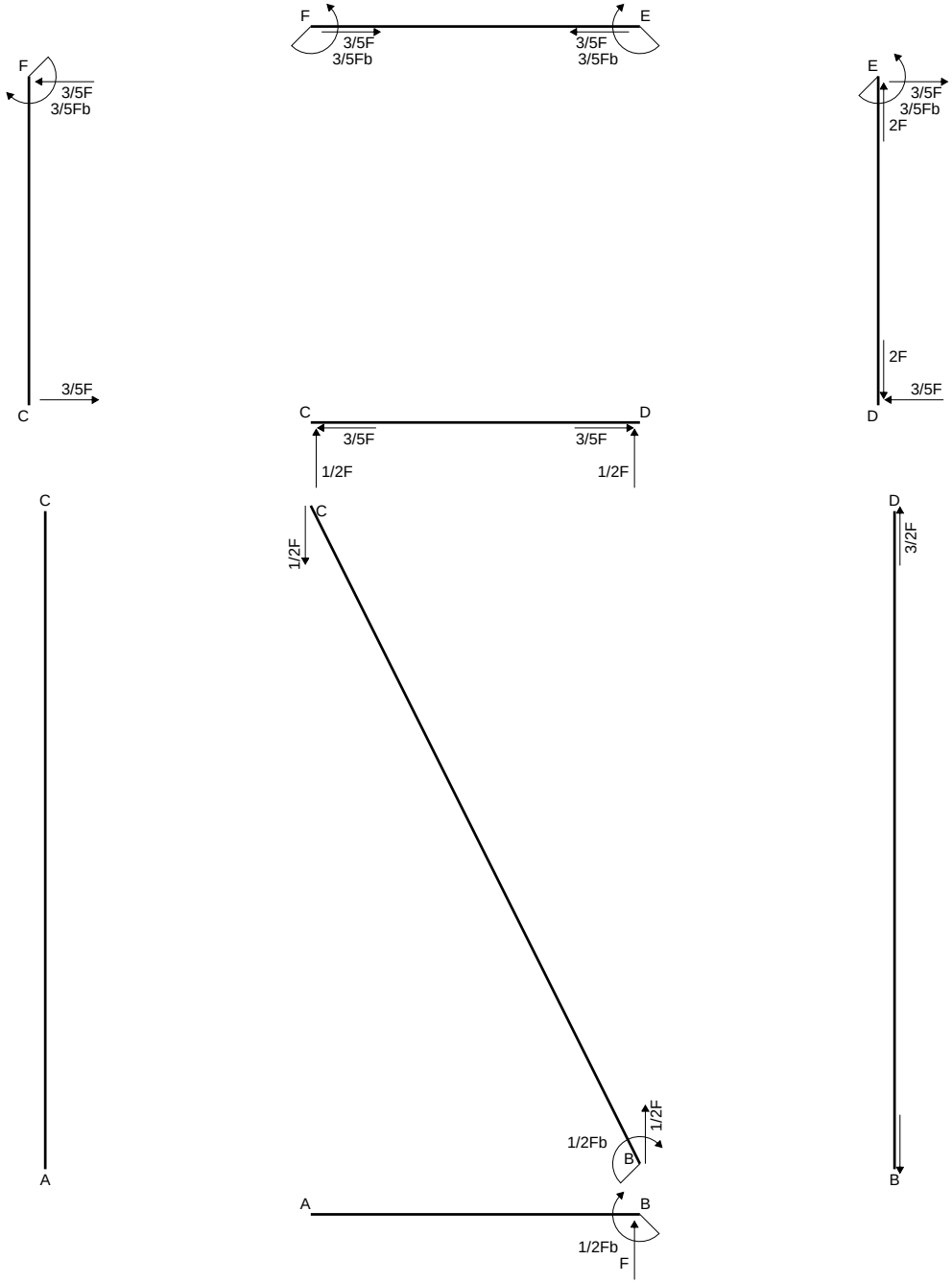
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3/5Fb
3/5F
3/5Fb
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3/5F
3/5Fb
E
3/5F
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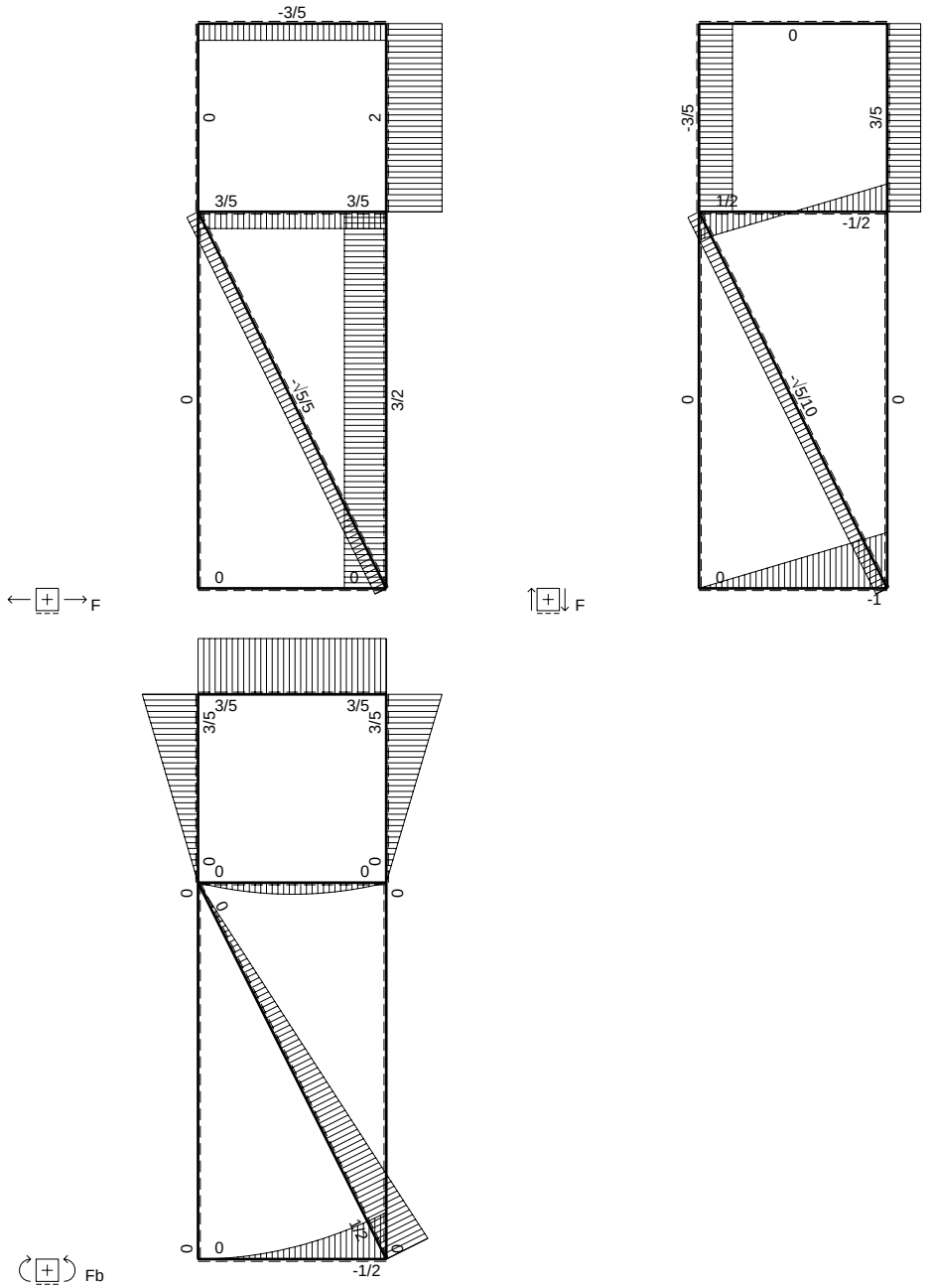
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F
A
B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-1/2
0
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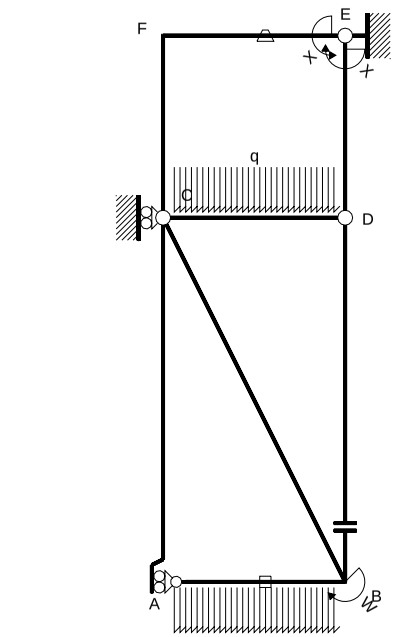
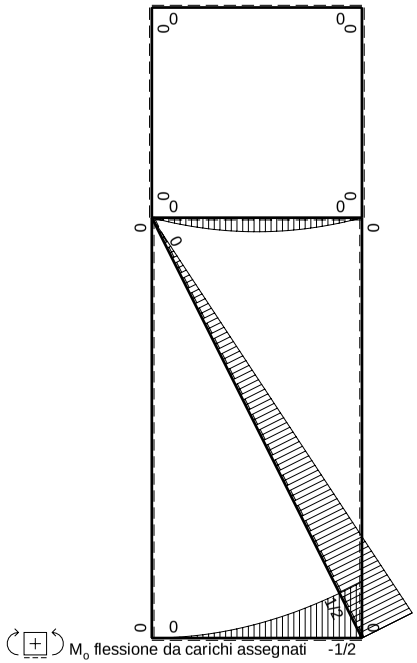
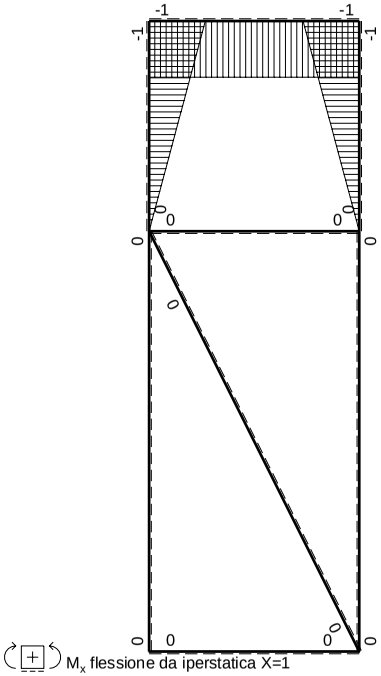
-1/2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

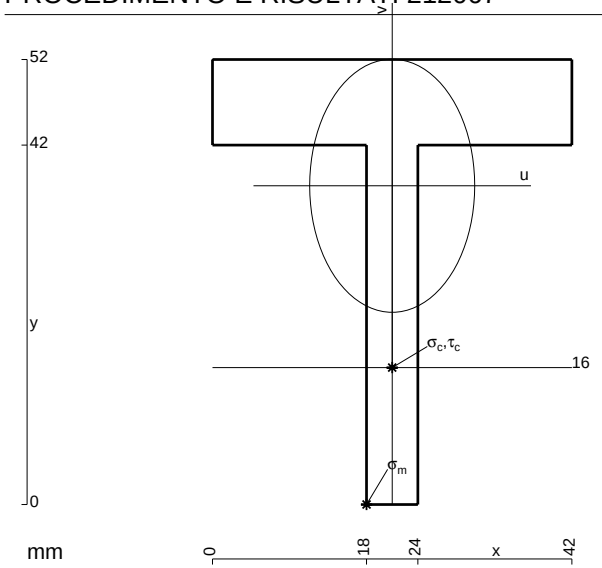
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 672. mm^2
J_u = 147014. mm^4
J_v = 62496. mm^4
y_g = 37.25 mm
T_y = -4260. N
M_x = -788100. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.25 mm
$\sigma_m = -Mv/J_u$ = -199.7 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.25 mm
$\sigma_c = -Mv/J_u$ = -113.9 N/mm^2
τ_c = 13.56 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 116.3 N/mm^2
S^* = 2808. mm^3

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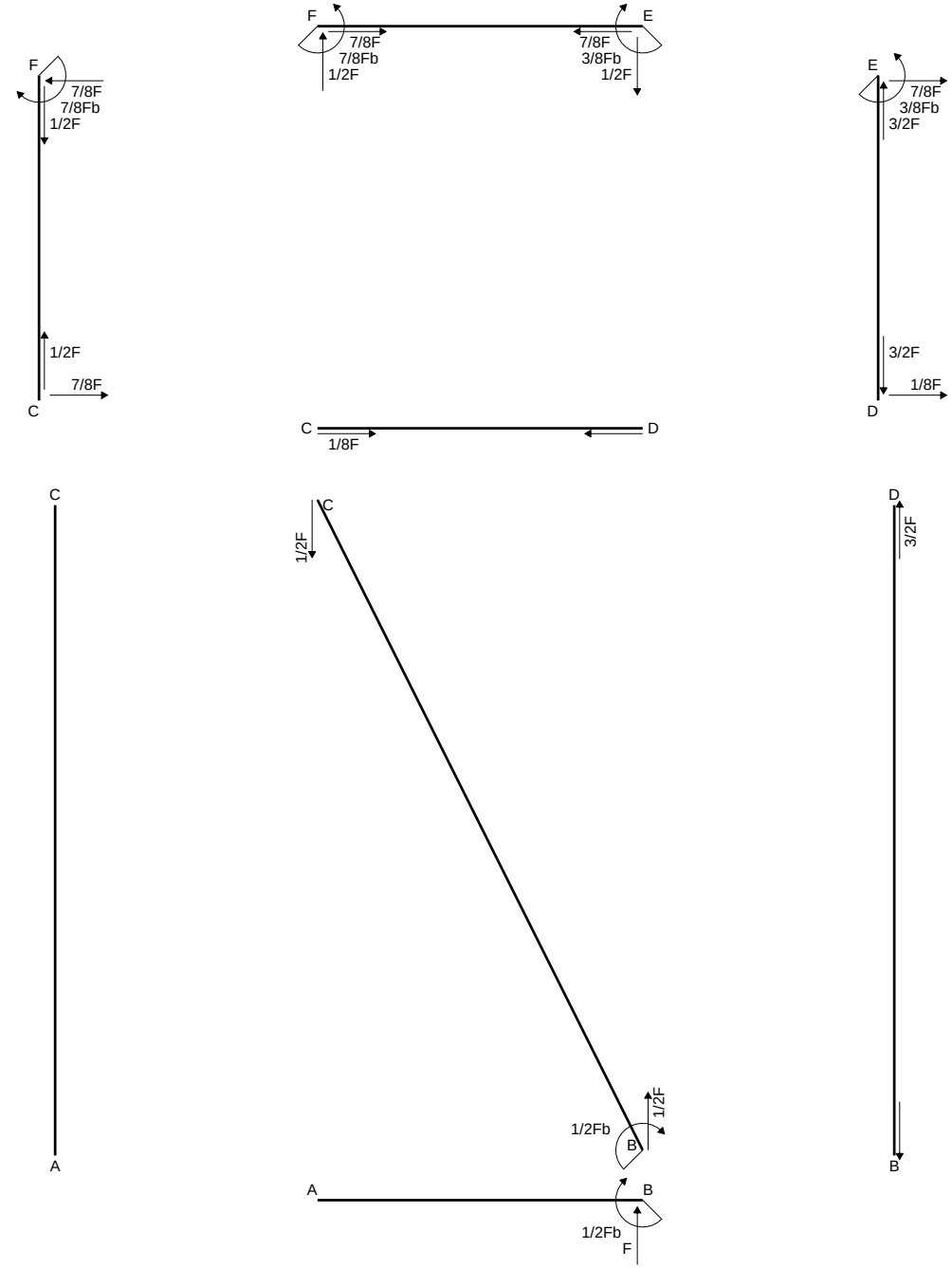
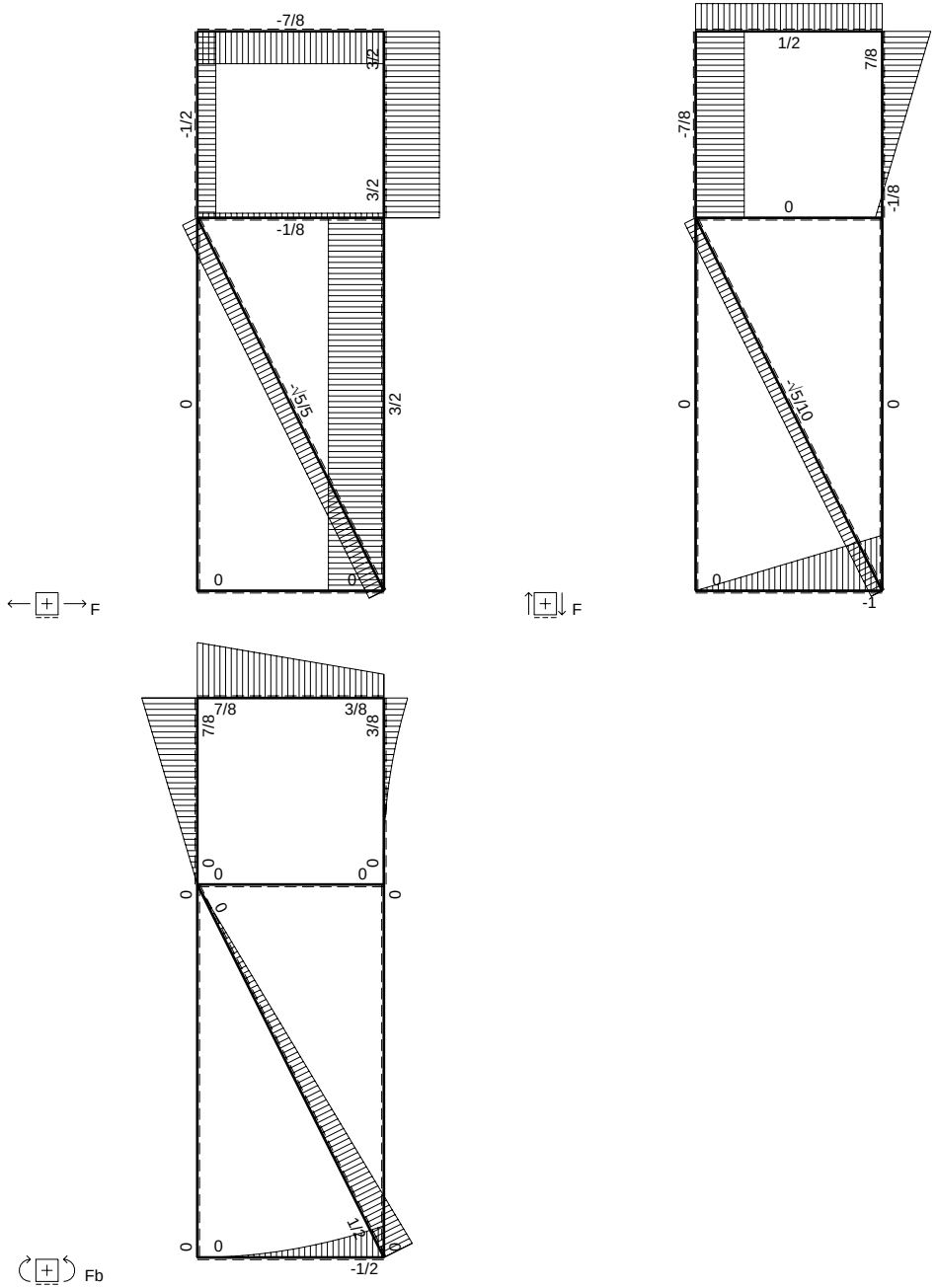

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
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7/8
-1/8
-√5/10
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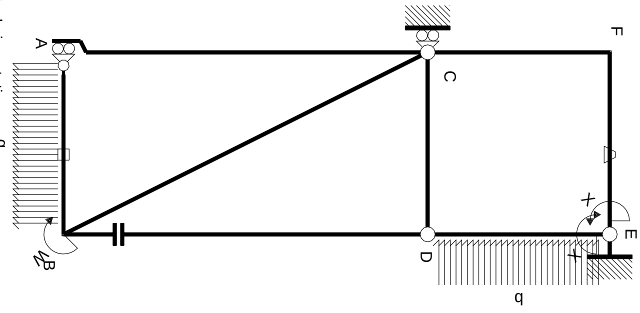
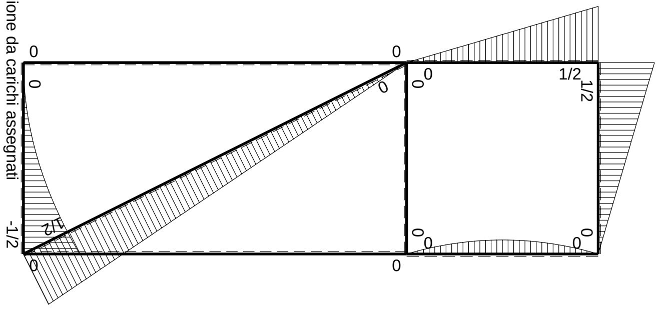
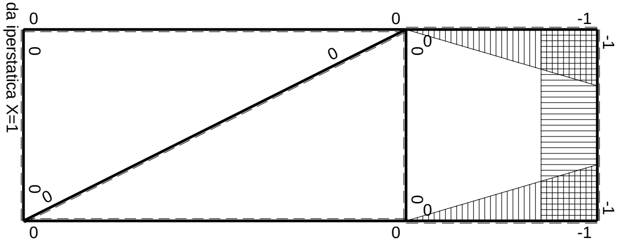
3/8
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

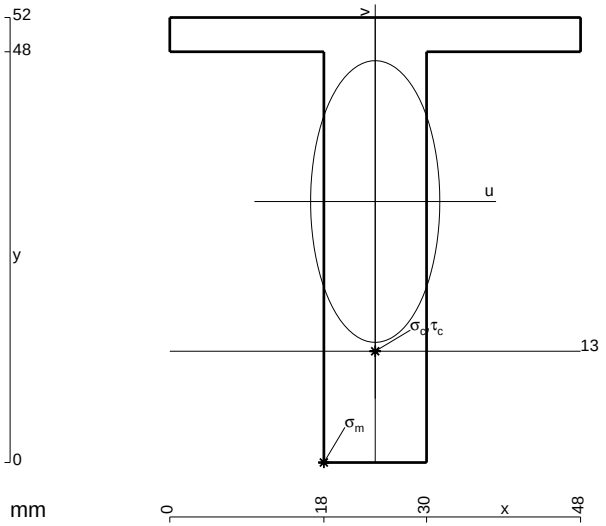
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 768. mm^2
J_u = 208192. mm^4
J_v = 43776. mm^4
y_g = 30.5 mm
T_y = -7160. N
M_x = -1432000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -30.5 mm
$\sigma_m = -Mv/J_u$ = -209.8 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.5 mm
$\sigma_c = -Mv/J_u$ = -120.4 N/mm^2
τ_c = 10.73 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 121.8 N/mm^2
S^* = 3744. mm^3

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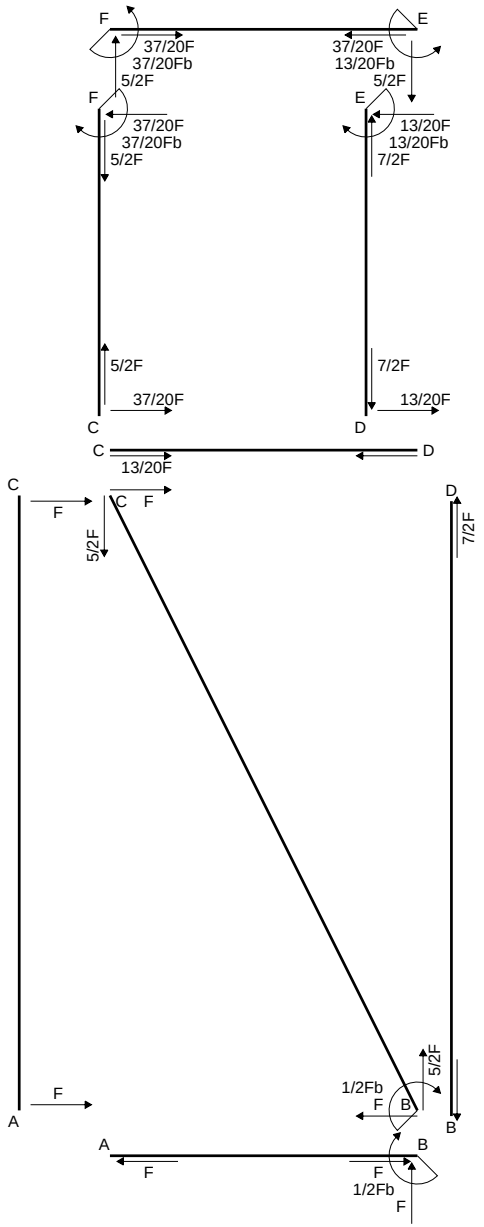
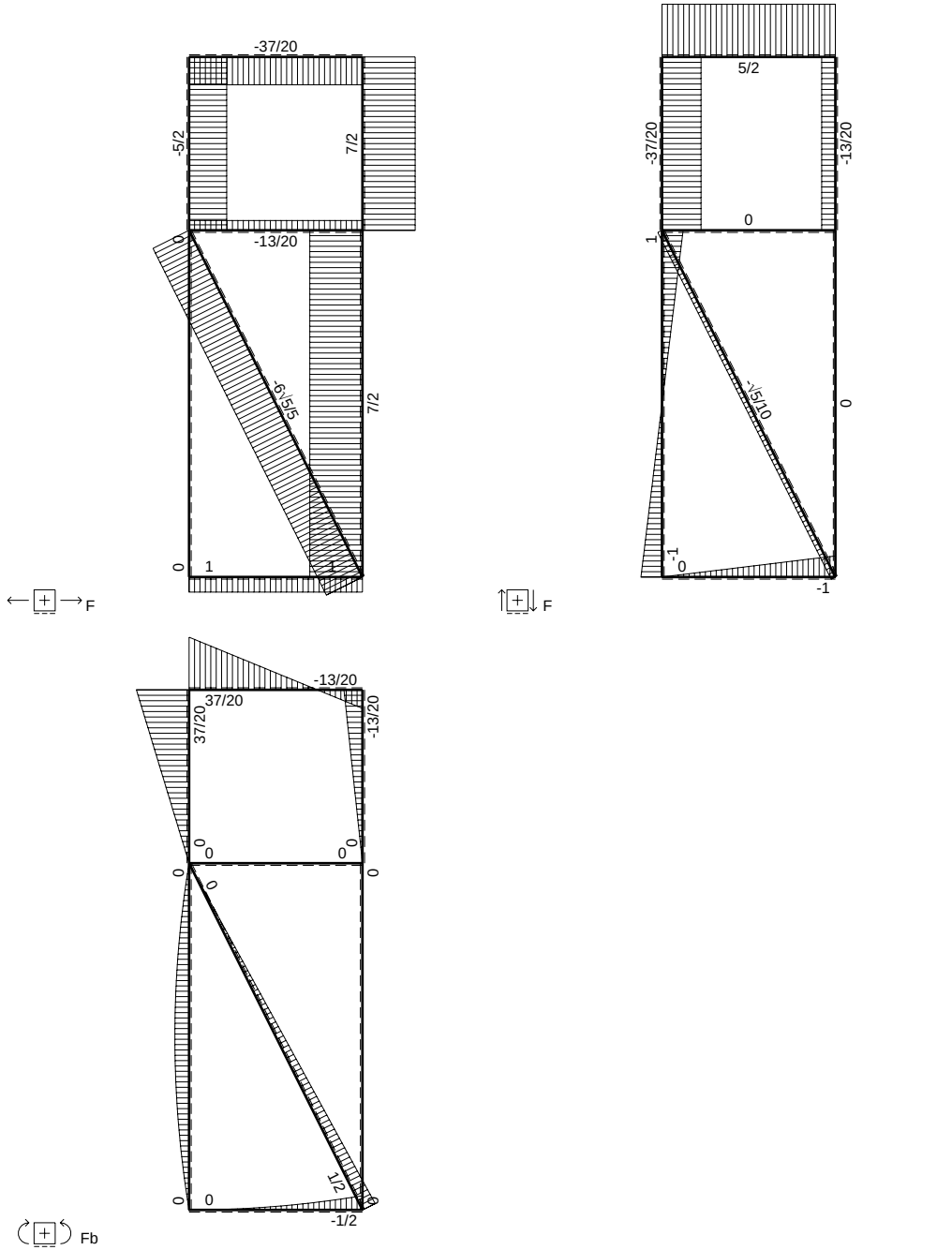
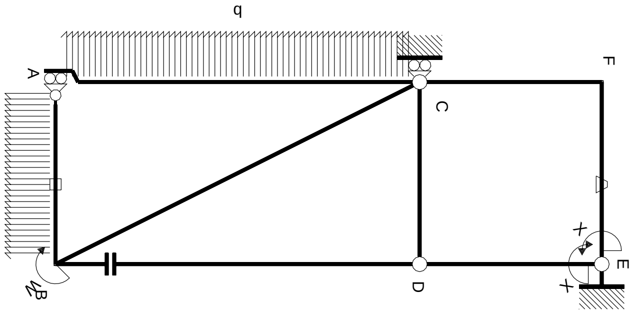
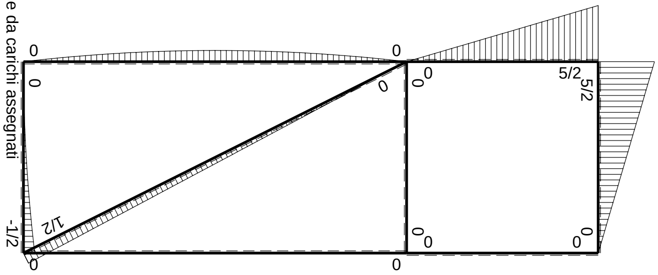

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

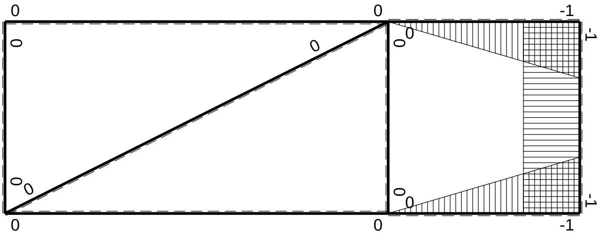

-37/20
-5/2
7/2
-13/20
-5\√5/5
1
0
F
5/2
-37/20
-13/20
-√5/10
-1
Fb
1/2
-1/2



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

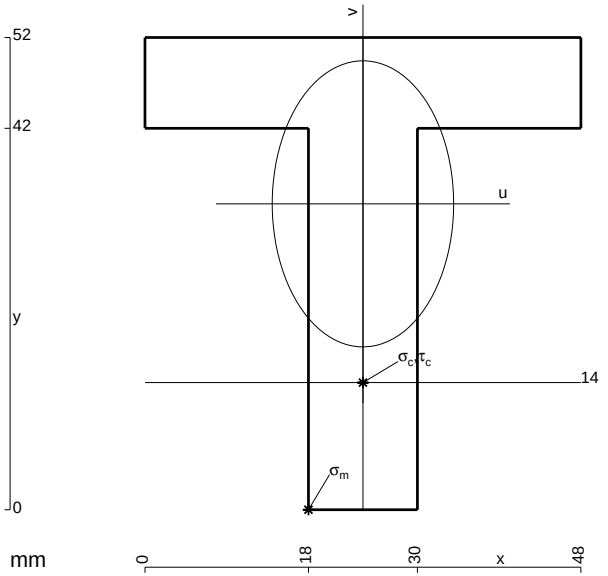
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 984. mm^2
J_u = 244285. mm^4
J_v = 98208. mm^4
y_g = 33.68 mm
N = 7500. N
T_y = -7500. N
M_x = -1650000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.68 mm
σ_m = N/A-Mv/J_u = -219.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.68 mm
σ_c = N/A-Mv/J_u = -125.3 N/mm^2
τ_c = 11.47 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.9 N/mm^2
S^* = 4483. mm^3

ESΣ09.xxxx.052

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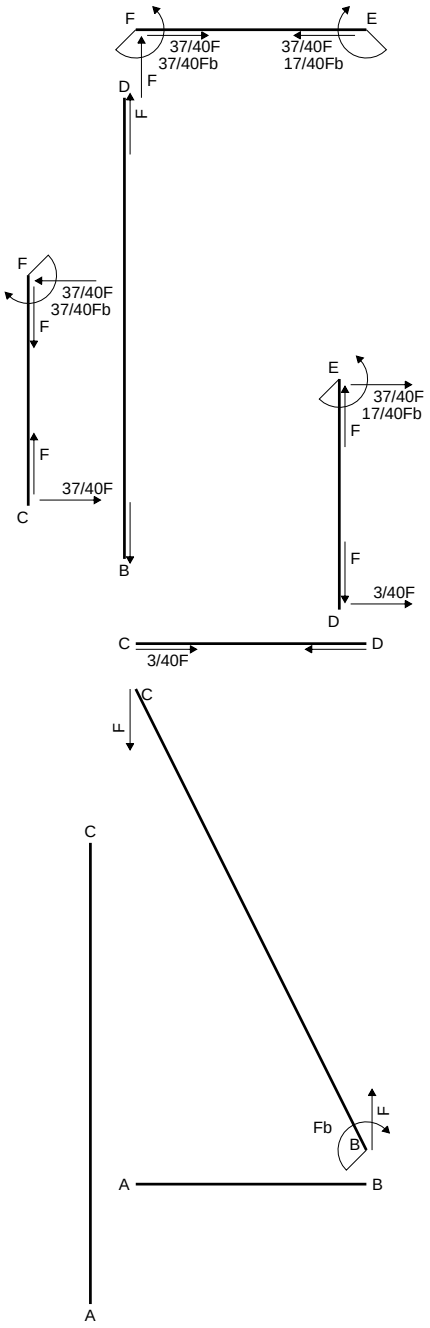
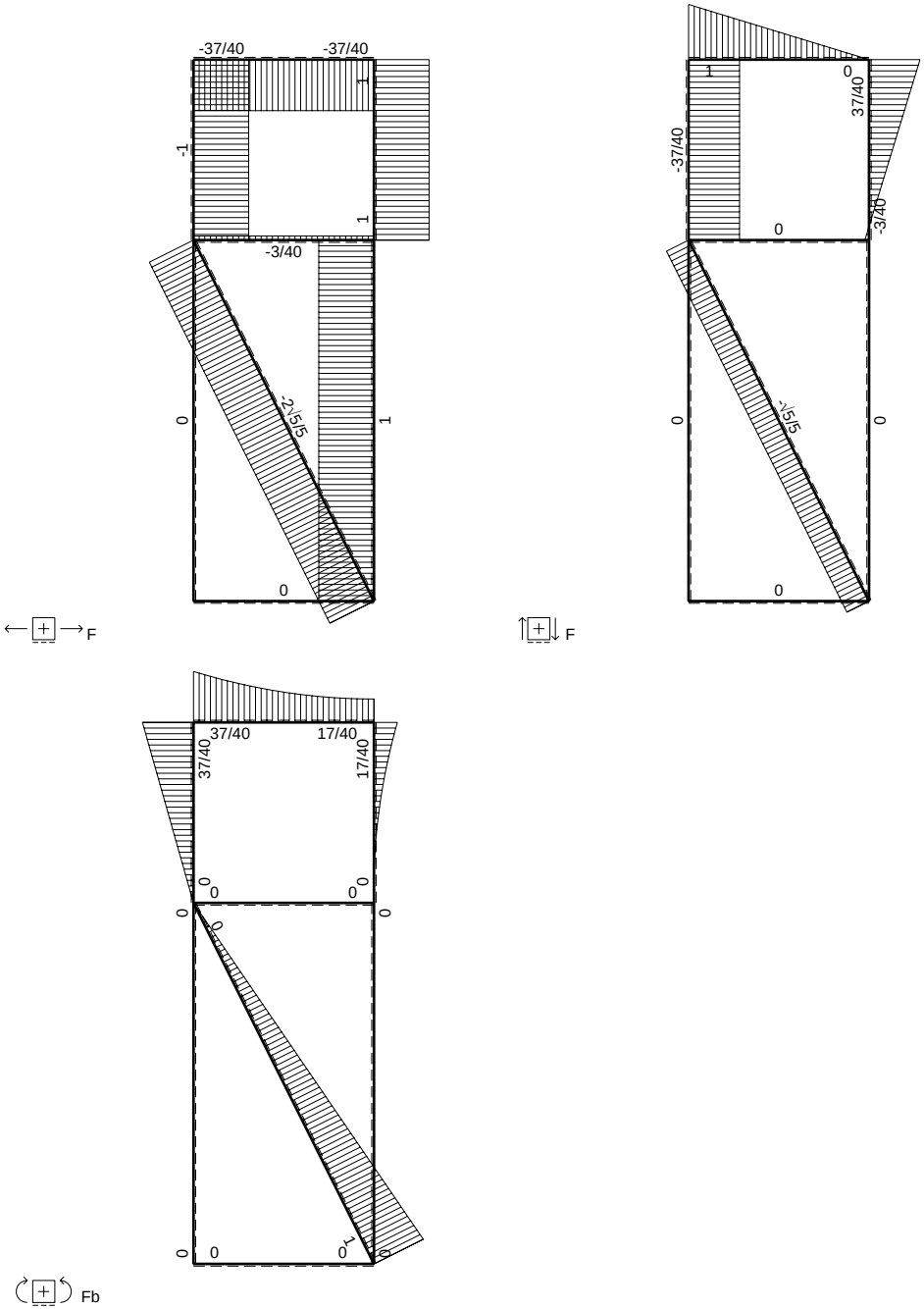

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D
C
F
C
Fb
B
F
A
B
A


-37/40
-37/40
1
-1
1
-3/40
-√5/5
0
1
0
F
1
0
37/40
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-3/40
0
-√5/5
0
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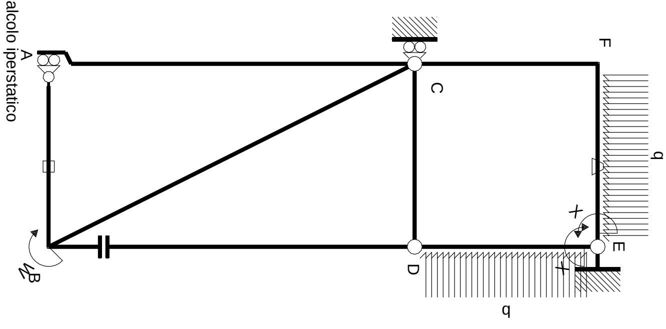
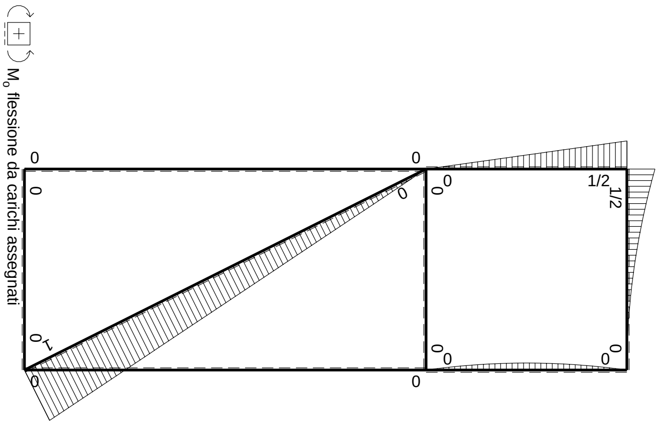
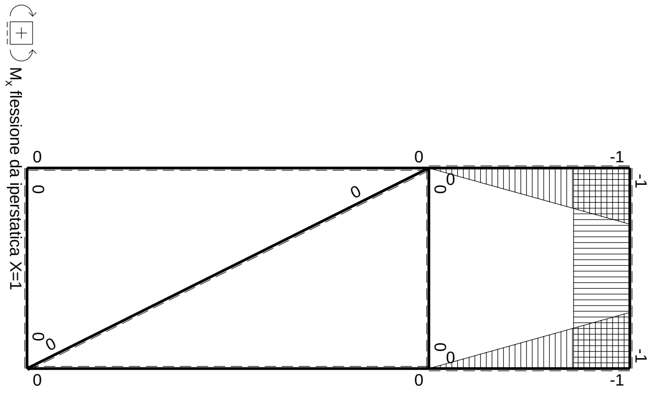
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$17/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-17/40Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

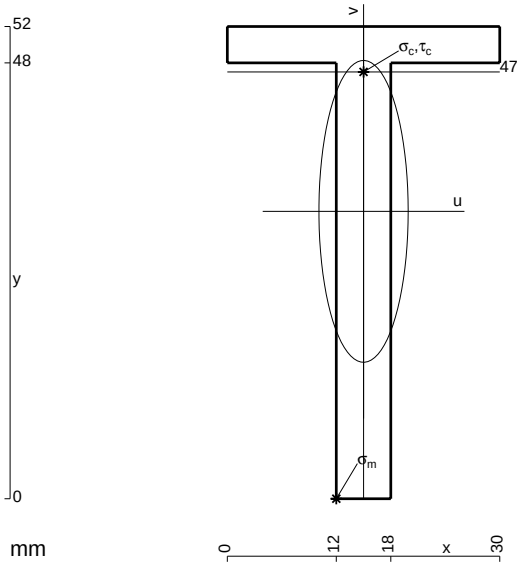
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 408. mm^2
J_u = 112717. mm^4
J_v = 9864. mm^4
y_g = 31.65 mm
N = -1547. N
T_y = -773.7 N
M_x = 830400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -31.65 mm
$\sigma_m = N/A - Mv/J_u$ = 229.4 N/mm^2
x_c = 15. mm
y_c = 47. mm
v_c = 15.35 mm
$\sigma_c = N/A - Mv/J_u$ = -116.9 N/mm^2
τ_c = 2.628 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117. N/mm^2
S^* = 2297. mm^3

ESΣ09.xxxx.053

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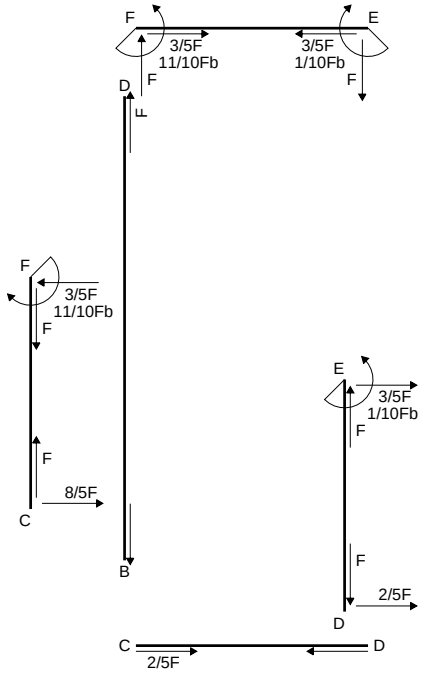
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
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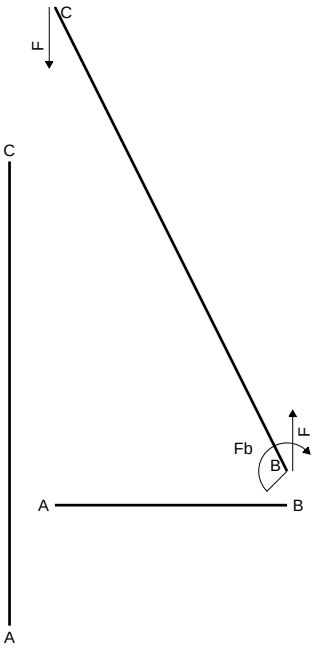
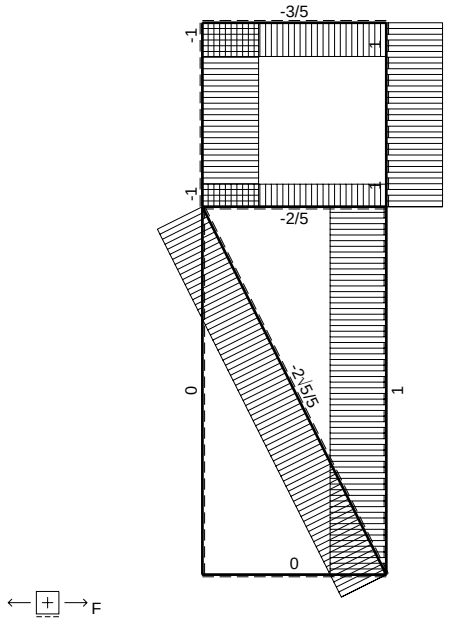
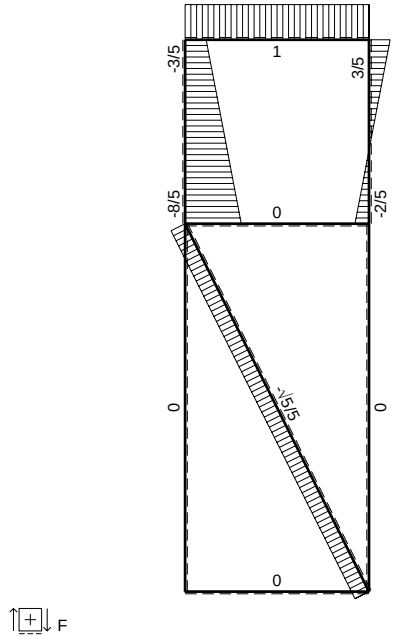
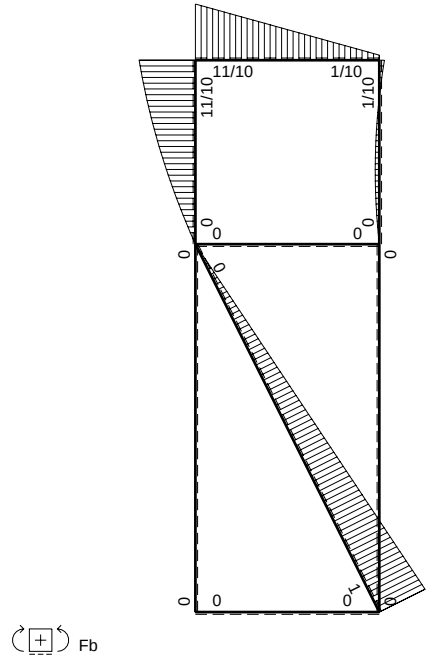
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						1/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982868 ESΣ09.xxxx.054

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

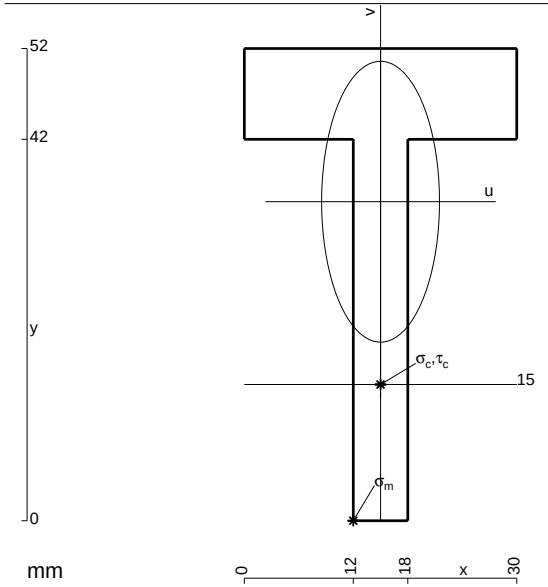
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 552. mm^2
J_u = 132127. mm^4
J_v = 23256. mm^4
y_g = 35.13 mm
N = -1601. N
T_y = -800.5 N
M_x = 912900. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.13 mm
$\sigma_m = N/A - Mv/J_u$ = 239.8 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.13 mm
$\sigma_c = N/A - Mv/J_u$ = 136.2 N/mm^2
τ_c = 2.511 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.3 N/mm^2
S^* = 2487. mm^3

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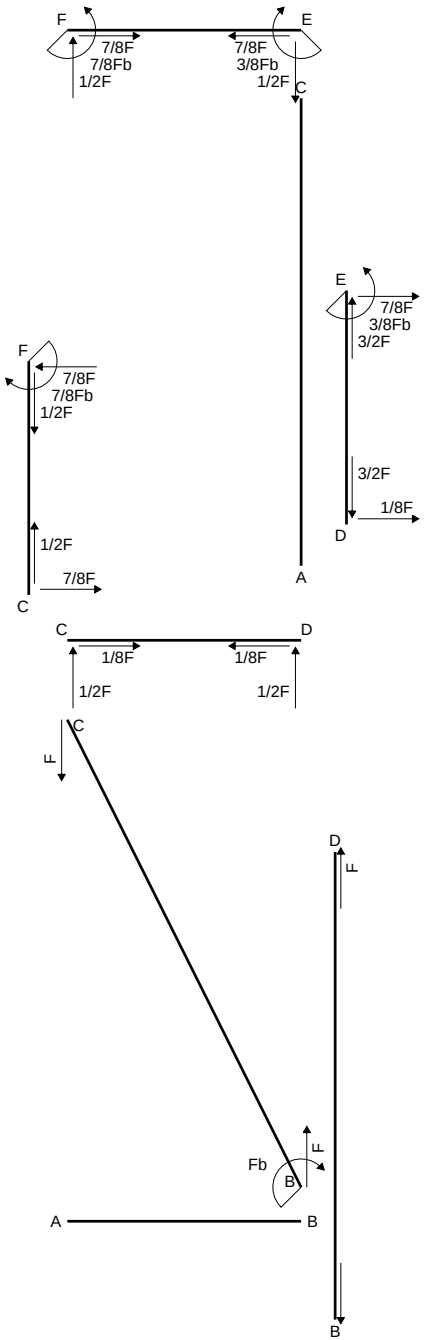
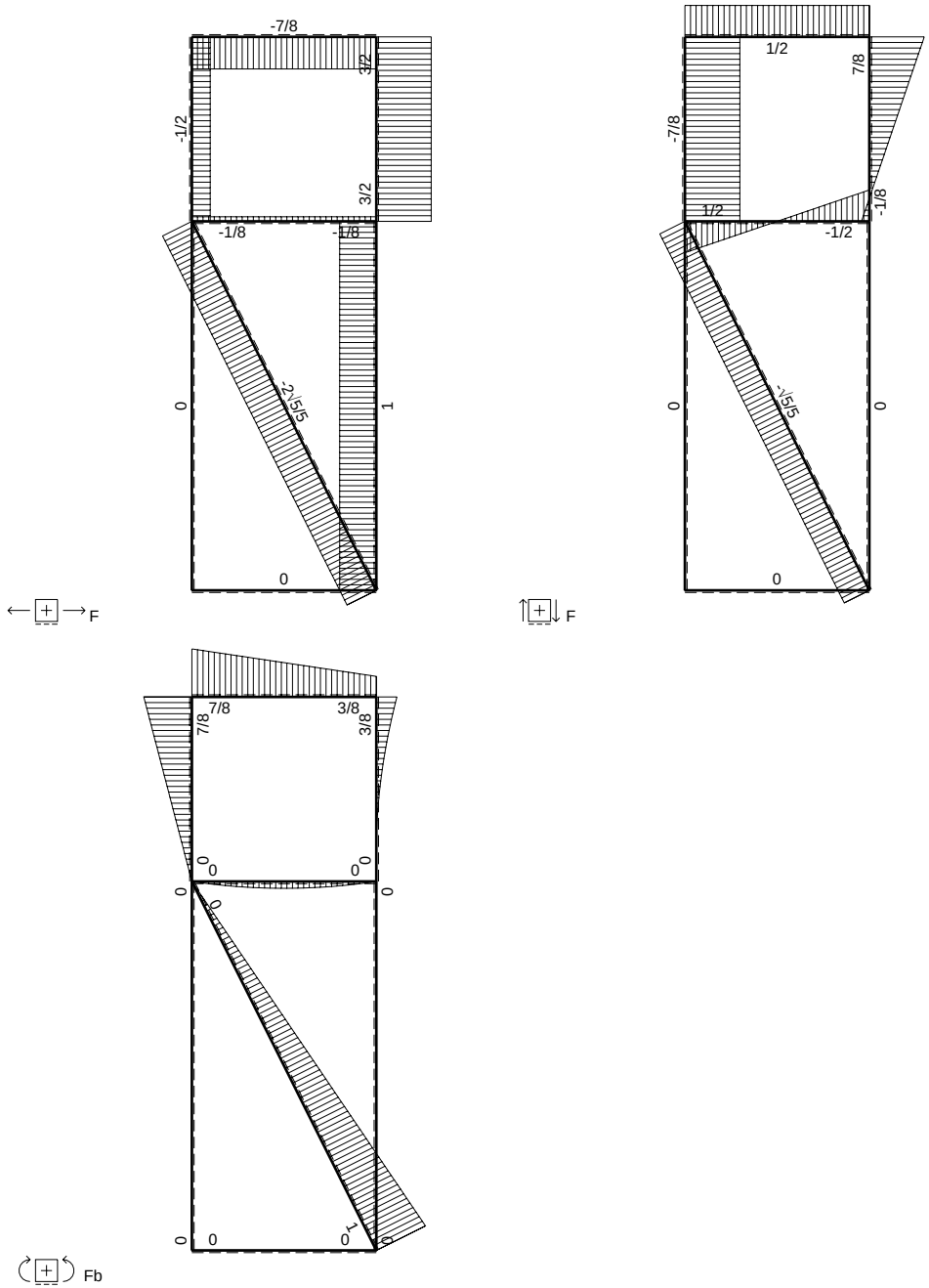
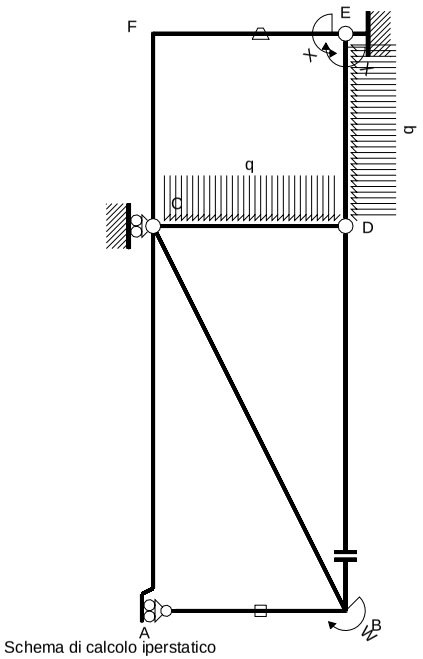
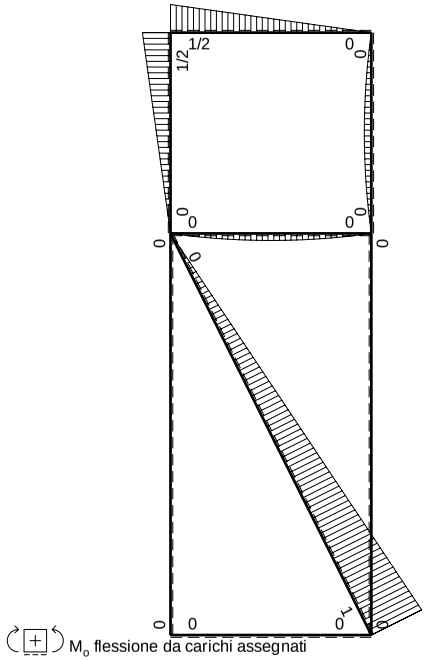

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B

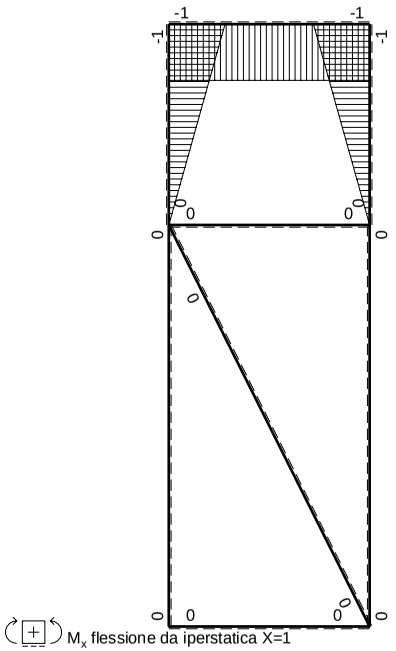

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
				totali			5/8Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

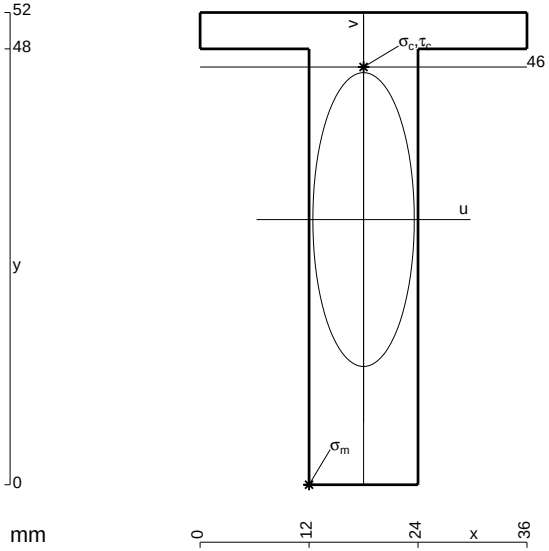
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 720. mm^2
J_u = 188659. mm^4
J_v = 22464. mm^4
y_g = 29.2 mm
N = -2129. N
T_y = -1064. N
M_x = 1309000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -29.2 mm
σ_m = N/A-Mv/J_u = 199.6 N/mm^2
x_c = 18. mm
y_c = 46. mm
v_c = 16.8 mm
σ_c = N/A-Mv/J_u = -119.5 N/mm^2
τ_c = 1.609 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.6 N/mm^2
S^* = 3422. mm^3

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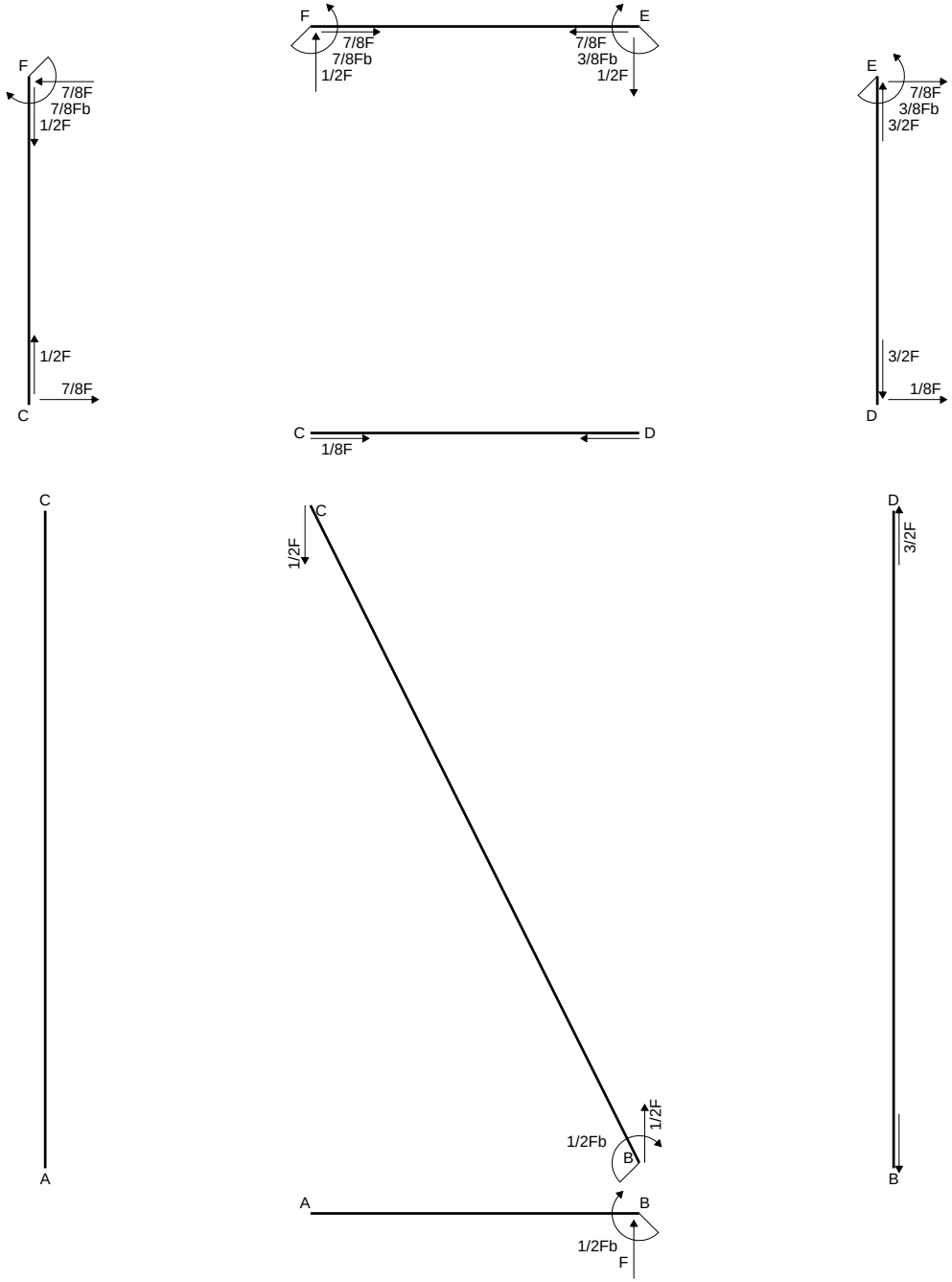
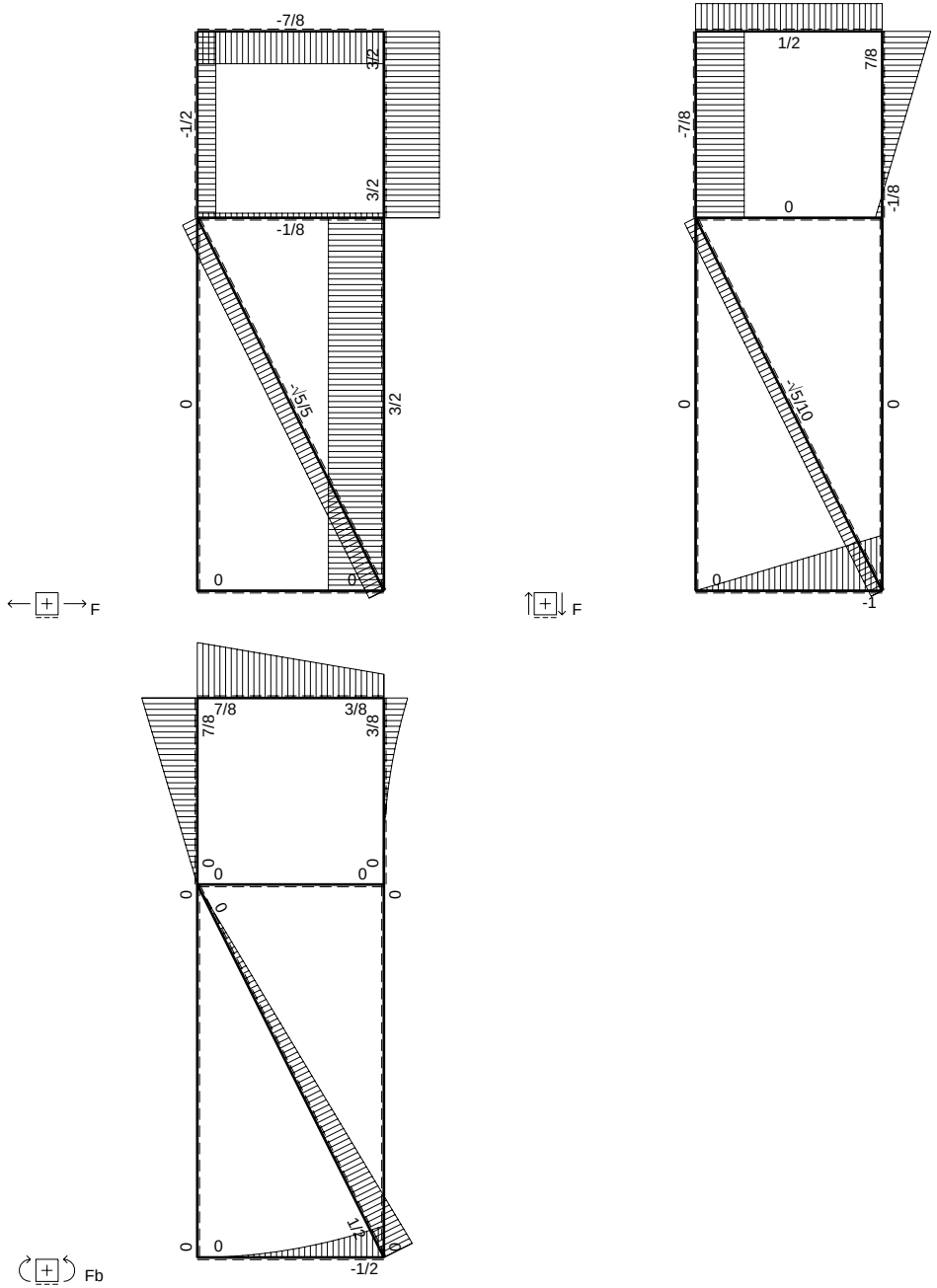

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
1/2
7/8
-1/8
-√5/10
-1
F
7/8
3/8
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212776 ESΣ09.xxxx.056

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

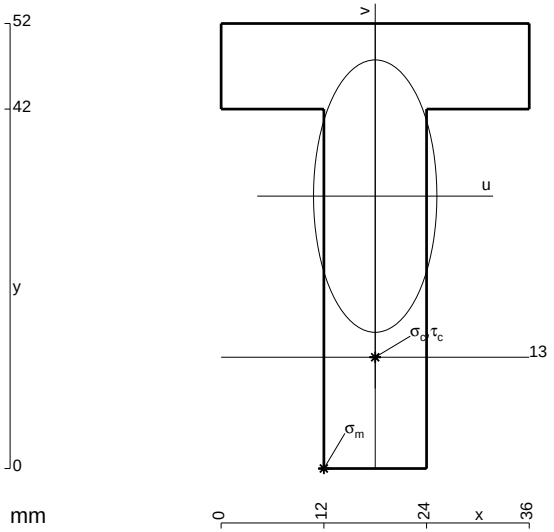
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 864. mm^2
J_u = 219048. mm^4
J_v = 44928. mm^4
y_g = 31.83 mm
T_y = -4890. N
M_x = -1442550. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.83 mm
$\sigma_m = -Mv/J_u$ = -209.6 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -18.83 mm
$\sigma_c = -Mv/J_u$ = -124. N/mm^2
τ_c = 7.352 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 124.7 N/mm^2
S^* = 3952. mm^3

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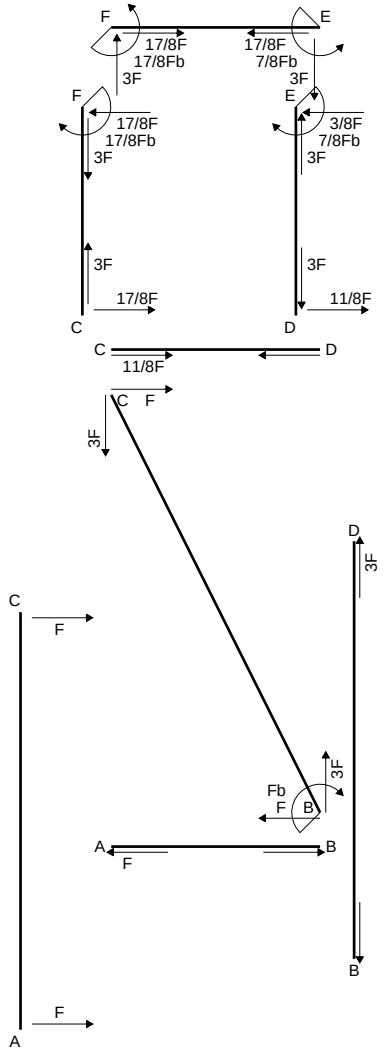
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E
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17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
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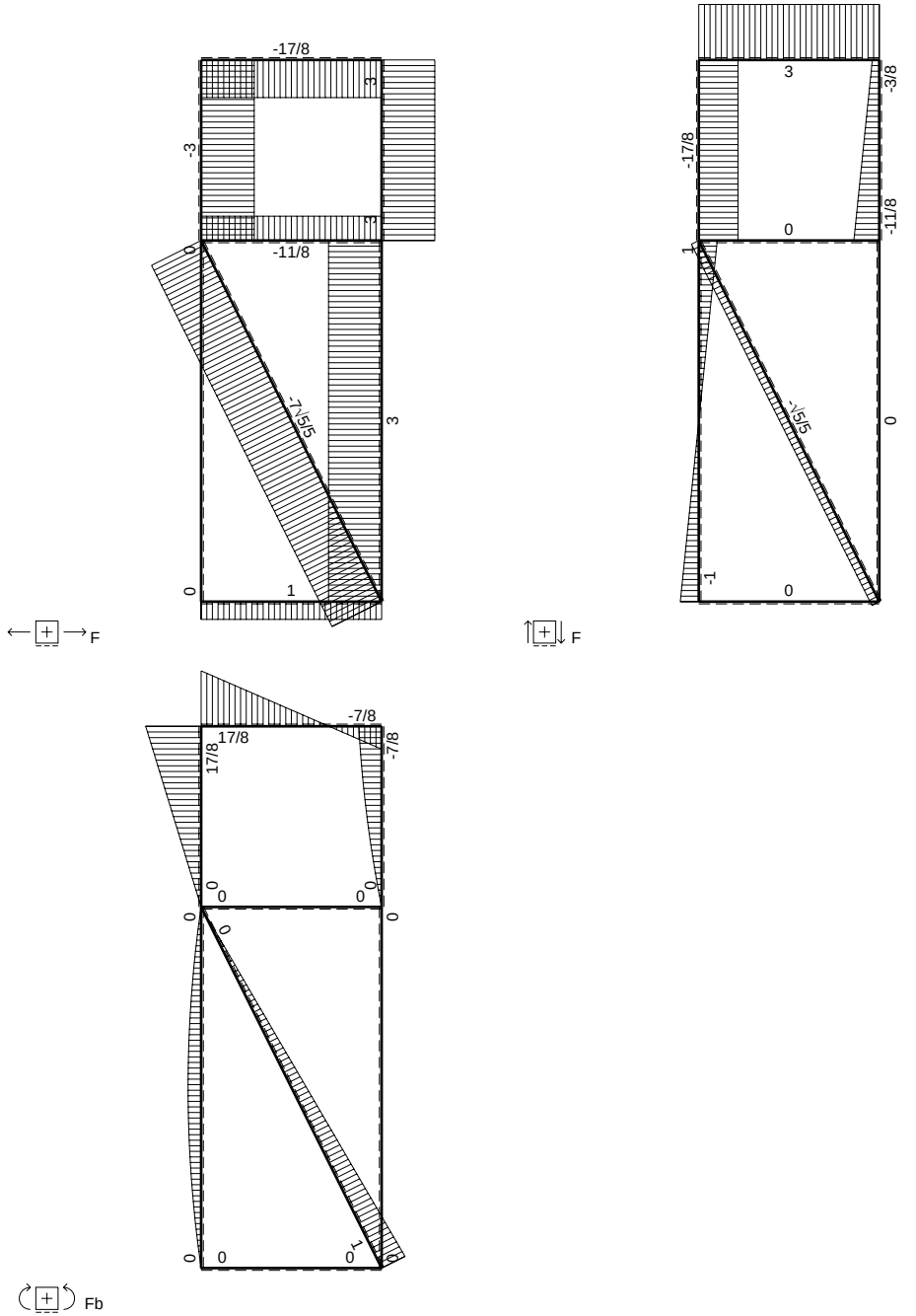
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
A
B
F


-17/8
-3
3
-11/8
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-7√5/5
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0
1
F
-17/8
3
-3/8
-11/8
0
-7√5/5
0
-1
0
F
-7/8
17/8
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-7/8
0
0
0
0
0
0
0
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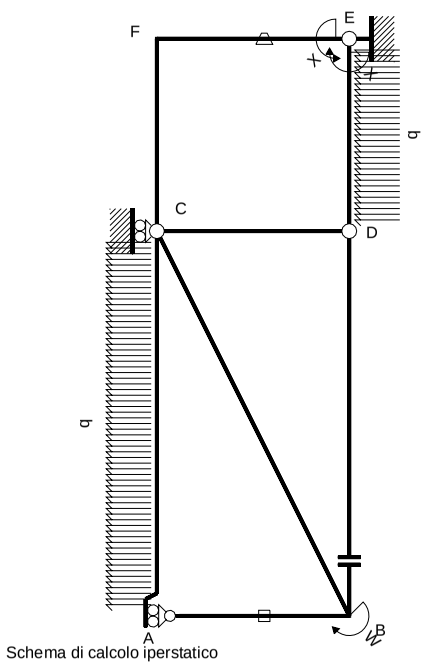
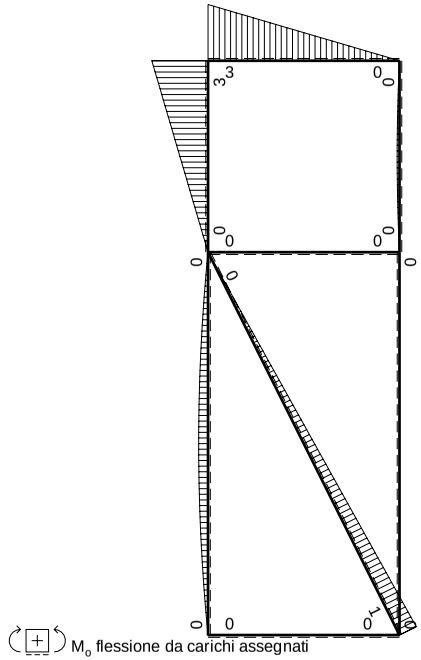
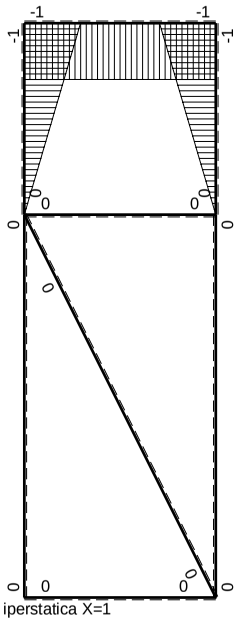
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
		totali					$-35/24Fb^2/EJ$	5/3Xb/EJ
		iperstatica $X=W_{EF}$					7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212737 ESΣ09.xxxx.057

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

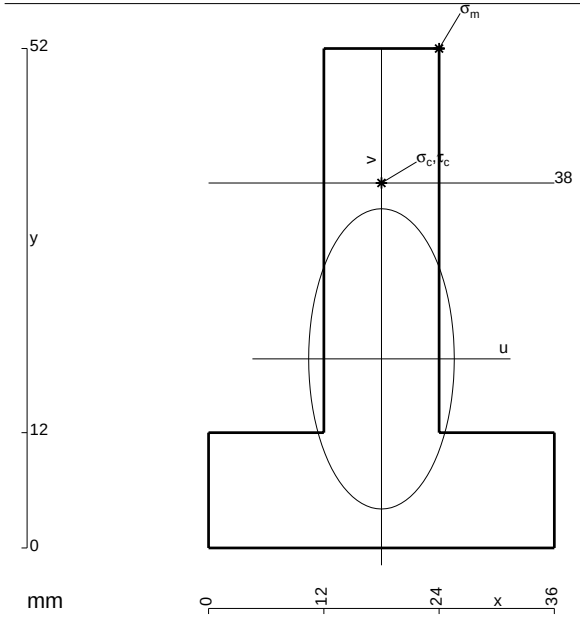
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 912. mm^2
J_u = 222885. mm^4
J_v = 52416. mm^4
y_g = 19.68 mm
N = -7357. N
T_y = -1051. N
M_x = 1457000. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 32.32 mm
$\sigma_m = N/A - Mv/J_u$ = -219.3 N/mm^2
x_c = 18. mm
y_c = 38. mm
v_c = 18.32 mm
$\sigma_c = N/A - Mv/J_u$ = -127.8 N/mm^2
τ_c = 1.671 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 127.8 N/mm^2
S^* = 4253. mm^3

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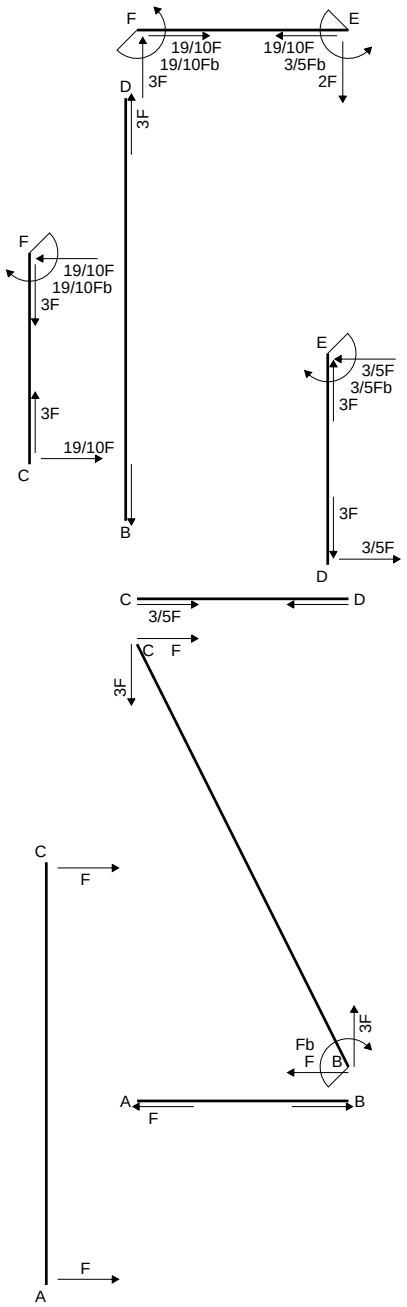
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E
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19/10Fb
19/10F
3/5Fb
D
3F
2F
3F
F
19/10F
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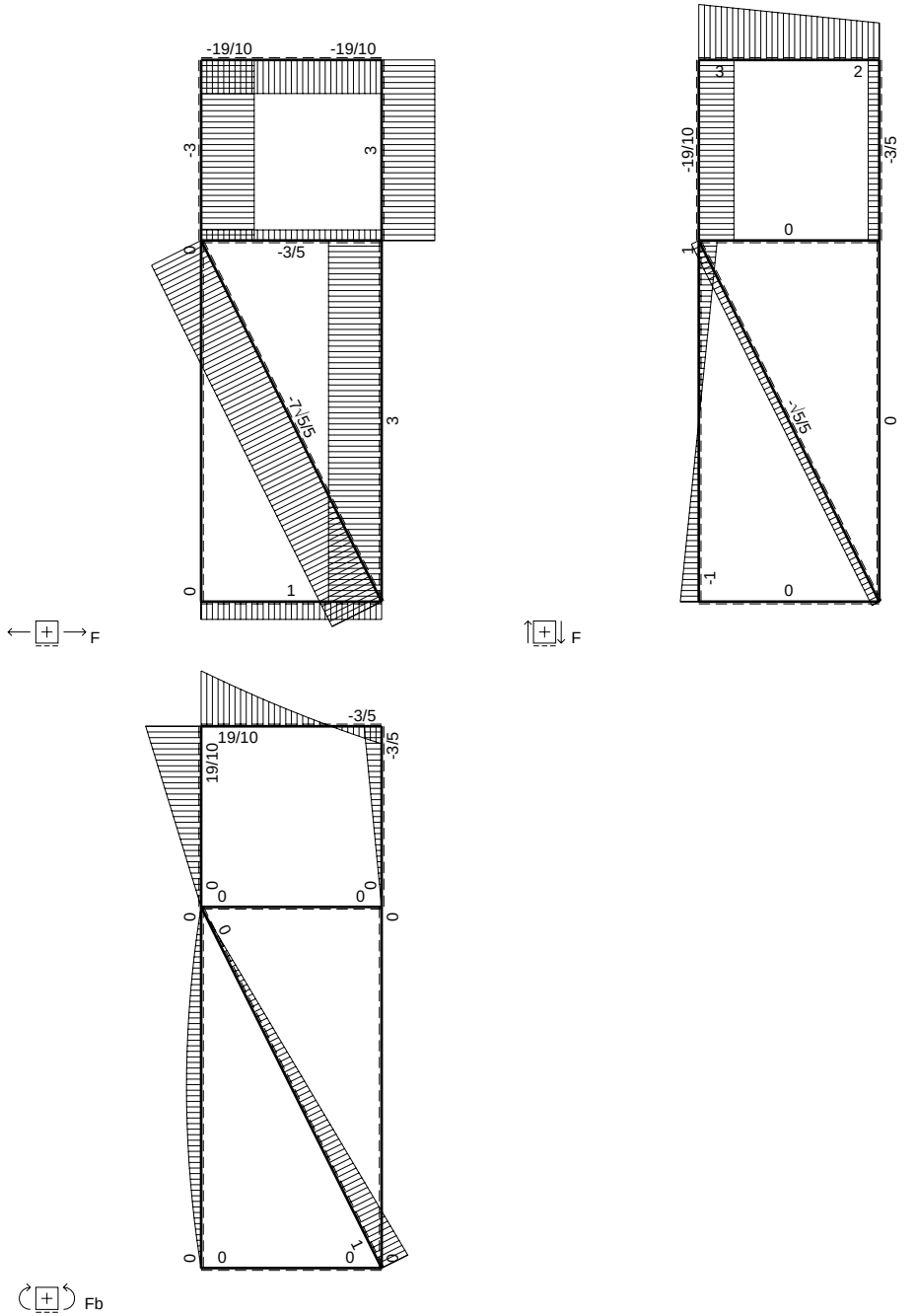
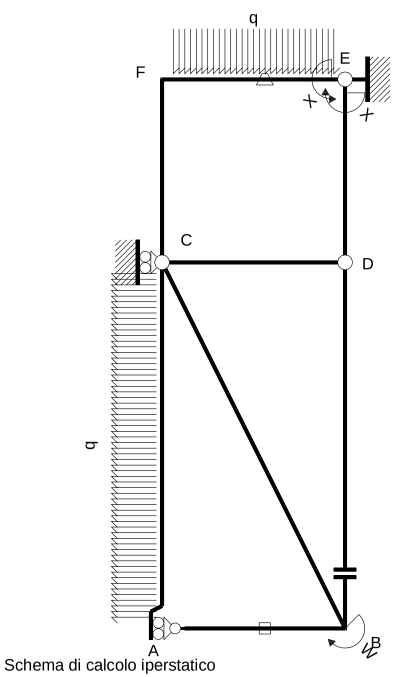
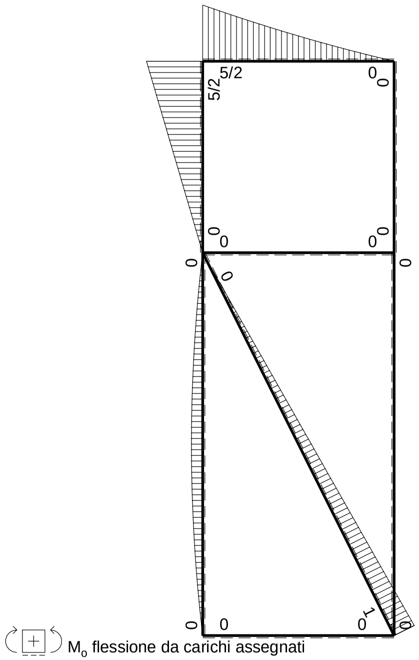
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A

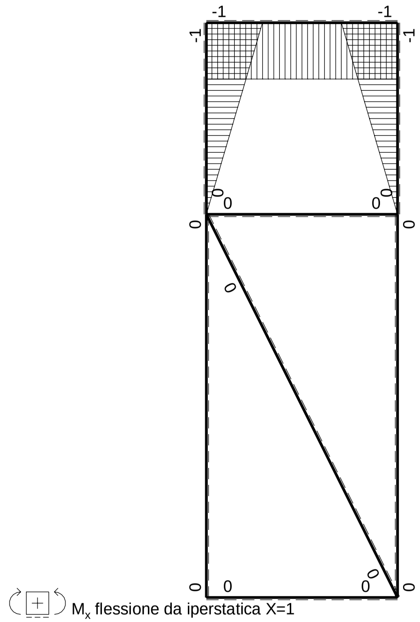

-19/10
-19/10
-3
3
-3/5
0
3
-7\5/5
3
0
1
F
-19/10
3
2
-3/5
0
1
-1\5/5
0
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

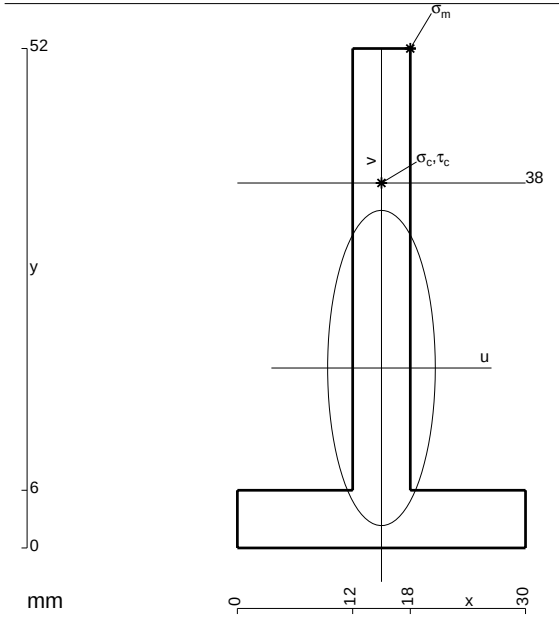
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 456. mm^2
J_u = 122856. mm^4
J_v = 14328. mm^4
y_g = 18.74 mm
N = -3851. N
T_y = -550.1 N
M_x = 811800. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 33.26 mm
$\sigma_m = N/A - Mv/J_u$ = -228.2 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 19.26 mm
$\sigma_c = N/A - Mv/J_u$ = -135.7 N/mm^2
τ_c = 1.646 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.8 N/mm^2
S^* = 2206. mm^3

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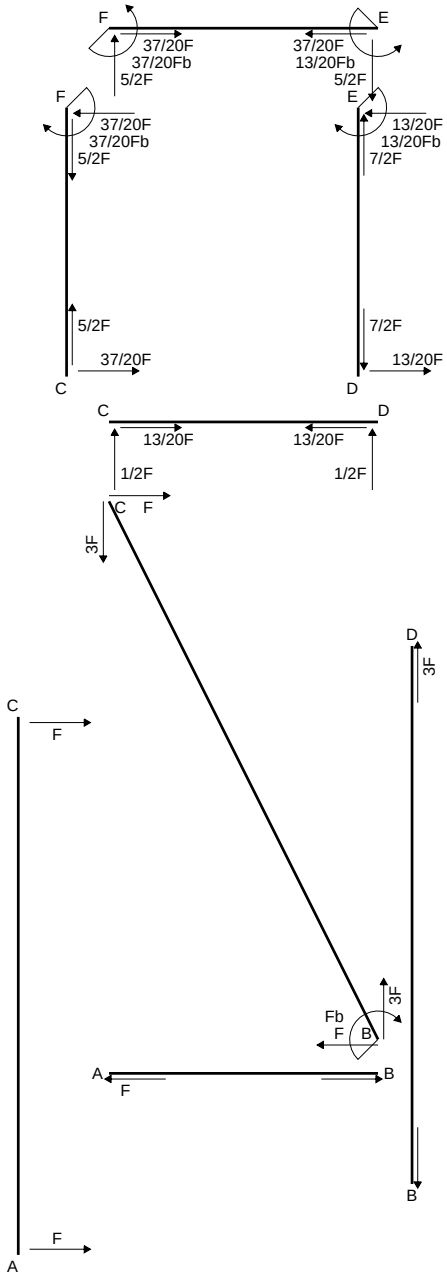
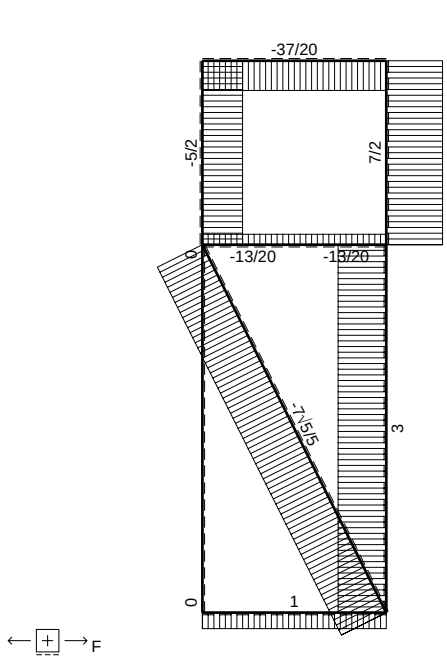
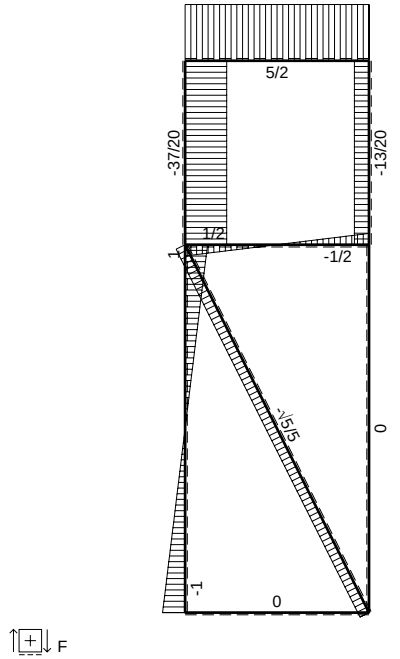
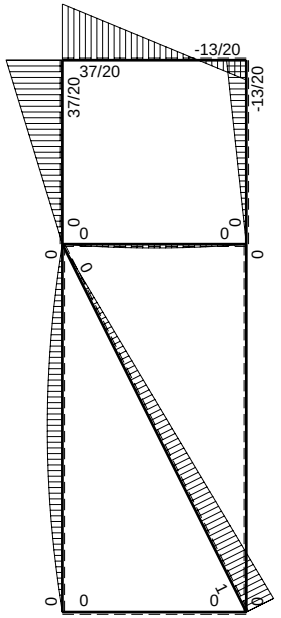
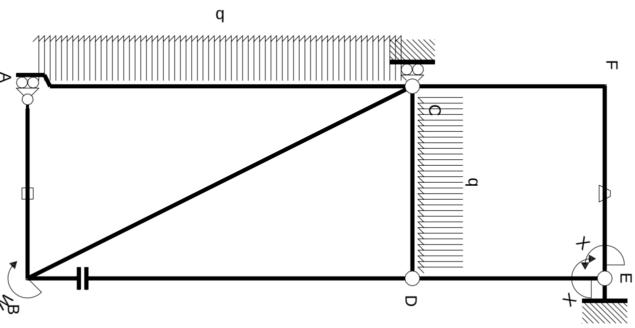

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B

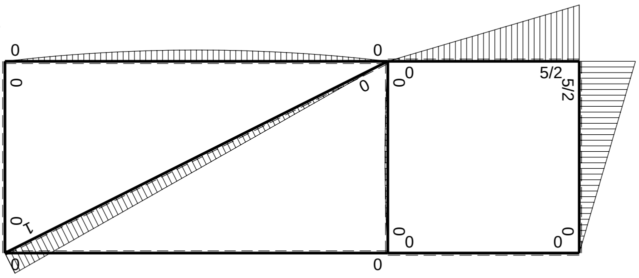

-37/20
-5/2
7/2
-13/20
-13/20
0
-7.5/5
3
0
1
F


5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F

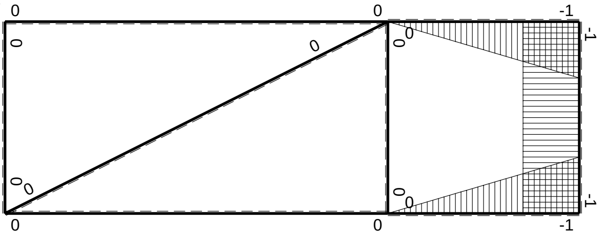

-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

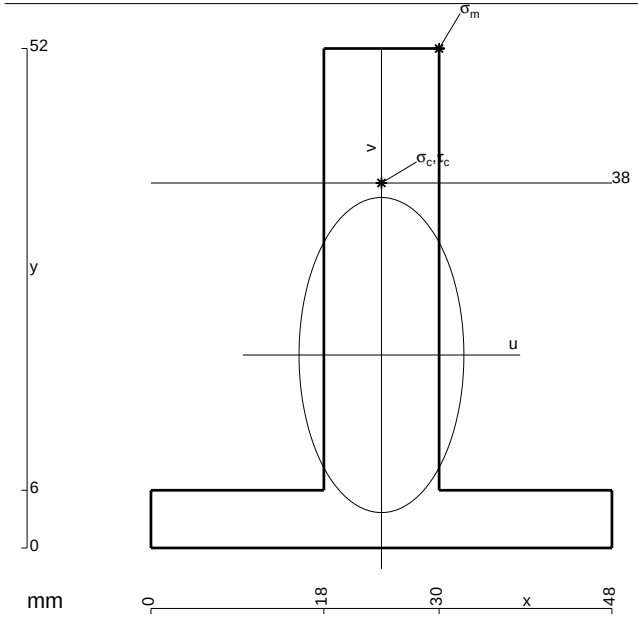
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 840. mm^2
J_u = 226138. mm^4
J_v = 61920. mm^4
y_g = 20.09 mm
N = -7325. N
T_y = -1046. N
M_x = 1638000. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 31.91 mm
$\sigma_m = N/A - Mv/J_u = -239.9\ N/mm^2$
x_c = 24. mm
y_c = 38. mm
v_c = 17.91 mm
$\sigma_c = N/A - Mv/J_u = -138.5\ N/mm^2$
$\tau_c = 1.614\ N/mm^2$
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 138.5\ N/mm^2$
$S^* = 4186.\ mm^3$

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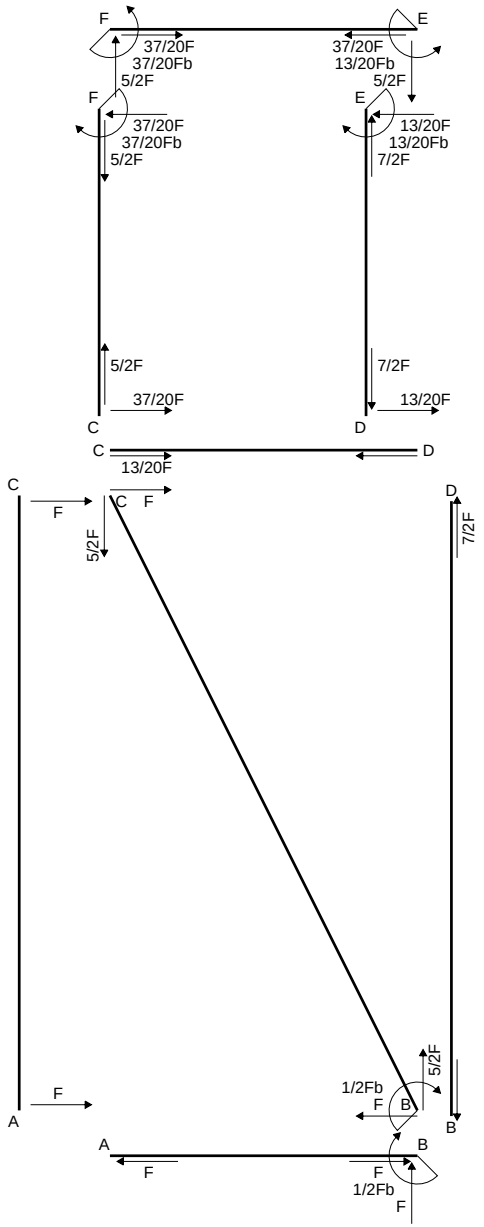
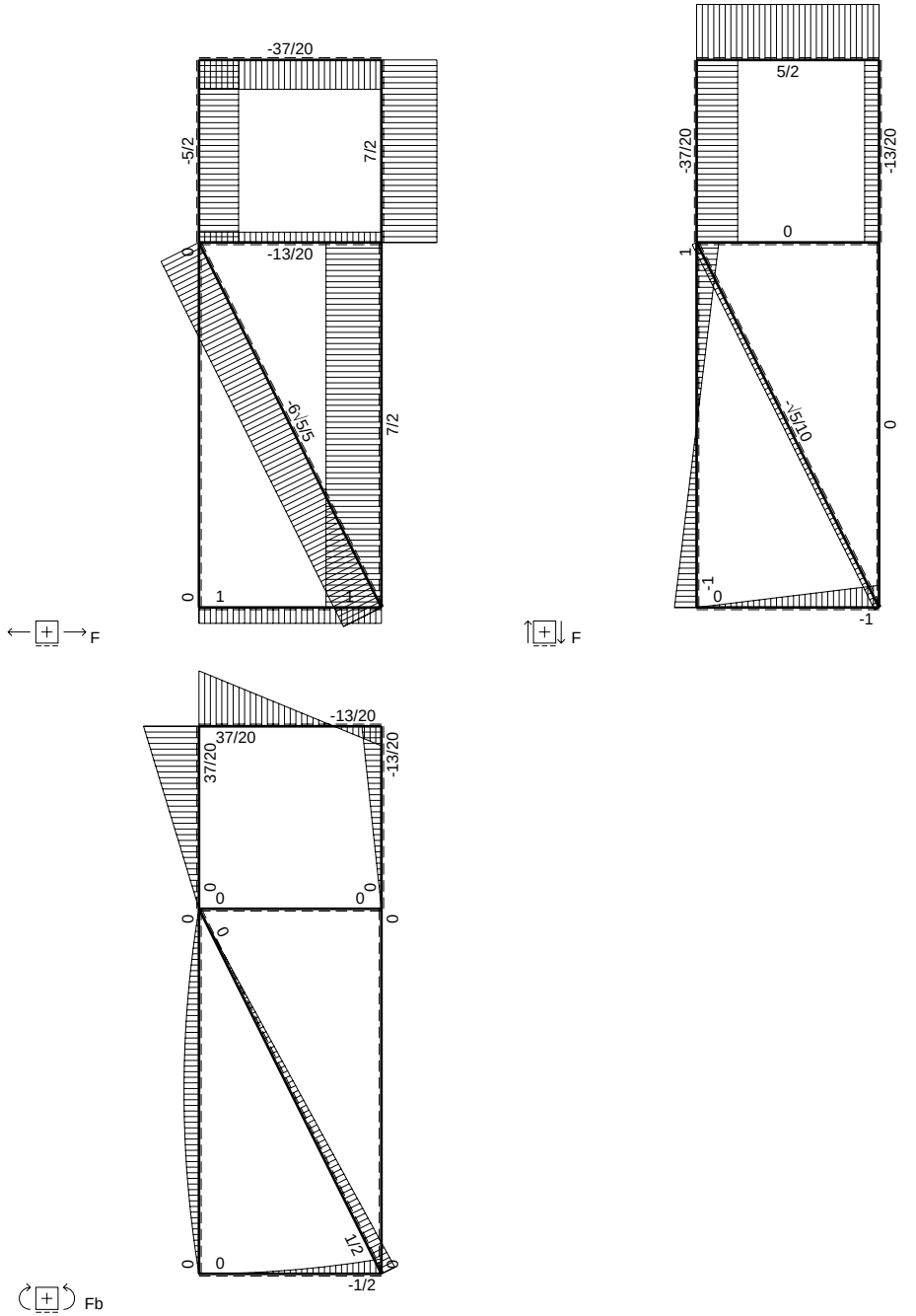
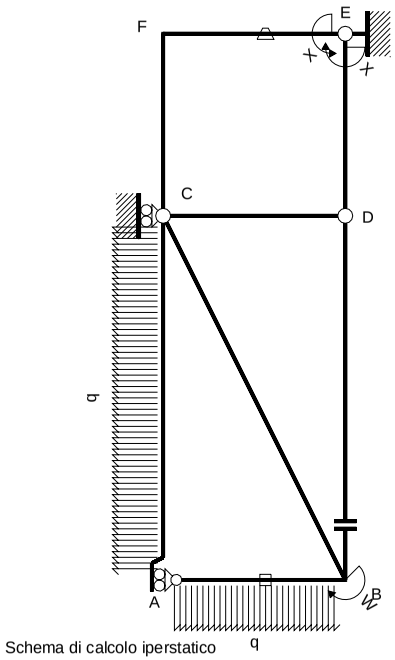
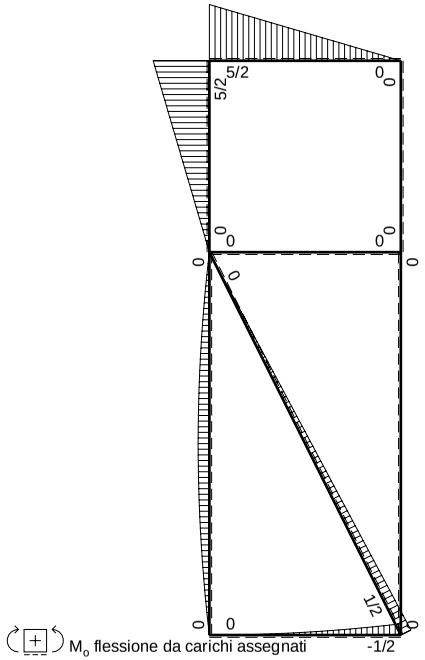

F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

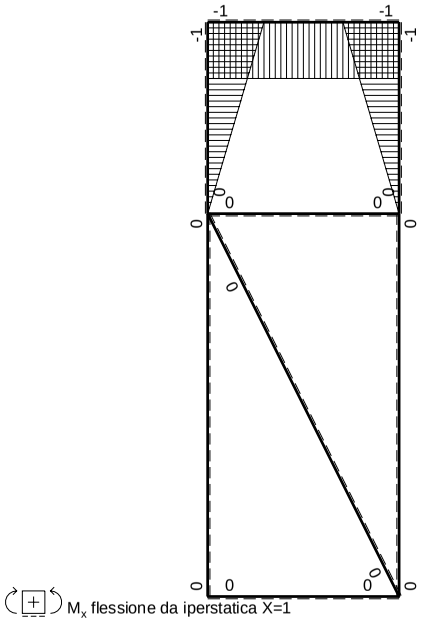

-37/20
-5/2
7/2
-13/20
-5\5/5
1
0
F
5/2
-37/20
-13/20
-\5/10
-1
37/20
Fb
-1/2
1/2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+5/2Fx$	Fb/EJ	$-5/2Fb+5/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$5/2Fb-5/2Fx$	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-5/2Fx$	0	$-5/2Fx^2/b$	0	x^2/b^2		
				totali			$-13/12Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210884 ESΣ09.xxxx.060

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

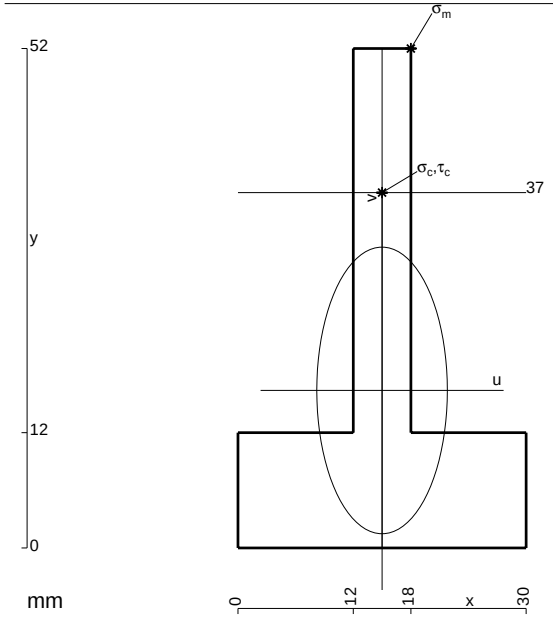
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 600. mm^2
J_u = 133664. mm^4
J_v = 27720. mm^4
y_g = 16.4 mm
N = 3920. N
T_y = -3920. N
M_x = -725200. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.6 mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 15. mm
y_c = 37. mm
v_c = 20.6 mm
$\sigma_c = N/A - Mv/J_u$ = 118.3 N/mm^2
τ_c = 12.36 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.2 N/mm^2
S^* = 2529. mm^3

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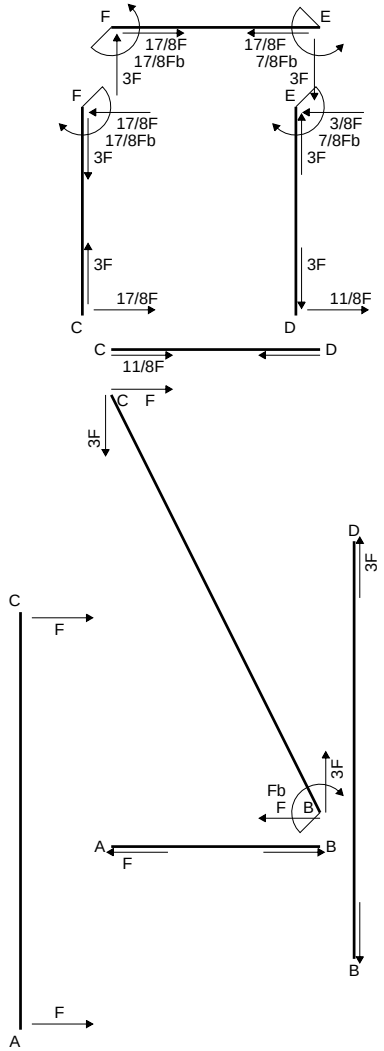
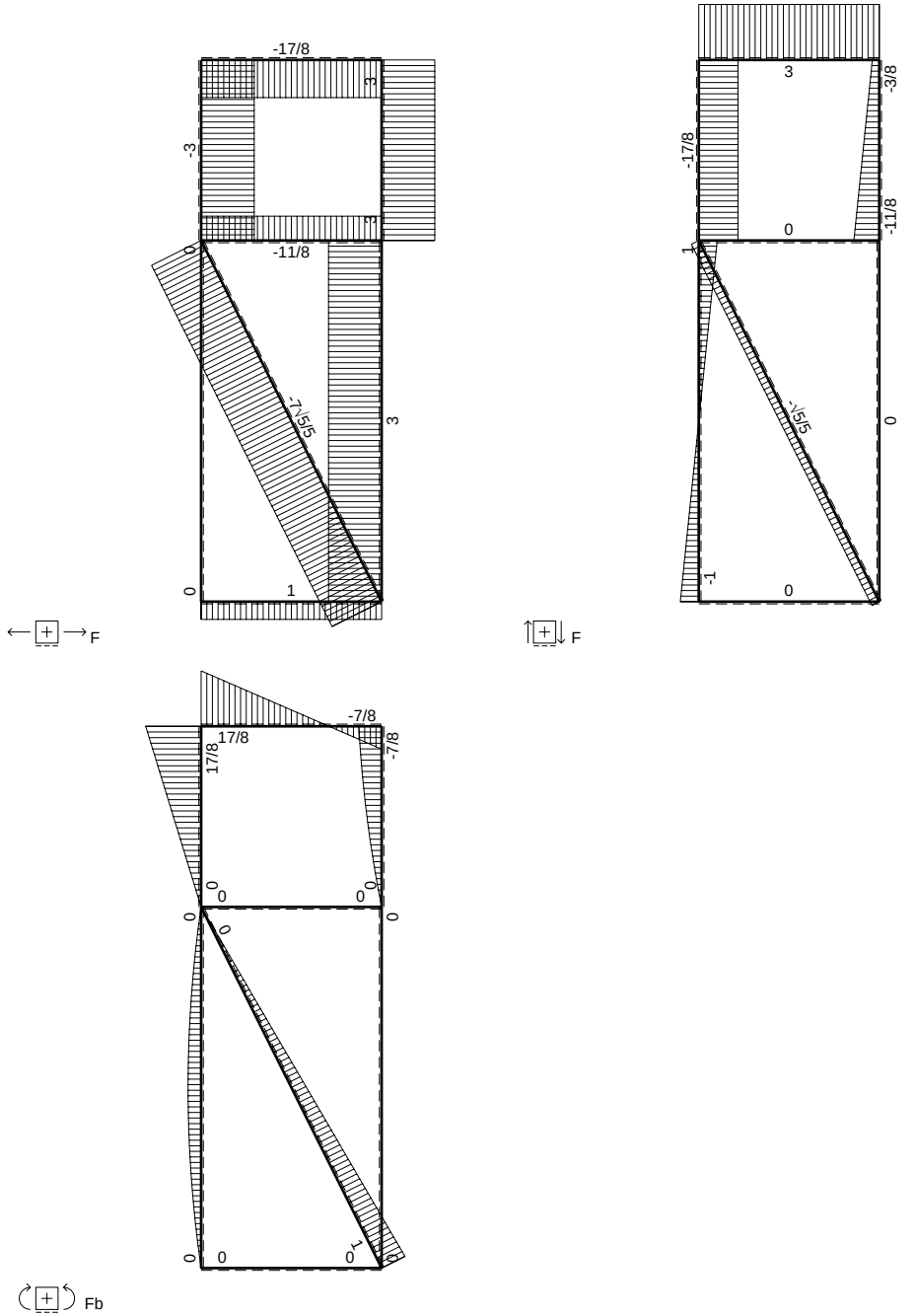

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A


-17/8
3
-3
-11/8
0
3
-7√5/5
1
F
3
-17/8
-3/8
-11/8
0
-√5/5
-1
F
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x^2/b^2	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x^2/b^2		
totali							-35/24Fb²/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

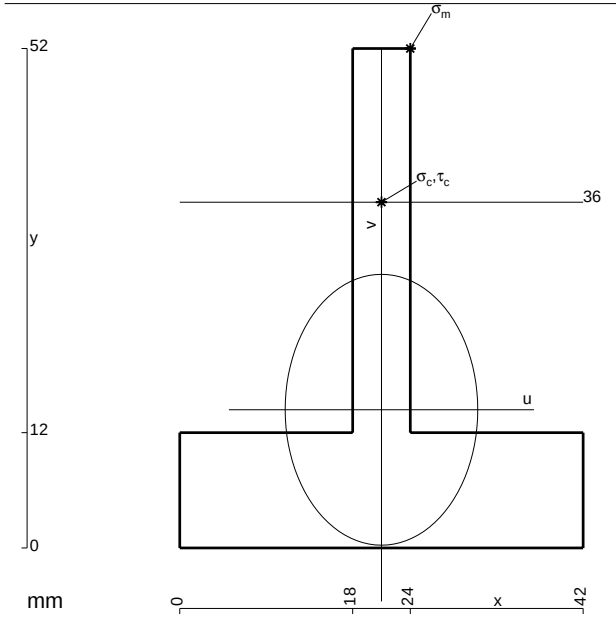
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 744. mm^2
J_u = 147953. mm^4
J_v = 74808. mm^4
y_g = 14.39 mm
N = -6198. N
T_y = -885.5 N
M_x = 792000. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 37.61 mm
$\sigma_m = N/A - Mv/J_u$ = -209.7 N/mm^2
x_c = 21. mm
y_c = 36. mm
v_c = 21.61 mm
$\sigma_c = N/A - Mv/J_u$ = -124. N/mm^2
τ_c = 2.836 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.1 N/mm^2
S^* = 2843. mm^3

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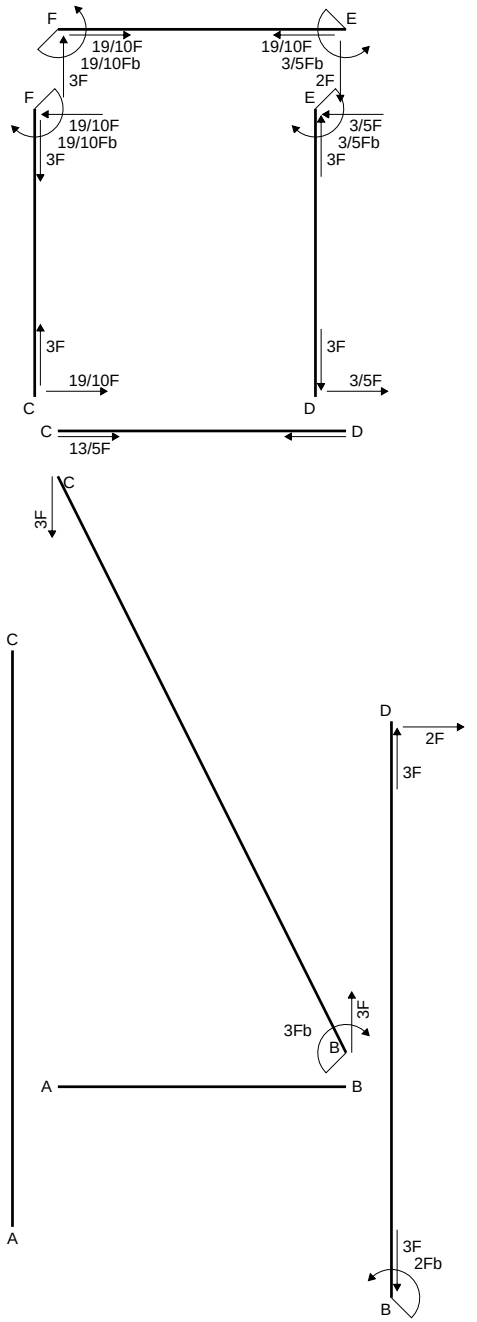
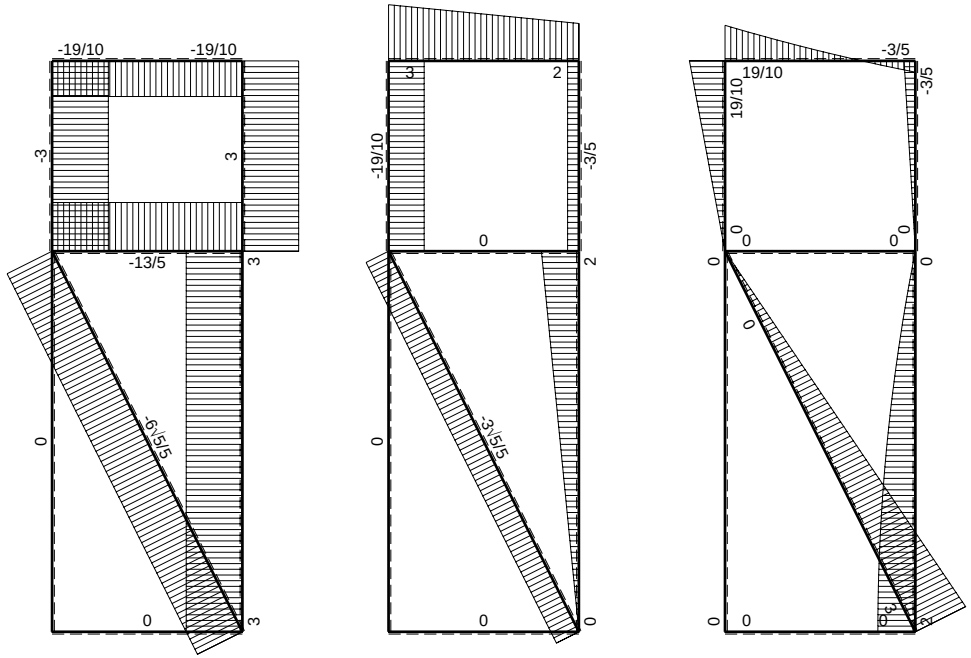

F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
19/10F
C
D
3F
3/5F
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-19/10
-19/10
-3
3
3
-13/5
0
-6√5/5
0
3
3
2
-19/10
-3/5
0
2
-3√5/5
0
0
19/10
-3/5
0
0
0
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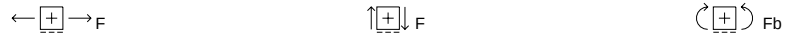
0
0
0
0


F
F
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx²	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx²	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx²	-Fb/EJ	-2Fx-1/2Fx²/b	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx²	Fb/EJ	-5/2Fb+3Fx-1/2Fx²/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

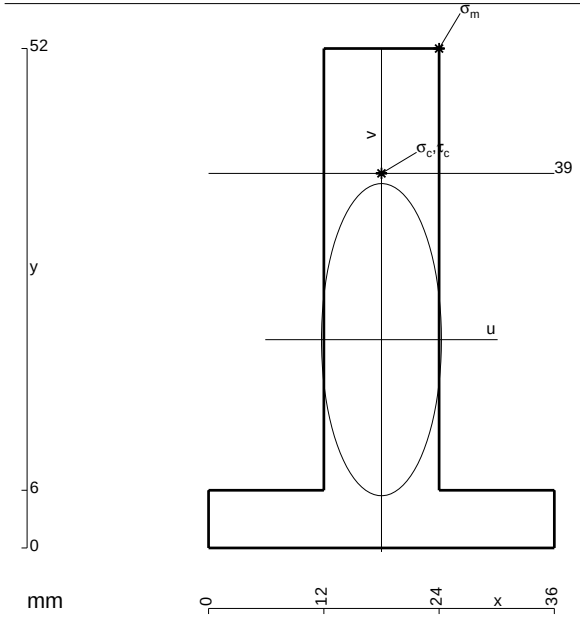
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 768. mm^2
J_u = 202933. mm^4
J_v = 29952. mm^4
y_g = 21.69 mm
N = -2925. N
T_y = -1462. N
M_x = 1438800. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 30.31 mm
$\sigma_m = N/A - Mv/J_u$ = -218.7 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 17.31 mm
$\sigma_c = N/A - Mv/J_u$ = -126.6 N/mm^2
τ_c = 2.231 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.6 N/mm^2
S^* = 3715. mm^3

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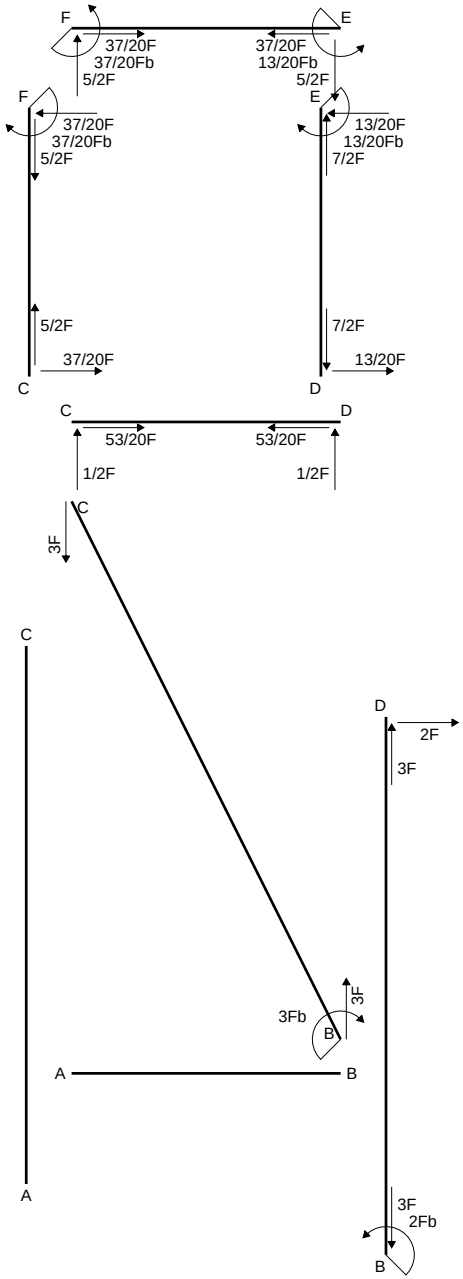
F
E
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37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
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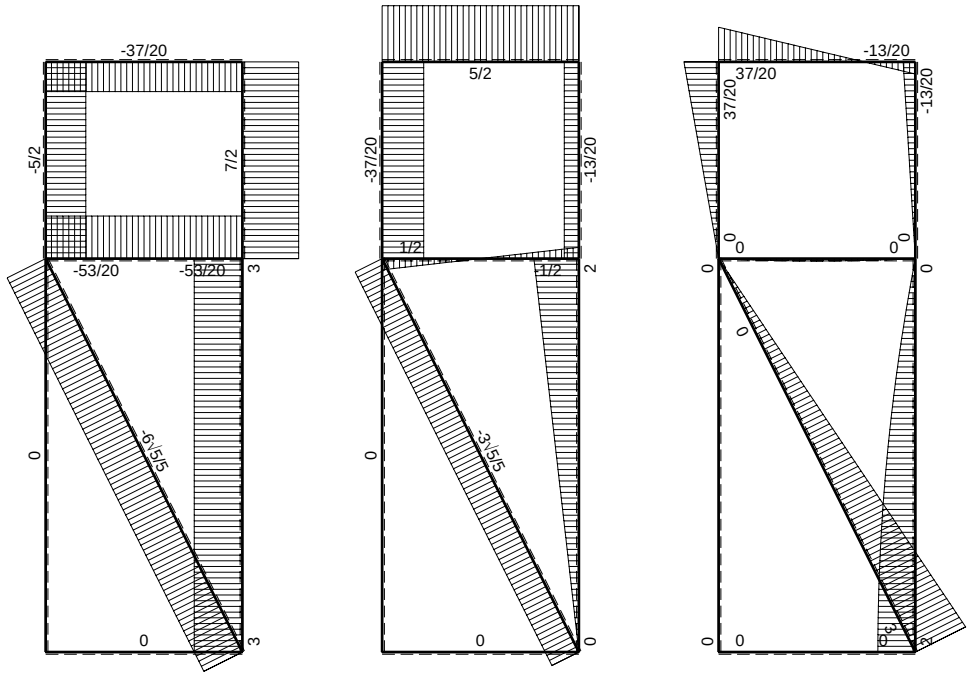
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
C
3F
C
D
2F
3F
3F
3Fb
B
A
B
3F
2Fb
A
B


-37/20
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7/2
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-53/20
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0
3
-6√5/5
5/2
-37/20
-13/20
1/2
1/2
2
0
0
-3√5/5
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
-3√5/5


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi\theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210286 ESΣ09.xxxx.063

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

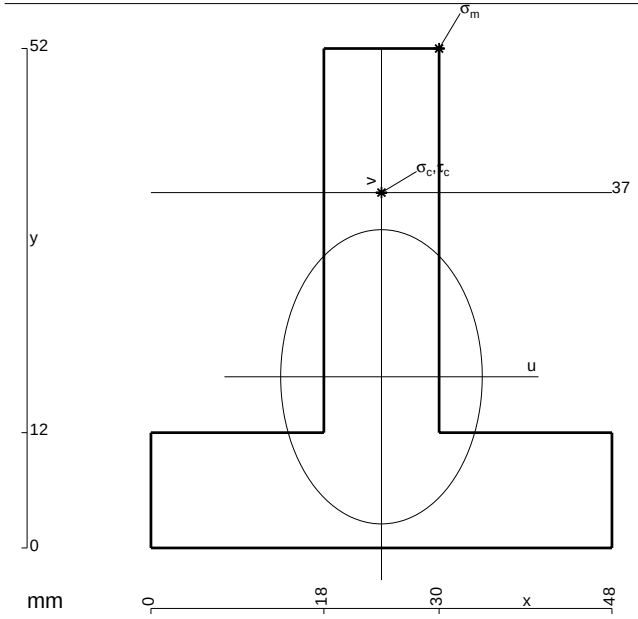
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1056. mm^2
J_u = 247901. mm^4
J_v = 116352. mm^4
y_g = 17.82 mm
N = -3059. N
T_y = -1529. N
M_x = 1641600. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 34.18 mm
$\sigma_m = N/A - Mv/J_u$ = -229.2 N/mm^2
x_c = 24. mm
y_c = 37. mm
v_c = 19.18 mm
$\sigma_c = N/A - Mv/J_u$ = -129.9 N/mm^2
τ_c = 2.469 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130. N/mm^2
S^* = 4803. mm^3

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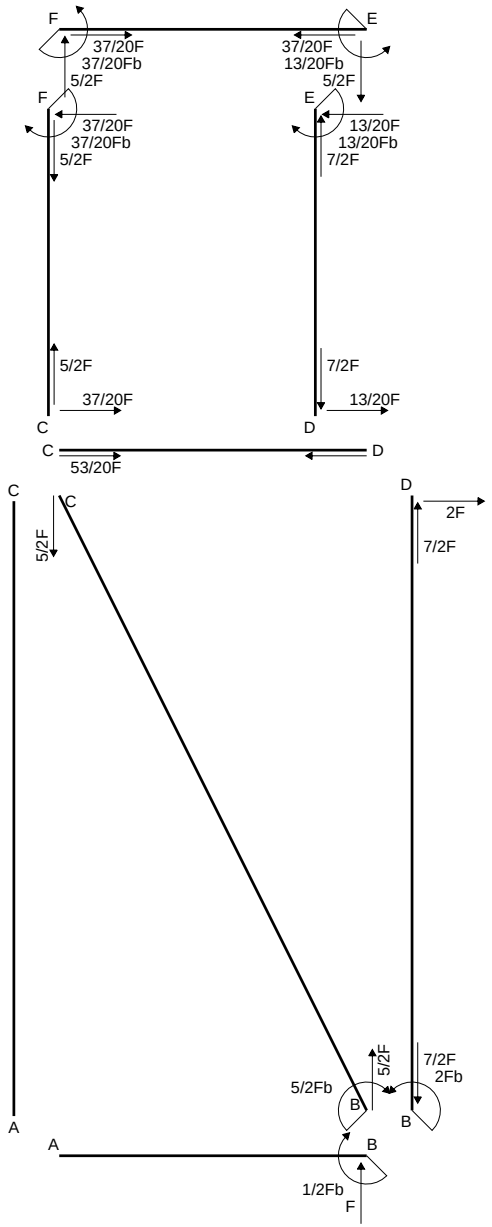
F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
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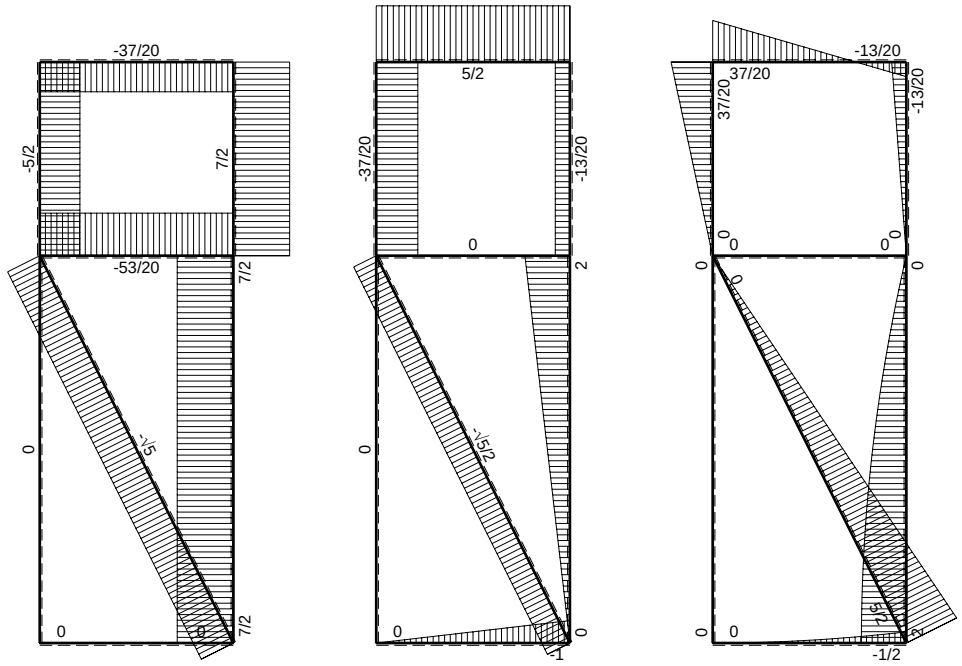
A
B
1/2Fb
F


-37/20
-5/2
7/2
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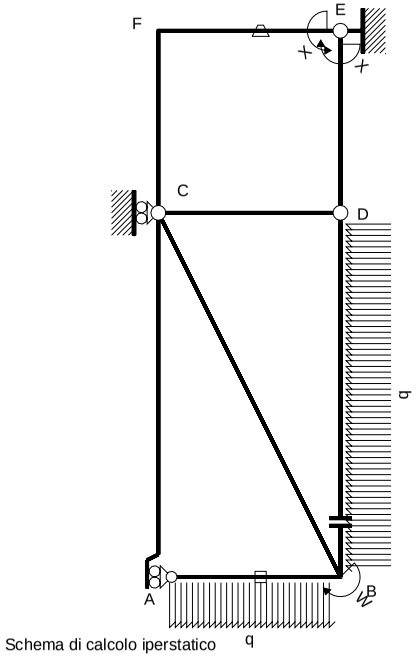
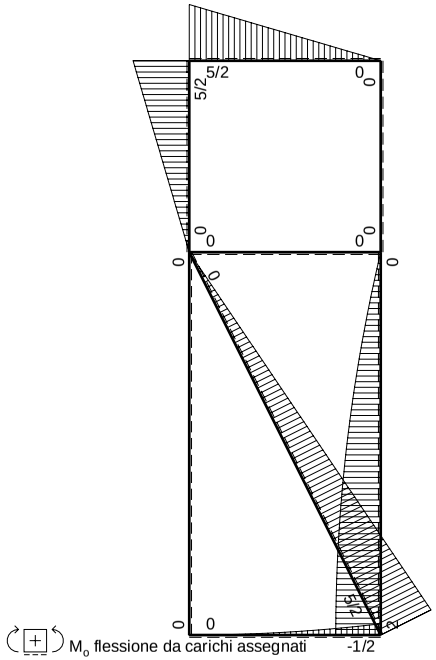
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2

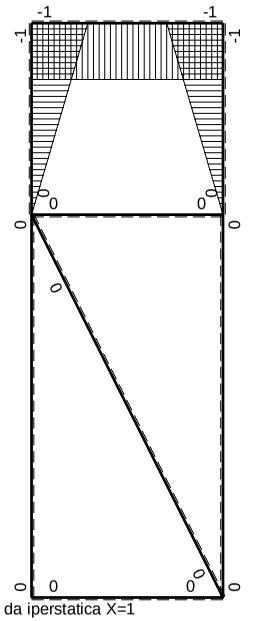

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

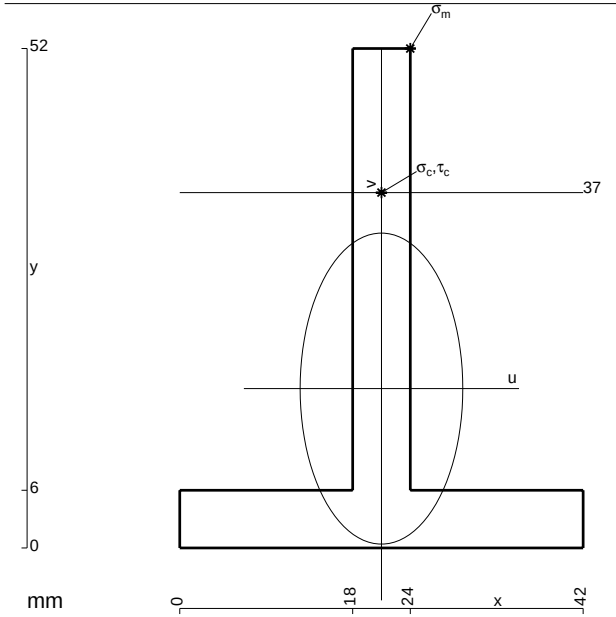
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 528. mm^2
J_u = 138472. mm^4
J_v = 37872. mm^4
y_g = 16.59 mm
N = -1610. N
T_y = -805. N
M_x = 918000. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.41 mm
$\sigma_m = N/A - Mv/J_u$ = -237.8 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 20.41 mm
$\sigma_c = N/A - Mv/J_u$ = -138.4 N/mm^2
τ_c = 2.434 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.4 N/mm^2
S^* = 2512. mm^3

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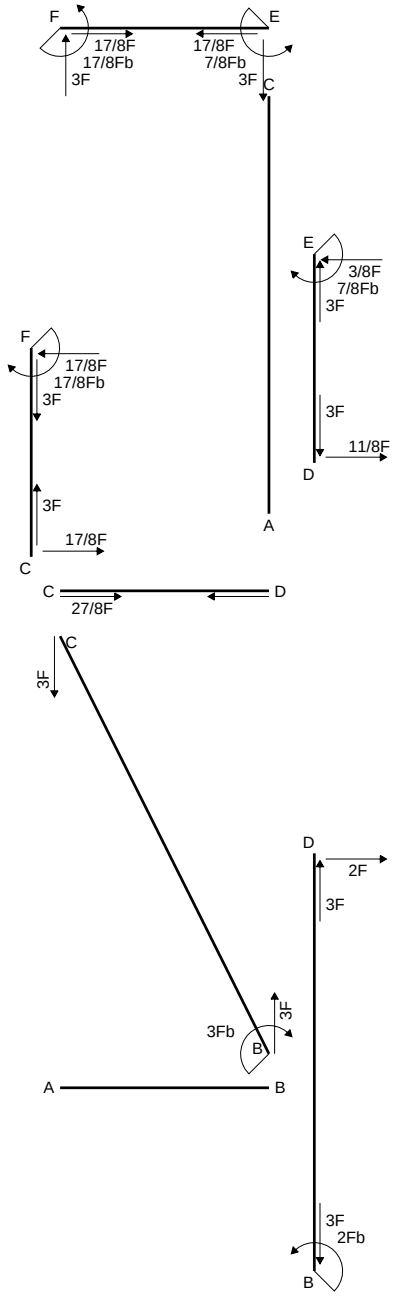
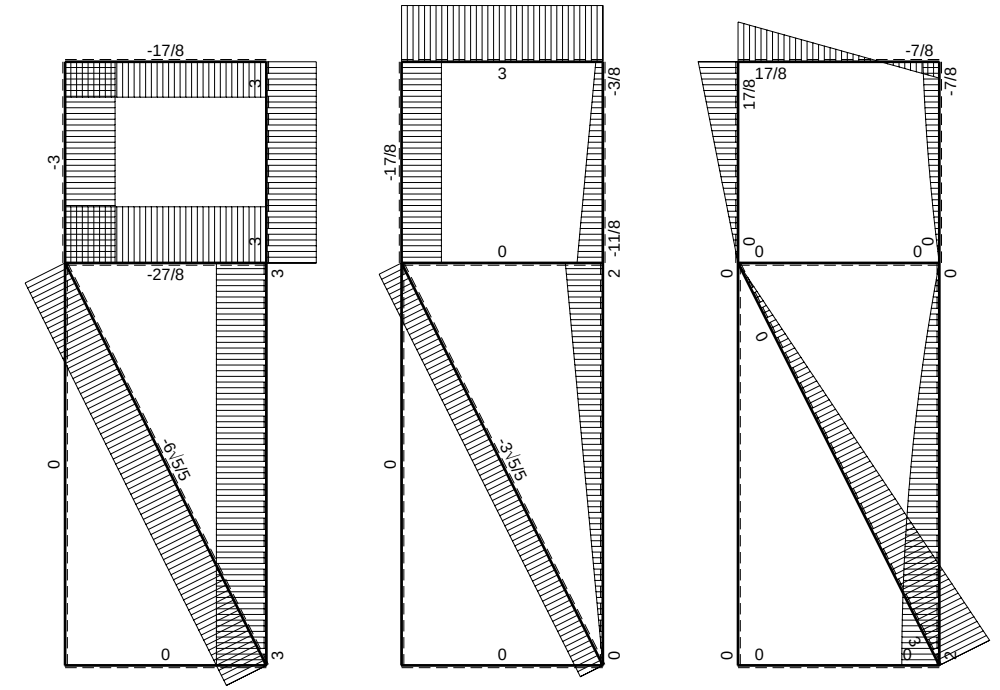

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6√5/5
3
0
0
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
3√5/5


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
totali							-35/24Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211392 ESΣ09.xxxx.065

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

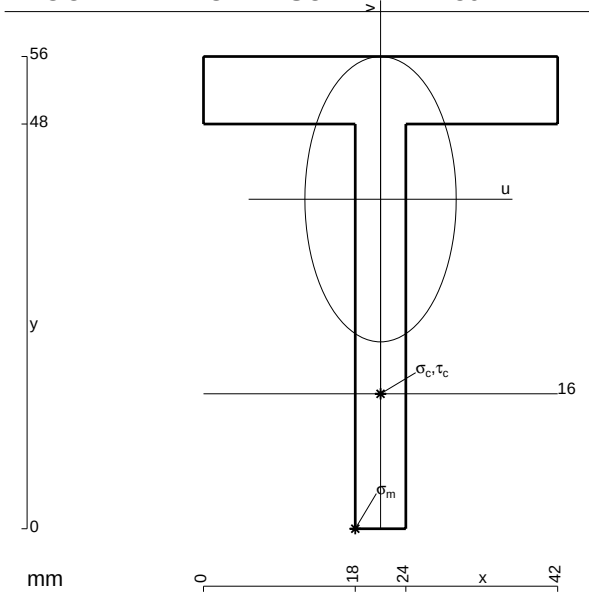
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 624. mm^2
J_u = 178668. mm^4
J_v = 50256. mm^4
y_g = 39.08 mm
N = -1422. N
T_y = -711.1 N
M_x = 922200. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.08 mm
$\sigma_m = N/A - Mv/J_u$ = 199.4 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -23.08 mm
$\sigma_c = N/A - Mv/J_u$ = 116.8 N/mm^2
τ_c = 1.979 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.9 N/mm^2
S^* = 2983. mm^3

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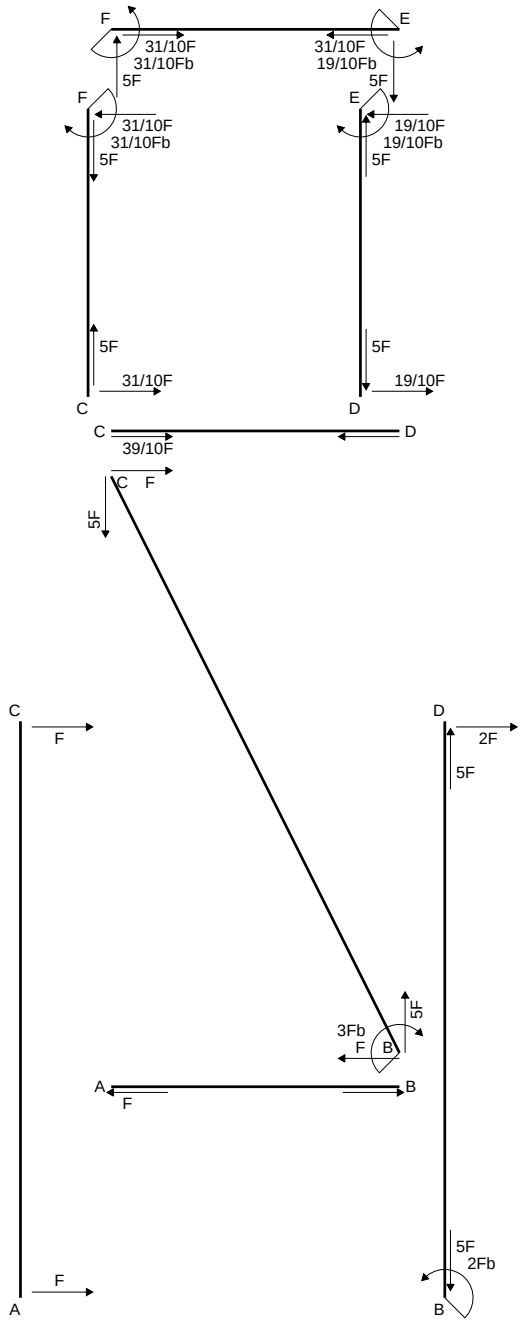
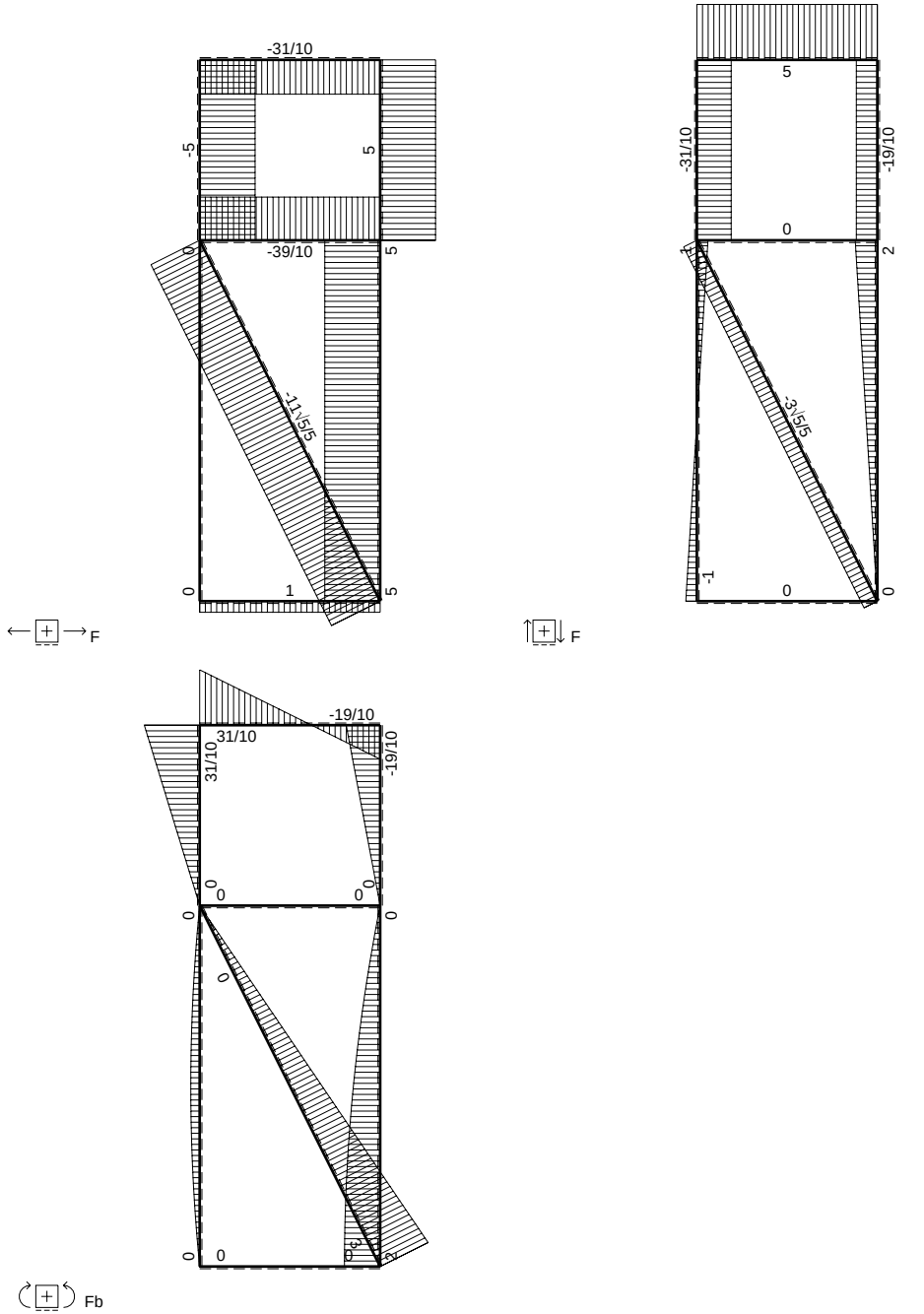

F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
totali							-19/6Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							19/10Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 983928 ESΣ09.xxxx.066

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

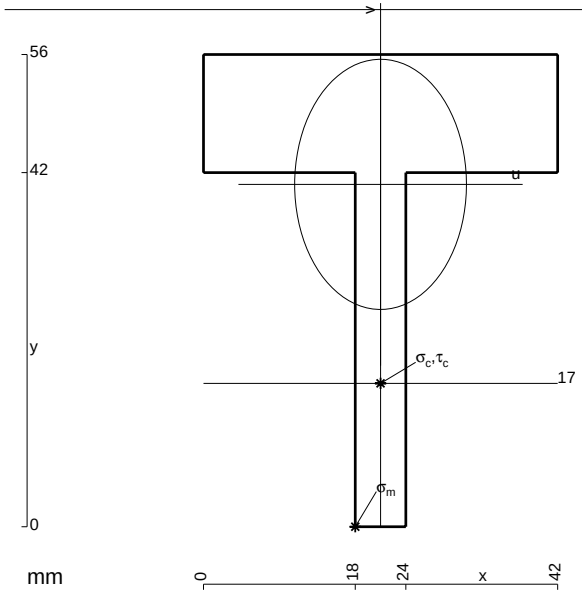
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$



A = 840. mm^2
J_u = 184946. mm^4
J_v = 87192. mm^4
y_g = 40.6 mm
N = -2558. N
T_y = -697.7 N
M_x = 967200. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -40.6 mm
$\sigma_m = N/A - Mv/J_u$ = 209.3 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -23.6 mm
$\sigma_c = N/A - Mv/J_u$ = 120.4 N/mm^2
τ_c = 2.058 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.4 N/mm^2
S^* = 3274. mm^3

ESΣ09.xxxx.066

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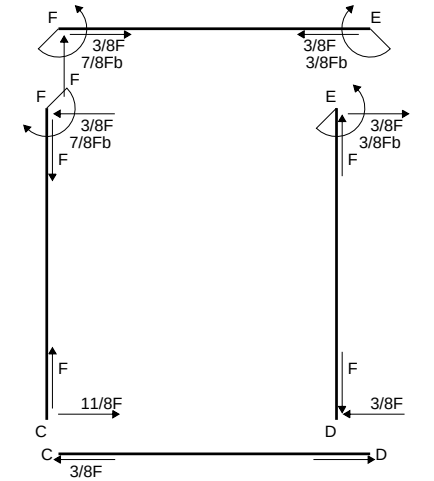
F
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7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
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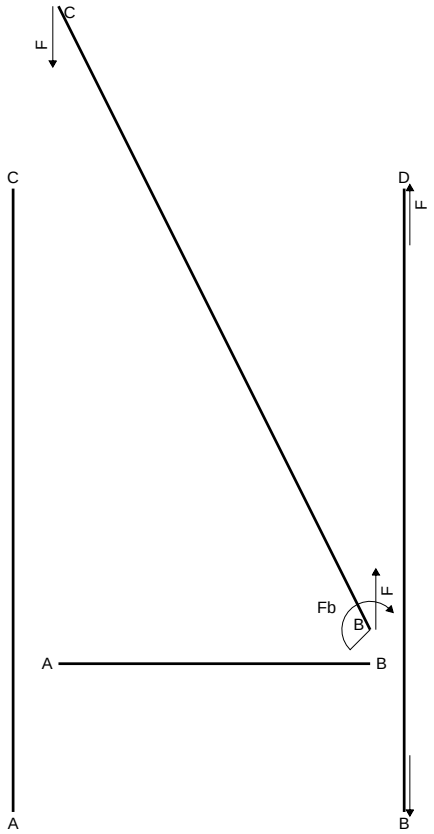
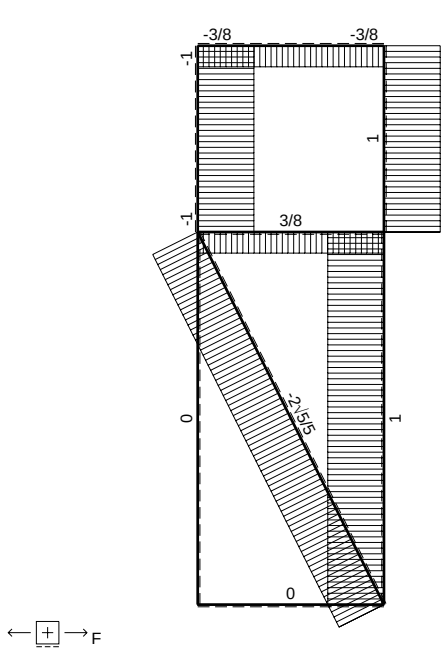
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
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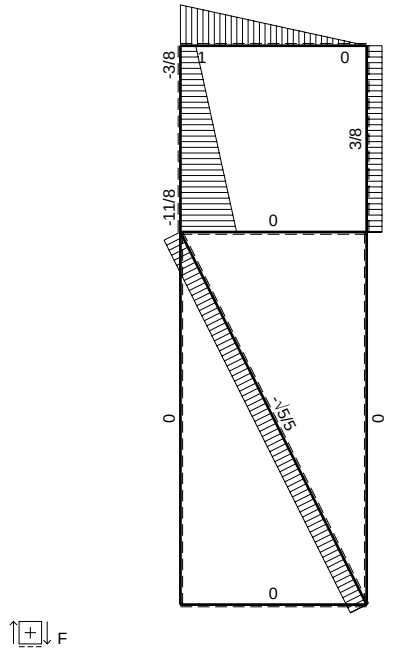
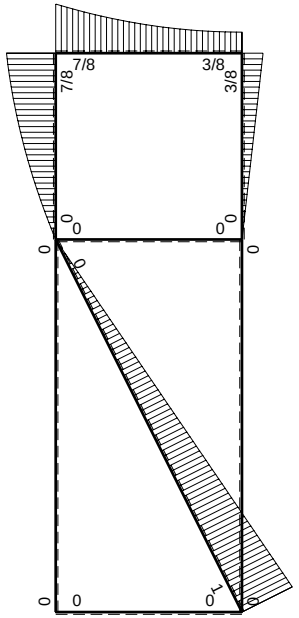
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

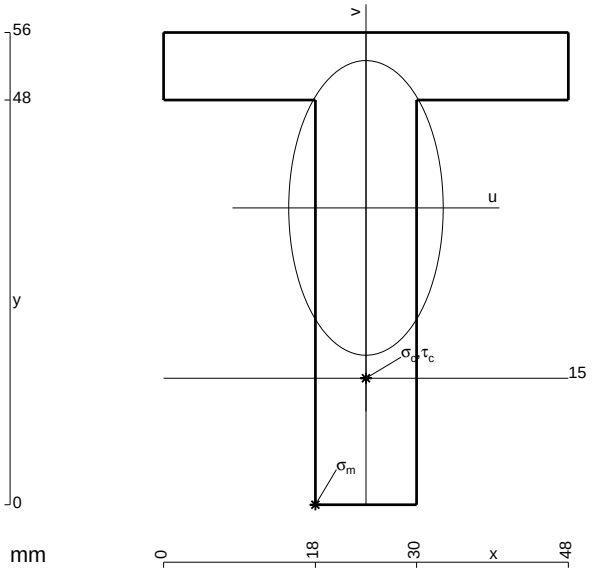
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 960. mm^2
J_u = 293274. mm^4
J_v = 80640. mm^4
y_g = 35.2 mm
N = -2513. N
T_y = -1257. N
M_x = 1854600. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.2 mm
$\sigma_m = N/A - Mv/J_u$ = 220. N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.2 mm
$\sigma_c = N/A - Mv/J_u$ = 125.1 N/mm^2
τ_c = 1.78 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 125.2 N/mm^2
S^* = 4986. mm^3

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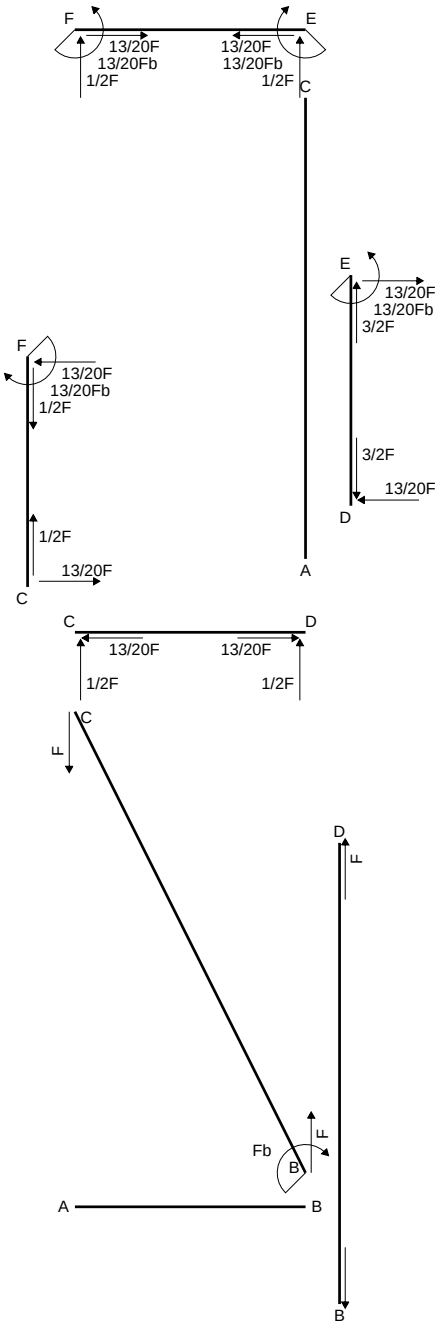
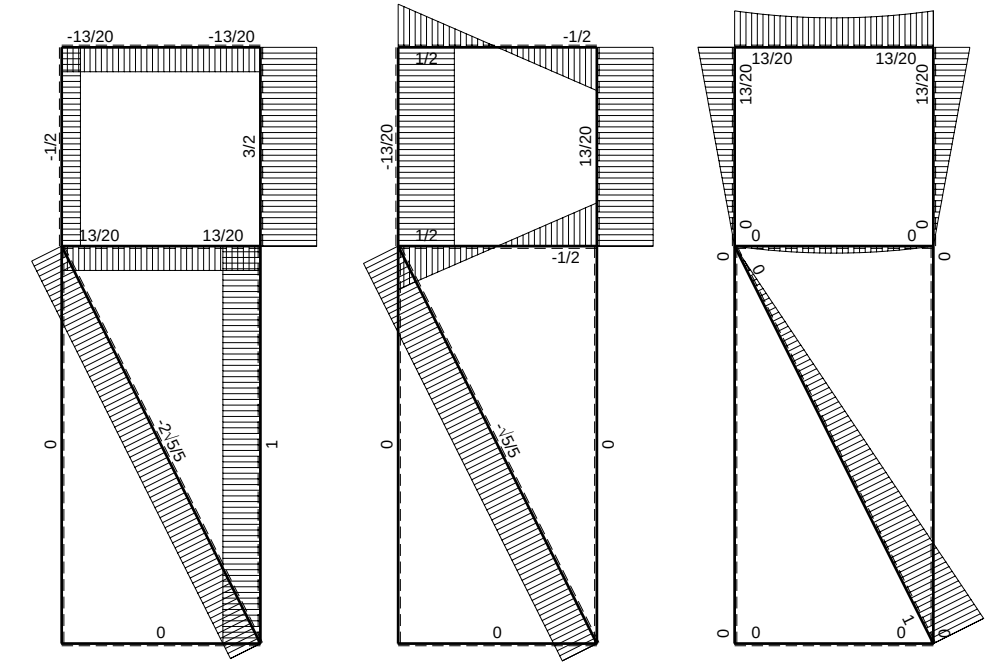
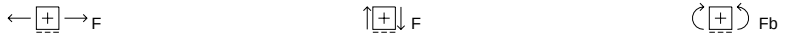
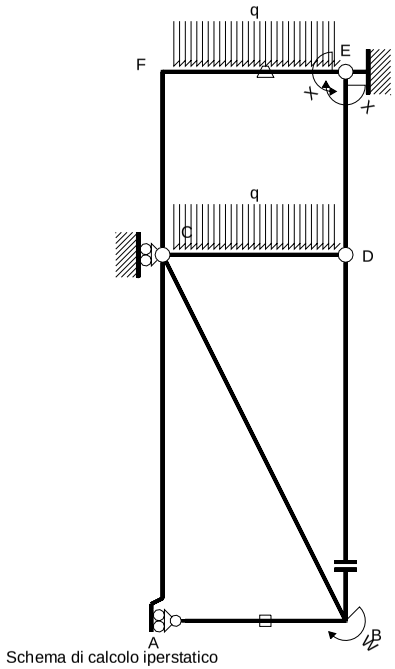
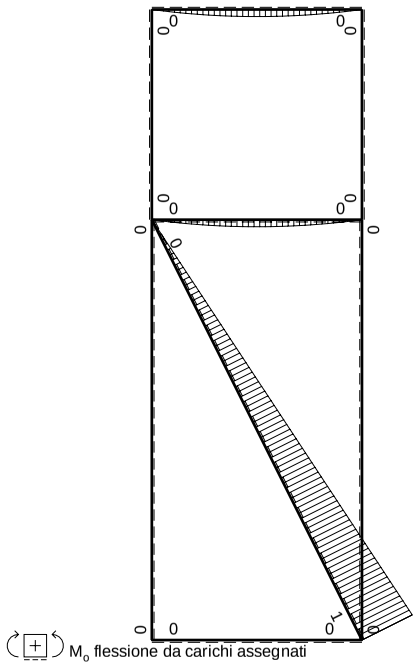

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

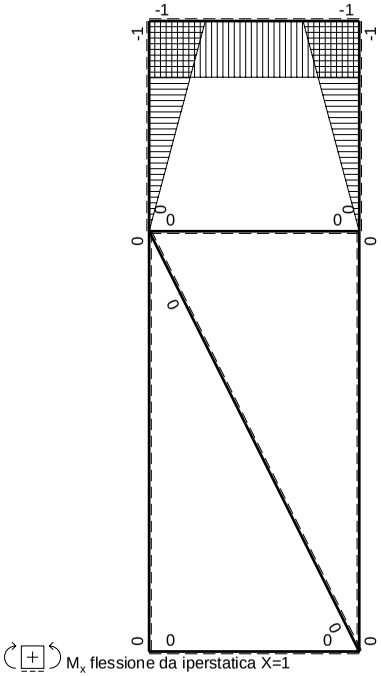

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957798 ESΣ09.xxxx.068

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

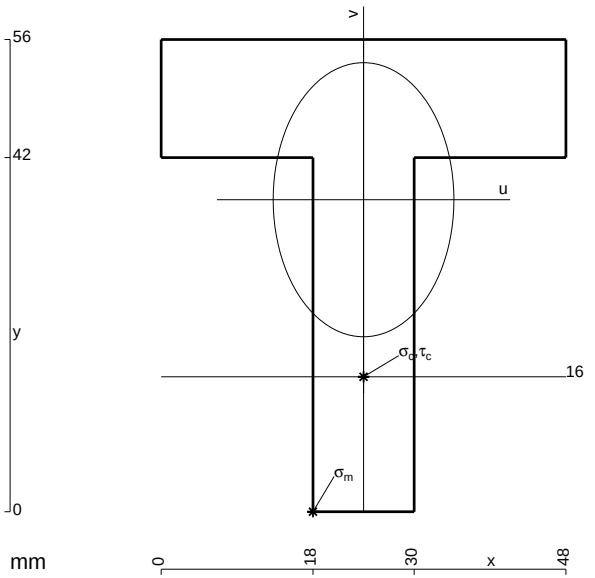
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 1176. mm^2
J_u = 310856. mm^4
J_v = 135072. mm^4
y_g = 37. mm
N = -2487. N
T_y = -1243. N
M_x = 1946000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -37. mm
$\sigma_m = N/A - Mv/J_u$ = 229.5 N/mm^2
x_c = 24. mm
y_c = 16. mm
v_c = -21. mm
$\sigma_c = N/A - Mv/J_u$ = 129.3 N/mm^2
τ_c = 1.856 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.4 N/mm^2
S^* = 5568. mm^3

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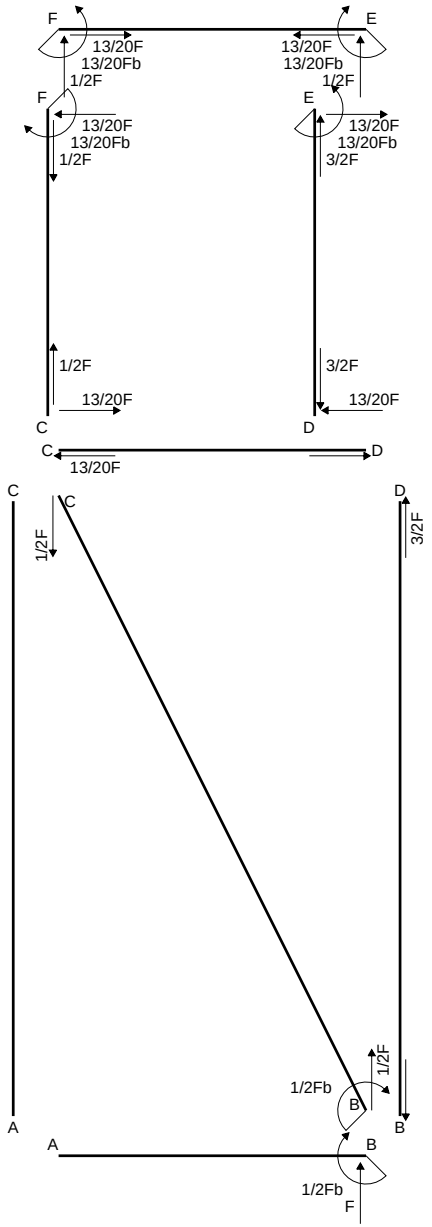
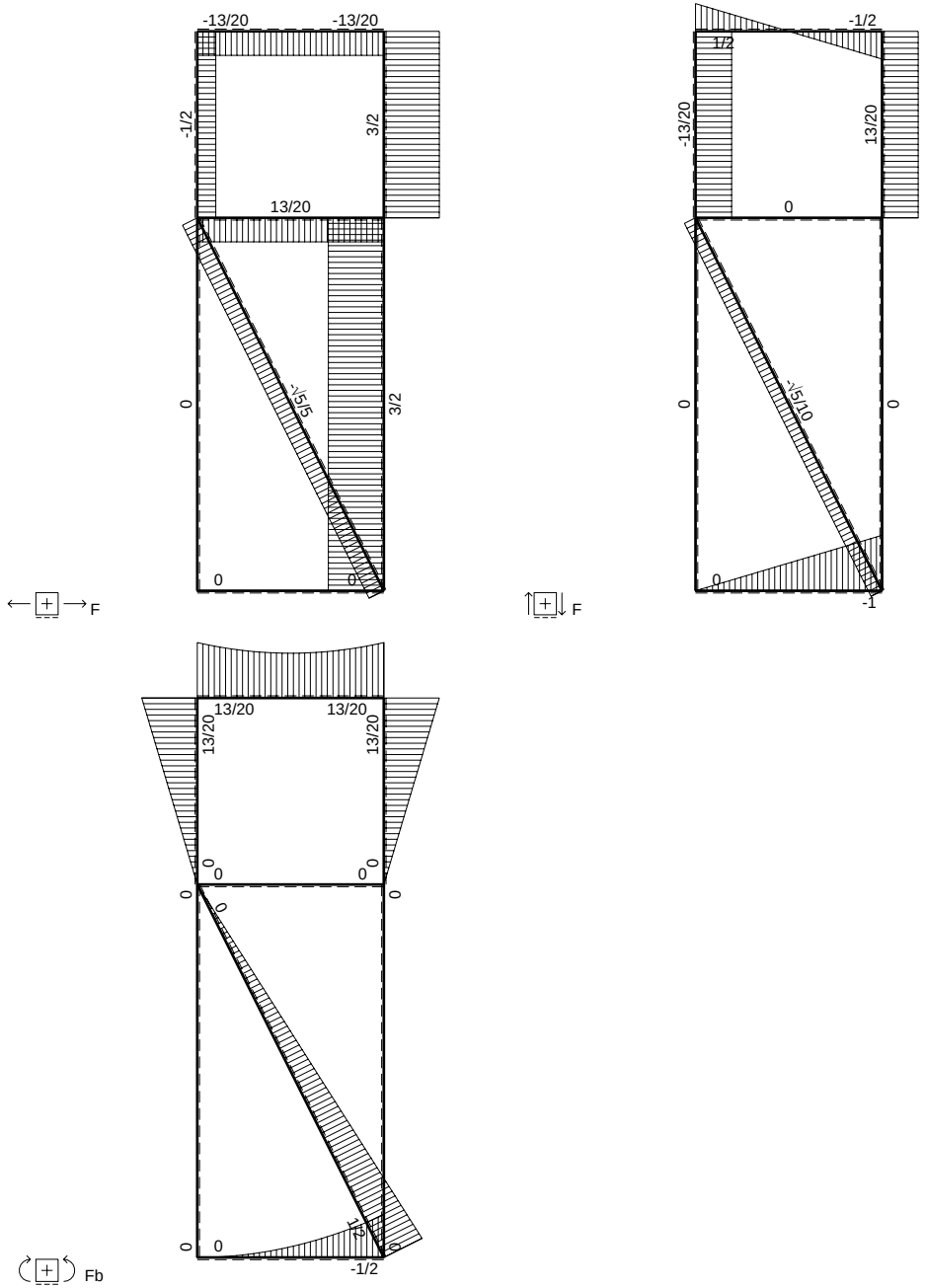
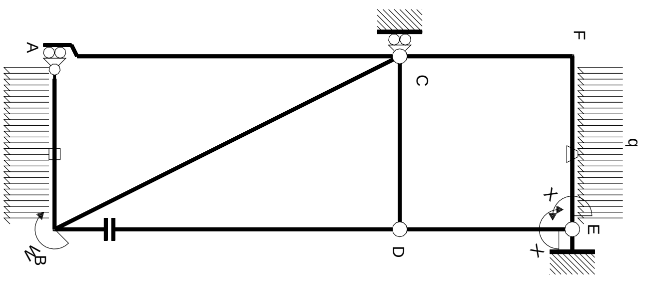
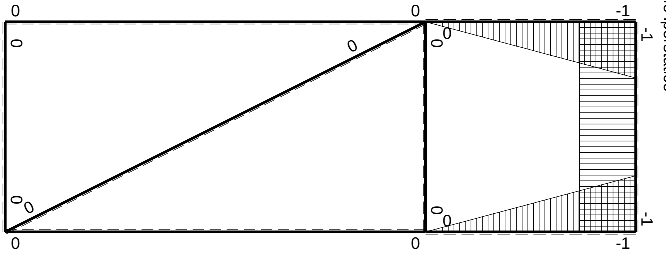

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

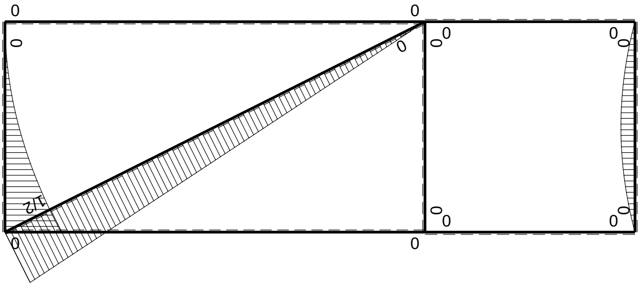

-13/20
-1/2
3/2
13/20
0
-√5/5
-√5/10
F
Fb



Schema di calcolo iperstatico



M_x flessione da iperstatica X=1



M_o flessione da carichi assegnati

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

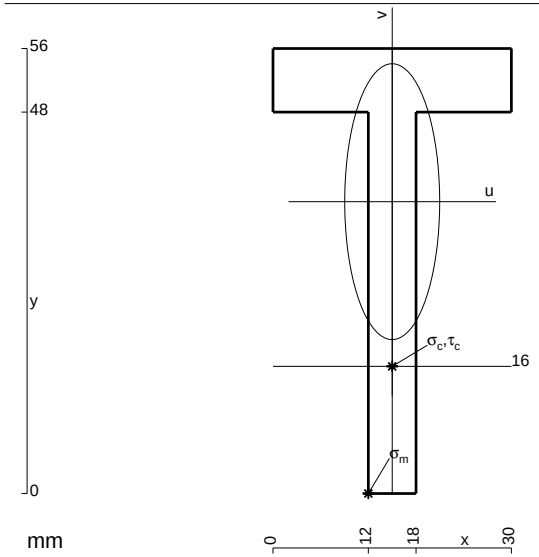
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 214815 ESΣ09.xxxx.069



A = 528. mm^2
J_u = 159209. mm^4
J_v = 18864. mm^4
y_g = 36.73 mm
T_y = -2810. N
M_x = -1039700. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.73 mm
$\sigma_m = -Mv/J_u$ = -239.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -20.73 mm
$\sigma_c = -Mv/J_u$ = -135.4 N/mm^2
τ_c = 8.112 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.1 N/mm^2
S^* = 2758. mm^3

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ESΣ09.xxxx.069

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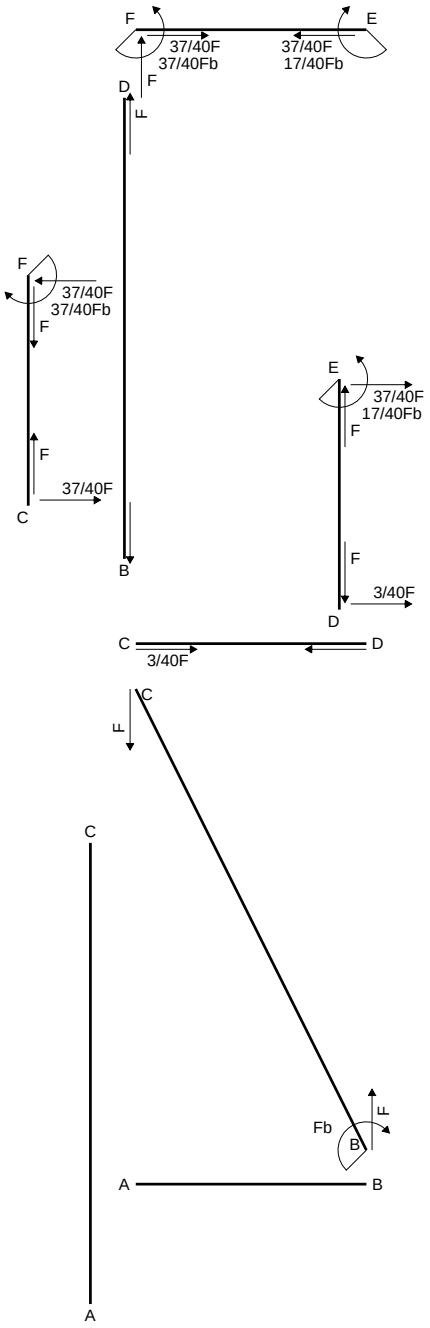
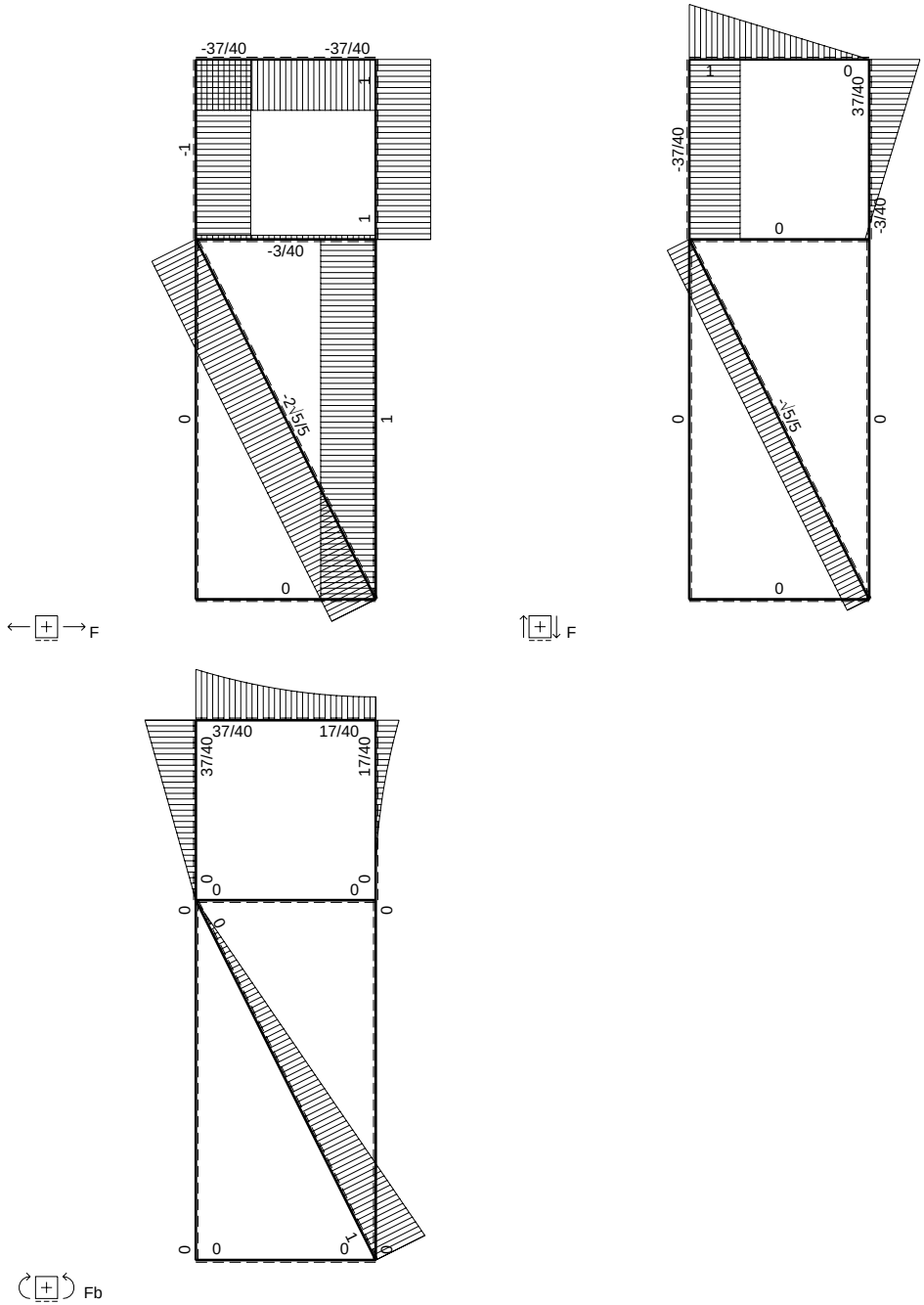

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D
C
F
C
Fb
B
F
A
B
A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-3/40
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1
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F
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37/40
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-3/40
0
-√5/5
0
0
F
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37/40
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0
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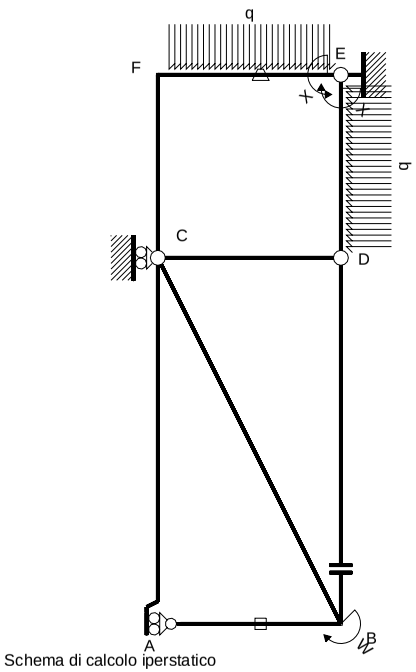
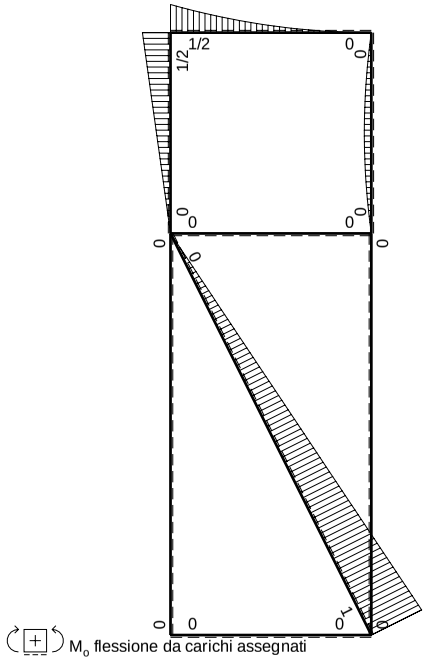
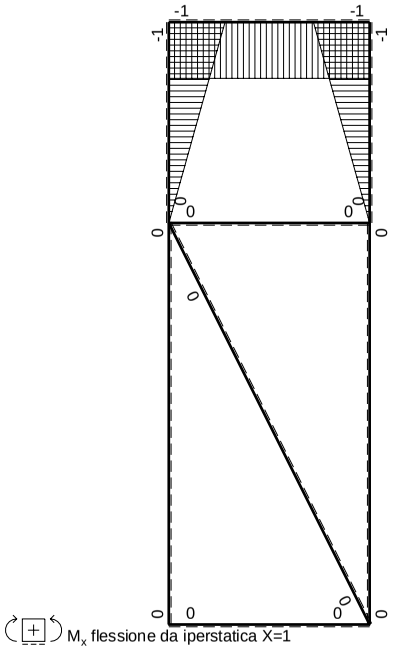
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212483 ESΣ09.xxxx.070

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

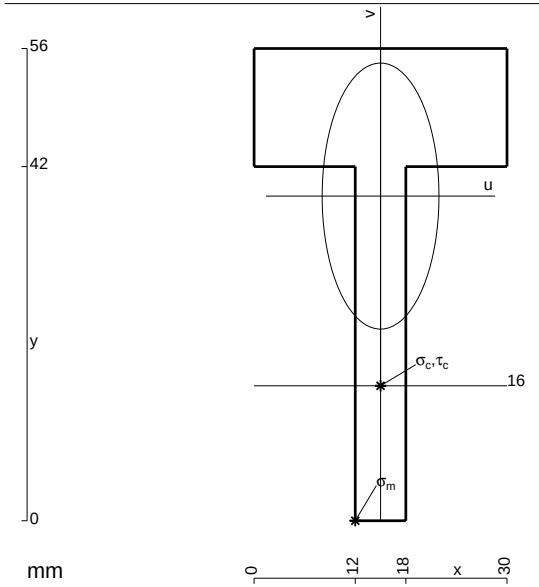
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 672. mm^2
J_u = 167384. mm^4
J_v = 32256. mm^4
y_g = 38.5 mm
N = -2021. N
T_y = -1011. N
M_x = 881400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -38.5 mm
$\sigma_m = N/A - Mv/J_u$ = 199.7 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -22.5 mm
$\sigma_c = N/A - Mv/J_u$ = 115.5 N/mm^2
τ_c = 2.947 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.6 N/mm^2
S^* = 2928. mm^3

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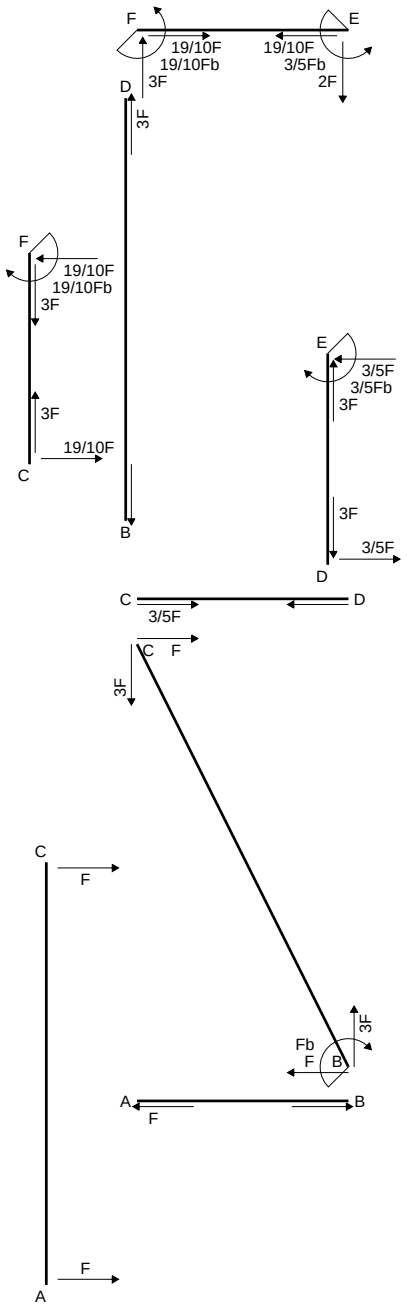
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19/10Fb
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3/5Fb
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2F
D
3F
F
19/10F
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3/5Fb
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3F
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3/5F
D
C
D
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C
F
3F
C
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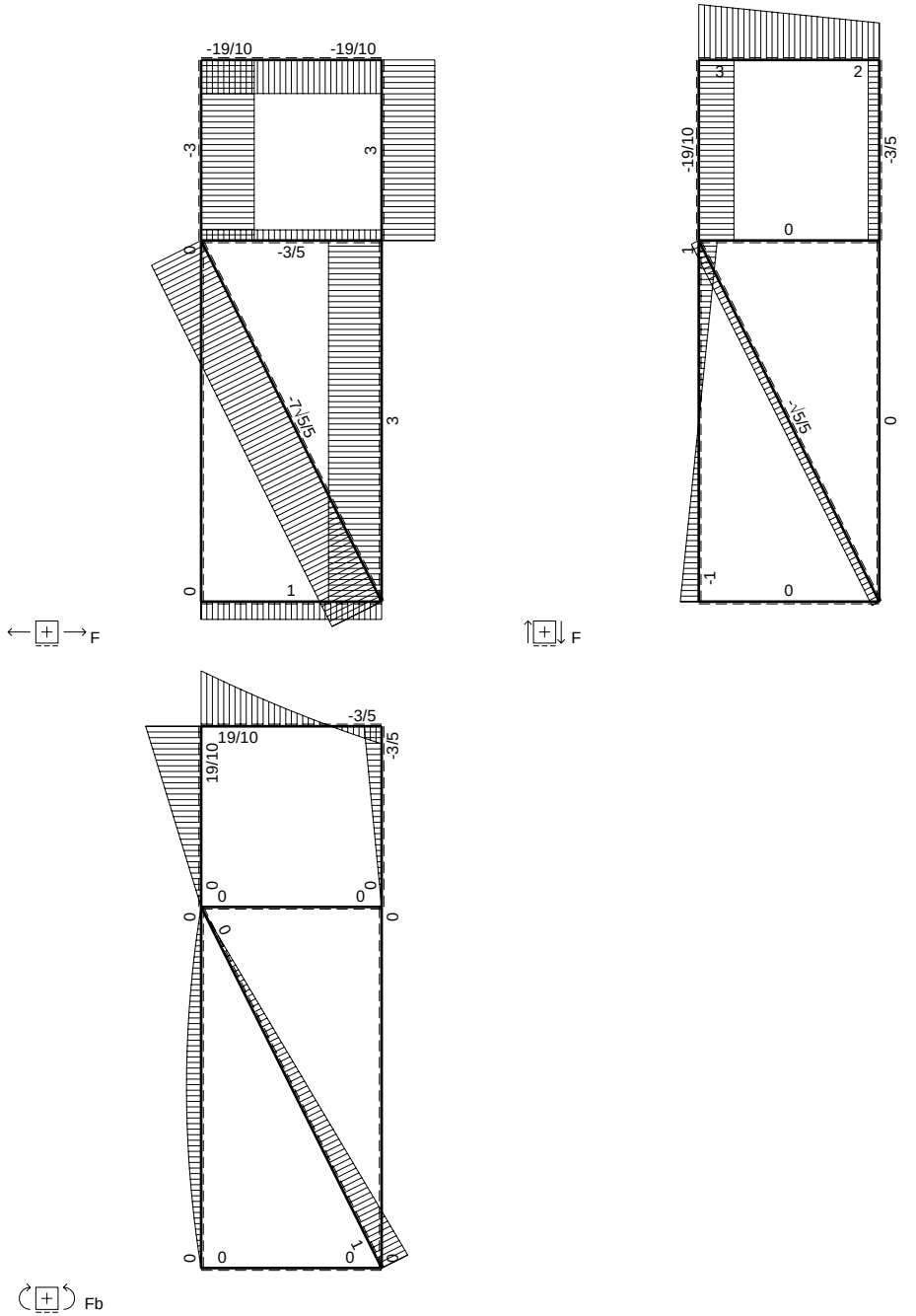
Fb
F
B
3F
A
B
F
F
A


