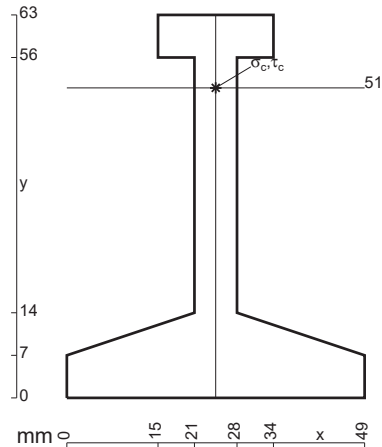
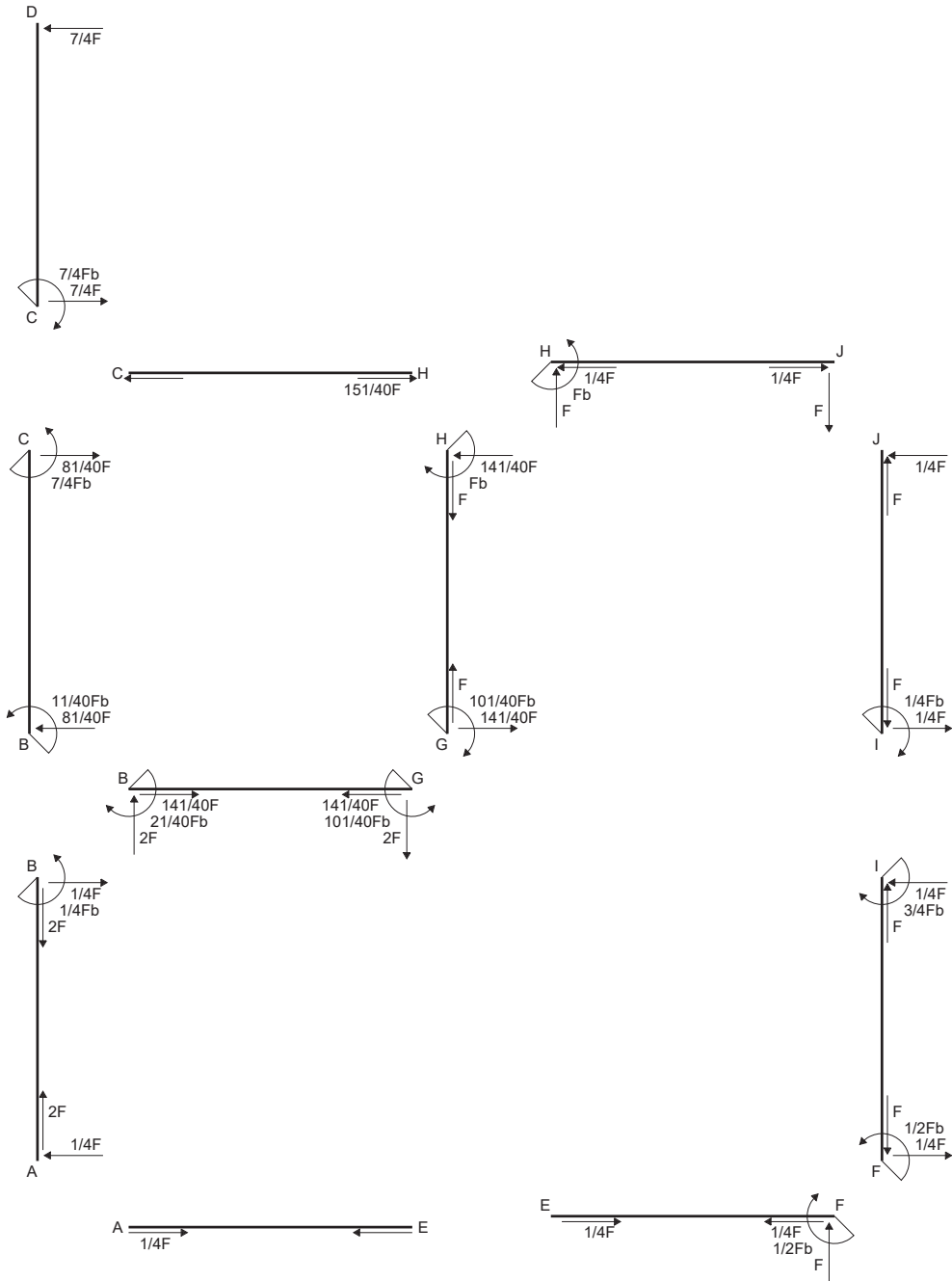


$H_G = -F$	$EJ_{BC} = EJ$	$EJ_{GH} = EJ$
$W_I = -W = -Fb$	$EJ_{CD} = EJ$	$EJ_{HC} = EJ$
$p_{EF} = -q = -F/b$	$EJ_{EF} = EJ$	$EJ_{FI} = EJ$
$\theta_{BC} = 4\theta = 4\alpha T/b = 4bF/EJ$	$EJ_{AE} = EJ$	$EJ_{IJ} = EJ$
$EJ_{AB} = EJ$	$EJ_{BG} = EJ$	$EJ_{JH} = EJ$

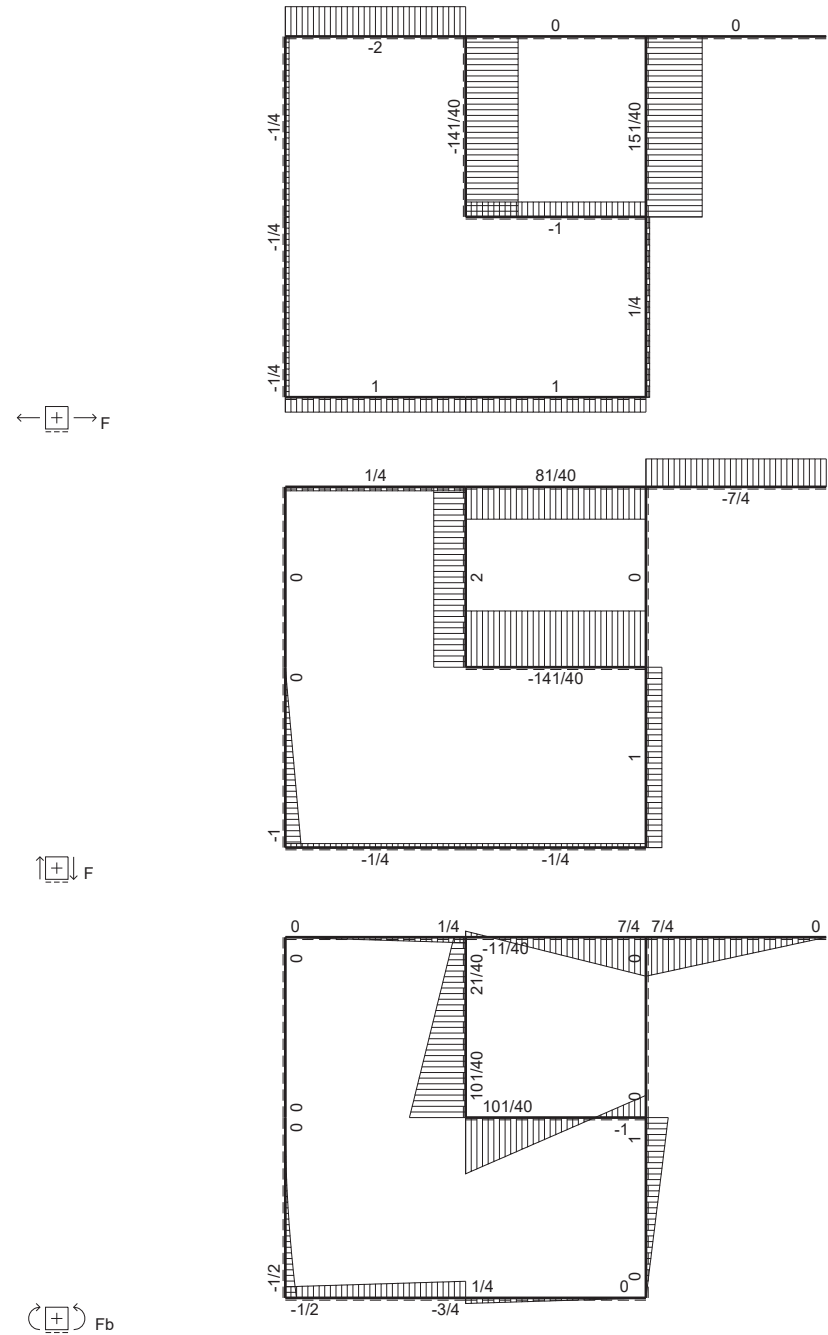
Reazioni iperstatiche in soluzione:  $X=W_{GB}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave CD ha la sezione riportata e dimensioni in mm, con:  
 $b = 690 \text{ mm}$ ,  $F = 1820 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da C a D  
 Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

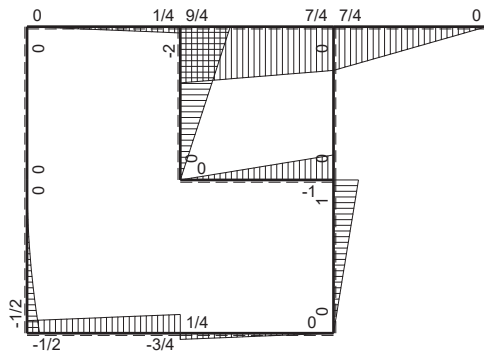
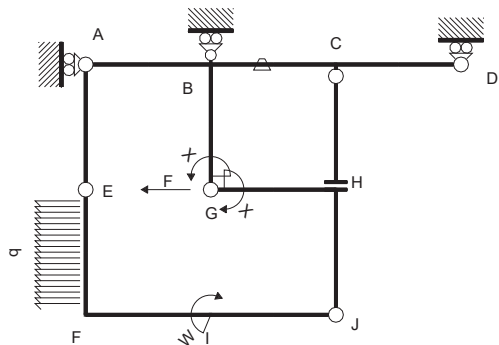


# REAZIONI



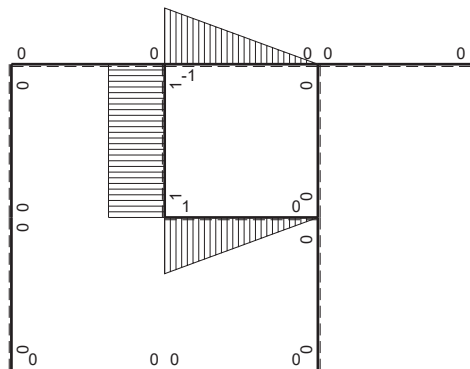
# AZIONI INTERNE





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{GB}$

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	0	1/4Fx	0	0	0	0	0+0	0	
BA b	0	-1/4Fb+1/4Fx	0	0	0	0			
BC b	-1+x/b	9/4Fb-1/2Fx	4Fb/EJ	-9/4Fb+11/4Fx-1/2Fx <sup>2</sup> /b	-4Fb/EJ+4Fx/EJ	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	(-25/24-2)Fb <sup>2</sup> /EJ	1/3Xb/EJ	
CB b	x/b	-7/4Fb-1/2Fx	-4Fb/EJ	-7/4Fx-1/2Fx <sup>2</sup> /b	-4Fx/EJ	x <sup>2</sup> /b <sup>2</sup>			
CD b	0	7/4Fb-7/4Fx	0	0	0	0	0+0	0	
DC b	0	-7/4Fx	0	0	0	0			
EF b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0	
FE b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0			
AE b	0	0	0	0	0	0	0+0	0	
EA b	0	0	0	0	0	0			
BG b	1	-2Fb+2Fx	0	-2Fb+2Fx	0	1	(-1+0)Fb <sup>2</sup> /EJ	Xb/EJ	
GB b	-1	2Fx	0	-2Fx	0	1			
GH b	1-x/b	-Fx	0	-Fx+Fx <sup>2</sup> /b	0	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	(-1/6+0)Fb <sup>2</sup> /EJ	1/3Xb/EJ	
HG b	-x/b	Fb-Fx	0	-Fx+Fx <sup>2</sup> /b	0	x <sup>2</sup> /b <sup>2</sup>			
HC b	0	0	0	0	0	0	0+0	0	
CH b	0	0	0	0	0	0			
FI b	0	-1/2Fb-1/4Fx	0	0	0	0	0+0	0	
IF b	0	3/4Fb-1/4Fx	0	0	0	0			
IJ b	0	1/4Fb-1/4Fx	0	0	0	0	0+0	0	
JI b	0	-1/4Fx	0	0	0	0			
JH b	0	Fx	0	0	0	0	0+0	0	
HJ b	0	-Fb+Fx	0	0	0	0			
	totali							-101/24Fb <sup>2</sup> /EJ	5/3Xb/EJ
	iperstatica $X=W_{GB}$							101/40Fb	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BG}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GB}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{GH}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{HG}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{x\theta} = \int_0^b (-9/4 + 11/4 x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-4 + 4x/b) \theta dx$$

$$= [-9/4 x + 11/8 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-4x + 2x^2/b]_0^b \theta$$

$$= (-9/4 b + 11/8 b - 1/6 b) Fb 1/EJ + (-4b + 2b) \theta = -73/24 Fb^2/EJ$$

$$L_{CB}^{x\theta} = \int_0^b (-7/4 x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (4x/b) \theta dx$$

$$= [-7/8 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [2x^2/b]_0^b \theta$$

$$= (-7/8 b - 1/6 b) Fb 1/EJ + (2b) \theta = -73/24 Fb^2/EJ$$

$$L_{BG}^{x\theta} = \int_0^b (-2 + 2x/b) Fb 1/EJ dx = [-2x + x^2/b]_0^b Fb 1/EJ$$

$$= (-2b + b) Fb 1/EJ = -Fb^2/EJ$$

$$L_{GB}^{x\theta} = \int_0^b (-2x/b) Fb 1/EJ dx = [-x^2/b]_0^b Fb 1/EJ$$

$$= (-b) Fb 1/EJ = -Fb^2/EJ$$

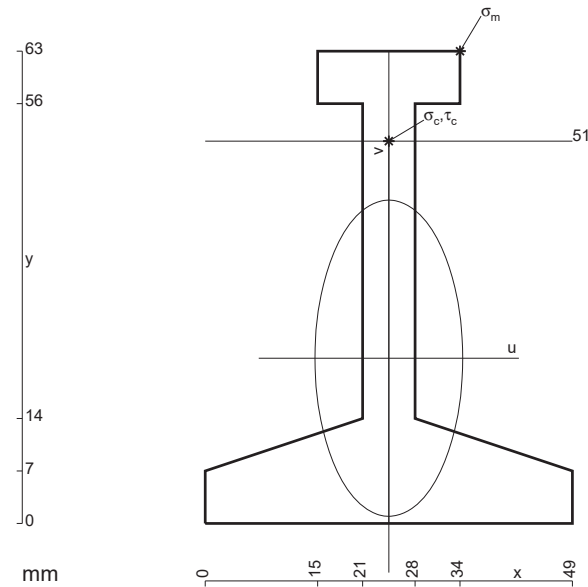
$$L_{GH}^{x\theta} = \int_0^b (-x/b + x^2/b^2) Fb 1/EJ dx = [-1/2 x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-1/2 b + 1/3 b) Fb 1/EJ = -1/6 Fb^2/EJ$$

$$L_{HG}^{x\theta} = \int_0^b (-x/b + x^2/b^2) Fb 1/EJ dx = [-1/2 x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-1/2 b + 1/3 b) Fb 1/EJ = -1/6 Fb^2/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 966. \text{ mm}^2$$

$$J_u = 429935. \text{ mm}^4$$

$$J_v = 93839. \text{ mm}^4$$

$$y_g = 22.04 \text{ mm}$$

$$T_y = -3185. \text{ N}$$

$$M_x = 2197650. \text{ Nmm}$$

$$x_m = 34. \text{ mm}$$

$$y_m = 63. \text{ mm}$$

$$u_m = 9.5 \text{ mm}$$

$$v_m = 40.96 \text{ mm}$$

$$\sigma_m = -Mv/J_u = -209.4 \text{ N/mm}^2$$

$$x_c = 24.5 \text{ mm}$$

$$y_c = 51. \text{ mm}$$

$$v_c = 28.96 \text{ mm}$$

$$\sigma_c = -Mv/J_u = -148. \text{ N/mm}^2$$

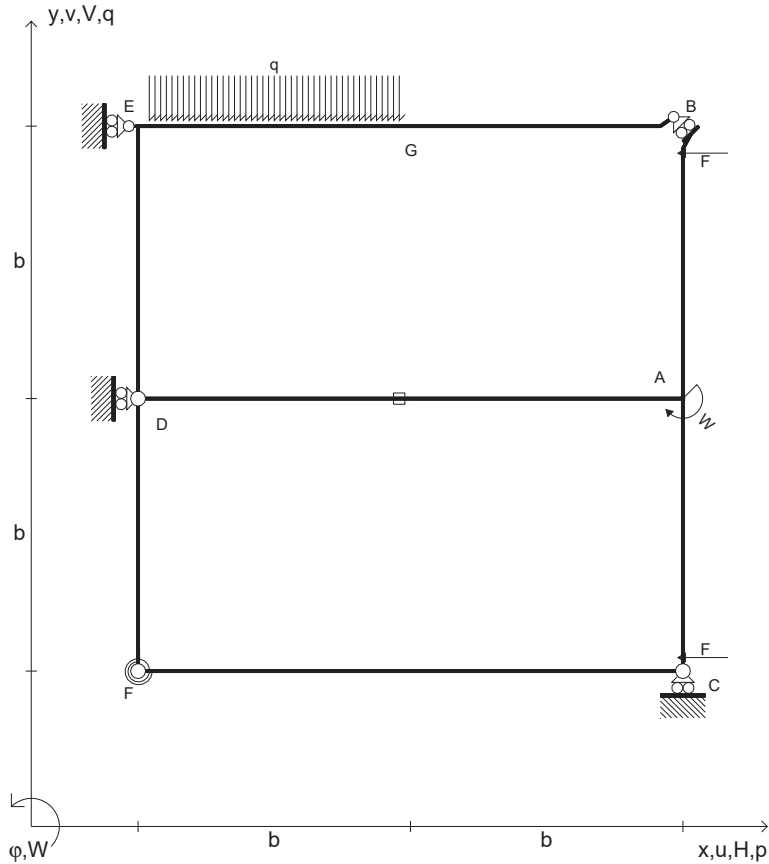
$$\tau_c = 6.438 \text{ N/mm}^2$$

$$\sigma_\theta = \sqrt{\sigma^2 + 3\tau^2} = 148.5 \text{ N/mm}^2$$

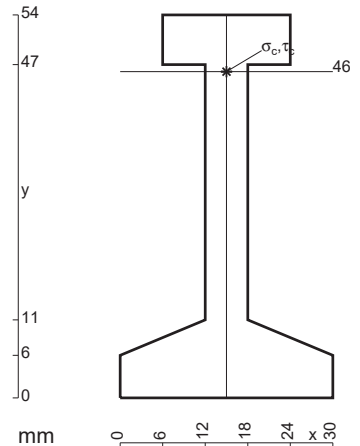
$$S = 6083. \text{ mm}^3$$

TEMA 02

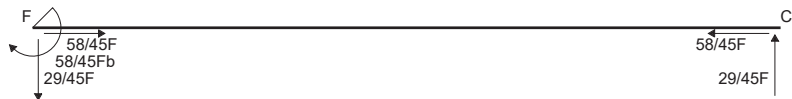
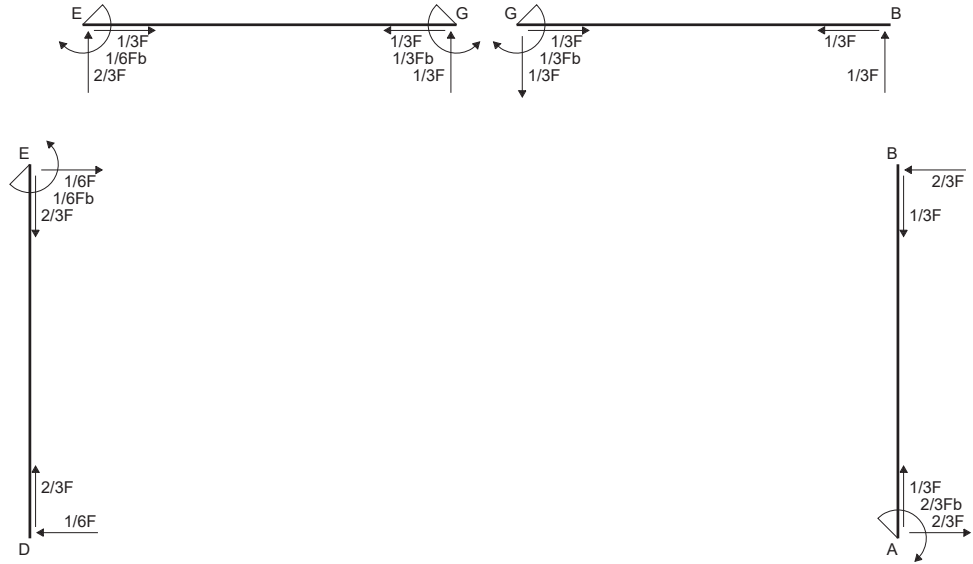
$H_{BA} = -F$   
 $H_{CA} = -F$   
 $W_A = -W = -Fb$   
 $q_{GE} = -q = -F/b$   
 $\varepsilon_{AD} = -\alpha T = -b^2 F/EJ$   
 $k_{FD} = 2EJ/b$   
 $EJ_{AB} = EJ$   
 $EJ_{CA} = EJ$   
 $EJ_{DE} = EJ$   
 $EJ_{FD} = EJ$   
 $EJ_{GE} = EJ$   
 $EJ_{BG} = EJ$   
 $EJ_{AD} = EJ$   
 $EJ_{CF} = EJ$



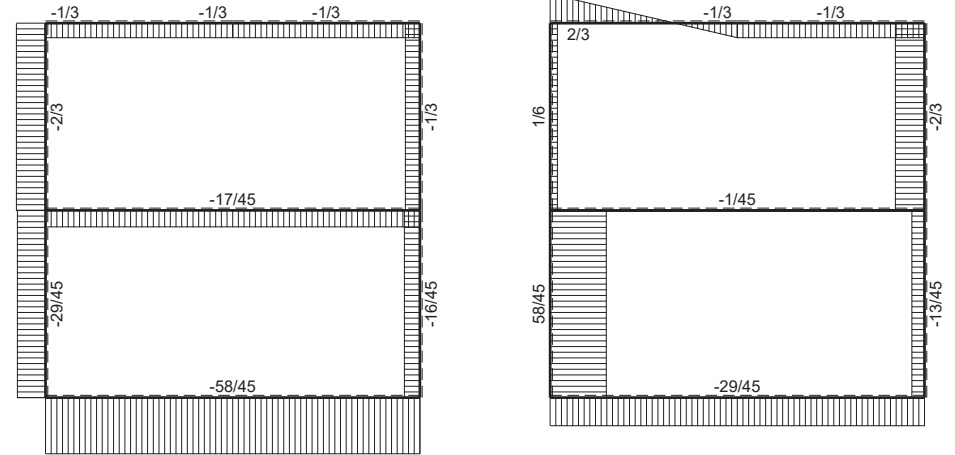
Reazioni iperstatiche in soluzione:  $X=W_{FC}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 Piano di scorrimento del vincolo con inclinazione assegnata.  
 La trave AB ha la sezione riportata e dimensioni in mm, con:  
 $b = 510 \text{ mm}$ ,  $F = 4490 \text{ N}$   
 Calcolare sulla sezione A la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da A a B  
 Elongazione termica specifica  $\varepsilon$  assegnata su asta AD.



# REAZIONI

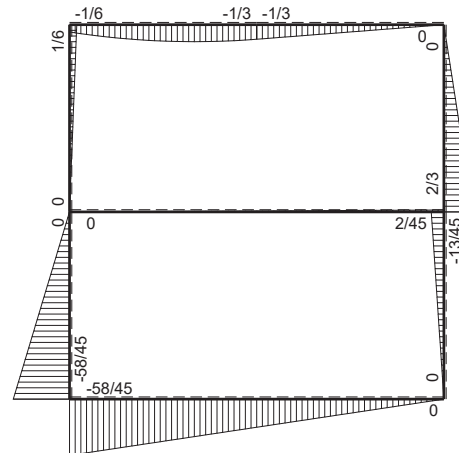


# AZIONI INTERNE

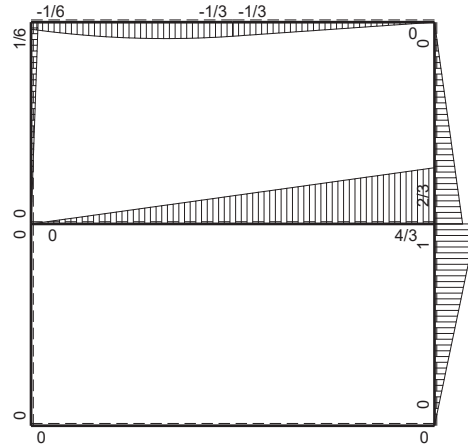
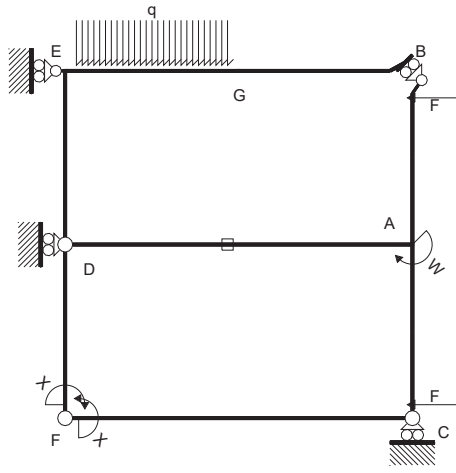


← ⊕ → F

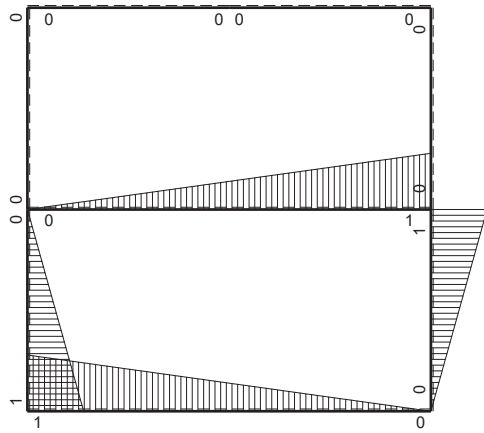
↑ ⊕ ↓ F



⊕ ⊖ F<sub>b</sub>



$\left(\begin{smallmatrix} + \\ - \end{smallmatrix}\right) M_0$  flessione da carichi assegnati



$\left(\begin{smallmatrix} + \\ - \end{smallmatrix}\right) M_x$  flessione da iperstatica  $X=1$



Quadro contributi PLV per iperstatica  $X=W_{FC}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	$2/3Fb - 2/3Fx$	0	0	0	0
BA b	0	$-2/3Fx$	0	0		
CA b	$x/b$	$Fx$	$Fx^2/b$	$x^2/b^2$	$1/3Fb^2/EJ$	$1/3Xb/EJ$
AC b	$-1+x/b$	$-Fb+Fx$	$Fb-2Fx+Fx^2/b$	$1-2x/b+x^2/b^2$		
DE b	0	$1/6Fx$	0	0	0	0
ED b	0	$-1/6Fb+1/6Fx$	0	0		
FD b	$1-x/b$	0	0	$1-2x/b+x^2/b^2$	0	$1/3Xb/EJ$
DF b	$-x/b$	0	0	$x^2/b^2$		
GE b	0	$-1/3Fb-1/3Fx+1/2qx^2$	0	0	0	0
EG b	0	$1/6Fb+2/3Fx-1/2qx^2$	0	0		
BG b	0	$-1/3Fx$	0	0	0	0
GB b	0	$1/3Fb-1/3Fx$	0	0		
AD 2b	$1-1/2x/b$	$4/3Fb-2/3Fx$	$4/3Fb-4/3Fx+1/3Fx^2/b$	$1-x/b+1/4x^2/b^2$	$8/9Fb^2/EJ$	$2/3Xb/EJ$
DA 2b	$-1/2x/b$	$-2/3Fx$	$1/3Fx^2/b$	$1/4x^2/b^2$		
CF 2b	$1/2x/b$	0	0	$1/4x^2/b^2$	0	$2/3Xb/EJ$
FC 2b	$-1+1/2x/b$	0	0	$1-x/b+1/4x^2/b^2$		
FD	molla asta $-W_{1FD}(W_{oFD}+XW_{1FD})/k_{FD}$					$1/2Xb/EJ$
AD	elongazione asta $N_{1AD}\epsilon_{AD}L_{AD}$				$2Fb^2/EJ$	
	totali				$29/9Fb^2/EJ$	$5/2Xb/EJ$
	iperstatica $X=W_{FC}$				$-58/45Fb$	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{CA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{AC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{FD}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx - 1 (-1) 1/2 b/EJ = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ - 1 (-1) 1/2 b/EJ$$

$$= (b - b + 1/3 b) 1/EJ - 1 (-1) 1/2 b/EJ = 5/6 b/EJ$$

$$L_{DF}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx - 1 (-1) 1/2 b/EJ = [1/3 x^3/b^2]_0^b 1/EJ - 1 (-1) 1/2 b/EJ$$

$$= (1/3 b) 1/EJ - 1 (-1) 1/2 b/EJ = 5/6 b/EJ$$

$$L_{AD}^{xx} = \int_0^{2b} (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^{2b} 1/EJ$$

$$= (2b - 2b + 2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{DA}^{xx} = \int_0^{2b} (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^{2b} 1/EJ$$

$$= (2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{CF}^{xx} = \int_0^{2b} (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^{2b} 1/EJ$$

$$= (2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{FC}^{xx} = \int_0^{2b} (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^{2b} 1/EJ$$

$$= (2b - 2b + 2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{CA}^{xo} = \int_0^b (x^2/b^2) Fb 1/EJ dx = [1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{AC}^{xo} = \int_0^b (1 - 2x/b + x^2/b^2) Fb 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b - b + 1/3 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{AD}^{xo} = \int_0^{2b} (4/3 - 4/3 x/b + 1/3 x^2/b^2) Fb 1/EJ dx + 1 (-1) (-2) Fb^2/EJ$$

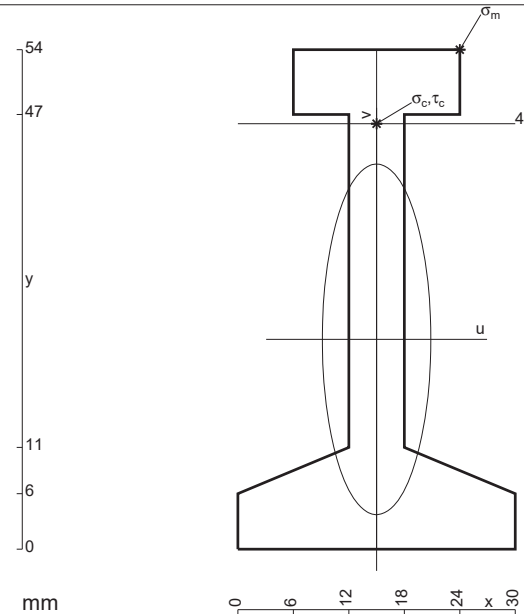
$$= [4/3 x - 2/3 x^2/b + 1/9 x^3/b^2]_0^{2b} Fb 1/EJ + 1 (-1) (-2) Fb^2/EJ$$

$$= (8/3 b - 8/3 b + 8/9 b) Fb 1/EJ + 1 (-1) (-2) Fb^2/EJ = 26/9 Fb^2/EJ$$

$$L_{DA}^{xo} = \int_0^{2b} (1/3 x^2/b^2) Fb 1/EJ dx + 1 (-1) (-2) Fb^2/EJ = [1/9 x^3/b^2]_0^{2b} Fb 1/EJ + 1 (-1) (-2) Fb^2/EJ$$

$$= (8/9 b) Fb 1/EJ + 1 (-1) (-2) Fb^2/EJ = 26/9 Fb^2/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 612. \text{ mm}^2$$

$$J_u = 219945. \text{ mm}^4$$

$$J_v = 21060. \text{ mm}^4$$

$$y_g = 22.68 \text{ mm}$$

$$N = -1497. \text{ N}$$

$$T_y = -2993. \text{ N}$$

$$M_x = 1526600. \text{ Nmm}$$

$$x_m = 24. \text{ mm}$$

$$y_m = 54. \text{ mm}$$

$$u_m = 9. \text{ mm}$$

$$v_m = 31.32 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = -219.8 \text{ N/mm}^2$$

$$x_c = 15. \text{ mm}$$

$$y_c = 46. \text{ mm}$$

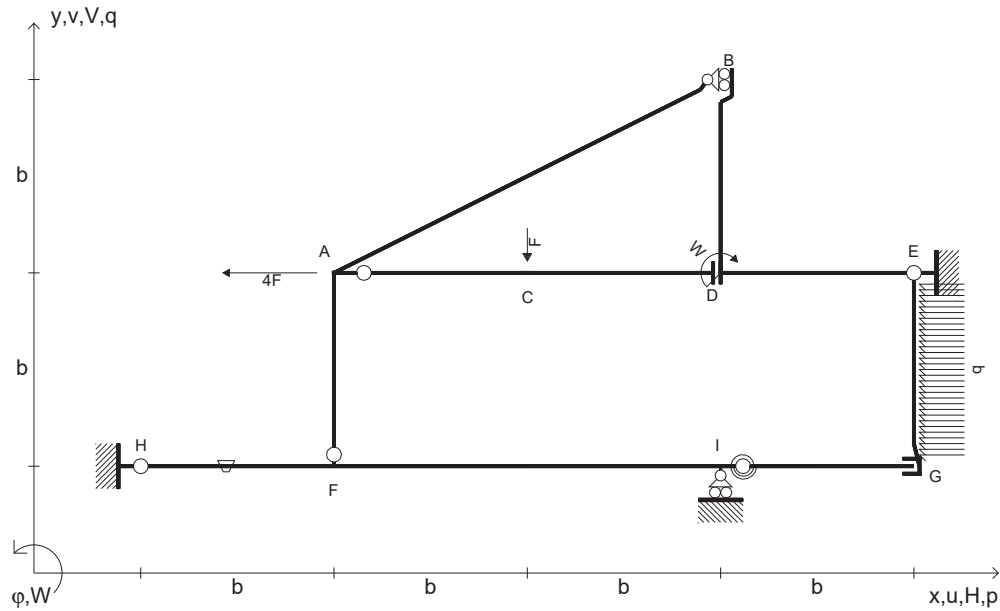
$$v_c = 23.32 \text{ mm}$$

$$\sigma_c = N/A - Mv/J_u = -164.3 \text{ N/mm}^2$$

$$\tau_c = 8.274 \text{ N/mm}^2$$

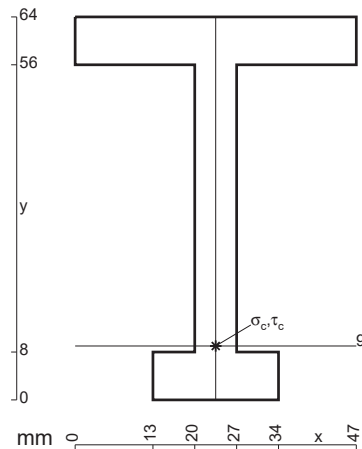
$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 164.9 \text{ N/mm}^2$$