-19/10
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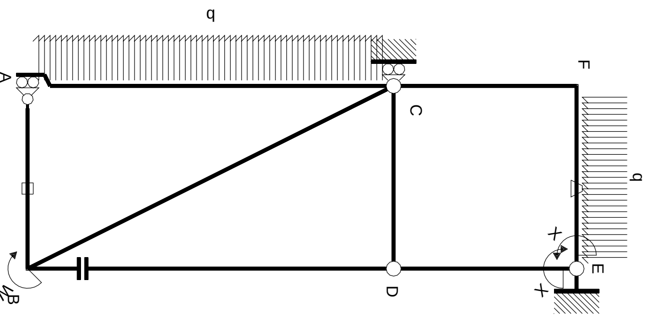
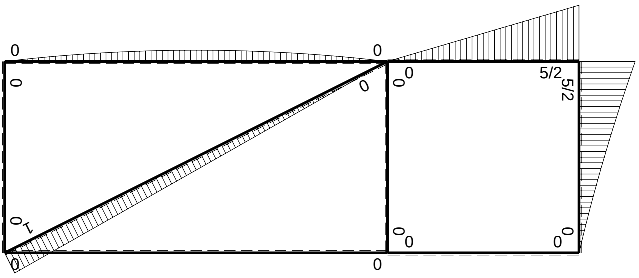
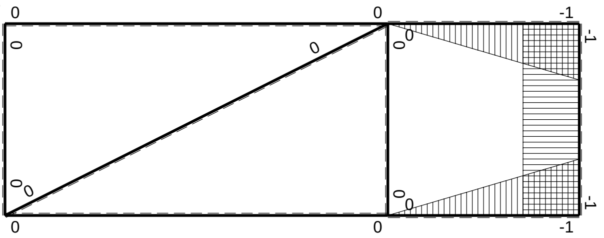
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{EF}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica X=W_{EF}						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

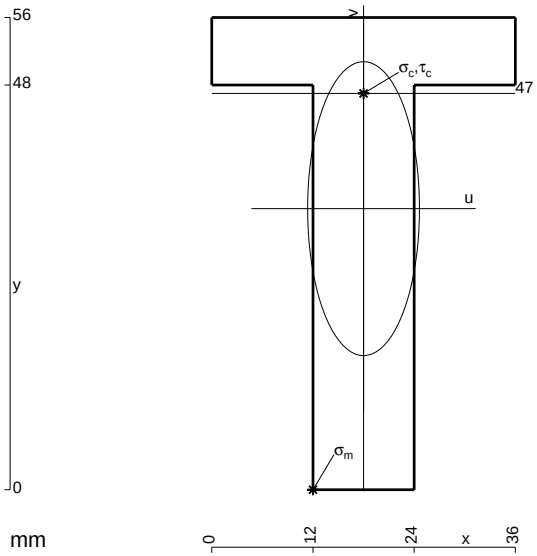
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 864. mm^2
J_u = 262656. mm^4
J_v = 38016. mm^4
y_g = 33.33 mm
N = -13211. N
T_y = -1887. N
M_x = 1772400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.33 mm
$\sigma_m = N/A - Mv/J_u$ = 209.6 N/mm^2
x_c = 18. mm
y_c = 47. mm
v_c = 13.67 mm
$\sigma_c = N/A - Mv/J_u$ = -107.5 N/mm^2
τ_c = 3.321 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 107.7 N/mm^2
S^* = 5546. mm^3

ESΣ09.xxxx.071

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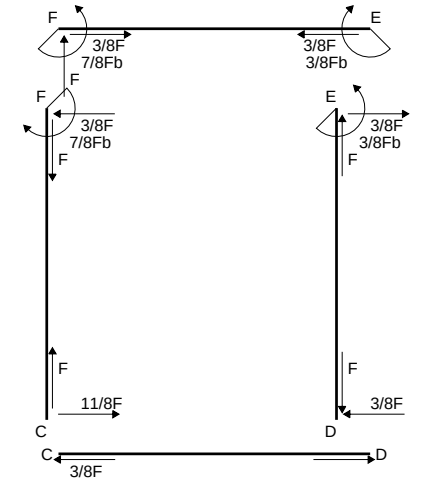
F
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7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
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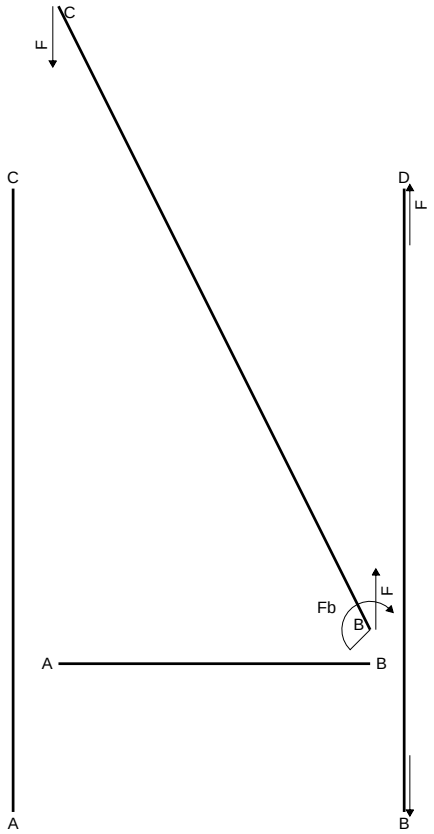
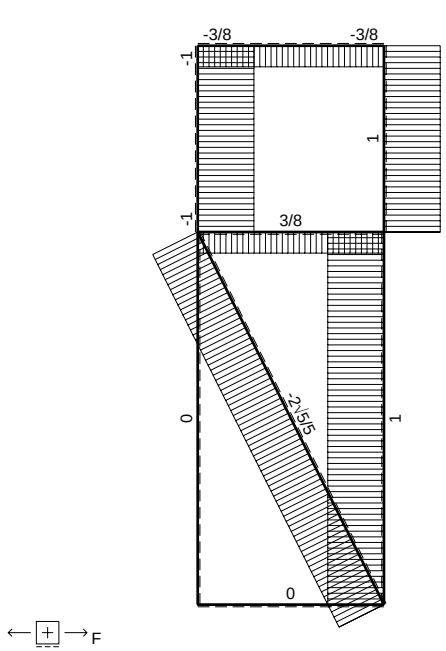
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
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-1
1
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1
-2√5/5
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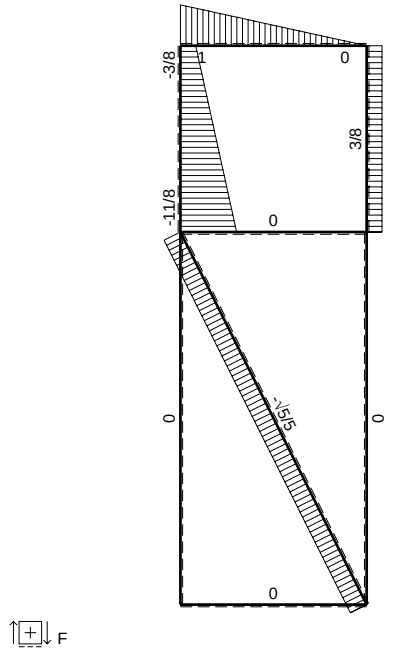
F


-3/8
1
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3/8
-11/8
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-√5/5
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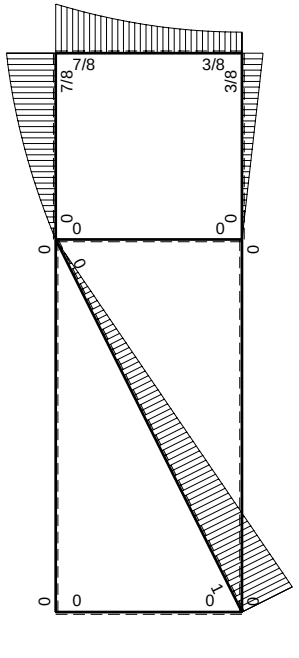
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

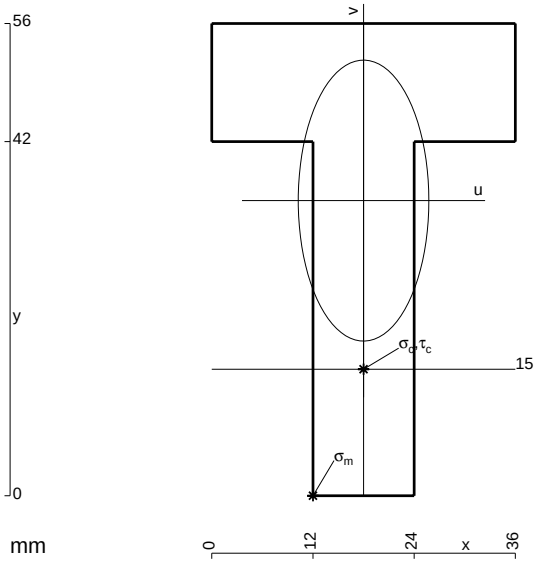
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 1008. mm^2
J_u = 279888. mm^4
J_v = 60480. mm^4
y_g = 35. mm
N = -3470. N
T_y = -1735. N
M_x = 1784800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -35. mm
$\sigma_m = N/A - Mv/J_u$ = 219.7 N/mm^2
x_c = 18. mm
y_c = 15. mm
v_c = -20. mm
$\sigma_c = N/A - Mv/J_u$ = 124.1 N/mm^2
τ_c = 2.557 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.2 N/mm^2
S^* = 4950. mm^3

ESΣ09.xxxx.072

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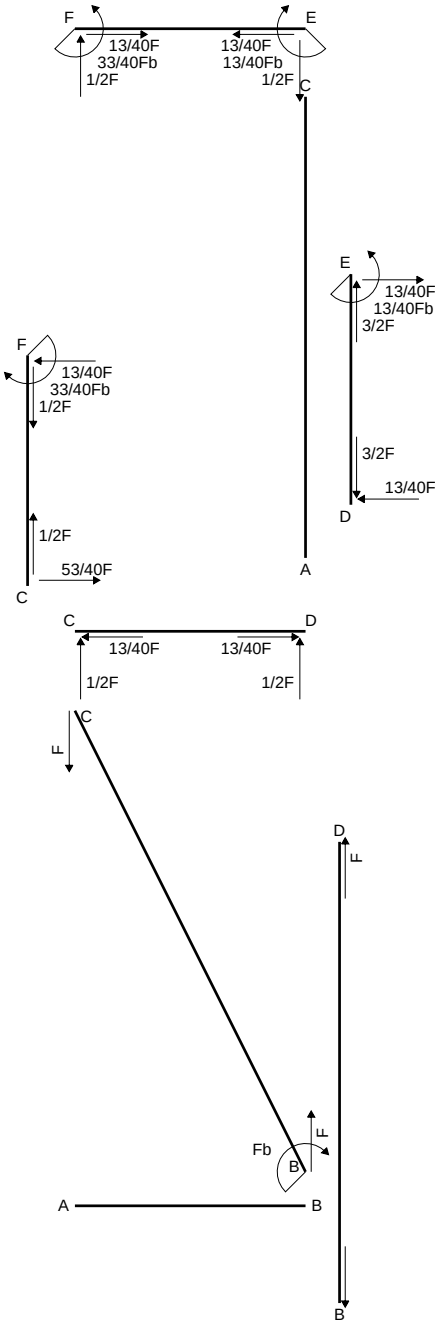
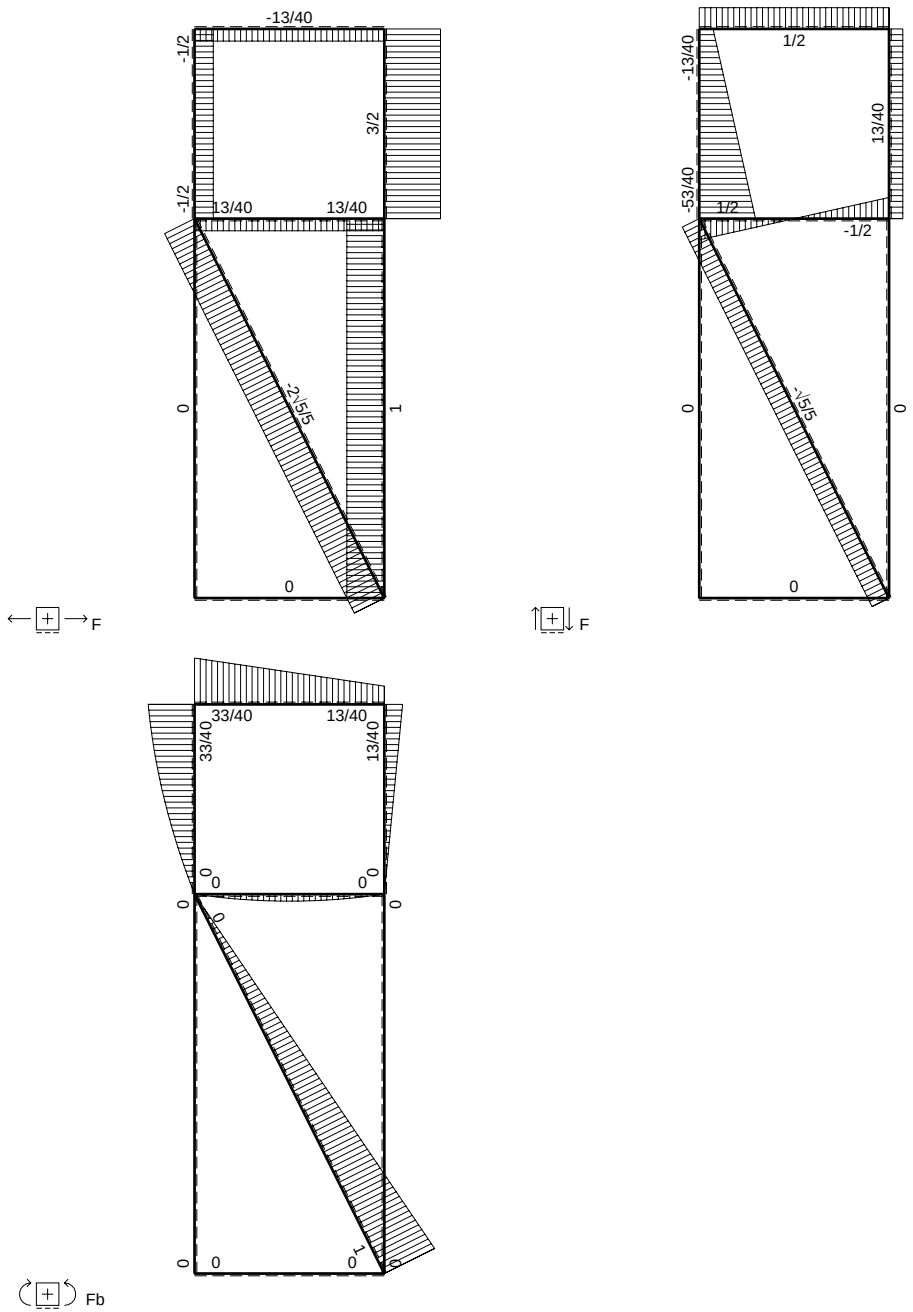

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F
-13/40
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13/40
-53/40
1/2
-1/2
-√5/5
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XkJ/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XbJ/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3XkJ/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

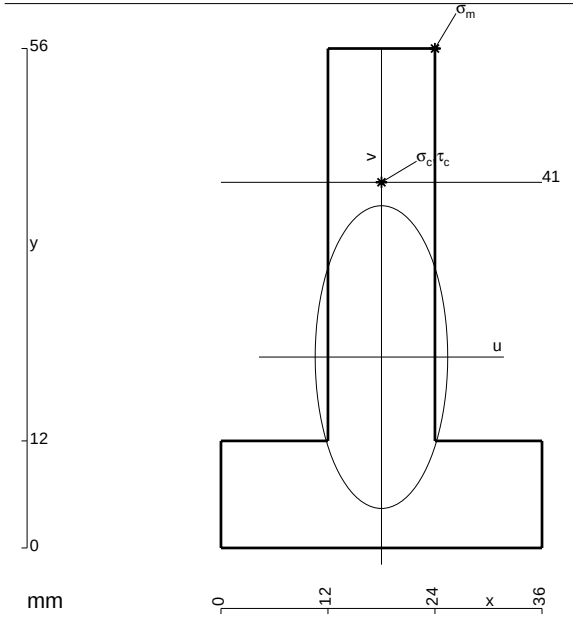
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 960. mm^2
J_u = 276646. mm^4
J_v = 52992. mm^4
y_g = 21.4 mm
N = -3238. N
T_y = -1619. N
M_x = 1810000. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 34.6 mm
$\sigma_m = N/A - Mv/J_u$ = -229.7 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.6 mm
$\sigma_c = N/A - Mv/J_u$ = -131.6 N/mm^2
τ_c = 2.379 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.7 N/mm^2
S^* = 4878. mm^3

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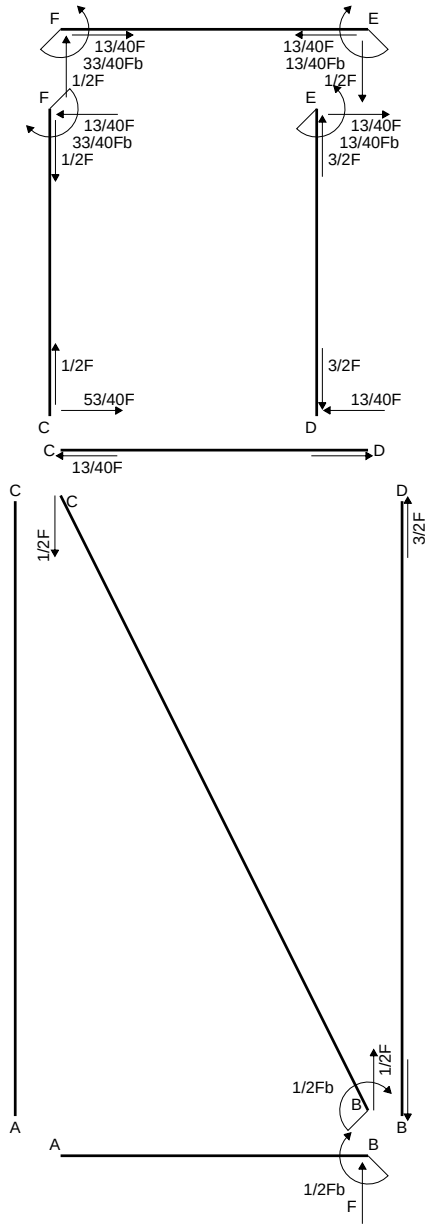
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
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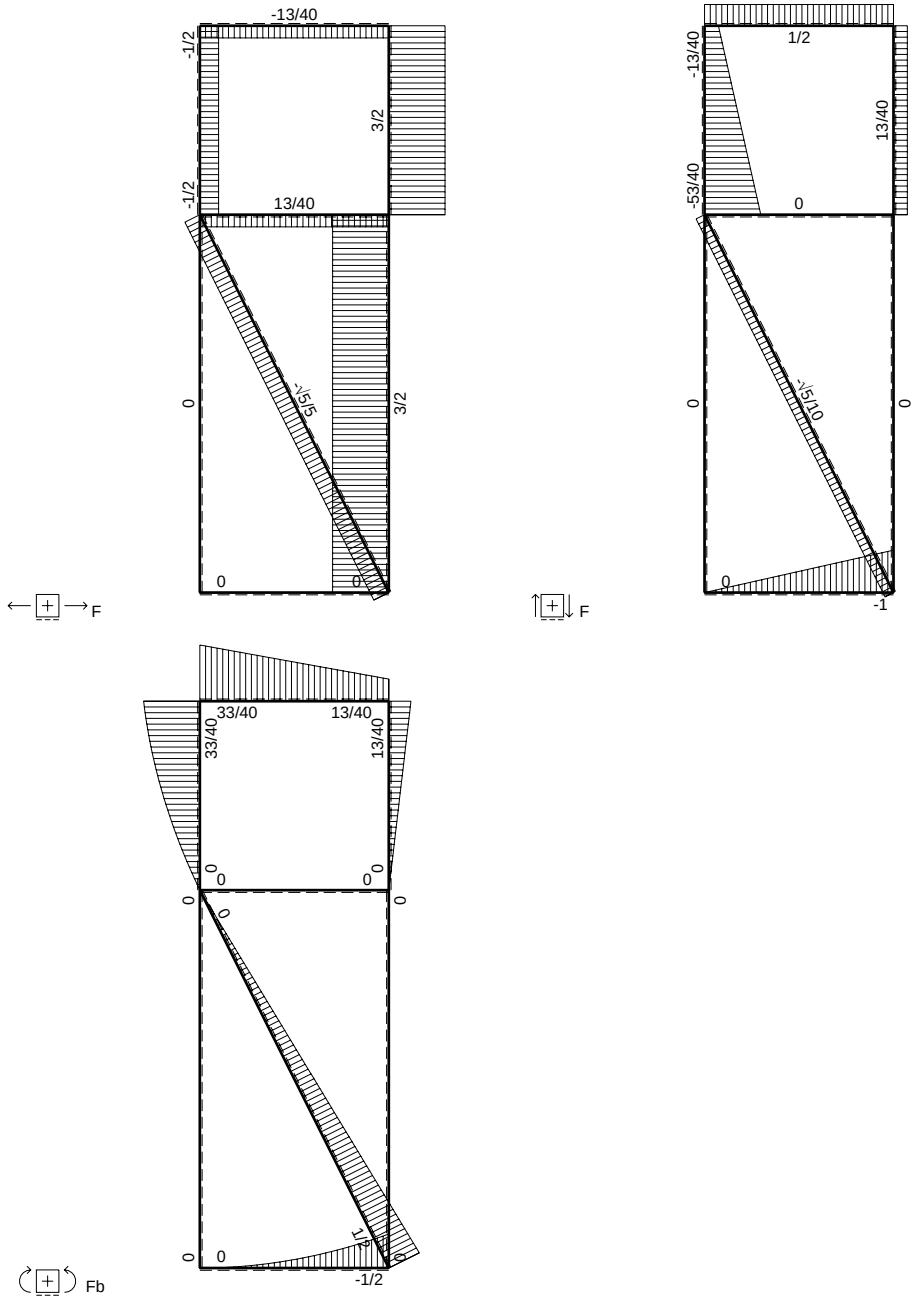
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211523 ESΣ09.xxxx.074

Schema di calcolo iperstatico

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

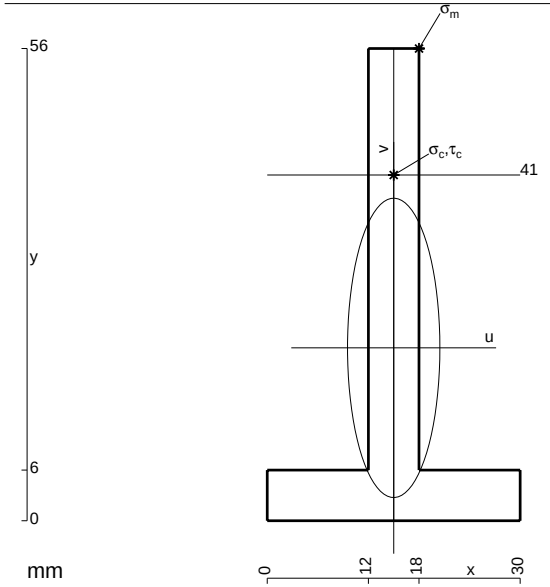
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211523 ESΣ09.xxxx.074



A = 480. mm^2
J_u = 151240. mm^4
J_v = 14400. mm^4
y_g = 20.5 mm
T_y = -3780. N
M_x = -1020600. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 35.5 mm
$\sigma_m = -Mv/J_u$ = 239.6 N/mm^2
x_c = 15. mm
y_c = 41. mm
v_c = 20.5 mm
$\sigma_c = -Mv/J_u$ = 138.3 N/mm^2
τ_c = 10.5 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.5 N/mm^2
S^* = 2520. mm^3

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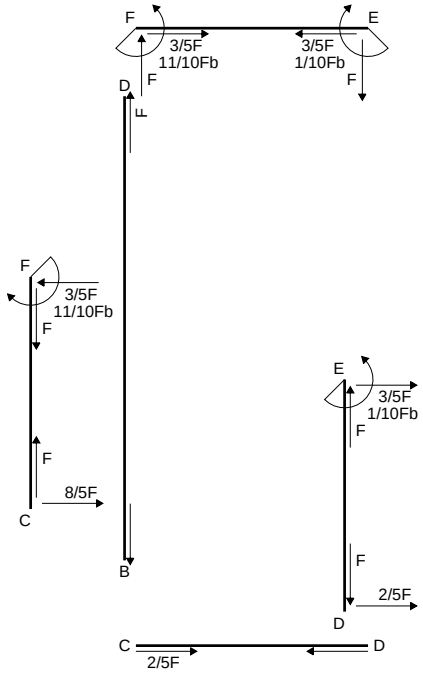
F
E
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11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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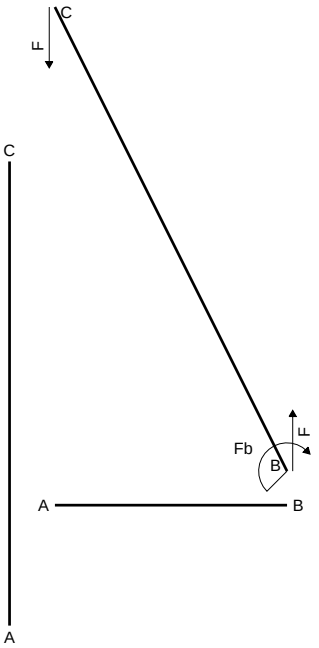
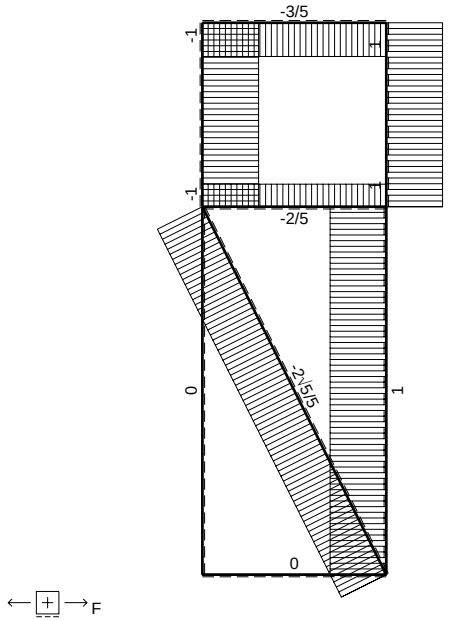
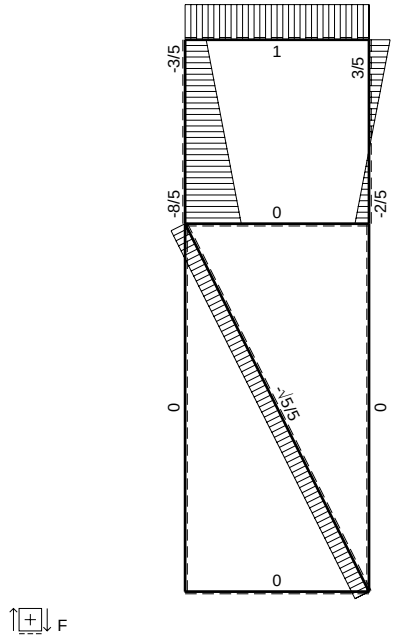
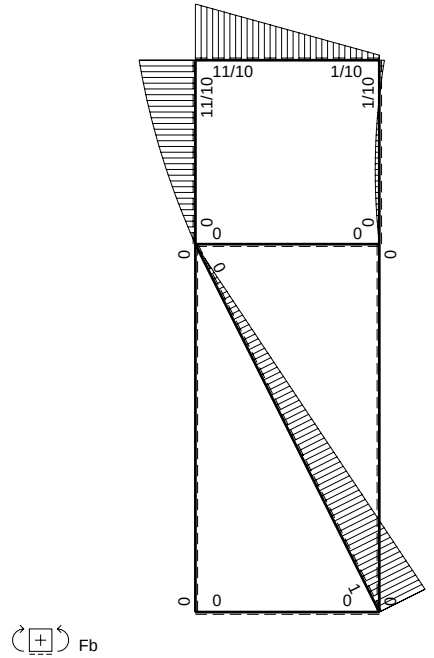
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3XMb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb²/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx²	0	-Fb+3/2Fx-1/2qx³/b	0	1-2x/b+x²/b²	(-3/8+0)Fb²/EJ	1/3XMb/EJ
CF b	x/b	-3/2Fx+1/2qx²	0	-3/2Fx²/b+1/2qx³/b	0	x²/b²		
	totali						1/6Fb²/EJ	5/3XMb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 211490 ESΣ09.xxxx.075

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

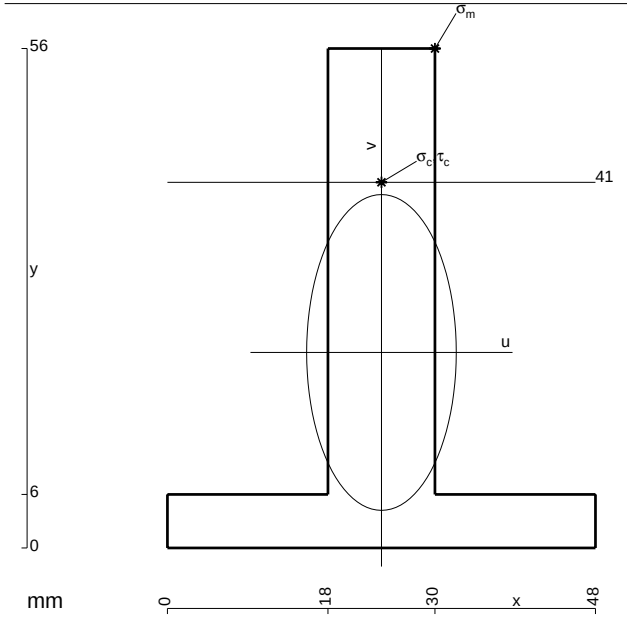
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 888. mm^2
J_u = 278426. mm^4
J_v = 62496. mm^4
y_g = 21.92 mm
N = -2478. N
T_y = -1239. N
M_x = 1606600. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 34.08 mm
$\sigma_m = N/A - Mv/J_u$ = -199.4 N/mm^2
x_c = 24. mm
y_c = 41. mm
v_c = 19.08 mm
$\sigma_c = N/A - Mv/J_u$ = -112.9 N/mm^2
τ_c = 1.774 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 112.9 N/mm^2
S^* = 4785. mm^3

ESΣ09.xxxx.075

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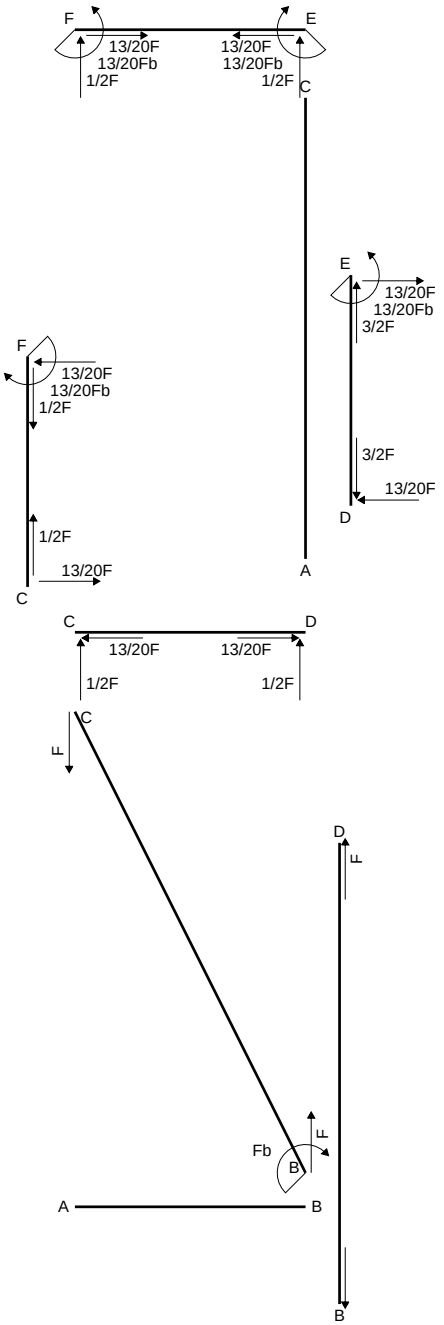
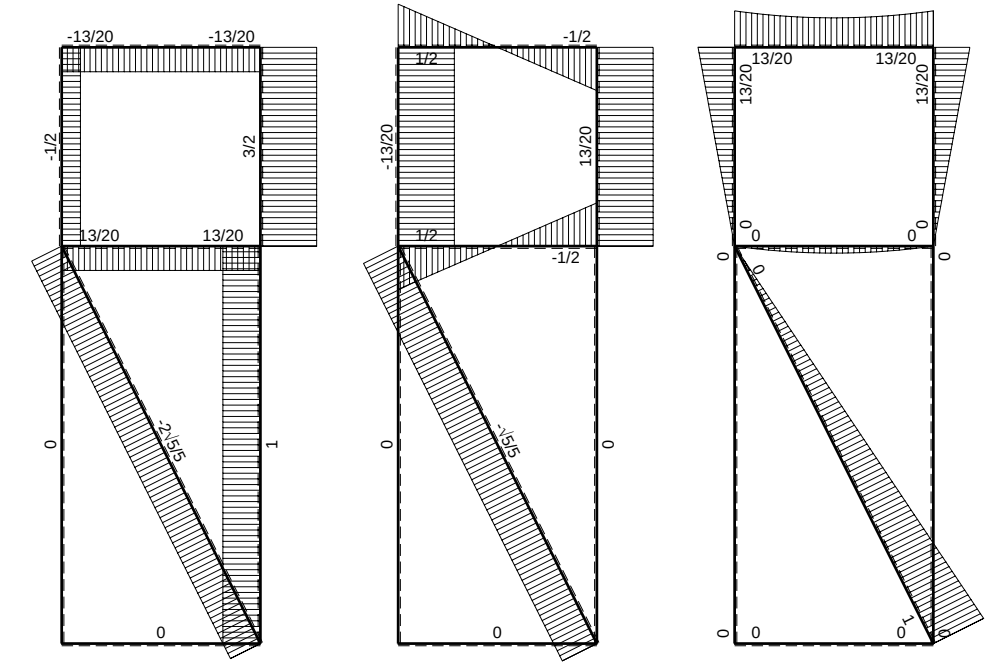
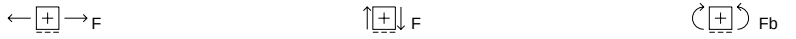
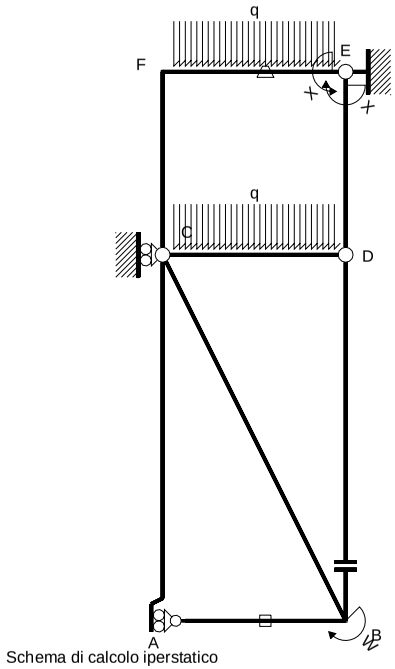
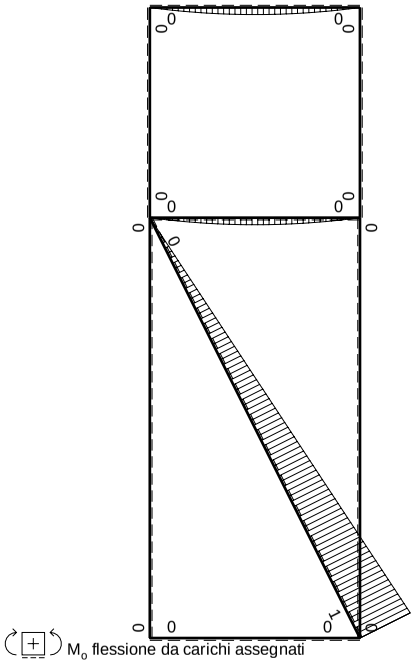

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

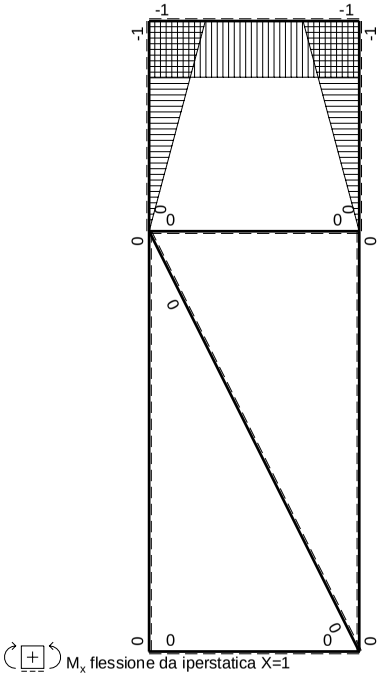

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984883 ESΣ09.xxxx.076

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

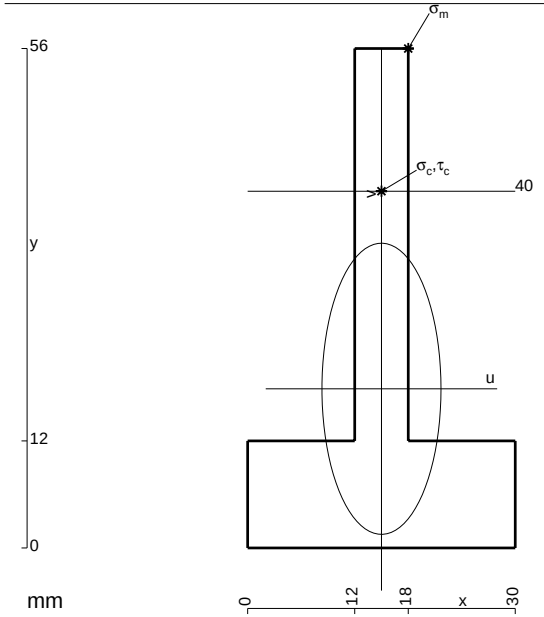
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 984883 ESΣ09.xxxx.076



A = 624. mm^2
J_u = 166321. mm^4
J_v = 27792. mm^4
y_g = 17.85 mm
N = -1306. N
T_y = -652.9 N
M_x = 905200. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 38.15 mm
$\sigma_m = N/A - Mv/J_u$ = -209.7 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 22.15 mm
$\sigma_c = N/A - Mv/J_u$ = -122.7 N/mm^2
τ_c = 1.894 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.7 N/mm^2
S^* = 2895. mm^3

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ESΣ09.xxxx.076

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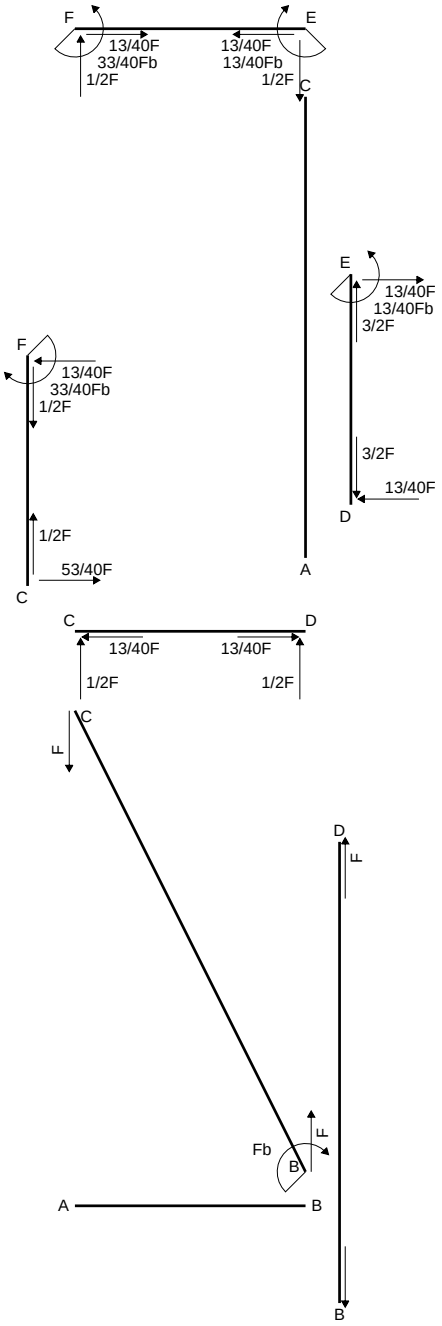
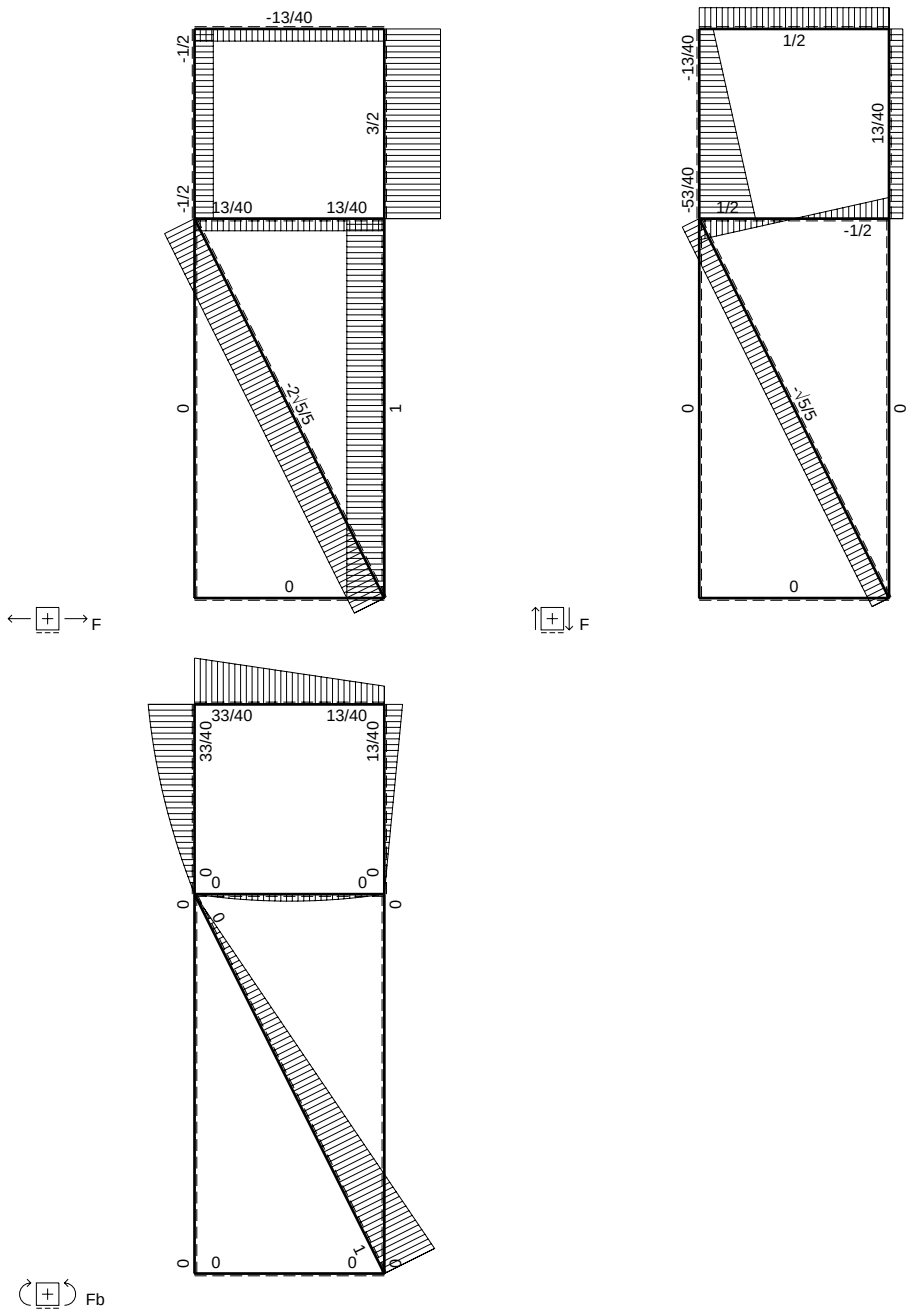

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
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-1/2
3/2
13/40
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1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
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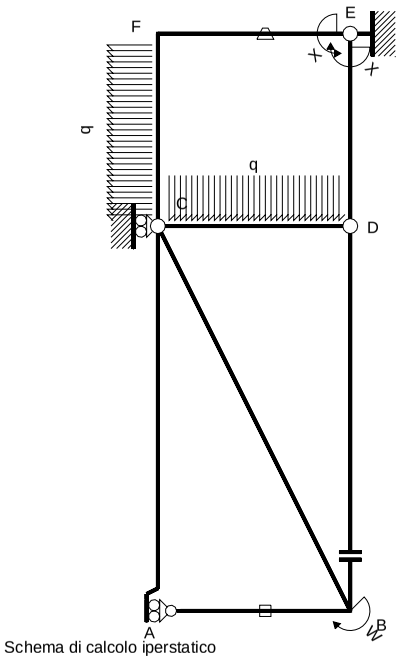
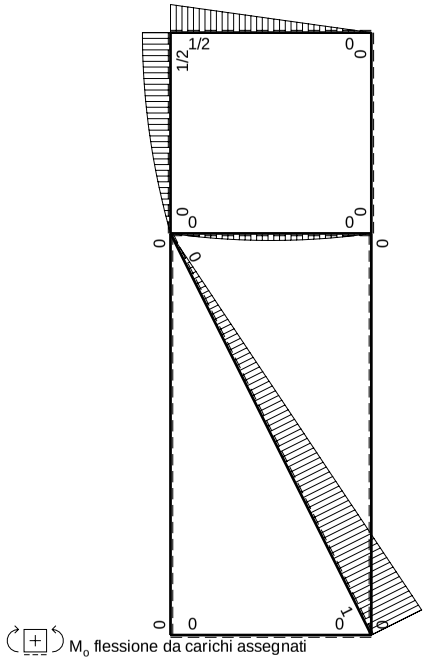
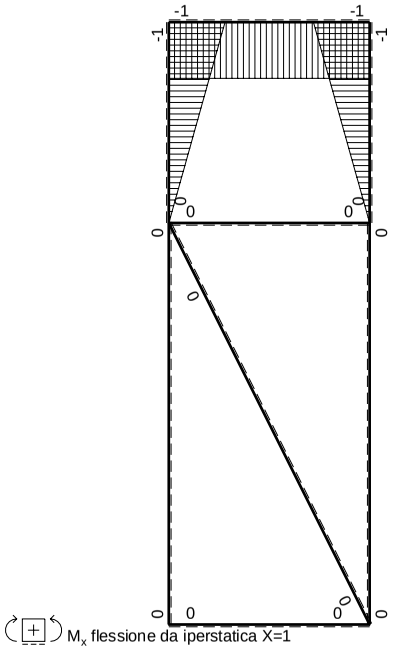
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 934507 ESΣ09.xxxx.077

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

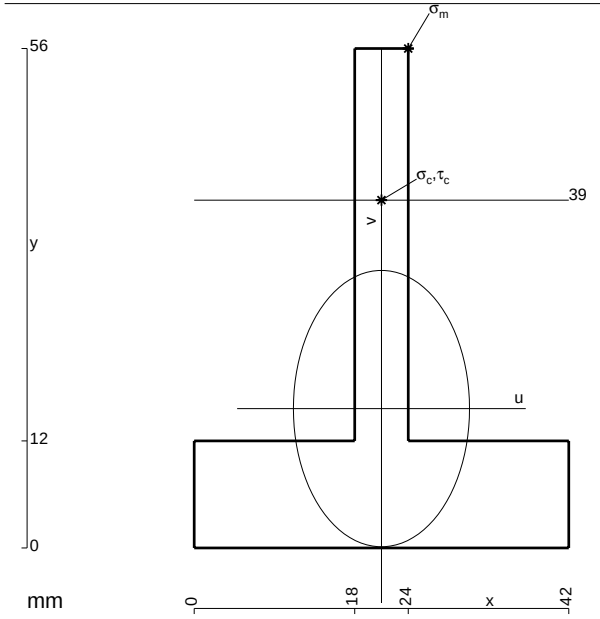
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 768. mm^2
J_u = 184468. mm^4
J_v = 74880. mm^4
y_g = 15.63 mm
N = -1351. N
T_y = -675.3 N
M_x = 996600. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 40.38 mm
$\sigma_m = N/A - Mv/J_u$ = -219.9 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 23.38 mm
$\sigma_c = N/A - Mv/J_u$ = -128. N/mm^2
τ_c = 1.984 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.1 N/mm^2
S^* = 3251. mm^3

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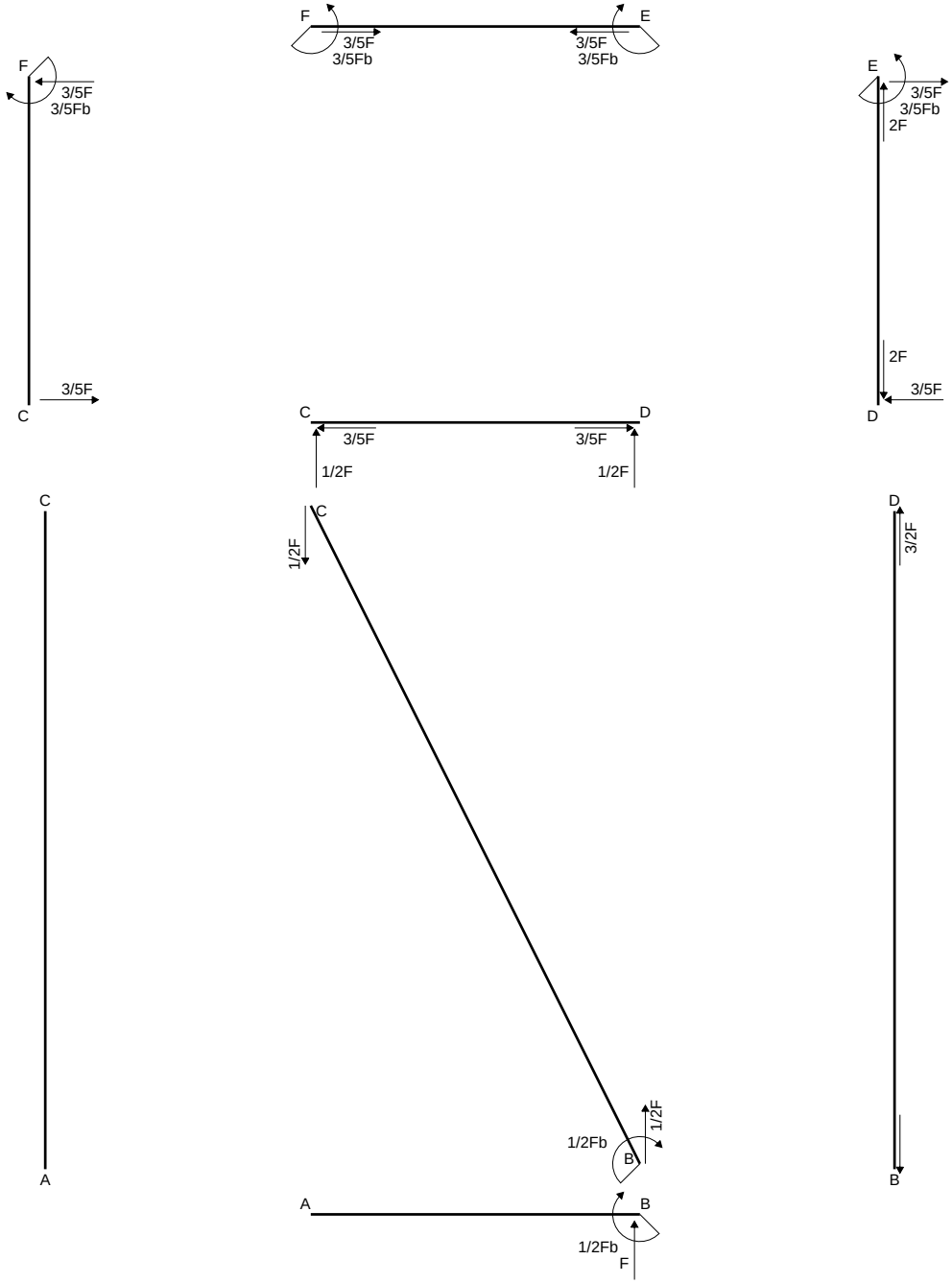
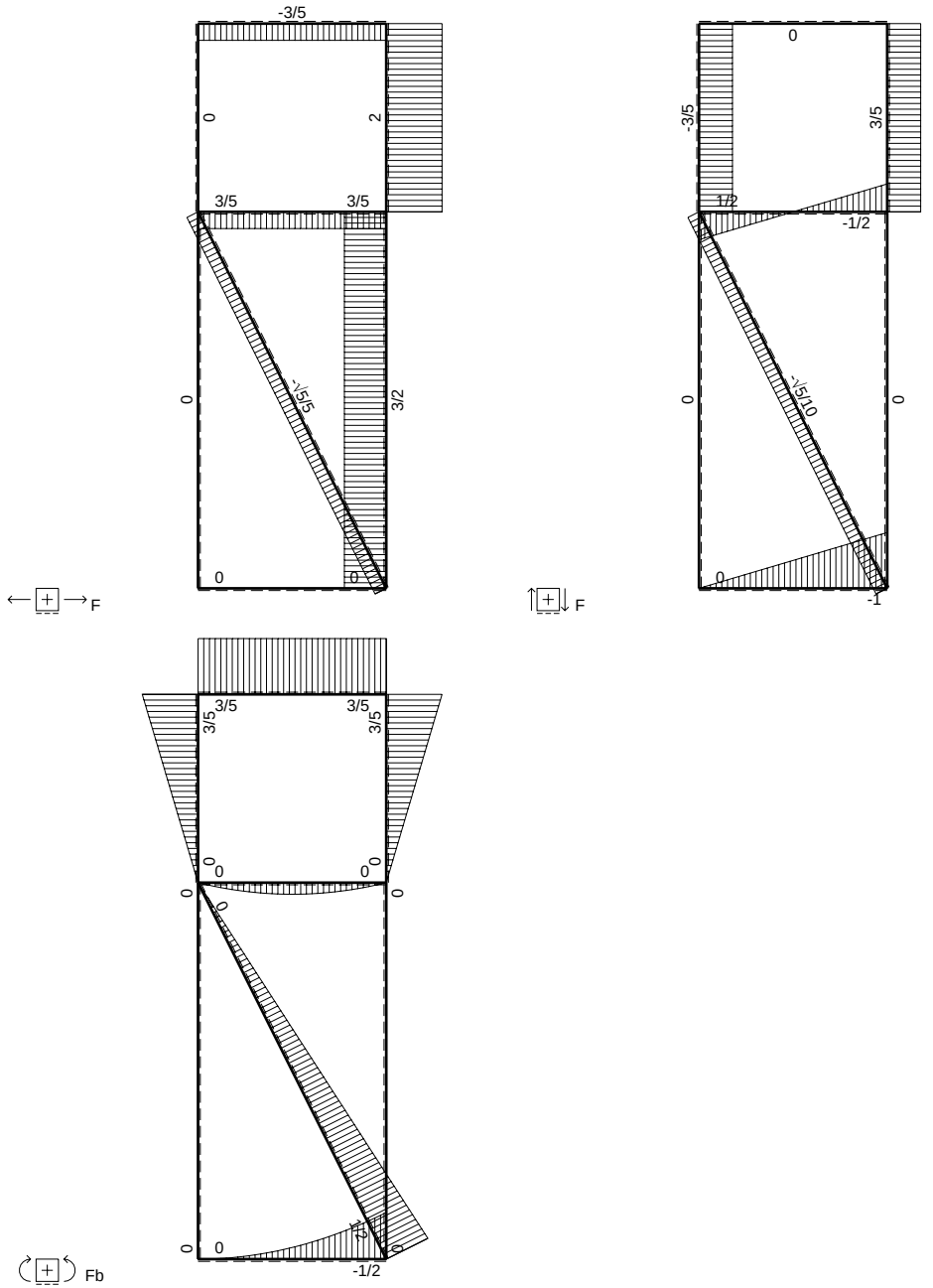

F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F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3/2
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0
F
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-3/5
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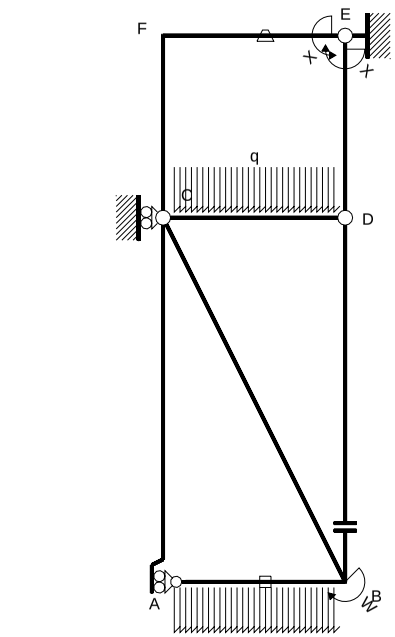
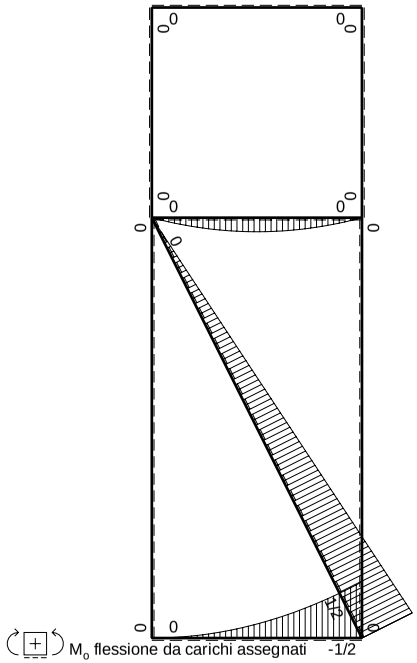
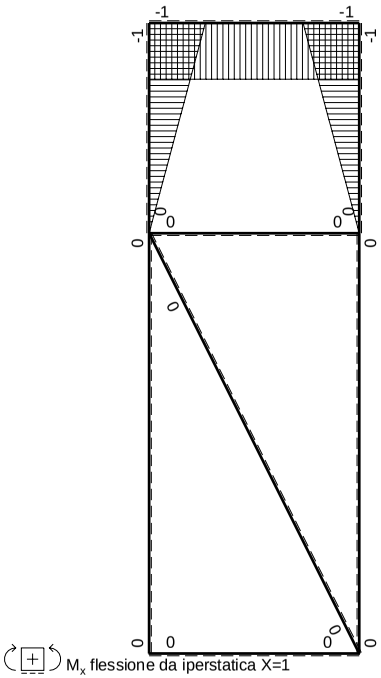
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

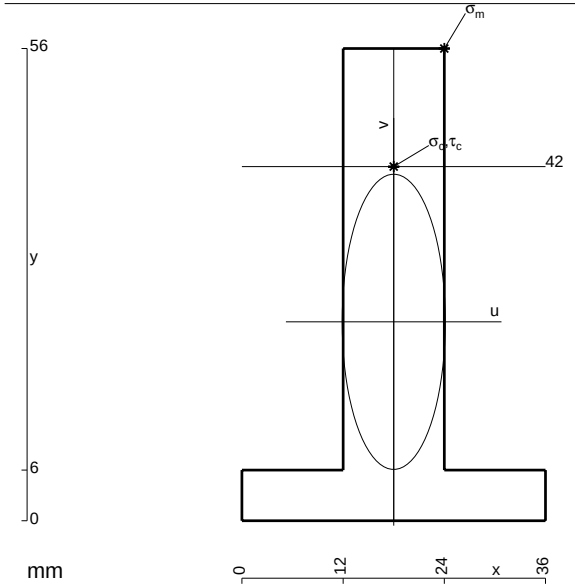
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 816. mm^2
J_u = 250166. mm^4
J_v = 30528. mm^4
y_g = 23.59 mm
T_y = -5070. N
M_x = -1774500. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 32.41 mm
$\sigma_m = -Mv/J_u$ = 229.9 N/mm^2
x_c = 18. mm
y_c = 42. mm
v_c = 18.41 mm
$\sigma_c = -Mv/J_u$ = 130.6 N/mm^2
τ_c = 7.21 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.2 N/mm^2
S^* = 4269. mm^3

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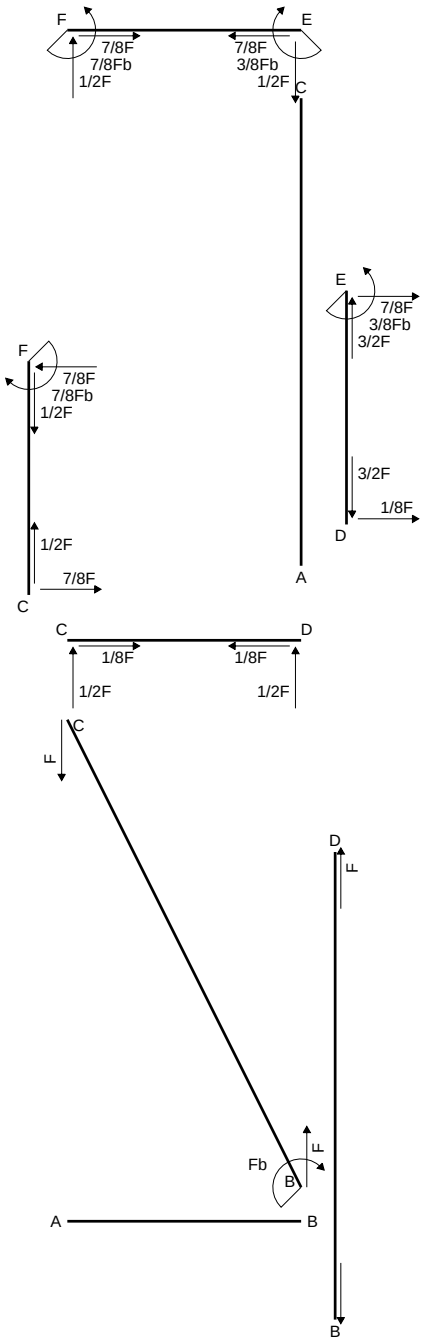
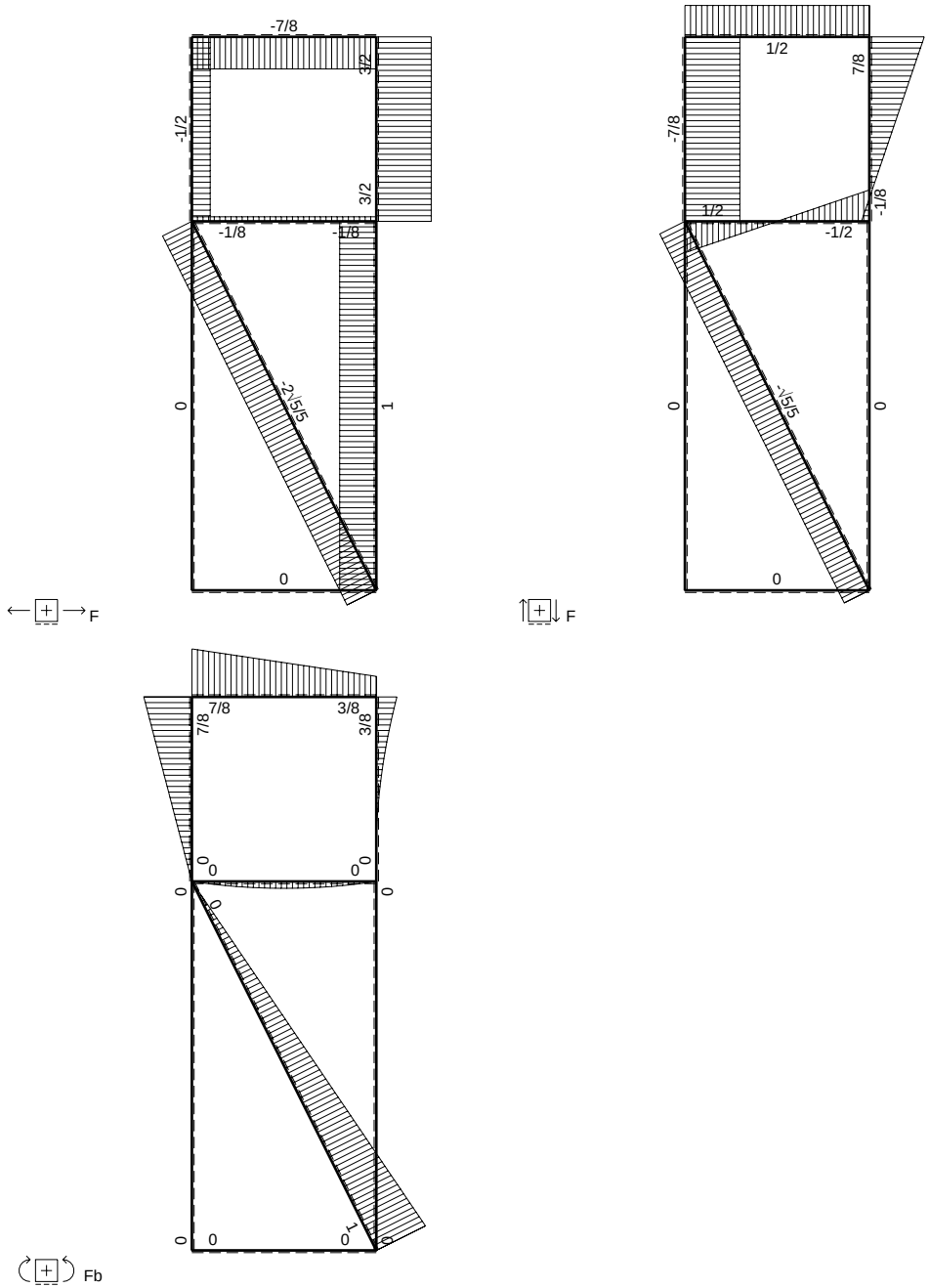

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
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3/2
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-1/8
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0
F
1/2
7/8
-7/8
1/2
-1/8
-1/2
0
-√5/5
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7/8
3/8
7/8
3/8
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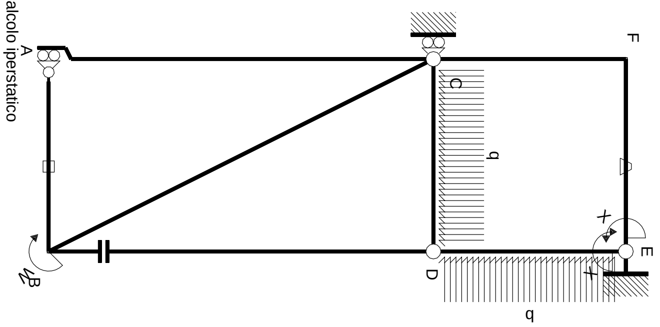
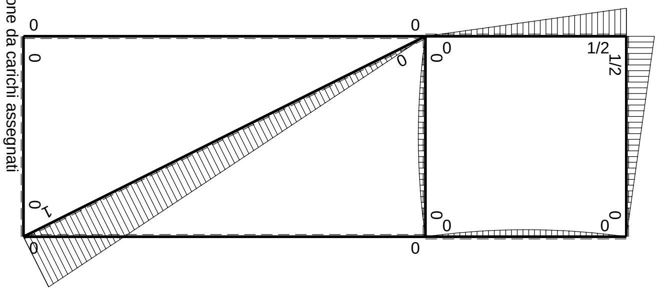
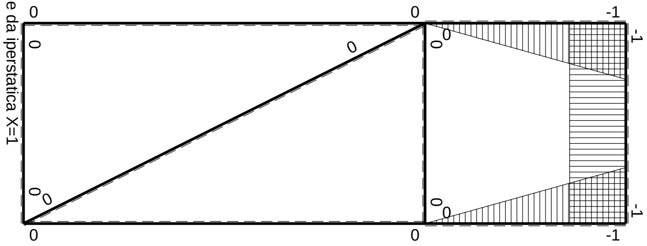
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{EF}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica X=W_{EF}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

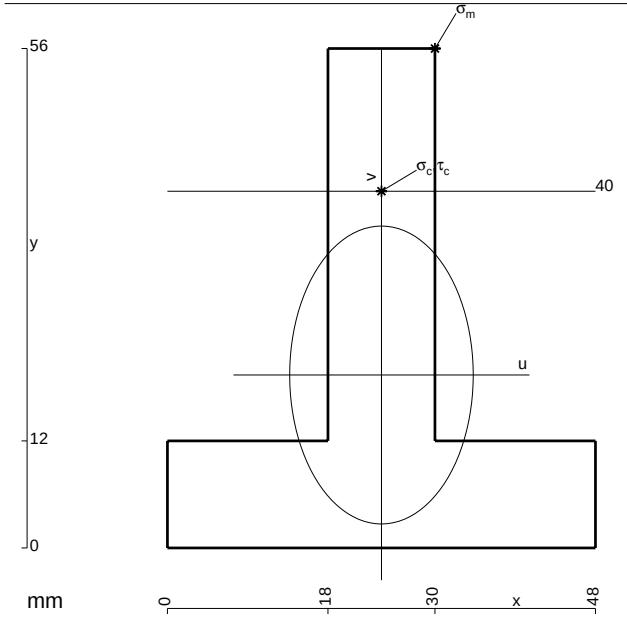
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 1104. mm^2
J_u = 308071. mm^4
J_v = 116928. mm^4
y_g = 19.39 mm
N = -2415. N
T_y = -1207. N
M_x = 1998000. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 36.61 mm
$\sigma_m = N/A - Mv/J_u$ = -239.6 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 20.61 mm
$\sigma_c = N/A - Mv/J_u$ = -135.8 N/mm^2
τ_c = 1.794 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.9 N/mm^2
S^* = 5493. mm^3

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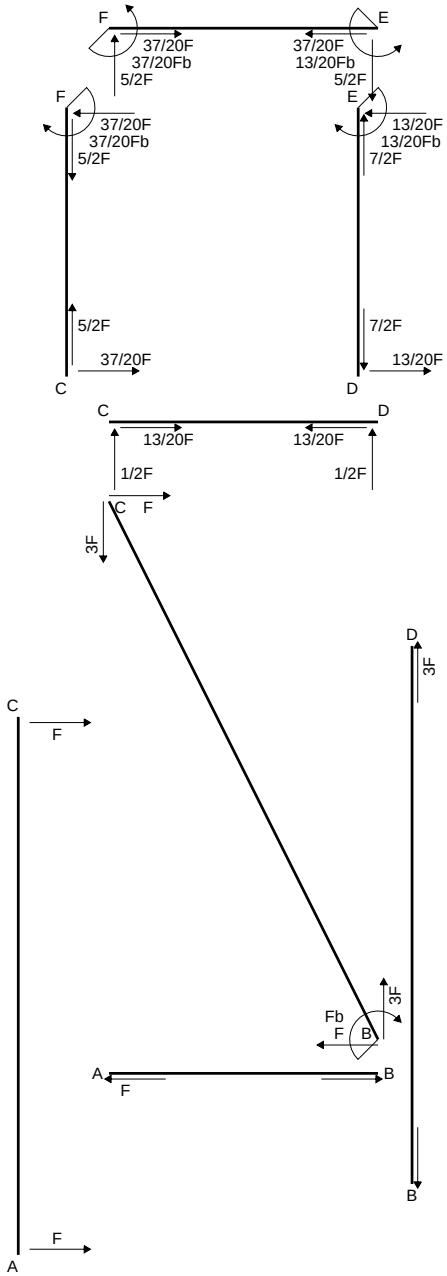
F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
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5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
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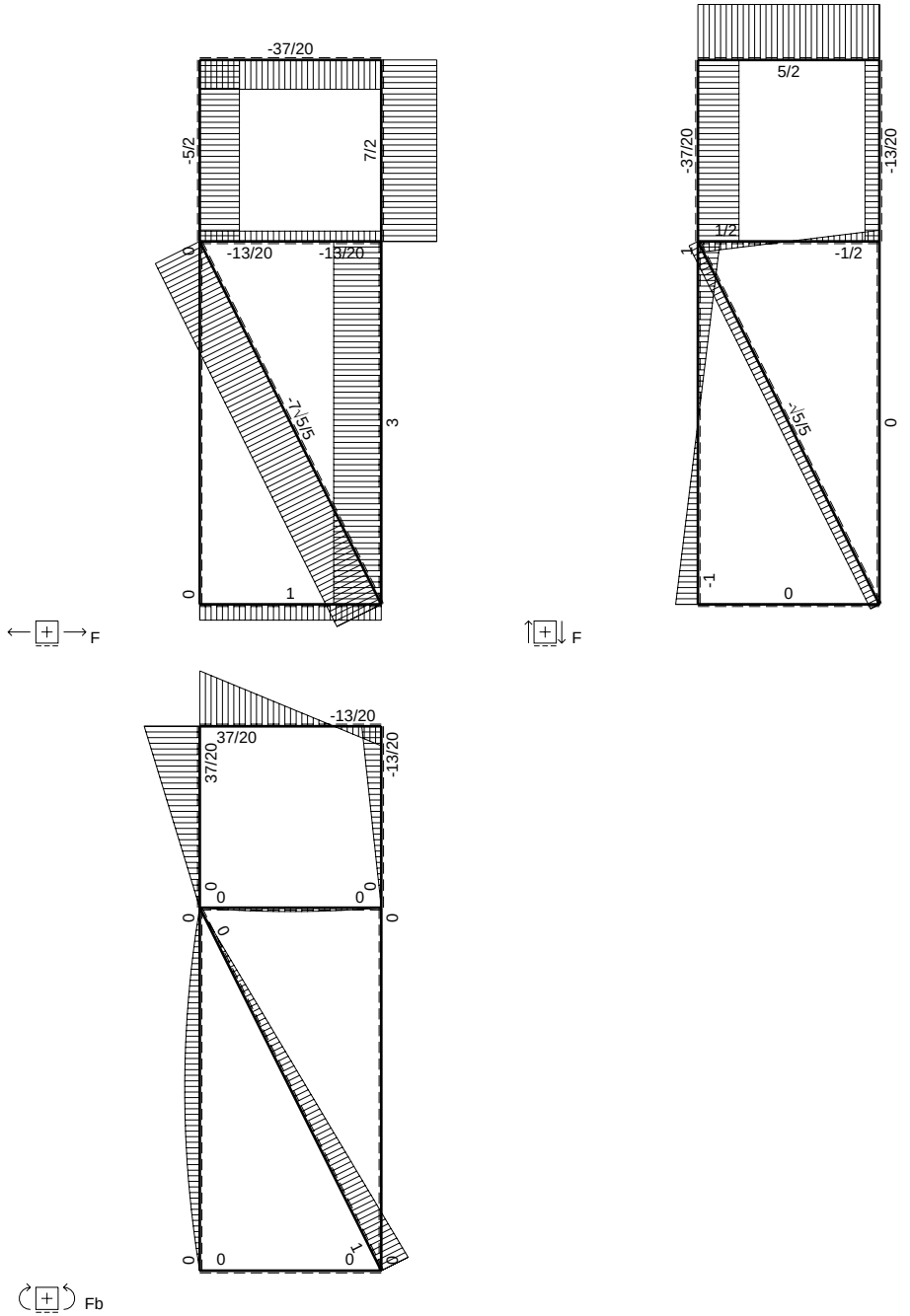
3F
Fb
F B
A
B
F
F
A
B


-37/20
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7/2
-13/20
-13/20
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-7√5/5
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F
5/2
-37/20
-13/20
1/2
-1/2
-7√5/5
0
-1
0
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-13/20
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37/20
-13/20
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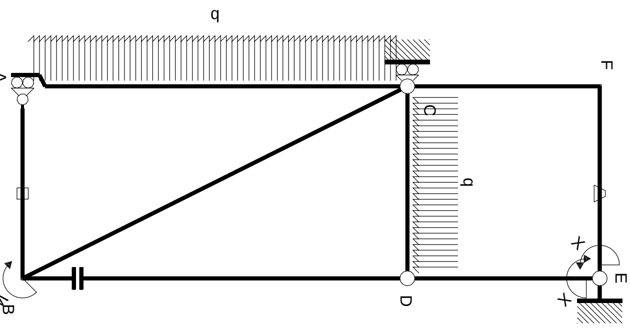
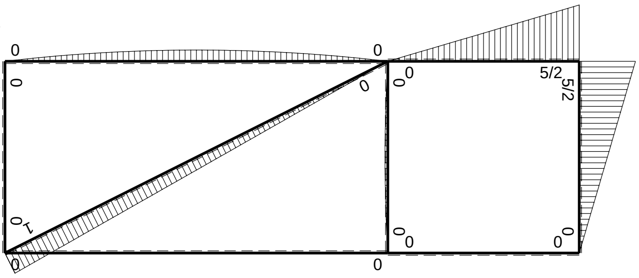
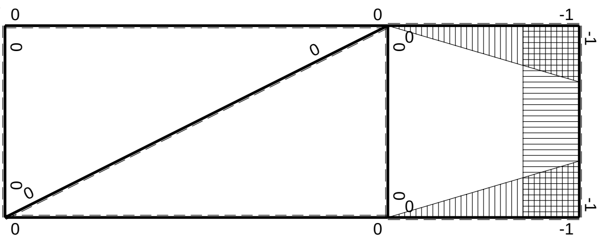
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

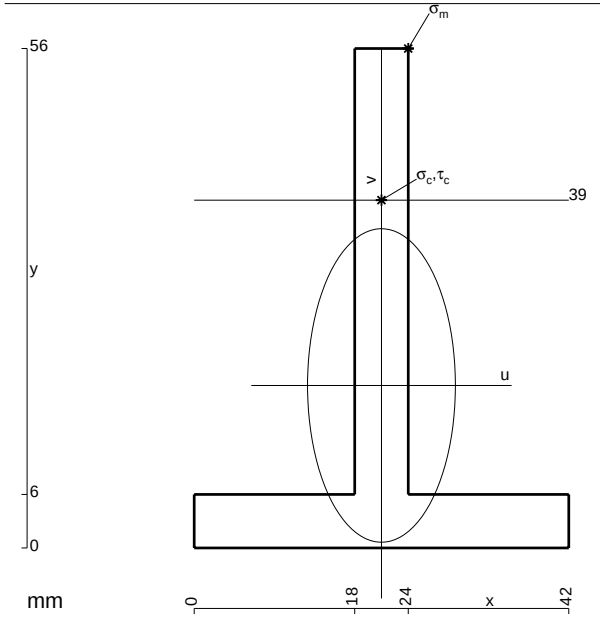
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 552. mm^2
J_u = 170630. mm^4
J_v = 37944. mm^4
y_g = 18.22 mm
N = -6793. N
T_y = -970.5 N
M_x = 846300. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 37.78 mm
$\sigma_m = N/A - Mv/J_u$ = -199.7 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 20.78 mm
$\sigma_c = N/A - Mv/J_u$ = -115.4 N/mm^2
τ_c = 2.831 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.5 N/mm^2
S^* = 2987. mm^3

ESΣ09.xxxx.080

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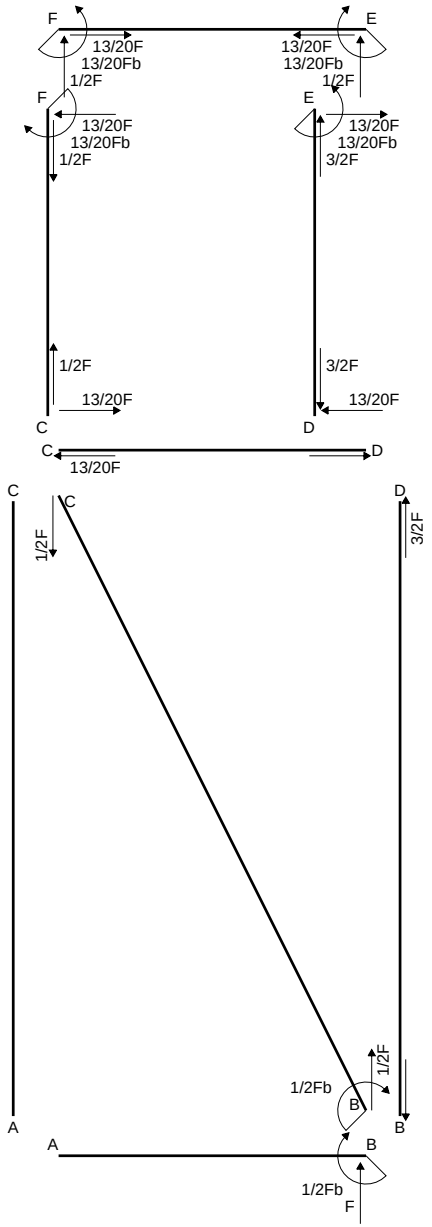
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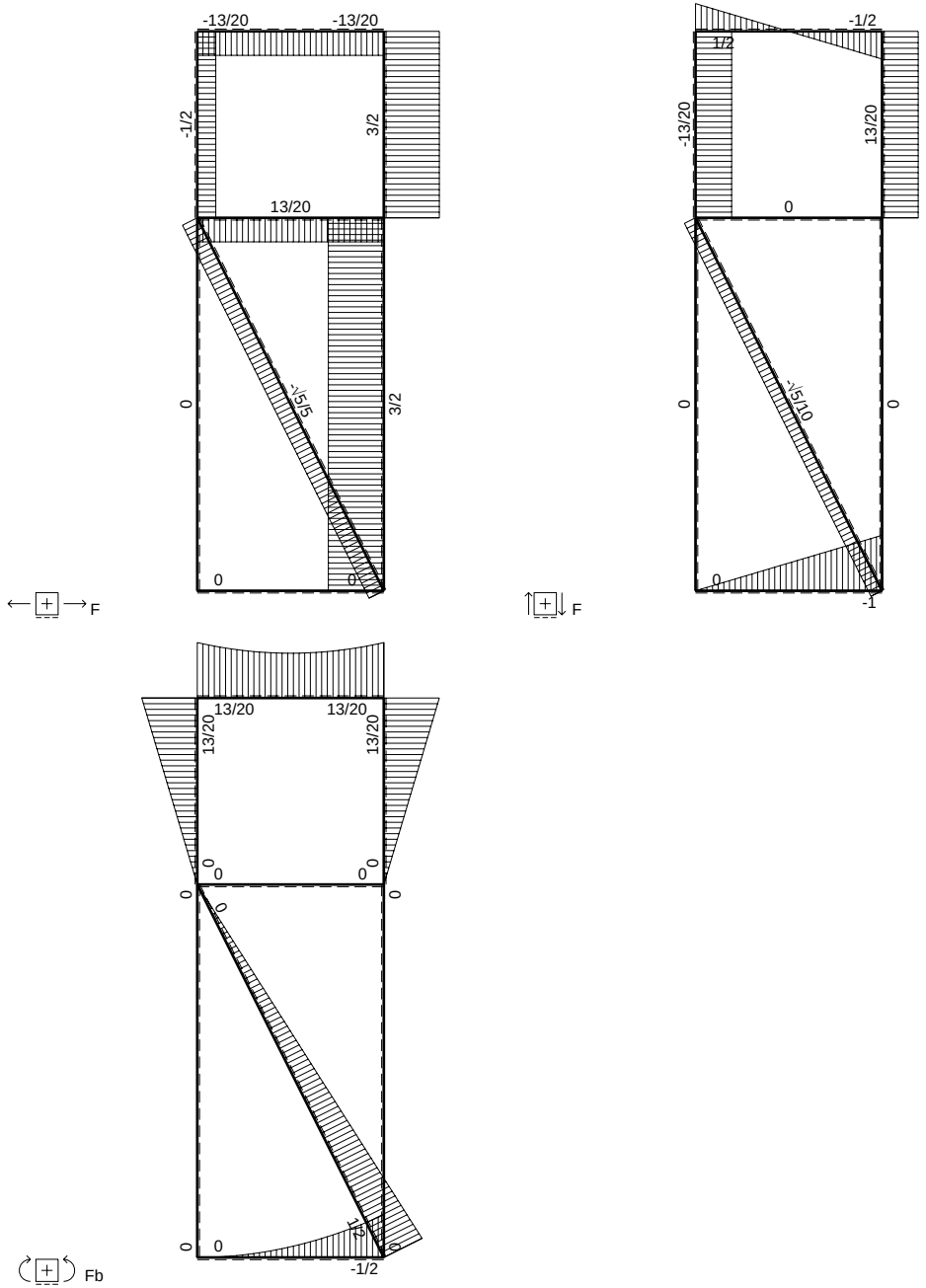
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb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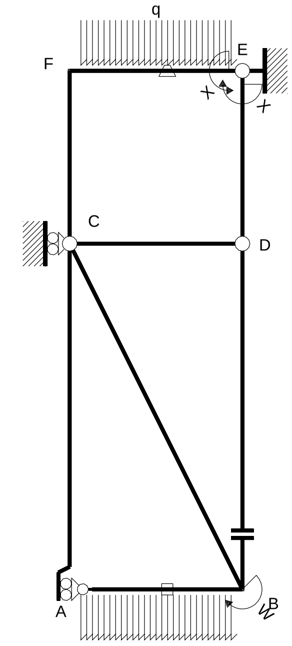
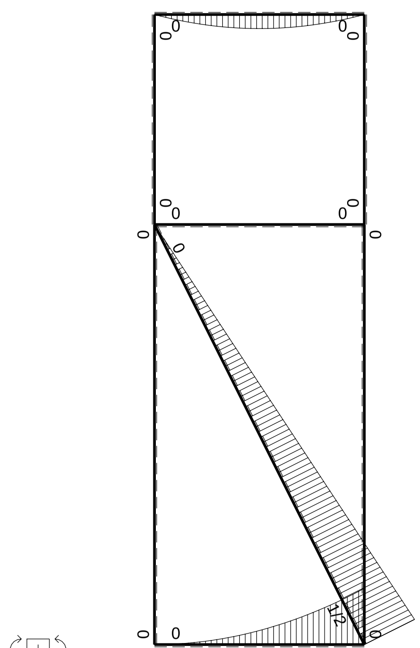
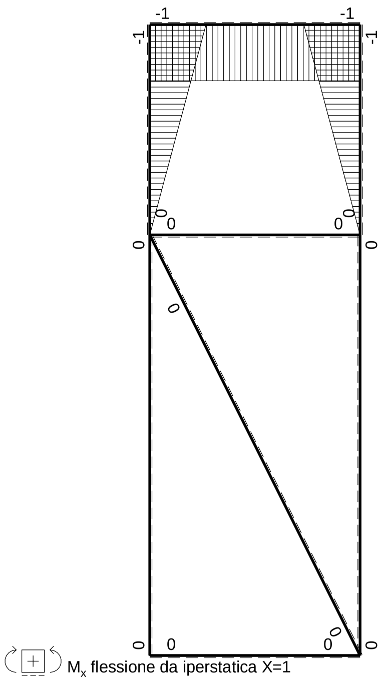
13/20
-1/2
3/2
0
-√5/5
-√5/10
1/2
-1
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

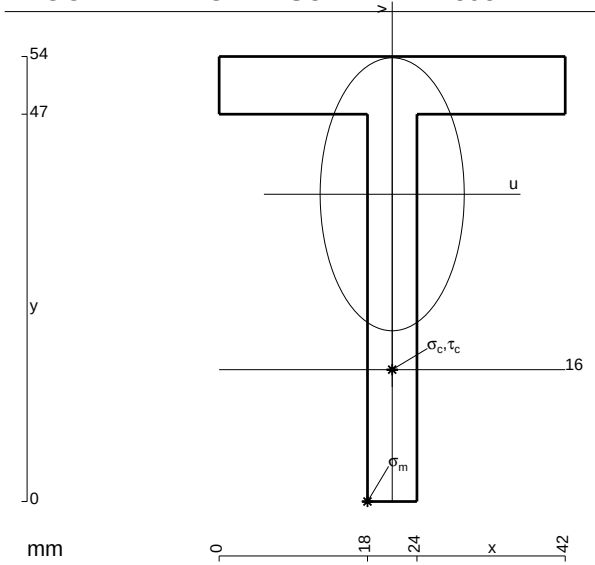
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 214395 ESΣ09.xxxx.081



A = 576. mm^2
J_u = 158042. mm^4
J_v = 44064. mm^4
y_g = 37.28 mm
T_y = -4340. N
M_x = -889700. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.28 mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.28 mm
$\sigma_c = -Mv/J_u$ = -119.8 N/mm^2
τ_c = 12.87 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.9 N/mm^2
S^* = 2811. mm^3

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ESΣ09.xxxx.081

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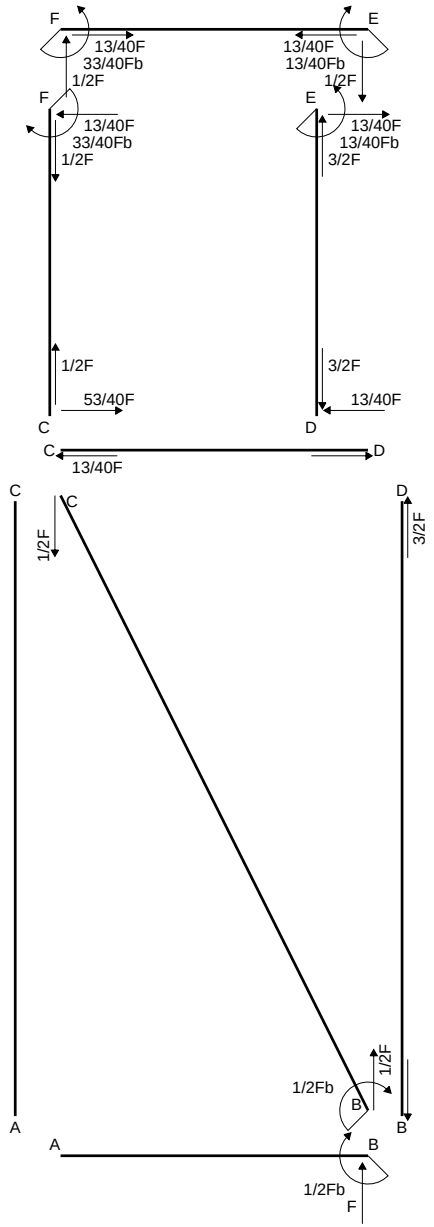
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
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1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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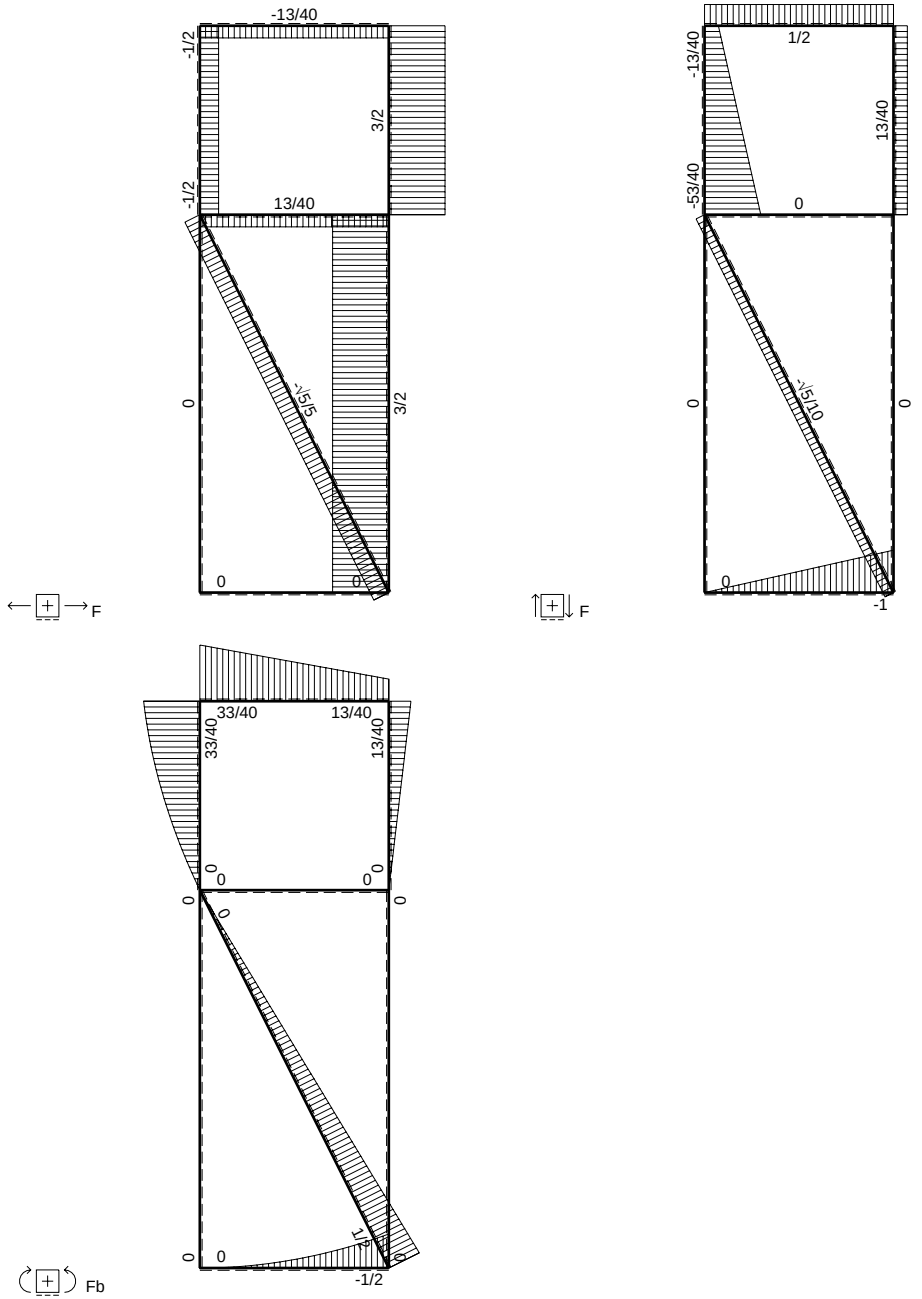
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
1/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211841 ESΣ09.xxxx.082

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

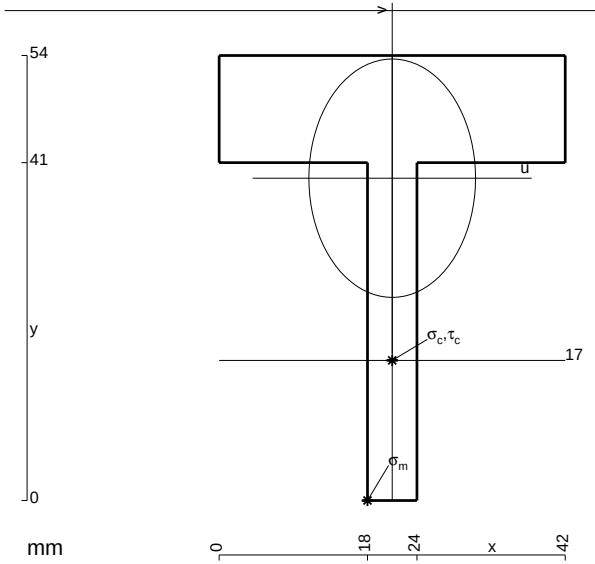
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 792. mm^2
J_u = 165782. mm^4
J_v = 81000. mm^4
y_g = 39.11 mm
T_y = -4140. N
M_x = -931500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.11 mm
$\sigma_m = -Mv/J_u$ = -219.8 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.11 mm
$\sigma_c = -Mv/J_u$ = -124.3 N/mm^2
τ_c = 13. N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.3 N/mm^2
S^* = 3123. mm^3

ESΣ09.xxxx.082

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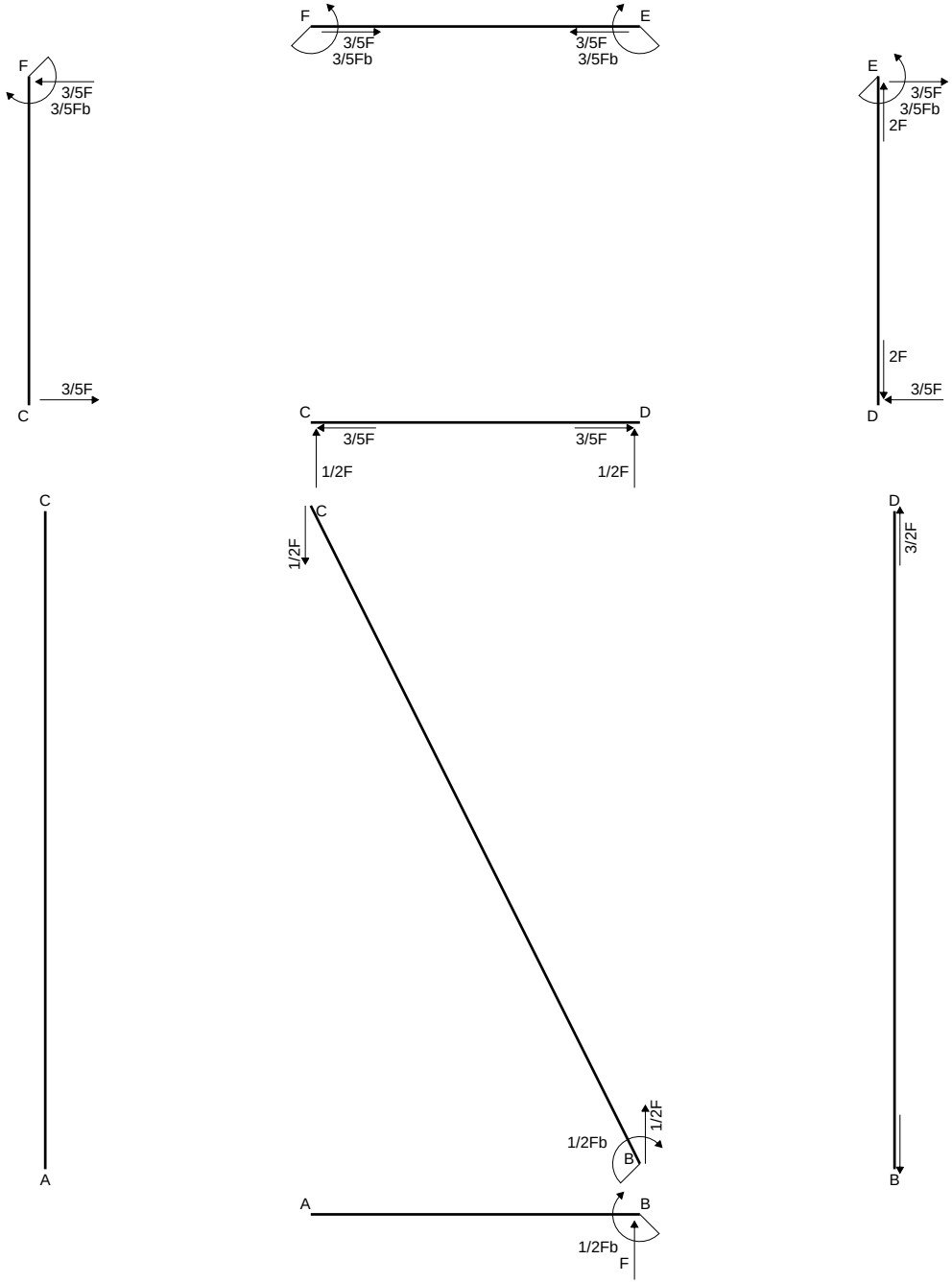
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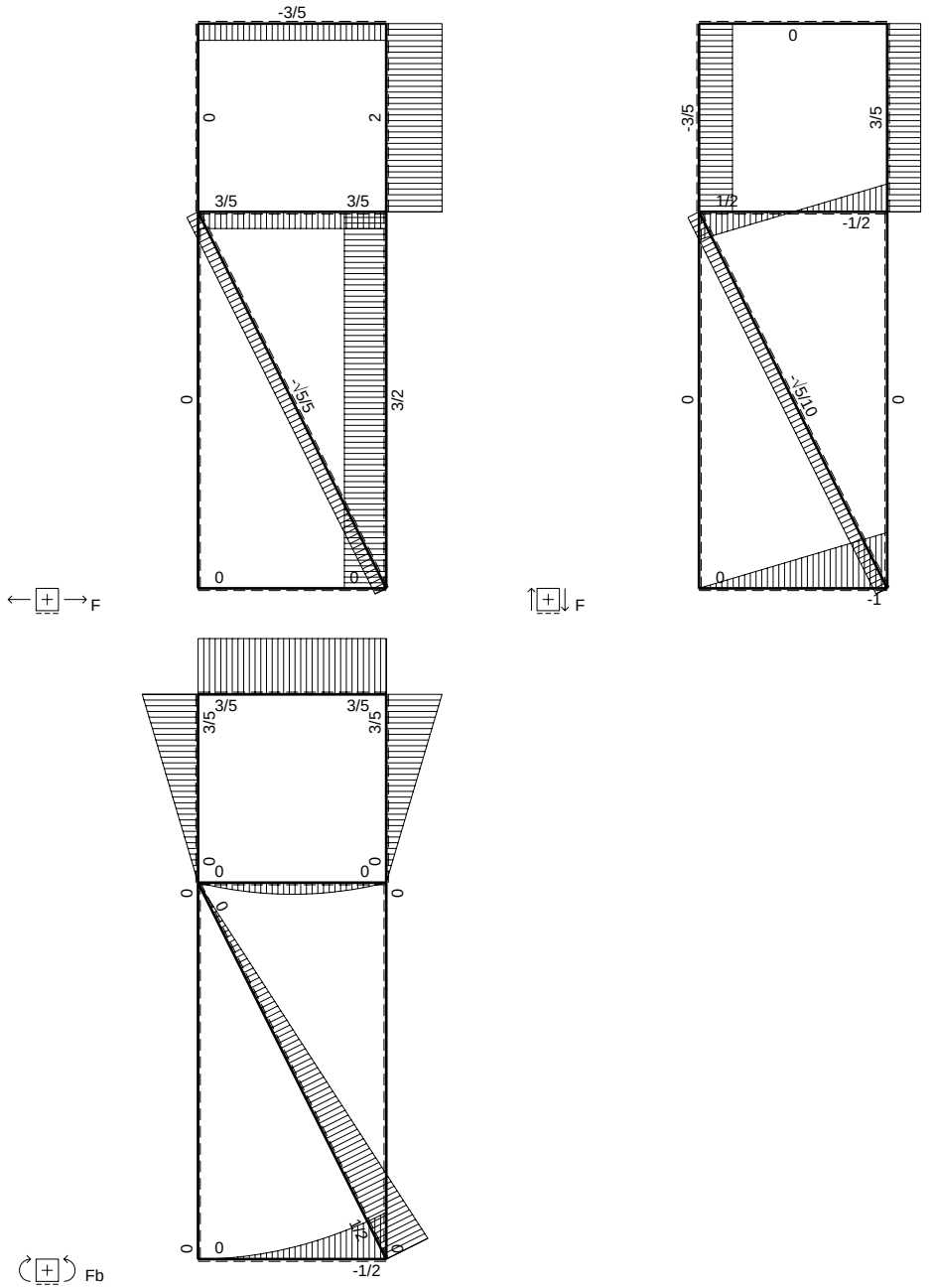
1/2Fb
F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-1/2
0
-√5/10
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0
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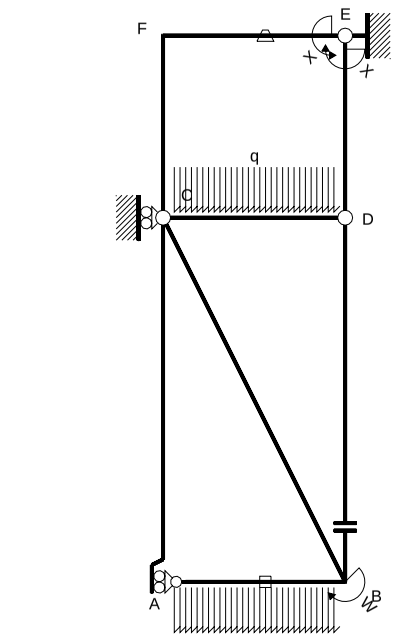
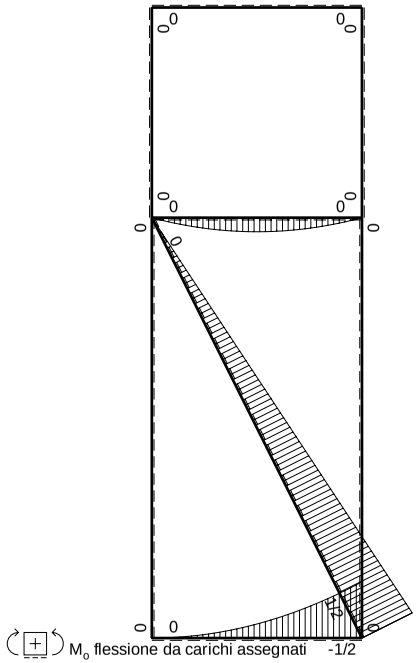
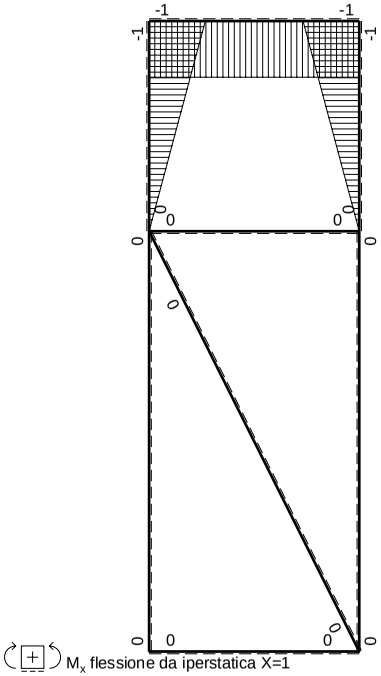
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

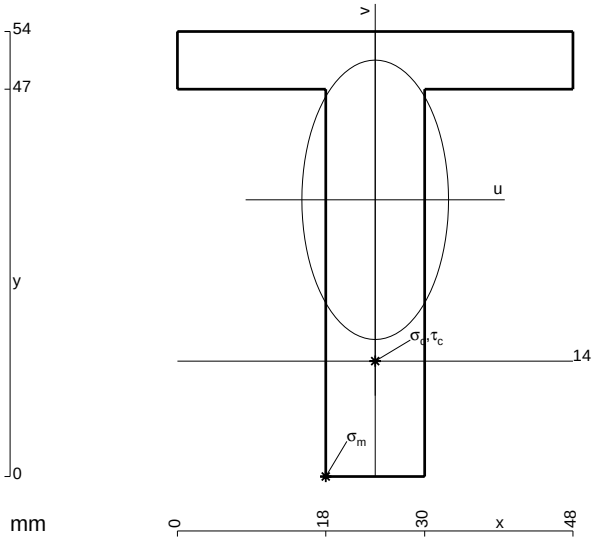
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 900. mm^2
J_u = 258693. mm^4
J_v = 71280. mm^4
y_g = 33.58 mm
T_y = -7230. N
M_x = -1771350. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.58 mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.58 mm
$\sigma_c = -Mv/J_u$ = -134.1 N/mm^2
τ_c = 10.4 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135.3 N/mm^2
S^* = 4465. mm^3

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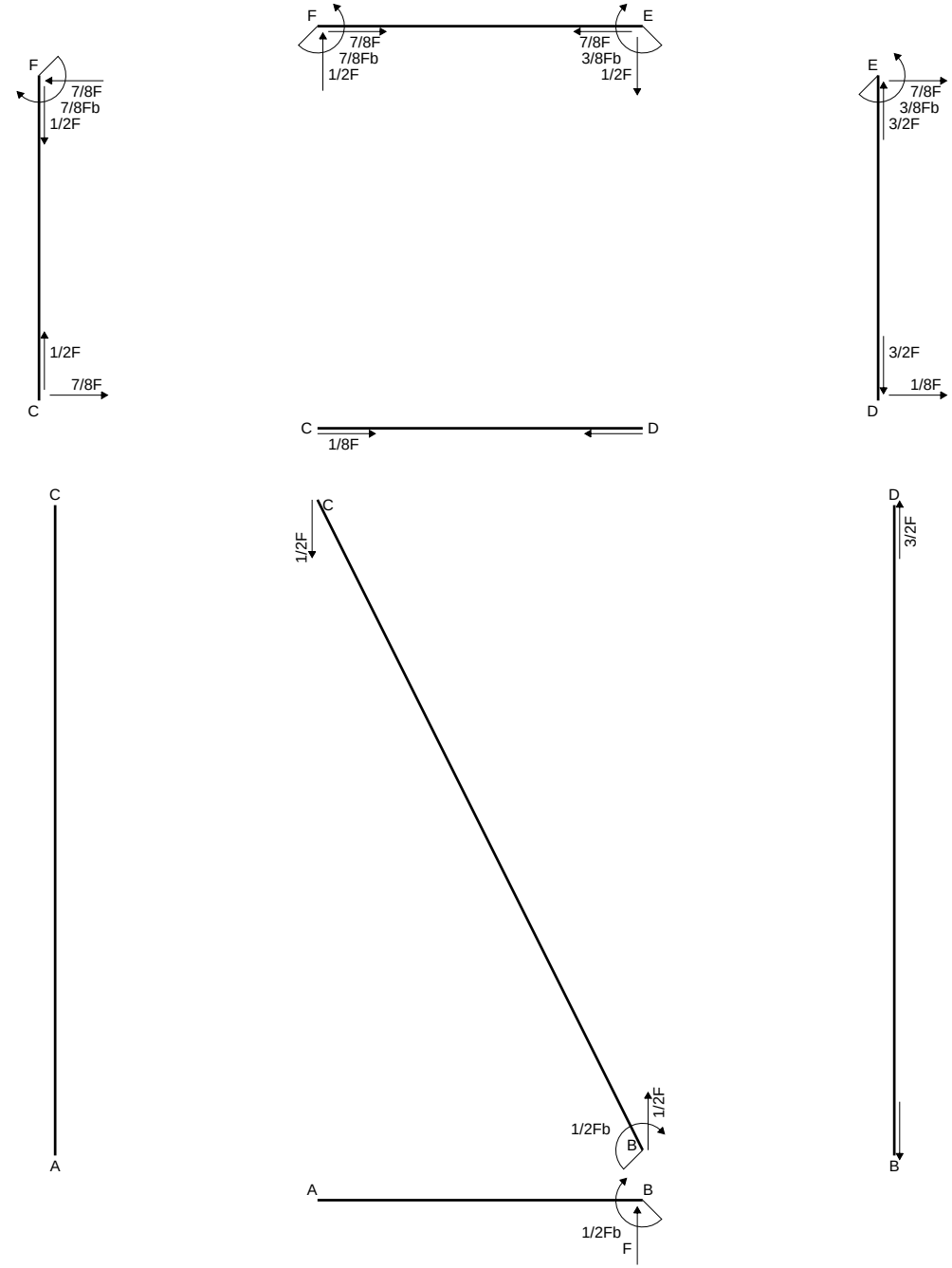
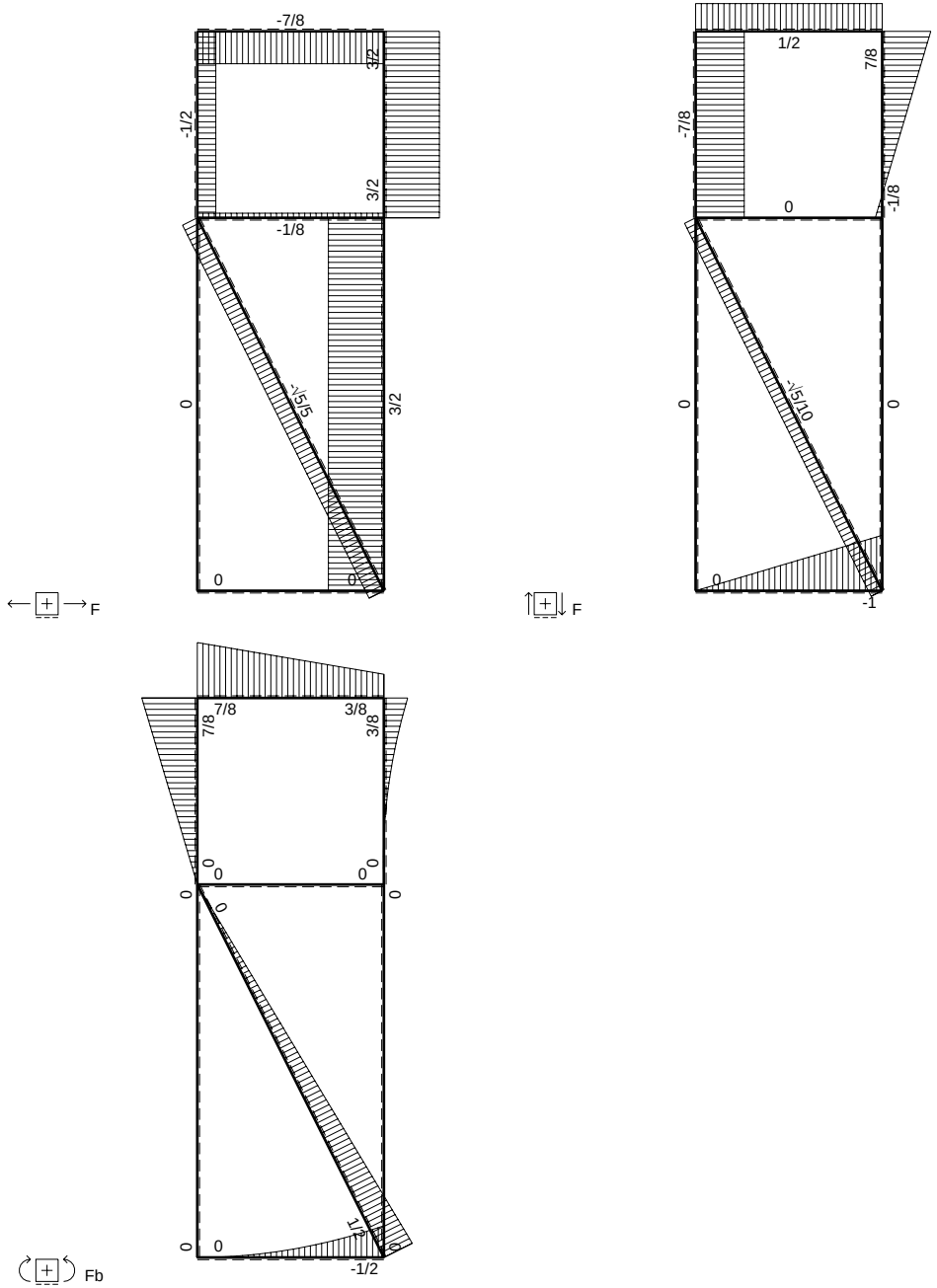

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
-1/2
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

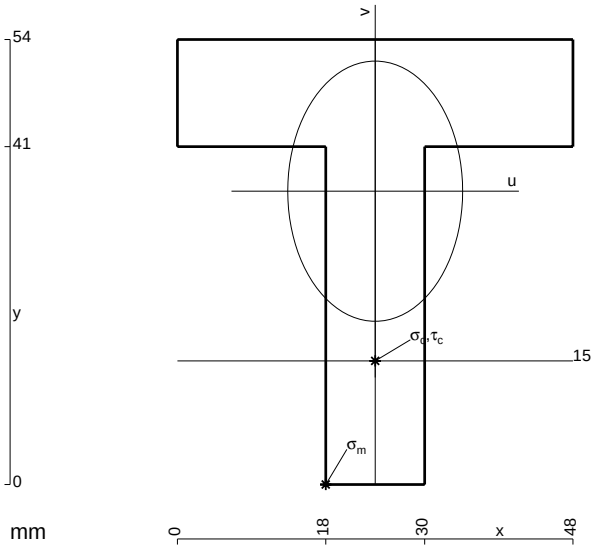
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 1116. mm^2
J_u = 278254. mm^4
J_v = 125712. mm^4
y_g = 35.6 mm
T_y = -7070. N
M_x = -1873550. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.6 mm
$\sigma_m = -Mv/J_u$ = -239.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.6 mm
$\sigma_c = -Mv/J_u$ = -138.7 N/mm^2
τ_c = 10.71 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 139.9 N/mm^2
S^* = 5057. mm^3

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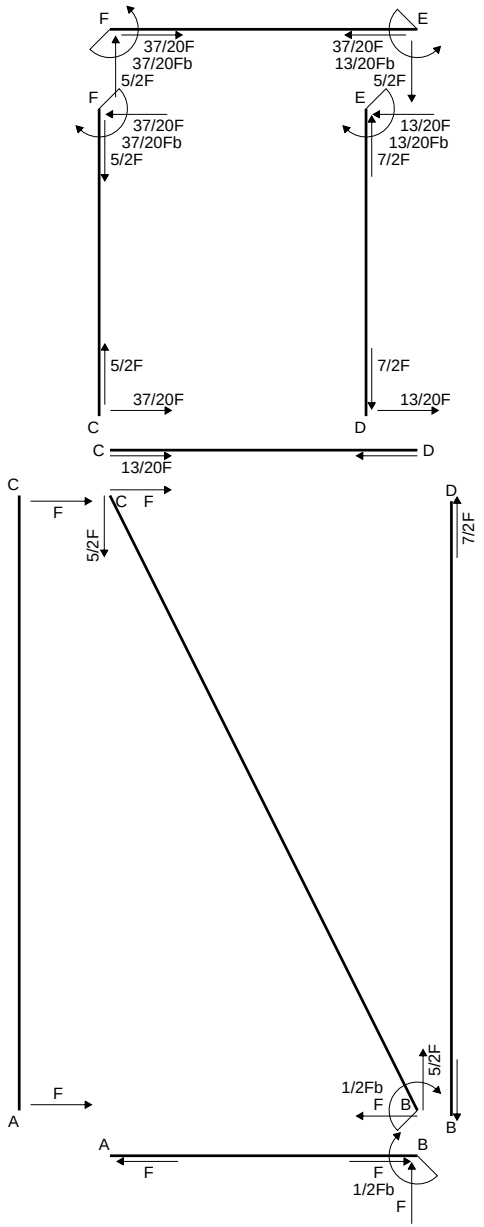
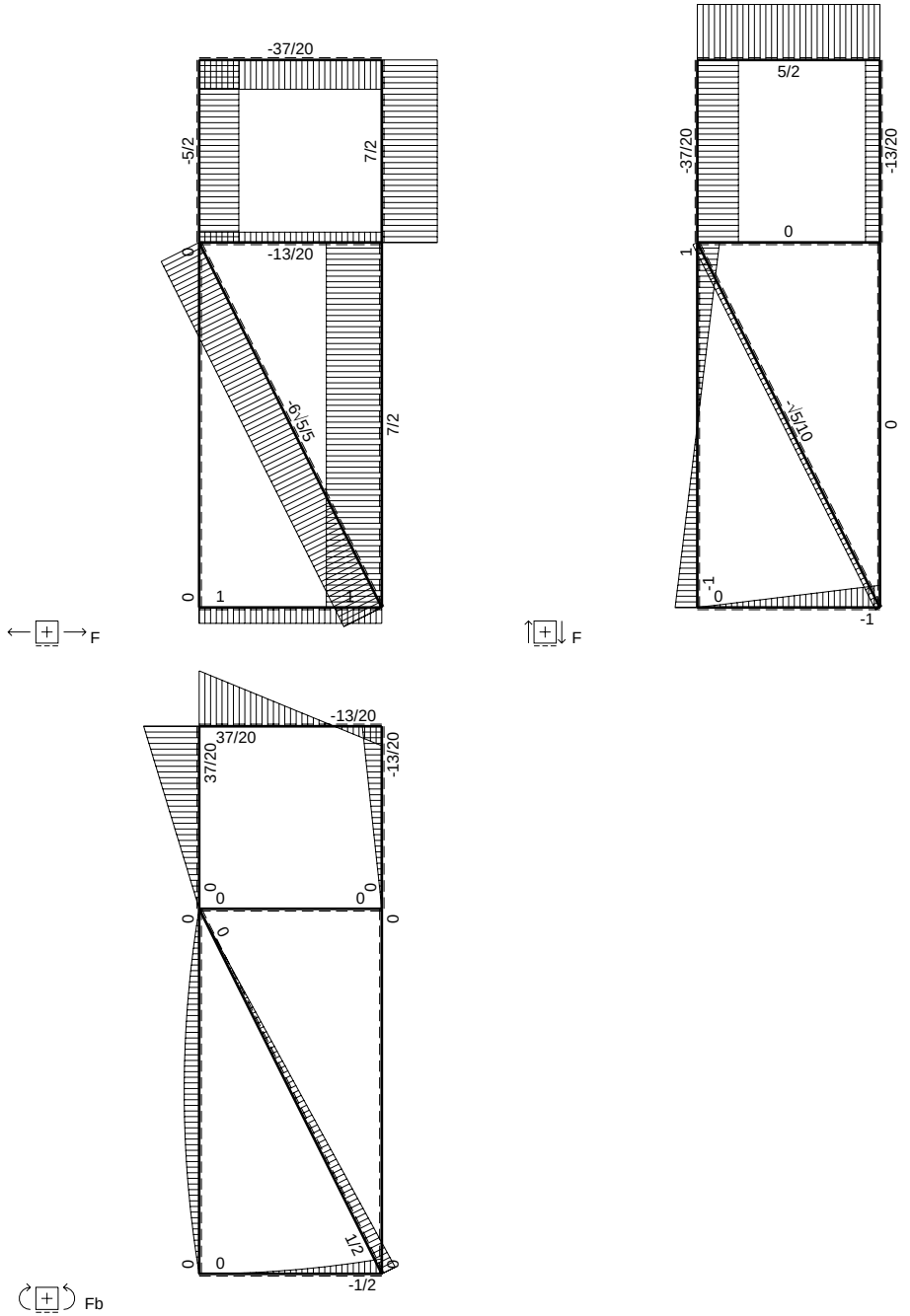
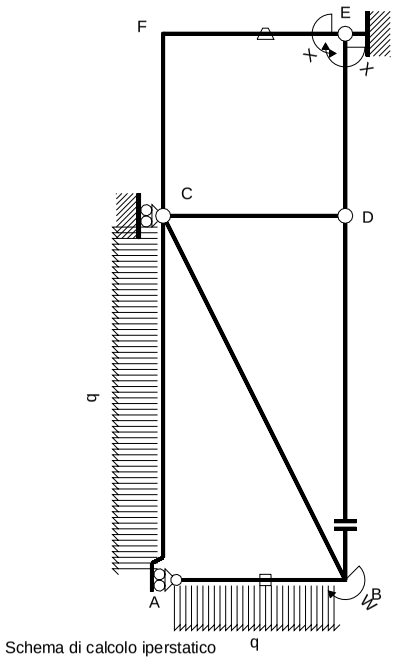
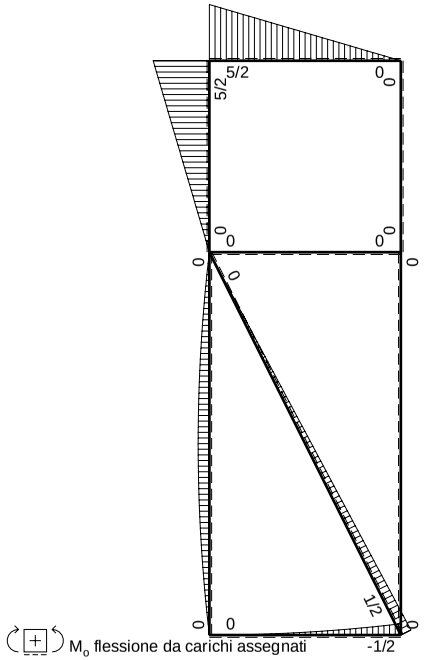

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

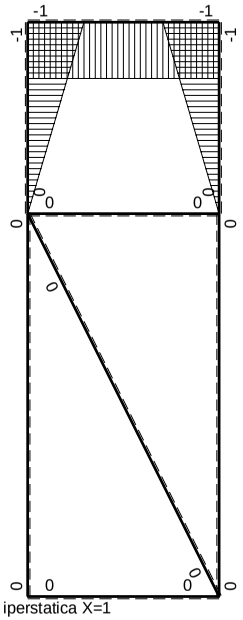

-37/20
5/2
7/2
-13/20
-5/√5
-√5/10
F
Fb
-1/2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
				totali			$-13/12Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 960387 ESΣ09.xxxx.085

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

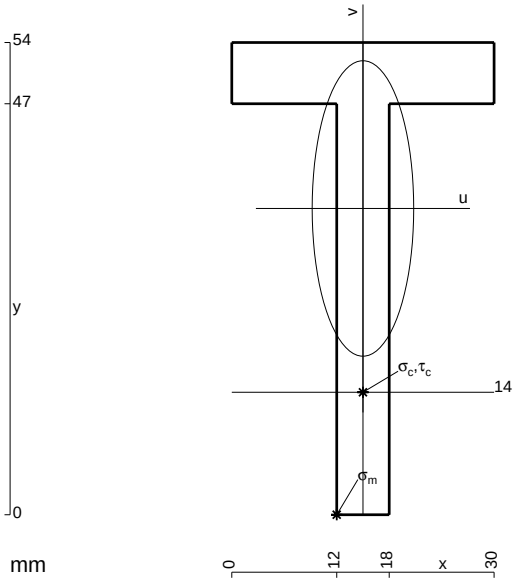
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 492. mm^2
J_u = 140516. mm^4
J_v = 16596. mm^4
y_g = 35.02 mm
N = 2890. N
T_y = -2890. N
M_x = -823650. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.02 mm
$\sigma_m = N/A - Mv/J_u$ = -199.4 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -21.02 mm
$\sigma_c = N/A - Mv/J_u$ = -117.4 N/mm^2
τ_c = 8.069 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.2 N/mm^2
S^* = 2354. mm^3

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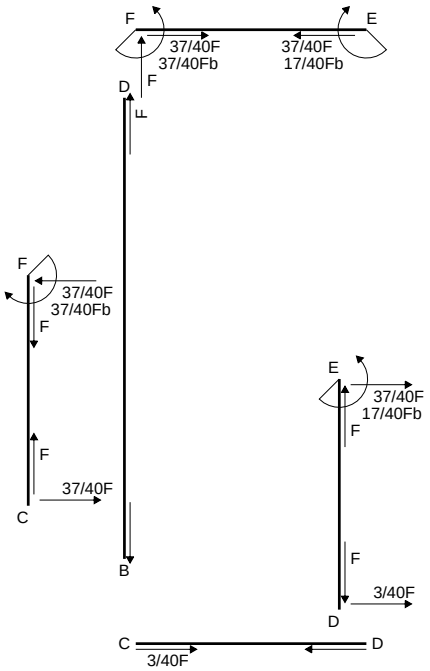
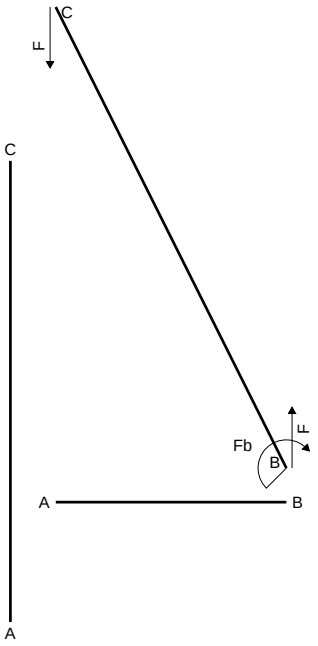
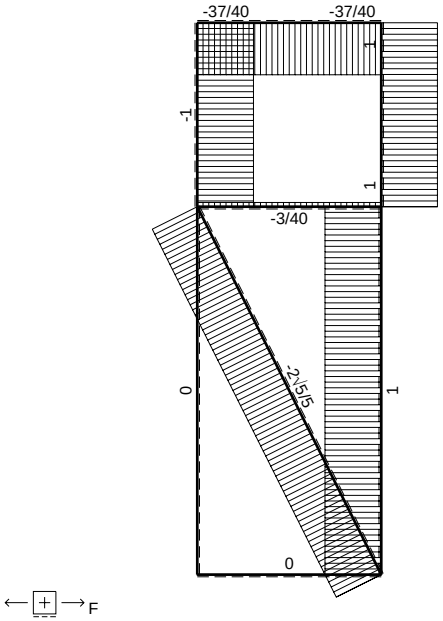
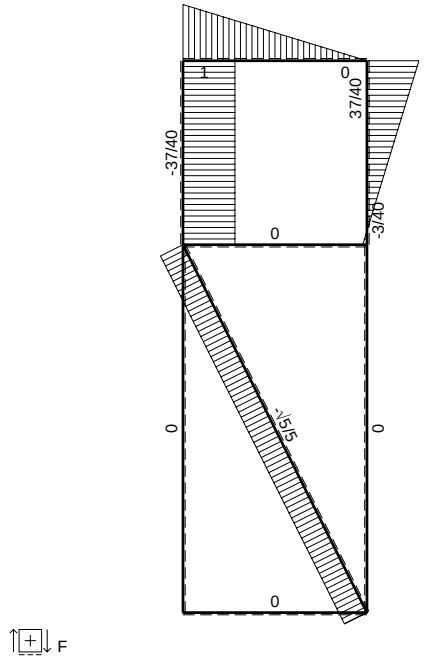
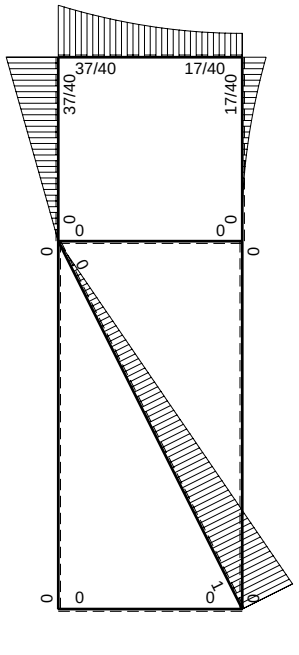
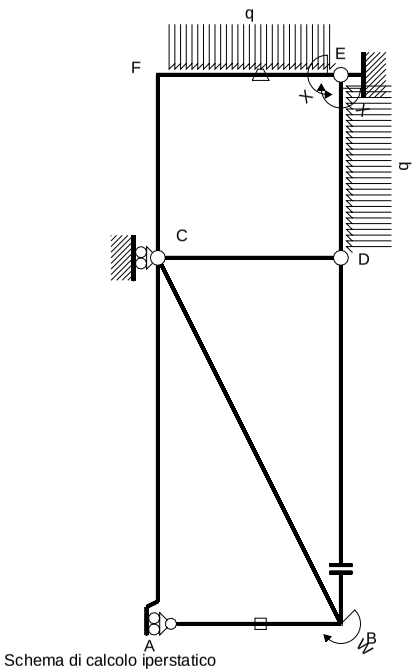

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D


C
F
C
Fb
F
B
A
B
A

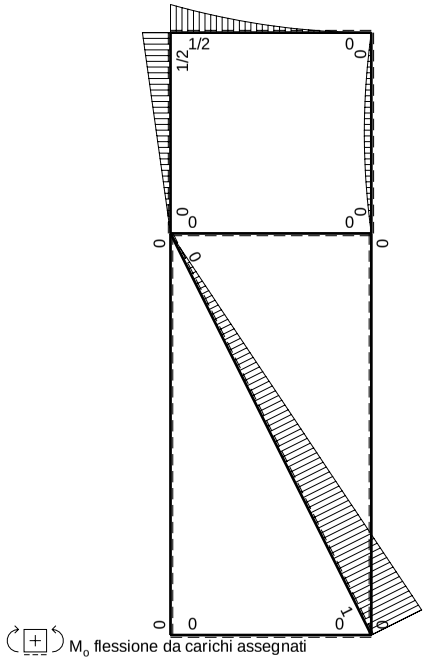

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F

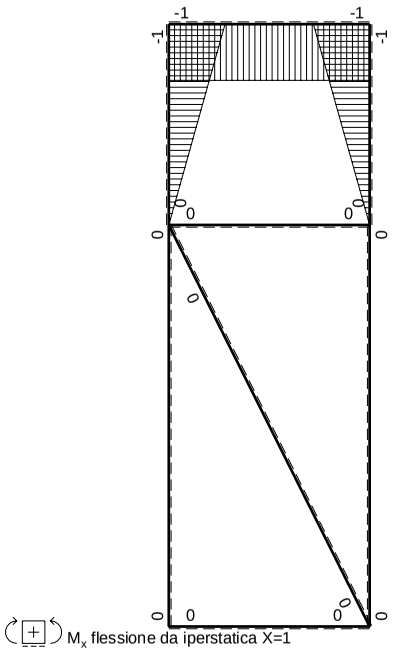

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 956389 ESΣ09.xxxx.086

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

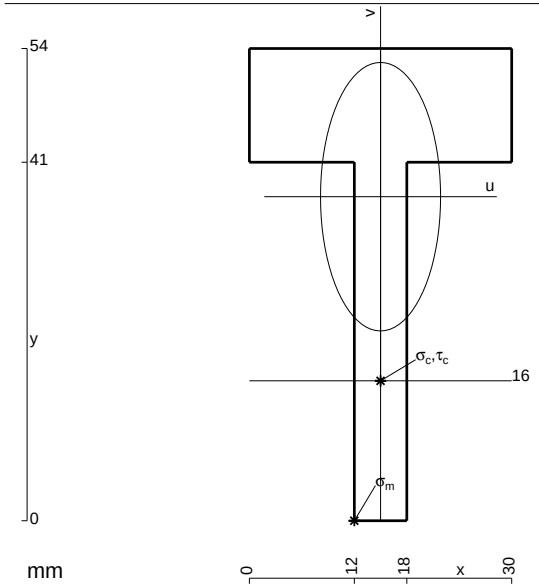
$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 636. mm^2
J_u = 149922. mm^4
J_v = 29988. mm^4
y_g = 37.06 mm
N = -1270. N
T_y = -635. N
M_x = 852000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.06 mm
$\sigma_m = N/A - Mv/J_u$ = 208.6 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.06 mm
$\sigma_c = N/A - Mv/J_u$ = 117.7 N/mm^2
τ_c = 1.969 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.7 N/mm^2
S^* = 2789. mm^3

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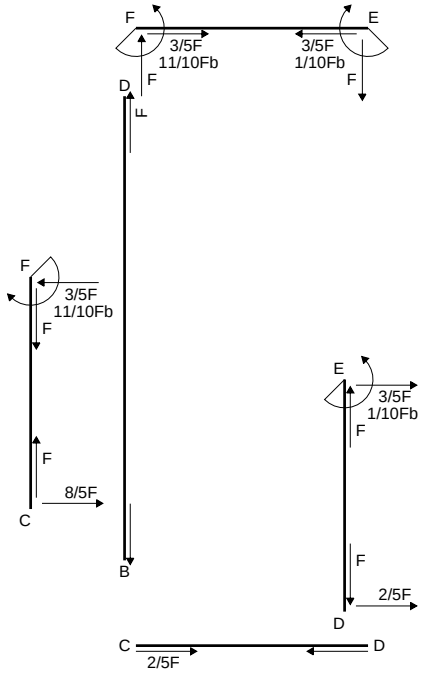
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
3/5F
1/10Fb
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8/5F
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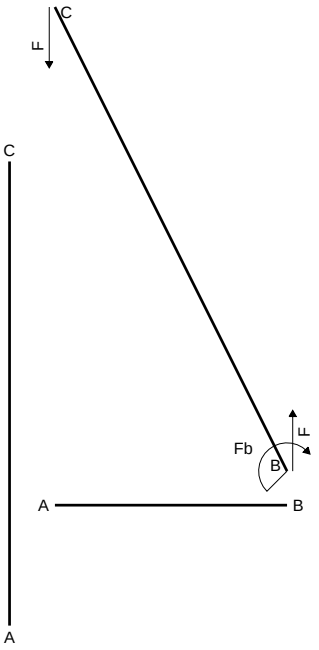
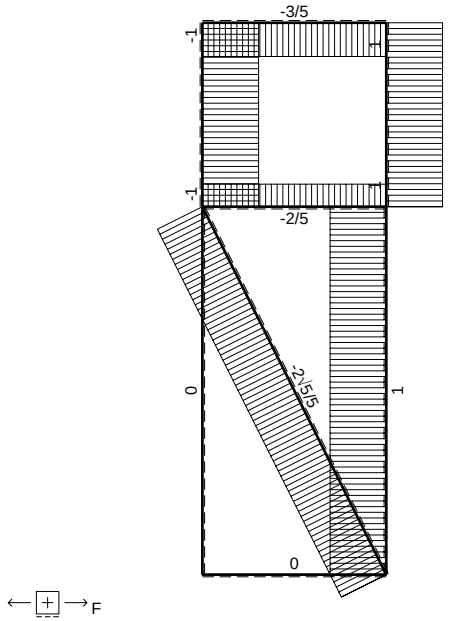
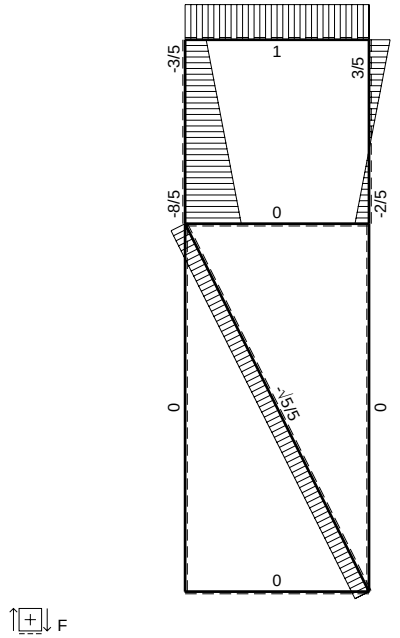
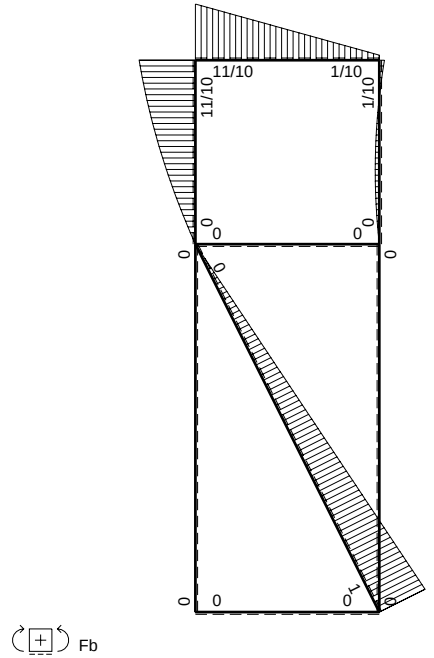
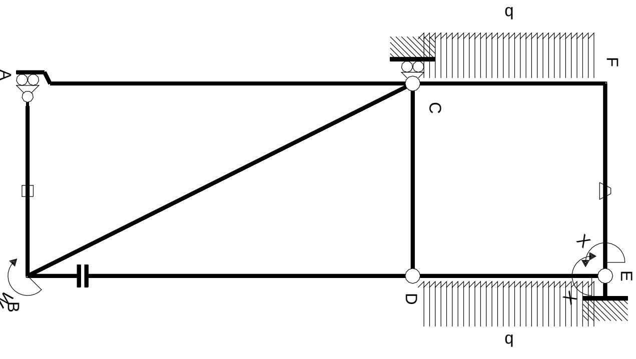
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

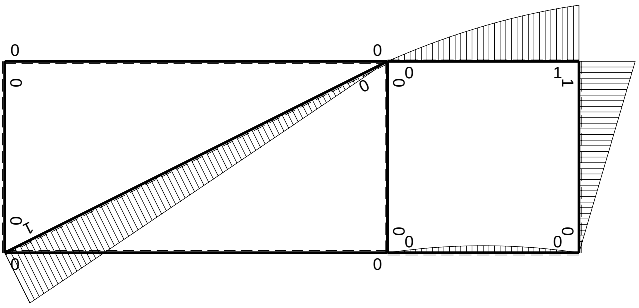

-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F

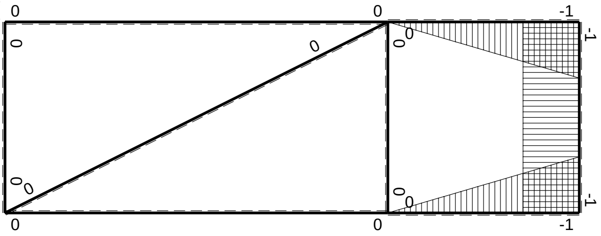

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

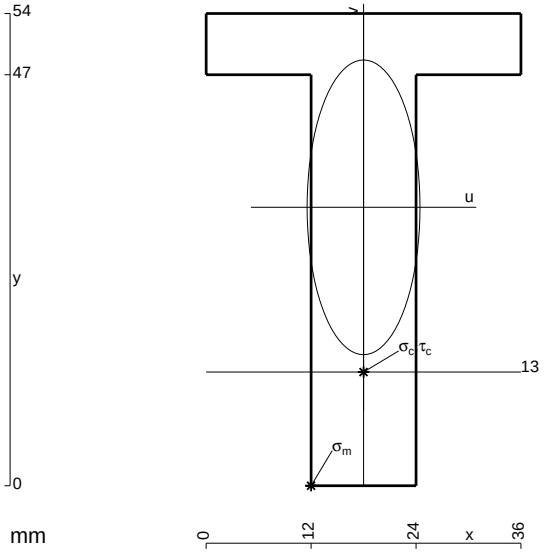
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 816. mm^2
J_u = 231827. mm^4
J_v = 33984. mm^4
y_g = 31.84 mm
N = -2263. N
T_y = -1131. N
M_x = 1619200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.84 mm
σ_m = N/A-Mv/J_u = 219.6 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -18.84 mm
σ_c = N/A-Mv/J_u = 128.8 N/mm^2
τ_c = 1.608 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.8 N/mm^2
S^* = 3953. mm^3

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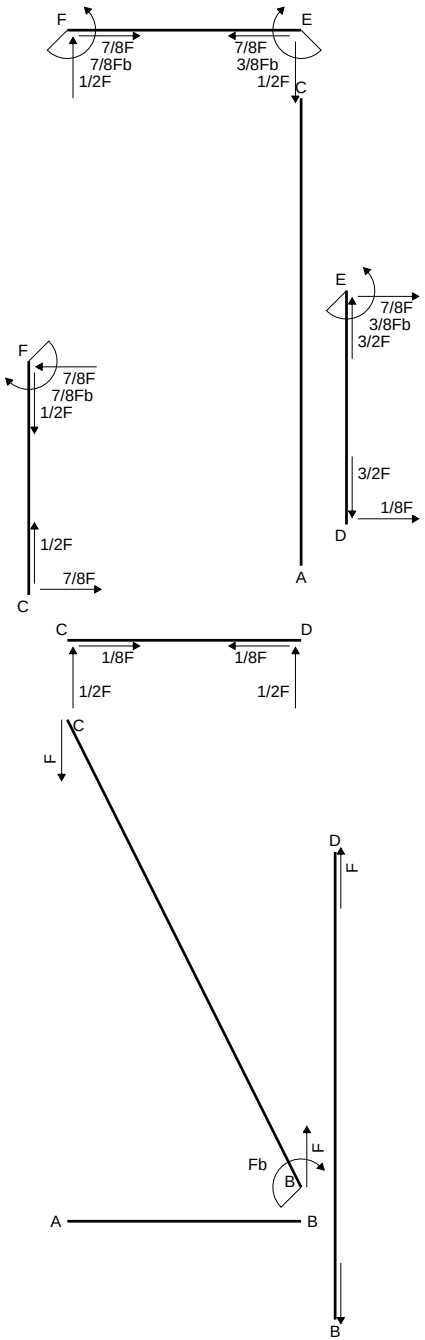
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
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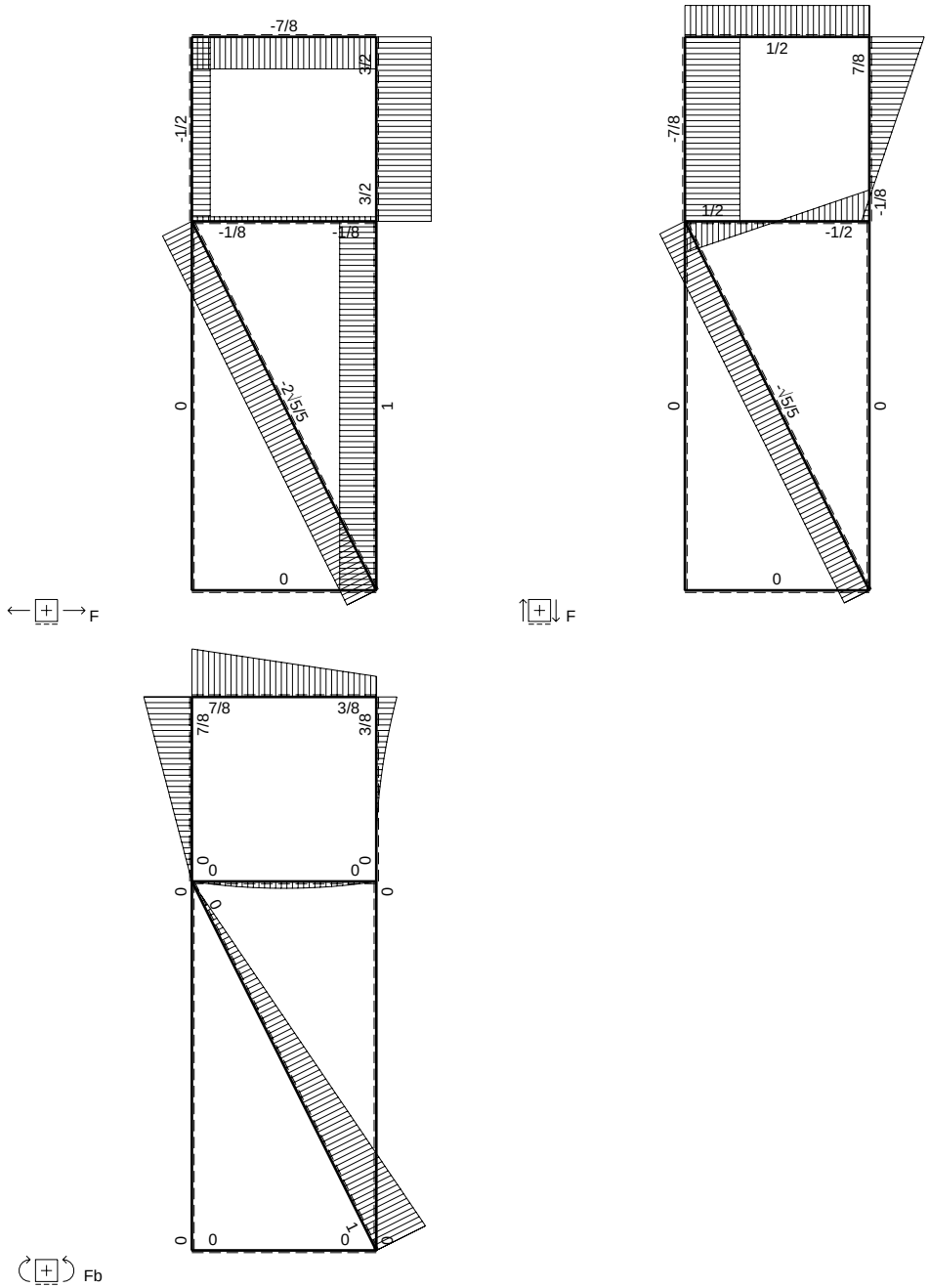
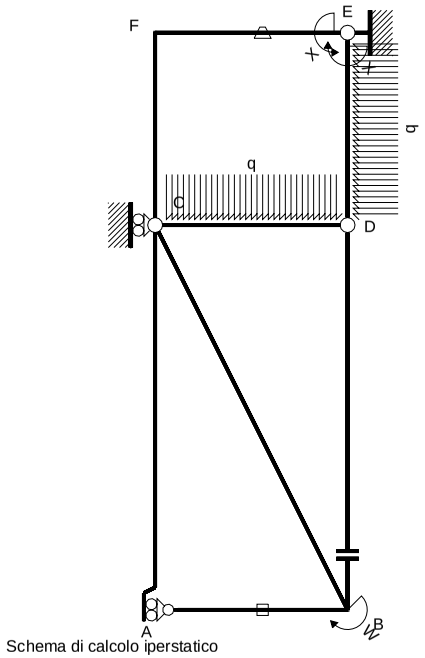
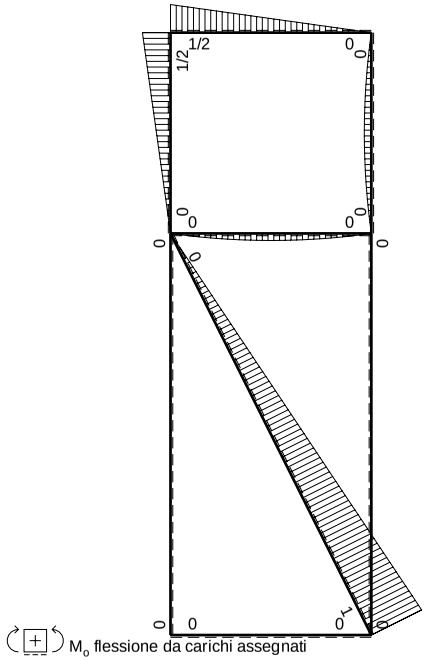
C
F
D
F
Fb
F
B
A
B
B

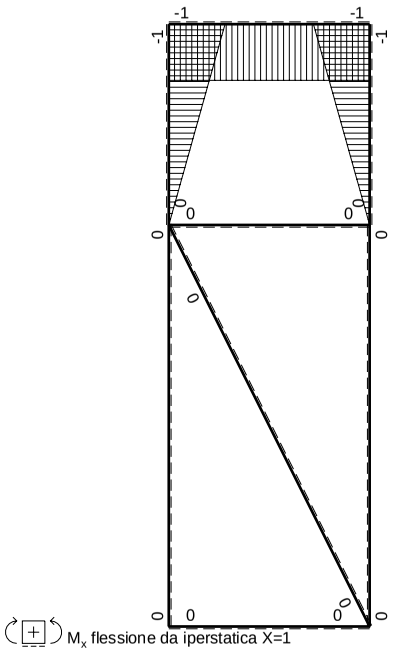

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
7/8
-7/8
1/2
-1/8
-1/2
0
-√5/5
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211720 ESΣ09.xxxx.088

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

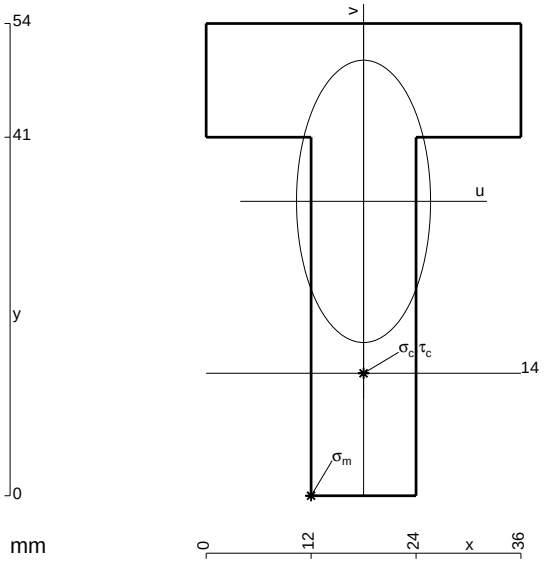
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 960. mm^2
J_u = 250363. mm^4
J_v = 56448. mm^4
y_g = 33.66 mm
N = -2272. N
T_y = -1136. N
M_x = 1727200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.66 mm
σ_m = N/A-Mv/J_u = 229.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.66 mm
σ_c = N/A-Mv/J_u = 133.3 N/mm^2
τ_c = 1.694 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.3 N/mm^2
S^* = 4479. mm^3

ESΣ09.xxxx.088

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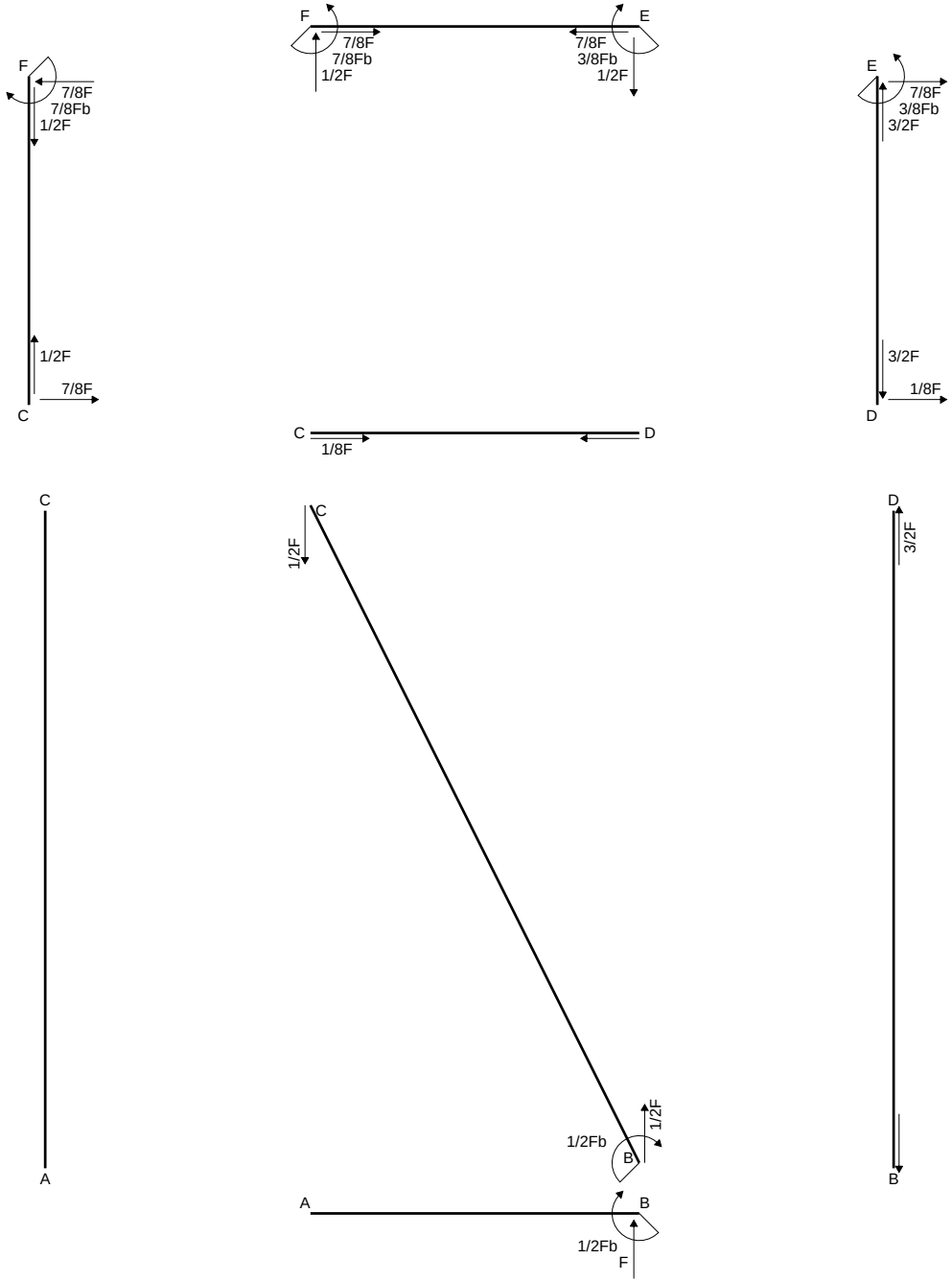
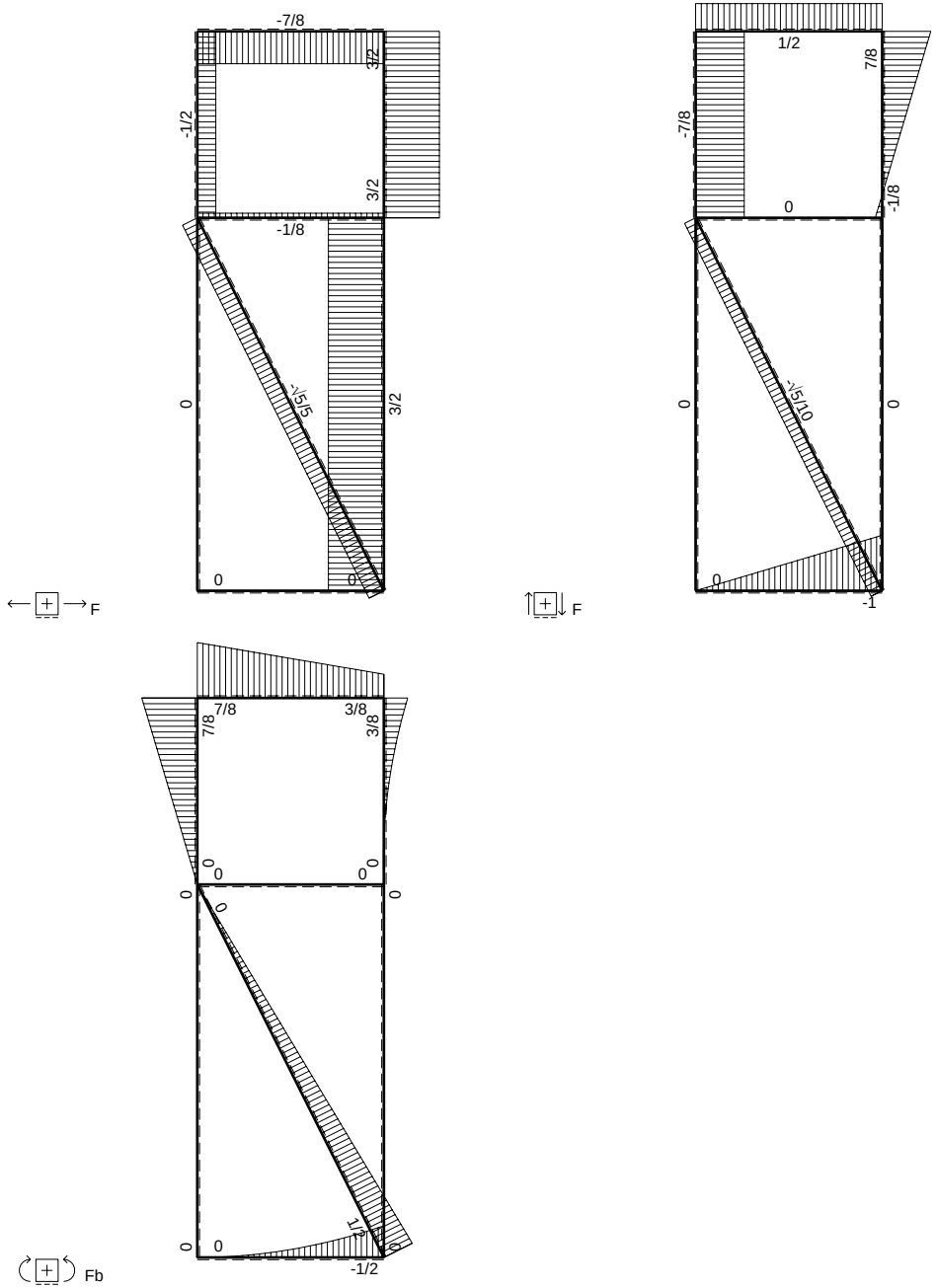

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
F
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
Fb
-1/2

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Schema di calcolo iperstatico

PROCEDIMENTO E RISULTATI 212062 ESΣ09.xxxx.089

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

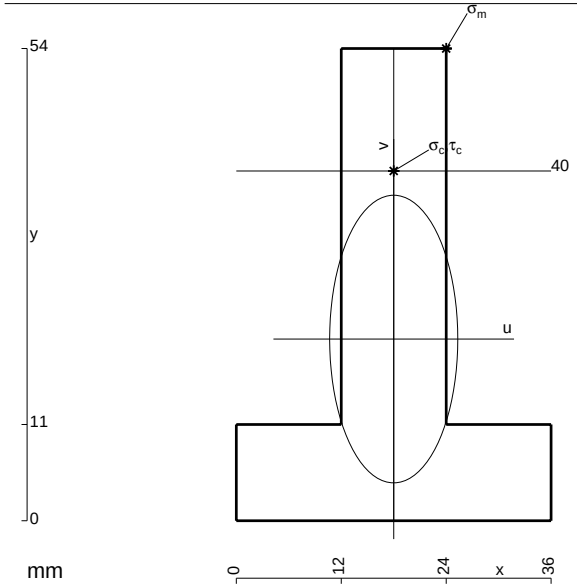
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 912. mm^2
J_u = 246834. mm^4
J_v = 48960. mm^4
y_g = 20.78 mm
T_y = -4950. N
M_x = -1782000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.22 mm
$\sigma_m = -Mv/J_u$ = 239.9 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.22 mm
$\sigma_c = -Mv/J_u$ = 138.8 N/mm^2
τ_c = 7.362 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.4 N/mm^2
S^* = 4406. mm^3

ESΣ09.xxxx.089

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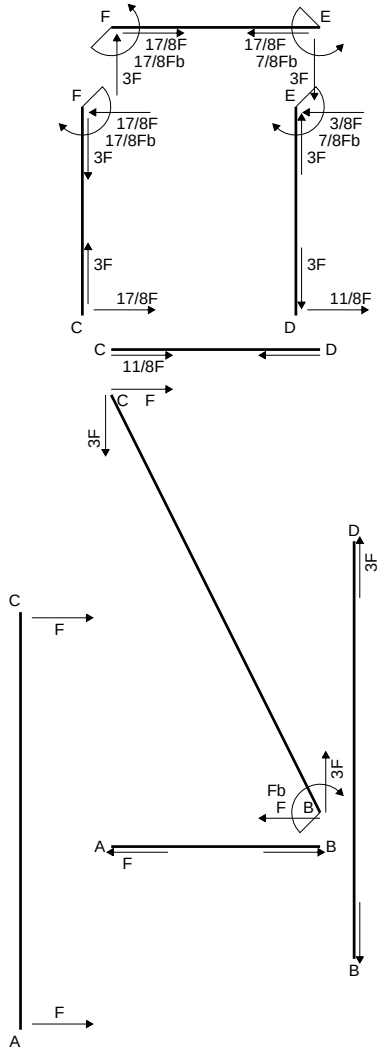
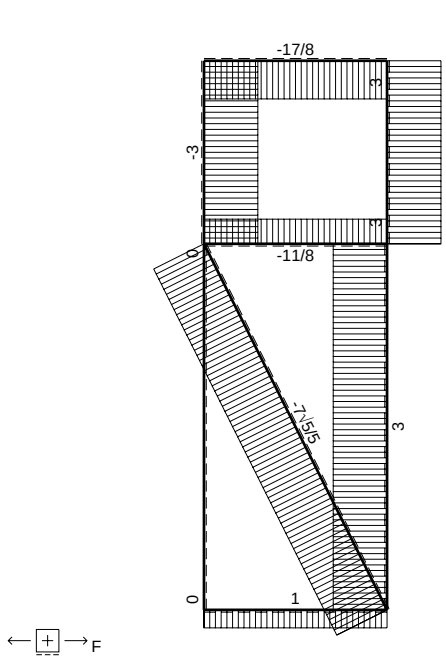
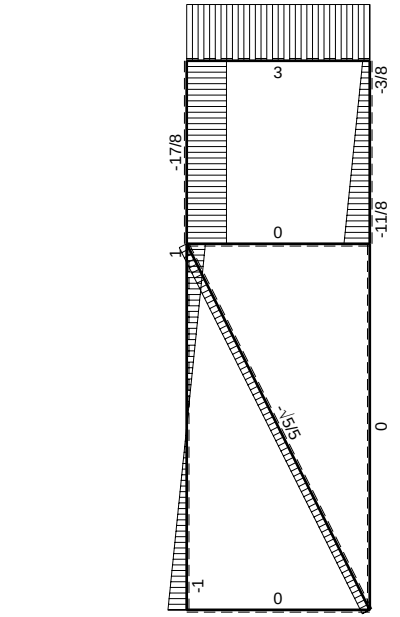
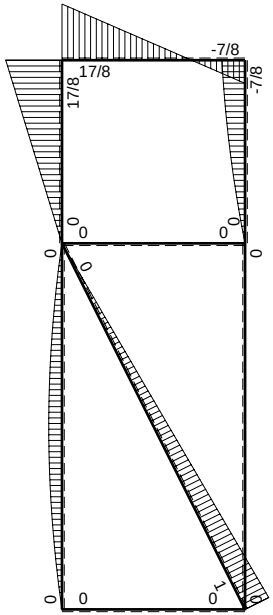

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A


-17/8
-3
-11/8
-7√5/5
3
0
1
F


3
-3/8
-17/8
-11/8
0
-√5/5
-1
F


-7/8
17/8
-7/8
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb²/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx²/b	0	1-2x/b+x²/b²	(-1+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx²/b	0	x²/b²		
	totali						-35/24Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

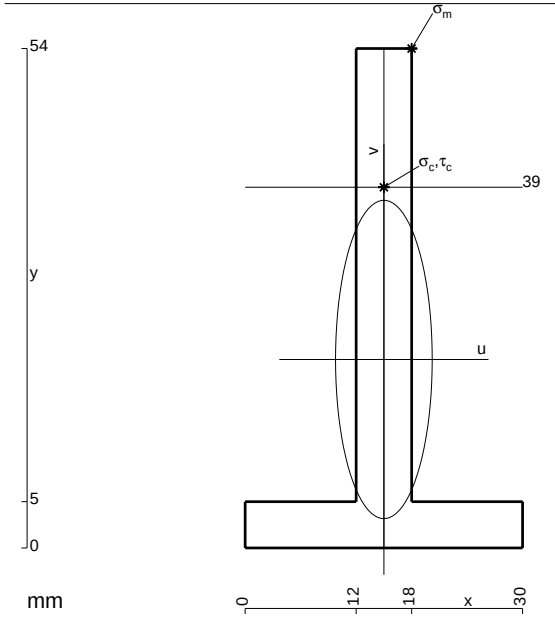
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 444. mm^2
J_u = 131544. mm^4
J_v = 12132. mm^4
y_g = 20.38 mm
N = -5979. N
T_y = -854.2 N
M_x = 725800. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 33.62 mm
$\sigma_m = N/A - Mv/J_u$ = -199. N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 18.62 mm
$\sigma_c = N/A - Mv/J_u$ = -116.2 N/mm^2
τ_c = 2.544 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.3 N/mm^2
S^* = 2351. mm^3

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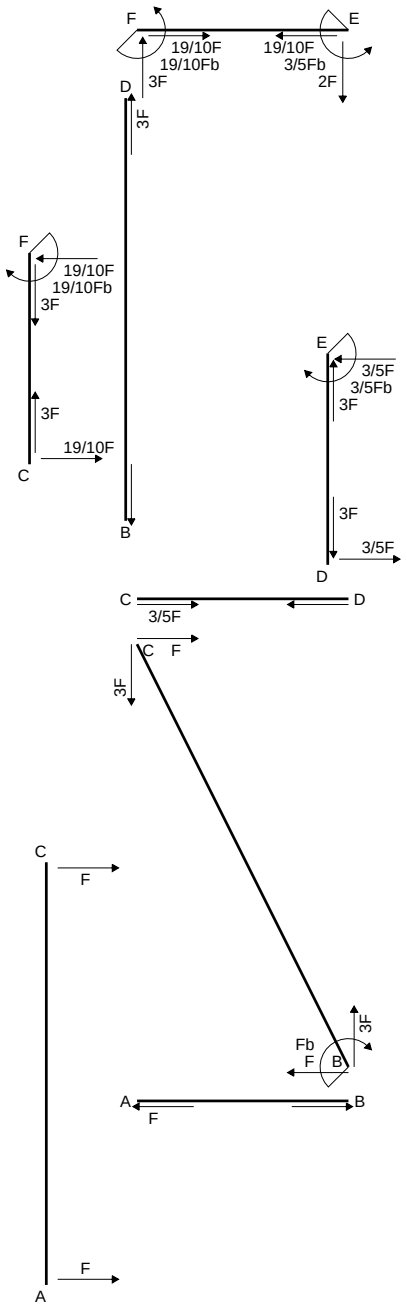
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E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
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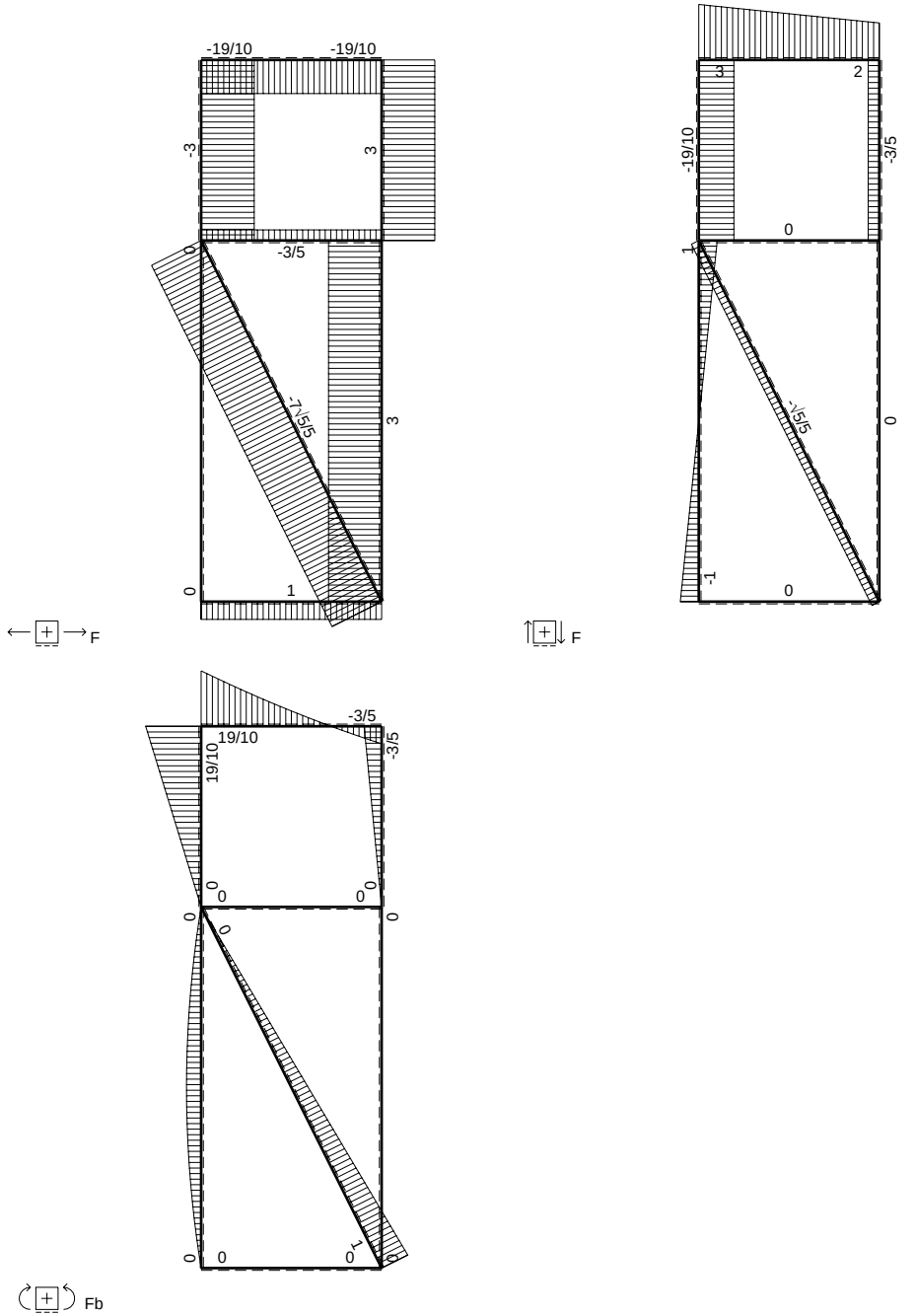
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C F
3F
C
F
Fb
F
B
3F
A
B
F
F
A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-3
3
-3/5
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3
-7\5/5
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F
-19/10
3
2
-3/5
0
1
0
-1\5/5
-1
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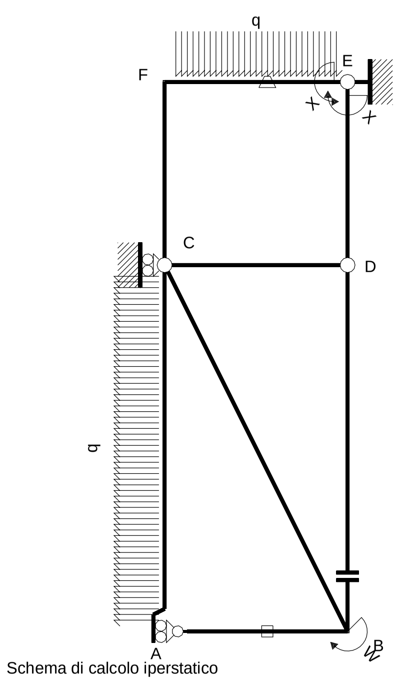
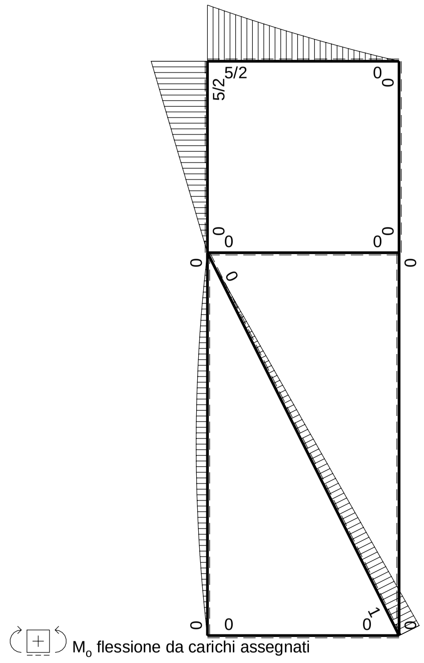
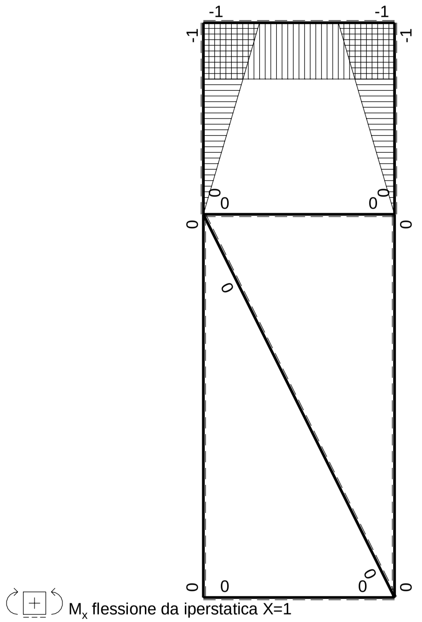
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

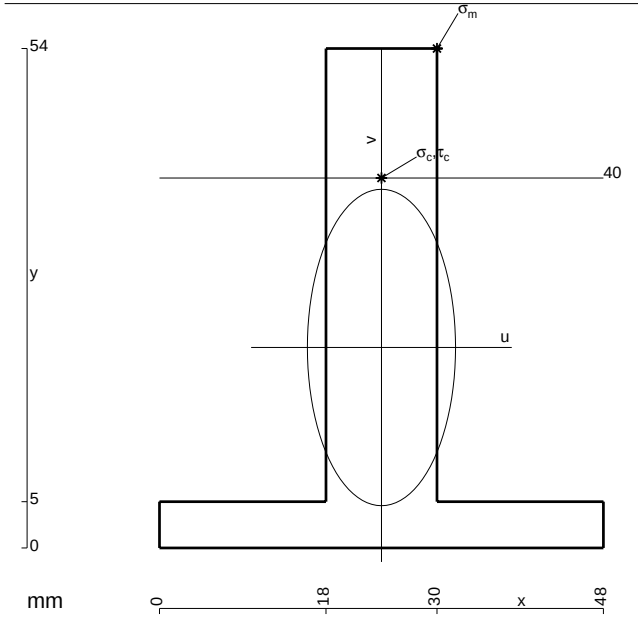
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 828. mm^2
J_u = 242396. mm^4
J_v = 53136. mm^4
y_g = 21.67 mm
N = -11239. N
T_y = -1606. N
M_x = 1471900. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 32.33 mm
$\sigma_m = N/A - Mv/J_u$ = -209.9 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 18.33 mm
$\sigma_c = N/A - Mv/J_u$ = -124.9 N/mm^2
τ_c = 2.348 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.9 N/mm^2
S^* = 4255. mm^3

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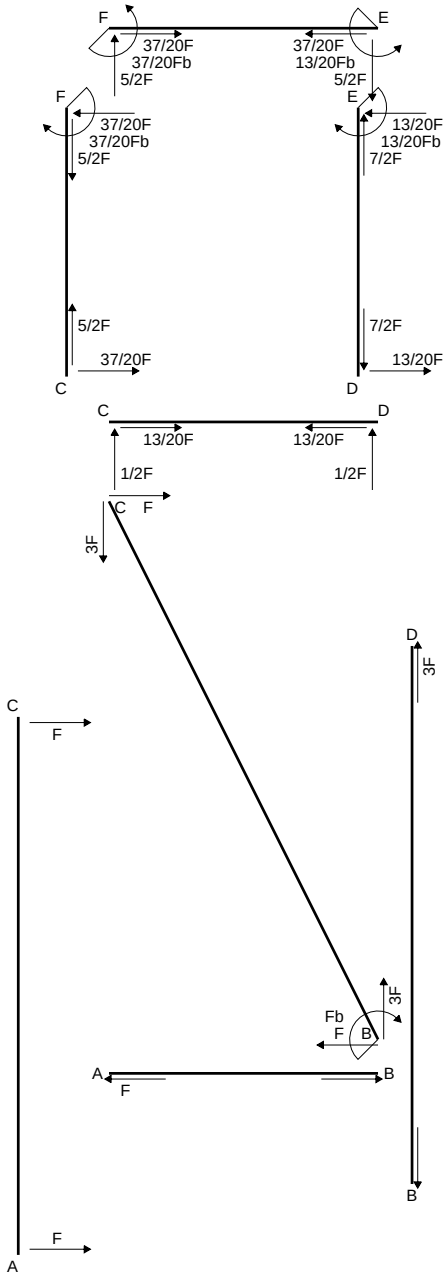
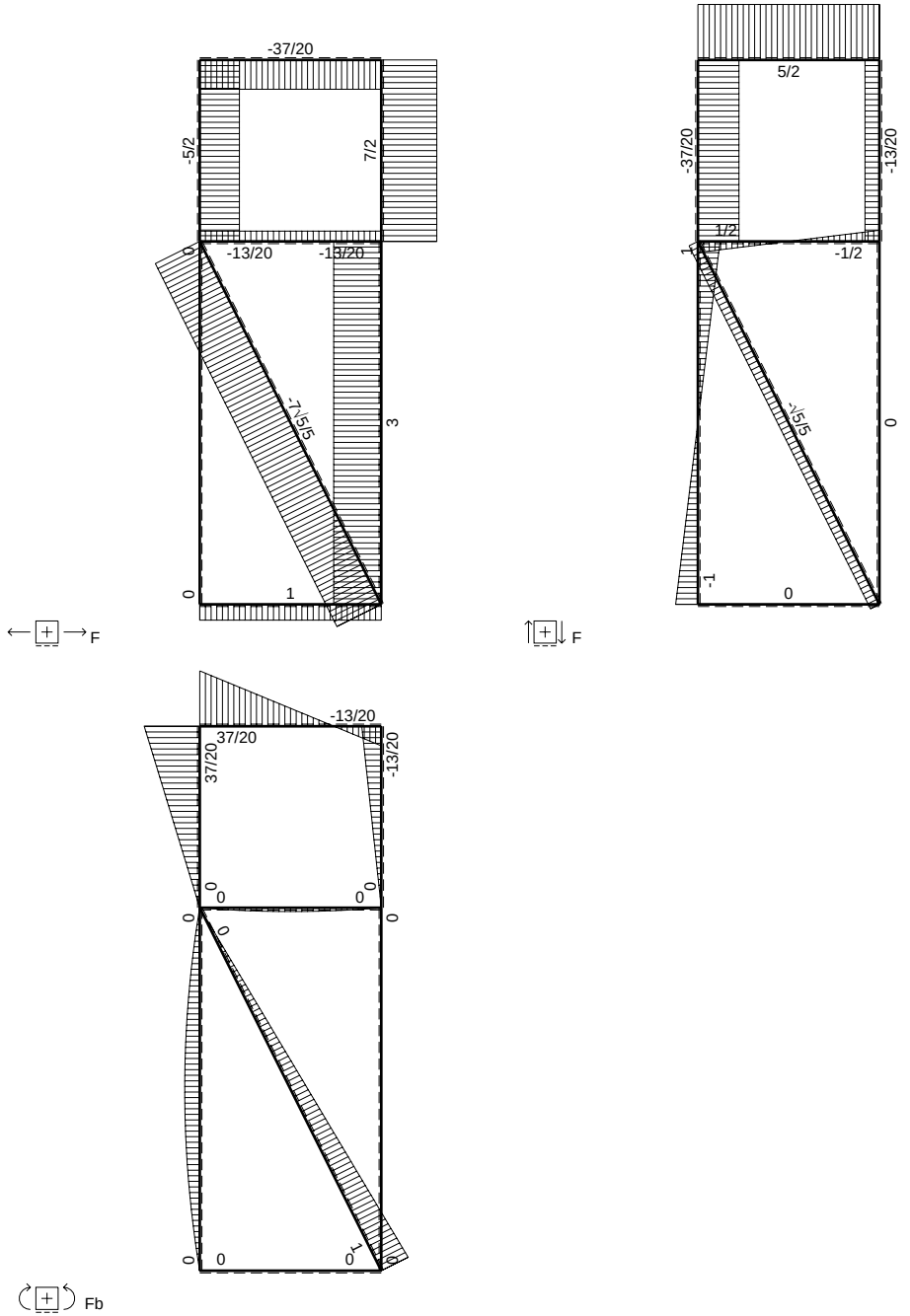

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B


-37/20
-5/2
7/2
-13/20
-13/20
0
3
-7.5/5
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
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0
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0
0
0
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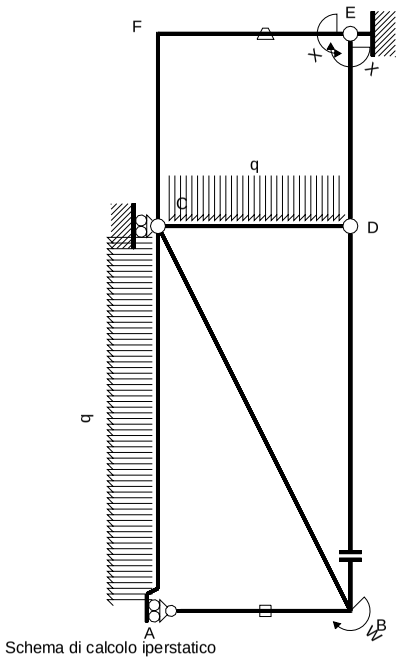
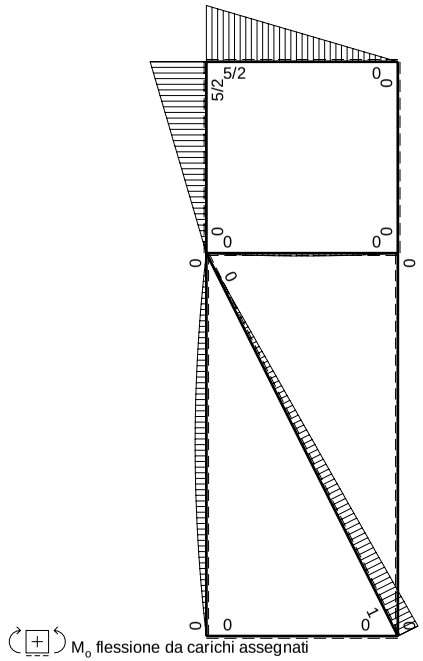
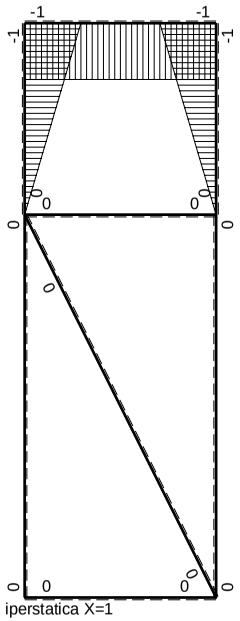
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
		totali					-13/12Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211282 ESΣ09.xxxx.092

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

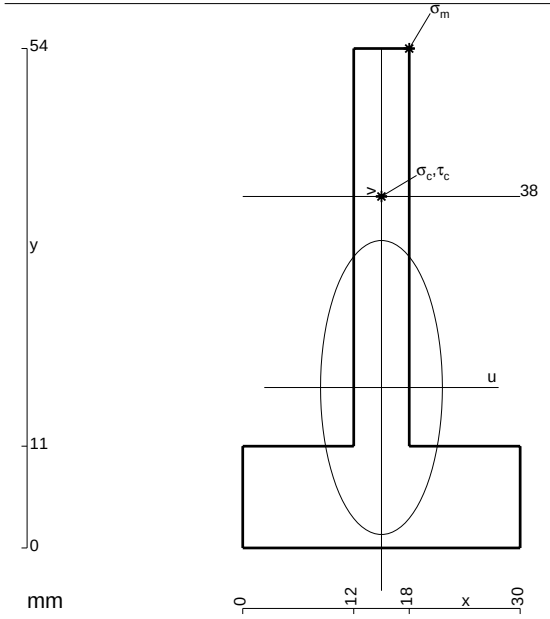
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 588. mm^2
J_u = 148637. mm^4
J_v = 25524. mm^4
y_g = 17.35 mm
N = -5917. N
T_y = -845.2 N
M_x = 850500. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 36.65 mm
$\sigma_m = N/A - Mv/J_u$ = -219.8 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 20.65 mm
$\sigma_c = N/A - Mv/J_u$ = -128.2 N/mm^2
τ_c = 2.607 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.3 N/mm^2
S^* = 2751. mm^3

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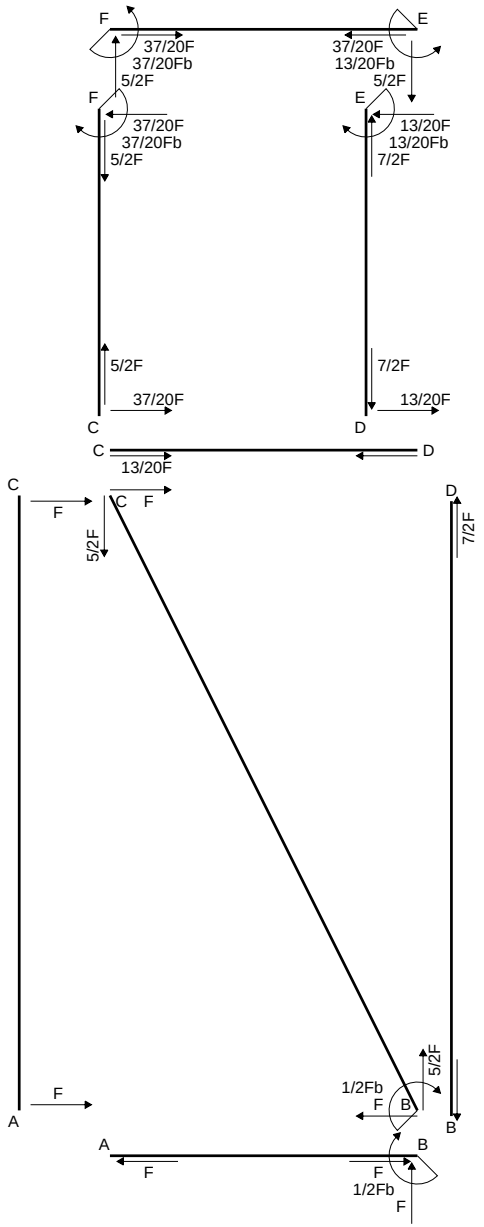
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13/20Fb
7/2F
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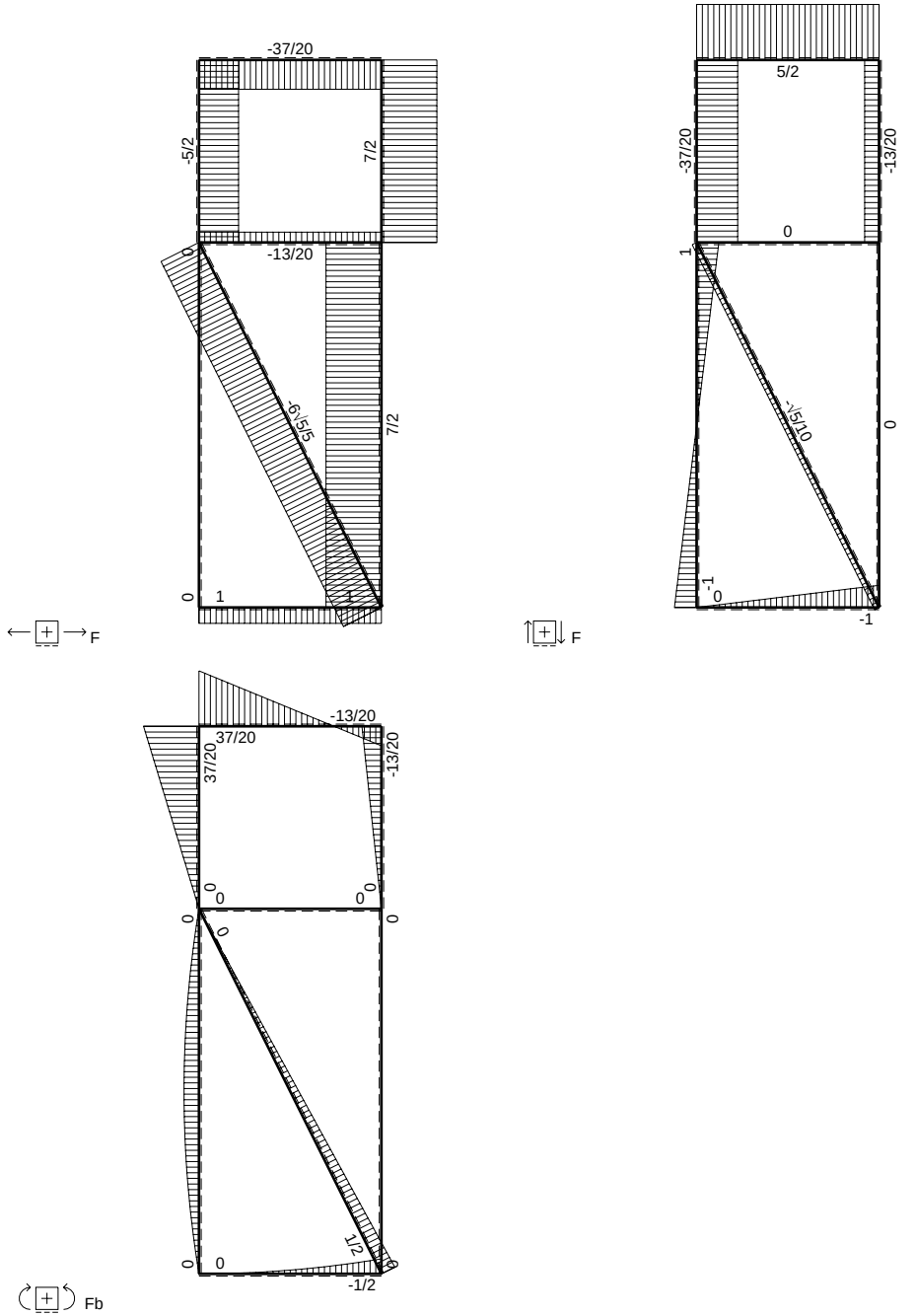
D
13/20F
A
B
1/2Fb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-13/20
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-\5/10
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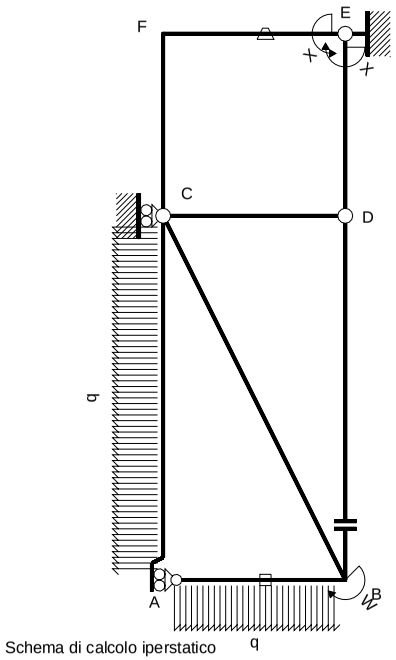
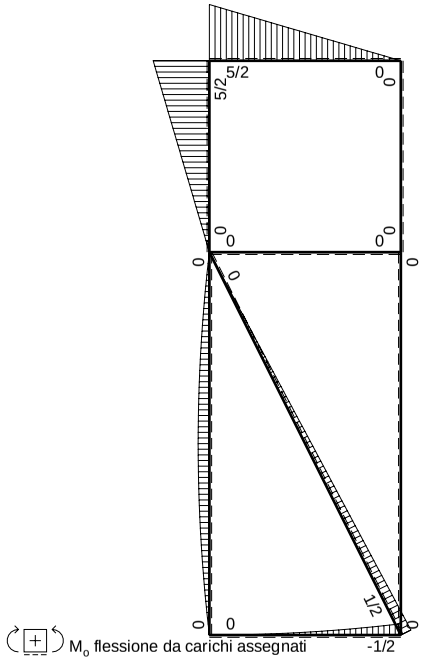
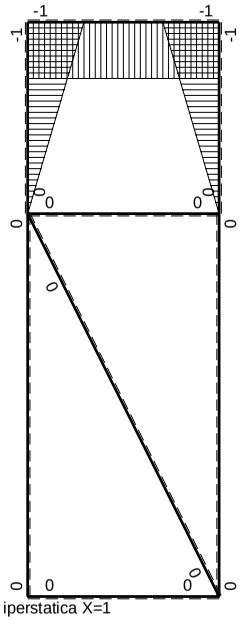
37/20
Fb
-1/2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
				totali			$-13/12Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212423 ESΣ09.xxxx.093

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

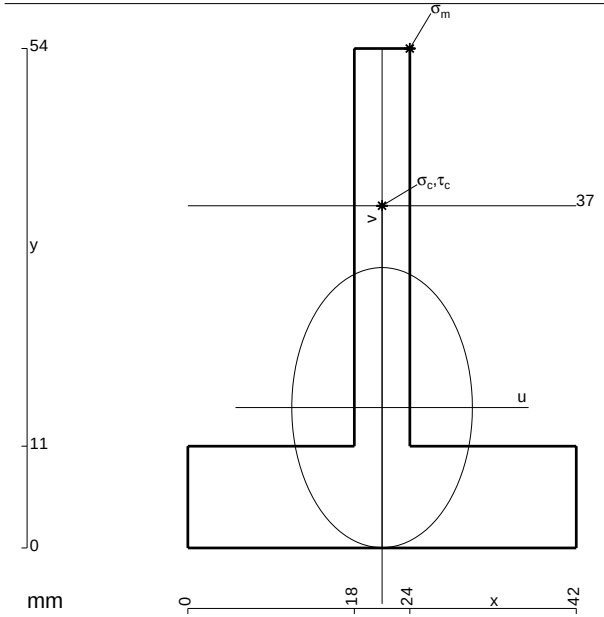
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 720. mm^2
J_u = 165098. mm^4
J_v = 68688. mm^4
y_g = 15.18 mm
N = 3890. N
T_y = -3890. N
M_x = -953050. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 38.83 mm
$\sigma_m = N/A - Mv/J_u$ = 229.5 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 21.83 mm
$\sigma_c = N/A - Mv/J_u$ = 131.4 N/mm^2
τ_c = 12.15 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.1 N/mm^2
S^* = 3093. mm^3

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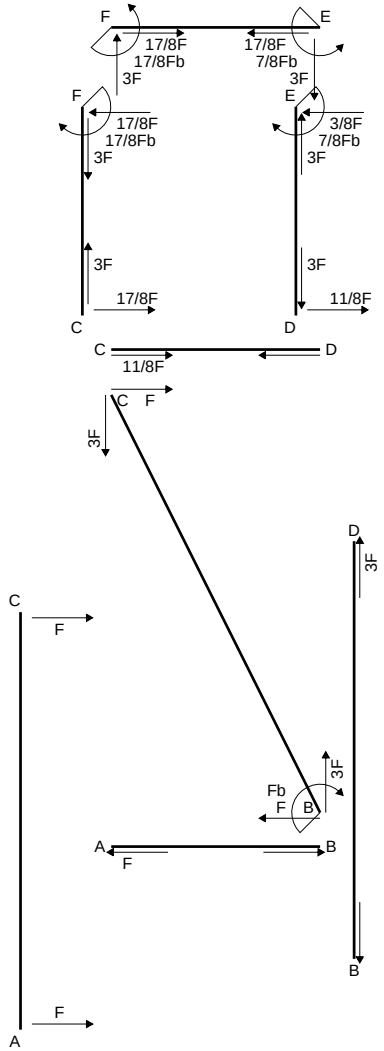
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17/8Fb
3F
17/8F
7/8Fb
3F
F
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17/8F
17/8Fb
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7/8Fb
3F
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3F
17/8F
11/8F
C
D
C
D
11/8F
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3F
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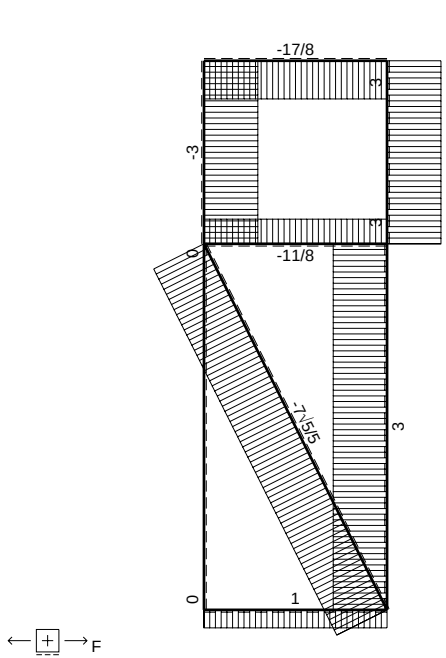
3F
C
F
3F
Fb
F
B
A
B
F
A
B
F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-11/8
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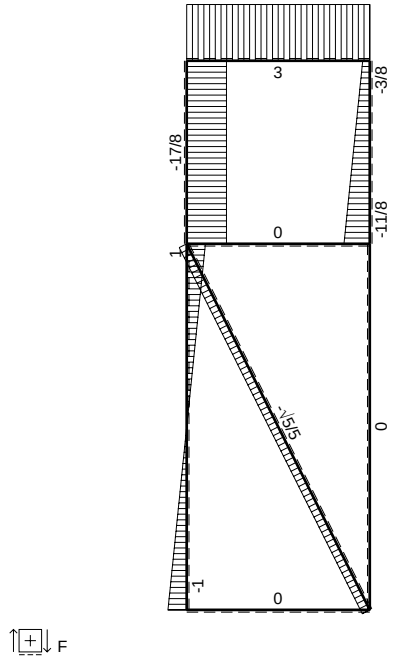
-7\5/5
3
0
1
F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-17/8
-11/8
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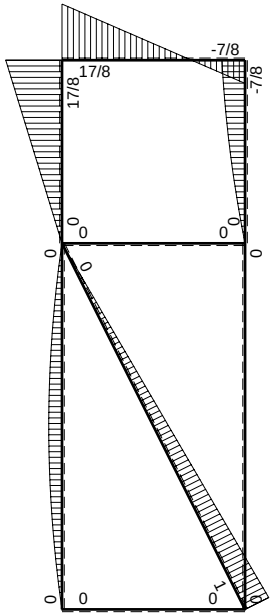
1
-1
-5/5
0
-1
0
F


-7/8
17/8
17/8
-7/8
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0
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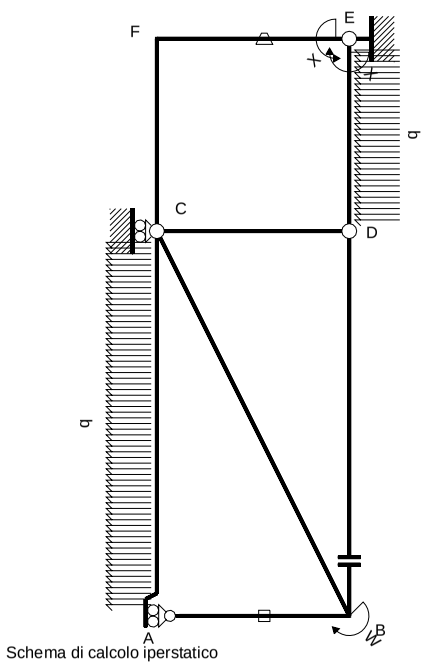
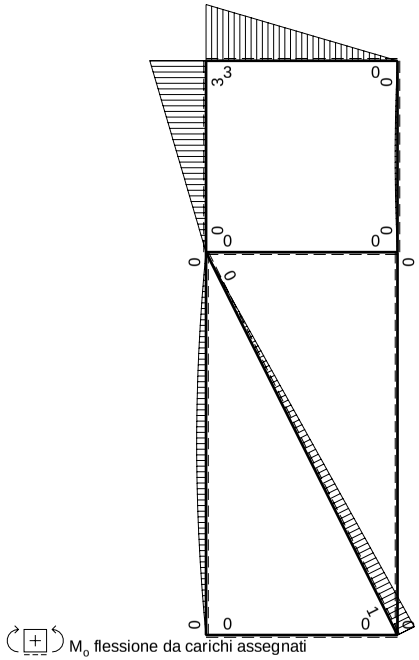
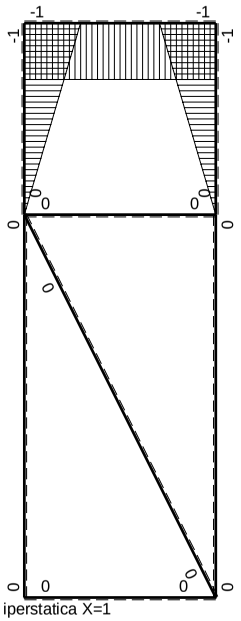
1
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
				totali			-35/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

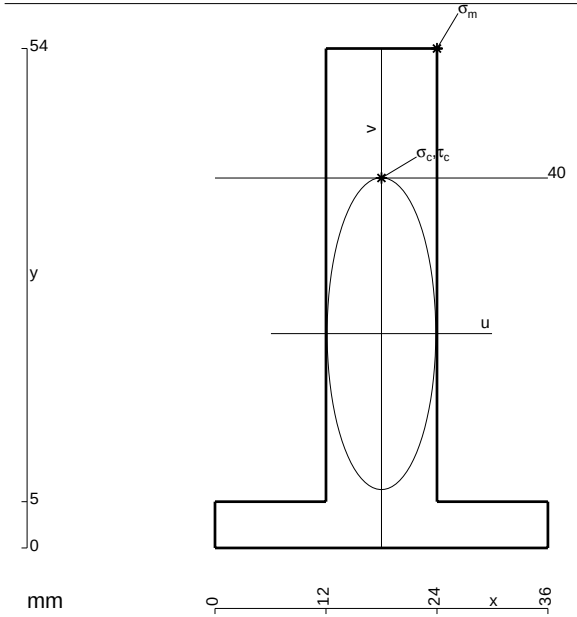
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 768. mm^2
J_u = 218489. mm^4
J_v = 26496. mm^4
y_g = 23.17 mm
N = -9517. N
T_y = -1360. N
M_x = 1611200. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 30.83 mm
$\sigma_m = N/A - Mv/J_u$ = -239.7 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 16.83 mm
$\sigma_c = N/A - Mv/J_u$ = -136.5 N/mm^2
τ_c = 2.076 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.5 N/mm^2
S^* = 4003. mm^3

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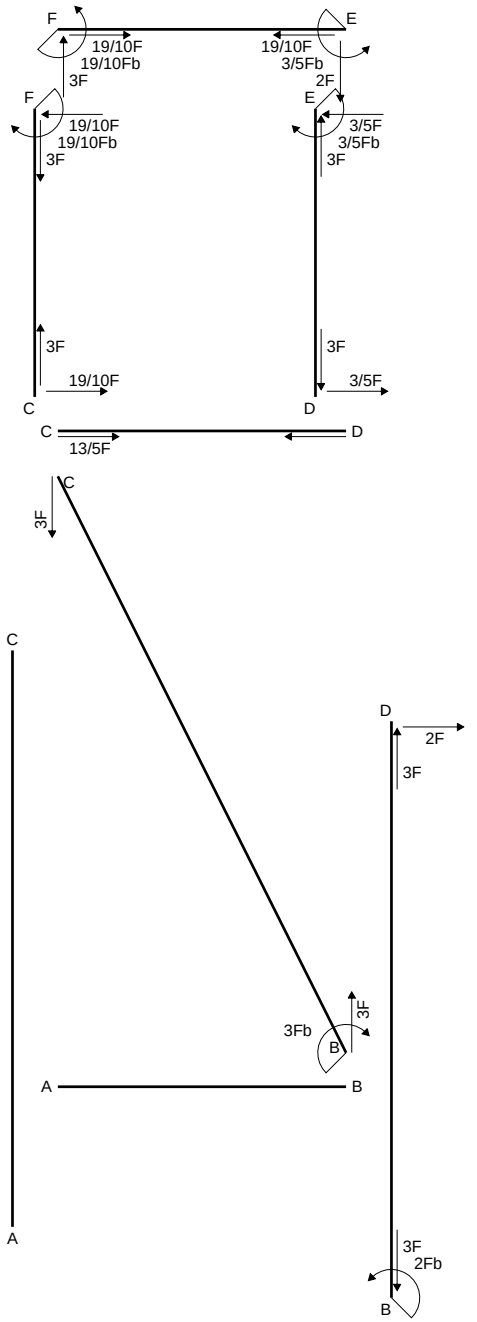
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E
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19/10Fb
19/10F
3/5Fb
3F
2F
F
E
19/10F
19/10Fb
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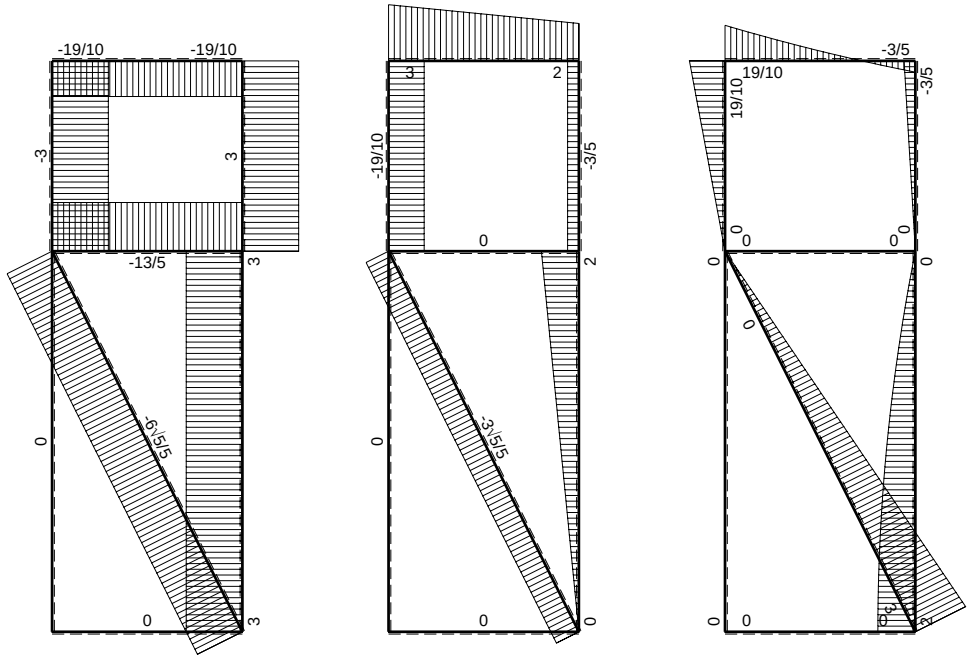
3F
3F
3F
19/10F
3/5F
C
D
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
3F
B
A
B
A
3F
2Fb
B


-19/10
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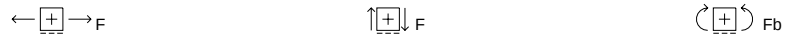
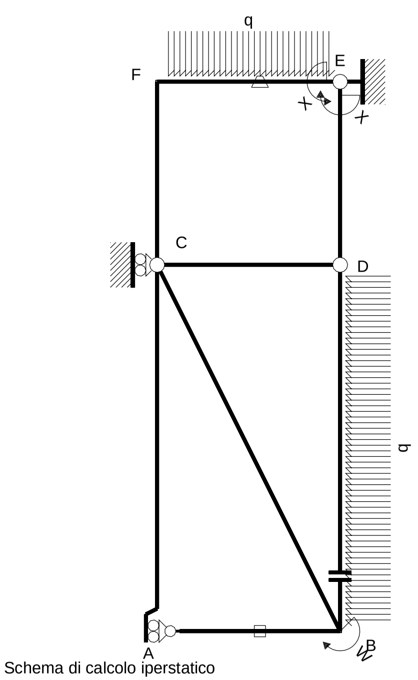
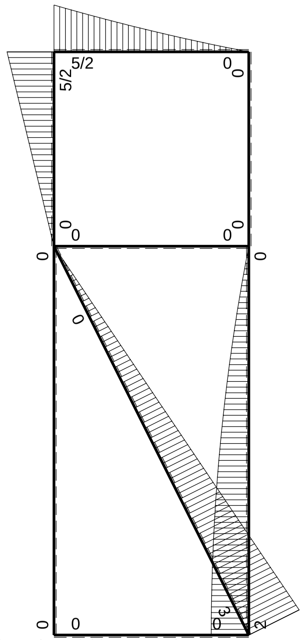
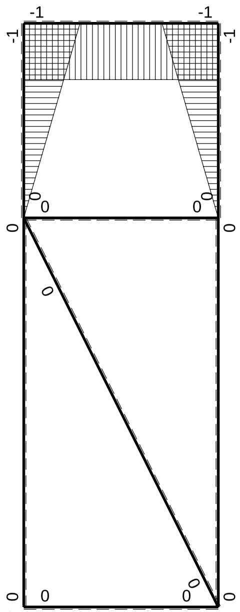

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

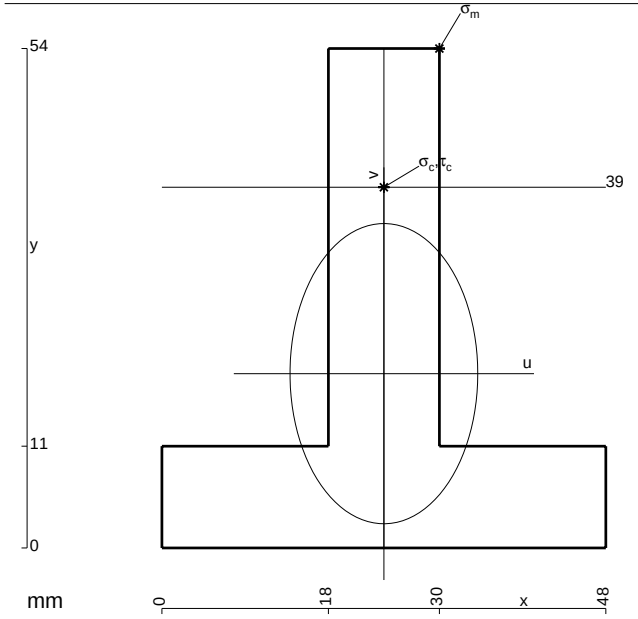
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1044. mm^2
J_u = 275075. mm^4
J_v = 107568. mm^4
y_g = 18.84 mm
N = -2415. N
T_y = -1207. N
M_x = 1539000. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 35.16 mm
$\sigma_m = N/A - Mv/J_u$ = -199. N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.16 mm
$\sigma_c = N/A - Mv/J_u$ = -115.1 N/mm^2
τ_c = 1.821 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.1 N/mm^2
S^* = 4978. mm^3

ESΣ09.xxxx.095

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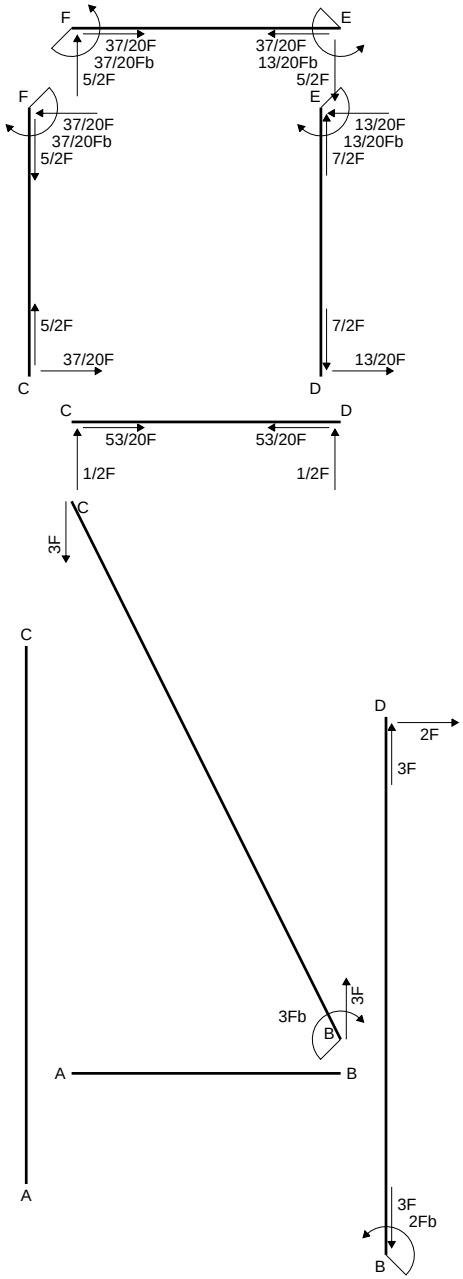
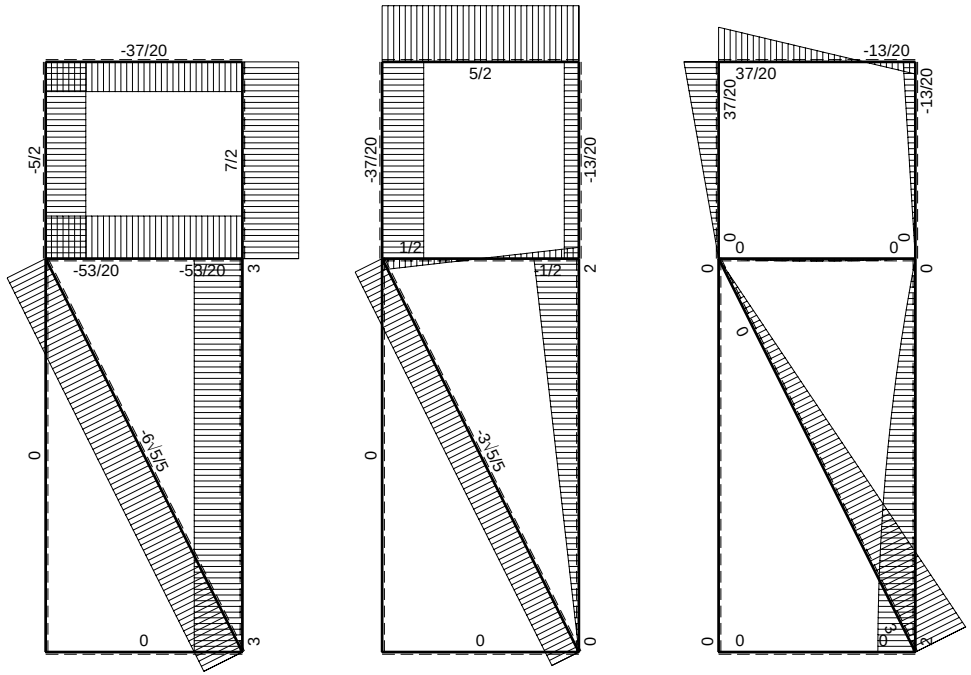

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-37/20
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7/2
-53/20
-53/20
3
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-6√5/5
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3
-37/20
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1/2
1/2
2
-3√5/5
0
0
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
3√5


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi\theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
totali							-13/12Fb^2/EJ	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 934988 ESΣ09.xxxx.096

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

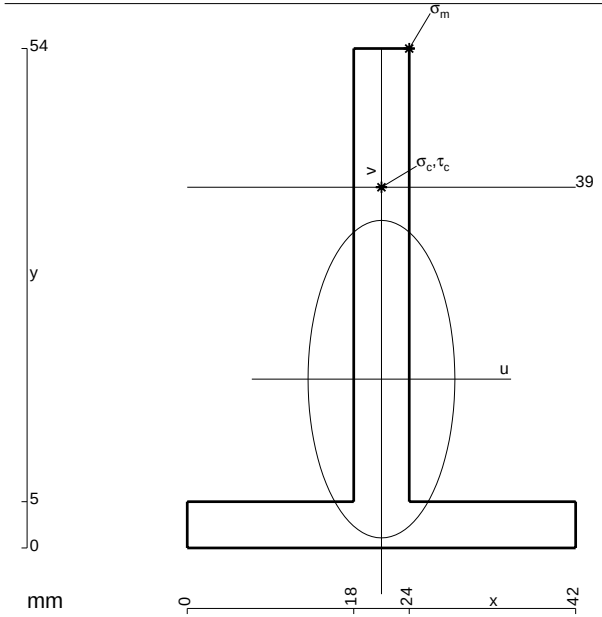
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 504. mm^2
J_u = 148565. mm^4
J_v = 31752. mm^4
y_g = 18.25 mm
N = -1261. N
T_y = -630.6 N
M_x = 846000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 35.75 mm
$\sigma_m = N/A - Mv/J_u$ = -206.1 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 20.75 mm
$\sigma_c = N/A - Mv/J_u$ = -120.7 N/mm^2
τ_c = 1.799 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.7 N/mm^2
S^* = 2543. mm^3

ESΣ09.xxxx.096

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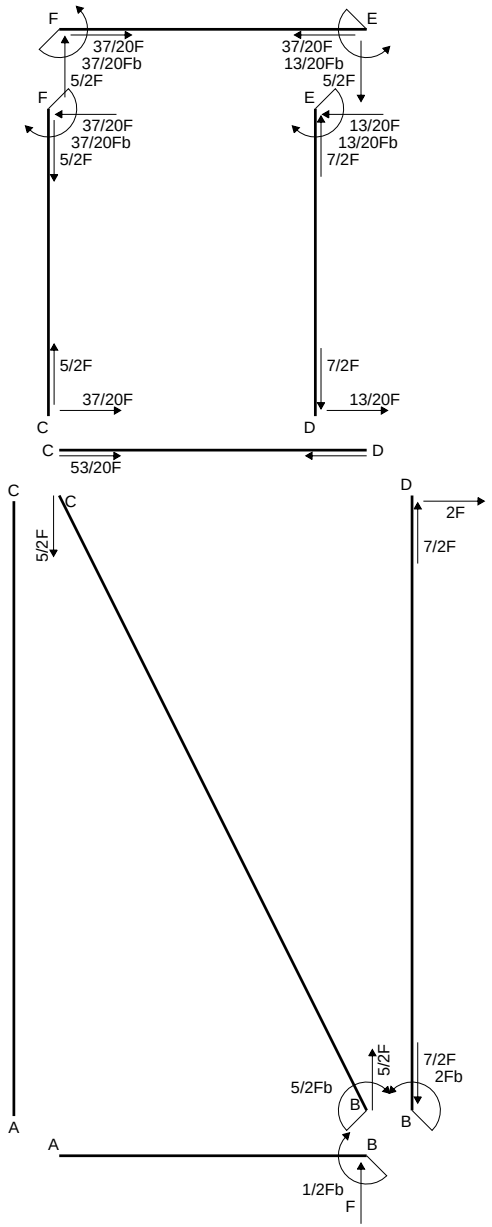
F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
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7/2F
5/2F
37/20F
7/2F
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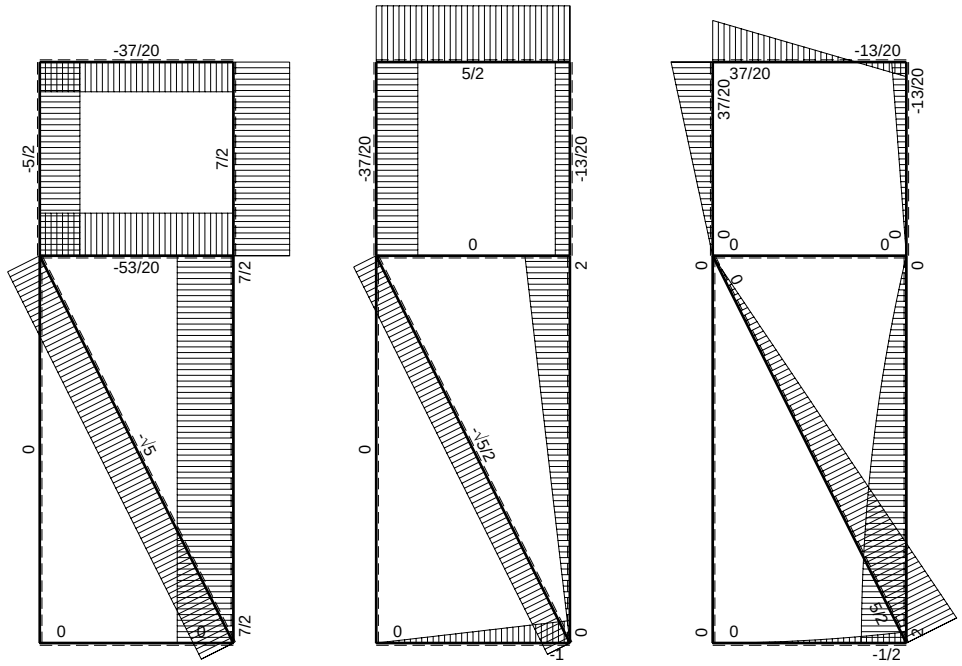
5/2Fb
2Fb
A
B
1/2Fb
F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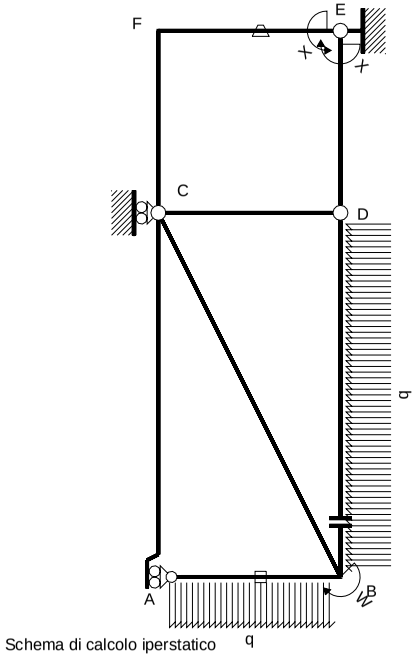
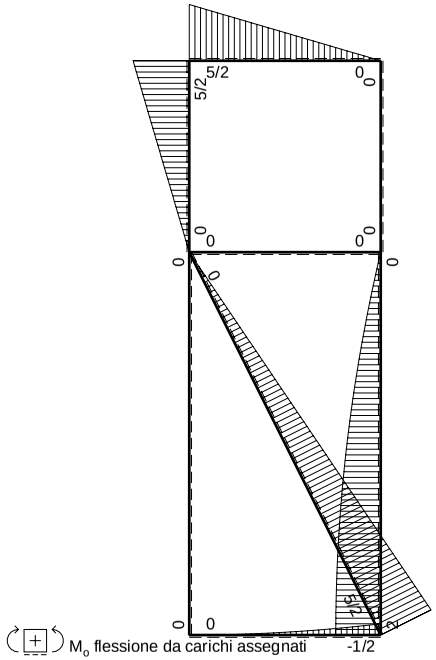
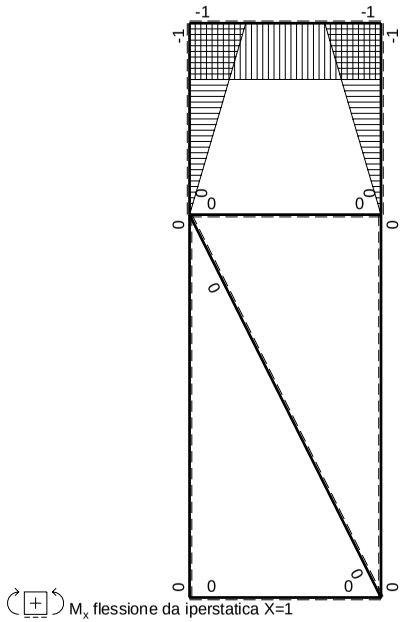
-5/2
7/2
-53/20
7/2
0
-5/2
5/2
-13/20
2
-1
37/20
-13/20
-1/2


F
F
Fb



Schema di calcolo iperstatico





Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
				totali			$-13/12Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209775 ESΣ09.xxxx.097

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

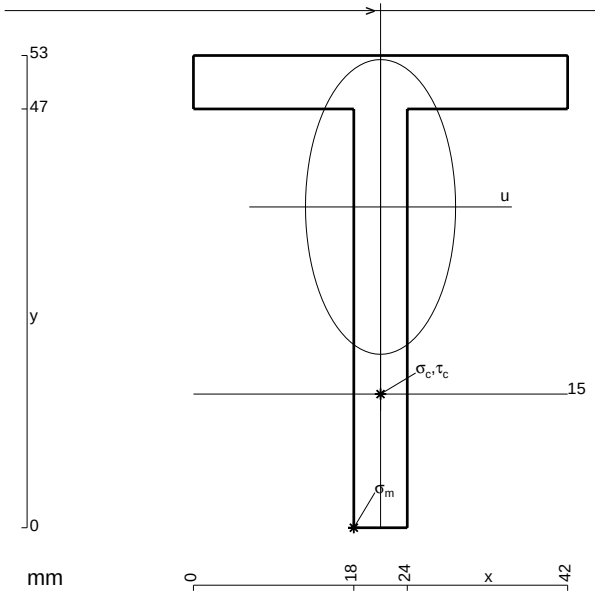
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 534. mm^2
J_u = 146122. mm^4
J_v = 37890. mm^4
y_g = 36.01 mm
N = -1297. N
T_y = -648.5 N
M_x = 899000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -36.01 mm
$\sigma_m = N/A - Mv/J_u$ = 219.1 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -21.01 mm
$\sigma_c = N/A - Mv/J_u$ = 126.8 N/mm^2
τ_c = 1.898 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.8 N/mm^2
S^* = 2566. mm^3

ESΣ09.xxxx.097

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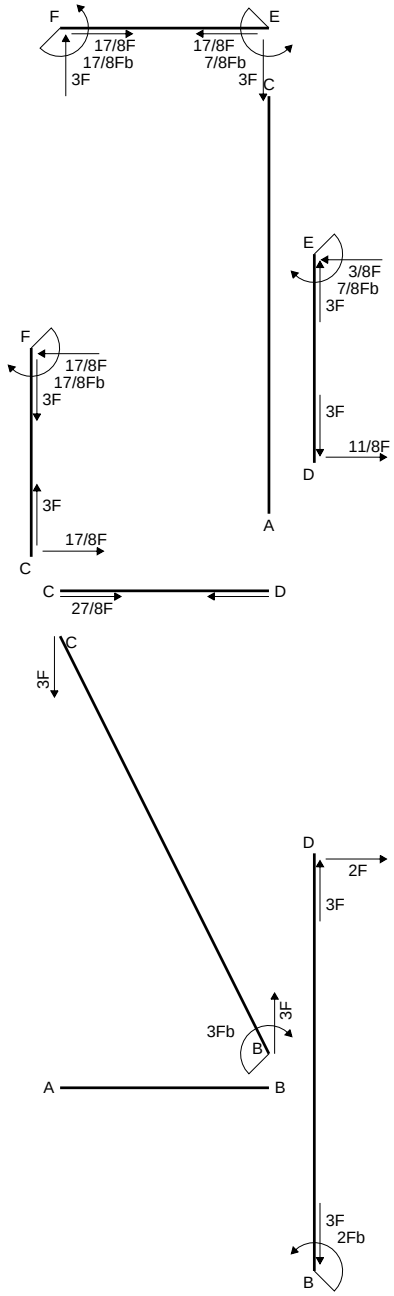
F
E
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17/8Fb
3F
17/8F
7/8Fb
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C
E
3/8F
7/8Fb
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17/8Fb
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11/8F
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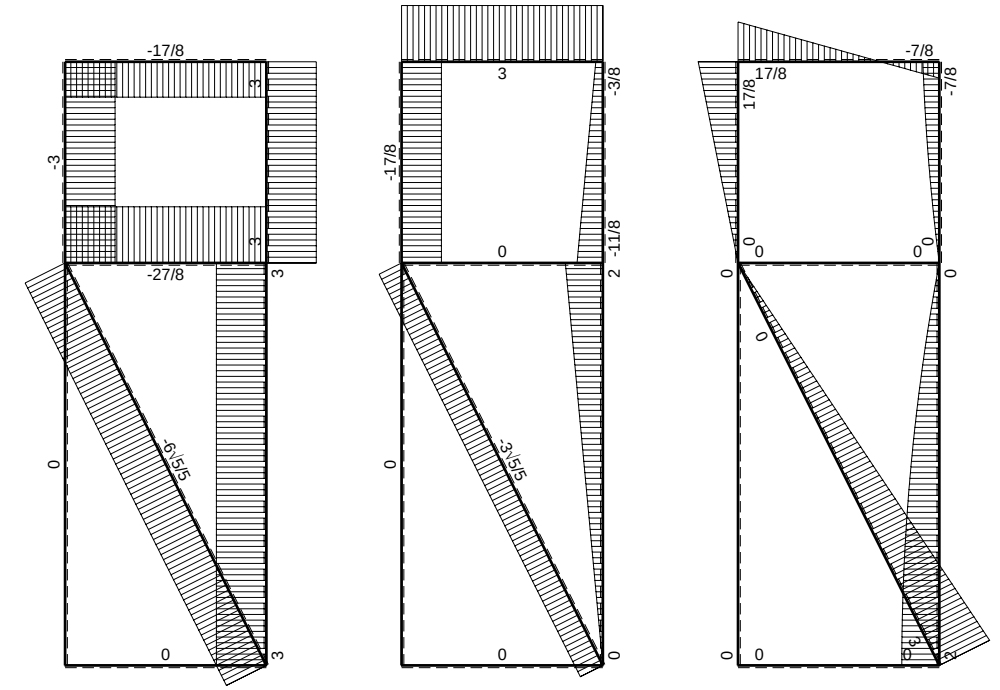
B
3F
2Fb
B


-17/8
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-17/8
3
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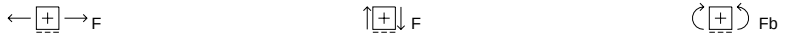
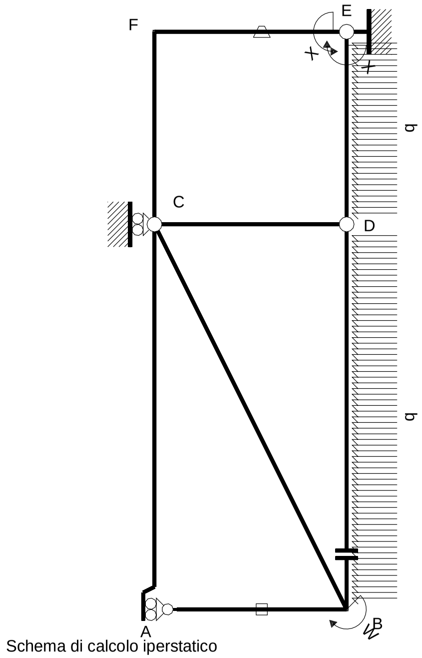
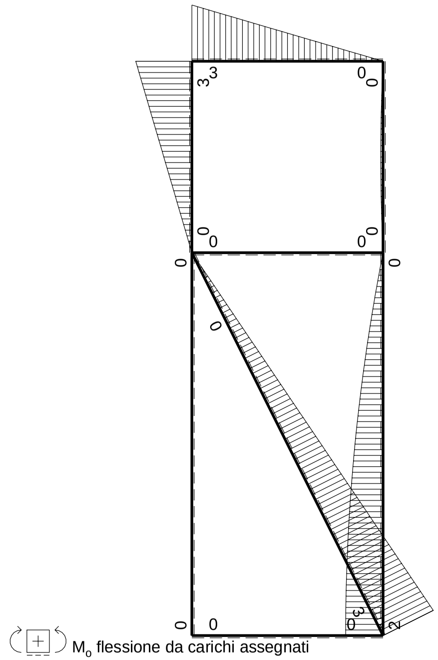
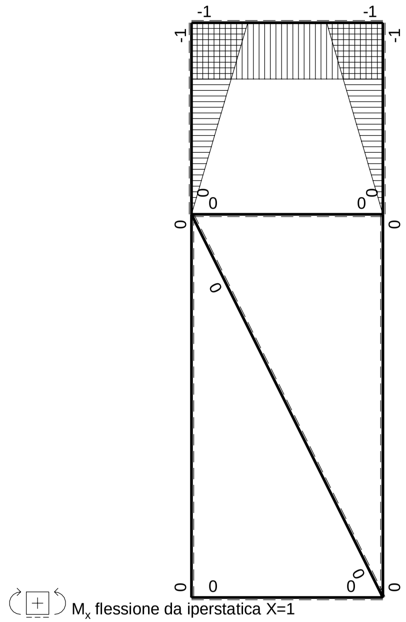

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

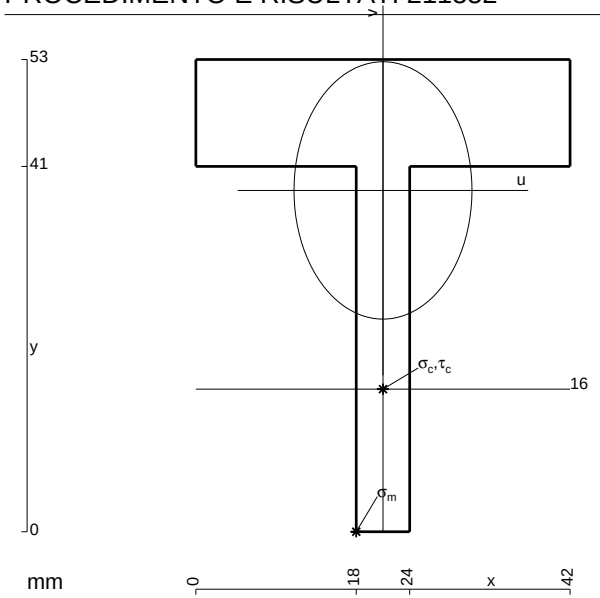
$= (-3\ b +3/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b\ )\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 750. mm^2
J_u = 156599. mm^4
J_v = 74826. mm^4
y_g = 38.31 mm
N = -1261. N
T_y = -630.6 N
M_x = 930600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.31 mm
$\sigma_m = N/A - Mv/J_u$ = 226. N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.31 mm
$\sigma_c = N/A - Mv/J_u$ = 130.9 N/mm^2
τ_c = 1.953 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.9 N/mm^2
S^* = 2910. mm^3

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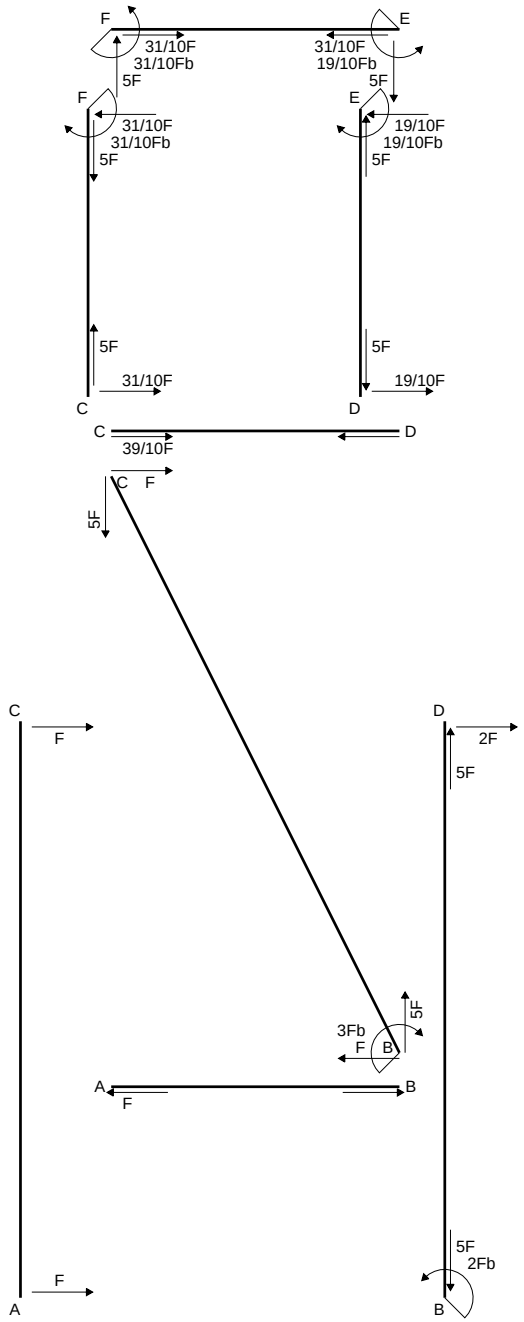
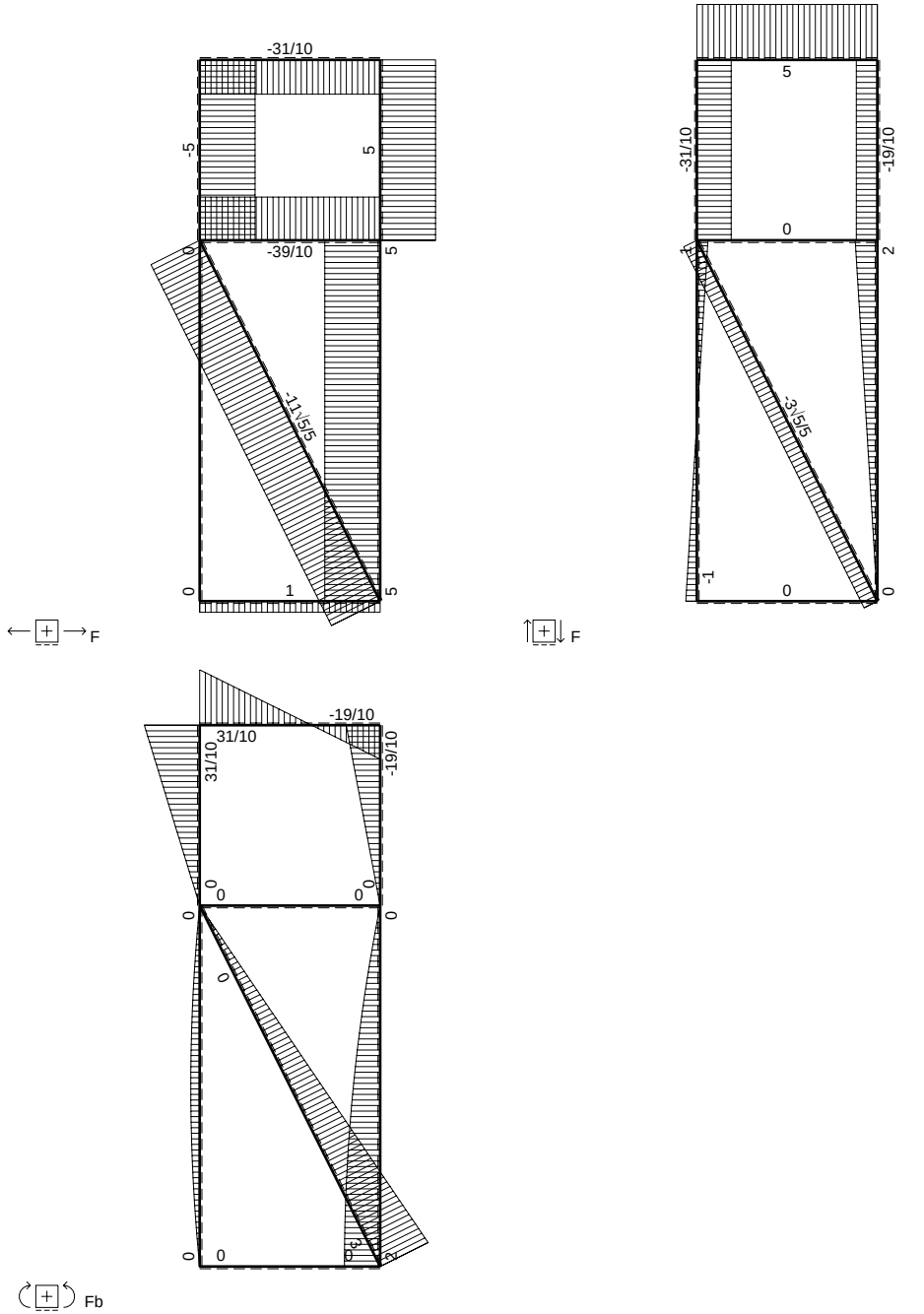
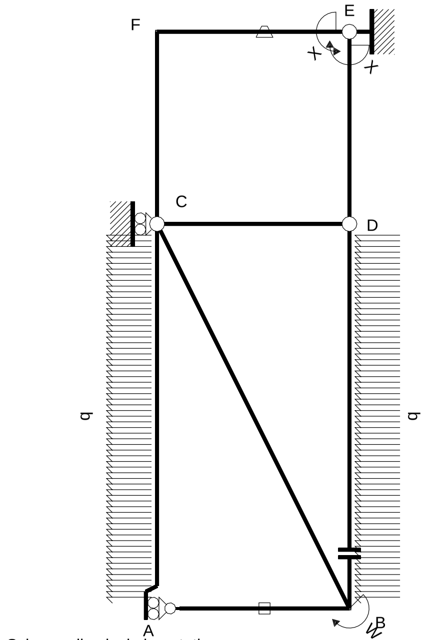
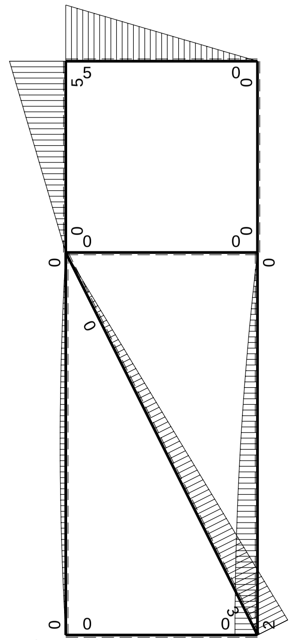

F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B

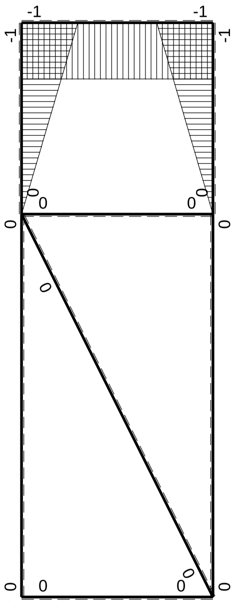

-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	$(-5/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	$-5Fb+10Fx-5Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/3+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5Fx	0	$-5Fx^2/b$	0	x^2/b^2		
	totali						$-19/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						19/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

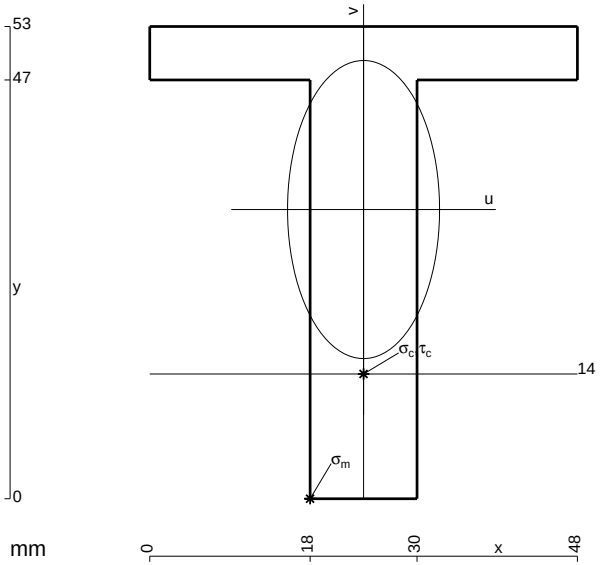
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$



A = 852. mm^2
J_u = 238570. mm^4
J_v = 62064. mm^4
y_g = 32.46 mm
N = -4181. N
T_y = -1140. N
M_x = 1785000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -32.46 mm
σ_m = N/A-Mv/J_u = 237.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -18.46 mm
σ_c = N/A-Mv/J_u = 133.2 N/mm^2
τ_c = 1.704 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.2 N/mm^2
S^* = 4277. mm^3

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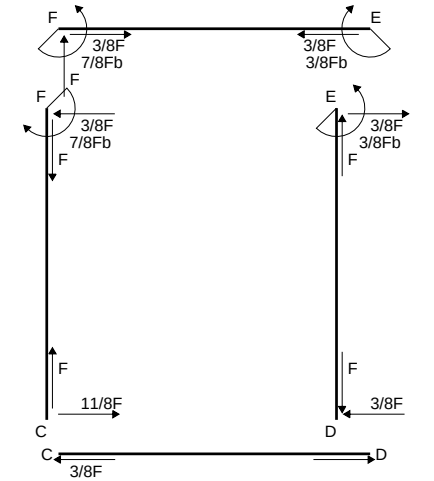
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
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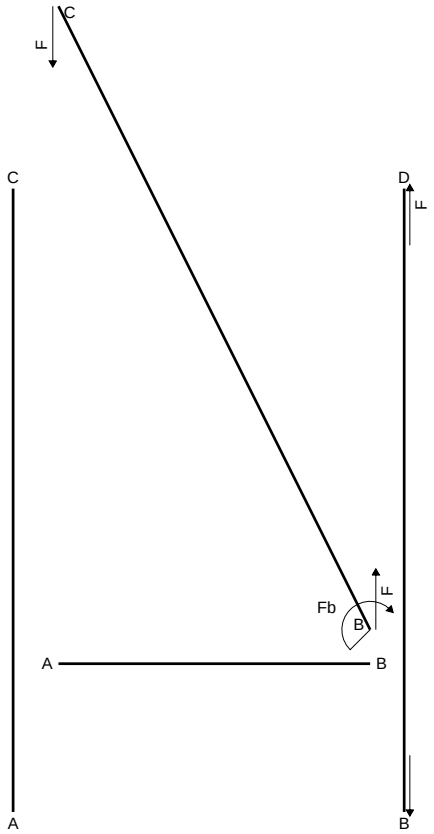
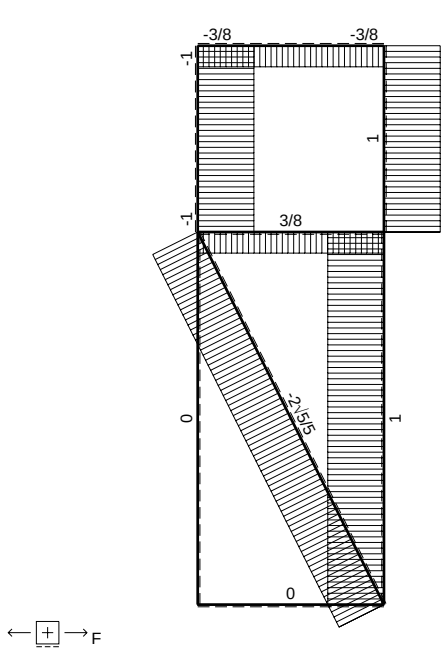
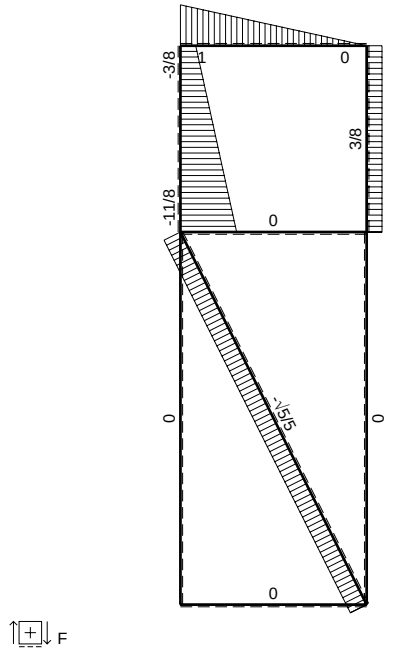
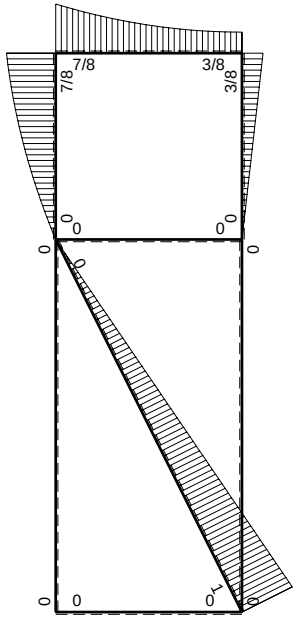
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

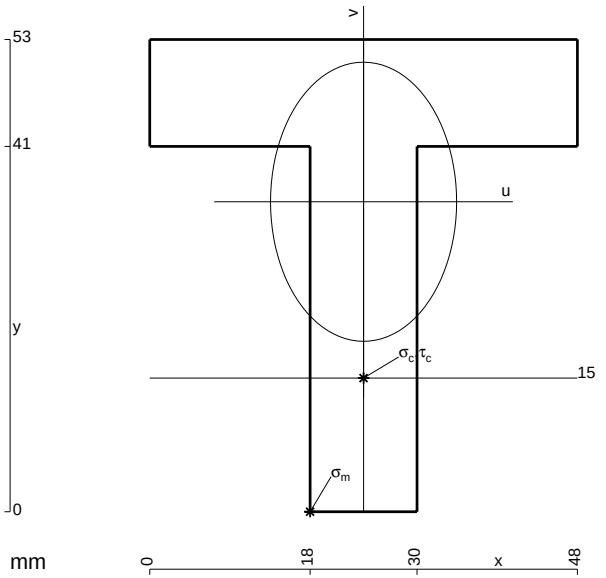
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 1068. mm^2
J_u = 262174. mm^4
J_v = 116496. mm^4
y_g = 34.79 mm
N = -3703. N
T_y = -1851. N
M_x = 1531800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.79 mm
σ_m = N/A-Mv/J_u = 199.8 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -19.79 mm
σ_c = N/A-Mv/J_u = 112.2 N/mm^2
τ_c = 2.891 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 112.3 N/mm^2
S^* = 4913. mm^3

ESΣ09.xxxx.100

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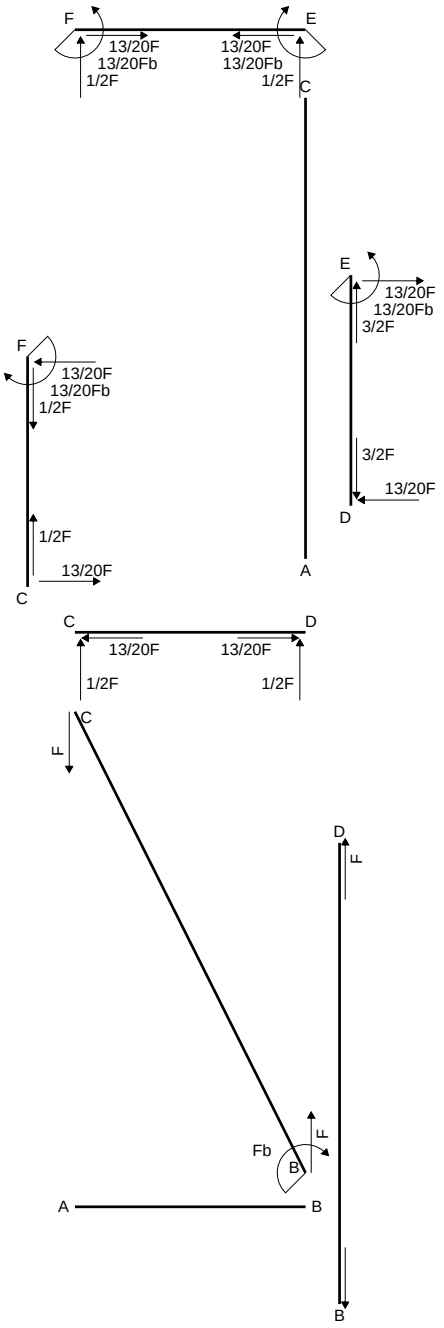
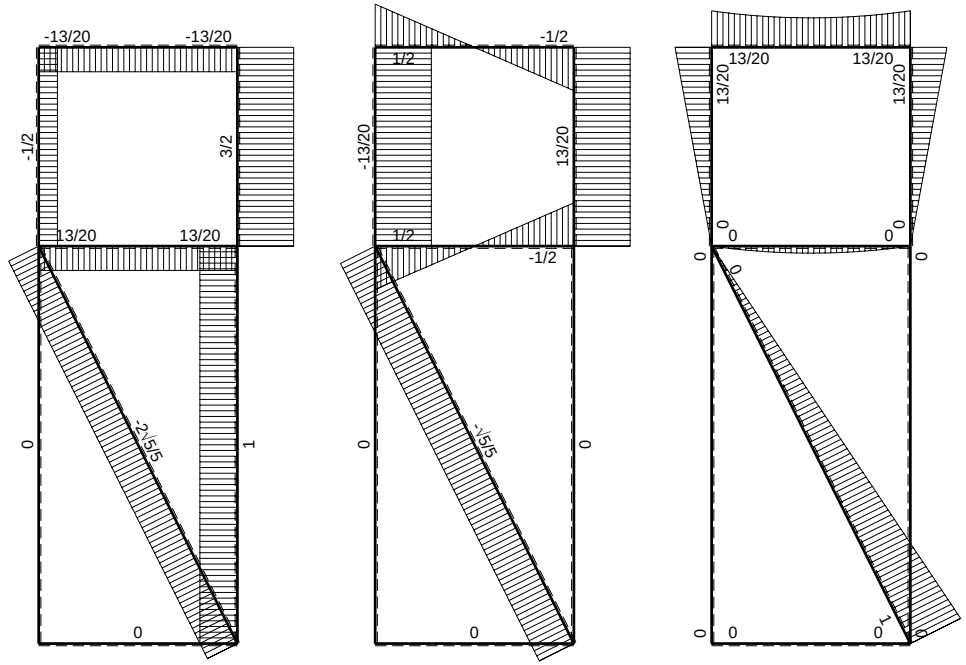
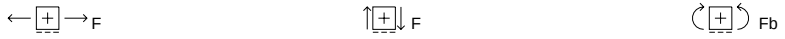
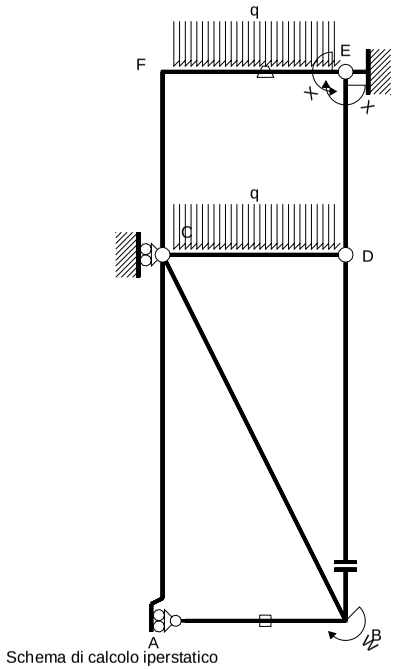
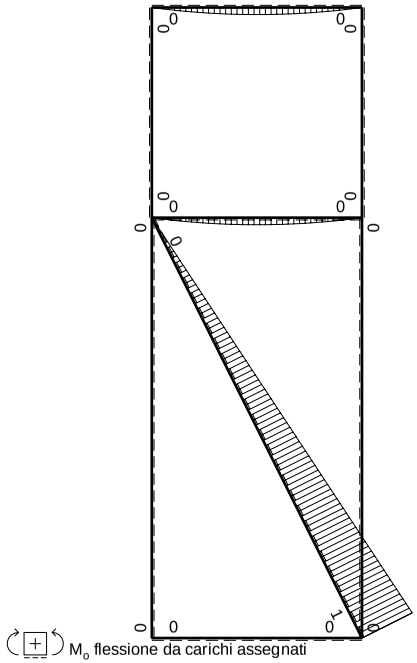

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

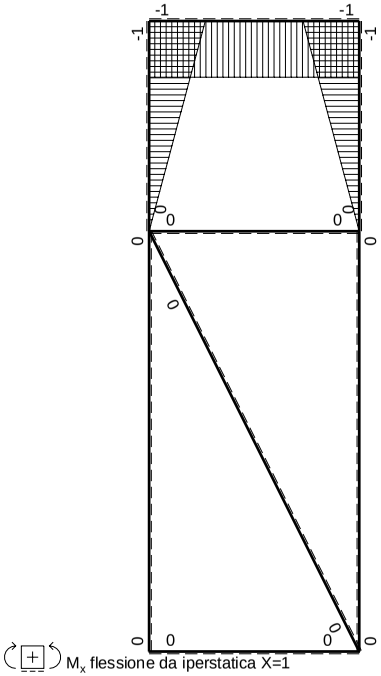

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982517 ESΣ09.xxxx.101

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$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

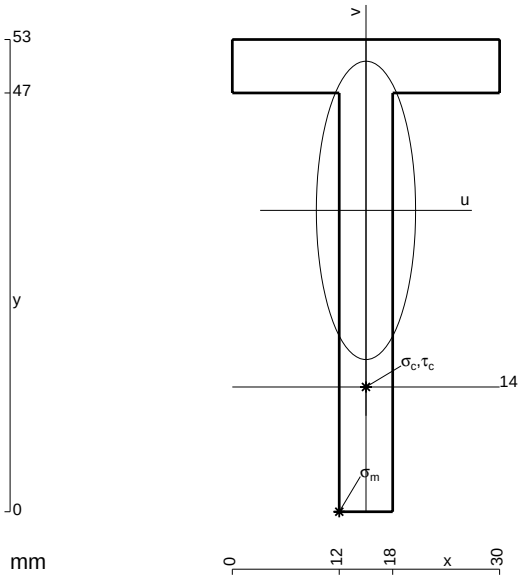
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (1/2\ x/b - 1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b - 1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b - 1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 462. mm^2
J_u = 129608. mm^4
J_v = 14346. mm^4
y_g = 33.82 mm
N = -1825. N
T_y = -912.3 N
M_x = 816000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -33.82 mm
$\sigma_m = N/A - Mv/J_u$ = 209. N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -19.82 mm
$\sigma_c = N/A - Mv/J_u$ = 120.9 N/mm^2
τ_c = 2.643 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 121. N/mm^2
S^* = 2253. mm^3

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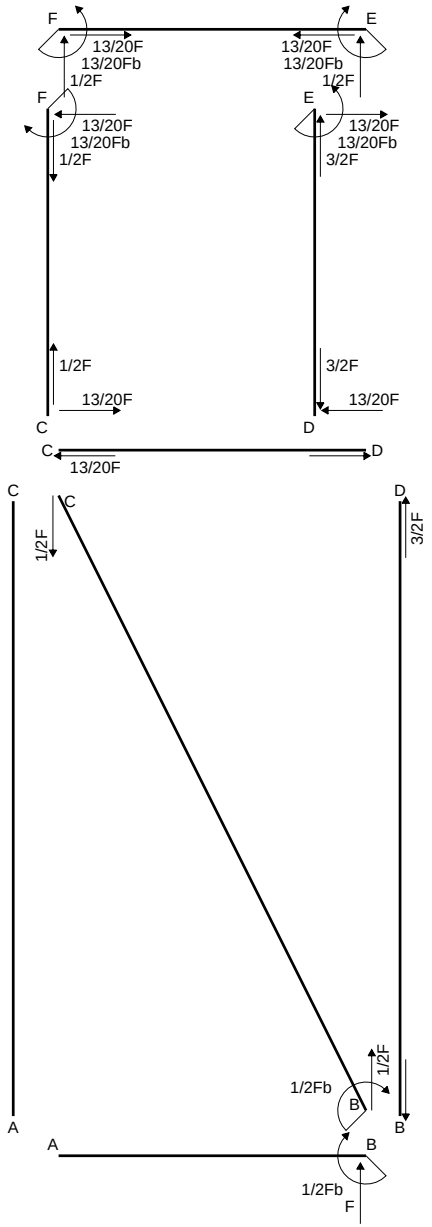
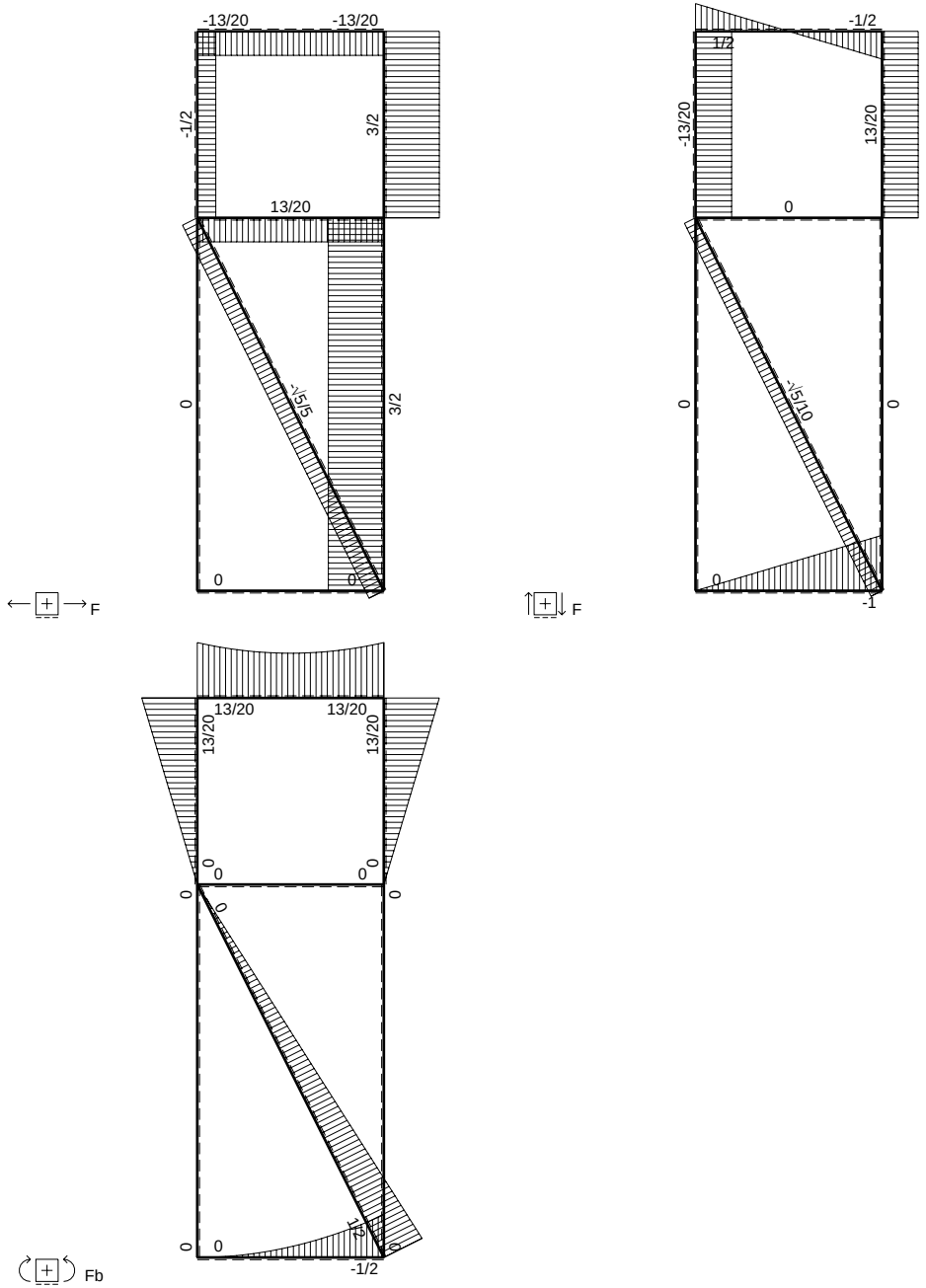

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
-1/2
3/2
13/20
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
totali							$13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/20Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

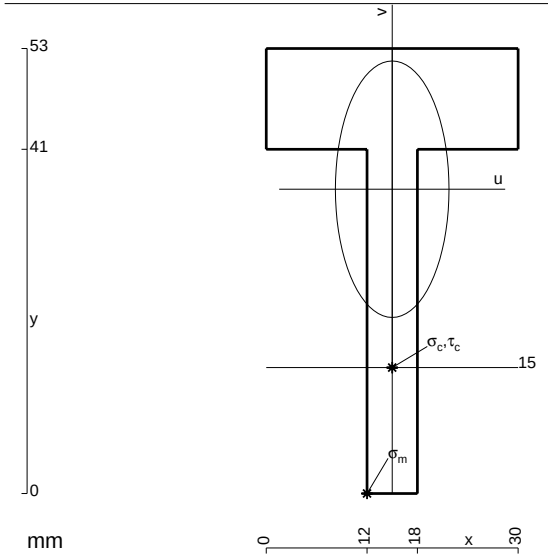
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 606. mm^2
J_u = 141406. mm^4
J_v = 27738. mm^4
y_g = 36.24 mm
T_y = -3900. N
M_x = -858000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.24 mm
$\sigma_m = -Mv/J_u$ = -219.9 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.24 mm
$\sigma_c = -Mv/J_u$ = -128.9 N/mm^2
τ_c = 11.89 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.5 N/mm^2
S^* = 2587. mm^3

ESΣ09.xxxx.102

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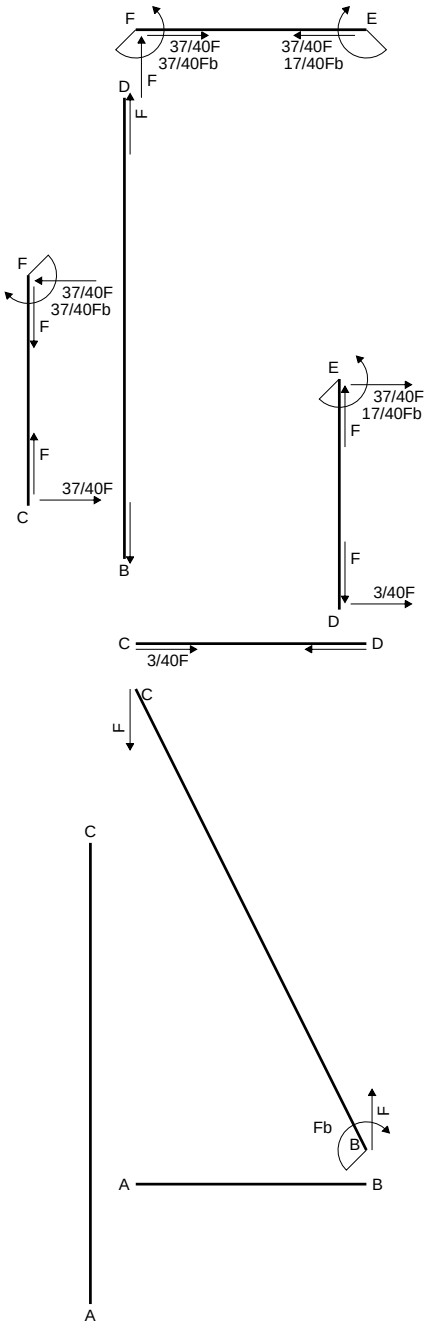
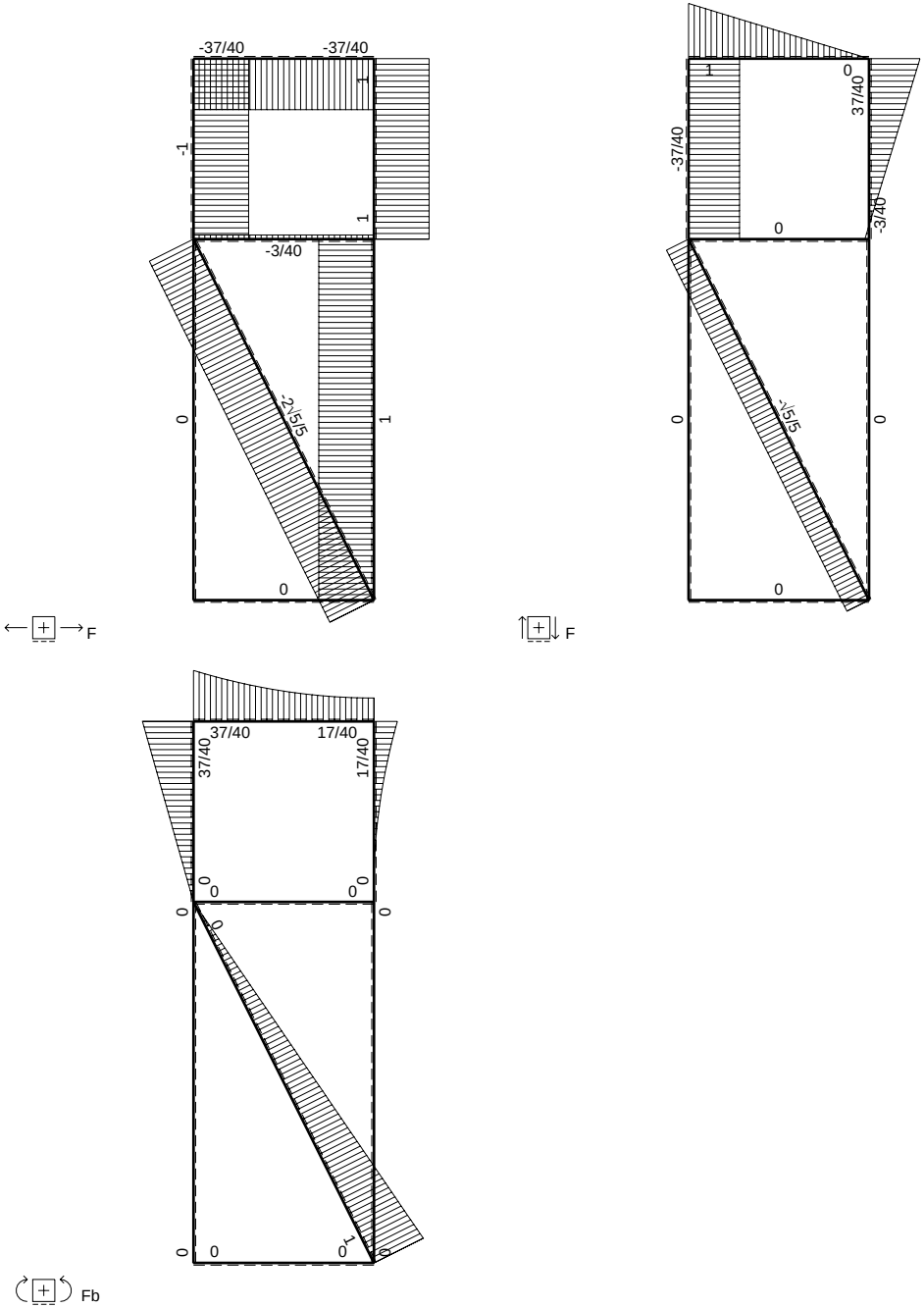

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
F
37/40F
C
B
E
37/40F
17/40Fb
F
F
3/40F
D
C
D
3/40F
C
F
C
Fb
B
F
A
B
A


-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
F
1
0
37/40
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-3/40
0
-√5/5
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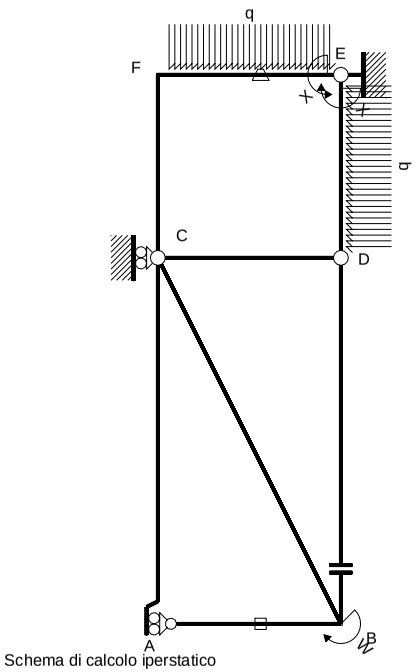
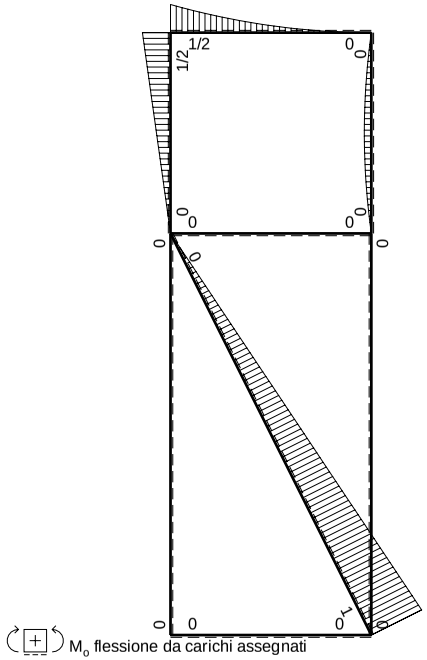
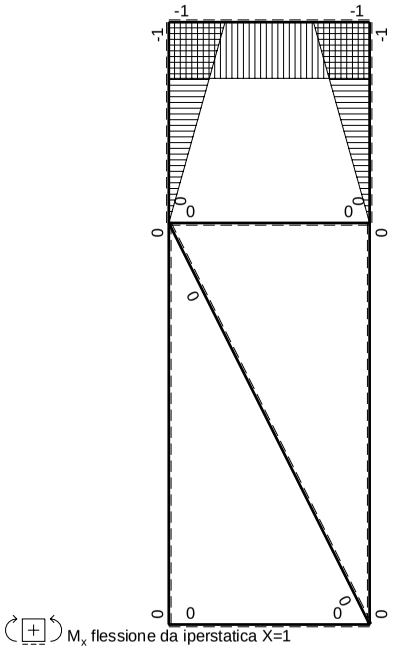
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213259 ESΣ09.xxxx.103

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

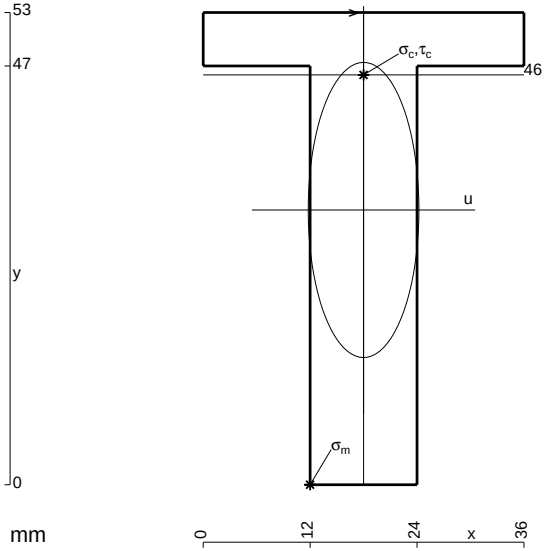
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213259 ESΣ09.xxxx.103



A = 780. mm^2
J_u = 214152. mm^4
J_v = 30096. mm^4
y_g = 30.84 mm
N = -3023. N
T_y = -1512. N
M_x = 1622400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -30.84 mm
$\sigma_m = N/A - Mv/J_u$ = 229.8 N/mm^2
x_c = 18. mm
y_c = 46. mm
v_c = 15.16 mm
$\sigma_c = N/A - Mv/J_u$ = -118.7 N/mm^2
τ_c = 2.545 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.8 N/mm^2
S^* = 4327. mm^3

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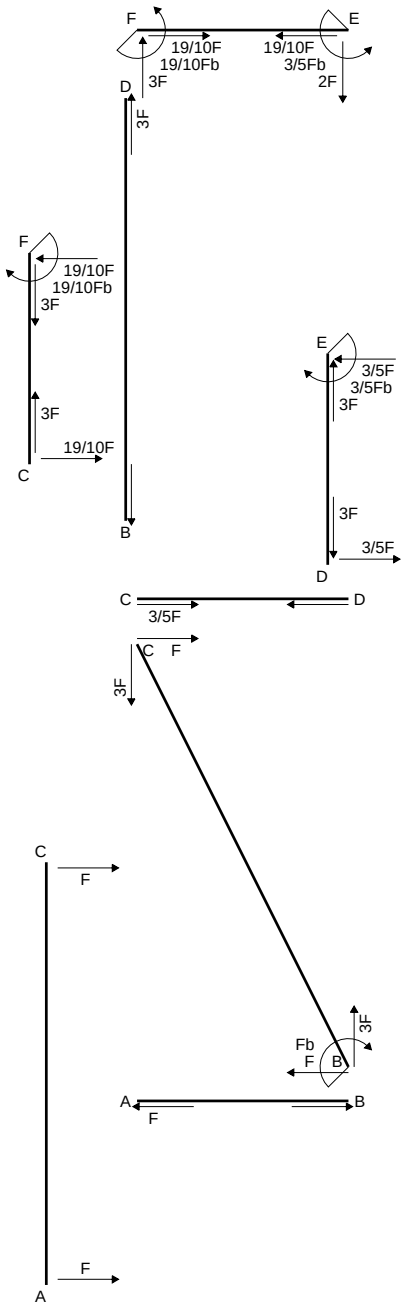
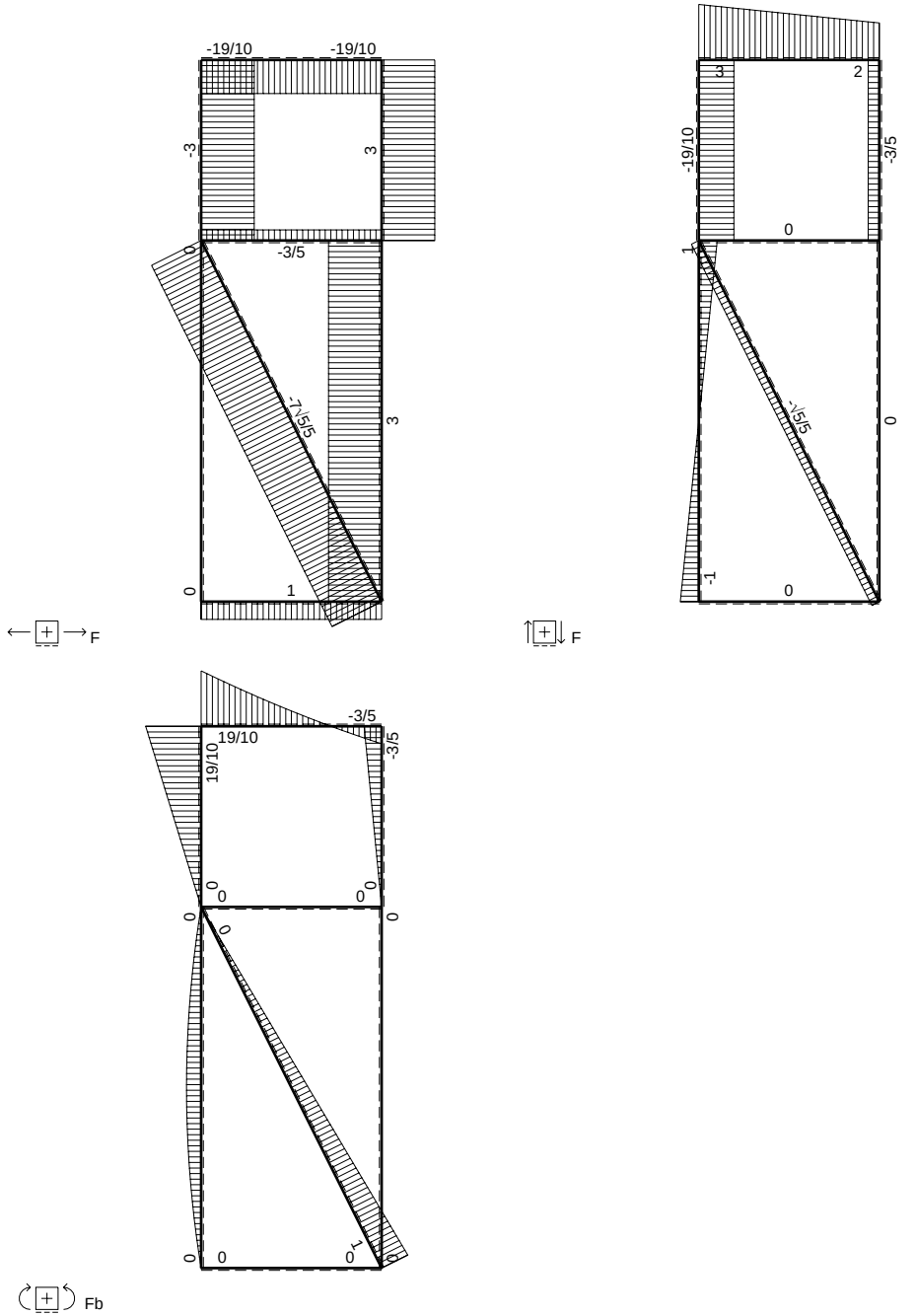

F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
-7\5/5
3
0
1
F
-19/10
3
2
-3/5
0
1
-1\5/5
0
-1
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F
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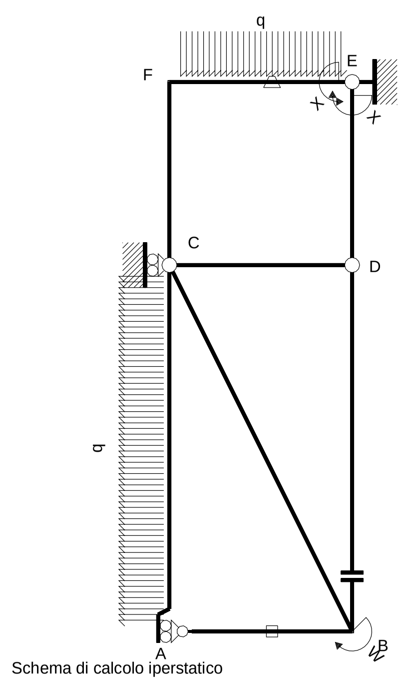
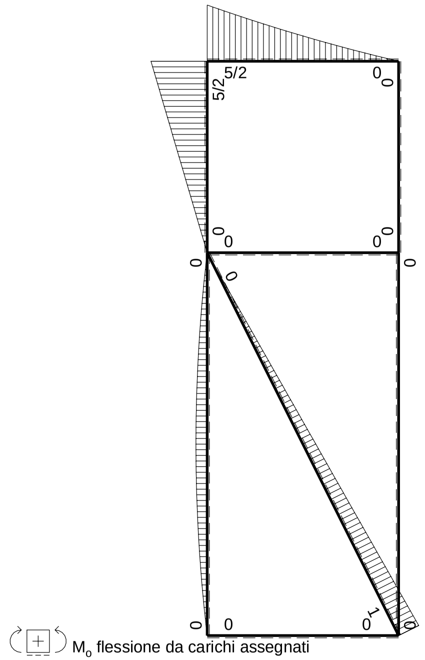
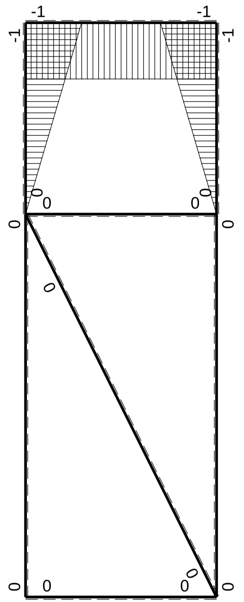
19/10
-3/5
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

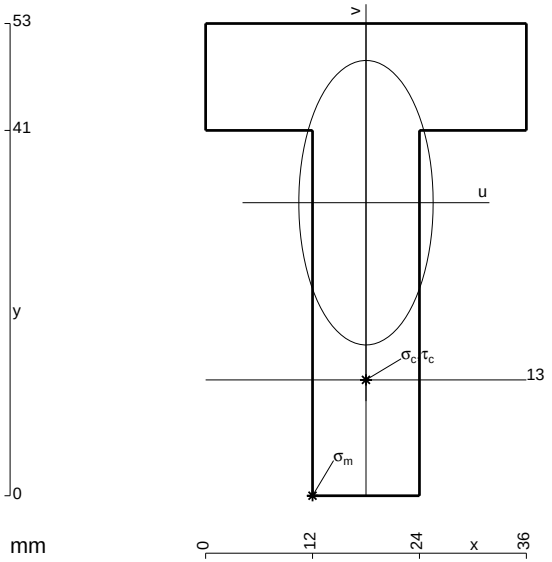
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 924. mm^2
J_u = 235641. mm^4
J_v = 52560. mm^4
y_g = 32.89 mm
N = -11082. N
T_y = -1583. N
M_x = 1805400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.89 mm
$\sigma_m = N/A - Mv/J_u$ = 240. N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -19.89 mm
$\sigma_c = N/A - Mv/J_u$ = 140.4 N/mm^2
τ_c = 2.305 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.5 N/mm^2
S^* = 4117. mm^3

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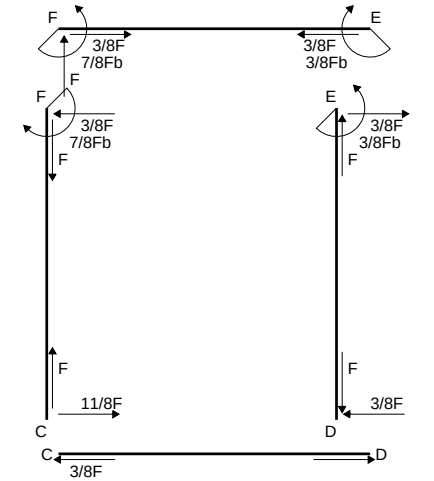
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7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
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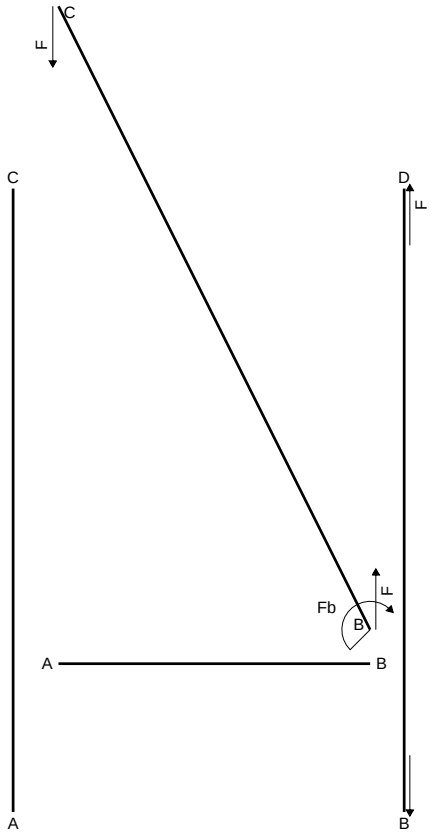
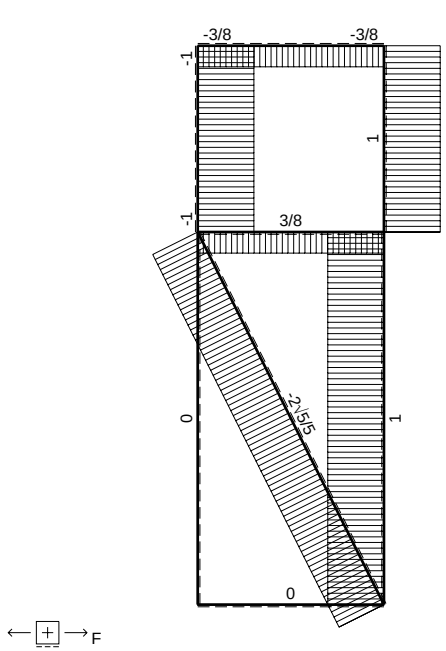
F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B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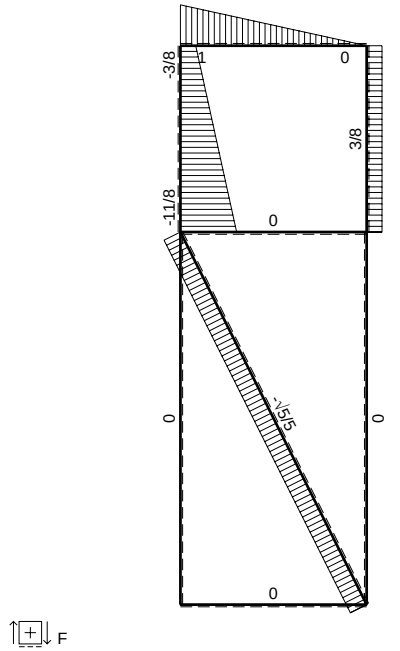
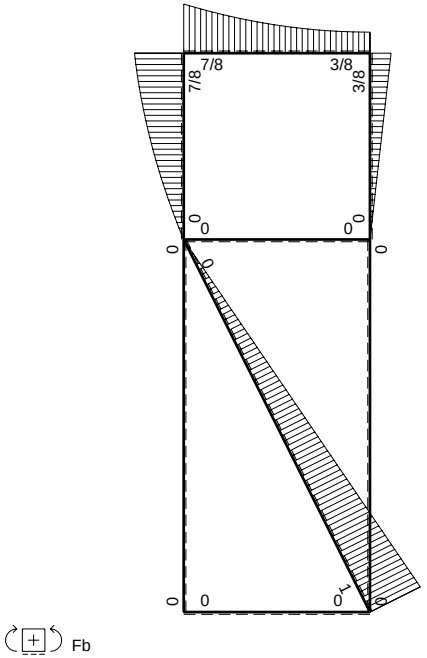
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

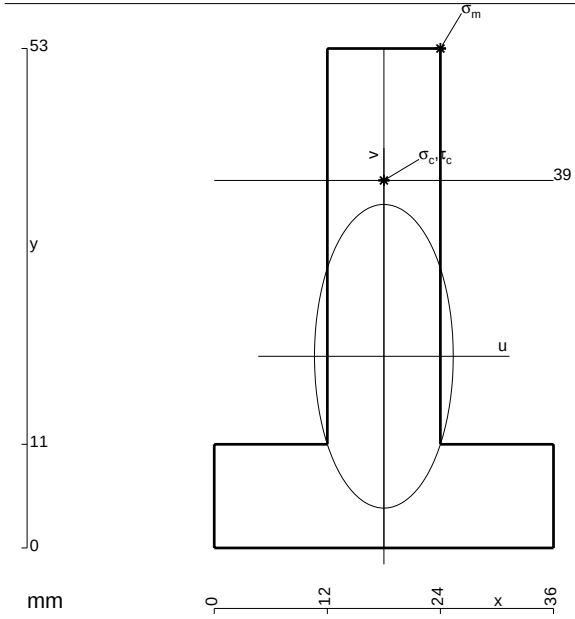
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 900. mm^2
J_u = 233812. mm^4
J_v = 48816. mm^4
y_g = 20.34 mm
N = -2299. N
T_y = -1149. N
M_x = 1413500. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 32.66 mm
$\sigma_m = N/A - Mv/J_u$ = -200. N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 18.66 mm
$\sigma_c = N/A - Mv/J_u$ = -115.4 N/mm^2
τ_c = 1.766 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.4 N/mm^2
S^* = 4311. mm^3

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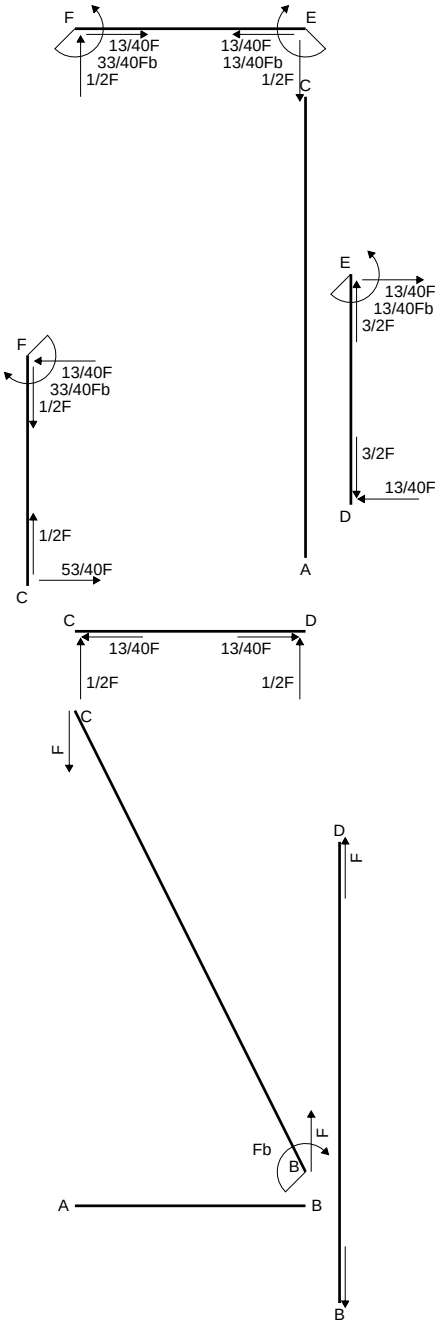
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E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
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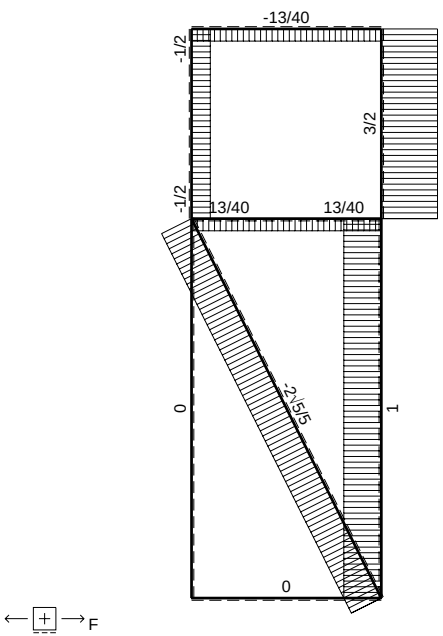
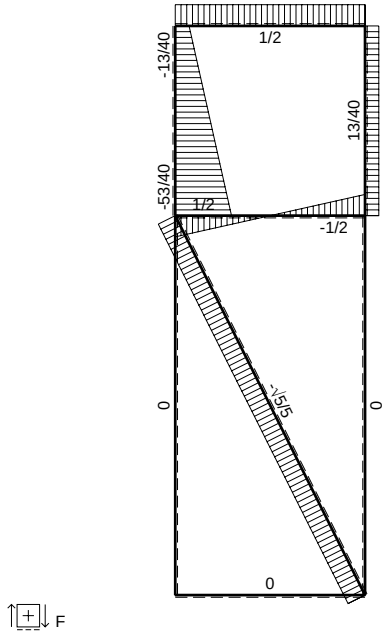
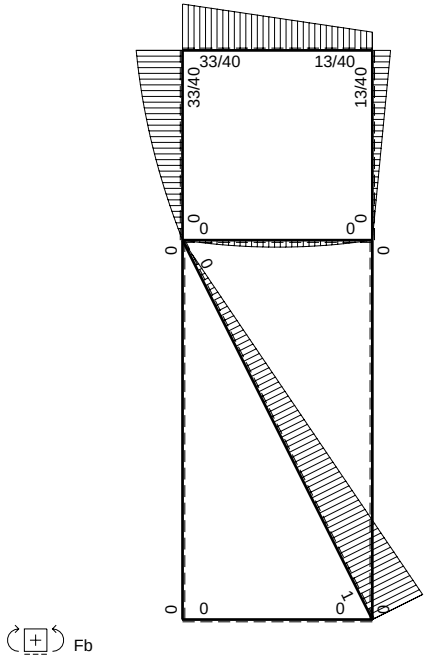
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F


-13/40
1/2
13/40
-53/40
1/2
-1/2
0
-√5/5
0
0
F


33/40
33/40
13/40
13/40
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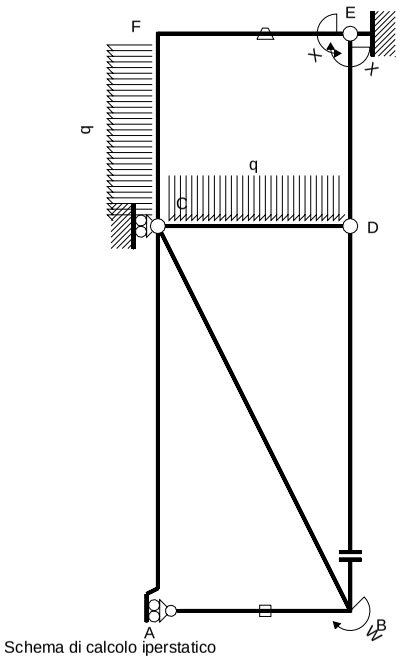
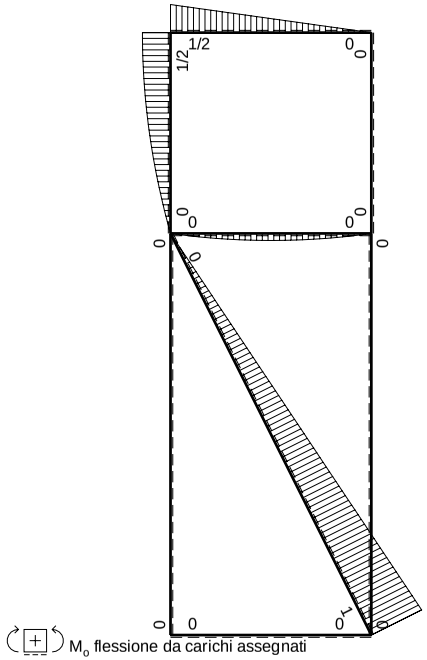
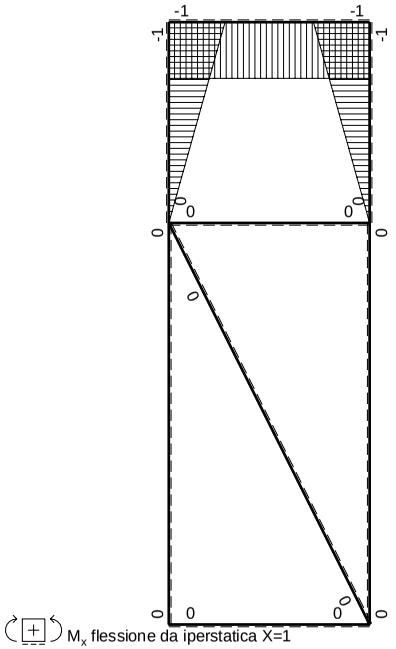
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 961051 ESΣ09.xxxx.106

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

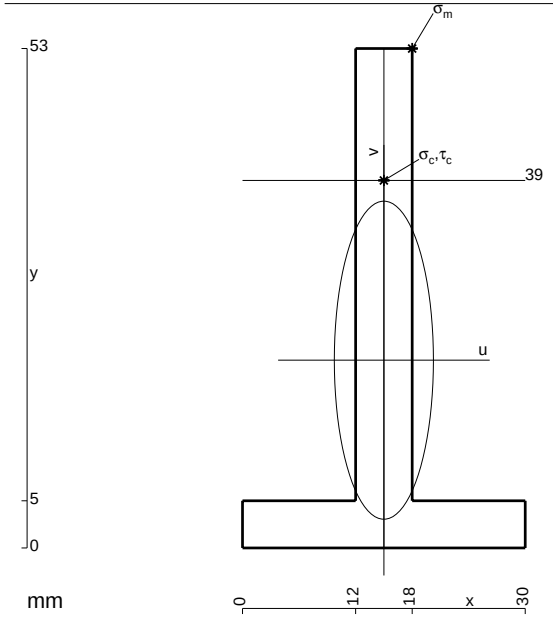
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 438. mm^2
J_u = 124872. mm^4
J_v = 12114. mm^4
y_g = 19.92 mm
N = -1181. N
T_y = -590.3 N
M_x = 778800. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 33.08 mm
$\sigma_m = N/A - Mv/J_u$ = -209. N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 19.08 mm
$\sigma_c = N/A - Mv/J_u$ = -121.7 N/mm^2
τ_c = 1.726 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.7 N/mm^2
S^* = 2190. mm^3

ESΣ09.xxxx.106

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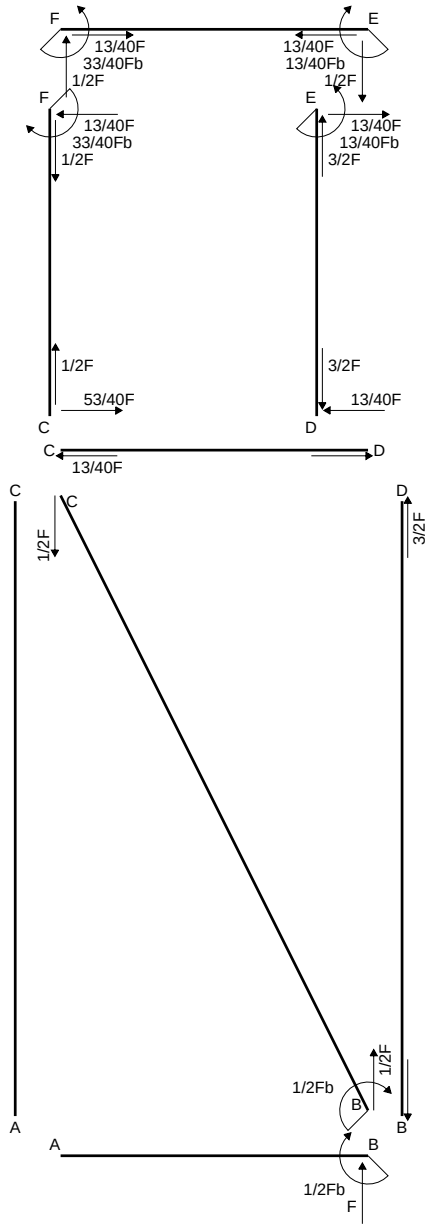
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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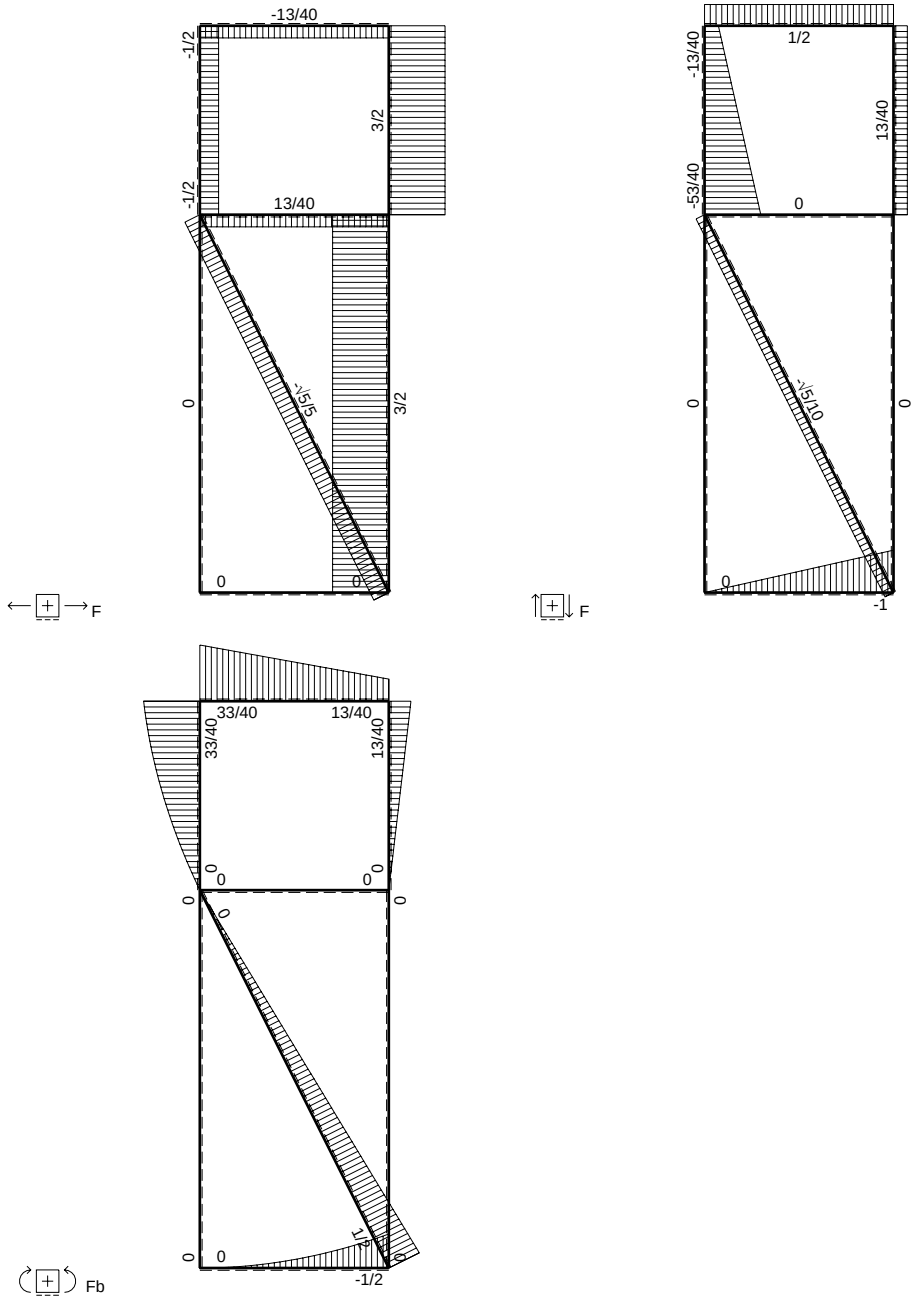
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
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13/40
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3/2
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0
0
F
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1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
0
0
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

PROCEDIMENTO E RISULTATI 212416 ESΣ09.xxxx.107

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

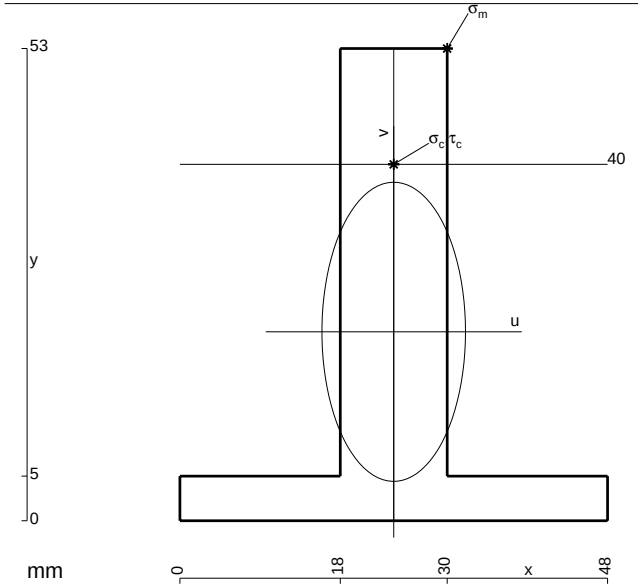
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 212416 ESΣ09.xxxx.107



A = 816. mm^2
J_u = 230061. mm^4
J_v = 52992. mm^4
y_g = 21.21 mm
T_y = -5130. N
M_x = -1590300. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 31.79 mm
$\sigma_m = -Mv/J_u$ = 219.8 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 18.79 mm
$\sigma_c = -Mv/J_u$ = 129.9 N/mm^2
τ_c = 7.332 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.5 N/mm^2
S^* = 3946. mm^3

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ESΣ09.xxxx.107

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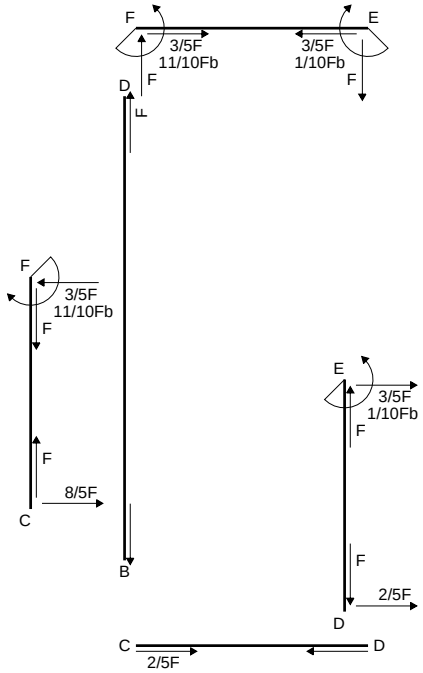
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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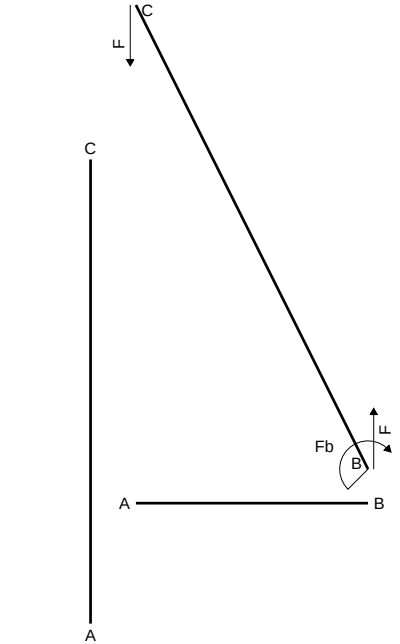
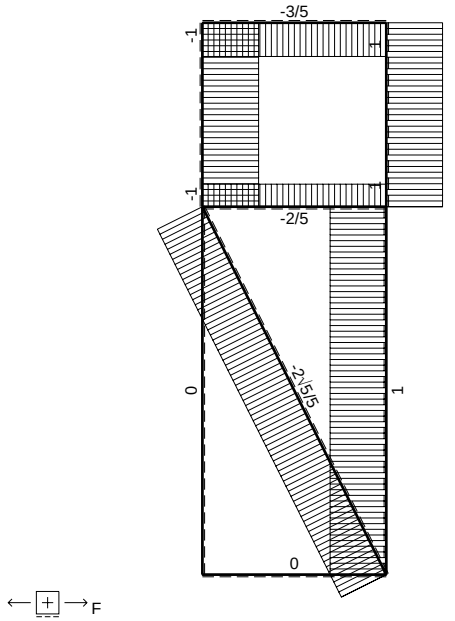
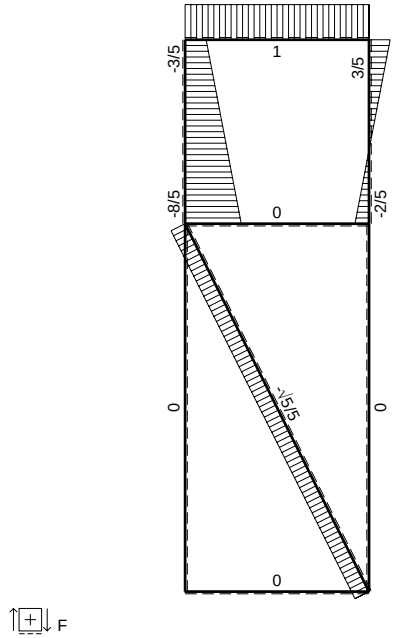
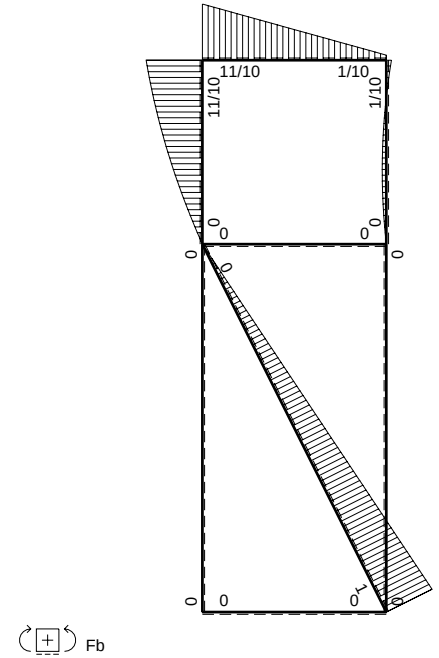
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
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0
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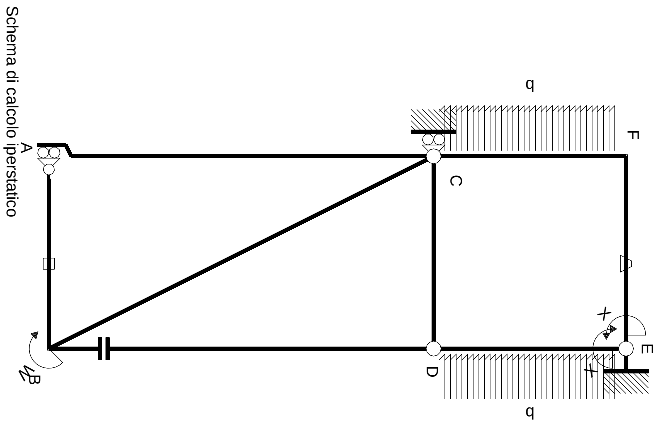
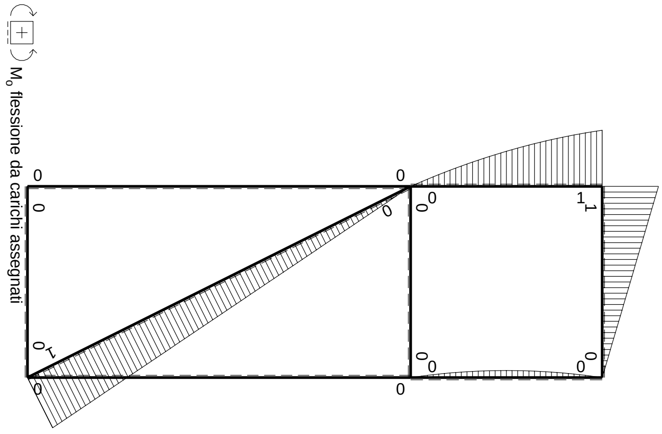
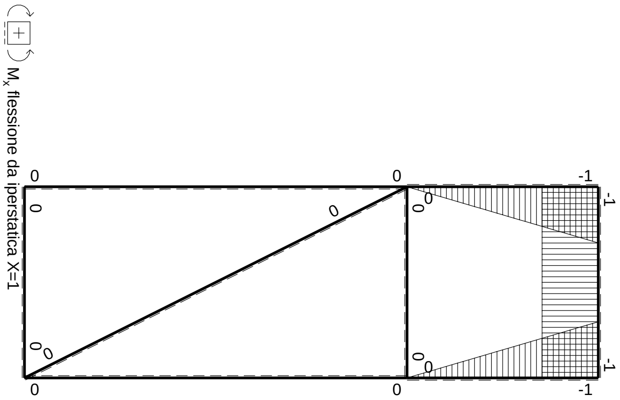
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

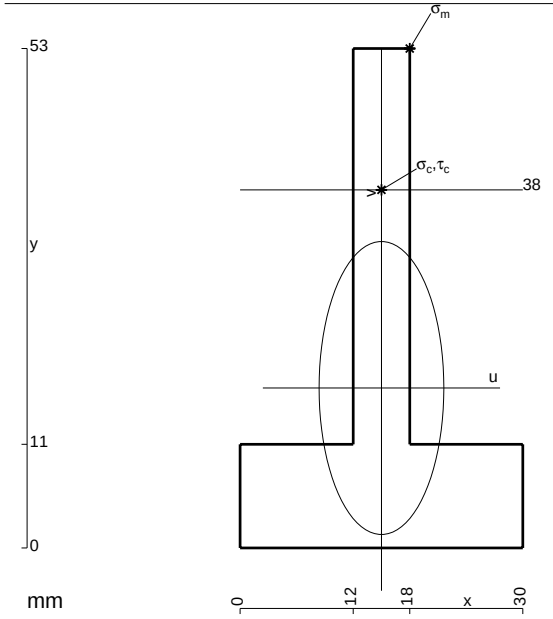
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 582. mm^2
J_u = 140714. mm^4
J_v = 25506. mm^4
y_g = 16.97 mm
N = -1199. N
T_y = -599.3 N
M_x = 884400. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.03 mm
$\sigma_m = N/A - Mv/J_u$ = -228.5 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.03 mm
$\sigma_c = N/A - Mv/J_u$ = -134.2 N/mm^2
τ_c = 1.822 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 134.2 N/mm^2
S^* = 2567. mm^3

ESΣ09.xxxx.108

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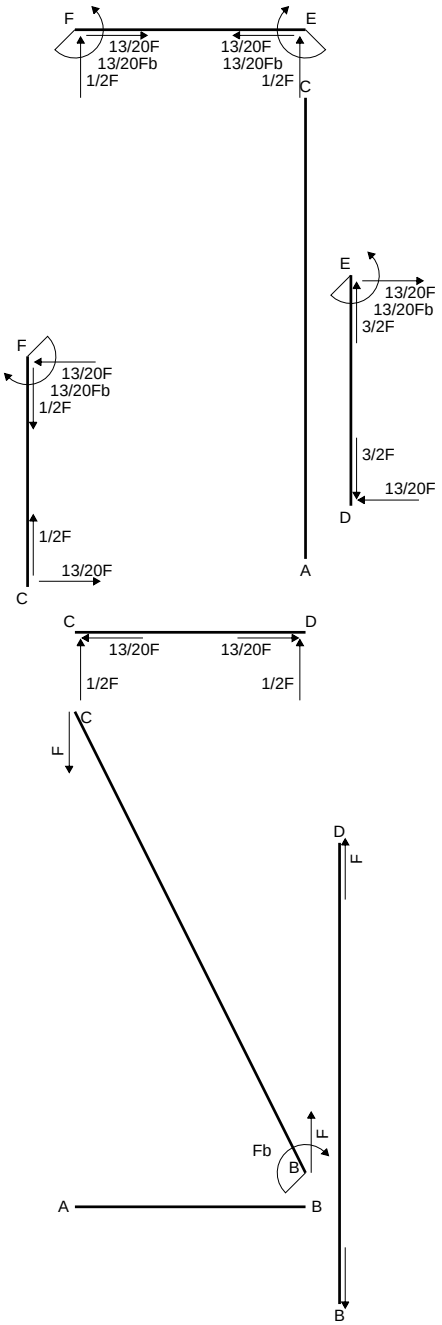
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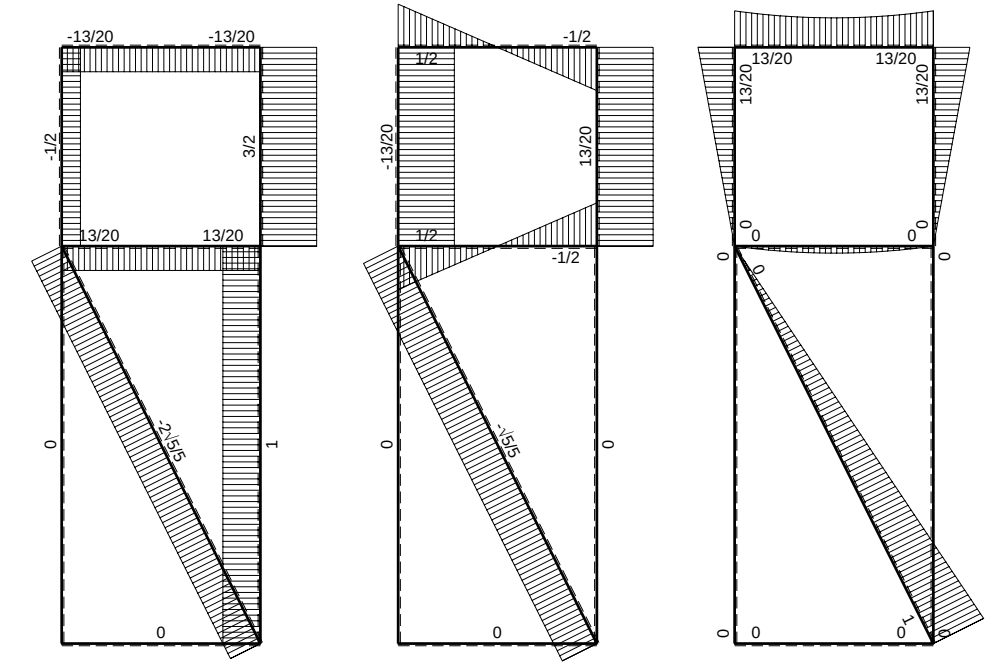
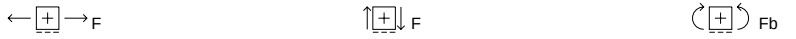
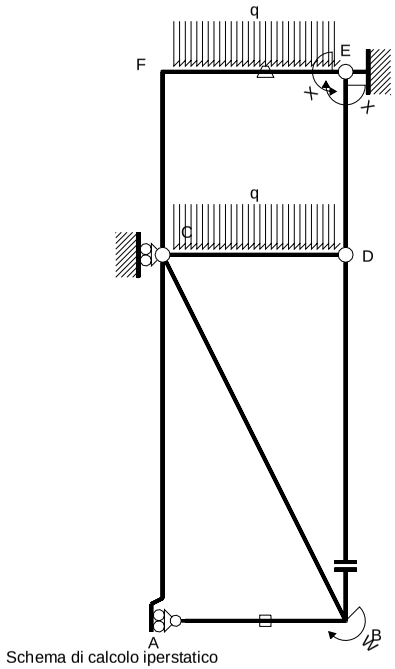
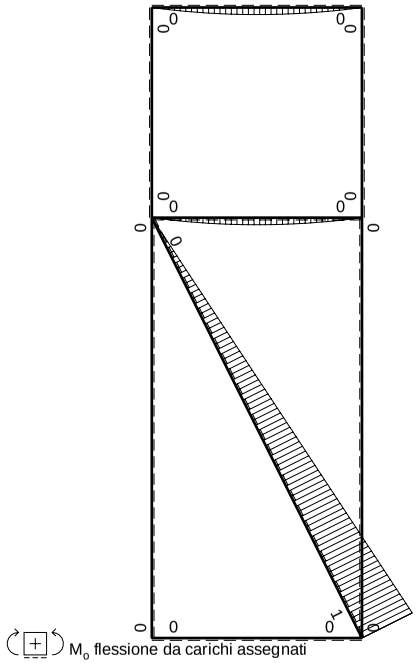
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

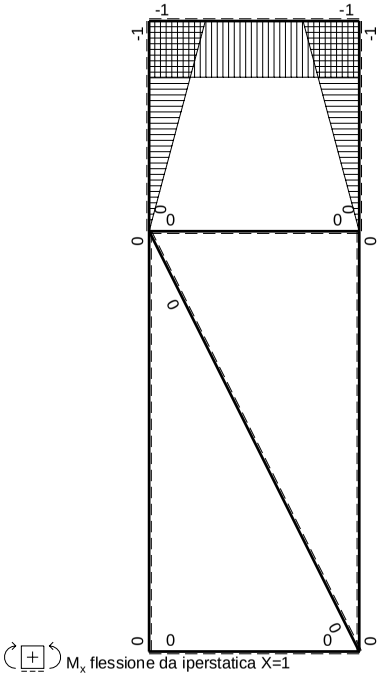

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957197 ESΣ09.xxxx.109

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

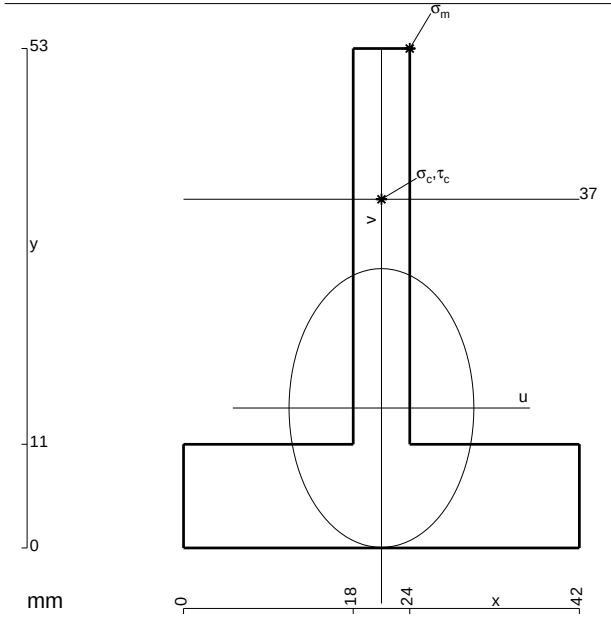
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 714. mm^2
J_u = 156211. mm^4
J_v = 68670. mm^4
y_g = 14.85 mm
N = -1243. N
T_y = -621.6 N
M_x = 973000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.15 mm
$\sigma_m = N/A - Mv/J_u$ = -239.4 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.15 mm
$\sigma_c = N/A - Mv/J_u$ = -139.7 N/mm^2
τ_c = 1.919 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.7 N/mm^2
S^* = 2894. mm^3

ESΣ09.xxxx.109

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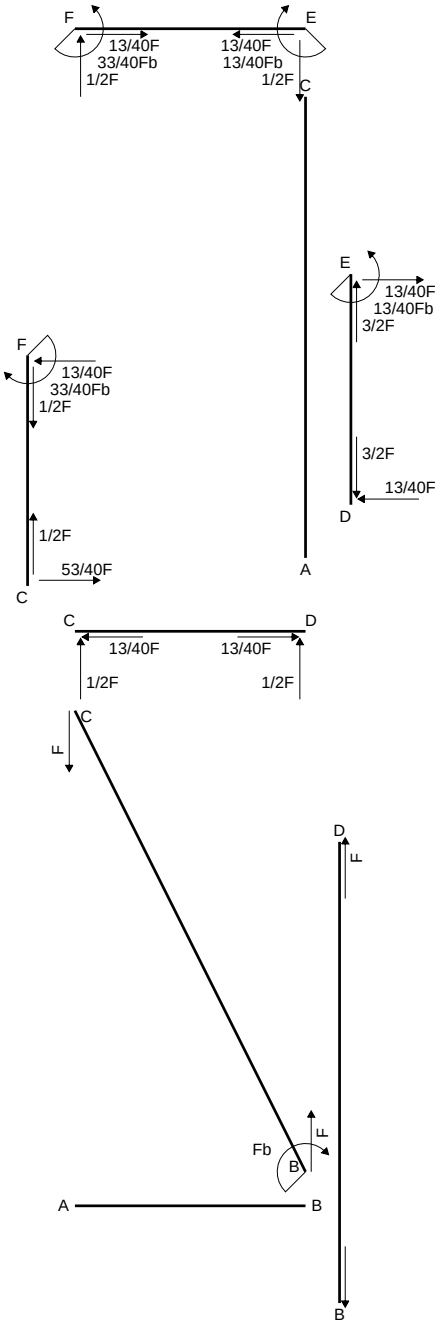
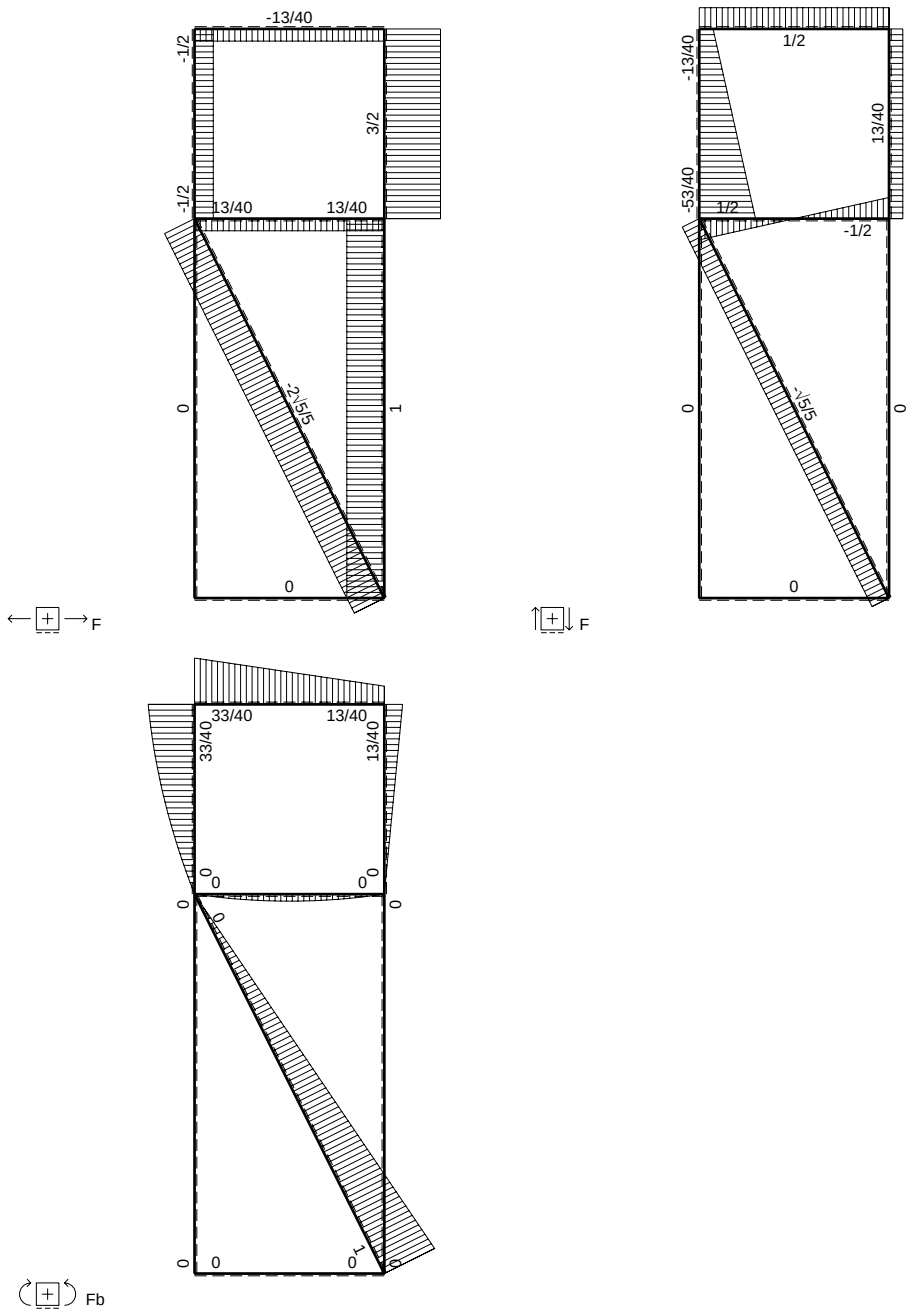

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
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3/2
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13/40
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0
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-FbEJ	-1/2Fx	FbEJ	1	$(-1/4+1)Fb^2/EJ$	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	FbEJ	$-1/2Fb+1/2Fx$	FbEJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210257 ESΣ09.xxxx.110

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$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

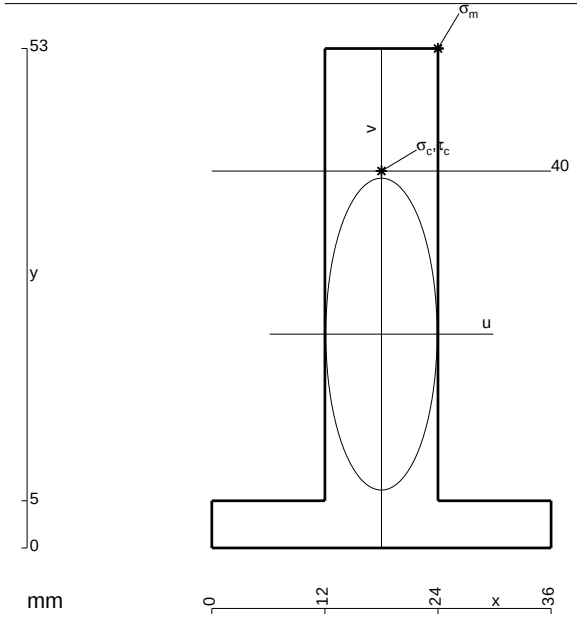
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210257 ESΣ09.xxxx.110



A = 756. mm^2
J_u = 207276. mm^4
J_v = 26352. mm^4
y_g = 22.69 mm
N = -3229. N
T_y = -1614. N
M_x = 1335700. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 30.31 mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 17.31 mm
$\sigma_c = N/A - Mv/J_u$ = -115.8 N/mm^2
τ_c = 2.411 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.9 N/mm^2
S^* = 3714. mm^3

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ESΣ09.xxxx.110

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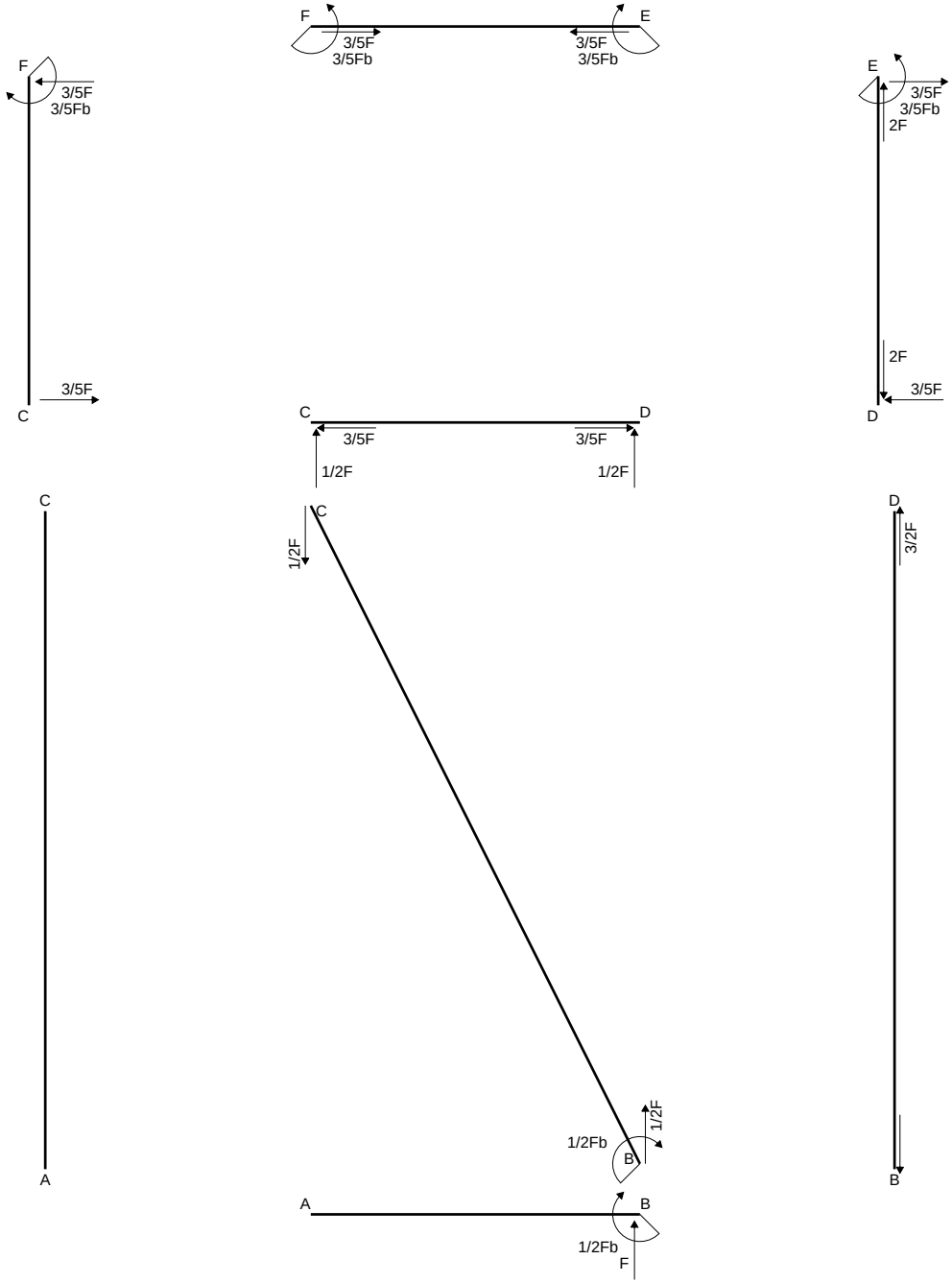
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E
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3/5Fb
3/5F
3/5Fb
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3/5F
3/5Fb
E
3/5F
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2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
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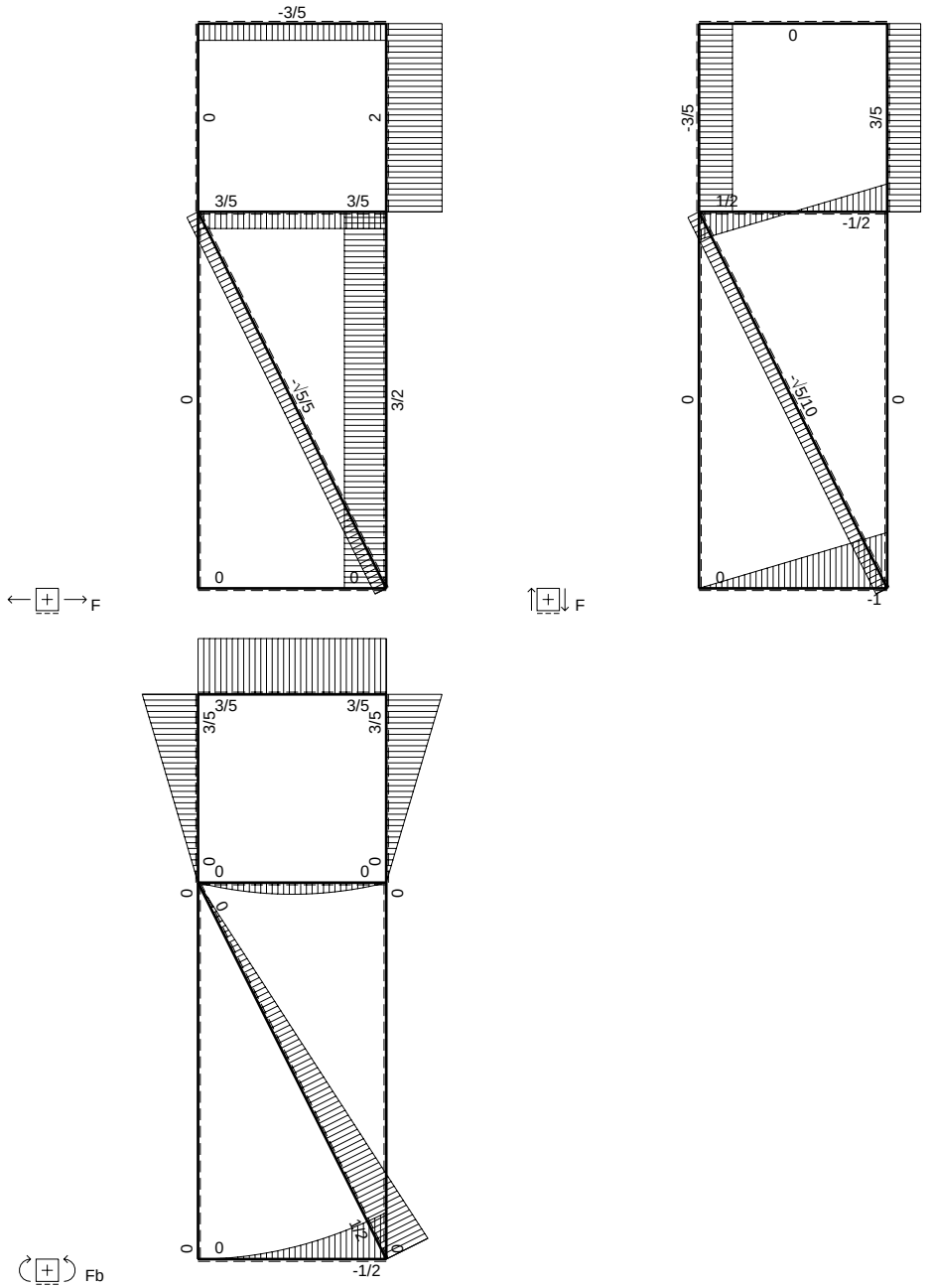
A
B
1/2Fb
F


-3/5
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3/5
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3/2
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F
0
-3/5
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-1/2
0
-√5/10
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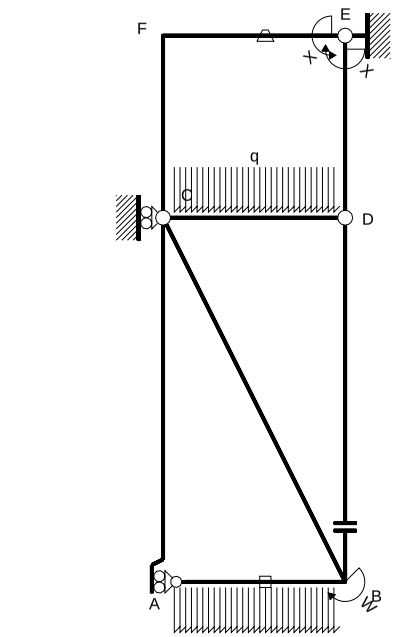
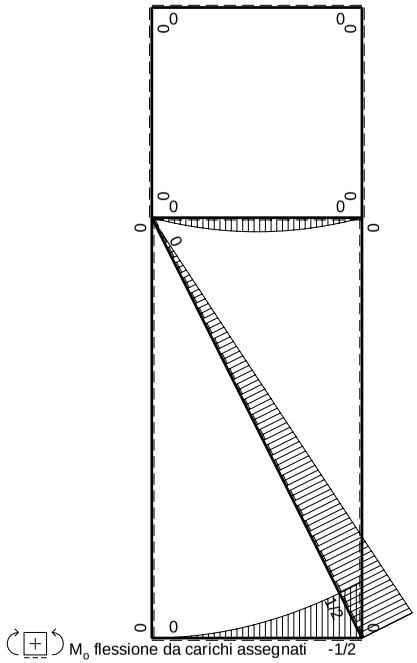
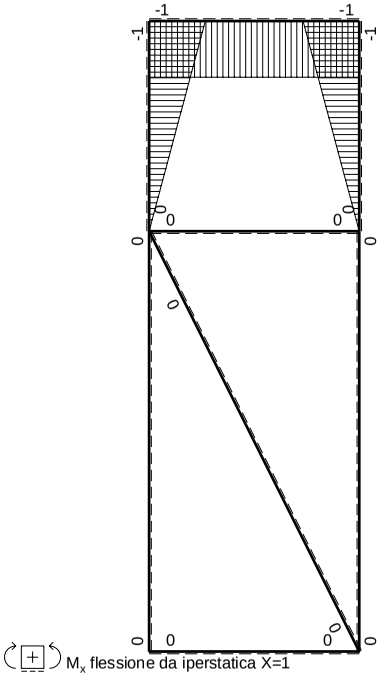
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

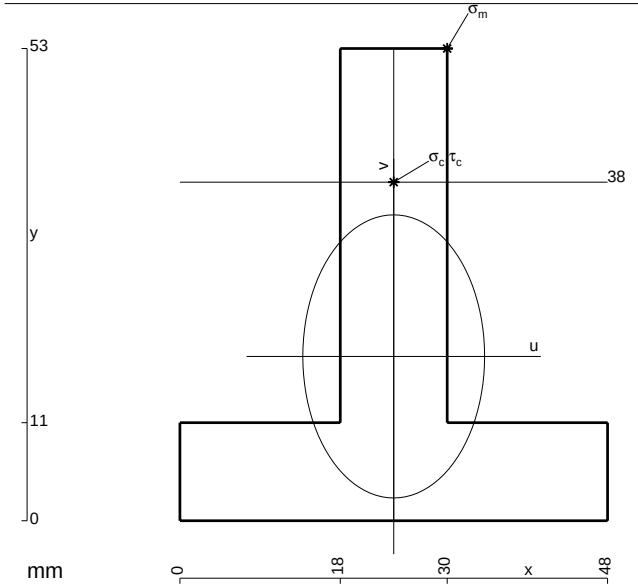
$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211522 ESΣ09.xxxx.111



A = 1032. mm^2
J_u = 260495. mm^4
J_v = 107424. mm^4
y_g = 18.44 mm
T_y = -7910. N
M_x = -1582000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.56 mm
$\sigma_m = -Mv/J_u$ = 209.9 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.56 mm
$\sigma_c = -Mv/J_u$ = 118.8 N/mm^2
τ_c = 12.32 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.7 N/mm^2
S^* = 4870. mm^3

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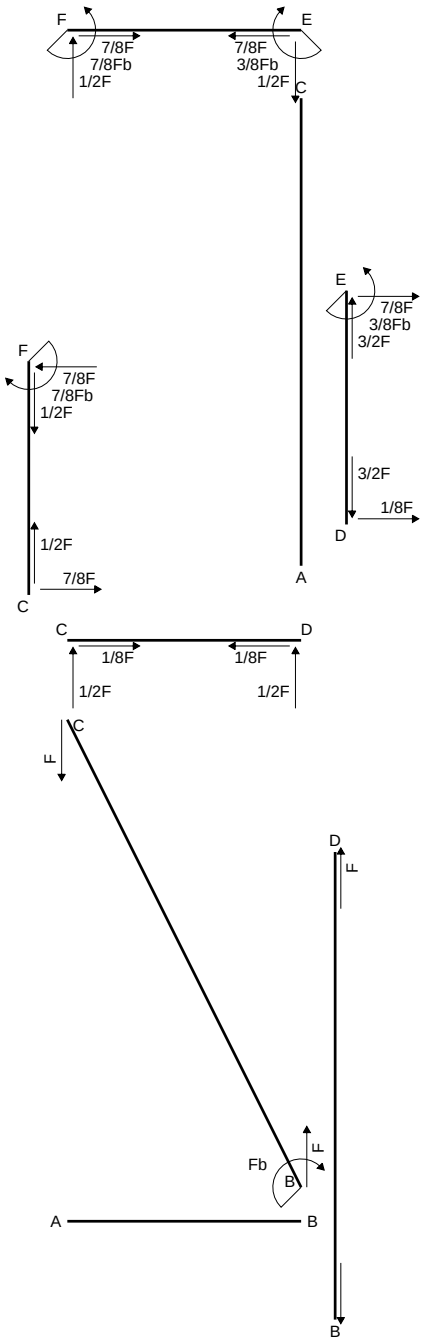
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
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1/2F
C
F
D
F
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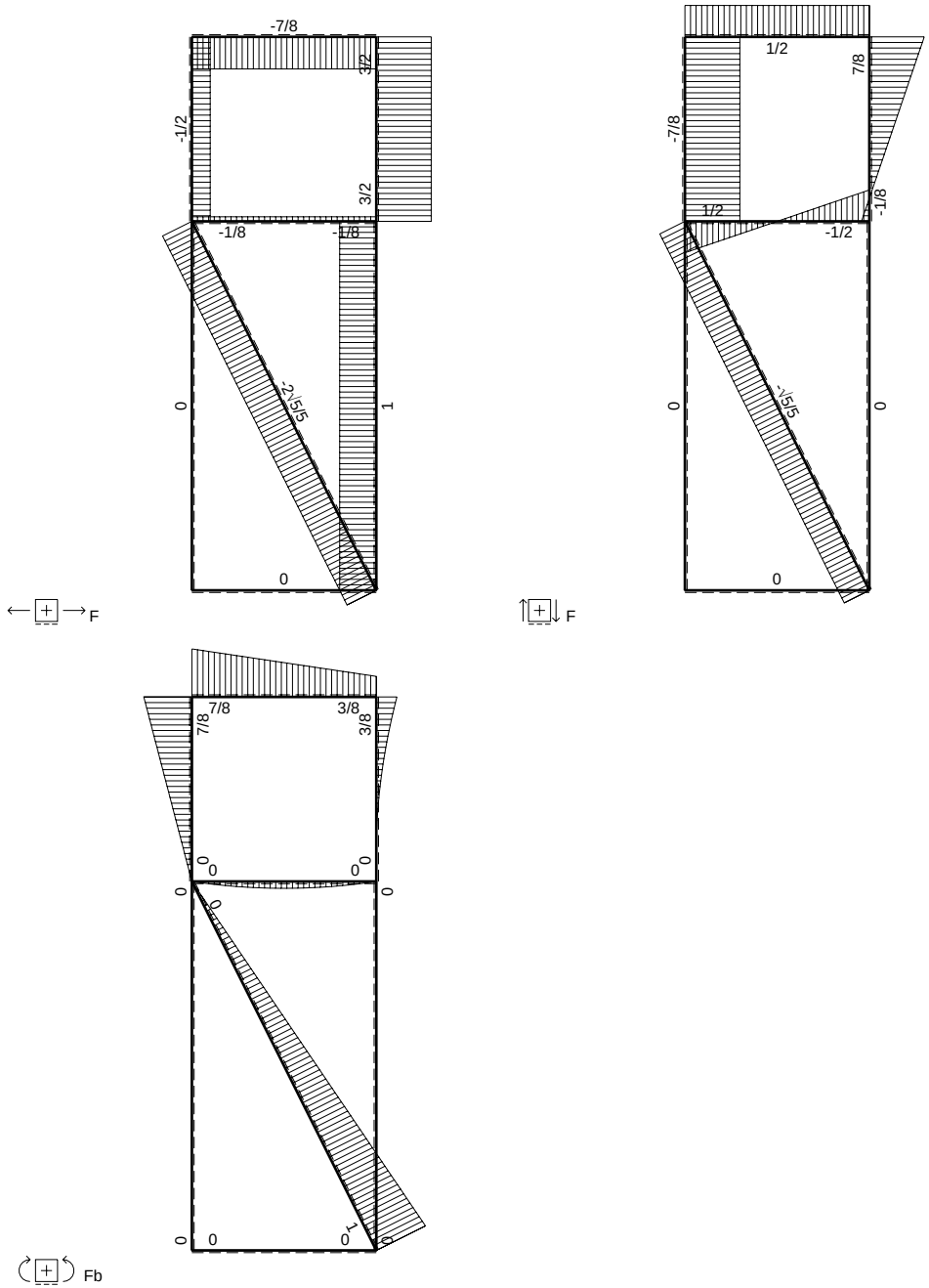
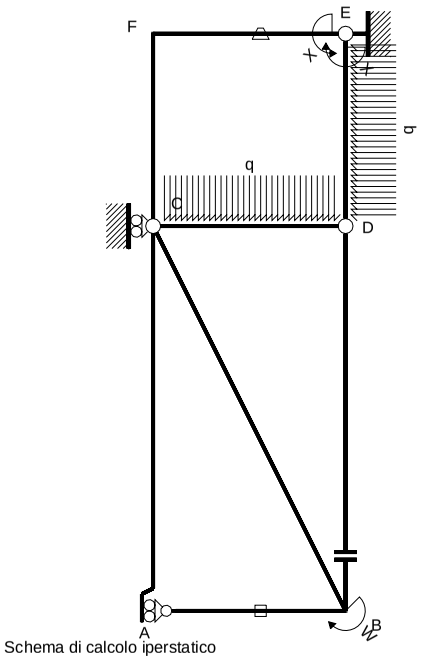
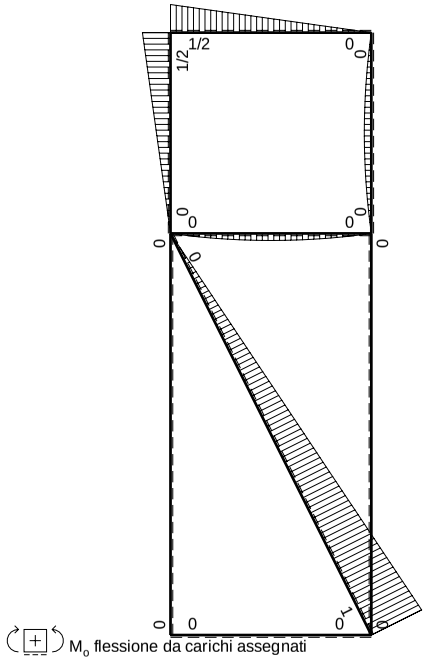
F
B
A
B
B

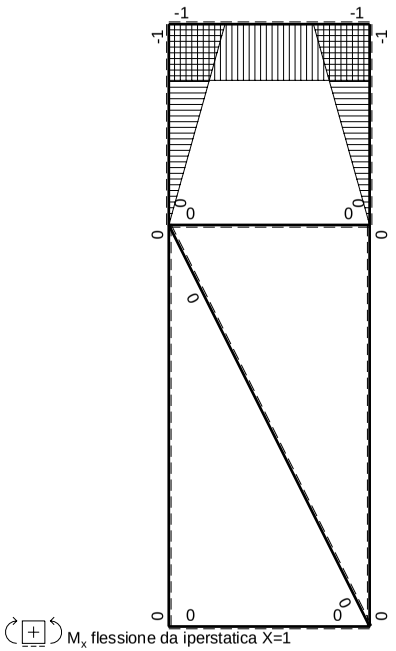

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

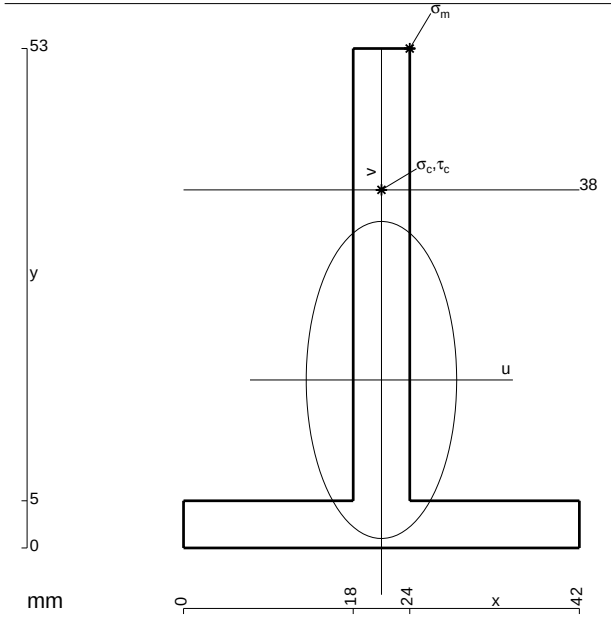
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 498. mm^2
J_u = 141019. mm^4
J_v = 31734. mm^4
y_g = 17.83 mm
N = -1762. N
T_y = -881. N
M_x = 866800. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 35.17 mm
$\sigma_m = N/A - Mv/J_u$ = -219.7 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 20.17 mm
$\sigma_c = N/A - Mv/J_u$ = -127.5 N/mm^2
τ_c = 2.593 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.6 N/mm^2
S^* = 2491. mm^3

ESΣ09.xxxx.112

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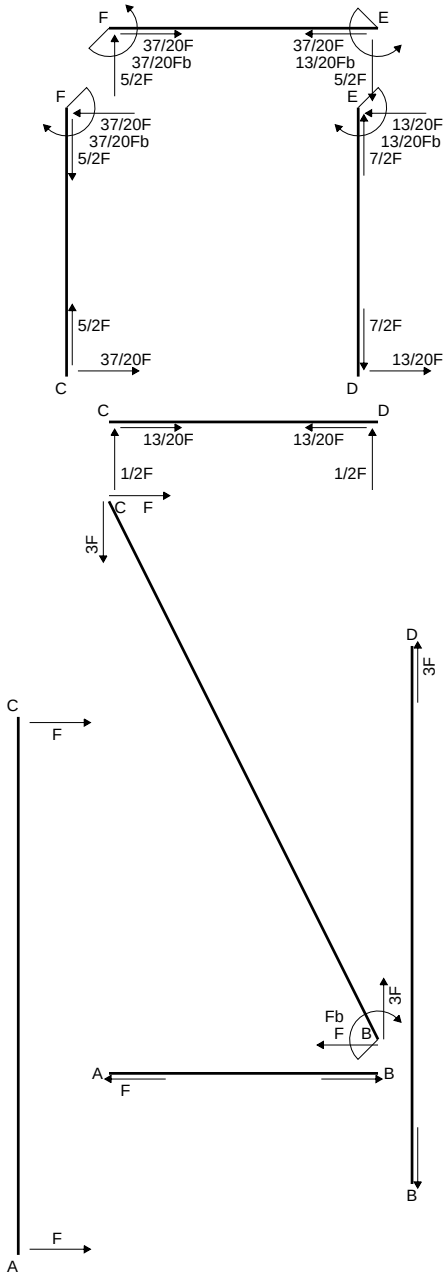
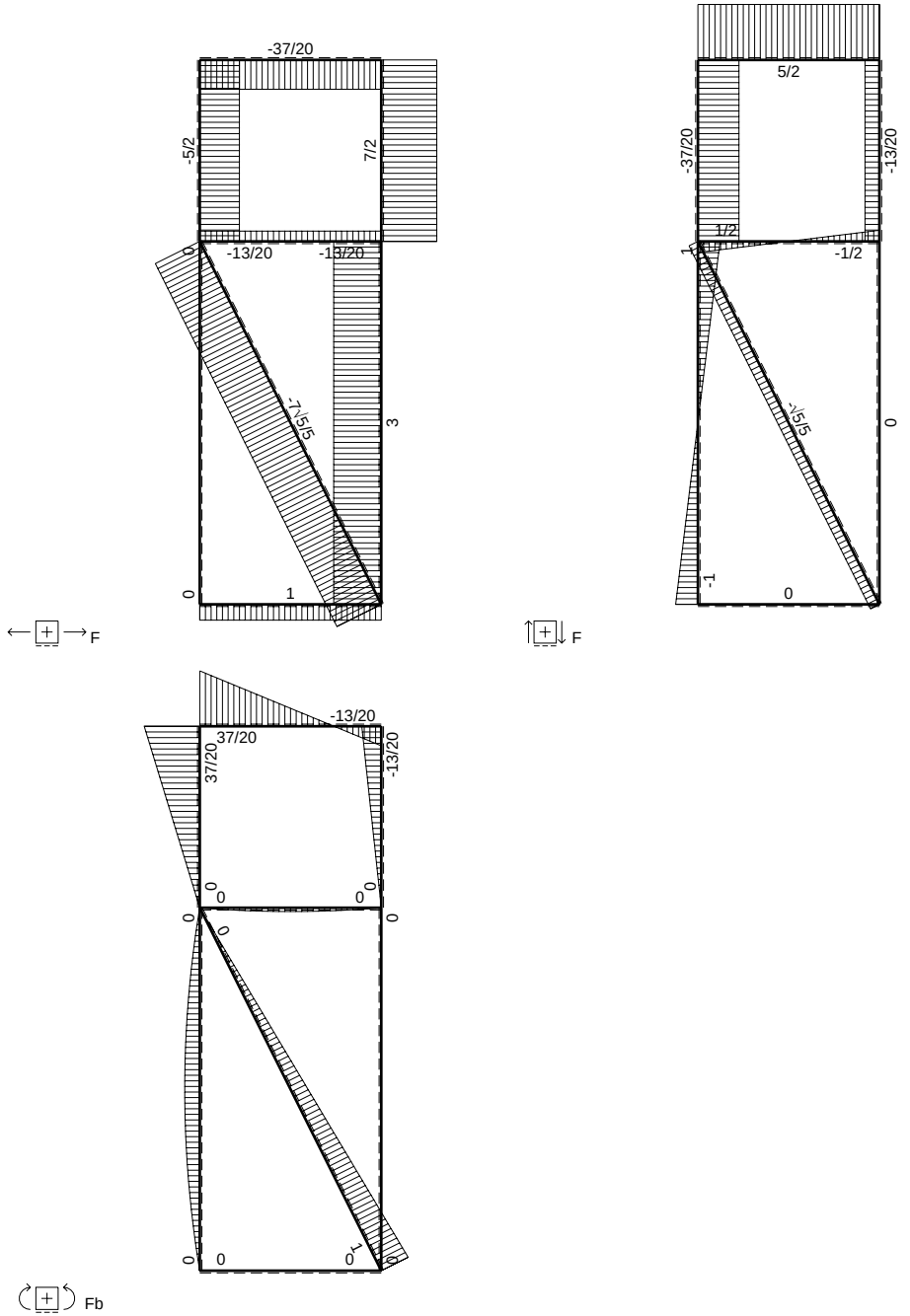