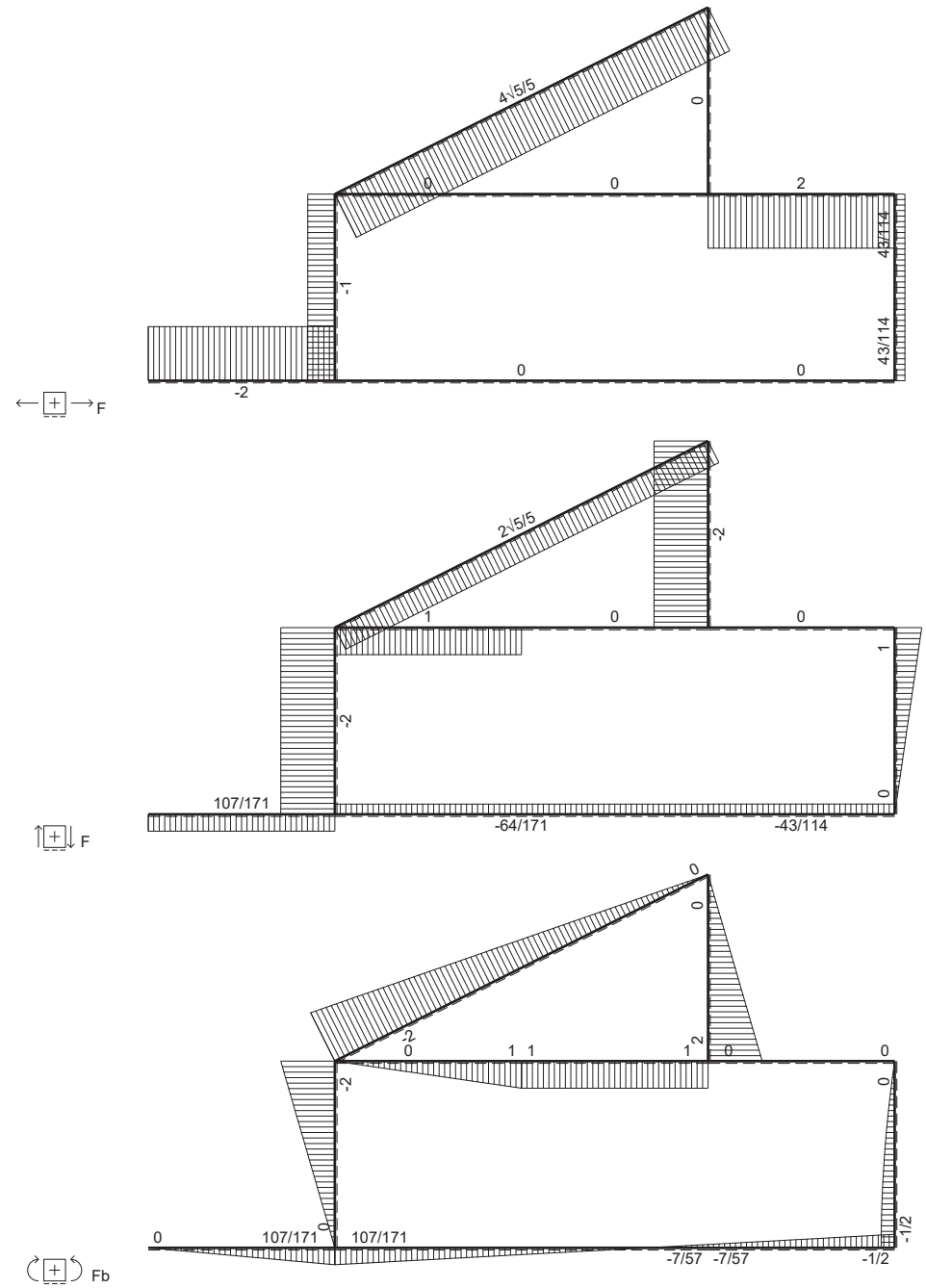
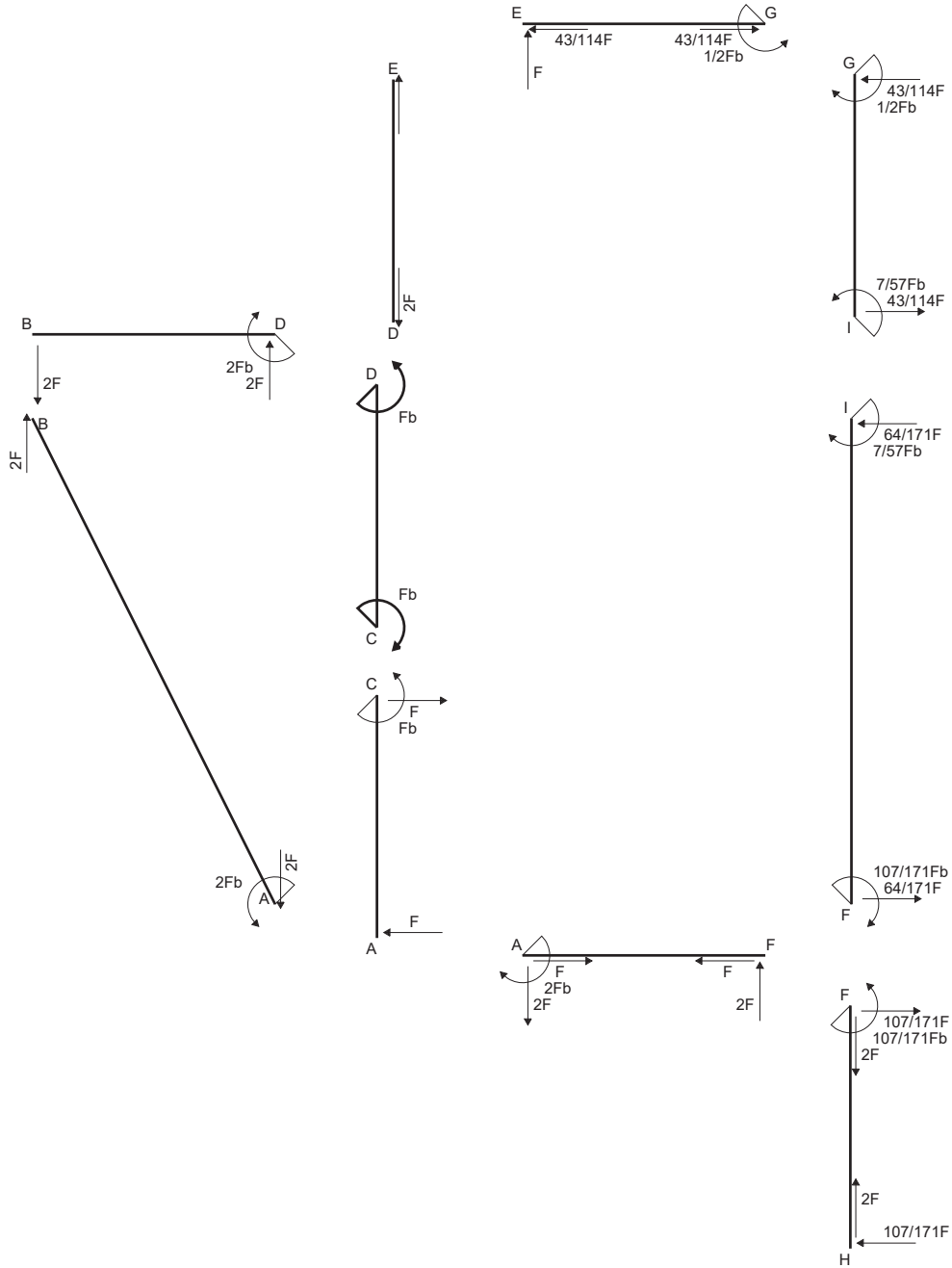
$$S = 3648. \text{ mm}^3$$

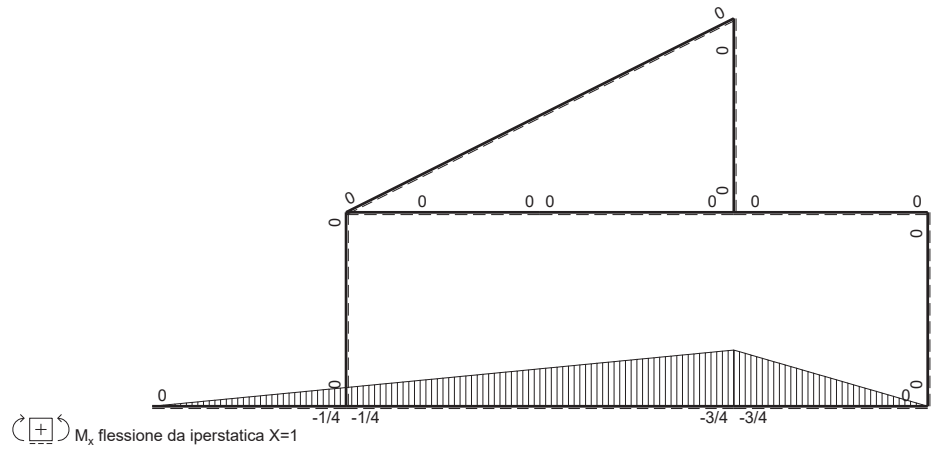
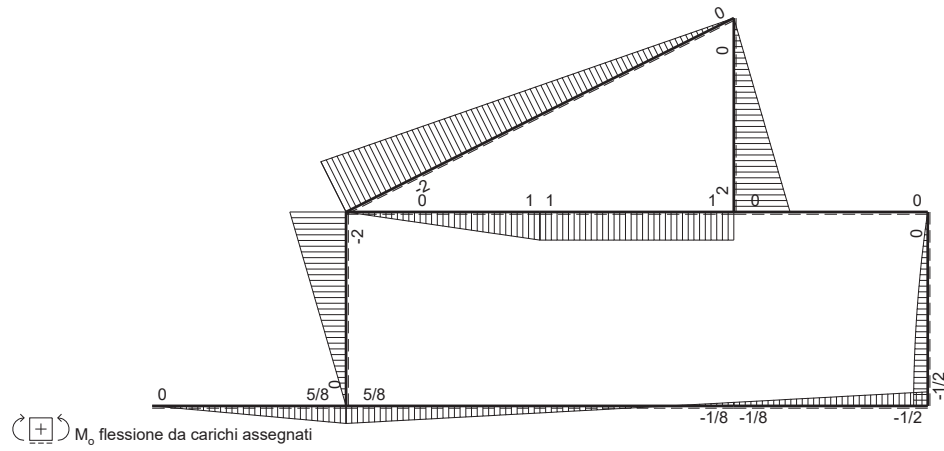
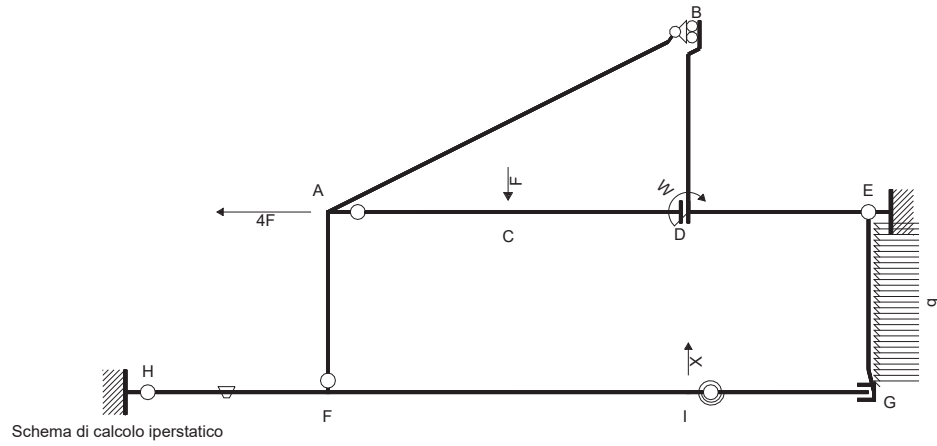


$H_A = -4F$	$EJ_{AB} = EJ$	$EJ_{GE} = EJ$
$V_C = -F$	$EJ_{AC} = EJ$	$EJ_{HF} = EJ$
$W_D = -W = -Fb$	$EJ_{CD} = EJ$	$EJ_{FI} = EJ$
$p_{GE} = -q = -F/b$	$EJ_{DB} = EJ$	$EJ_{IG} = EJ$
$\theta_{HF} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{DE} = EJ$	
$K_{IG} = 4EJ/b$	$EJ_{FA} = EJ$	

Reazioni iperstatiche in soluzione:  $X=V_I$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 Carichi di aste curve misurati in proiezione sugli assi  $x, y$ .  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave FA ha la sezione riportata e dimensioni in mm, con:  
 $b = 620$  mm,  $F = 2060$  N  
 Calcolare sulla sezione A la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da F a A  
 Curvatura  $\theta$  asta HF positiva se convessa a destra con inizio H.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13







Quadro contributi PLV per iperstatica  $X=V_1$

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB $\sqrt{5}b$	0	$-2Fb+2\sqrt{5/5}Fx$	0	0	0	0	0	0
AC b	0	Fx	0	0	0	0	0+0	0
CA b	0	$-Fb+Fx$	0	0	0	0	0+0	0
CD b	0	Fb	0	0	0	0	0+0	0
DC b	0	$-Fb$	0	0	0	0	0+0	0
DB b	0	$2Fb-2Fx$	0	0	0	0	0+0	0
BD b	0	$-2Fx$	0	0	0	0	0+0	0
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0	0+0	0
FA b	0	$-2Fx$	0	0	0	0	0+0	0
AF b	0	$2Fb-2Fx$	0	0	0	0	0+0	0
GE b	0	$-1/2Fb+1/2qx^2$	0	0	0	0	0+0	0
EG b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
HF b	$-1/4x$	$5/8Fx$	$-Fb/EJ$	$-5/32Fx^2$	$1/4Fxb/EJ$	$1/16x^2$	$(-5/96+1/8)Fb^3/EJ$	$1/48Xb^3/EJ$
FH b	$1/4b-1/4x$	$-5/8Fb+5/8Fx$	$Fb/EJ$	$-5/32Fb^2+5/16Fbx-5/32Fx^2$	$1/4Fb^2/EJ-1/4Fxb/EJ$	$1/16b^2-1/8bx+1/16x^2$	$(-3/16+0)Fb^3/EJ$	$13/24Xb^3/EJ$
FI 2b	$-1/4b-1/4x$	$5/8Fb-3/8Fx$	0	$-5/32Fb^2-1/16Fbx+3/32Fx^2$	0	$1/16b^2+1/8bx+1/16x^2$	$(3/32+0)Fb^3/EJ$	$3/16Xb^3/EJ$
IF 2b	$3/4b-1/4x$	$1/8Fb-3/8Fx$	0	$3/32Fb^2-5/16Fbx+3/32Fx^2$	0	$9/16b^2-3/8bx+1/16x^2$	$(3/32+0)Fb^3/EJ$	$3/16Xb^3/EJ$
IG b	$-3/4b+3/4x$	$-1/8Fb-3/8Fx$	0	$3/32Fb^2+3/16Fbx-9/32Fx^2$	0	$9/16b^2-9/8bx+9/16x^2$	$(3/32+0)Fb^3/EJ$	$3/16Xb^3/EJ$
GI b	$3/4x$	$1/2Fb-3/8Fx$	0	$3/8Fbx-9/32Fx^2$	0	$9/16x^2$	$(3/32+0)Fb^3/EJ$	$3/16Xb^3/EJ$
IG	molla asta $-W_{1IG}(W_{oIG}+XW_{1IG})/k_{1IG}$						$3/128Fb^3/EJ$	$9/64Xb^3/EJ$
	totali						$1/384Fb^3/EJ$	$57/64Xb^3/EJ$
	iperstatica $X=V_1$						$-1/342F$	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{HF}^{xx} = \int_0^b (1/16 x^2/b^2) b^2 1/EJ dx = [1/48 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/48 b) b^2 1/EJ = 1/48 b^3/EJ$$

$$L_{FH}^{xx} = \int_0^b (1/16 - 1/8 x/b + 1/16 x^2/b^2) b^2 1/EJ dx = [1/16 x - 1/16 x^2/b + 1/48 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/16 b - 1/16 b + 1/48 b) b^2 1/EJ = 1/48 b^3/EJ$$

$$L_{FI}^{xx} = \int_0^{2b} (1/16 + 1/8 x/b + 1/16 x^2/b^2) b^2 1/EJ dx = [1/16 x + 1/16 x^2/b + 1/48 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (1/8 b + 1/4 b + 1/6 b) b^2 1/EJ = 13/24 b^3/EJ$$

$$L_{IF}^{xx} = \int_0^{2b} (9/16 - 3/8 x/b + 1/16 x^2/b^2) b^2 1/EJ dx = [9/16 x - 3/16 x^2/b + 1/48 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (9/8 b - 3/4 b + 1/6 b) b^2 1/EJ = 13/24 b^3/EJ$$

$$L_{IG}^{xx} = \int_0^b (9/16 - 9/8 x/b + 9/16 x^2/b^2) b^2 1/EJ dx + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ$$

$$= [9/16 x - 9/16 x^2/b + 3/16 x^3/b^2]_0^b b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ$$

$$= (9/16 b - 9/16 b + 3/16 b) b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ = 21/64 b^3/EJ$$

$$L_{GI}^{xx} = \int_0^b (9/16 x^2/b^2) b^2 1/EJ dx + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ = [3/16 x^3/b^2]_0^b b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ$$

$$= (3/16 b) b^2 1/EJ + 3/4 \cdot 3/4 \cdot 1/4 b^3/EJ = 21/64 b^3/EJ$$

$$L_{HF}^{x\theta} = \int_0^b (-5/32 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1/4 x/b) \theta dx = [-5/96 x^3/b^2]_0^b Fb^2 1/EJ + [1/8 x^2/b]_0^b \theta$$

$$= (-5/96 b) Fb^2 1/EJ + (1/8 b) \theta = 7/96 Fb^3/EJ$$

$$L_{FH}^{x\theta} = \int_0^b (-5/32 + 5/16 x/b - 5/32 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-1/4 + 1/4 x/b) \theta dx$$

$$= [-5/32 x + 5/32 x^2/b - 5/96 x^3/b^2]_0^b Fb^2 1/EJ + [-1/4 x + 1/8 x^2/b]_0^b \theta$$

$$= (-5/32 b + 5/32 b - 5/96 b) Fb^2 1/EJ + (-1/4 b + 1/8 b) \theta = 7/96 Fb^3/EJ$$

$$L_{FI}^{x\theta} = \int_0^{2b} (-5/32 - 1/16 x/b + 3/32 x^2/b^2) Fb^2 1/EJ dx = [-5/32 x - 1/32 x^2/b + 1/32 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-5/16 b - 1/8 b + 1/4 b) Fb^2 1/EJ = -3/16 Fb^3/EJ$$

$$L_{IF}^{x\theta} = \int_0^{2b} (3/32 - 5/16 x/b + 3/32 x^2/b^2) Fb^2 1/EJ dx = [3/32 x - 5/32 x^2/b + 1/32 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (3/16 b - 5/8 b + 1/4 b) Fb^2 1/EJ = -3/16 Fb^3/EJ$$

$$L_{IG}^{x\theta} = \int_0^b (3/32 + 3/16 x/b - 9/32 x^2/b^2) Fb^2 1/EJ dx + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

$$= [3/32 x + 3/32 x^2/b - 3/32 x^3/b^2]_0^b Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

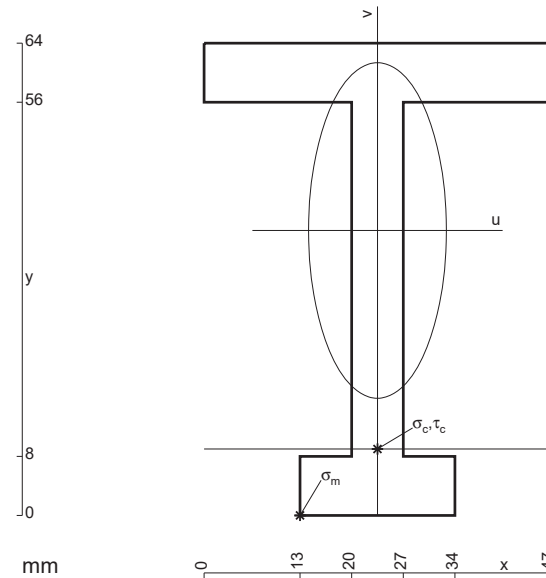
$$= (3/32 b + 3/32 b - 3/32 b) Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ = 15/128 Fb^3/EJ$$

$$L_{GI}^{x\theta} = \int_0^b (3/8 x/b - 9/32 x^2/b^2) Fb^2 1/EJ dx + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

$$= [3/16 x^2/b - 3/32 x^3/b^2]_0^b Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ$$

$$= (3/16 b - 3/32 b) Fb^2 1/EJ + 3/4 \cdot 1/8 \cdot 1/4 Fb^3/EJ = 15/128 Fb^3/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 880. \text{ mm}^2$$

$$J_u = 455365. \text{ mm}^4$$

$$J_v = 76761. \text{ mm}^4$$

$$y_g = 38.62 \text{ mm}$$

$$N = -2060. \text{ N}$$

$$T_y = -4120. \text{ N}$$

$$M_x = -2554400. \text{ Nmm}$$

$$x_m = 13. \text{ mm}$$

$$u_m = -10.5 \text{ mm}$$

$$v_m = -38.62 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = -219. \text{ N/mm}^2$$

$$x_c = 23.5 \text{ mm}$$

$$y_c = 9. \text{ mm}$$

$$v_c = -29.62 \text{ mm}$$

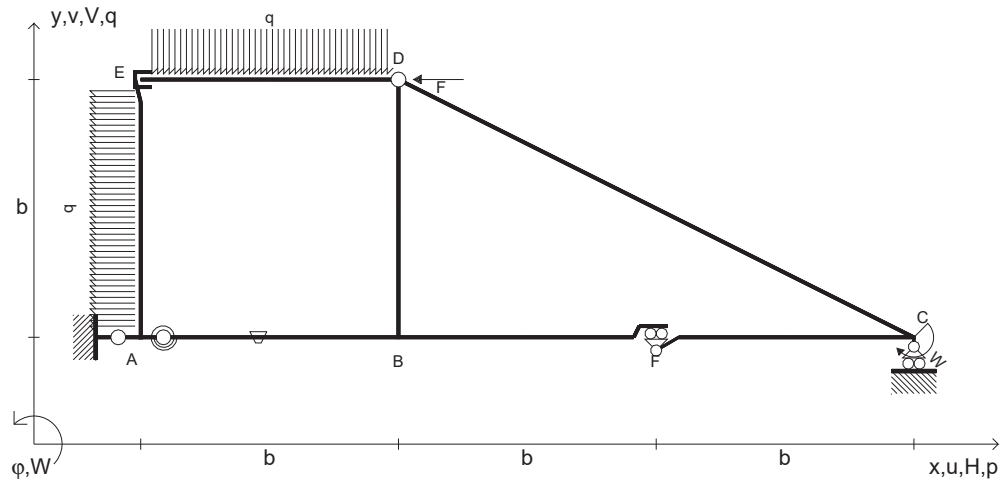
$$\sigma_c = N/A - Mv/J_u = -168.5 \text{ N/mm}^2$$

$$\tau_c = 7.79 \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 169. \text{ N/mm}^2$$

$$S = 6027. \text{ mm}^3$$

TEMA 04



$H_D = -F$	$K_{AB} = 3/2EJ/b$	$EJ_{BD} = EJ$
$W_C = -W = -Fb$	$EJ_{AB} = EJ$	$EJ_{BF} = EJ$
$P_{EA} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{FC} = EJ$
$q_{DE} = -q = -F/b$	$EJ_{DE} = EJ$	
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EA} = EJ$	

Reazioni iperstatiche in soluzione:  $X=W_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

La trave CD ha la sezione riportata e dimensioni in mm, con:

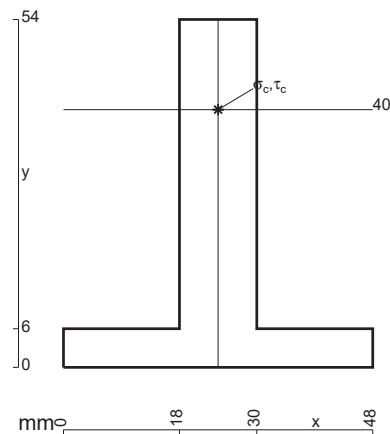
$b = 490 \text{ mm}$ ,  $F = 1620 \text{ N}$

Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .

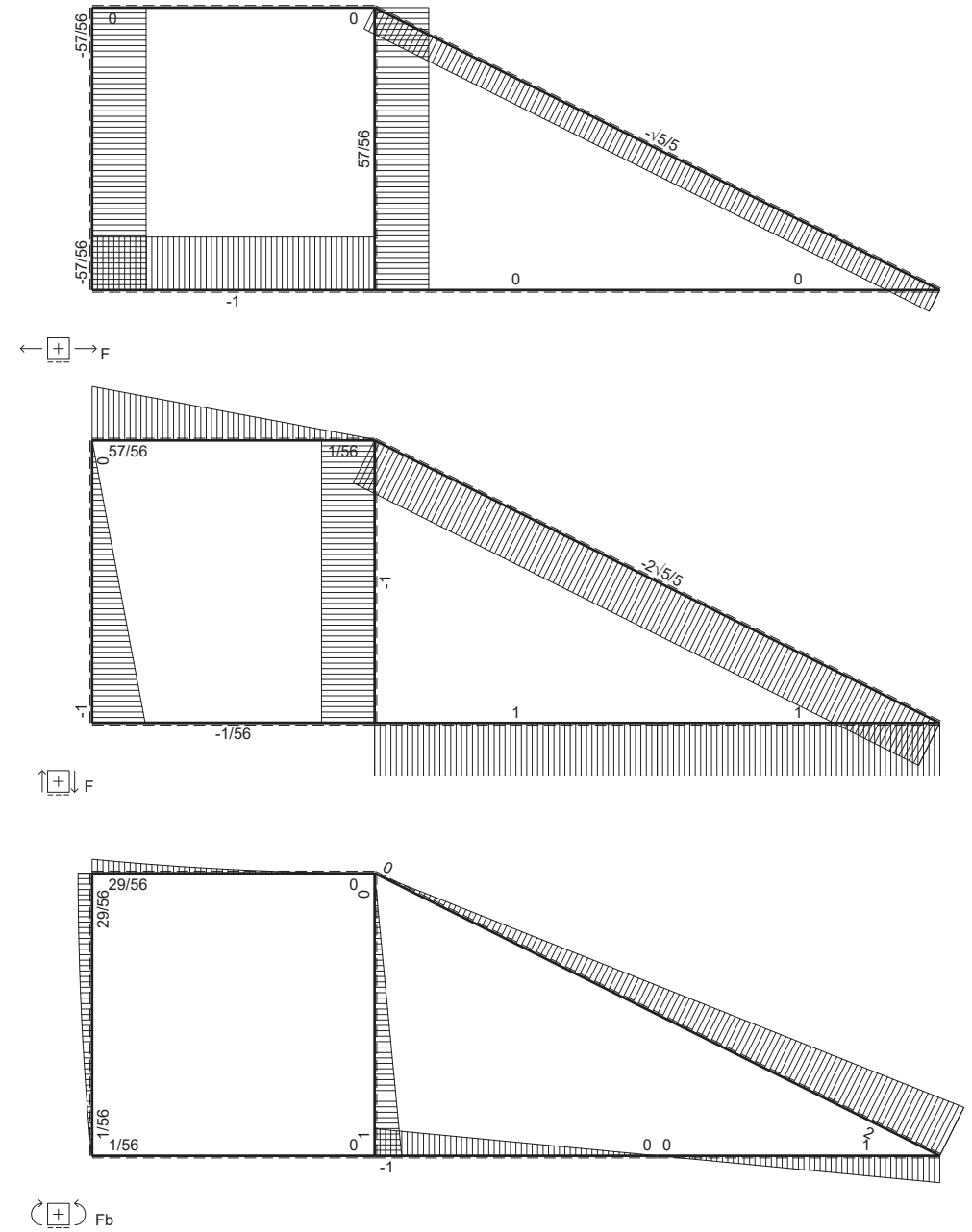
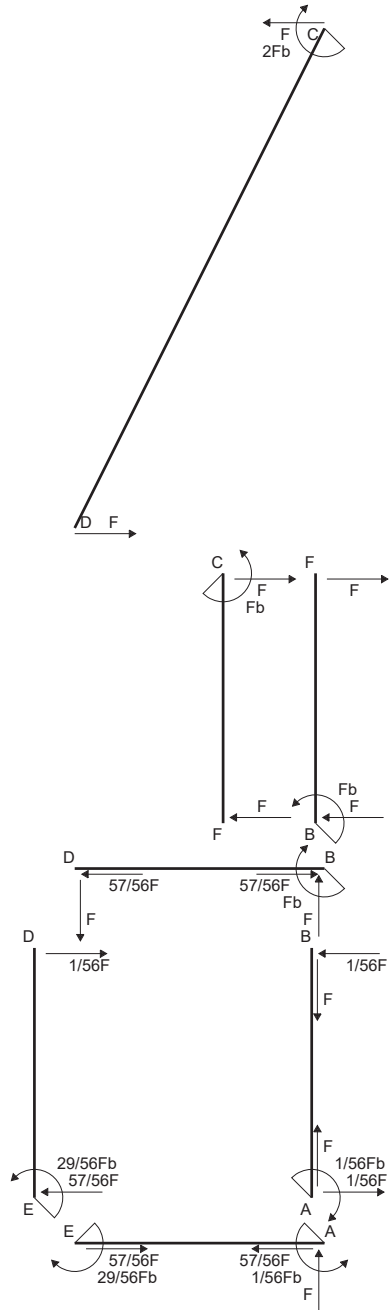
Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

Leombo inferiore sezione su tratteggio trave, a destra da C a D  
Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.

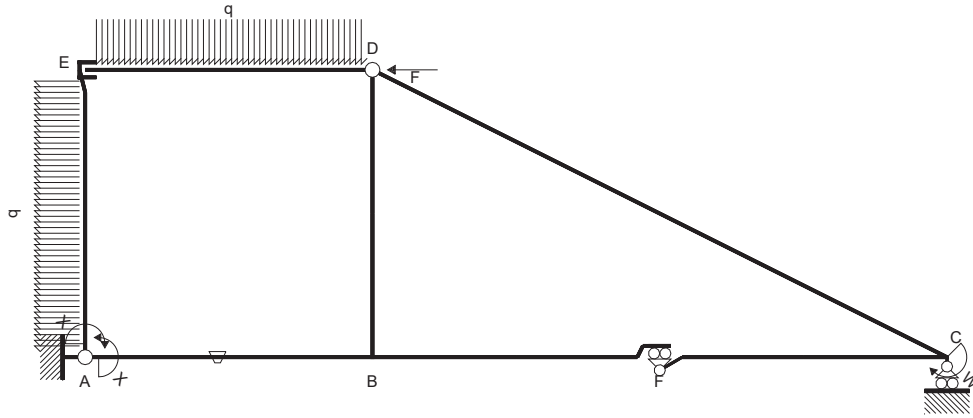
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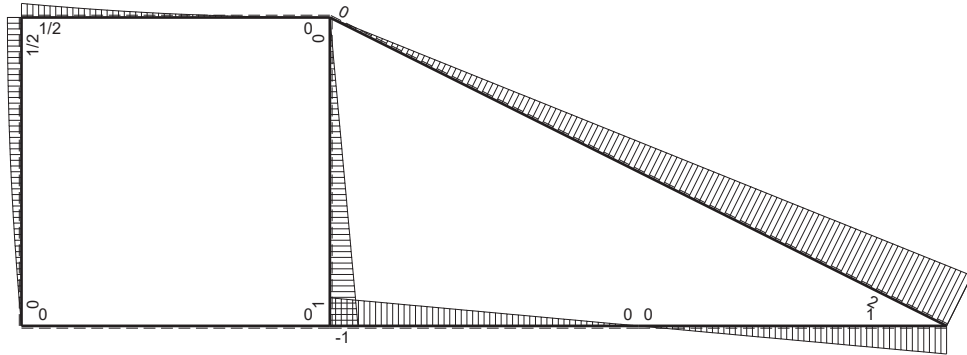




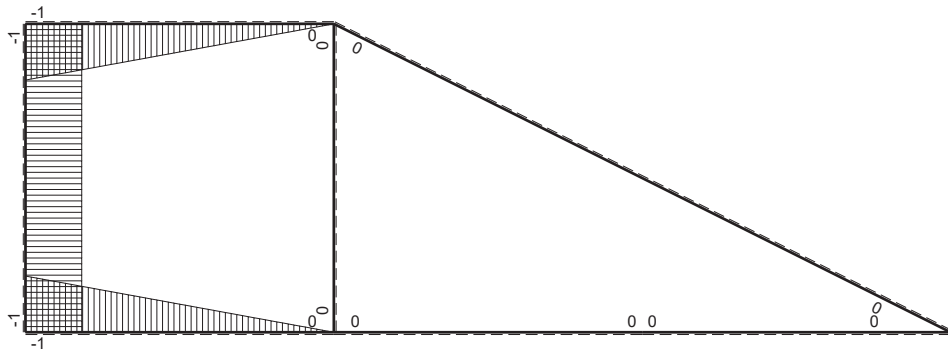
PROCEDIMENTO E RISULTATI



Schema di calcolo iperstatico



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI

Quadro contributi PLV per iperstatica X=W<sub>AB</sub>

→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ+\theta) dx$	$\int X M_x M_x / EJ dx$
AB b	$-1+x/b$	0	$-Fb/EJ$	0	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$	$(0+1/2)Fb^2/EJ$	$1/3Xb/EJ$
BA b	$x/b$	0	$Fb/EJ$	0	$Fx/EJ$	$x^2/b^2$	0	0
CD $\sqrt{5}b$	0	$2Fb-2\sqrt{5}Fx$	0	0	0	0	0	0
DE b	$-x/b$	$1/2qx^2$	0	$-1/2qx^3/b$	0	$x^2/b^2$	$(-1/8+0)Fb^2/EJ$	$1/3Xb/EJ$
ED b	$1-x/b$	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb+3/2Fx-3/2Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(-1/3+0)Fb^2/EJ$	$Xb/EJ$
EA b	-1	$1/2Fb-1/2qx^2$	0	$-1/2Fb+1/2Fx^2/b$	0	1	$(-1/3+0)Fb^2/EJ$	0
AE b	1	$-Fx+1/2qx^2$	0	$-Fx+1/2Fx^2/b$	0	1	0+0	0
BD b	0	$Fb-Fx$	0	0	0	0	0+0	0
DB b	0	$-Fx$	0	0	0	0	0+0	0
BF b	0	$-Fb+Fx$	0	0	0	0	0+0	0
FB b	0	$Fx$	0	0	0	0	0+0	0
FC b	0	$Fx$	0	0	0	0	0+0	0
CF b	0	$-Fb+Fx$	0	0	0	0	0+0	0
AB	molla asta $-W_{1AB}(W_{0AB}+XW_{1AB})/k_{AB}$							$2/3Xb/EJ$
	totali						$1/24Fb^2/EJ$	$7/3Xb/EJ$
	iperstatica X=W <sub>AB</sub>						$-1/56Fb$	