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B


-37/20
-5/2
7/2
-13/20
-13/20
0
-7.5/5
3
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211767 ESΣ09.xxxx.113

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

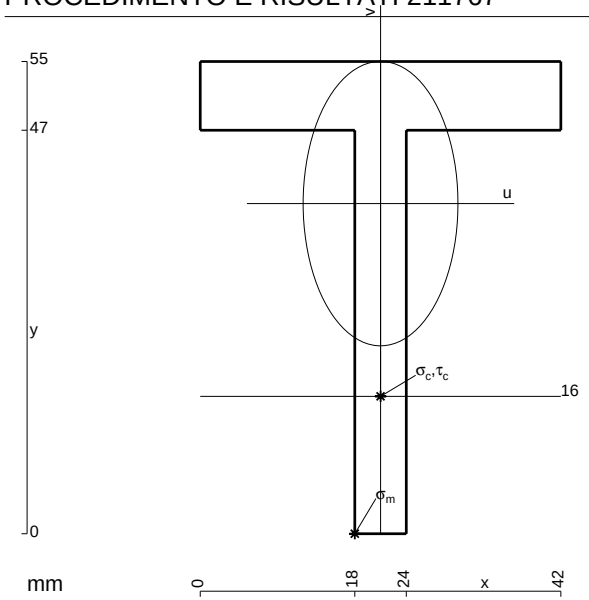
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 618. mm^2
J_u = 169652. mm^4
J_v = 50238. mm^4
y_g = 38.45 mm
N = -6637. N
T_y = -948.1 N
M_x = 1060000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.45 mm
$\sigma_m = N/A - Mv/J_u$ = 229.5 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.45 mm
$\sigma_c = N/A - Mv/J_u$ = 129.5 N/mm^2
τ_c = 2.723 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.6 N/mm^2
S^* = 2923. mm^3

ESΣ09.xxxx.113

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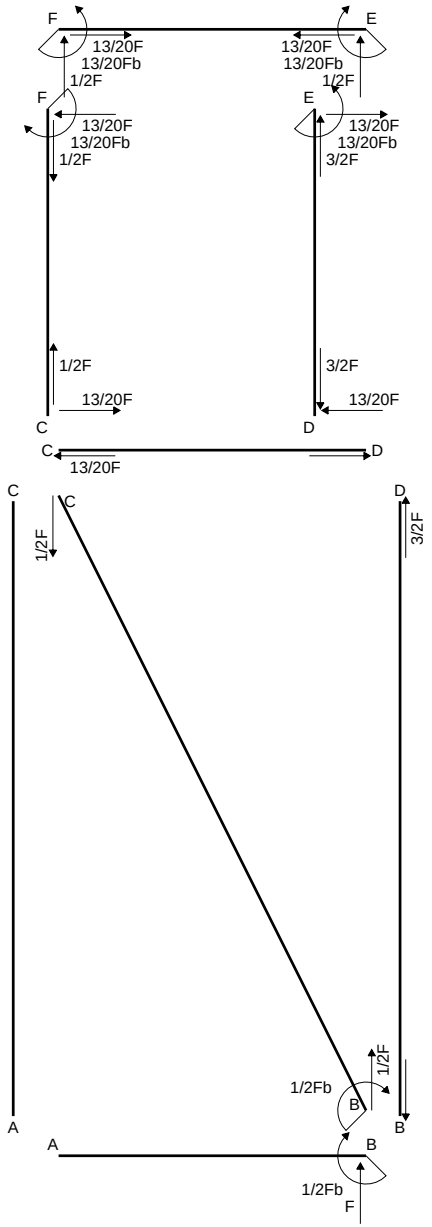
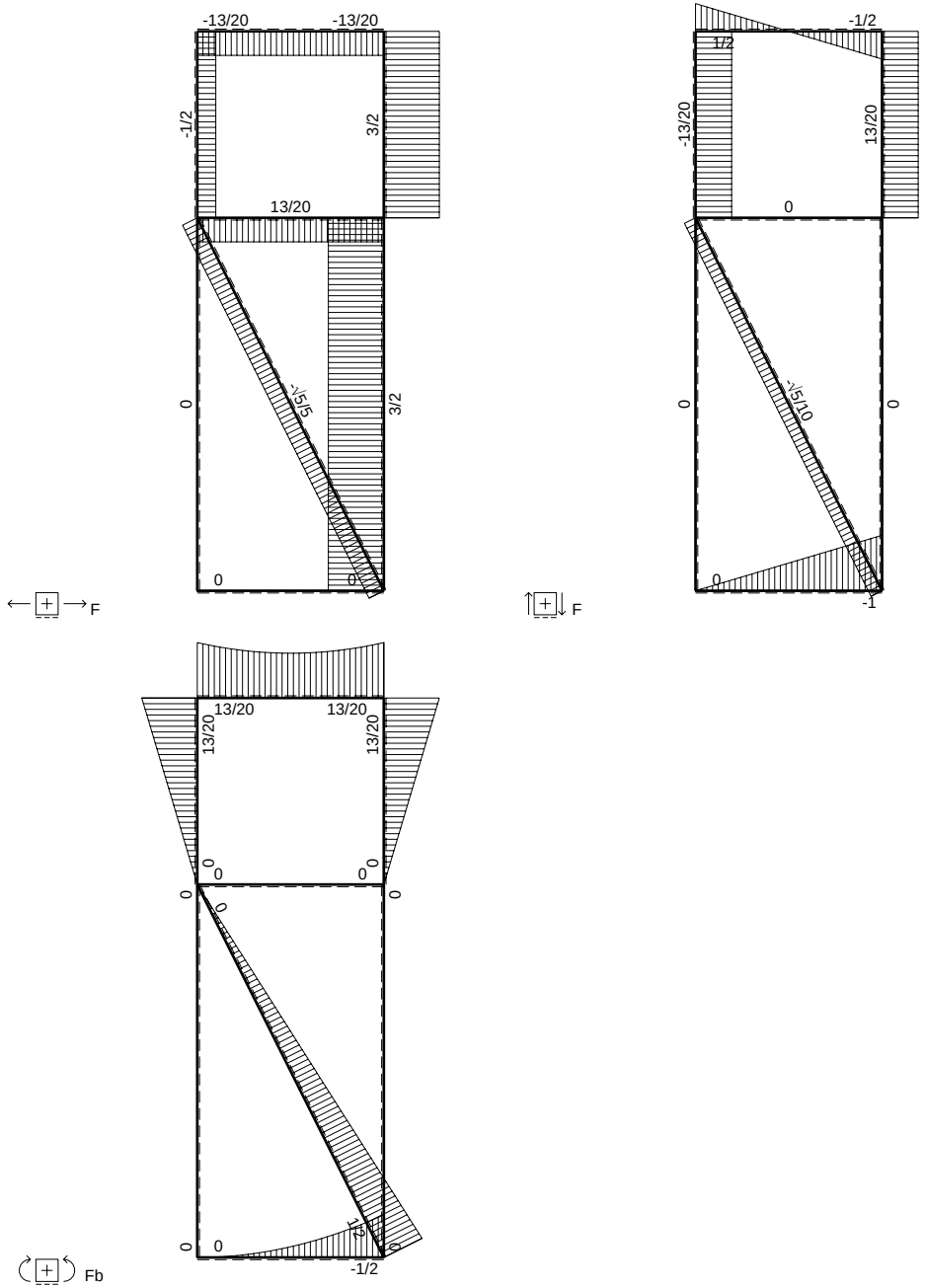
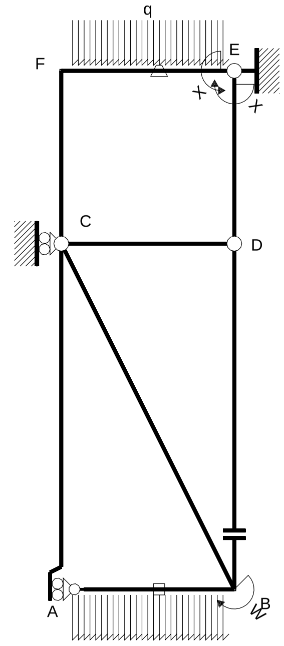
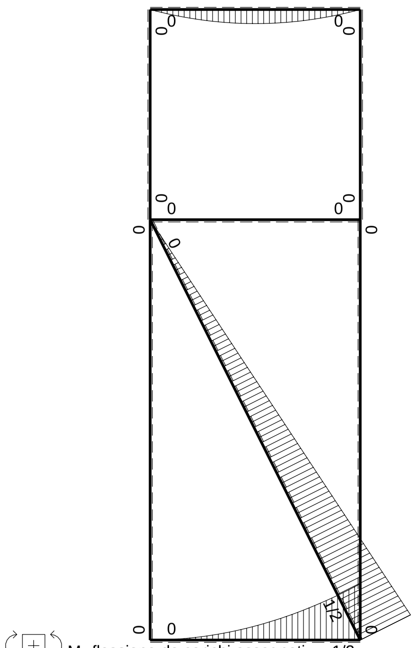

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

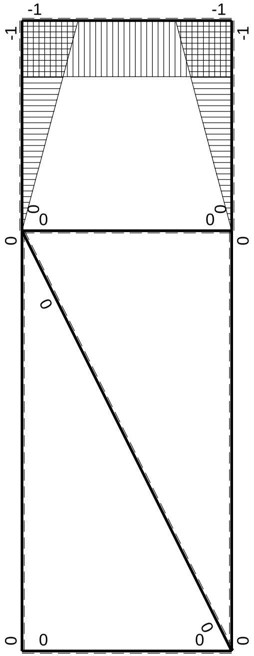

-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
1/2
-1
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

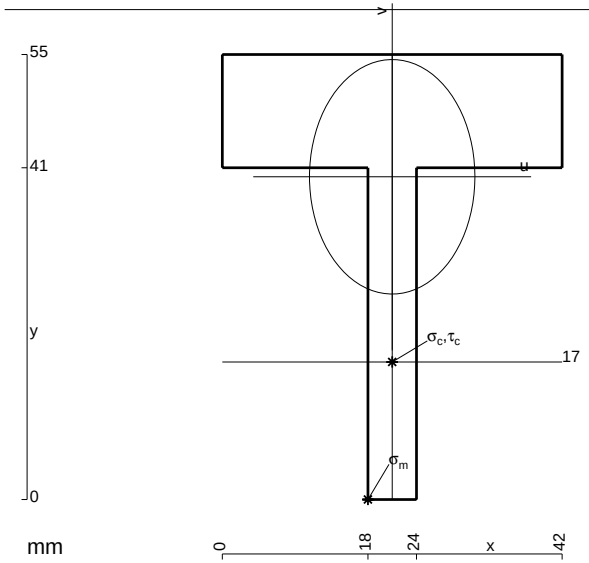
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210301 ESΣ09.xxxx.114



A = 834. mm^2
J_u = 175228. mm^4
J_v = 87174. mm^4
y_g = 39.89 mm
T_y = -3900. N
M_x = -1053000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.89 mm
$\sigma_m = -Mv/J_u$ = -239.7 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.89 mm
$\sigma_c = -Mv/J_u$ = -137.5 N/mm^2
τ_c = 11.88 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.1 N/mm^2
S^* = 3202. mm^3

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ESΣ09.xxxx.114

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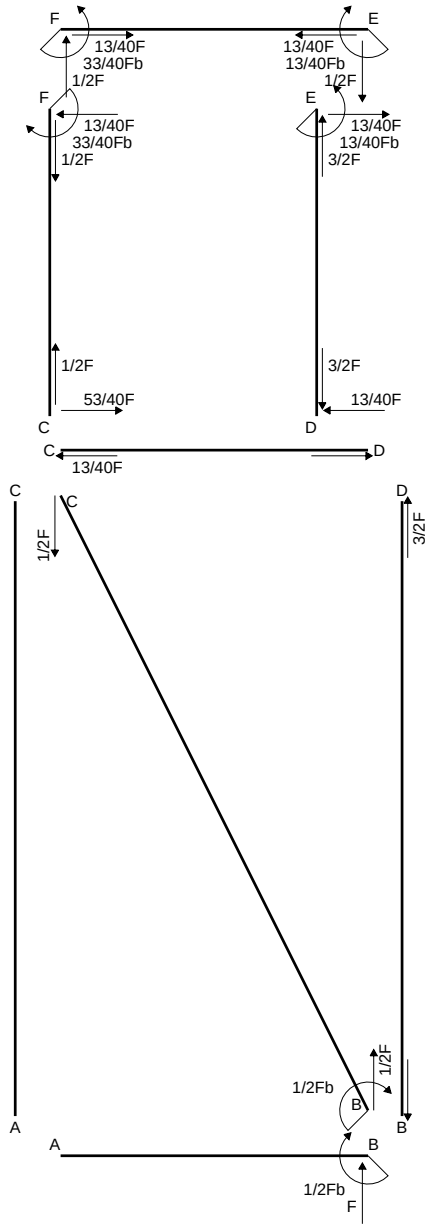
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
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3/2F
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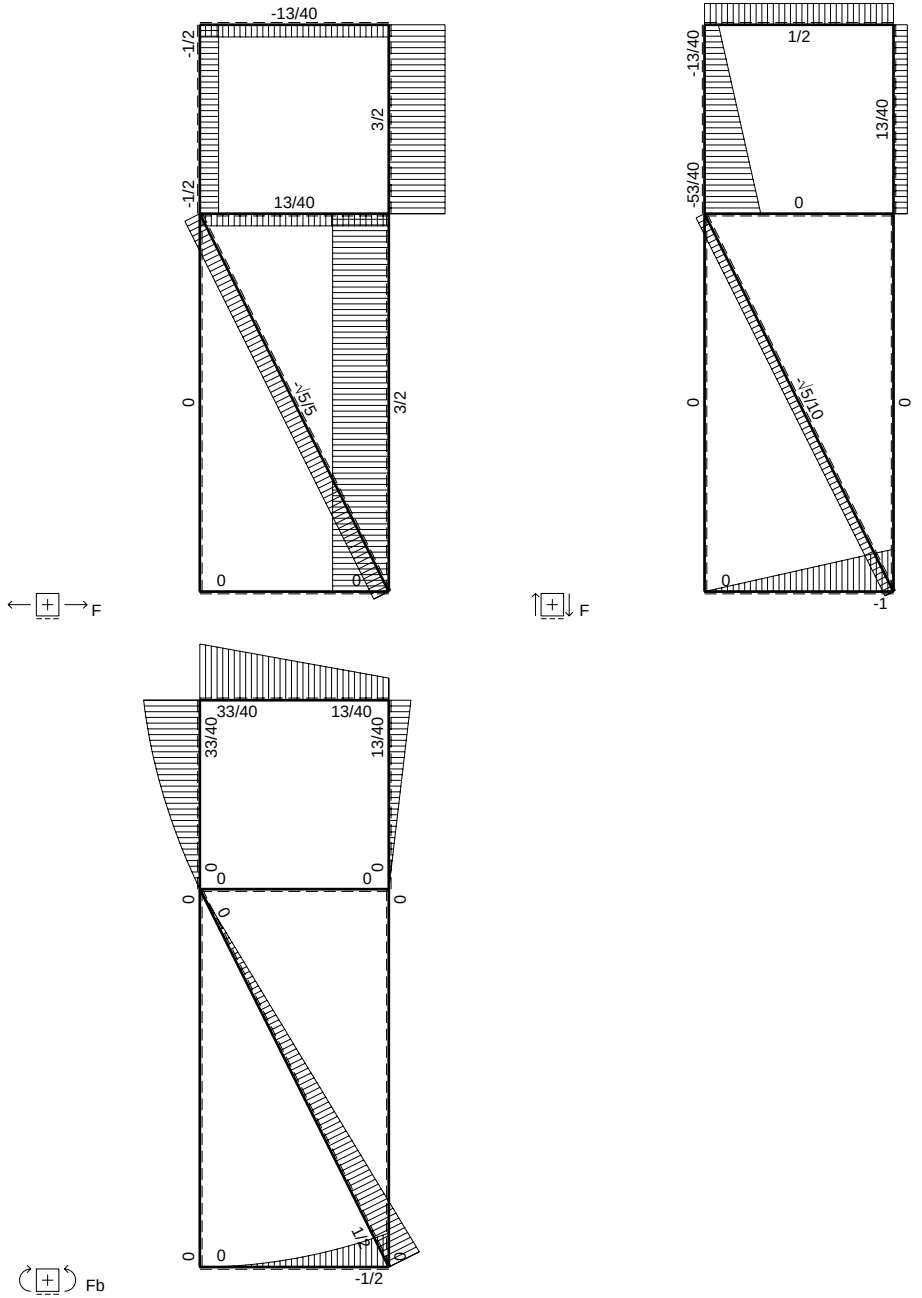
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
1/2
0
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

PROCEDIMENTO E RISULTATI 212148 ESΣ09.xxxx.115

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

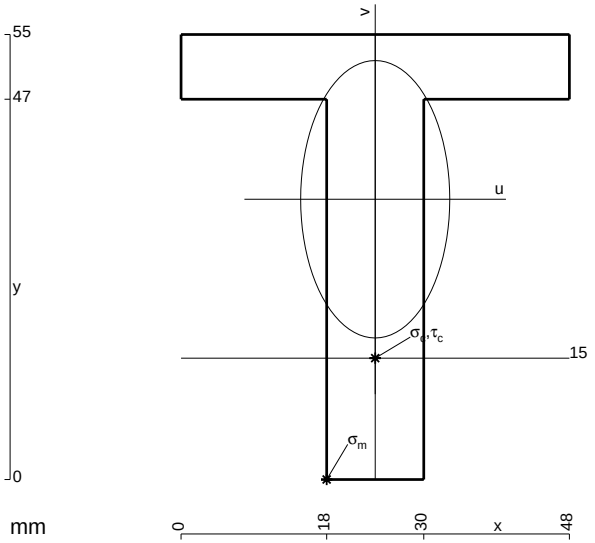
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 948. mm^2
J_u = 278641. mm^4
J_v = 80496. mm^4
y_g = 34.64 mm
T_y = -5540. N
M_x = -1606600. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.64 mm
$\sigma_m = -Mv/J_u$ = -199.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -19.64 mm
$\sigma_c = -Mv/J_u$ = -113.2 N/mm^2
τ_c = 8.094 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.1 N/mm^2
S^* = 4885. mm^3

ESΣ09.xxxx.115

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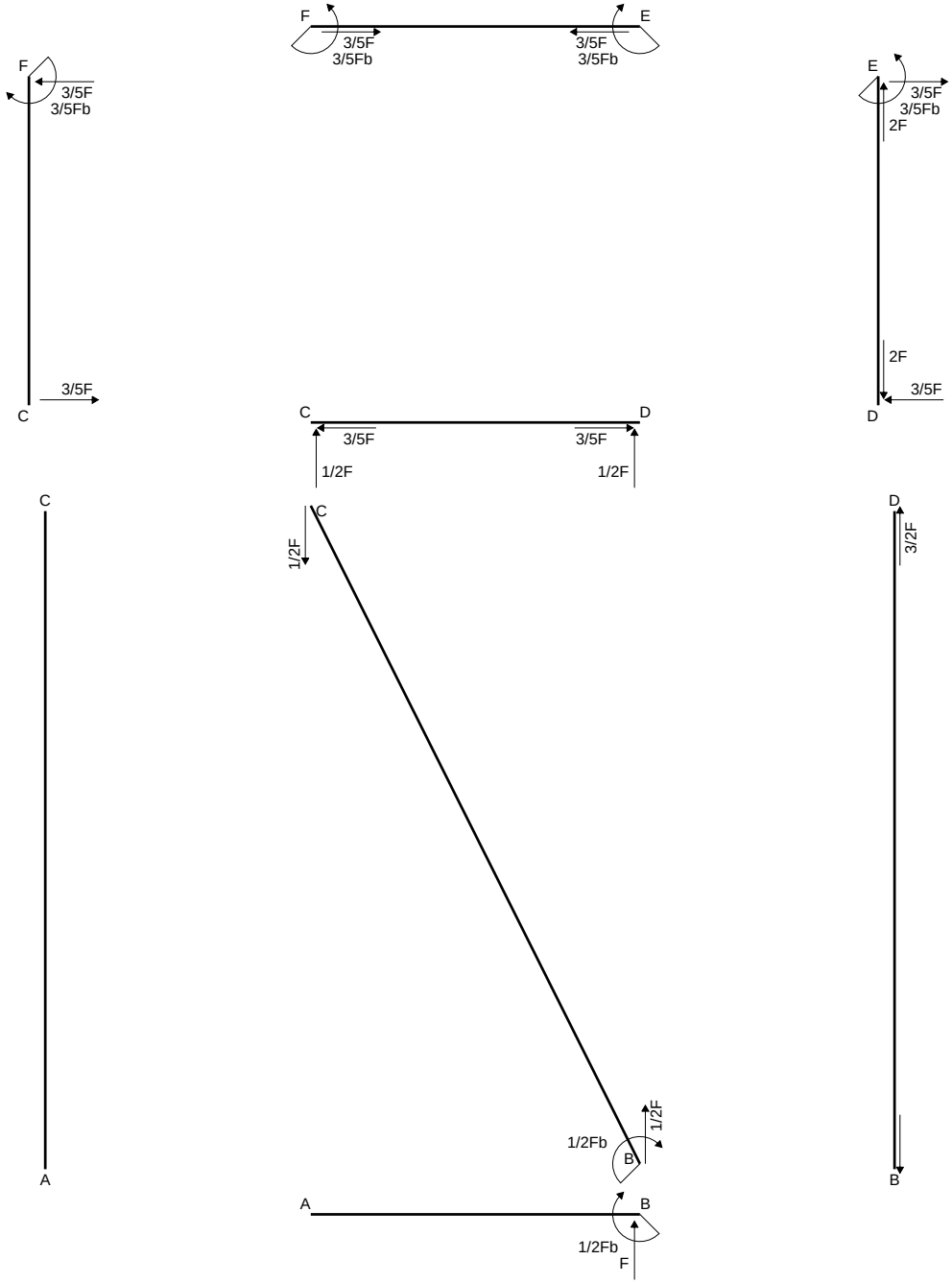
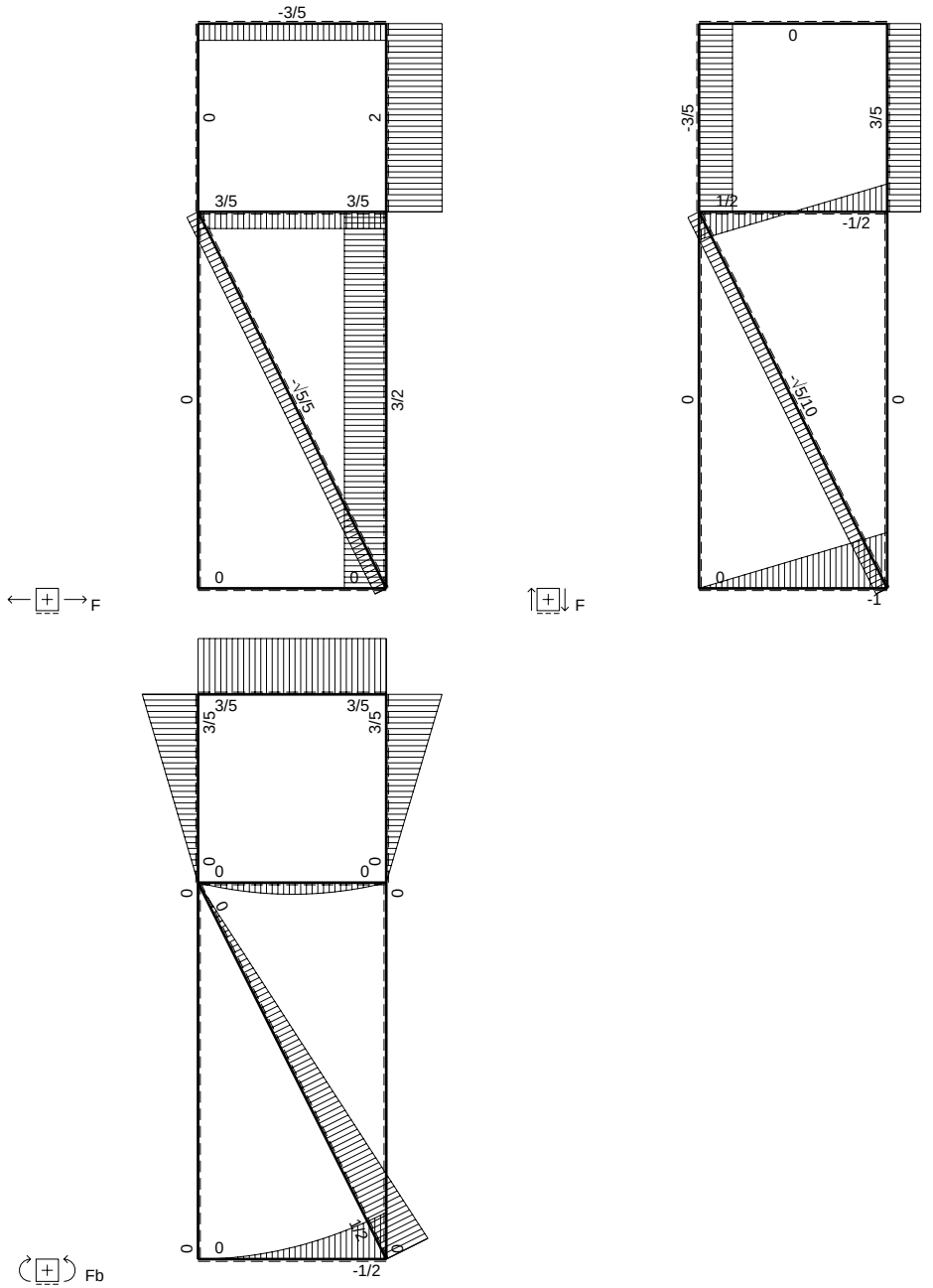
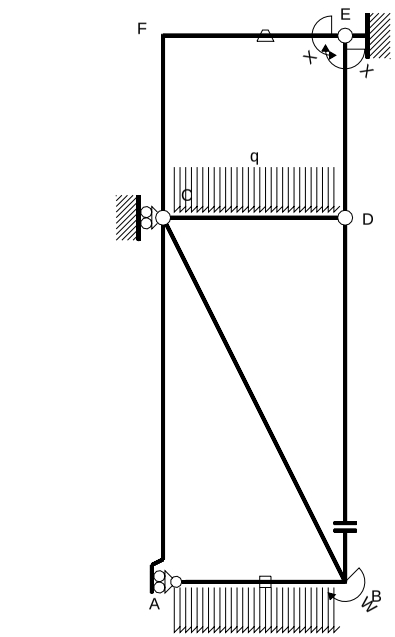
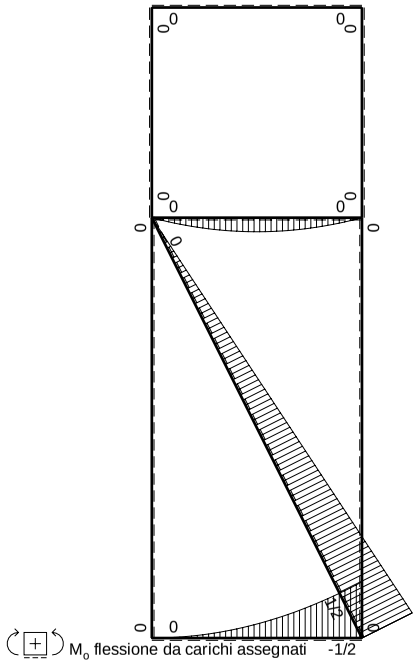

F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F

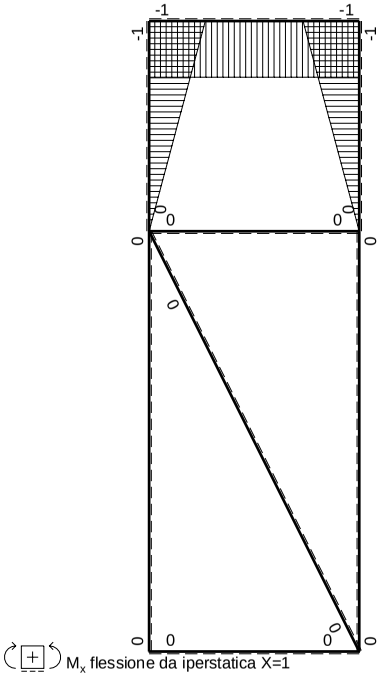

-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

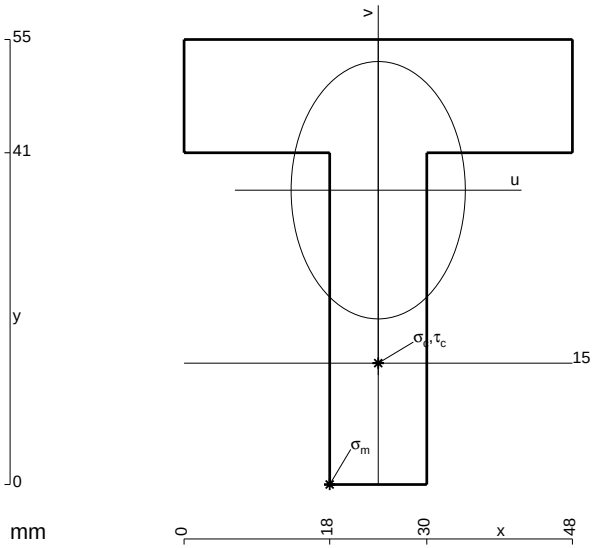
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1164. mm^2
J_u = 294703. mm^4
J_v = 134928. mm^4
y_g = 36.38 mm
T_y = -5480. N
M_x = -1698800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -36.38 mm
$\sigma_m = -Mv/J_u$ = -209.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -21.38 mm
$\sigma_c = -Mv/J_u$ = -123.2 N/mm^2
τ_c = 8.054 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 124. N/mm^2
S^* = 5198. mm^3

ESΣ09.xxxx.116

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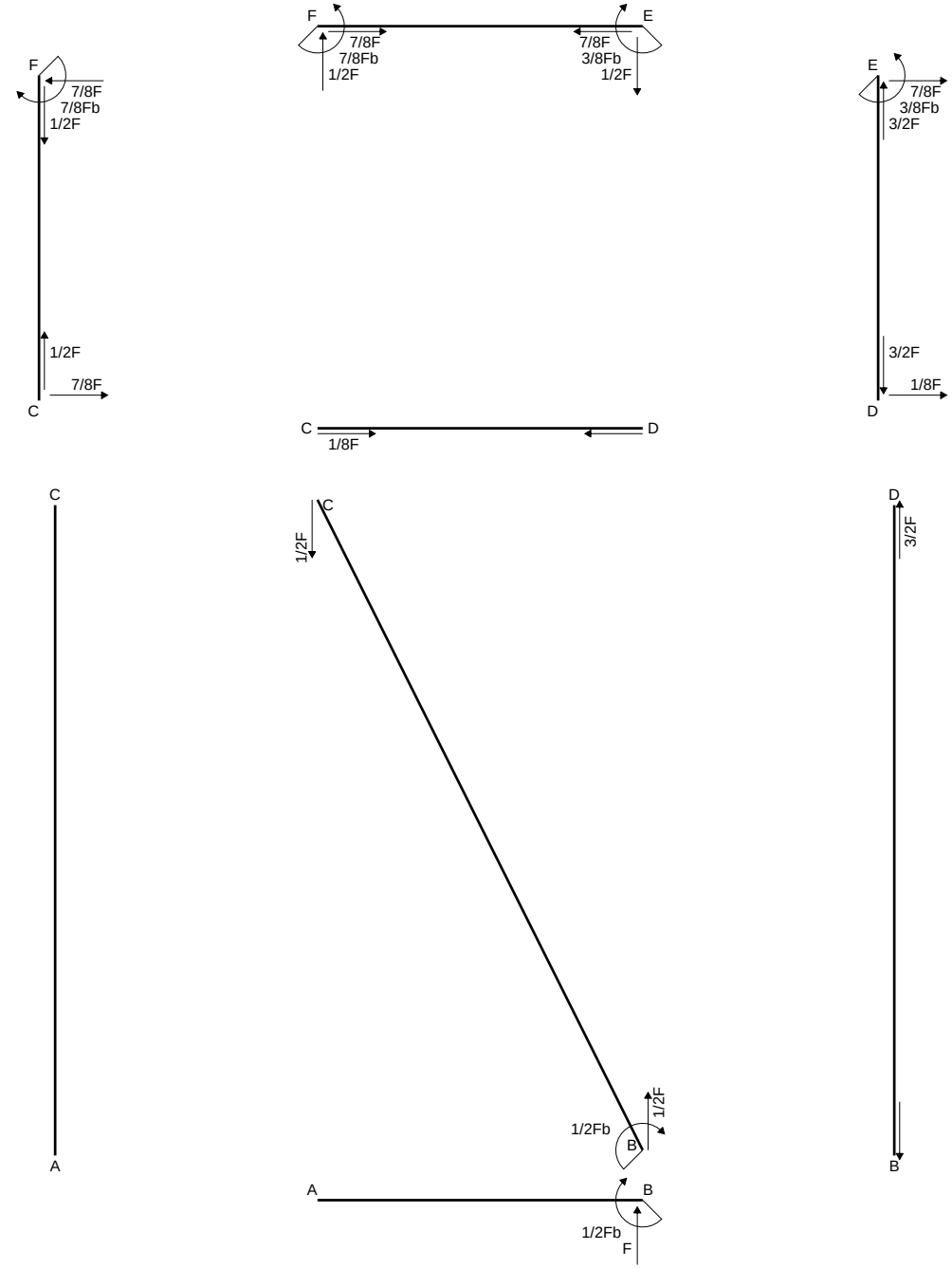
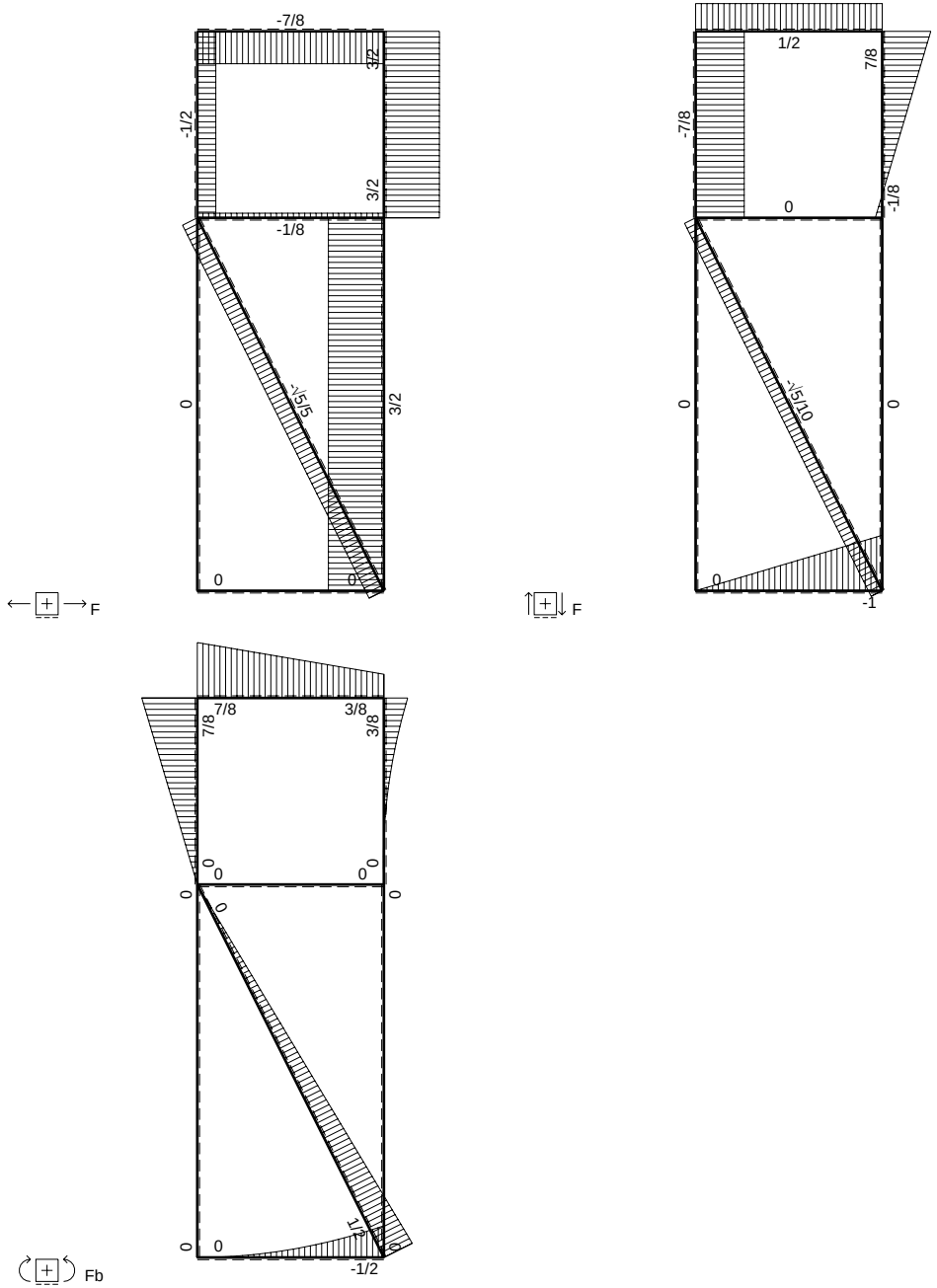

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
-1/2
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981196 ESΣ09.xxxx.117

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

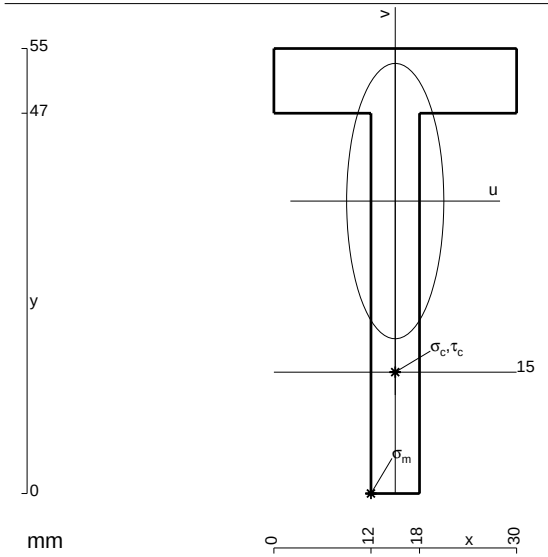
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 522. mm^2
J_u = 151243. mm^4
J_v = 18846. mm^4
y_g = 36.14 mm
T_y = -2780. N
M_x = -917400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.14 mm
$\sigma_m = -Mv/J_u$ = -219.2 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.14 mm
$\sigma_c = -Mv/J_u$ = -128.3 N/mm^2
τ_c = 7.897 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129. N/mm^2
S^* = 2578. mm^3

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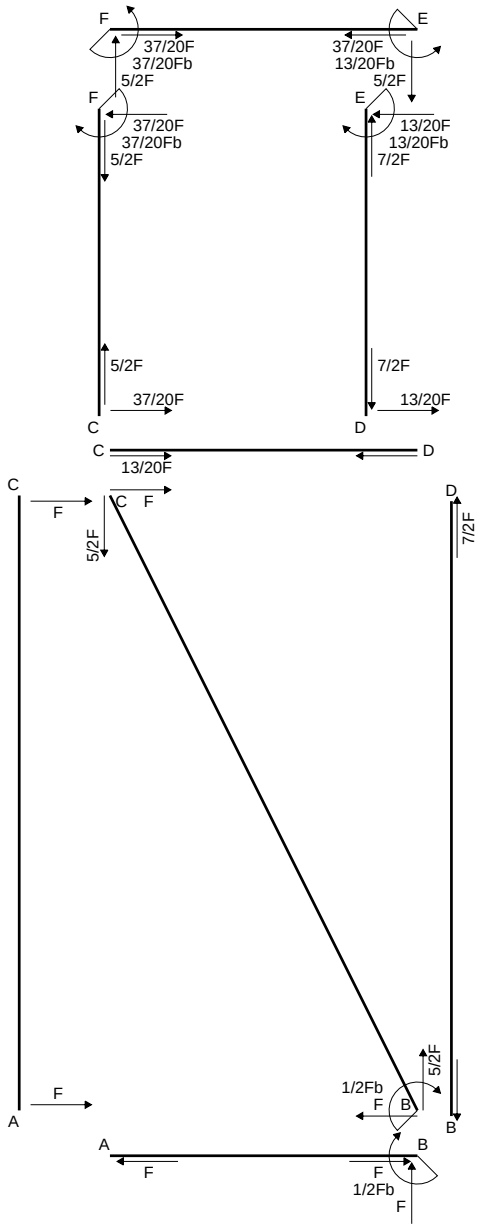
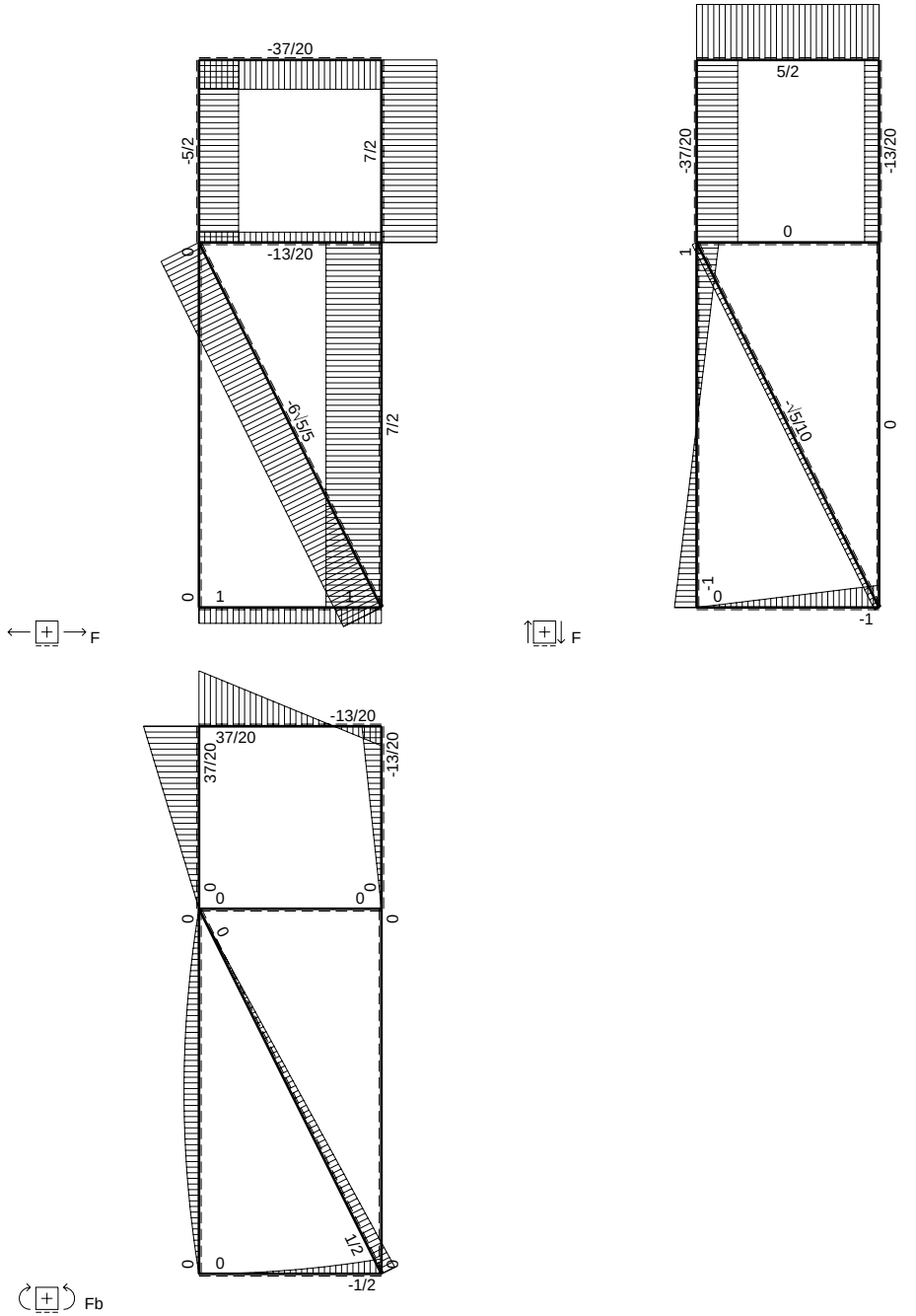

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb


-37/20
5/2
7/2
-13/20
-5/√5
-√5/10
1/2
-1/2
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957454 ESΣ09.xxxx.118

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$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{Xo} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

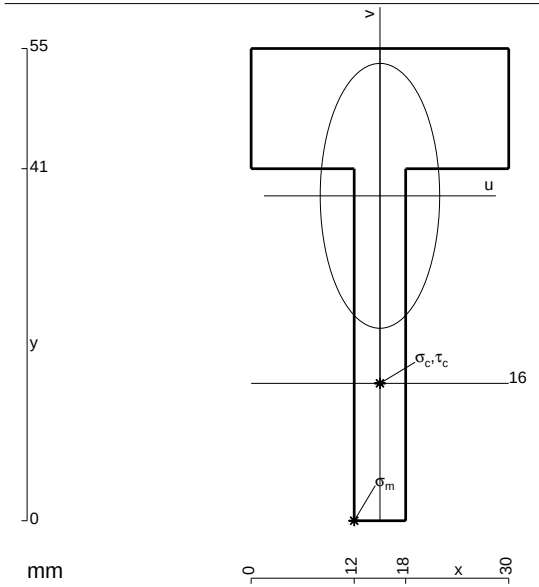
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L_{FC}^{Xo} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L_{CF}^{Xo} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 666. mm^2
J_u = 158641. mm^4
J_v = 32238. mm^4
y_g = 37.84 mm
N = 2800. N
T_y = -2800. N
M_x = -980000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.84 mm
$\sigma_m = N/A - Mv/J_u$ = -229.6 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.84 mm
$\sigma_c = N/A - Mv/J_u$ = -130.7 N/mm^2
τ_c = 8.427 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.5 N/mm^2
S^* = 2865. mm^3

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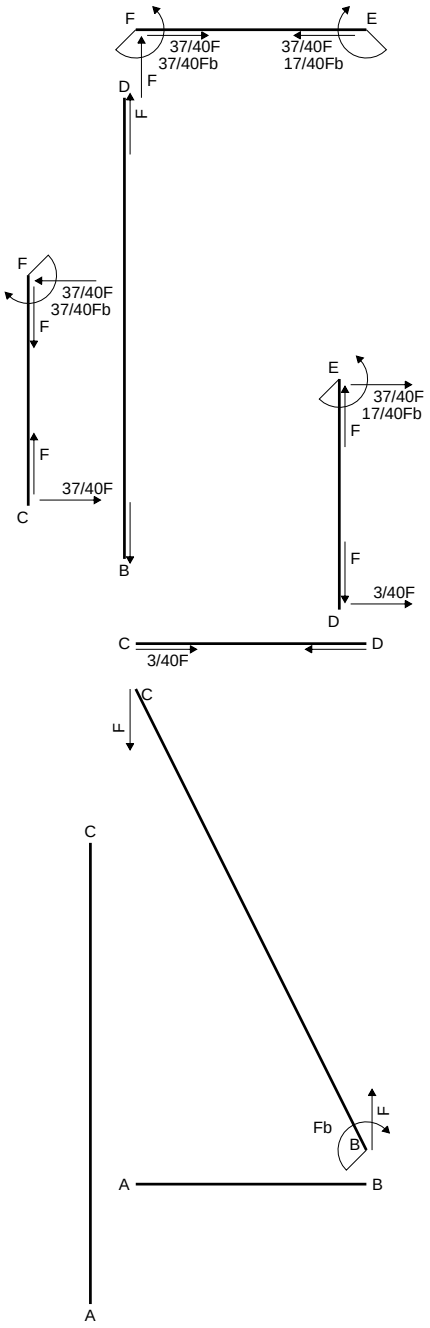
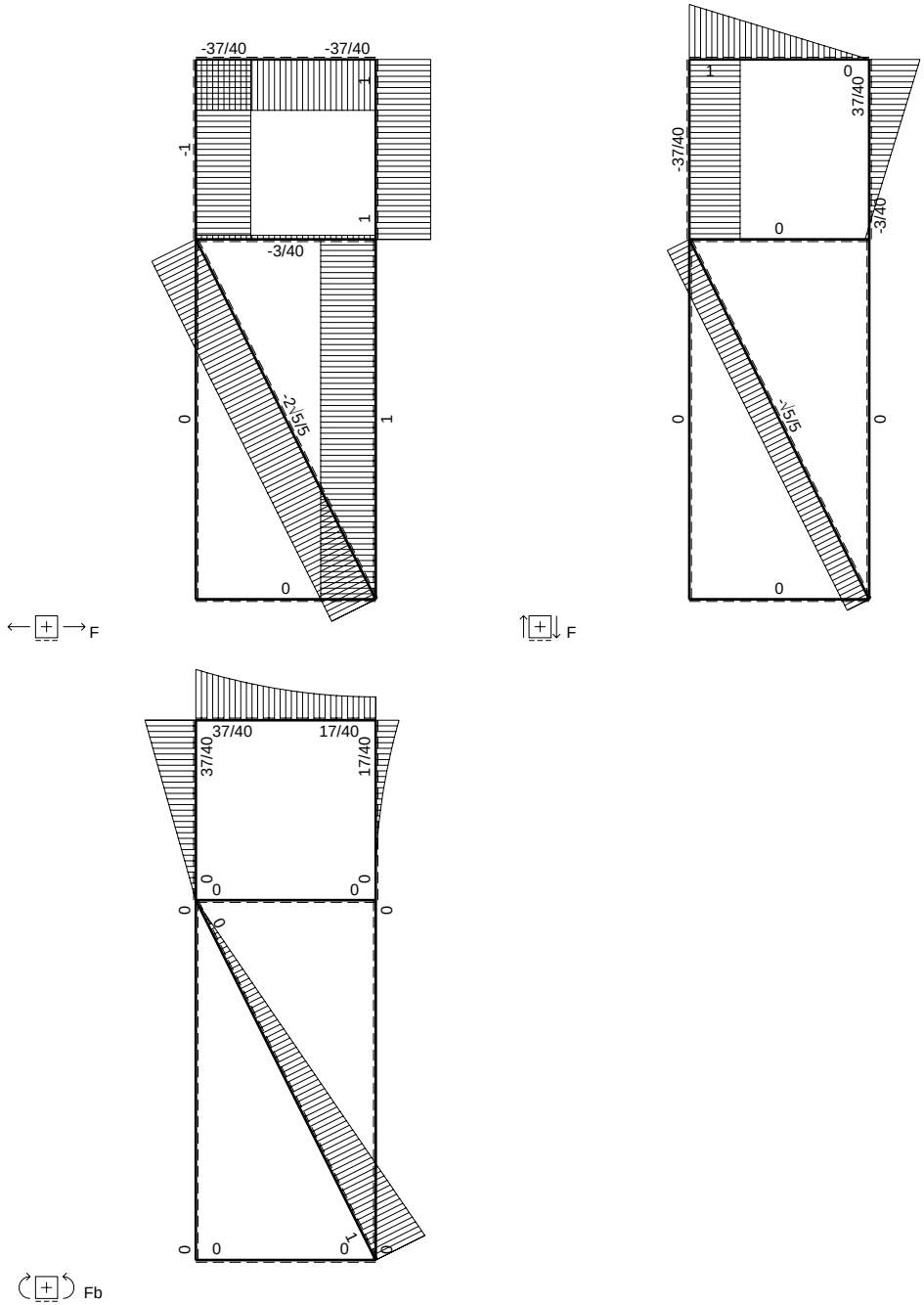

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D
C
F
C
Fb
F
B
A
B
A


-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F
1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F
37/40
17/40
37/40
17/40
0
0
0
0
0
0
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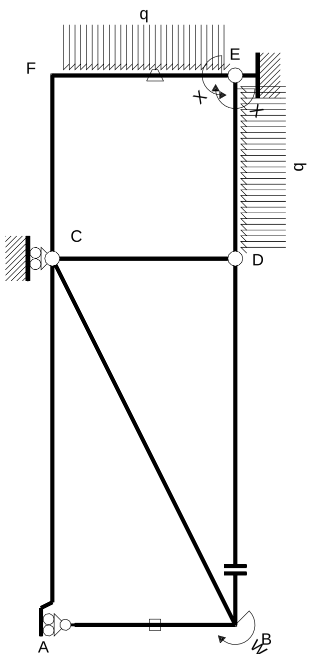
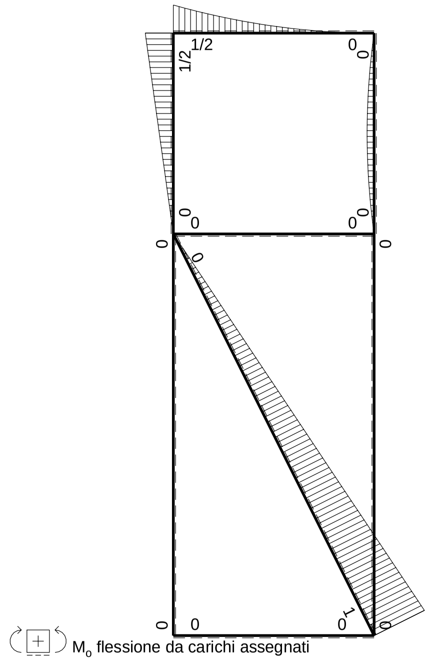
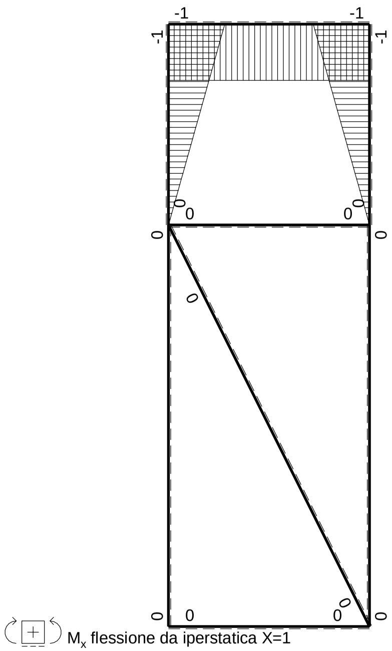
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$17/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-17/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

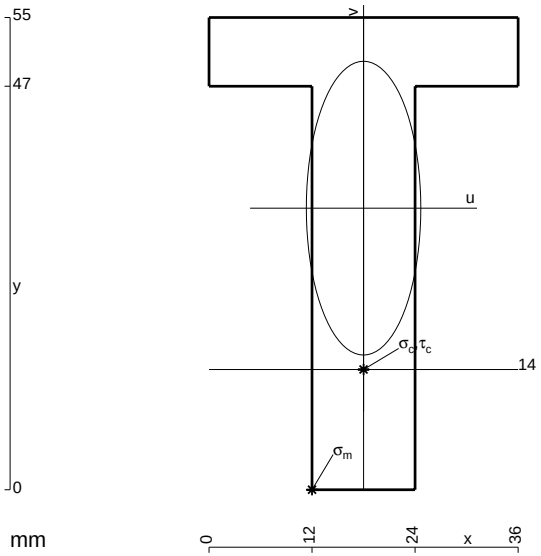
$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 852. mm^2
J_u = 249536. mm^4
J_v = 37872. mm^4
y_g = 32.8 mm
N = -2227. N
T_y = -1114. N
M_x = 1842600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.8 mm
σ_m = N/A-Mv/J_u = 239.6 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.8 mm
σ_c = N/A-Mv/J_u = 136.2 N/mm^2
τ_c = 1.612 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.2 N/mm^2
S^* = 4334. mm^3

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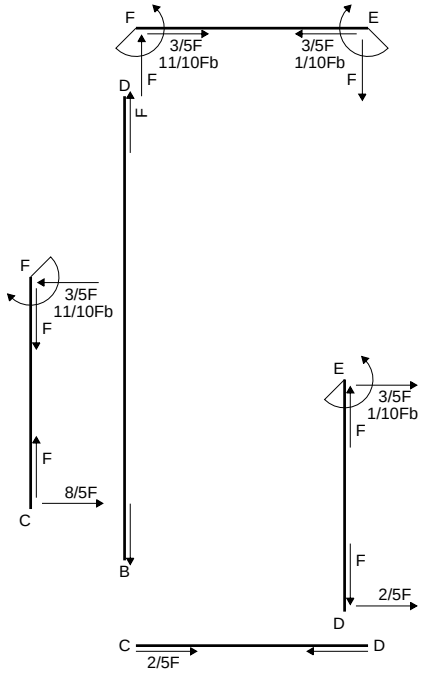
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
F
E
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1/10Fb
F
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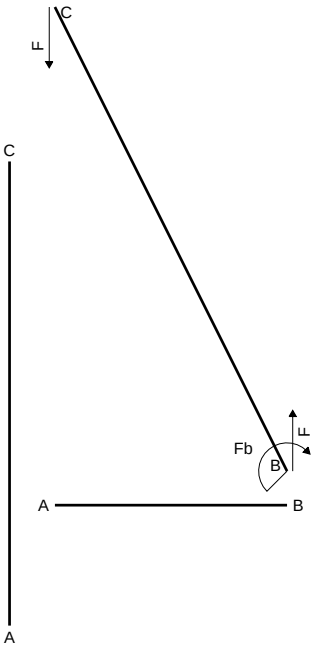
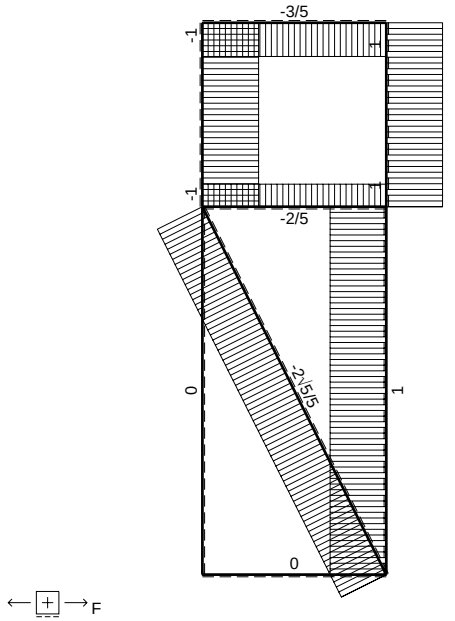
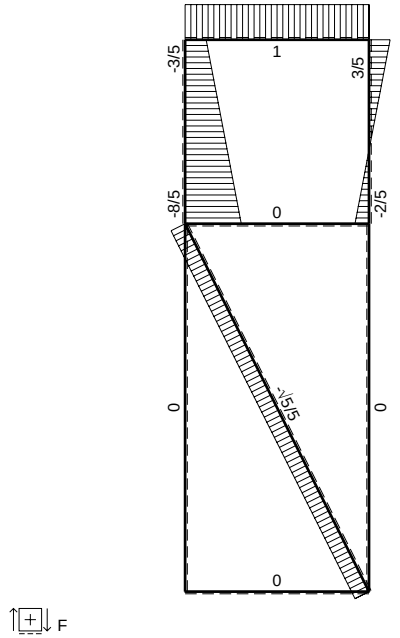
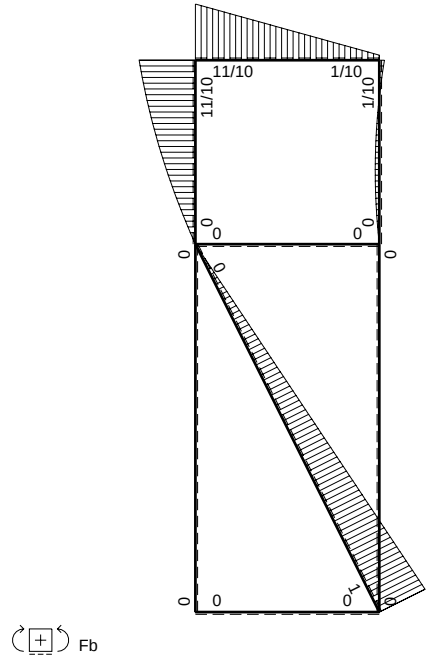
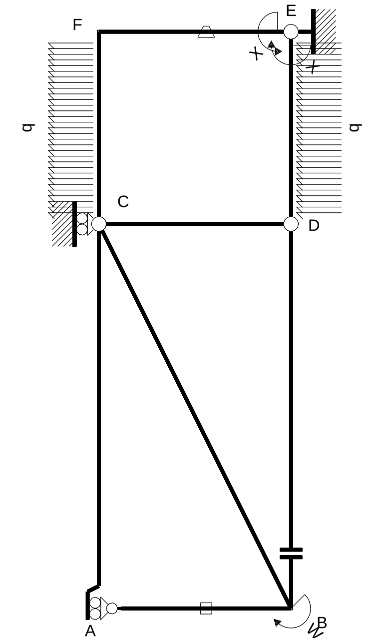
B
F
2/5F
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

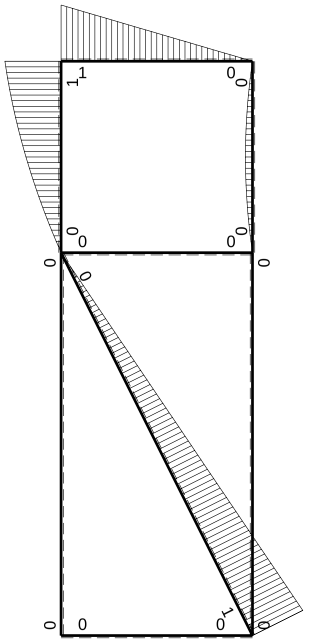

-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F

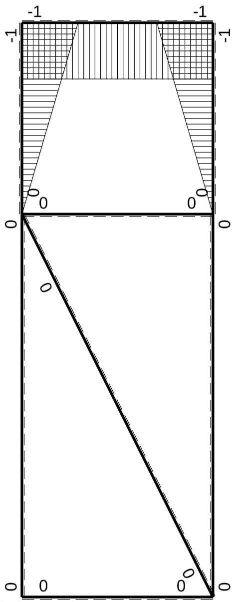

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

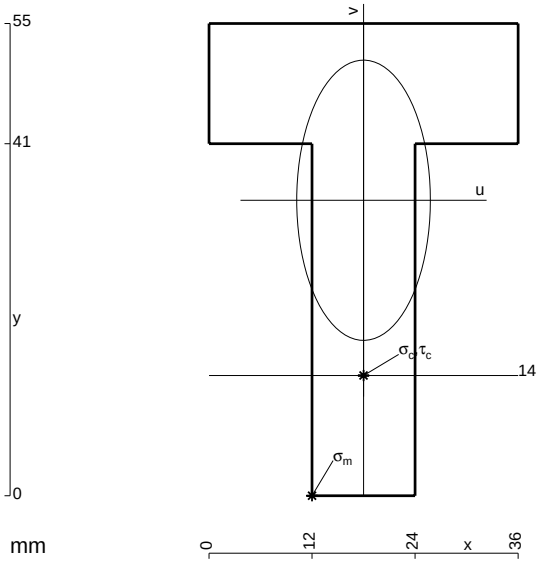
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 996. mm^2
J_u = 265432. mm^4
J_v = 60336. mm^4
y_g = 34.42 mm
N = -3596. N
T_y = -1798. N
M_x = 1567800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.42 mm
σ_m = N/A-Mv/J_u = 199.7 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -20.42 mm
σ_c = N/A-Mv/J_u = 117. N/mm^2
τ_c = 2.6 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.1 N/mm^2
S^* = 4606. mm^3

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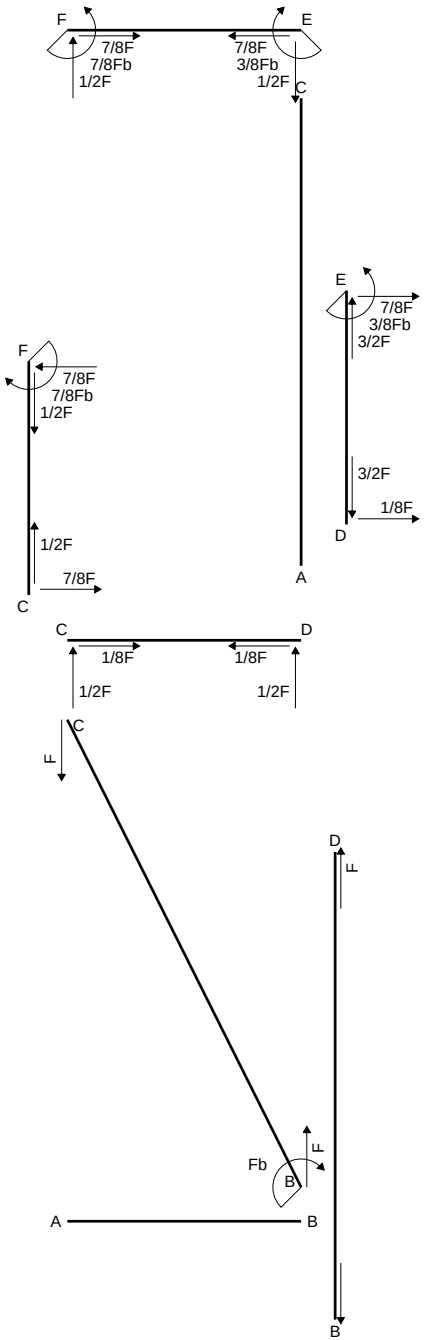
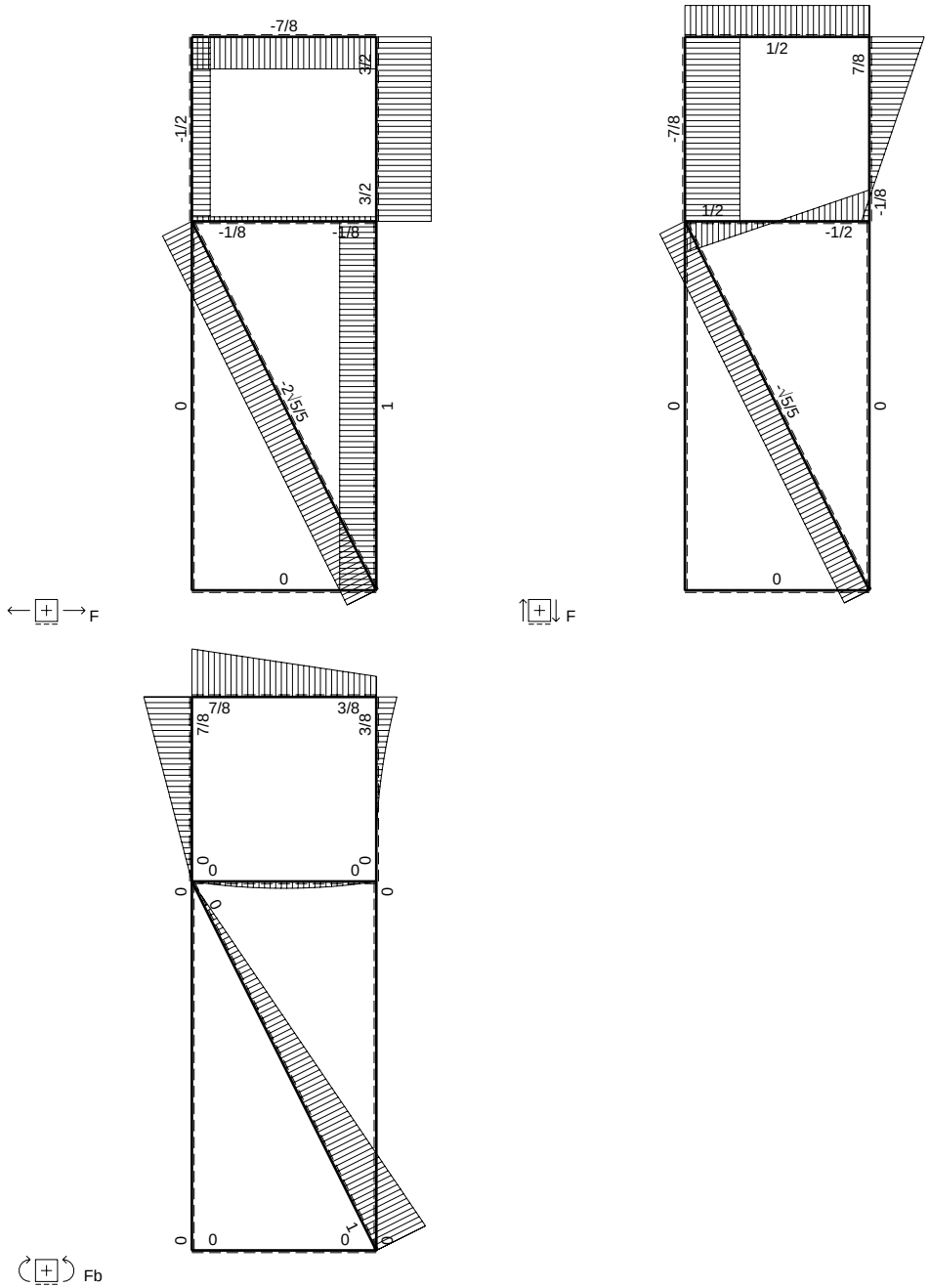
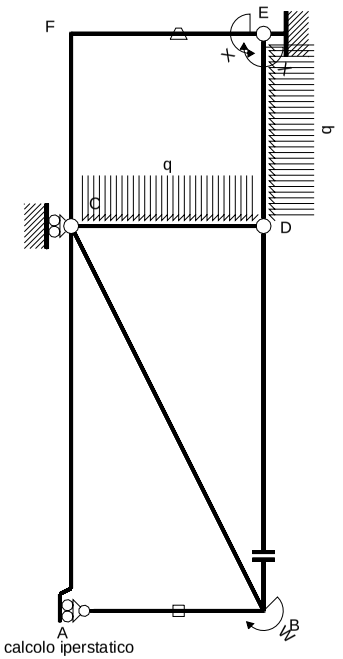
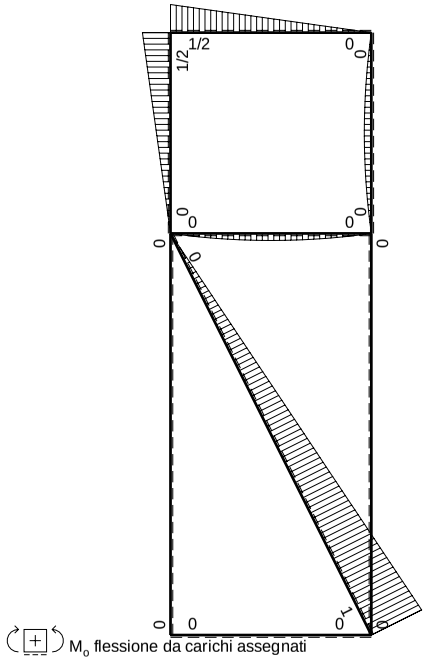

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B

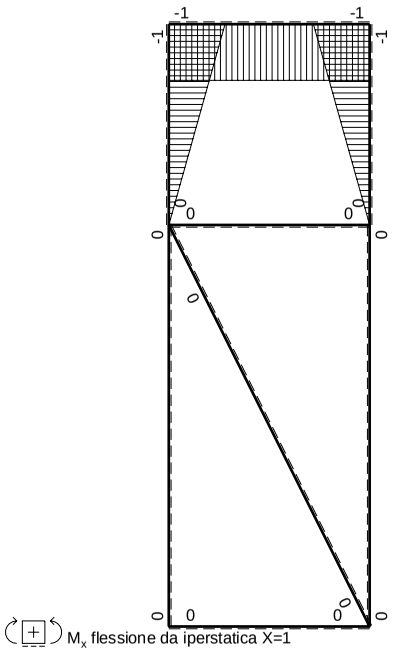

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
				totali			5/8Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211210 ESΣ09.xxxx.121

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

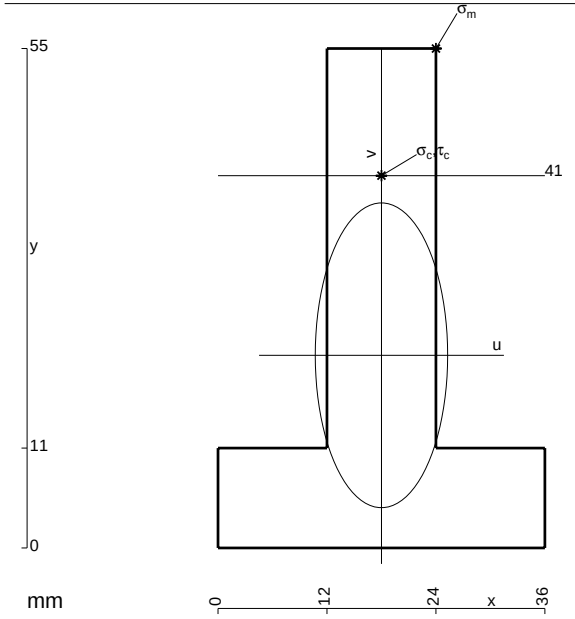
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 924. mm^2
J_u = 260306. mm^4
J_v = 49104. mm^4
y_g = 21.21 mm
N = -3381. N
T_y = -1690. N
M_x = 1587600. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 33.79 mm
$\sigma_m = N/A - Mv/J_u$ = -209.7 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.79 mm
$\sigma_c = N/A - Mv/J_u$ = -124.3 N/mm^2
τ_c = 2.435 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.4 N/mm^2
S^* = 4500. mm^3

ESΣ09.xxxx.121

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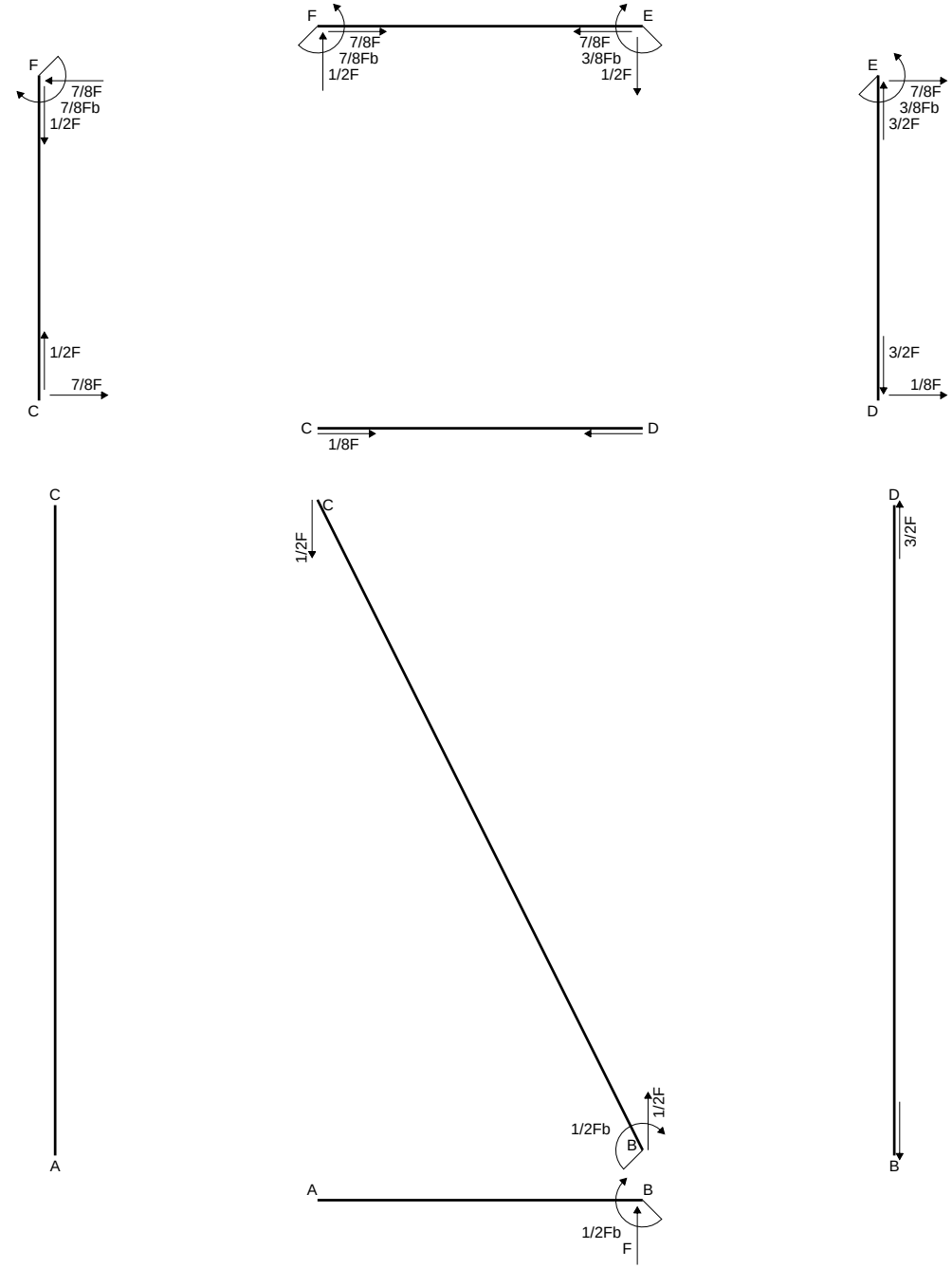
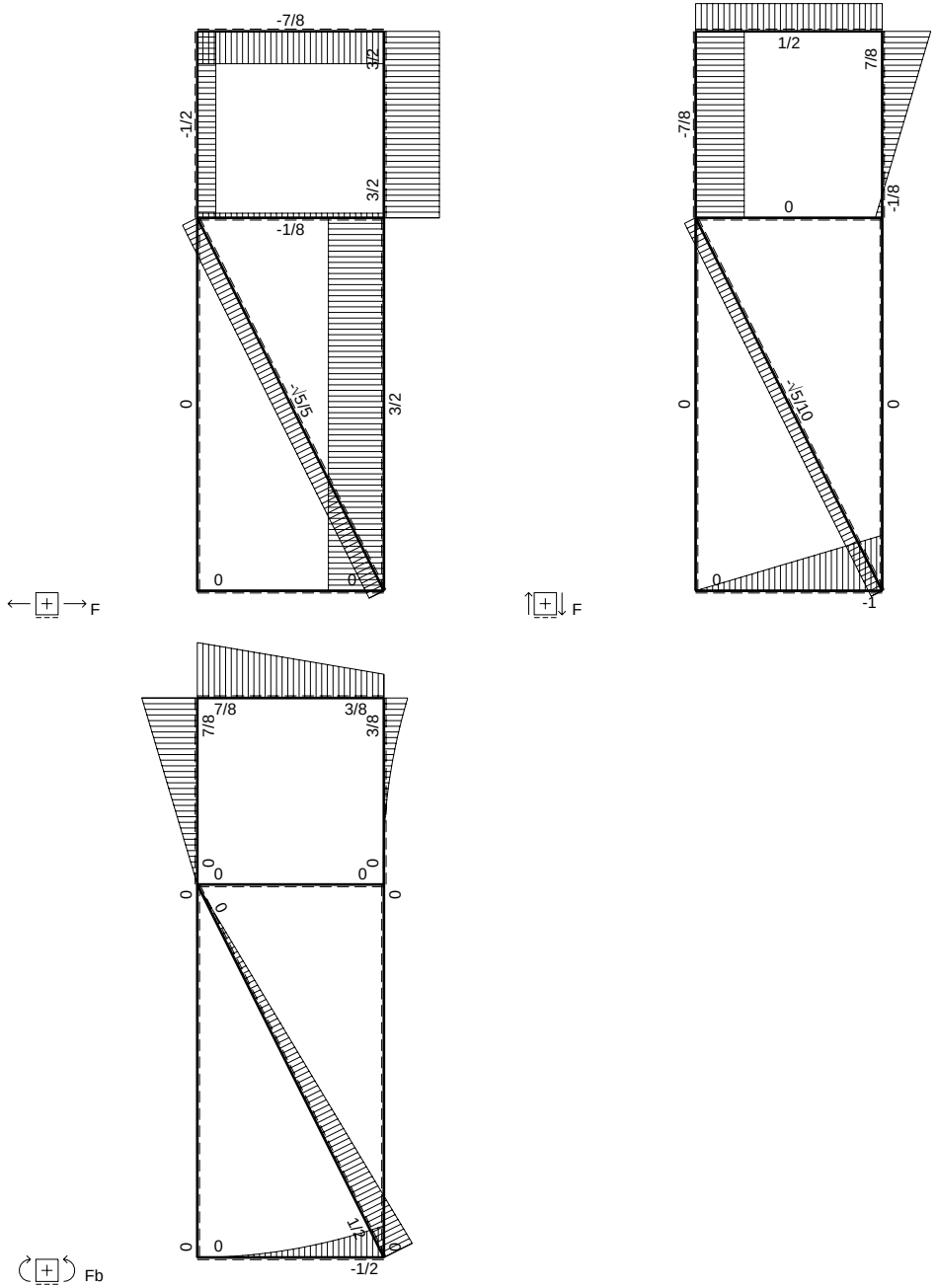
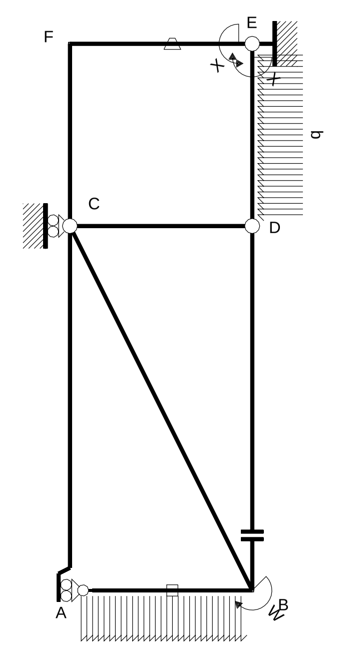
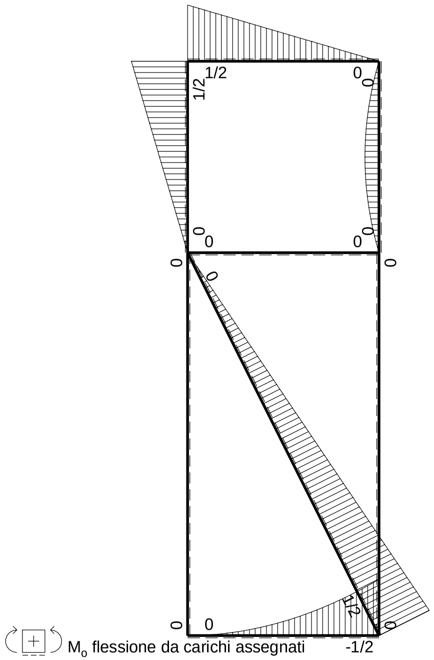

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
D
3/2F
1/8F
A
B
1/2Fb

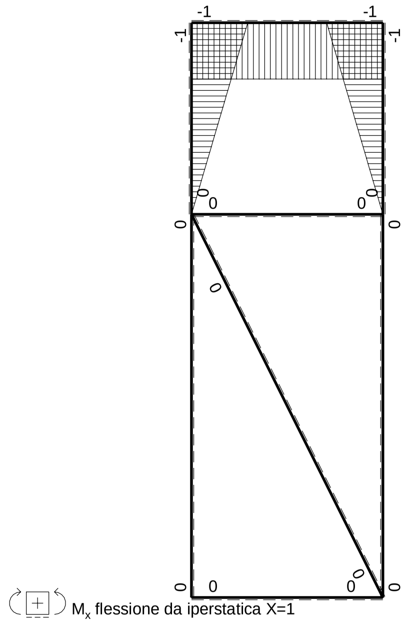

-7/8
3/2
-1/2
-1/8
0
-√5/5
F
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
-1/2
Fb



Schema di calcolo iperstatico q



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$1-x/b$	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	$-Fb/EJ$	$-1/2Fx$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	$1/3Xb/EJ$
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						$-3/8Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

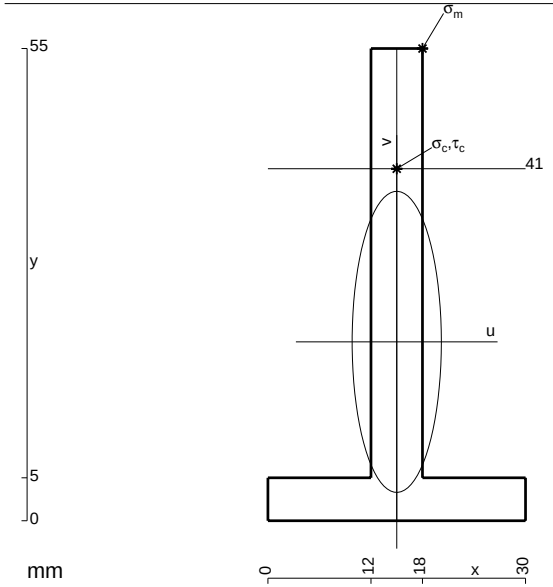
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 450. mm^2
J_u = 138437. mm^4
J_v = 12150. mm^4
y_g = 20.83 mm
T_y = -3870. N
M_x = -890100. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 34.17 mm
$\sigma_m = -Mv/J_u$ = 219.7 N/mm^2
x_c = 15. mm
y_c = 41. mm
v_c = 20.17 mm
$\sigma_c = -Mv/J_u$ = 129.7 N/mm^2
τ_c = 10.63 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131. N/mm^2
S^* = 2282. mm^3

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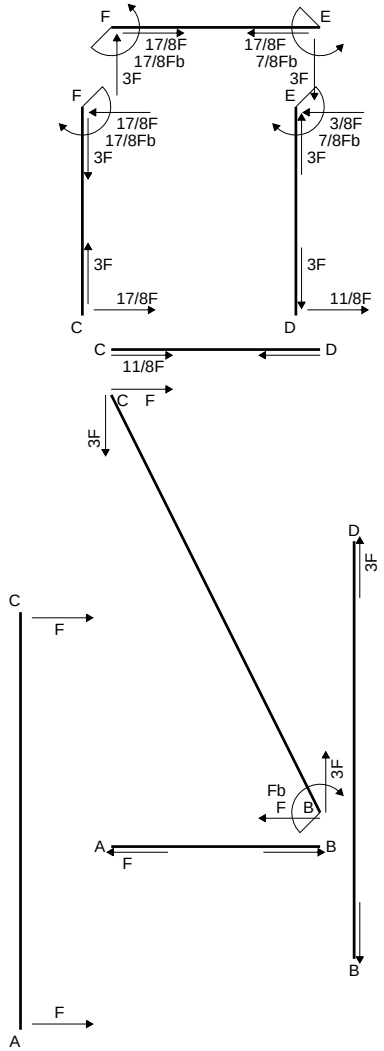
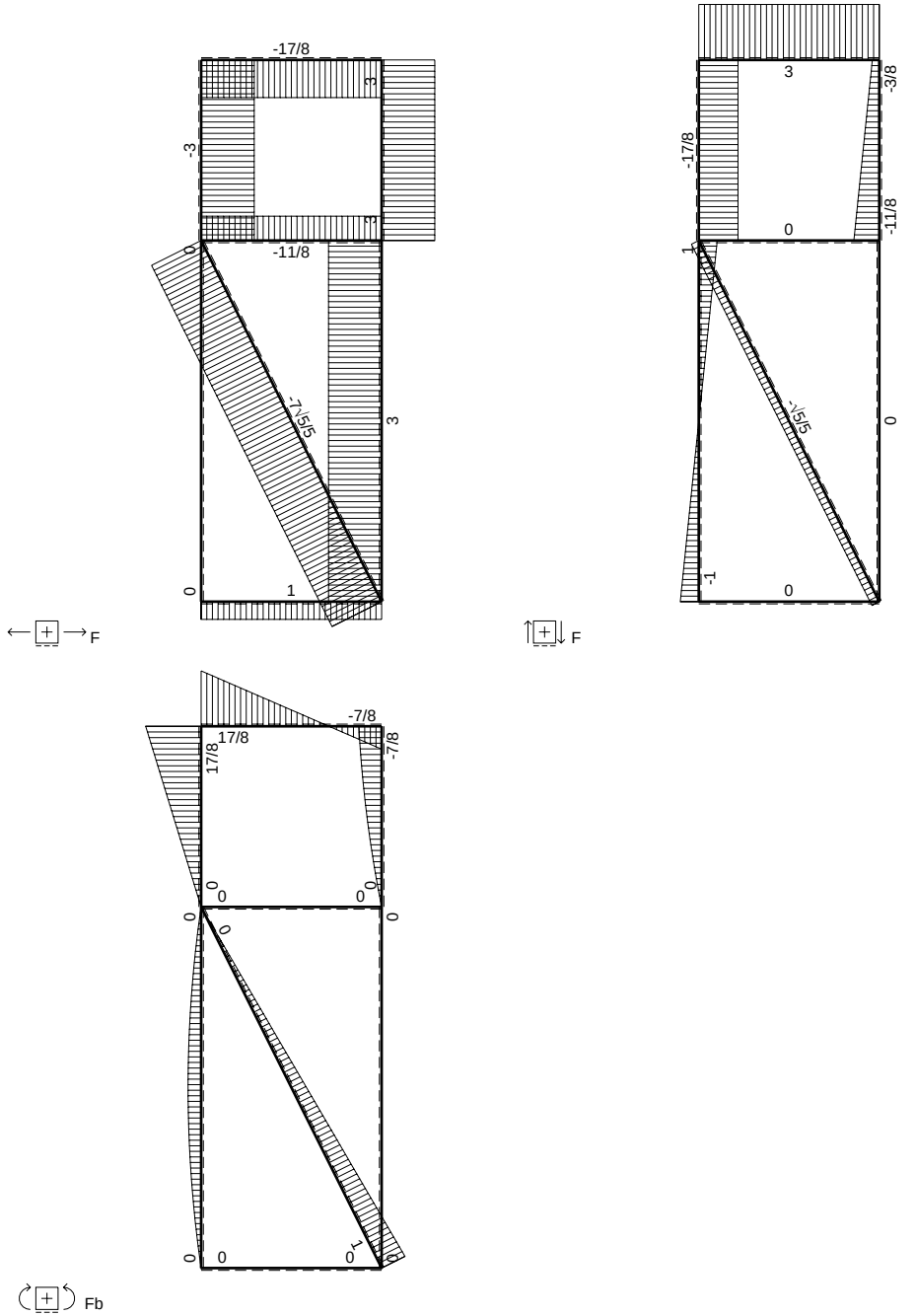

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
B
F
A


-17/8
-3
3
-11/8
0
1
-7√5/5
3
0
1
F
-17/8
3
-3/8
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1
-7√5/5
0
-1
0
F
-7/8
17/8
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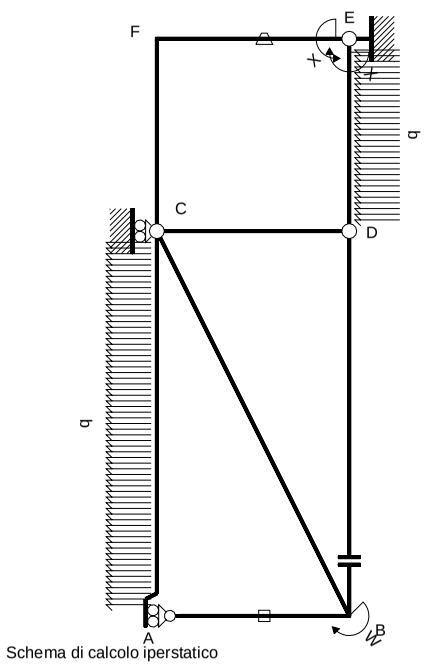
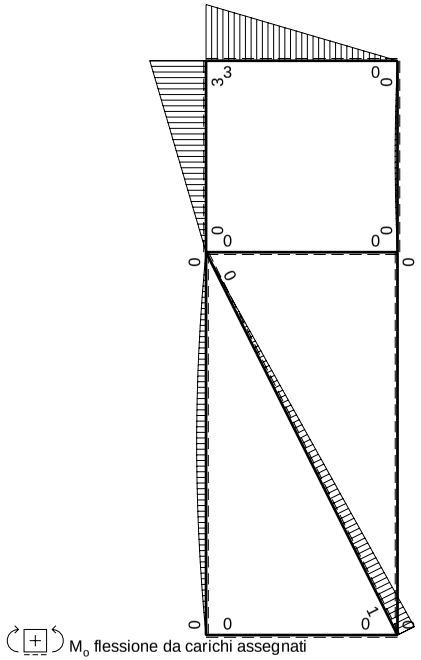
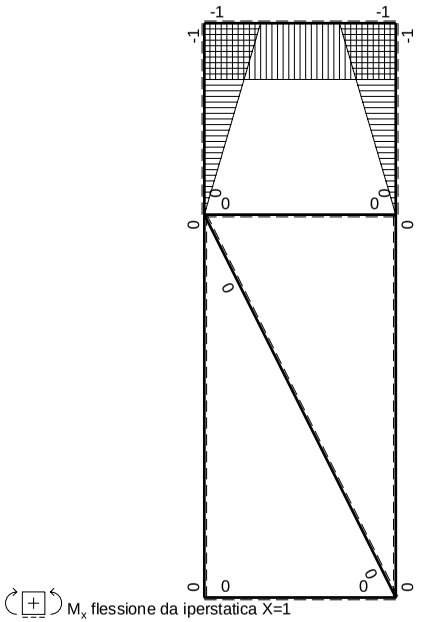
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
				totali			-35/24Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209852 ESΣ09.xxxx.123

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

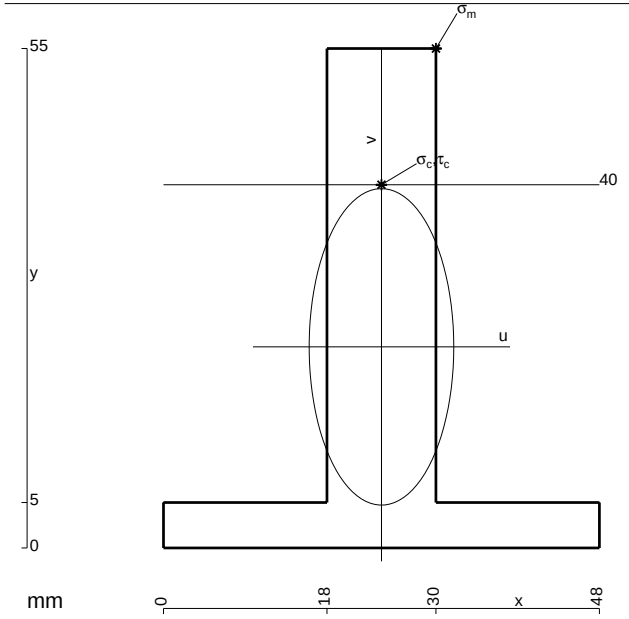
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 840. mm^2
J_u = 255143. mm^4
J_v = 53280. mm^4
y_g = 22.14 mm
N = -10550. N
T_y = -1507. N
M_x = 1685000. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 32.86 mm
$\sigma_m = N/A - Mv/J_u = -229.6\ N/mm^2$
x_c = 24. mm
y_c = 40. mm
v_c = 17.86 mm
$\sigma_c = N/A - Mv/J_u = -130.5\ N/mm^2$
τ_c = 2.247 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 130.5 N/mm^2
S^* = 4564. mm^3

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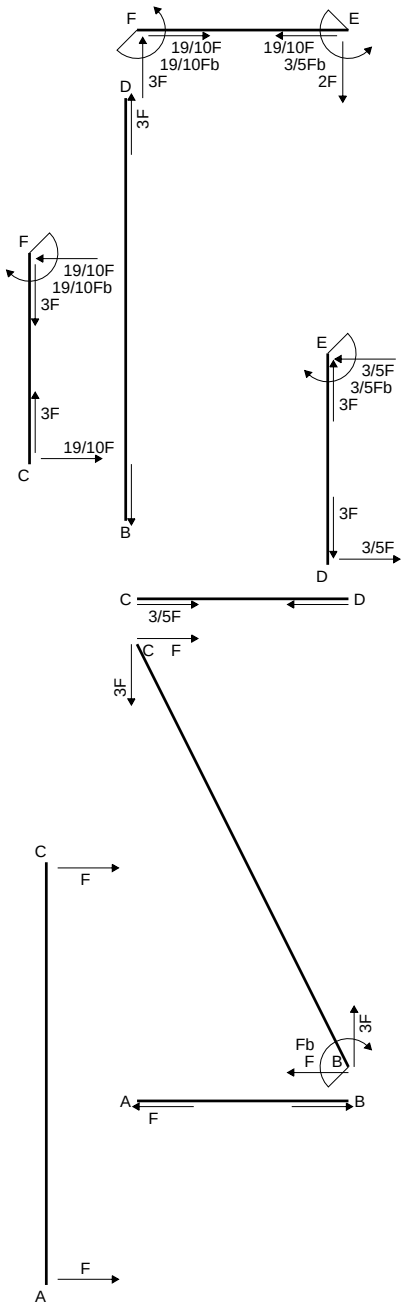
F
E
19/10F
19/10Fb
19/10F
3/5Fb
D
3F
2F
3F
F
19/10F
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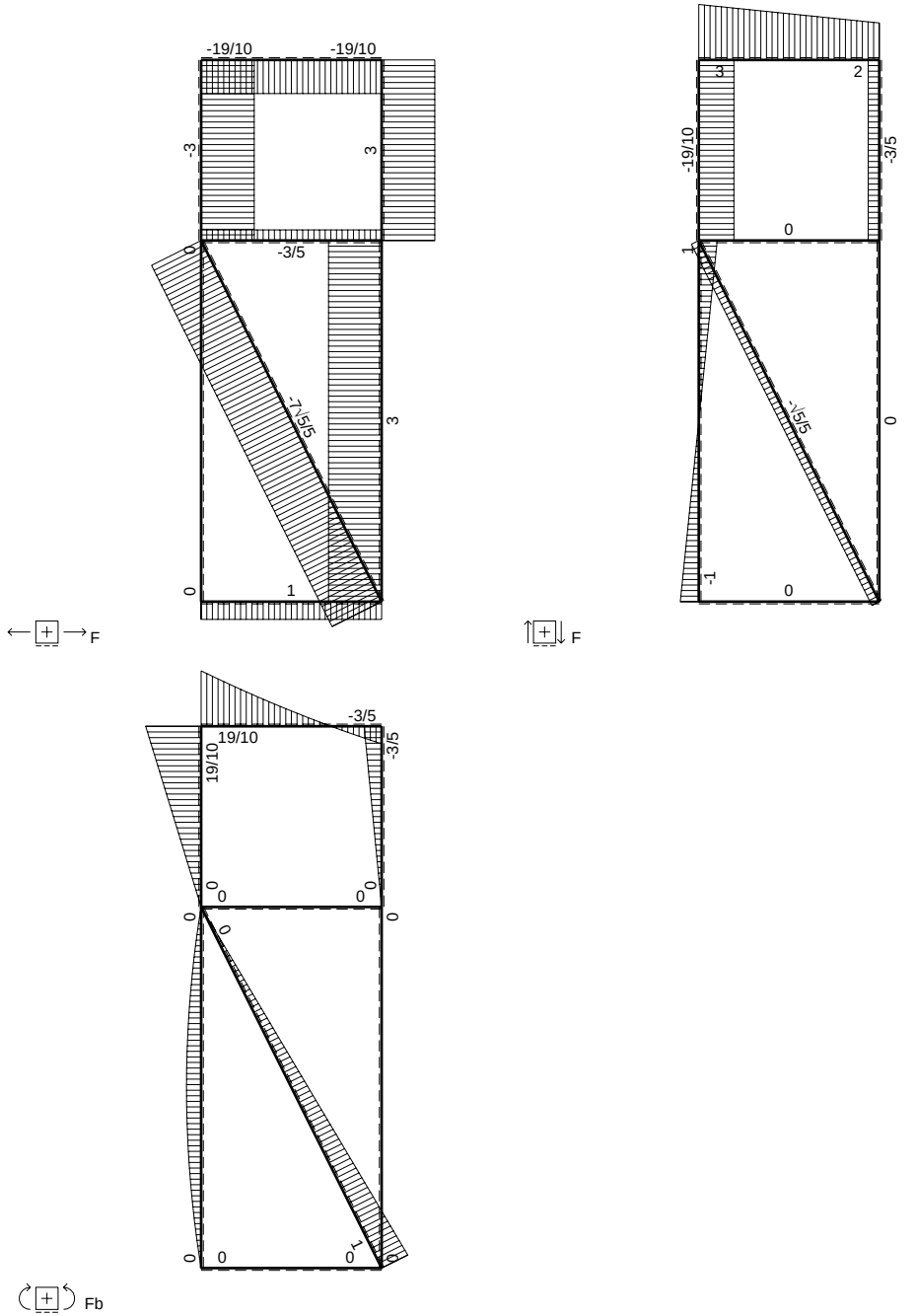
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
3F
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
0
-7\5/5
3
0
1
F
3
2
-19/10
-3/5
0
1
-1\5/5
0
-1
0
F
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-3/5
0
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0
0
0
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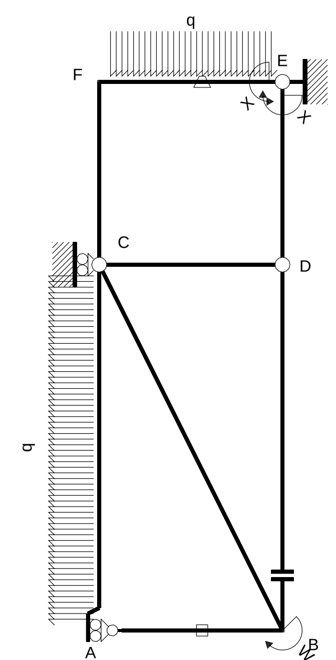
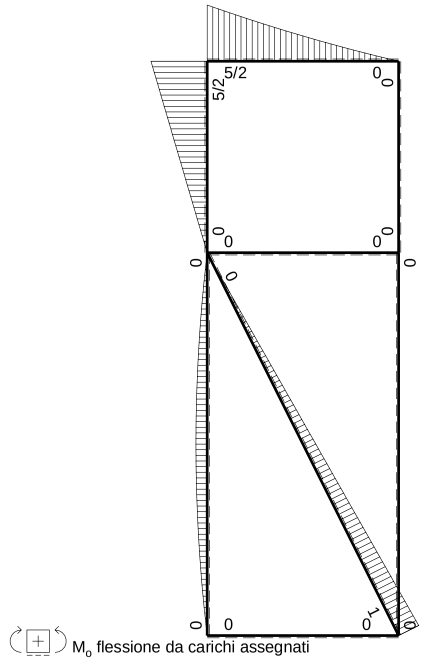
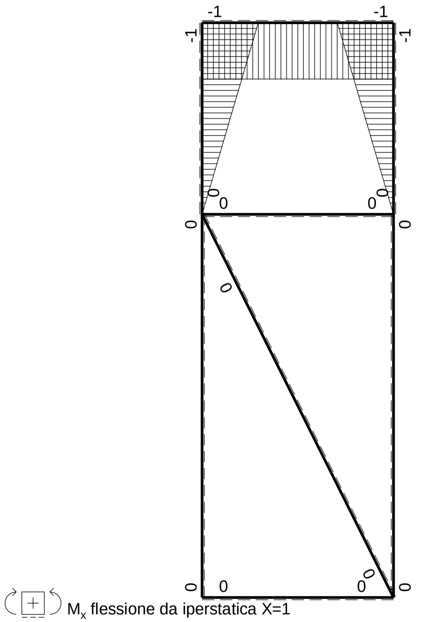
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	$Fb-\sqrt{5}/5Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	$5/2Fb-5/2Fx$	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-5/2Fx$	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

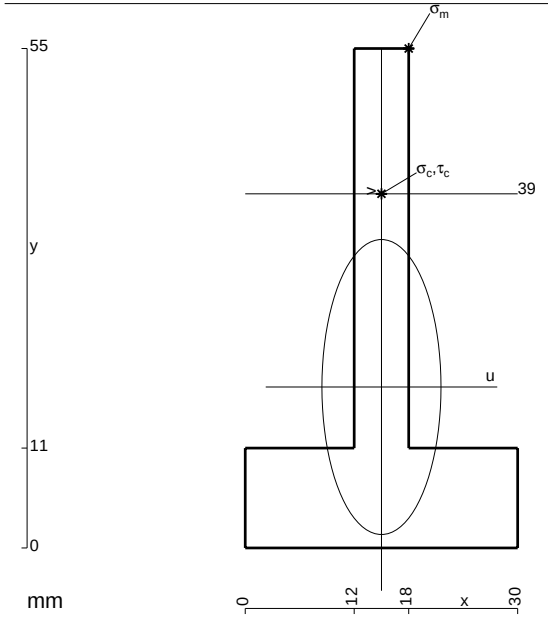
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 594. mm^2
J_u = 156836. mm^4
J_v = 25542. mm^4
y_g = 17.72 mm
N = -5604. N
T_y = -800.5 N
M_x = 966600. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.28 mm
$\sigma_m = N/A - Mv/J_u$ = -239.2 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 21.28 mm
$\sigma_c = N/A - Mv/J_u$ = -140.6 N/mm^2
τ_c = 2.391 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.6 N/mm^2
S^* = 2811. mm^3

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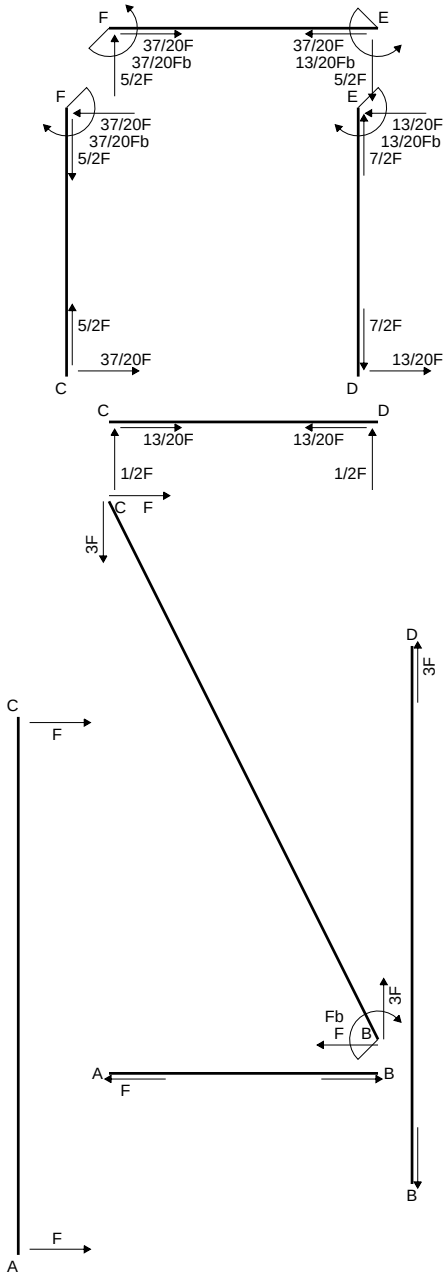
F
E
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37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
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5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
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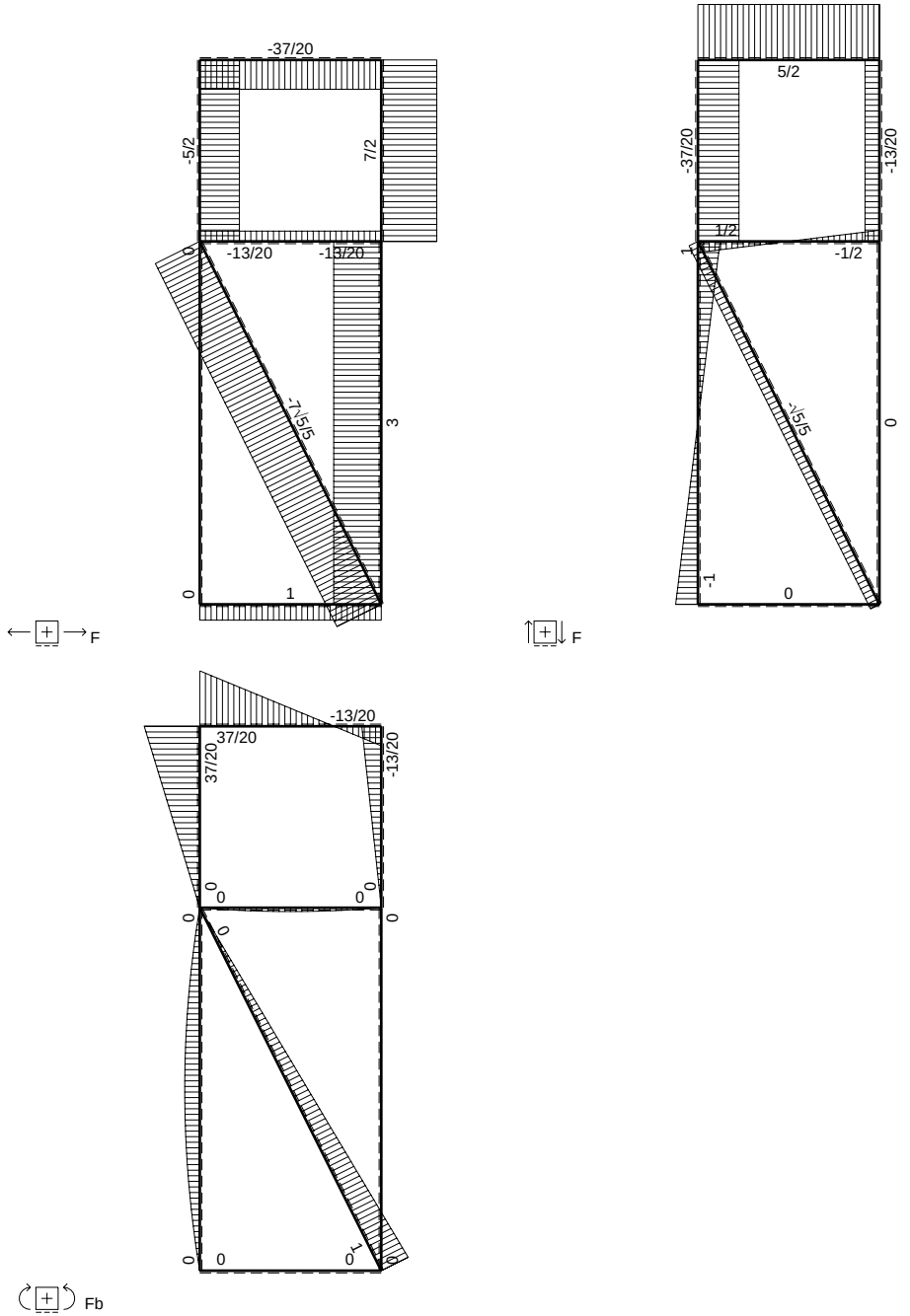
A
B
F
F
A
B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7/2
-13/20
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-7/5/5
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0
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F
5/2
-37/20
-13/20
1/2
-1/2
-7/5/5
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-1
0
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0
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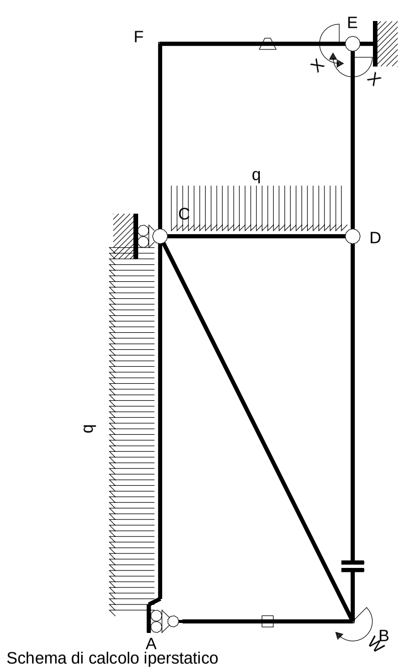
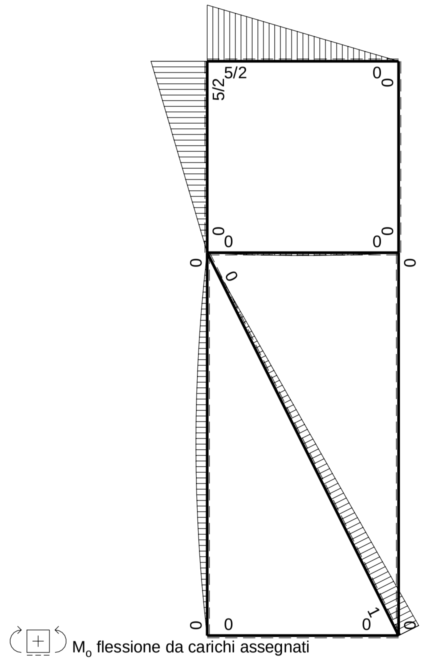
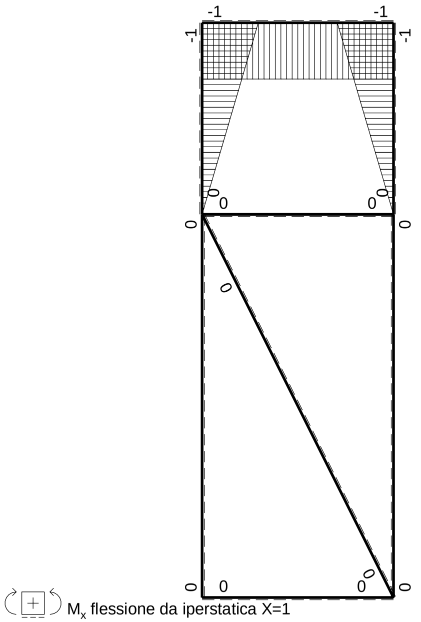
1
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

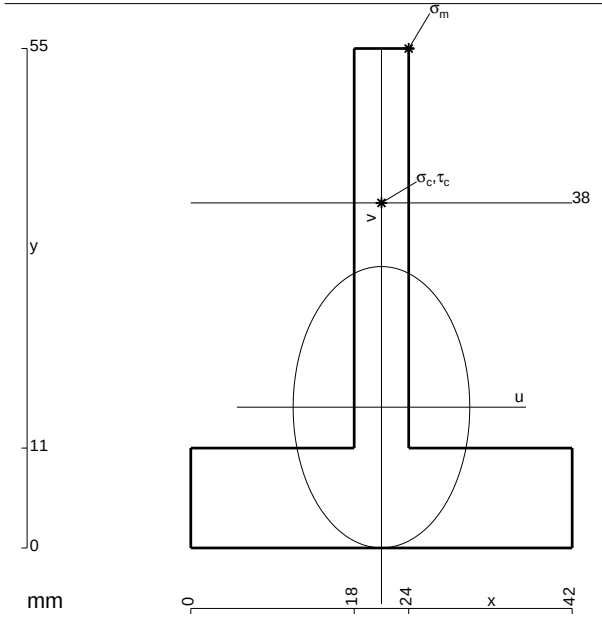
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 726. mm^2
J_u = 174301. mm^4
J_v = 68706. mm^4
y_g = 15.5 mm
N = -4602. N
T_y = -657.4 N
M_x = 852600. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 39.5 mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 22.5 mm
$\sigma_c = N/A - Mv/J_u$ = -116.4 N/mm^2
τ_c = 1.988 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.4 N/mm^2
S^* = 3162. mm^3

ESΣ09.xxxx.125

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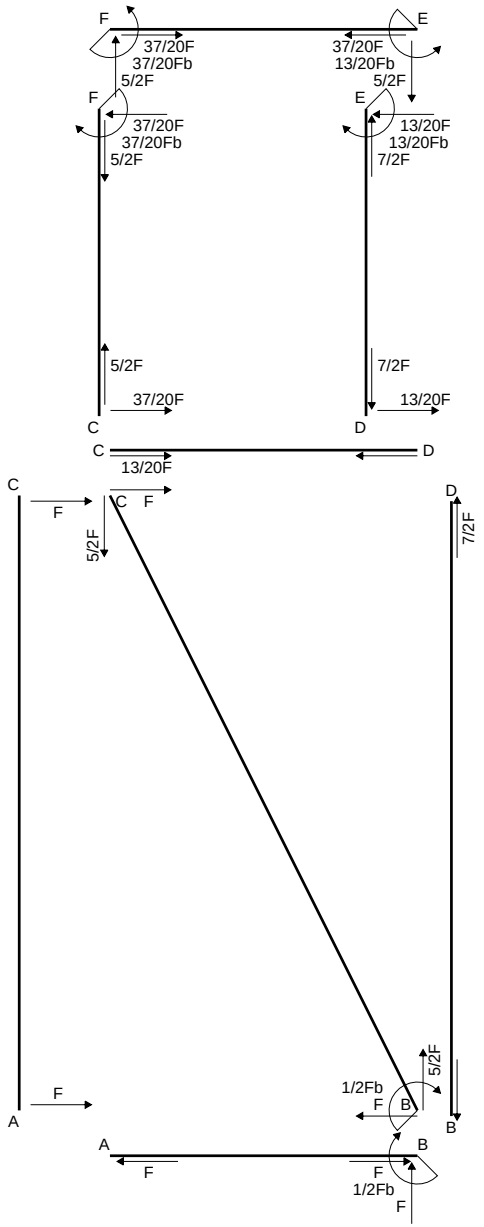
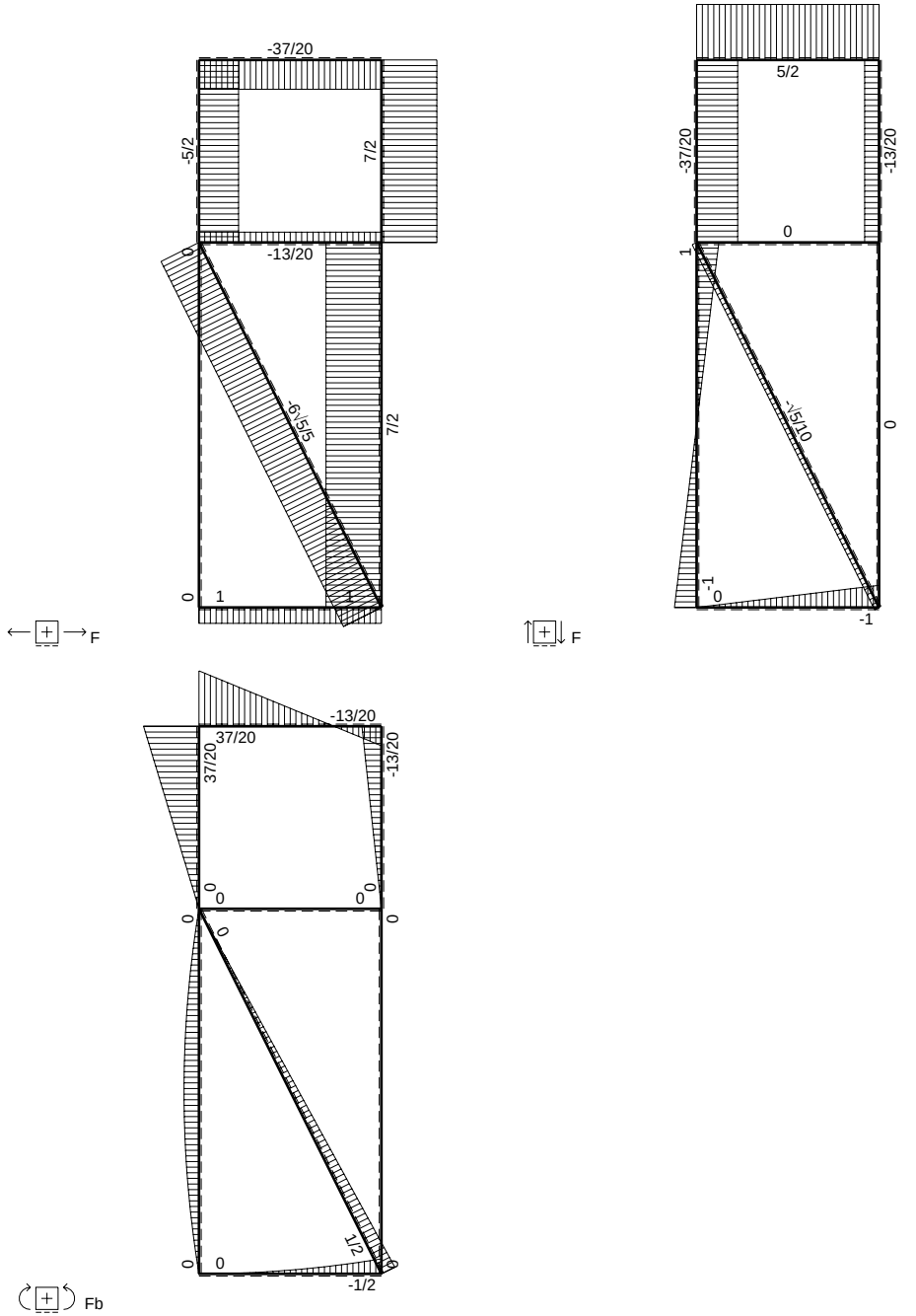
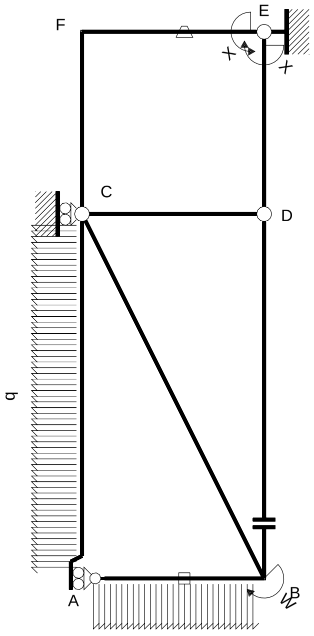
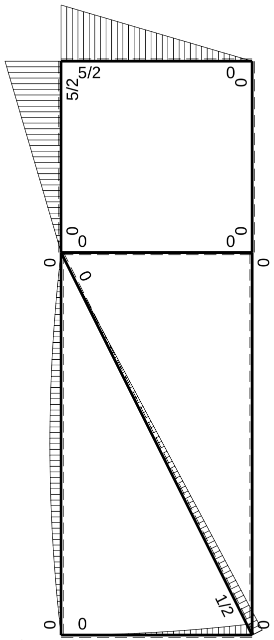

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

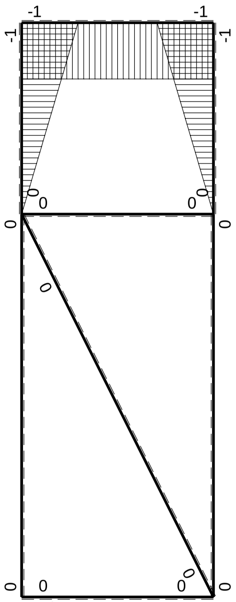

-37/20
-5/2
7/2
-13/20
-5\5/5
1
0
F
5/2
-37/20
-13/20
-5\5/10
-1
Fb
1/2
-1/2



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{EF}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica X=W_{EF}							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

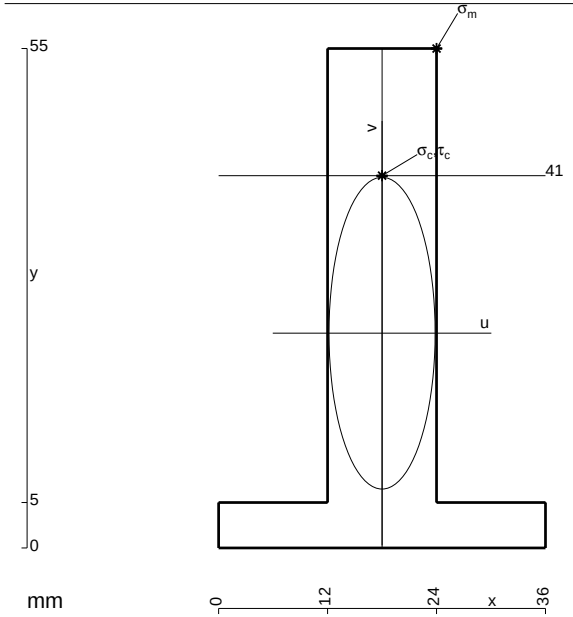
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 780. mm^2
J_u = 230087. mm^4
J_v = 26640. mm^4
y_g = 23.65 mm
N = 4820. N
T_y = -4820. N
M_x = -1494200. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 31.35 mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 17.35 mm
$\sigma_c = N/A - Mv/J_u$ = 118.8 N/mm^2
τ_c = 7.14 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.5 N/mm^2
S^* = 4090. mm^3

ESΣ09.xxxx.126

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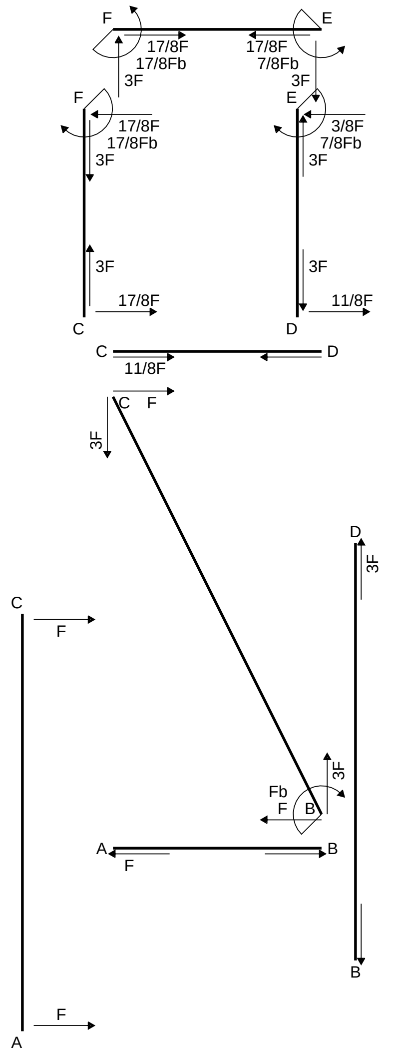
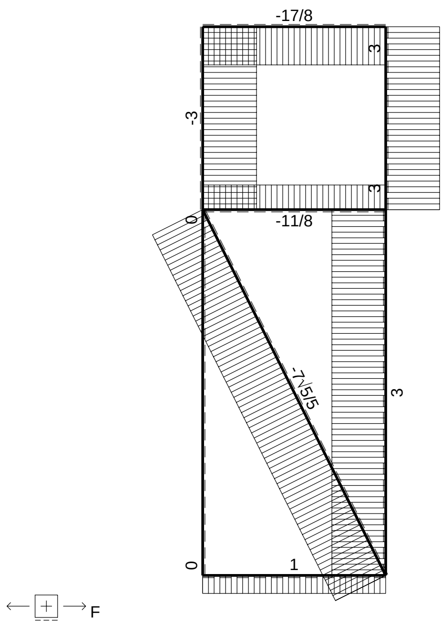
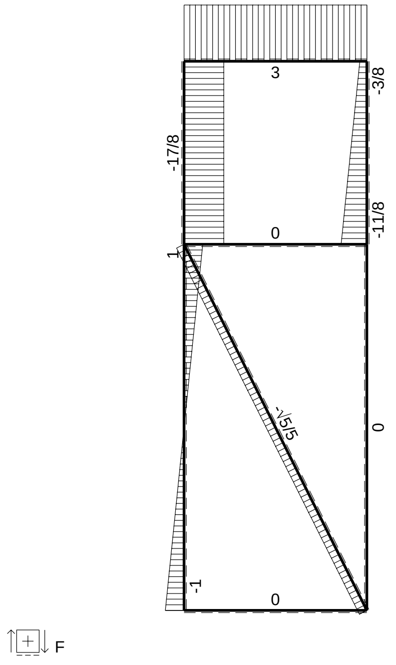
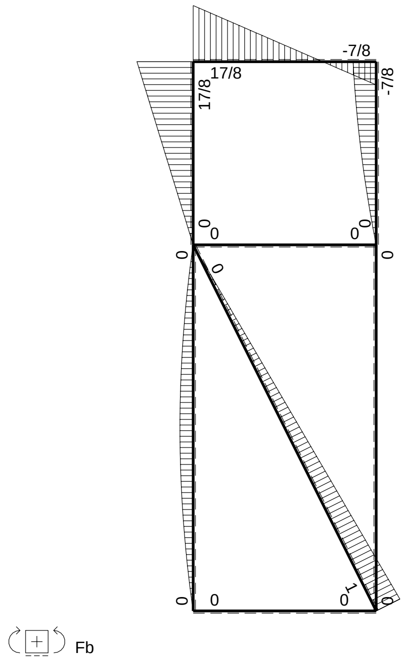
REAZIONI 981315

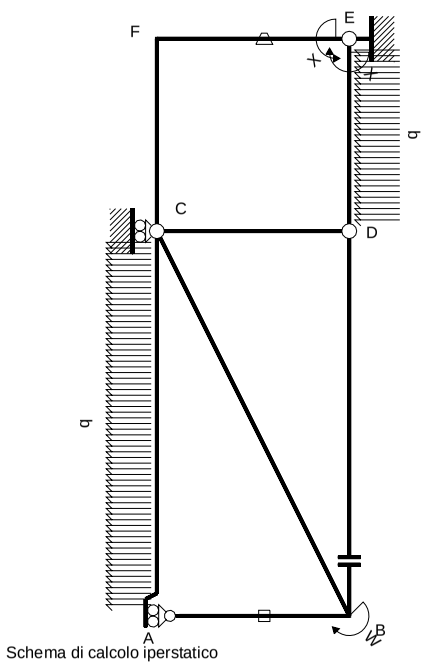


AZIONI INTERNE 981315

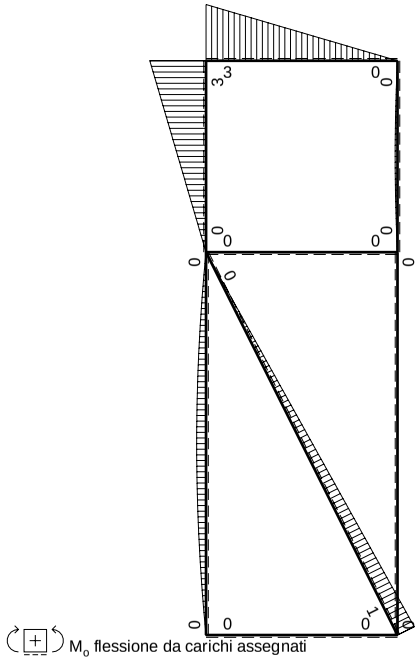




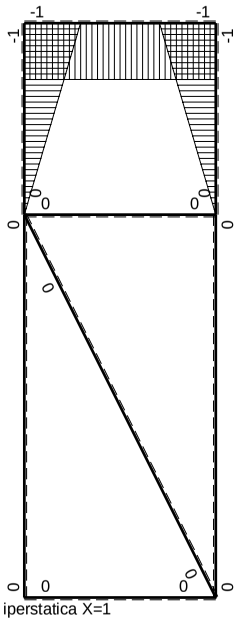




Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
		totali					$-35/24Fb^2/EJ$	5/3Xb/EJ
		iperstatica $X=W_{EF}$					7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981315 ESΣ09.xxxx.127

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

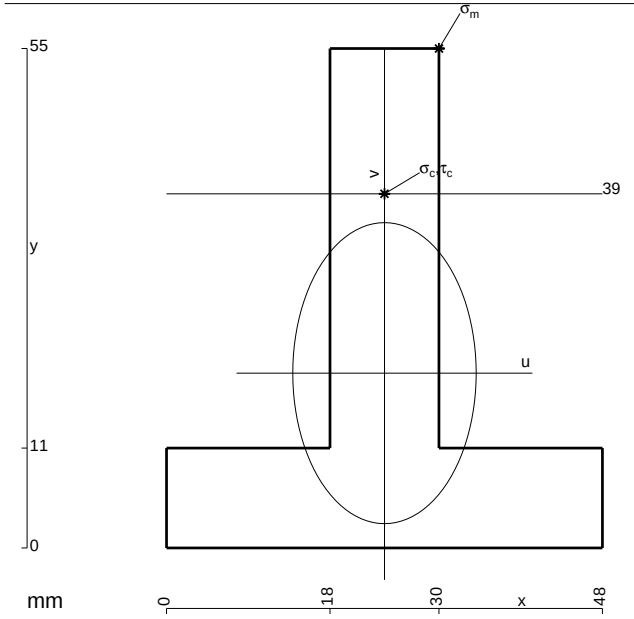
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 1056. mm^2
J_u = 290158. mm^4
J_v = 107712. mm^4
y_g = 19.25 mm
N = -8171. N
T_y = -1167. N
M_x = 1722600. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 35.75 mm
$\sigma_m = N/A - Mv/J_u$ = -220. N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 19.75 mm
$\sigma_c = N/A - Mv/J_u$ = -125. N/mm^2
τ_c = 1.786 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125. N/mm^2
S^* = 5328. mm^3

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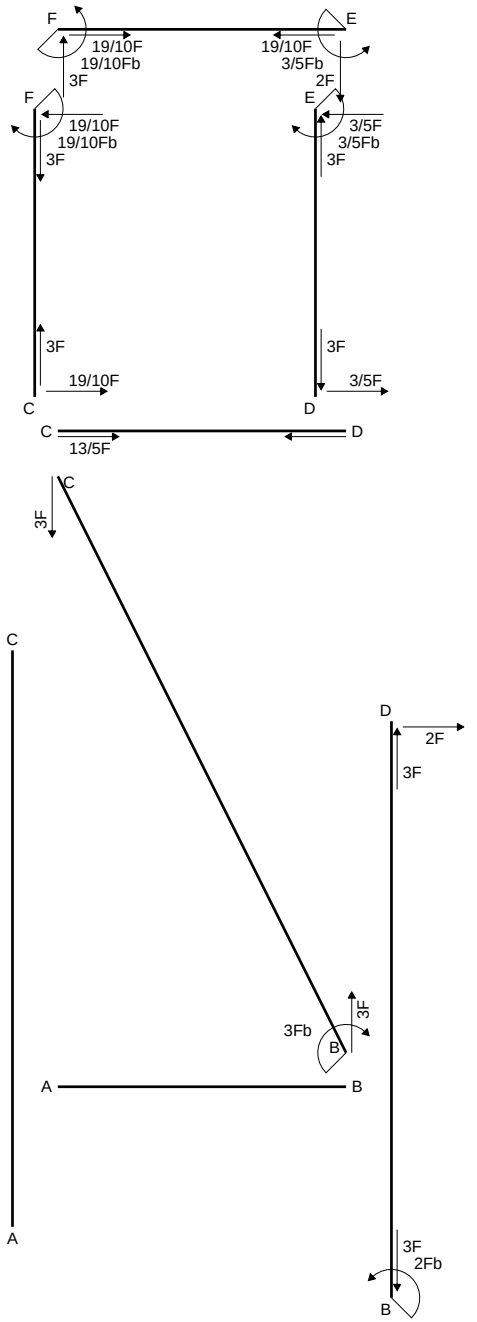
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E
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19/10Fb
19/10F
3/5Fb
3F
2F
F
E
19/10F
19/10Fb
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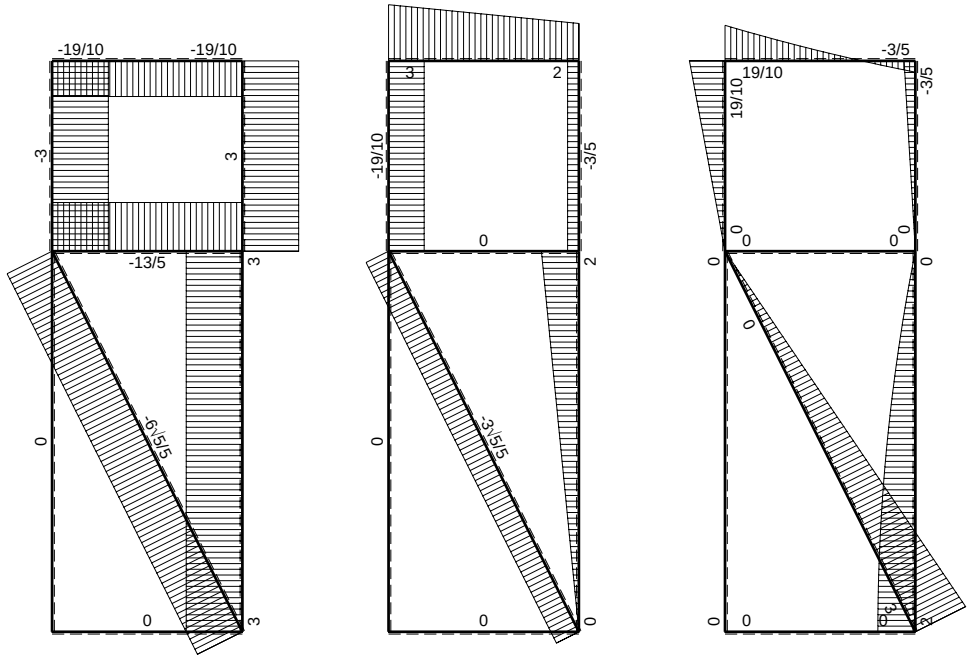
3/5Fb
3F
3F
3F
19/10F
3/5F
C
D
C
D
13/5F
C
3F
C
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
A
B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-3
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-6√5/5
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3
3
2
-19/10
-3/5
0
2
-3√5/5
0
0
-3/5
19/10
0
0
0
0
0
0
0
0
0
0
0


F
F
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
totali							-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

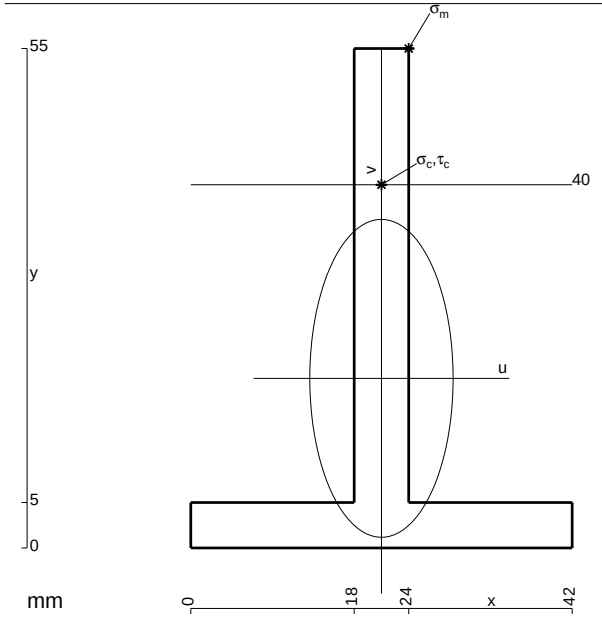
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 510. mm^2
J_u = 156357. mm^4
J_v = 31770. mm^4
y_g = 18.68 mm
N = -1234. N
T_y = -617.2 N
M_x = 966000. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 36.32 mm
$\sigma_m = N/A - Mv/J_u$ = -226.8 N/mm^2
x_c = 21. mm
y_c = 40. mm
v_c = 21.32 mm
$\sigma_c = N/A - Mv/J_u$ = -134.2 N/mm^2
τ_c = 1.707 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.2 N/mm^2
S^* = 2594. mm^3

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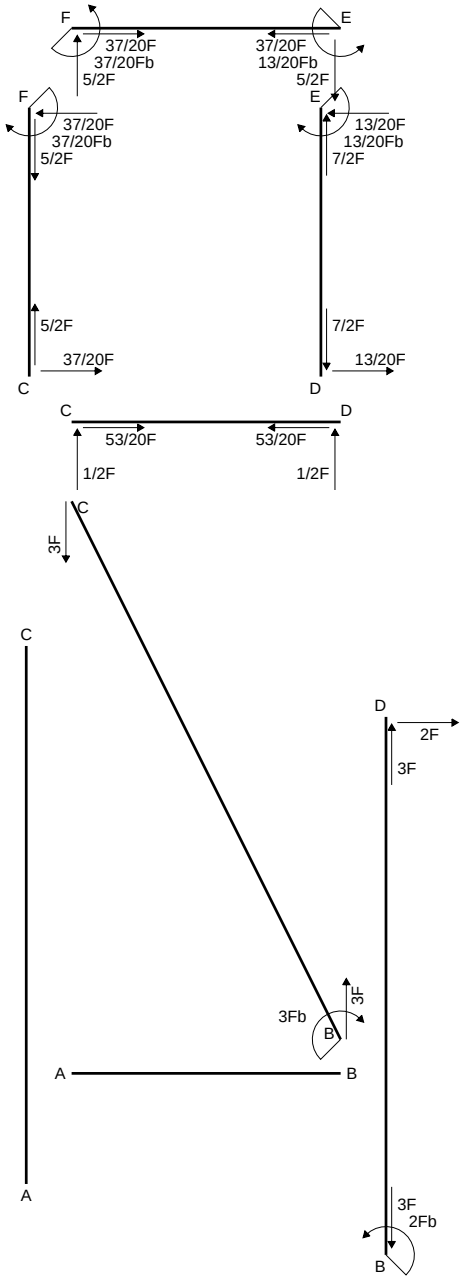
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13/20F
13/20Fb
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1/2F
1/2F
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3F
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3F
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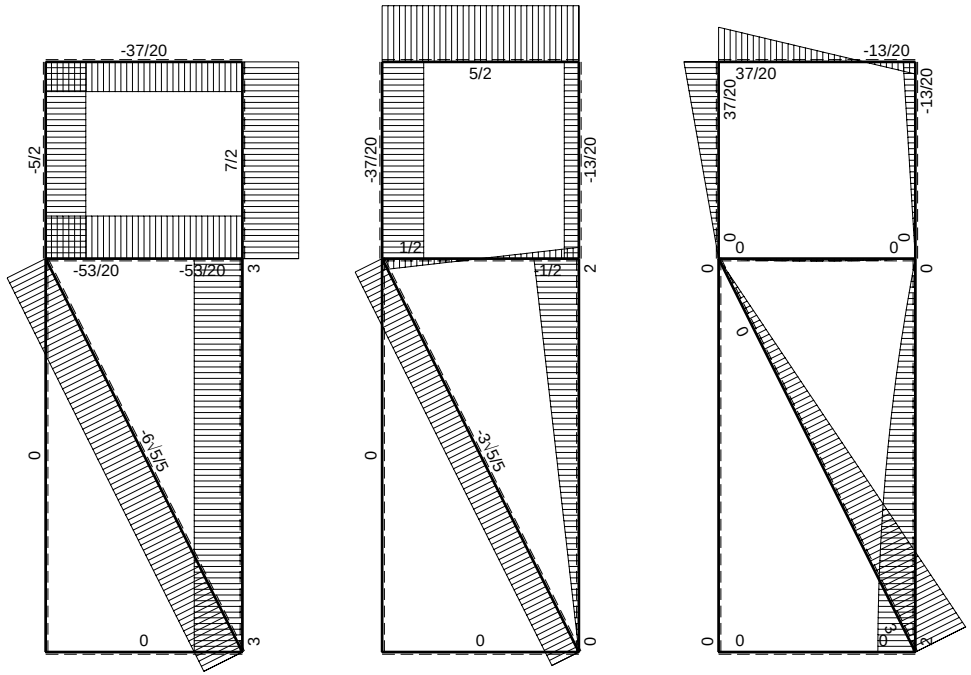
B
3F
A
B
A
3F
2Fb
B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2
0
0
-3√5/5
-13/20
37/20
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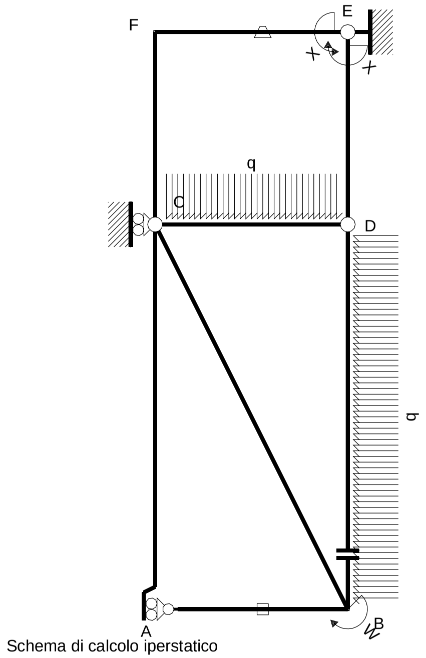
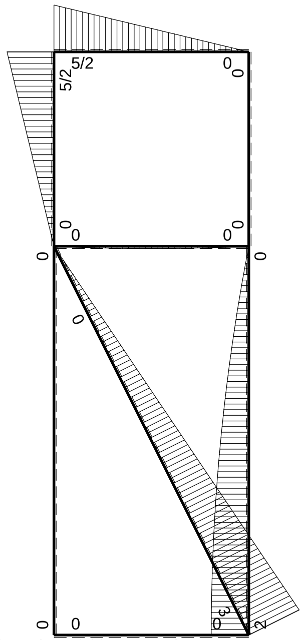
0
0
0
0
0
0
0
-3√5/5

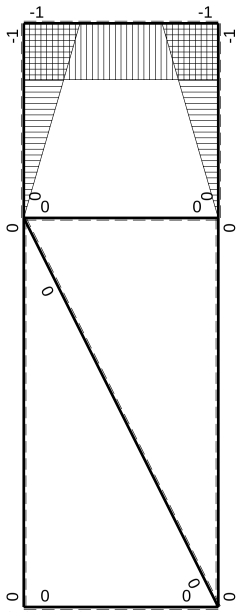

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

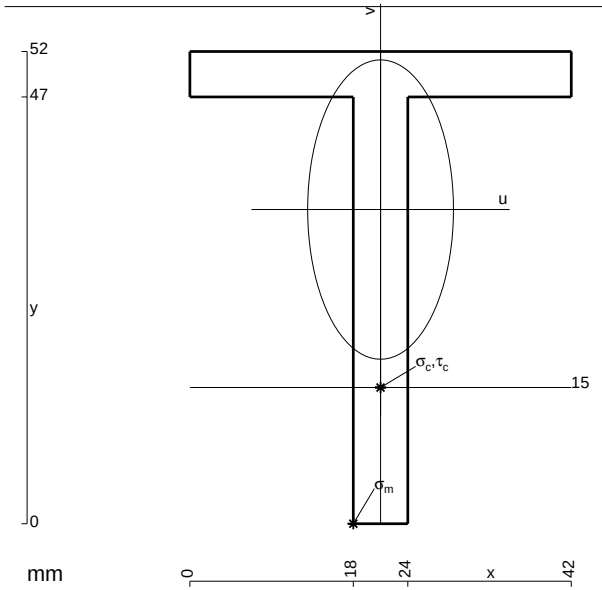
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 492. mm^2
J_u = 133716. mm^4
J_v = 31716. mm^4
y_g = 34.6 mm
N = -1181. N
T_y = -590.3 N
M_x = 924000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -34.6 mm
$\sigma_m = N/A - Mv/J_u$ = 236.7 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -19.6 mm
$\sigma_c = N/A - Mv/J_u$ = 133. N/mm^2
τ_c = 1.794 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.1 N/mm^2
S^* = 2439. mm^3

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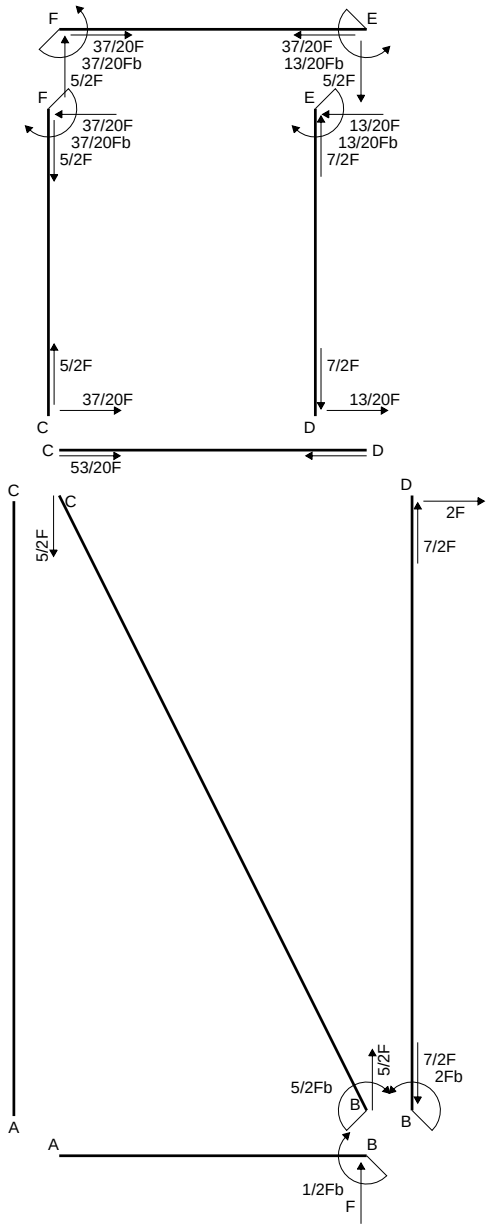
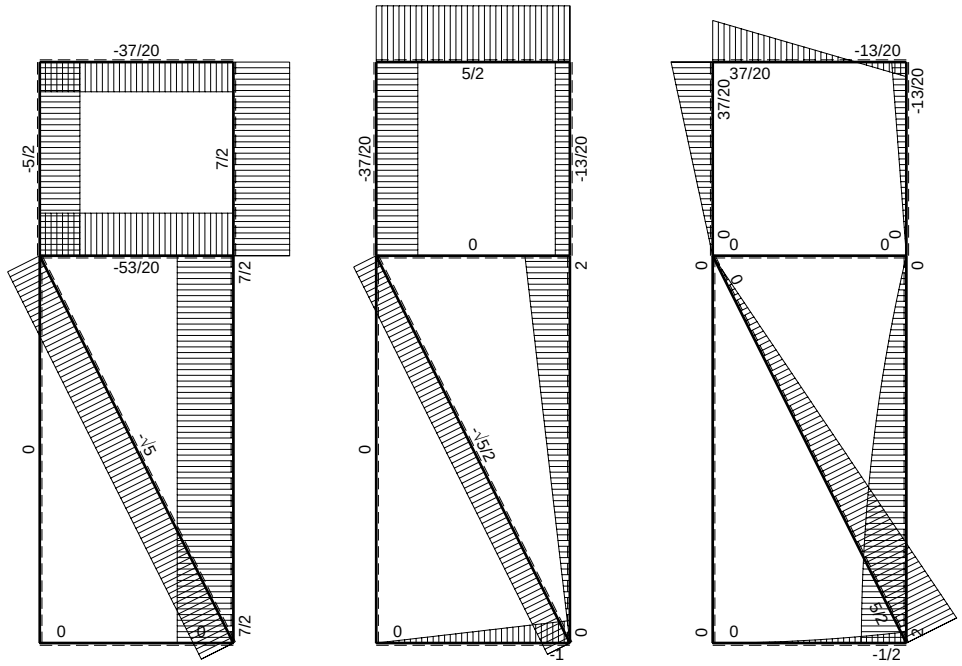

F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
53/20F
2F
7/2F
5/2F
5/2Fb
2Fb
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	5/2Fb-√5/2Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

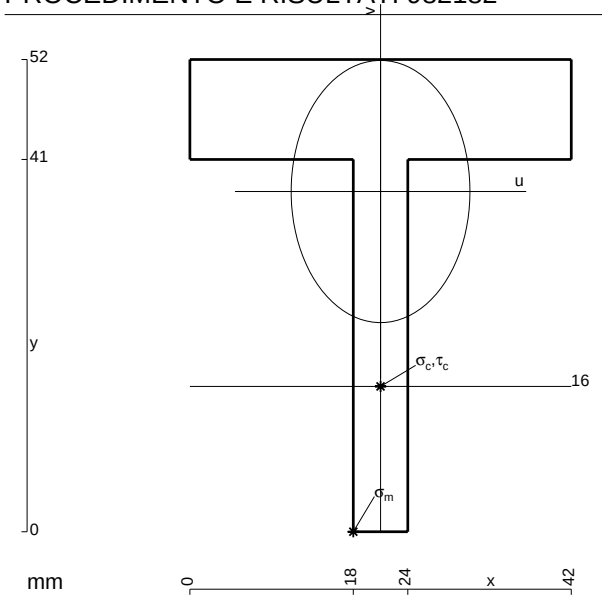
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 708. mm^2
J_u = 147634. mm^4
J_v = 68652. mm^4
y_g = 37.47 mm
N = -1923. N
T_y = -961.5 N
M_x = 795500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.47 mm
$\sigma_m = N/A - Mv/J_u$ = 199.2 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.47 mm
$\sigma_c = N/A - Mv/J_u$ = 112.9 N/mm^2
τ_c = 3.07 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.1 N/mm^2
S^* = 2829. mm^3

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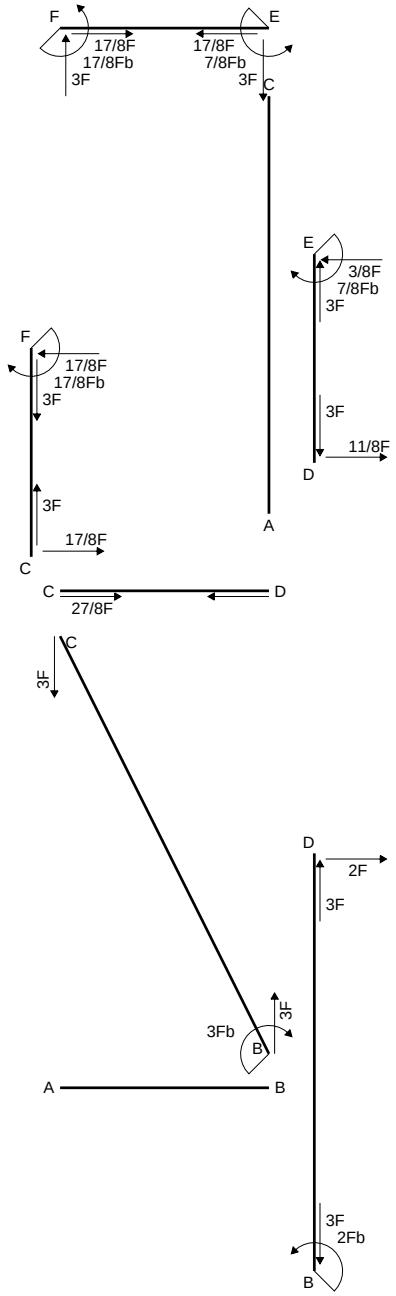
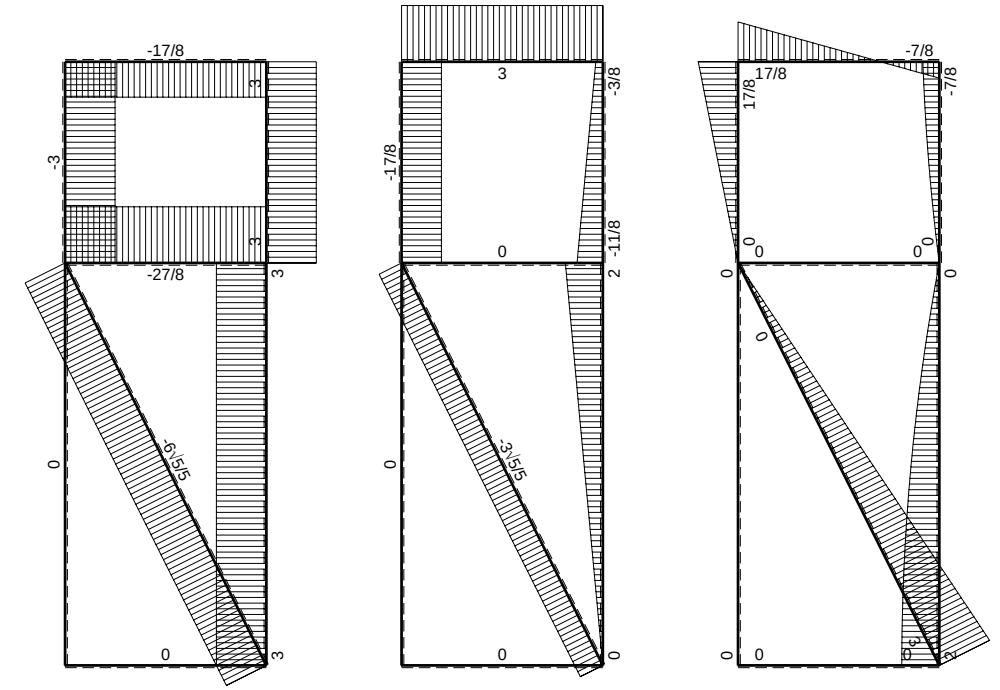

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6√5/5
0
3
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0

F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^{2}b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XkJ/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	XkJ/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3XkJ/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
	totali						-35/24Fb^2/EJ	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [\ x]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b]_o^b\ Fb\ 1/EJ\ + [- x]_o^b\ \theta$

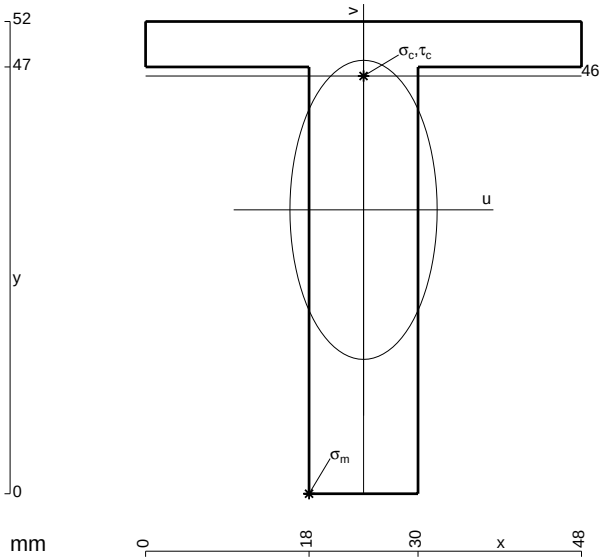
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [- x^3/b^2]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 804. mm^2
J_u = 218133. mm^4
J_v = 52848. mm^4
y_g = 31.26 mm
N = -3327. N
T_y = -1664. N
M_x = 1488000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -31.26 mm
$\sigma_m = N/A - Mv/J_u$ = 209.1 N/mm^2
x_c = 24. mm
y_c = 46. mm
v_c = 14.74 mm
$\sigma_c = N/A - Mv/J_u$ = -104.7 N/mm^2
τ_c = 2.898 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 104.8 N/mm^2
S^* = 4560. mm^3

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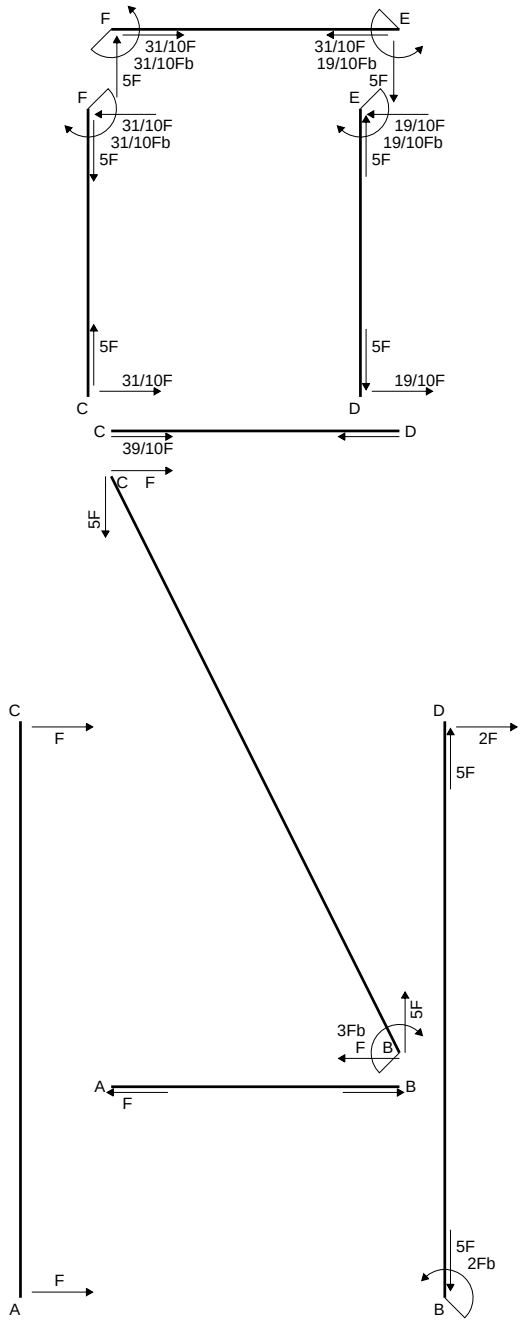
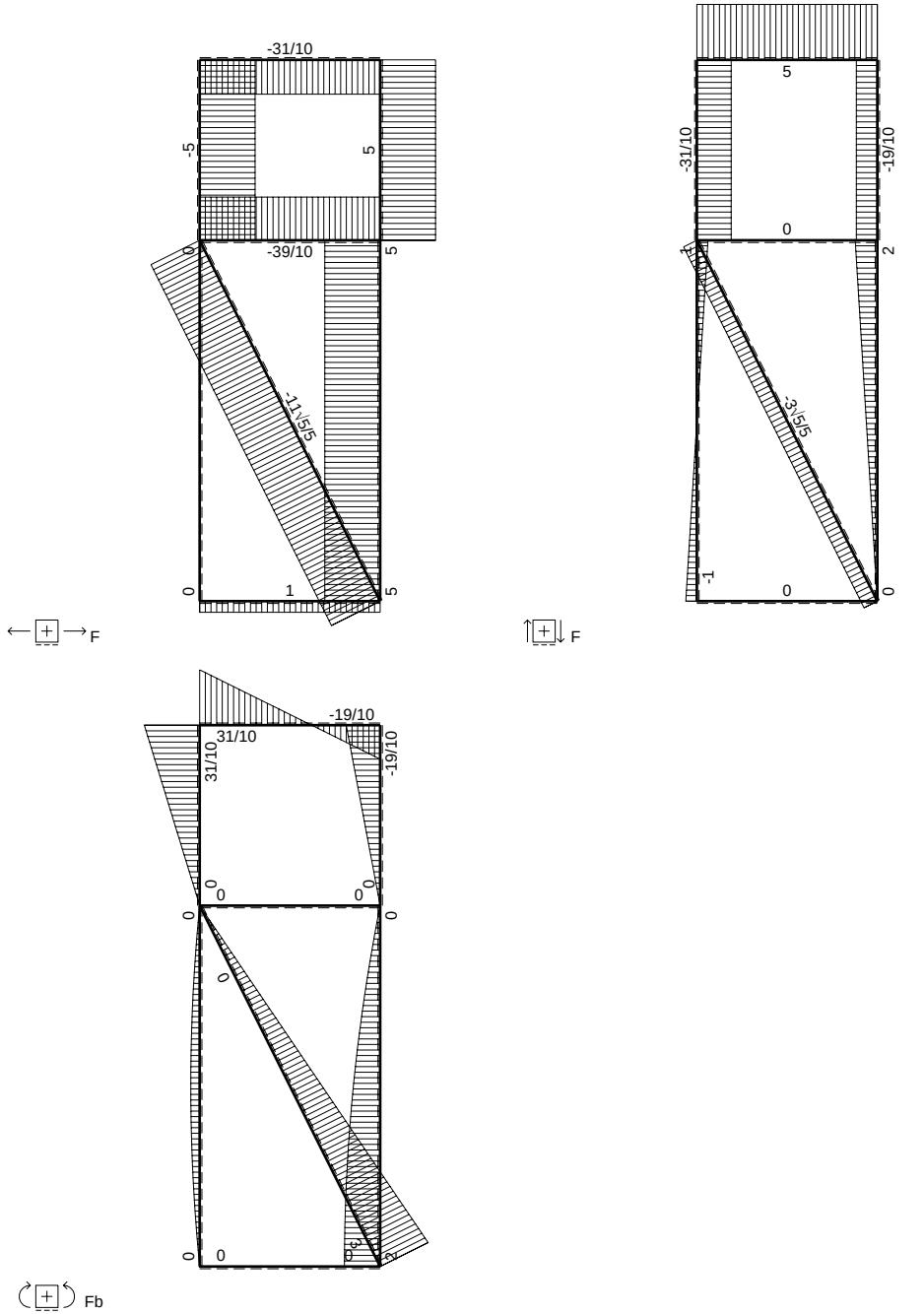

F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B


-31/10
-39/10
-11√5/5
-3√5/5
-19/10
31/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
	totali						-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211673 ESΣ09.xxxx.132

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

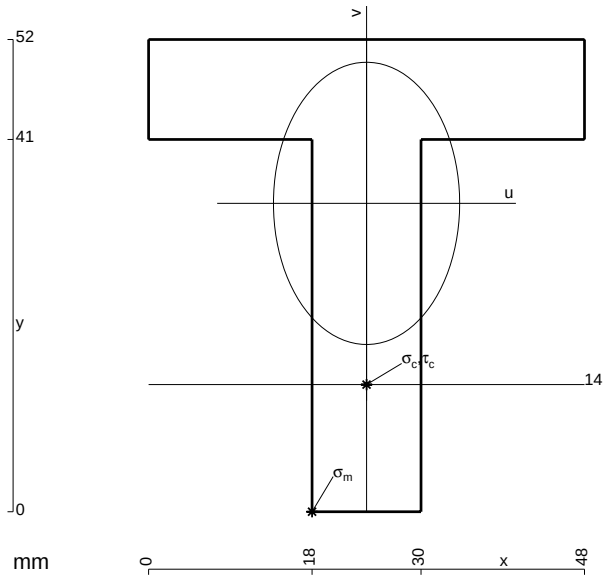
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$



A = 1020. mm^2
J_u = 246410. mm^4
J_v = 107280. mm^4
y_g = 33.96 mm
N = -6100. N
T_y = -1664. N
M_x = 1636800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.96 mm
$\sigma_m = N/A - Mv/J_u$ = 219.6 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.96 mm
$\sigma_c = N/A - Mv/J_u$ = 126.6 N/mm^2
τ_c = 2.548 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.7 N/mm^2
S^* = 4529. mm^3

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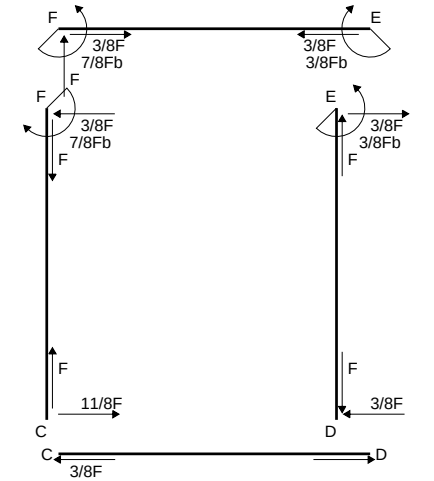
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
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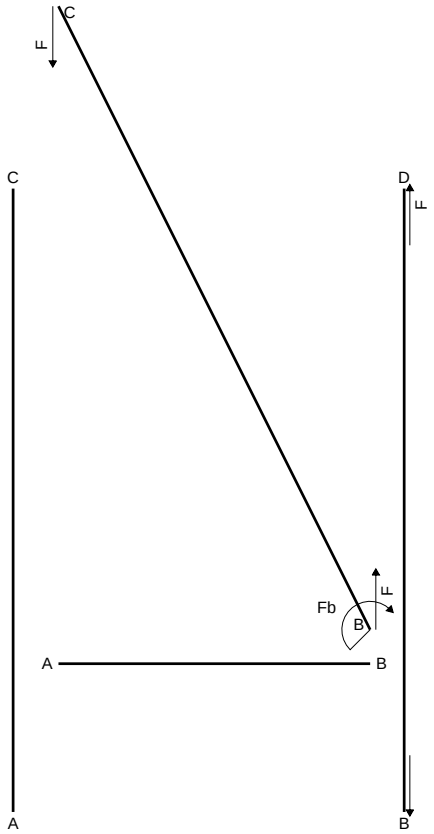
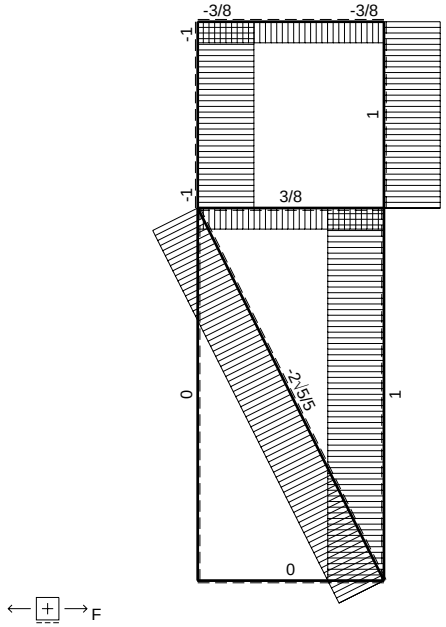
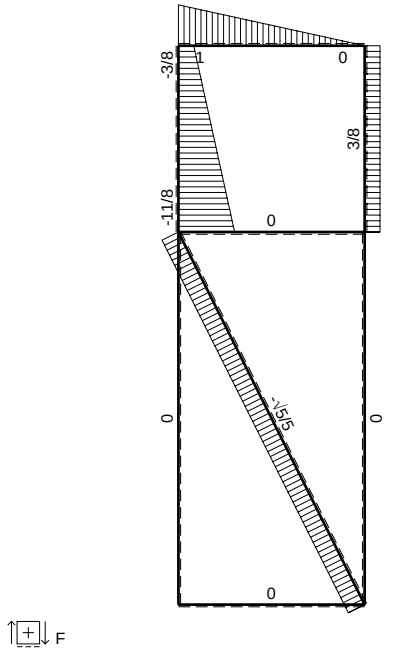
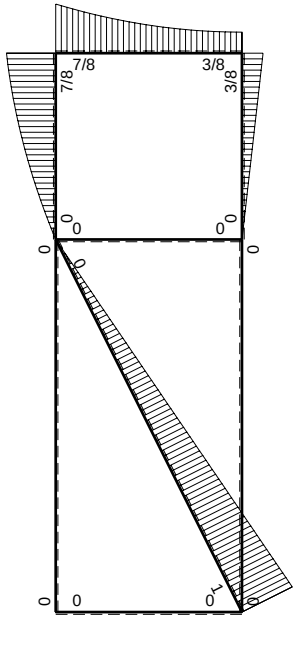
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B


-3/8
-3/8
-1
1
-1
3/8
0
-2√5/5
1
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

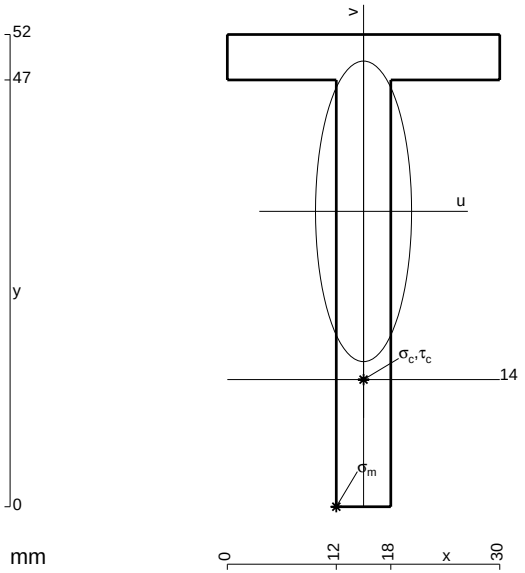
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 432. mm^2
J_u = 118416. mm^4
J_v = 12096. mm^4
y_g = 32.53 mm
N = -1583. N
T_y = -791.6 N
M_x = 849600. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -32.53 mm
σ_m = N/A-Mv/J_u = 229.7 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -18.53 mm
σ_c = N/A-Mv/J_u = 129.3 N/mm^2
τ_c = 2.389 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.3 N/mm^2
S^* = 2144. mm^3

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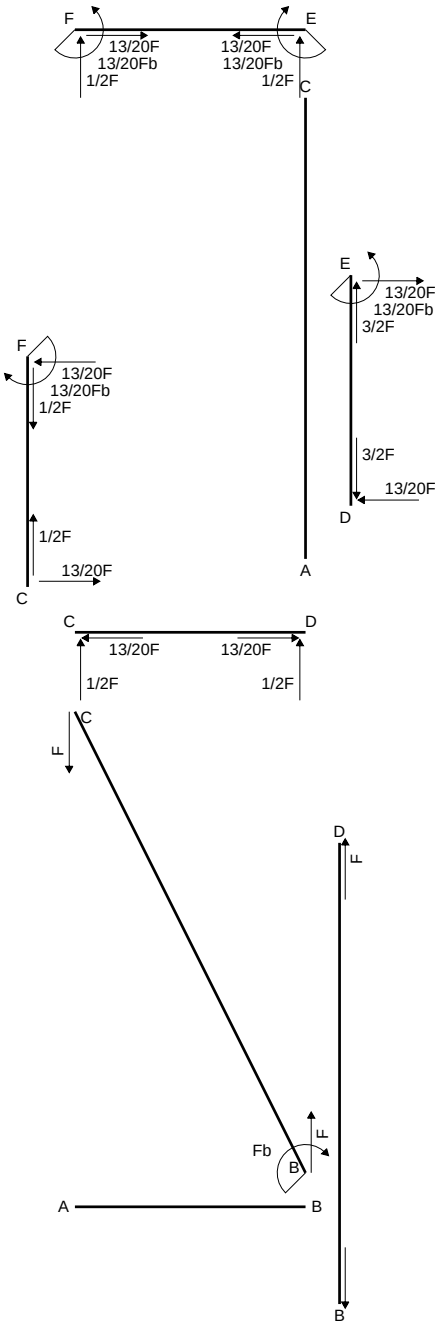
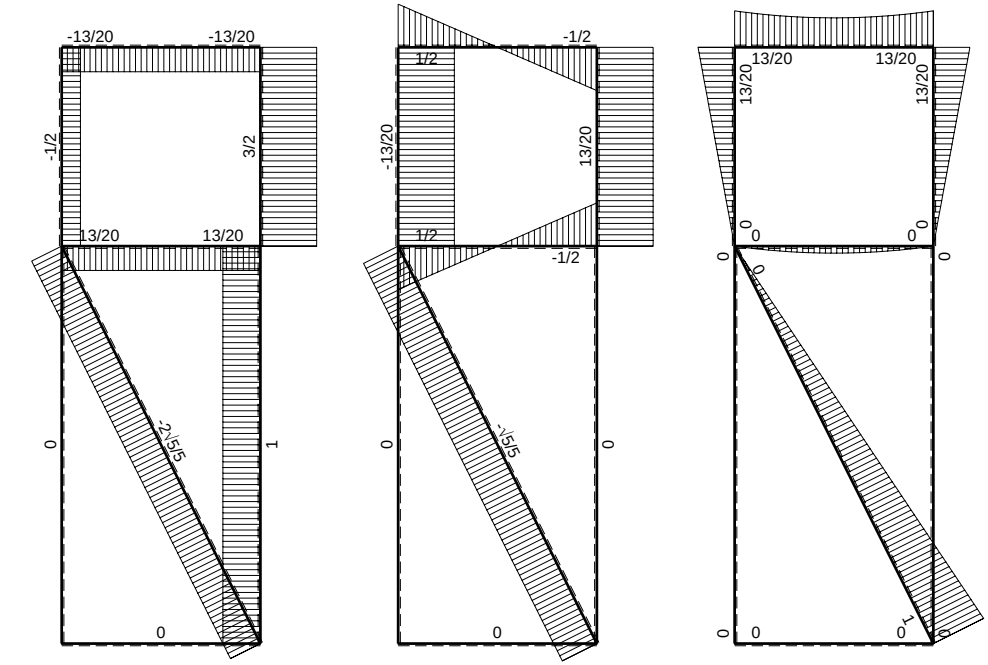

F
E
13/20F
13/20Fb
1/2F
C
E
13/20F
13/20Fb
3/2F
F
13/20F
13/20Fb
1/2F
3/2F
13/20F
D
1/2F
13/20F
C
A
C
D
13/20F
13/20F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/20
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-2√5/5
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-1/2
1/2
-13/20
13/20
-1/2
-√5/5
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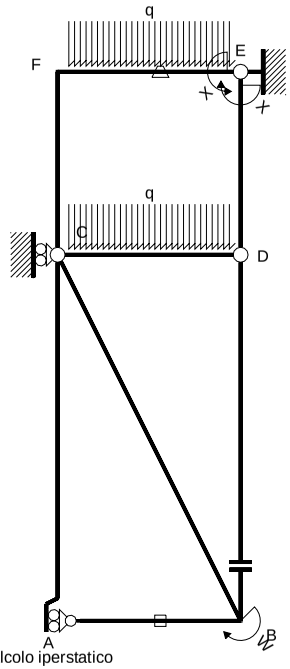
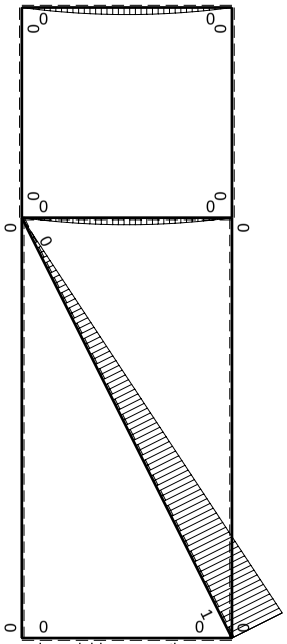
13/20
13/20
0
0
0
0
0
0
0
1
0

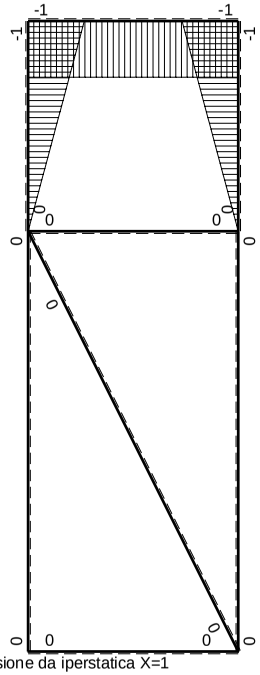

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

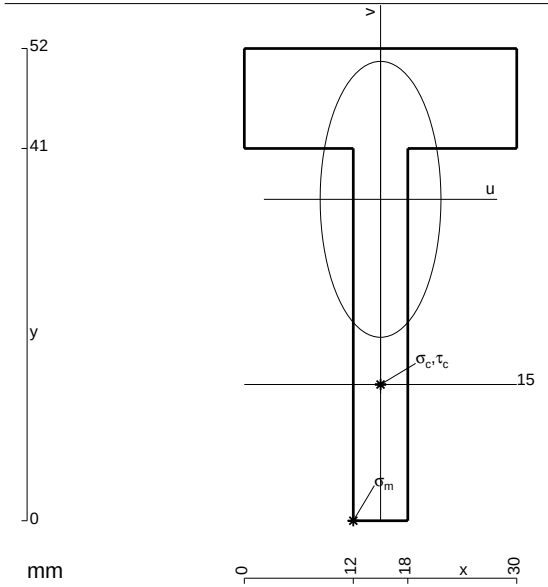
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211805 ESΣ09.xxxx.134



A = 576. mm^2
J_u = 133062. mm^4
J_v = 25488. mm^4
y_g = 35.4 mm
N = -1592. N
T_y = -796. N
M_x = 907800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.4 mm
$\sigma_m = N/A - Mv/J_u$ = 238.7 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.4 mm
$\sigma_c = N/A - Mv/J_u$ = 136.4 N/mm^2
τ_c = 2.503 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.5 N/mm^2
S^* = 2511. mm^3