## PROCEDIMENTO E RISULTATI

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx + 1 \cdot 1/3 \cdot b/EJ = \left[ x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ + 1 \cdot 1/3 \cdot b/EJ$$

$$= (b - b + 1/3 b) 1/EJ + 1 \cdot 1/3 \cdot b/EJ = b/EJ$$

$$L_{BA}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx + 1 \cdot 1/3 \cdot b/EJ = \left[ 1/3 x^3/b^2 \right]_0^b 1/EJ + 1 \cdot 1/3 \cdot b/EJ$$

$$= (1/3 b) 1/EJ + 1 \cdot 1/3 \cdot b/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = \left[ x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EA}^{xx} = \int_0^b (1) 1/EJ dx = \left[ x \right]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{AE}^{xx} = \int_0^b (1) 1/EJ dx = \left[ x \right]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1 - x/b) \theta dx + 1 \cdot 0 \cdot 2/3 \cdot Fb^2/EJ = \left[ x - 1/2 x^2/b \right]_0^b \theta + 1 \cdot 0 \cdot 2/3 \cdot Fb^2/EJ$$

$$= (b - 1/2 b) \theta + 1 \cdot 0 \cdot 2/3 \cdot Fb^2/EJ = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-x/b) \theta dx + 1 \cdot 0 \cdot 2/3 \cdot Fb^2/EJ = \left[ -1/2 x^2/b \right]_0^b \theta + 1 \cdot 0 \cdot 2/3 \cdot Fb^2/EJ$$

$$= (-1/2 b) \theta + 1 \cdot 0 \cdot 2/3 \cdot Fb^2/EJ = 1/2 Fb^2/EJ$$

$$L_{DE}^{xo} = \int_0^b (-1/2 x^3/b^3) Fb 1/EJ dx = \left[ -1/8 x^4/b^3 \right]_0^b Fb 1/EJ$$

$$= (-1/8 b) Fb 1/EJ = -1/8 Fb^2/EJ$$

$$L_{ED}^{xo} = \int_0^b (-1/2 + 3/2 x/b - 3/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx$$

$$= \left[ -1/2 x + 3/4 x^2/b - 1/2 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb 1/EJ$$

$$= (-1/2 b + 3/4 b - 1/2 b + 1/8 b) Fb 1/EJ = -1/8 Fb^2/EJ$$

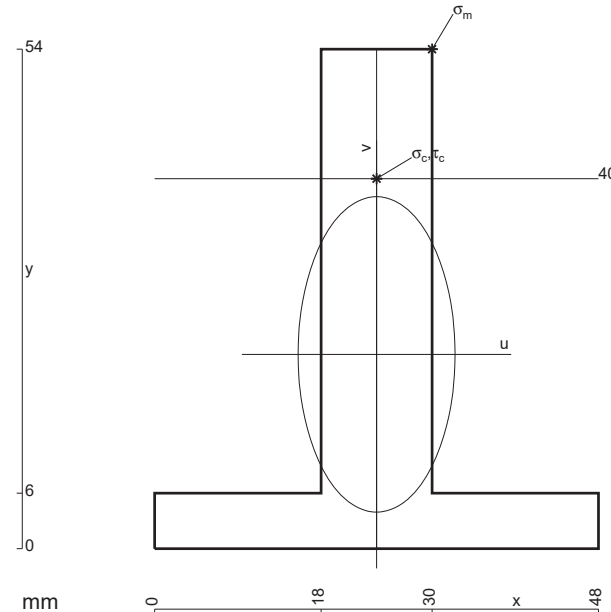
$$L_{EA}^{xo} = \int_0^b (-1/2 + 1/2 x^2/b^2) Fb 1/EJ dx = \left[ -1/2 x + 1/6 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-1/2 b + 1/6 b) Fb 1/EJ = -1/3 Fb^2/EJ$$

$$L_{AE}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2) Fb 1/EJ dx = \left[ -1/2 x^2/b + 1/6 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-1/2 b + 1/6 b) Fb 1/EJ = -1/3 Fb^2/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 864. \text{ mm}^2$$

$$J_u = 251424. \text{ mm}^4$$

$$J_v = 62208. \text{ mm}^4$$

$$y_g = 21. \text{ mm}$$

$$N = -724.5 \text{ N}$$

$$T_y = -1449. \text{ N}$$

$$M_x = 1587600. \text{ Nmm}$$

$$x_m = 30. \text{ mm}$$

$$y_m = 54. \text{ mm}$$

$$u_m = 6. \text{ mm}$$

$$v_m = 33. \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = -209.2 \text{ N/mm}^2$$

$$x_c = 24. \text{ mm}$$

$$y_c = 40. \text{ mm}$$

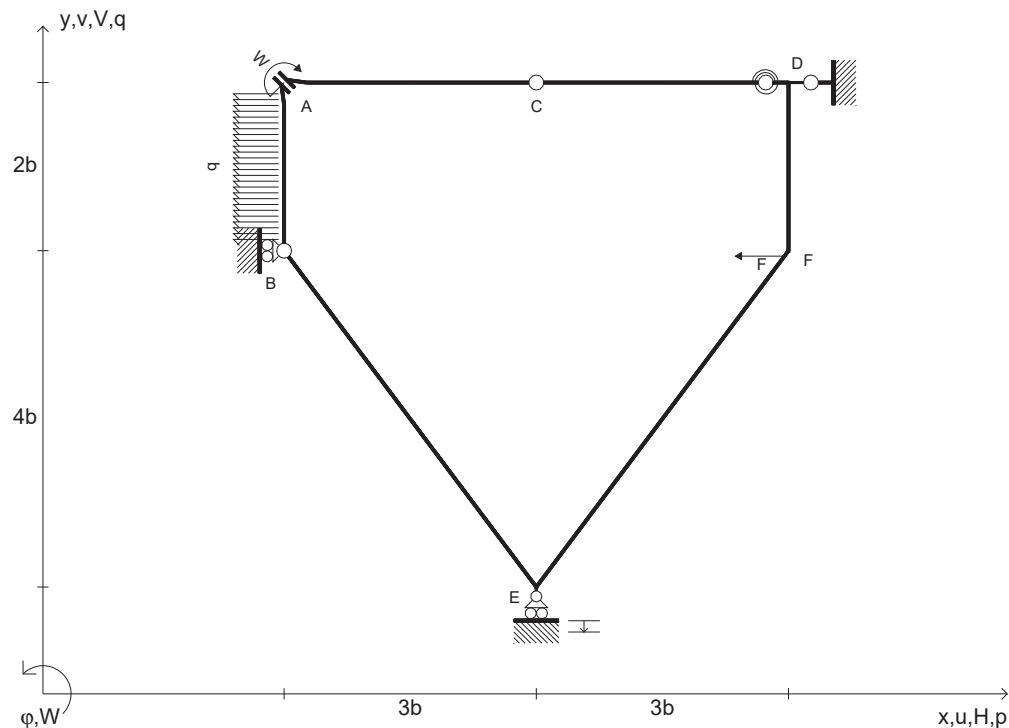
$$v_c = 19. \text{ mm}$$

$$\sigma_c = N/A - Mv/J_u = -120.8 \text{ N/mm}^2$$

$$\tau_c = 2.098 \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 120.9 \text{ N/mm}^2$$

$$S = 4368. \text{ mm}^3$$



$H_{FE} = -F$	$k_{DC} = 4EJ/b$	$EJ_{EF} = EJ$
$W_A = -W = -Fb$	$EJ_{AB} = EJ$	$EJ_{BE} = EJ$
$p_{AB} = -q = -F/b$	$EJ_{CD} = EJ$	$EJ_{DF} = EJ$
$v_E = -\delta = -b^3F/EJ$	$EJ_{AC} = EJ$	



Reazioni iperstatiche in soluzione:  $X=H_B$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

$J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Piano di scorrimento del vincolo con inclinazione assegnata.

La trave AB ha la sezione riportata e dimensioni in mm, con:

$b = 190 \text{ mm}$ ,  $F = 4990 \text{ N}$

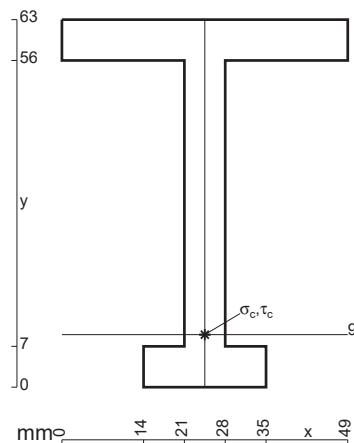
Calcolare sulla sezione mediana la massima tensione normale  $\sigma_m$

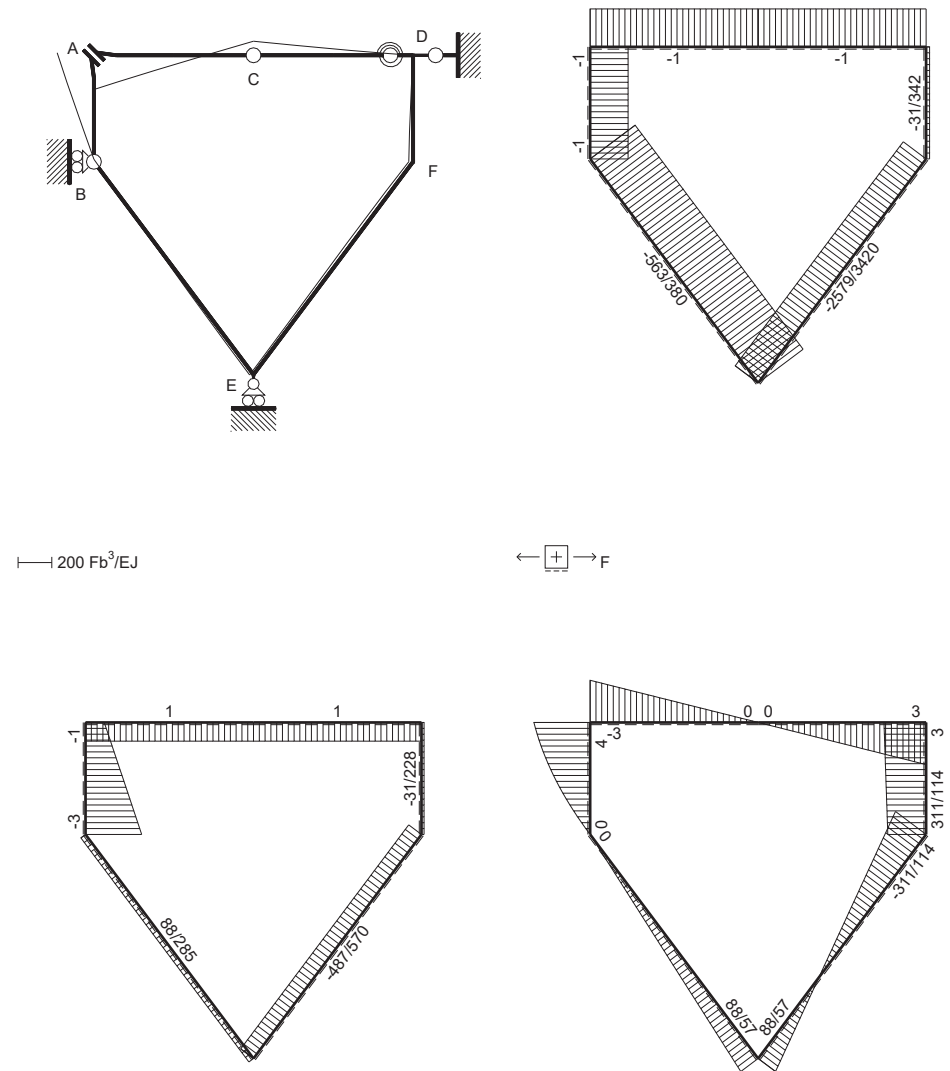
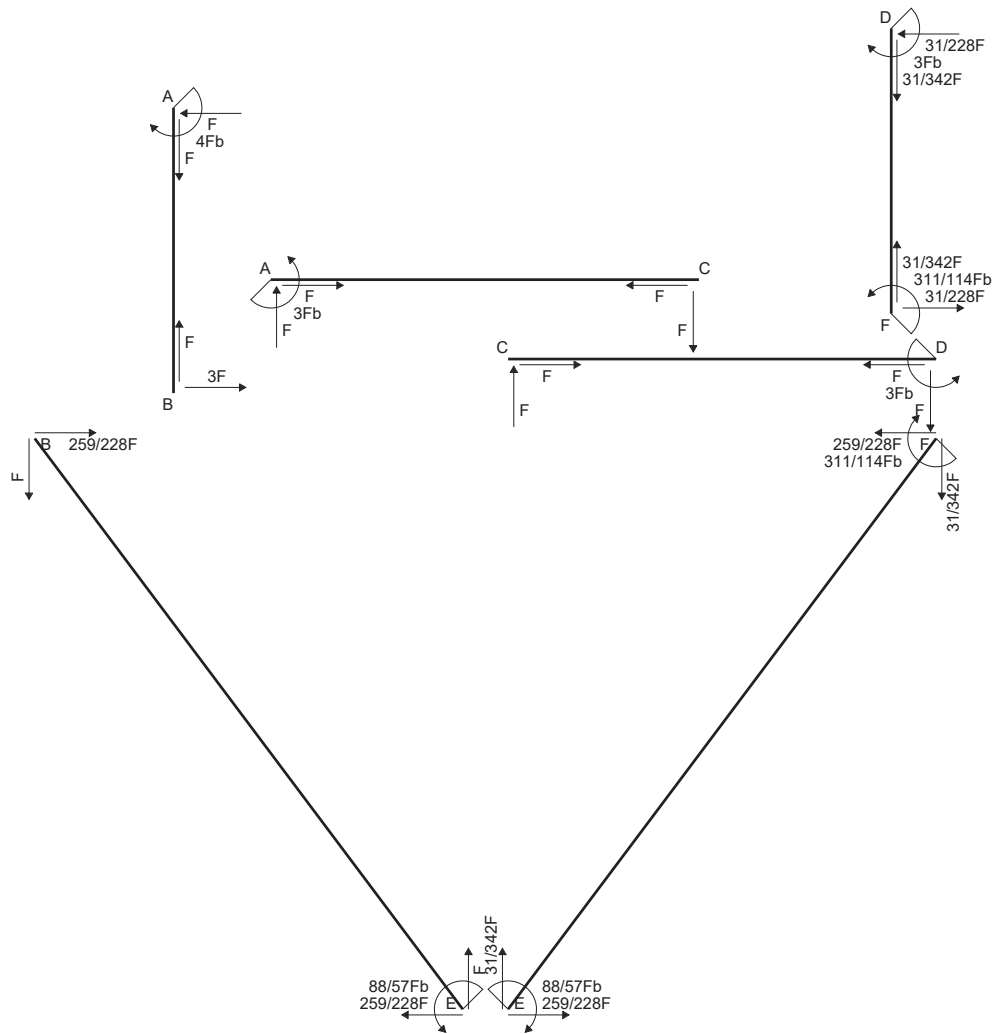
Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

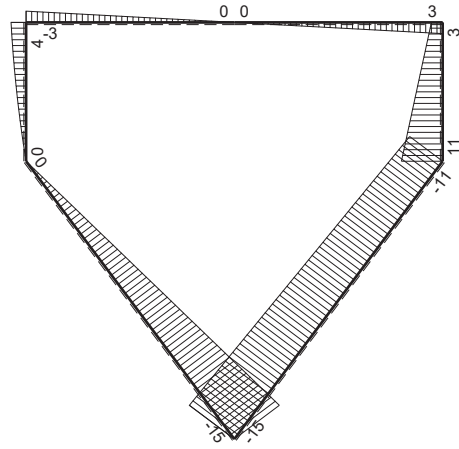
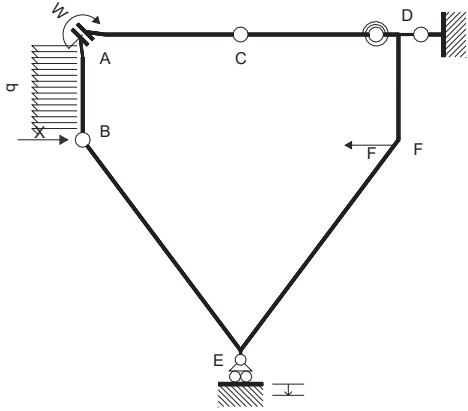
Leombo inferiore sezione su tratteggio trave, a destra da A a B

Spostamento verticale assoluto v imposto al nodo E.

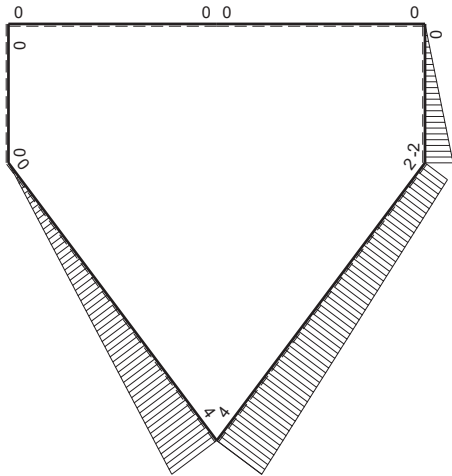
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$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contribuiti PLV per iperstatica X=H<sub>B</sub>

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB 2b	0	$4Fb-Fx-1/2qx^2$	0	0	0	0
BA 2b	0	$-3Fx+1/2qx^2$	0	0	0	0
CD 3b	0	Fx	0	0	0	0
DC 3b	0	$-3Fb+Fx$	0	0	0	0
AC 3b	0	$-3Fb+Fx$	0	0	0	0
CA 3b	0	Fx	0	0	0	0
EF 5b	$4b-2/5x$	$-15Fb+4/5Fx$	$-60Fb^2+46/5Fbx-8/25Fx^2$	$16b^2-16/5bx+4/25x^2$	$-595/3Fb^3/EJ$	$140/3Xb^3/EJ$
FE 5b	$-2b-2/5x$	$11Fb+4/5Fx$	$-22Fb^2-6Fbx-8/25Fx^2$	$4b^2+8/5bx+4/25x^2$	$-100Fb^3/EJ$	$80/3Xb^3/EJ$
BE 5b	$4/5x$	$-3Fx$	$-12/5Fx^2$	$16/25x^2$		
EB 5b	$-4b+4/5x$	$15Fb-3Fx$	$-60Fb^2+24Fbx-12/5Fx^2$	$16b^2-32/5bx+16/25x^2$		
DF 2b	-x	$3Fb+4Fx$	$-3Fbx-4Fx^2$	$x^2$	$-50/3Fb^3/EJ$	$8/3Xb^3/EJ$
FD 2b	$2b-x$	$-11Fb+4Fx$	$-22Fb^2+19Fbx-4Fx^2$	$4b^2-4bx+x^2$	$2/3Fb^3/EJ$	
E	cedimento nodo $-V_{iE}u_{iE}$					
	totali					
	iperstatica X=H <sub>B</sub>					
					$943/228F$	$76Xb^3/EJ$

Sviluppi di calcolo iperstatica

$$L_{EF}^{XX} = \int_0^{5b} (16 - 16/5 x/b + 4/25 x^2/b^2) b^2 1/EJ dx = [16x - 8/5 x^2/b + 4/75 x^3/b^2]_0^{5b} b^2 1/EJ$$

$$= (80b - 40b + 20/3 b) b^2 1/EJ = 140/3 b^3/EJ$$

$$L_{FE}^{XX} = \int_0^{5b} (4 + 8/5 x/b + 4/25 x^2/b^2) b^2 1/EJ dx = [4x + 4/5 x^2/b + 4/75 x^3/b^2]_0^{5b} b^2 1/EJ$$

$$= (20b + 20b + 20/3 b) b^2 1/EJ = 140/3 b^3/EJ$$

$$L_{BE}^{XX} = \int_0^{5b} (16/25 x^2/b^2) b^2 1/EJ dx = [16/75 x^3/b^2]_0^{5b} b^2 1/EJ$$

$$= (80/3 b) b^2 1/EJ = 80/3 b^3/EJ$$

$$L_{EB}^{XX} = \int_0^{5b} (16 - 32/5 x/b + 16/25 x^2/b^2) b^2 1/EJ dx = [16x - 16/5 x^2/b + 16/75 x^3/b^2]_0^{5b} b^2 1/EJ$$

$$= (80b - 80b + 80/3 b) b^2 1/EJ = 80/3 b^3/EJ$$

$$L_{DF}^{XX} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{FD}^{XX} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{EF}^{Xo} = \int_0^{5b} (-60 + 46/5 x/b - 8/25 x^2/b^2) Fb^2 1/EJ dx = [-60x + 23/5 x^2/b - 8/75 x^3/b^2]_0^{5b} Fb^2 1/EJ$$

$$= (-300b + 115b - 40/3 b) Fb^2 1/EJ = -595/3 Fb^3/EJ$$

$$L_{FE}^{Xo} = \int_0^{5b} (-22 - 6x/b - 8/25 x^2/b^2) Fb^2 1/EJ dx = [-22x - 3x^2/b - 8/75 x^3/b^2]_0^{5b} Fb^2 1/EJ$$

$$= (-110b - 75b - 40/3 b) Fb^2 1/EJ = -595/3 Fb^3/EJ$$

$$L_{BE}^{Xo} = \int_0^{5b} (-12/5 x^2/b^2) Fb^2 1/EJ dx = [-4/5 x^3/b^2]_0^{5b} Fb^2 1/EJ$$

$$= (-100b) Fb^2 1/EJ = -100 Fb^3/EJ$$

$$L_{EB}^{Xo} = \int_0^{5b} (-60 + 24x/b - 12/5 x^2/b^2) Fb^2 1/EJ dx = [-60x + 12x^2/b - 4/5 x^3/b^2]_0^{5b} Fb^2 1/EJ$$

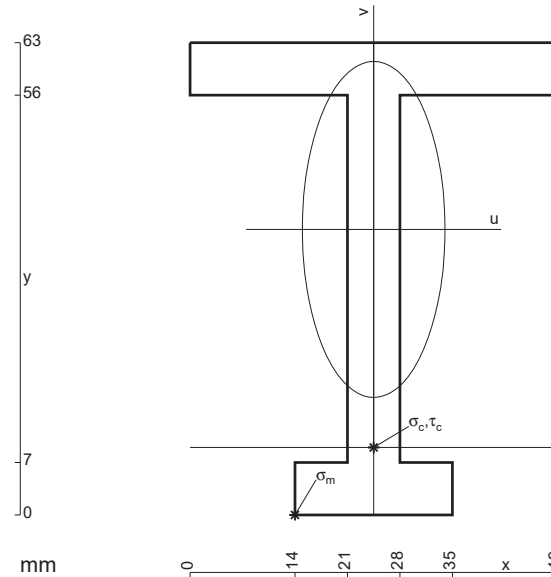
$$= (-300b + 300b - 100b) Fb^2 1/EJ = -100 Fb^3/EJ$$

$$L_{DF}^{Xo} = \int_0^{2b} (-3x/b - 4x^2/b^2) Fb^2 1/EJ dx = [-3/2 x^2/b - 4/3 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-6b - 32/3 b) Fb^2 1/EJ = -50/3 Fb^3/EJ$$

$$L_{FD}^{Xo} = \int_0^{2b} (-22 + 19x/b - 4x^2/b^2) Fb^2 1/EJ dx = [-22x + 19/2 x^2/b - 4/3 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-44b + 38b - 32/3 b) Fb^2 1/EJ = -50/3 Fb^3/EJ$$



$$A = 833. \text{ mm}^2$$

$$J_u = 418633. \text{ mm}^4$$

$$J_v = 75431. \text{ mm}^4$$

$$y_g = 38.09 \text{ mm}$$

$$N = -4990. \text{ N}$$

$$T_y = -9980. \text{ N}$$

$$M_x = 2370250. \text{ Nmm}$$

$$x_m = 14. \text{ mm}$$

$$u_m = -10.5 \text{ mm}$$

$$v_m = -38.09 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = 209.7 \text{ N/mm}^2$$

$$x_c = 24.5 \text{ mm}$$

$$y_c = 9. \text{ mm}$$

$$v_c = -29.09 \text{ mm}$$

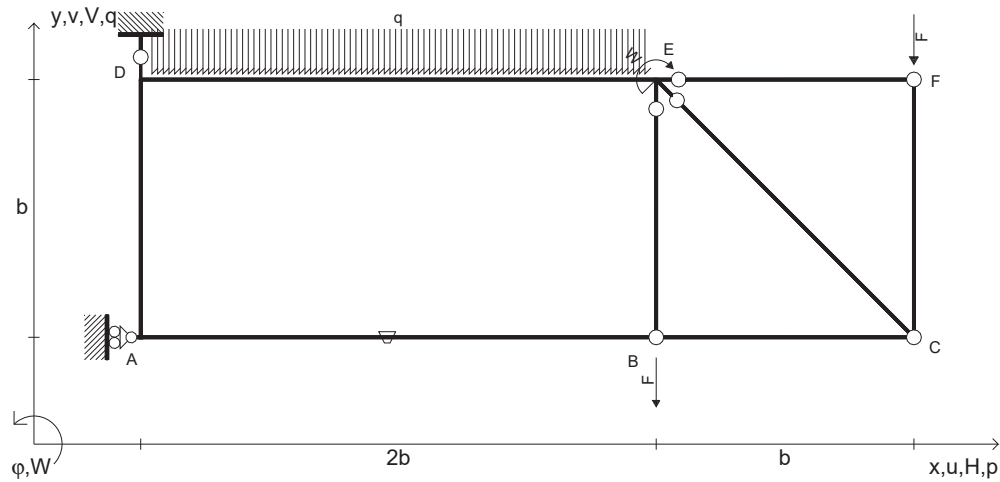
$$\sigma_c = N/A - Mv/J_u = 158.7 \text{ N/mm}^2$$

$$\tau_c = 18.75 \text{ N/mm}^2$$

$$\sigma_p = \sqrt{\sigma^2 + 3\tau^2} = 162. \text{ N/mm}^2$$

$$S = 5506. \text{ mm}^3$$

TEMA 06



$V_F = -F$	$EJ_{AB} = EJ$	$EJ_{BE} = EJ$
$V_B = -F$	$EJ_{BC} = EJ$	$EJ_{CE} = EJ$
$W_E = -W = -Fb$	$EJ_{DE} = EJ$	$EJ_{CF} = EJ$
$q_{DE} = -q = -F/b$	$EJ_{EF} = EJ$	
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{AD} = EJ$	

Reazioni iperstatiche in soluzione:  $X=V_{EB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

La trave AD ha la sezione riportata e dimensioni in mm, con:

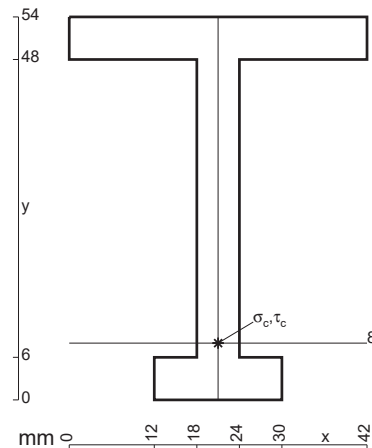
$b = 450 \text{ mm}$ ,  $F = 830 \text{ N}$

Calcolare sulla sezione D la massima tensione normale  $\sigma_m$ .

Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da A a D  
Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.

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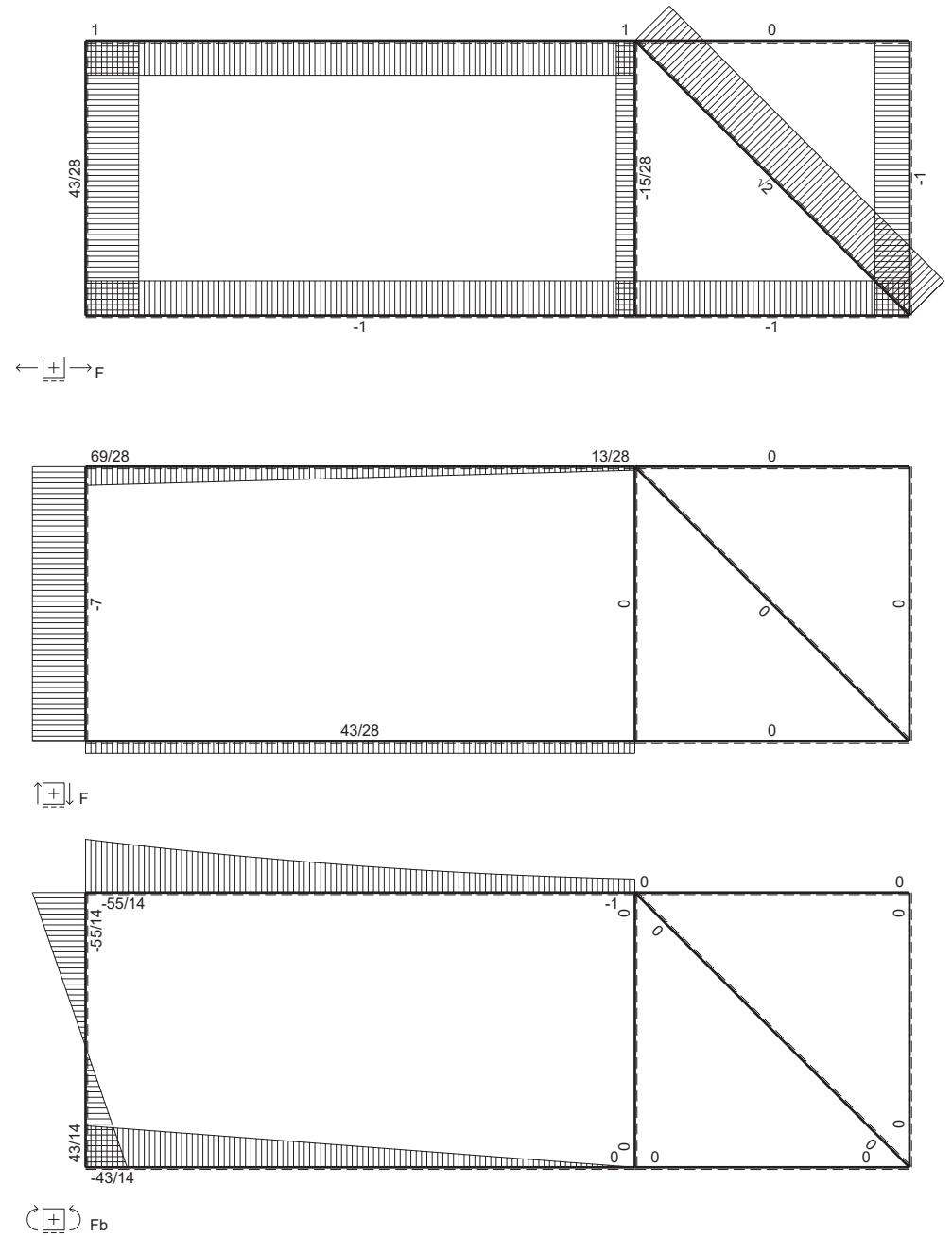
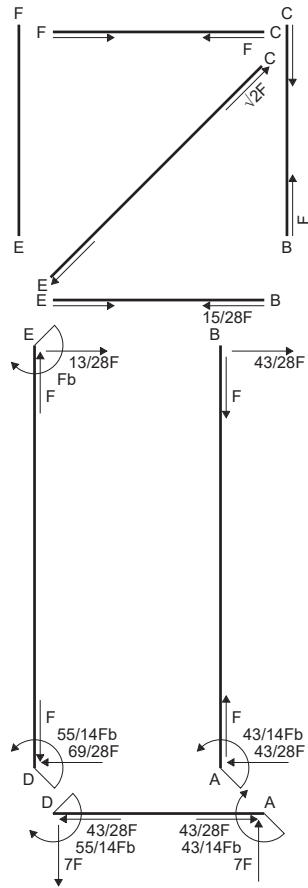


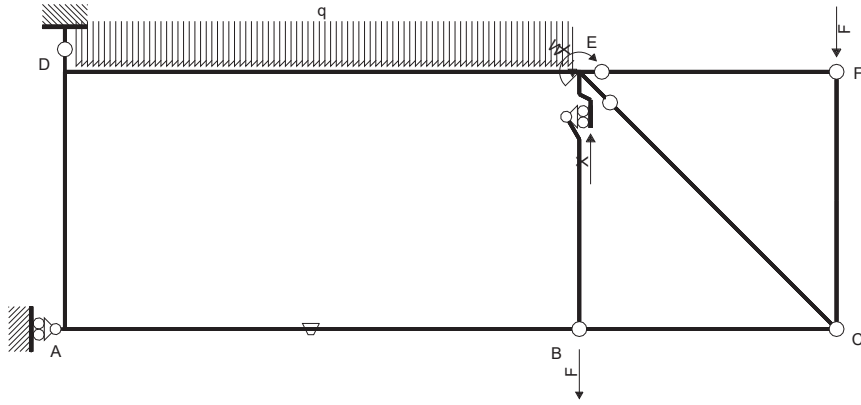
29.06.22

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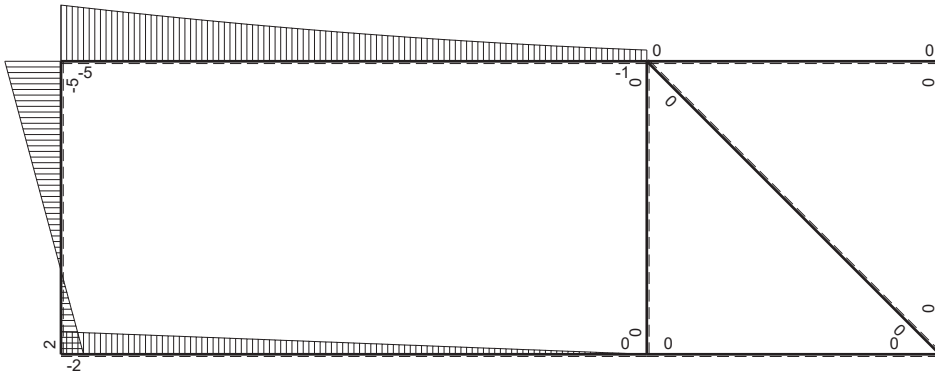
29.06.22



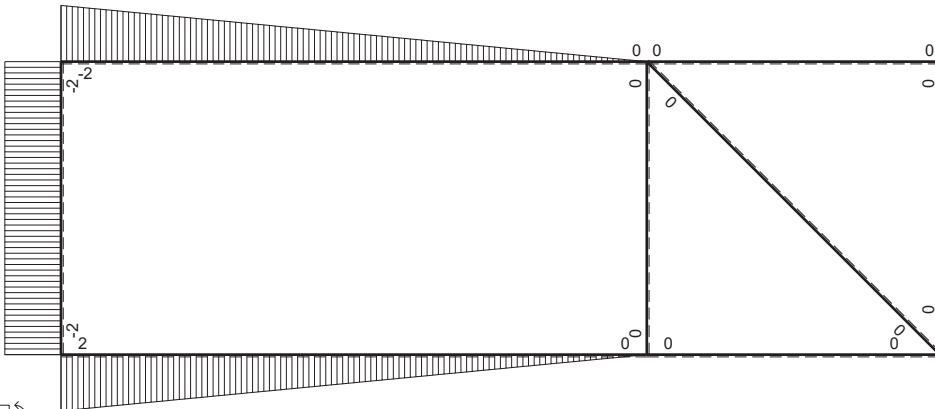




Schema di calcolo iperstatico



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contribuiti PLV per iperstatica X=V<sub>EB</sub>