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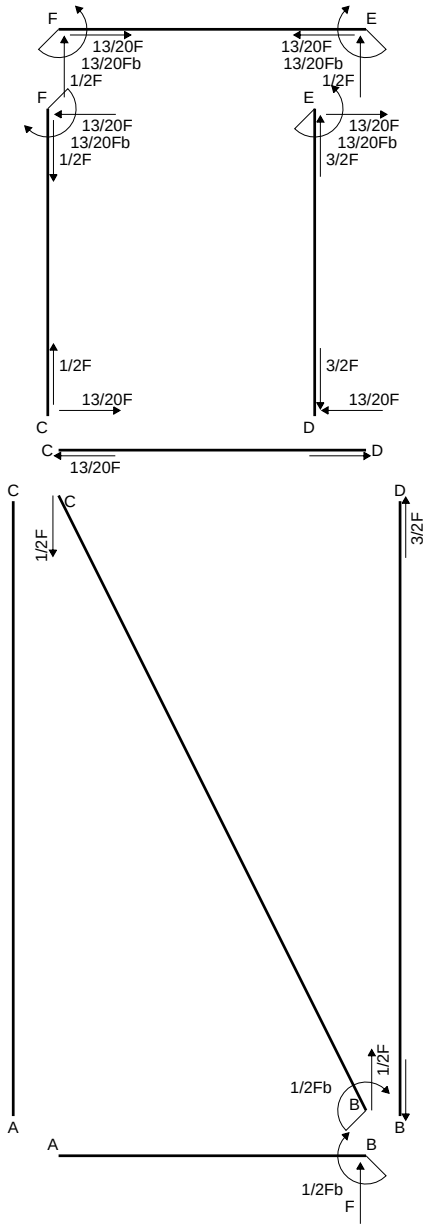
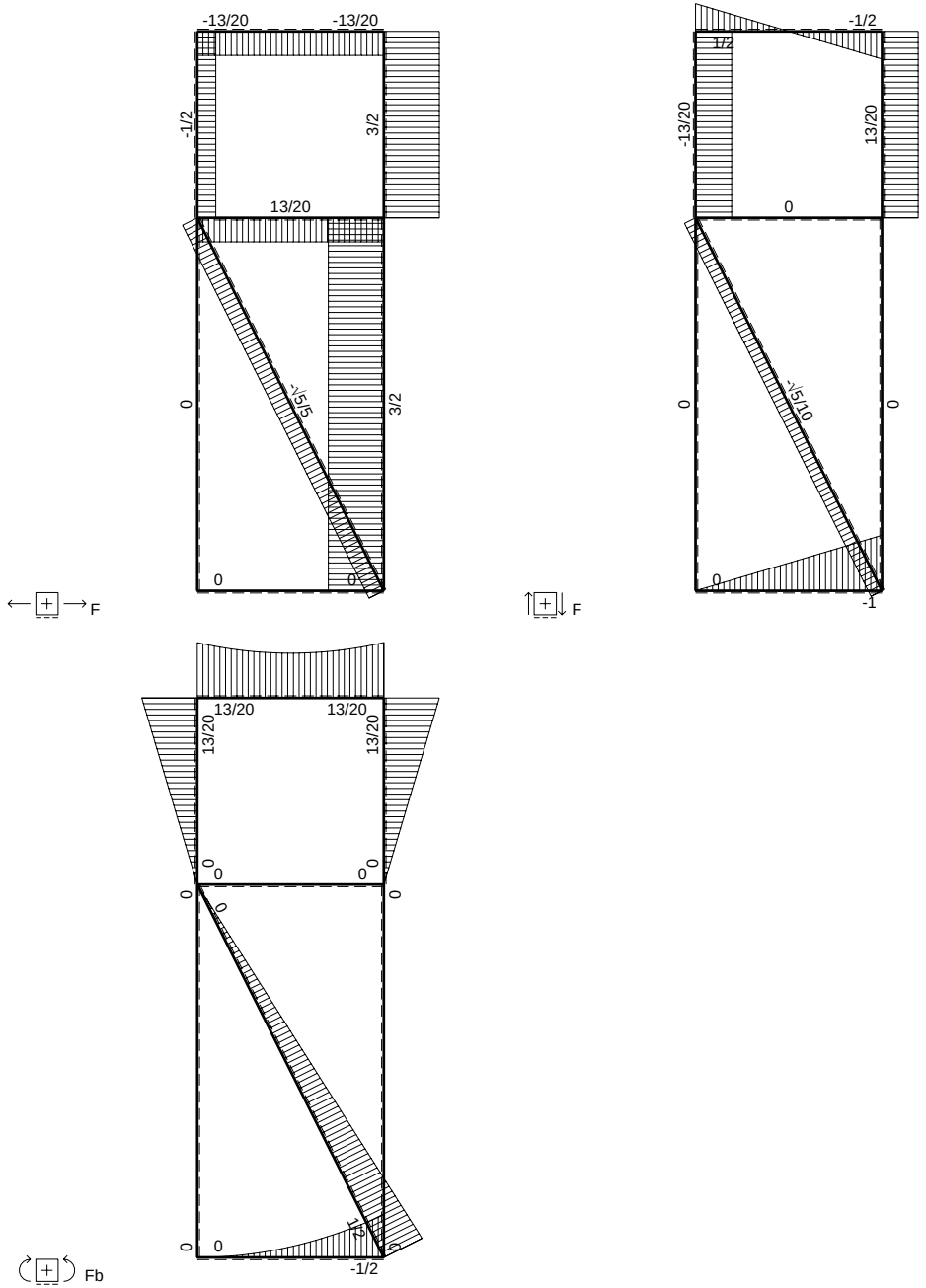
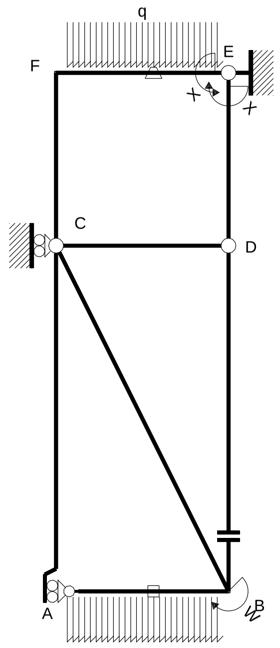
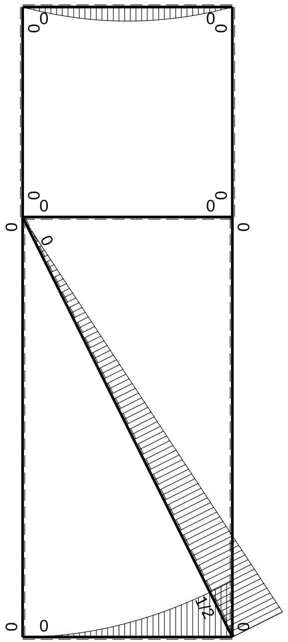

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

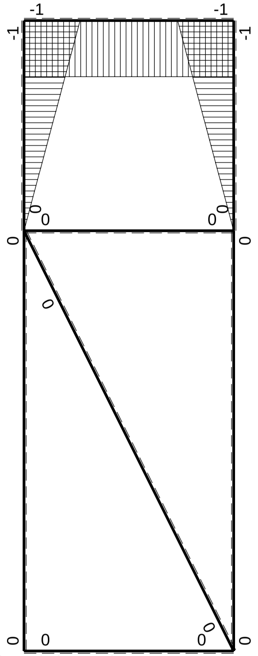

-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
13/20
Fb
-1/2



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

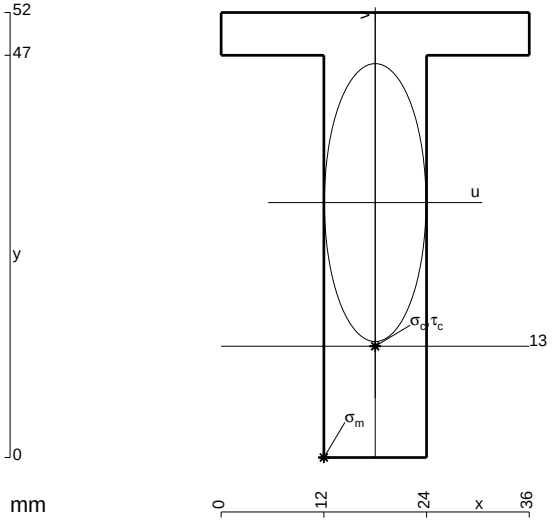
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 744. mm^2
J_u = 196439. mm^4
J_v = 26208. mm^4
y_g = 29.79 mm
T_y = -4790. N
M_x = -1317250. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -29.79 mm
$\sigma_m = -Mv/J_u$ = -199.8 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -16.79 mm
$\sigma_c = -Mv/J_u$ = -112.6 N/mm^2
τ_c = 7.383 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.3 N/mm^2
S^* = 3633. mm^3

ESΣ09.xxxx.135

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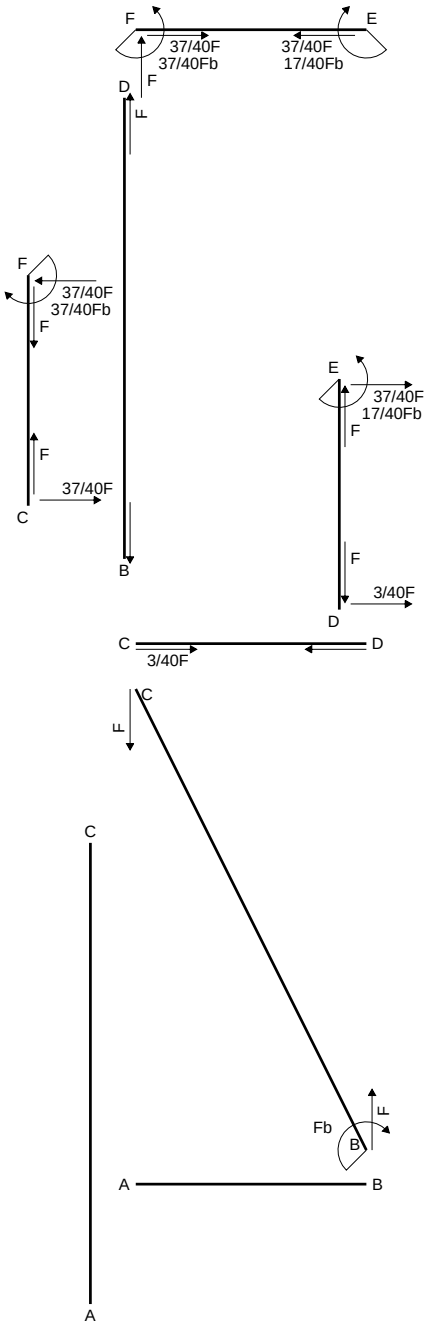
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E
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37/40Fb
37/40F
17/40Fb
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F
F
37/40F
37/40Fb
F
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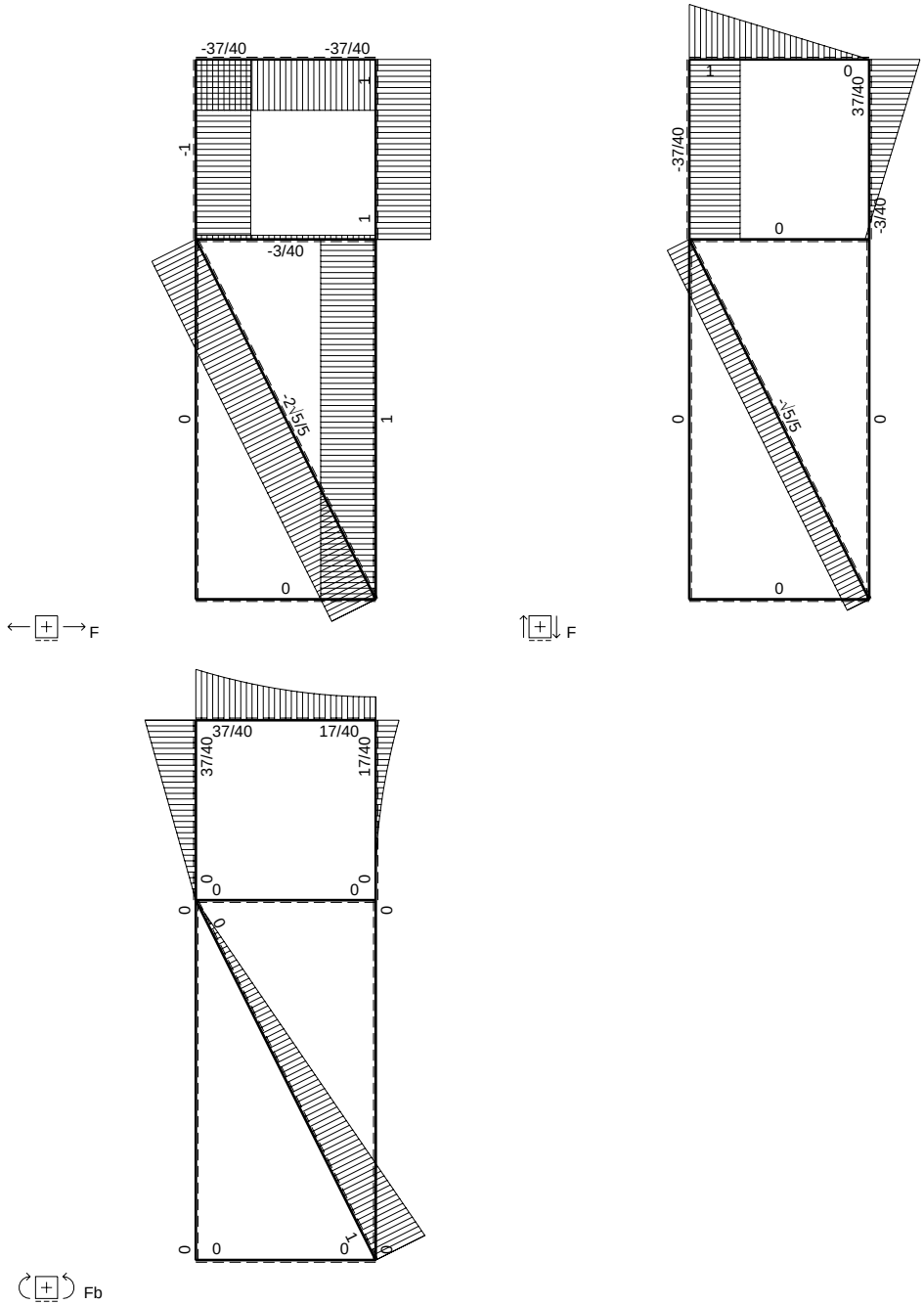
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D
C
F
C
Fb
B
F
A
B
A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1
-1
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-3/40
-2√5/5
0
1
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F
1
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0
-√5/5
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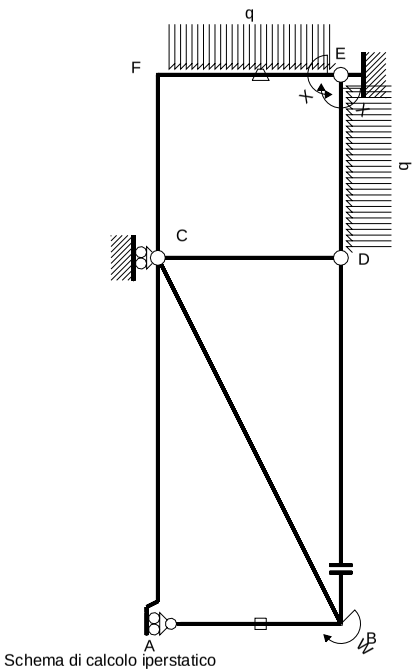
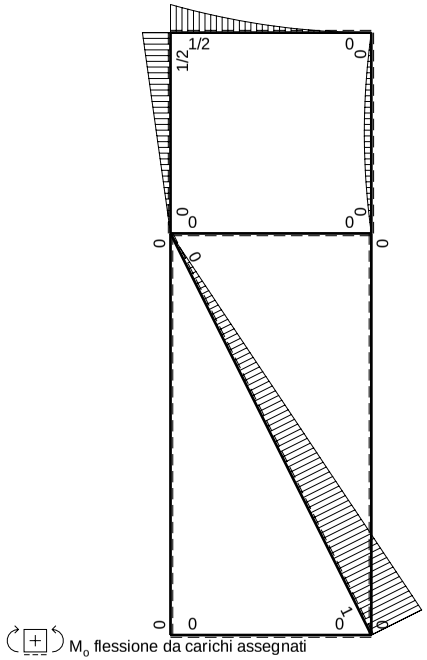
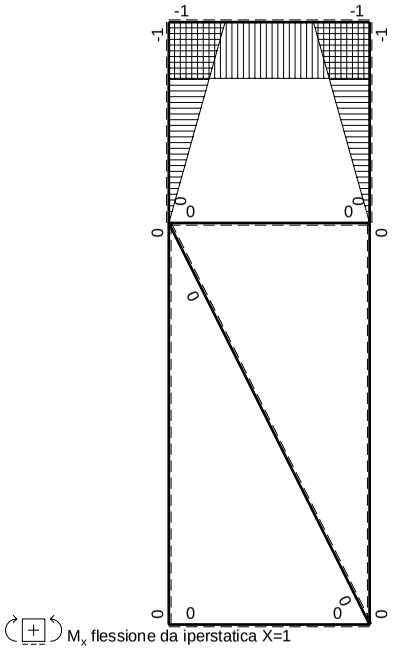
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957828 ESΣ09.xxxx.136

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

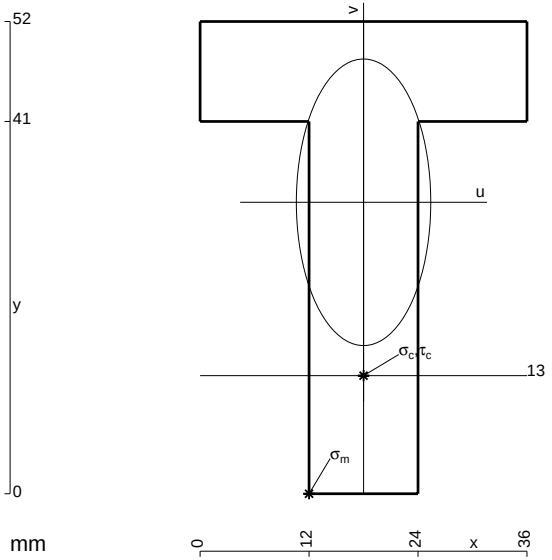
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 957828 ESΣ09.xxxx.136



A = 888. mm^2
J_u = 221232. mm^4
J_v = 48672. mm^4
y_g = 32.09 mm
N = -2218. N
T_y = -1109. N
M_x = 1463200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.09 mm
σ_m = N/A-Mv/J_u = 209.8 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -19.09 mm
σ_c = N/A-Mv/J_u = 123.8 N/mm^2
τ_c = 1.668 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.8 N/mm^2
S^* = 3993. mm^3

ESΣ09.xxxx.136

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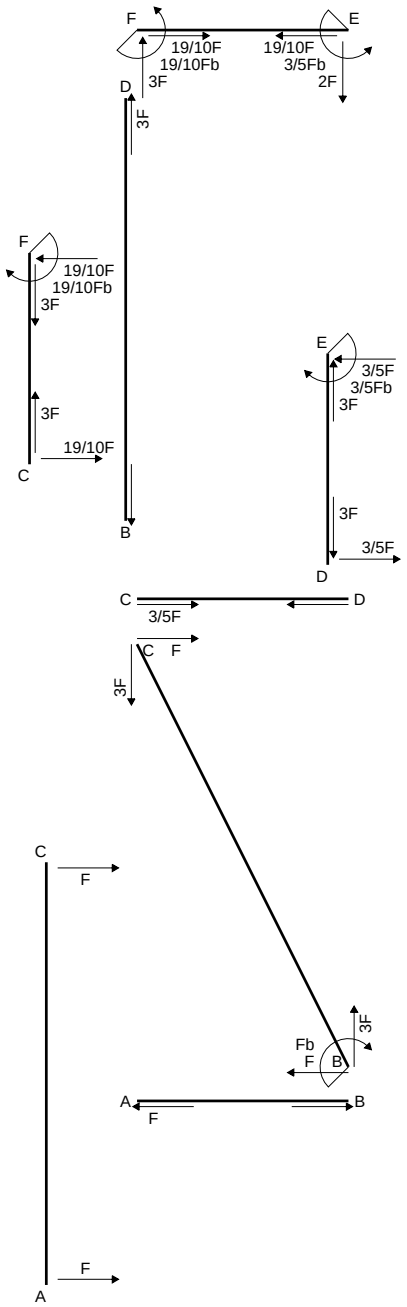
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19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
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E
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3/5Fb
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3F
19/10F
C
B
3F
3/5F
D
C
D
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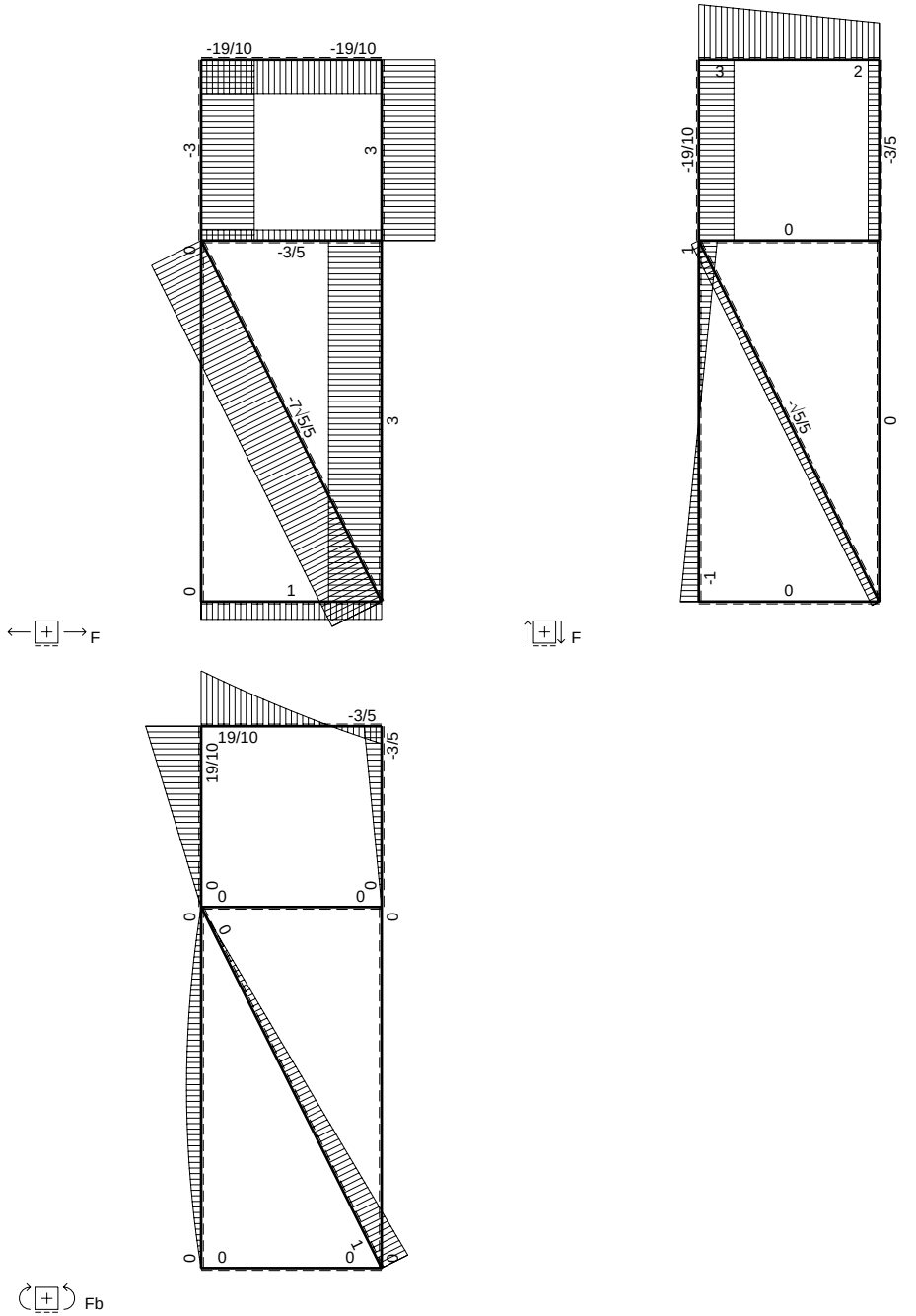
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
3
-7\5/5
1
F
-19/10
3
2
-3/5
0
1
-1\5/5
-1
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19/10
-3/5
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0
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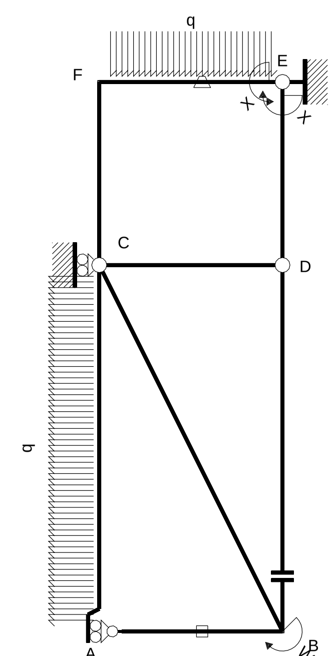
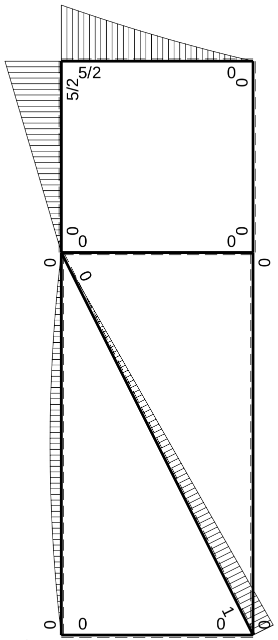
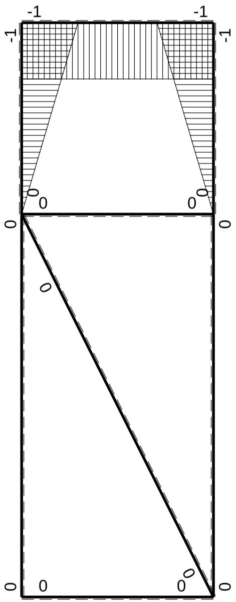
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

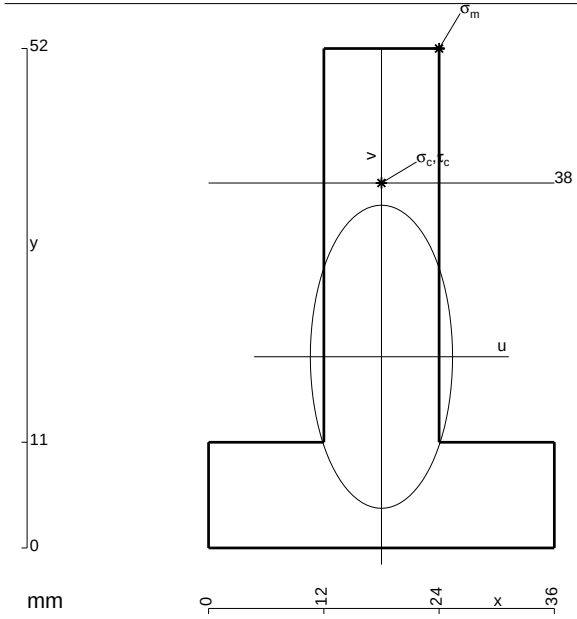
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 888. mm^2
J_u = 221232. mm^4
J_v = 48672. mm^4
y_g = 19.91 mm
N = -7357. N
T_y = -1051. N
M_x = 1457000. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 32.09 mm
$\sigma_m = N/A - Mv/J_u$ = -219.7 N/mm^2
x_c = 18. mm
y_c = 38. mm
v_c = 18.09 mm
$\sigma_c = N/A - Mv/J_u$ = -127.5 N/mm^2
τ_c = 1.669 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.5 N/mm^2
S^* = 4216. mm^3

ESΣ09.xxxx.137

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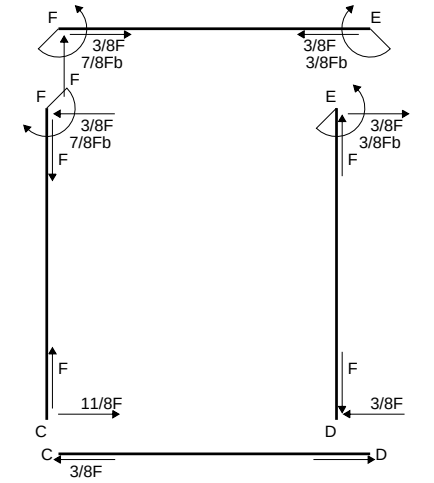
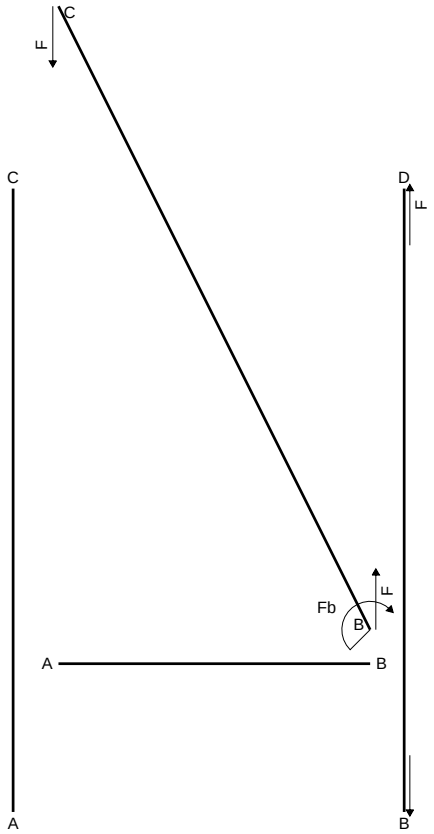
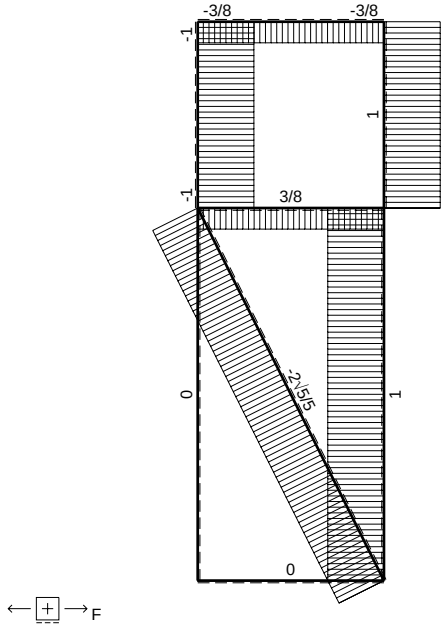
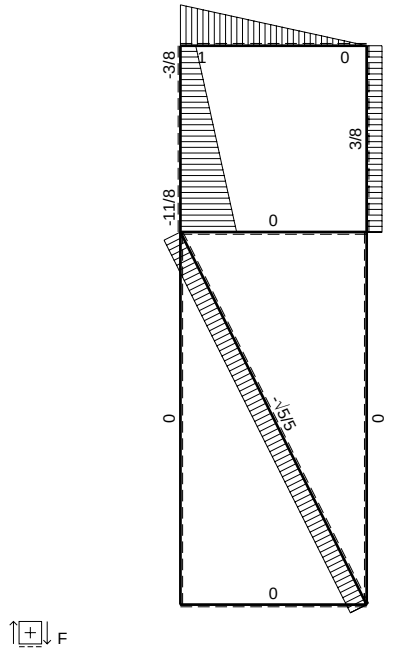
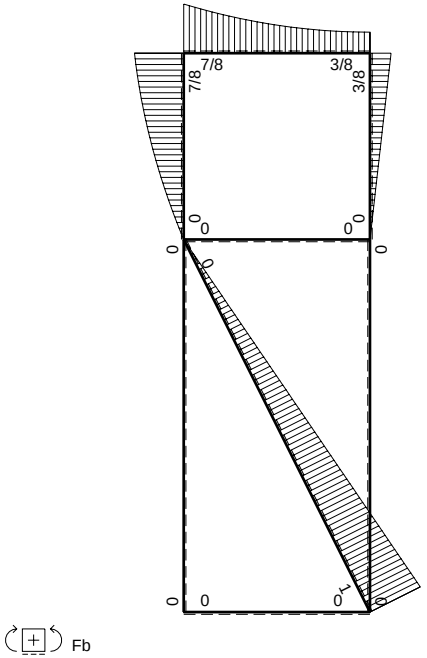
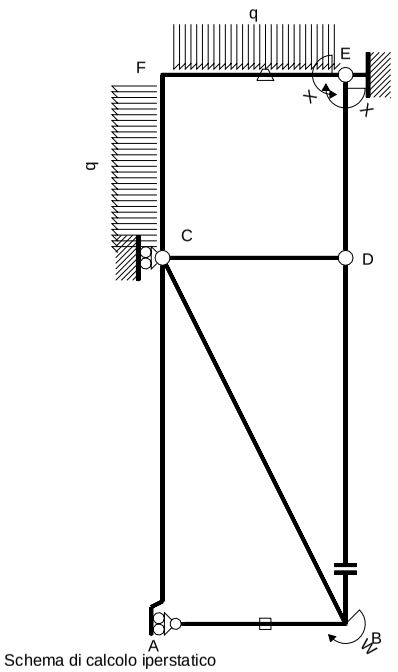

F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B

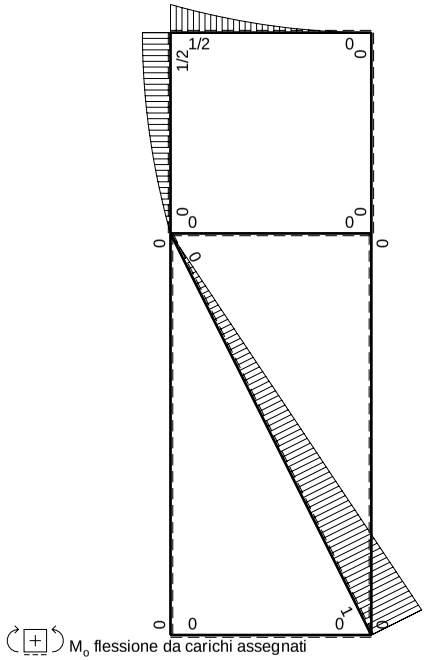

-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F

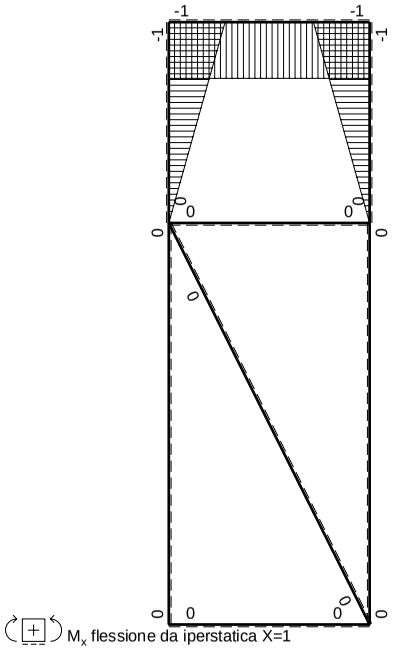

7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

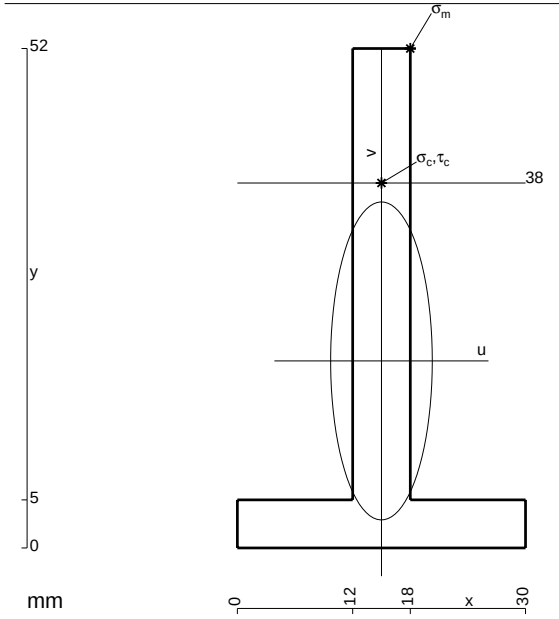
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 432. mm^2
J_u = 118416. mm^4
J_v = 12096. mm^4
y_g = 19.47 mm
N = -1118. N
T_y = -559. N
M_x = 825000. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 32.53 mm
$\sigma_m = N/A - Mv/J_u$ = -229.2 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 18.53 mm
$\sigma_c = N/A - Mv/J_u$ = -131.7 N/mm^2
τ_c = 1.687 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.7 N/mm^2
S^* = 2144. mm^3

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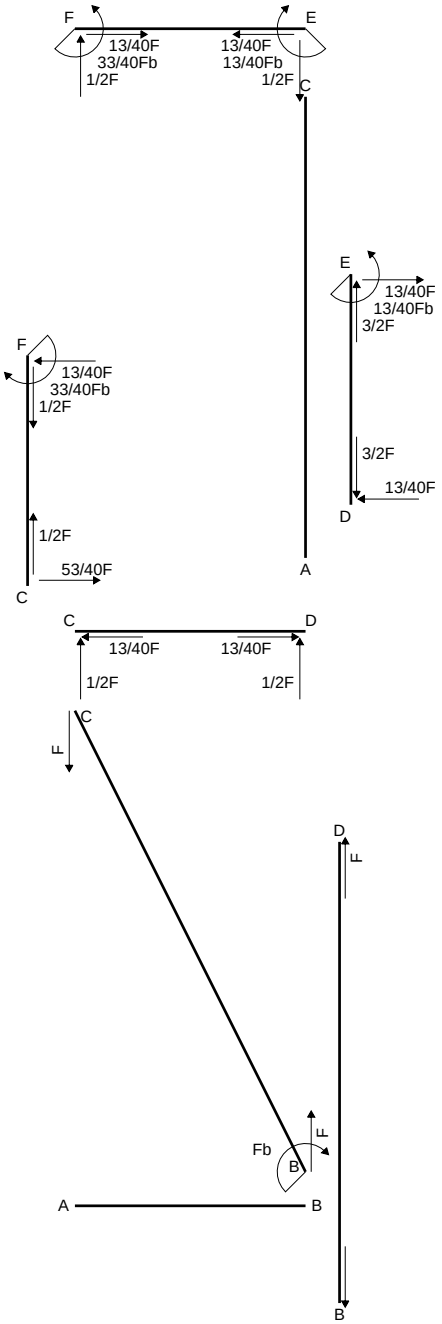
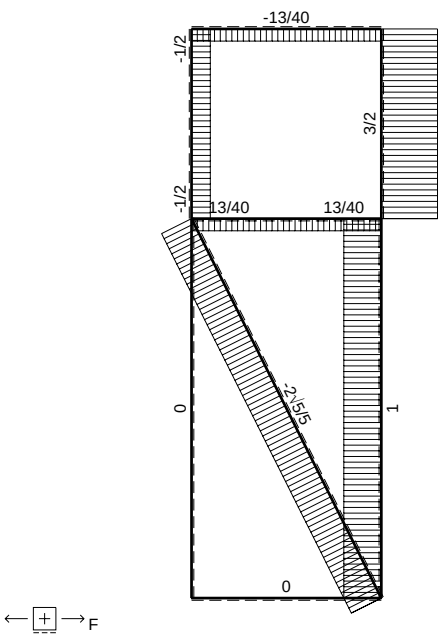
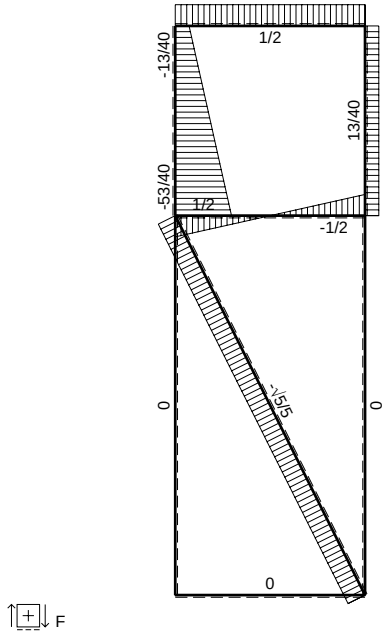
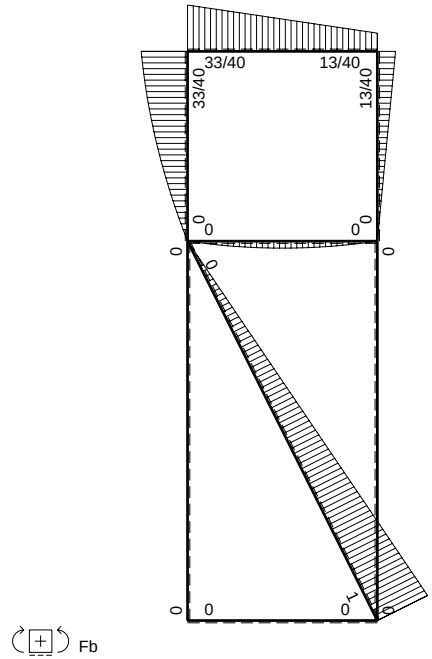

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F


-13/40
1/2
13/40
-53/40
1/2
-1/2
0
-√5/5
0
0
F


33/40
33/40
13/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210905 ESΣ09.xxxx.139

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14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

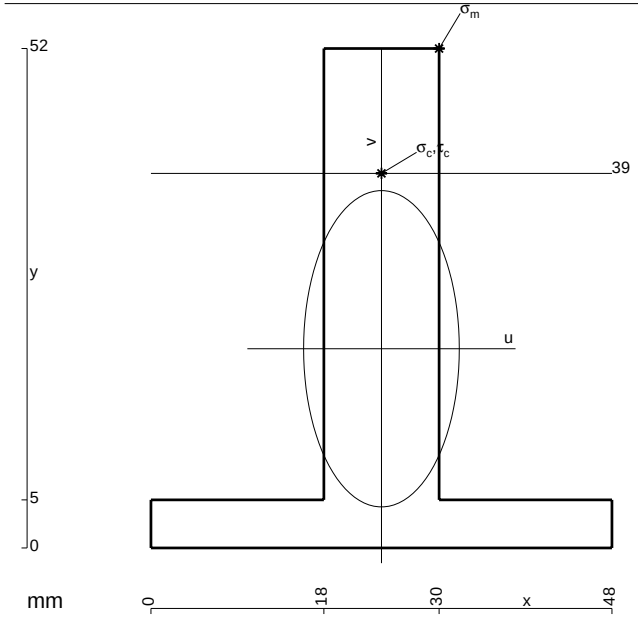
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210905 ESΣ09.xxxx.139



A = 804. mm^2
J_u = 218133. mm^4
J_v = 52848. mm^4
y_g = 20.74 mm
N = -2111. N
T_y = -1055. N
M_x = 1652000. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 31.26 mm
$\sigma_m = N/A - Mv/J_u$ = -239.4 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 18.26 mm
$\sigma_c = N/A - Mv/J_u$ = -140.9 N/mm^2
τ_c = 1.557 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.9 N/mm^2
S^* = 3863. mm^3

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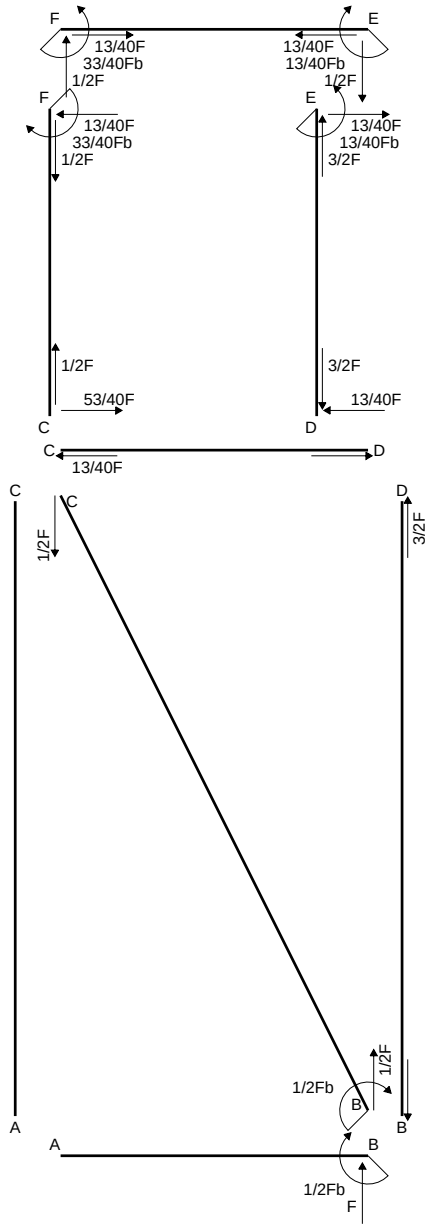
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
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1/2Fb
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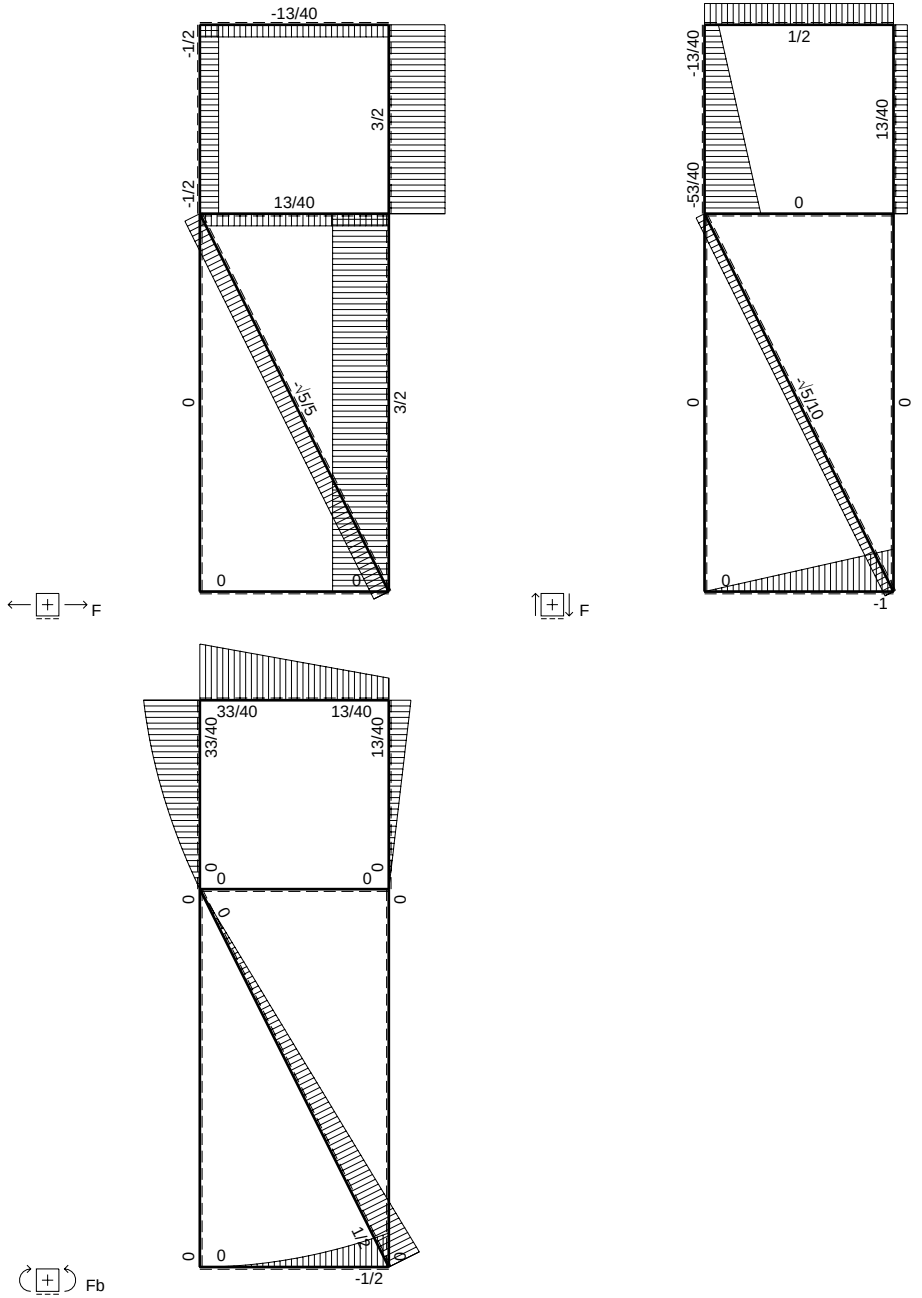
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212073 ESΣ09.xxxx.140

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

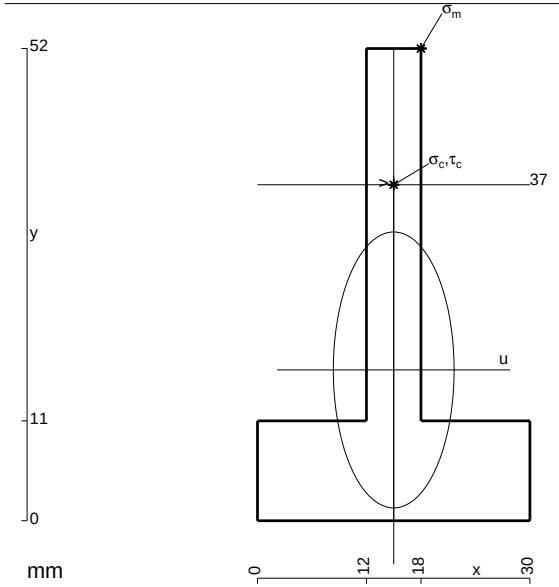
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 576. mm^2
J_u = 133062. mm^4
J_v = 25488. mm^4
y_g = 16.6 mm
T_y = -4060. N
M_x = -751100. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.4 mm
$\sigma_m = -Mv/J_u$ = 199.8 N/mm^2
x_c = 15. mm
y_c = 37. mm
v_c = 20.4 mm
$\sigma_c = -Mv/J_u$ = 115.1 N/mm^2
τ_c = 12.77 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.2 N/mm^2
S^* = 2511. mm^3

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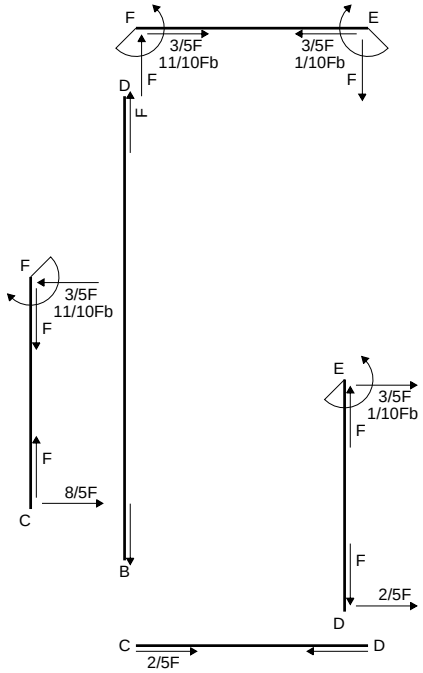
F
E
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11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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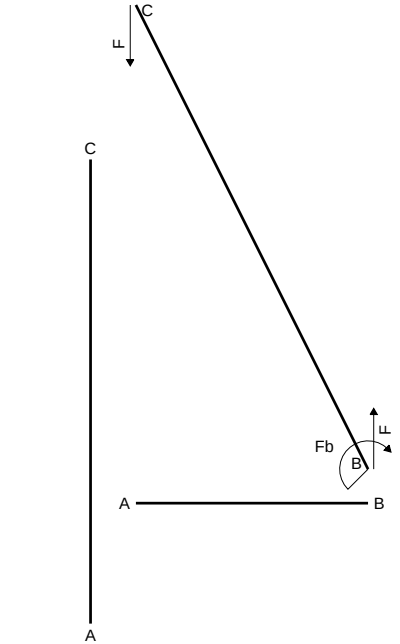
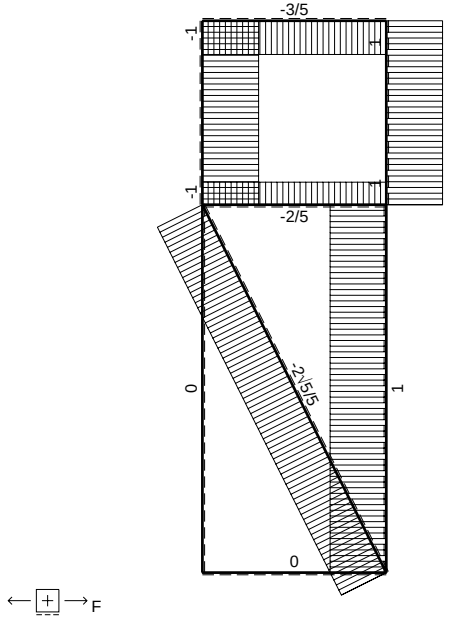
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A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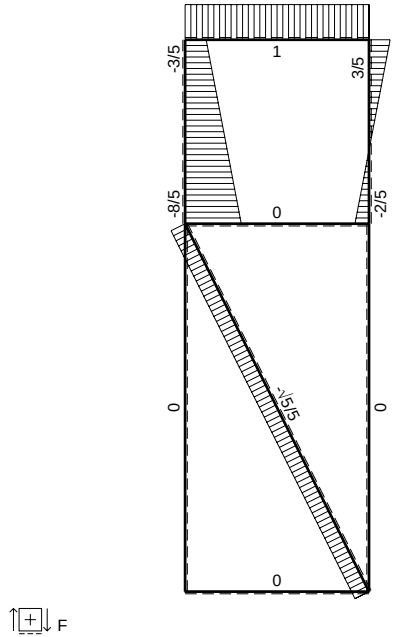
-1
1
-2/5
0
-2/5/5
1
0
F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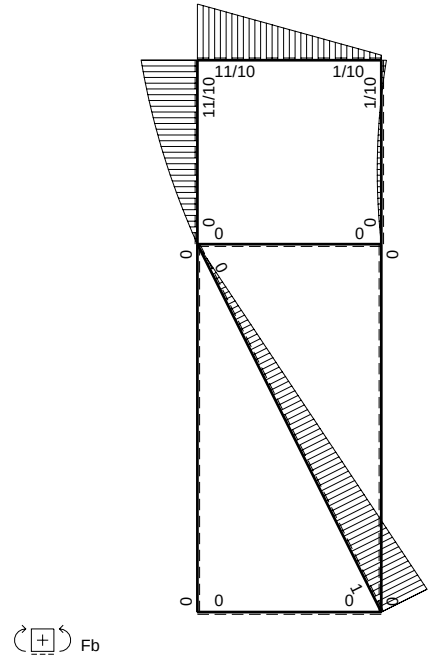
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
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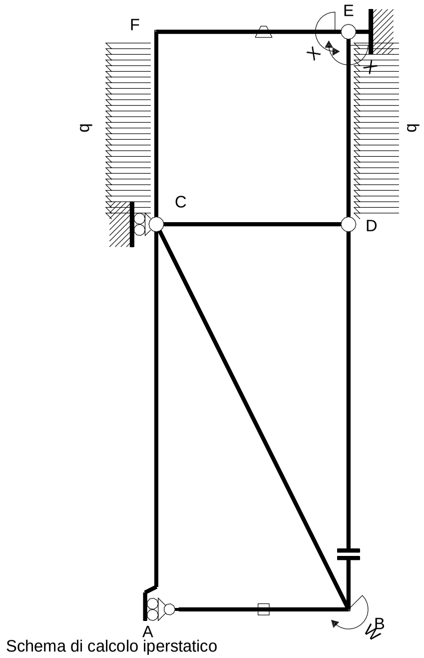
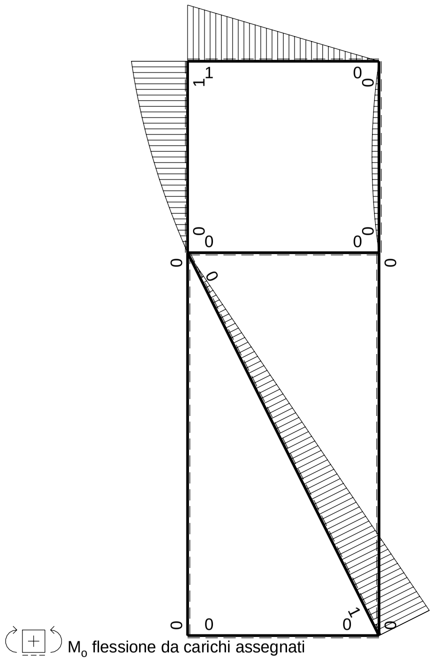
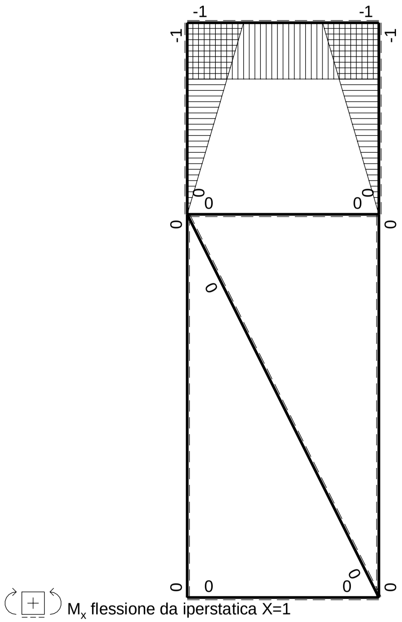
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

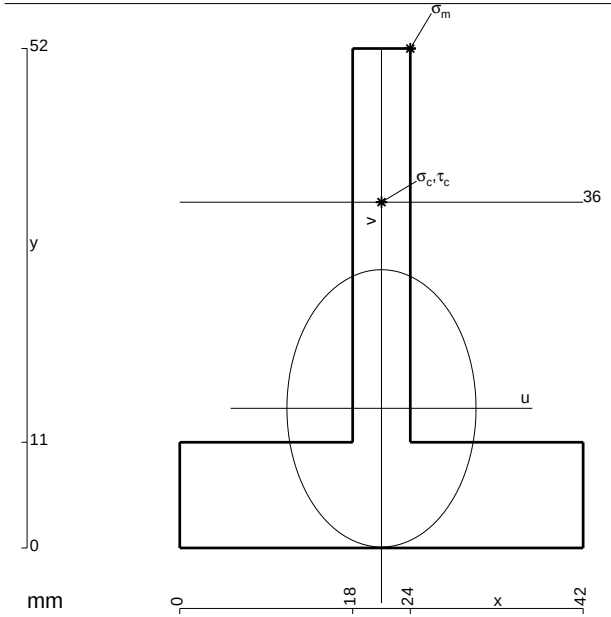
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 708. mm^2
J_u = 147634. mm^4
J_v = 68652. mm^4
y_g = 14.53 mm
N = -1825. N
T_y = -912.3 N
M_x = 816000. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 37.47 mm
$\sigma_m = N/A - Mv/J_u$ = -209.7 N/mm^2
x_c = 21. mm
y_c = 36. mm
v_c = 21.47 mm
$\sigma_c = N/A - Mv/J_u$ = -121.2 N/mm^2
τ_c = 2.913 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 121.3 N/mm^2
S^* = 2829. mm^3

ESΣ09.xxxx.141

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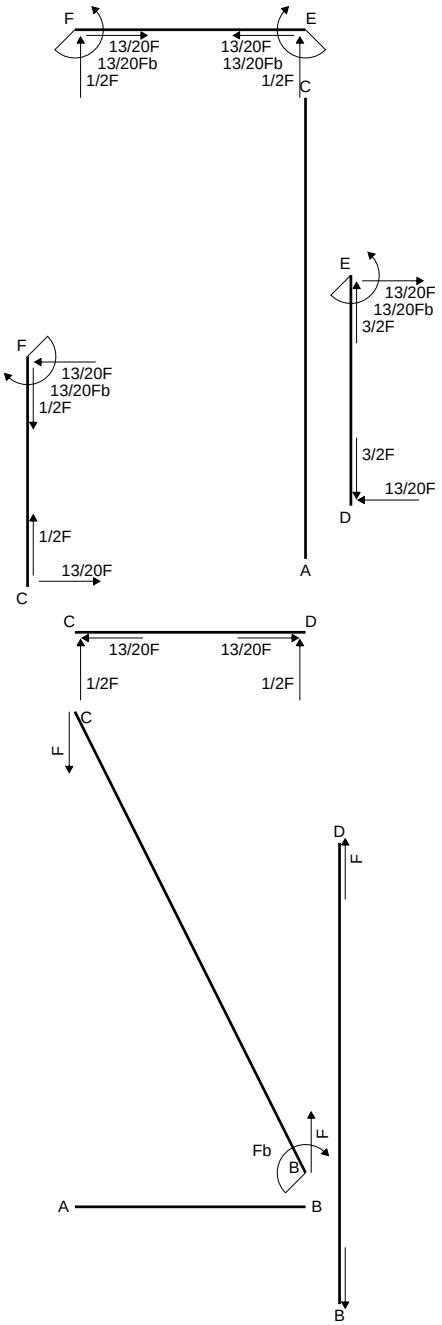
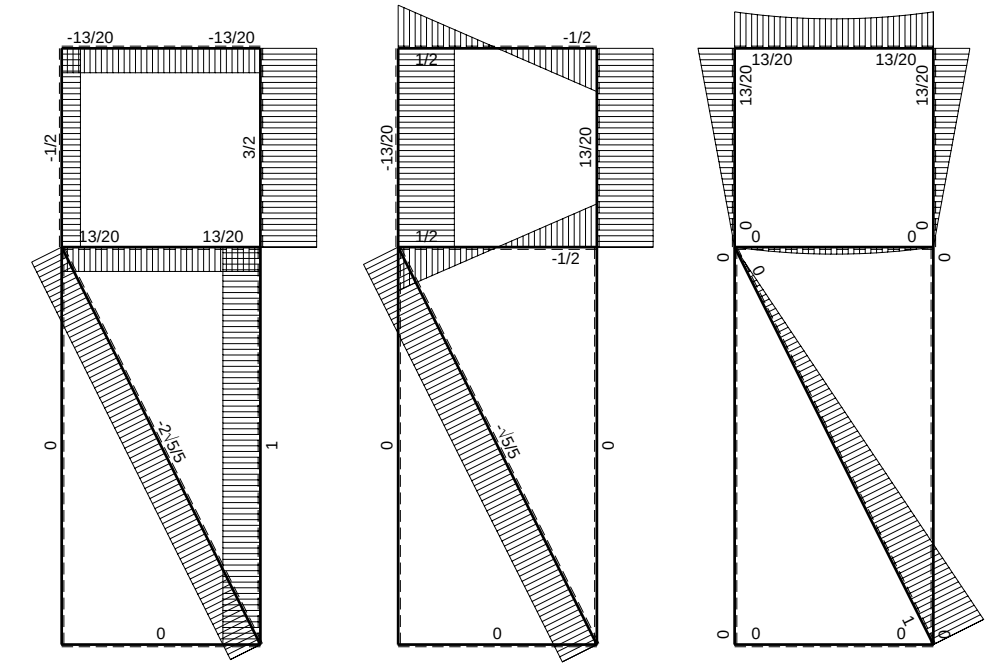

F
E
13/20F
13/20Fb
1/2F
C
E
13/20F
13/20Fb
3/2F
3/2F
13/20F
D
F
13/20F
13/20Fb
1/2F
1/2F
13/20F
C
A
C
D
13/20F
13/20F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-13/20
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3/2
13/20
13/20
-2√5/5
1
0
0
1/2
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13/20
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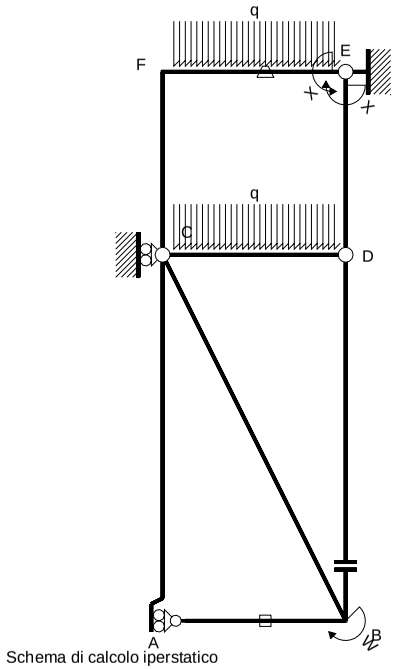
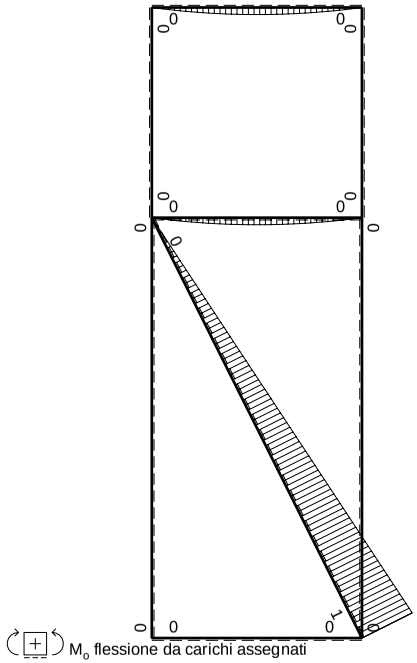
1
0

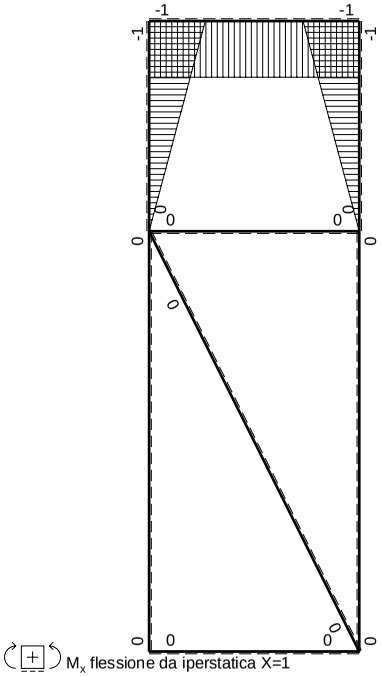

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982088 ESΣ09.xxxx.142

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

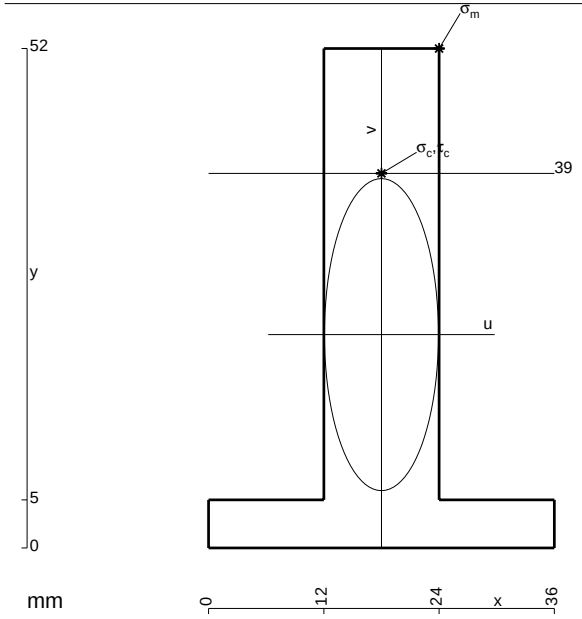
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 982088 ESΣ09.xxxx.142



A = 744. mm^2
J_u = 196439. mm^4
J_v = 26208. mm^4
y_g = 22.21 mm
N = -2889. N
T_y = -1445. N
M_x = 1421200. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 29.79 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 16.79 mm
$\sigma_c = N/A - Mv/J_u$ = -125.4 N/mm^2
τ_c = 2.226 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.4 N/mm^2
S^* = 3633. mm^3

ESΣ09.xxxx.142

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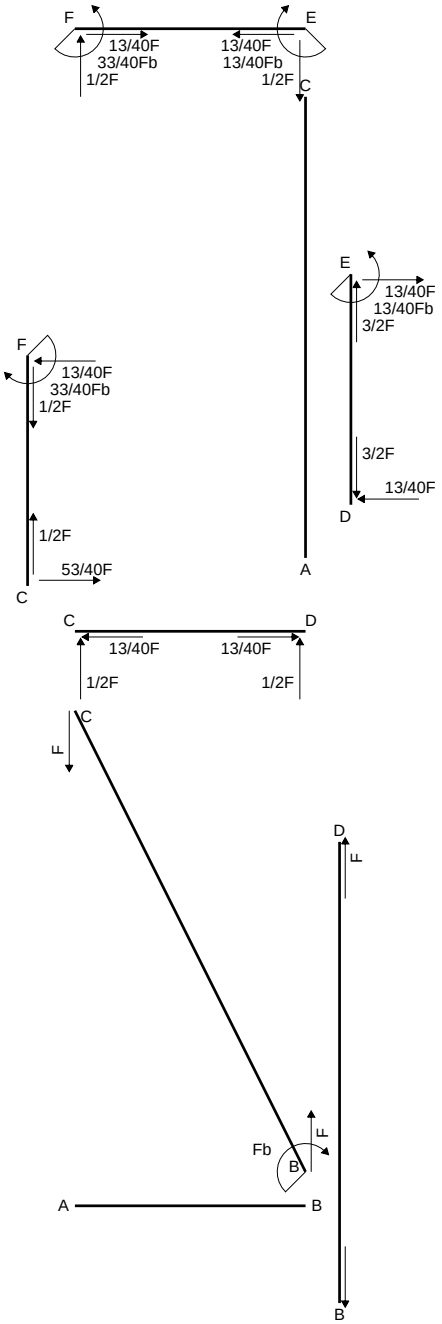
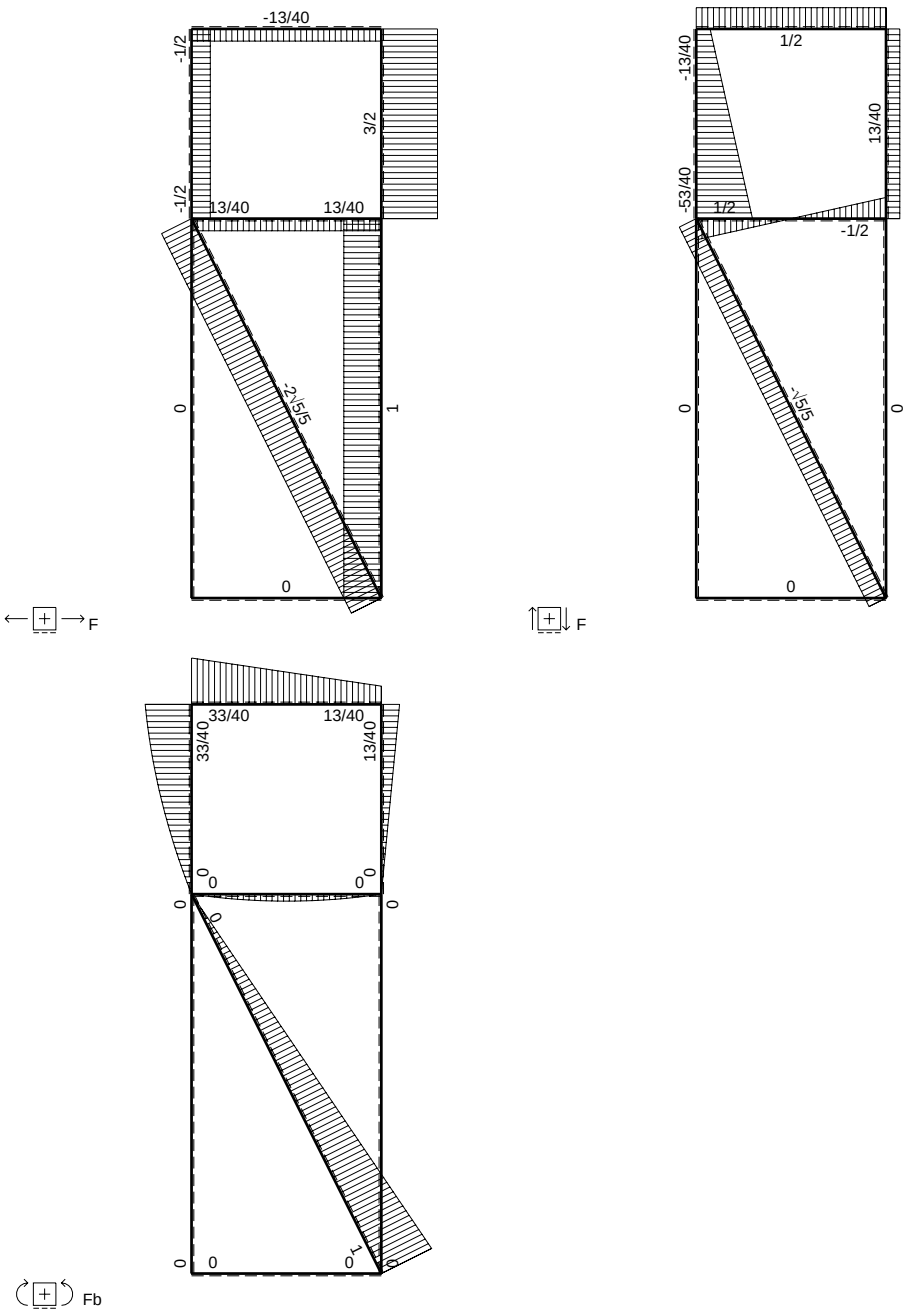

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
F
13/40F
33/40Fb
1/2F
3/2F
13/40F
D
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XkJ/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	XbJ/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XkJ/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XkJ/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213310 ESΣ09.xxxx.143

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

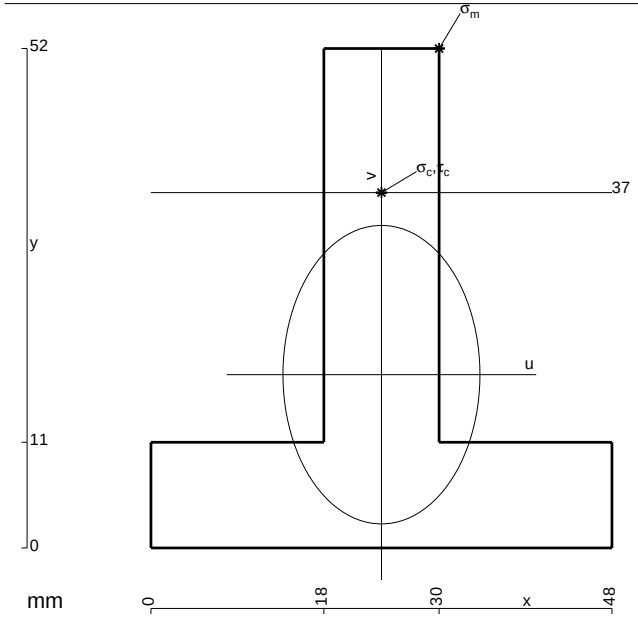
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213310 ESΣ09.xxxx.143



A = 1020. mm^2
J_u = 246410. mm^4
J_v = 107280. mm^4
y_g = 18.04 mm
N = -3068. N
T_y = -1534. N
M_x = 1646400. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 33.96 mm
$\sigma_m = N/A - Mv/J_u$ = -229.9 N/mm^2
x_c = 24. mm
y_c = 37. mm
v_c = 18.96 mm
$\sigma_c = N/A - Mv/J_u$ = -129.7 N/mm^2
τ_c = 2.471 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.8 N/mm^2
S^* = 4763. mm^3

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ESΣ09.xxxx.143

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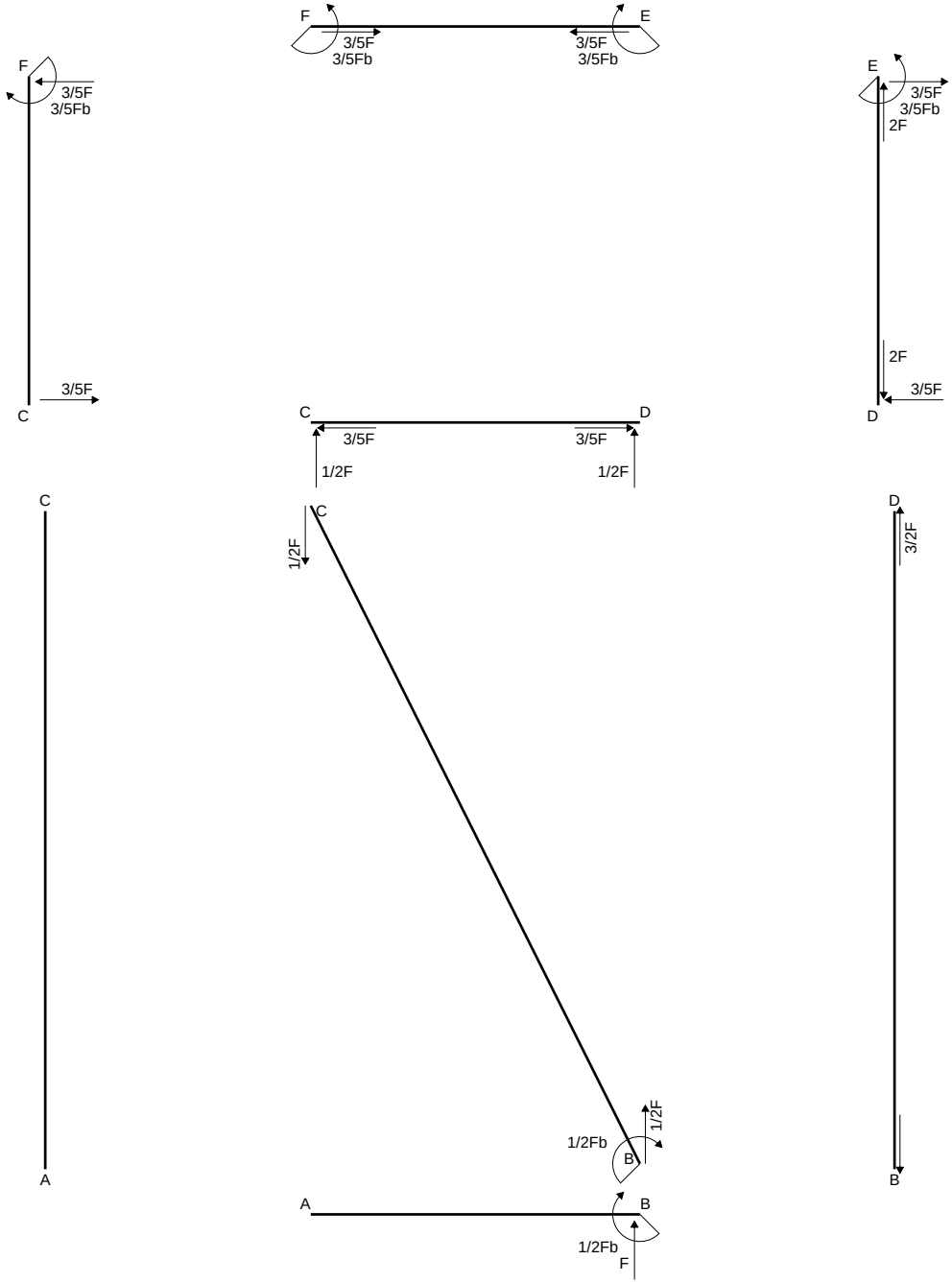
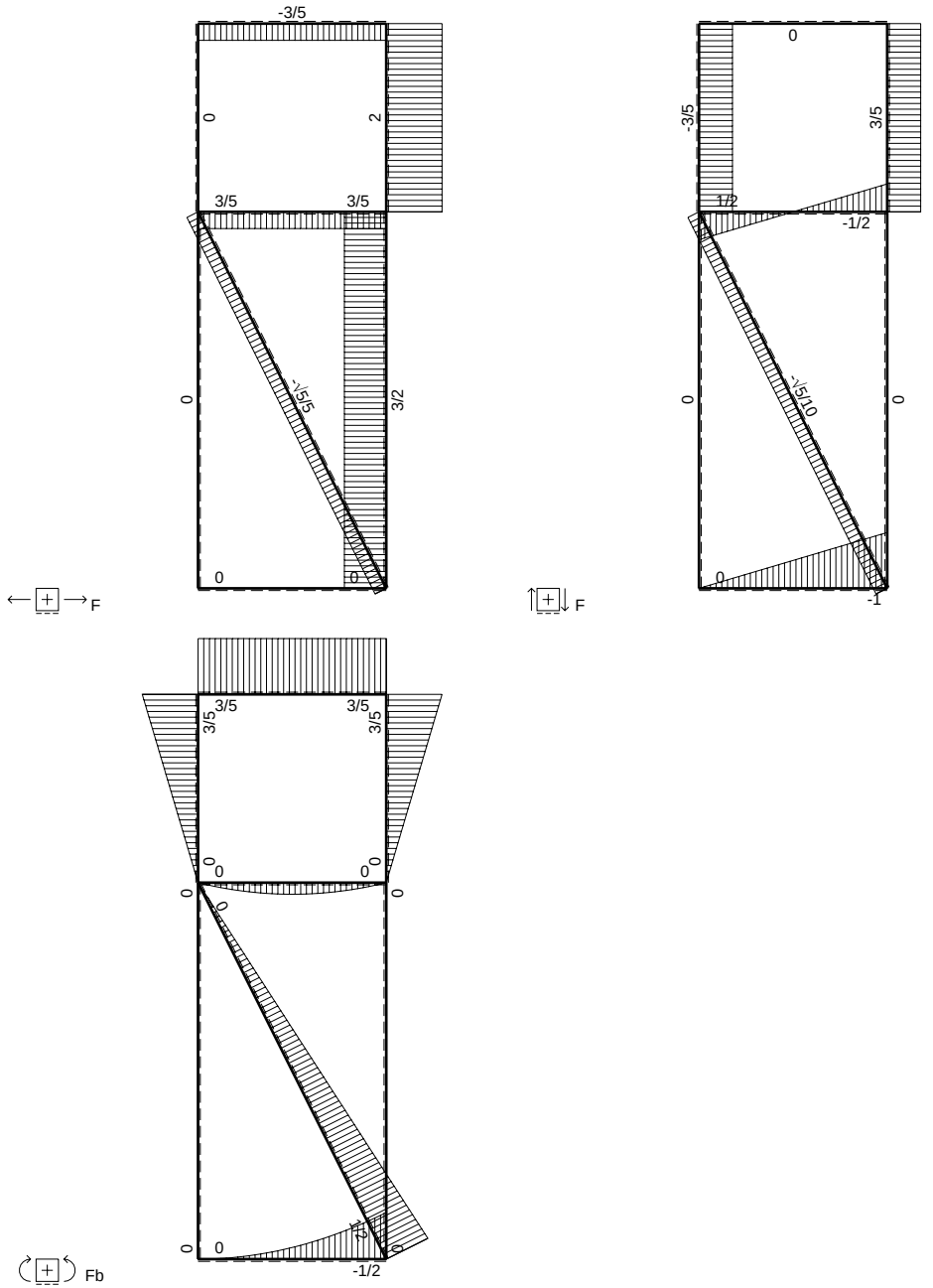

F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F


-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
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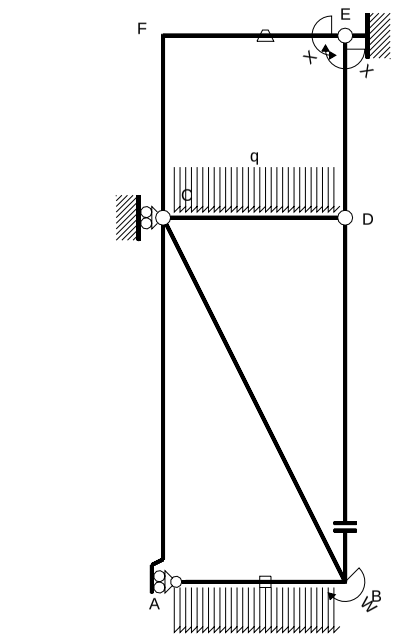
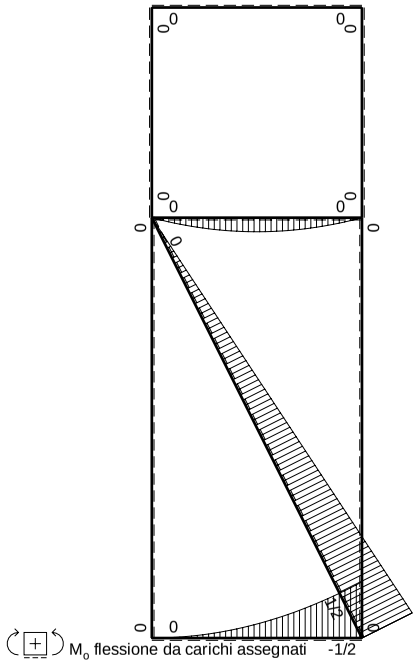
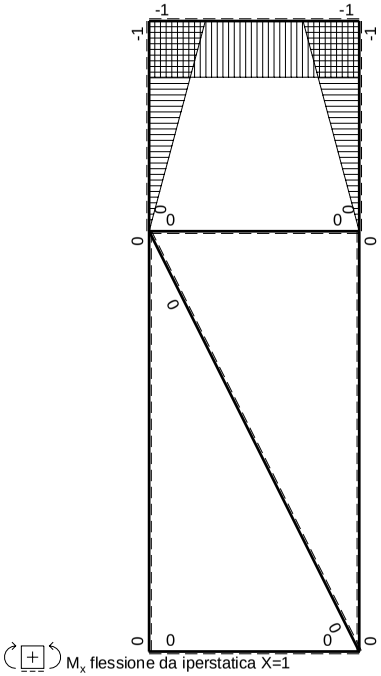
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

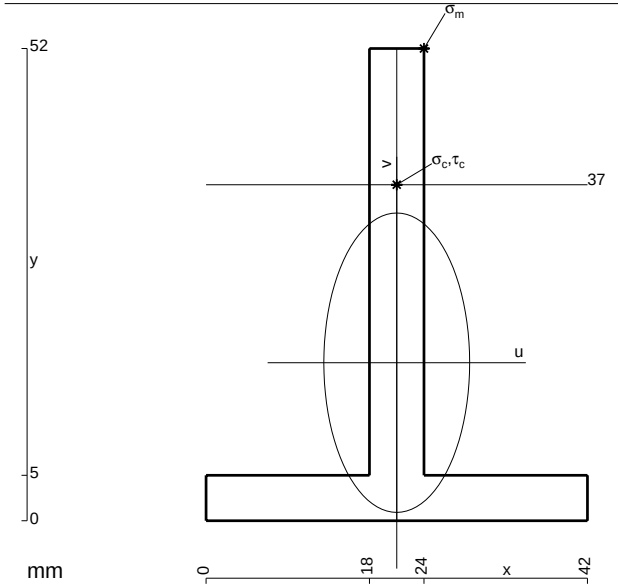
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 492. mm^2
J_u = 133716. mm^4
J_v = 31716. mm^4
y_g = 17.4 mm
T_y = -3630. N
M_x = -925650. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 34.6 mm
$\sigma_m = -Mv/J_u$ = 239.5 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 19.6 mm
$\sigma_c = -Mv/J_u$ = 135.7 N/mm^2
τ_c = 11.03 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137. N/mm^2
S^* = 2439. mm^3

ESΣ09.xxxx.144

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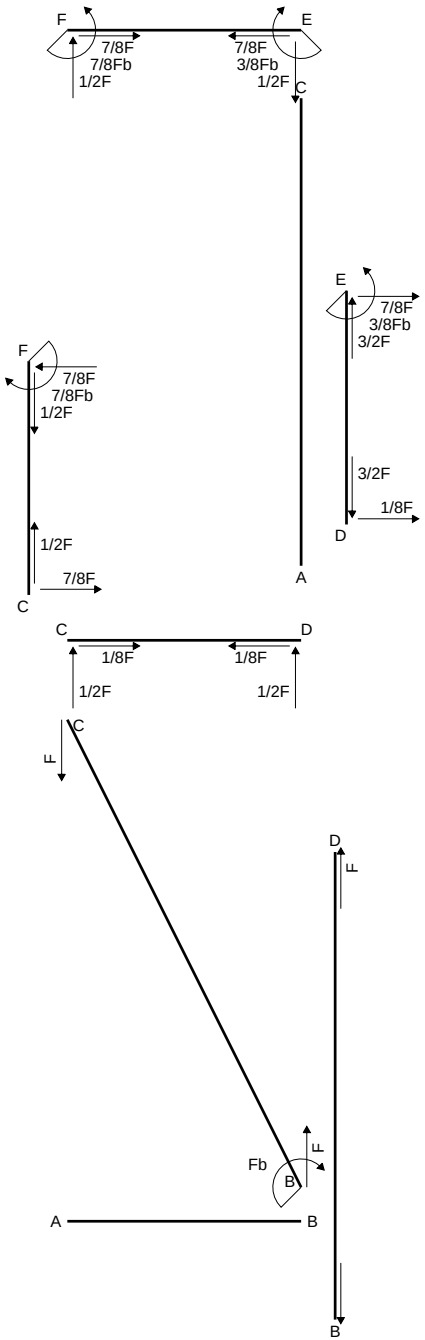
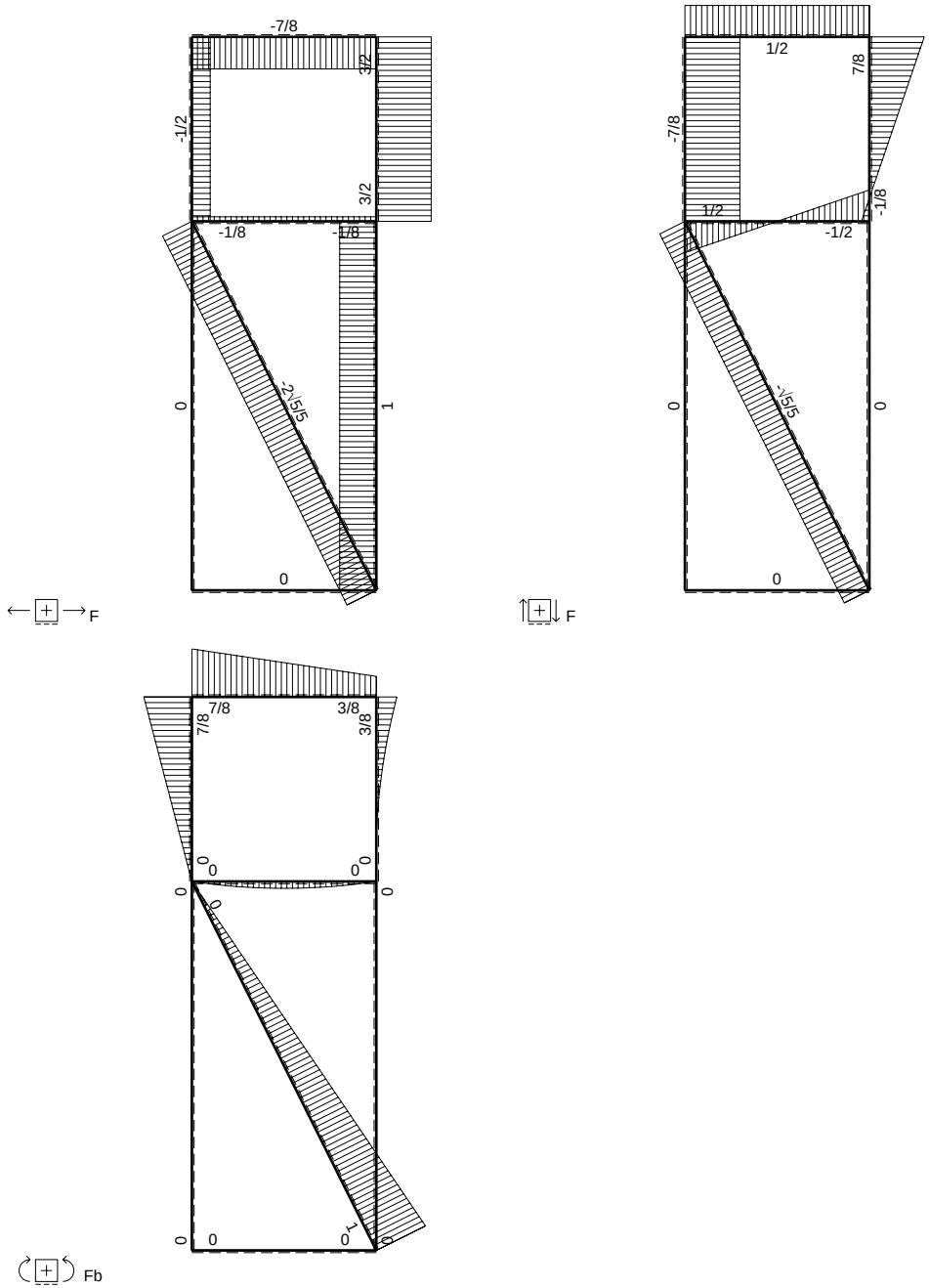

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210204 ESΣ09.xxxx.145

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

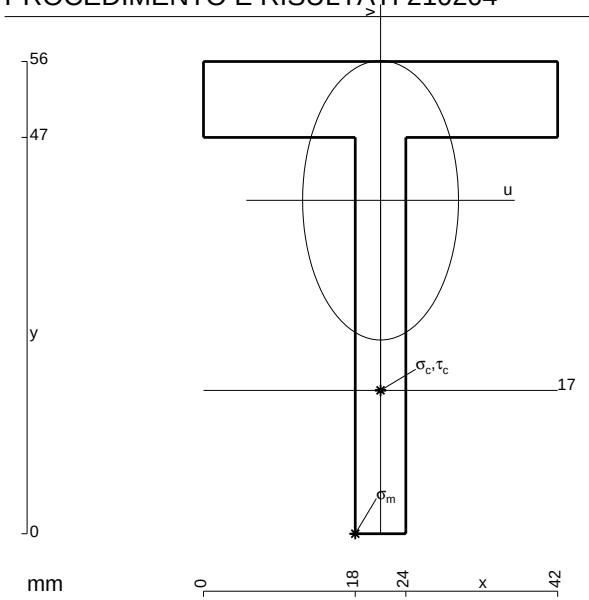
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 660. mm^2
J_u = 181086. mm^4
J_v = 56412. mm^4
y_g = 39.54 mm
N = -1422. N
T_y = -711.1 N
M_x = 922200. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.54 mm
$\sigma_m = N/A - Mv/J_u$ = 199.2 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.54 mm
$\sigma_c = N/A - Mv/J_u$ = 112.6 N/mm^2
τ_c = 2.072 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 112.7 N/mm^2
S^* = 3166. mm^3

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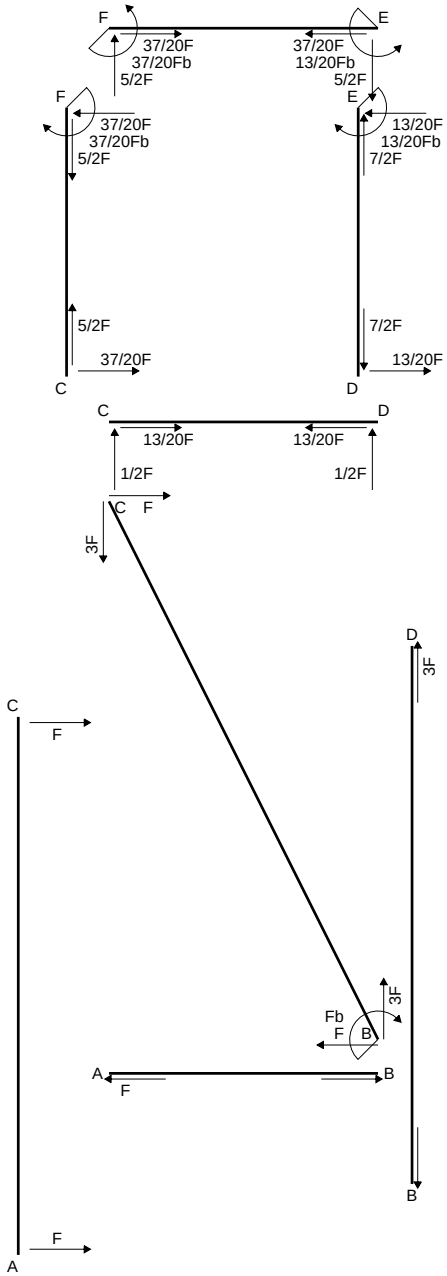
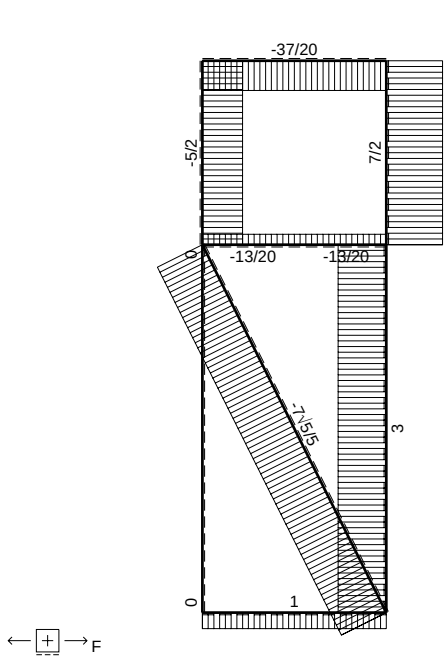
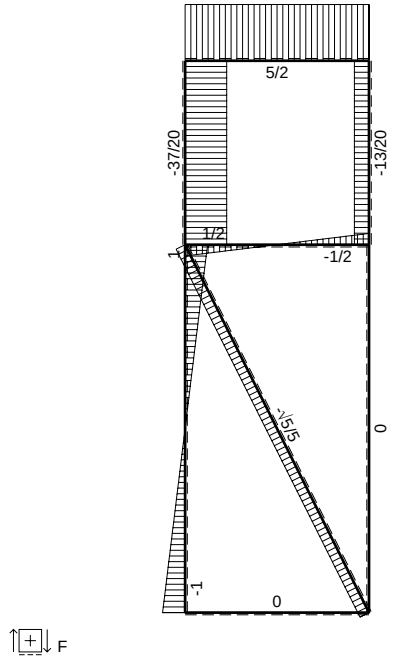
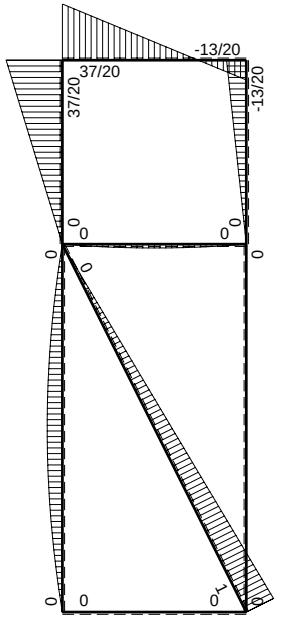
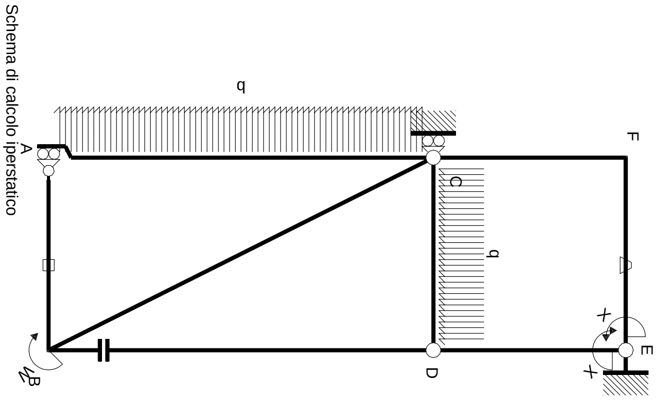

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B

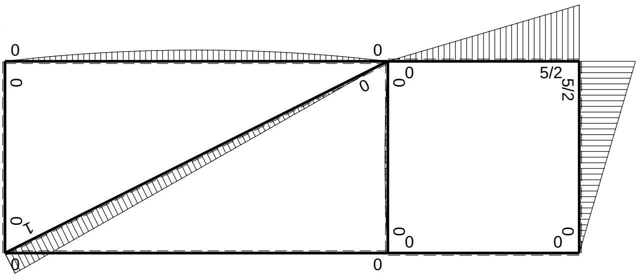

-37/20
-5/2
7/2
-13/20
-13/20
0
-7.5/5
3
0
1
F


5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F

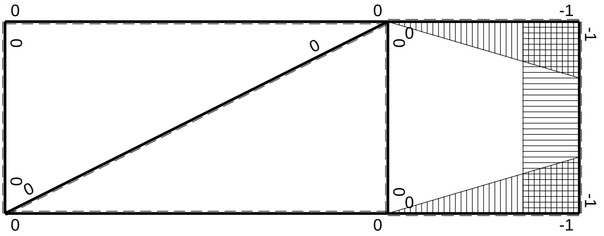

-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{EF}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica X=W_{EF}						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

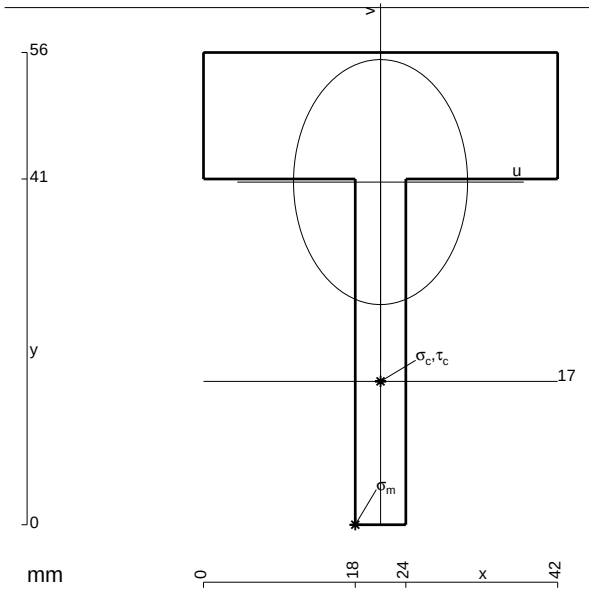
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 876. mm^2
J_u = 184977. mm^4
J_v = 93348. mm^4
y_g = 40.64 mm
N = -4946. N
T_y = -706.6 N
M_x = 979600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -40.64 mm
$\sigma_m = N/A - Mv/J_u$ = 209.6 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -23.64 mm
$\sigma_c = N/A - Mv/J_u$ = 119.5 N/mm^2
τ_c = 2.087 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.6 N/mm^2
S^* = 3278. mm^3

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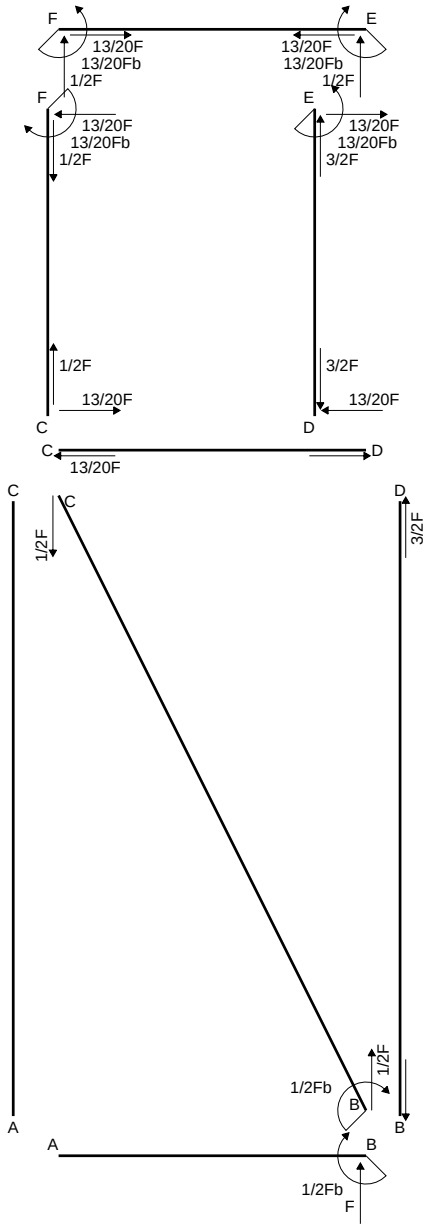
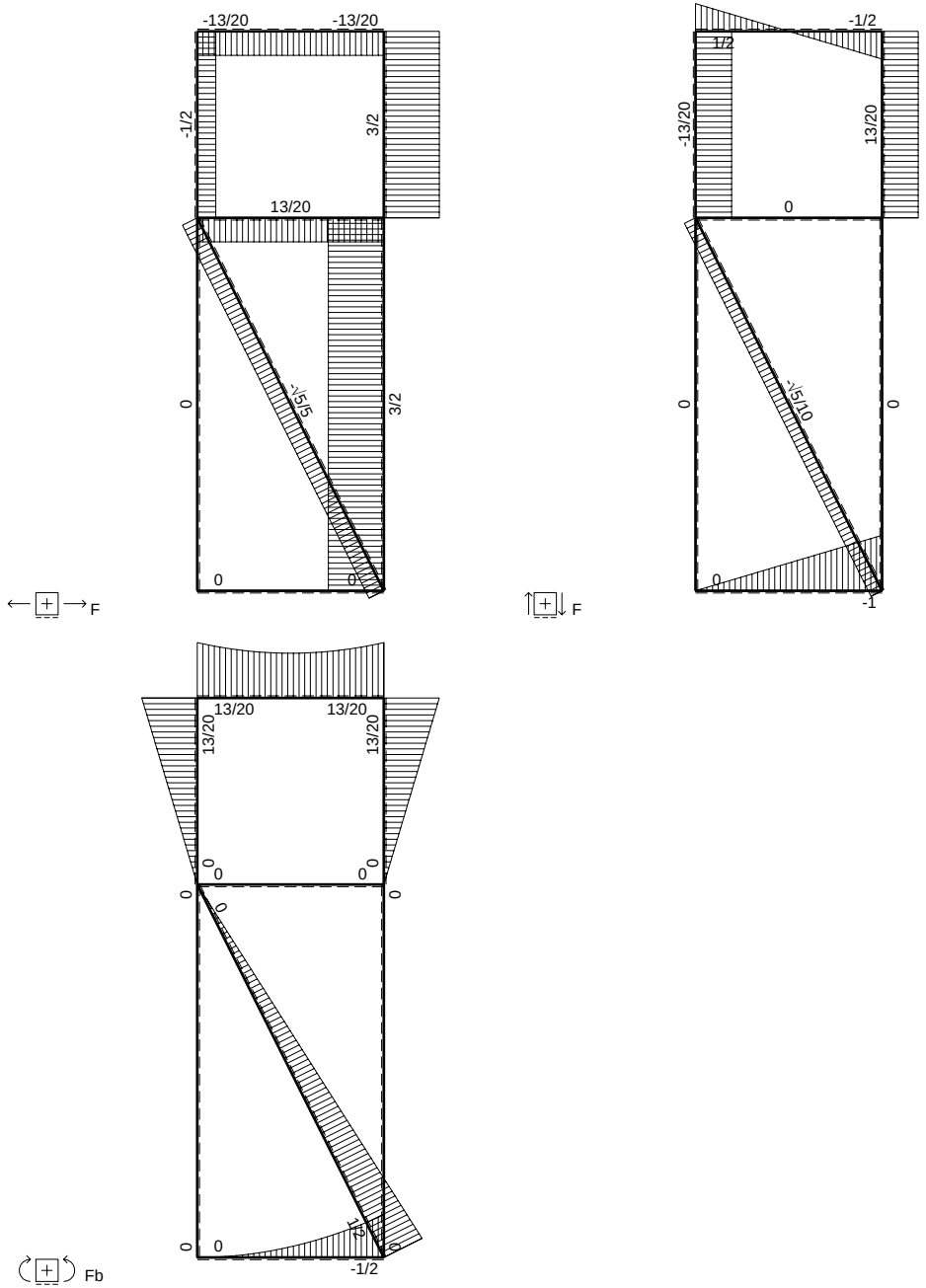
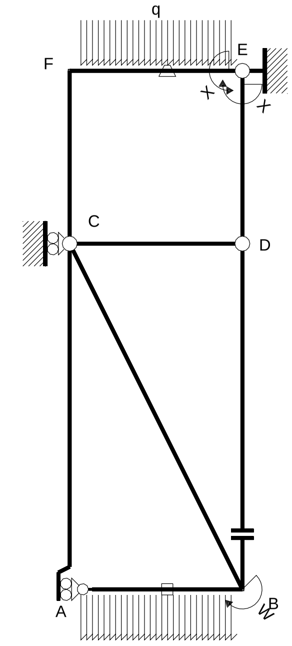
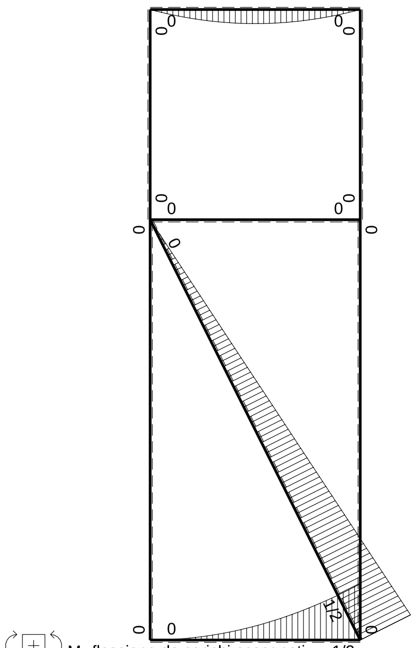

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

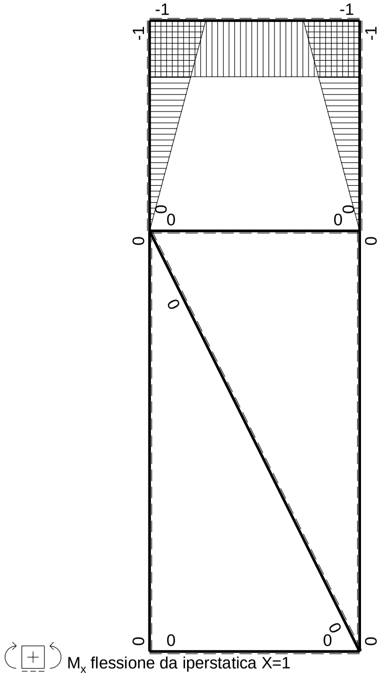

-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
1/2
-1
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

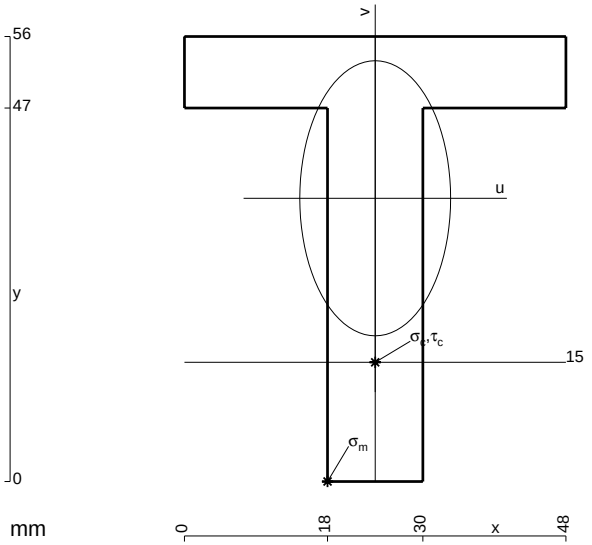
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 981630 ESΣ09.xxxx.147



A = 996. mm^2
J_u = 298526. mm^4
J_v = 89712. mm^4
y_g = 35.64 mm
T_y = -5580. N
M_x = -1841400. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.64 mm
$\sigma_m = -Mv/J_u$ = -219.9 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.64 mm
$\sigma_c = -Mv/J_u$ = -127.3 N/mm^2
τ_c = 7.891 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.1 N/mm^2
S^* = 5066. mm^3

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ESΣ09.xxxx.147

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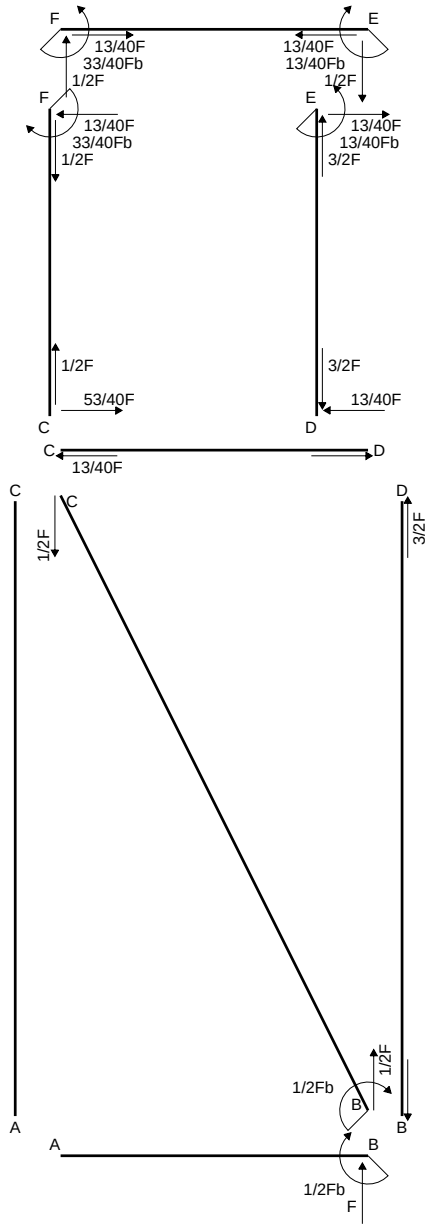
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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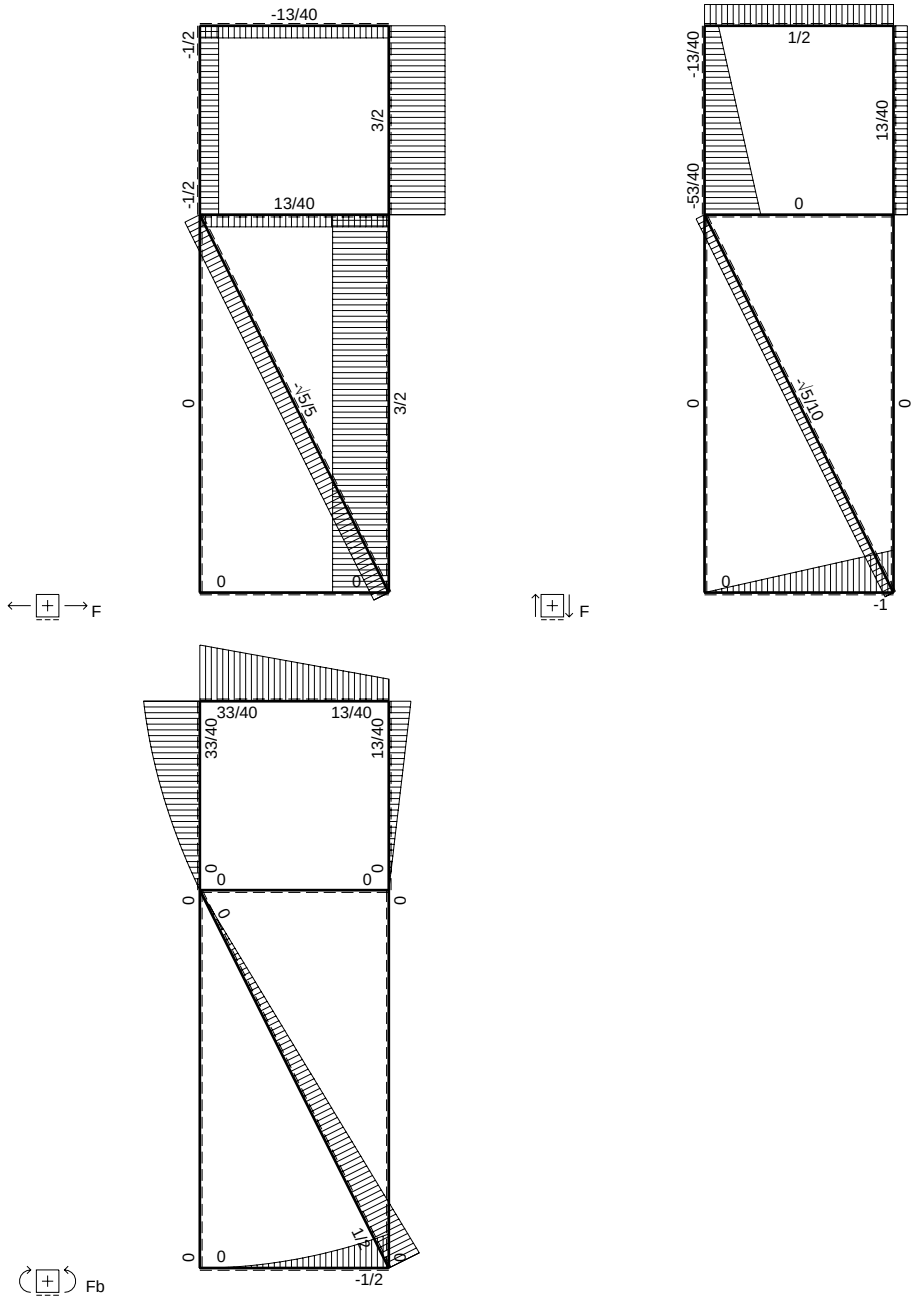
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_X(x)$	$M_o(x)$	θ	M_XM_o	$M_X\theta$	M_XM_X	$\int M_X(M_o/EJ+\theta)dx$	$\int XM_XM_X/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 983430 ESΣ09.xxxx.148

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

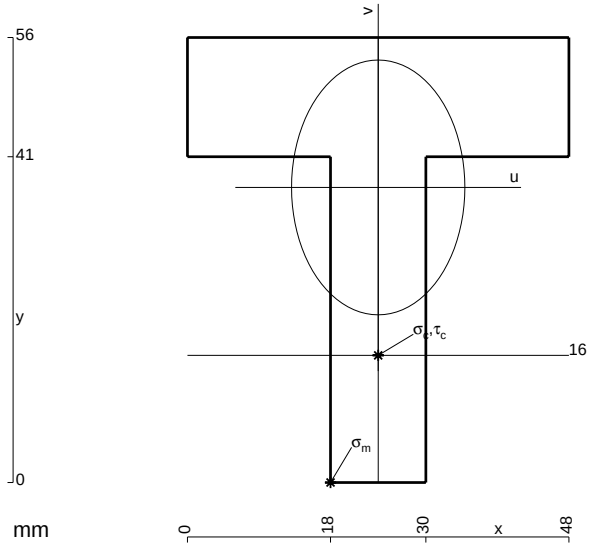
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 983430 ESΣ09.xxxx.148



A = 1212. mm^2
J_u = 311566. mm^4
J_v = 144144. mm^4
y_g = 37.13 mm
T_y = -5510. N
M_x = -1928500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -37.13 mm
$\sigma_m = -Mv/J_u$ = -229.8 N/mm^2
x_c = 24. mm
y_c = 16. mm
v_c = -21.13 mm
$\sigma_c = -Mv/J_u$ = -130.8 N/mm^2
τ_c = 8.244 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.6 N/mm^2
S^* = 5594. mm^3

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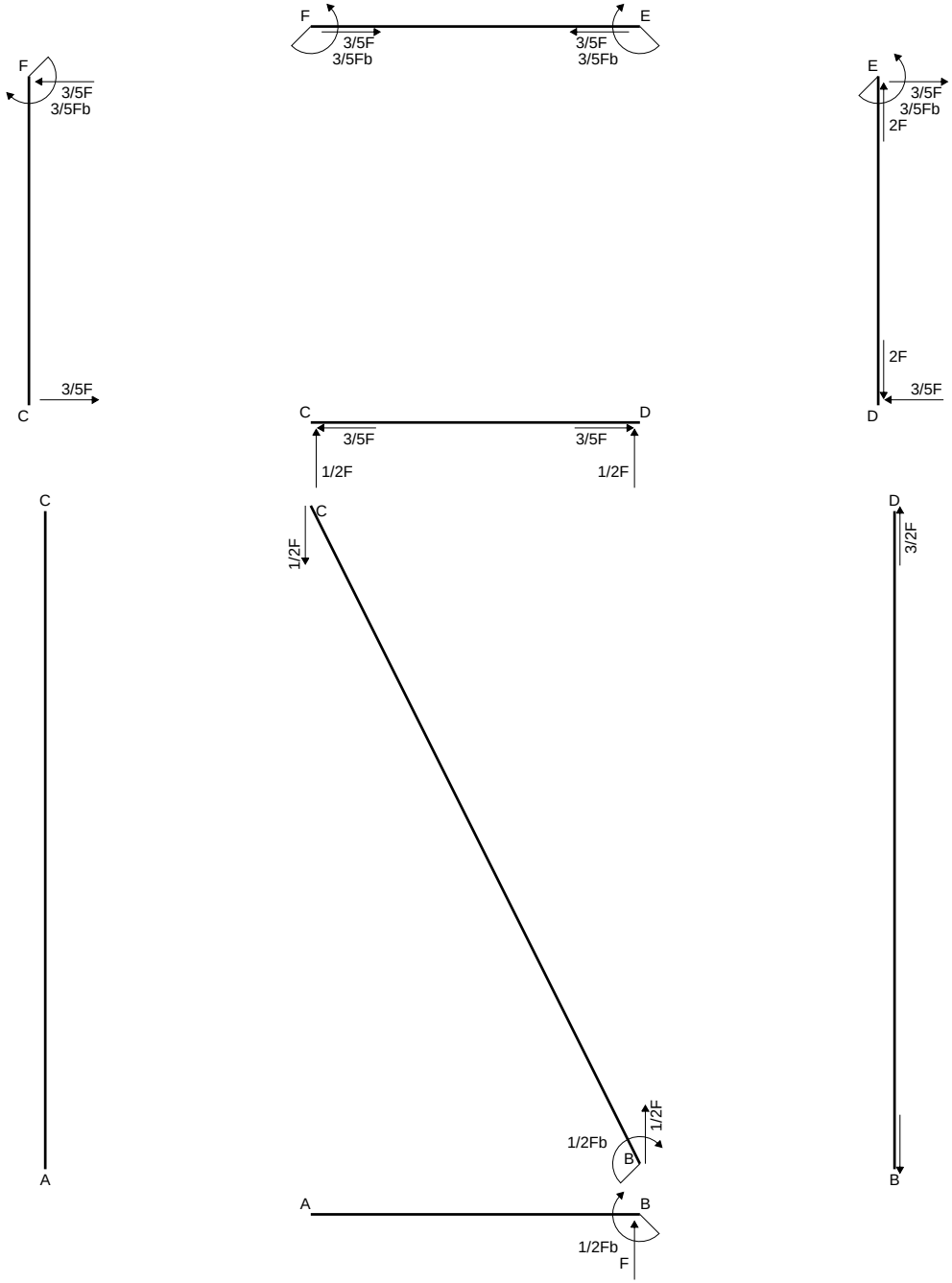
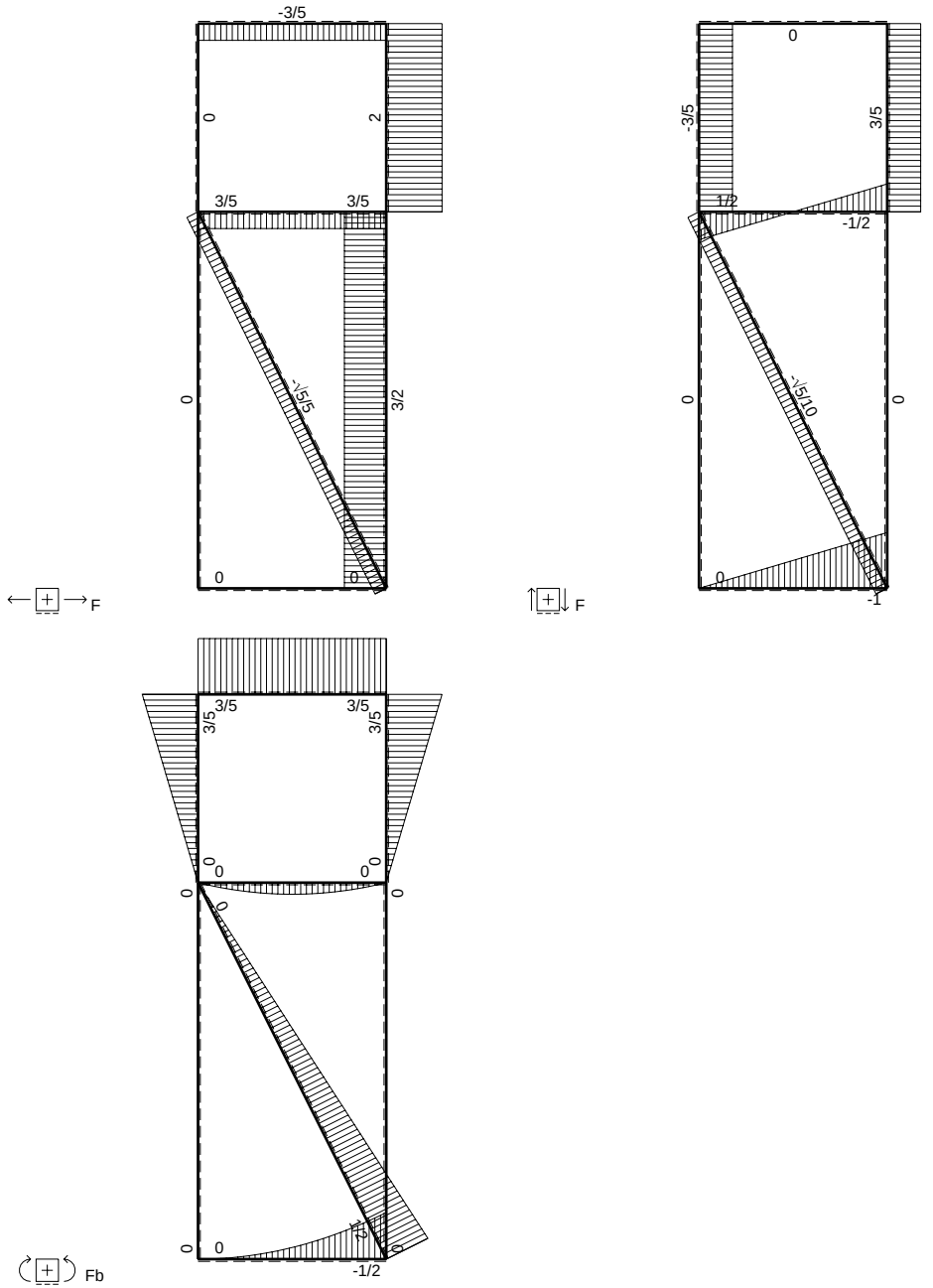

F
E
3/5F
3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
3/5Fb
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F


-3/5
0
2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
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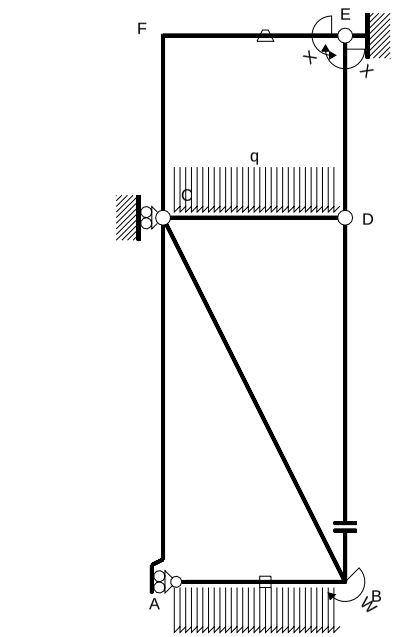
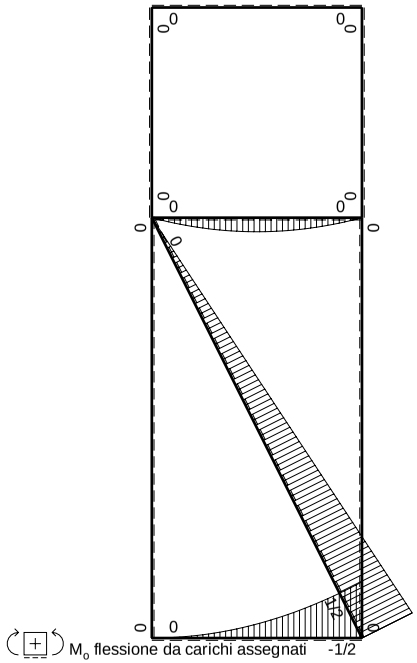
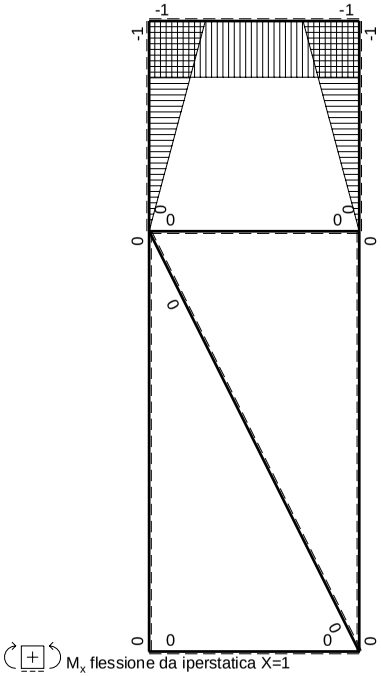
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

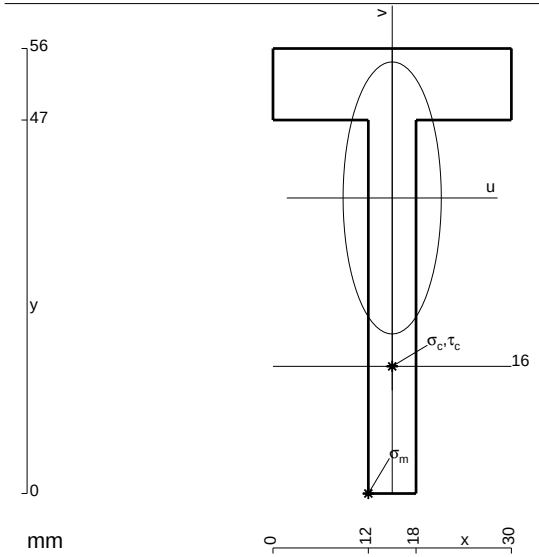
$L_{EF}^{Xo} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 935514 ESΣ09.xxxx.149



A = 552. mm^2
J_u = 161875. mm^4
J_v = 21096. mm^4
y_g = 37.2 mm
T_y = -2820. N
M_x = -1043400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.2 mm
$\sigma_m = -Mv/J_u$ = -239.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.2 mm
$\sigma_c = -Mv/J_u$ = -136.6 N/mm^2
τ_c = 8.138 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 137.3 N/mm^2
S^* = 2803. mm^3

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ESΣ09.xxxx.149

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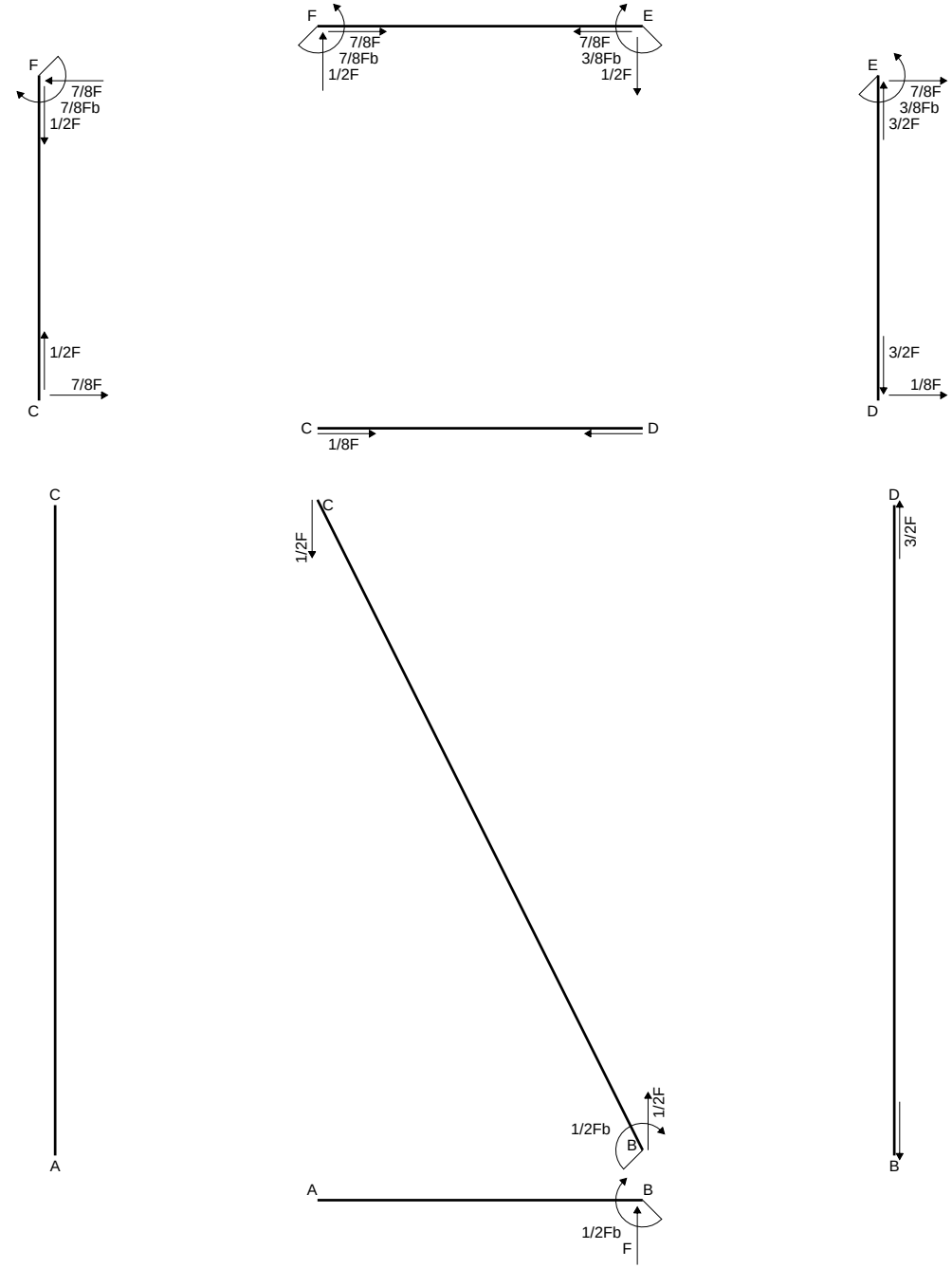
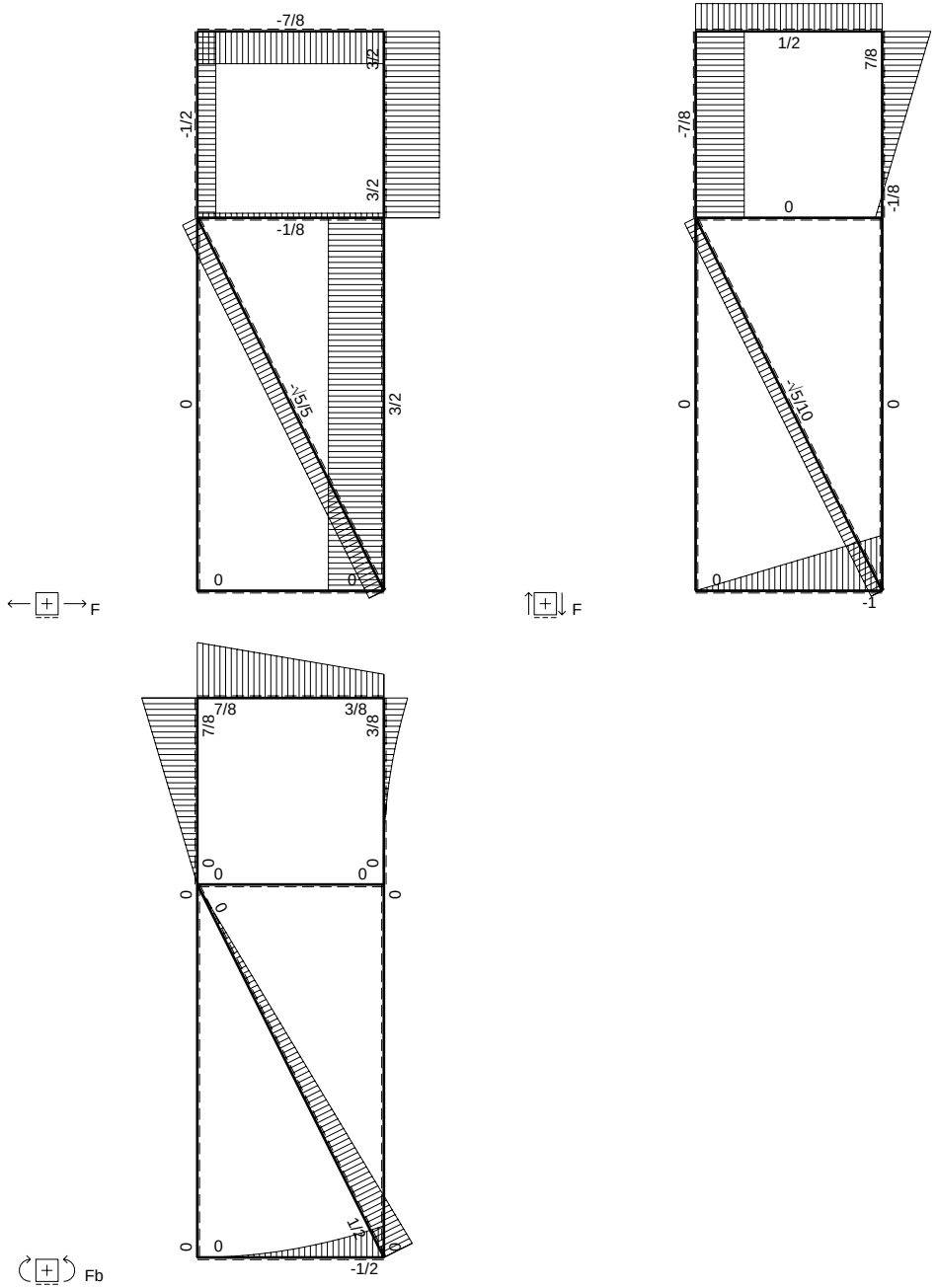

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
1/2
7/8
-1/8
-√5/10
-1
F
7/8
3/8
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

PROCEDIMENTO E RISULTATI 245588 ESΣ09.xxxx.150

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

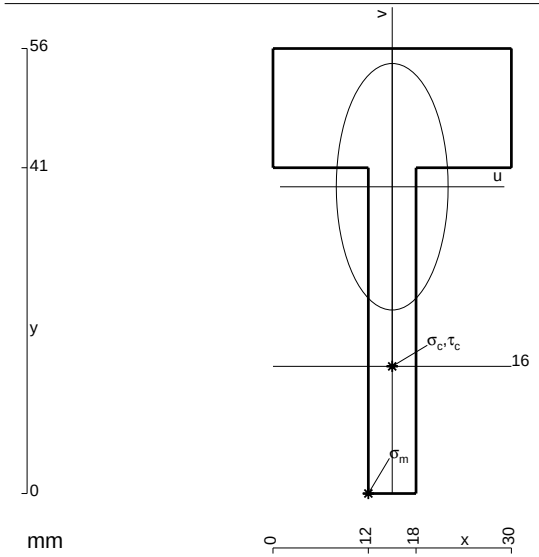
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 696. mm^2
J_u = 167595. mm^4
J_v = 34488. mm^4
y_g = 38.6 mm
T_y = -4450. N
M_x = -867750. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -38.6 mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -22.6 mm
$\sigma_c = -Mv/J_u$ = -117. N/mm^2
τ_c = 13. N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 119.2 N/mm^2
S^* = 2938. mm^3

ESΣ09.xxxx.150

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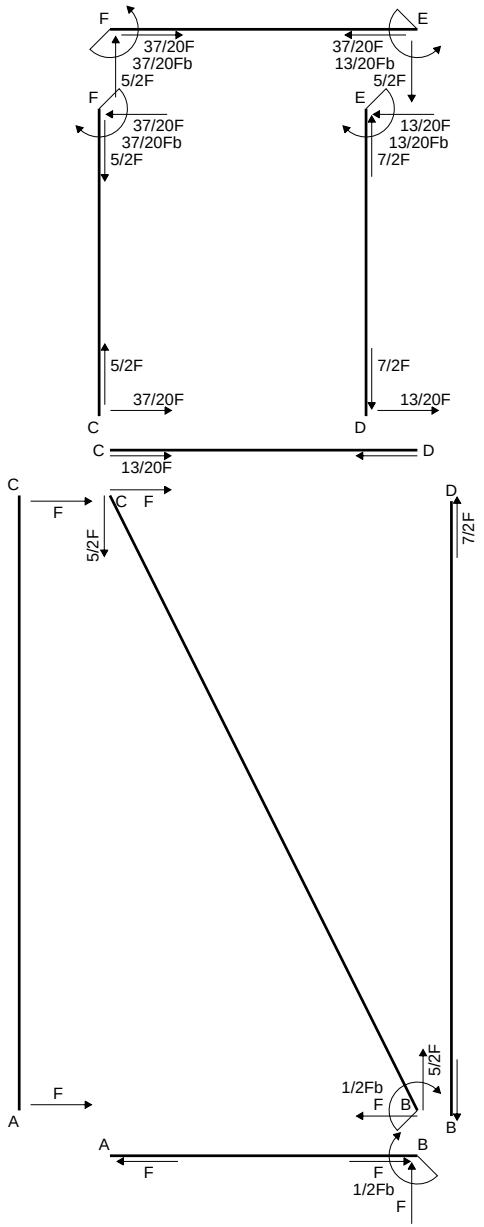
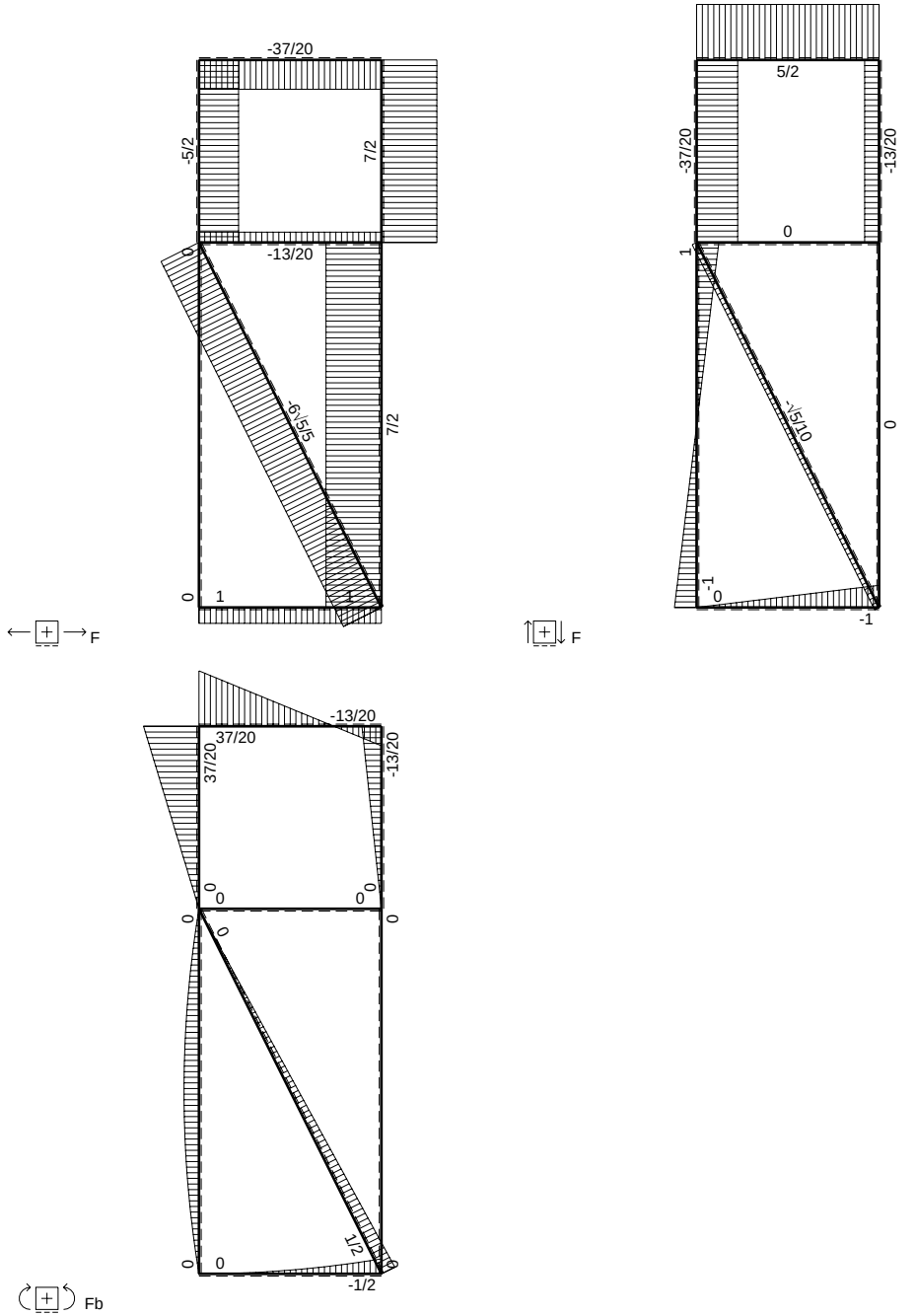
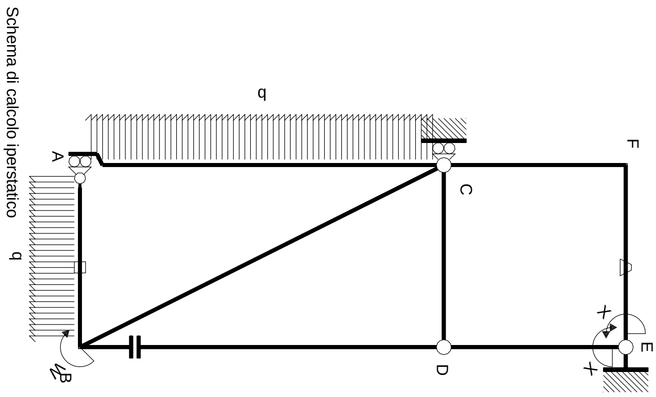
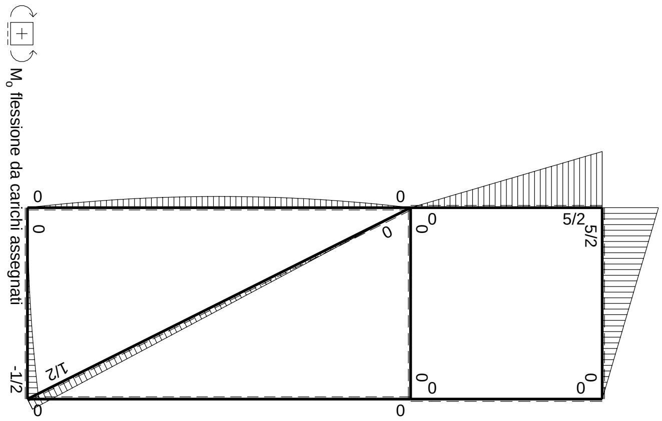

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

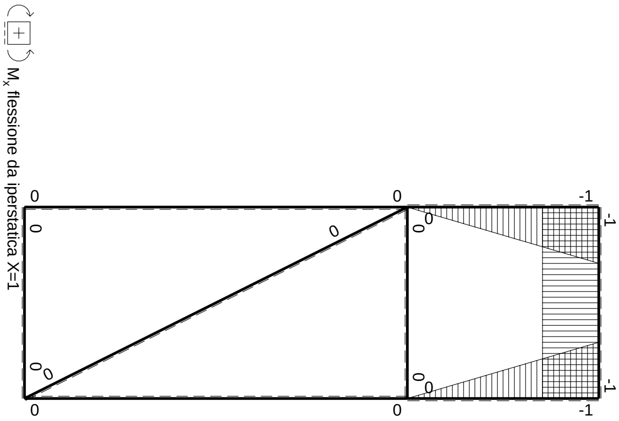

-37/20
5/2
7/2
-13/20
-5/√5
-√5/10
1/2
-1/2
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

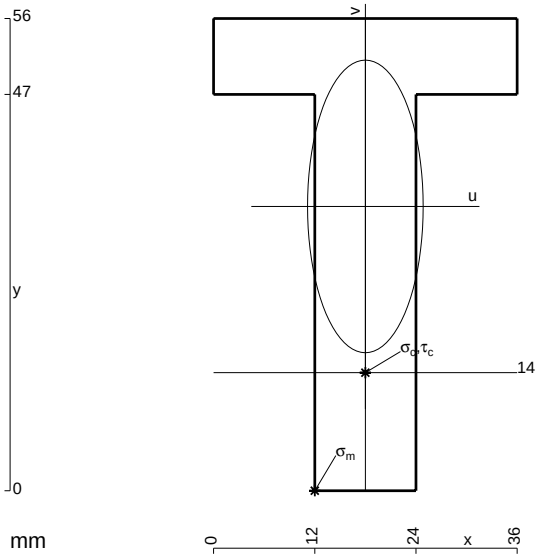
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 888. mm^2
J_u = 267344. mm^4
J_v = 41760. mm^4
y_g = 33.72 mm
N = 8280. N
T_y = -8280. N
M_x = -1738800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.72 mm
$\sigma_m = N/A - Mv/J_u$ = -210. N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.72 mm
$\sigma_c = N/A - Mv/J_u$ = -118.9 N/mm^2
τ_c = 11.58 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 120.6 N/mm^2
S^* = 4488. mm^3

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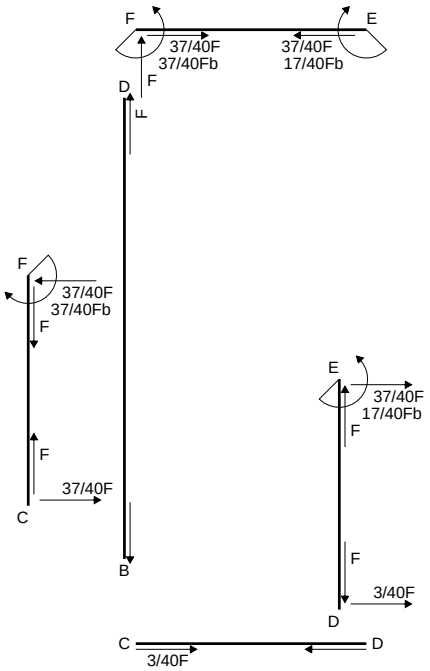
F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
F
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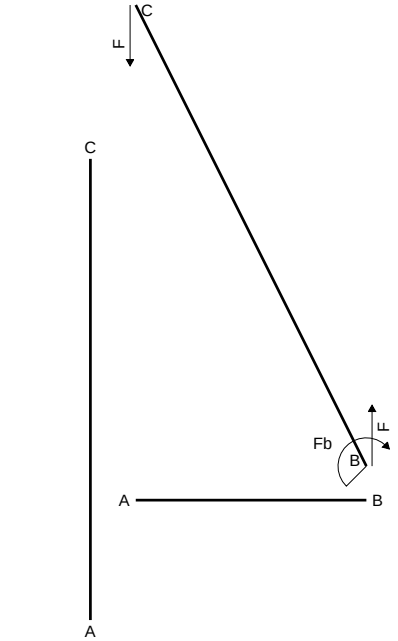
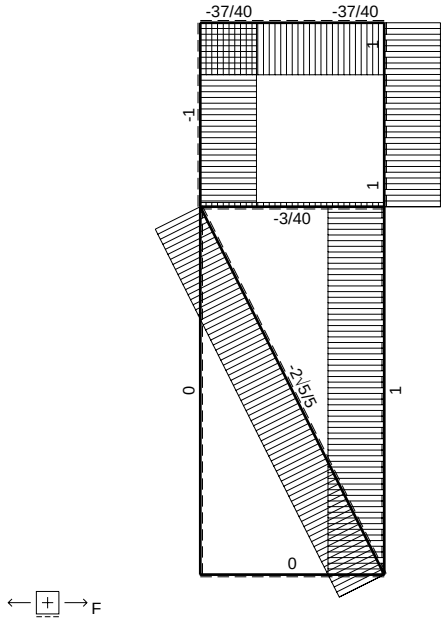
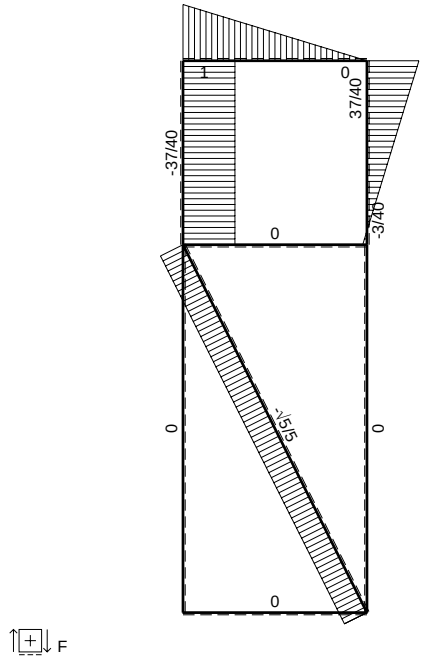
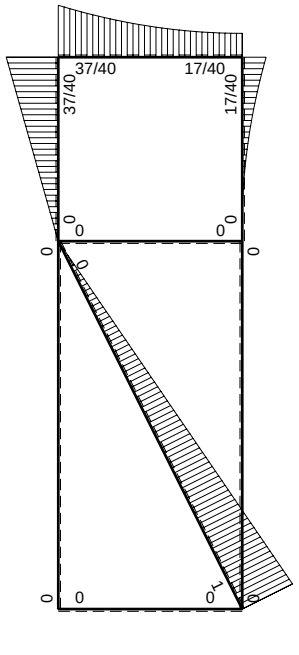
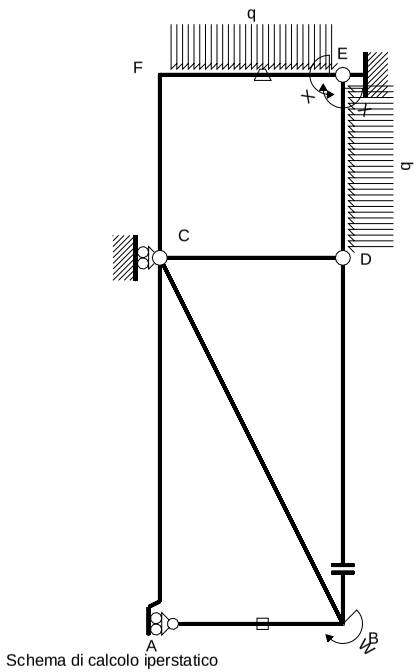
D
C
D
3/40F


C
F
C
Fb
F
B
A
B
A

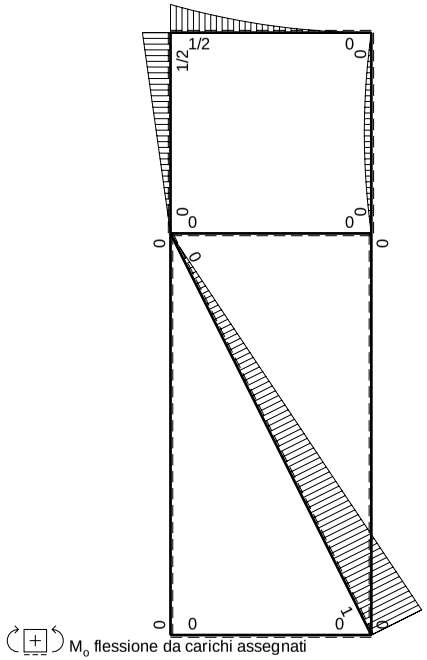

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
F

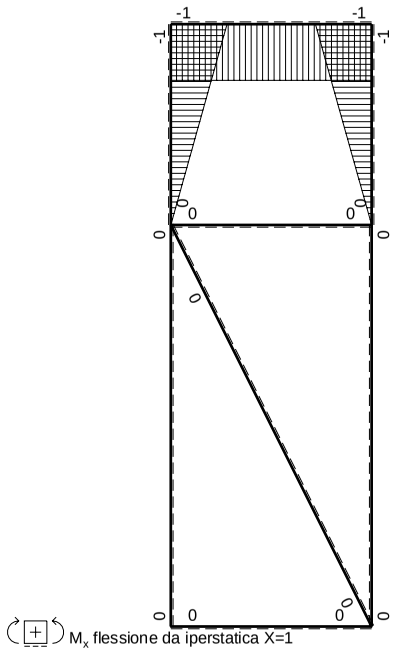

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
	totali						17/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-17/40Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

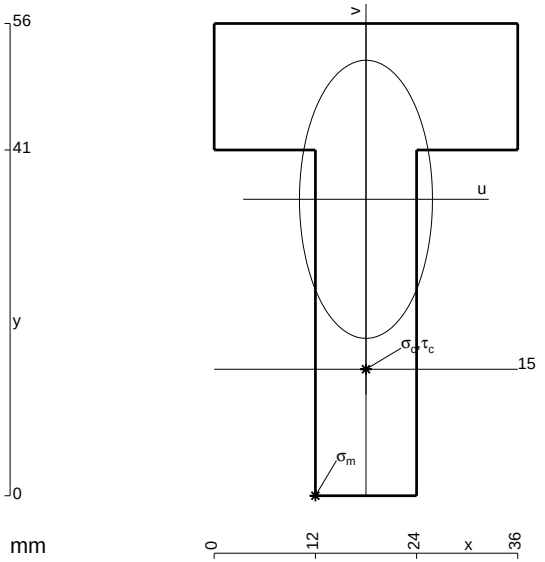
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210947 ESΣ09.xxxx.152



A = 1032. mm^2
J_u = 280881. mm^4
J_v = 64224. mm^4
y_g = 35.15 mm
N = -3470. N
T_y = -1735. N
M_x = 1784800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -35.15 mm
$\sigma_m = N/A - Mv/J_u$ = 220. N/mm^2
x_c = 18. mm
y_c = 15. mm
v_c = -20.15 mm
$\sigma_c = N/A - Mv/J_u$ = 124.7 N/mm^2
τ_c = 2.562 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.8 N/mm^2
S^* = 4977. mm^3

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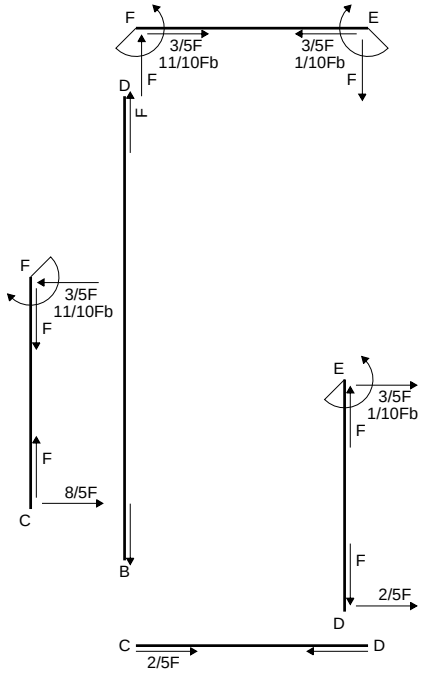
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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E
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C
B
F
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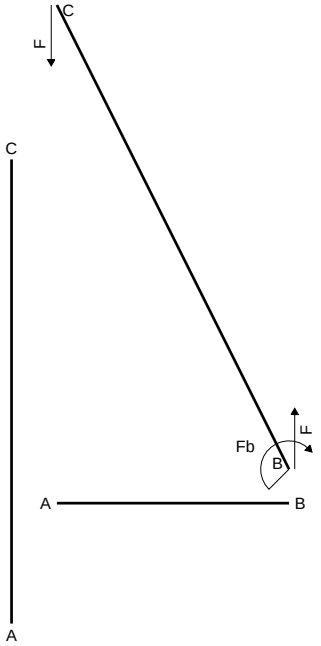
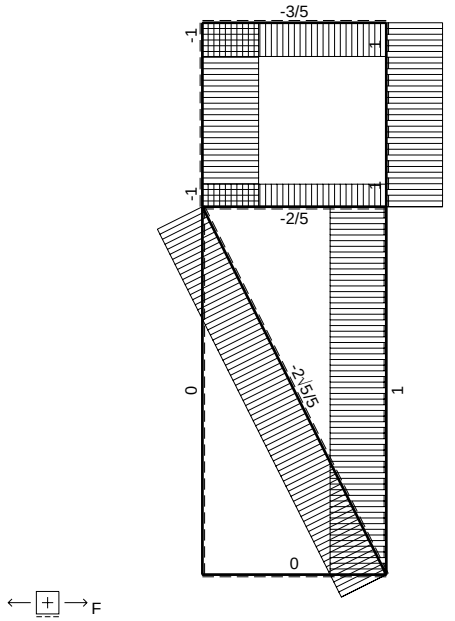
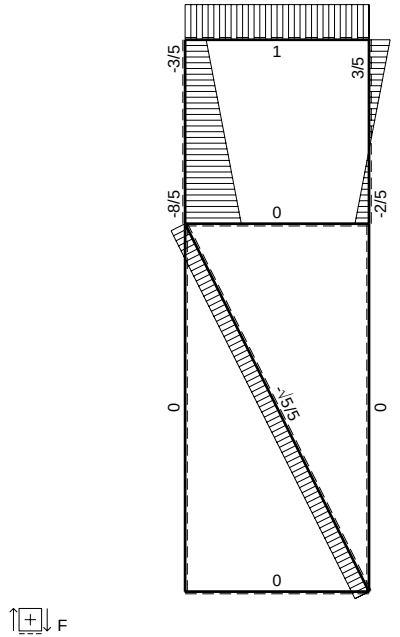
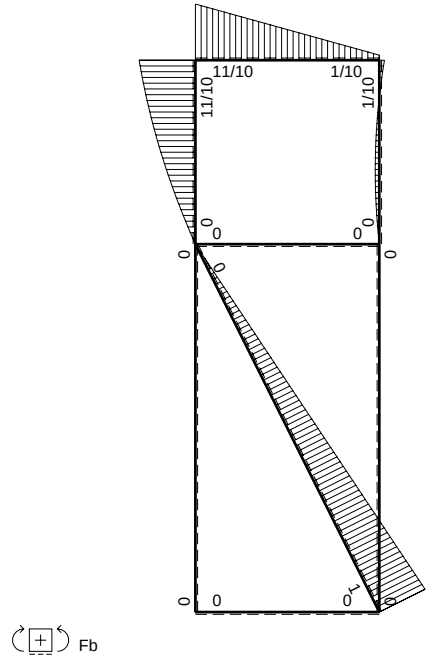
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx²	0	1/2Fx²b-1/2qx³/b	0	x²/b²	(1/24+0)Fb²/EJ	1/3XMb/EJ
ED b	1-x/b	1/2Fx-1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb²/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx²	0	-Fb+3/2Fx-1/2qx³/b	0	1-2x/b+x²/b²	(-3/8+0)Fb²/EJ	1/3XMb/EJ
CF b	x/b	-3/2Fx+1/2qx²	0	-3/2Fx²/b+1/2qx³/b	0	x²/b²		
	totali						1/6Fb²/EJ	5/3XMb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

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PROCEDIMENTO E RISULTATI 212450 ESΣ09.xxxx.153

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

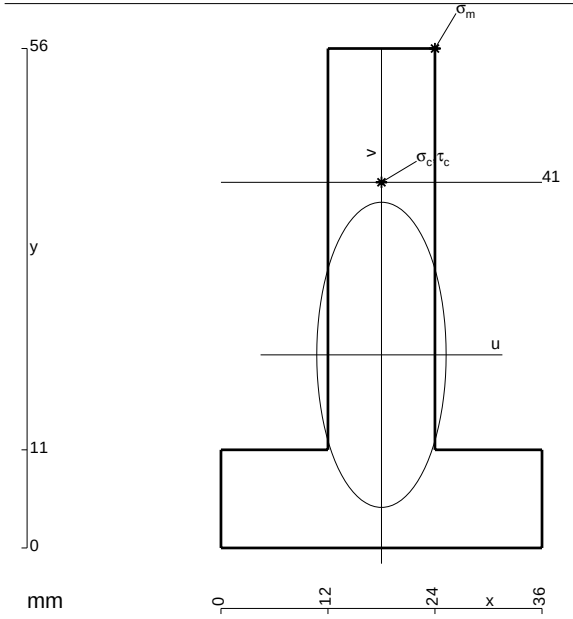
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 936. mm^2
J_u = 274232. mm^4
J_v = 49248. mm^4
y_g = 21.65 mm
N = -3229. N
T_y = -1614. N
M_x = 1805000. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 34.35 mm
$\sigma_m = N/A - Mv/J_u$ = -229.5 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.35 mm
$\sigma_c = N/A - Mv/J_u$ = -130.8 N/mm^2
τ_c = 2.371 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.9 N/mm^2
S^* = 4832. mm^3

ESΣ09.xxxx.153

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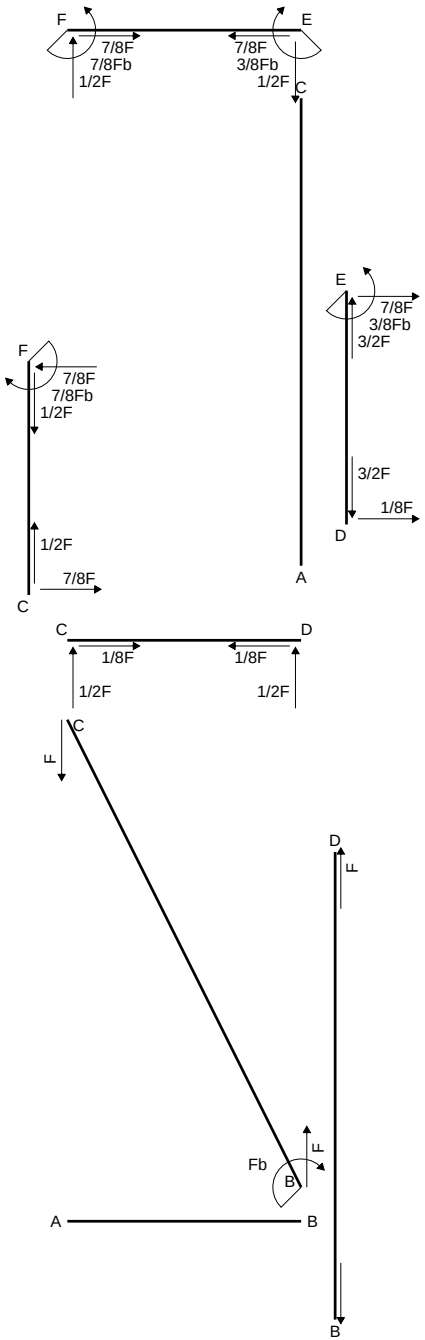
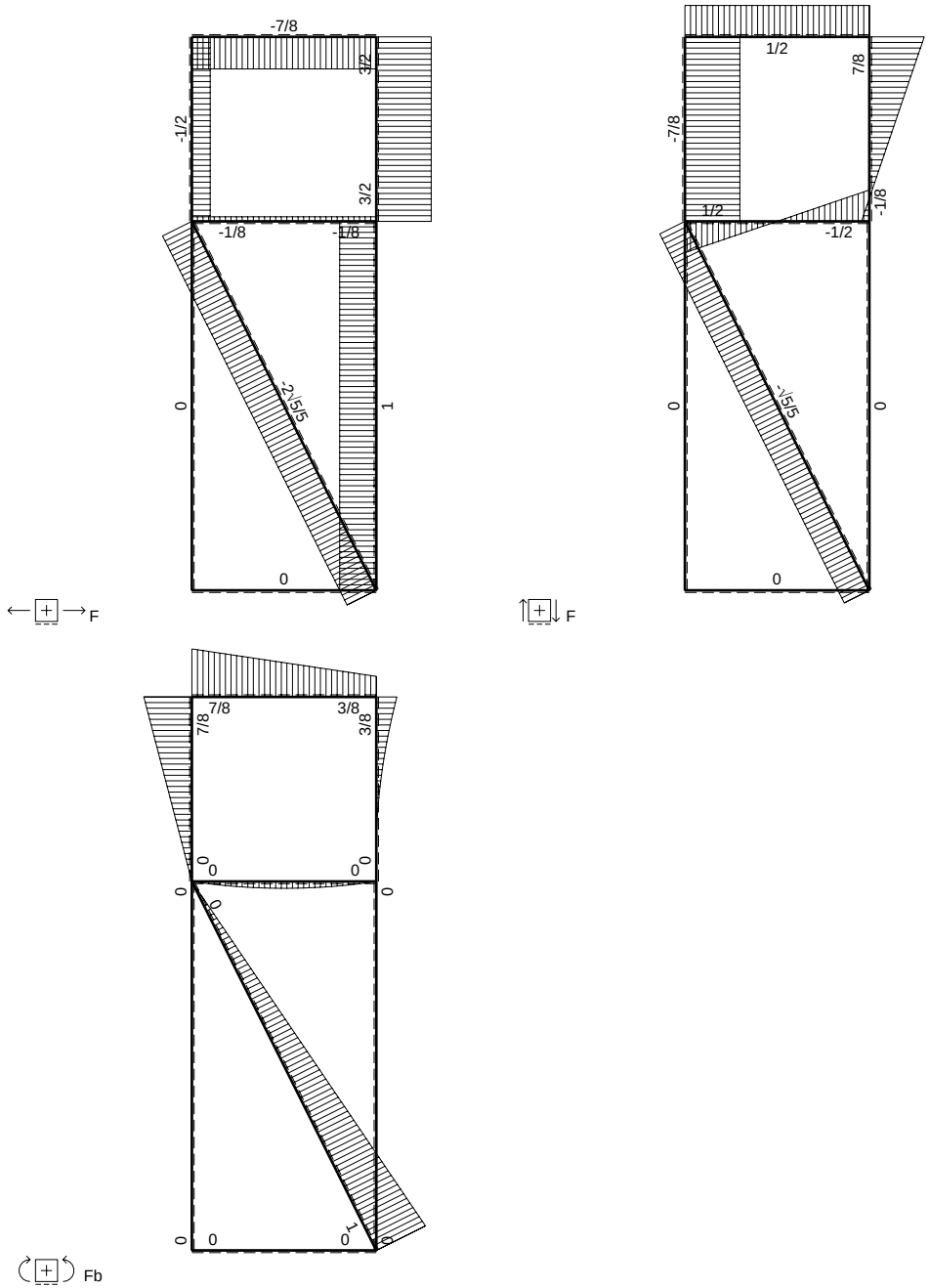

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
0
F
7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984367 ESΣ09.xxxx.154

14.11.24

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

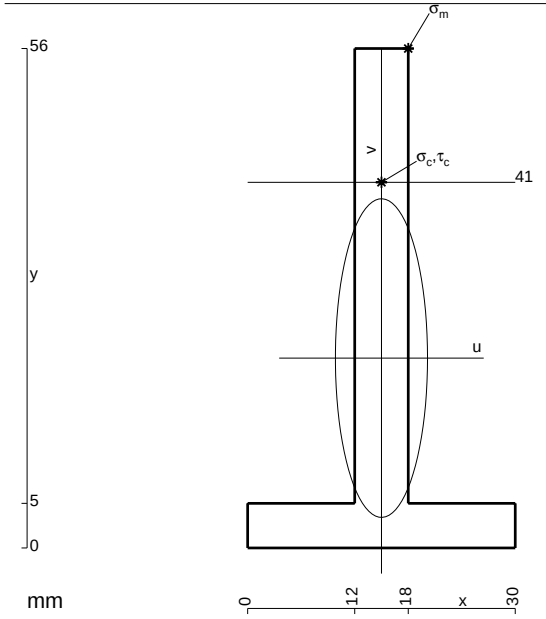
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 456. mm^2
J_u = 145554. mm^4
J_v = 12168. mm^4
y_g = 21.29 mm
N = -1637. N
T_y = -818.4 N
M_x = 988200. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 34.71 mm
$\sigma_m = N/A - Mv/J_u$ = -239.2 N/mm^2
x_c = 15. mm
y_c = 41. mm
v_c = 19.71 mm
$\sigma_c = N/A - Mv/J_u$ = -137.4 N/mm^2
τ_c = 2.295 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.5 N/mm^2
S^* = 2449. mm^3

ESΣ09.xxxx.154

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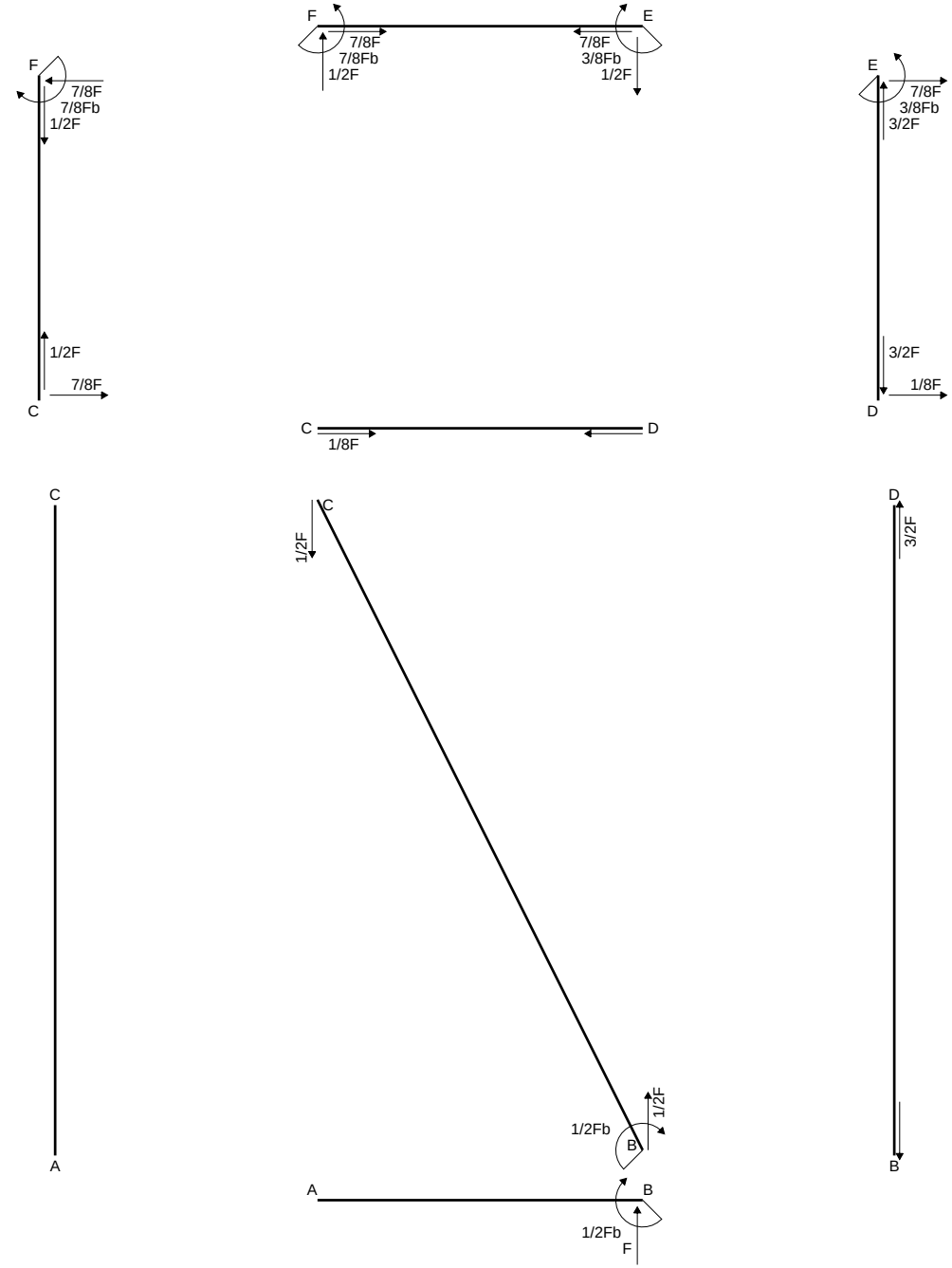
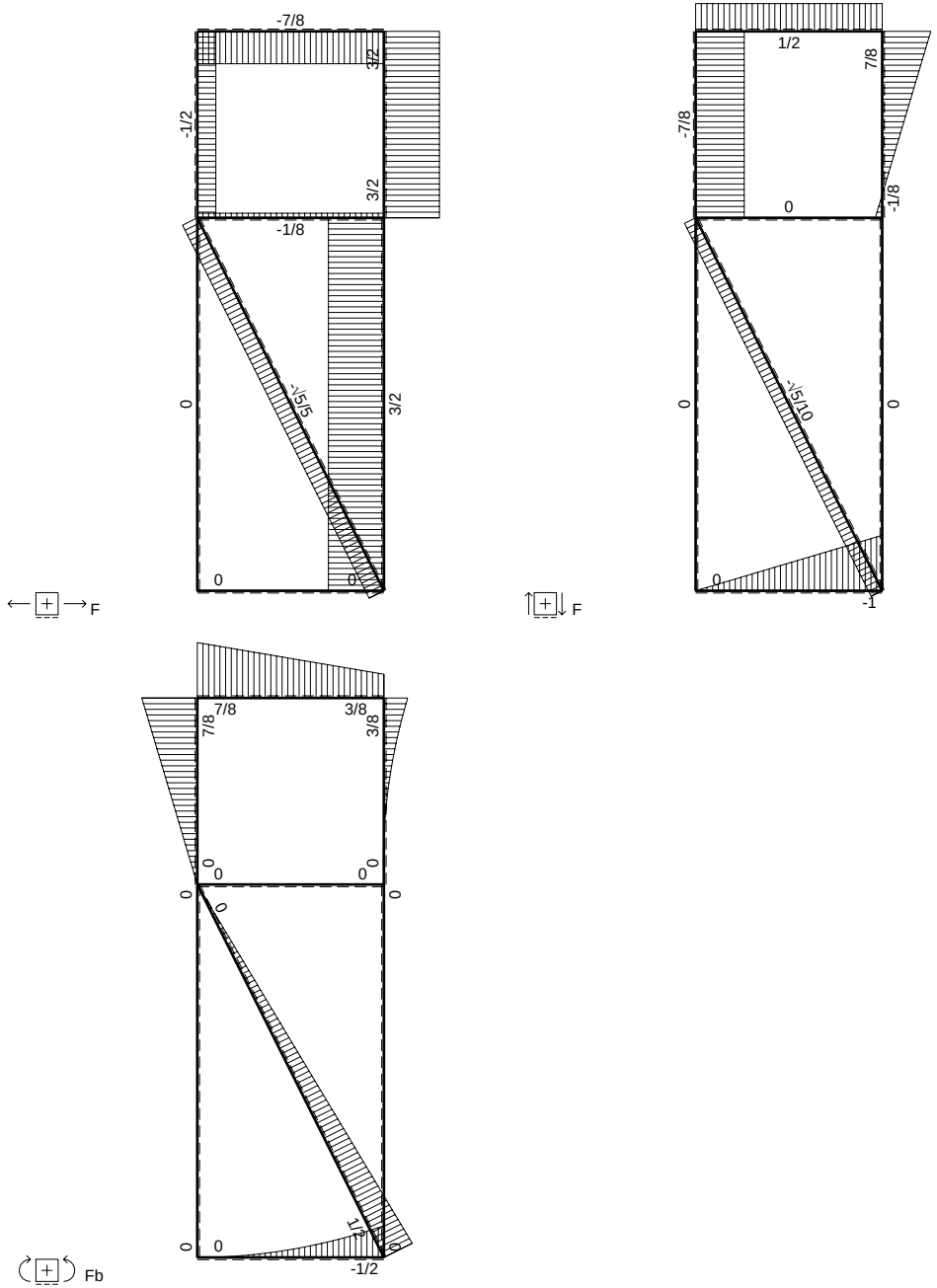

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
0
-√5/5
F
1/2
7/8
-1/8
-√5/10
-1
7/8
3/8
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx^2+1/2qx^2$	0	$1/2Fx^2b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	-Fb/EJ	$-1/2Fx$	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 983679 ESΣ09.xxxx.155

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

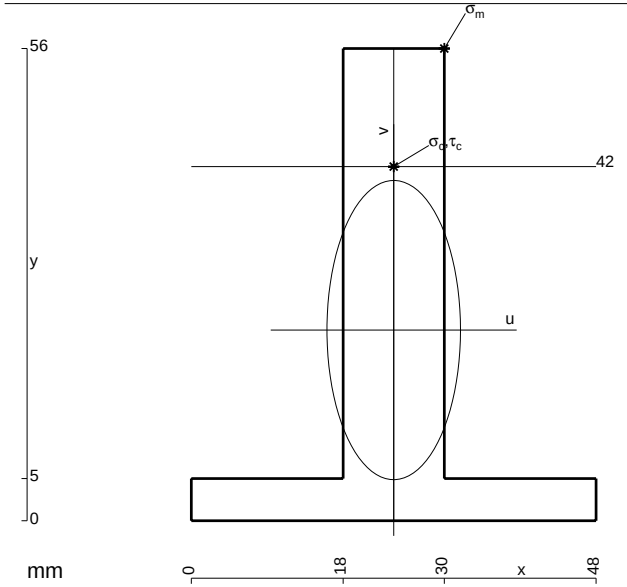
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 852. mm^2
J_u = 268308. mm^4
J_v = 53424. mm^4
y_g = 22.61 mm
T_y = -5540. N
M_x = -1606600. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 33.39 mm
$\sigma_m = -Mv/J_u$ = 199.9 N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 19.39 mm
$\sigma_c = -Mv/J_u$ = 116.1 N/mm^2
τ_c = 7.628 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.8 N/mm^2
S^* = 4433. mm^3

ESΣ09.xxxx.155

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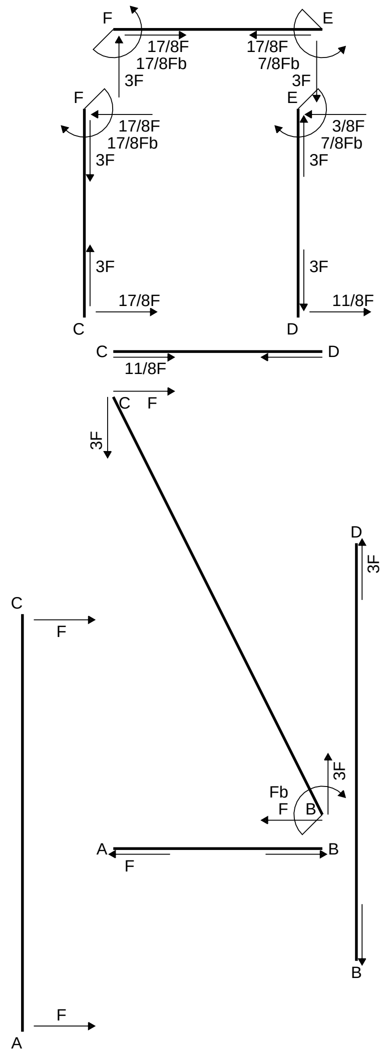
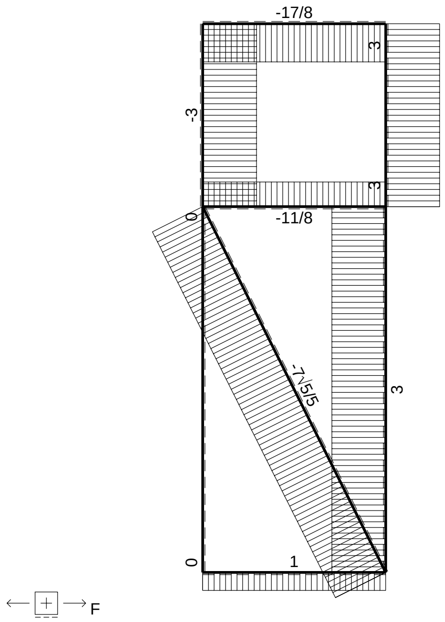
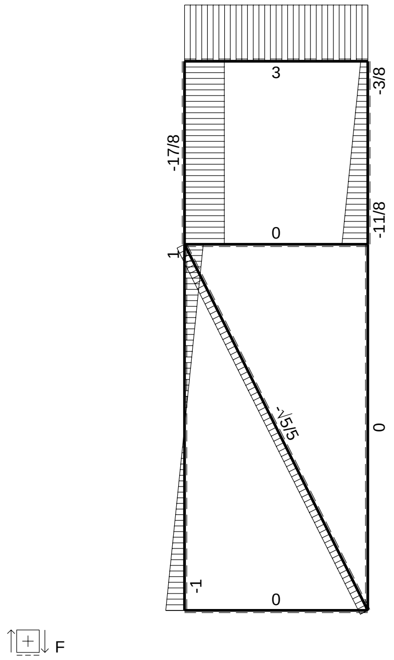
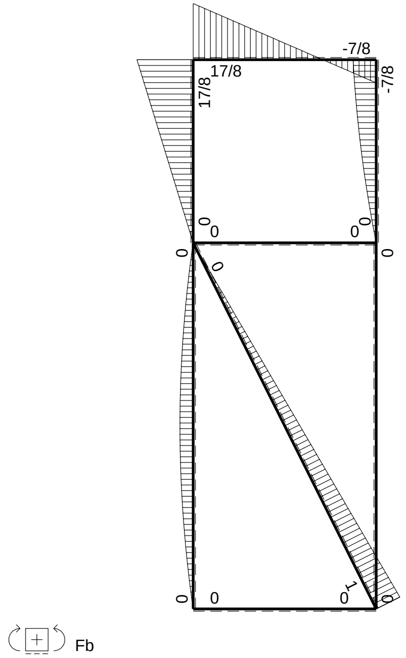
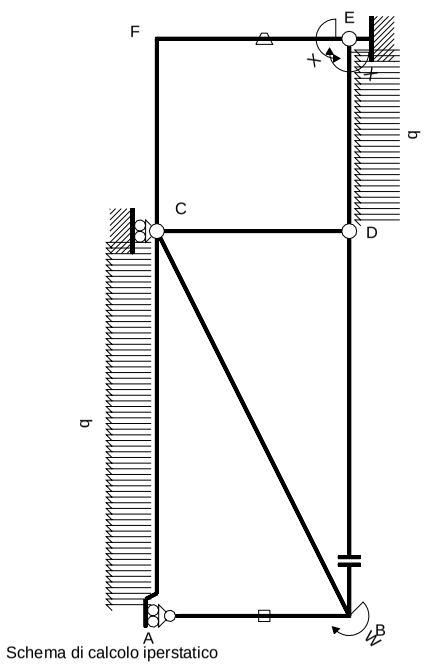

F
E
17/8F
17/8Fb
17/8F
7/8Fb
3F
3F
F
E
17/8F
17/8Fb
3/8F
7/8Fb
3F
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C
F
3F
D
3F
C
F
Fb
F
B
3F
A
B
F
F
A
B

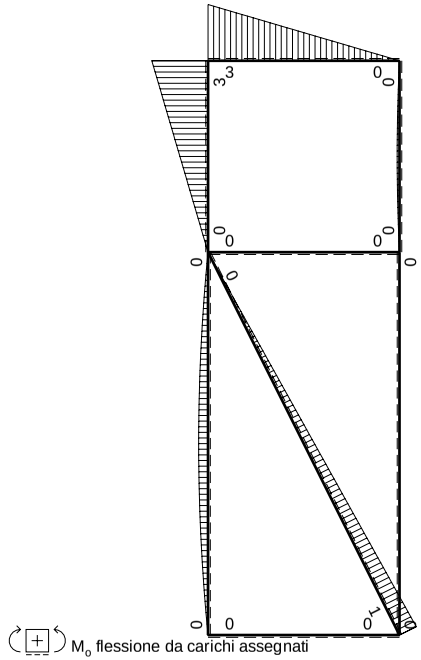

-17/8
3
-3
3
-11/8
0
-7√5/5
3
0
1
F


3
-3/8
-17/8
-11/8
0
1
-√5/5
0
-1
0
F

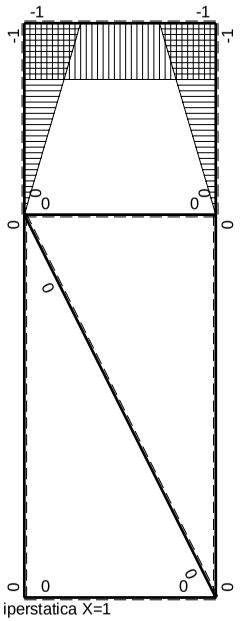

-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
1
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
		totali					-35/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 957491 ESΣ09.xxxx.156

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

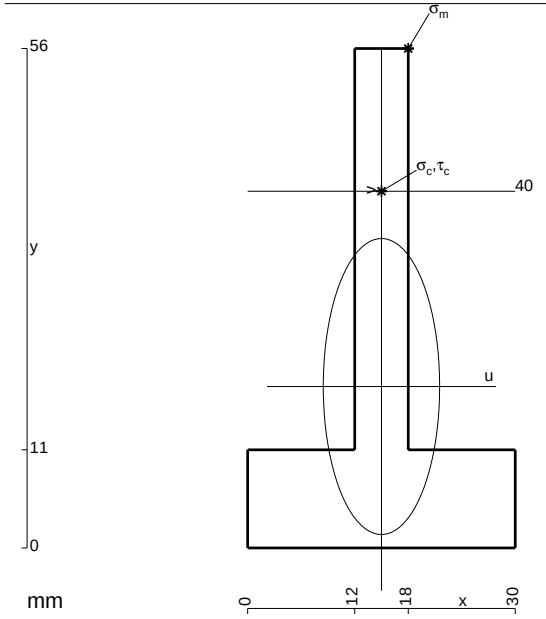
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 600. mm^2
J_u = 165314. mm^4
J_v = 25560. mm^4
y_g = 18.1 mm
N = -4445. N
T_y = -635. N
M_x = 880400. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 37.9 mm
$\sigma_m = N/A - Mv/J_u$ = -209.3 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 21.9 mm
$\sigma_c = N/A - Mv/J_u$ = -124. N/mm^2
τ_c = 1.838 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.1 N/mm^2
S^* = 2870. mm^3

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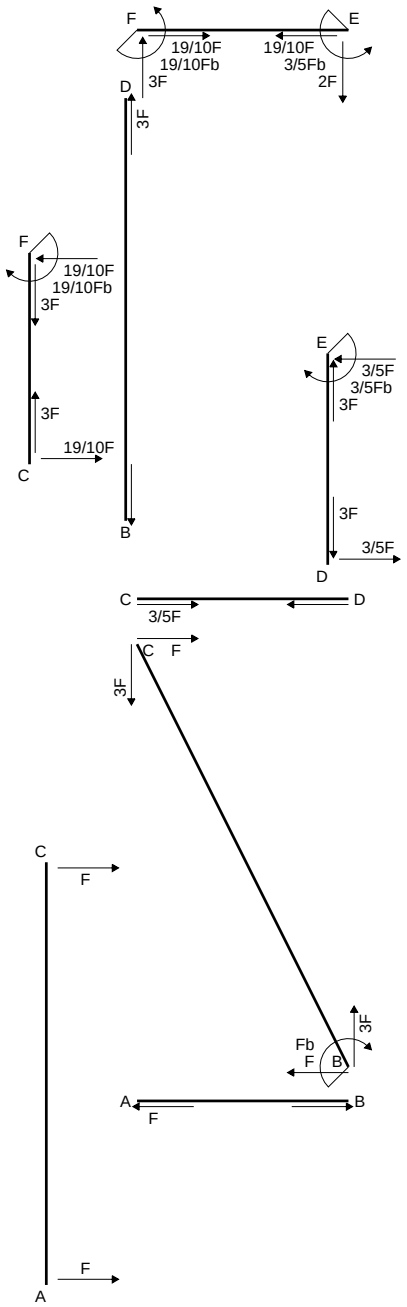
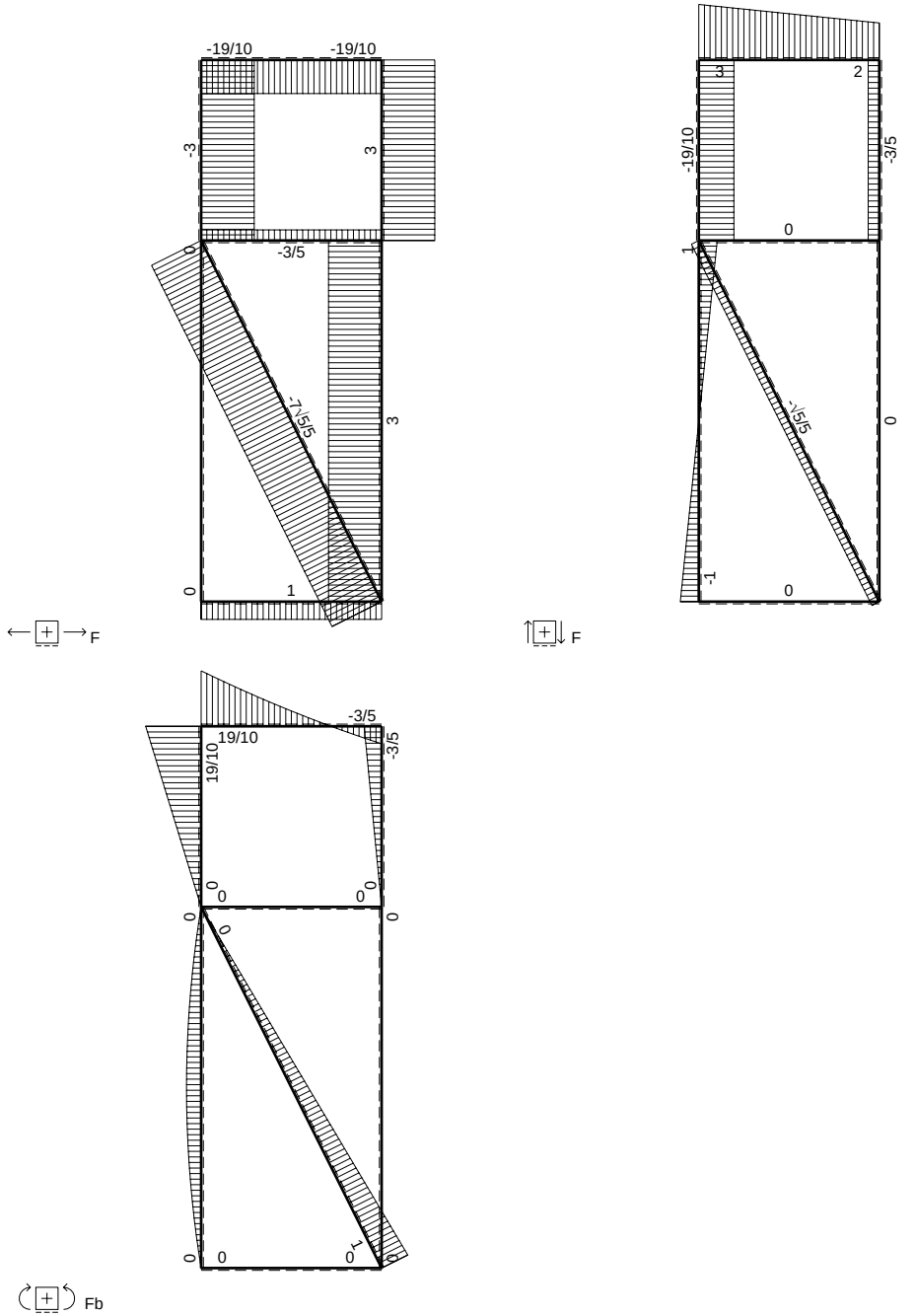

F
E
19/10F
19/10Fb
19/10F
3/5Fb
D
3F
2F
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
B
3F
3/5F
D
C
D
3/5F
C
F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
-19/10
-3
3
-3/5
0
3
-7\5/5
1
F
-19/10
3
2
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0
1
-1\5/5
-1
0
F
-3/5
19/10
19/10
-3/5
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx²	-Fb/EJ	-2Fx-1/2Fx²/b	Fb/EJ	1	(-7/6+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx²	Fb/EJ	-5/2Fb+3Fx-1/2Fx²/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
totali							-Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 209949 ESΣ09.xxxx.157

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

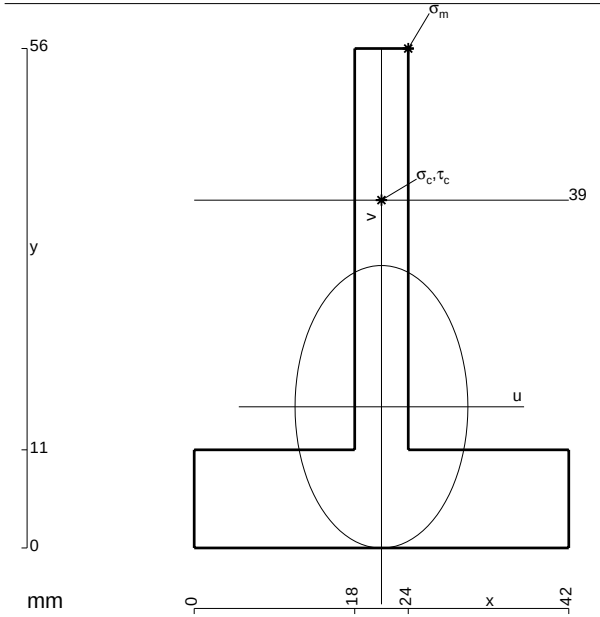
$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 209949 ESΣ09.xxxx.157



A = 732. mm^2
J_u = 183822. mm^4
J_v = 68724. mm^4
y_g = 15.83 mm
N = -4633. N
T_y = -661.9 N
M_x = 976800. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 40.17 mm
$\sigma_m = N/A - Mv/J_u$ = -219.8 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 23.17 mm
$\sigma_c = N/A - Mv/J_u$ = -129.5 N/mm^2
τ_c = 1.939 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.5 N/mm^2
S^* = 3231. mm^3

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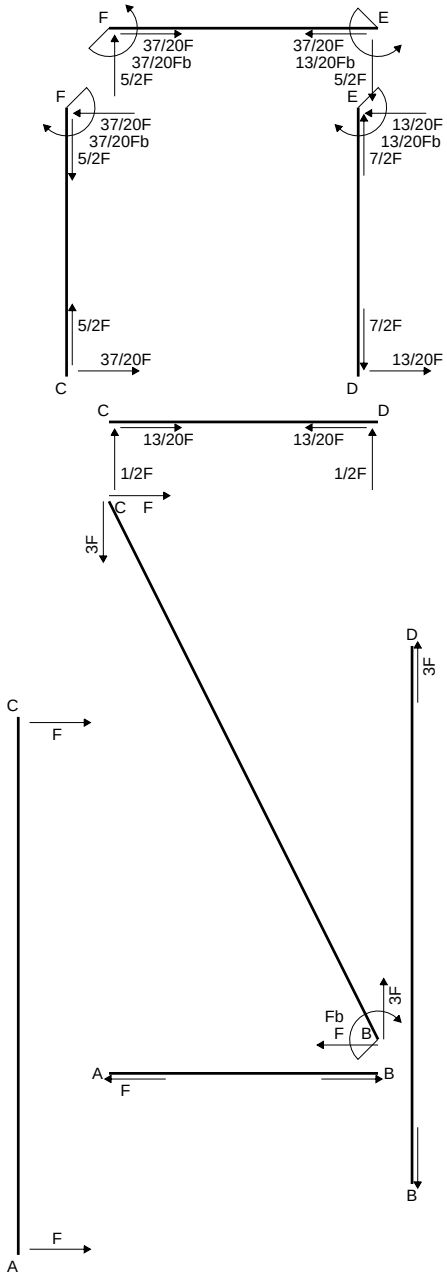
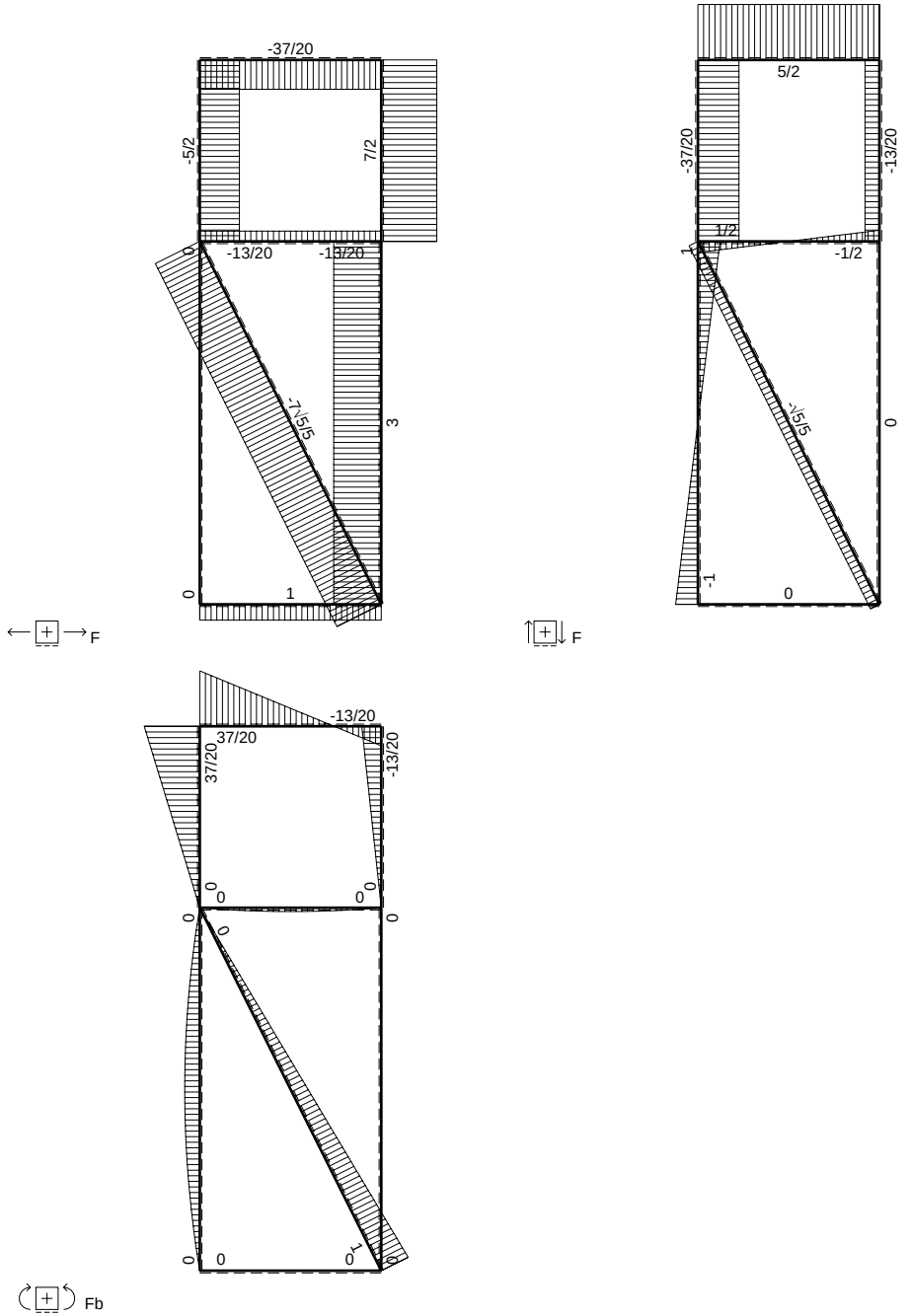

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B


-37/20
-5/2
7/2
-13/20
-13/20
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3
-7.5/5
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
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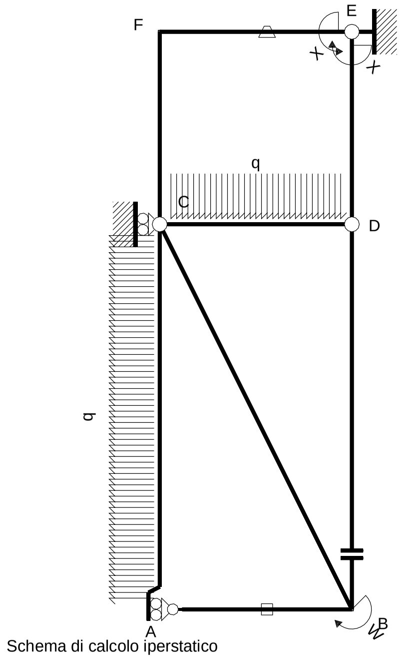
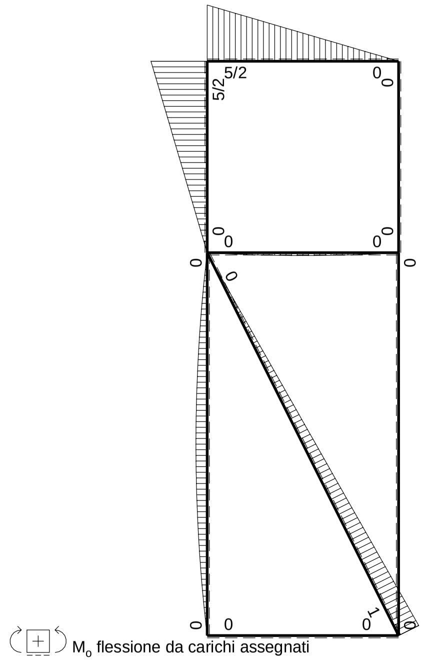
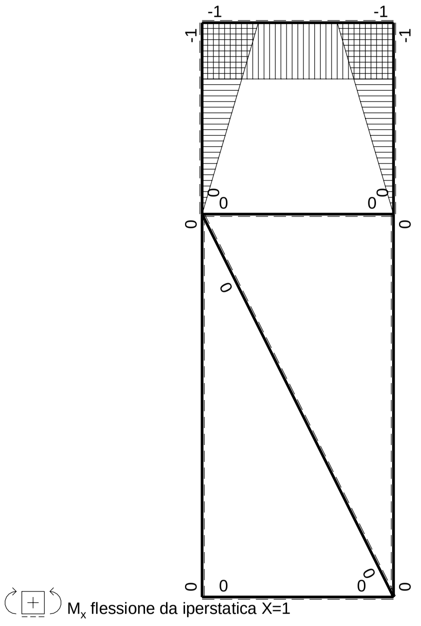
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

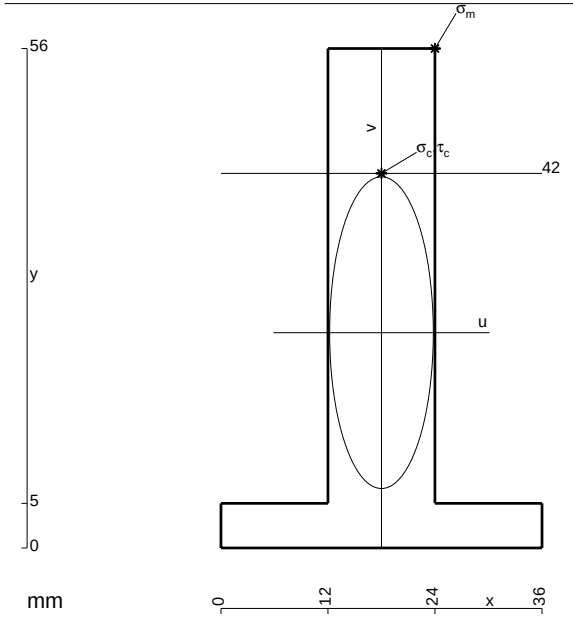
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 792. mm^2
J_u = 242073. mm^4
J_v = 26784. mm^4
y_g = 24.14 mm
N = -7482. N
T_y = -1069. N
M_x = 1673000. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 31.86 mm
$\sigma_m = N/A - Mv/J_u$ = -229.7 N/mm^2
x_c = 18. mm
y_c = 42. mm
v_c = 17.86 mm
$\sigma_c = N/A - Mv/J_u$ = -132.9 N/mm^2
τ_c = 1.537 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 132.9 N/mm^2
S^* = 4177. mm^3

ESΣ09.xxxx.158

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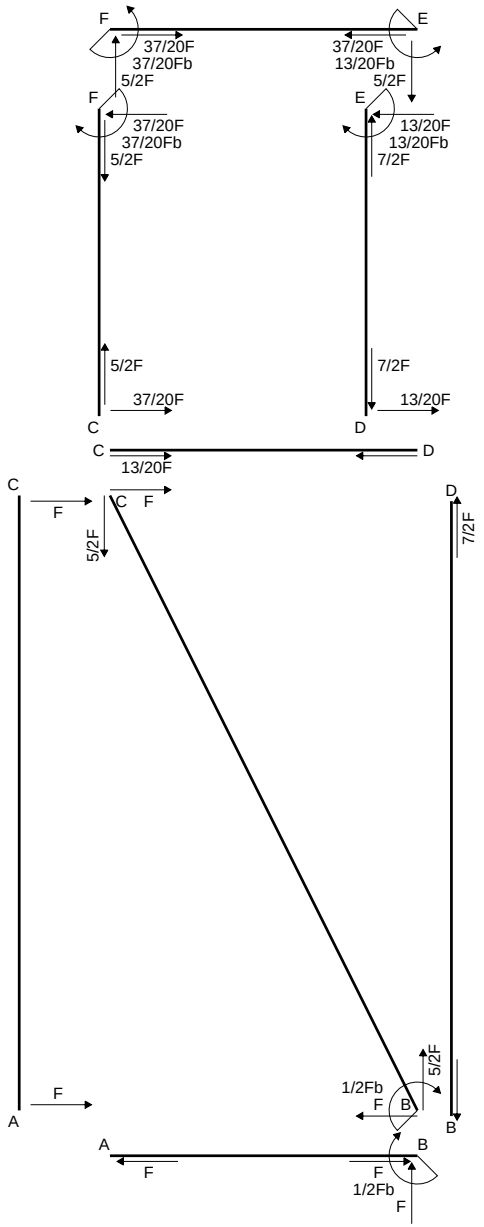
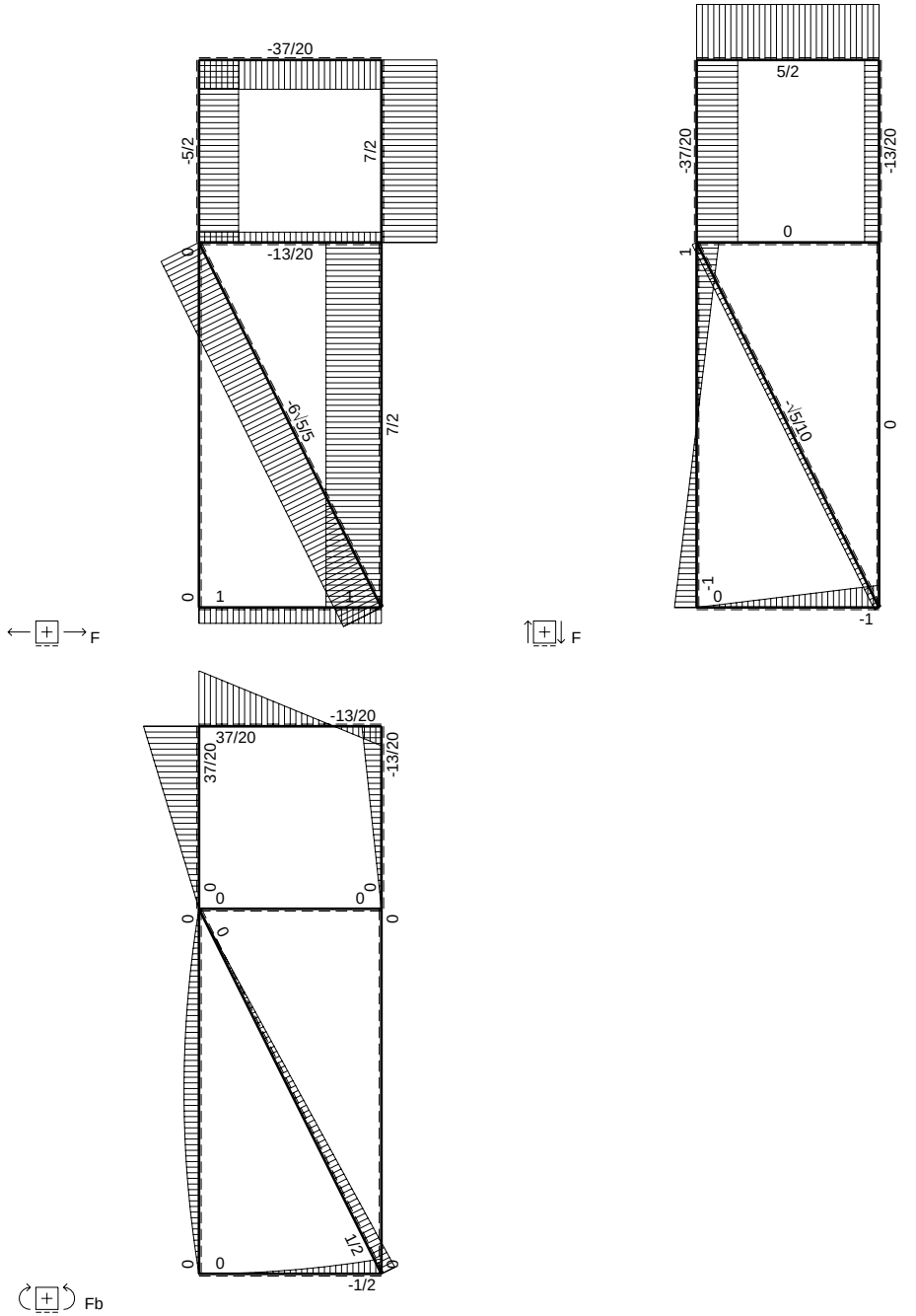

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb


-37/20
-13/20
5/2
7/2
-5/2
-5\5/5
-5\5/10
37/20
1/2
-1/2
F
Fb

PROCEDIMENTO E RISULTATI 212891 ESΣ09.xxxx.159

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

14.11.24

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

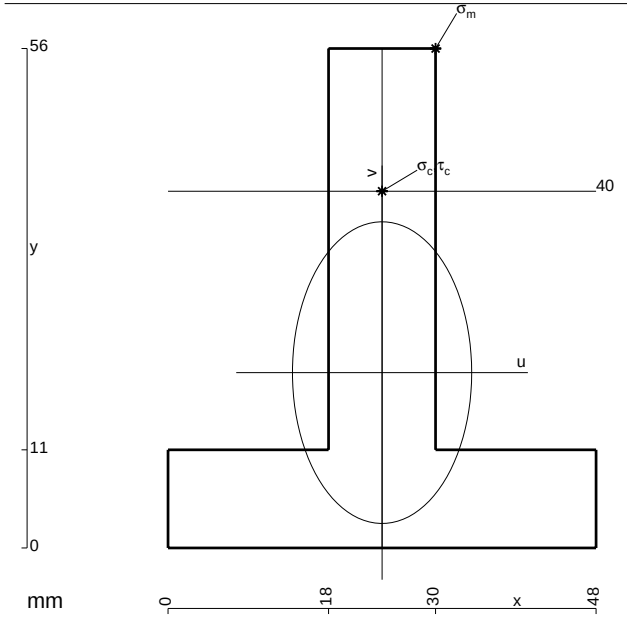
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1068. mm^2
J_u = 305751. mm^4
J_v = 107856. mm^4
y_g = 19.66 mm
N = 5340. N
T_y = -5340. N
M_x = -1975800. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 36.34 mm
$\sigma_m = N/A - Mv/J_u$ = 239.9 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 20.34 mm
$\sigma_c = N/A - Mv/J_u$ = 136.5 N/mm^2
τ_c = 7.92 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.1 N/mm^2
S^* = 5442. mm^3

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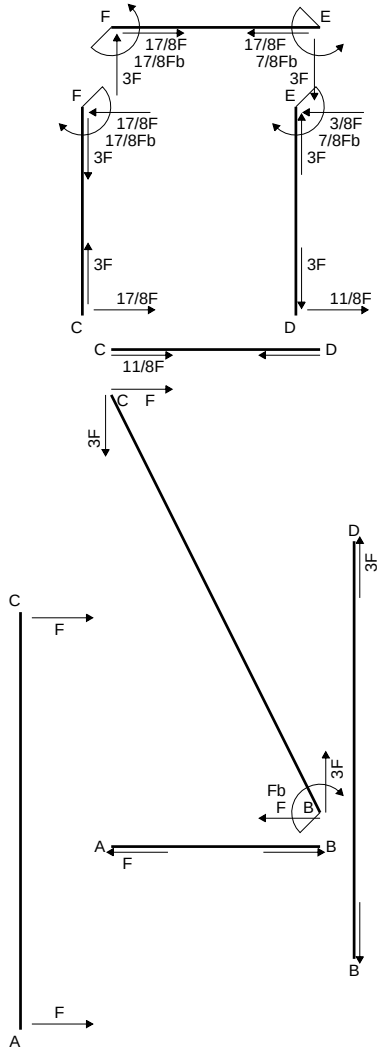
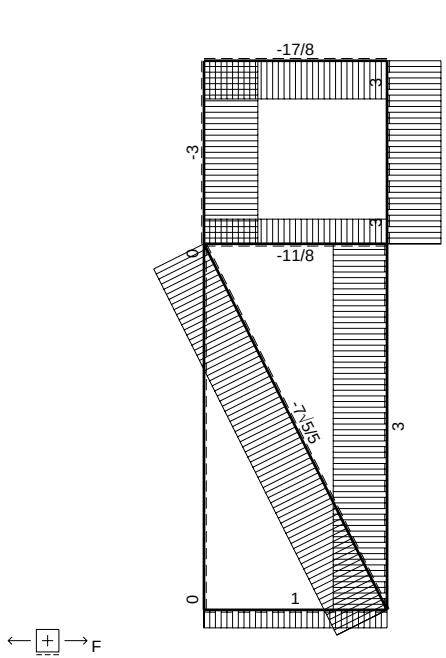
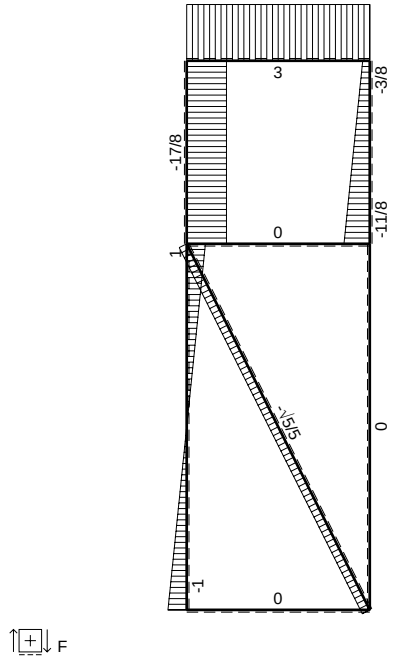
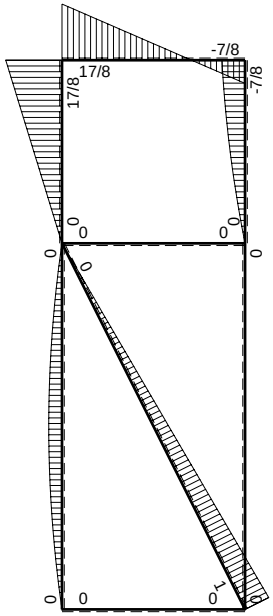
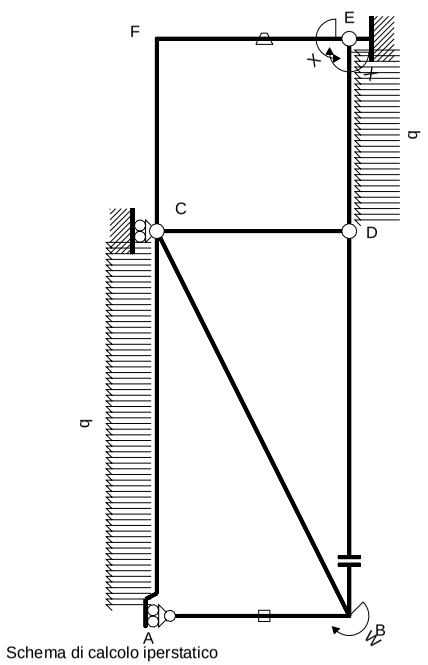

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
3F
3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
F
3F
Fb
F
B
A
B
F
A
F
B

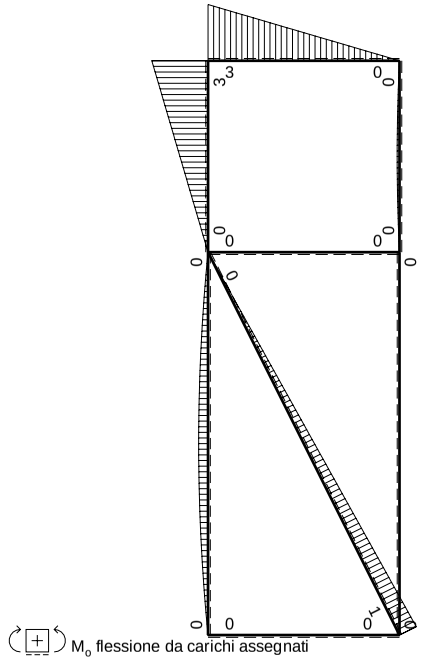

-17/8
-3
-11/8
-7√5/5
3
0
1
F


3
-3/8
-17/8
-11/8
0
-√5/5
-1
F

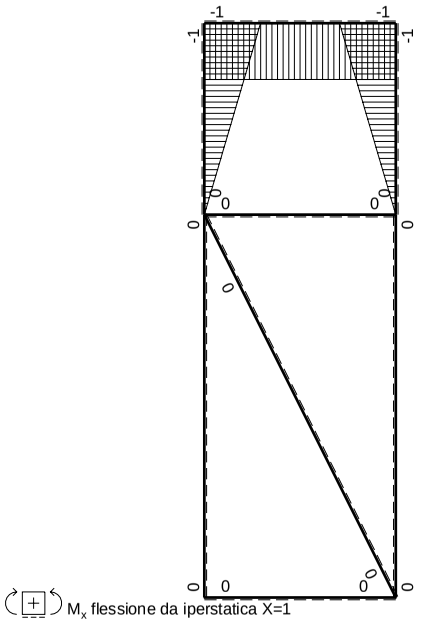

-7/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
		totali					$-35/24Fb^2/EJ$	5/3Xb/EJ
		iperstatica $X=W_{EF}$					7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 959992 ESΣ09.xxxx.160

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

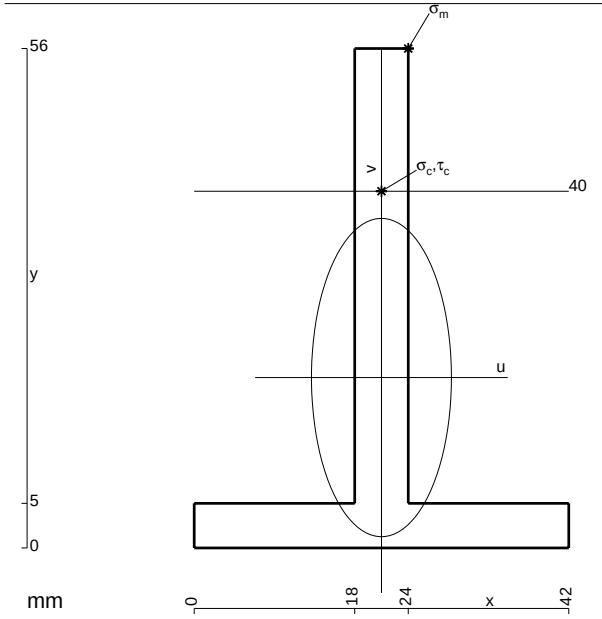
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 516. mm^2
J_u = 164398. mm^4
J_v = 31788. mm^4
y_g = 19.1 mm
N = -6668. N
T_y = -952.6 N
M_x = 830700. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 36.9 mm
$\sigma_m = N/A - Mv/J_u$ = -199.4 N/mm^2
x_c = 21. mm
y_c = 40. mm
v_c = 20.9 mm
$\sigma_c = N/A - Mv/J_u$ = -118.5 N/mm^2
τ_c = 2.679 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.6 N/mm^2
S^* = 2774. mm^3

ESΣ09.xxxx.160

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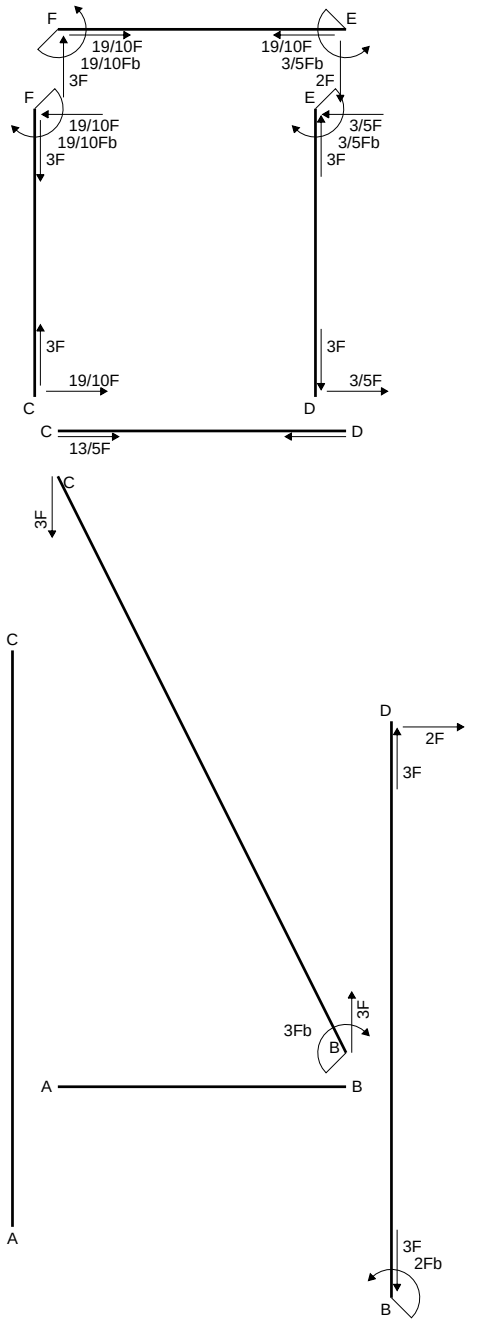
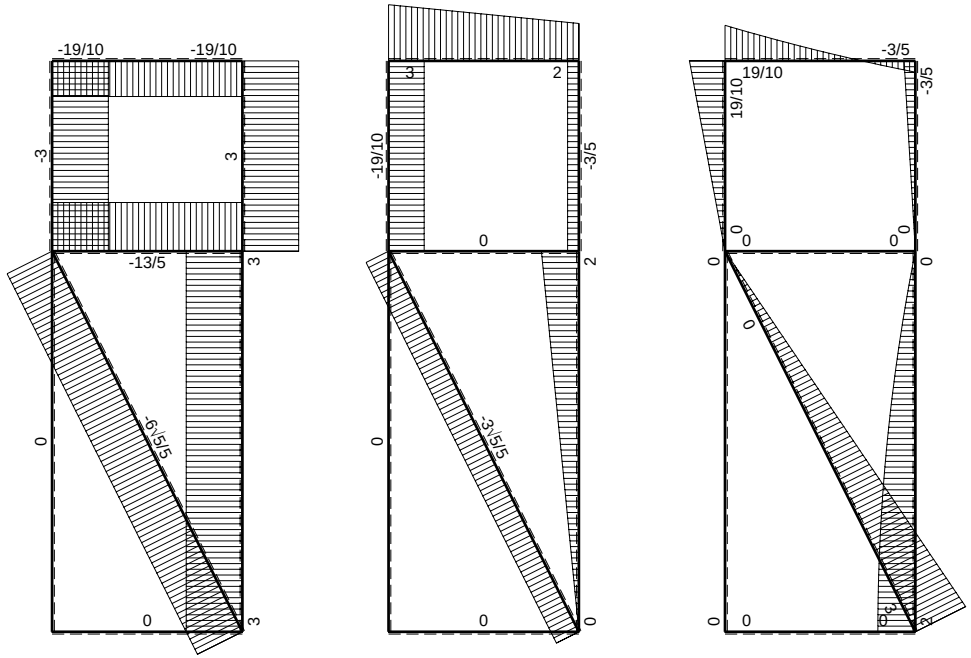

F
E
19/10F
19/10Fb
3F
19/10F
3/5Fb
2F
F
E
19/10F
19/10Fb
3F
3/5F
3/5Fb
3F
3F
3F
19/10F
3/5F
C
D
C
D
13/5F
C
3F
C
2F
D
3F
3Fb
3F
B
A
B
3F
2Fb
B
A


-19/10
-19/10
-3
3
-13/5
3
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-6√5/5
0
3
3
2
-19/10
-3/5
0
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-3/5
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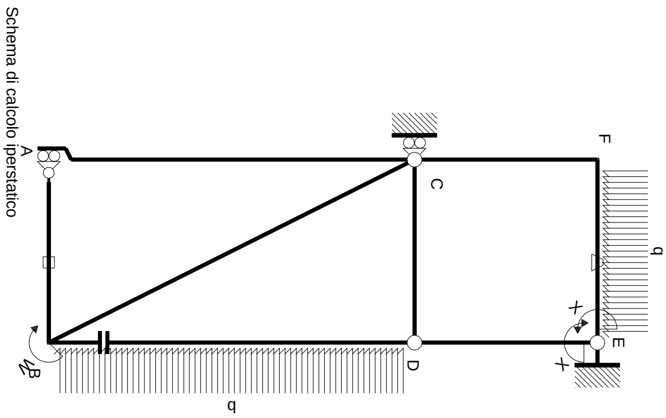
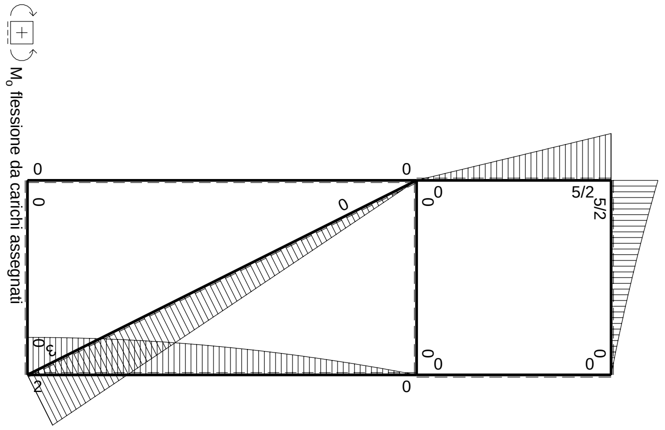
0

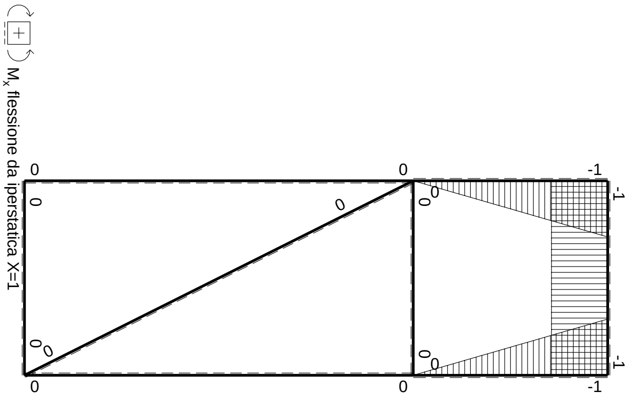
F F Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (- b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

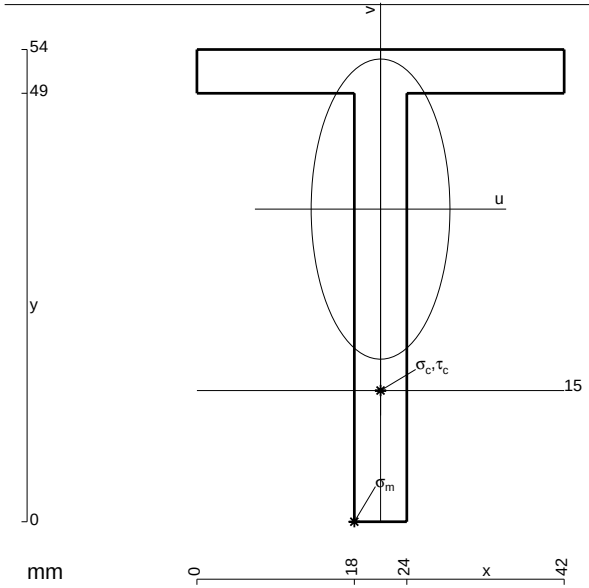
$= (-5/2\ b +3/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b)\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b)\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 504. mm^2
J_u = 148565. mm^4
J_v = 31752. mm^4
y_g = 35.75 mm
N = -1932. N
T_y = -966. N
M_x = 885600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -35.75 mm
$\sigma_m = N/A - Mv/J_u$ = 209.3 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -20.75 mm
$\sigma_c = N/A - Mv/J_u$ = 119.9 N/mm^2
τ_c = 2.755 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120. N/mm^2
S^* = 2543. mm^3

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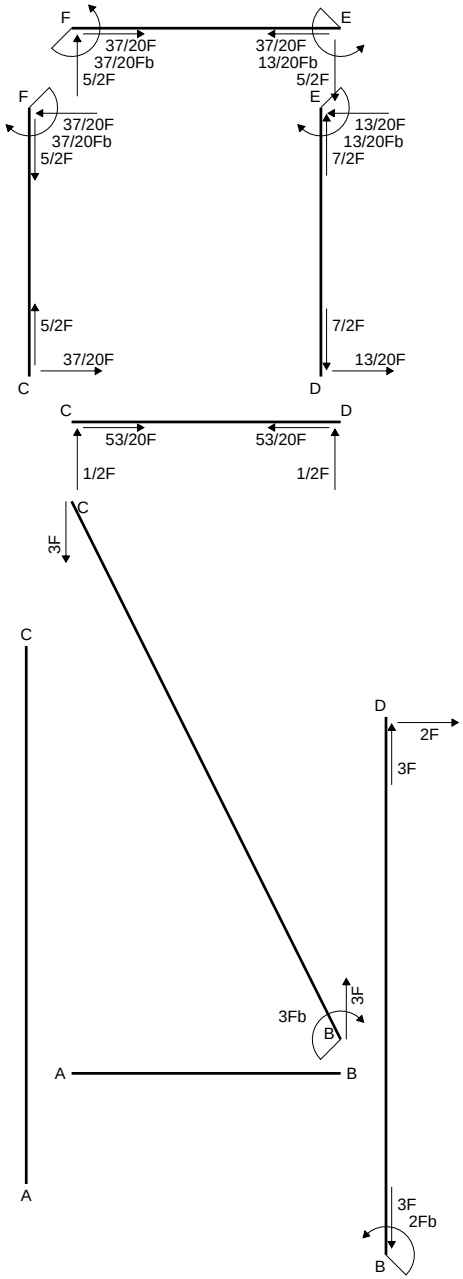
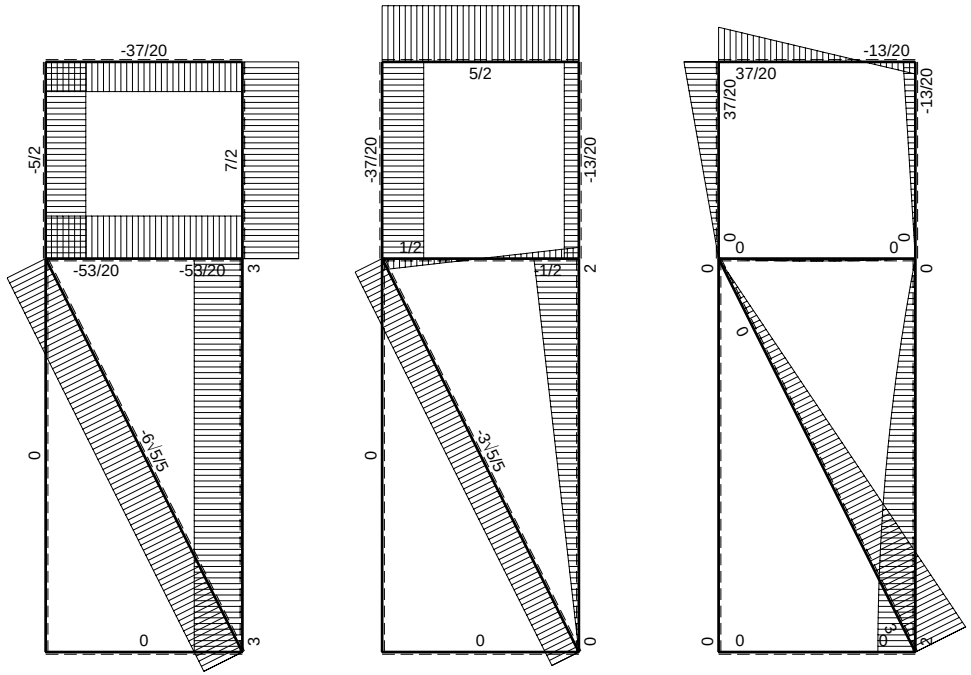

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
53/20F
1/2F
1/2F
C
3F
C
D
2F
3F
3Fb
B
3F
A
B
A
3F
2Fb
B


-37/20
-5/2
7/2
-53/20
-53/20
3
0
0
3
-6√5/5
5/2
-37/20
-13/20
1/2
1/2
2
0
0
-3√5/5
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
3√5
3

F F Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi\theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982812 ESΣ09.xxxx.162

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

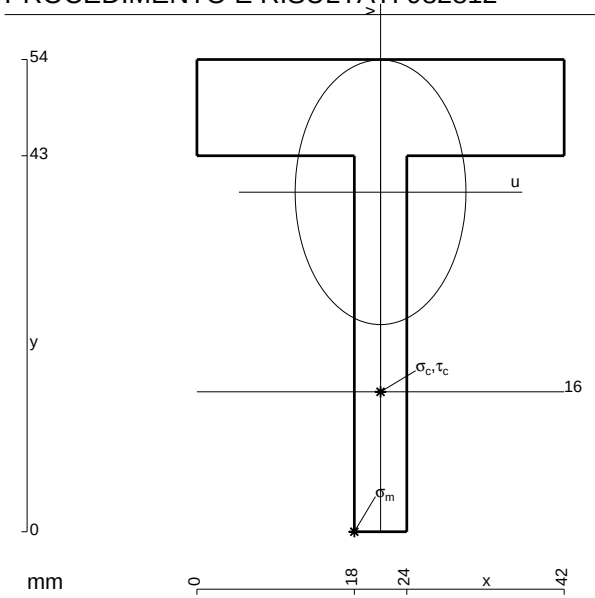
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 720. mm^2
J_u = 165098. mm^4
J_v = 68688. mm^4
y_g = 38.83 mm
N = -1878. N
T_y = -939.1 N
M_x = 945000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.83 mm
$\sigma_m = N/A - Mv/J_u$ = 219.6 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.83 mm
$\sigma_c = N/A - Mv/J_u$ = 128. N/mm^2
τ_c = 2.806 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.1 N/mm^2
S^* = 2959. mm^3

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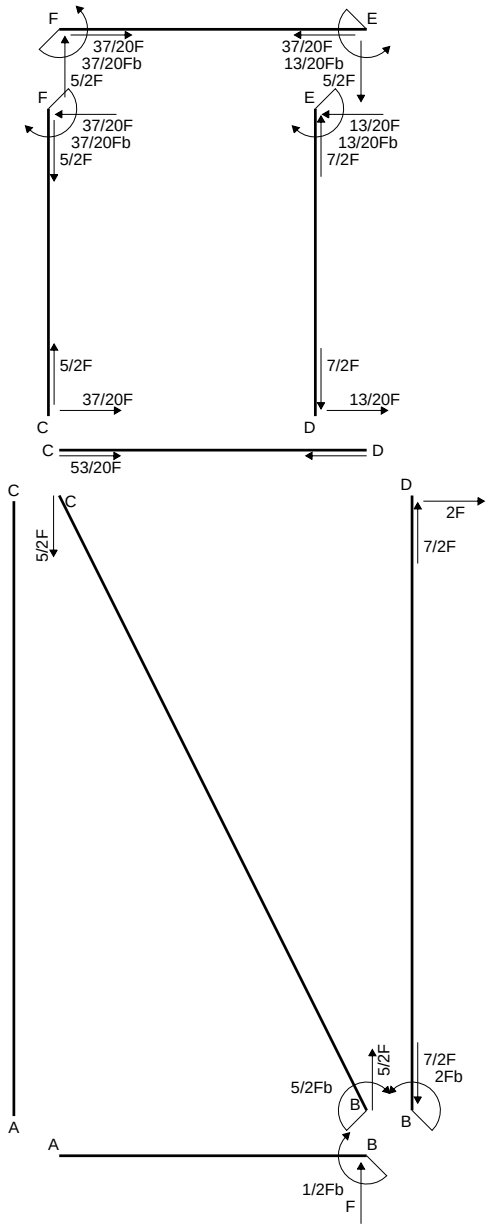
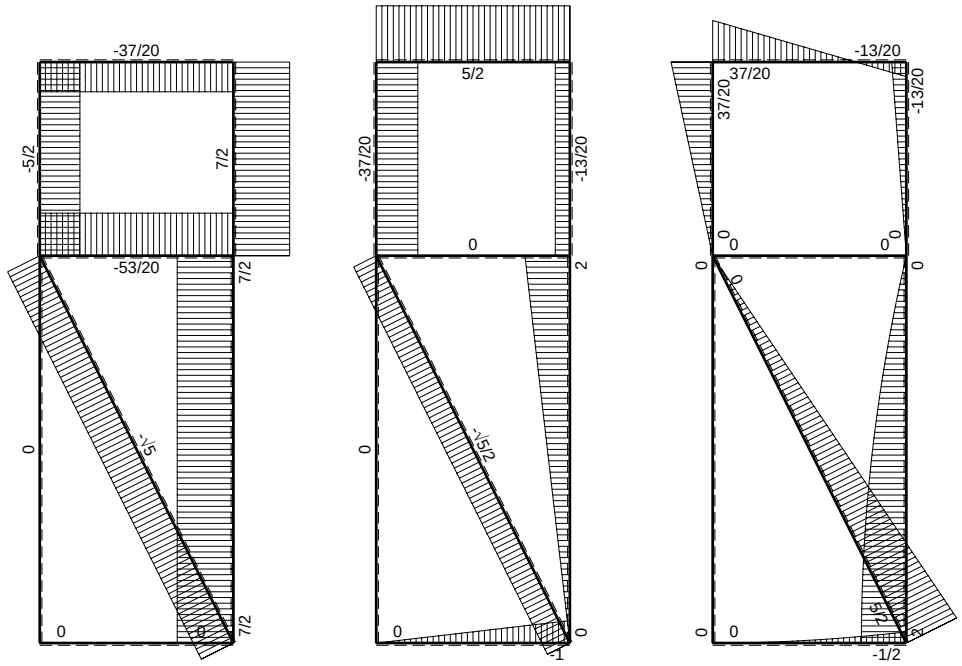

F
E
37/20F
37/20Fb
5/2F
13/20Fb
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
53/20F
2F
7/2F
5/2F
5/2Fb
2Fb
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
0
-5/2
5/2
-13/20
2
-1
37/20
-1/2


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$5/2Fb-\sqrt{5}/2Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	1/3Xb/EJ	
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2	(-5/6+0)Fb²/EJ	
	totali						-13/12Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 280177 ESΣ09.xxxx.163

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

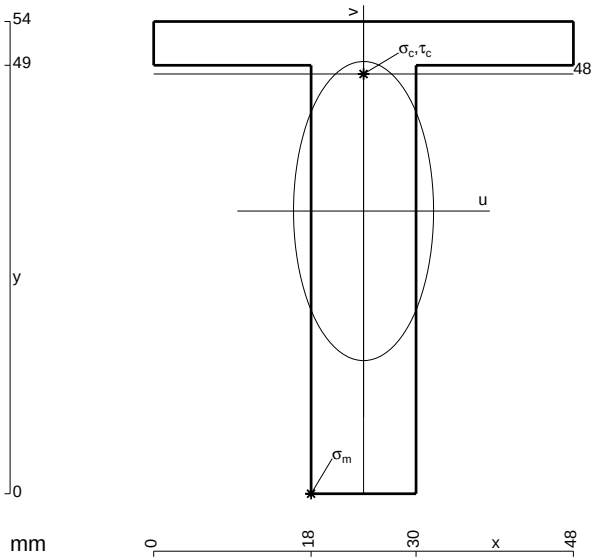
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 828. mm^2
J_u = 242396. mm^4
J_v = 53136. mm^4
y_g = 32.33 mm
N = -3198. N
T_y = -1599. N
M_x = 1751750. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -32.33 mm
σ_m = N/A-Mv/J_u = 229.8 N/mm^2
x_c = 24. mm
y_c = 48. mm
v_c = 15.67 mm
σ_c = N/A-Mv/J_u = -117.1 N/mm^2
τ_c = 2.636 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.2 N/mm^2
S^* = 4796. mm^3

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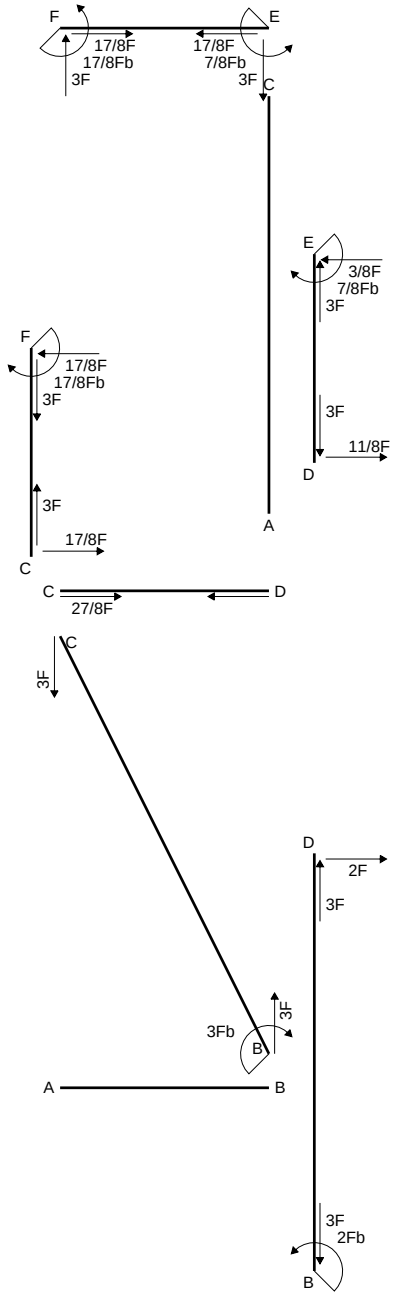
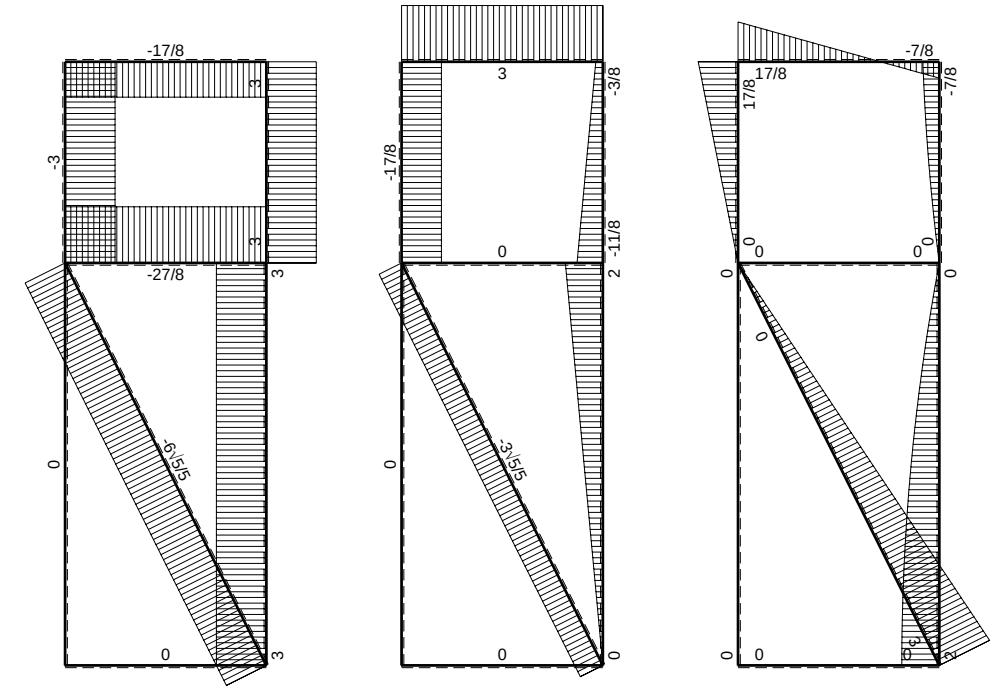
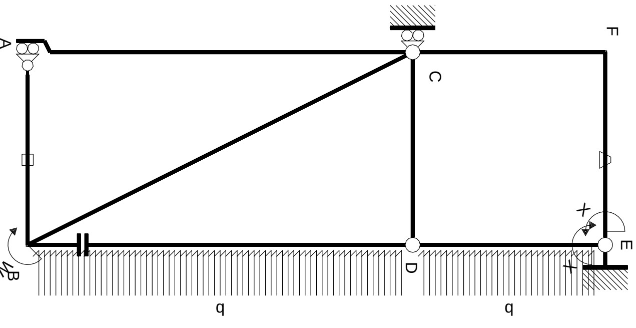
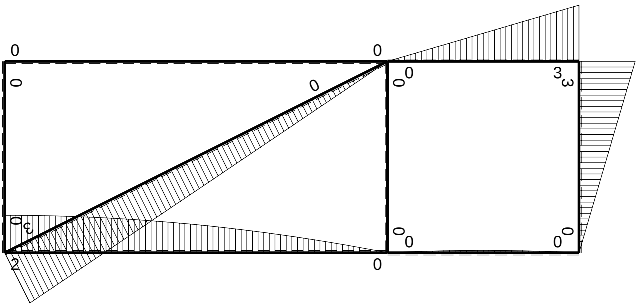

F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6√5/5
3
0
0
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
3√5/5

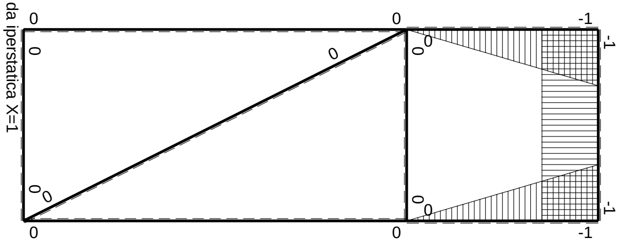

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

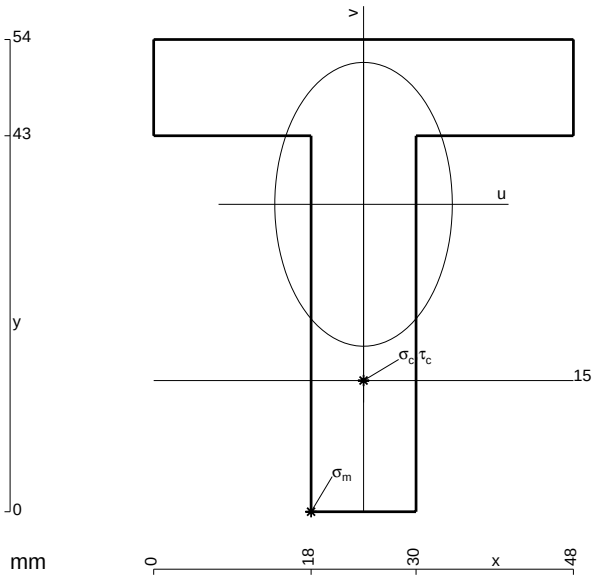
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 1044. mm^2
J_u = 275075. mm^4
J_v = 107568. mm^4
y_g = 35.16 mm
N = -3193. N
T_y = -1597. N
M_x = 1892100. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.16 mm
$\sigma_m = N/A - Mv/J_u$ = 238.8 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.16 mm
$\sigma_c = N/A - Mv/J_u$ = 135.6 N/mm^2
τ_c = 2.408 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135.6 N/mm^2
S^* = 4978. mm^3

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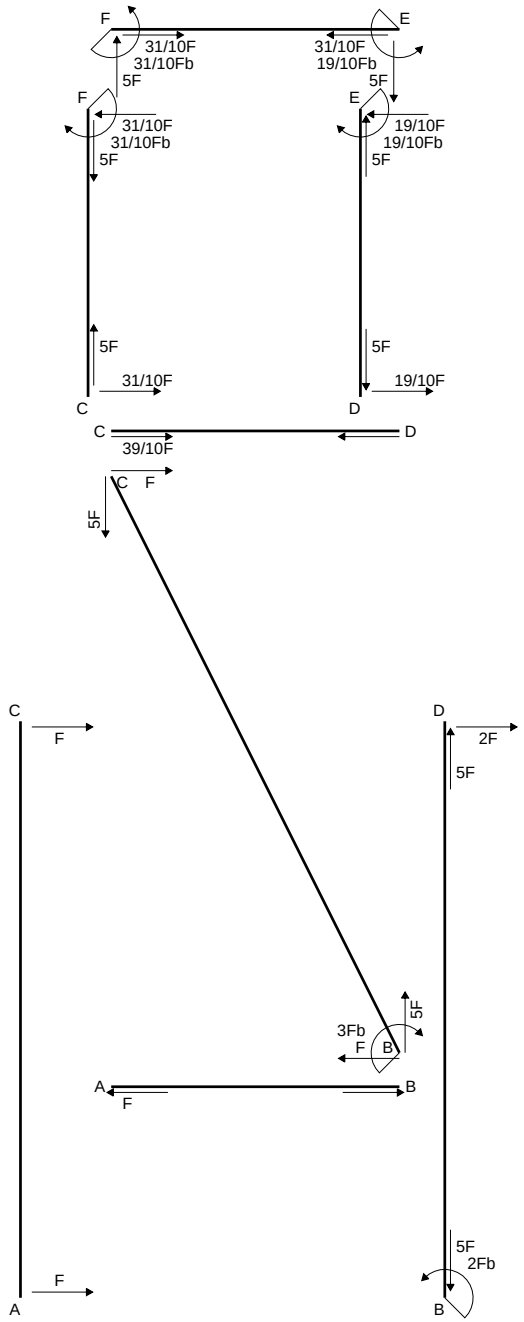
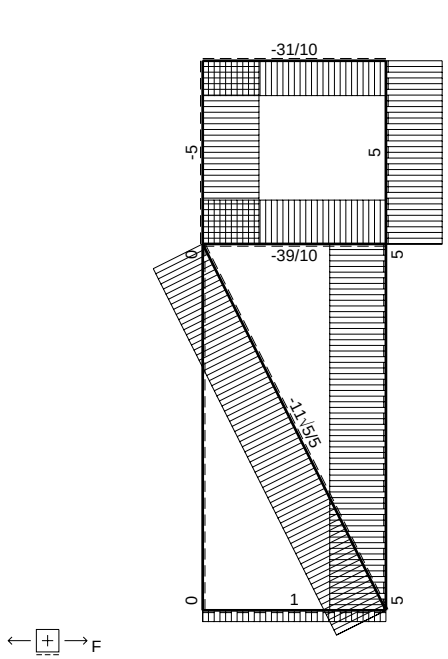
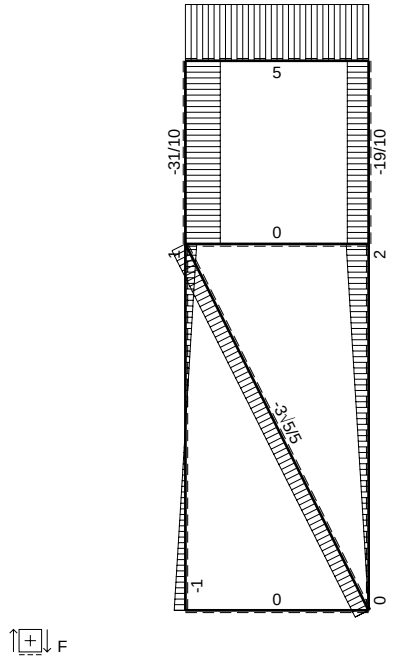
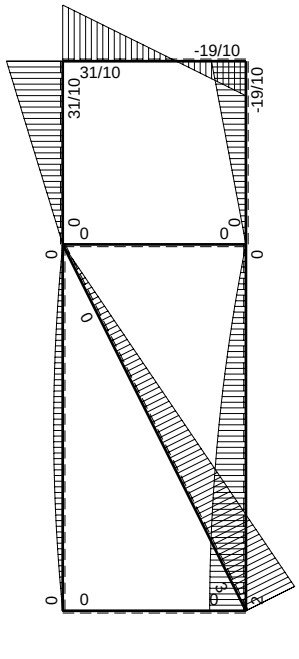
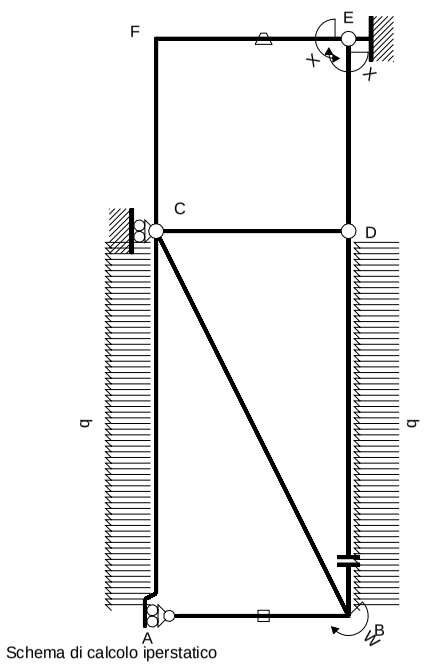

F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B

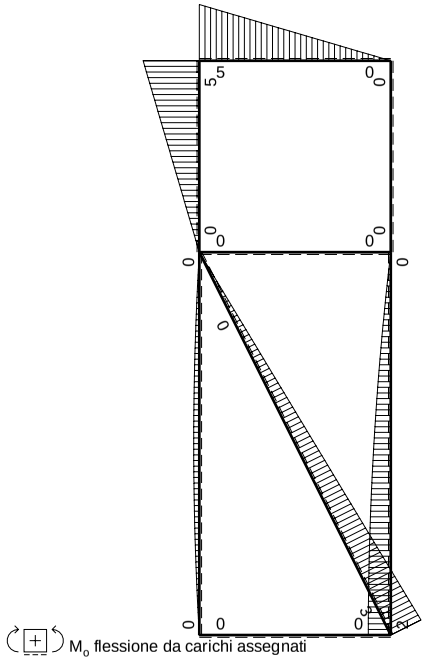

-31/10
-39/10
-11√5/5
F


-31/10
-19/10
-3√5/5
F

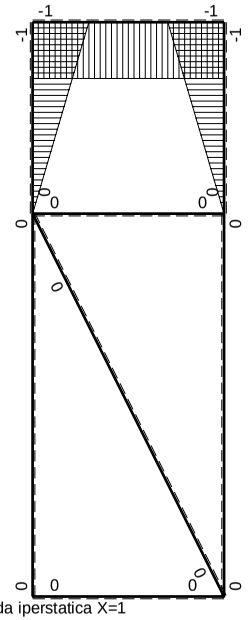

-19/10
31/10
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
		totali					-19/6Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212323 ESΣ09.xxxx.165

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

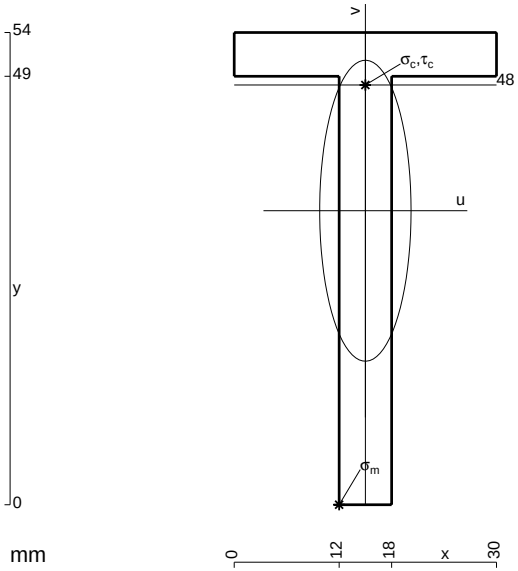
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$



A = 444. mm^2
J_u = 131544. mm^4
J_v = 12132. mm^4
y_g = 33.62 mm
N = -2263. N
T_y = -617.2 N
M_x = 786600. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -33.62 mm
$\sigma_m = N/A - Mv/J_u$ = 196. N/mm^2
x_c = 15. mm
y_c = 48. mm
v_c = 14.38 mm
$\sigma_c = N/A - Mv/J_u$ = -91.08 N/mm^2
τ_c = 2.167 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 91.15 N/mm^2
S^* = 2771. mm^3

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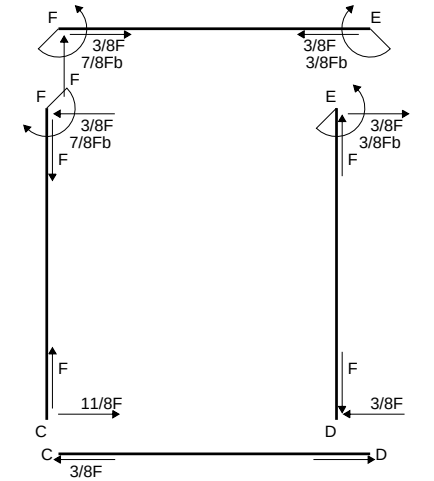
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
F
11/8F
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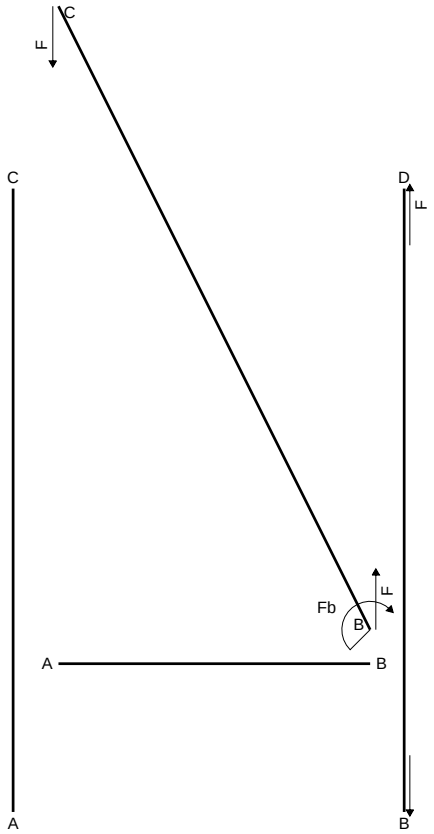
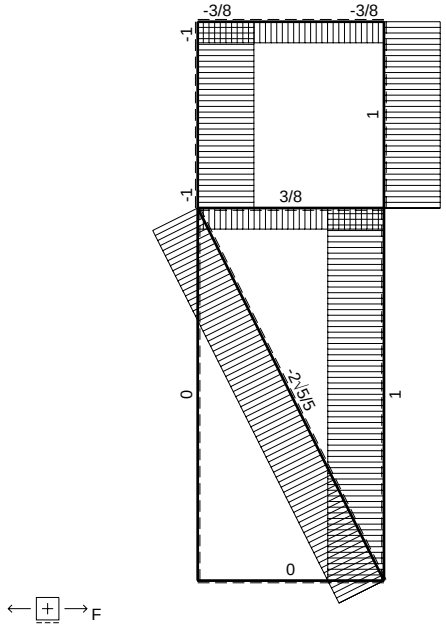
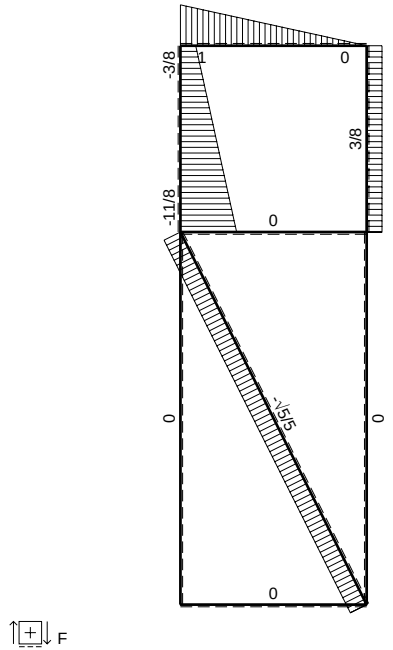
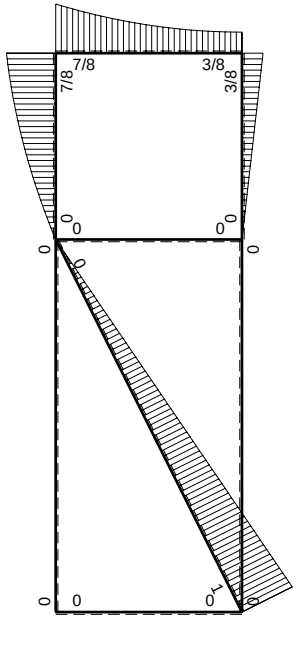
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
F
B
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

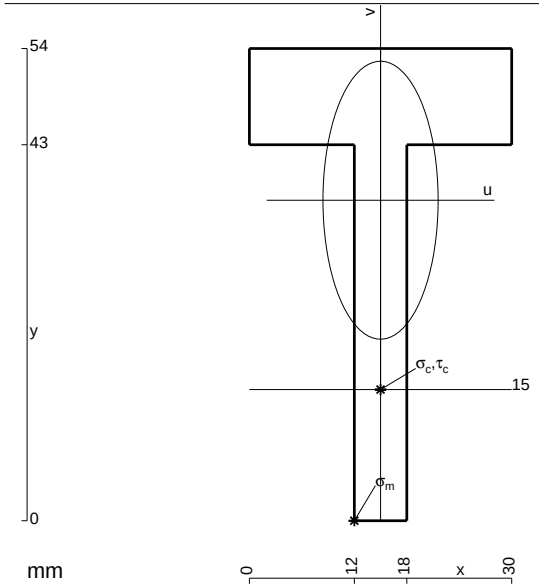
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 588. mm^2
J_u = 148637. mm^4
J_v = 25524. mm^4
y_g = 36.65 mm
N = -1279. N
T_y = -639.5 N
M_x = 858000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.65 mm
σ_m = N/A-Mv/J_u = 209.4 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.65 mm
σ_c = N/A-Mv/J_u = 122.8 N/mm^2
τ_c = 1.881 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.9 N/mm^2
S^* = 2624. mm^3

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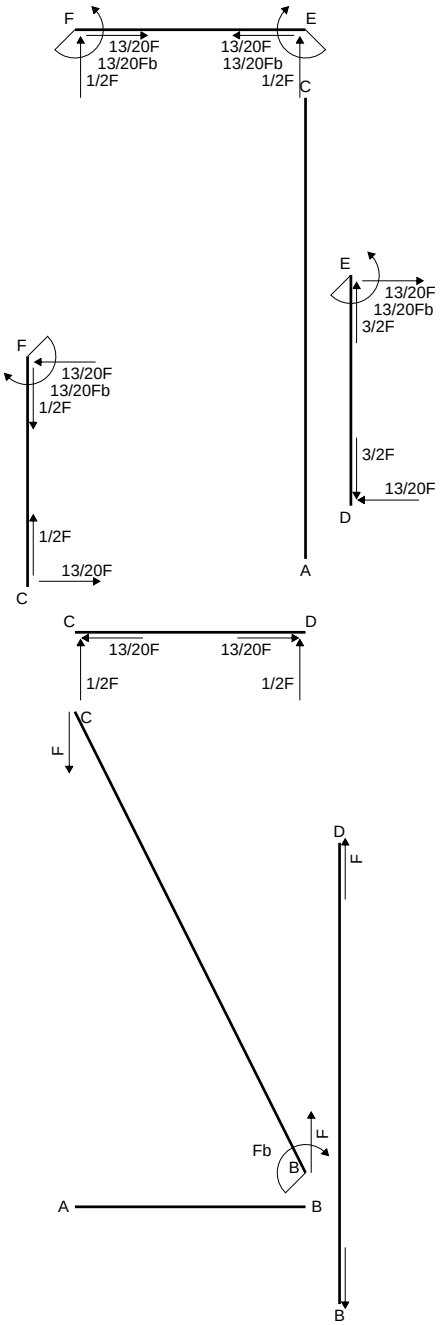
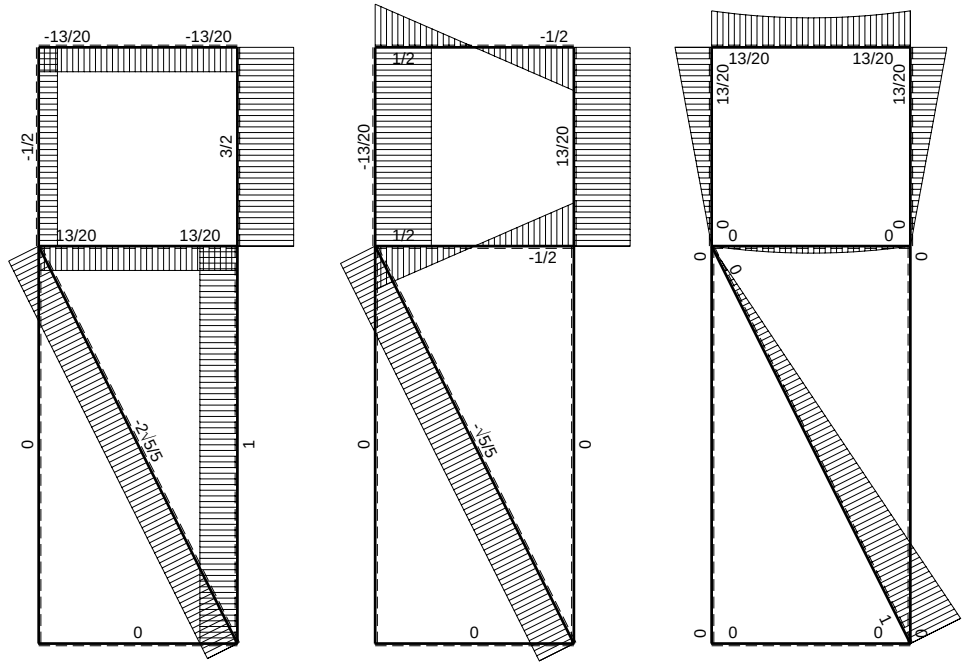
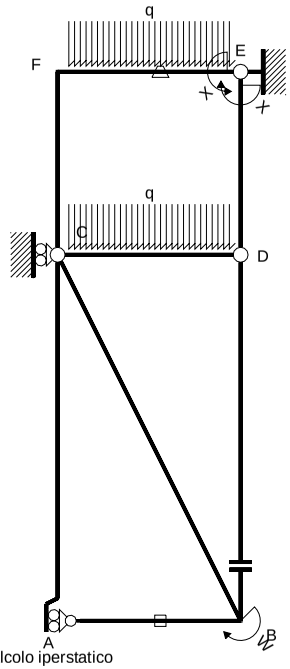
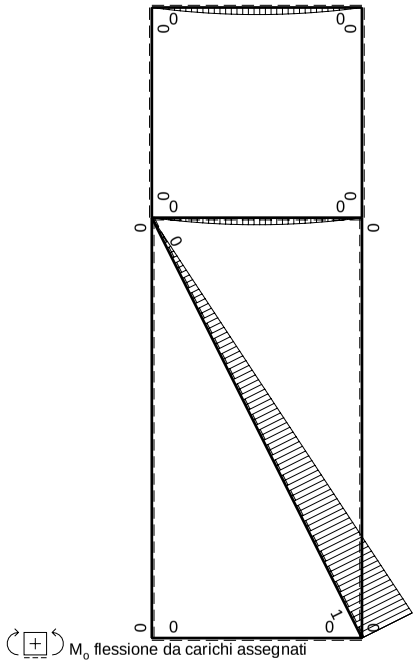

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

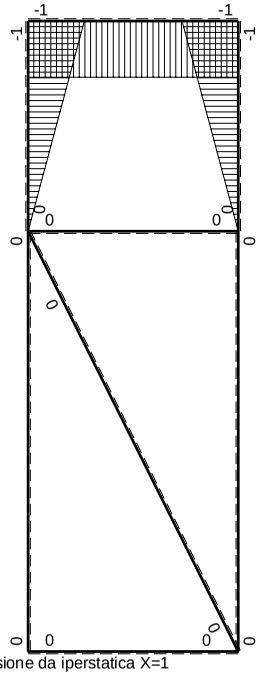

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

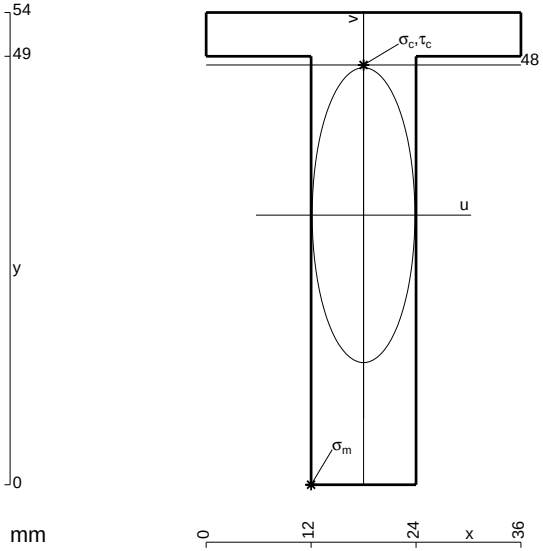
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 768. mm^2
J_u = 218489. mm^4
J_v = 26496. mm^4
y_g = 30.83 mm
N = -2200. N
T_y = -1100. N
M_x = 1574400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -30.83 mm
$\sigma_m = N/A - Mv/J_u$ = 219.3 N/mm^2
x_c = 18. mm
y_c = 48. mm
v_c = 17.17 mm
$\sigma_c = N/A - Mv/J_u$ = -126.6 N/mm^2
τ_c = 1.65 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.6 N/mm^2
S^* = 3933. mm^3

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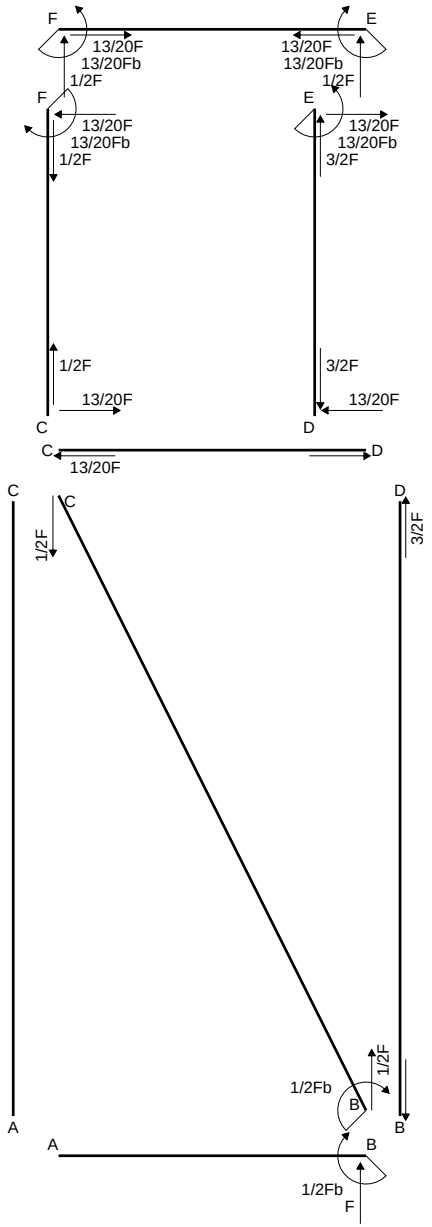
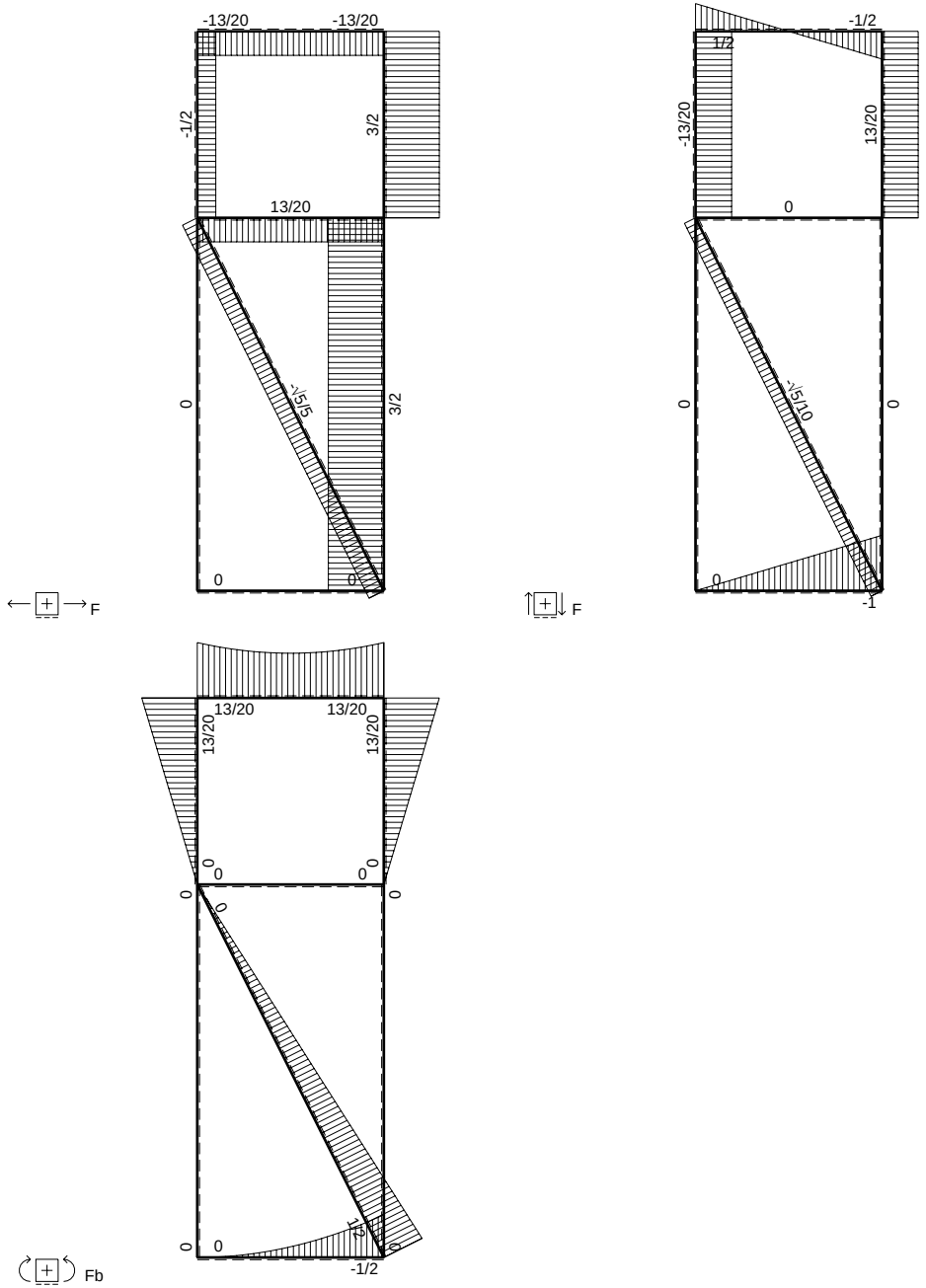
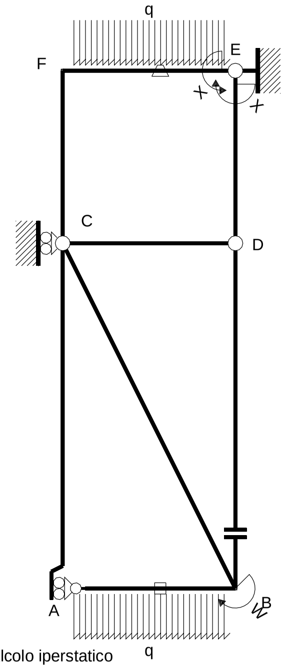
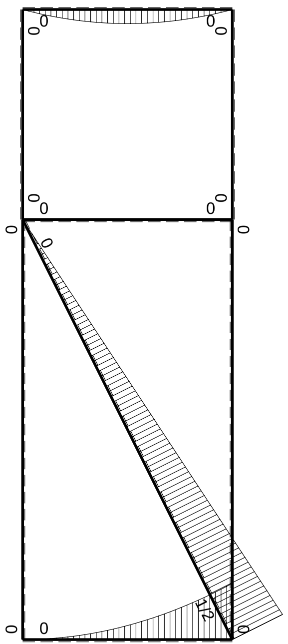

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

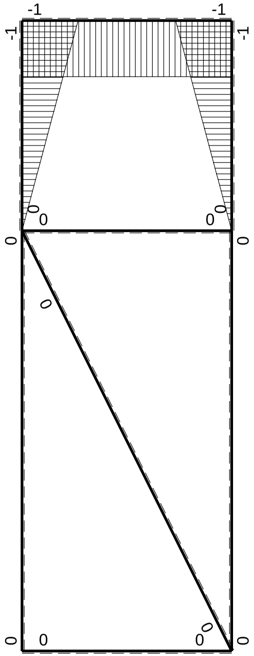

-13/20
13/20
-1/2
3/2
0
-√5/5
-√5/10
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

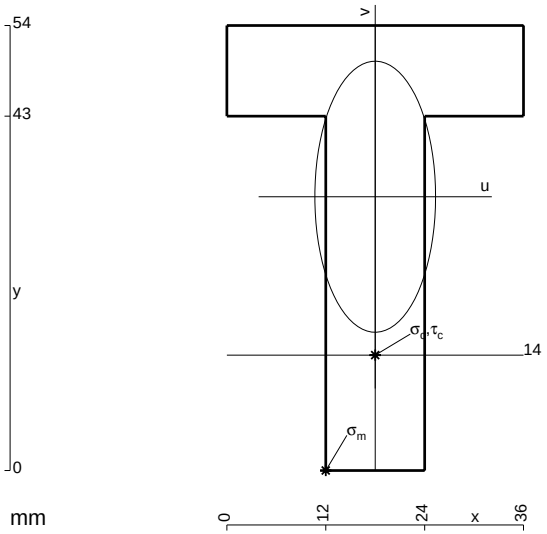
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 912. mm^2
J_u = 246834. mm^4
J_v = 48960. mm^4
y_g = 33.22 mm
T_y = -5020. N
M_x = -1706800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.22 mm
$\sigma_m = -Mv/J_u$ = -229.7 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.22 mm
$\sigma_c = -Mv/J_u$ = -132.9 N/mm^2
τ_c = 7.467 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.6 N/mm^2
S^* = 4406. mm^3

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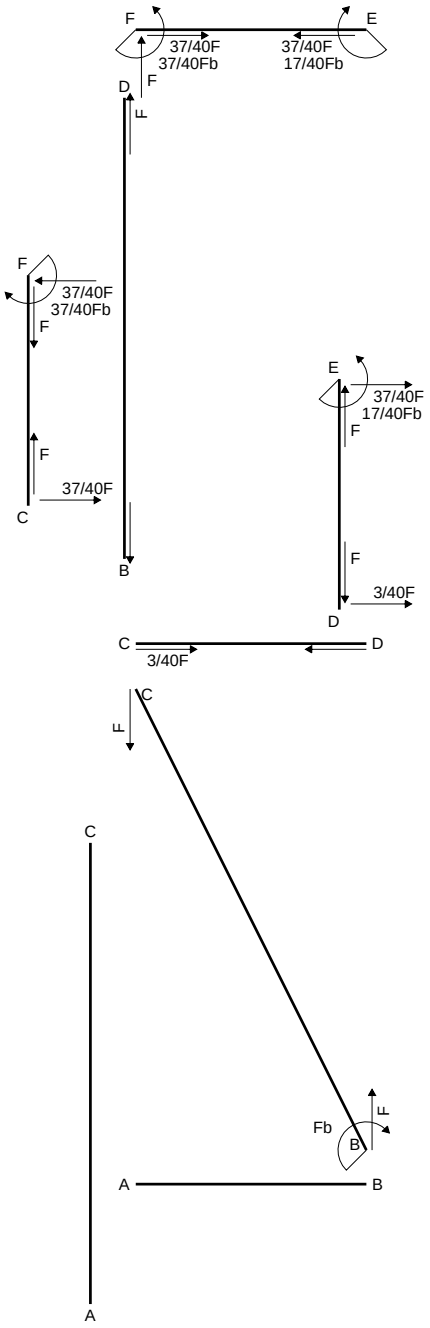
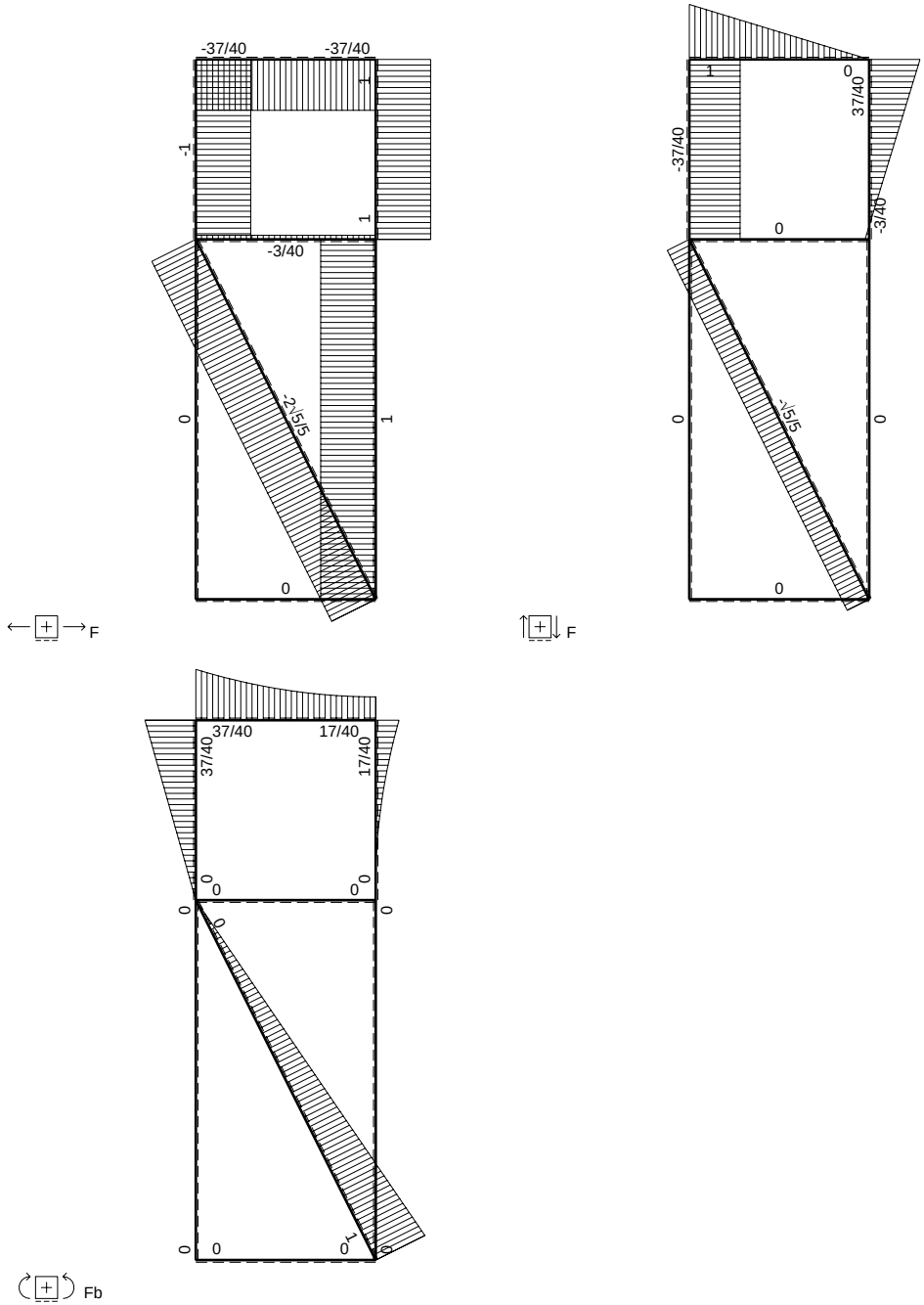

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D
C
F
C
Fb
B
F
A
B
A


-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
F
1
0
37/40
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-√5/5
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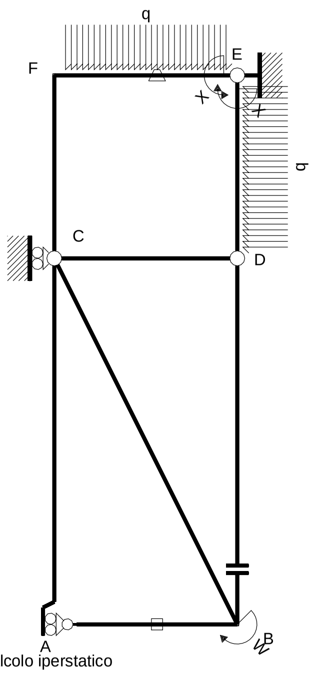
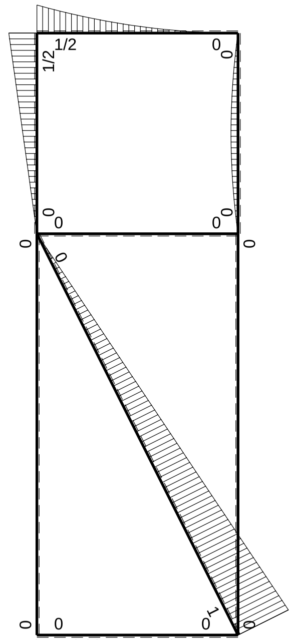
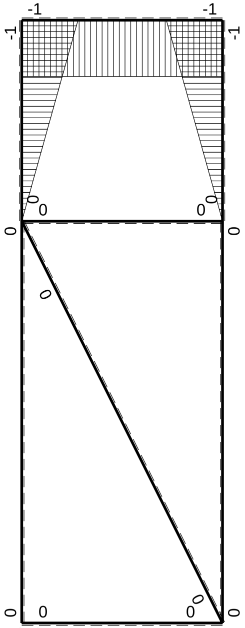
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$17/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-17/40Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

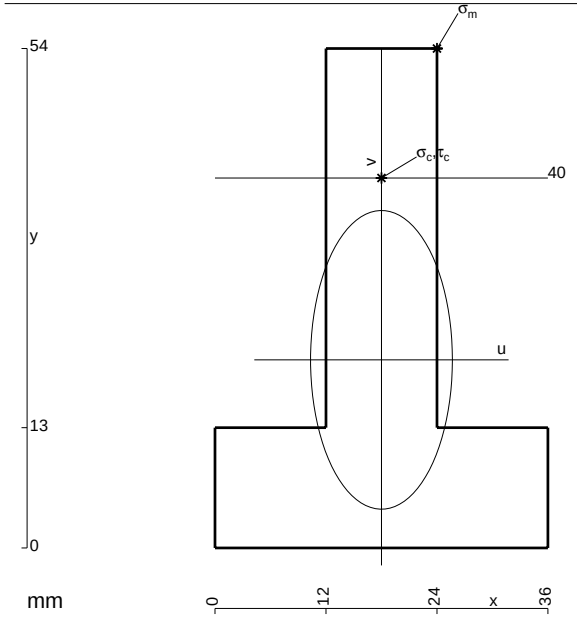
$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 960. mm^2
J_u = 250363. mm^4
J_v = 56448. mm^4
y_g = 20.34 mm
N = -2191. N
T_y = -1096. N
M_x = 1764000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.66 mm
$\sigma_m = N/A - Mv/J_u$ = -239.5 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.66 mm
$\sigma_c = N/A - Mv/J_u$ = -140.8 N/mm^2
τ_c = 1.634 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.8 N/mm^2
S^* = 4479. mm^3

ESΣ09.xxxx.169

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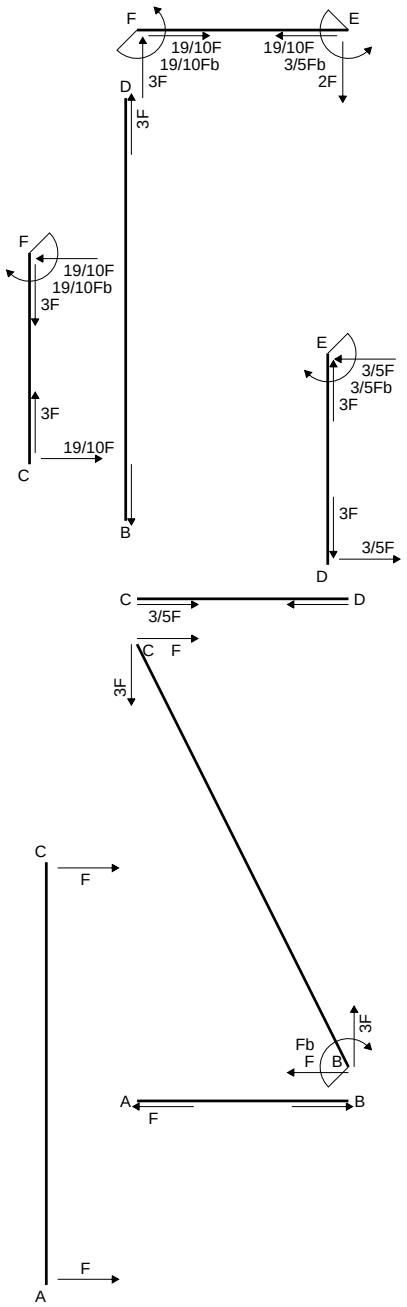
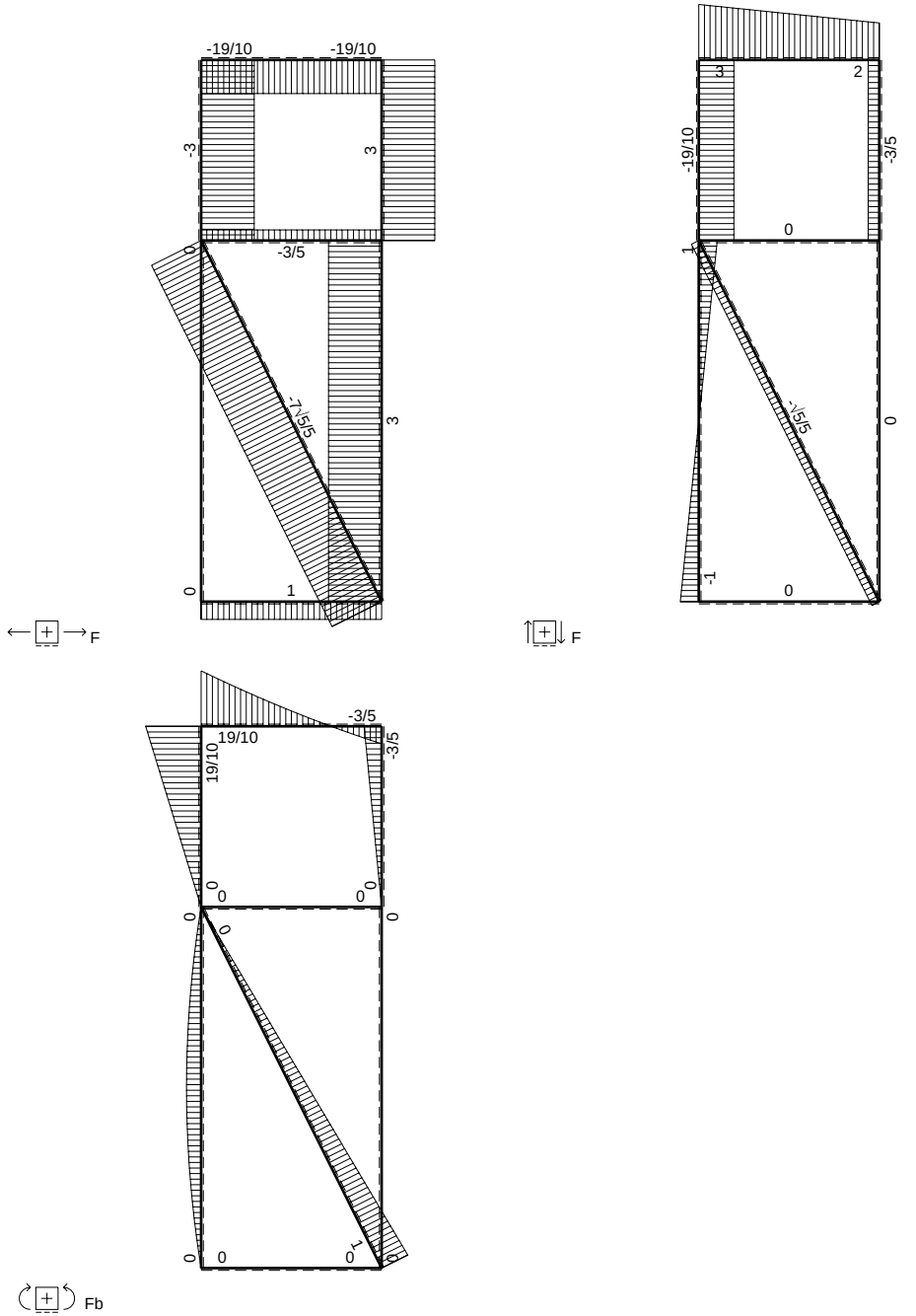

F
E
19/10F
19/10Fb
19/10F
3/5Fb
3F
2F
D
3F
F
19/10F
19/10Fb
3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
3F
B
D
3/5F
C
D
3/5F
C F
3F
C
F
Fb
F
B
3F
A
B
F
F
A


-19/10
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-3
3
-3/5
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-7\5/5
1
0
F
-19/10
3
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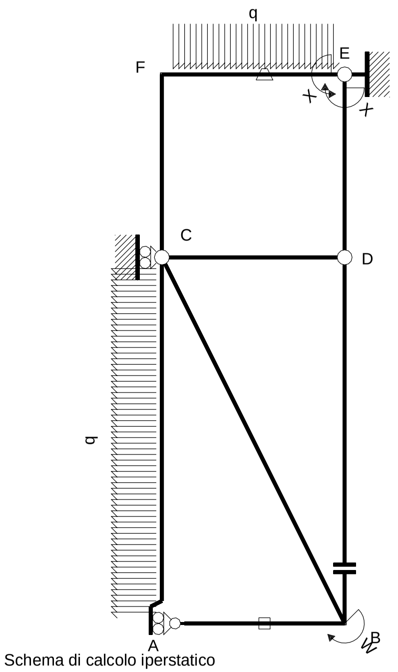
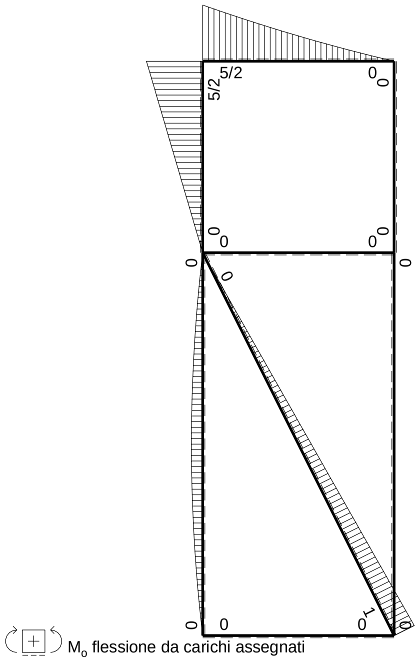
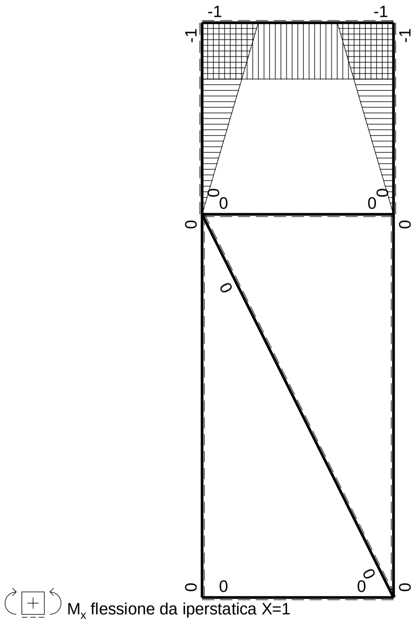
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

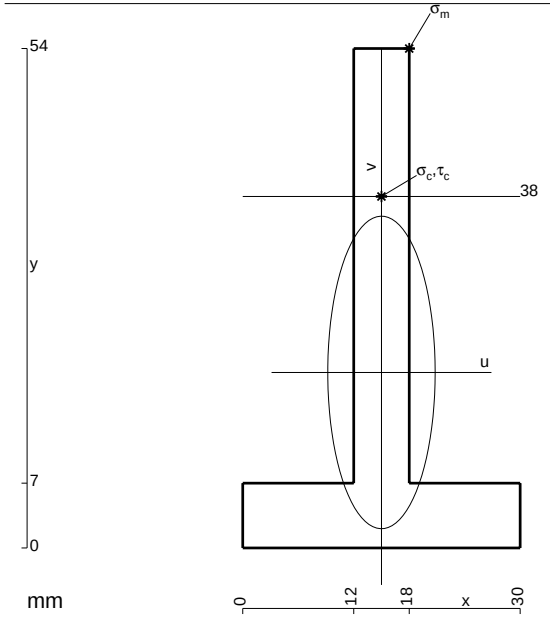
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 492. mm^2
J_u = 140516. mm^4
J_v = 16596. mm^4
y_g = 18.98 mm
N = -6167. N
T_y = -881. N
M_x = 748600. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 35.02 mm
$\sigma_m = N/A - Mv/J_u$ = -199.1 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 19.02 mm
$\sigma_c = N/A - Mv/J_u$ = -113.9 N/mm^2
τ_c = 2.711 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114. N/mm^2
S^* = 2594. mm^3

ESΣ09.xxxx.170

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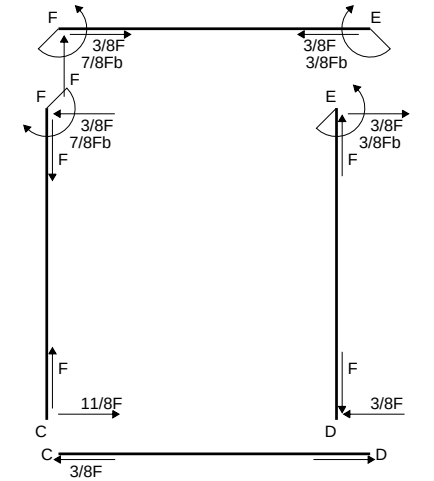
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E
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7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
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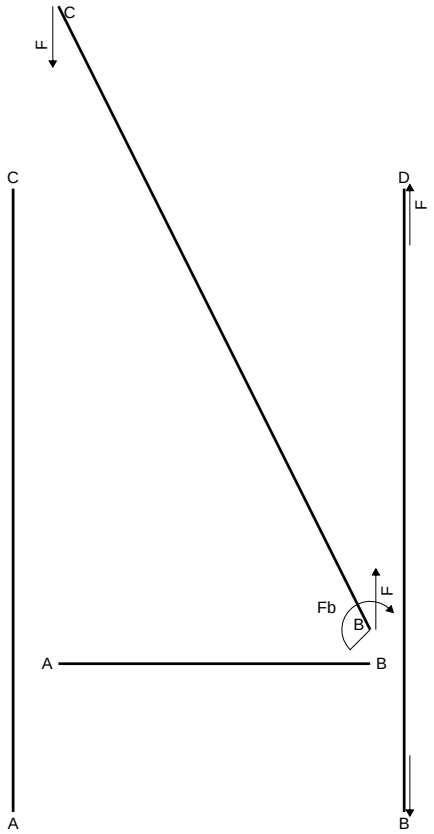
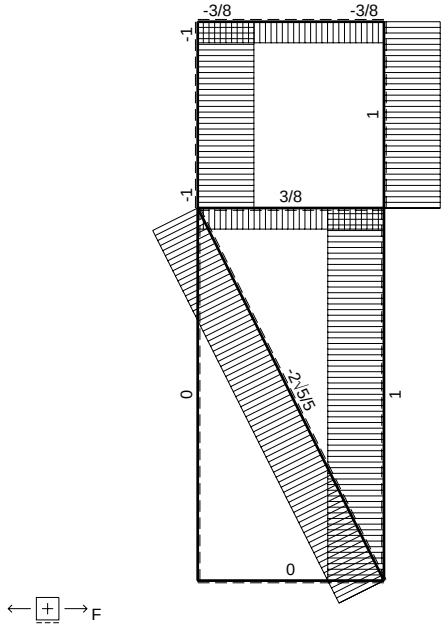
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B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-3/8
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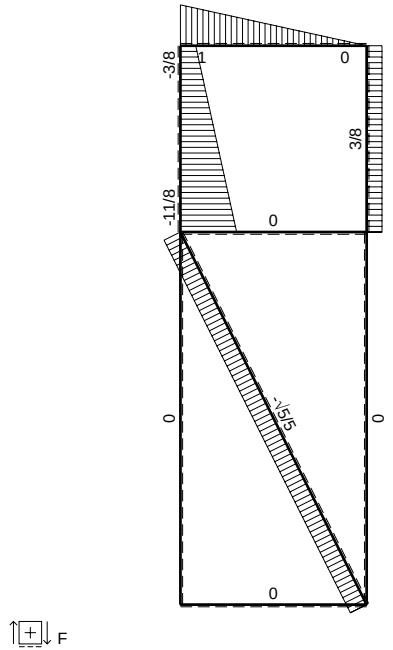
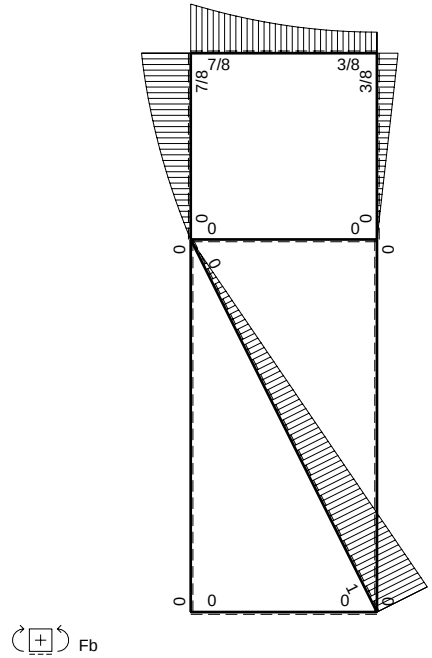
-1
1
3/8
0
-2√5/5
1
1
0
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 280952 ESΣ09.xxxx.171

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

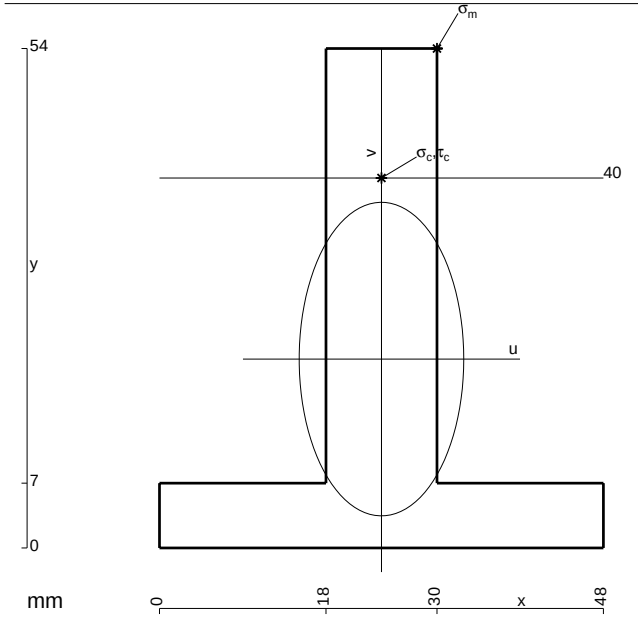
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 900. mm^2
J_u = 258693. mm^4
J_v = 71280. mm^4
y_g = 20.42 mm
N = -3461. N
T_y = -1731. N
M_x = 1586700. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.58 mm
$\sigma_m = N/A - Mv/J_u$ = -209.8 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 19.58 mm
$\sigma_c = N/A - Mv/J_u$ = -123.9 N/mm^2
τ_c = 2.49 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124. N/mm^2
S^* = 4465. mm^3

ESΣ09.xxxx.171

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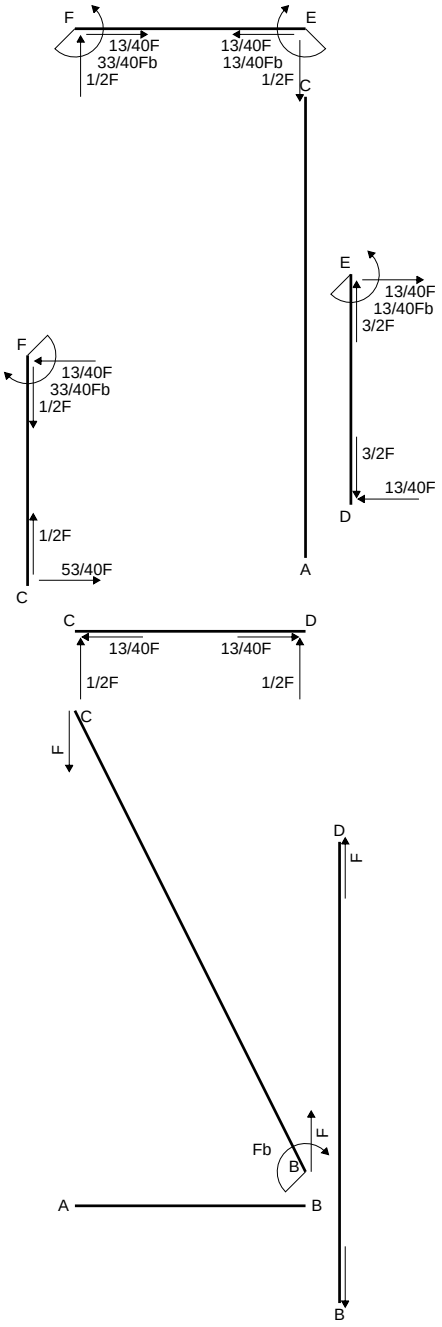
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
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13/40F
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1/2F
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D
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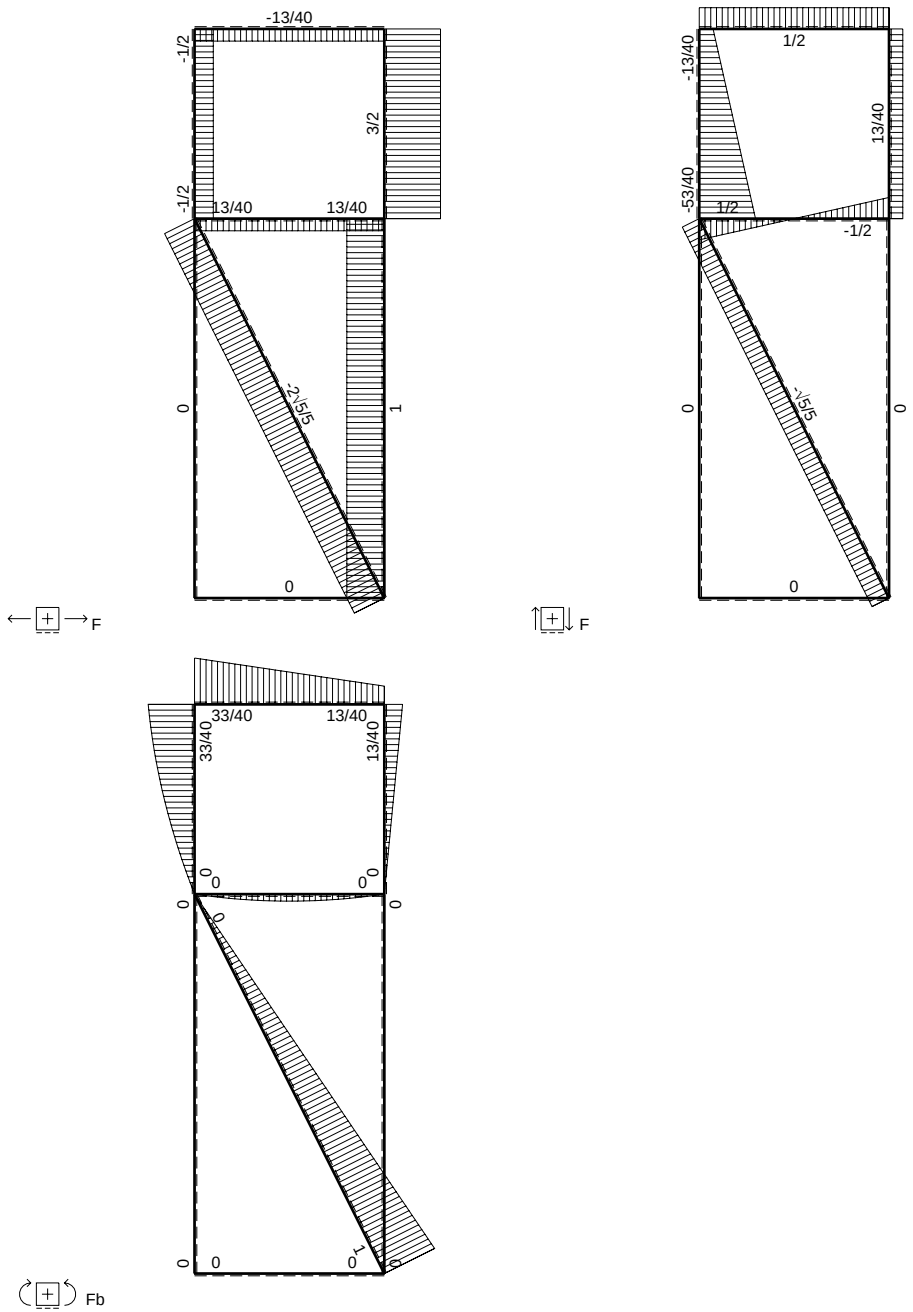
B
B


-13/40
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3/2
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0
1
0
F
-13/40
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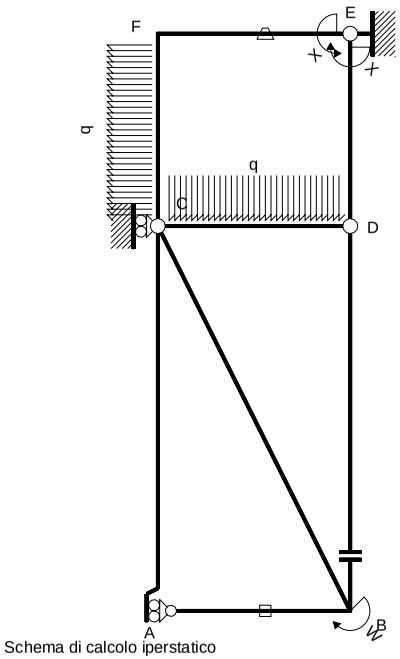
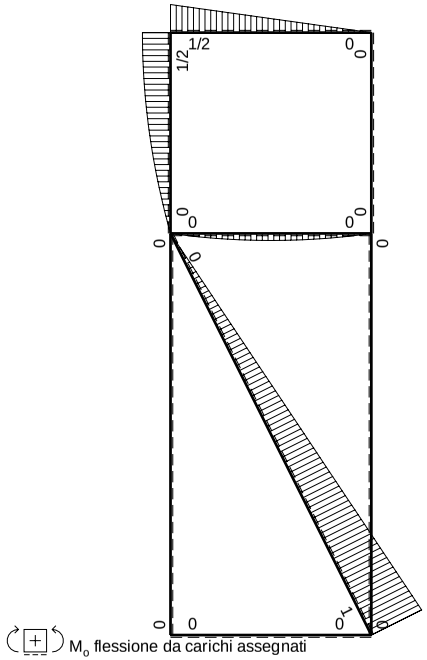
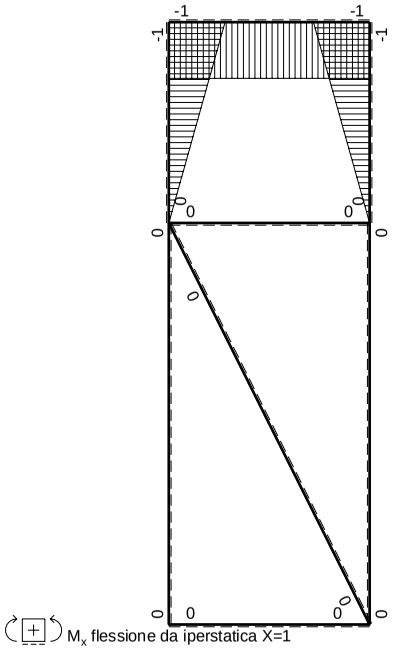
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211554 ESΣ09.xxxx.172

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

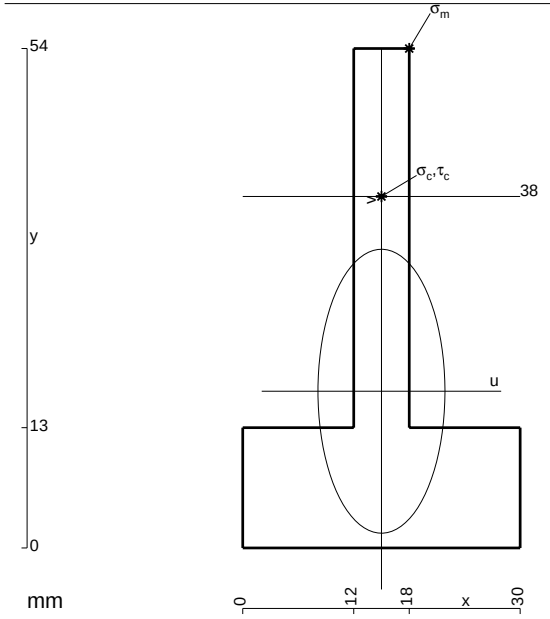
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211554 ESΣ09.xxxx.172



A = 636. mm^2
J_u = 149922. mm^4
J_v = 29988. mm^4
y_g = 16.94 mm
N = -1744. N
T_y = -872.1 N
M_x = 877500. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 37.06 mm
$\sigma_m = N/A - Mv/J_u$ = -219.6 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.06 mm
$\sigma_c = N/A - Mv/J_u$ = -126. N/mm^2
τ_c = 2.704 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.1 N/mm^2
S^* = 2789. mm^3

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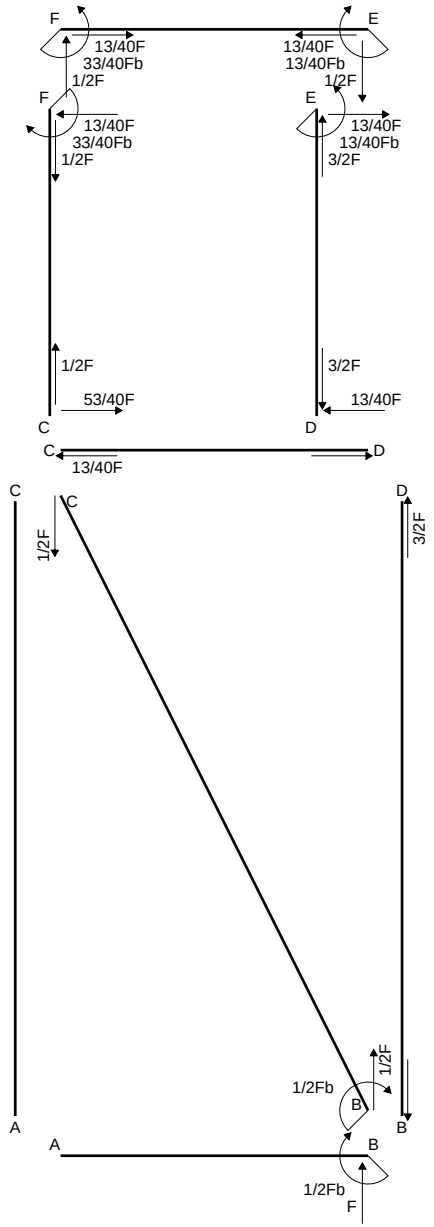
F
E
13/40F
33/40Fb
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13/40F
13/40Fb
1/2F
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E
13/40F
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13/40Fb
3/2F
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53/40F
3/2F
13/40F
C
D
C
D
13/40F
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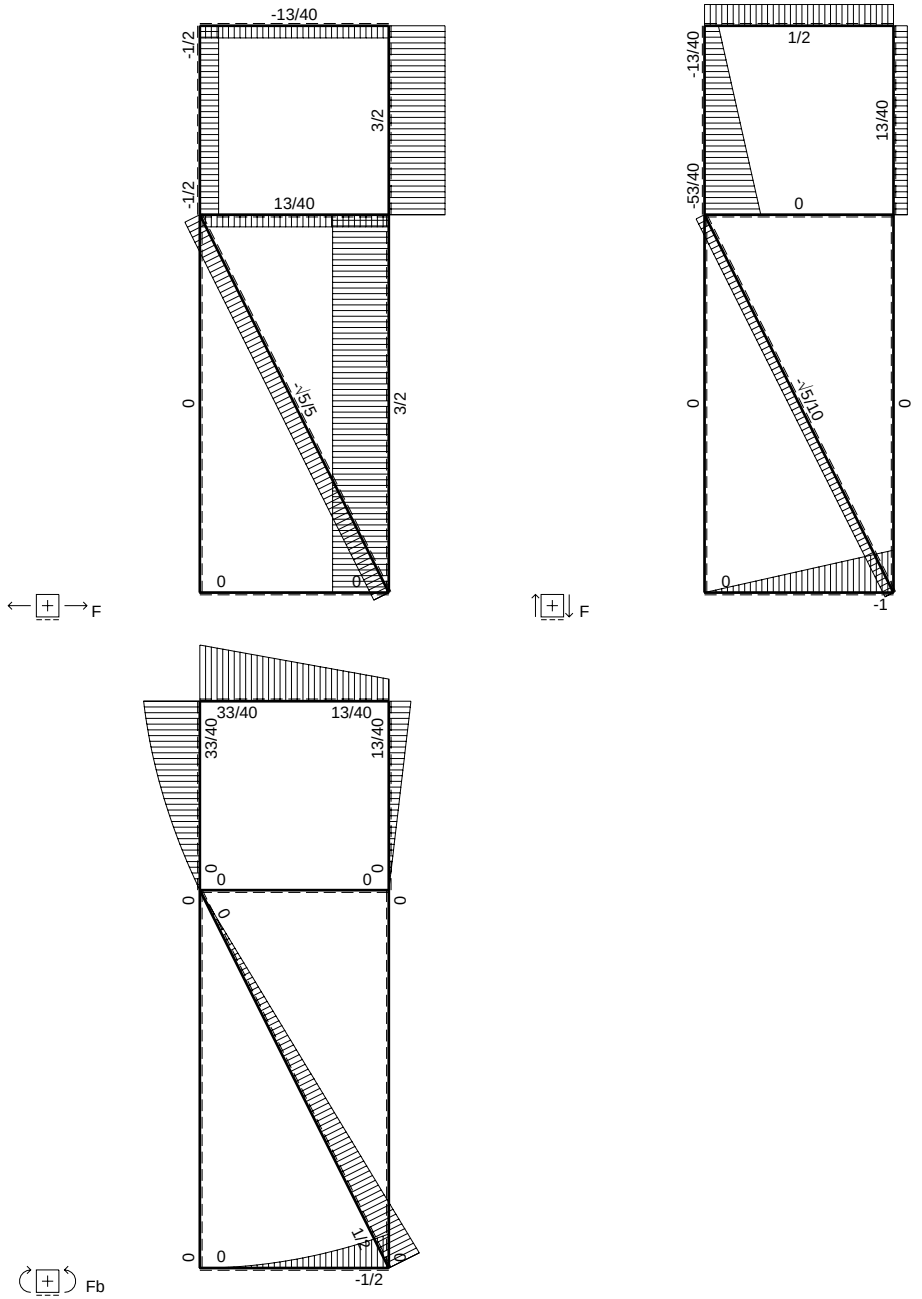
1/2F
A
B
A
B
1/2Fb
F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3/2
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3/2
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0
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1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212585 ESΣ09.xxxx.173

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

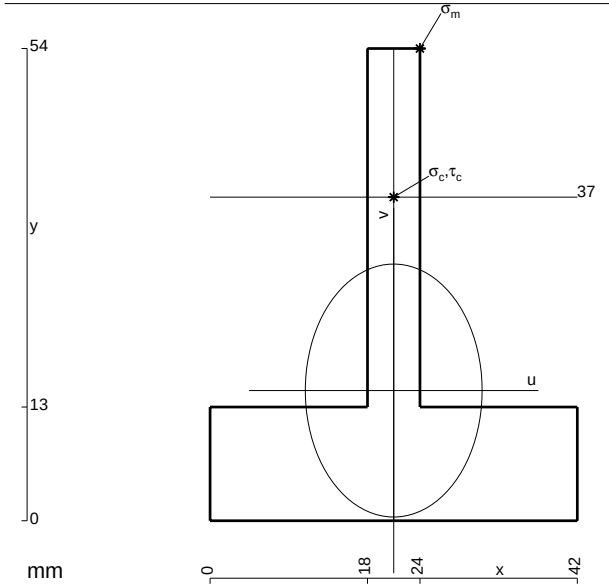
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 212585 ESΣ09.xxxx.173



A = 792. mm^2
J_u = 165782. mm^4
J_v = 81000. mm^4
y_g = 14.89 mm
T_y = -3970. N
M_x = -972650. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 39.11 mm
$\sigma_m = -Mv/J_u$ = 229.5 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.11 mm
$\sigma_c = -Mv/J_u$ = 129.7 N/mm^2
τ_c = 12.46 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.5 N/mm^2
S^* = 3123. mm^3

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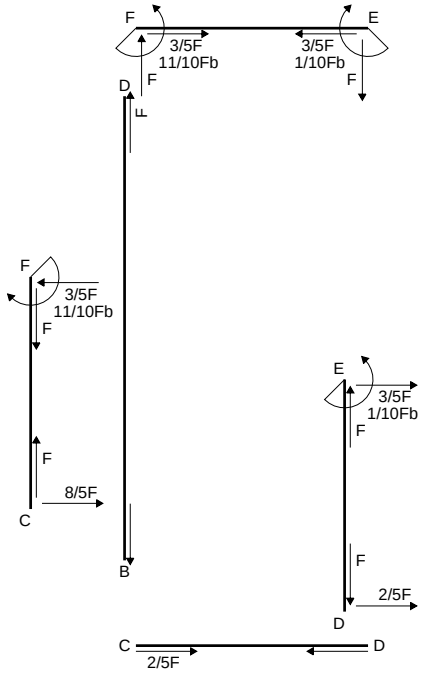
F
E
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11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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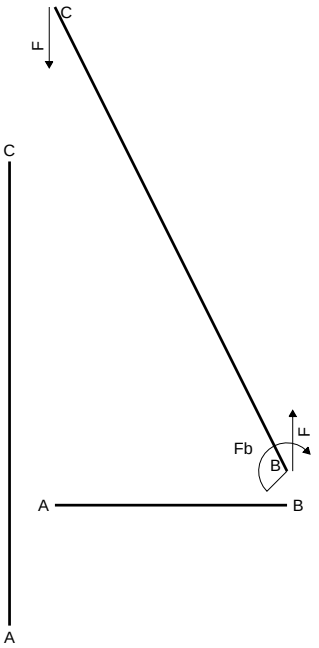
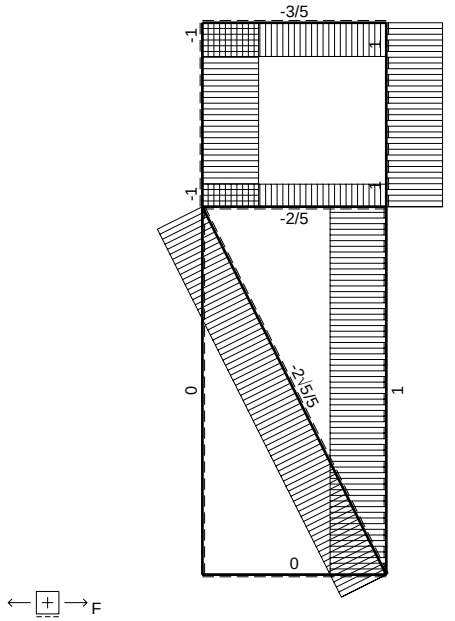
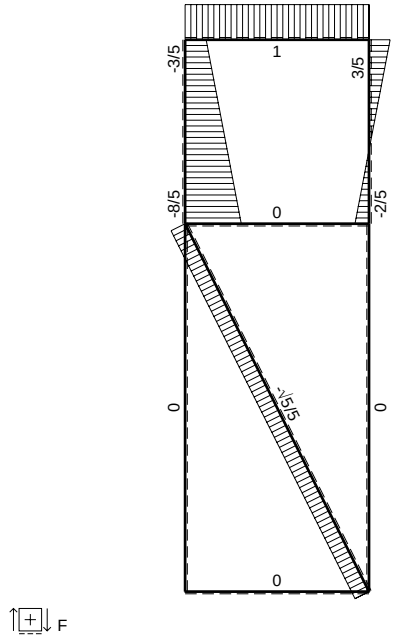
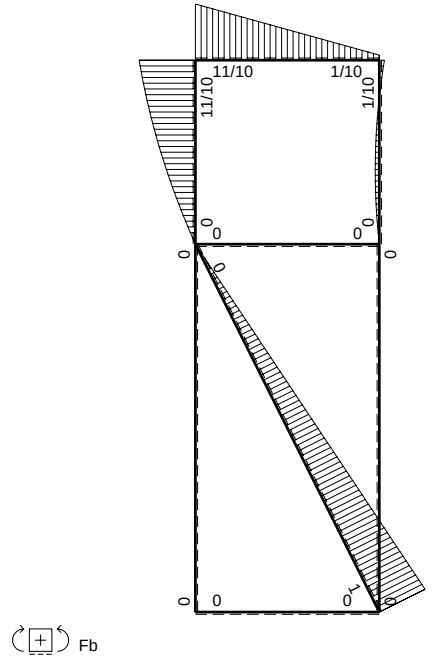
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5
0
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3XLb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	XLb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3XLb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
totali							1/6Fb^2/EJ	5/3XLb/EJ
iperstatica $X=W_{EF}$							-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212046 ESΣ09.xxxx.174

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

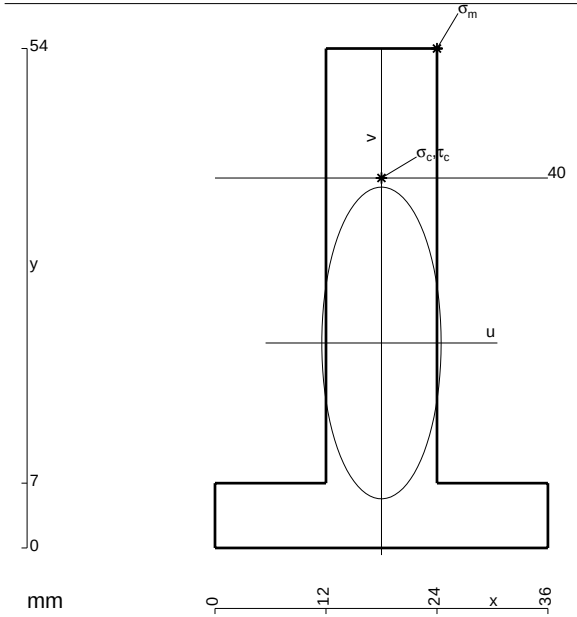
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 816. mm^2
J_u = 231827. mm^4
J_v = 33984. mm^4
y_g = 22.16 mm
N = -2898. N
T_y = -1449. N
M_x = 1717200. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 31.84 mm
$\sigma_m = N/A - Mv/J_u$ = -239.4 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 17.84 mm
$\sigma_c = N/A - Mv/J_u$ = -135.7 N/mm^2
τ_c = 2.173 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.7 N/mm^2
S^* = 4173. mm^3

ESΣ09.xxxx.174

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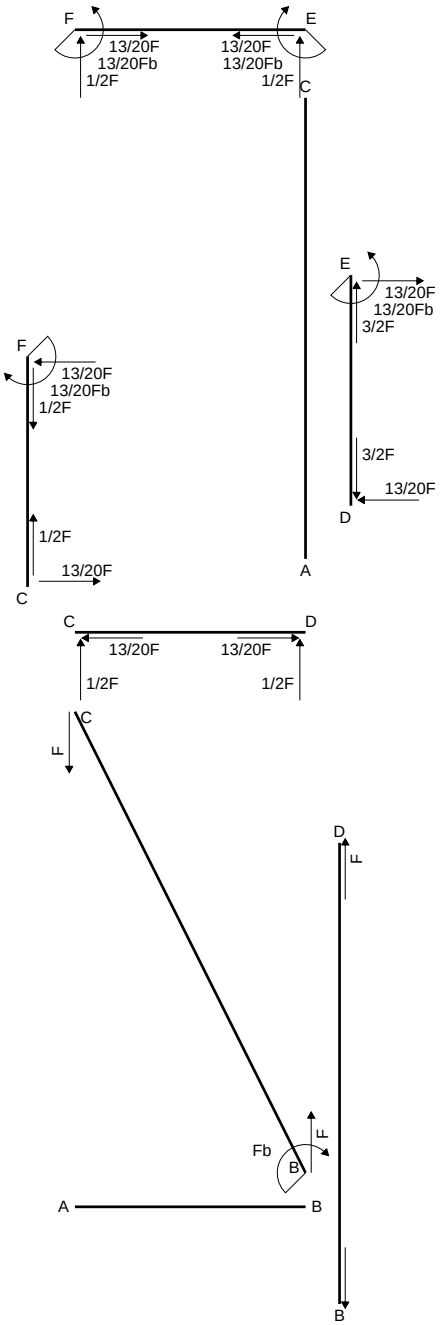
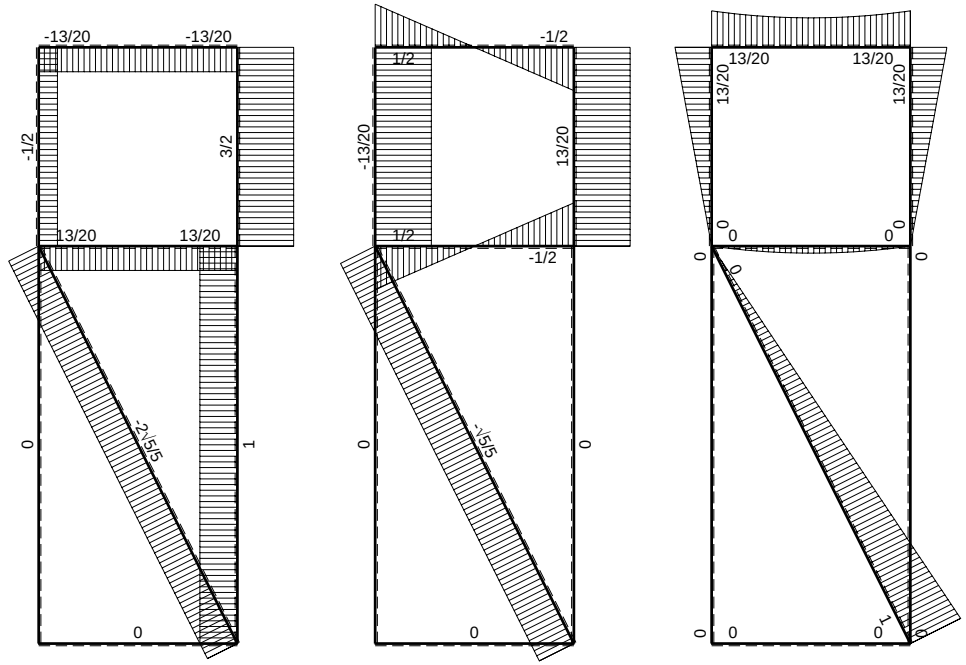
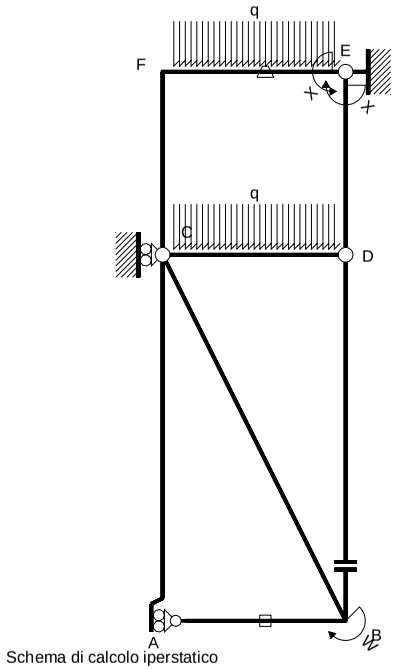
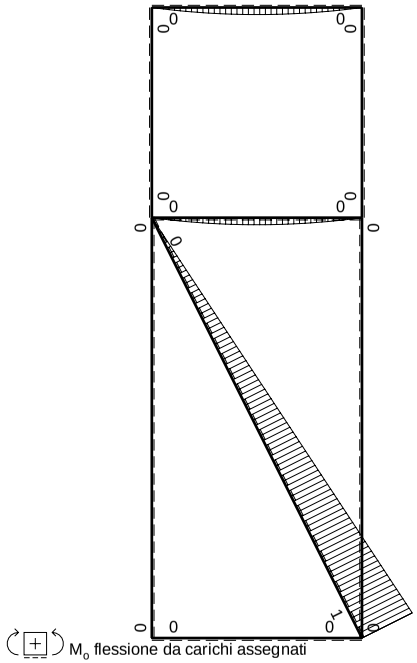

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

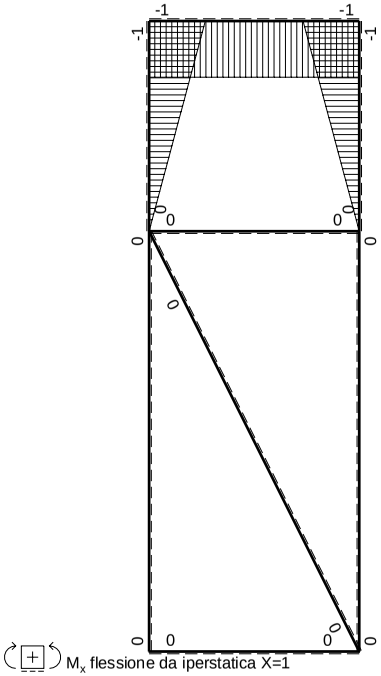

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981992 ESΣ09.xxxx.175

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

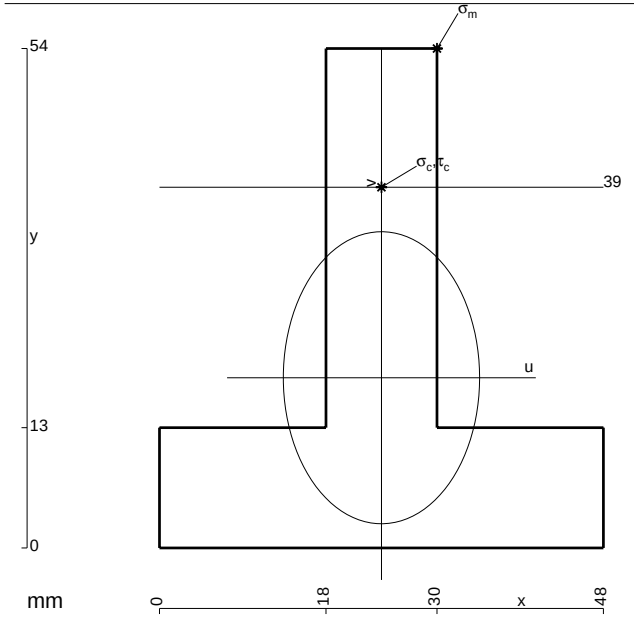
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 981992 ESΣ09.xxxx.175



A = 1116. mm^2
J_u = 278255. mm^4
J_v = 125712. mm^4
y_g = 18.4 mm
N = -2424. N
T_y = -1212. N
M_x = 1544700. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 35.6 mm
$\sigma_m = N/A - Mv/J_u$ = -199.8 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.6 mm
$\sigma_c = N/A - Mv/J_u$ = -116.5 N/mm^2
τ_c = 1.836 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.6 N/mm^2
S^* = 5057. mm^3

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ESΣ09.xxxx.175

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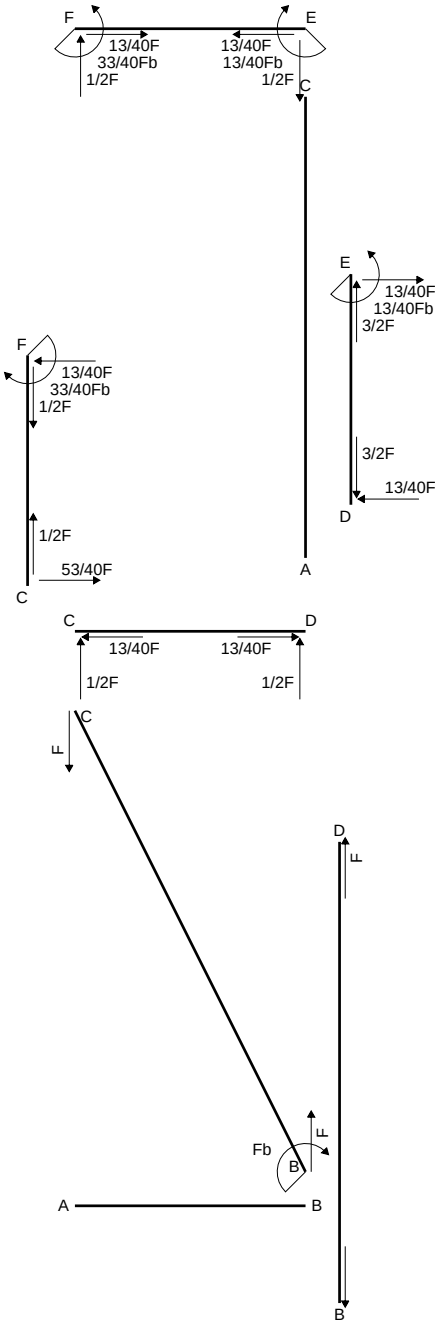
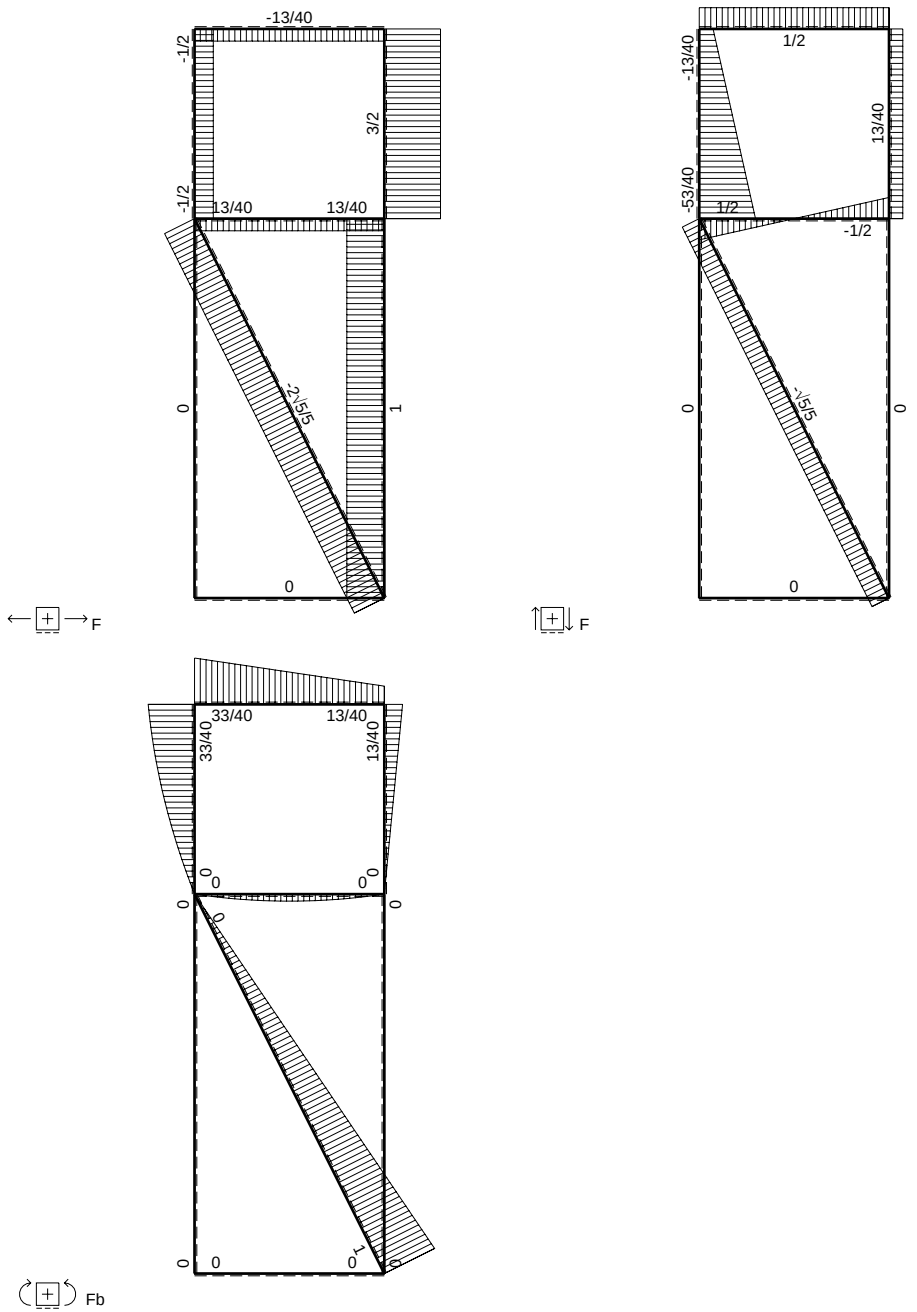

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B


-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
0
1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XLb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb²/EJ	XLb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb²/EJ	1/3XLb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							13/24Fb²/EJ	5/3XLb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 959735 ESΣ09.xxxx.176

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

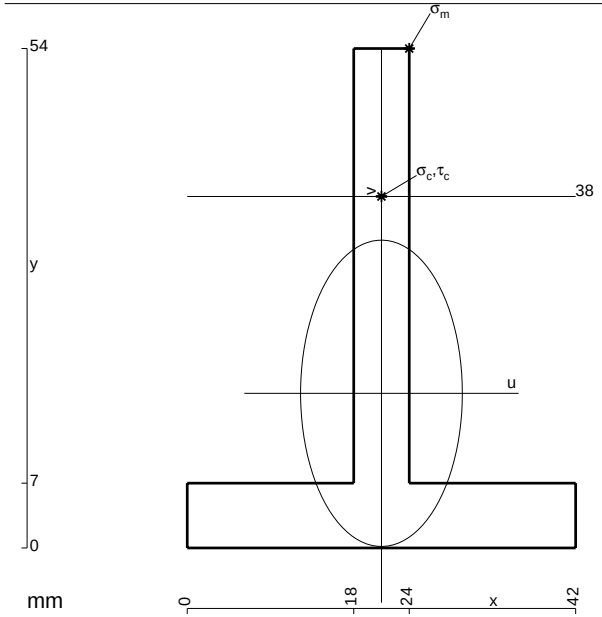
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 576. mm^2
J_u = 158042. mm^4
J_v = 44064. mm^4
y_g = 16.72 mm
N = -1306. N
T_y = -652.9 N
M_x = 876000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 37.28 mm
$\sigma_m = N/A - Mv/J_u$ = -208.9 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 21.28 mm
$\sigma_c = N/A - Mv/J_u$ = -120.2 N/mm^2
τ_c = 1.936 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.3 N/mm^2
S^* = 2811. mm^3

ESΣ09.xxxx.176

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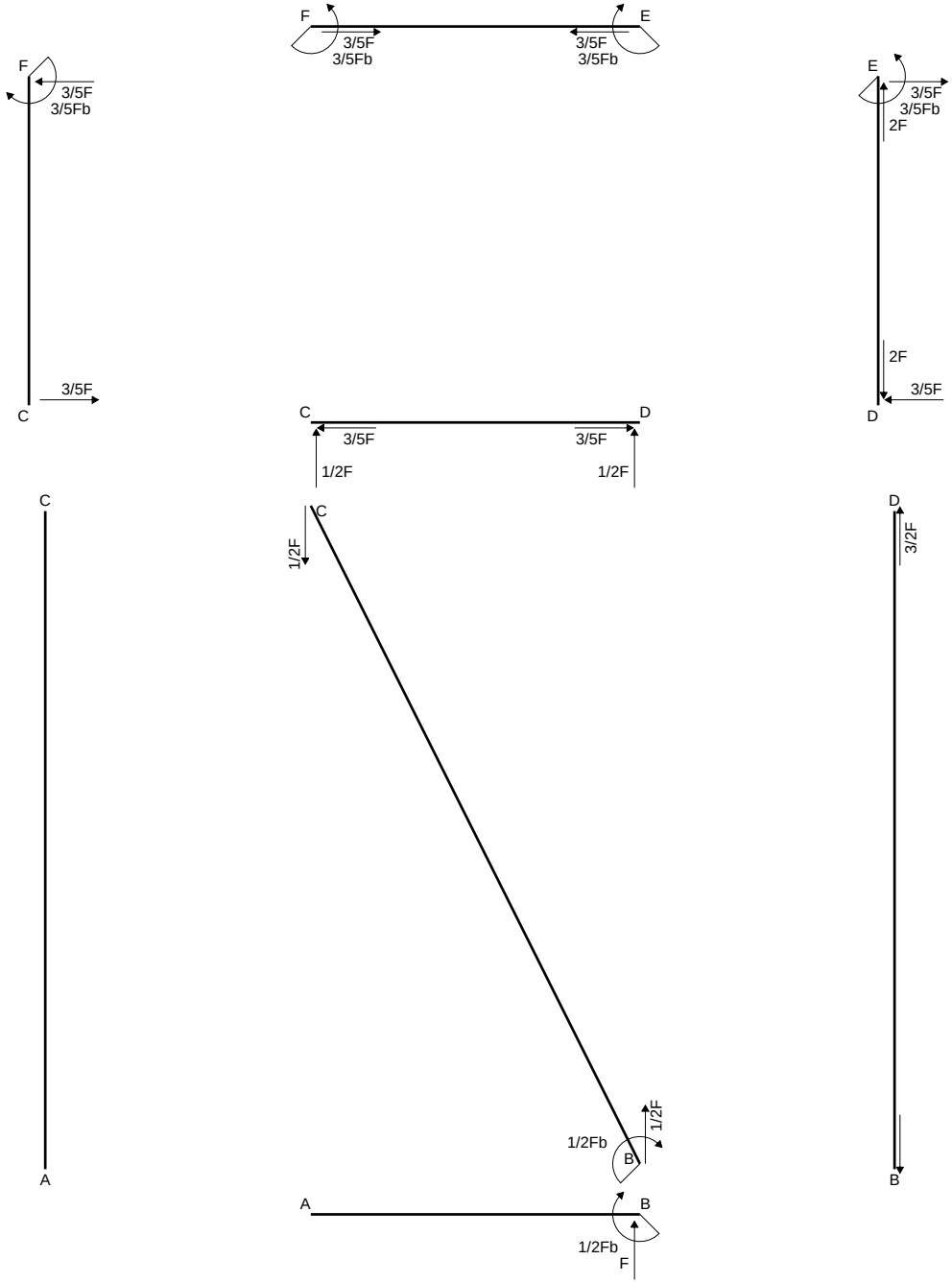
F
E
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3/5Fb
3/5F
3/5Fb
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3/5F
3/5Fb
E
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C
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2F
D
3/5F
C
D
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3/5F
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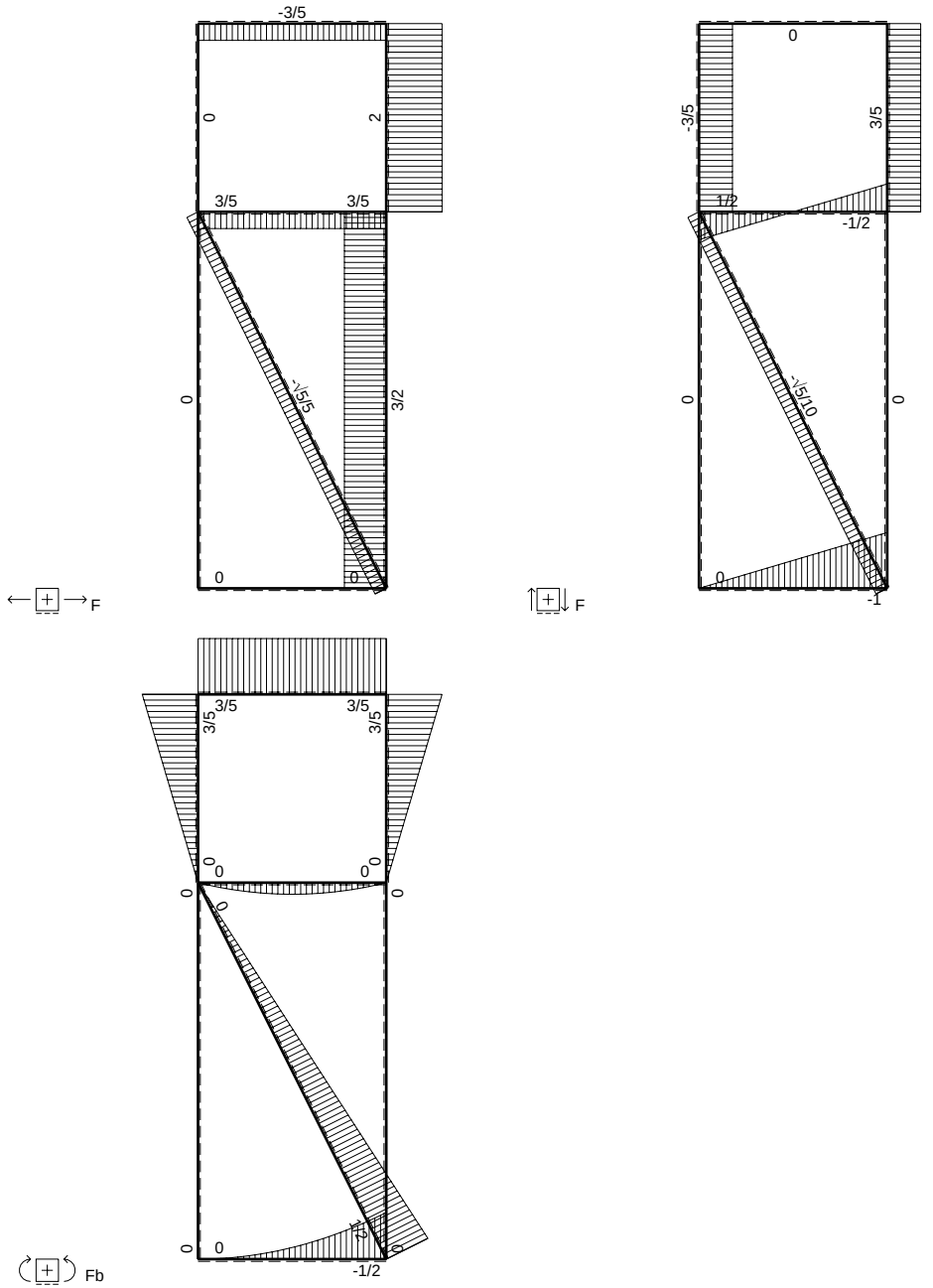
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F


-3/5
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3/5
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0
0
F
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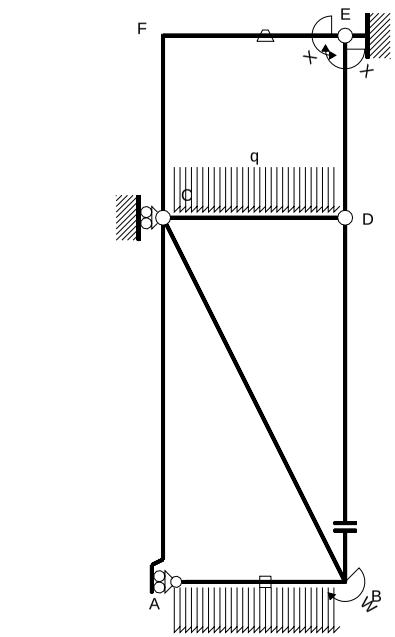
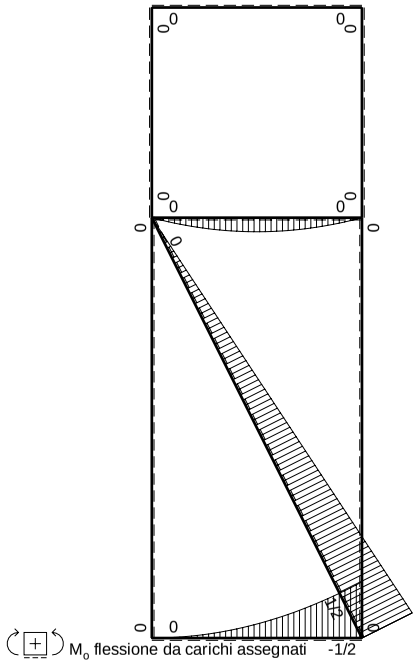
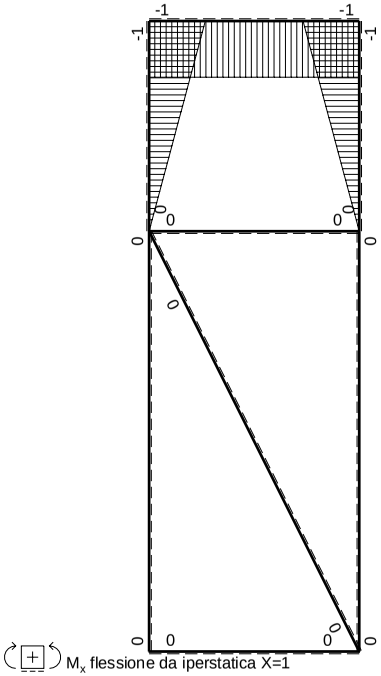
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

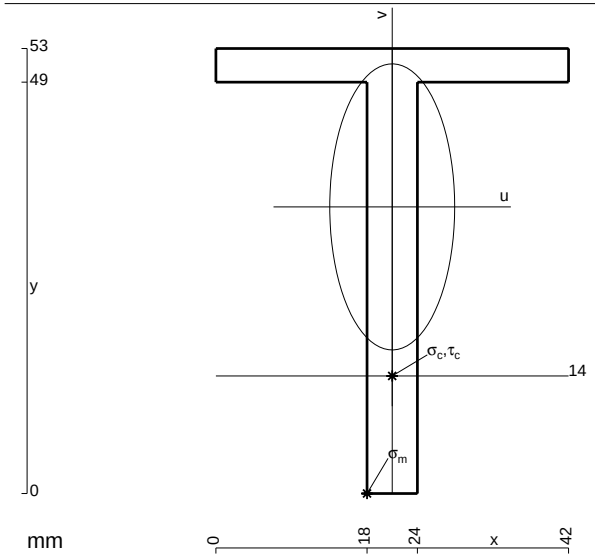
$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1\ )\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1\ )\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 462. mm^2
J_u = 134125. mm^4
J_v = 25578. mm^4
y_g = 34.14 mm
T_y = -2780. N
M_x = -861800. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -34.14 mm
$\sigma_m = -Mv/J_u$ = -219.3 N/mm^2
x_c = 21. mm
y_c = 14. mm
v_c = -20.14 mm
$\sigma_c = -Mv/J_u$ = -129.4 N/mm^2
τ_c = 7.874 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.1 N/mm^2
S^* = 2279. mm^3