→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ+\theta) dx$	$\int X M_x M_x / EJ dx$
AB 2b	$2b-x$	$-2Fb+Fx$	$-Fb/EJ$	$-4Fb^2+4Fbx-Fx^2$	$-2Fb^2/EJ+Fx b/EJ$	$4b^2-4bx+x^2$	$(-8/3-2)Fb^3/EJ$	$8/3Xb^3/EJ$
BA 2b	$-x$	$Fx$	$Fb/EJ$	$-Fx^2$	$-Fx b/EJ$	$x^2$	$0+0$	$0$
BC b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
CB b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
DE 2b	$-2b+x$	$-5Fb+3Fx-1/2qx^2$	$0$	$10Fb^2-11Fbx+4Fx^2-1/2qx^3$	$0$	$4b^2-4bx+x^2$	$(20/3+0)Fb^3/EJ$	$8/3Xb^3/EJ$
ED 2b	$x$	$Fb+Fx+1/2qx^2$	$0$	$Fbx+Fx^2+1/2qx^3$	$0$	$x^2$	$0+0$	$0$
EF b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
FE b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
AD b	$-2b$	$2Fb-7Fx$	$0$	$-4Fb^2+14Fbx$	$0$	$4b^2$	$(3+0)Fb^3/EJ$	$4Xb^3/EJ$
DA b	$2b$	$5Fb-7Fx$	$0$	$10Fb^2-14Fbx$	$0$	$4b^2$	$0+0$	$0$
BE b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
EB b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
CE $\sqrt{2}b$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
CF b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
FC b	$0$	$0$	$0$	$0$	$0$	$0$	$0+0$	$0$
totali							$5Fb^3/EJ$	$28/3Xb^3/EJ$
iperstatica X=V <sub>EB</sub>							$-15/28F$	

## PROCEDIMENTO E RISULTATI

Sviluppi di calcolo iperstatica

$$L_{AB}^{XX} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{BA}^{XX} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{DE}^{XX} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{ED}^{XX} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{AD}^{XX} = \int_0^b (4) b^2 1/EJ dx = [4x]_0^b b^2 1/EJ$$

$$= (4b) b^2 1/EJ = 4 b^3/EJ$$

$$L_{DA}^{XX} = \int_0^b (4) b^2 1/EJ dx = [4x]_0^b b^2 1/EJ$$

$$= (4b) b^2 1/EJ = 4 b^3/EJ$$

$$L_{AB}^{X\theta} = \int_0^{2b} (-4 + 4x/b - x^2/b^2) Fb^2 1/EJ dx + \int_0^{2b} (-2 + x/b) \theta dx$$

$$= [-4x + 2x^2/b - 1/3 x^3/b^2]_0^{2b} Fb^2 1/EJ + [-2x + 1/2 x^2/b]_0^{2b} \theta$$

$$= (-8b + 8b - 8/3 b) Fb^2 1/EJ + (-4b + 2b) \theta = -14/3 Fb^3/EJ$$

$$L_{BA}^{X\theta} = \int_0^{2b} (-x^2/b^2) Fb^2 1/EJ dx + \int_0^{2b} (x/b) \theta dx = [-1/3 x^3/b^2]_0^{2b} Fb^2 1/EJ + [1/2 x^2/b]_0^{2b} \theta$$

$$= (-8/3 b) Fb^2 1/EJ + (2b) \theta = -14/3 Fb^3/EJ$$

$$L_{DE}^{X\theta} = \int_0^{2b} (10 - 11x/b + 4x^2/b^2 - 1/2 x^3/b^3) Fb^2 1/EJ dx$$

$$= [10x - 11/2 x^2/b + 4/3 x^3/b^2 - 1/8 x^4/b^3]_0^{2b} Fb^2 1/EJ$$

$$= (20b - 22b + 32/3 b - 2b) Fb^2 1/EJ = 20/3 Fb^3/EJ$$

$$L_{ED}^{X\theta} = \int_0^{2b} (x/b + x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = [1/2 x^2/b + 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^{2b} Fb^2 1/EJ$$

$$= (2b + 8/3 b + 2b) Fb^2 1/EJ = 20/3 Fb^3/EJ$$

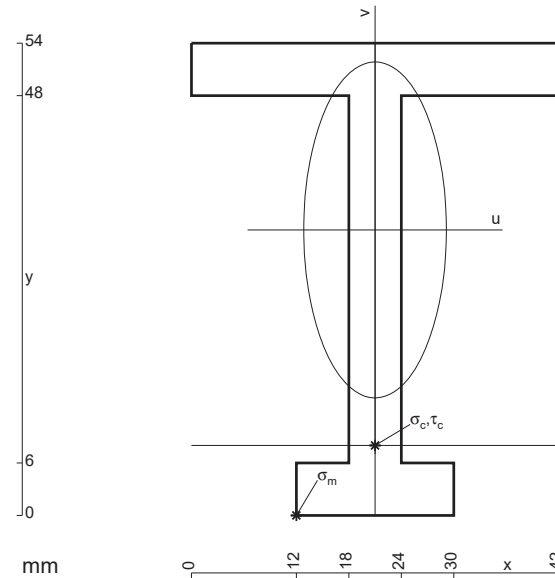
$$L_{AD}^{X\theta} = \int_0^b (-4 + 14x/b) Fb^2 1/EJ dx = [-4x + 7x^2/b]_0^b Fb^2 1/EJ$$

$$= (-4b + 7b) Fb^2 1/EJ = 3 Fb^3/EJ$$

$$L_{DA}^{X\theta} = \int_0^b (10 - 14x/b) Fb^2 1/EJ dx = [10x - 7x^2/b]_0^b Fb^2 1/EJ$$

$$= (10b - 7b) Fb^2 1/EJ = 3 Fb^3/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 612. \text{ mm}^2$$

$$J_u = 225968. \text{ mm}^4$$

$$J_v = 40716. \text{ mm}^4$$

$$y_g = 32.65 \text{ mm}$$

$$N = 1275. \text{ N}$$

$$T_y = -5810. \text{ N}$$

$$M_x = -1467320. \text{ Nmm}$$

$$x_m = 12. \text{ mm}$$

$$u_m = -9. \text{ mm}$$

$$v_m = -32.65 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = -209.9 \text{ N/mm}^2$$

$$x_c = 21. \text{ mm}$$

$$y_c = 8. \text{ mm}$$

$$v_c = -24.65 \text{ mm}$$

$$\sigma_c = N/A - Mv/J_u = -158. \text{ N/mm}^2$$

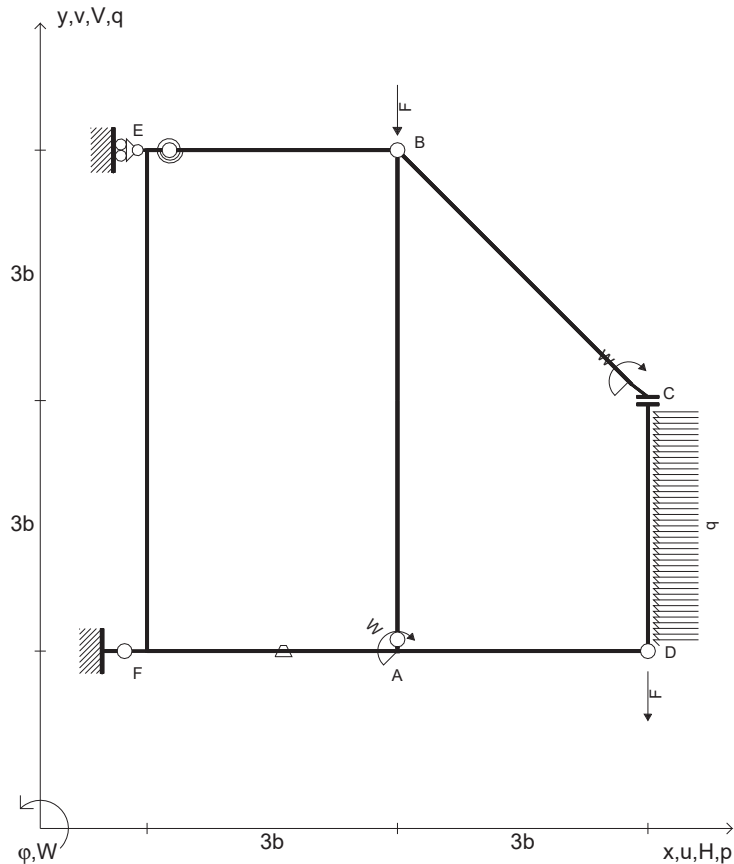
$$\tau_c = 15.04 \text{ N/mm}^2$$

$$\sigma_r = \sqrt{\sigma^2 + 3\tau^2} = 160.1 \text{ N/mm}^2$$

$$S = 3510. \text{ mm}^3$$

# TEMA 07

- $V_D = -F$
- $V_B = -F$
- $W_{CB} = -W = -Fb$
- $W_A = -W = -Fb$
- $p_{CD} = -q = -F/b$
- $\theta_{AF} = -\theta = -\alpha T/b = -bF/EJ$
- $k_{EB} = 4EJ/b$
- $EJ_{AB} = EJ$
- $EJ_{CD} = EJ$
- $EJ_{BC} = EJ$
- $EJ_{EB} = EJ$
- $EJ_{FE} = EJ$
- $EJ_{AF} = EJ$
- $EJ_{DA} = EJ$



Reazioni iperstatiche in soluzione:  $X=V_{AB}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

La trave CD ha la sezione riportata e dimensioni in mm, con:

$b = 150 \text{ mm}$ ,  $F = 2830 \text{ N}$

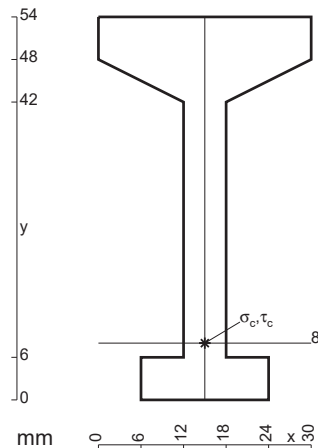
Calcolare sulla sezione mediana la massima tensione normale  $\sigma_m$

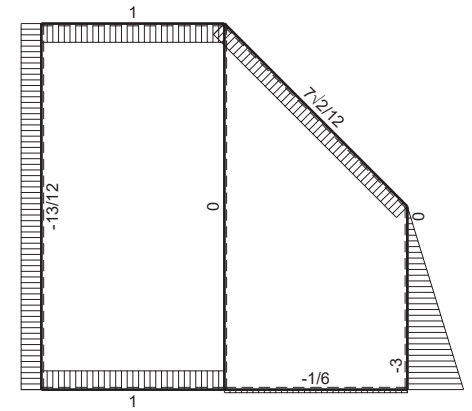
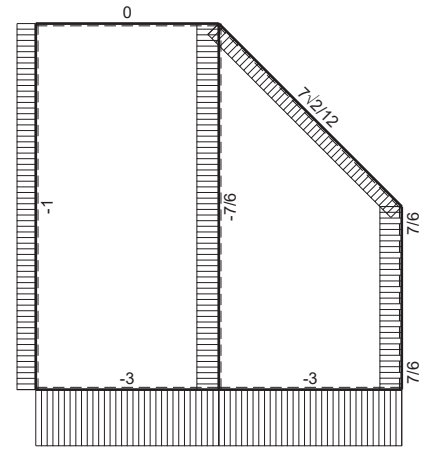
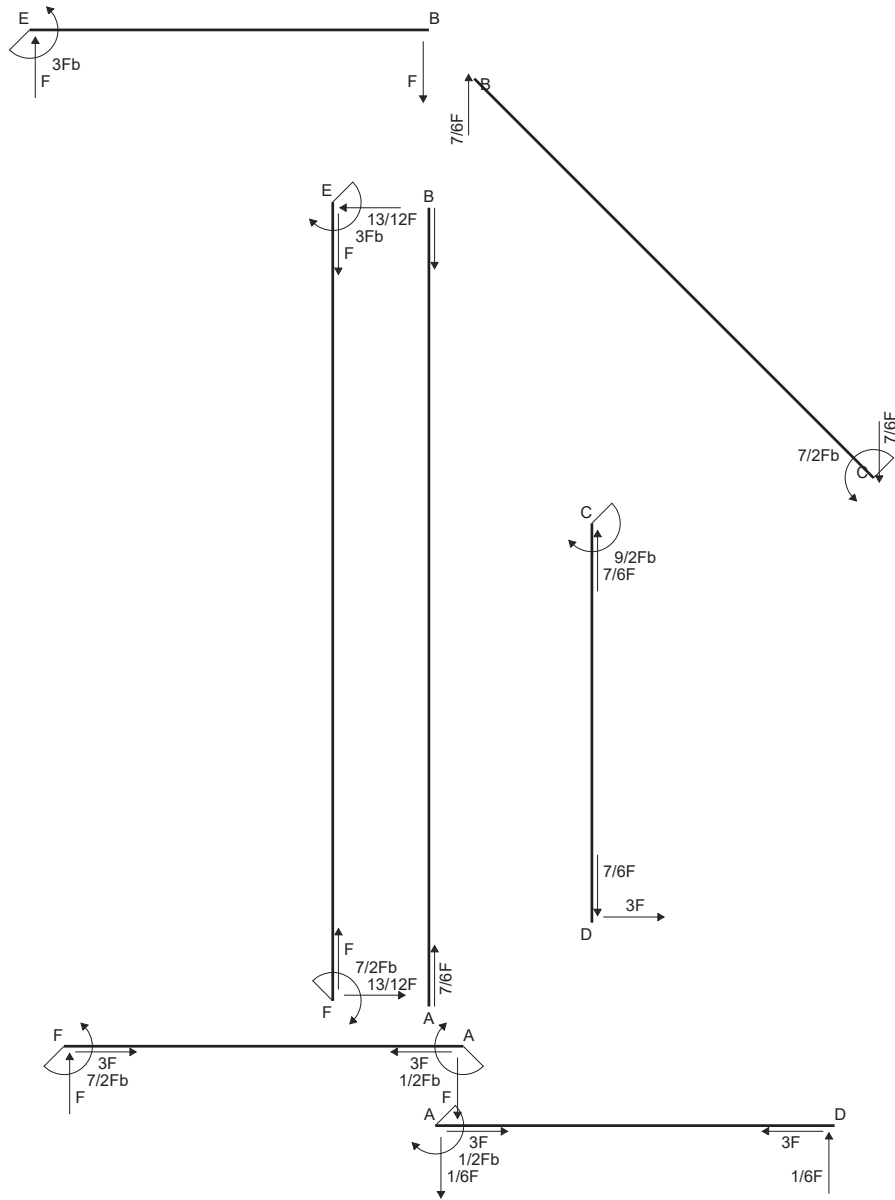
Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

Leombo inferiore sezione su tratteggio trave, a destra da C a D

Curvatura  $\theta$  asta AF positiva se convessa a destra con inizio A.

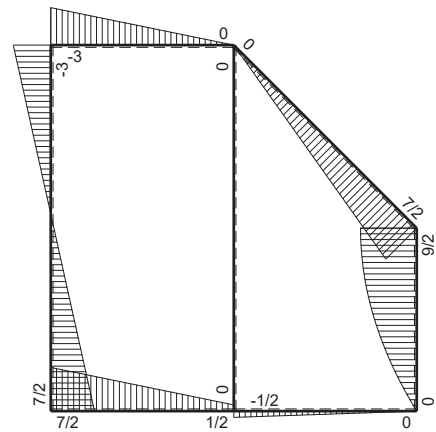
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13



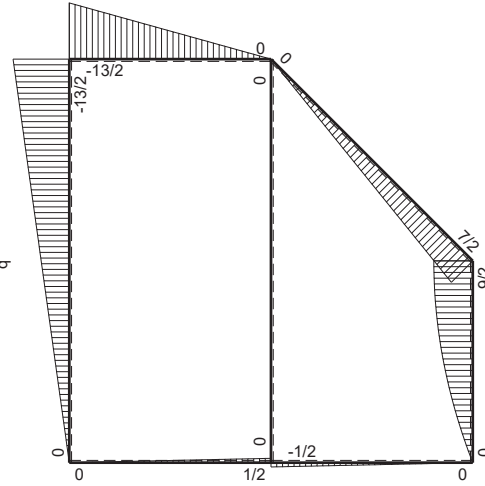
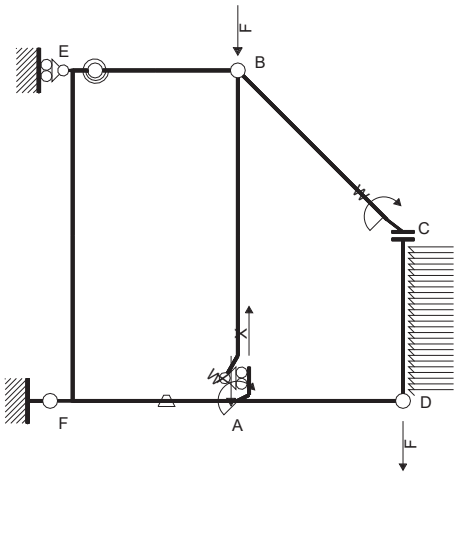


← ⊕ → F

↑ ⊕ ↓ F

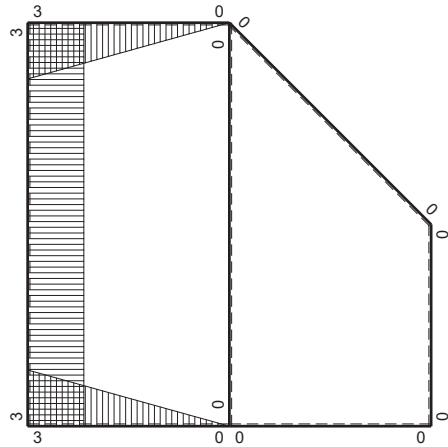


⊕ F<sub>b</sub>



$M_x$  flessione da carichi assegnati

Schema di calcolo iperstatico



$M_x$  flessione da iperstatica X=1

Quadro contribuiti PLV per iperstatica X=V<sub>AB</sub>

→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ + \theta) dx$	$\int X M_x M_x / EJ dx$
AB 6b	0	0	0	0	0	0	0+0	0
BA 6b	0	0	0	0	0	0	0+0	0
CD 3b	0	$9/2Fb - 1/2qx^2$	0	0	0	0	0+0	0
DC 3b	0	$-3Fx + 1/2qx^2$	0	0	0	0	0	0
BC 3√2b	0	$7√2/12Fx$	0	0	0	0	0	0
EB 3b	$3b-x$	$-13/2Fb + 13/6Fx$	0	$-39/2Fb^2 + 13Fbx - 13/6Fx^2$	0	$9b^2 - 6bx + x^2$	$(-39/2+0)Fb^3/EJ$	$9Xb^3/EJ$
BE 3b	-x	$13/6Fx$	0	$-13/6Fx^2$	0	$x^2$	0	0
FE 6b	3b	$-13/12Fx$	0	$-13/4Fbx$	0	$9b^2$	$(-117/2+0)Fb^3/EJ$	$54Xb^3/EJ$
EF 6b	-3b	$13/2Fb - 13/12Fx$	0	$-39/2Fb^2 + 13/4Fbx$	0	$9b^2$	0	0
AF 3b	x	$1/2Fb - 1/6Fx$	$-Fb/EJ$	$1/2Fbx - 1/6Fx^2$	$-Fbx/EJ$	$x^2$	$(3/4-9/2)Fb^3/EJ$	$9Xb^3/EJ$
FA 3b	$-3b+x$	$-1/6Fx$	$Fb/EJ$	$1/2Fbx - 1/6Fx^2$	$-3Fb^2/EJ + Fxb/EJ$	$9b^2 - 6bx + x^2$	0	0
DA 3b	0	$-1/6Fx$	0	0	0	0	0+0	0
AD 3b	0	$1/2Fb - 1/6Fx$	0	0	0	0	$-39/8Fb^3/EJ$	$9/4Xb^3/EJ$
EB	molla asta $-W_{TEB}(W_{0EB} + XW_{1EB})/k_{EB}$							
	totali						$-693/8Fb^3/EJ$	$297/4Xb^3/EJ$
	iperstatica X=V <sub>AB</sub>						$7/6F$	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{EB}^{XX} = \int_0^{3b} (9 - 6x/b + x^2/b^2) b^2 1/EJ dx - 3 (-3) 1/4 b^3/EJ$$

$$= [9x - 3x^2/b + 1/3 x^3/b^2]_0^{3b} b^2 1/EJ - 3 (-3) 1/4 b^3/EJ$$

$$= (27b - 27b + 9b) b^2 1/EJ - 3 (-3) 1/4 b^3/EJ = 45/4 b^3/EJ$$

$$L_{BE}^{XX} = \int_0^{3b} (x^2/b^2) b^2 1/EJ dx - 3 (-3) 1/4 b^3/EJ = [1/3 x^3/b^2]_0^{3b} b^2 1/EJ - 3 (-3) 1/4 b^3/EJ$$

$$= (9b) b^2 1/EJ - 3 (-3) 1/4 b^3/EJ = 45/4 b^3/EJ$$

$$L_{FE}^{XX} = \int_0^{6b} (9) b^2 1/EJ dx = [9x]_0^{6b} b^2 1/EJ$$

$$= (54b) b^2 1/EJ = 54 b^3/EJ$$

$$L_{EF}^{XX} = \int_0^{6b} (9) b^2 1/EJ dx = [9x]_0^{6b} b^2 1/EJ$$

$$= (54b) b^2 1/EJ = 54 b^3/EJ$$

$$L_{AF}^{XX} = \int_0^{3b} (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^{3b} b^2 1/EJ$$

$$= (9b) b^2 1/EJ = 9 b^3/EJ$$

$$L_{FA}^{XX} = \int_0^{3b} (9 - 6x/b + x^2/b^2) b^2 1/EJ dx = [9x - 3x^2/b + 1/3 x^3/b^2]_0^{3b} b^2 1/EJ$$

$$= (27b - 27b + 9b) b^2 1/EJ = 9 b^3/EJ$$

$$L_{EB}^{Xo} = \int_0^{3b} (-39/2 + 13x/b - 13/6 x^2/b^2) Fb^2 1/EJ dx - 3 13/2 1/4 Fb^3/EJ$$

$$= [-39/2 x + 13/2 x^2/b - 13/18 x^3/b^2]_0^{3b} Fb^2 1/EJ - 3 13/2 1/4 Fb^3/EJ$$

$$= (-117/2 b + 117/2 b - 39/2 b) Fb^2 1/EJ - 3 13/2 1/4 Fb^3/EJ = -195/8 Fb^3/EJ$$

$$L_{BE}^{Xo} = \int_0^{3b} (-13/6 x^2/b^2) Fb^2 1/EJ dx - 3 13/2 1/4 Fb^3/EJ$$

$$= [-13/18 x^3/b^2]_0^{3b} Fb^2 1/EJ - 3 13/2 1/4 Fb^3/EJ$$

$$= (-39/2 b) Fb^2 1/EJ - 3 13/2 1/4 Fb^3/EJ = -195/8 Fb^3/EJ$$

$$L_{FE}^{Xo} = \int_0^{6b} (-13/4 x/b) Fb^2 1/EJ dx = [-13/8 x^2/b]_0^{6b} Fb^2 1/EJ$$

$$= (-117/2 b) Fb^2 1/EJ = -117/2 Fb^3/EJ$$

$$L_{EF}^{Xo} = \int_0^{6b} (-39/2 + 13/4 x/b) Fb^2 1/EJ dx = [-39/2 x + 13/8 x^2/b]_0^{6b} Fb^2 1/EJ$$

$$= (-117b + 117/2 b) Fb^2 1/EJ = -117/2 Fb^3/EJ$$

$$L_{AF}^{Xo} = \int_0^{3b} (1/2 x/b - 1/6 x^2/b^2) Fb^2 1/EJ dx + \int_0^{3b} (-x/b) \theta dx$$

$$= [1/4 x^2/b - 1/18 x^3/b^2]_0^{3b} Fb^2 1/EJ + [-1/2 x^2/b]_0^{3b} \theta$$

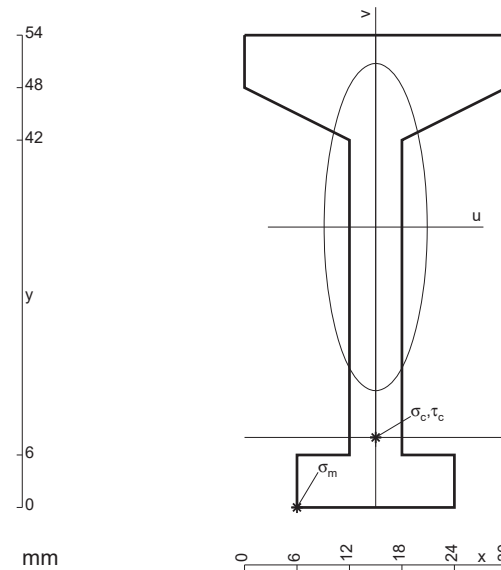
$$= (9/4 b - 3/2 b) Fb^2 1/EJ + (-9/2 b) \theta = -15/4 Fb^3/EJ$$

$$L_{FA}^{Xo} = \int_0^{3b} (1/2 x/b - 1/6 x^2/b^2) Fb^2 1/EJ dx + \int_0^{3b} (3 - x/b) \theta dx$$

$$= [1/4 x^2/b - 1/18 x^3/b^2]_0^{3b} Fb^2 1/EJ + [3x - 1/2 x^2/b]_0^{3b} \theta$$

$$= (9/4 b - 3/2 b) Fb^2 1/EJ + (9b - 9/2 b) \theta = -15/4 Fb^3/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 612. \text{ mm}^2$$

$$J_u = 214270. \text{ mm}^4$$

$$J_v = 21276. \text{ mm}^4$$

$$y_g = 32.06 \text{ mm}$$

$$N = 3302. \text{ N}$$

$$T_y = -4245. \text{ N}$$

$$M_x = 1432690. \text{ Nmm}$$

$$x_m = 6. \text{ mm}$$

$$u_m = -9. \text{ mm}$$

$$v_m = -32.06 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = 219.8 \text{ N/mm}^2$$

$$x_c = 15. \text{ mm}$$

$$y_c = 8. \text{ mm}$$

$$v_c = -24.06 \text{ mm}$$

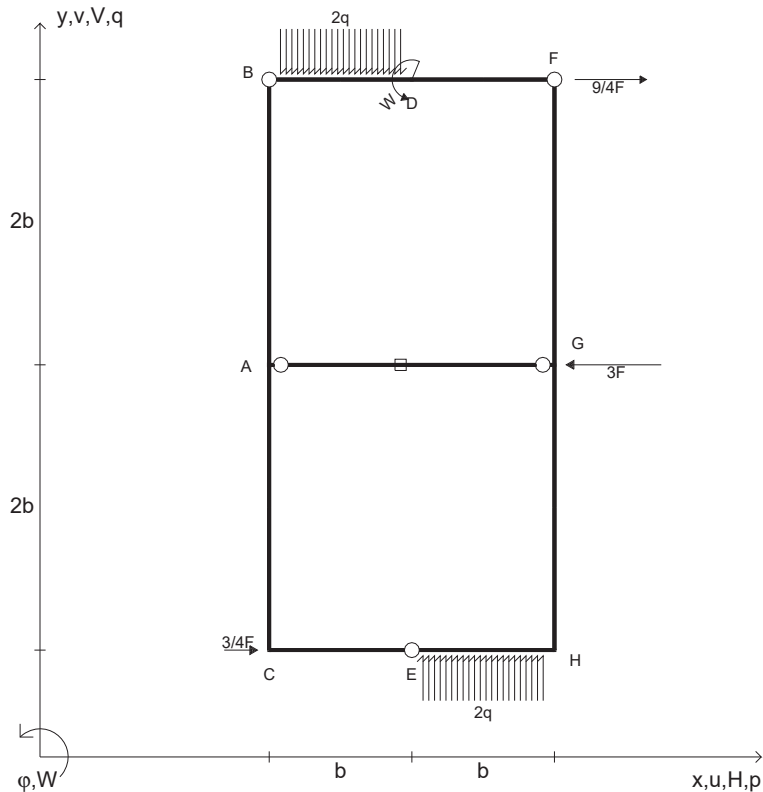
$$\sigma_c = N/A - Mv/J_u = 166.3 \text{ N/mm}^2$$

$$\tau_c = 11.36 \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 167.4 \text{ N/mm}^2$$

$$S = 3439. \text{ mm}^3$$

- $H_G = -3F$
- $H_C = 3/4F$
- $H_F = 9/4F$
- $W_D = W = Fb$
- $q_{DB} = -2q = -2F/b$
- $q_{HE} = 2q = 2F/b$
- $\varepsilon_{GA} = -5\alpha T = -5b^2 F/EJ$
- $EJ_{AB} = EJ$
- $EJ_{CA} = EJ$
- $EJ_{DB} = EJ$
- $EJ_{EC} = EJ$
- $EJ_{FD} = EJ$
- $EJ_{GA} = EJ$
- $EJ_{HE} = EJ$
- $EJ_{GF} = EJ$
- $EJ_{HG} = EJ$



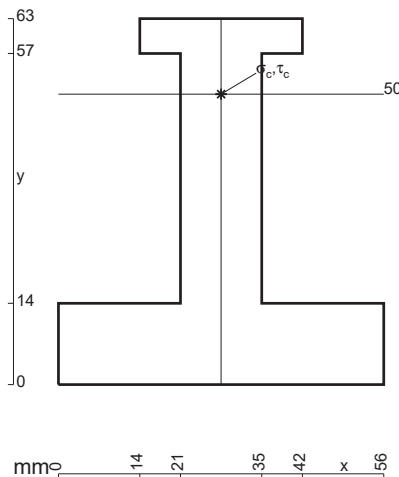
Sul fronte di questo foglio:

- indicare chiaramente eventuali reazioni iperstatiche.
- Riportare graficamente le reazioni vincolari calcolate.
- Tracciare i diagrammi finali con quote frazionarie.

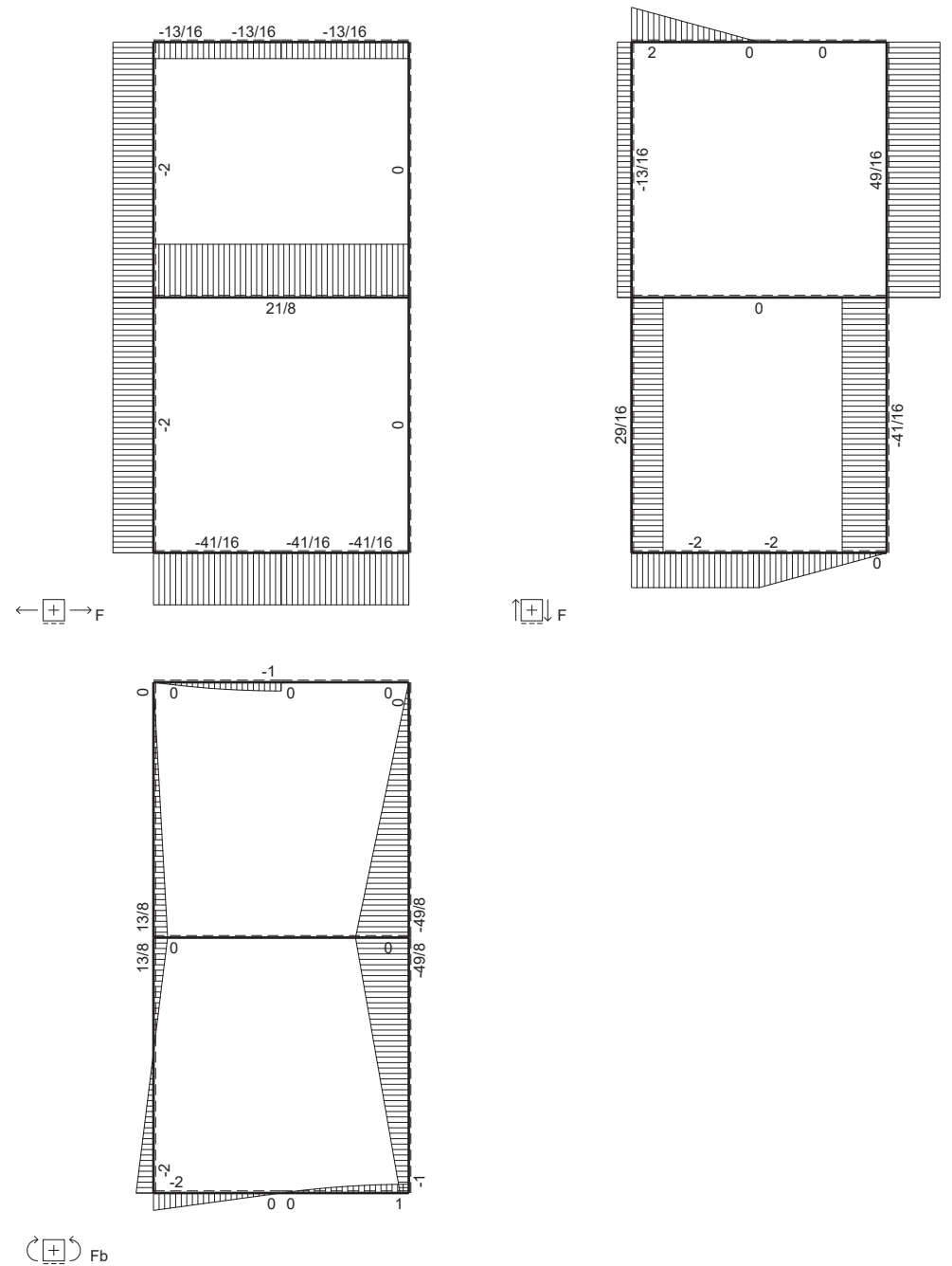
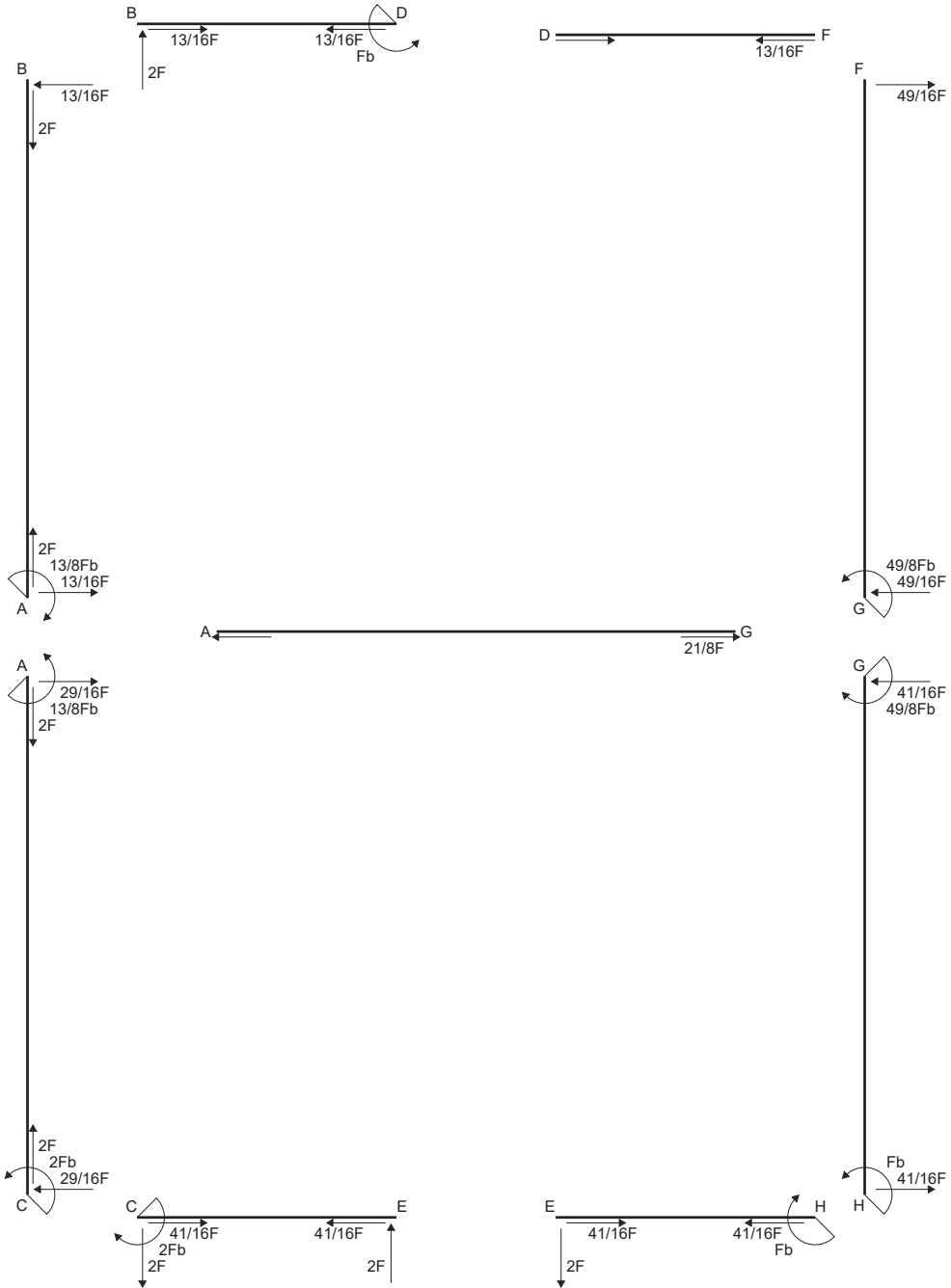
Sul retro di questo foglio:

- illustrare l'analisi cinematica.
  - Tracciare i diagrammi della struttura ausiliaria.
  - Scrivere l'espressione del PLV con funzioni  $M_0$  e  $M^*$ .
- Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave EC ha la sezione riportata e dimensioni in mm, con:  
 $b = 440 \text{ mm}$ ,  $F = 3950 \text{ N}$

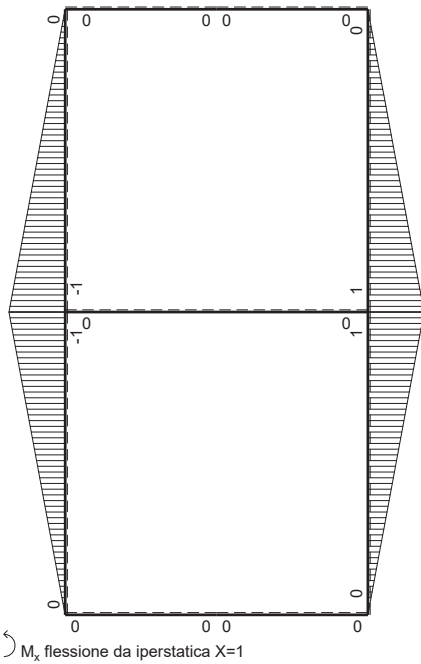
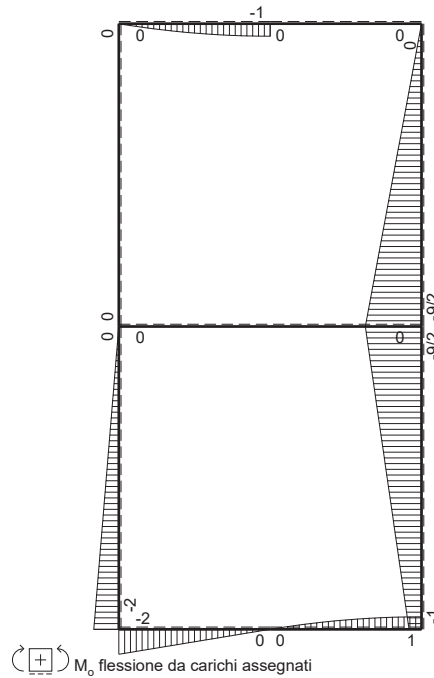
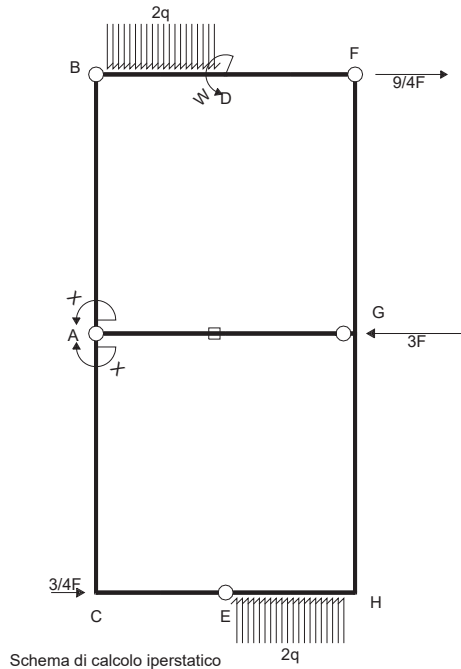
- Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .
- Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.
- Leombo inferiore sezione su tratteggio trave, a destra da E a C
- Elongazione termica specifica  $\varepsilon$  assegnata su asta GA.