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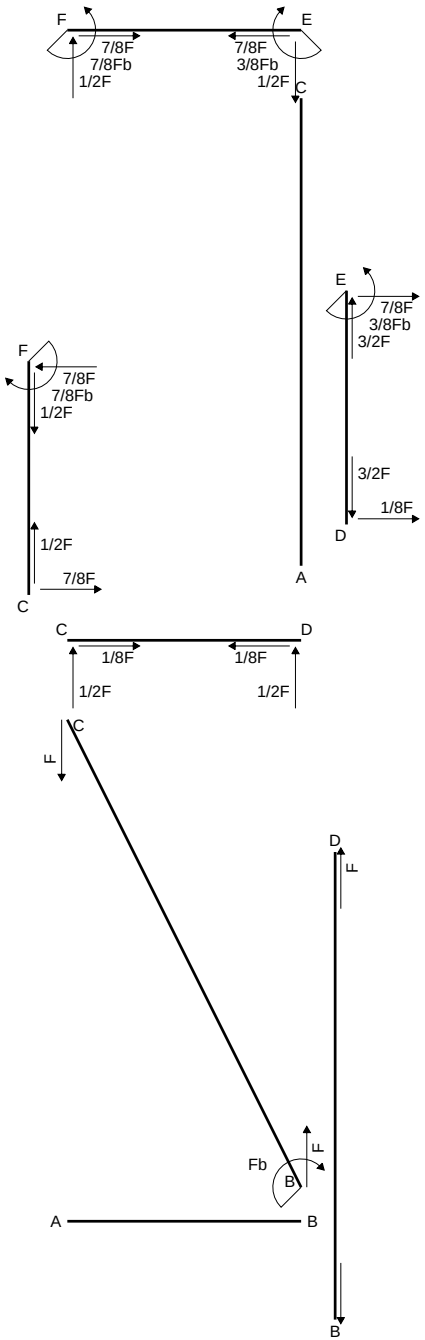
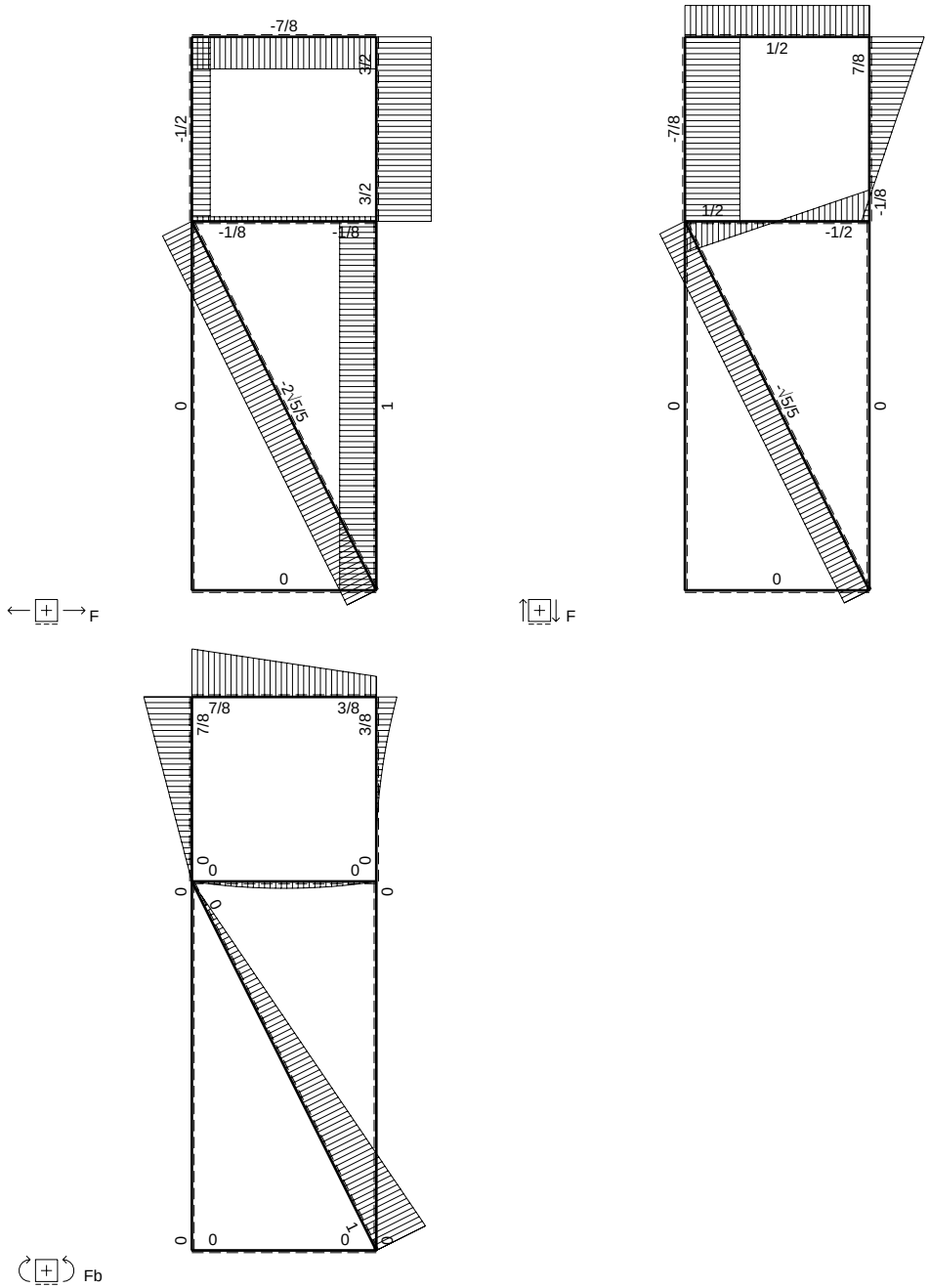

F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
3/8Fb
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
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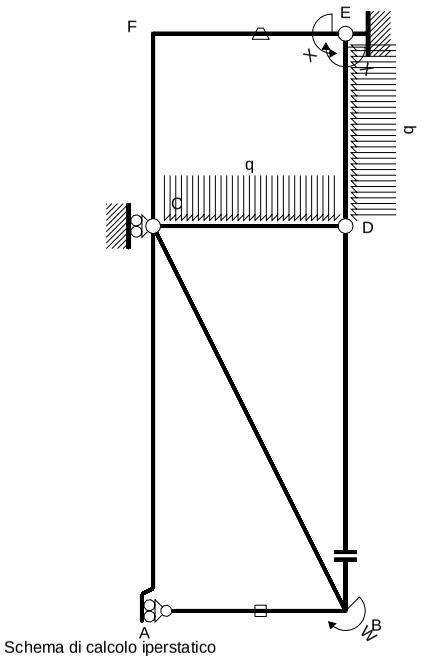
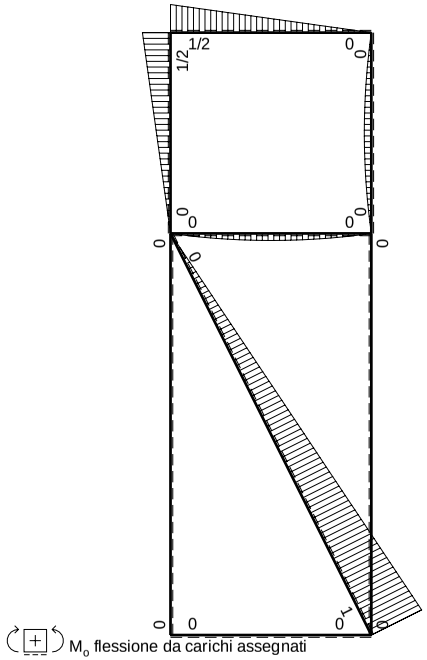
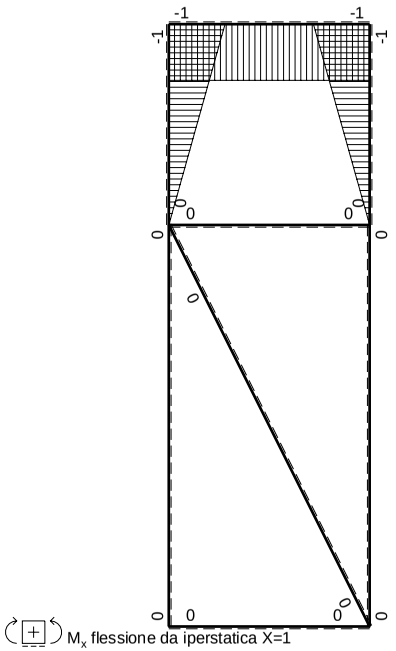
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x\theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
				totali			5/8Fb^2/EJ	5/3Xb/EJ
				iperstatica $X=W_{EF}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212007 ESΣ09.xxxx.178

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

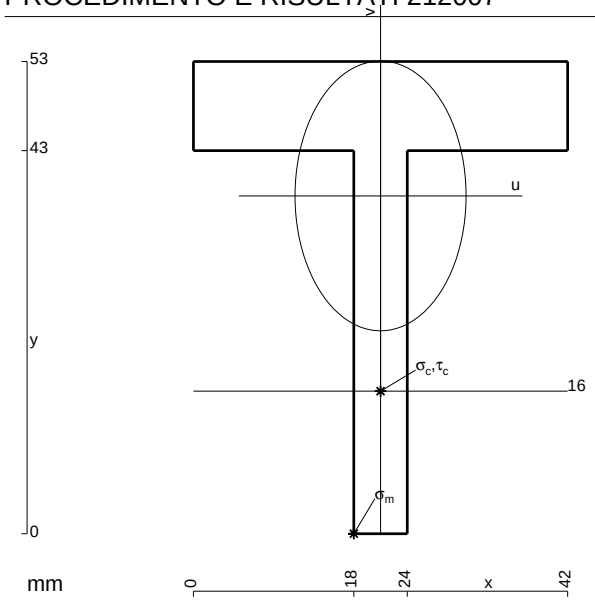
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 678. mm^2
J_u = 155489. mm^4
J_v = 62514. mm^4
y_g = 37.92 mm
N = -1288. N
T_y = -644. N
M_x = 950400. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.92 mm
$\sigma_m = N/A - Mv/J_u$ = 229.9 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.92 mm
$\sigma_c = N/A - Mv/J_u$ = 132.1 N/mm^2
τ_c = 1.982 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.1 N/mm^2
S^* = 2872. mm^3

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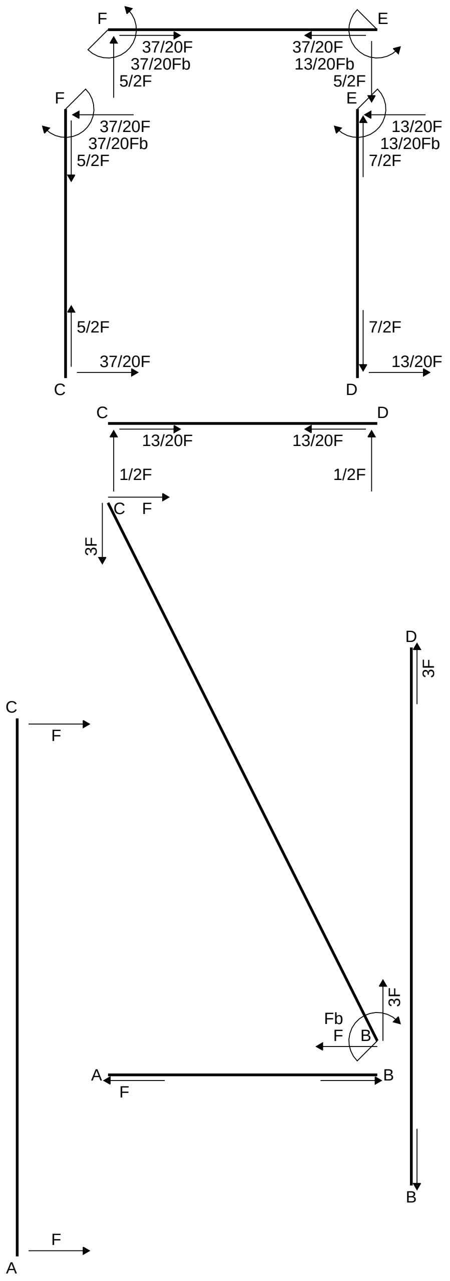
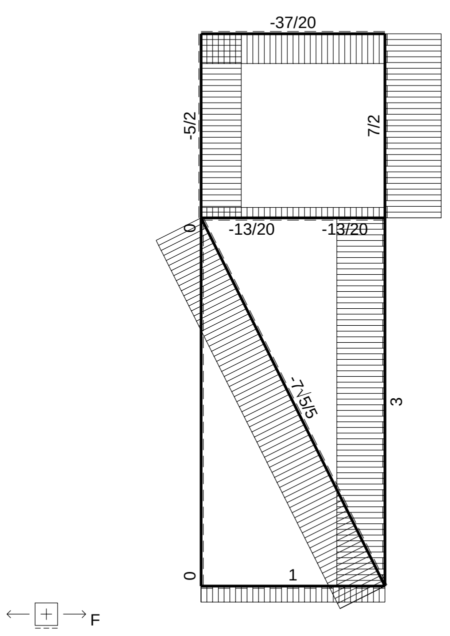
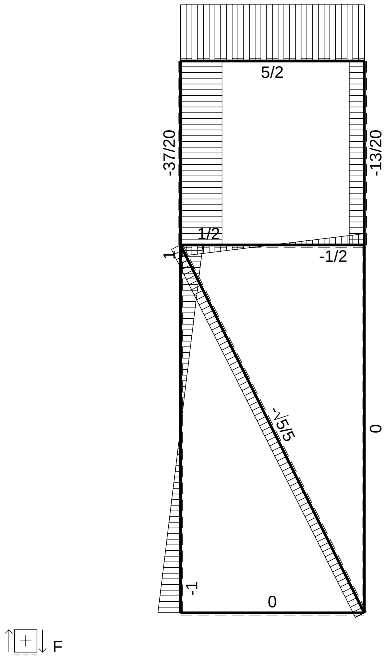
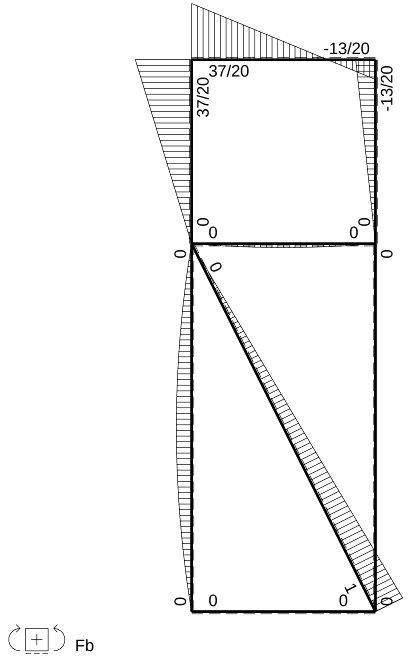
REAZIONI 211990

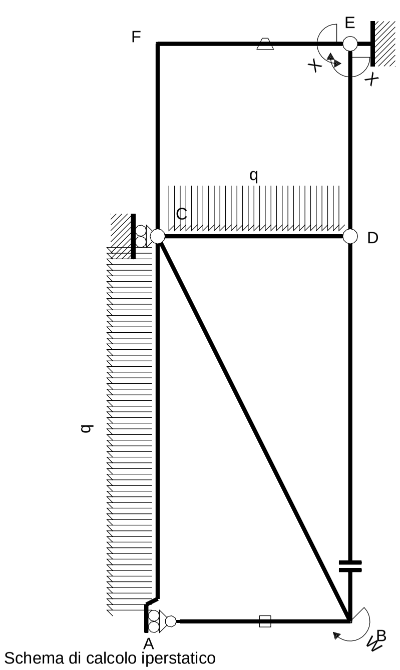


AZIONI INTERNE 211990

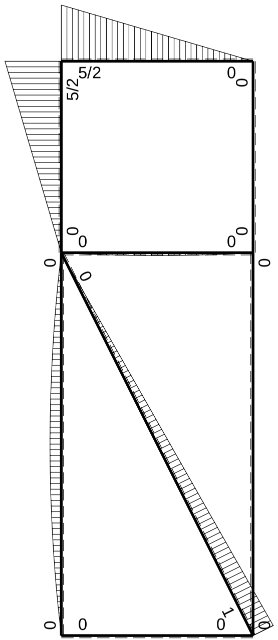




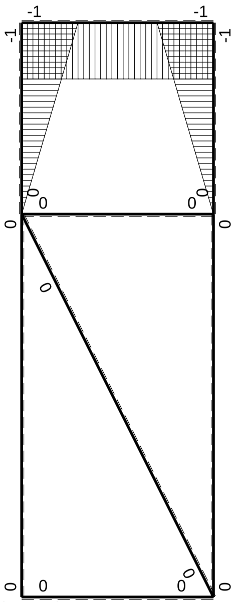




Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

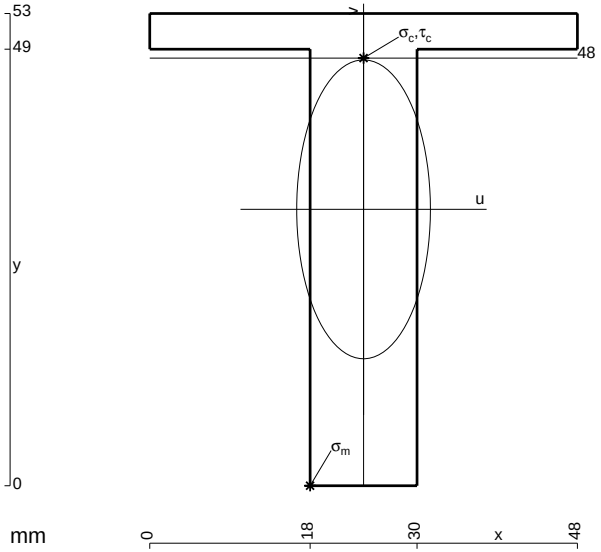
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 780. mm^2
J_u = 219548. mm^4
J_v = 43920. mm^4
y_g = 31.02 mm
N = -7889. N
T_y = -1127. N
M_x = 1764000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -31.02 mm
$\sigma_m = N/A - Mv/J_u$ = 239.1 N/mm^2
x_c = 24. mm
y_c = 48. mm
v_c = 16.98 mm
$\sigma_c = N/A - Mv/J_u$ = -146.5 N/mm^2
τ_c = 1.73 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 146.5 N/mm^2
S^* = 4045. mm^3

ESΣ09.xxxx.179

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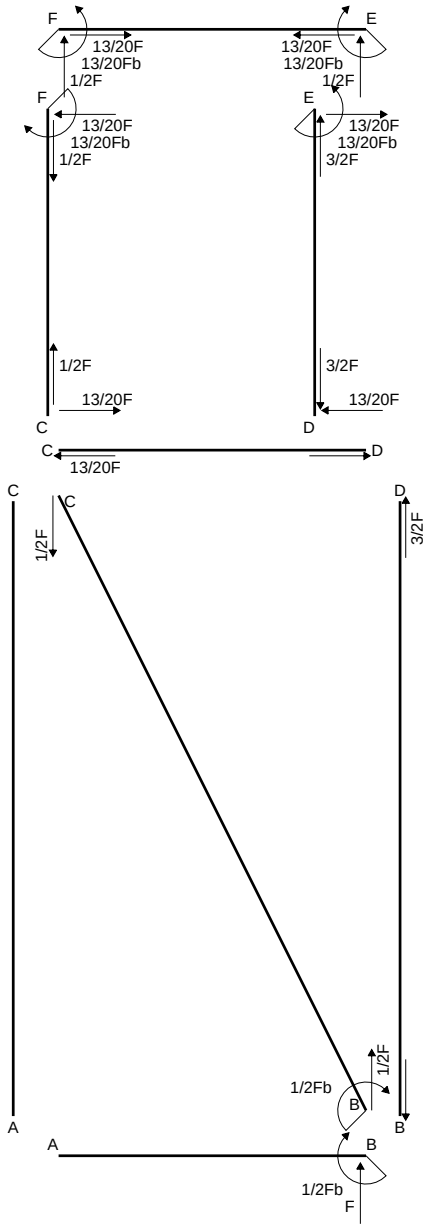
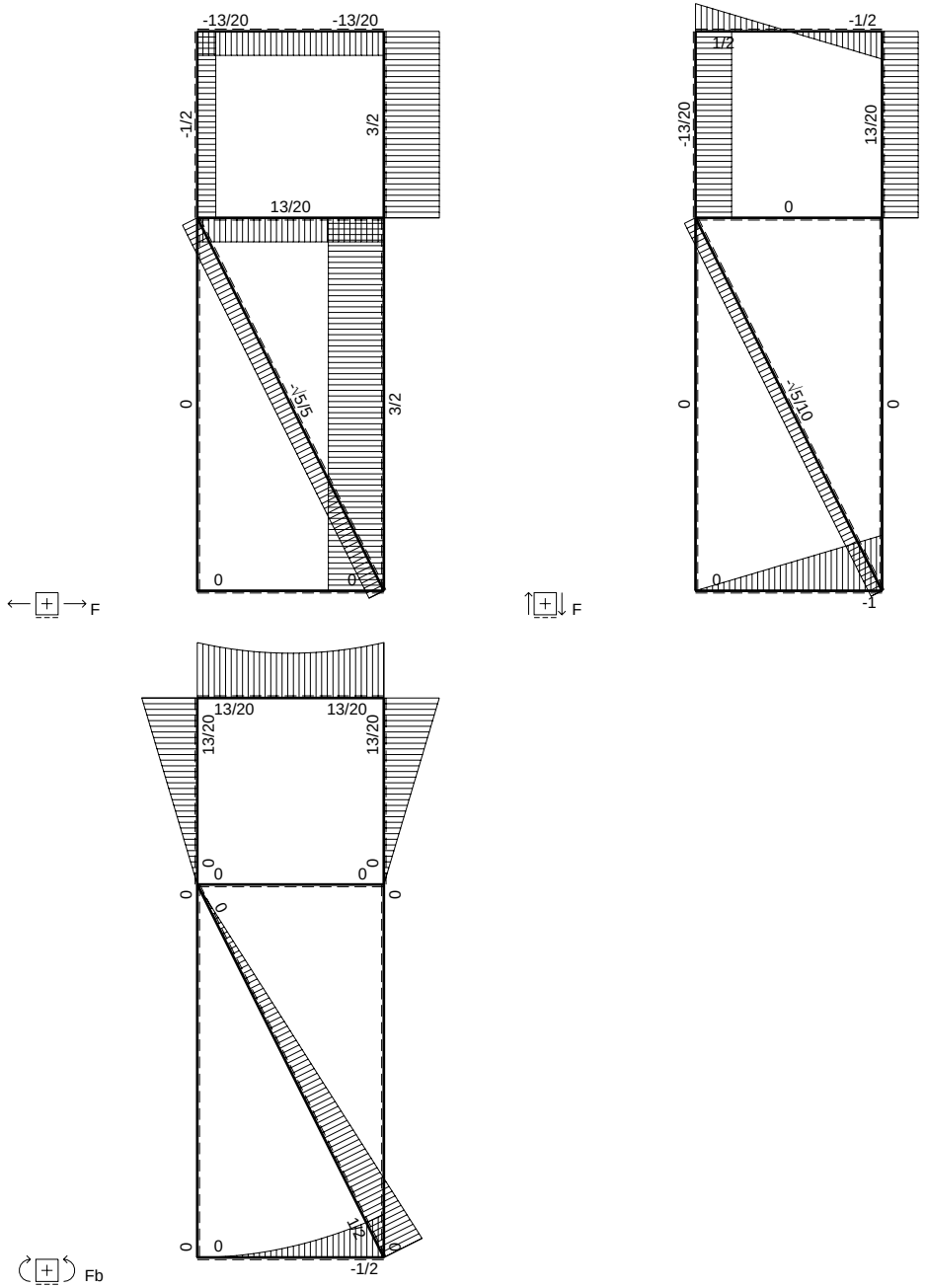
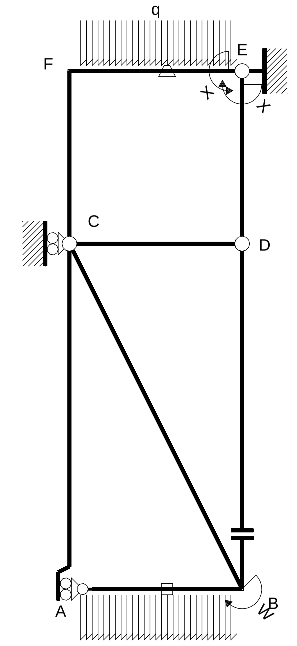
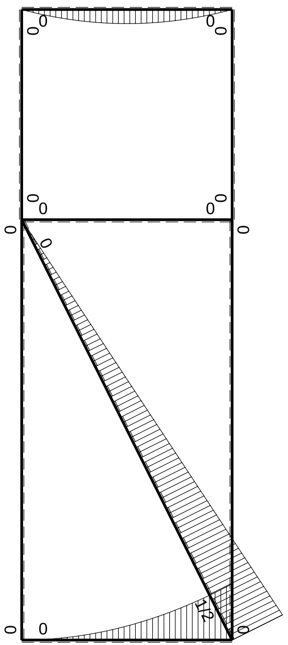

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

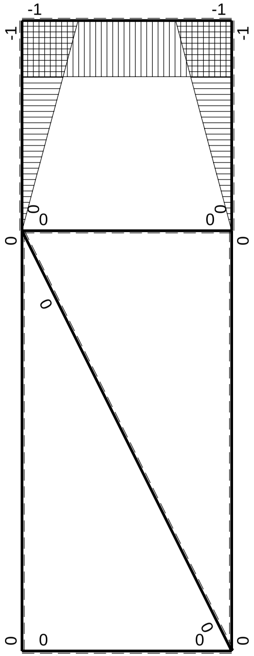

-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	AB b	BA b	BC √5b	AC 2b	CA 2b	DB 2b	BD 2b	DE b	ED b	CD b	DC b	EF b	FE b	FC b	CF b	totali	iperstatica $X=W_{EF}$
$M_x(x)$	0	0	0	0	0	0	0	-x/b	1-x/b	0	0	-1	1	-1+x/b	x/b		
$M_o(x)$	$-1/2qx^2$	$1/2Fb-Fx+1/2qx^2$	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0	0	0	$-1/2Fx+1/2qx^2$	$1/2Fx-1/2qx^2$	0	0		
θ	0	0	0	0	0	0	0	0	0	0	0	-Fb/EJ	Fb/EJ	0	0		
M_xM_o	0	0	0	0	0	0	0	0	0	0	0	$1/2Fx-1/2Fx^2/b$	$1/2Fx-1/2Fx^2/b$	0	0		
$M_x\theta$	0	0	0	0	0	0	0	0	0	0	0	Fb/EJ	Fb/EJ	0	0		
M_xM_x	0	0	0	0	0	0	0	x^2/b^2	$1-2x/b+x^2/b^2$	0	0	1	1	$1-2x/b+x^2/b^2$	x^2/b^2		
$\int M_x(M_o/EJ+\theta)dx$	0+0		0	0+0		0+0		0+0		0+0		$(1/12+1)Fb^2/EJ$		0+0		$13/12Fb^2/EJ$	-13/20Fb
$\int XM_xM_x/EJdx$	0		0	0		0		1/3Xb/EJ		0		Xb/EJ		1/3Xb/EJ		5/3Xb/EJ	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

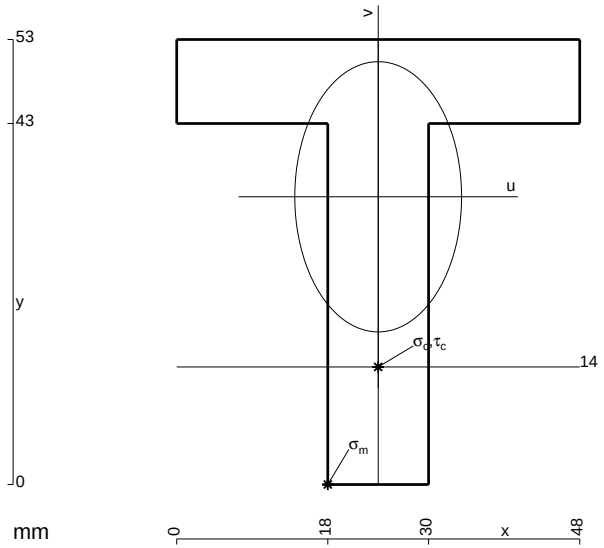
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 981112 ESΣ09.xxxx.180



A = 996. mm^2
J_u = 258139. mm^4
J_v = 98352. mm^4
y_g = 34.27 mm
T_y = -8140. N
M_x = -1505900. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.27 mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -20.27 mm
$\sigma_c = -Mv/J_u$ = -118.3 N/mm^2
τ_c = 12.04 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.1 N/mm^2
S^* = 4582. mm^3

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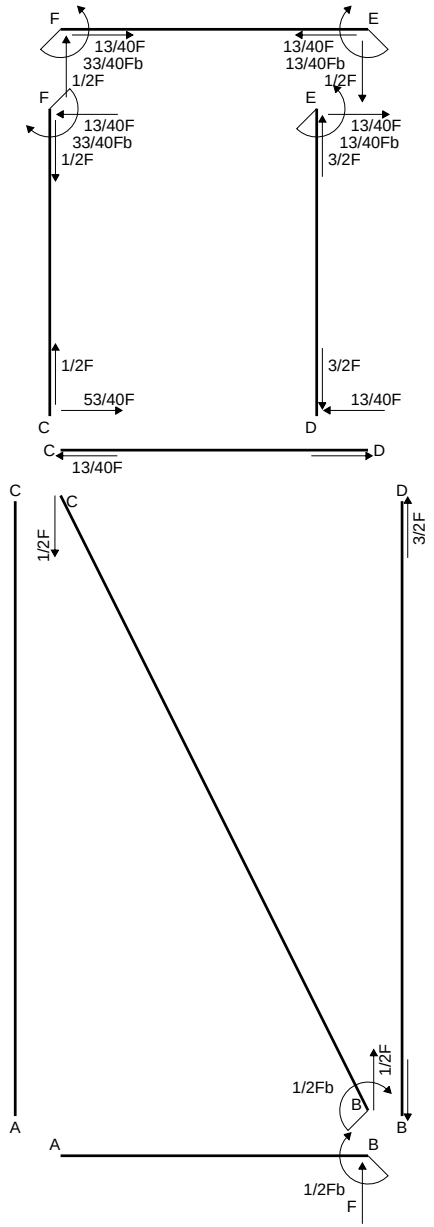
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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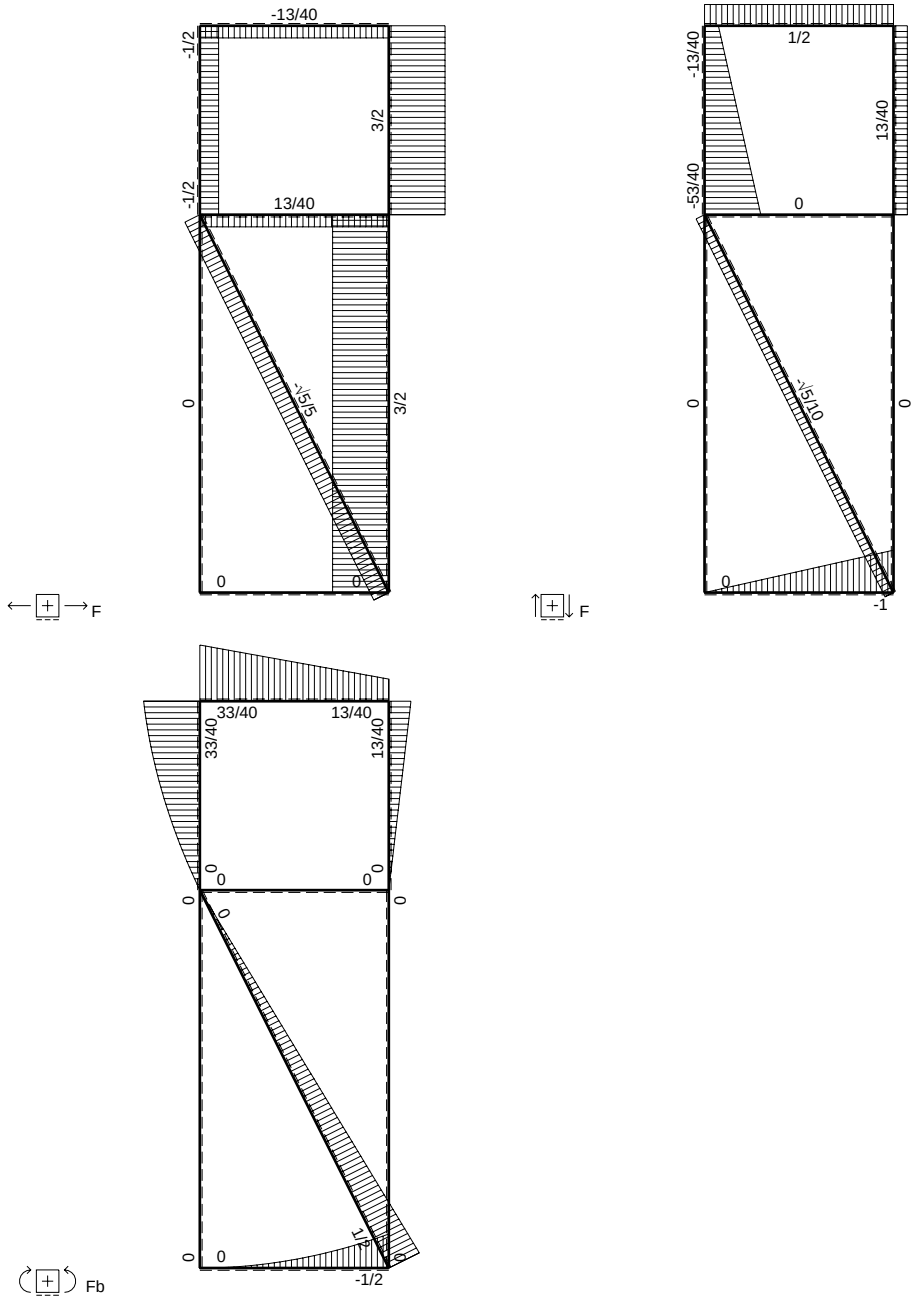
B
1/2F
A
B
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
0
0
0
0
-1/2
Fb

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2F_X$	-Fb/EJ	$-1/2F_X$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2F_X$	Fb/EJ	$-1/2Fb+1/2F_X$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

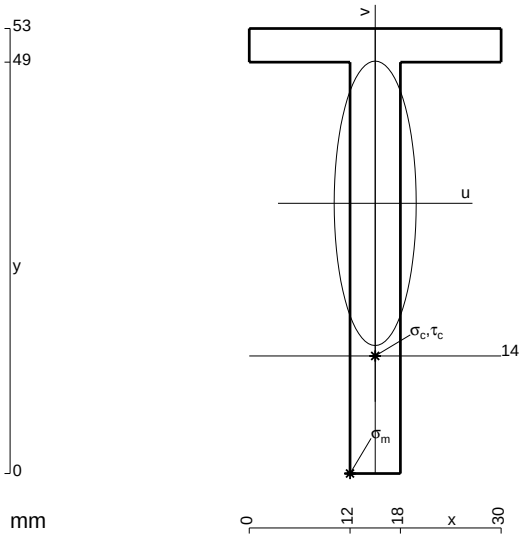
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 983273 ESΣ09.xxxx.181



A = 414. mm^2
J_u = 118828. mm^4
J_v = 9882. mm^4
y_g = 32.18 mm
T_y = -3870. N
M_x = -774000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -32.18 mm
$\sigma_m = -Mv/J_u$ = -209.6 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -18.18 mm
$\sigma_c = -Mv/J_u$ = -118.4 N/mm^2
τ_c = 11.48 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.1 N/mm^2
S^* = 2115. mm^3

ESΣ09.xxxx.181

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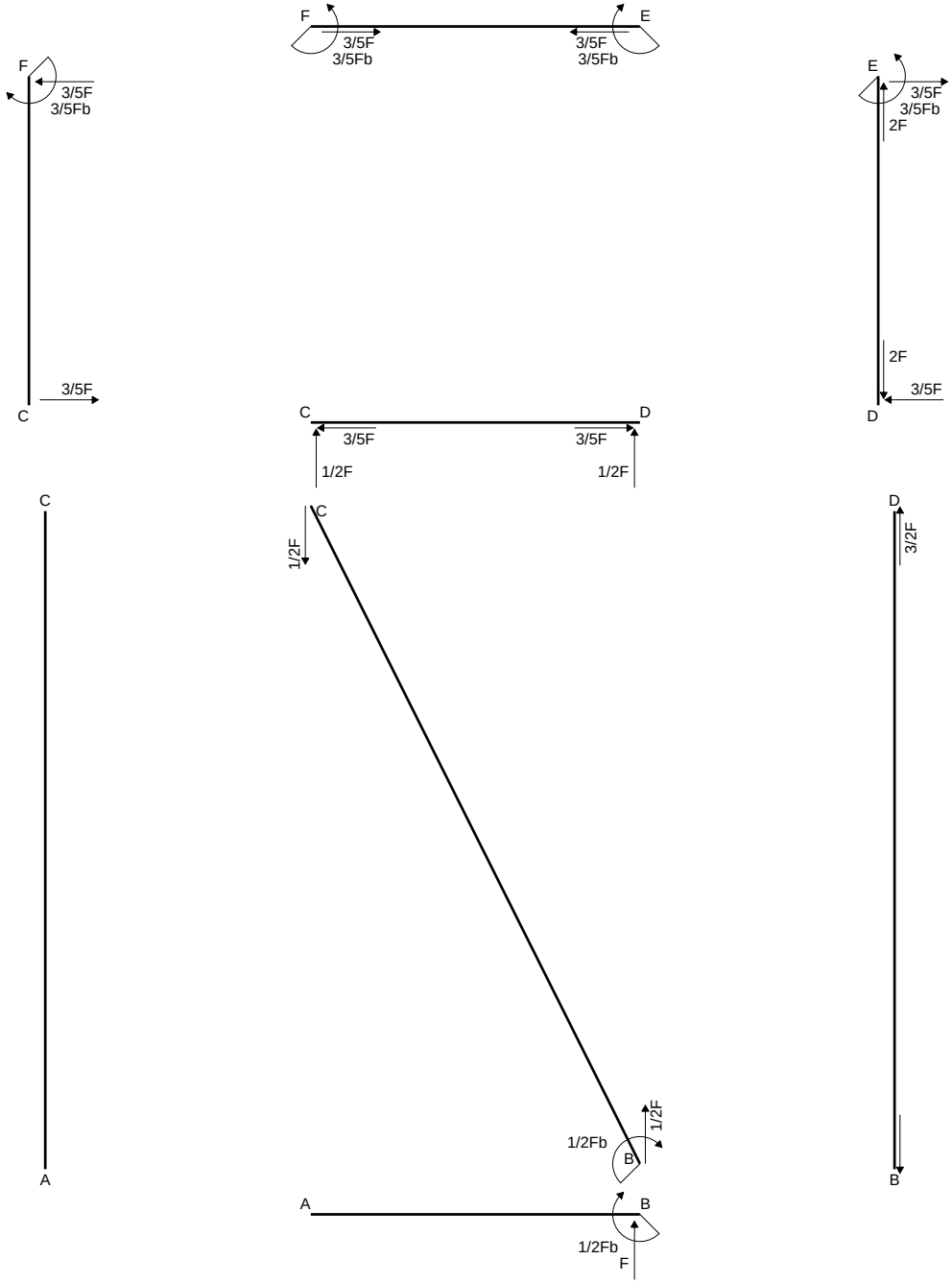
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E
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3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
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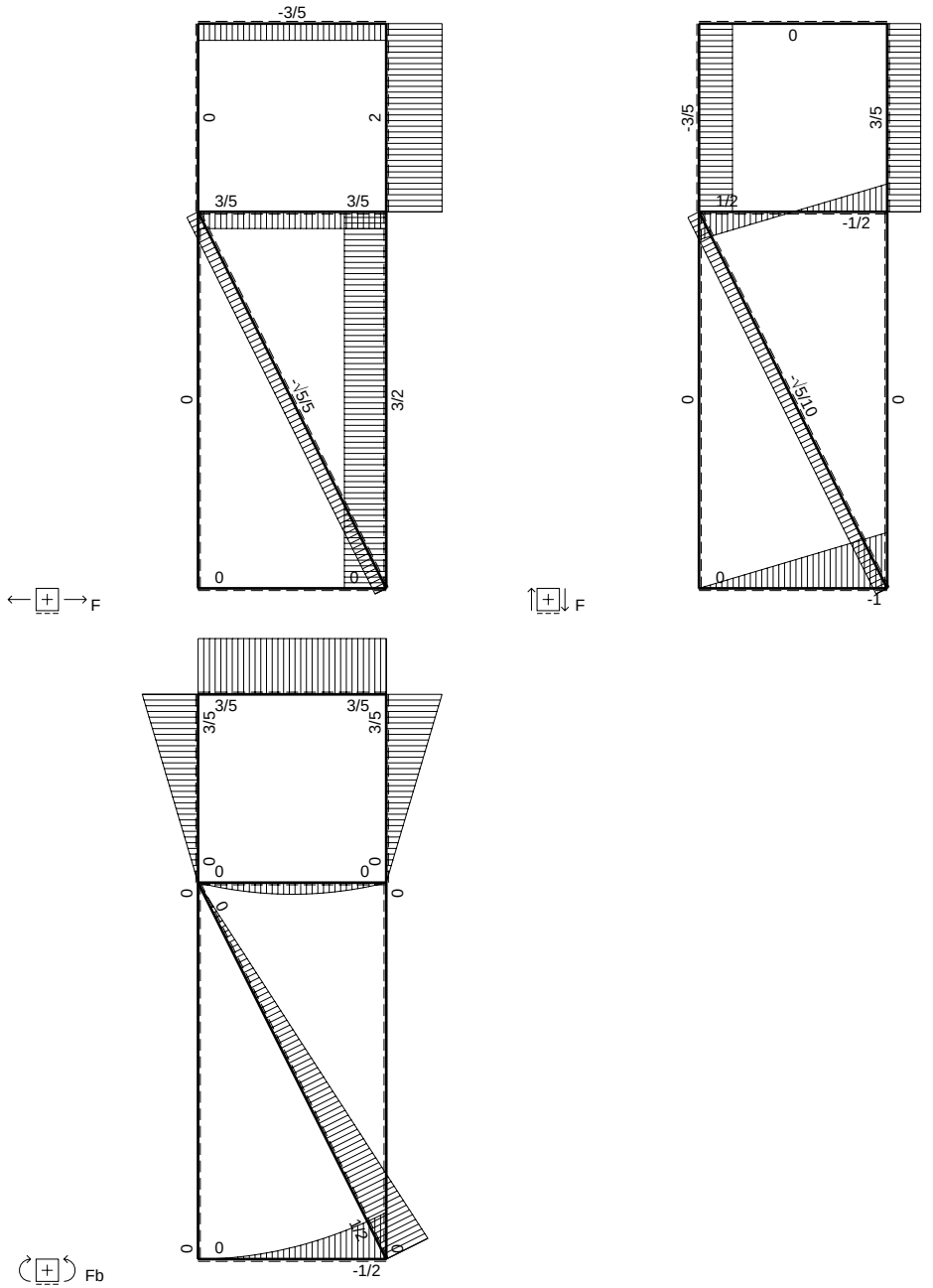
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F


-3/5
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3/5
3/5
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3/2
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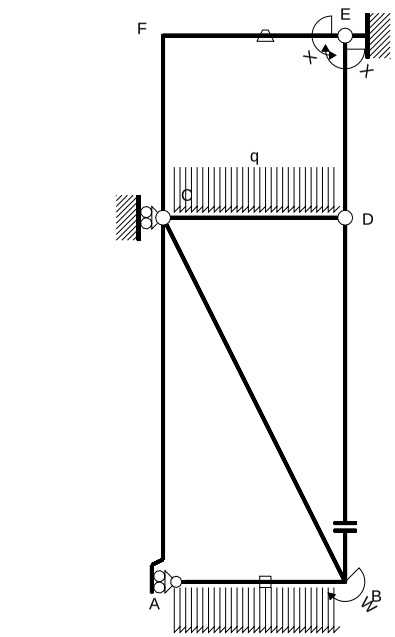
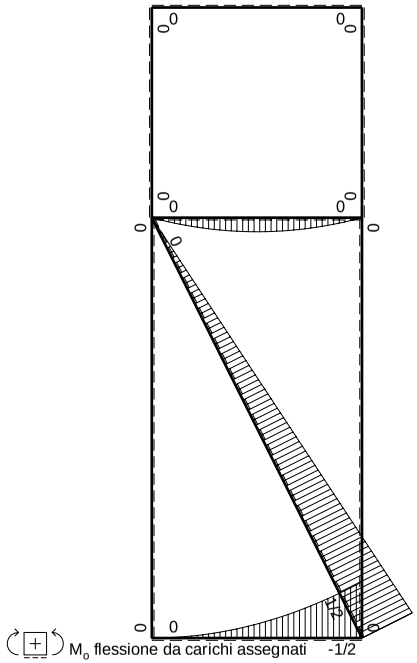
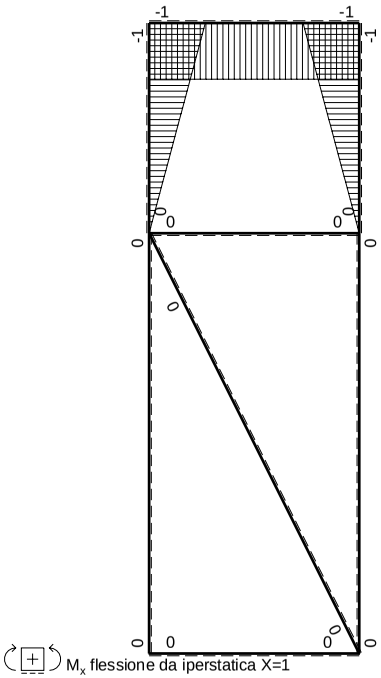
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
0
0
0
0
0
0
0
-√2
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

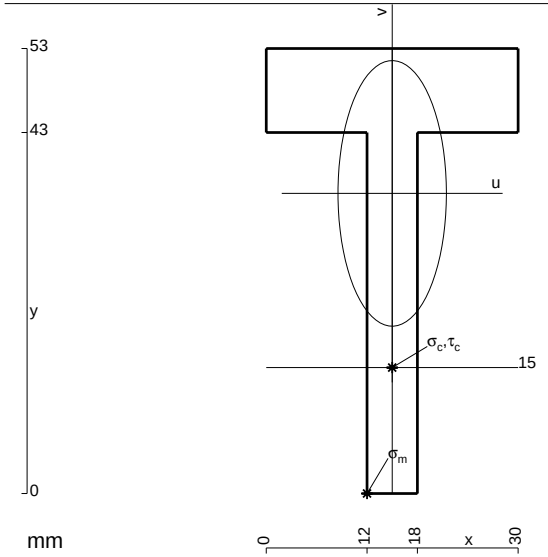
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 558. mm^2
J_u = 139662. mm^4
J_v = 23274. mm^4
y_g = 35.75 mm
T_y = -3900. N
M_x = -858000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.75 mm
$\sigma_m = -Mv/J_u$ = -219.6 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.75 mm
$\sigma_c = -Mv/J_u$ = -127.5 N/mm^2
τ_c = 11.83 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 129.1 N/mm^2
S^* = 2542. mm^3

ESΣ09.xxxx.182

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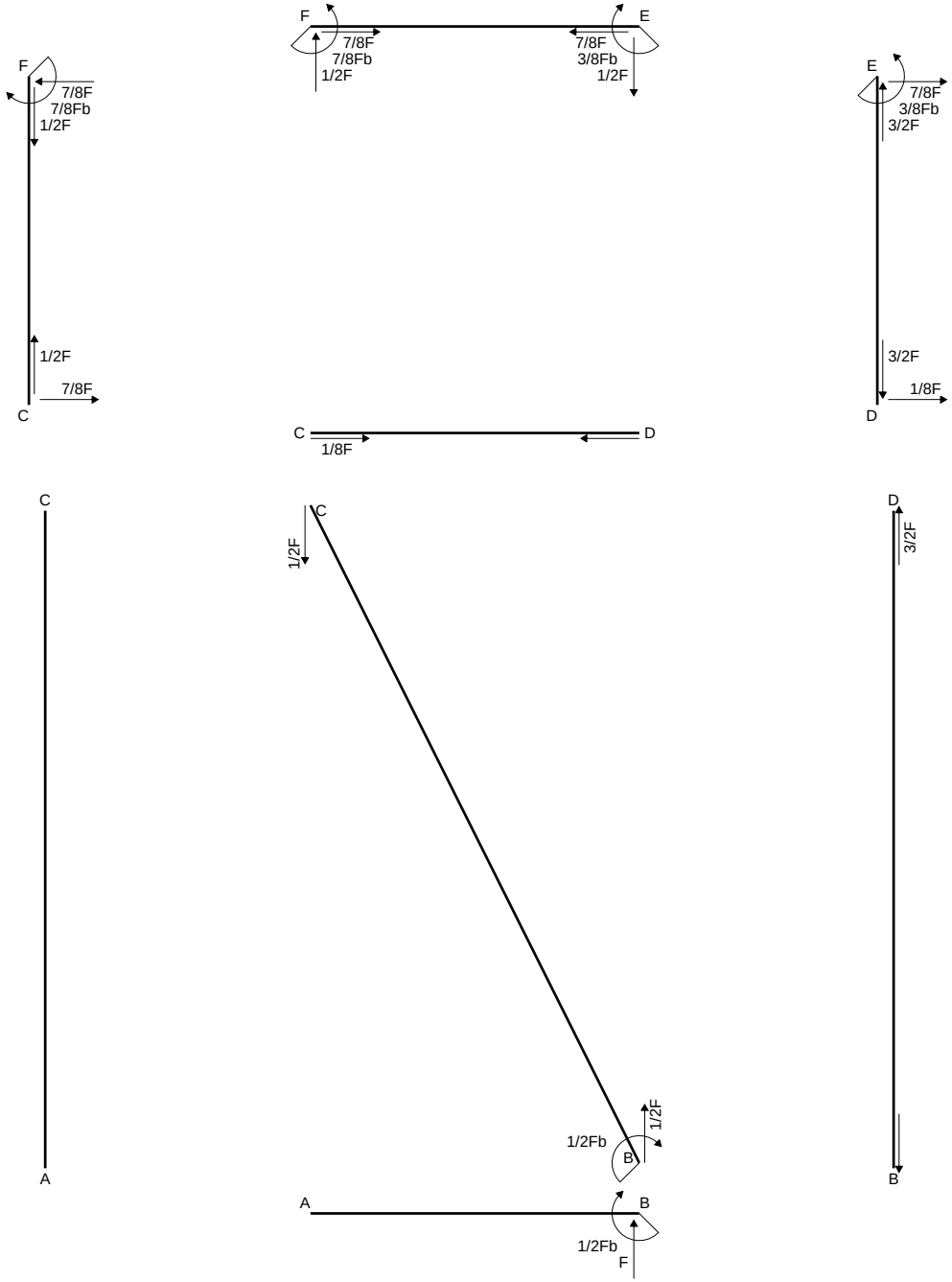
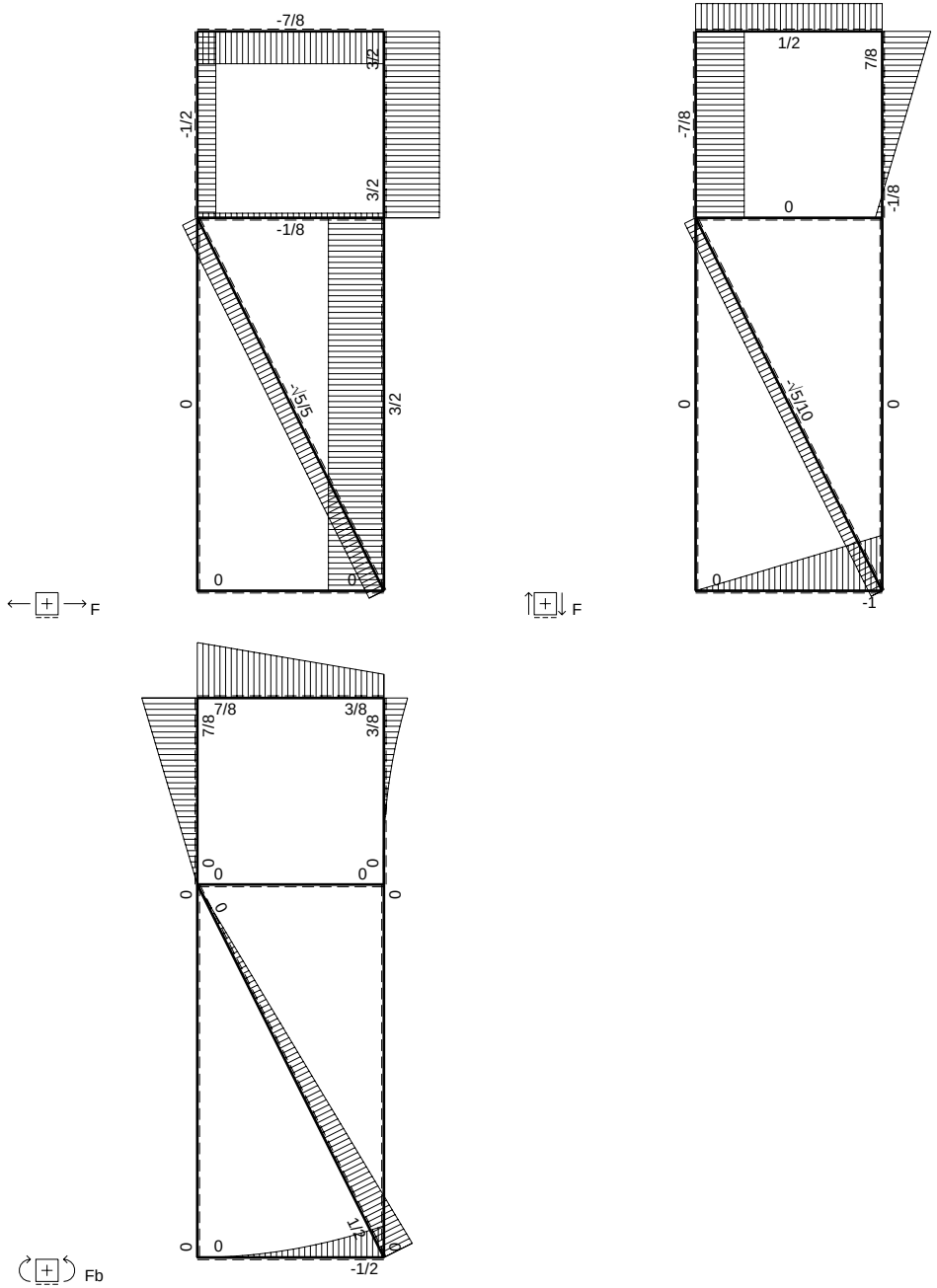

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
F
1/2
7/8
-1/8
-√5/10
7/8
3/8
Fb
-1/2

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2$/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982030 ESΣ09.xxxx.183

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

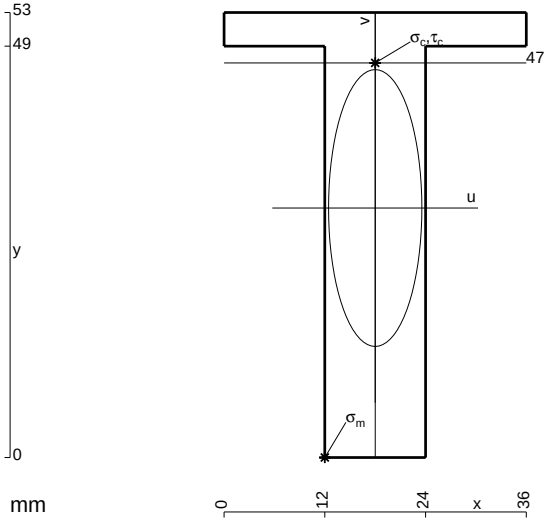
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 3/4\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 732. mm^2
J_u = 199072. mm^4
J_v = 22608. mm^4
y_g = 29.71 mm
T_y = -6420. N
M_x = -1540800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -29.71 mm
$\sigma_m = -Mv/J_u$ = -230. N/mm^2
x_c = 18. mm
y_c = 47. mm
v_c = 17.29 mm
$\sigma_c = -Mv/J_u$ = 133.8 N/mm^2
τ_c = 9.417 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 134.8 N/mm^2
S^* = 3504. mm^3

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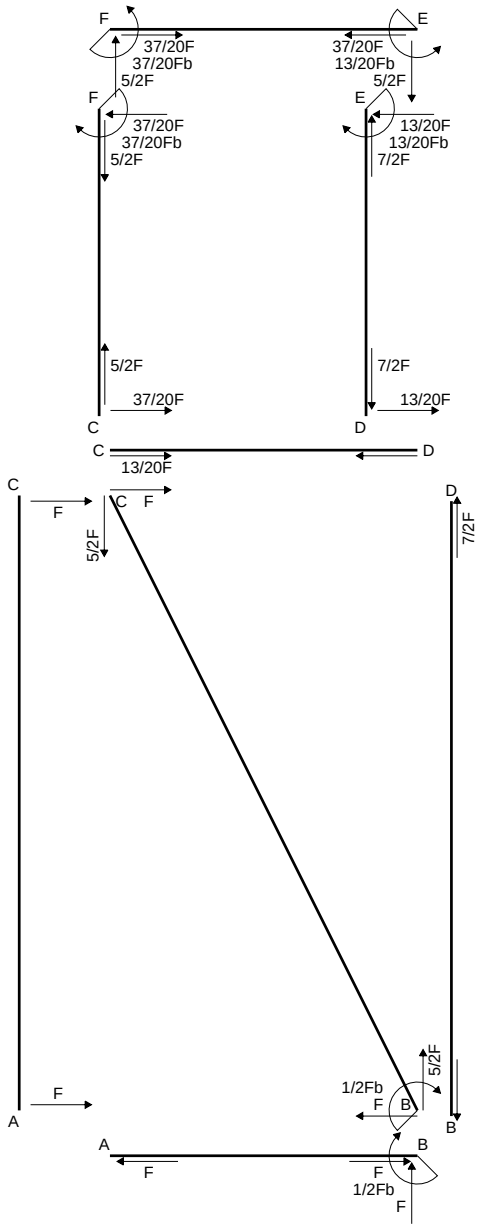
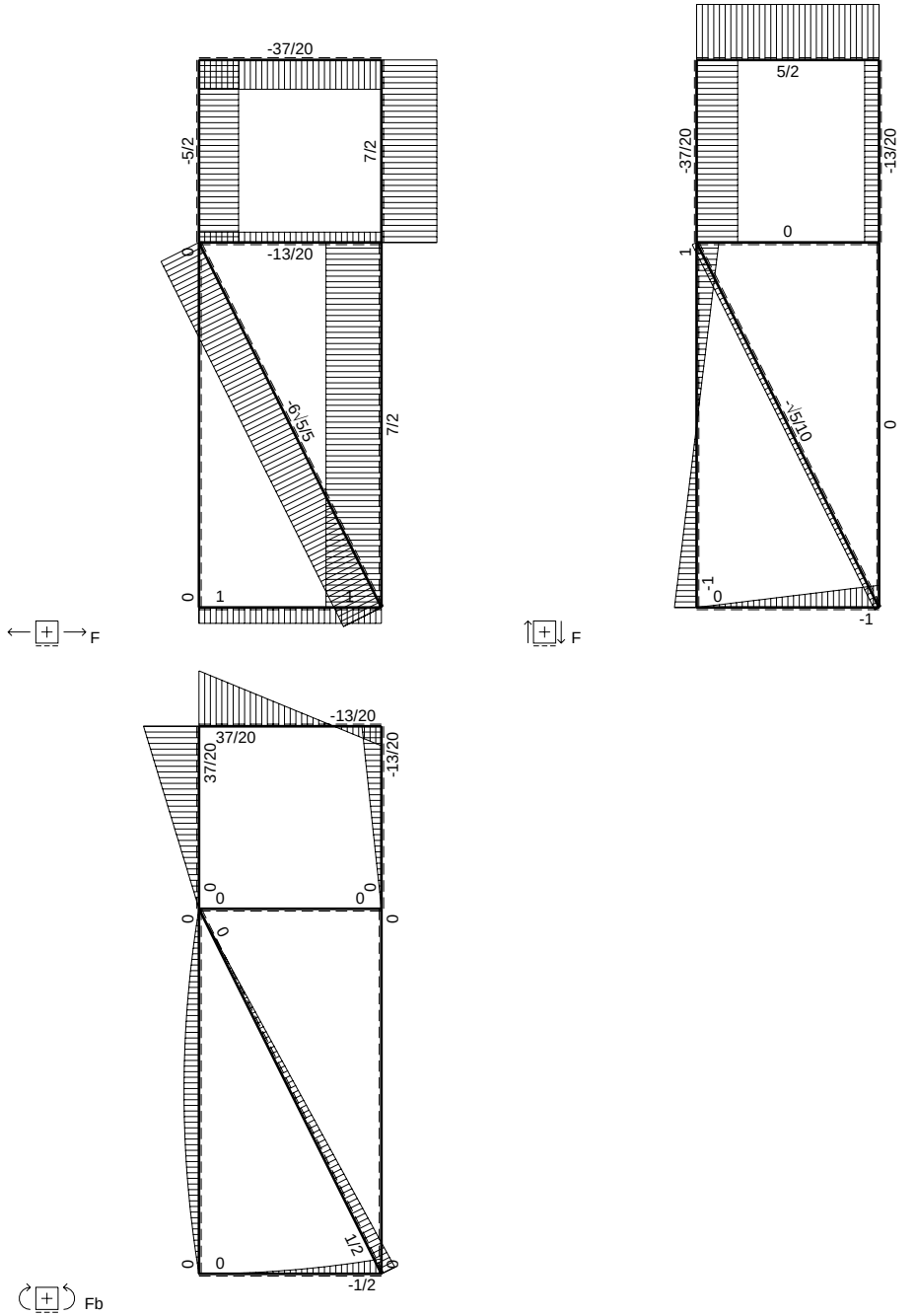
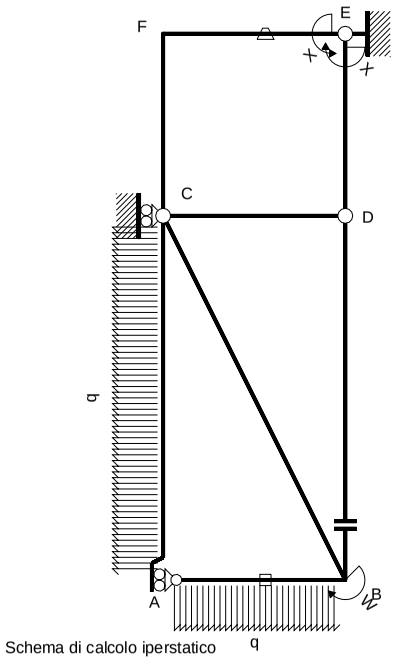
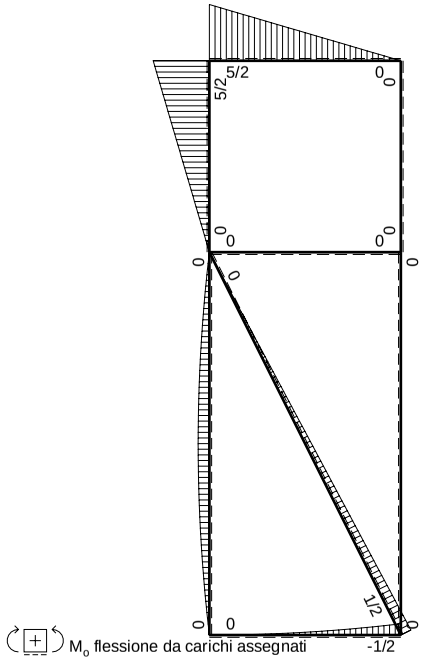

F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
C
D
A
B
1/2Fb

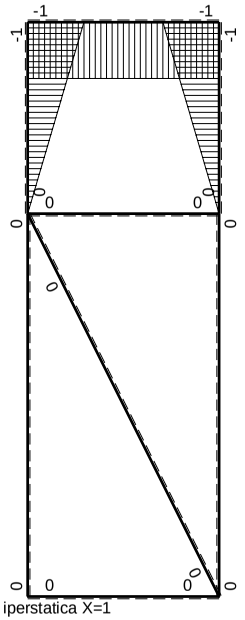

-37/20
-5/2
7/2
-13/20
-5\5/5
1
0
F
5/2
-37/20
-13/20
-5\5/10
-1
Fb
1/2
-1/2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
				totali			$-13/12Fb^2/EJ$	5/3Xb/EJ
				iperstatica $X=W_{EF}$			13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982534 ESΣ09.xxxx.184

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

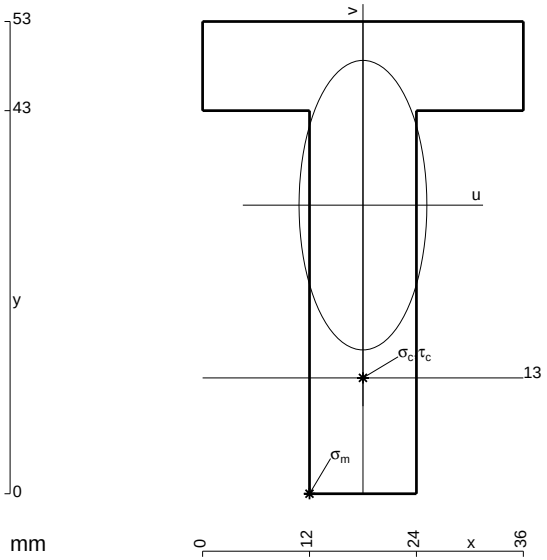
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 876. mm^2
J_u = 231422. mm^4
J_v = 45072. mm^4
y_g = 32.39 mm
N = 6940. N
T_y = -6940. N
M_x = -1769700. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.39 mm
$\sigma_m = N/A - Mv/J_u$ = -239.8 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -19.39 mm
$\sigma_c = N/A - Mv/J_u$ = -140.4 N/mm^2
τ_c = 10.09 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 141.4 N/mm^2
S^* = 4039. mm^3

ESΣ09.xxxx.184

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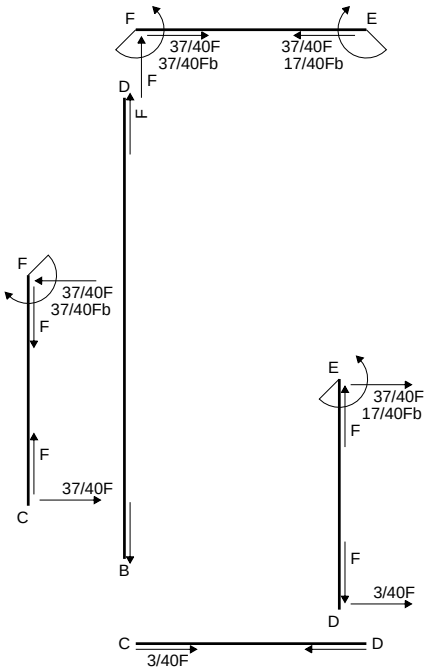
F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
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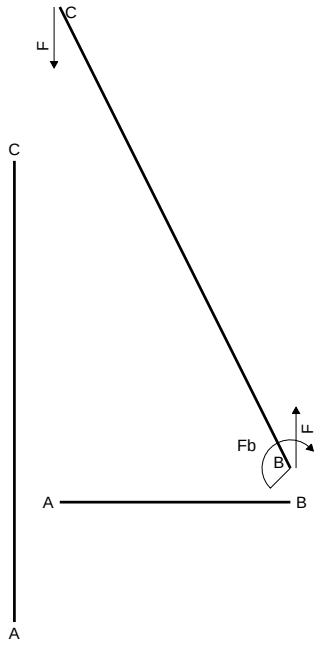
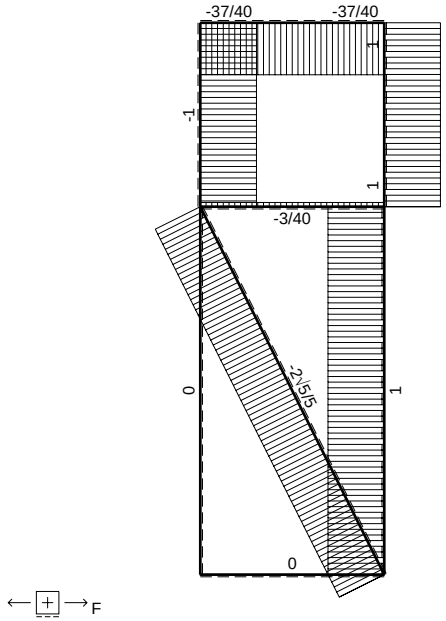
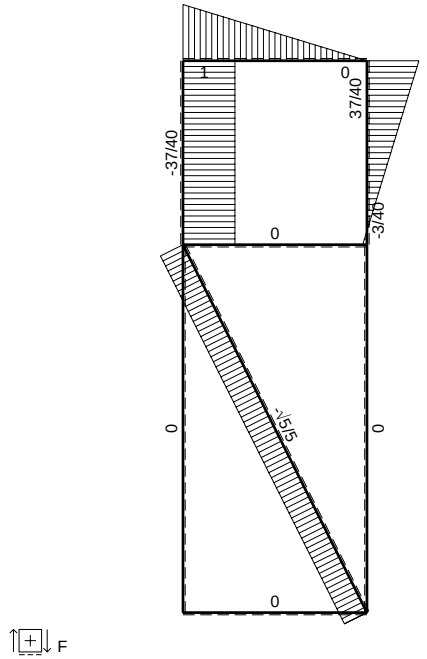
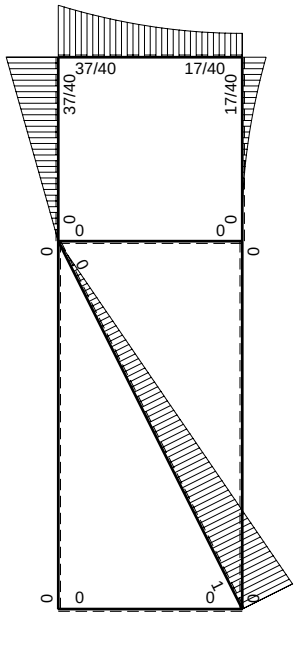
D
C
3/40F
D


C
F
C
Fb
F
B
A
B
A


-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
1
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
0
F


37/40
17/40
37/40
17/40
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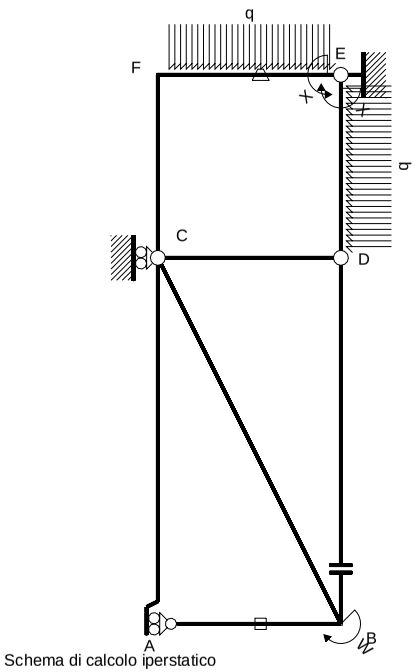
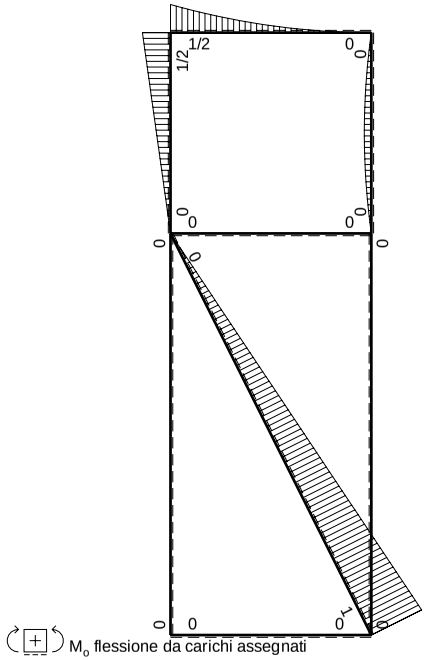
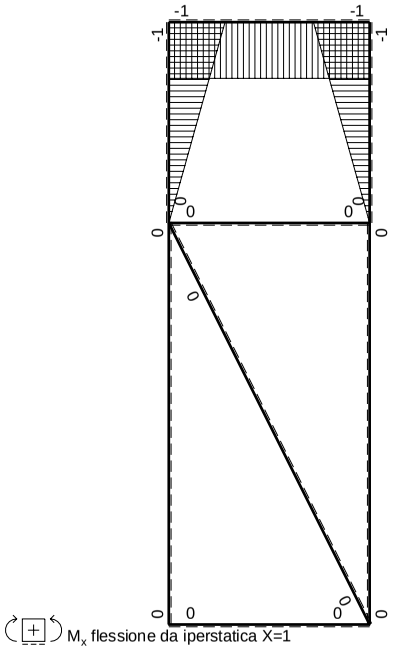
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 211254 ESΣ09.xxxx.185

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

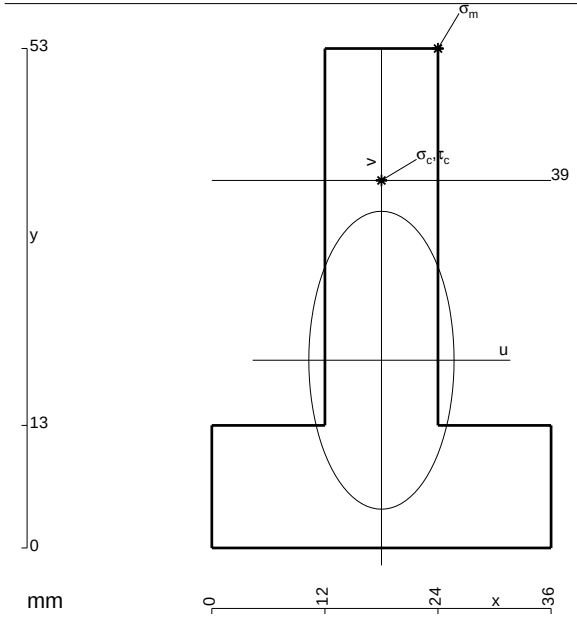
$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 211254 ESΣ09.xxxx.185



A = 948. mm^2
J_u = 236998. mm^4
J_v = 56304. mm^4
y_g = 19.92 mm
N = -2299. N
T_y = -1149. N
M_x = 1413500. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 33.08 mm
$\sigma_m = N/A - Mv/J_u$ = -199.7 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 19.08 mm
$\sigma_c = N/A - Mv/J_u$ = -116.2 N/mm^2
τ_c = 1.771 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.3 N/mm^2
S^* = 4382. mm^3

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ESΣ09.xxxx.185

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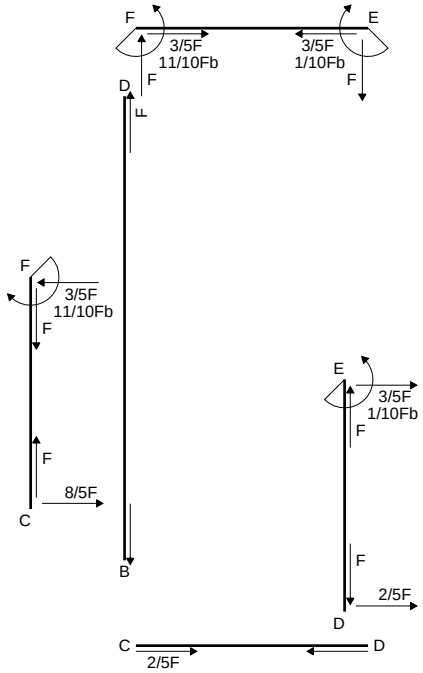
F
E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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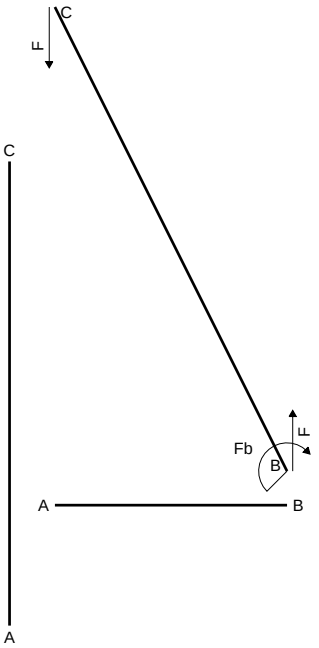
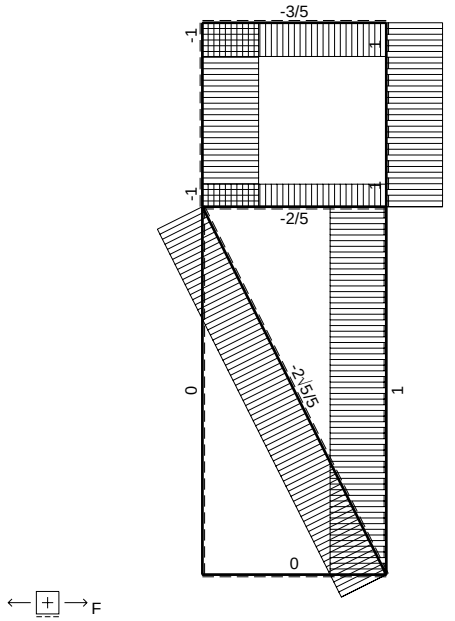
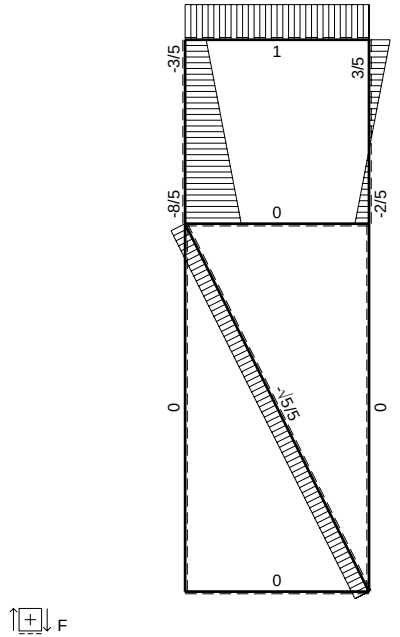
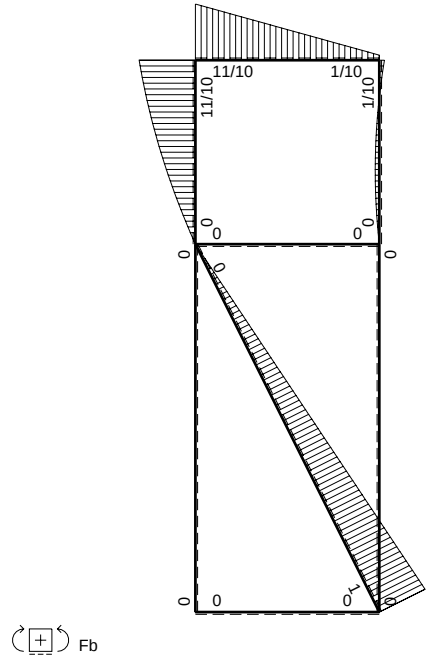
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A


-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
0
F


11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	(-1/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	Fb-1/2Fx-1/2qx^2	0	-Fb+3/2Fx-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-3/8+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3/2Fx+1/2qx^2	0	-3/2Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						1/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210898 ESΣ09.xxxx.186

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

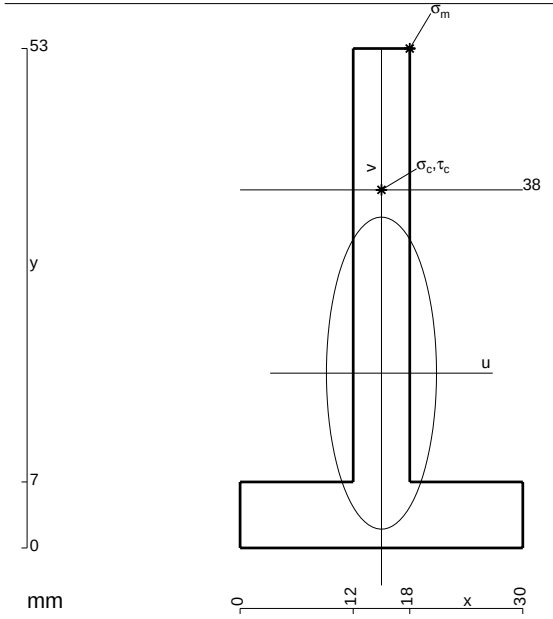
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

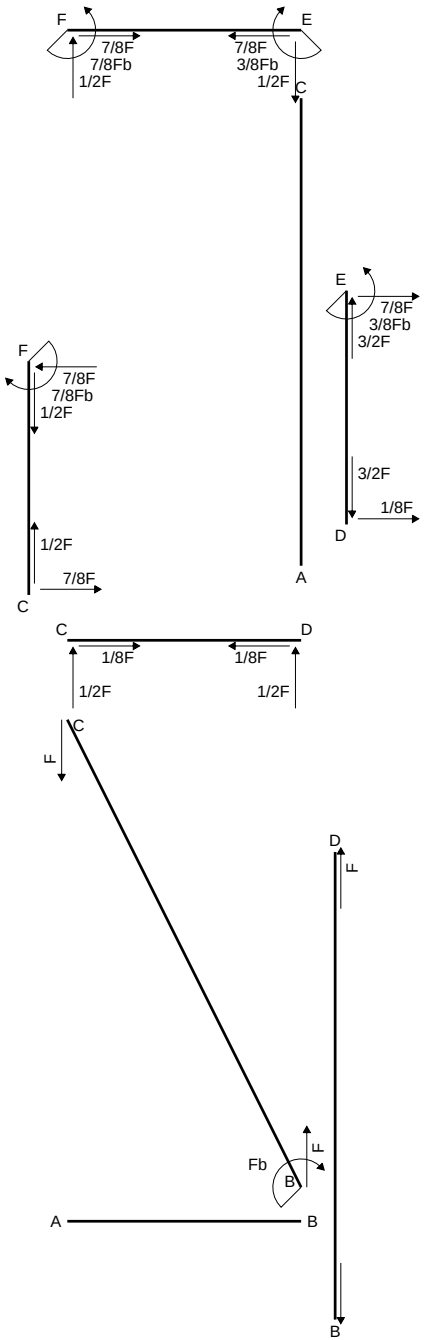
$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 486. mm^2
J_u = 133275. mm^4
J_v = 16578. mm^4
y_g = 18.55 mm
N = -1216. N
T_y = -608.2 N
M_x = 802400. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 34.45 mm
$\sigma_m = N/A - Mv/J_u$ = -209.9 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 19.45 mm
$\sigma_c = N/A - Mv/J_u$ = -119.6 N/mm^2
τ_c = 1.845 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.7 N/mm^2
S^* = 2426. mm^3

ESΣ09.xxxx.186

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24


F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
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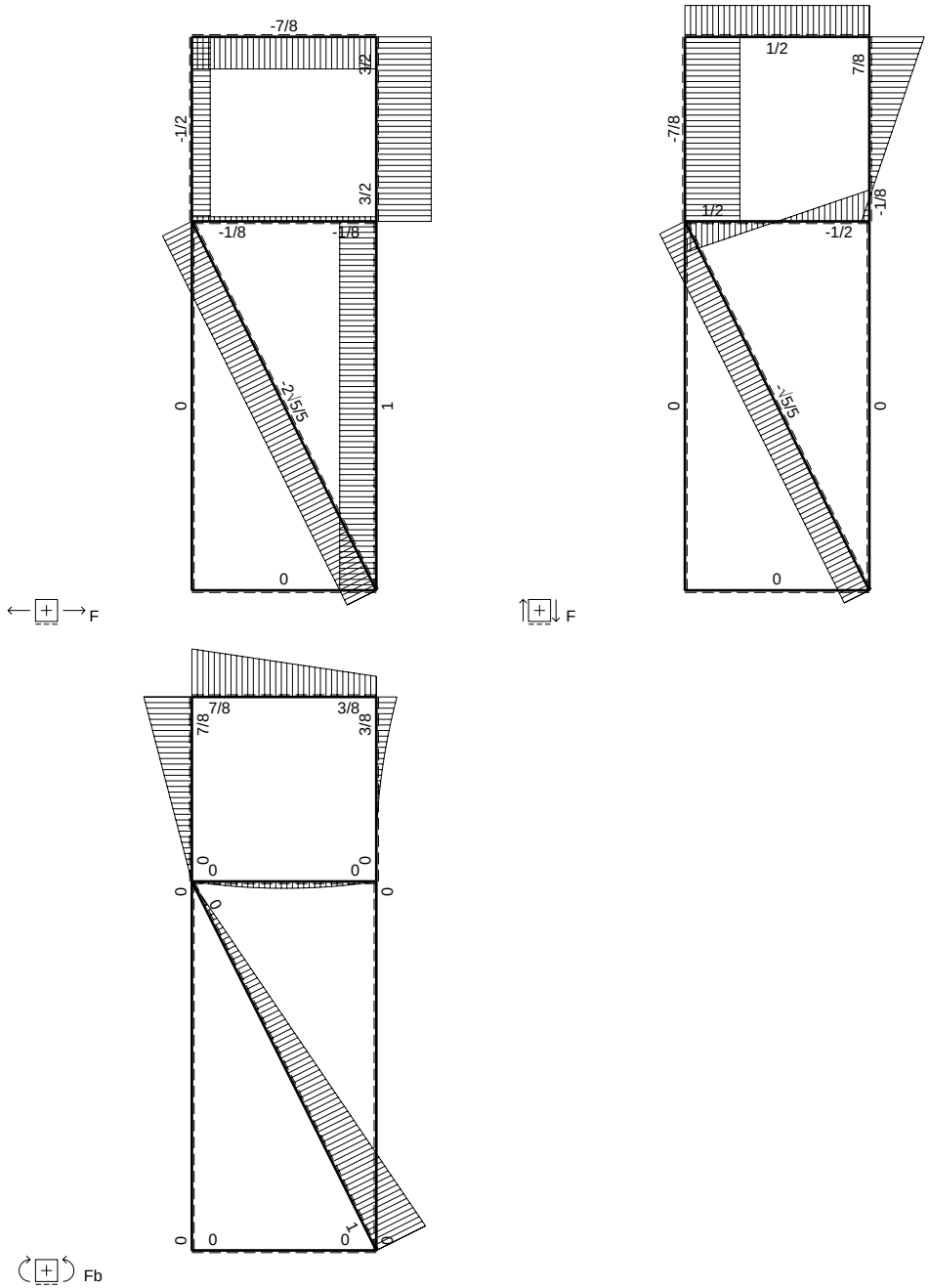
3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
C
A
C
D
1/8F
1/8F
1/2F
1/2F
C
F
D
F
Fb
F
B
A
B
B


-7/8
3/2
-1/2
3/2
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-2√5/5
1
0
F
1/2
7/8
-7/8
-1/8
1/2
-1/2
0
-√5/5
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7/8
3/8
3/8
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0
0
0
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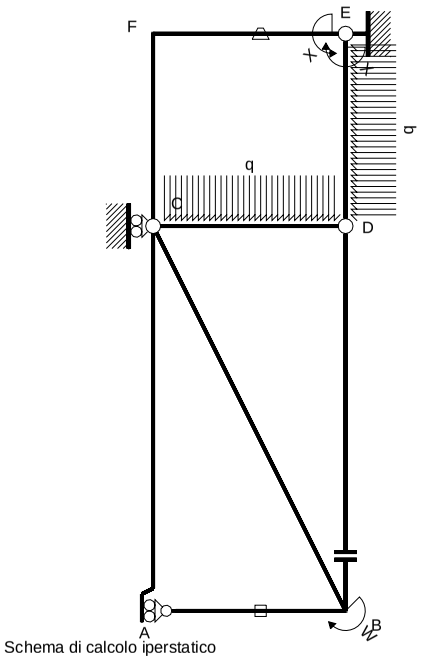
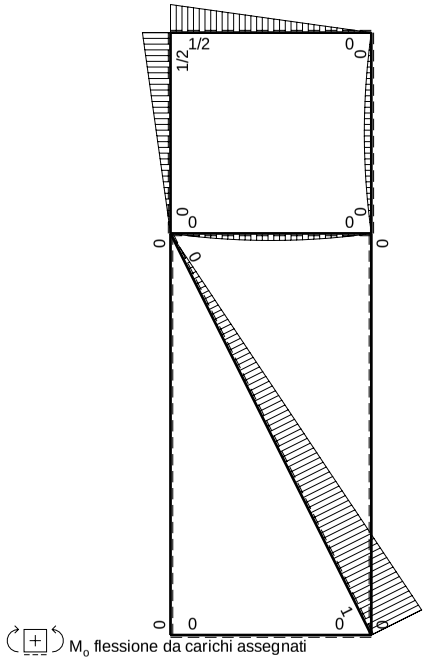
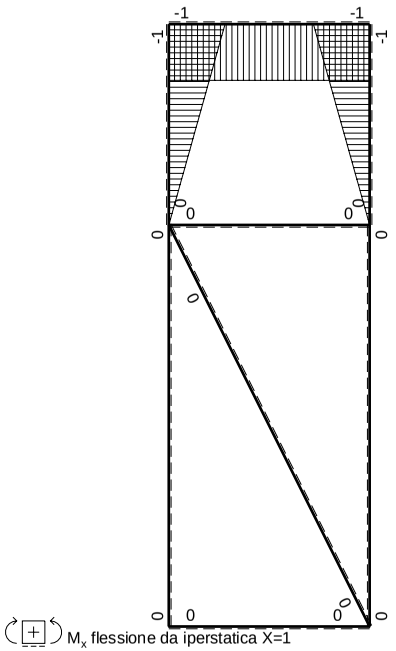
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x\theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212719 ESΣ09.xxxx.187

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

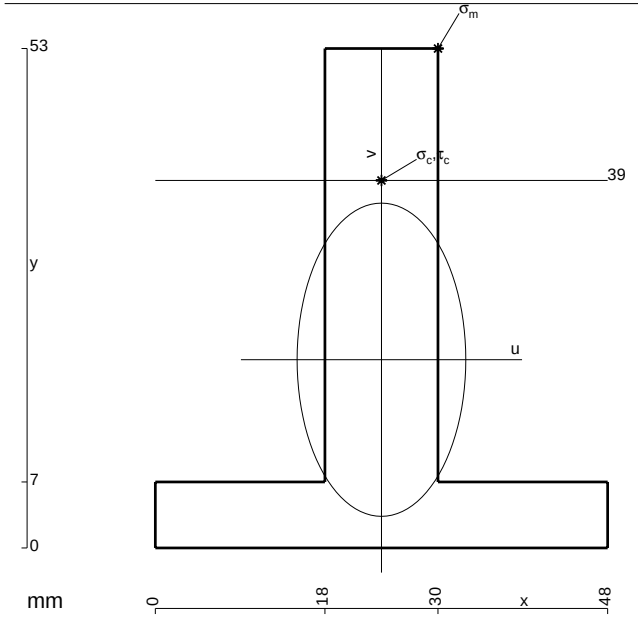
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 888. mm^2
J_u = 245383. mm^4
J_v = 71136. mm^4
y_g = 19.97 mm
N = -2326. N
T_y = -1163. N
M_x = 1612000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 33.03 mm
$\sigma_m = N/A - Mv/J_u$ = -219.6 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 19.03 mm
$\sigma_c = N/A - Mv/J_u$ = -127.6 N/mm^2
τ_c = 1.727 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.6 N/mm^2
S^* = 4373. mm^3

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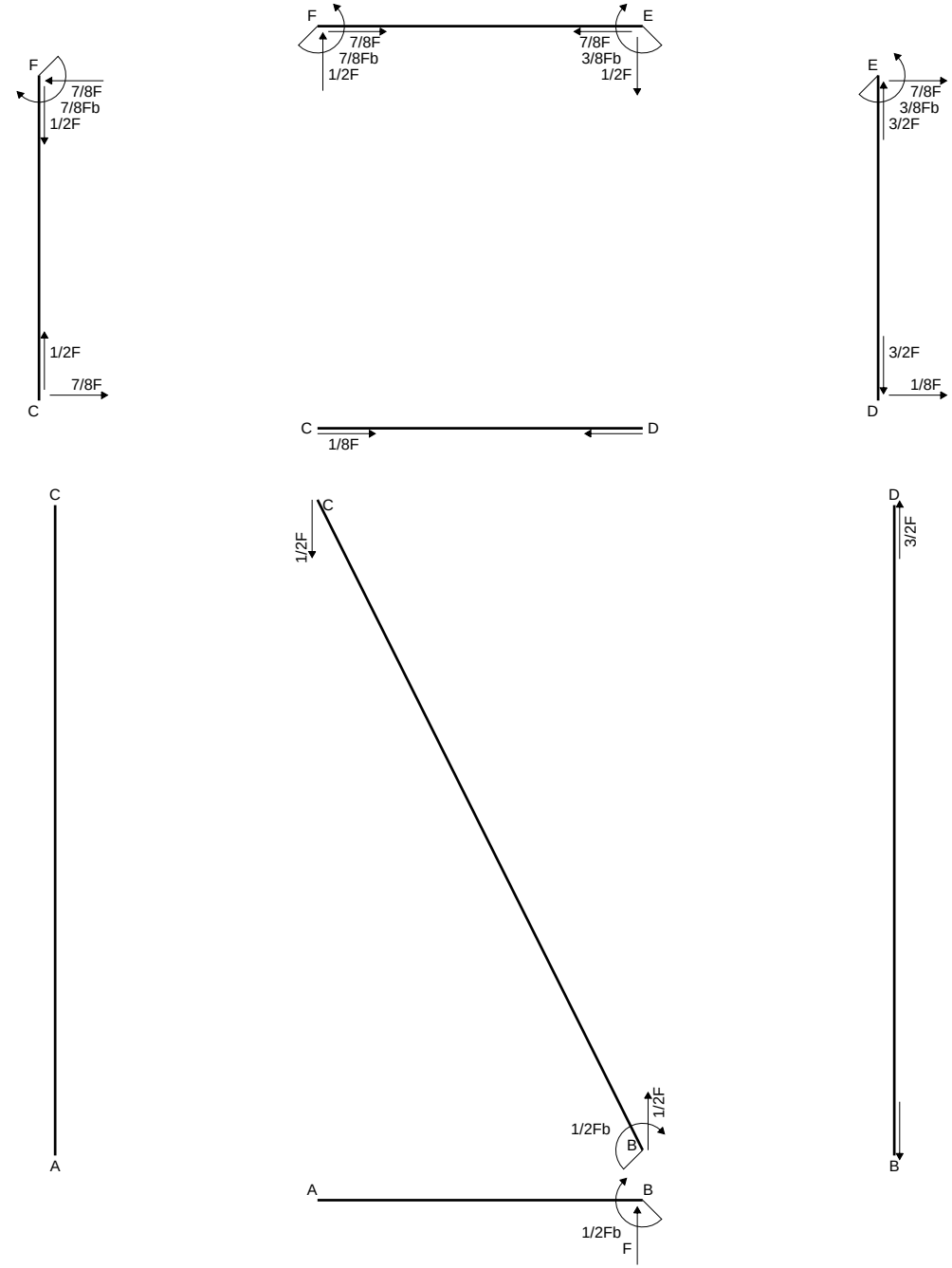
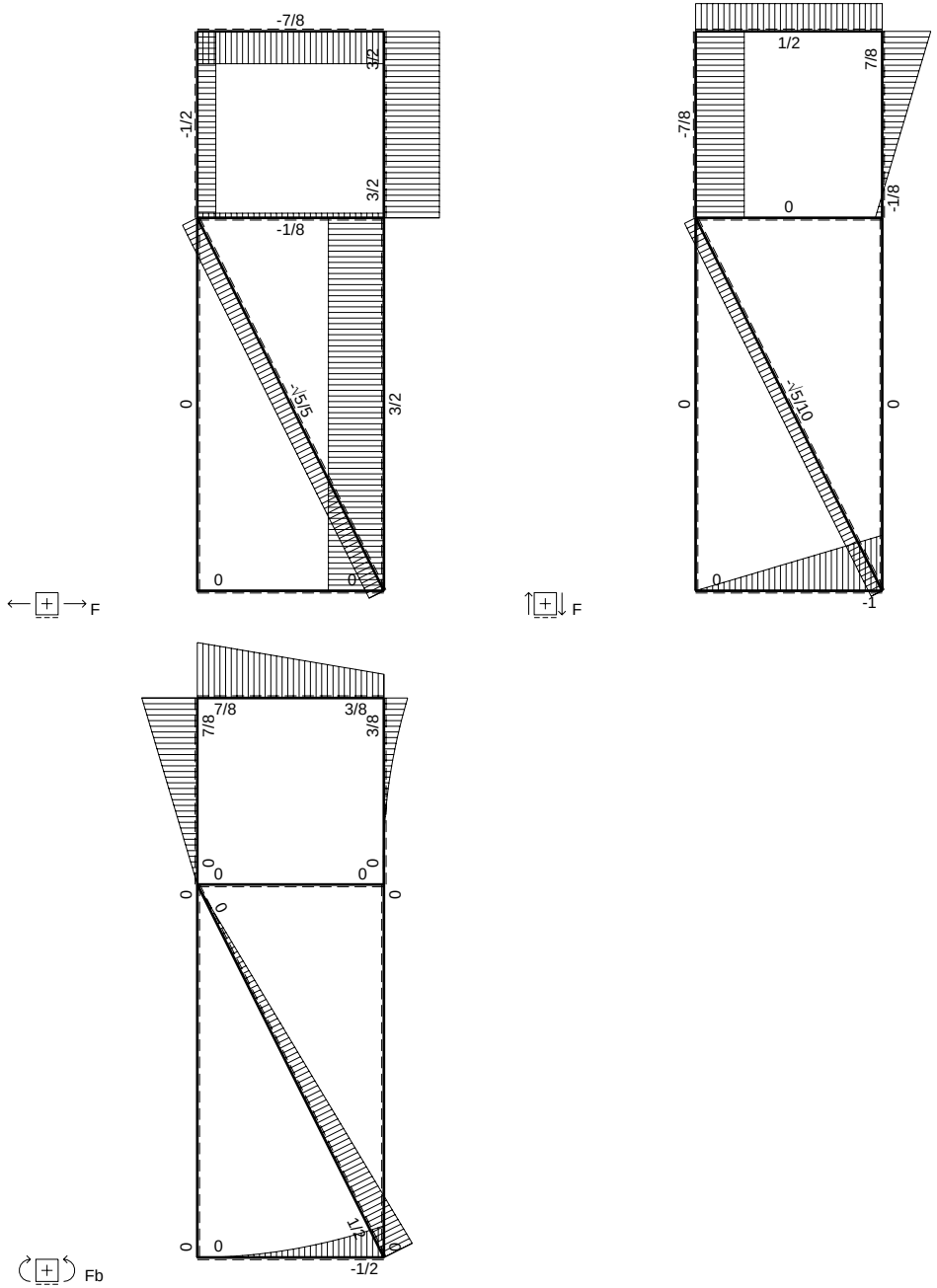

F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
3/2F
1/8F
D
1/8F
A
1/2Fb
B


-7/8
3/2
-1/2
-1/8
-√5/5
0
1/2
7/8
-1/8
-√5/10
-1
F
7/8
3/8
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$							-3/8Fb

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 212162 ESΣ09.xxxx.188

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

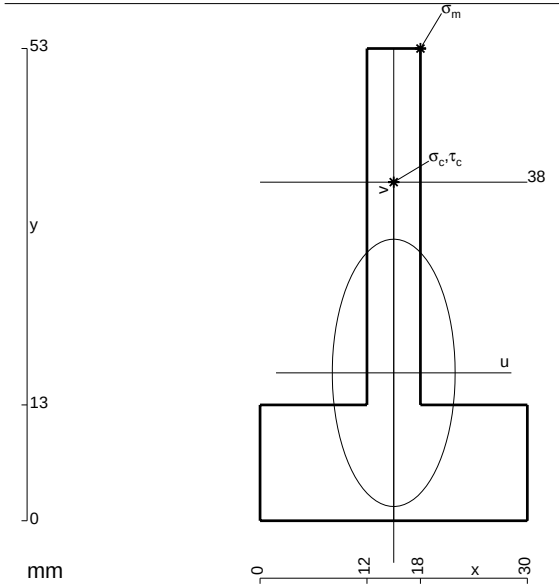
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 630. mm^2
J_u = 141827. mm^4
J_v = 29970. mm^4
y_g = 16.6 mm
T_y = -2710. N
M_x = -894300. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.4 mm
$\sigma_m = -Mv/J_u$ = 229.6 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.4 mm
$\sigma_c = -Mv/J_u$ = 135. N/mm^2
τ_c = 8.285 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.7 N/mm^2
S^* = 2601. mm^3

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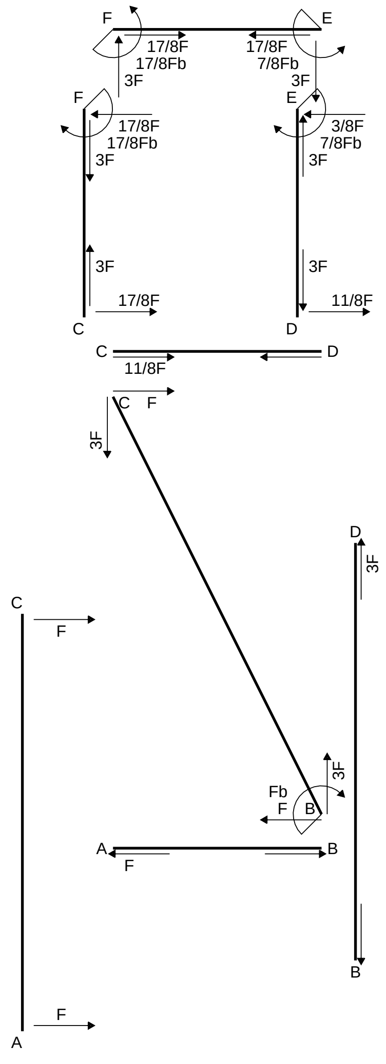
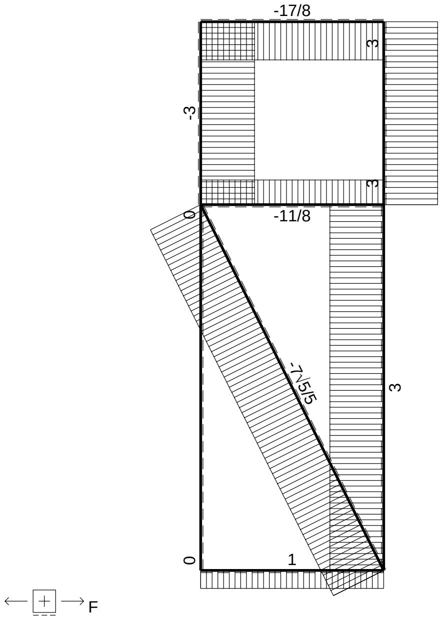
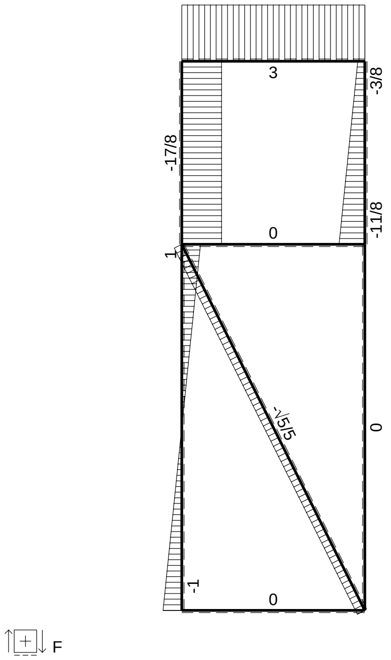
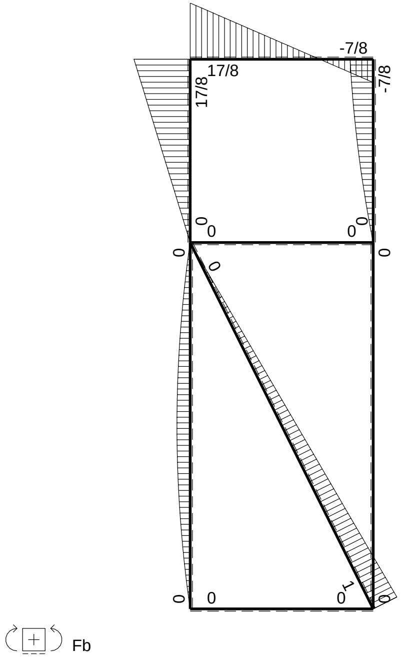
REAZIONI 212317

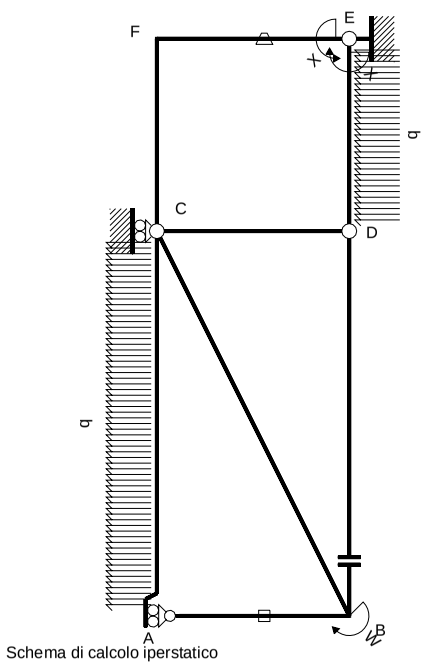


AZIONI INTERNE 212317

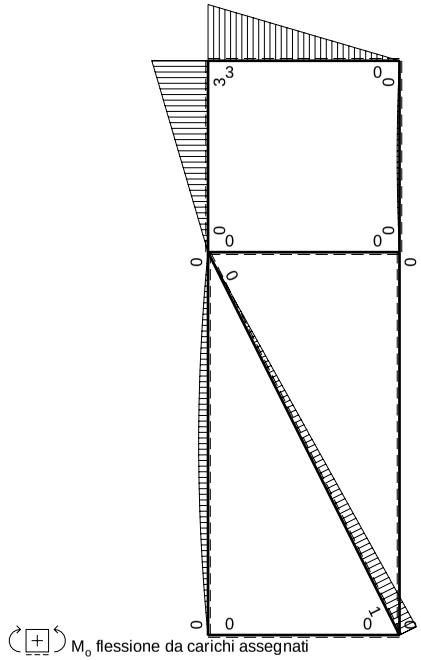




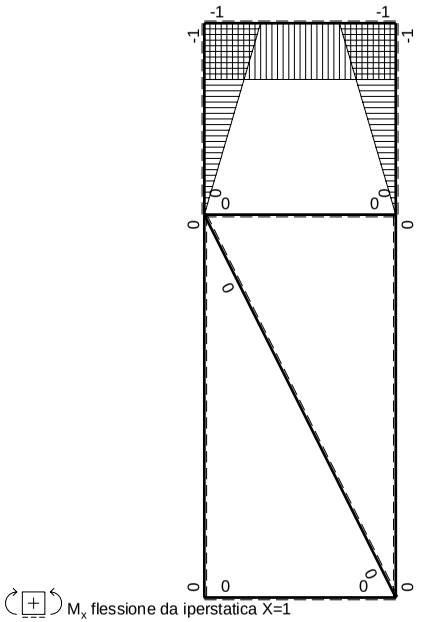




Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
		totali					$-35/24Fb^2/EJ$	5/3Xb/EJ
		iperstatica $X=W_{EF}$					7/8Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

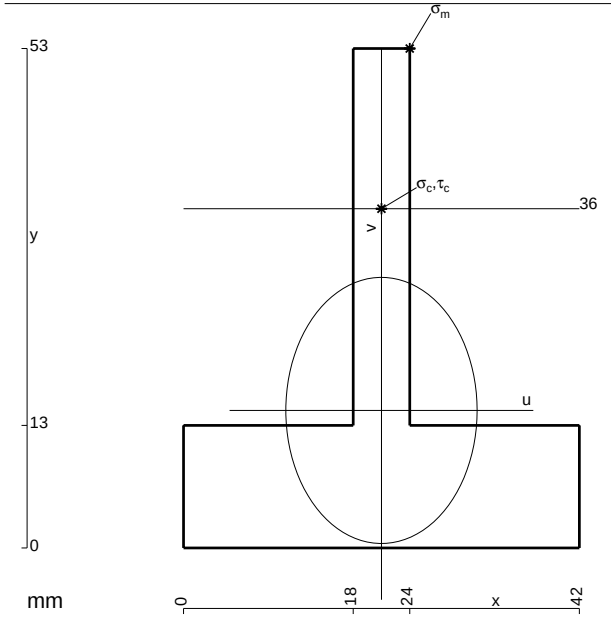
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 786. mm^2
J_u = 156767. mm^4
J_v = 80982. mm^4
y_g = 14.59 mm
N = -4257. N
T_y = -608.2 N
M_x = 952000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.41 mm
$\sigma_m = N/A - Mv/J_u$ = -238.7 N/mm^2
x_c = 21. mm
y_c = 36. mm
v_c = 21.41 mm
$\sigma_c = N/A - Mv/J_u$ = -135.4 N/mm^2
τ_c = 1.973 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.5 N/mm^2
S^* = 3051. mm^3

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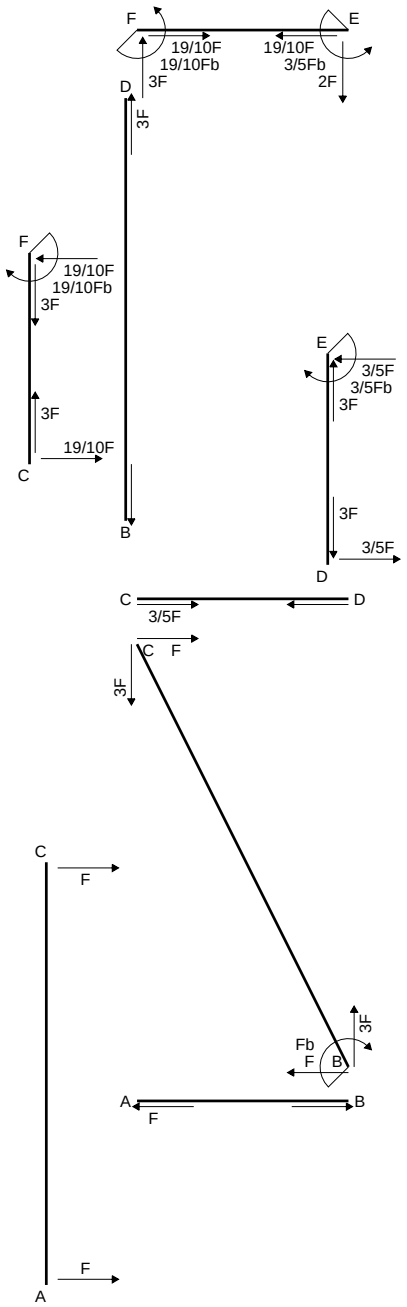
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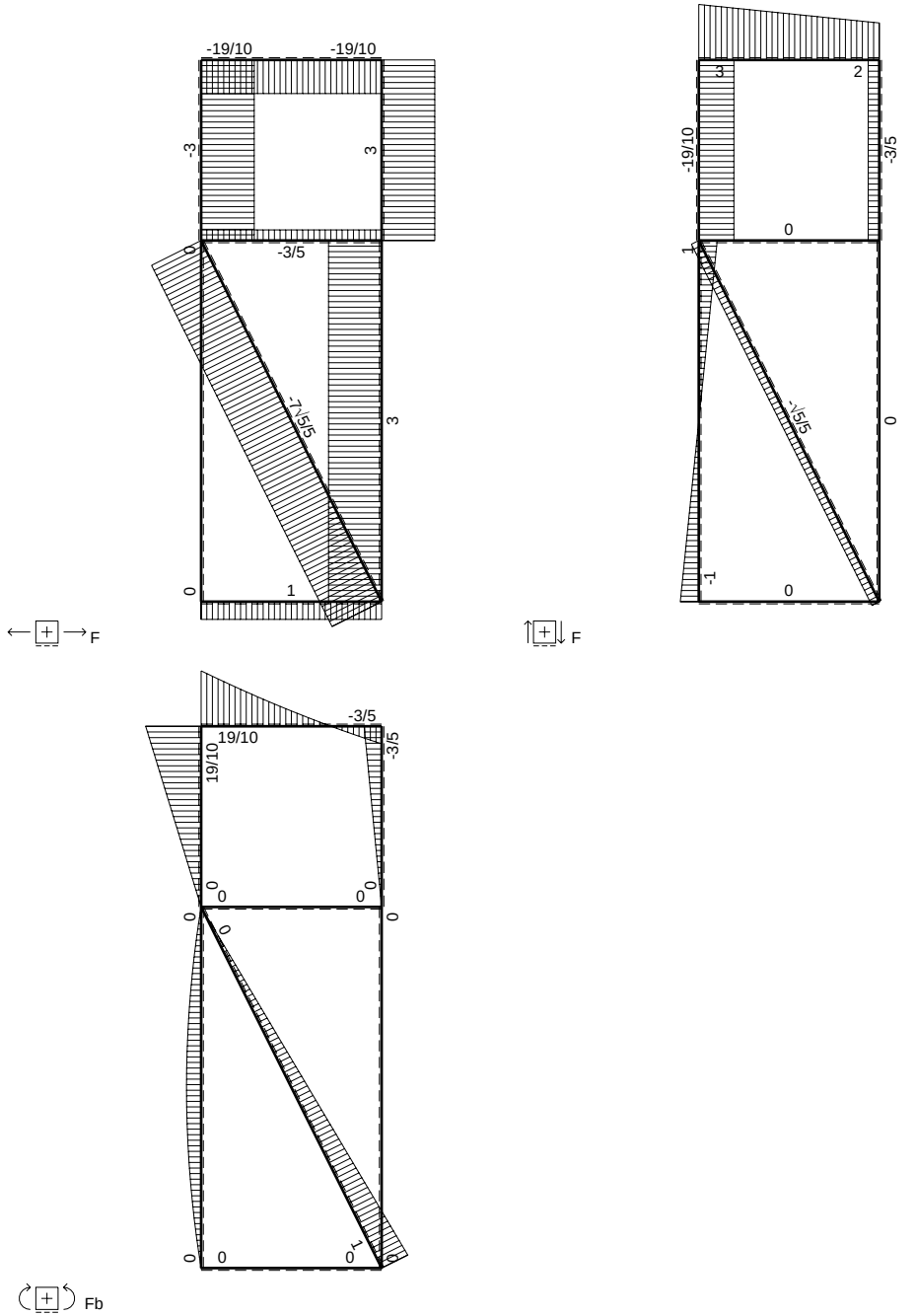
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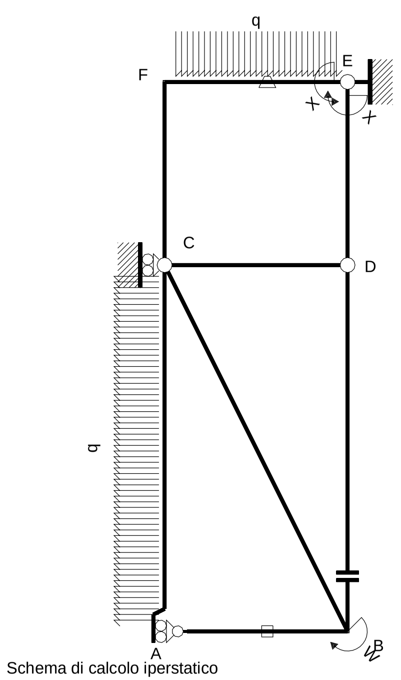
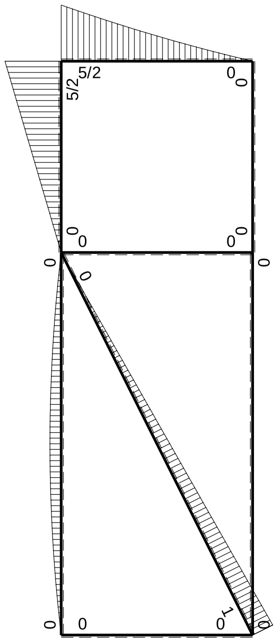
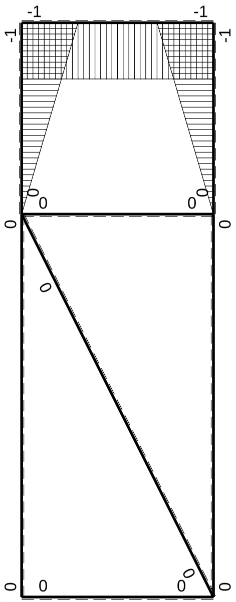
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

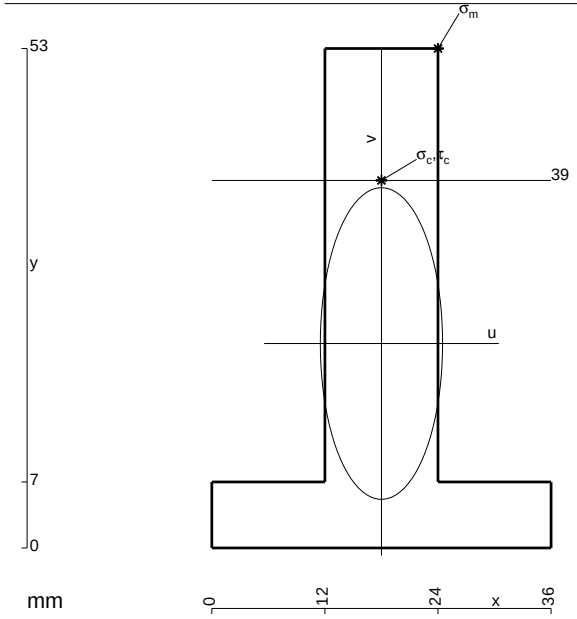
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 804. mm^2
J_u = 219865. mm^4
J_v = 33840. mm^4
y_g = 21.69 mm
N = -11051. N
T_y = -1579. N
M_x = 1306100. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 31.31 mm
$\sigma_m = N/A - Mv/J_u$ = -199.7 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 17.31 mm
$\sigma_c = N/A - Mv/J_u$ = -116.6 N/mm^2
τ_c = 2.443 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.6 N/mm^2
S^* = 4083. mm^3

ESΣ09.xxxx.190

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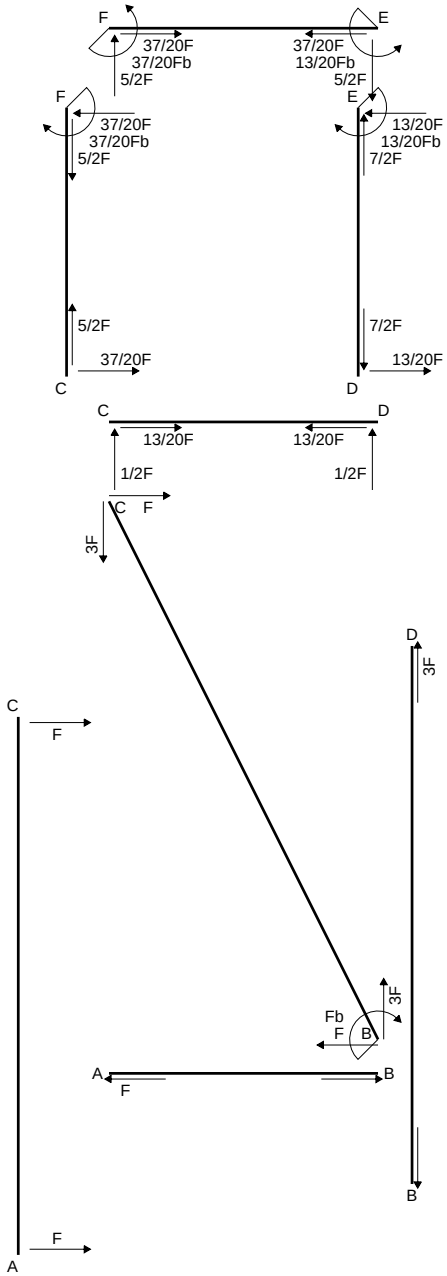
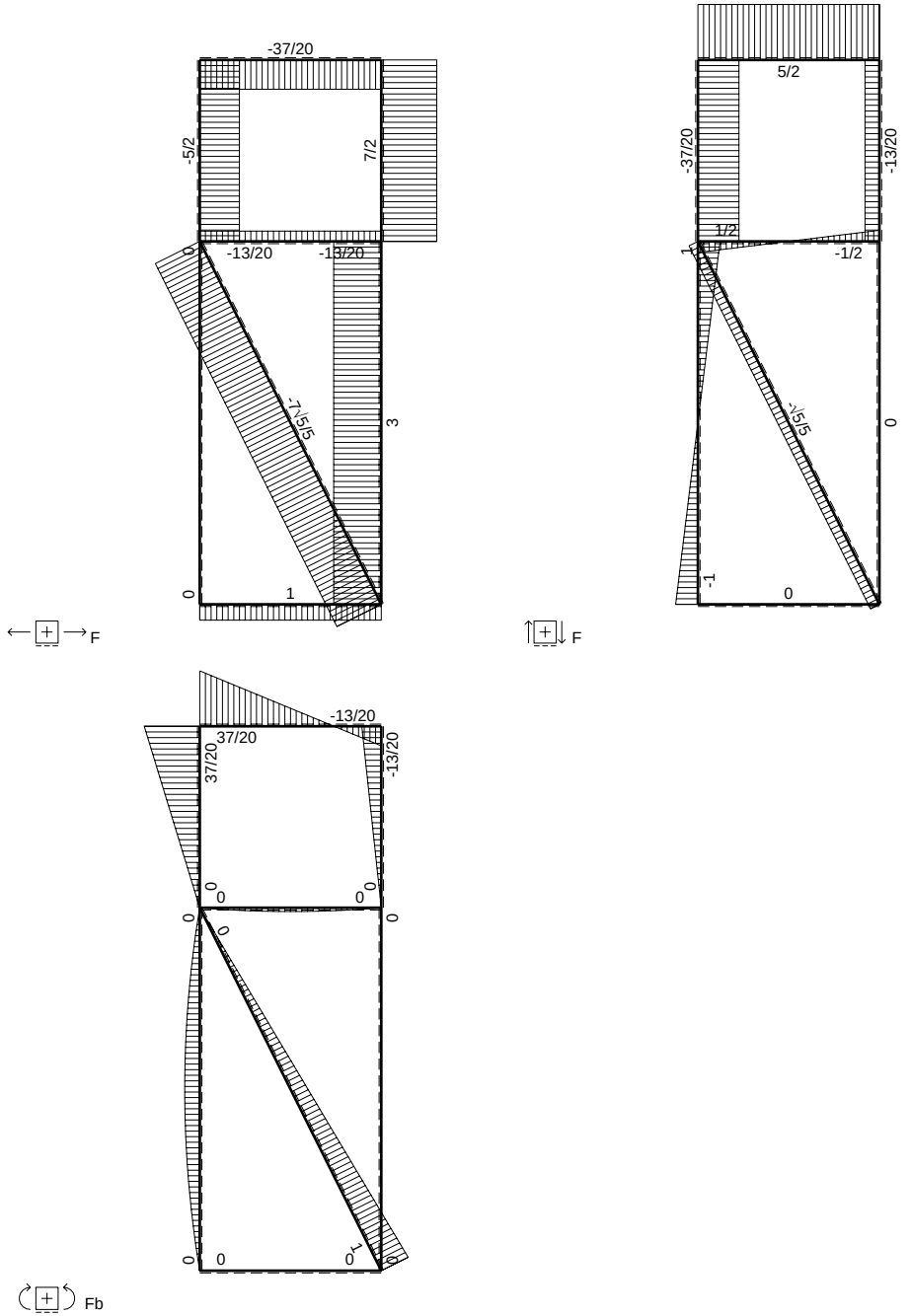

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B


-37/20
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7/2
-13/20
-13/20
0
-7.5/5
3
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
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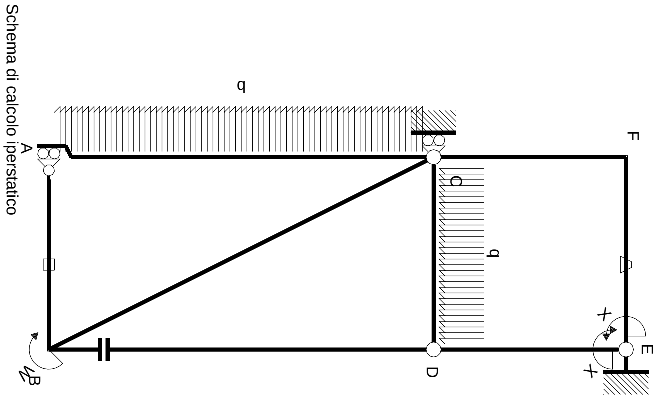
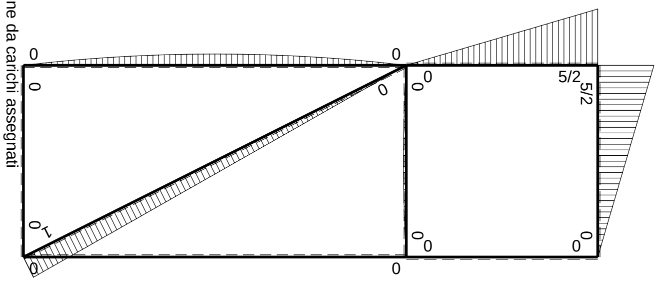
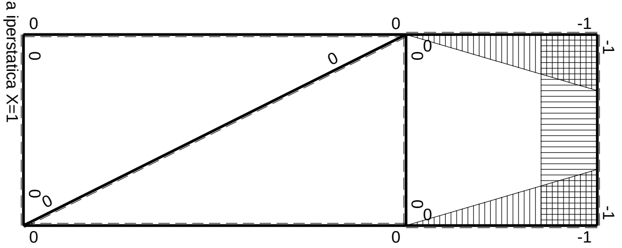
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

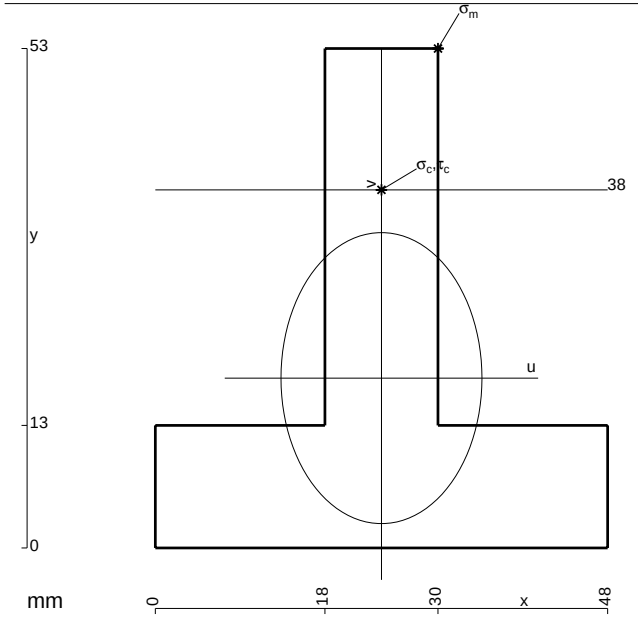
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1104. mm^2
J_u = 263311. mm^4
J_v = 125568. mm^4
y_g = 18.02 mm
N = -11739. N
T_y = -1677. N
M_x = 1500000. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.98 mm
$\sigma_m = N/A - Mv/J_u$ = -209.9 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.98 mm
$\sigma_c = N/A - Mv/J_u$ = -124.4 N/mm^2
τ_c = 2.625 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.5 N/mm^2
S^* = 4946. mm^3

ESΣ09.xxxx.191

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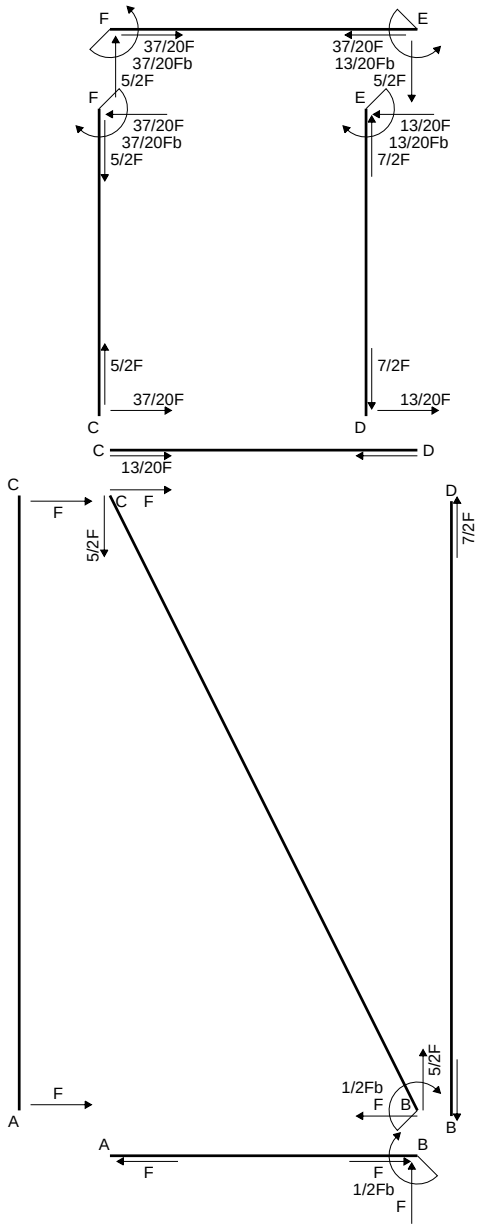
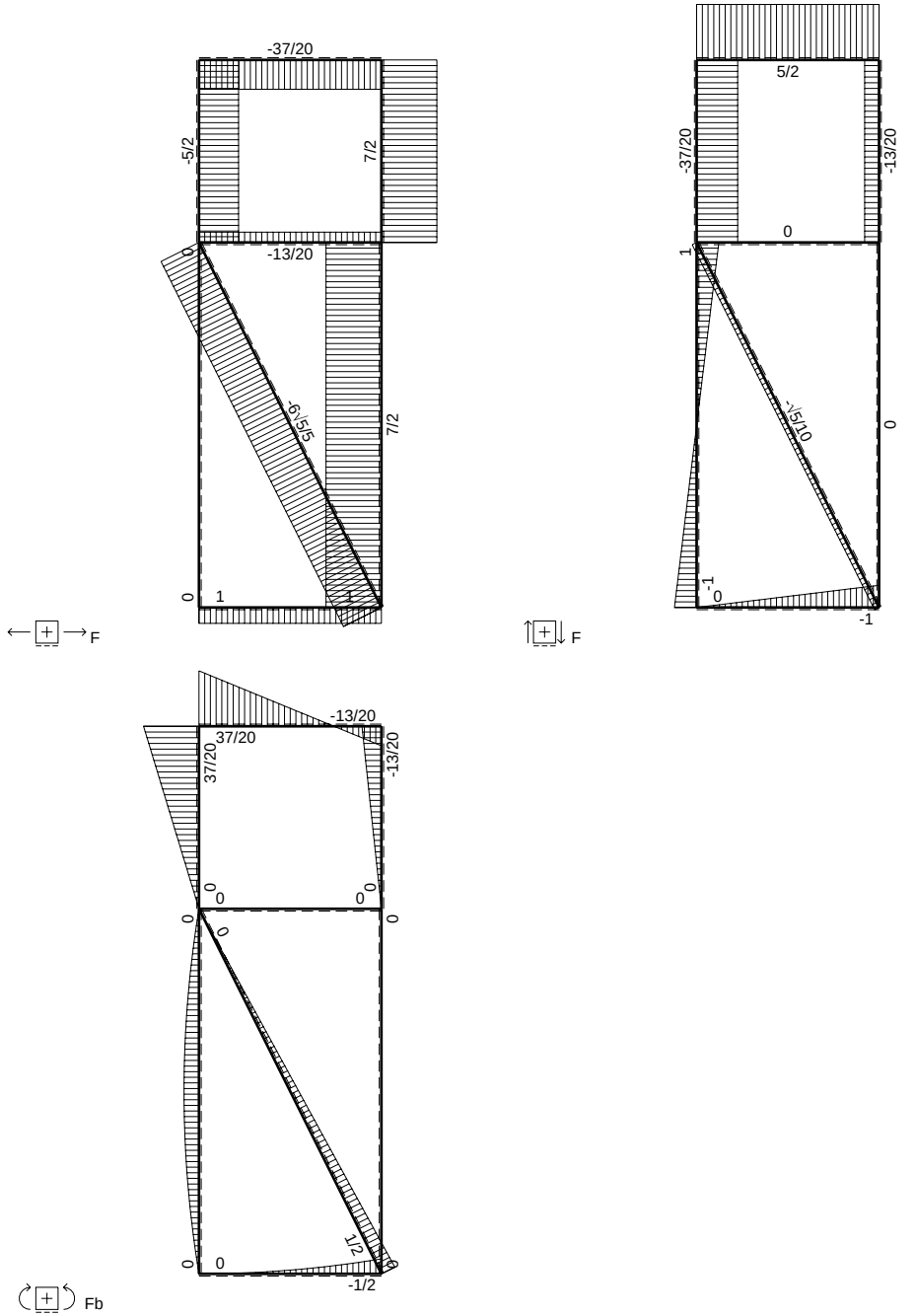

F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb


-37/20
-5/2
7/2
-13/20
-5\5/5
1
0
F
5/2
-37/20
-13/20
-5\5/10
-1
Fb
-1/2

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

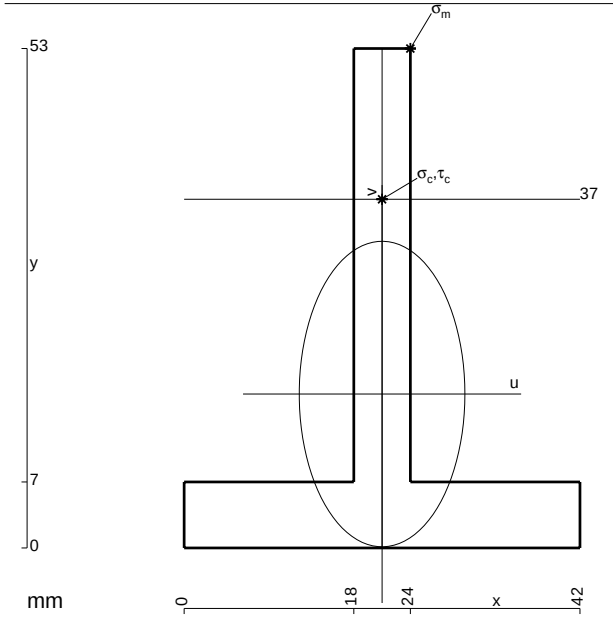
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 570. mm^2
J_u = 149839. mm^4
J_v = 44046. mm^4
y_g = 16.33 mm
N = 3950. N
T_y = -3950. N
M_x = -869000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.67 mm
$\sigma_m = N/A - Mv/J_u$ = 219.6 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 20.67 mm
$\sigma_c = N/A - Mv/J_u$ = 126.8 N/mm^2
τ_c = 12.09 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.5 N/mm^2
S^* = 2752. mm^3

ESΣ09.xxxx.192

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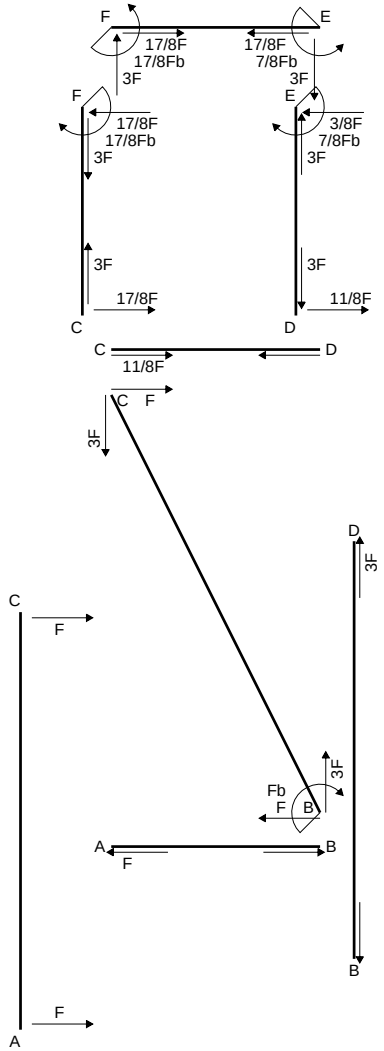
F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
F
E
17/8F
17/8Fb
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3/8F
7/8Fb
3F
3F
3F
17/8F
11/8F
C
D
C
D
11/8F
C F
3F
D
3F
C
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3F
Fb
F
B
A
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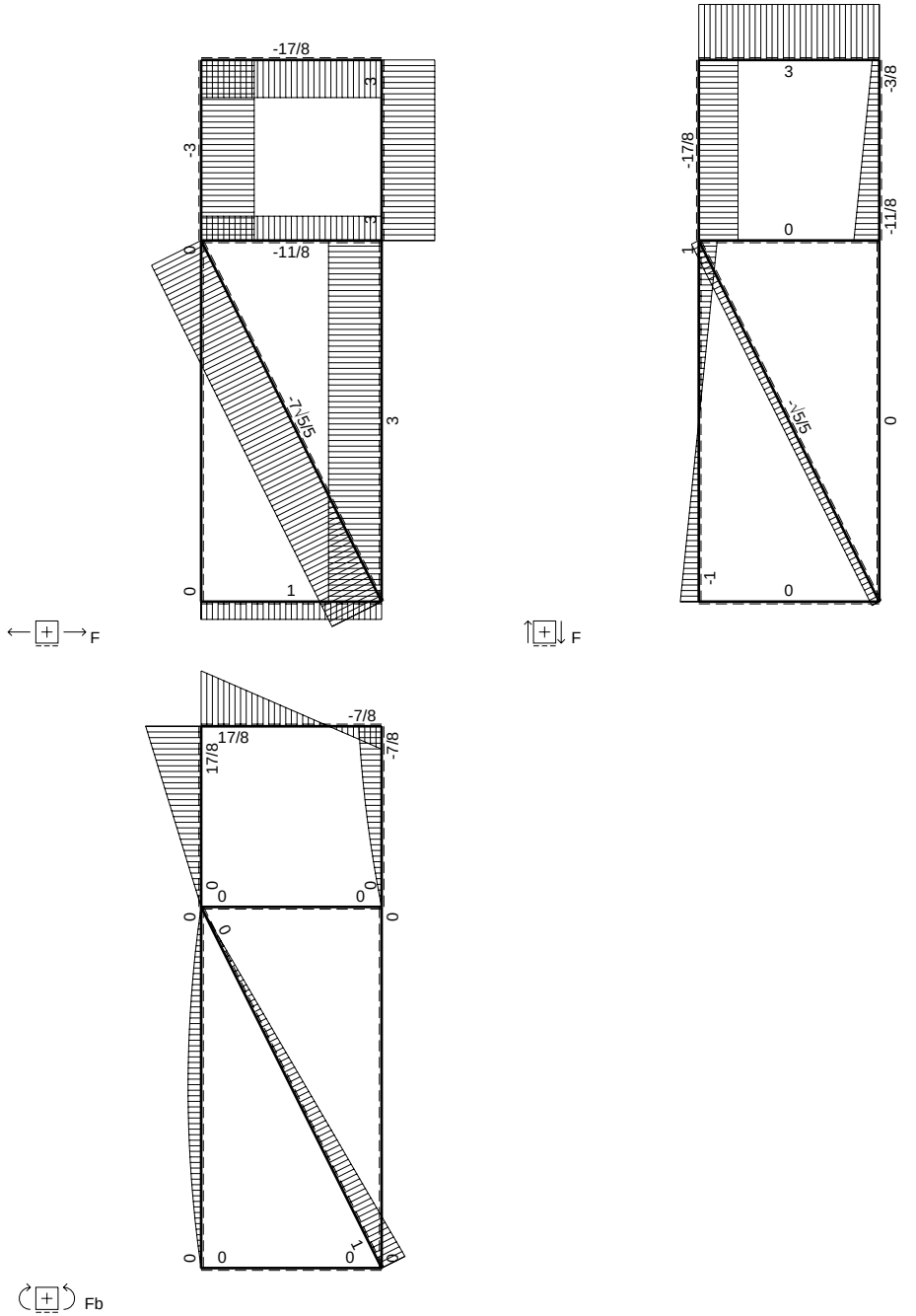
F
B
F
A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1
F
-17/8
3
-3/8
-11/8
0
-7√5/5
0
-1
0
F
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
0
0
0
0
0
1
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	(-3/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	-3Fb+6Fx-3Fx^2/b	0	1-2x/b+x^2/b^2	(-1+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-3Fx	0	-3Fx^2/b	0	x^2/b^2		
	totali						-35/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210932 ESΣ09.xxxx.193

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

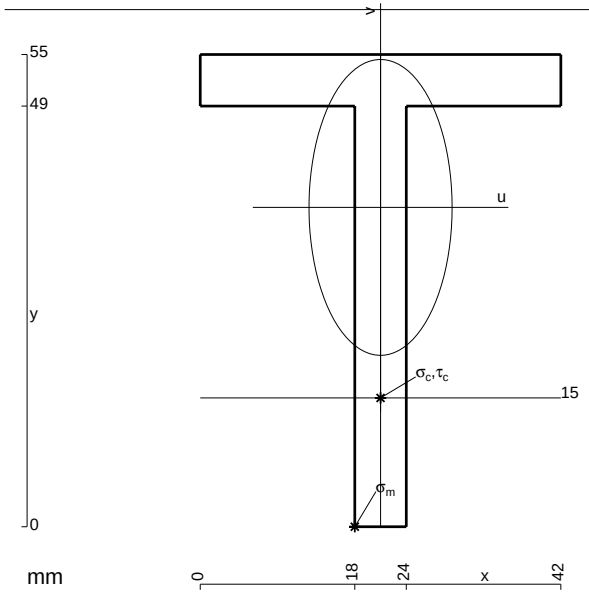
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 546. mm^2
J_u = 162198. mm^4
J_v = 37926. mm^4
y_g = 37.19 mm
N = -6605. N
T_y = -943.6 N
M_x = 1055000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.19 mm
σ_m = N/A-Mv/J_u = 229.8 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -22.19 mm
σ_c = N/A-Mv/J_u = 132.3 N/mm^2
τ_c = 2.591 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.3 N/mm^2
S^* = 2672. mm^3

ESΣ09.xxxx.193

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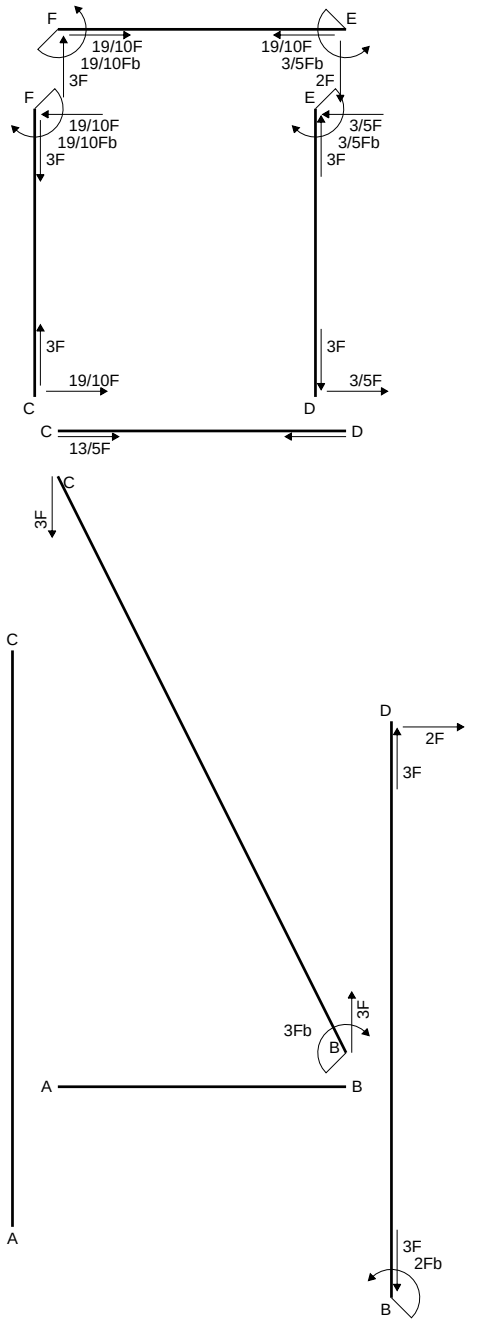
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19/10Fb
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19/10F
19/10Fb
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3/5F
C
D
C
D
13/5F
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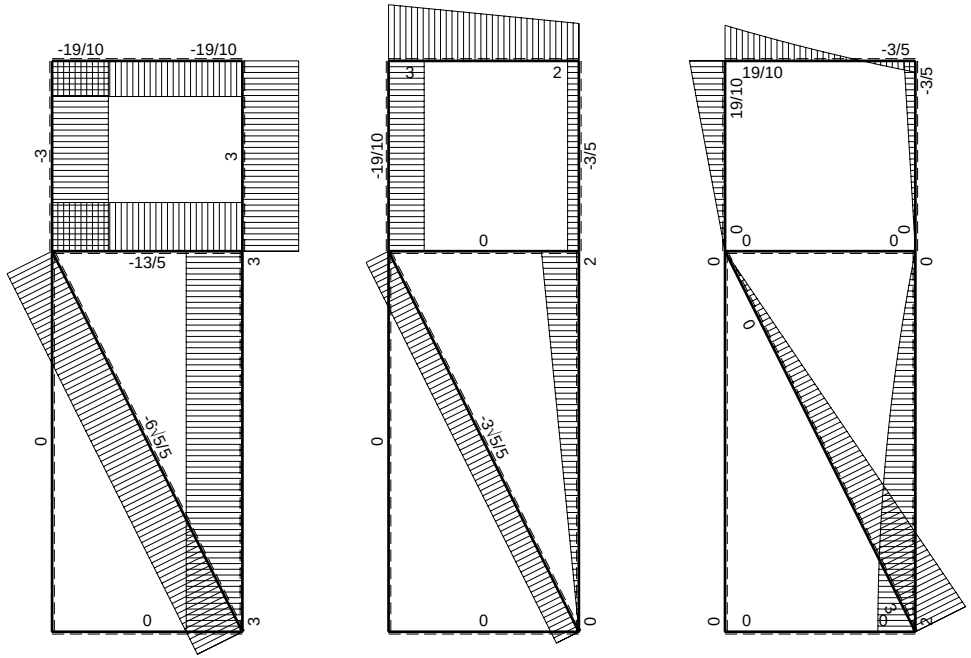
3F
C
D
2F
3F
3Fb
3F
B
A
B
3F
2Fb
A
B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-3/5
0
2
0
0
-3√5/5
-3/5
19/10
-3/5
0
0
0
0
0
0
0
0


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	2Fx+1/2qx^2	-Fb/EJ	-2Fx-1/2Fx^2/b	Fb/EJ	1	(-7/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+3Fx-1/2qx^2	Fb/EJ	-5/2Fb+3Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
totali							-Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$							3/5Fb

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 960114 ESΣ09.xxxx.194

14.11.24

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

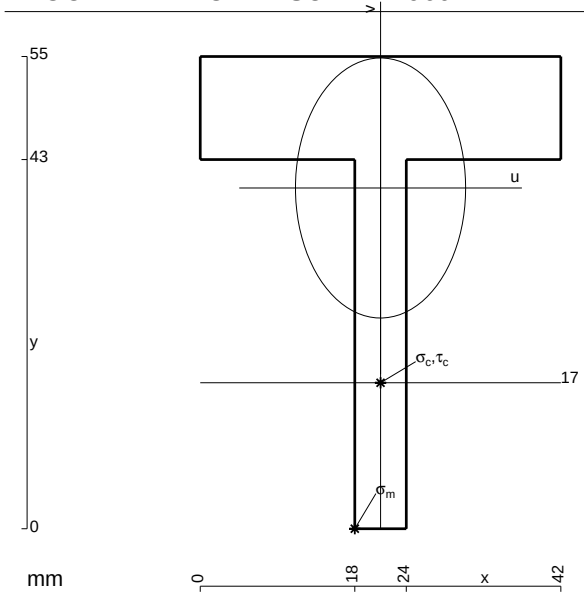
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 762. mm^2
J_u = 174852. mm^4
J_v = 74862. mm^4
y_g = 39.69 mm
N = -1744. N
T_y = -872.1 N
M_x = 1053000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.69 mm
$\sigma_m = N/A - Mv/J_u$ = 236.7 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.69 mm
$\sigma_c = N/A - Mv/J_u$ = 134.3 N/mm^2
τ_c = 2.644 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.4 N/mm^2
S^* = 3181. mm^3

ESΣ09.xxxx.194

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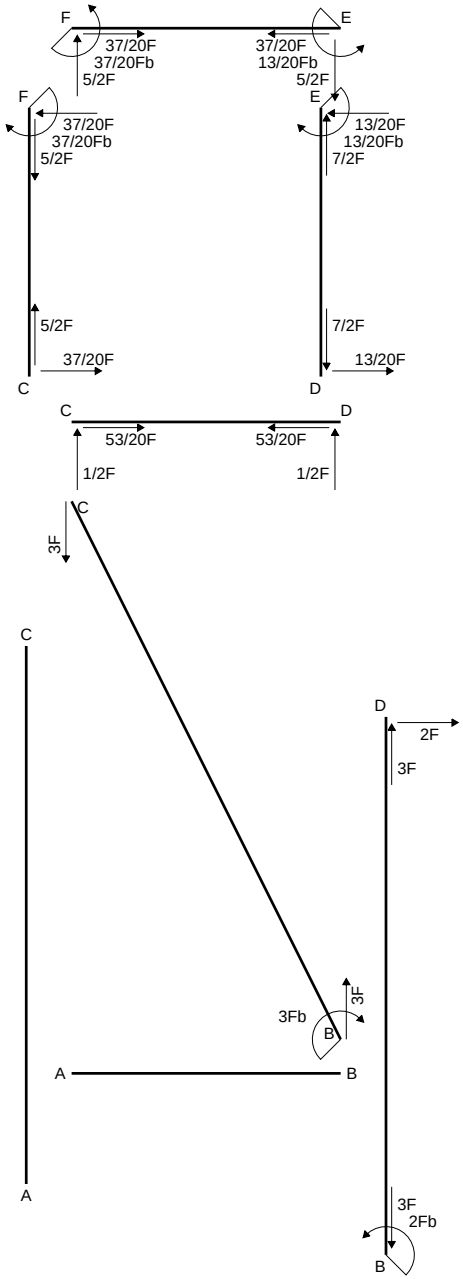
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37/20Fb
5/2F
37/20F
13/20Fb
5/2F
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13/20F
13/20Fb
7/2F
5/2F
37/20F
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13/20F
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D
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D
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53/20F
1/2F
1/2F
C
3F
C
D
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3F
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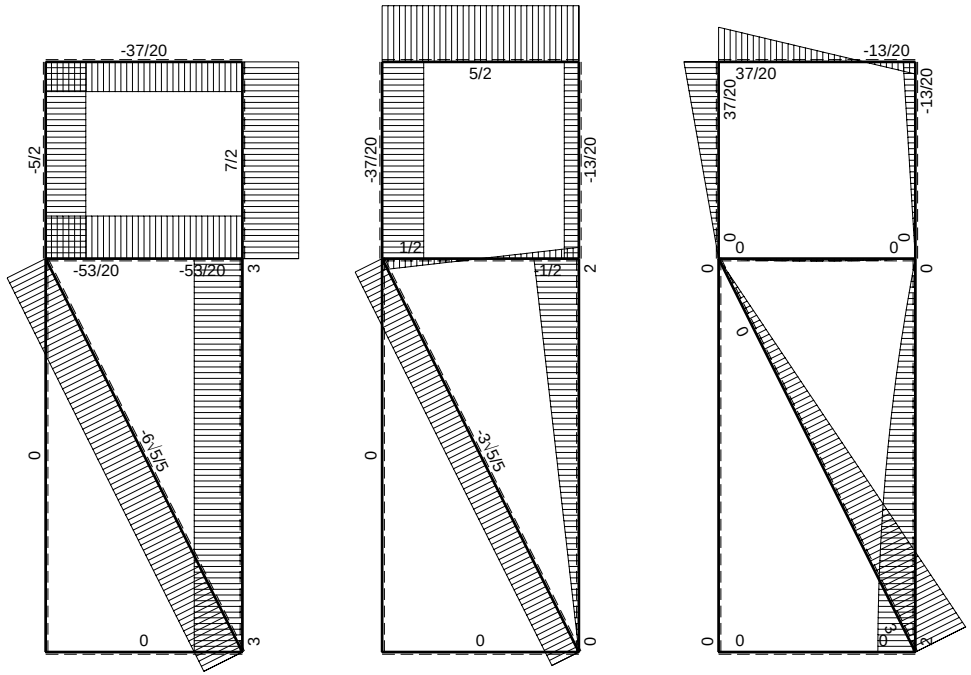
B
A
3F
2Fb
B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-37/20
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1/2
1/2
2
0
0
-3√5/5
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
3√5
3


F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx^2/b	0	1-2x/b+x^2/b^2	(-5/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx^2/b	0	x^2/b^2		
	totali						-13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

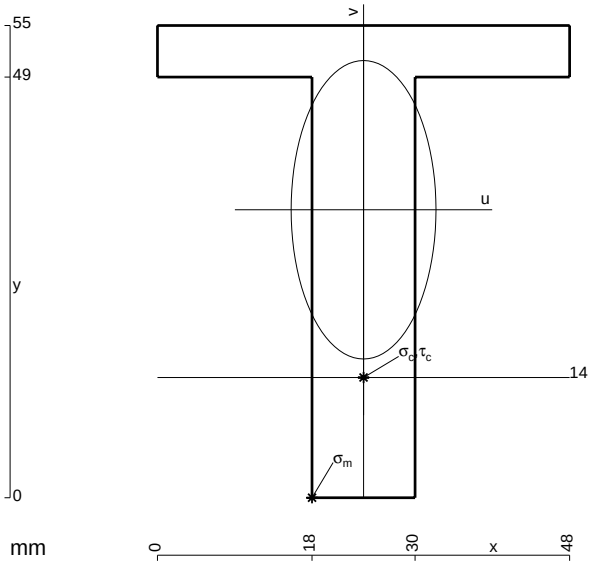
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 876. mm^2
J_u = 264708. mm^4
J_v = 62352. mm^4
y_g = 33.54 mm
N = -2442. N
T_y = -1221. N
M_x = 1583400. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.54 mm
σ_m = N/A-Mv/J_u = 197.8 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.54 mm
σ_c = N/A-Mv/J_u = 114.1 N/mm^2
τ_c = 1.714 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.1 N/mm^2
S^* = 4459. mm^3

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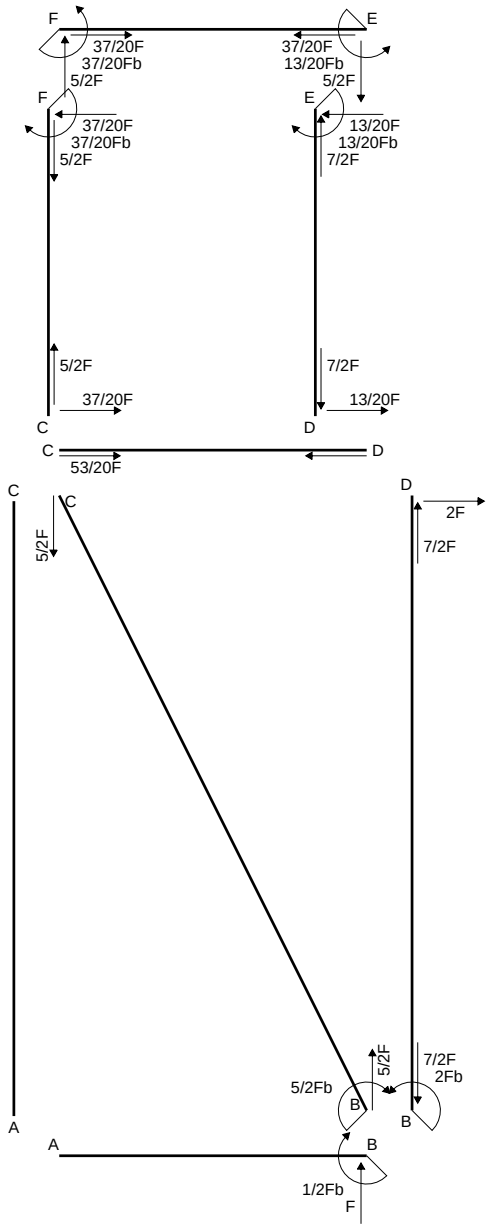
F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
7/2F
13/20F
C
D
C
D
53/20F
C
C
D
D
2F
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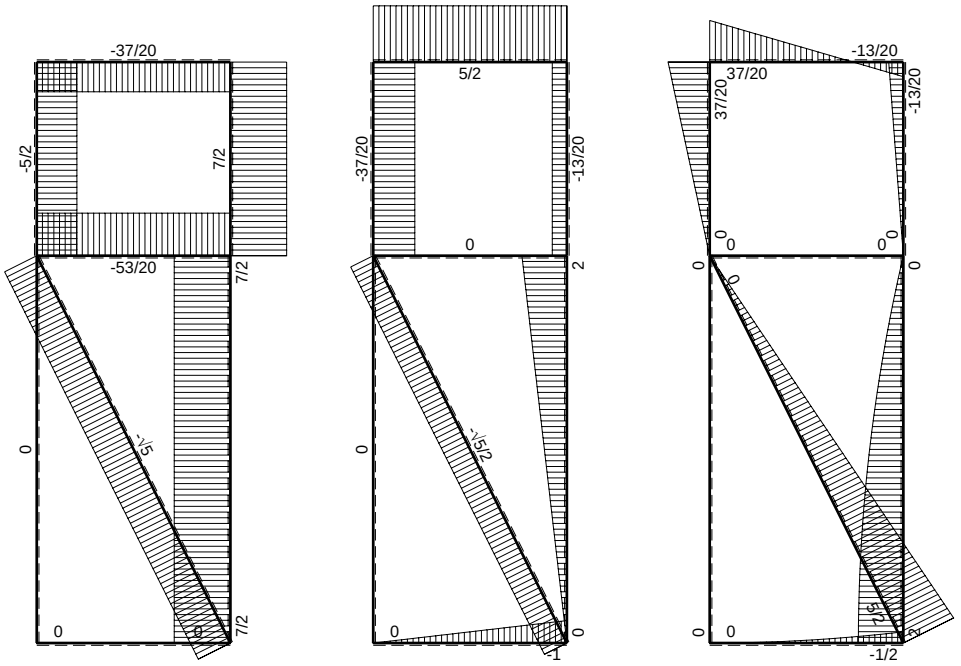
A
B
1/2Fb
F


-37/20
-5/2
7/2
-53/20
7/2
0
-√5
0
7/2
5/2
-37/20
-13/20
0
2
0
-√5/2
0
-1
0
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
-√5/2
0
-1/2

F
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	5/2Fb-√5/2Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 956625 ESΣ09.xxxx.196

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

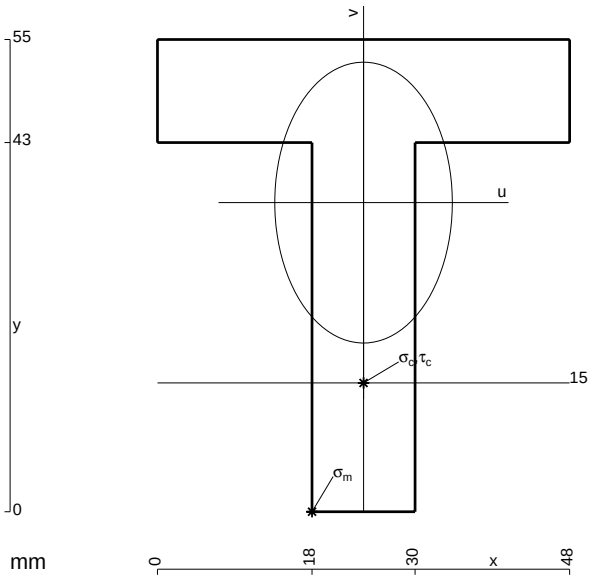
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 1092. mm^2
J_u = 292252. mm^4
J_v = 116784. mm^4
y_g = 36.01 mm
N = -2482. N
T_y = -1241. N
M_x = 1720500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -36.01 mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -21.01 mm
$\sigma_c = N/A - Mv/J_u$ = 121.4 N/mm^2
τ_c = 1.816 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 121.4 N/mm^2
S^* = 5131. mm^3

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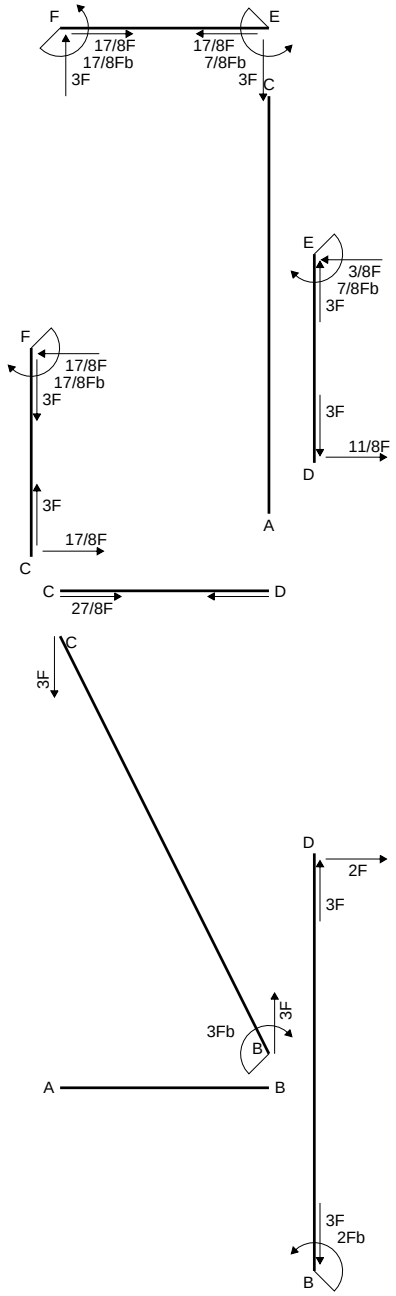
F
E
17/8F
17/8Fb
3F
17/8F
7/8Fb
3F
C
E
3/8F
7/8Fb
3F
F
17/8F
17/8Fb
3F
3F
11/8F
D
3F
A
17/8F
C
C
D
27/8F
C
3F
D
2F
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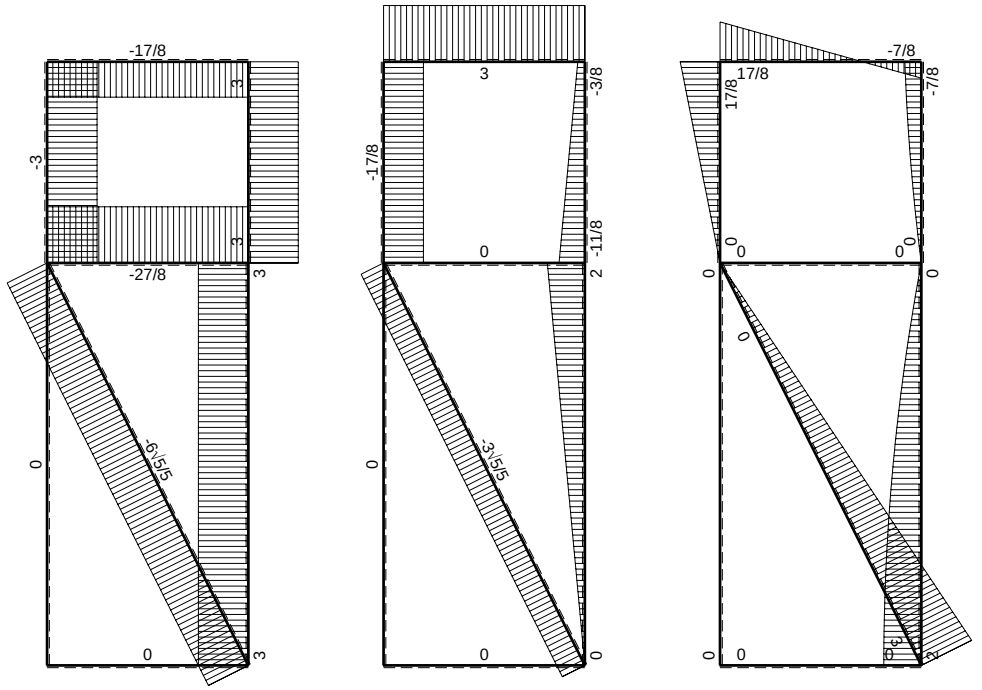
3Fb
3F
B
A
B
3F
2Fb
B


-17/8
3
-3
3
-27/8
3
0
-6√5/5
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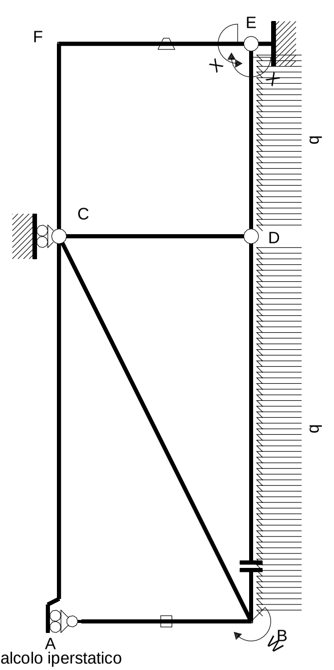
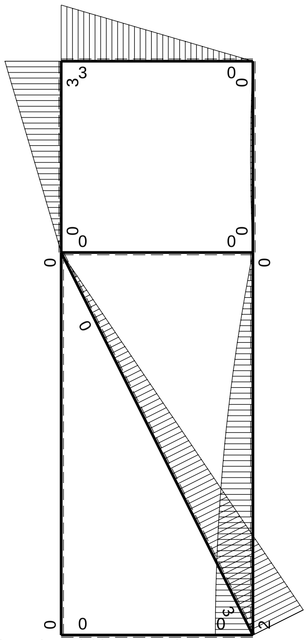
3
-17/8
3
-3/8
0
-11/8
2
-3√5/5
0
0
-7/8
17/8
17/8
-7/8
0
0
0
0
0
0
3√5/5

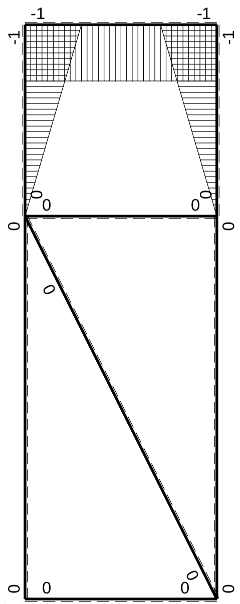

F
F
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	3Fb-3√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
BD 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	3Fx	-Fb/EJ	-3Fx	Fb/EJ	1	$(-3/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-3Fb+3Fx	Fb/EJ	-3Fb+3Fx	Fb/EJ	1		
FC b	-1+x/b	3Fb-3Fx	0	$-3Fb+6Fx-3Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-3Fx	0	$-3Fx^2/b$	0	x^2/b^2		
	totali						$-35/24Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						7/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-3/2\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-3 +3\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-3\ x +3/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

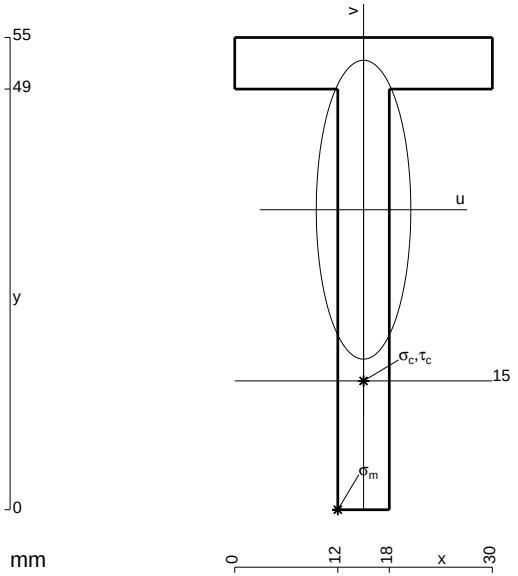
$= (-3\ b +3/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = -1/2\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-3 +6\ x/b -3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-3\ x +3\ x^2/b - x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-3\ b +3\ b - b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b)\ Fb\ 1/EJ\ = -\ Fb^2/EJ$



A = 474. mm^2
J_u = 143796. mm^4
J_v = 14382. mm^4
y_g = 34.94 mm
N = -1234. N
T_y = -617.2 N
M_x = 910800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -34.94 mm
$\sigma_m = N/A - Mv/J_u$ = 218.7 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -19.94 mm
$\sigma_c = N/A - Mv/J_u$ = 123.7 N/mm^2
τ_c = 1.767 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.8 N/mm^2
S^* = 2470. mm^3

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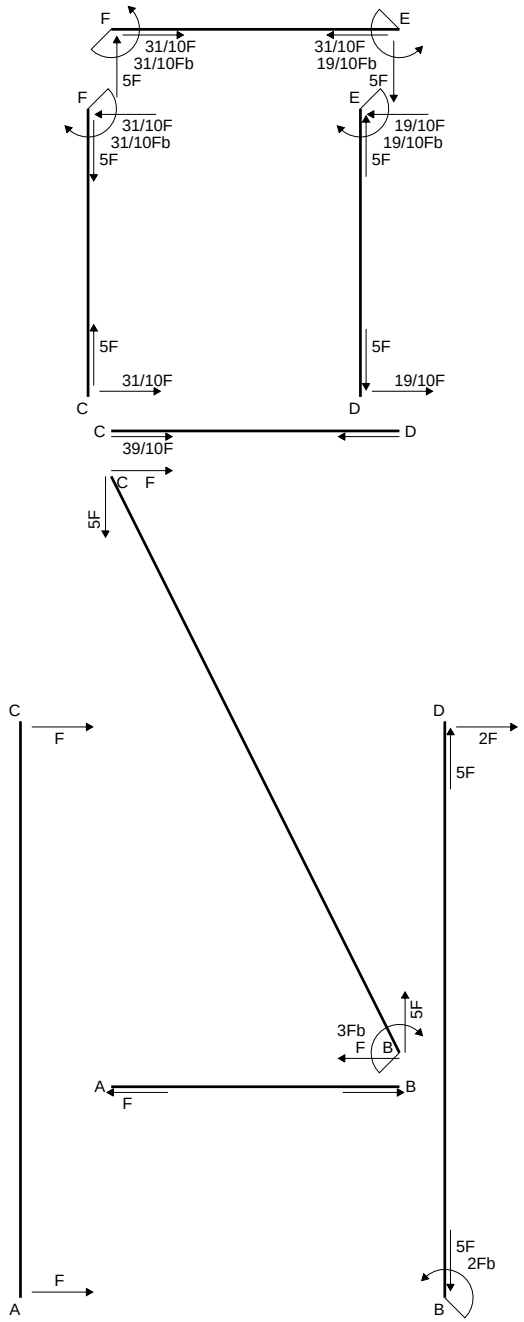
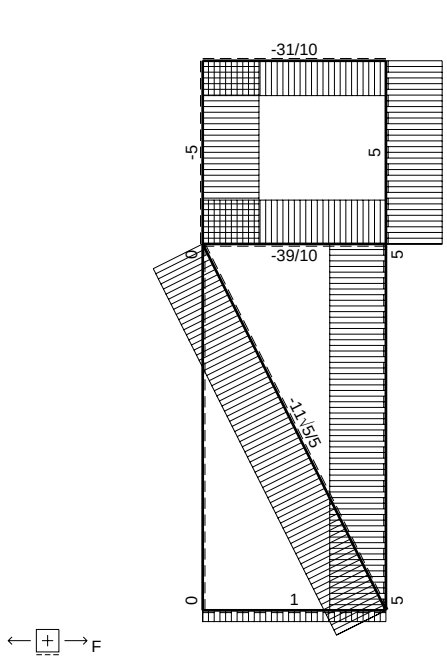
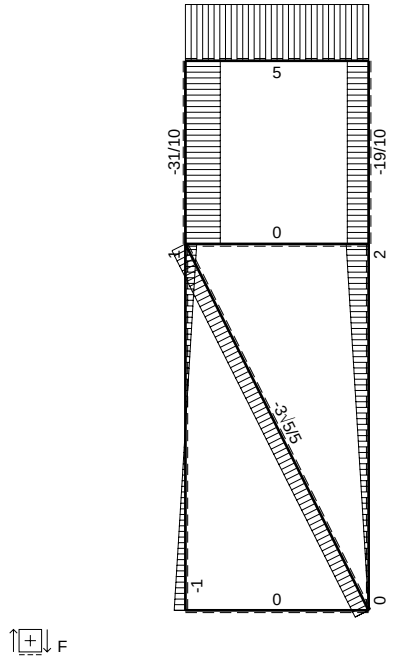
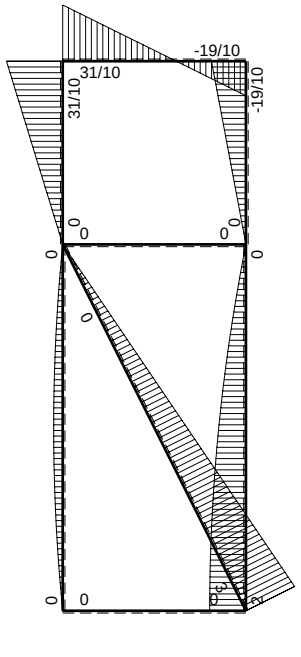
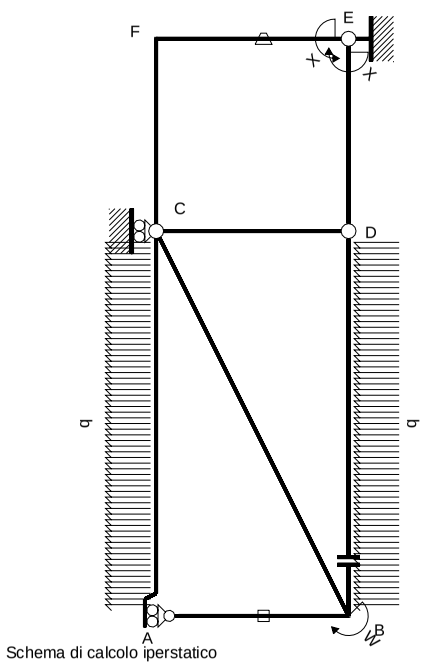

F
E
31/10F
31/10Fb
19/10Fb
5F
31/10F
19/10F
C
D
39/10F
2F
2Fb
3Fb
A
B

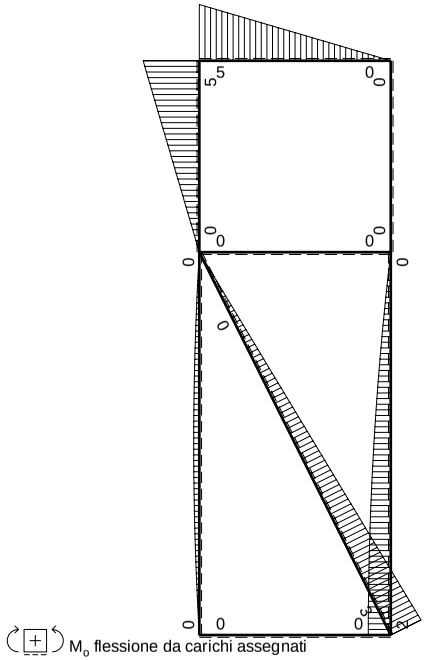

-31/10
-39/10
-11√5/5
F


-31/10
-19/10
-3√5/5
F

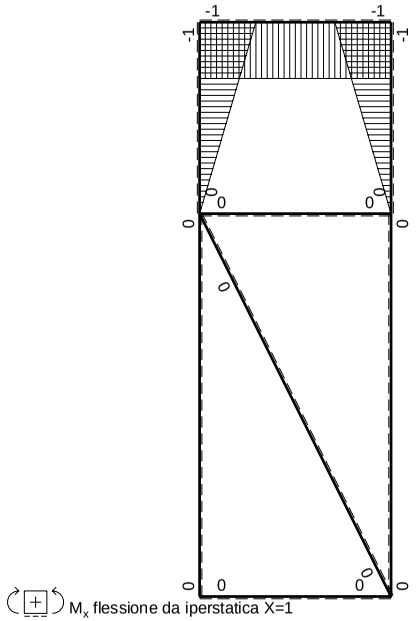

-19/10
31/10
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	3Fb-3$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx^2	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx^2	0	0	0	0		
DB 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
BD 2b	0	-2Fb+1/2qx^2	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	5Fx	-Fb/EJ	-5Fx	Fb/EJ	1	(-5/2+1)Fb^2/EJ	Xb/EJ
FE b	1	-5Fb+5Fx	Fb/EJ	-5Fb+5Fx	Fb/EJ	1		
FC b	-1+x/b	5Fb-5Fx	0	-5Fb+10Fx-5Fx^2/b	0	1-2x/b+x^2/b^2	(-5/3+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-5Fx	0	-5Fx^2/b	0	x^2/b^2		
	totali						-19/6Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						19/10Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 280320 ESΣ09.xxxx.198

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/2\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5 +5\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-5\ x +5/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

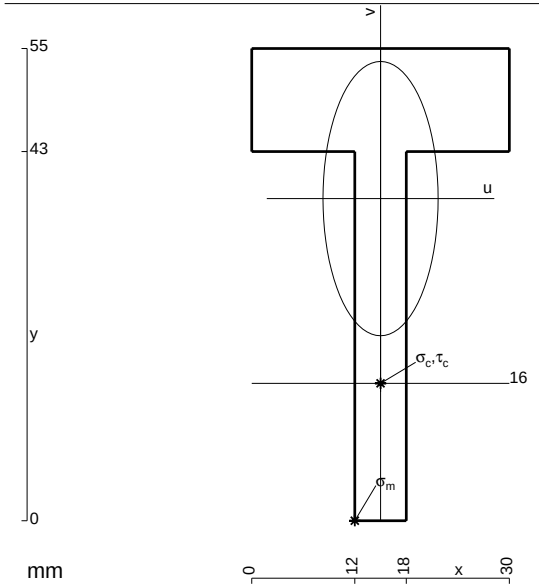
$= (-5\ b +5/2\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -3/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5 +10\ x/b -5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5\ x +5\ x^2/b -5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5\ b +5\ b -5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/3\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/3\ b\ )\ Fb\ 1/EJ\ = -5/3\ Fb^2/EJ$



A = 618. mm^2
J_u = 157731. mm^4
J_v = 27774. mm^4
y_g = 37.52 mm
N = -2263. N
T_y = -617.2 N
M_x = 966000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.52 mm
$\sigma_m = N/A - Mv/J_u$ = 226.1 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.52 mm
$\sigma_c = N/A - Mv/J_u$ = 128.1 N/mm^2
τ_c = 1.848 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.2 N/mm^2
S^* = 2834. mm^3

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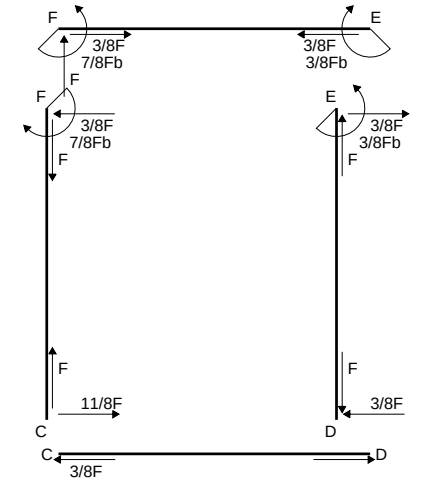
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
3/8Fb
F
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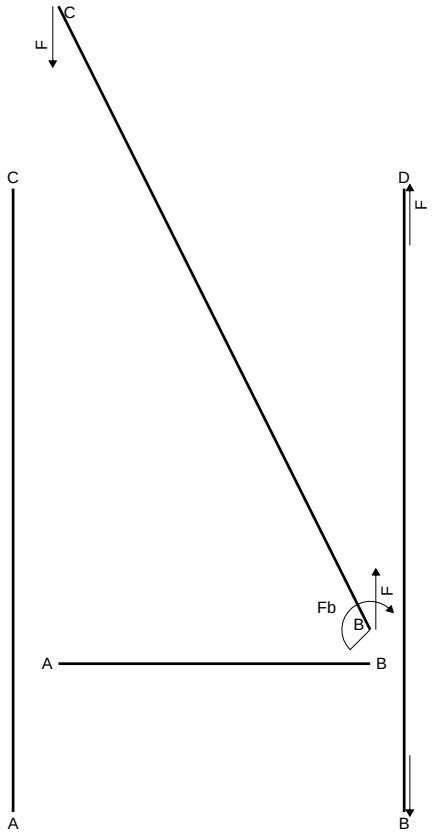
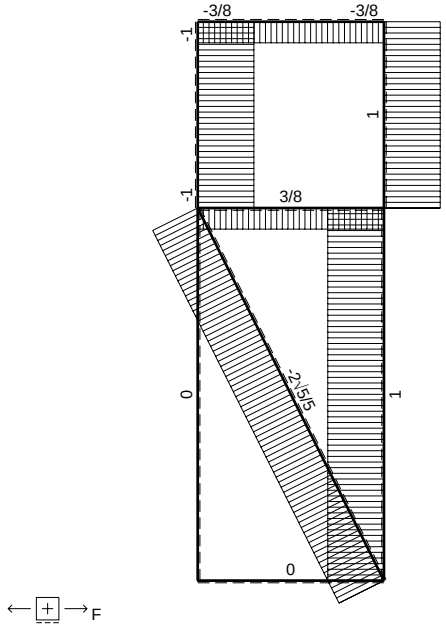
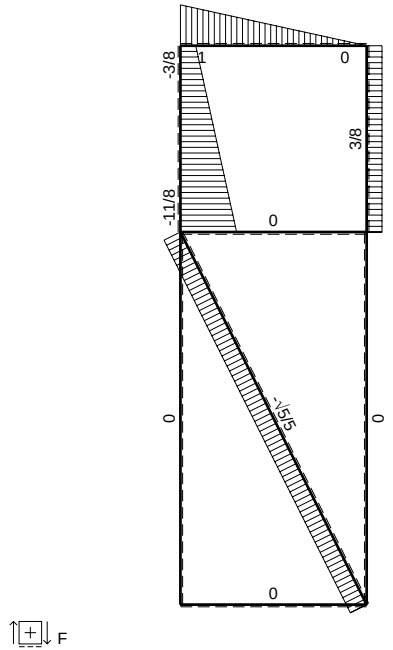
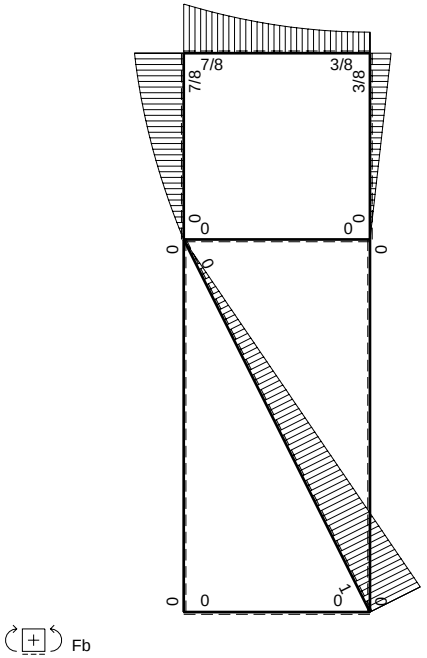
11/8F
F
3/8F
C
D
C
D
3/8F