PROCEDIMENTO E RISULTATI



PROCEDIMENTO E RISULTATI

Quadro contributi PLV per iperstatica  $X=W_{AB}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB 2b	$-1+1/2x/b$	0	0	$1-x/b+1/4x^2/b^2$	0	$2/3Xb/EJ$
BA 2b	$1/2x/b$	0	0	$1/4x^2/b^2$	0	$2/3Xb/EJ$
CA 2b	$-1/2x/b$	$-2Fb+Fx$	$Fx-1/2Fx^2/b$	$1/4x^2/b^2$	$2/3Fb^2/EJ$	$2/3Xb/EJ$
AC 2b	$1-1/2x/b$	$Fx$	$Fx-1/2Fx^2/b$	$1-x/b+1/4x^2/b^2$	$2/3Fb^2/EJ$	$2/3Xb/EJ$
DB b	0	$-Fb+qx^2$	0	0	0	0
BD b	0	$2Fx-qx^2$	0	0	0	0
EC b	0	$-2Fx$	0	0	0	0
CE b	0	$2Fb-2Fx$	0	0	0	0
FD b	0	0	0	0	0	0
DF b	0	0	0	0	0	0
GA 2b	0	0	0	0	0	0
AG 2b	0	0	0	0	0	0
HE b	0	$Fb-qx^2$	0	0	0	0
EH b	0	$-2Fx+qx^2$	0	0	0	0
GF 2b	$1-1/2x/b$	$-9/2Fb+9/4Fx$	$-9/2Fb+9/2Fx-9/8Fx^2/b$	$1-x/b+1/4x^2/b^2$	$-3Fb^2/EJ$	$2/3Xb/EJ$
FG 2b	$-1/2x/b$	$9/4Fx$	$-9/8Fx^2/b$	$1/4x^2/b^2$	$-3Fb^2/EJ$	$2/3Xb/EJ$
HG 2b	$1/2x/b$	$-Fb-7/4Fx$	$-1/2Fx-7/8Fx^2/b$	$1/4x^2/b^2$	$-10/3Fb^2/EJ$	$2/3Xb/EJ$
GH 2b	$-1+1/2x/b$	$9/2Fb-7/4Fx$	$-9/2Fb+4Fx-7/8Fx^2/b$	$1-x/b+1/4x^2/b^2$	$-10/3Fb^2/EJ$	$2/3Xb/EJ$
GA	elongazione asta $N_{IGA} \epsilon_{GA} L_{GA}$				$10Fb^2/EJ$	
	totali				$13/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{AB}$				$-13/8Fb$	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{AB}^{XX} = \int_0^{2b} (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[ x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2b - 2b + 2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{BA}^{XX} = \int_0^{2b} (1/4 x^2/b^2) 1/EJ dx = \left[ 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{CA}^{XX} = \int_0^{2b} (1/4 x^2/b^2) 1/EJ dx = \left[ 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{AC}^{XX} = \int_0^{2b} (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[ x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2b - 2b + 2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{GF}^{XX} = \int_0^{2b} (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[ x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2b - 2b + 2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{FG}^{XX} = \int_0^{2b} (1/4 x^2/b^2) 1/EJ dx = \left[ 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{HG}^{XX} = \int_0^{2b} (1/4 x^2/b^2) 1/EJ dx = \left[ 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{GH}^{XX} = \int_0^{2b} (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = \left[ x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (2b - 2b + 2/3 b) 1/EJ = 2/3 b/EJ$$

$$L_{CA}^{Xo} = \int_0^{2b} (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = \left[ 1/2 x^2/b - 1/6 x^3/b^2 \right]_0^{2b} Fb 1/EJ$$

$$= (2b - 4/3 b) Fb 1/EJ = 2/3 Fb^2/EJ$$

$$L_{AC}^{Xo} = \int_0^{2b} (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = \left[ 1/2 x^2/b - 1/6 x^3/b^2 \right]_0^{2b} Fb 1/EJ$$

$$= (2b - 4/3 b) Fb 1/EJ = 2/3 Fb^2/EJ$$

$$L_{GF}^{Xo} = \int_0^{2b} (-9/2 + 9/2 x/b - 9/8 x^2/b^2) Fb 1/EJ dx = \left[ -9/2 x + 9/4 x^2/b - 3/8 x^3/b^2 \right]_0^{2b} Fb 1/EJ$$

$$= (-9b + 9b - 3b) Fb 1/EJ = -3 Fb^2/EJ$$

$$L_{FG}^{Xo} = \int_0^{2b} (-9/8 x^2/b^2) Fb 1/EJ dx = \left[ -3/8 x^3/b^2 \right]_0^{2b} Fb 1/EJ$$

$$= (-3b) Fb 1/EJ = -3 Fb^2/EJ$$

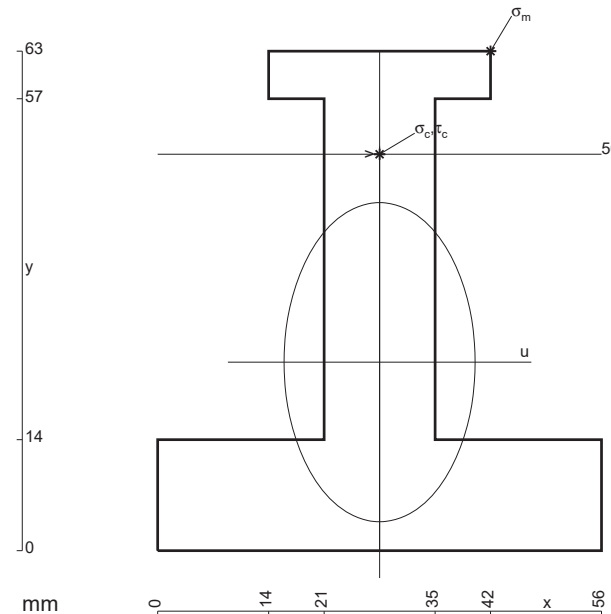
$$L_{HG}^{Xo} = \int_0^{2b} (-1/2 x/b - 7/8 x^2/b^2) Fb 1/EJ dx = \left[ -1/4 x^2/b - 7/24 x^3/b^2 \right]_0^{2b} Fb 1/EJ$$

$$= (-b - 7/3 b) Fb 1/EJ = -10/3 Fb^2/EJ$$

$$L_{GH}^{Xo} = \int_0^{2b} (-9/2 + 4x/b - 7/8 x^2/b^2) Fb 1/EJ dx = \left[ -9/2 x + 2x^2/b - 7/24 x^3/b^2 \right]_0^{2b} Fb 1/EJ$$

$$= (-9b + 8b - 7/3 b) Fb 1/EJ = -10/3 Fb^2/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 1554. \text{ mm}^2$$

$$J_u = 629904. \text{ mm}^4$$

$$J_v = 225694. \text{ mm}^4$$

$$y_g = 23.77 \text{ mm}$$

$$N = -10122. \text{ N}$$

$$T_y = -7900. \text{ N}$$

$$M_x = -3476000. \text{ Nmm}$$

$$x_m = 42. \text{ mm}$$

$$y_m = 63. \text{ mm}$$

$$u_m = 14. \text{ mm}$$

$$v_m = 39.23 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = 210. \text{ N/mm}^2$$

$$x_c = 28. \text{ mm}$$

$$y_c = 50. \text{ mm}$$

$$v_c = 26.23 \text{ mm}$$

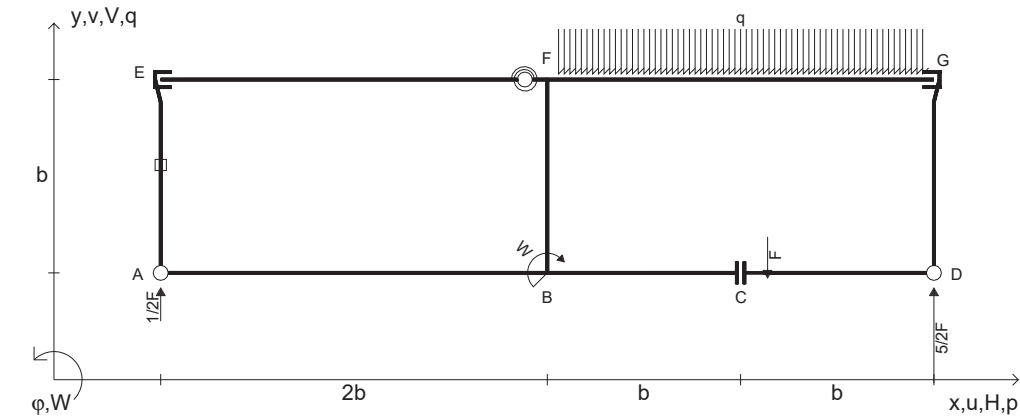
$$\sigma_c = N/A - Mv/J_u = 138.2 \text{ N/mm}^2$$

$$\tau_c = 8.063 \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 138.9 \text{ N/mm}^2$$

$$S = 9000. \text{ mm}^3$$

# TEMA 09



$V_{CD} = -F$	$q_{FG} = -q = -F/b$	$EJ_{BC} = EJ$	$EJ_{FG} = EJ$
$V_A = 1/2F$	$\varepsilon_{AE} = -\alpha T = -b^2 F/EJ$	$EJ_{CD} = EJ$	$EJ_{BF} = EJ$
$V_D = 5/2F$	$k_{FE} = 4EJ/b$	$EJ_{AE} = EJ$	$EJ_{DG} = EJ$
$W_B = -W = -Fb$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$	

Reazioni iperstatiche in soluzione:  $X = V_{AE}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

La trave CD ha la sezione riportata e dimensioni in mm,

$b = 620 \text{ mm}$ ,  $F = 5820 \text{ N}$

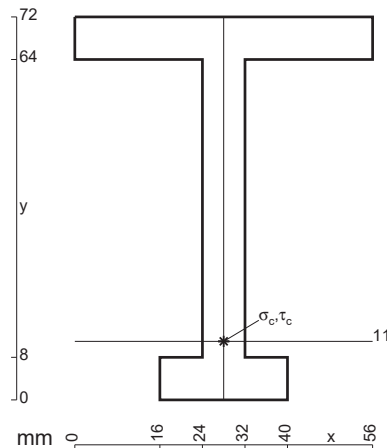
Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .

Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

Leombo inferiore sezione su tratteggio trave, a destra da C a D

Elongazione termica specifica  $\varepsilon$  assegnata su asta AE.

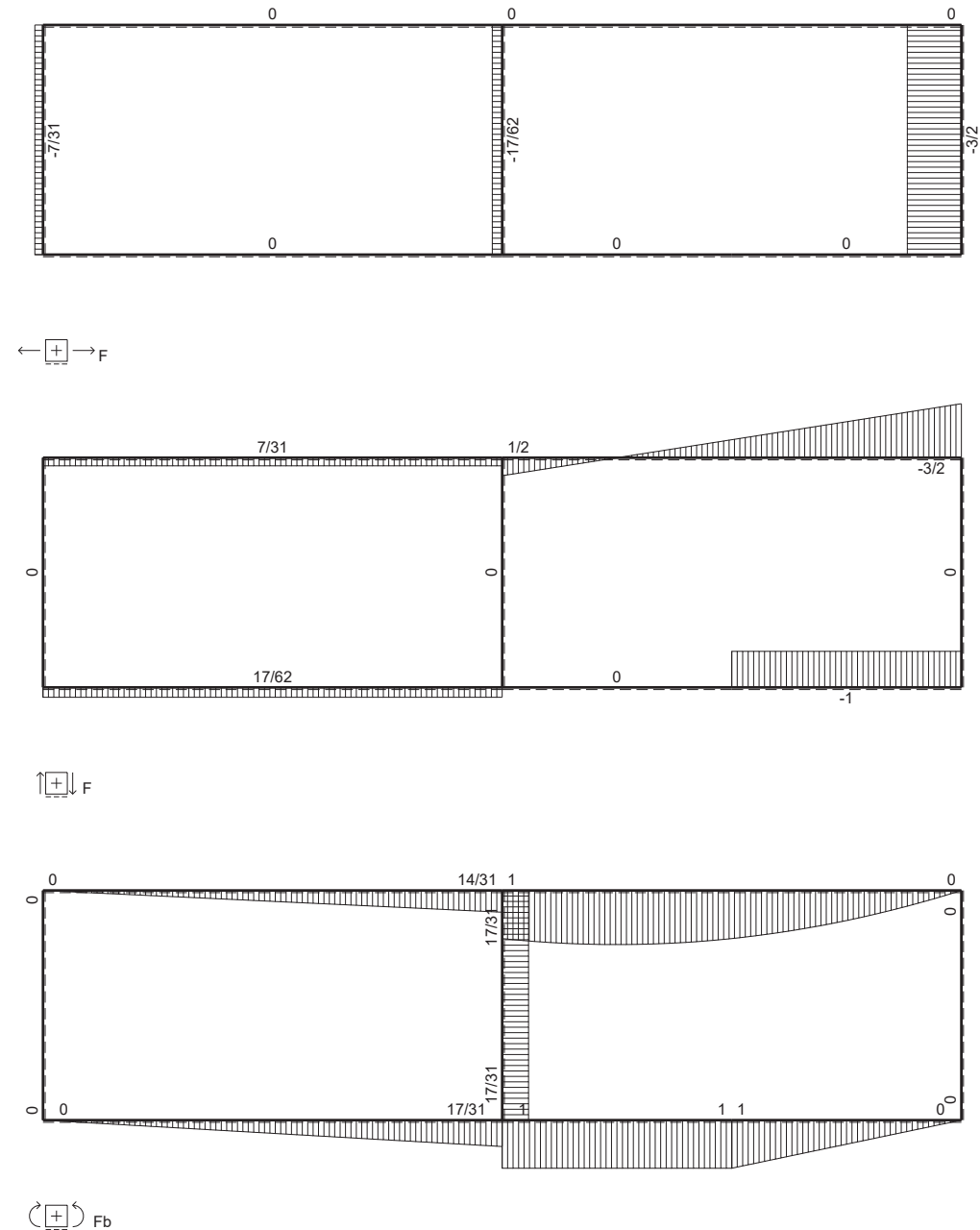
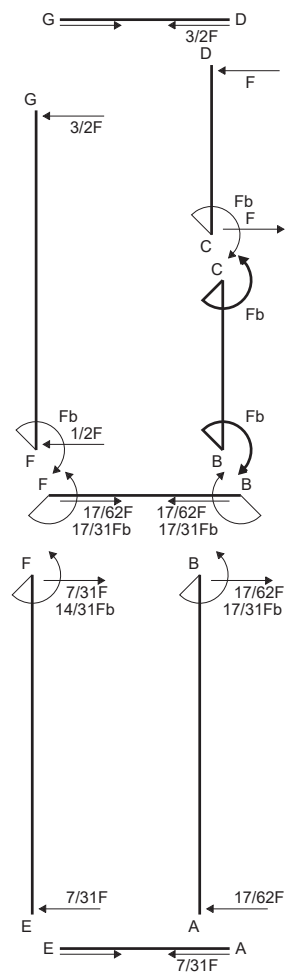
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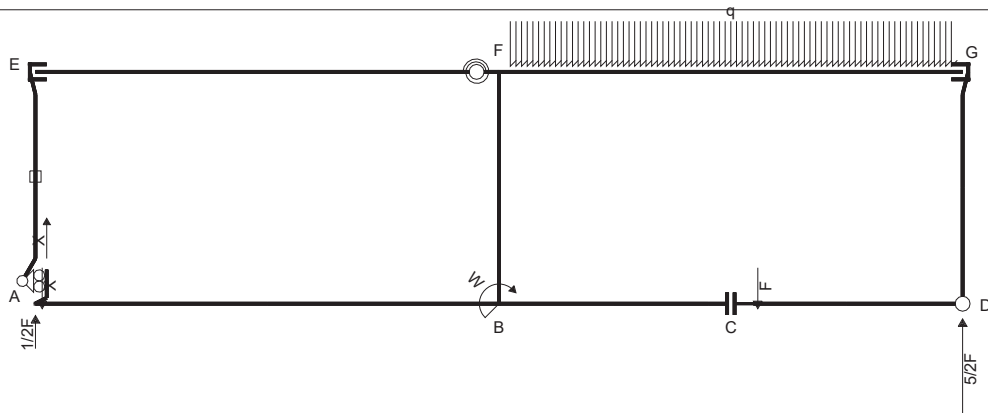
06.09.22

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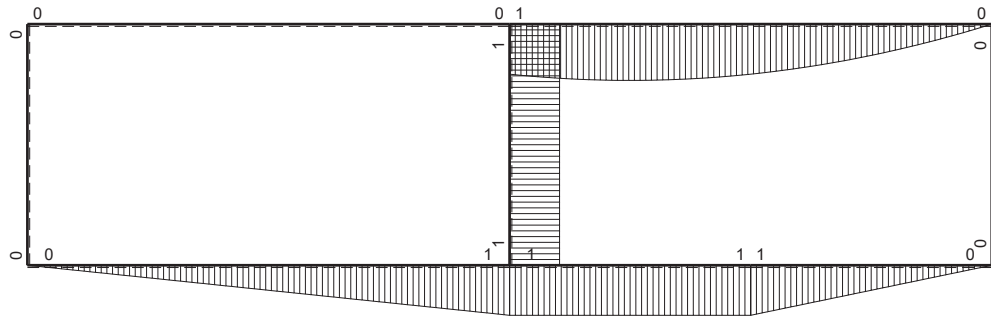
06.09.22



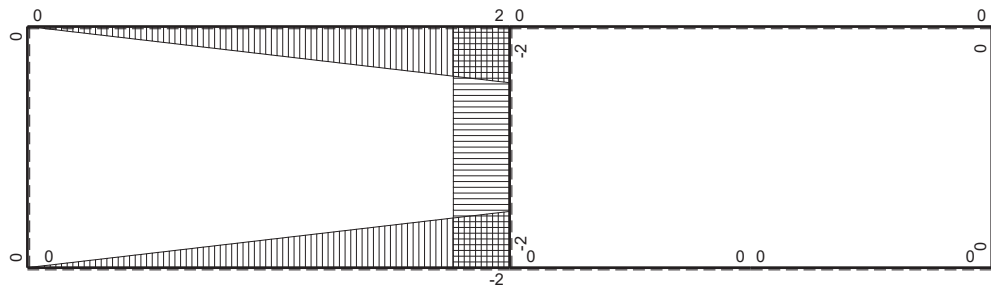
PROCEDIMENTO E RISULTATI



Schema di calcolo iperstatico



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI

Quadro contributi PLV per iperstatica  $X=V_{AE}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB 2b	-x	1/2Fx	-1/2Fx <sup>2</sup>	x <sup>2</sup>	-4/3Fb <sup>3</sup> /EJ	8/3Xb <sup>3</sup> /EJ
BA 2b	2b-x	-Fb+1/2Fx	-2Fb <sup>2</sup> +2Fbx-1/2Fx <sup>2</sup>	4b <sup>2</sup> -4bx+x <sup>2</sup>	-4/3Fb <sup>3</sup> /EJ	8/3Xb <sup>3</sup> /EJ
BC b	0	Fb	0	0	0	0
CB b	0	-Fb	0	0	0	0
CD b	0	Fb-Fx	0	0	0	0
DC b	0	-Fx	0	0	0	0
AE b	0	0	0	0	0	0
EA b	0	0	0	0	0	0
EF 2b	x	0	0	x <sup>2</sup>	0	8/3Xb <sup>3</sup> /EJ
FE 2b	-2b+x	0	0	4b <sup>2</sup> -4bx+x <sup>2</sup>	0	8/3Xb <sup>3</sup> /EJ
FG 2b	0	Fb+1/2Fx-1/2qx <sup>2</sup>	0	0	0	0
GF 2b	0	-3/2Fx+1/2qx <sup>2</sup>	0	0	0	0
BF b	-2b	Fb	-2Fb <sup>2</sup>	4b <sup>2</sup>	-2Fb <sup>3</sup> /EJ	4Xb <sup>3</sup> /EJ
FB b	2b	-Fb	-2Fb <sup>2</sup>	4b <sup>2</sup>	-2Fb <sup>3</sup> /EJ	4Xb <sup>3</sup> /EJ
DG b	0	0	0	0	0	0
GD b	0	0	0	0	0	0
AE	elongazione asta $N_{1AE} \epsilon_{AE} L_{AE}$				Fb <sup>3</sup> /EJ	
FE	molla asta $-W_{1FE}(W_{0FE} + XW_{1FE})/k_{FE}$					Xb <sup>3</sup> /EJ
	totali				-7/3Fb <sup>3</sup> /EJ	31/3Xb <sup>3</sup> /EJ
	iperstatica $X=V_{AE}$				7/31F	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{AB}^{xx} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^{2b} b^2 1/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{EF}^{xx} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx + 2 \cdot 2 \cdot 1/4 b^3/EJ = [1/3 x^3/b^2]_0^{2b} b^2 1/EJ + 2 \cdot 2 \cdot 1/4 b^3/EJ$$

$$= (8/3 b) b^2 1/EJ + 2 \cdot 2 \cdot 1/4 b^3/EJ = 11/3 b^3/EJ$$

$$L_{FE}^{xx} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx + 2 \cdot 2 \cdot 1/4 b^3/EJ$$

$$= [4x - 2x^2/b + 1/3 x^3/b^2]_0^{2b} b^2 1/EJ + 2 \cdot 2 \cdot 1/4 b^3/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ + 2 \cdot 2 \cdot 1/4 b^3/EJ = 11/3 b^3/EJ$$

$$L_{BF}^{xx} = \int_0^b (4) b^2 1/EJ dx = [4x]_0^b b^2 1/EJ$$

$$= (4b) b^2 1/EJ = 4 b^3/EJ$$

$$L_{FB}^{xx} = \int_0^b (4) b^2 1/EJ dx = [4x]_0^b b^2 1/EJ$$

$$= (4b) b^2 1/EJ = 4 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^{2b} (-1/2 x^2/b^2) Fb^2 1/EJ dx = [-1/6 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-4/3 b) Fb^2 1/EJ = -4/3 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^{2b} (-2 + 2x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = [-2x + x^2/b - 1/6 x^3/b^2]_0^{2b} Fb^2 1/EJ$$

$$= (-4b + 4b - 4/3 b) Fb^2 1/EJ = -4/3 Fb^3/EJ$$

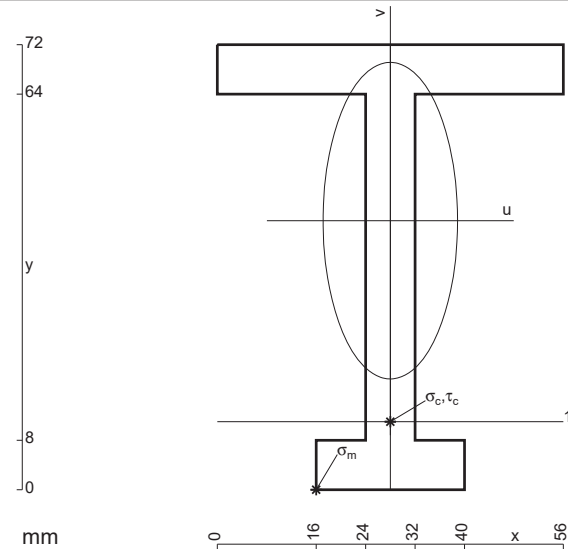
$$L_{BF}^{xo} = \int_0^b (-2) Fb^2 1/EJ dx = [-2x]_0^b Fb^2 1/EJ$$

$$= (-2b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{FB}^{xo} = \int_0^b (-2) Fb^2 1/EJ dx = [-2x]_0^b Fb^2 1/EJ$$

$$= (-2b) Fb^2 1/EJ = -2 Fb^3/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 1088. \text{ mm}^2$$

$$J_u = 714170. \text{ mm}^4$$

$$J_v = 128683. \text{ mm}^4$$

$$y_g = 43.53 \text{ mm}$$

$$T_y = -5820. \text{ N}$$

$$M_x = 3608400. \text{ Nmm}$$

$$x_m = 16. \text{ mm}$$

$$u_m = -12. \text{ mm}$$

$$v_m = -43.53 \text{ mm}$$

$$\sigma_m = -Mv/J_u = 219.9 \text{ N/mm}^2$$

$$x_c = 28. \text{ mm}$$

$$y_c = 11. \text{ mm}$$

$$v_c = -32.53 \text{ mm}$$

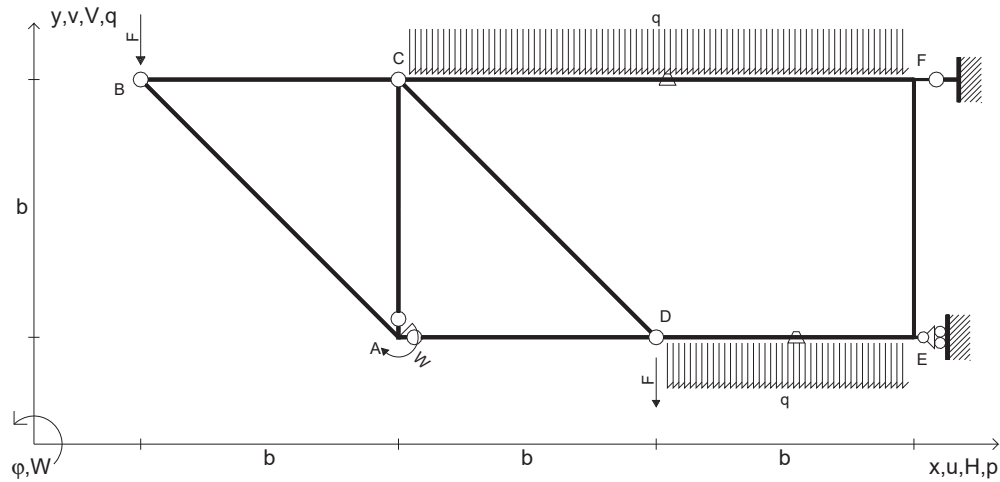
$$\sigma_c = -Mv/J_u = 164.4 \text{ N/mm}^2$$

$$\tau_c = 8.563 \text{ N/mm}^2$$

$$\sigma_p = \sqrt{\sigma^2 + 3\tau^2} = 165. \text{ N/mm}^2$$

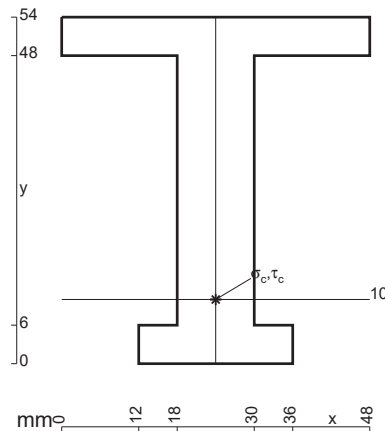
$$S = 8406. \text{ mm}^3$$

TEMA 10

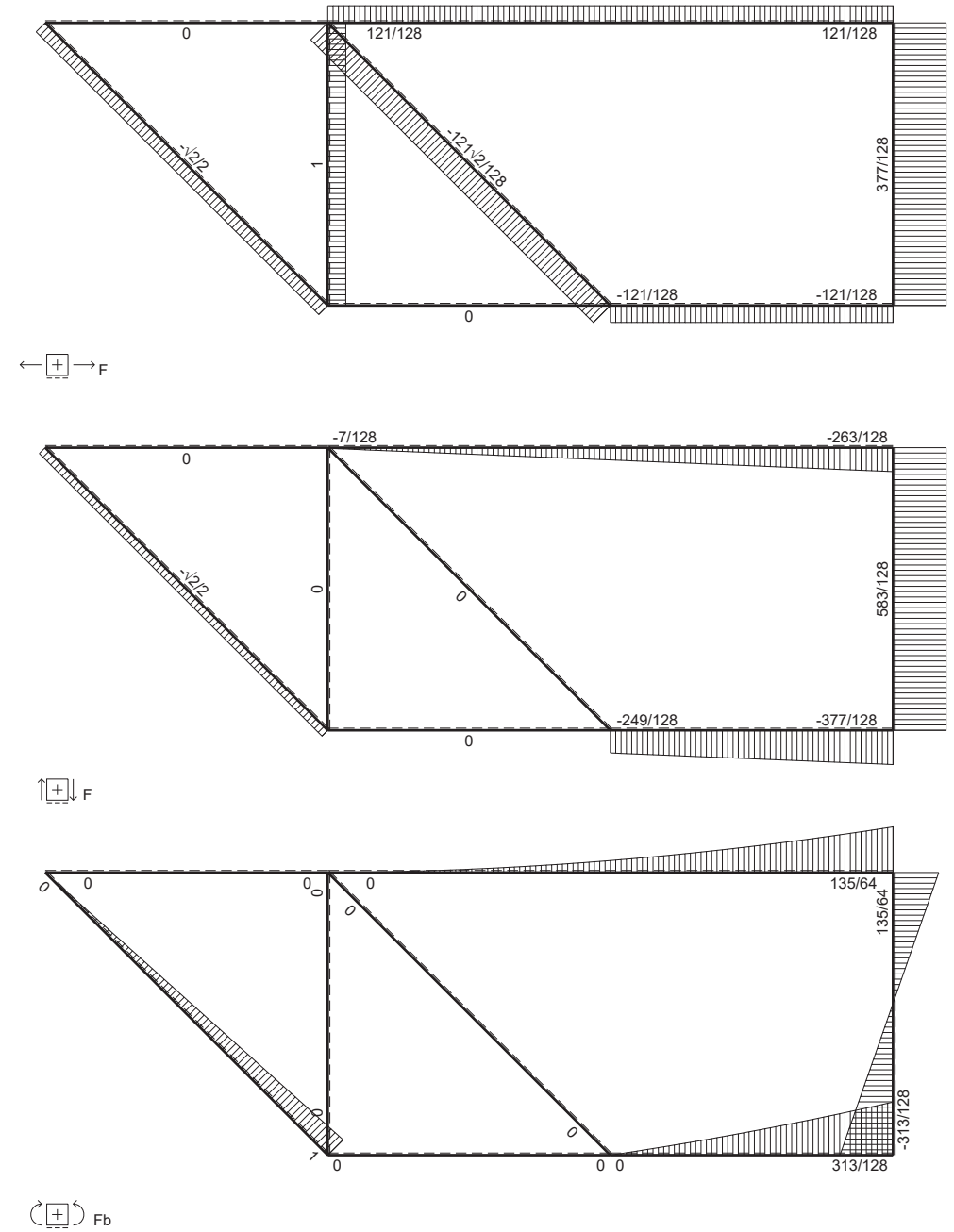
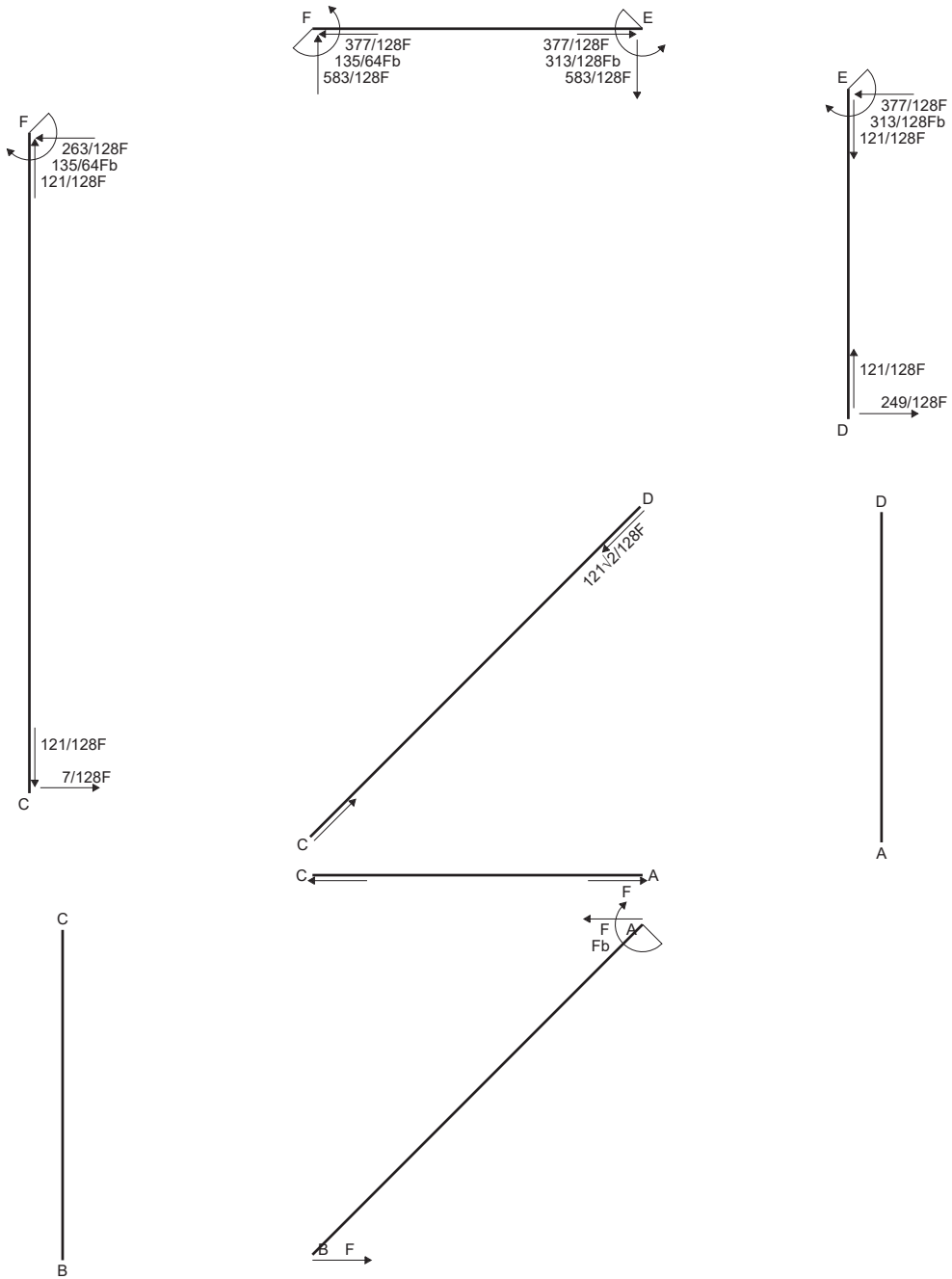


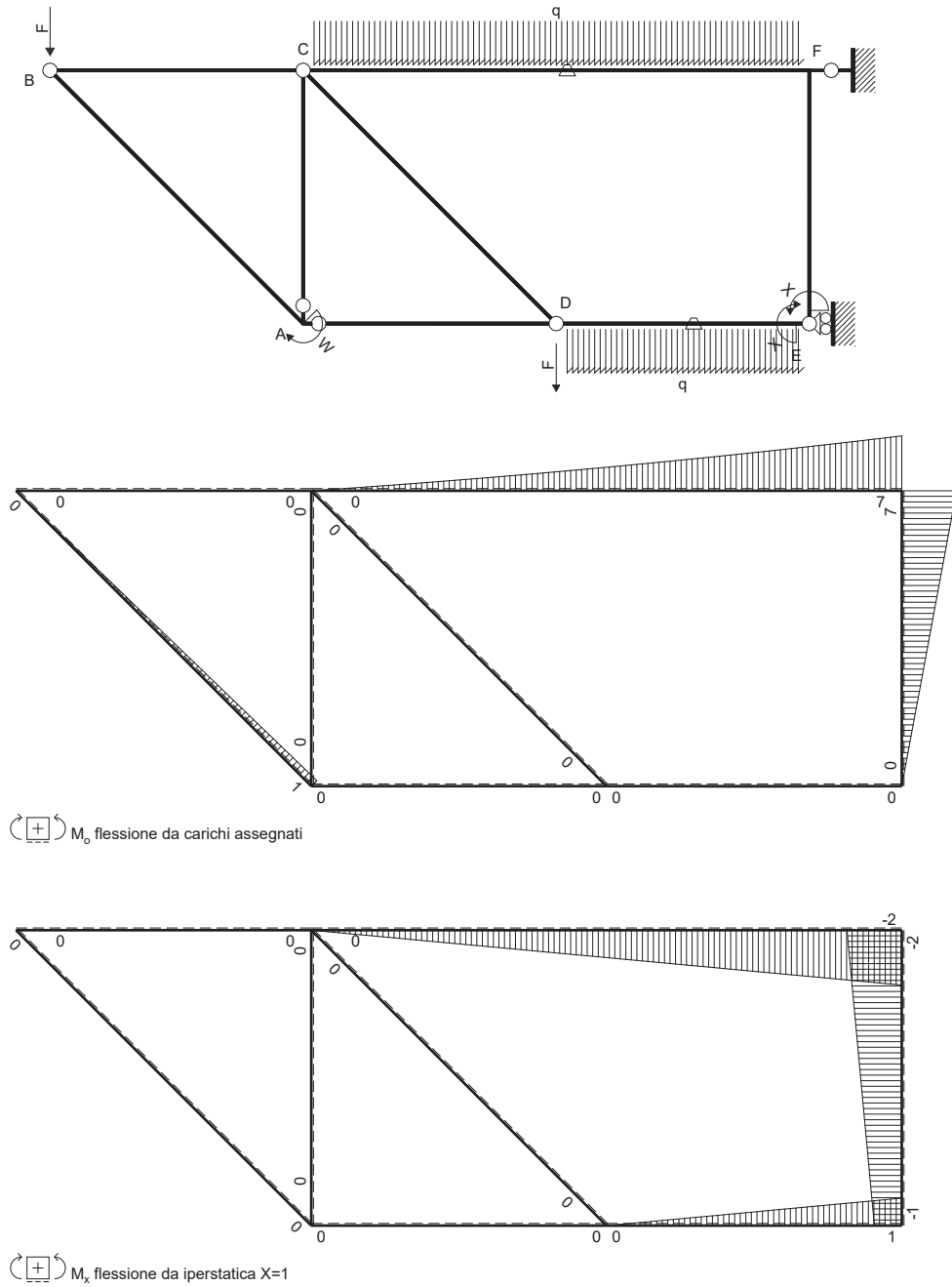
$V_D = -F$	$\theta_{ED} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EF} = EJ$
$V_B = -F$	$\theta_{FC} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{CB} = EJ$
$W_A = -W = -Fb$	$EJ_{AB} = EJ$	$EJ_{FC} = EJ$
$q_{FC} = -q = -F/b$	$EJ_{AC} = EJ$	$EJ_{DA} = EJ$
$q_{ED} = -q = -F/b$	$EJ_{DC} = EJ$	$EJ_{ED} = EJ$

Reazioni iperstatiche in soluzione:  $X=W_{EF}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - X_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave AB ha la sezione riportata e dimensioni in mm, con:  
 $b = 550 \text{ mm}$ ,  $F = 4300 \text{ N}$   
 Calcolare sulla sezione A la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da A a B  
 Curvatura  $\theta$  asta ED positiva se convessa a destra con inizio E.  
 Curvatura  $\theta$  asta FC positiva se convessa a destra con inizio F.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13









Quadro contributi PLV per iperstatica  $X=W_{EF}$

→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ + \theta) dx$	$\int X M_x M_x / EJ dx$
AB $\sqrt{2}b$	0	$Fb - \sqrt{2}/2Fx$	0	0	0	0	0	0
AC b	0	0	0	0	0	0	0+0	0
CA b	0	0	0	0	0	0	0	0
DC $\sqrt{2}b$	0	0	0	0	0	0	0	0
EF b	$-1-x/b$	$7Fx$	0	$-7Fx - 7Fx^2/b$	0	$1+2x/b+x^2/b^2$	$(-35/6+0)Fb^2/EJ$	$7/3Xb/EJ$
FE b	$2-x/b$	$-7Fb+7Fx$	0	$-14Fb+21Fx-7Fx^2/b$	0	$4-4x/b+x^2/b^2$	0	0
CB b	0	0	0	0	0	0	0+0	0
BC b	0	0	0	0	0	0	0	0
FC 2b	$-2+x/b$	$7Fb-9/2Fx+1/2qx^2$	$-Fb/EJ$	$-14Fb+16Fx-11/2Fx^2/b+1/2qx^3/b$	$2Fb/EJ-Fx/EJ$	$4-4x/b+x^2/b^2$	$(-26/3+2)Fb^2/EJ$	$8/3Xb/EJ$
CF 2b	$x/b$	$-5/2Fx-1/2qx^2$	$Fb/EJ$	$-5/2Fx^2/b-1/2qx^3/b$	$Fx/EJ$	$x^2/b^2$	0	0
DA b	0	0	0	0	0	0	0+0	0
AD b	0	0	0	0	0	0	0	0
ED b	$1-x/b$	$-1/2Fx+1/2qx^2$	$-Fb/EJ$	$-1/2Fx+Fx^2/b-1/2qx^3/b$	$-Fb/EJ+Fx/EJ$	$1-2x/b+x^2/b^2$	$(-1/24-1/2)Fb^2/EJ$	$1/3Xb/EJ$
DE b	$-x/b$	$1/2Fx-1/2qx^2$	$Fb/EJ$	$-1/2Fx^2/b+1/2qx^3/b$	$-Fx/EJ$	$x^2/b^2$	0	0
totali							$-313/24Fb^2/EJ$	$16/3Xb/EJ$
iperstatica $X=W_{EF}$							313/128Fb	

## PROCEDIMENTO E RISULTATI

Sviluppi di calcolo iperstatica

$$L_{EF}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) 1/EJ dx = \left[ x + x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b + b + 1/3 b) 1/EJ = 7/3 b/EJ$$

$$L_{FE}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) 1/EJ dx = \left[ 4x - 2x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (4b - 2b + 1/3 b) 1/EJ = 7/3 b/EJ$$

$$L_{FC}^{xx} = \int_0^{2b} (4 - 4x/b + x^2/b^2) 1/EJ dx = \left[ 4x - 2x^2/b + 1/3 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (8b - 8b + 8/3 b) 1/EJ = 8/3 b/EJ$$

$$L_{CF}^{xx} = \int_0^{2b} (x^2/b^2) 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^{2b} 1/EJ$$

$$= (8/3 b) 1/EJ = 8/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = \left[ x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DE}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{EF}^{x_0} = \int_0^b (-7x/b - 7x^2/b^2) Fb 1/EJ dx = \left[ -7/2 x^2/b - 7/3 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-7/2 b - 7/3 b) Fb 1/EJ = -35/6 Fb^2/EJ$$

$$L_{FE}^{x_0} = \int_0^b (-14 + 21x/b - 7x^2/b^2) Fb 1/EJ dx = \left[ -14x + 21/2 x^2/b - 7/3 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (-14b + 21/2 b - 7/3 b) Fb 1/EJ = -35/6 Fb^2/EJ$$

$$L_{FC}^{x_0} = \int_0^{2b} (-14 + 16x/b - 11/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^{2b} (2 - x/b) \theta dx$$

$$= \left[ -14x + 8x^2/b - 11/6 x^3/b^2 + 1/8 x^4/b^3 \right]_0^{2b} Fb 1/EJ + \left[ 2x - 1/2 x^2/b \right]_0^{2b} \theta$$

$$= (-28b + 32b - 44/3 b + 2b) Fb 1/EJ + (4b - 2b) \theta = -20/3 Fb^2/EJ$$

$$L_{CF}^{x_0} = \int_0^{2b} (-5/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^{2b} (-x/b) \theta dx$$

$$= \left[ -5/6 x^3/b^2 - 1/8 x^4/b^3 \right]_0^{2b} Fb 1/EJ + \left[ -1/2 x^2/b \right]_0^{2b} \theta$$

$$= (-20/3 b - 2b) Fb 1/EJ + (-2b) \theta = -20/3 Fb^2/EJ$$

$$L_{ED}^{x_0} = \int_0^b (-1/2 x/b + x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= \left[ -1/4 x^2/b + 1/3 x^3/b^2 - 1/8 x^4/b^3 \right]_0^b Fb 1/EJ + \left[ -x + 1/2 x^2/b \right]_0^b \theta$$

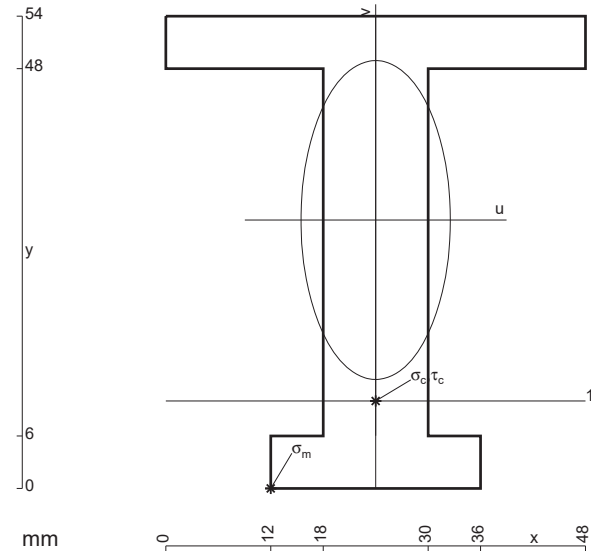
$$= (-1/4 b + 1/3 b - 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = -13/24 Fb^2/EJ$$

$$L_{DE}^{x_0} = \int_0^b (-1/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= \left[ -1/6 x^3/b^2 + 1/8 x^4/b^3 \right]_0^b Fb 1/EJ + \left[ 1/2 x^2/b \right]_0^b \theta$$

$$= (-1/6 b + 1/8 b) Fb 1/EJ + (1/2 b) \theta = -13/24 Fb^2/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 936. \text{ mm}^2$$

$$J_u = 311455. \text{ mm}^4$$

$$J_v = 68256. \text{ mm}^4$$

$$y_g = 30.69 \text{ mm}$$

$$N = -3041. \text{ N}$$

$$T_y = -3041. \text{ N}$$

$$M_x = 2365000. \text{ Nmm}$$

$$x_m = 12. \text{ mm}$$

$$u_m = -12. \text{ mm}$$

$$v_m = -30.69 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = 229.8 \text{ N/mm}^2$$

$$x_c = 24. \text{ mm}$$

$$y_c = 10. \text{ mm}$$

$$v_c = -20.69 \text{ mm}$$

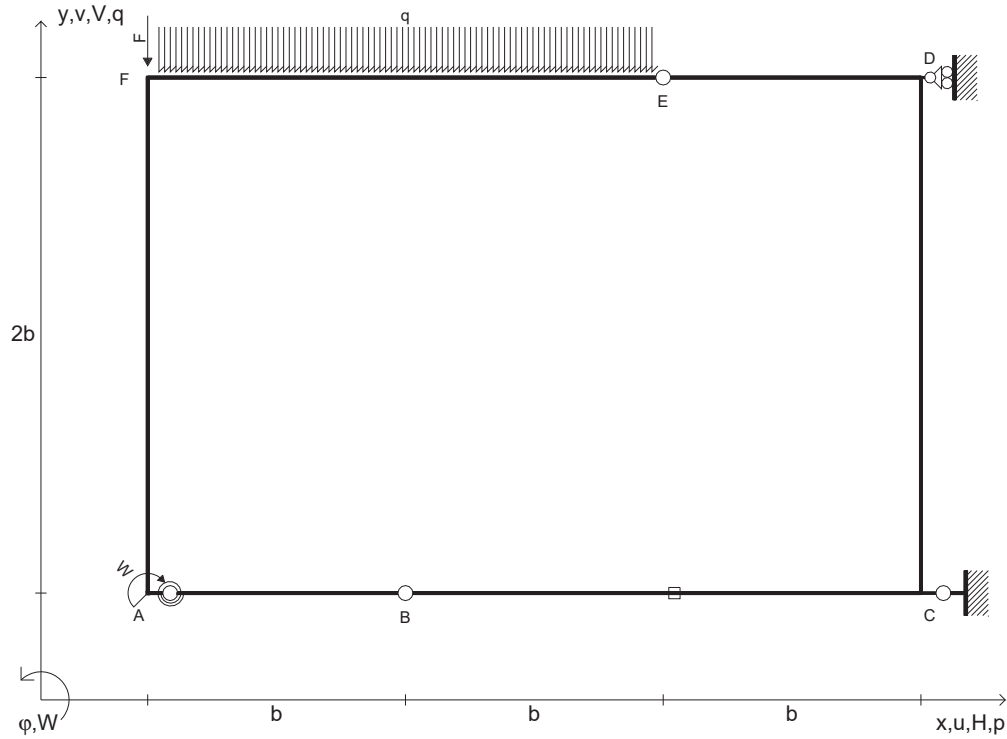
$$\sigma_c = N/A - Mv/J_u = 153.9 \text{ N/mm}^2$$

$$\tau_c = 4.13 \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 154. \text{ N/mm}^2$$

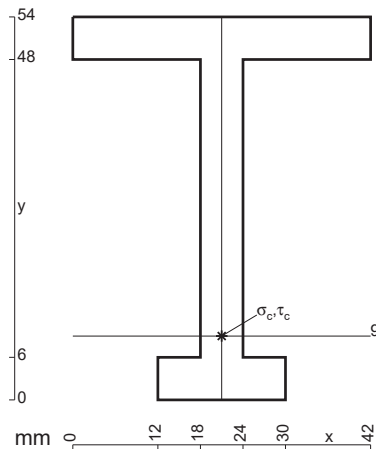
$$S = 5077. \text{ mm}^3$$

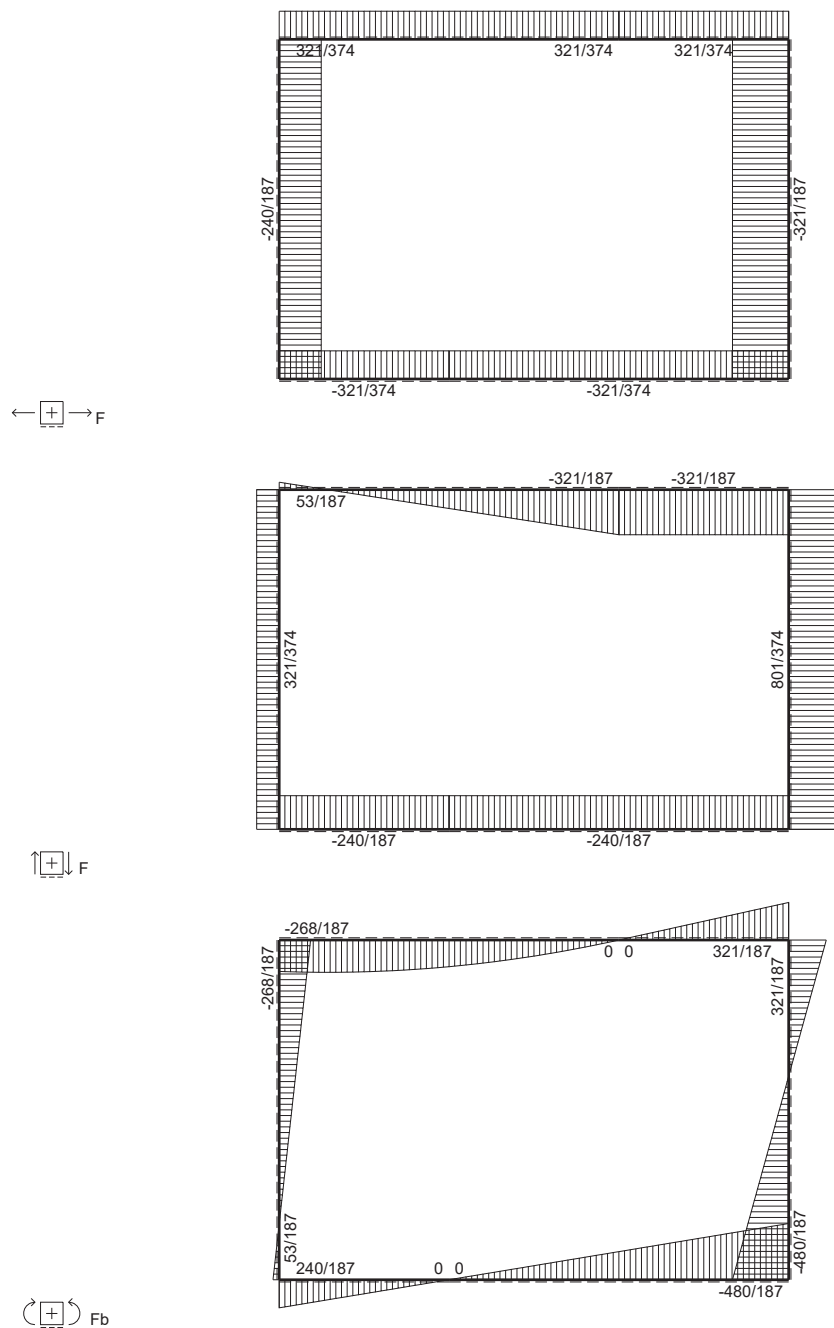
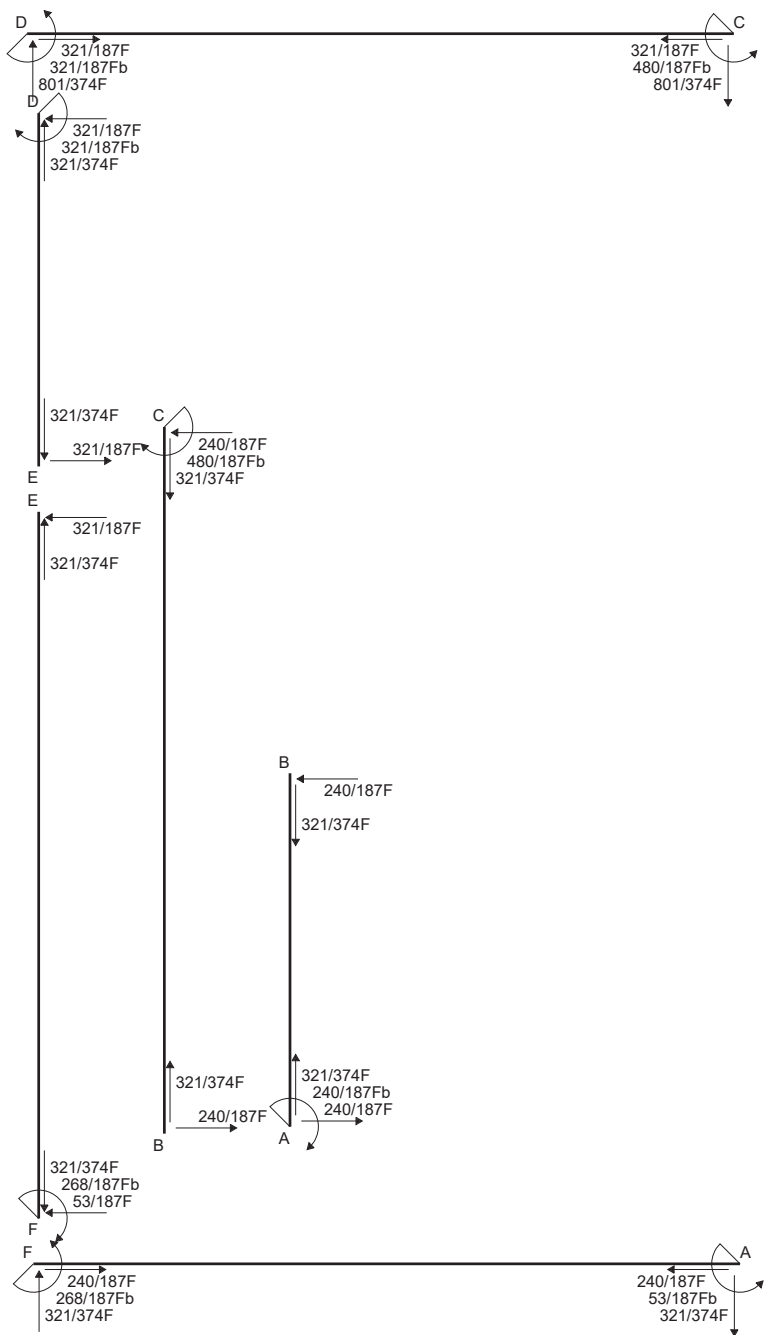
TEMA 11



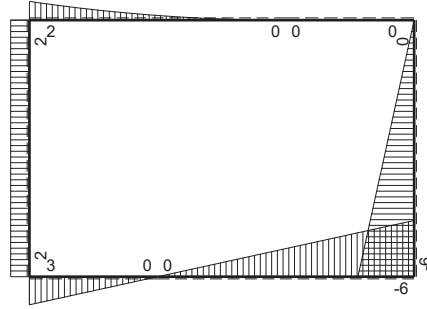
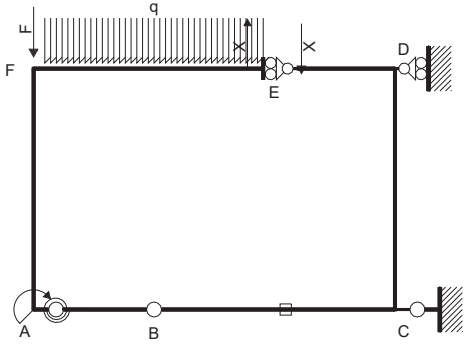
$V_F = -F$	$\epsilon_{BC} = -\alpha T = -b^2 F/EJ$	$EJ_{BC} = EJ$	$EJ_{EF} = EJ$
$W_A = -W = -Fb$	$k_{AB} = 4EJ/b$	$EJ_{CD} = EJ$	$EJ_{FA} = EJ$
$q_{EF} = -q = -F/b$	$EJ_{AB} = EJ$	$EJ_{DE} = EJ$	

Reazioni iperstatiche in soluzione:  $X=V_{EF}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave CD ha la sezione riportata e dimensioni in mm, con:  
 $b = 350 \text{ mm}$ ,  $F = 1580 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da C a D  
 Elongazione termica specifica  $\epsilon$  assegnata su asta BC.

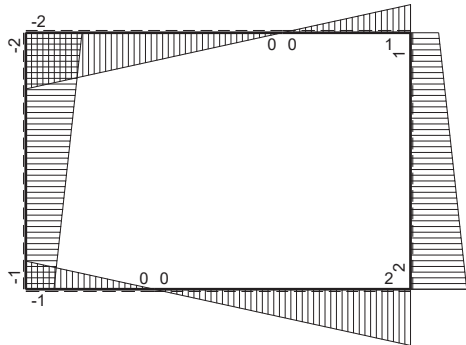




PROCEDIMENTO E RISULTATI



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI

Quadro contribuiti PLV per iperstatica X=V<sub>EF</sub>

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	-b+x	3Fb-3Fx	-3Fb <sup>2</sup> +6Fbx-3Fx <sup>2</sup>	b <sup>2</sup> -2bx+x <sup>2</sup>	-Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
BA b	x	-3Fx	-3Fx <sup>2</sup>	x <sup>2</sup>	-8Fb <sup>3</sup> /EJ	8/3Xb <sup>3</sup> /EJ
BC 2b	x	-3Fx	-3Fx <sup>2</sup>	x <sup>2</sup>	-10Fb <sup>3</sup> /EJ	14/3Xb <sup>3</sup> /EJ
CB 2b	-2b+x	6Fb-3Fx	-12Fb <sup>2</sup> +12Fbx-3Fx <sup>2</sup>	4b <sup>2</sup> -4bx+x <sup>2</sup>	-10Fb <sup>3</sup> /EJ	14/3Xb <sup>3</sup> /EJ
CD 2b	2b-1/2x	-6Fb+3Fx	-12Fb <sup>2</sup> +9Fbx-3/2Fx <sup>2</sup>	4b <sup>2</sup> -2bx+1/4x <sup>2</sup>	-10Fb <sup>3</sup> /EJ	14/3Xb <sup>3</sup> /EJ
DC 2b	-b-1/2x	3Fx	-3Fbx-3/2Fx <sup>2</sup>	b <sup>2</sup> +bx+1/4x <sup>2</sup>	-10Fb <sup>3</sup> /EJ	14/3Xb <sup>3</sup> /EJ
DE b	b-x	0	0	b <sup>2</sup> -2bx+x <sup>2</sup>	0	1/3Xb <sup>3</sup> /EJ
ED b	-x	0	0	x <sup>2</sup>	0	1/3Xb <sup>3</sup> /EJ
EF 2b	-x	1/2qx <sup>2</sup>	-1/2qx <sup>3</sup>	x <sup>2</sup>	-2Fb <sup>3</sup> /EJ	8/3Xb <sup>3</sup> /EJ
FE 2b	2b-x	-2Fb+2Fx-1/2qx <sup>2</sup>	-4Fb <sup>2</sup> +6Fbx-3Fx <sup>2</sup> +1/2qx <sup>3</sup>	4b <sup>2</sup> -4bx+x <sup>2</sup>	-2Fb <sup>3</sup> /EJ	8/3Xb <sup>3</sup> /EJ
FA 2b	-2b+1/2x	2Fb	-4Fb <sup>2</sup> +Fbx	4b <sup>2</sup> -2bx+1/4x <sup>2</sup>	-6Fb <sup>3</sup> /EJ	14/3Xb <sup>3</sup> /EJ
AF 2b	b+1/2x	-2Fb	-2Fb <sup>2</sup> -Fbx	b <sup>2</sup> +bx+1/4x <sup>2</sup>	-6Fb <sup>3</sup> /EJ	14/3Xb <sup>3</sup> /EJ
AB	molla asta $-W_{iAB}(W_{oAB}+XW_{iAB})/k_{iAB}$				-3/4Fb <sup>3</sup> /EJ	1/4Xb <sup>3</sup> /EJ
BC	elongazione asta $N_{iBC} \epsilon_{BC-BC}$				Fb <sup>3</sup> /EJ	
	totali				-107/4Fb <sup>3</sup> /EJ	187/12Xb <sup>3</sup> /EJ
	iperstatica X=V <sub>EF</sub>				321/187F	

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{AB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx + 1 \cdot 1/4 \cdot b^3/EJ$$

$$= \left[ x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ + 1 \cdot 1/4 \cdot b^3/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ + 1 \cdot 1/4 \cdot b^3/EJ = 7/12 b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx + 1 \cdot 1 \cdot 1/4 \cdot b^3/EJ = \left[ 1/3 x^3/b^2 \right]_0^b b^2 1/EJ + 1 \cdot 1/4 \cdot b^3/EJ$$

$$= (1/3 b) b^2 1/EJ + 1 \cdot 1/4 \cdot b^3/EJ = 7/12 b^3/EJ$$

$$L_{BC}^{xx} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = \left[ 4x - 2x^2/b + 1/3 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^{2b} (4 - 2x/b + 1/4 x^2/b^2) b^2 1/EJ dx = \left[ 4x - x^2/b + 1/12 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (8b - 4b + 2/3 b) b^2 1/EJ = 14/3 b^3/EJ$$

$$L_{DC}^{xx} = \int_0^{2b} (1 + x/b + 1/4 x^2/b^2) b^2 1/EJ dx = \left[ x + 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (2b + 2b + 2/3 b) b^2 1/EJ = 14/3 b^3/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = \left[ x - x^2/b + 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{EF}^{xx} = \int_0^{2b} (x^2/b^2) b^2 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{FE}^{xx} = \int_0^{2b} (4 - 4x/b + x^2/b^2) b^2 1/EJ dx = \left[ 4x - 2x^2/b + 1/3 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (8b - 8b + 8/3 b) b^2 1/EJ = 8/3 b^3/EJ$$

$$L_{FA}^{xx} = \int_0^{2b} (4 - 2x/b + 1/4 x^2/b^2) b^2 1/EJ dx = \left[ 4x - x^2/b + 1/12 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (8b - 4b + 2/3 b) b^2 1/EJ = 14/3 b^3/EJ$$

$$L_{AF}^{xx} = \int_0^{2b} (1 + x/b + 1/4 x^2/b^2) b^2 1/EJ dx = \left[ x + 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^{2b} b^2 1/EJ$$

$$= (2b + 2b + 2/3 b) b^2 1/EJ = 14/3 b^3/EJ$$

$$L_{AB}^{xo} = \int_0^b (-3 + 6x/b - 3x^2/b^2) Fb^2 1/EJ dx + 1 \cdot (-3) \cdot 1/4 \cdot Fb^3/EJ$$

$$= \left[ -3x + 3x^2/b - x^3/b^2 \right]_0^b Fb^2 1/EJ + 1 \cdot (-3) \cdot 1/4 \cdot Fb^3/EJ$$

$$= (-3b + 3b - b) Fb^2 1/EJ + 1 \cdot (-3) \cdot 1/4 \cdot Fb^3/EJ = -7/4 Fb^3/EJ$$

$$L_{BA}^{xo} = \int_0^b (-3x^2/b^2) Fb^2 1/EJ dx + 1 \cdot (-3) \cdot 1/4 \cdot Fb^3/EJ = \left[ -x^3/b^2 \right]_0^b Fb^2 1/EJ + 1 \cdot (-3) \cdot 1/4 \cdot Fb^3/EJ$$