C
F
C
D
F
Fb
F
B
A
B
A
B


-3/8
-3/8
-1
-1
1
3/8
-1
0
1
-2√5/5
0
1
0
F


-3/8
1
0
3/8
-11/8
0
-√5/5
0
0
F


7/8
7/8
3/8
3/8
0
0
0
0
0
0
0
0
0
0
-1
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	$-1/2Fx^2/b$	Fb/EJ	1	$(-1/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

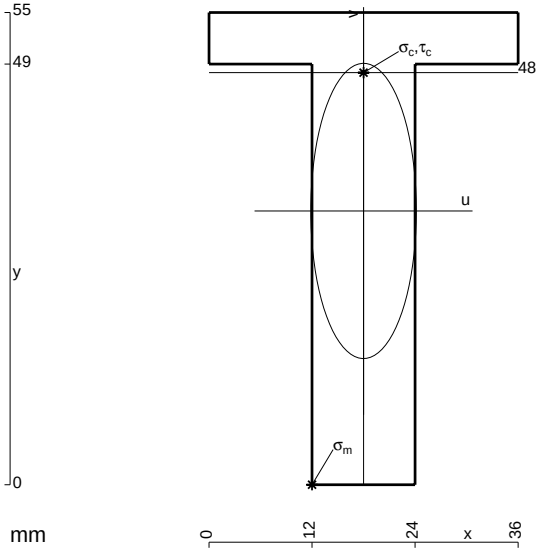
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 804. mm^2
J_u = 237762. mm^4
J_v = 30384. mm^4
y_g = 31.89 mm
N = -2182. N
T_y = -1091. N
M_x = 1805600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.89 mm
σ_m = N/A-Mv/J_u = 239.4 N/mm^2
x_c = 18. mm
y_c = 48. mm
v_c = 16.11 mm
σ_c = N/A-Mv/J_u = -125.1 N/mm^2
τ_c = 1.738 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.1 N/mm^2
S^* = 4544. mm^3

ESΣ09.xxxx.199

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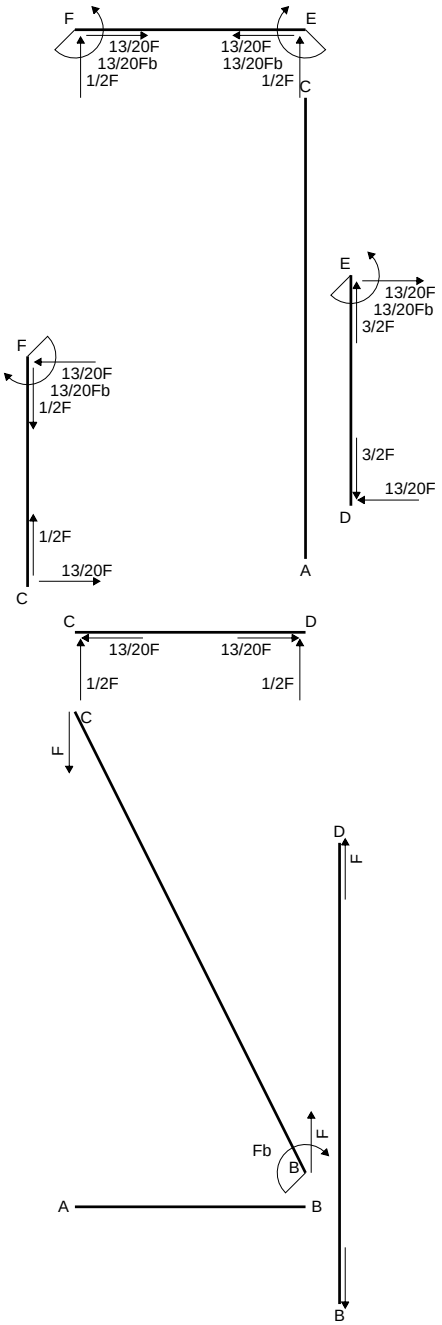
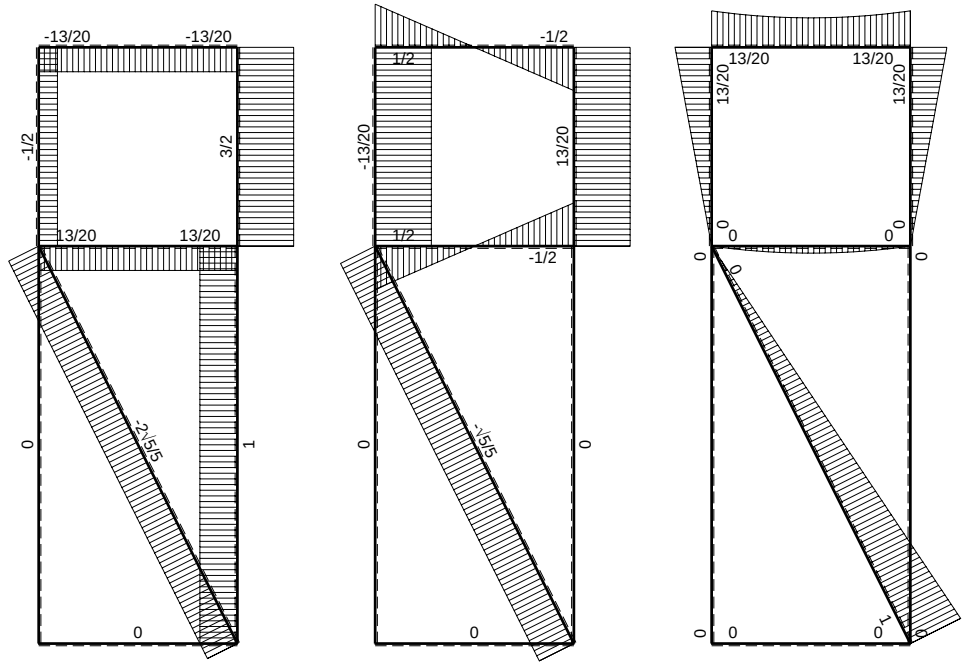
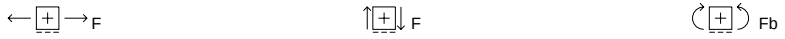
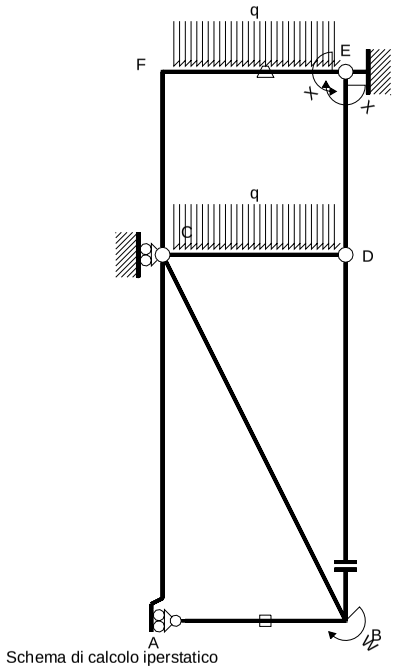
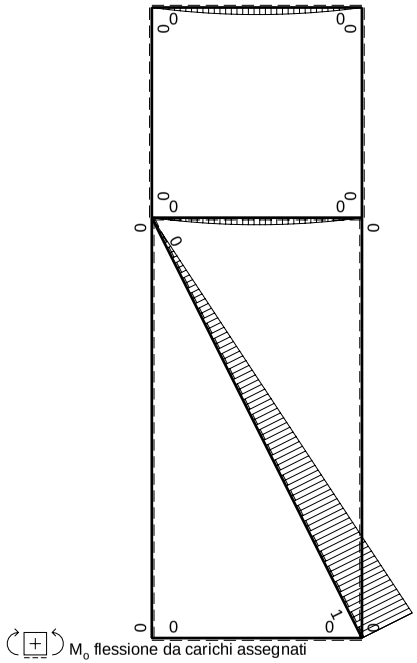

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

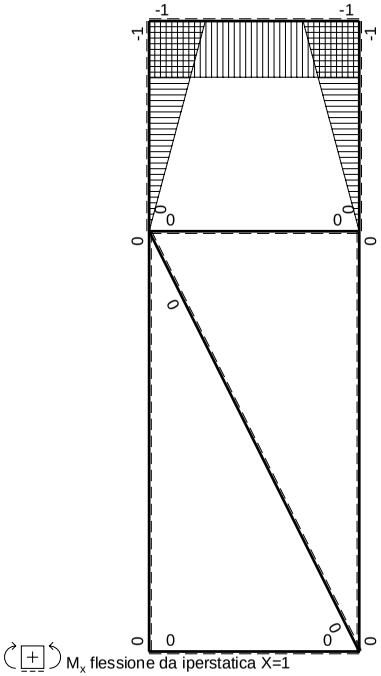

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 215530 ESΣ09.xxxx.200

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

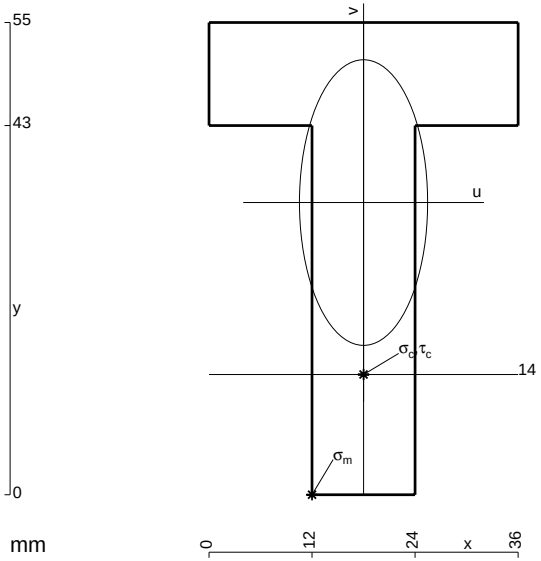
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 948. mm^2
J_u = 262515. mm^4
J_v = 52848. mm^4
y_g = 34.03 mm
N = -3605. N
T_y = -1802. N
M_x = 1571700. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.03 mm
$\sigma_m = N/A - Mv/J_u$ = 199.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -20.03 mm
$\sigma_c = N/A - Mv/J_u$ = 116.1 N/mm^2
τ_c = 2.598 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.2 N/mm^2
S^* = 4541. mm^3

ESΣ09.xxxx.200

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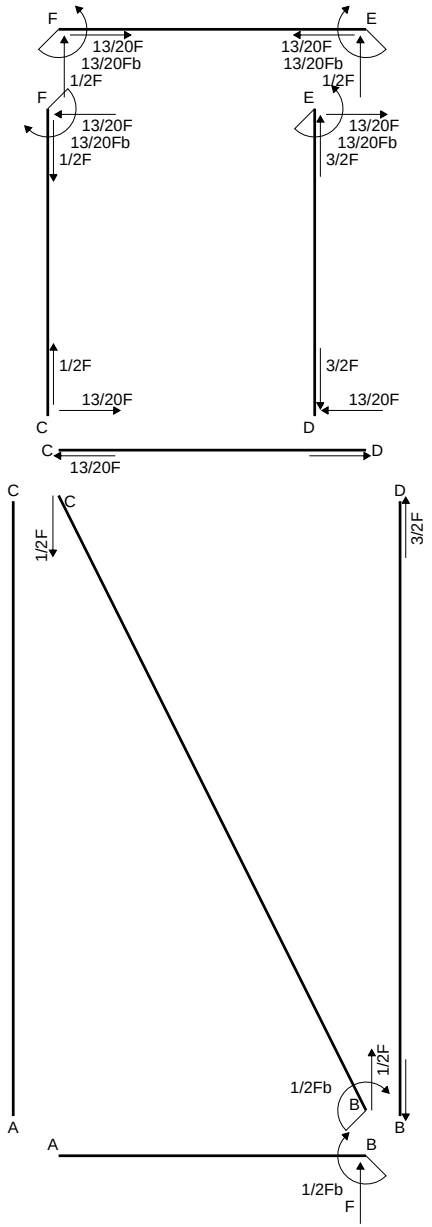
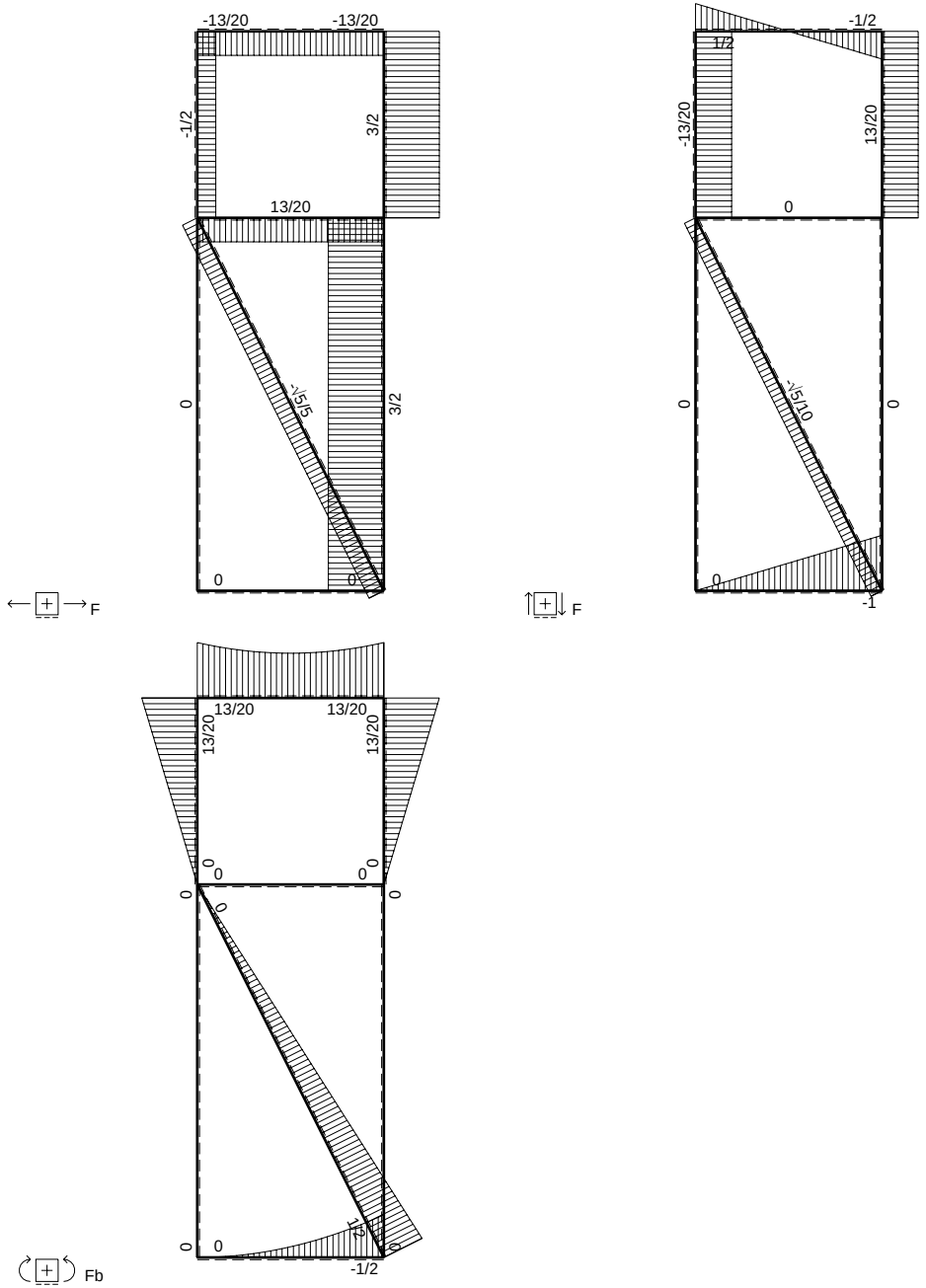
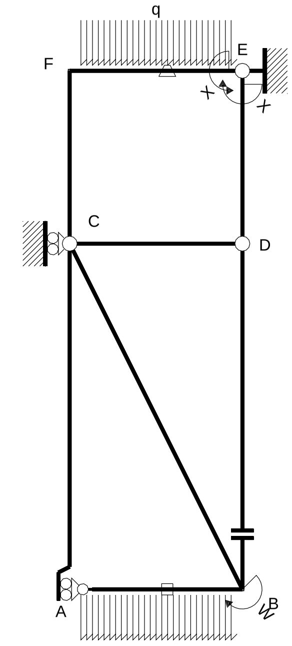
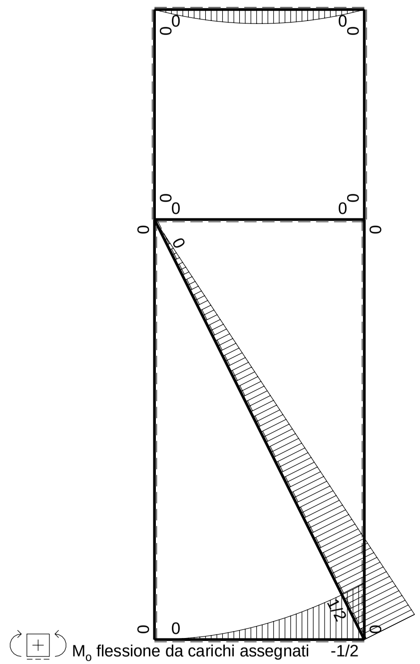

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb

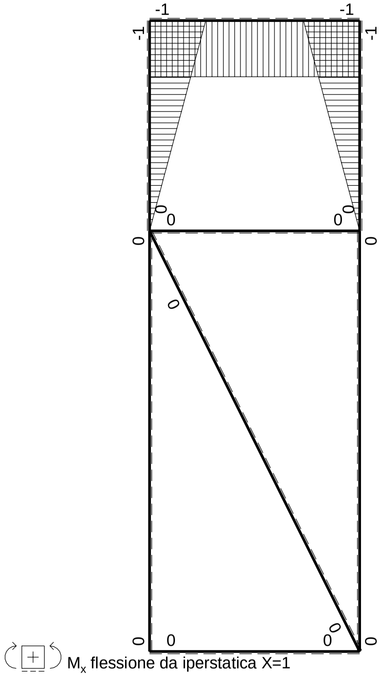

-13/20
13/20
-1/2
3/2
0
-√5/5
F
1/2
-√5/10
-1
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

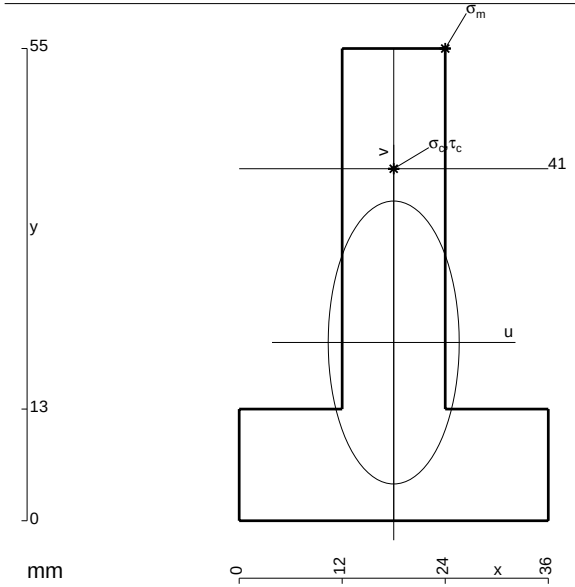
$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$



A = 972. mm^2
J_u = 264196. mm^4
J_v = 56592. mm^4
y_g = 20.76 mm
T_y = -7710. N
M_x = -1619100. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 34.24 mm
$\sigma_m = -Mv/J_u$ = 209.8 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 20.24 mm
$\sigma_c = -Mv/J_u$ = 124. N/mm^2
τ_c = 11.13 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.5 N/mm^2
S^* = 4576. mm^3

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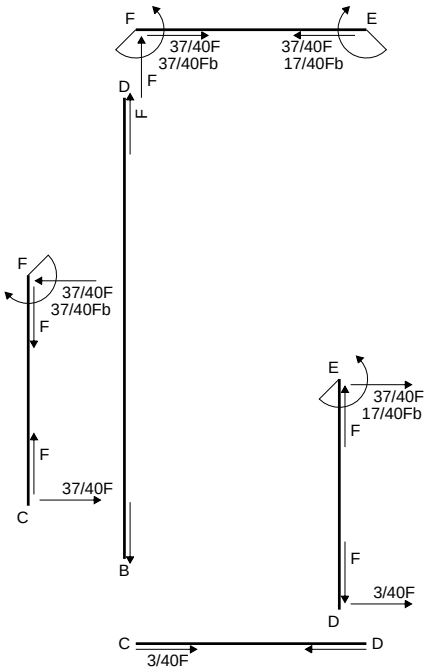
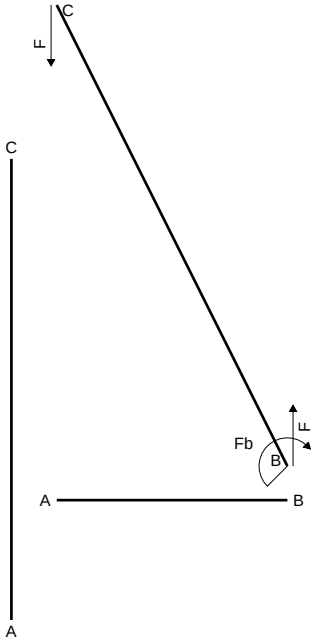
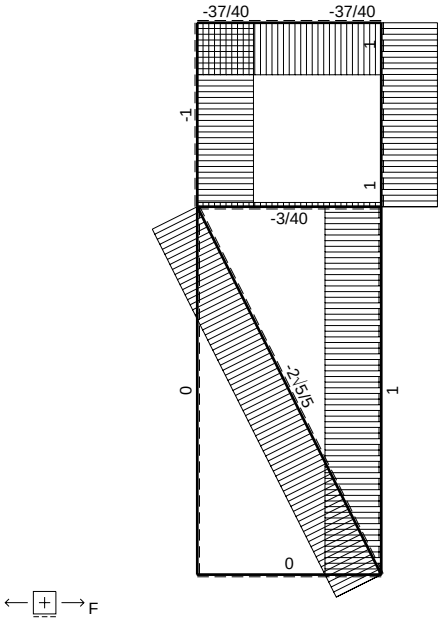
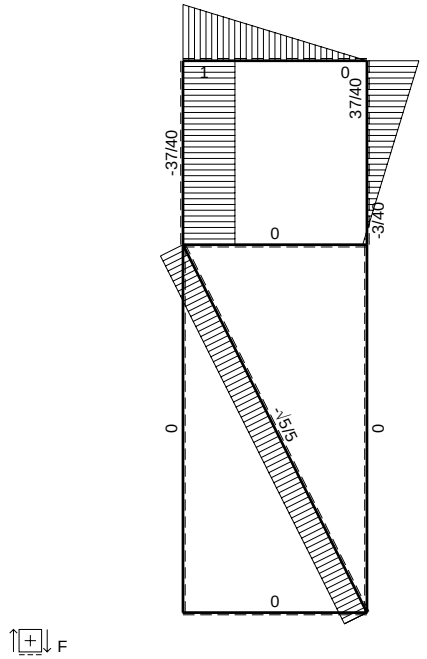
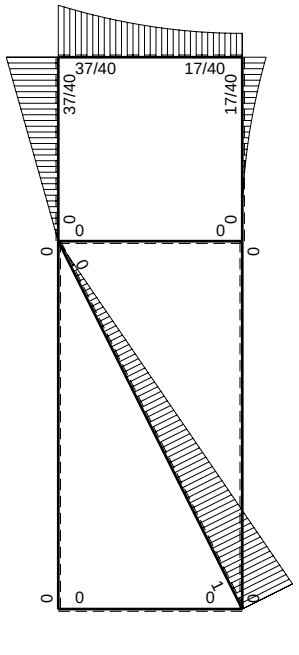
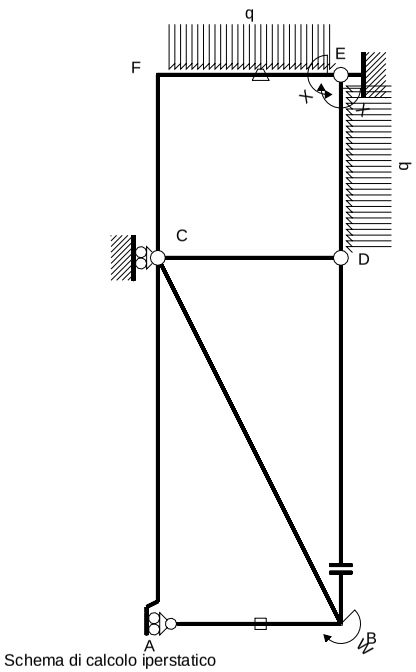

F
E
37/40F
37/40Fb
37/40F
17/40Fb
D
F
F
37/40F
37/40Fb
F
E
37/40F
17/40Fb
F
F
37/40F
C
B
F
3/40F
D
C
3/40F
D


C
F
C
Fb
F
B
A
B
A

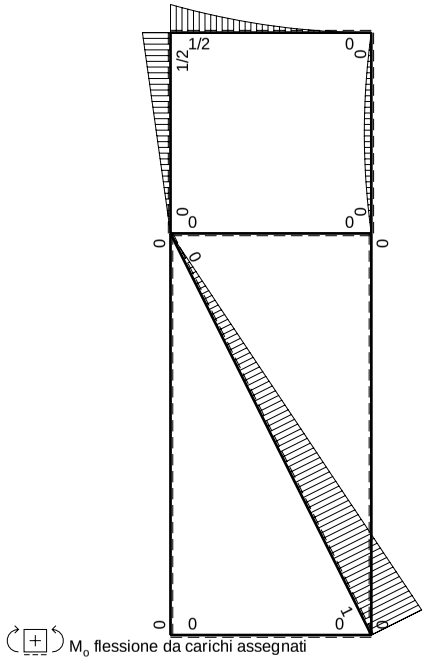

-37/40
-37/40
1
-1
1
-3/40
-2√5/5
0
1
0
F


1
0
37/40
-37/40
-3/40
0
-√5/5
0
0
F

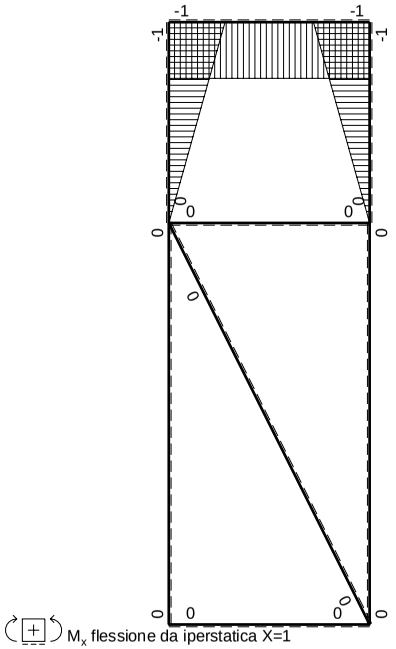

37/40
17/40
37/40
17/40
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2qx^2	-Fb/EJ	-1/2Fx^2/b	Fb/EJ	1	(-1/6+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+Fx-1/2qx^2	Fb/EJ	-1/2Fb+Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					17/24Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-17/40Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ \ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

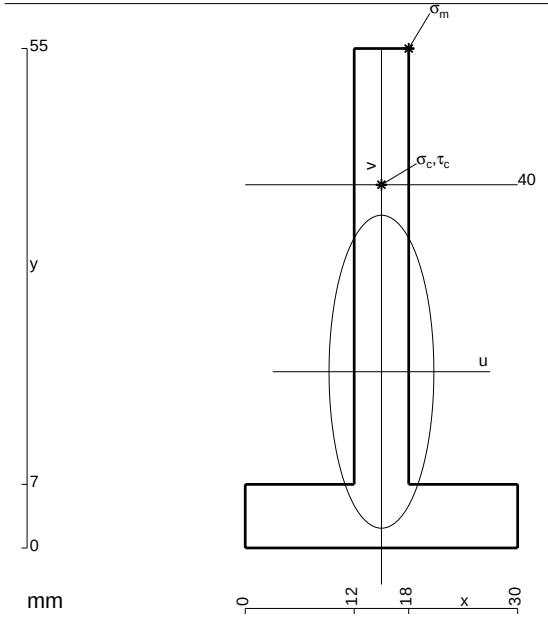
$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ \ = 5/6\ \ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ \ Fb^2/EJ$



A = 498. mm^2
J_u = 147997. mm^4
J_v = 16614. mm^4
y_g = 19.4 mm
N = -1744. N
T_y = -872.1 N
M_x = 897000. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 35.6 mm
$\sigma_m = N/A - Mv/J_u$ = -219.3 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 20.6 mm
$\sigma_c = N/A - Mv/J_u$ = -128.3 N/mm^2
τ_c = 2.483 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.4 N/mm^2
S^* = 2529. mm^3

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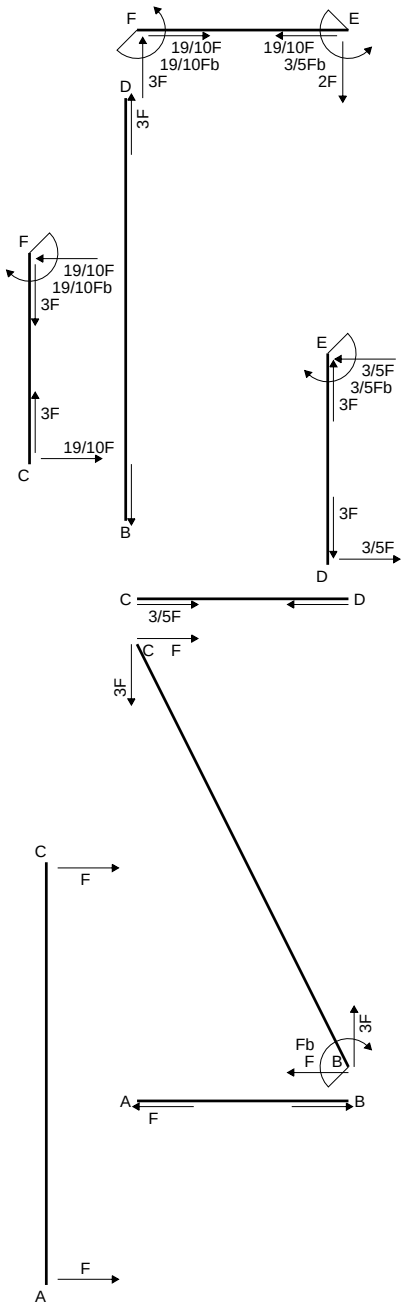
F
E
19/10F
19/10Fb
19/10F
3/5Fb
D
3F
2F
3F
F
19/10F
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3F
E
3/5F
3/5Fb
3F
3F
19/10F
C
3F
B
D
3/5F
C
D
3/5F
C
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3F
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Fb
F
B
3F
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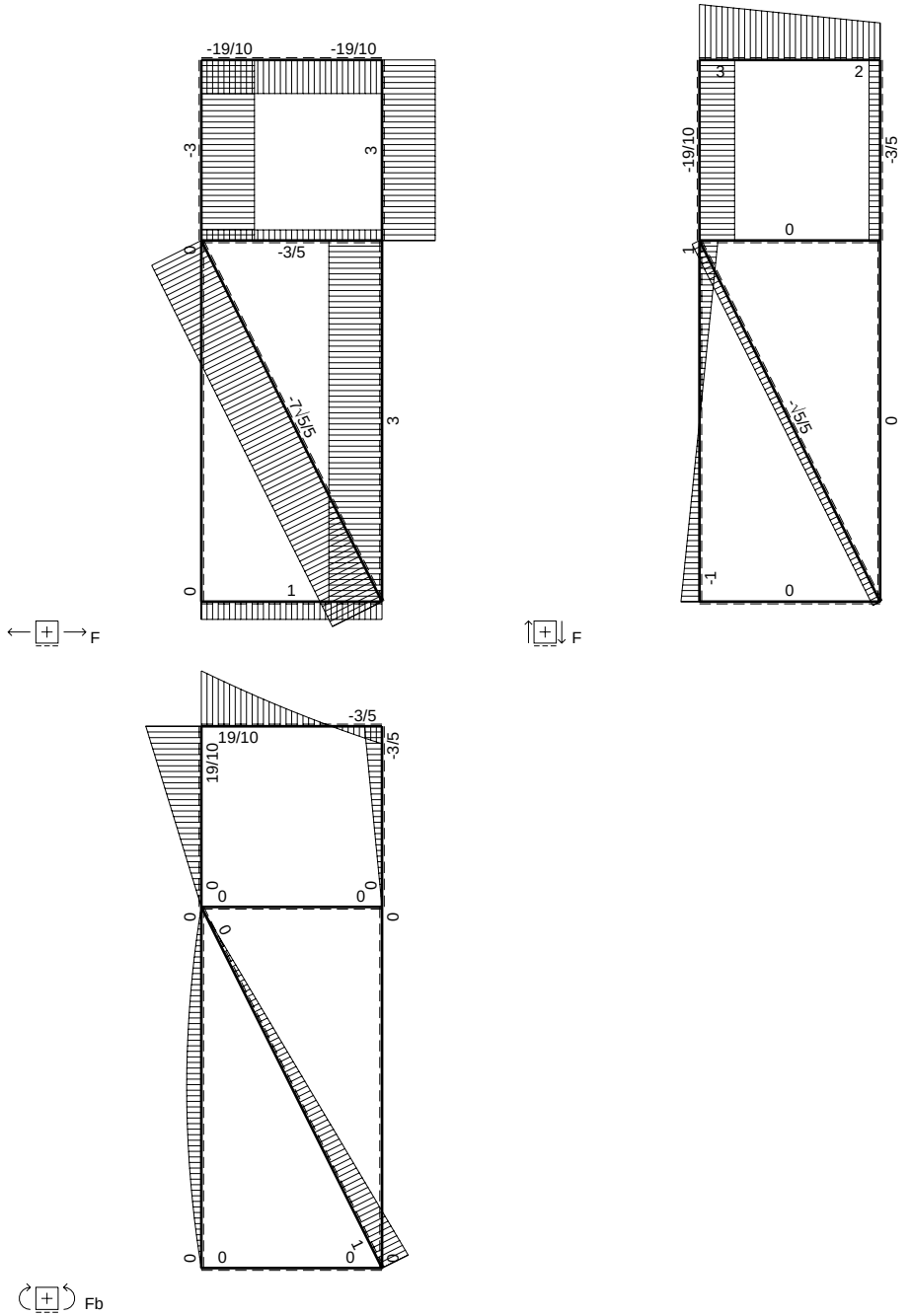
B
F
F
A


-19/10
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-3/5
0
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-7\5/5
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F
3
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-19/10
-3/5
0
1
-1\5/5
0
-1
0
F
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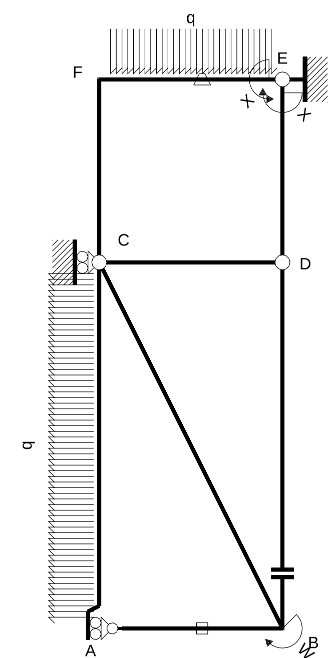
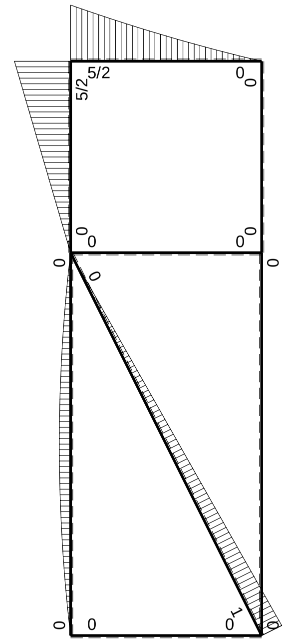
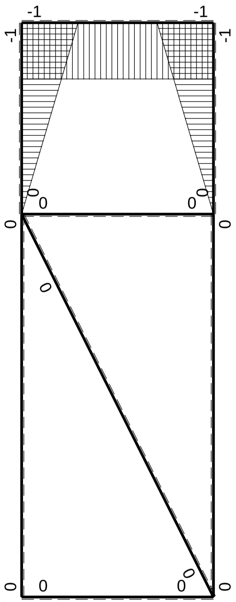
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$2Fx+1/2qx^2$	-Fb/EJ	$-2Fx-1/2Fx^2/b$	Fb/EJ	1	$(-7/6+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+3Fx-1/2qx^2$	Fb/EJ	$-5/2Fb+3Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	$-5/2Fx^2/b$	0	x^2/b^2		
	totali						$-Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [- x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +3\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-5/2\ x +3/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

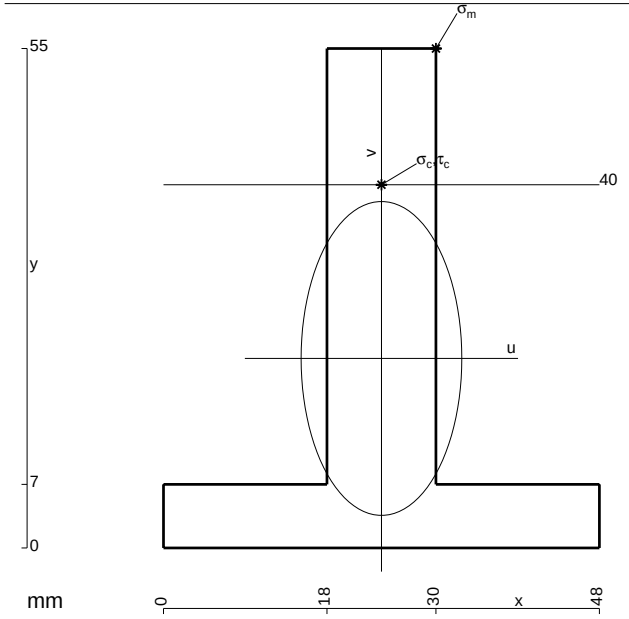
$= (-5/2\ b +3/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 912. mm^2
J_u = 272448. mm^4
J_v = 71424. mm^4
y_g = 20.87 mm
N = -10894. N
T_y = -1556. N
M_x = 1740000. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 34.13 mm
$\sigma_m = N/A - Mv/J_u$ = -229.9 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 19.13 mm
$\sigma_c = N/A - Mv/J_u$ = -134.1 N/mm^2
τ_c = 2.282 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 134.2 N/mm^2
S^* = 4794. mm^3

ESΣ09.xxxx.203

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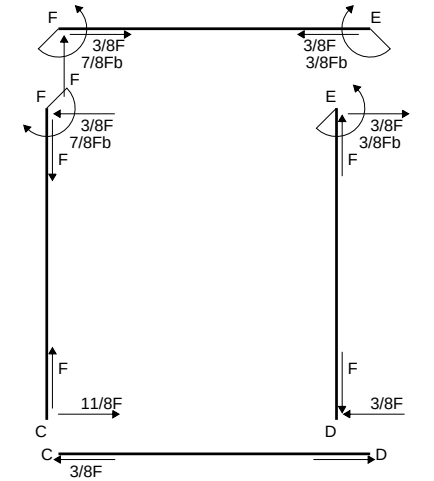
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E
3/8F
7/8Fb
3/8F
3/8Fb
F
F
E
3/8F
7/8Fb
3/8F
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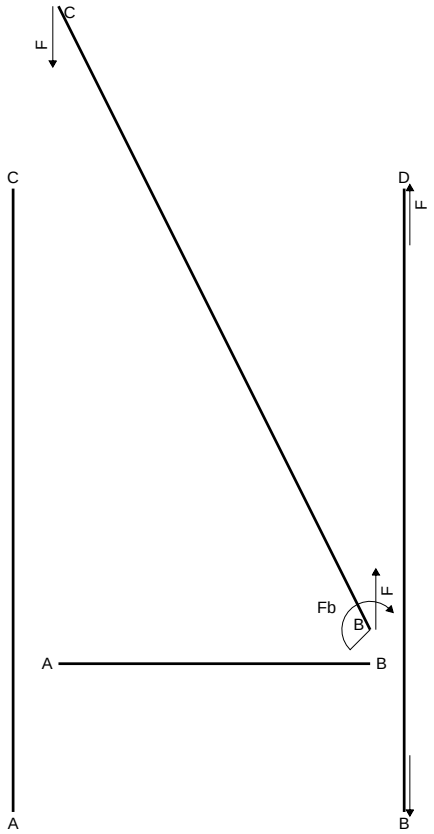
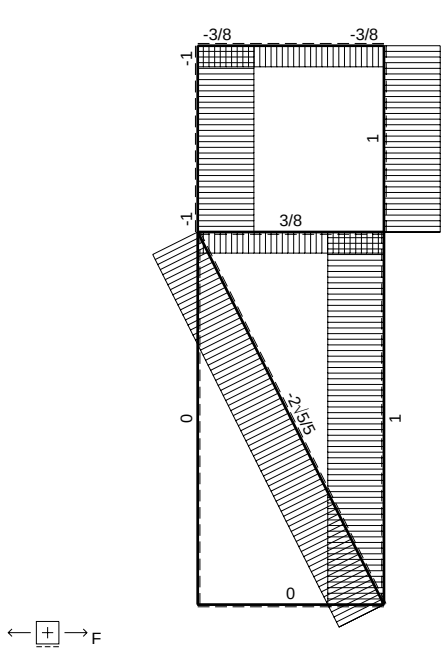
C
D
C
D
3/8F


C
F
C
D
F
Fb
B
F
A
B
A
B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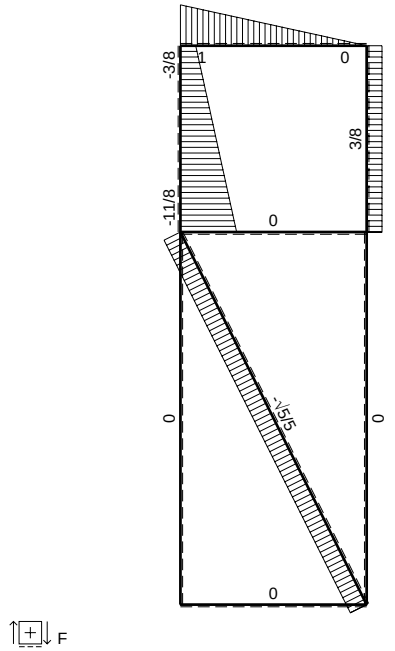
-3/8
-1
-1
1
3/8
0
1
-2√5/5
0
1
0
F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0
3/8
-11/8
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-√5/5
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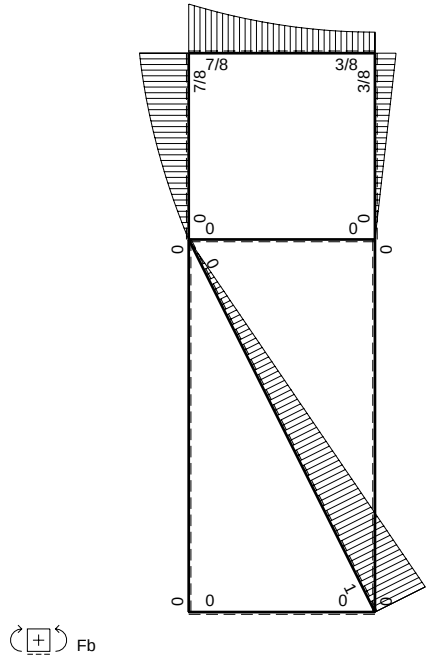
0
F


7/8
7/8
3/8
3/8
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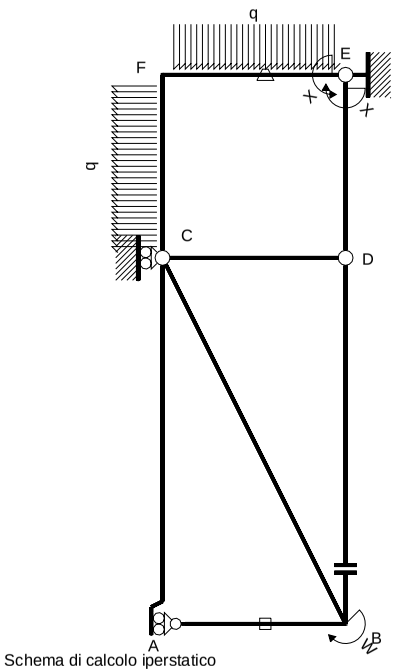
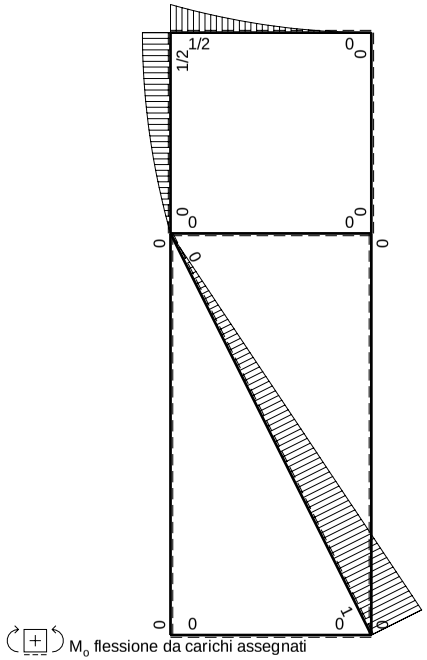
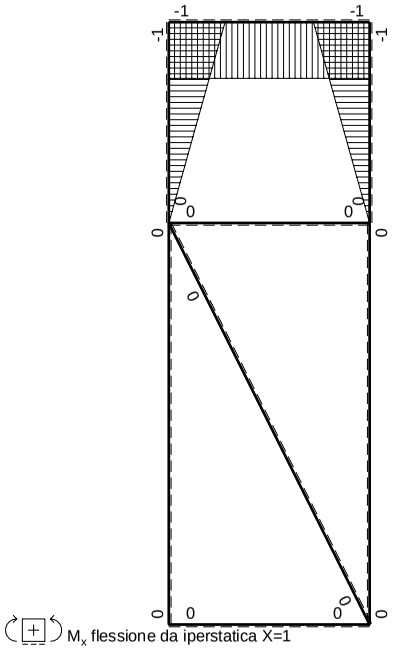
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2qx^2$	-Fb/EJ	-1/2Fx²/b	Fb/EJ	1	(-1/6+1)Fb²/EJ	Xb/EJ
FE b	1	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						5/8Fb²/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 966407 ESΣ09.xxxx.204

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 5/6\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 5/6\ Fb^2/EJ$

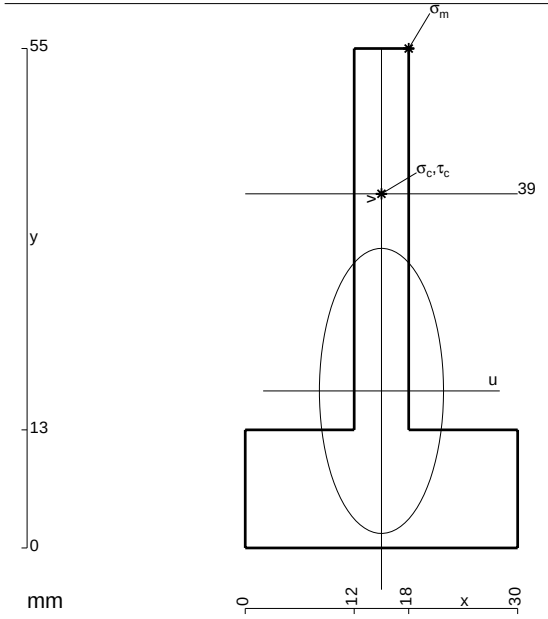
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 642. mm^2
J_u = 158306. mm^4
J_v = 30006. mm^4
y_g = 17.29 mm
N = -1646. N
T_y = -822.9 N
M_x = 993600. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.71 mm
$\sigma_m = N/A - Mv/J_u$ = -239.2 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 21.71 mm
$\sigma_c = N/A - Mv/J_u$ = -138.8 N/mm^2
τ_c = 2.471 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.9 N/mm^2
S^* = 2852. mm^3

ESΣ09.xxxx.204

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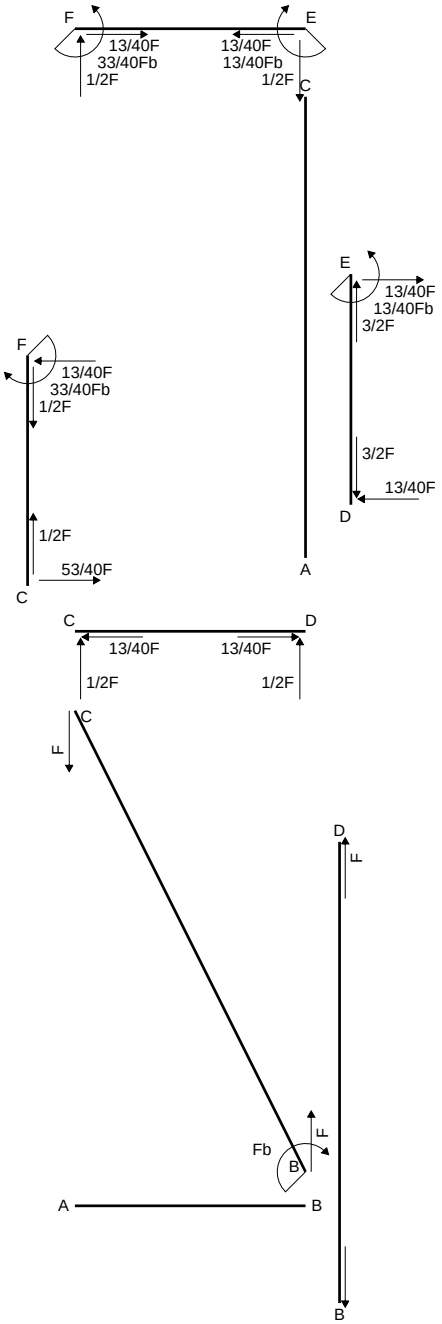
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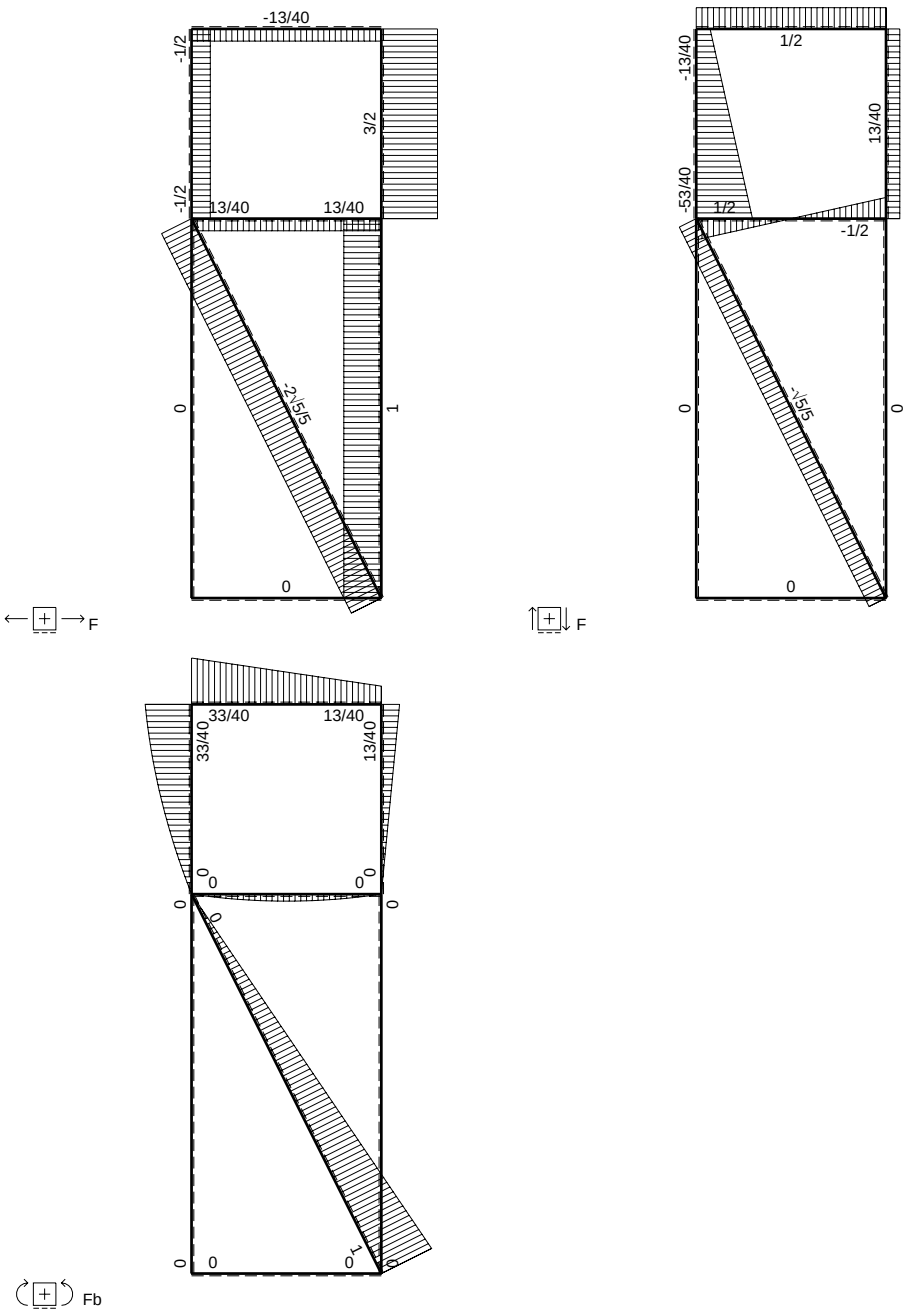
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B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-2√5/5
1
0
F
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-1/2
-√5/5
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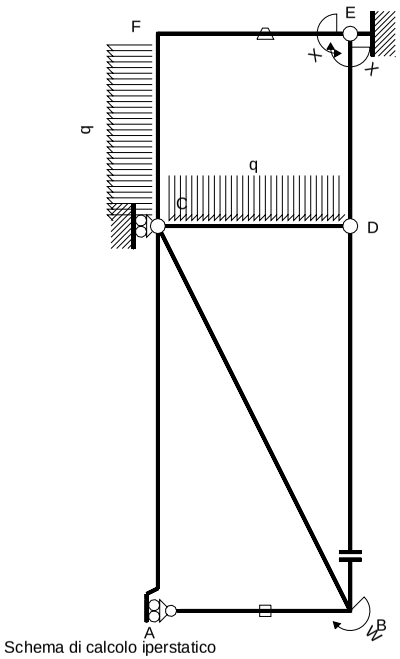
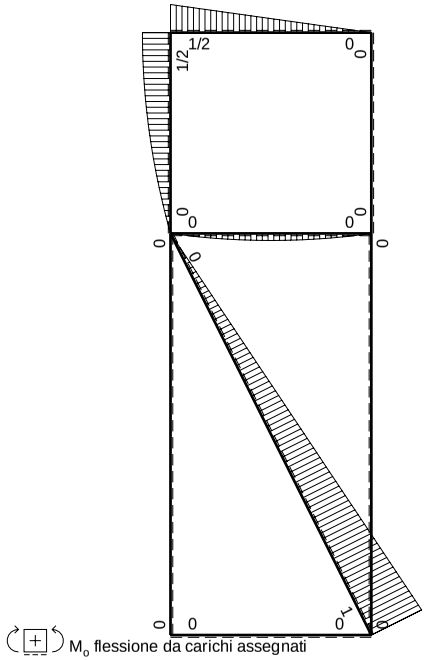
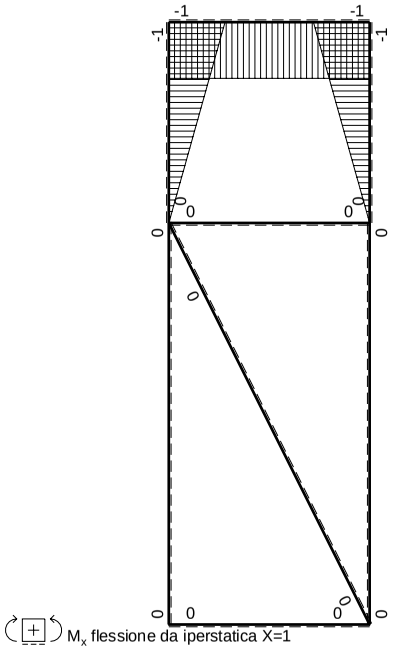
33/40
33/40
13/40
0
0
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 213325 ESΣ09.xxxx.205

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

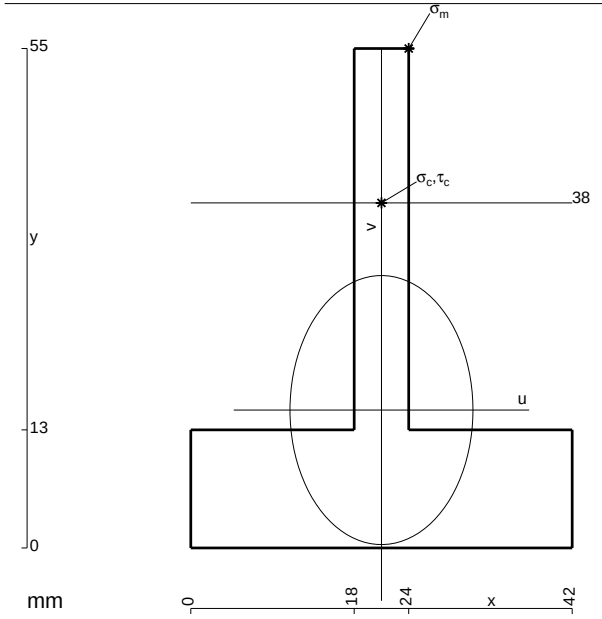
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213325 ESΣ09.xxxx.205



A = 798. mm^2
J_u = 175127. mm^4
J_v = 81018. mm^4
y_g = 15.18 mm
N = -1342. N
T_y = -670.8 N
M_x = 870000. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 39.82 mm
$\sigma_m = N/A - Mv/J_u$ = -199.5 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 22.82 mm
$\sigma_c = N/A - Mv/J_u$ = -115. N/mm^2
τ_c = 2.039 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.1 N/mm^2
S^* = 3194. mm^3

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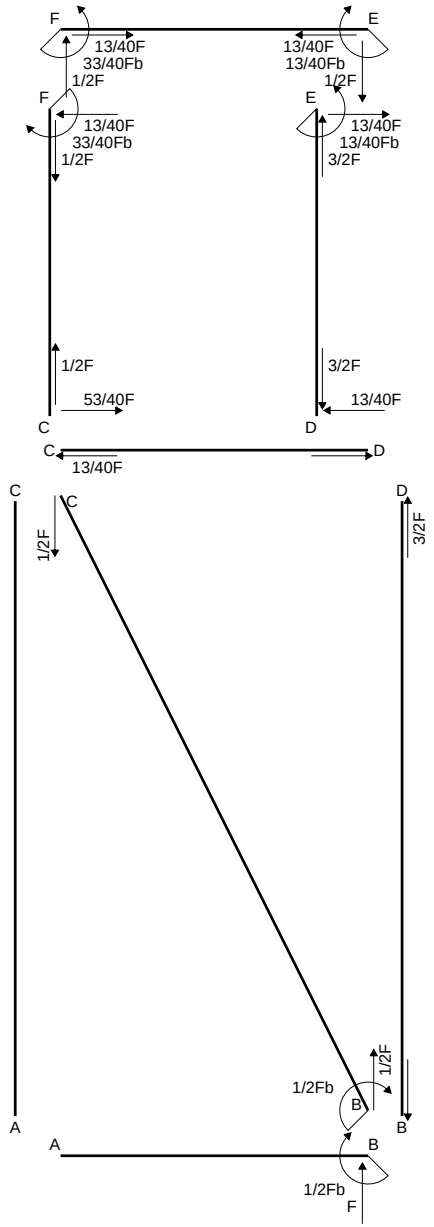
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
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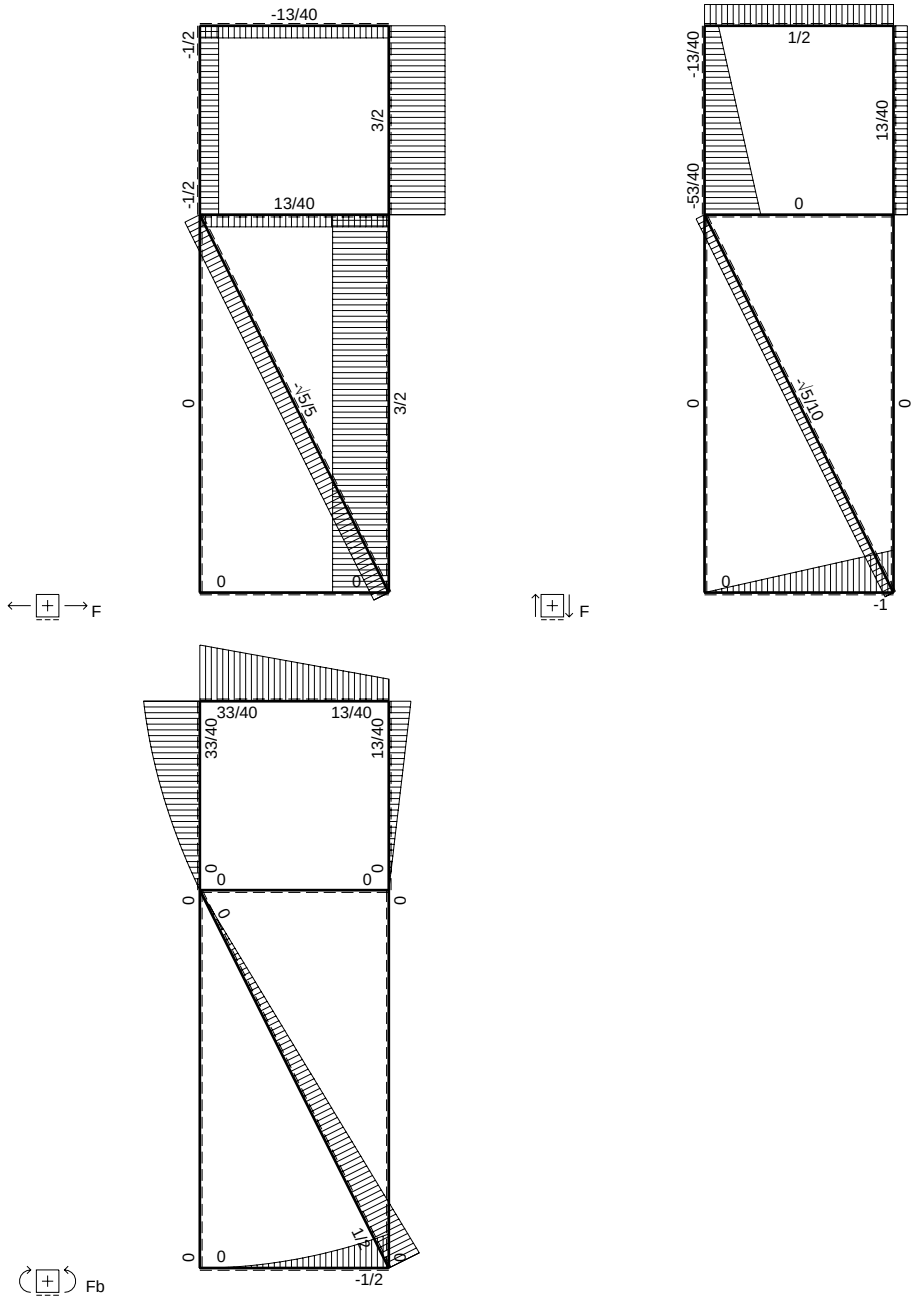
A
B
1/2Fb
F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-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-1/2
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	-1+x/b	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
totali							$13/24Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 210484 ESΣ09.xxxx.206

14.11.24

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

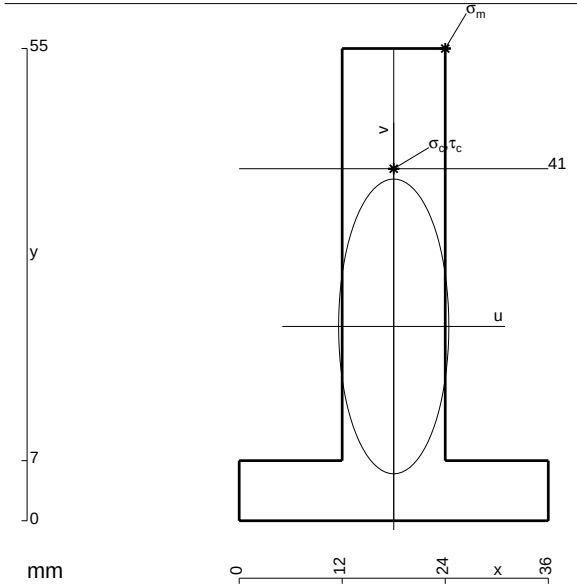
$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 210484 ESΣ09.xxxx.206



A = 828. mm^2
J_u = 244195. mm^4
J_v = 34128. mm^4
y_g = 22.63 mm
T_y = -5110. N
M_x = -1584100. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 32.37 mm
$\sigma_m = -Mv/J_u$ = 210. N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 18.37 mm
$\sigma_c = -Mv/J_u$ = 119.2 N/mm^2
τ_c = 7.432 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.9 N/mm^2
S^* = 4262. mm^3

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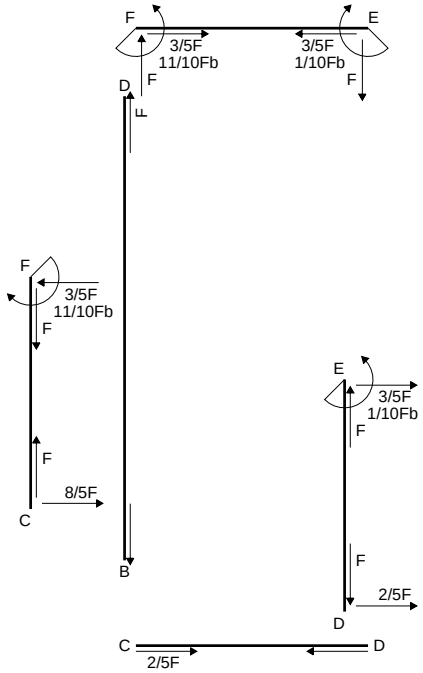
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E
3/5F
11/10Fb
3/5F
1/10Fb
D
F
F
F
3/5F
11/10Fb
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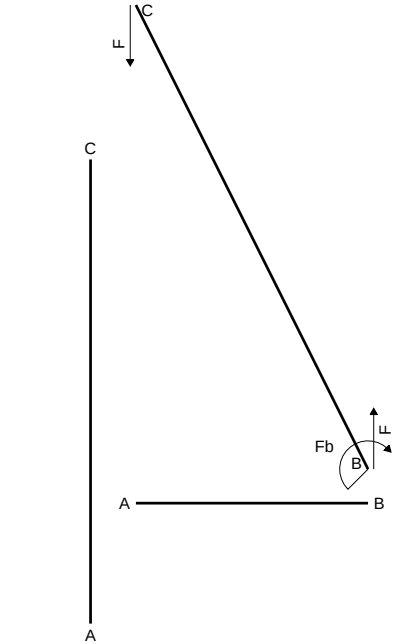
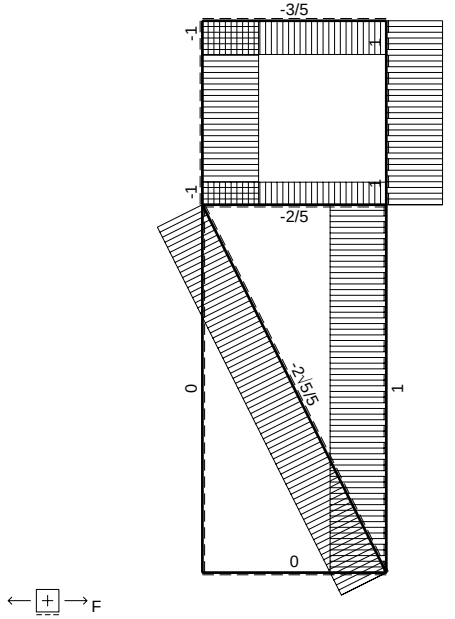
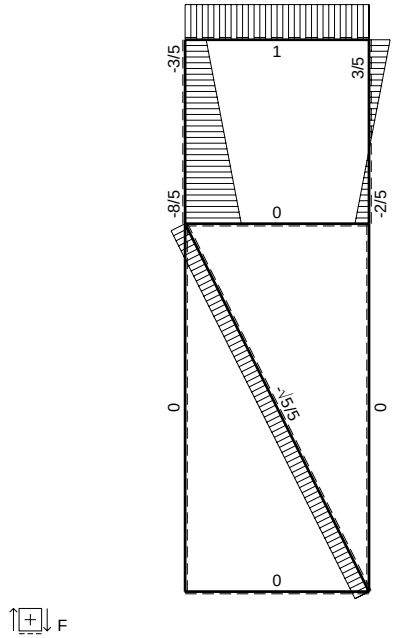
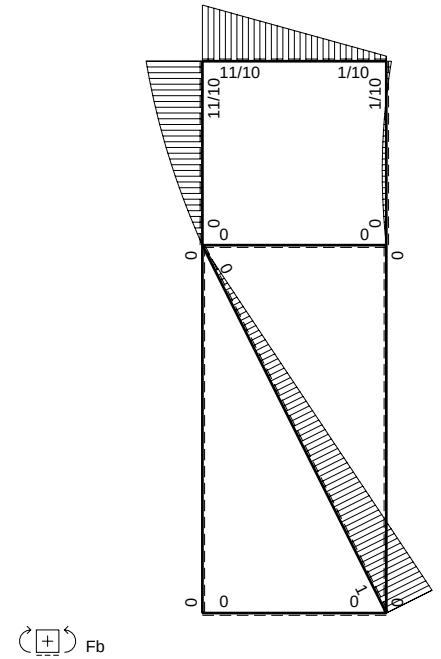
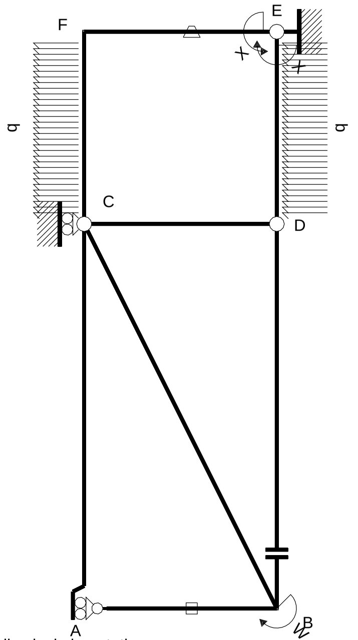
D
C
2/5F
D


C
F
C
Fb
F
B
A
B
A

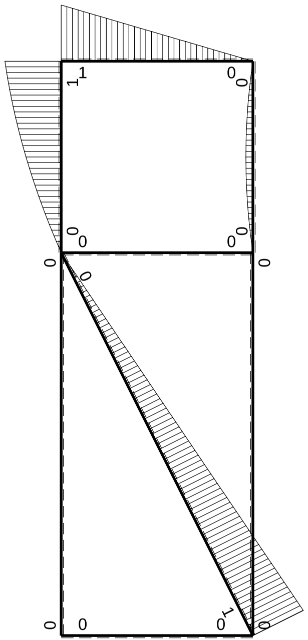

-3/5
-1
1
-1
1
-2/5
0
-2/5/5
1
0
F


-3/5
1
3/5
-8/5
0
-2/5
0
-2/5/5
0
0
F

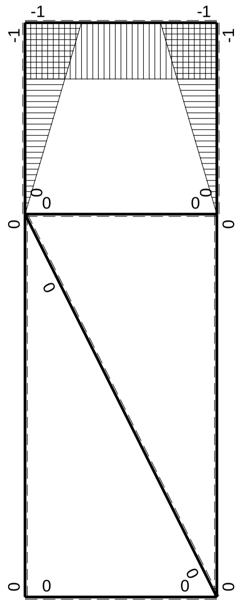

11/10
1/10
11/10
1/10
0
0
0
0
0
0
0
0
0
0
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	Fx	-Fb/EJ	-Fx	Fb/EJ	1	$(-1/2+1)Fb^2/EJ$	Xb/EJ
FE b	1	-Fb+Fx	Fb/EJ	-Fb+Fx	Fb/EJ	1		
FC b	-1+x/b	$Fb-1/2Fx-1/2qx^2$	0	$-Fb+3/2Fx-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-3/8+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-3/2Fx+1/2qx^2$	0	$-3/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$1/6Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-1/10Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b)\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/2\ b)\ Fb\ 1/EJ\ + (\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1 + x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

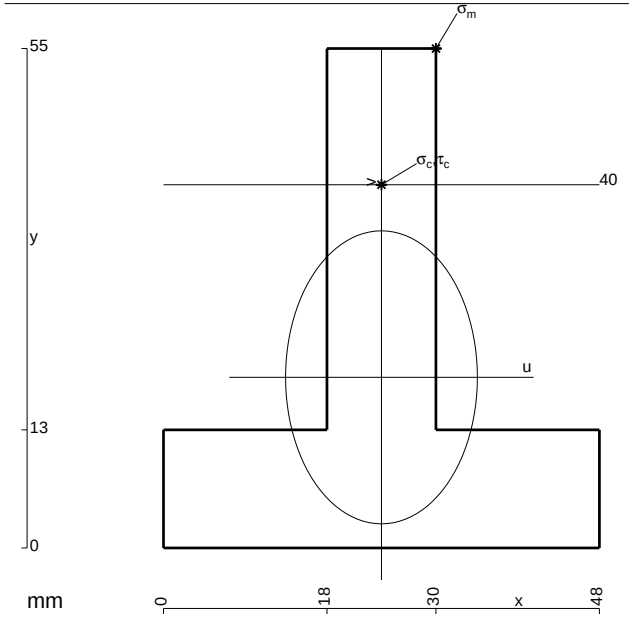
$= (-\ b +1/2\ b)\ Fb\ 1/EJ\ + (-\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1 +3/2\ x/b -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-\ x +3/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-\ b +3/4\ b -1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-3/2\ x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [-1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/8\ b)\ Fb\ 1/EJ\ = -3/8\ Fb^2/EJ$



A = 1128. mm^2
J_u = 293725. mm^4
J_v = 125856. mm^4
y_g = 18.79 mm
N = -2388. N
T_y = -1194. N
M_x = 1762200. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 36.21 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 21.21 mm
$\sigma_c = N/A - Mv/J_u$ = -129.4 N/mm^2
τ_c = 1.751 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 129.4 N/mm^2
S^* = 5168. mm^3

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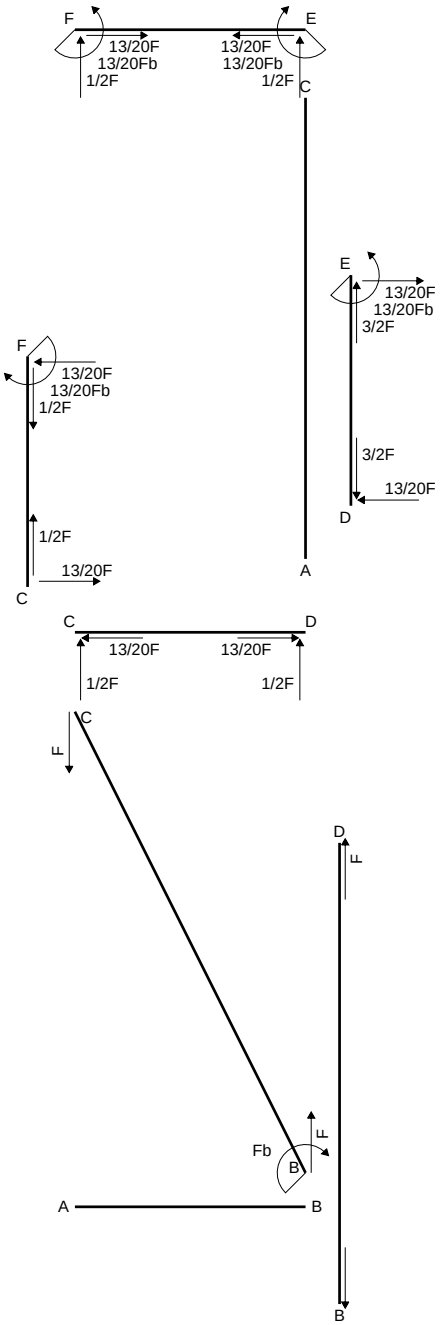
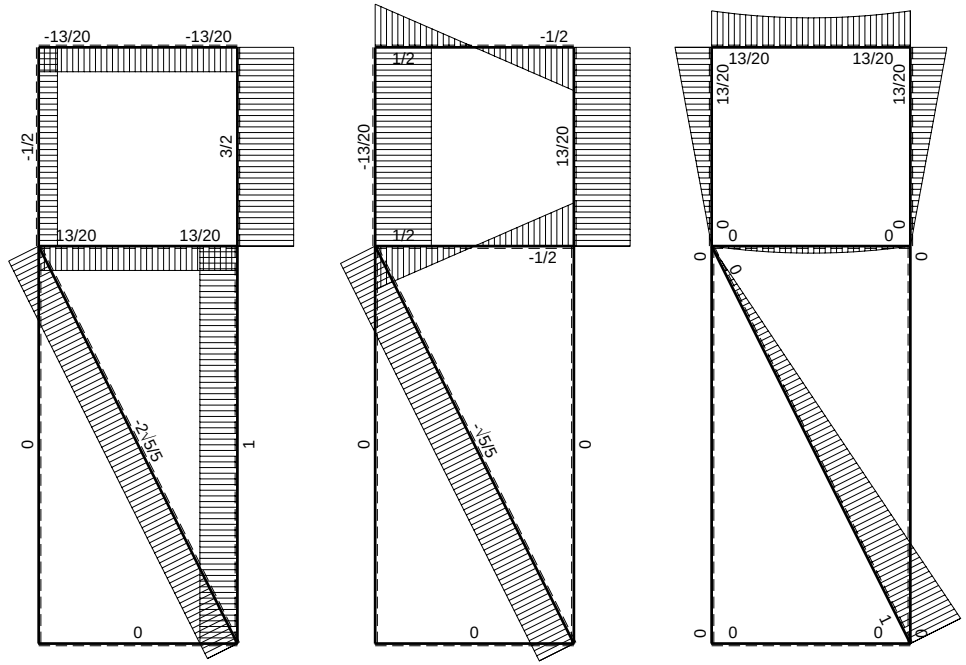
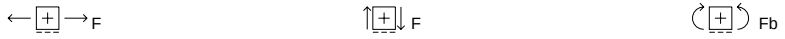
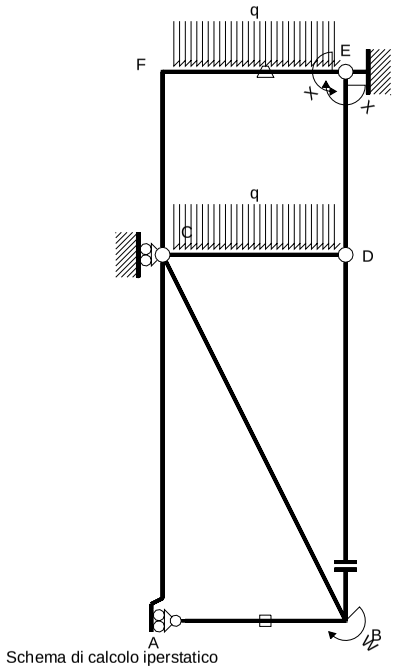
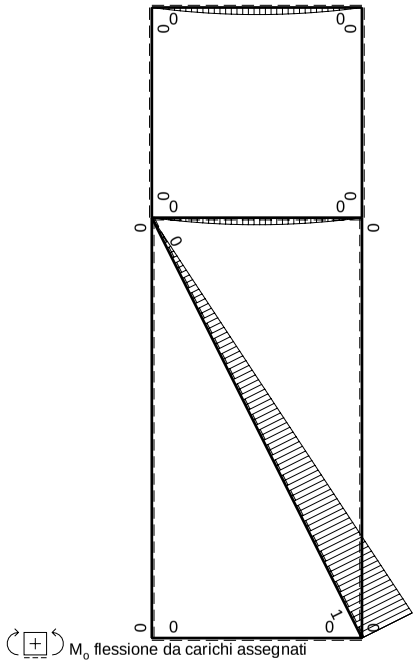

F
E
13/20F
13/20Fb
1/2F
C
A
D
3/2F
B
Fb


-13/20
-1/2
3/2
13/20
1/2
0
1
-2√5/5
-√5/5

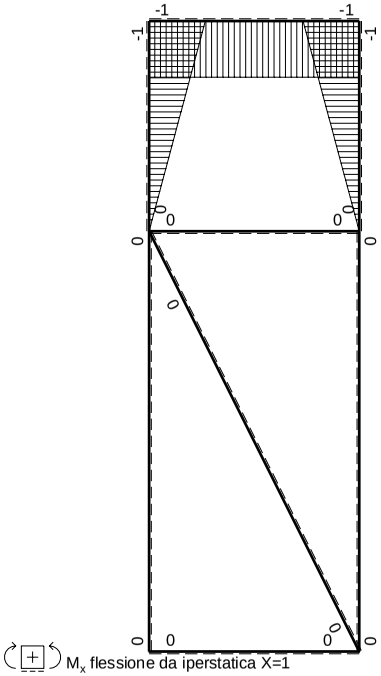

F
F
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	-1/2Fx+1/2qx^2	-Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1	(1/12+1)Fb^2/EJ	Xb/EJ
FE b	1	1/2Fx-1/2qx^2	Fb/EJ	1/2Fx-1/2Fx^2/b	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						13/12Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 983859 ESΣ09.xxxx.208

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

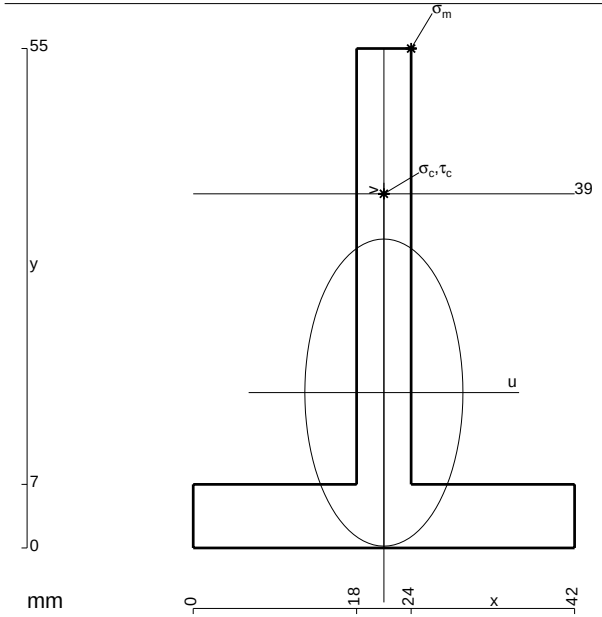
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 983859 ESΣ09.xxxx.208



A = 582. mm^2
J_u = 166519. mm^4
J_v = 44082. mm^4
y_g = 17.11 mm
N = -1279. N
T_y = -639.5 N
M_x = 1001000. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.89 mm
$\sigma_m = N/A - Mv/J_u$ = -230. N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 21.89 mm
$\sigma_c = N/A - Mv/J_u$ = -133.8 N/mm^2
τ_c = 1.837 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.8 N/mm^2
S^* = 2870. mm^3

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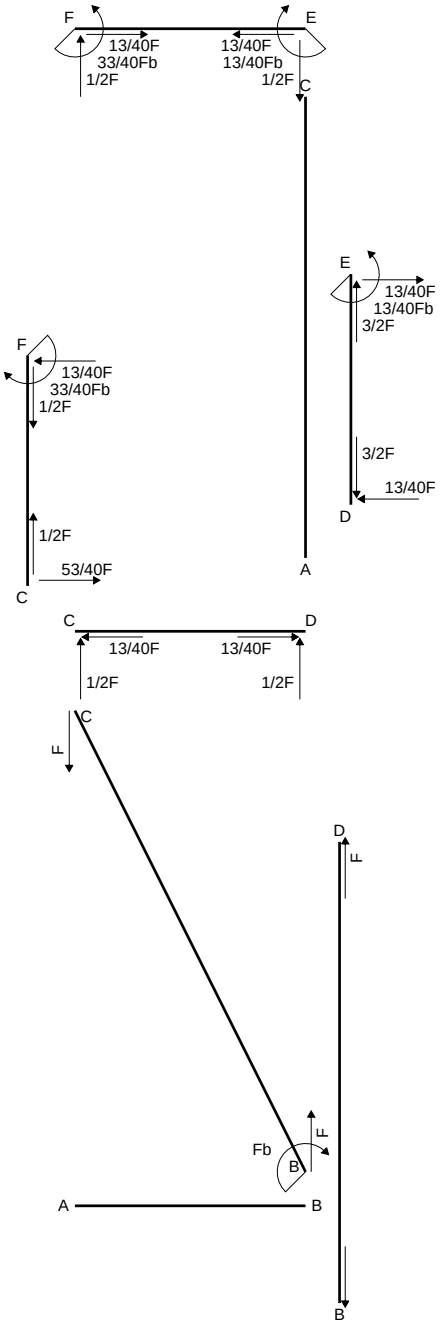
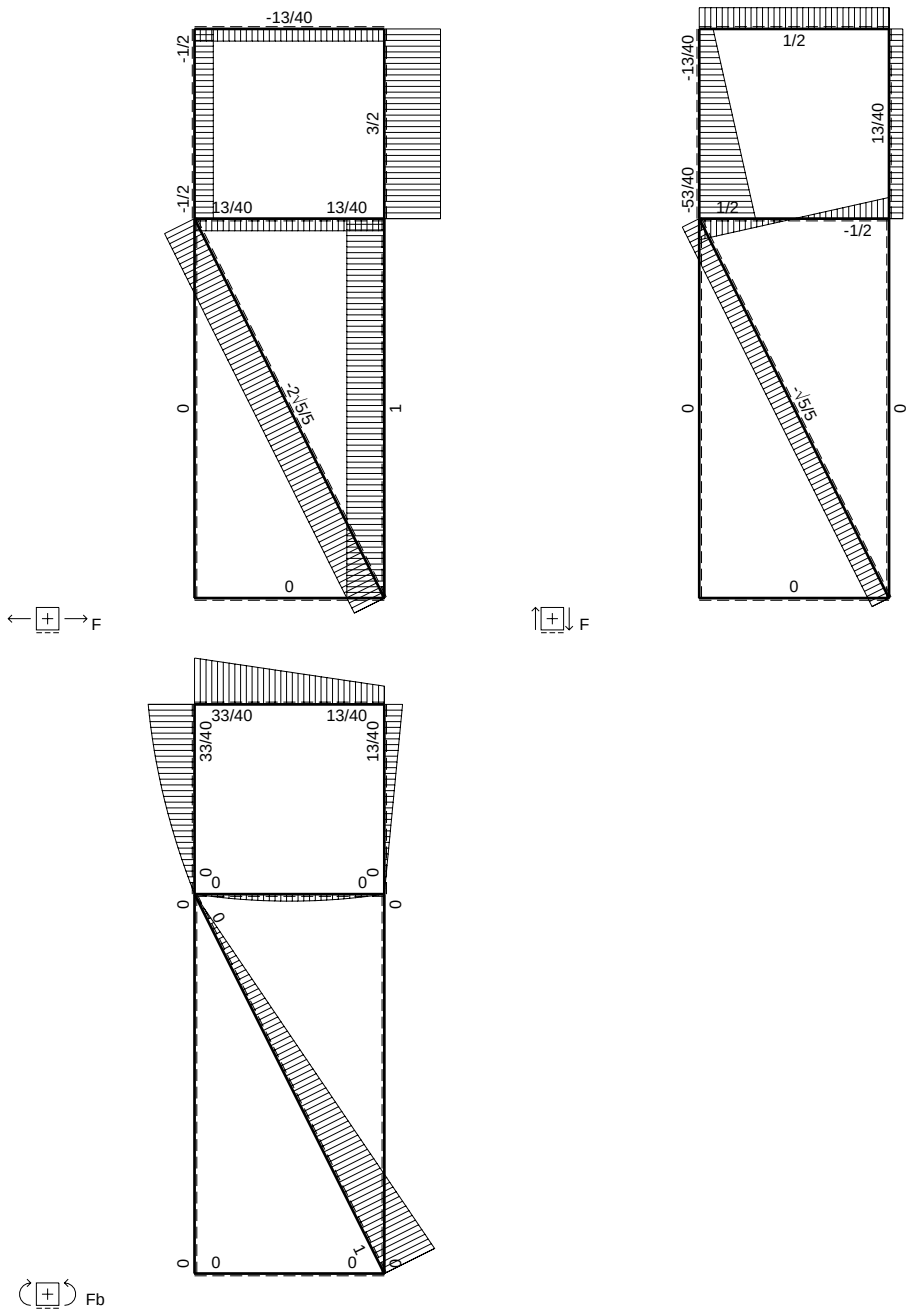
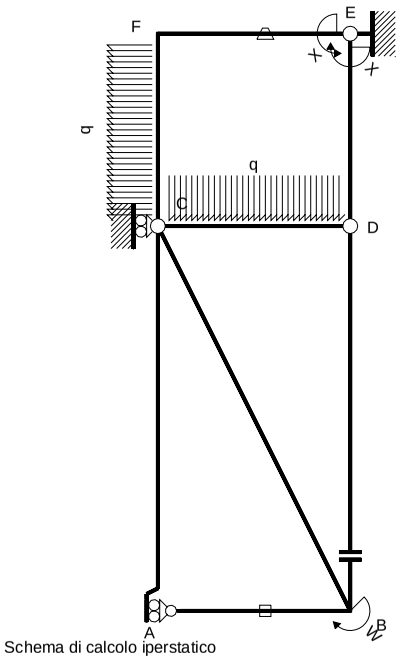
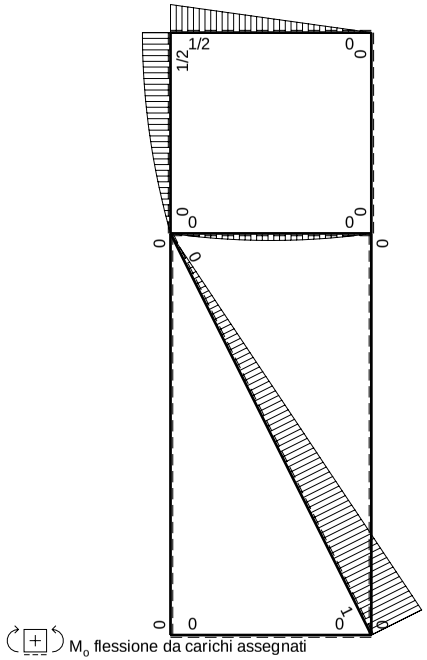

F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
C
E
13/40F
13/40Fb
3/2F
3/2F
13/40F
D
F
13/40F
33/40Fb
1/2F
1/2F
53/40F
C
A
C
D
13/40F
13/40F
1/2F
1/2F
C
F
D
F
Fb
B
F
A
B
B

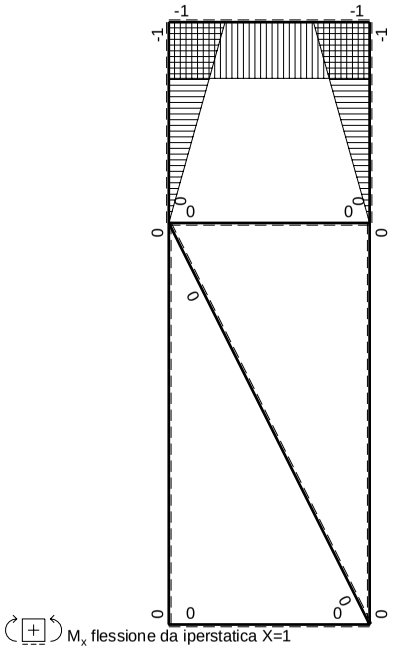

-13/40
-1/2
-1/2
3/2
13/40
13/40
0
-2√5/5
1
0
F
-13/40
1/2
13/40
-53/40
1/2
-1/2
-√5/5
0
0
0
F
33/40
33/40
13/40
0
0
0
0
0
0
0
1
0
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC $\sqrt{5}$b	0	Fb-$\sqrt{5}$/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2qx^2	0	-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b	0	1-2x/b+x^2/b^2	(-5/24+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-Fx+1/2qx^2	0	-Fx^2/b+1/2qx^3/b	0	x^2/b^2		
	totali						13/24Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982827 ESΣ09.xxxx.209

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 3/4\ Fb^2/EJ$

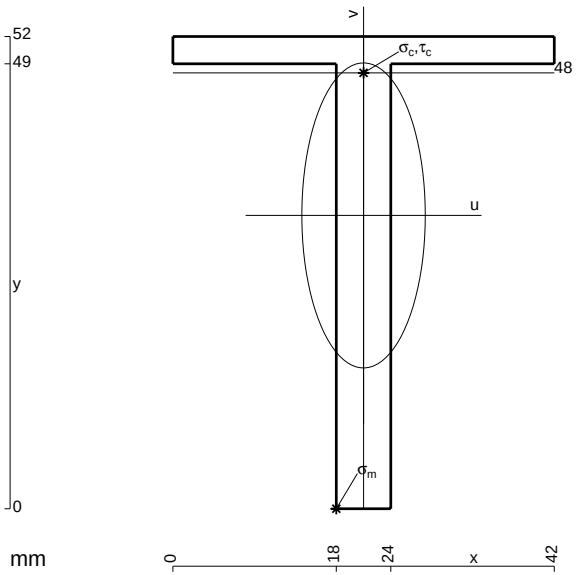
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (- x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 420. mm^2
J_u = 118542. mm^4
J_v = 19404. mm^4
y_g = 32.3 mm
N = -1136. N
T_y = -568. N
M_x = 889000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -32.3 mm
$\sigma_m = N/A - Mv/J_u$ = 239.5 N/mm^2
x_c = 21. mm
y_c = 48. mm
v_c = 15.7 mm
$\sigma_c = N/A - Mv/J_u$ = -120.4 N/mm^2
τ_c = 1.909 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.5 N/mm^2
S^* = 2390. mm^3

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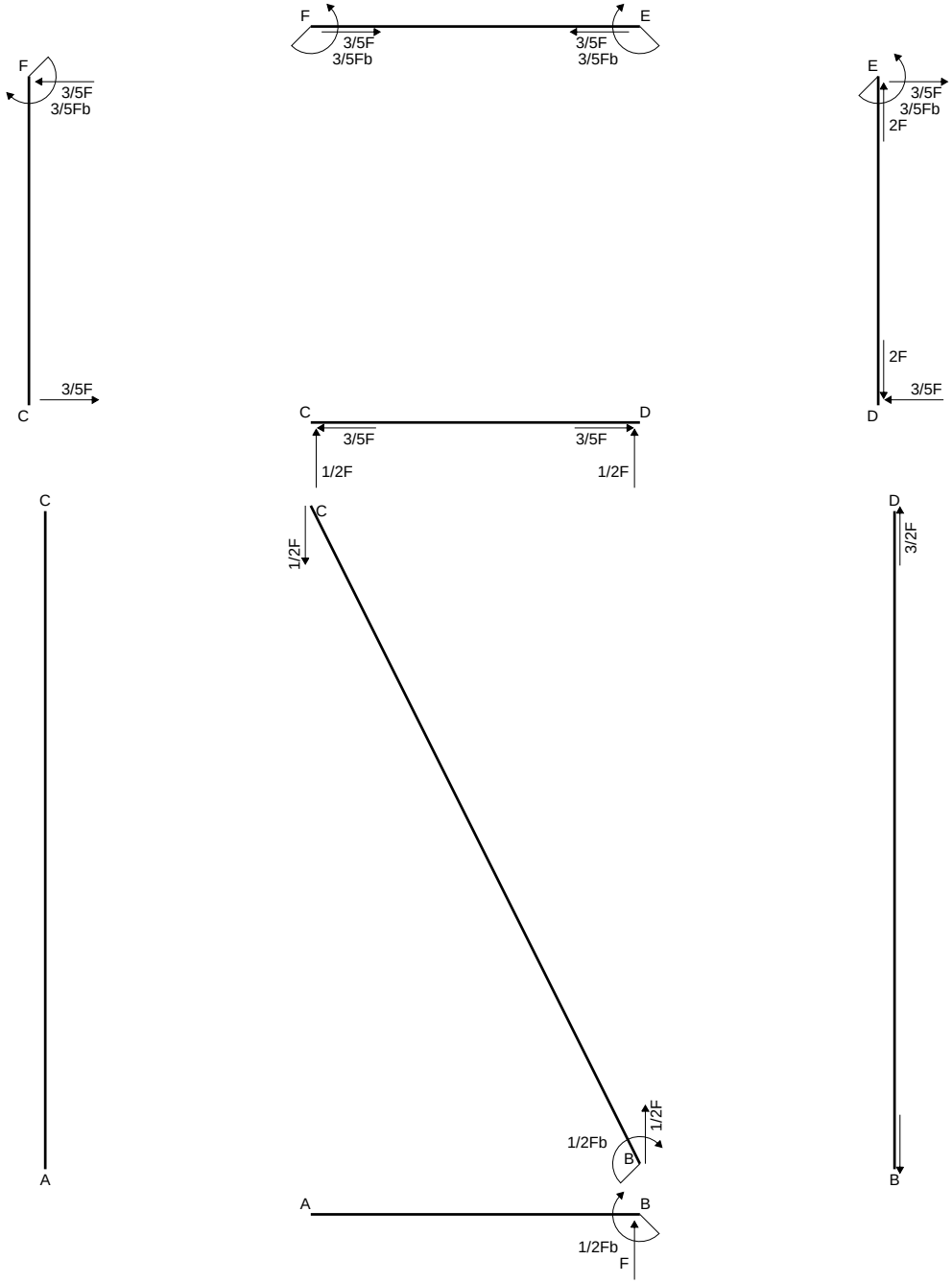
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E
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3/5Fb
3/5F
3/5Fb
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3/5F
3/5Fb
E
3/5F
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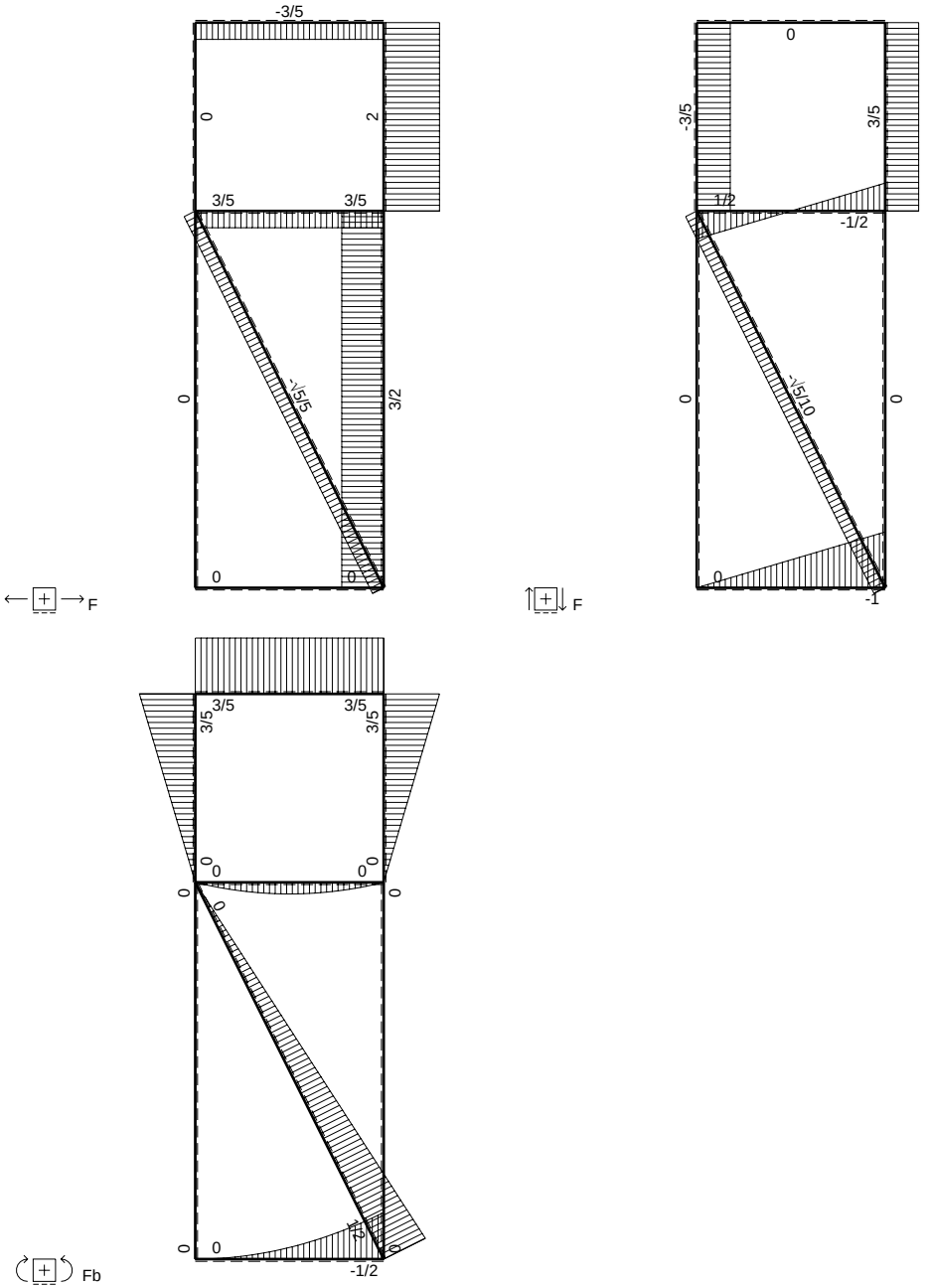
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F


-3/5
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2
3/5
3/5
0
3/2
-√5/5
0
0
F
0
-3/5
3/5
1/2
-1/2
0
-√5/10
0
0
-1
F
3/5
3/5
3/5
3/5
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0
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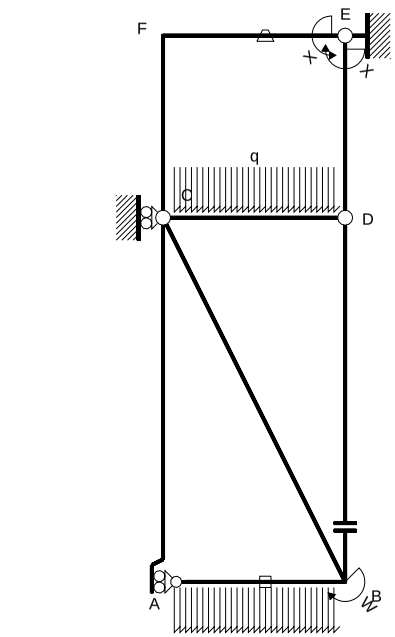
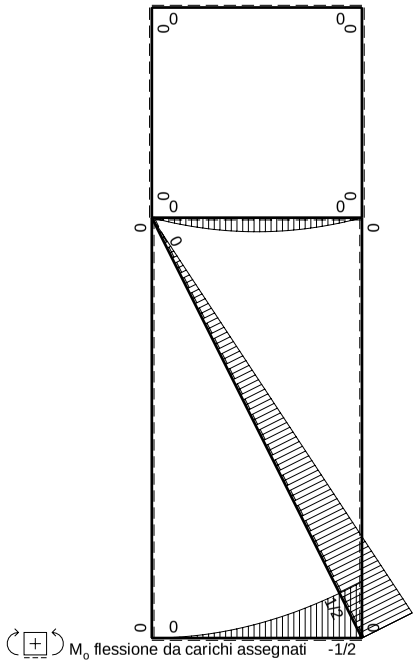
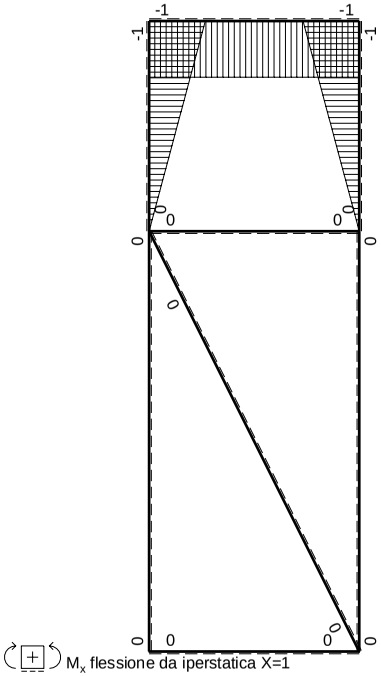
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

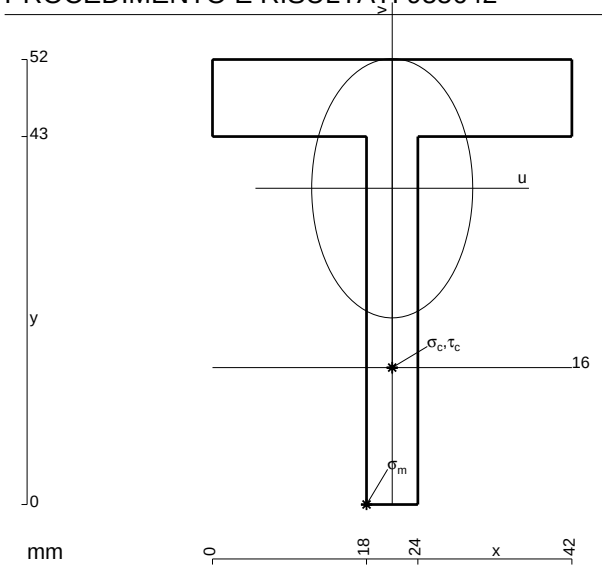
$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (1\ )\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1\ )\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 636. mm^2
J_u = 145963. mm^4
J_v = 56340. mm^4
y_g = 36.95 mm
T_y = -4270. N
M_x = -789950. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -36.95 mm
$\sigma_m = -Mv/J_u$ = -200. N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -20.95 mm
$\sigma_c = -Mv/J_u$ = -113.4 N/mm^2
τ_c = 13.55 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.8 N/mm^2
S^* = 2779. mm^3

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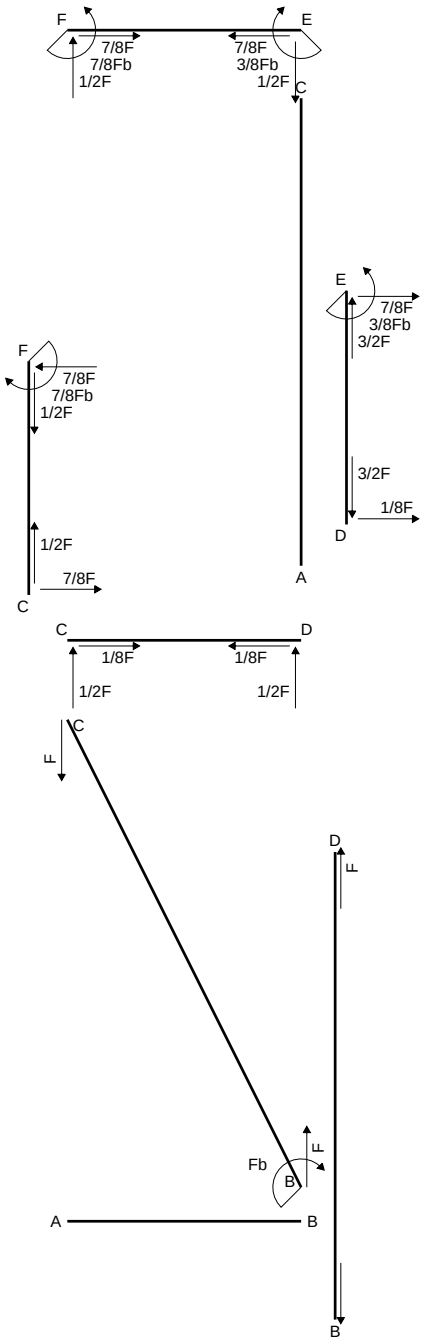
F
E
7/8F
7/8Fb
1/2F
7/8F
3/8Fb
1/2F
C
E
7/8F
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3/2F
F
7/8F
7/8Fb
1/2F
3/2F
1/8F
D
1/2F
7/8F
A
C
C
D
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1/8F
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1/2F
C
F
D
F
Fb
F
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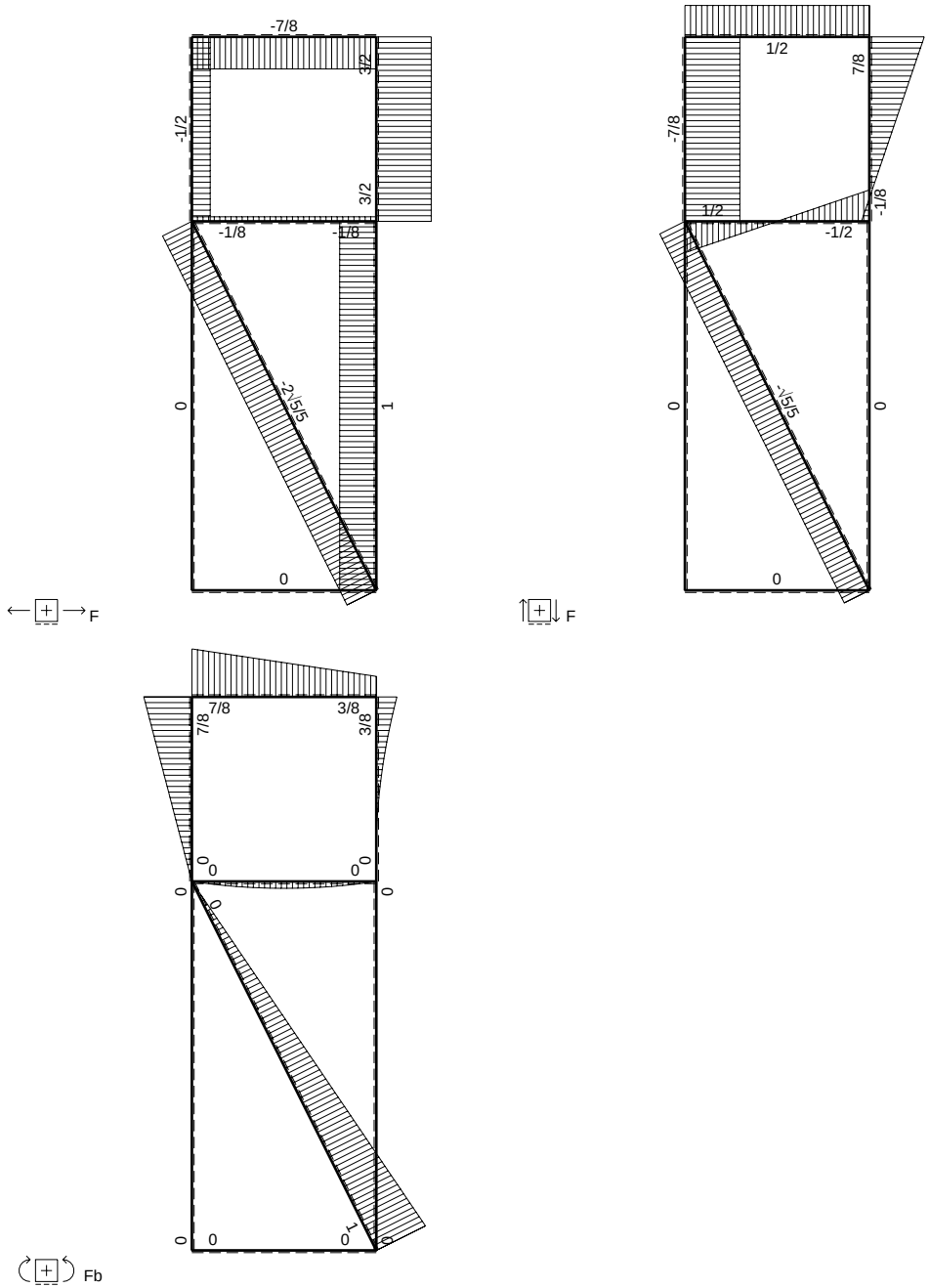
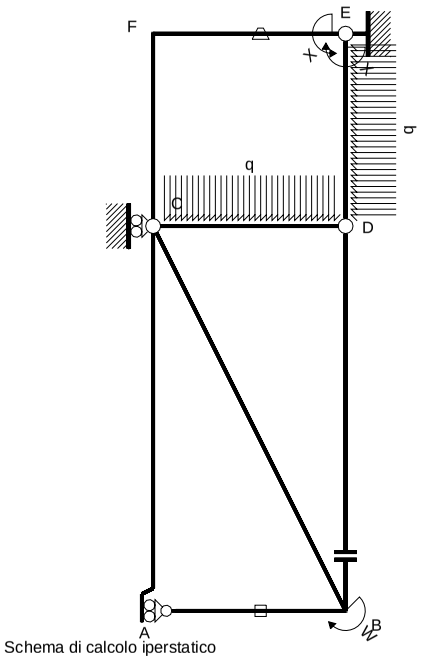
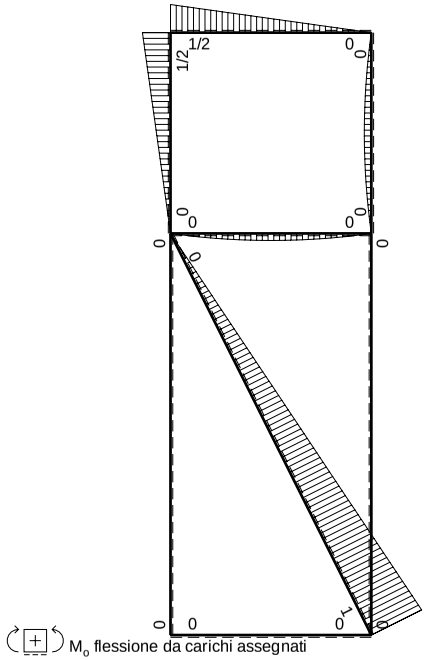
A
B
B

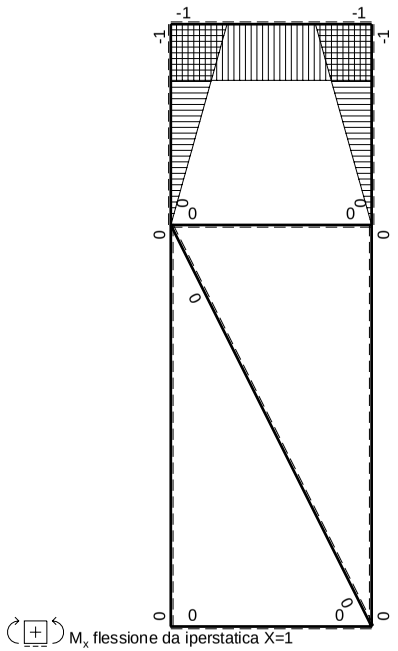

-7/8
3/2
-1/2
3/2
-1/8
-1/8
0
-√5/5
1
0
F
1/2
7/8
-7/8
1/2
-1/8
-1/2
0
-√5/5
0
F
7/8
3/8
7/8
3/8
0
0
0
0
0
0
0
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	-1/2Fx+1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
ED b	1-x/b	1/2Fx-1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2		
CD b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx^2	0	0	0	0		
EF b	-1	1/2Fx	-Fb/EJ	-1/2Fx	Fb/EJ	1	(-1/4+1)Fb^2/EJ	Xb/EJ
FE b	1	-1/2Fb+1/2Fx	Fb/EJ	-1/2Fb+1/2Fx	Fb/EJ	1		
FC b	-1+x/b	1/2Fb-1/2Fx	0	-1/2Fb+Fx-1/2Fx^2/b	0	1-2x/b+x^2/b^2	(-1/6+0)Fb^2/EJ	1/3Xb/EJ
CF b	x/b	-1/2Fx	0	-1/2Fx^2/b	0	x^2/b^2		
		totali					5/8Fb^2/EJ	5/3Xb/EJ
		iperstatica $X=W_{EF}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981051 ESΣ09.xxxx.211

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b)\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

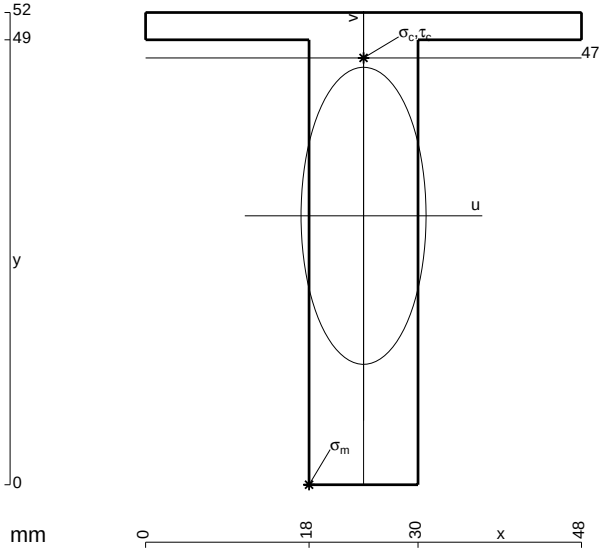
$= (-1/2\ b +1/4\ b)\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b)\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 732. mm^2
J_u = 195951. mm^4
J_v = 34704. mm^4
y_g = 29.61 mm
N = -3166. N
T_y = -1583. N
M_x = 1416000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -29.61 mm
σ_m = N/A-Mv/J_u = 209.7 N/mm^2
x_c = 24. mm
y_c = 47. mm
v_c = 17.39 mm
σ_c = N/A-Mv/J_u = -130. N/mm^2
τ_c = 2.322 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130. N/mm^2
S^* = 3449. mm^3

ESΣ09.xxxx.211

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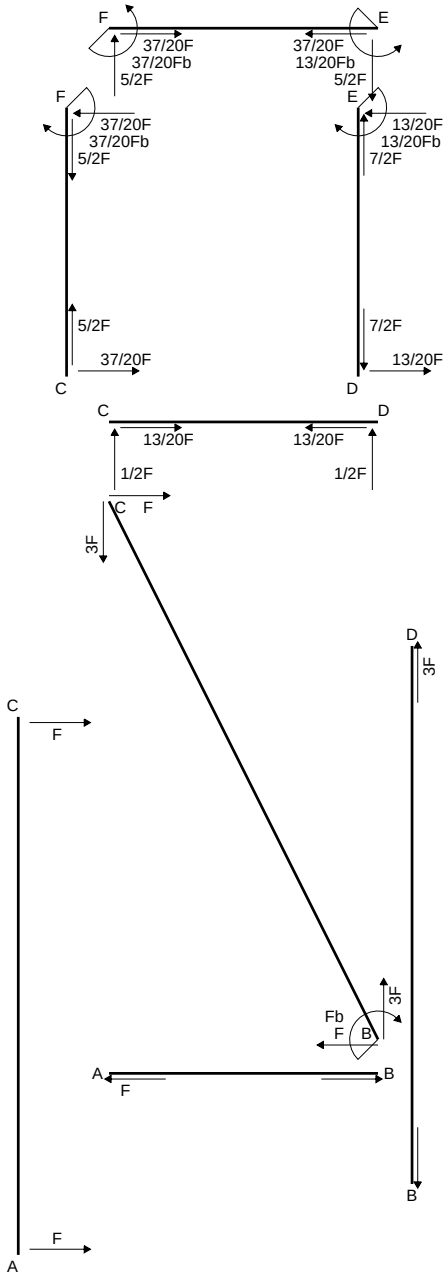
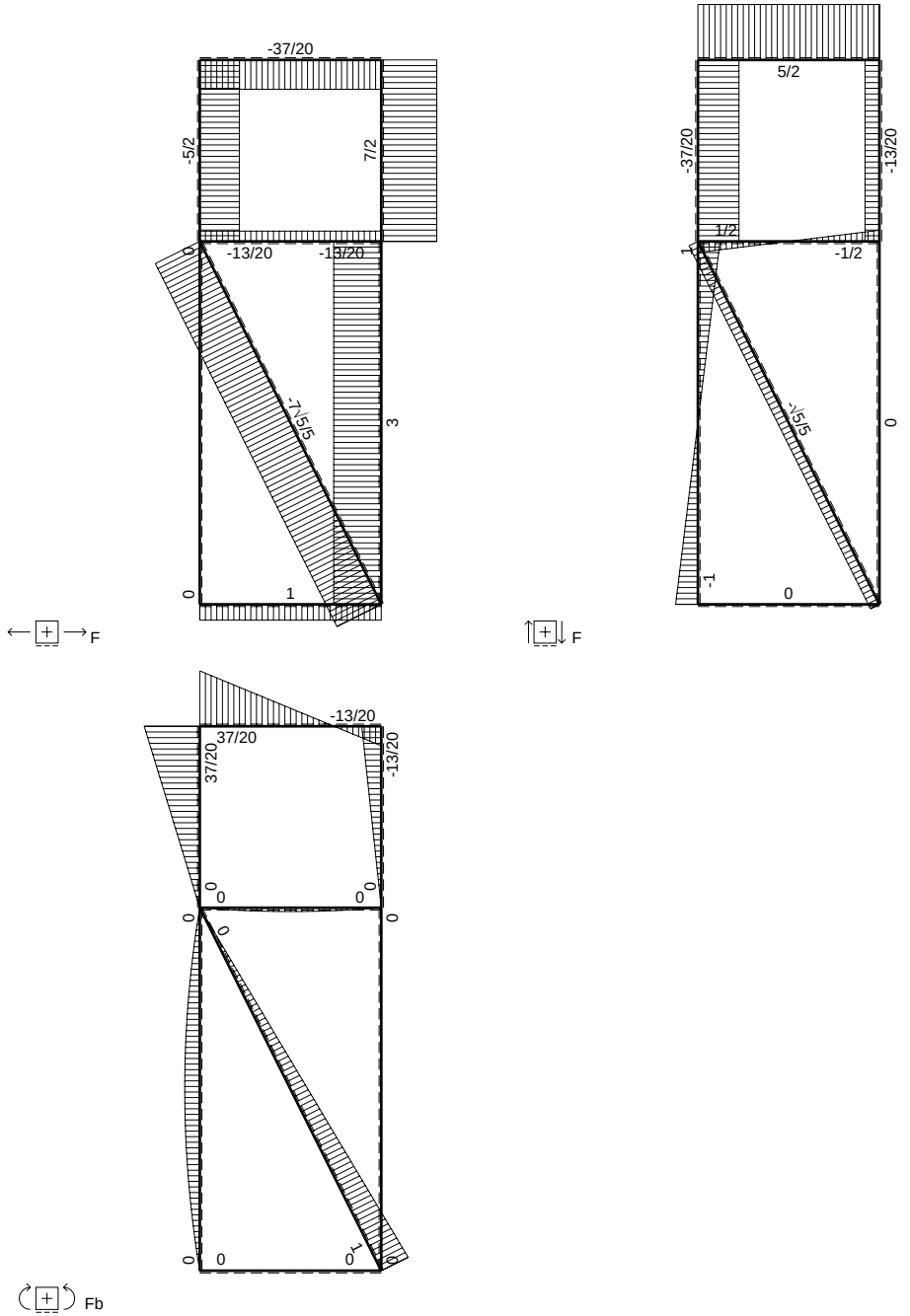

F
E
37/20F
37/20Fb
5/2F
37/20F
13/20Fb
5/2F
F
E
37/20F
37/20Fb
5/2F
13/20F
13/20Fb
7/2F
5/2F
37/20F
C
7/2F
13/20F
D
C
D
13/20F
13/20F
1/2F
1/2F
C F
3F
D
3F
C
F
3F
Fb
F B
A
B
F
F
A
B


-37/20
-5/2
7/2
-13/20
-13/20
0
-7.5/5
3
0
1
F
5/2
-37/20
-13/20
1/2
-1/2
-7.5/5
0
-1
0
F
-13/20
37/20
37/20
-13/20
0
0
0
0
0
0
0
0
0
0
0
0
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
BC √5b	0	Fb-√5/5Fx	0	0	0	0	0	0
AC 2b	0	-Fx+1/2qx²	0	0	0	0	0+0	0
CA 2b	0	Fx-1/2qx²	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	1/2Fx-1/2qx²	0	0	0	0	0+0	0
DC b	0	-1/2Fx+1/2qx²	0	0	0	0		
EF b	-1	5/2Fx	-Fb/EJ	-5/2Fx	Fb/EJ	1	(-5/4+1)Fb²/EJ	Xb/EJ
FE b	1	-5/2Fb+5/2Fx	Fb/EJ	-5/2Fb+5/2Fx	Fb/EJ	1		
FC b	-1+x/b	5/2Fb-5/2Fx	0	-5/2Fb+5Fx-5/2Fx²/b	0	$1-2x/b+x^2/b^2$	(-5/6+0)Fb²/EJ	1/3Xb/EJ
CF b	x/b	-5/2Fx	0	-5/2Fx²/b	0	x^2/b^2		
	totali						$-13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{EF}$						13/20Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 984497 ESΣ09.xxxx.212

14.11.24

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

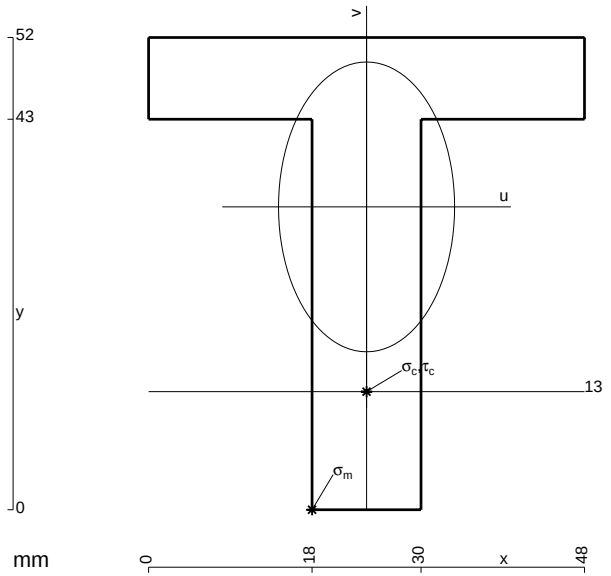
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 948. mm^2
J_u = 241377. mm^4
J_v = 89136. mm^4
y_g = 33.35 mm
N = -11959. N
T_y = -1708. N
M_x = 1680800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.35 mm
σ_m = N/A-Mv/J_u = 219.6 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -20.35 mm
σ_c = N/A-Mv/J_u = 129.1 N/mm^2
τ_c = 2.47 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.1 N/mm^2
S^* = 4188. mm^3

ESΣ09.xxxx.212

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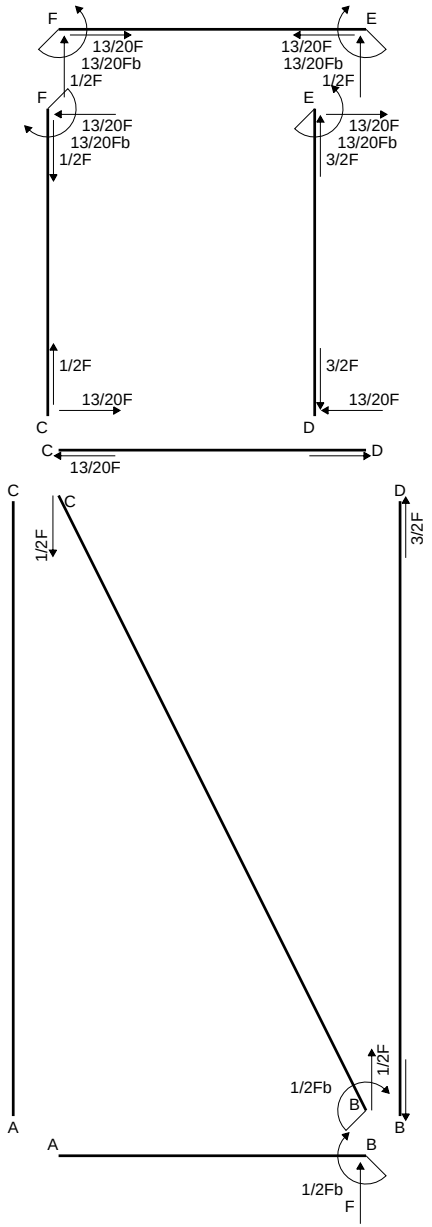
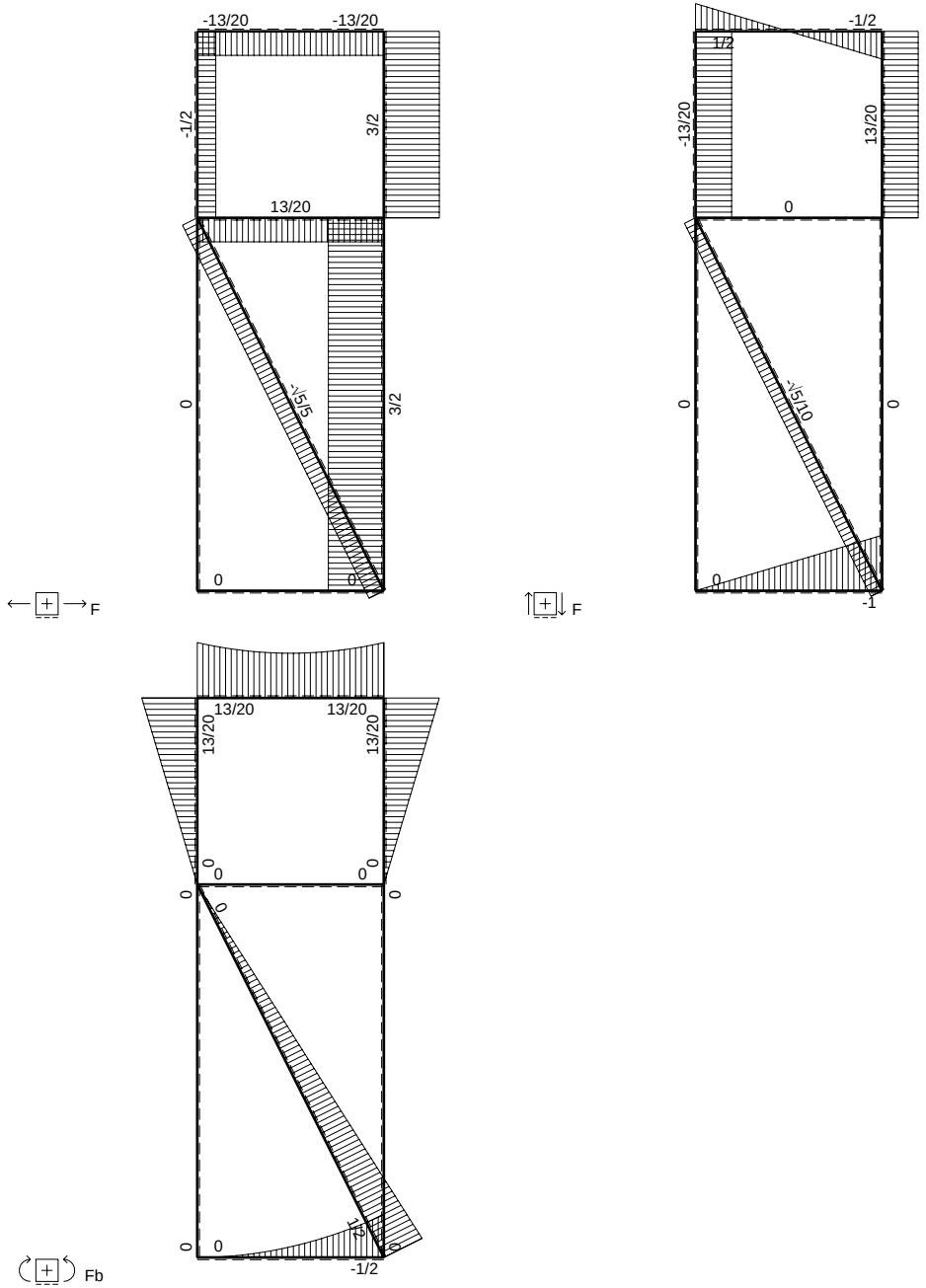

F
E
13/20F
13/20Fb
1/2F
3/2F
C
D
A
B
1/2Fb


-13/20
-1/2
3/2
13/20
0
-√5/5
-√5/10
F
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_\chi(x)$	$M_o(x)$	θ	$M_\chi M_o$	$M_\chi \theta$	$M_\chi M_\chi$	$\int M_\chi(M_o/EJ+\theta)dx$	$\int XM_\chi M_\chi/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	1/2Fb-√5/10Fx	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1	$(1/12+1)Fb^2/EJ$	Xb/EJ
FE b	1	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-1/2Fx^2/b$	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						$13/12Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

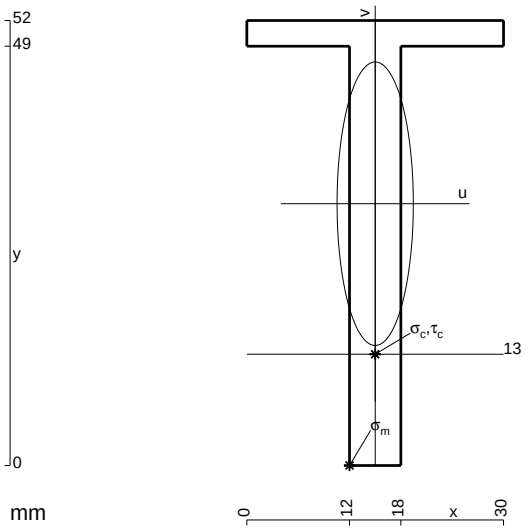
$L^{Xo}_{EF} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

$= (1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b\ )\ \theta\ = 13/12\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 213144 ESΣ09.xxxx.213



A = 384. mm^2
J_u = 105473. mm^4
J_v = 7632. mm^4
y_g = 30.59 mm
T_y = -3300. N
M_x = -792000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -30.59 mm
$\sigma_m = -Mv/J_u$ = -229.7 N/mm^2
x_c = 15. mm
y_c = 13. mm
v_c = -17.59 mm
$\sigma_c = -Mv/J_u$ = -132.1 N/mm^2
τ_c = 9.8 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.2 N/mm^2
S^* = 1879. mm^3

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.24

ESΣ09.xxxx.213

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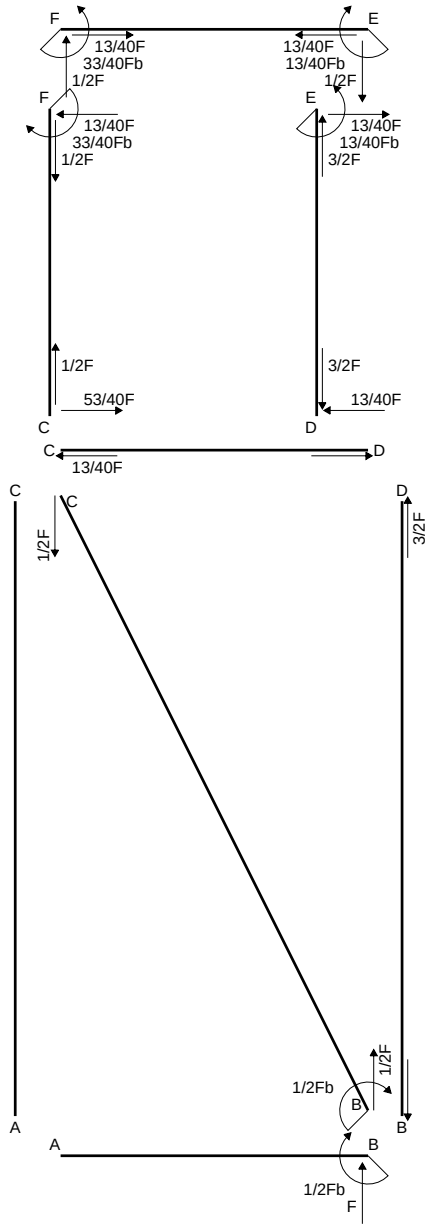
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
1/2F
F
E
13/40F
33/40Fb
1/2F
13/40F
13/40Fb
3/2F
1/2F
53/40F
3/2F
13/40F
C
D
C
D
13/40F
C
D
C
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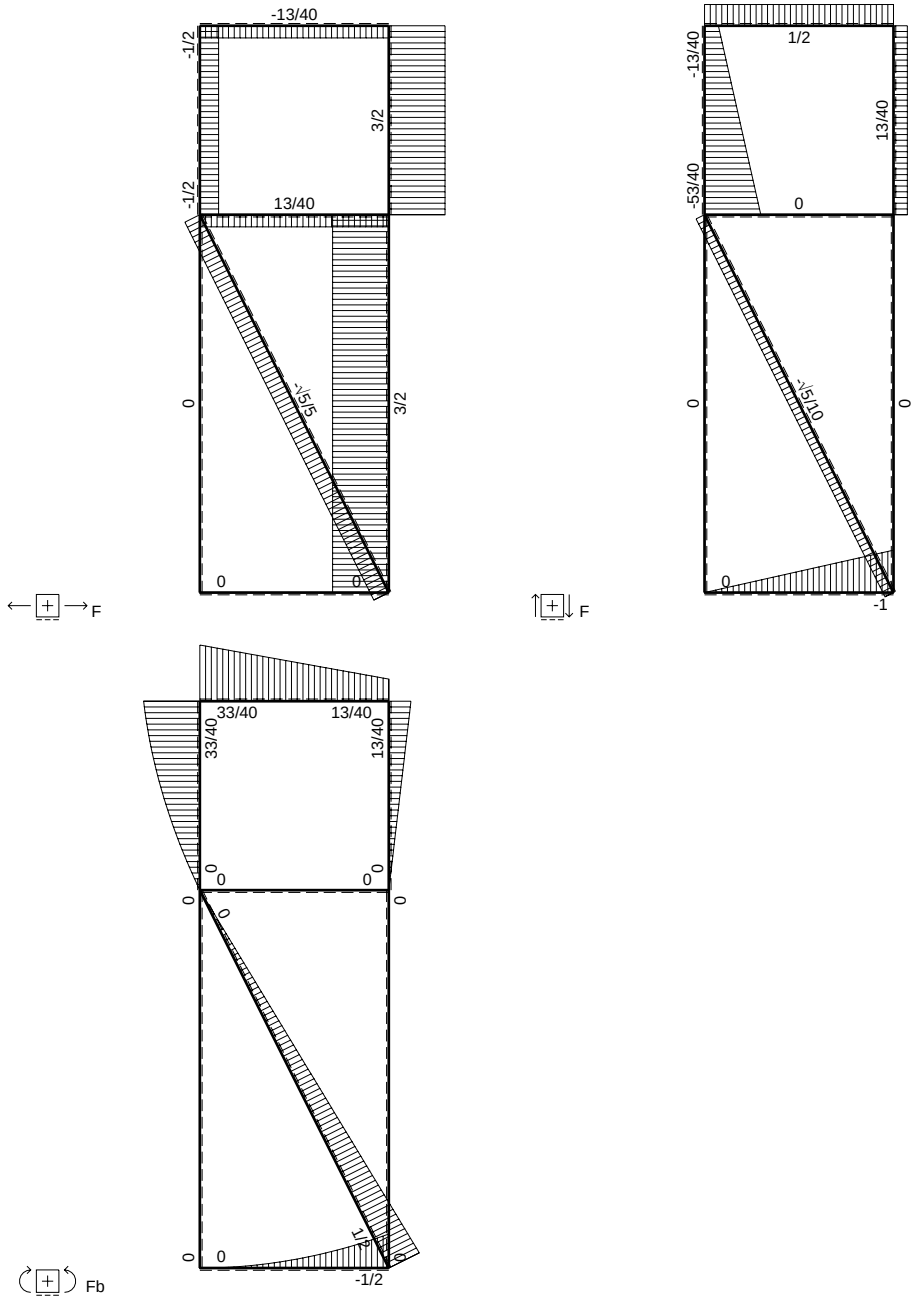
A
B
1/2Fb
F


-13/40
-1/2
3/2
-1/2
13/40
0
-√5/5
3/2
0
0
F
-13/40
1/2
13/40
-53/40
0
0
-√5/10
0
0
-1
F
33/40
13/40
33/40
13/40
0
0
0
0
0
0
0
-√2/2
0
-1/2
Fb

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3XbEJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	1/2Fx	-FbEJ	-1/2Fx	FbEJ	1	$(-1/4+1)Fb^2/EJ$	XbEJ
FE b	1	$-1/2Fb+1/2Fx$	FbEJ	$-1/2Fb+1/2Fx$	FbEJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx+1/2Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-5/24+0)Fb^2/EJ$	1/3XbEJ
CF b	x/b	$-Fx+1/2qx^2$	0	$-Fx^2/b+1/2qx^3/b$	0	x^2/b^2		
	totali						$13/24Fb^2/EJ$	5/3XbEJ
	iperstatica $X=W_{EF}$						-13/40Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982319 ESΣ09.xxxx.214

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$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

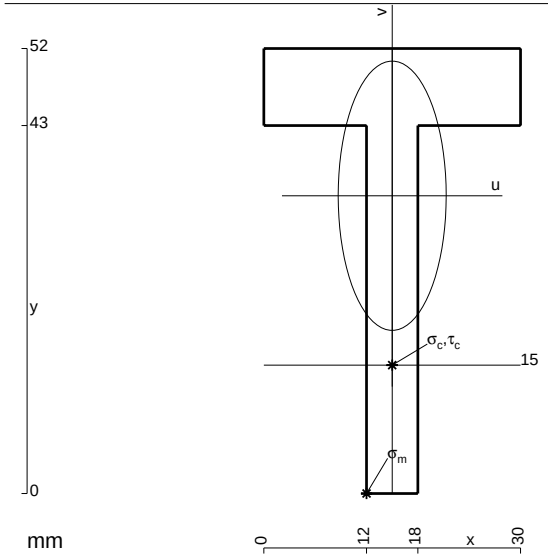
$L^{Xo}_{FC} = \int_o^b (-1/2 +1/2\ x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx$

$= [-1/2\ x +1/4\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/4\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -5/24\ Fb^2/EJ$



A = 528. mm^2
J_u = 130762. mm^4
J_v = 21024. mm^4
y_g = 34.8 mm
T_y = -3530. N
M_x = -900150. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -34.8 mm
$\sigma_m = -Mv/J_u$ = -239.5 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -19.8 mm
$\sigma_c = -Mv/J_u$ = -136.3 N/mm^2
τ_c = 11.05 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 137.6 N/mm^2
S^* = 2457. mm^3

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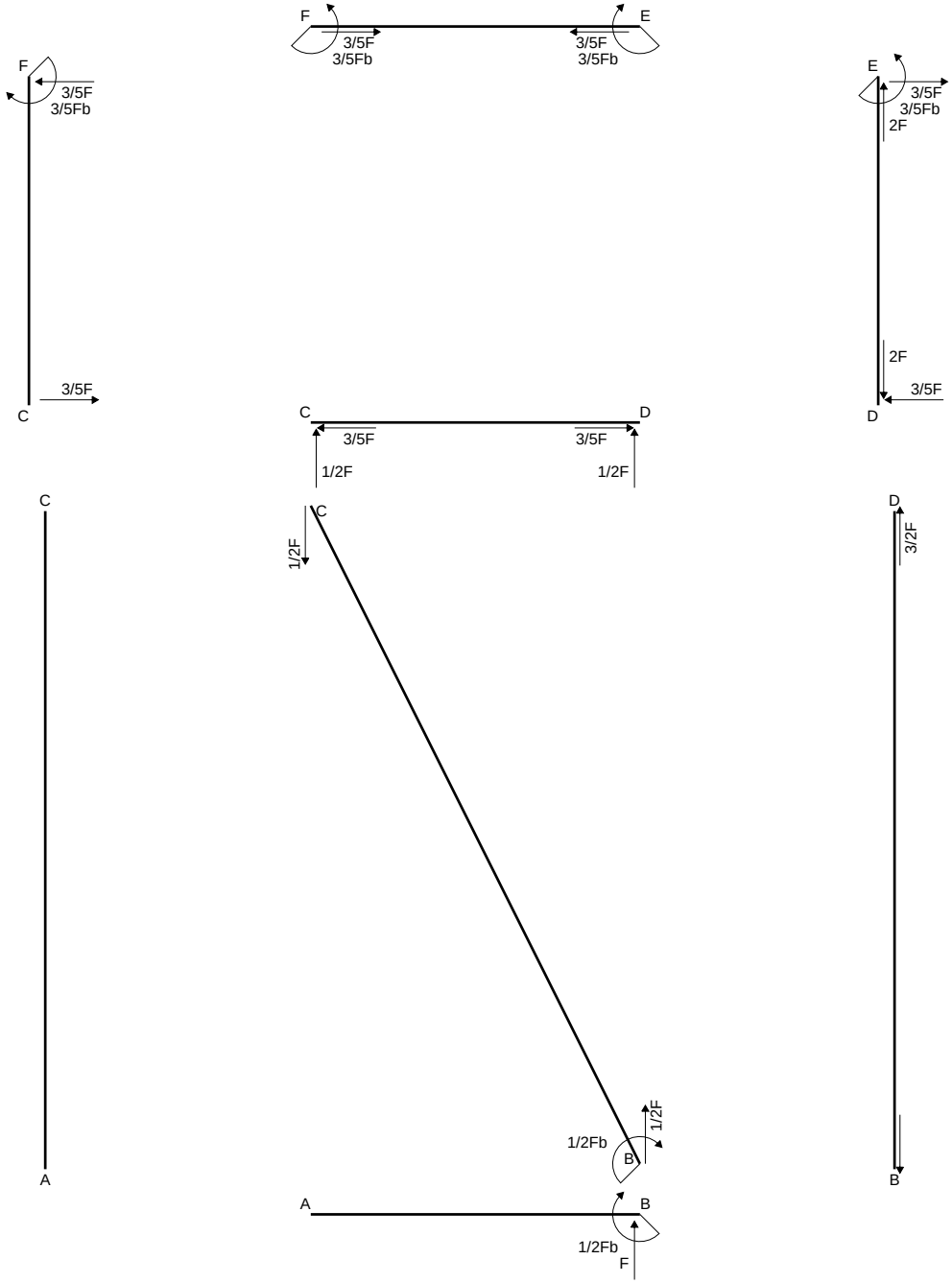
F
E
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3/5Fb
3/5F
3/5Fb
F
3/5F
3/5Fb
E
3/5F
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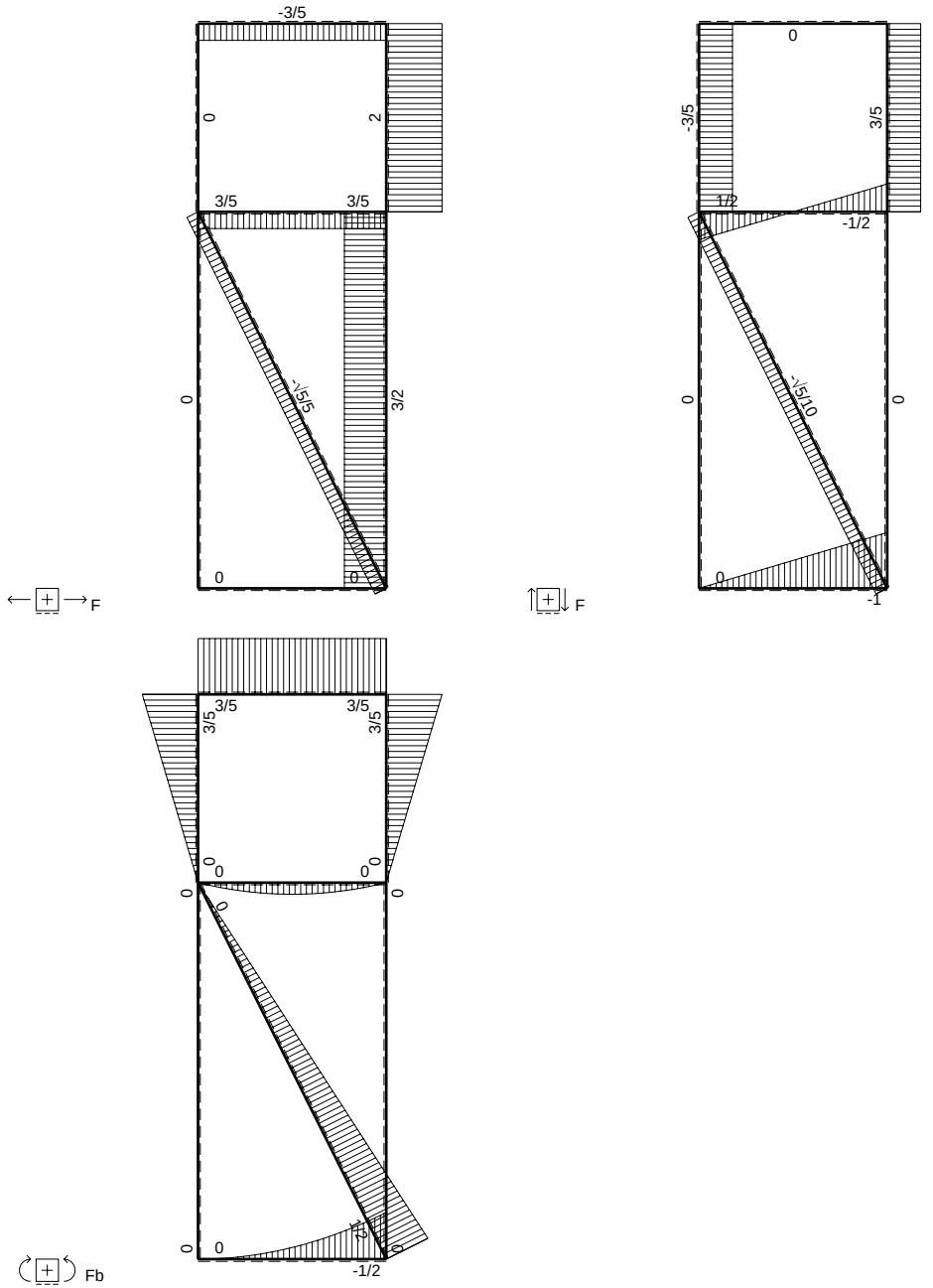
2F
C
3/5F
2F
D
3/5F
C
D
3/5F
3/5F
1/2F
1/2F
C
D
1/2F
3/2F
1/2F
1/2Fb
B
A
B
1/2Fb
F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-3/10
0
0
-1
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3/5
3/5
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0
0
0
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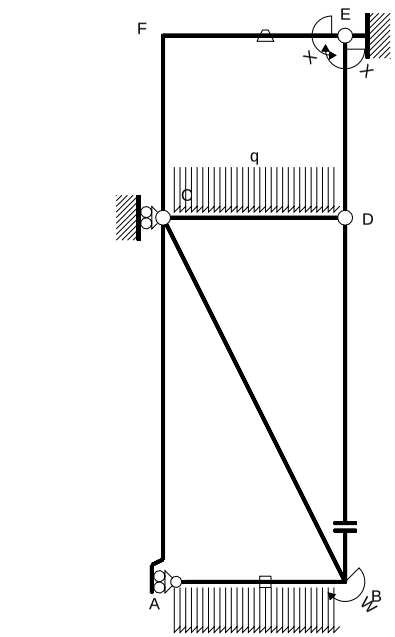
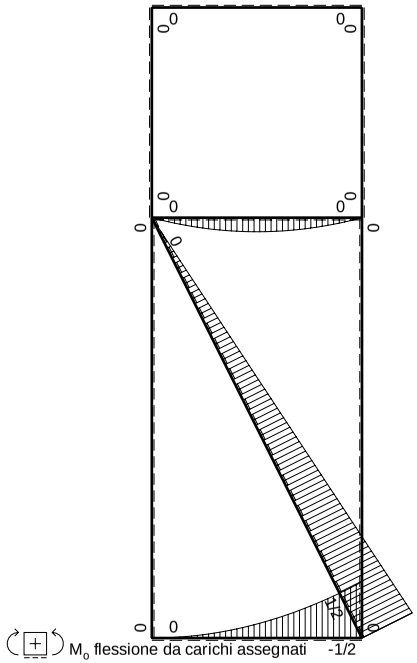
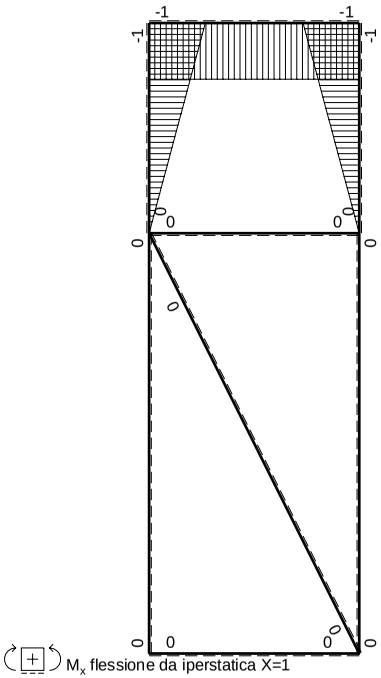
0
0
-1/2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC $\sqrt{5}$b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	-x/b	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	1-x/b	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	-1	0	-Fb/EJ	0	Fb/EJ	1	$(0+1)Fb^2/EJ$	Xb/EJ
FE b	1	0	Fb/EJ	0	Fb/EJ	1		
FC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CF b	x/b	0	0	0	0	x^2/b^2		
	totali						Fb^2/EJ	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/5Fb	

Sviluppi di calcolo iperstatica

$L_{DE}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{ED}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{EF}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FE}^{XX} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L_{FC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

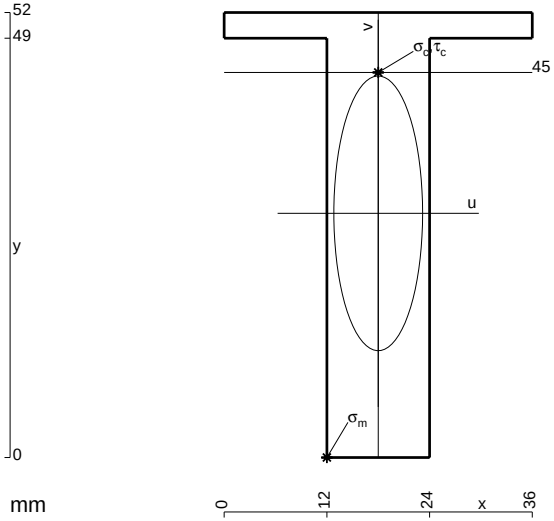
$L_{EF}^{Xo} = \int_o^b (1)\ \theta\ dx = [\ x\ ]_o^b\ \theta$

$= (\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FE}^{Xo} = \int_o^b (-1)\ \theta\ dx = [-\ x\ ]_o^b\ \theta$

$= (-\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI 956361 ESΣ09.xxxx.215



A = 696. mm^2
J_u = 179409. mm^4
J_v = 18720. mm^4
y_g = 28.53 mm
T_y = -4570. N
M_x = -1256750. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -28.53 mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 18. mm
y_c = 45. mm
v_c = 16.47 mm
$\sigma_c = -Mv/J_u$ = 115.3 N/mm^2
τ_c = 6.917 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 116. N/mm^2
S^* = 3259. mm^3

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ESΣ09.xxxx.215

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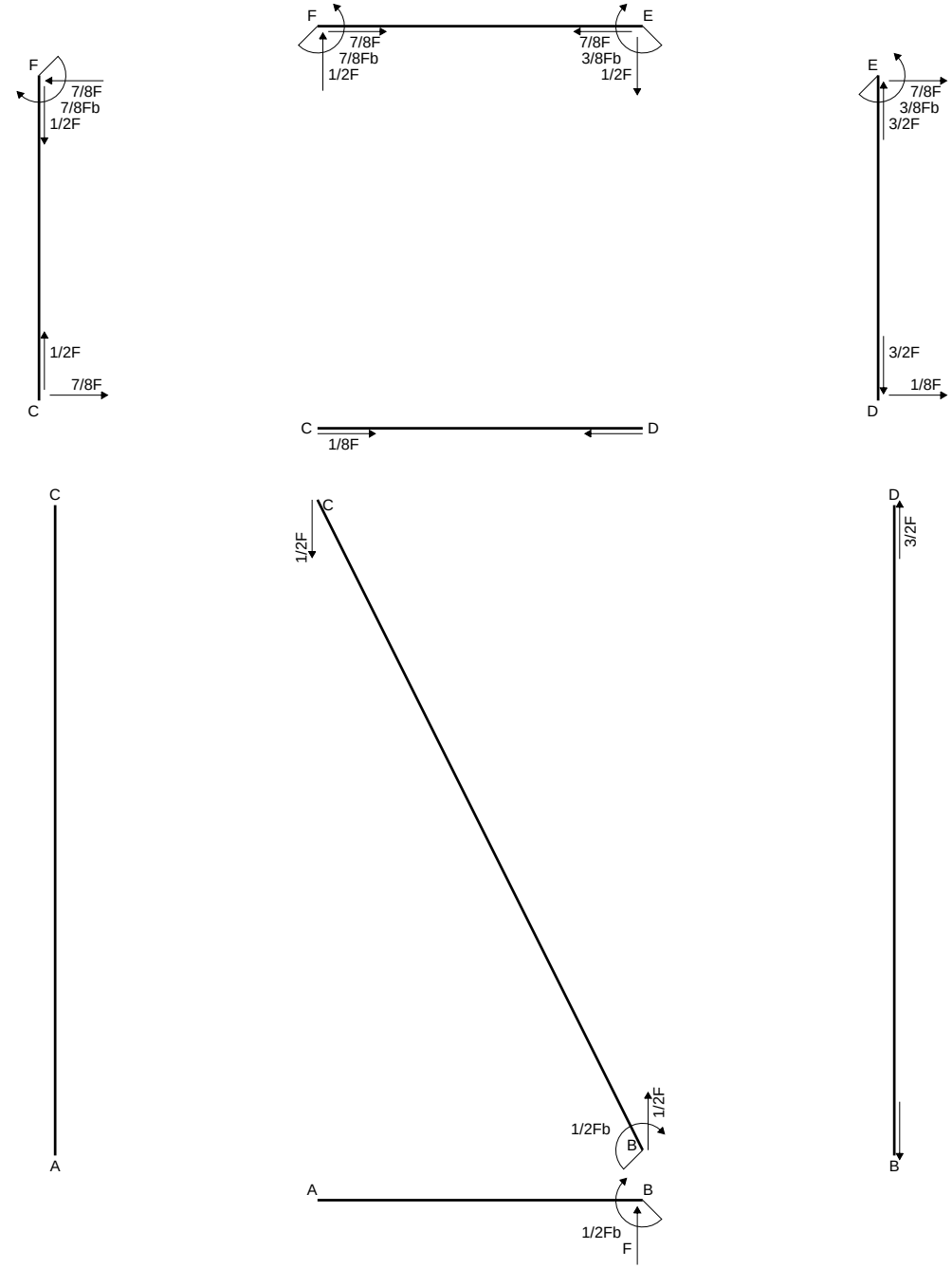
F
7/8F
7/8Fb
1/2F
E
7/8F
3/8Fb
1/2F
1/2F
7/8F
C
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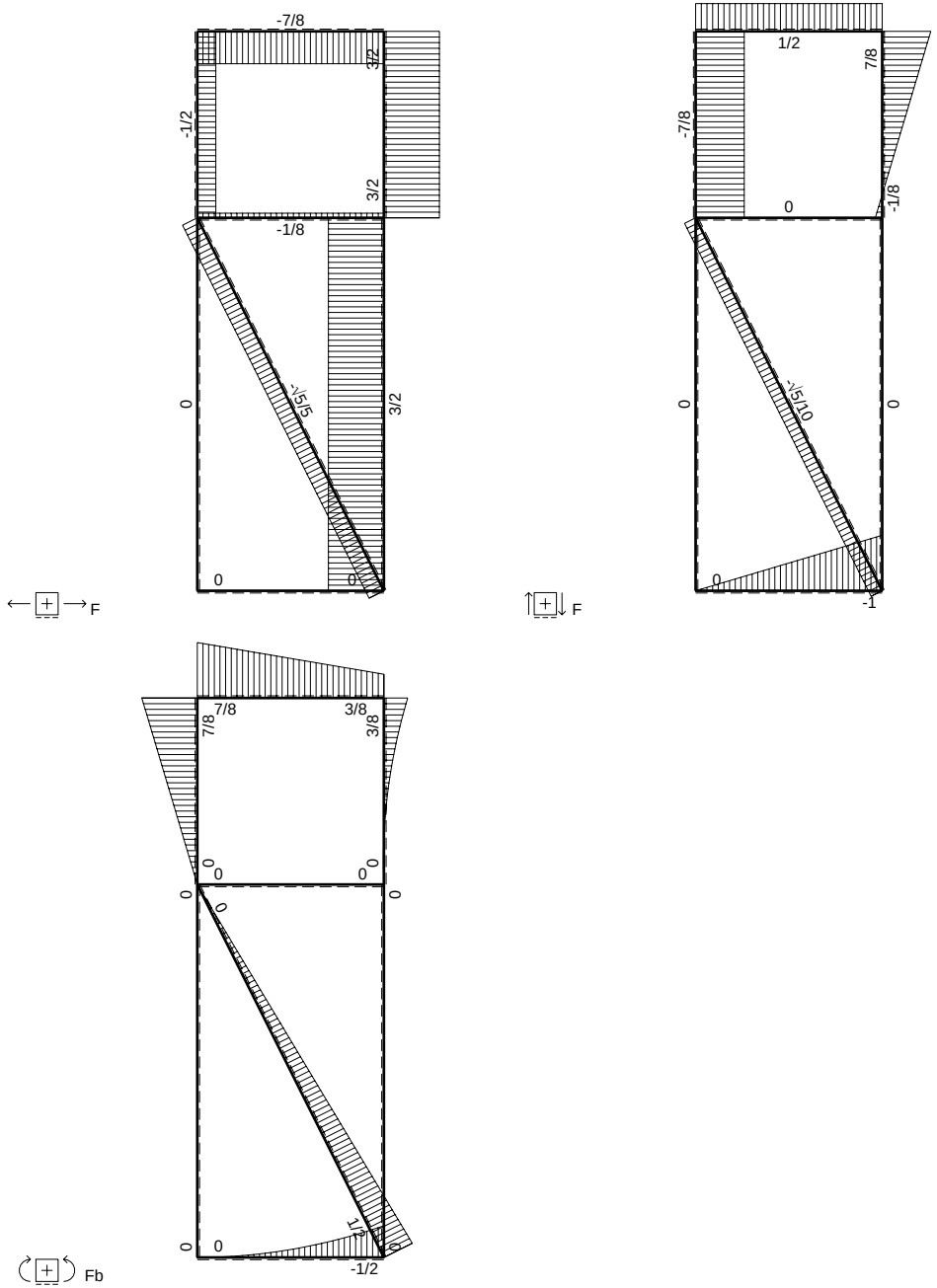
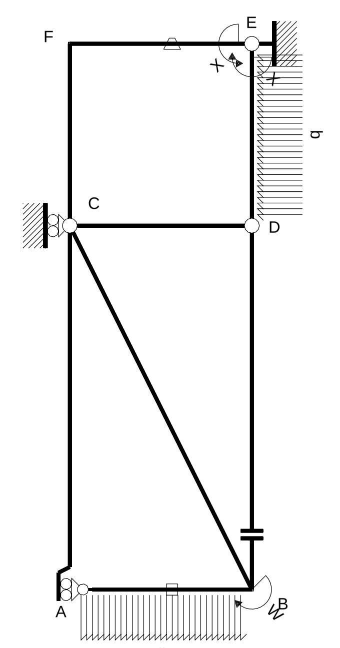
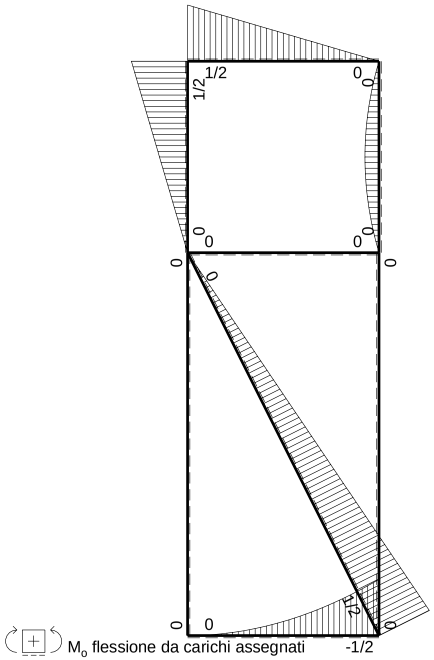
3/2F
1/8F
A
B
1/2Fb

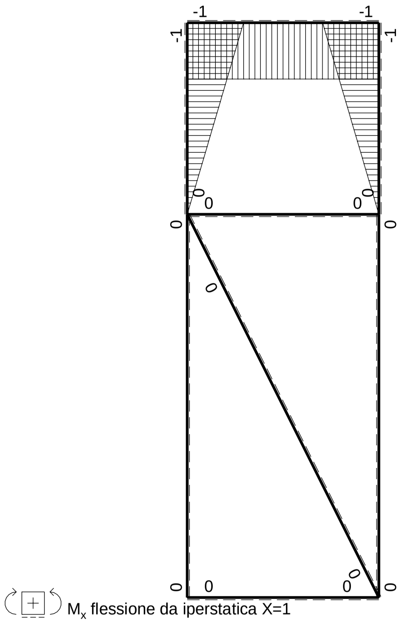

-7/8
3/2
-1/2
-1/8
√5/5
F
1/2
7/8
-1/8
√5/10
7/8
3/8
Fb
-1/2



Schema di calcolo iperstatico q



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	0	0	0	0	0	0+0	0
CA 2b	0	0	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
ED b	$1-x/b$	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$1/2Fx$	-Fb/EJ	$-1/2Fx$	Fb/EJ	1	$(-1/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-1/2Fb+1/2Fx$	Fb/EJ	$-1/2Fb+1/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$1/2Fb-1/2Fx$	0	$-1/2Fb+Fx-1/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-1/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-1/2Fx$	0	$-1/2Fx^2/b$	0	x^2/b^2		
	totali						$5/8Fb^2/EJ$	5/3Xb/EJ
	iperstatica $X=W_{EF}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1\ )\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{DE} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{ED} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{EF} = \int_o^b (-1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1\ )\ \theta\ dx = [-1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-1/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-1/2 +1/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1\ )\ \theta\ dx = [-1/2\ x +1/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x\ ]_o^b\ \theta$

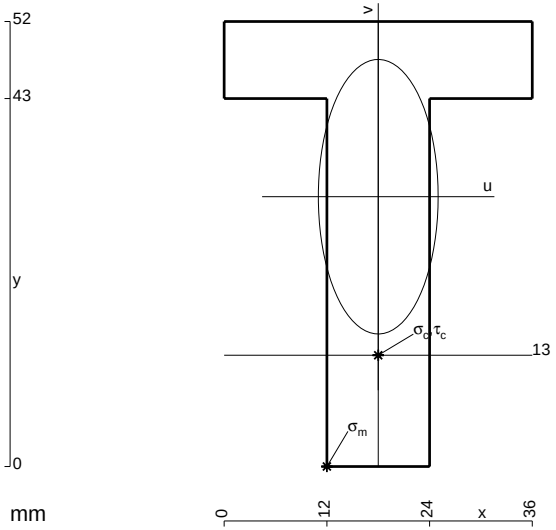
$= (-1/2\ b +1/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = 3/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ = -1/6\ Fb^2/EJ$



A = 840. mm^2
J_u = 216237. mm^4
J_v = 41184. mm^4
y_g = 31.53 mm
T_y = -4880. N
M_x = -1439600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.53 mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -18.53 mm
$\sigma_c = -Mv/J_u$ = -123.4 N/mm^2
τ_c = 7.343 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124. N/mm^2
S^* = 3904. mm^3

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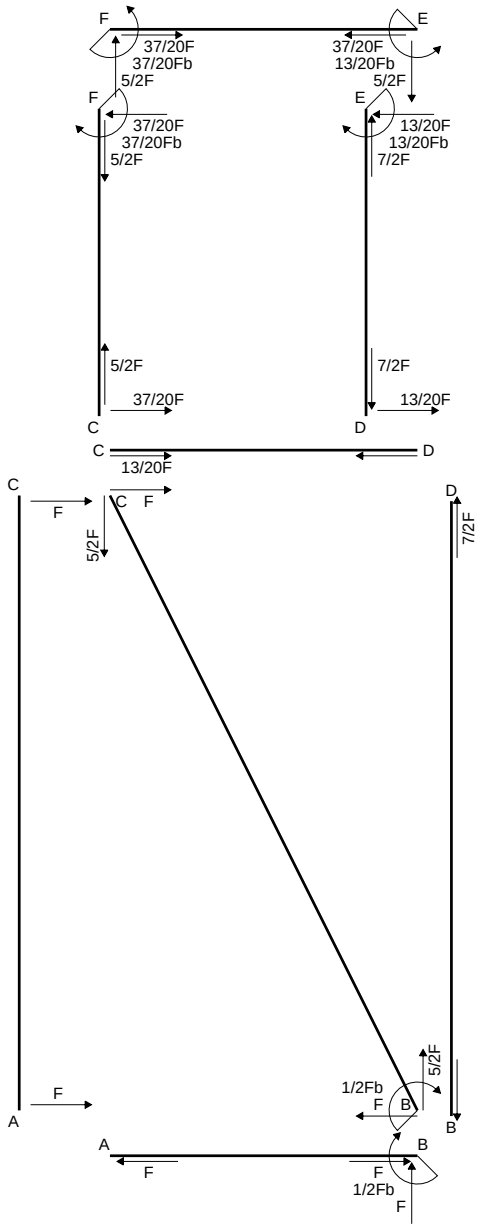
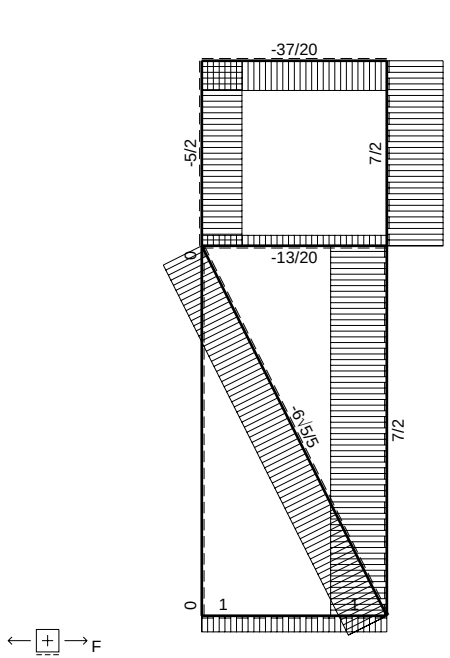
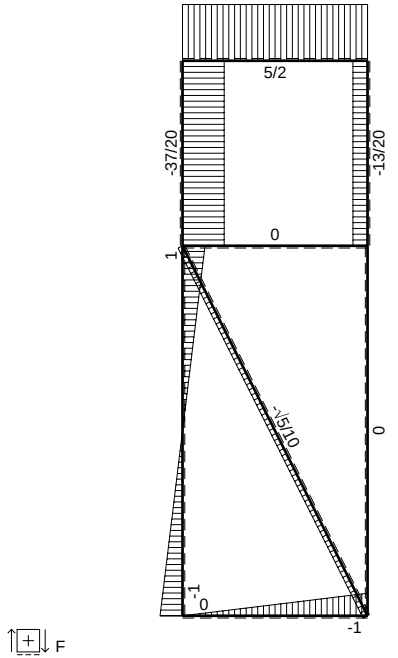
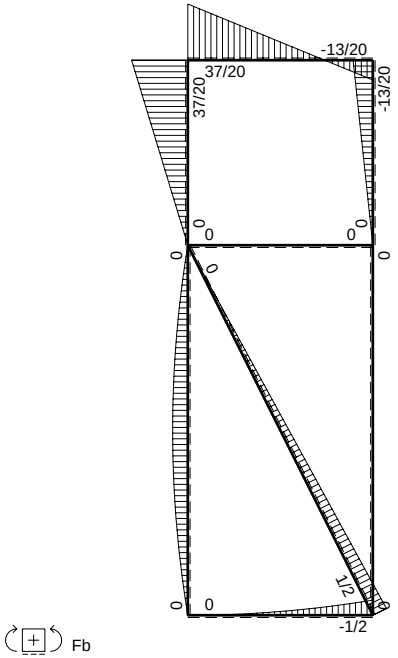
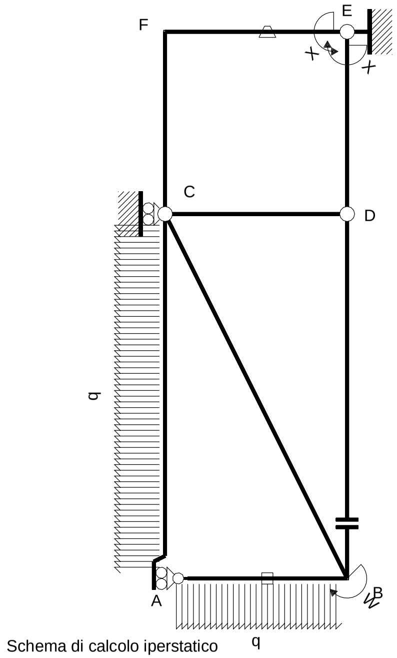

F
E
37/20F
37/20Fb
5/2F
13/20Fb
7/2F
C
D
13/20F
A
B
1/2Fb

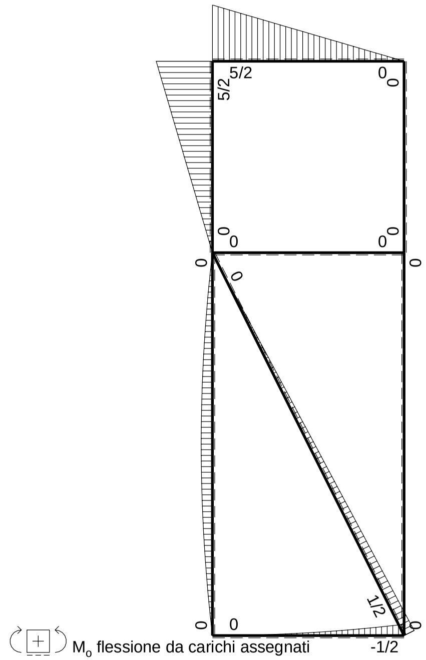

-37/20
-5/2
7/2
-13/20
-5\5/5
F


5/2
-37/20
-13/20
-5\5/10
F

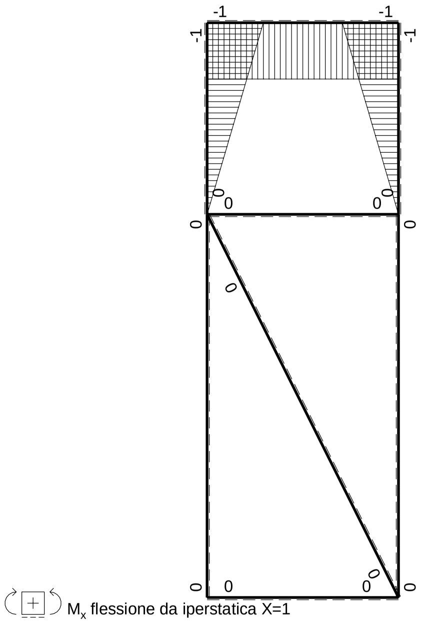

-13/20
37/20
-13/20
1/2
-1/2
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{EF}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BC √5b	0	$1/2Fb-\sqrt{5}/10Fx$	0	0	0	0	0	0
AC 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
CA 2b	0	$Fx-1/2qx^2$	0	0	0	0		
DB 2b	0	0	0	0	0	0	0+0	0
BD 2b	0	0	0	0	0	0		
DE b	$-x/b$	0	0	0	0	x^2/b^2	0+0	1/3Xb/EJ
ED b	$1-x/b$	0	0	0	0	$1-2x/b+x^2/b^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
EF b	-1	$5/2Fx$	$-Fb/EJ$	$-5/2Fx$	Fb/EJ	1	$(-5/4+1)Fb^2/EJ$	Xb/EJ
FE b	1	$-5/2Fb+5/2Fx$	Fb/EJ	$-5/2Fb+5/2Fx$	Fb/EJ	1		
FC b	$-1+x/b$	$5/2Fb-5/2Fx$	0	$-5/2Fb+5Fx-5/2Fx^2/b$	0	$1-2x/b+x^2/b^2$	$(-5/6+0)Fb^2/EJ$	1/3Xb/EJ
CF b	x/b	$-5/2Fx$	0	$-5/2Fx^2/b$	0	x^2/b^2		
totali							$-13/12Fb^2/EJ$	5/3Xb/EJ
iperstatica $X=W_{EF}$							13/20Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{DE} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{ED} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{EF} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FE} = \int_o^b (1)\ 1/EJ\ dx = [\ x\ ]_o^b\ 1/EJ$

$= (\ b\ )\ 1/EJ\ =\ b/EJ$

$L^{XX}_{FC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{Xo}_{EF} = \int_o^b (-5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (1)\ \theta\ dx = [-5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [\ x\ ]_o^b\ \theta$

$= (-5/4\ b\ )\ Fb\ 1/EJ\ + (\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FE} = \int_o^b (-5/2 +5/2\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1)\ \theta\ dx = [-5/2\ x +5/4\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x\ ]_o^b\ \theta$

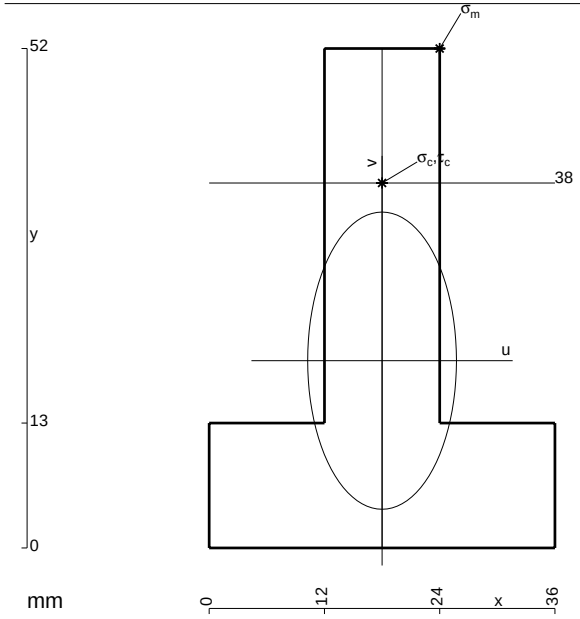
$= (-5/2\ b +5/4\ b\ )\ Fb\ 1/EJ\ + (-\ b\ )\ \theta\ = -1/4\ Fb^2/EJ$

$L^{Xo}_{FC} = \int_o^b (-5/2 +5\ x/b -5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/2\ x +5/2\ x^2/b -5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/2\ b +5/2\ b -5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$

$L^{Xo}_{CF} = \int_o^b (-5/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-5/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-5/6\ b\ )\ Fb\ 1/EJ\ = -5/6\ Fb^2/EJ$



A = 936. mm^2
J_u = 224094. mm^4
J_v = 56160. mm^4
y_g = 19.5 mm
N = 4770. N
T_y = -4770. N
M_x = -1478700. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 32.5 mm
$\sigma_m = N/A - Mv/J_u$ = 219.6 N/mm^2
x_c = 18. mm
y_c = 38. mm
v_c = 18.5 mm
$\sigma_c = N/A - Mv/J_u$ = 127.2 N/mm^2
τ_c = 7.599 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.8 N/mm^2
S^* = 4284. mm^3

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