## PROCEDIMENTO E RISULTATI

$$= (-b) Fb^2 1/EJ + 1 \cdot (-3) \cdot 1/4 \cdot Fb^3/EJ = -7/4 Fb^3/EJ$$

$$L_{BC}^{xo} = \int_0^{2b} (-3x^2/b^2) Fb^2 1/EJ dx - 1/2 \cdot (-1) \cdot 2 \cdot Fb^3/EJ = \left[ -x^3/b^2 \right]_0^{2b} Fb^2 1/EJ - 1/2 \cdot (-1) \cdot 2 \cdot Fb^3/EJ$$

$$= (-8b) Fb^2 1/EJ - 1/2 \cdot (-1) \cdot 2 \cdot Fb^3/EJ = -7 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^{2b} (-12 + 12x/b - 3x^2/b^2) Fb^2 1/EJ dx - 1/2 \cdot (-1) \cdot 2 \cdot Fb^3/EJ$$

$$= \left[ -12x + 6x^2/b - x^3/b^2 \right]_0^{2b} Fb^2 1/EJ - 1/2 \cdot (-1) \cdot 2 \cdot Fb^3/EJ$$

$$= (-24b + 24b - 8b) Fb^2 1/EJ - 1/2 \cdot (-1) \cdot 2 \cdot Fb^3/EJ = -7 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^{2b} (-12 + 9x/b - 3/2 x^2/b^2) Fb^2 1/EJ dx = \left[ -12x + 9/2 x^2/b - 1/2 x^3/b^2 \right]_0^{2b} Fb^2 1/EJ$$

$$= (-24b + 18b - 4b) Fb^2 1/EJ = -10 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^{2b} (-3x/b - 3/2 x^2/b^2) Fb^2 1/EJ dx = \left[ -3/2 x^2/b - 1/2 x^3/b^2 \right]_0^{2b} Fb^2 1/EJ$$

$$= (-6b - 4b) Fb^2 1/EJ = -10 Fb^3/EJ$$

$$L_{EF}^{xo} = \int_0^{2b} (-1/2 x^3/b^3) Fb^2 1/EJ dx = \left[ -1/8 x^4/b^3 \right]_0^{2b} Fb^2 1/EJ$$

$$= (-2b) Fb^2 1/EJ = -2 Fb^3/EJ$$

$$L_{FE}^{xo} = \int_0^{2b} (-4 + 6x/b - 3x^2/b^2 + 1/2 x^3/b^3) Fb^2 1/EJ dx = \left[ -4x + 3x^2/b - x^3/b^2 + 1/8 x^4/b^3 \right]_0^{2b} Fb^2 1/EJ$$

$$= (-8b + 12b - 8b + 2b) Fb^2 1/EJ = -2 Fb^3/EJ$$

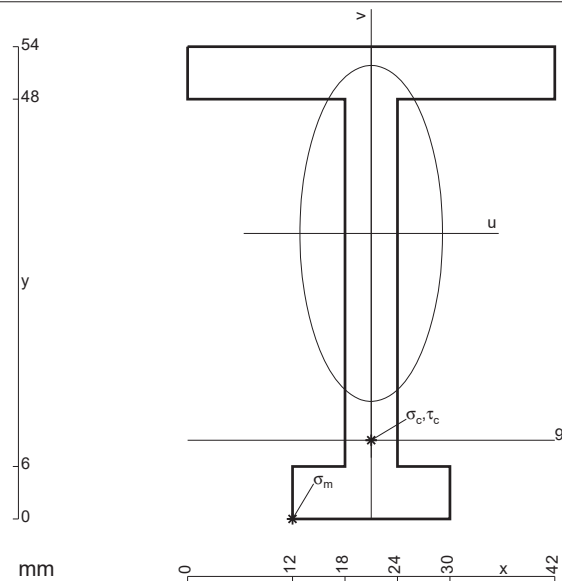
$$L_{FA}^{xo} = \int_0^{2b} (-4 + x/b) Fb^2 1/EJ dx = \left[ -4x + 1/2 x^2/b \right]_0^{2b} Fb^2 1/EJ$$

$$= (-8b + 2b) Fb^2 1/EJ = -6 Fb^3/EJ$$

$$L_{AF}^{xo} = \int_0^{2b} (-2 - x/b) Fb^2 1/EJ dx = \left[ -2x - 1/2 x^2/b \right]_0^{2b} Fb^2 1/EJ$$

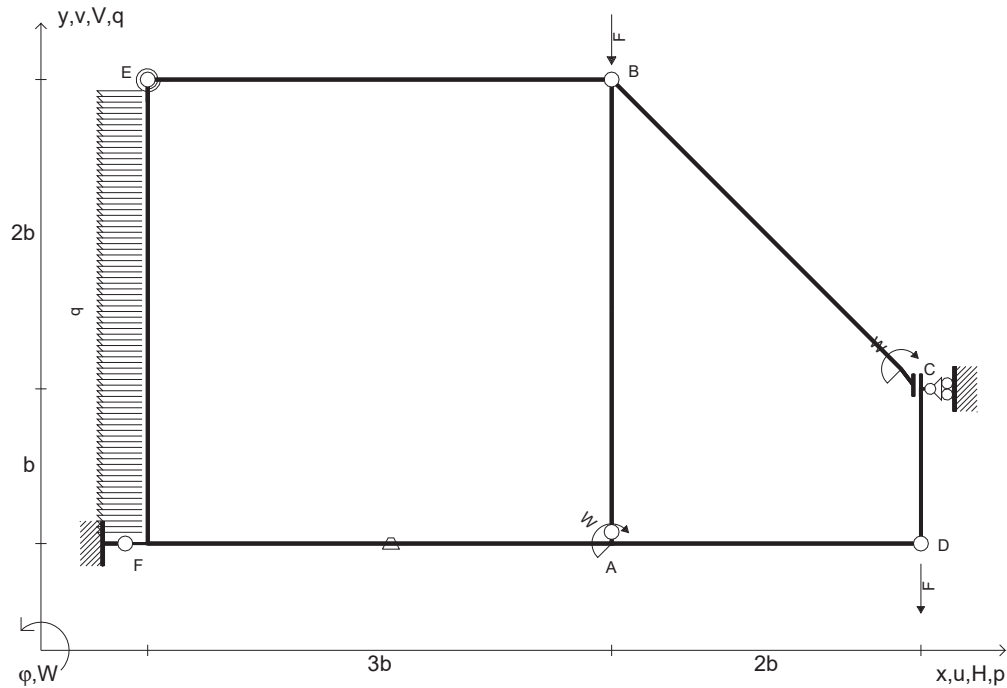
$$= (-4b - 2b) Fb^2 1/EJ = -6 Fb^3/EJ$$

## PROCEDIMENTO E RISULTATI



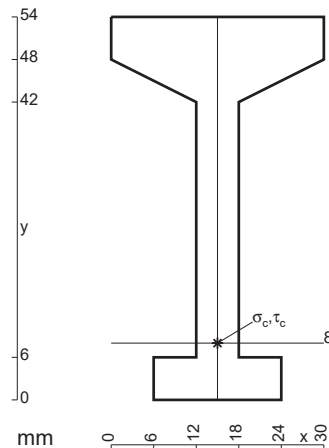
$$\begin{aligned}
 A &= 612. \text{ mm}^2 \\
 J_u &= 225968. \text{ mm}^4 \\
 J_v &= 40716. \text{ mm}^4 \\
 y_g &= 32.65 \text{ mm} \\
 N &= -2712. \text{ N} \\
 T_y &= 3384. \text{ N} \\
 M_x &= -1419470. \text{ Nmm} \\
 x_m &= 12. \text{ mm} \\
 u_m &= -9. \text{ mm} \\
 v_m &= -32.65 \text{ mm} \\
 \sigma_m &= N/A - Mv/J_u = -209.5 \text{ N/mm}^2 \\
 x_c &= 21. \text{ mm} \\
 y_c &= 9. \text{ mm} \\
 v_c &= -23.65 \text{ mm} \\
 \sigma_c &= N/A - Mv/J_u = -153. \text{ N/mm}^2 \\
 \tau_c &= 9.121 \text{ N/mm}^2 \\
 \sigma_\phi &= \sqrt{\sigma^2 + 3\tau^2} = 153.8 \text{ N/mm}^2 \\
 S^* &= 3655. \text{ mm}^3
 \end{aligned}$$

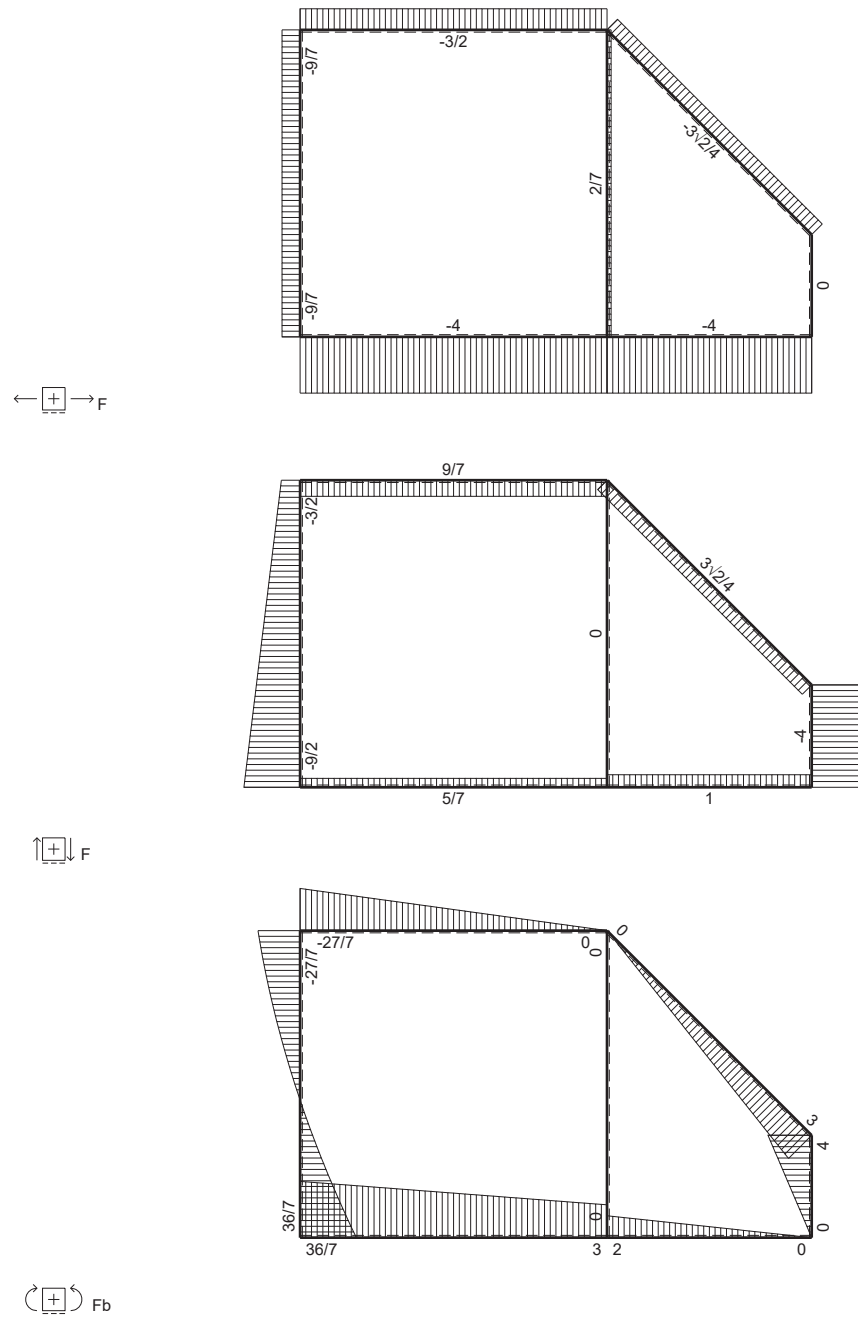
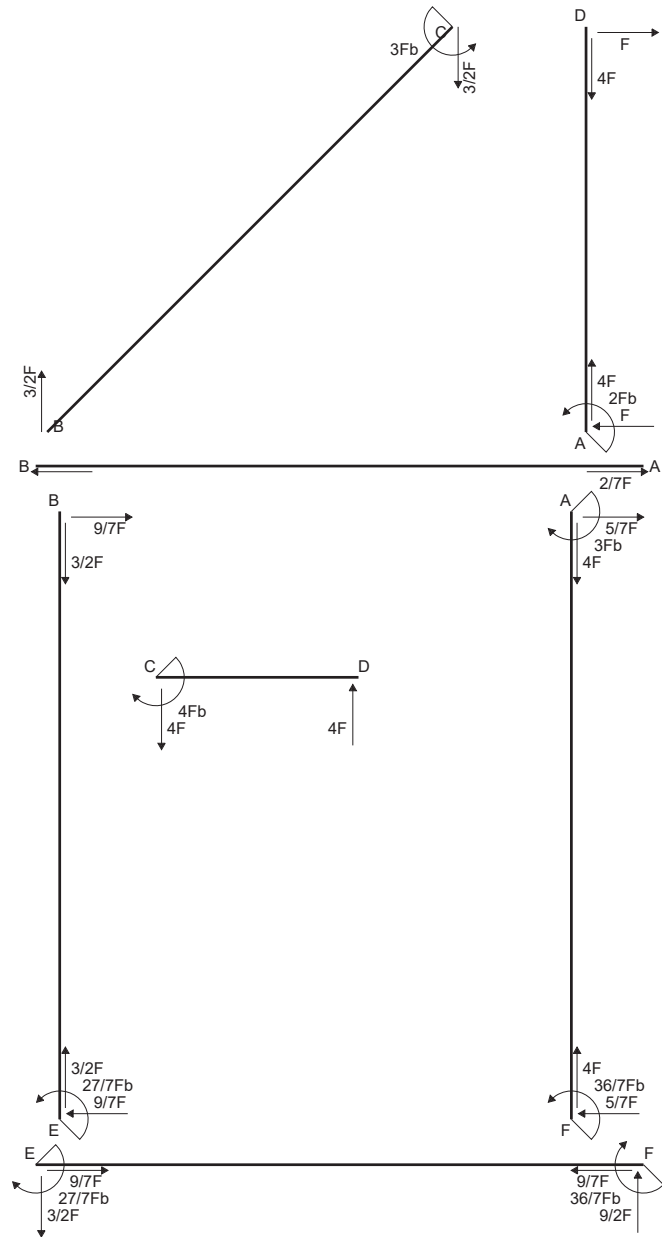




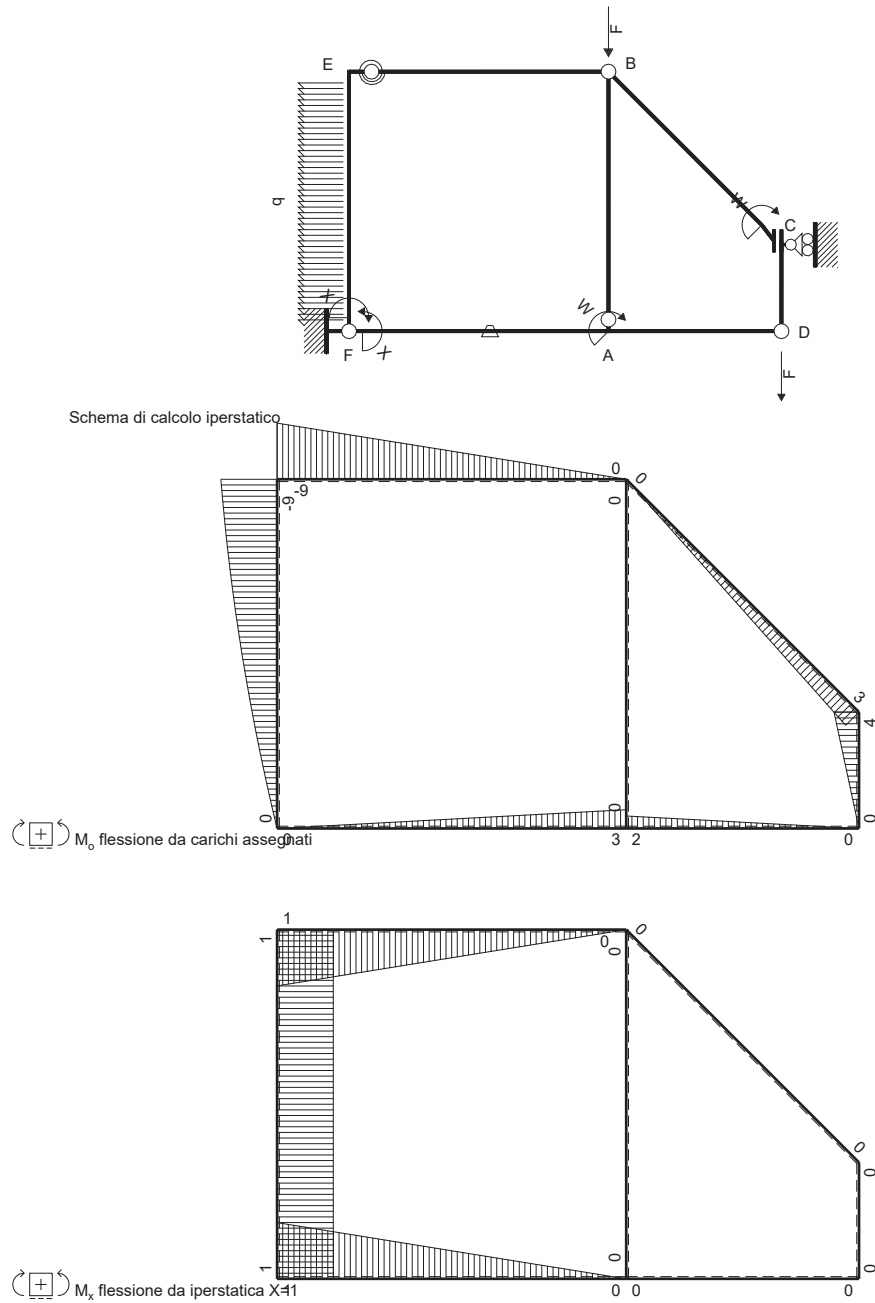
$V_D = -F$	$\theta_{AF} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{EB} = EJ$
$V_B = -F$	$K_{EB} = 4EJ/b$	$EJ_{FE} = EJ$
$W_{CB} = -W = -Fb$	$EJ_{AB} = EJ$	$EJ_{AF} = EJ$
$W_A = -W = -Fb$	$EJ_{CD} = EJ$	$EJ_{DA} = EJ$
$p_{FE} = -q = -F/b$	$EJ_{BC} = EJ$	

Reazioni iperstatiche in soluzione:  $X=W_{FA}$   
 Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
 La trave CD ha la sezione riportata e dimensioni in mm, con:  
 $b = 250 \text{ mm}$ ,  $F = 1470 \text{ N}$   
 Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .  
 Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.  
 Lembo inferiore sezione su tratteggio trave, a destra da C a D  
 Curvatura  $\theta$  asta AF positiva se convessa a destra con inizio A.  
 @ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13





$\left[ \begin{array}{c} + \\ \leftarrow \end{array} \right] F$



Quadro contribuiti PLV per iperstatica  $X=W_{FA}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ+\theta) dx$	$\int X M_x M_x / EJ dx$
AB 3b	0	0	0	0	0	0	0+0	0
BA 3b	0	0	0	0	0	0	0+0	0
CD b	0	$4Fb-4Fx$	0	0	0	0	0	0
DC b	0	$-4Fx$	0	0	0	0	0	0
BC $2\sqrt{2}b$	0	$3\sqrt{2}/4Fx$	0	0	0	0	0	0
EB 3b	$1-1/3x/b$	$-9Fb+3Fx$	0	$-9Fb+6Fx-Fx^2/b$	0	$1-2/3x/b+1/9x^2/b^2$	$(-9+0)Fb^2/EJ$	$Xb/EJ$
BE 3b	$-1/3x/b$	$3Fx$	0	$-Fx^2/b$	0	$1/9x^2/b^2$	0	0
FE 3b	1	$-9/2Fx+1/2qx^2$	0	$-9/2Fx+1/2Fx^2/b$	0	1	$(-63/4+0)Fb^2/EJ$	$3Xb/EJ$
EF 3b	-1	$9Fb-3/2Fx-1/2qx^2$	0	$-9Fb+3/2Fx+1/2Fx^2/b$	0	1	0	0
AF 3b	$1/3x/b$	$3Fb-Fx$	$-Fb/EJ$	$Fx-1/3Fx^2/b$	$-1/3Fx/EJ$	$1/9x^2/b^2$	$(3/2-3/2)Fb^2/EJ$	$Xb/EJ$
FA 3b	$-1+1/3x/b$	$-Fx$	$Fb/EJ$	$Fx-1/3Fx^2/b$	$-Fb/EJ+1/3Fx/EJ$	$1-2/3x/b+1/9x^2/b^2$	0	0
DA 2b	0	$Fx$	0	0	0	0	0+0	0
AD 2b	0	$-2Fb+Fx$	0	0	0	0	0	0
EB	molla asta $-W_{IEB}(W_{0EB}+XW_{IEB})/k_{EB}$						$-9/4Fb^2/EJ$	$1/4Xb/EJ$
	totali						$-27Fb^2/EJ$	$21/4Xb/EJ$
	iperstatica $X=W_{FA}$						$36/7Fb$	

## PROCEDIMENTO E RISULTATI

Sviluppi di calcolo iperstatica

$$L_{-EB}^{XX} = \int_0^{3b} (1 - 2/3 x/b + 1/9 x^2/b^2) 1/EJ dx - 1 (-1) 1/4 b/EJ$$

$$= [x - 1/3 x^2/b + 1/27 x^3/b^2]_0^{3b} 1/EJ - 1 (-1) 1/4 b/EJ$$

$$= (3b - 3b + b) 1/EJ - 1 (-1) 1/4 b/EJ = 5/4 b/EJ$$

$$L_{-BE}^{XX} = \int_0^{3b} (1/9 x^2/b^2) 1/EJ dx - 1 (-1) 1/4 b/EJ = [1/27 x^3/b^2]_0^{3b} 1/EJ - 1 (-1) 1/4 b/EJ$$

$$= (b) 1/EJ - 1 (-1) 1/4 b/EJ = 5/4 b/EJ$$

$$L_{-FE}^{XX} = \int_0^{3b} (1) 1/EJ dx = [x]_0^{3b} 1/EJ$$

$$= (3b) 1/EJ = 3 b/EJ$$

$$L_{-EF}^{XX} = \int_0^{3b} (1) 1/EJ dx = [x]_0^{3b} 1/EJ$$

$$= (3b) 1/EJ = 3 b/EJ$$

$$L_{-AF}^{XX} = \int_0^{3b} (1/9 x^2/b^2) 1/EJ dx = [1/27 x^3/b^2]_0^{3b} 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{-FA}^{XX} = \int_0^{3b} (1 - 2/3 x/b + 1/9 x^2/b^2) 1/EJ dx = [x - 1/3 x^2/b + 1/27 x^3/b^2]_0^{3b} 1/EJ$$

$$= (3b - 3b + b) 1/EJ = b/EJ$$

$$L_{-EB}^{Xo} = \int_0^{3b} (-9 + 6x/b - x^2/b^2) Fb 1/EJ dx - 1 \cdot 9 \cdot 1/4 Fb^2/EJ$$

$$= [-9x + 3x^2/b - 1/3 x^3/b^2]_0^{3b} Fb 1/EJ - 1 \cdot 9 \cdot 1/4 Fb^2/EJ$$

$$= (-27b + 27b - 9b) Fb 1/EJ - 1 \cdot 9 \cdot 1/4 Fb^2/EJ = -45/4 Fb^2/EJ$$

$$L_{-BE}^{Xo} = \int_0^{3b} (-x^2/b^2) Fb 1/EJ dx - 1 \cdot 9 \cdot 1/4 Fb^2/EJ = [-1/3 x^3/b^2]_0^{3b} Fb 1/EJ - 1 \cdot 9 \cdot 1/4 Fb^2/EJ$$

$$= (-9b) Fb 1/EJ - 1 \cdot 9 \cdot 1/4 Fb^2/EJ = -45/4 Fb^2/EJ$$

$$L_{-FE}^{Xo} = \int_0^{3b} (-9/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [-9/4 x^2/b + 1/6 x^3/b^2]_0^{3b} Fb 1/EJ$$

$$= (-81/4 b + 9/2 b) Fb 1/EJ = -63/4 Fb^2/EJ$$

$$L_{-EF}^{Xo} = \int_0^{3b} (-9 + 3/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [-9x + 3/4 x^2/b + 1/6 x^3/b^2]_0^{3b} Fb 1/EJ$$

$$= (-27b + 27/4 b + 9/2 b) Fb 1/EJ = -63/4 Fb^2/EJ$$

$$L_{-AF}^{Xo} = \int_0^{3b} (x/b - 1/3 x^2/b^2) Fb 1/EJ dx + \int_0^{3b} (-1/3 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/9 x^3/b^2]_0^{3b} Fb 1/EJ + [-1/6 x^2/b]_0^{3b} \theta$$

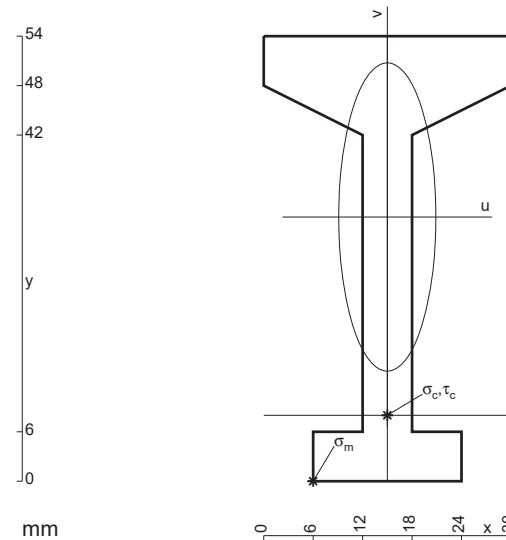
$$= (9/2 b - 3b) Fb 1/EJ + (-3/2 b) \theta = 0$$

$$L_{-FA}^{Xo} = \int_0^{3b} (x/b - 1/3 x^2/b^2) Fb 1/EJ dx + \int_0^{3b} (1 - 1/3 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/9 x^3/b^2]_0^{3b} Fb 1/EJ + [x - 1/6 x^2/b]_0^{3b} \theta$$

$$= (9/2 b - 3b) Fb 1/EJ + (3b - 3/2 b) \theta = 0$$

## PROCEDIMENTO E RISULTATI



$$A = 612. \text{ mm}^2$$

$$J_u = 214270. \text{ mm}^4$$

$$J_v = 21276. \text{ mm}^4$$

$$y_g = 32.06 \text{ mm}$$

$$T_y = -5880. \text{ N}$$

$$M_x = 1470000. \text{ Nmm}$$

$$x_m = 6. \text{ mm}$$

$$u_m = -9. \text{ mm}$$

$$v_m = -32.06 \text{ mm}$$

$$\sigma_m = -Mv/J_u = 219.9 \text{ N/mm}^2$$

$$x_c = 15. \text{ mm}$$

$$y_c = 8. \text{ mm}$$

$$v_c = -24.06 \text{ mm}$$

$$\sigma_c = -Mv/J_u = 165.1 \text{ N/mm}^2$$

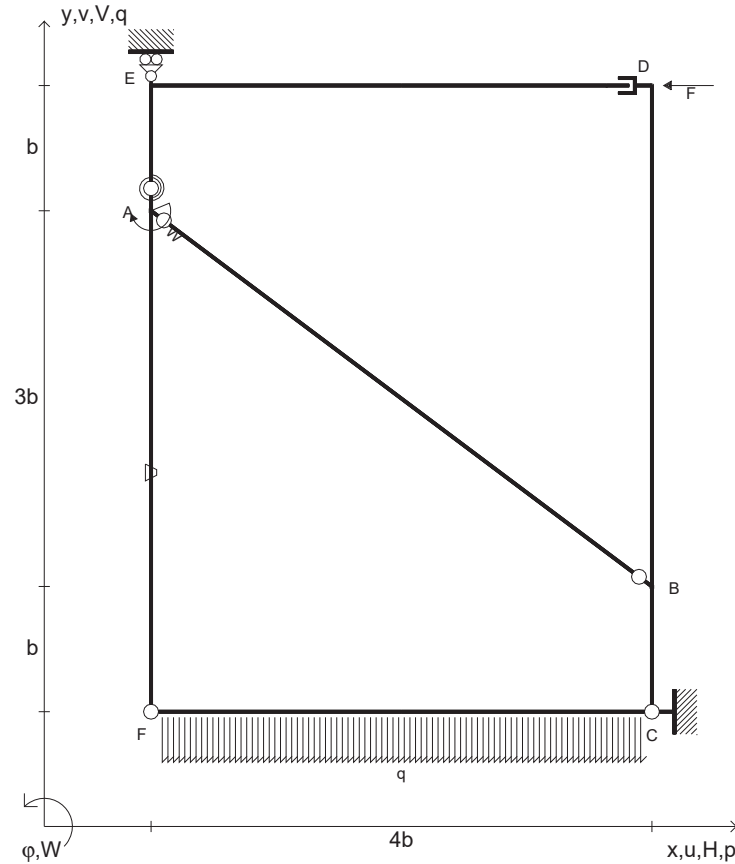
$$\tau_c = 15.73 \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 167.3 \text{ N/mm}^2$$

$$S = 3439. \text{ mm}^3$$

# TEMA 13

$$\begin{aligned}
 H_{DB} &= -F \\
 W_A &= -W = -Fb \\
 q_{CF} &= -q = -F/b \\
 \theta_{FA} &= -\theta = -\alpha T/b = -bF/EJ \\
 k_{AE} &= 4EJ/b \\
 EJ_{AB} &= EJ \\
 EJ_{BC} &= EJ \\
 EJ_{DB} &= EJ \\
 EJ_{ED} &= EJ \\
 EJ_{AE} &= EJ \\
 EJ_{FA} &= EJ \\
 EJ_{CF} &= EJ
 \end{aligned}$$



Reazioni iperstatiche in soluzione:  $X = V_{DE}$

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

La trave CF ha la sezione riportata e dimensioni in mm, con:

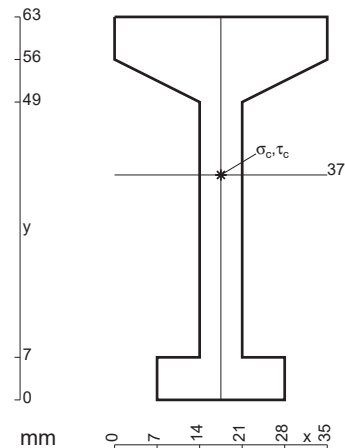
$b = 220 \text{ mm}$ ,  $F = 382070 \text{ N}$

Calcolare sulla sezione C la massima tensione normale  $\sigma_m$ .

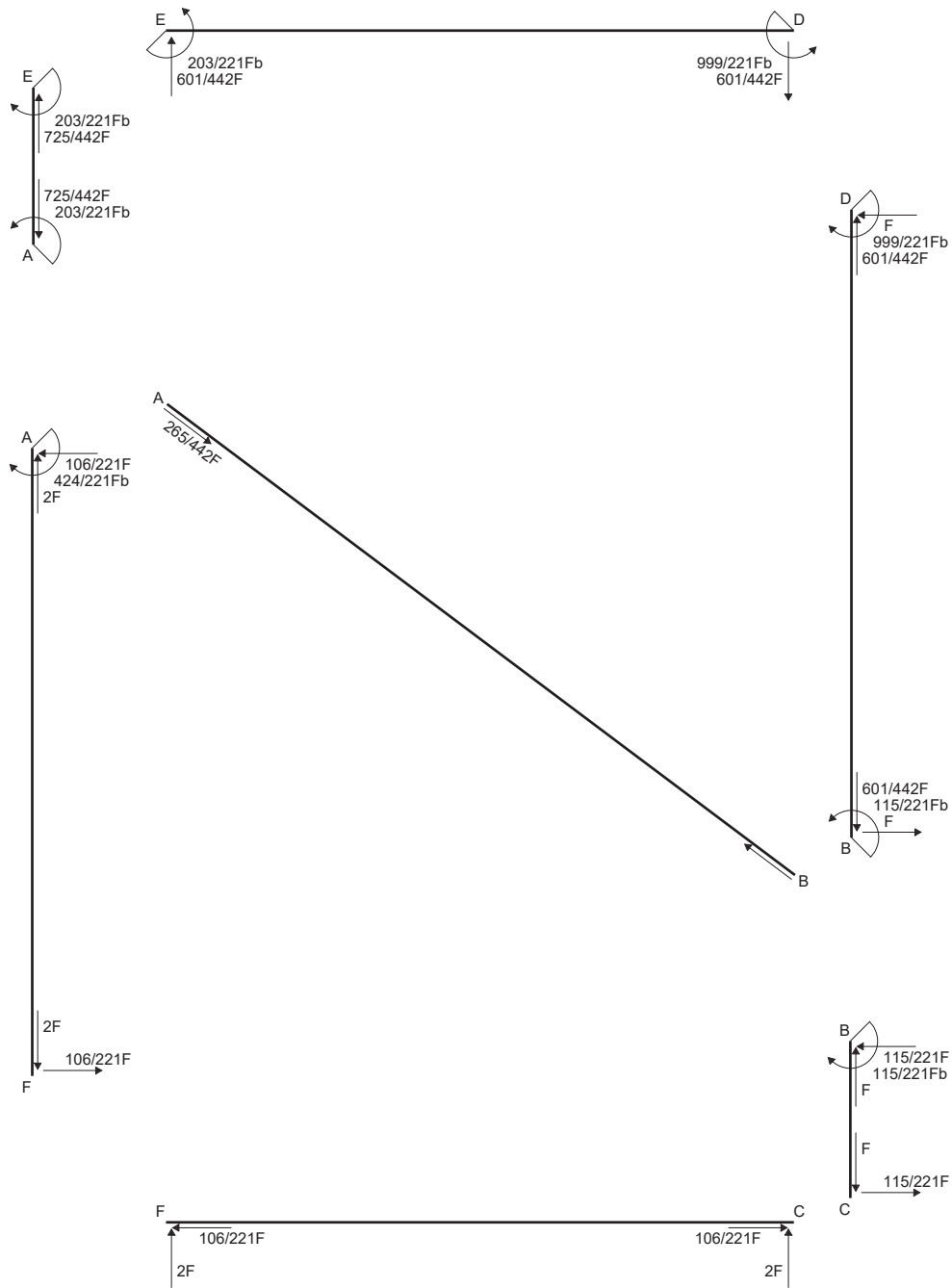
Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

Lembo inferiore sezione su tratteggio trave, a destra da C a F  
Curvatura  $\theta$  asta FA positiva se convessa a destra con inizio F.

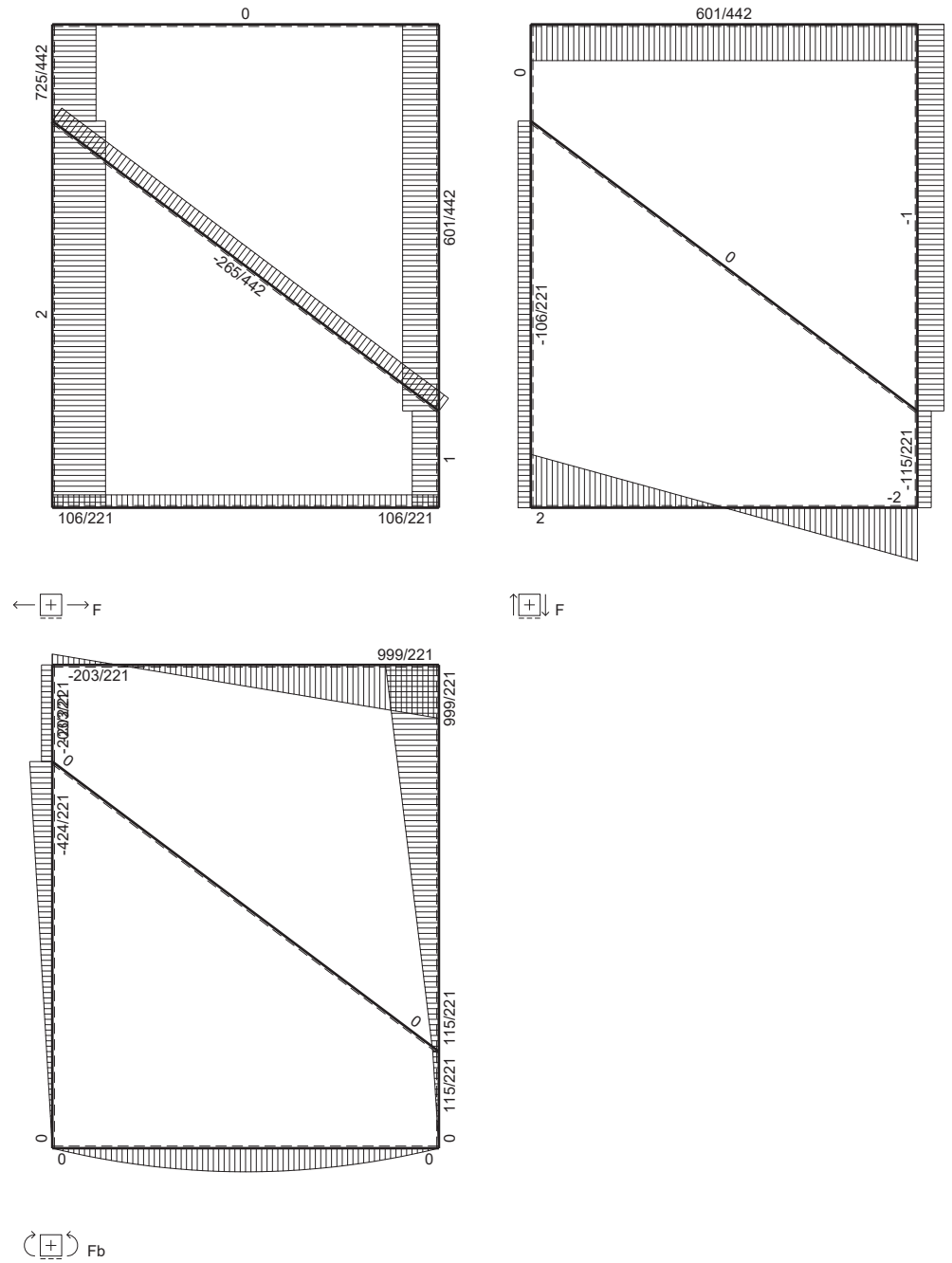
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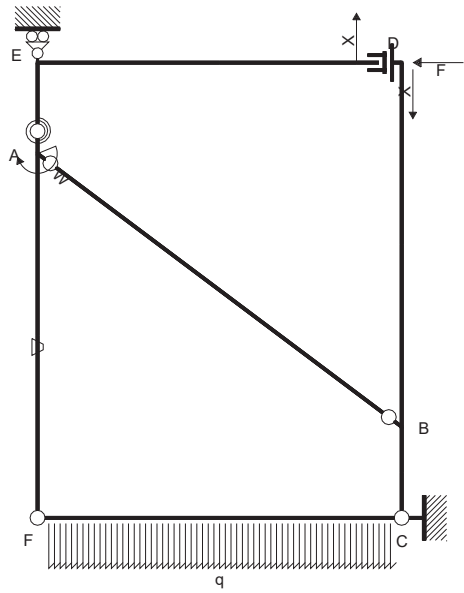


# REAZIONI

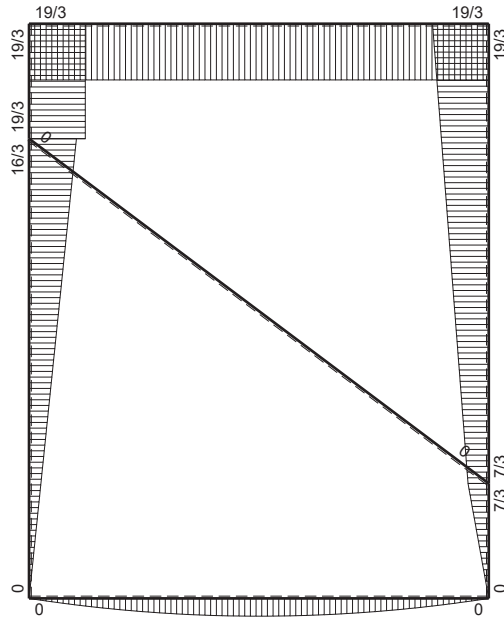


# AZIONI INTERNE

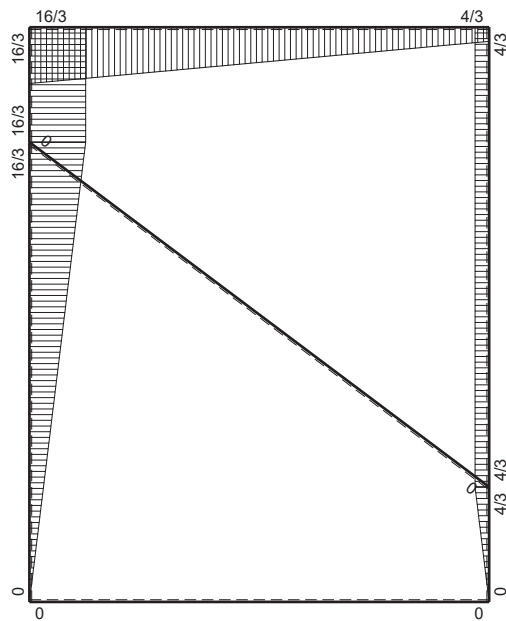




Schema di calcolo iperstatico



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=V_{DE}$

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJdx$
AB 5b	0	0	0	0	0	0	0+0	0
BA 5b	0	0	0	0	0	0		
BC b	$4/3b-4/3x$	$7/3Fb-7/3Fx$	0	$28/9Fb^2-56/9Fbx+28/9Fx^2$	0	$16/9b^2-32/9bx+16/9x^2$	$(28/27+0)Fb^3/EJ$	$16/27Xb^3/EJ$
CB b	$-4/3x$	$-7/3Fx$	0	$28/9Fx^2$	0	$16/9x^2$		
DB 4b	$4/3b$	$19/3Fb-Fx$	0	$76/9Fb^2-4/3Fbx$	0	$16/9b^2$	$(208/9+0)Fb^3/EJ$	$64/9Xb^3/EJ$
BD 4b	$-4/3b$	$-7/3Fb-Fx$	0	$28/9Fb^2+4/3Fbx$	0	$16/9b^2$		
ED 4b	$16/3b-x$	$19/3Fb$	0	$304/9Fb^2-19/3Fbx$	0	$256/9b^2-32/3bx+x^2$	$(760/9+0)Fb^3/EJ$	$448/9Xb^3/EJ$
DE 4b	$-4/3b-x$	$-19/3Fb$	0	$76/9Fb^2+19/3Fbx$	0	$16/9b^2+8/3bx+x^2$		
AE b	$16/3b$	$19/3Fb$	0	$304/9Fb^2$	0	$256/9b^2$	$(304/9+0)Fb^3/EJ$	$256/9Xb^3/EJ$
EA b	$-16/3b$	$-19/3Fb$	0	$304/9Fb^2$	0	$256/9b^2$		
FA 4b	$4/3x$	$4/3Fx$	$-Fb/EJ$	$16/9Fx^2$	$-4/3Fxb/EJ$	$16/9x^2$	$(1024/27-32/3)Fb^3/EJ$	$1024/27Xb^3/EJ$
AF 4b	$-16/3b+4/3x$	$-16/3Fb+4/3Fx$	$Fb/EJ$	$256/9Fb^2-128/9Fbx+16/9Fx^2$	$-16/3Fb^2/EJ+4/3Fxb/EJ$	$256/9b^2-128/9bx+16/9x^2$		
CF 4b	0	$-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FC 4b	0	$2Fx-1/2qx^2$	0	0	0	0		
AE	molla asta $-W_{1AE}(W_{0AE}+XW_{1AE})/k_{AE}$						$76/9Fb^3/EJ$	$64/9Xb^3/EJ$
	totali						$4808/27Fb^3/EJ$	$3536/27Xb^3/EJ$
	iperstatica $X=V_{DE}$						$-601/442F$	

Sviluppi di calcolo iperstatica



## PROCEDIMENTO E RISULTATI

$$L_{BC}^{xx} = \int_0^b (16/9 - 32/9 x/b + 16/9 x^2/b^2) b^2 1/EJ dx = [16/9 x - 16/9 x^2/b + 16/27 x^3/b^2]_0^b b^2 1/EJ$$

$$= (16/9 b - 16/9 b + 16/27 b) b^2 1/EJ = 16/27 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (16/9 x^2/b^2) b^2 1/EJ dx = [16/27 x^3/b^2]_0^b b^2 1/EJ$$

$$= (16/27 b) b^2 1/EJ = 16/27 b^3/EJ$$

$$L_{DB}^{xx} = \int_0^{4b} (16/9) b^2 1/EJ dx = [16/9 x]_0^{4b} b^2 1/EJ$$

$$= (64/9 b) b^2 1/EJ = 64/9 b^3/EJ$$

$$L_{BD}^{xx} = \int_0^{4b} (16/9) b^2 1/EJ dx = [16/9 x]_0^{4b} b^2 1/EJ$$

$$= (64/9 b) b^2 1/EJ = 64/9 b^3/EJ$$

$$L_{ED}^{xx} = \int_0^{4b} (256/9 - 32/3 x/b + x^2/b^2) b^2 1/EJ dx = [256/9 x - 16/3 x^2/b + 1/3 x^3/b^2]_0^{4b} b^2 1/EJ$$

$$= (1024/9 b - 256/3 b + 64/3 b) b^2 1/EJ = 448/9 b^3/EJ$$

$$L_{DE}^{xx} = \int_0^{4b} (16/9 + 8/3 x/b + x^2/b^2) b^2 1/EJ dx = [16/9 x + 4/3 x^2/b + 1/3 x^3/b^2]_0^{4b} b^2 1/EJ$$

$$= (64/9 b + 64/3 b + 64/3 b) b^2 1/EJ = 448/9 b^3/EJ$$

$$L_{AE}^{xx} = \int_0^b (256/9) b^2 1/EJ dx - 16/3 (-16/3) 1/4 b^3/EJ = [256/9 x]_0^b b^2 1/EJ - 16/3 (-16/3) 1/4 b^3/EJ$$

$$= (256/9 b) b^2 1/EJ - 16/3 (-16/3) 1/4 b^3/EJ = 320/9 b^3/EJ$$

$$L_{EA}^{xx} = \int_0^b (256/9) b^2 1/EJ dx - 16/3 (-16/3) 1/4 b^3/EJ = [256/9 x]_0^b b^2 1/EJ - 16/3 (-16/3) 1/4 b^3/EJ$$

$$= (256/9 b) b^2 1/EJ - 16/3 (-16/3) 1/4 b^3/EJ = 320/9 b^3/EJ$$

$$L_{FA}^{xx} = \int_0^{4b} (16/9 x^2/b^2) b^2 1/EJ dx = [16/27 x^3/b^2]_0^{4b} b^2 1/EJ$$

$$= (1024/27 b) b^2 1/EJ = 1024/27 b^3/EJ$$

$$L_{AF}^{xx} = \int_0^{4b} (256/9 - 128/9 x/b + 16/9 x^2/b^2) b^2 1/EJ dx = [256/9 x - 64/9 x^2/b + 16/27 x^3/b^2]_0^{4b} b^2 1/EJ$$

$$= (1024/9 b - 1024/9 b + 1024/27 b) b^2 1/EJ = 1024/27 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (28/9 - 56/9 x/b + 28/9 x^2/b^2) Fb^2 1/EJ dx = [28/9 x - 28/9 x^2/b + 28/27 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (28/9 b - 28/9 b + 28/27 b) Fb^2 1/EJ = 28/27 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (28/9 x^2/b^2) Fb^2 1/EJ dx = [28/27 x^3/b^2]_0^b Fb^2 1/EJ$$

$$= (28/27 b) Fb^2 1/EJ = 28/27 Fb^3/EJ$$

$$L_{DB}^{xo} = \int_0^{4b} (76/9 - 4/3 x/b) Fb^2 1/EJ dx = [76/9 x - 2/3 x^2/b]_0^{4b} Fb^2 1/EJ$$

$$= (304/9 b - 32/3 b) Fb^2 1/EJ = 208/9 Fb^3/EJ$$

$$L_{BD}^{xo} = \int_0^{4b} (28/9 + 4/3 x/b) Fb^2 1/EJ dx = [28/9 x + 2/3 x^2/b]_0^{4b} Fb^2 1/EJ$$

$$= (112/9 b + 32/3 b) Fb^2 1/EJ = 208/9 Fb^3/EJ$$

$$L_{ED}^{xo} = \int_0^{4b} (304/9 - 19/3 x/b) Fb^2 1/EJ dx = [304/9 x - 19/6 x^2/b]_0^{4b} Fb^2 1/EJ$$

## PROCEDIMENTO E RISULTATI

$$= (1216/9 b - 152/3 b) Fb^2 1/EJ = 760/9 Fb^3/EJ$$

$$L_{DE}^{xo} = \int_0^{4b} (76/9 + 19/3 x/b) Fb^2 1/EJ dx = [76/9 x + 19/6 x^2/b]_0^{4b} Fb^2 1/EJ$$

$$= (304/9 b + 152/3 b) Fb^2 1/EJ = 760/9 Fb^3/EJ$$

$$L_{AE}^{xo} = \int_0^b (304/9) Fb^2 1/EJ dx - 16/3 (-19/3) 1/4 Fb^3/EJ$$

$$= [304/9 x]_0^b Fb^2 1/EJ - 16/3 (-19/3) 1/4 Fb^3/EJ$$

$$= (304/9 b) Fb^2 1/EJ - 16/3 (-19/3) 1/4 Fb^3/EJ = 380/9 Fb^3/EJ$$

$$L_{EA}^{xo} = \int_0^b (304/9) Fb^2 1/EJ dx - 16/3 (-19/3) 1/4 Fb^3/EJ$$

$$= [304/9 x]_0^b Fb^2 1/EJ - 16/3 (-19/3) 1/4 Fb^3/EJ$$

$$= (304/9 b) Fb^2 1/EJ - 16/3 (-19/3) 1/4 Fb^3/EJ = 380/9 Fb^3/EJ$$

$$L_{FA}^{xo} = \int_0^{4b} (16/9 x^2/b^2) Fb^2 1/EJ dx + \int_0^{4b} (-4/3 x/b) \theta dx$$

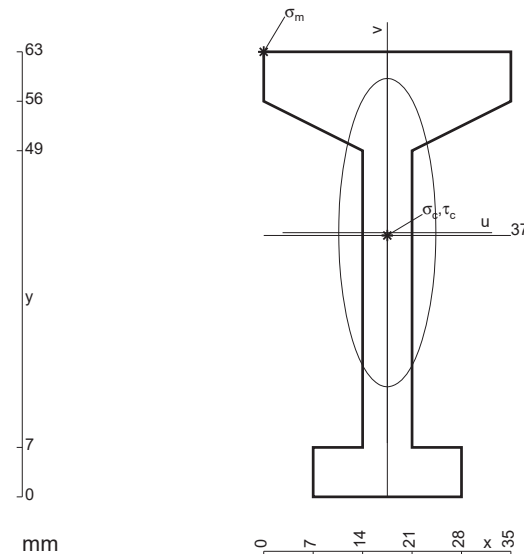
$$= [16/27 x^3/b^2]_0^{4b} Fb^2 1/EJ + [-2/3 x^2/b]_0^{4b} \theta$$

$$= (1024/27 b) Fb^2 1/EJ + (-32/3 b) \theta = 736/27 Fb^3/EJ$$

$$L_{AF}^{xo} = \int_0^{4b} (256/9 - 128/9 x/b + 16/9 x^2/b^2) Fb^2 1/EJ dx + \int_0^{4b} (16/3 - 4/3 x/b) \theta dx$$

$$= [256/9 x - 64/9 x^2/b + 16/27 x^3/b^2]_0^{4b} Fb^2 1/EJ + [16/3 x - 2/3 x^2/b]_0^{4b} \theta$$

$$= (1024/9 b - 1024/9 b + 1024/27 b) Fb^2 1/EJ + (64/3 b - 32/3 b) \theta = 736/27 Fb^3/EJ$$



$$A = 833. \text{ mm}^2$$

$$J_u = 396962. \text{ mm}^4$$

$$J_v = 39416. \text{ mm}^4$$

$$y_g = 37.4 \text{ mm}$$

$$N = 183255. \text{ N}$$

$$T_y = -764140. \text{ N}$$

$$y_m = 63. \text{ mm}$$

$$u_m = -17.5 \text{ mm}$$

$$v_m = 25.6 \text{ mm}$$

$$\sigma_m = N/A = 220. \text{ N/mm}^2$$

$$x_c = 17.5 \text{ mm}$$

$$y_c = 37. \text{ mm}$$

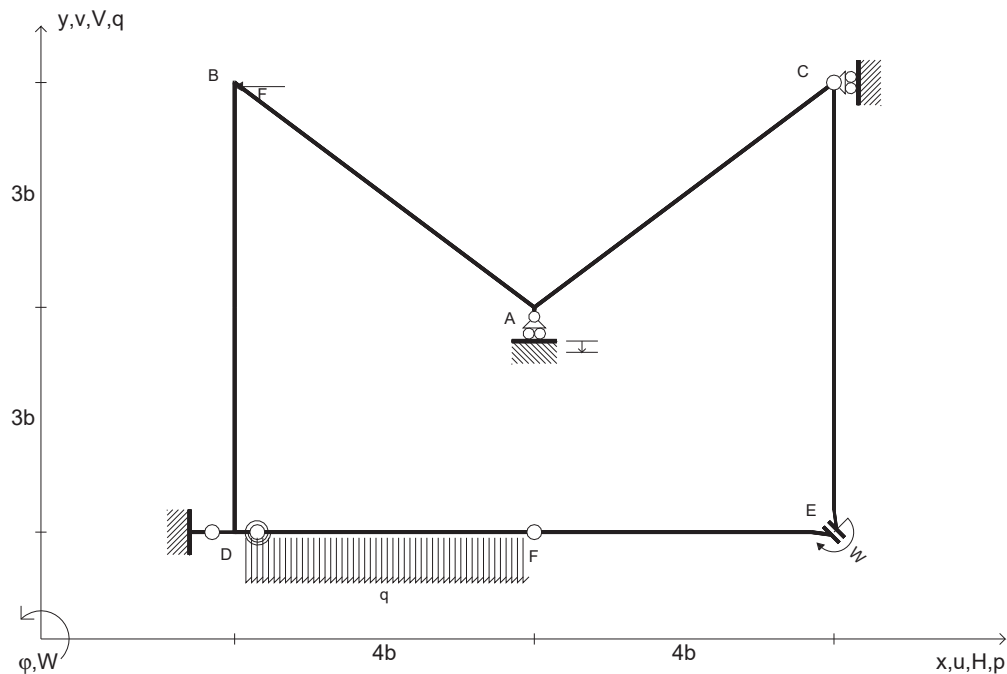
$$v_c = -4020 \text{ mm}$$

$$\sigma_c = N/A = 220. \text{ N/mm}^2$$

$$\tau_c = 2260. \text{ N/mm}^2$$

$$\sigma_o = \sqrt{\sigma^2 + 3\tau^2} = 3920. \text{ N/mm}^2$$

$$S = 8218. \text{ mm}^3$$



$H_{BA} = -F$	$k_{DF} = 4EJ/b$	$EJ_{EC} = EJ$
$W_E = -W = -Fb$	$EJ_{AB} = EJ$	$EJ_{FD} = EJ$
$q_{FD} = -q = -F/b$	$EJ_{CA} = EJ$	$EJ_{EF} = EJ$
$V_A = -\delta = -b^3 F/EJ$	$EJ_{DB} = EJ$	



Consegnare SOLO questo foglio con quote frazionarie intere sui diagrammi. Indicare l'iperstatica scelta e i risultati ( $y_g, J_u, S_u^*, s_c, s_m, t_c$ ) richiesti per la sezione.

Sul retro: analisi cinematica, diagrammi  $M_0, M^*, N_0, N^*$ , equazione PLV con funzioni esplicite dei momenti.

Carichi e deformazioni date hanno verso efficace in disegno. Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Diagrammi di carico con valori riferiti ad asse della trave.

Componenti di carico distribuito riferiti ad assi ortogonali.

$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Piano di scorrimento del vincolo con inclinazione assegnata.

La trave FD ha la sezione riportata e dimensioni in mm, con:

$b = 140 \text{ mm}, F = 1730 \text{ N}$

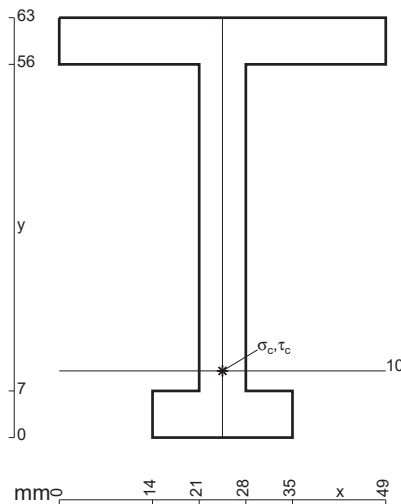
Calcolare sulla sezione D la massima tensione normale  $\sigma_m$ .

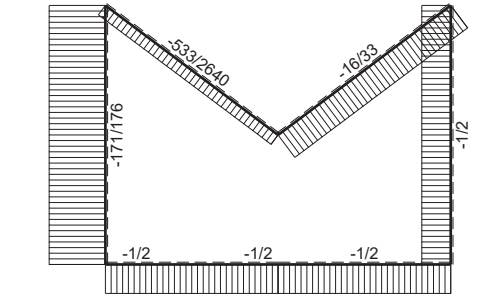
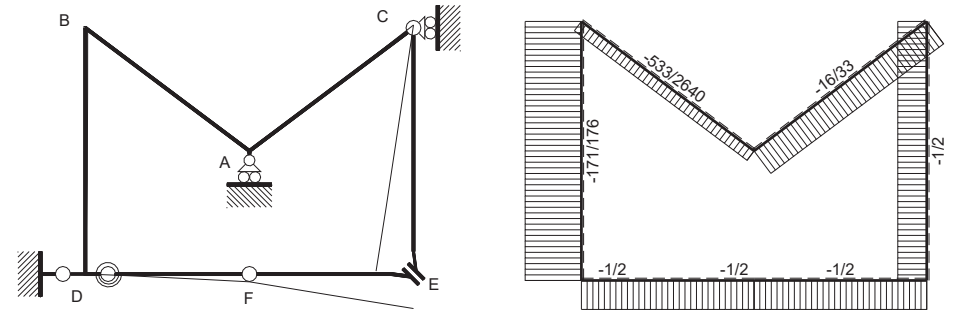
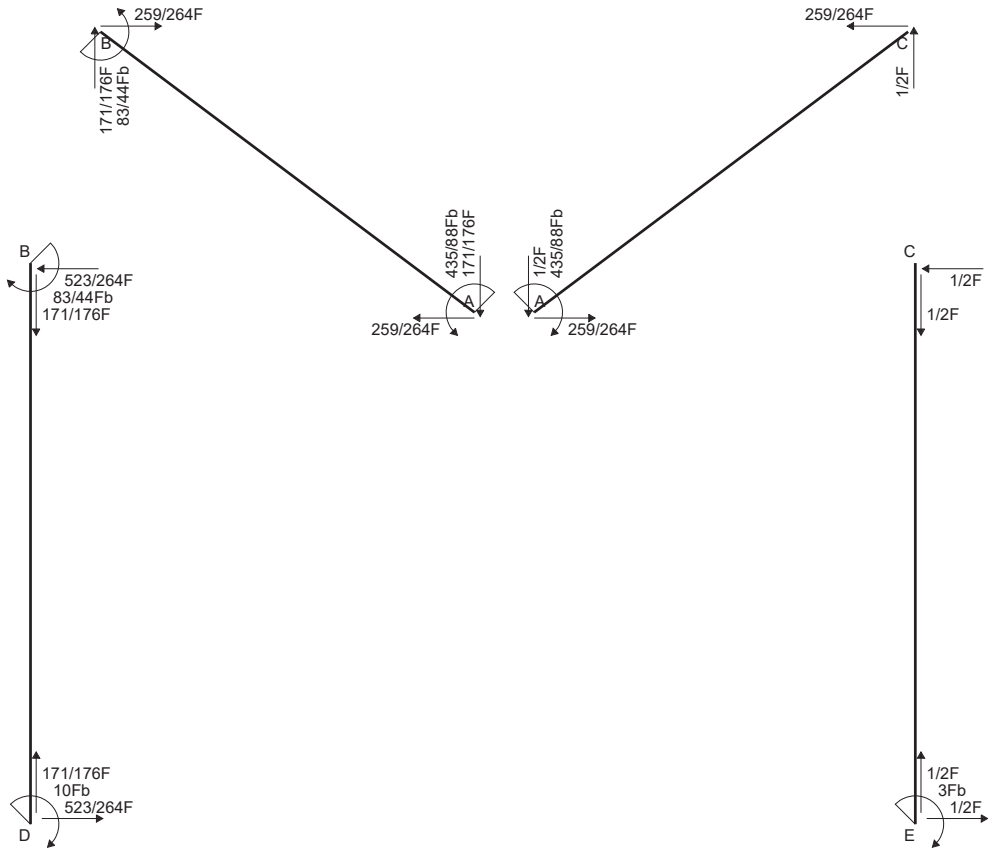
Calcolare in \* le tensioni  $\sigma_c, \tau_c$  e la tensione di von Mises.

Leombo inferiore sezione su tratteggio trave, a destra da F a D

Spostamento verticale assoluto  $v$  imposto al nodo A.

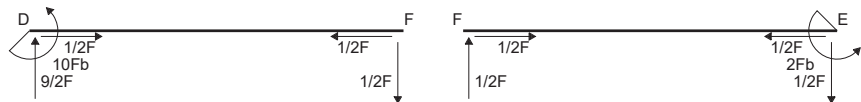
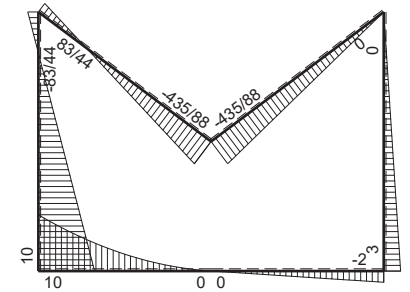
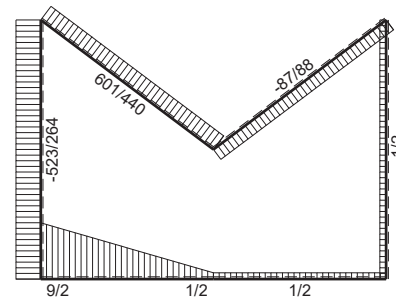
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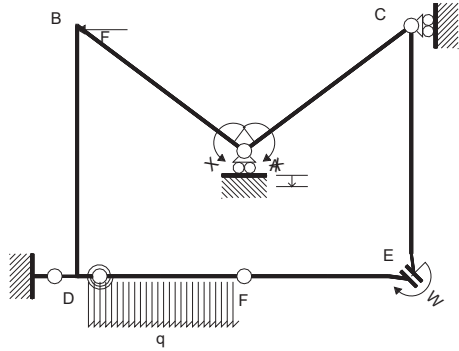
$1500 Fb^3/EJ$

$\leftarrow \oplus \rightarrow F$

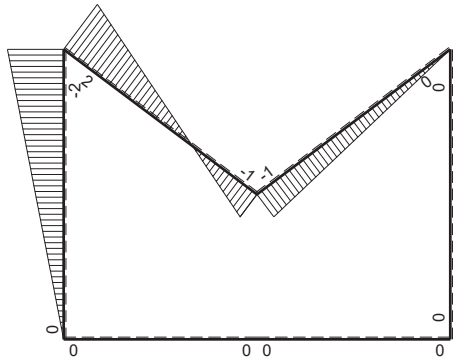


$\leftarrow \oplus \rightarrow F$

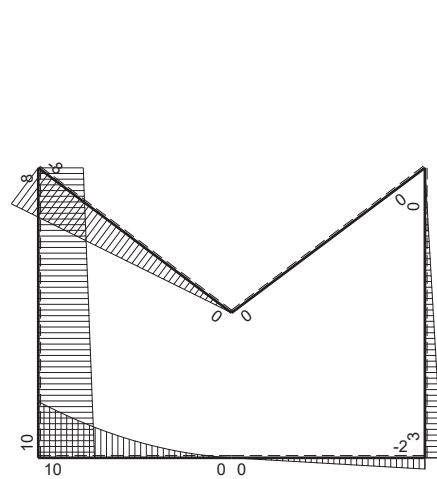
$\leftarrow \oplus \rightarrow Fb$



Schema di calcolo iperstatico



$M_x$  flessione da iperstatica  $X=1$



$M_0$  flessione da carichi assegnati

Quadro contributi PLV per iperstatica  $X=W_{AB}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x^2 M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$	
AB 5b	$-1+3/5x/b$	$-8/5Fx$	$8/5Fx - 24/25Fx^2/b$	$1-6/5x/b+9/25x^2/b^2$	$-20Fb^2/EJ$	$5Xb/EJ$	
BA 5b	$-2+3/5x/b$	$8Fb-8/5Fx$	$-16Fb+8Fx-24/25Fx^2/b$	$4-12/5x/b+9/25x^2/b^2$	0	$5/3Xb/EJ$	
CA 5b	$-1/5x/b$	0	0	$1/25x^2/b^2$	0	0	
AC 5b	$1-1/5x/b$	0	0	$1-2/5x/b+1/25x^2/b^2$	$-52Fb^2/EJ$	$8Xb/EJ$	
DB 6b	$-1/3x/b$	$10Fb-1/3Fx$	$-10/3Fx+1/9Fx^2/b$	$1/9x^2/b^2$	0	0	
BD 6b	$2-1/3x/b$	$-8Fb-1/3Fx$	$-16Fb+2Fx+1/9Fx^2/b$	$4-4/3x/b+1/9x^2/b^2$	0	0	
EC 6b	0	$3Fb-1/2Fx$	0	0	0	0	
CE 6b	0	$-1/2Fx$	0	0	0	0	
FD 4b	0	$1/2Fx+1/2qx^2$	0	0	0	0	
DF 4b	0	$-10Fb+9/2Fx-1/2qx^2$	0	0	0	0	
EF 4b	0	$-2Fb+1/2Fx$	0	0	0	0	
FE 4b	0	$1/2Fx$	0	0	0	0	
A	cedimento nodo $-V_{1A}U_A$						
	totali						$-1/2Fb^2/EJ$
	iperstatica $X=W_{AB}$						$-145/2Fb^2/EJ$
							$44/3Xb/EJ$
							$435/88Fb$

Sviluppi di calcolo iperstatica

## PROCEDIMENTO E RISULTATI

$$L_{AB}^{XX} = \int_0^{5b} (1 - 6/5 x/b + 9/25 x^2/b^2) 1/EJ dx = \left[ x - 3/5 x^2/b + 3/25 x^3/b^2 \right]_0^{5b} 1/EJ$$

$$= (5b - 15b + 15b) 1/EJ = 5b/EJ$$

$$L_{BA}^{XX} = \int_0^{5b} (4 - 12/5 x/b + 9/25 x^2/b^2) 1/EJ dx = \left[ 4x - 6/5 x^2/b + 3/25 x^3/b^2 \right]_0^{5b} 1/EJ$$

$$= (20b - 30b + 15b) 1/EJ = 5b/EJ$$

$$L_{CA}^{XX} = \int_0^{5b} (1/25 x^2/b^2) 1/EJ dx = \left[ 1/75 x^3/b^2 \right]_0^{5b} 1/EJ$$

$$= (5/3b) 1/EJ = 5/3b/EJ$$

$$L_{AC}^{XX} = \int_0^{5b} (1 - 2/5 x/b + 1/25 x^2/b^2) 1/EJ dx = \left[ x - 1/5 x^2/b + 1/75 x^3/b^2 \right]_0^{5b} 1/EJ$$

$$= (5b - 5b + 5/3b) 1/EJ = 5/3b/EJ$$

$$L_{DB}^{XX} = \int_0^{6b} (1/9 x^2/b^2) 1/EJ dx = \left[ 1/27 x^3/b^2 \right]_0^{6b} 1/EJ$$

$$= (8b) 1/EJ = 8b/EJ$$

$$L_{BD}^{XX} = \int_0^{6b} (4 - 4/3 x/b + 1/9 x^2/b^2) 1/EJ dx = \left[ 4x - 2/3 x^2/b + 1/27 x^3/b^2 \right]_0^{6b} 1/EJ$$

$$= (24b - 24b + 8b) 1/EJ = 8b/EJ$$

$$L_{AB}^{Xo} = \int_0^{5b} (8/5 x/b - 24/25 x^2/b^2) Fb 1/EJ dx = \left[ 4/5 x^2/b - 8/25 x^3/b^2 \right]_0^{5b} Fb 1/EJ$$

$$= (20b - 40b) Fb 1/EJ = -20 Fb^2/EJ$$

$$L_{BA}^{Xo} = \int_0^{5b} (-16 + 8x/b - 24/25 x^2/b^2) Fb 1/EJ dx = \left[ -16x + 4x^2/b - 8/25 x^3/b^2 \right]_0^{5b} Fb 1/EJ$$

$$= (-80b + 100b - 40b) Fb 1/EJ = -20 Fb^2/EJ$$

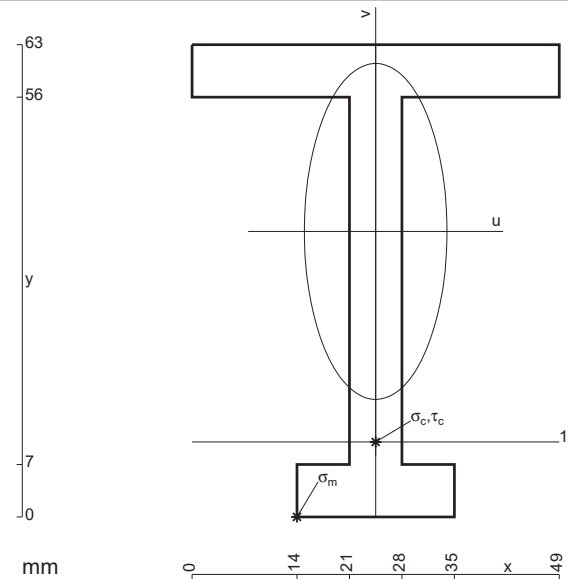
$$L_{DB}^{Xo} = \int_0^{6b} (-10/3 x/b + 1/9 x^2/b^2) Fb 1/EJ dx = \left[ -5/3 x^2/b + 1/27 x^3/b^2 \right]_0^{6b} Fb 1/EJ$$

$$= (-60b + 8b) Fb 1/EJ = -52 Fb^2/EJ$$

$$L_{BD}^{Xo} = \int_0^{6b} (-16 + 2x/b + 1/9 x^2/b^2) Fb 1/EJ dx = \left[ -16x + x^2/b + 1/27 x^3/b^2 \right]_0^{6b} Fb 1/EJ$$

$$= (-96b + 36b + 8b) Fb 1/EJ = -52 Fb^2/EJ$$

## PROCEDIMENTO E RISULTATI



$$A = 833. \text{ mm}^2$$

$$J_u = 418633. \text{ mm}^4$$

$$J_v = 75431. \text{ mm}^4$$

$$y_g = 38.09 \text{ mm}$$

$$N = -865. \text{ N}$$

$$T_y = 7785. \text{ N}$$

$$M_x = 2422000. \text{ Nmm}$$

$$x_m = 14. \text{ mm}$$

$$u_m = -10.5 \text{ mm}$$

$$v_m = -38.09 \text{ mm}$$

$$\sigma_m = N/A - Mv/J_u = 219.3 \text{ N/mm}^2$$

$$x_c = 24.5 \text{ mm}$$

$$y_c = 10. \text{ mm}$$

$$v_c = -28.09 \text{ mm}$$

$$\sigma_c = N/A - Mv/J_u = 161.5 \text{ N/mm}^2$$

$$\tau_c = 15.16 \text{ N/mm}^2$$

$$\sigma_p = \sqrt{\sigma^2 + 3\tau^2} = 163.6 \text{ N/mm}^2$$

$$S = 5706. \text{ mm}^3$$