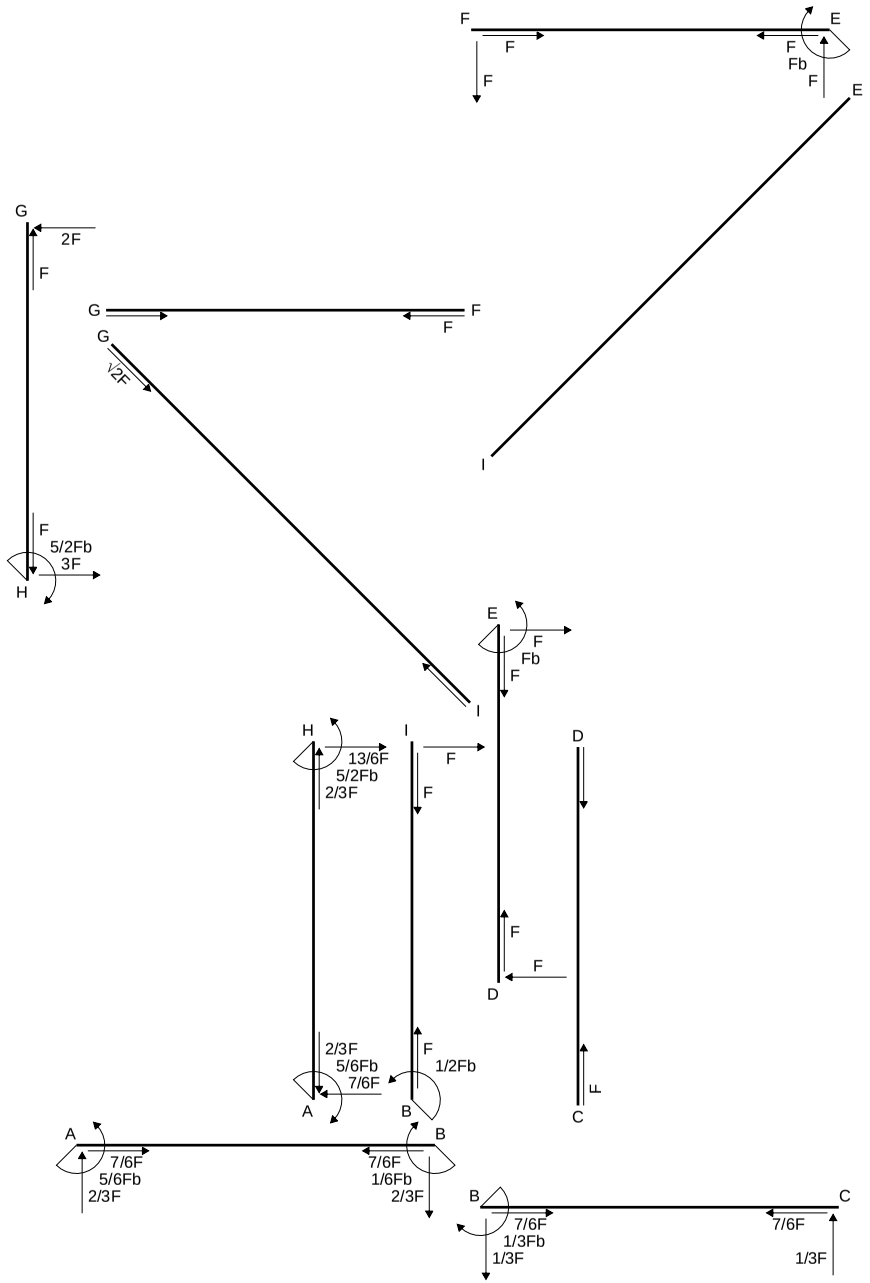
F
E
F
F
Fb
F
E
G
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F
G
F
F
G
½F
I
F
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E
F
Fb
F
I
H
13/6F
5/2Fb
2/3F
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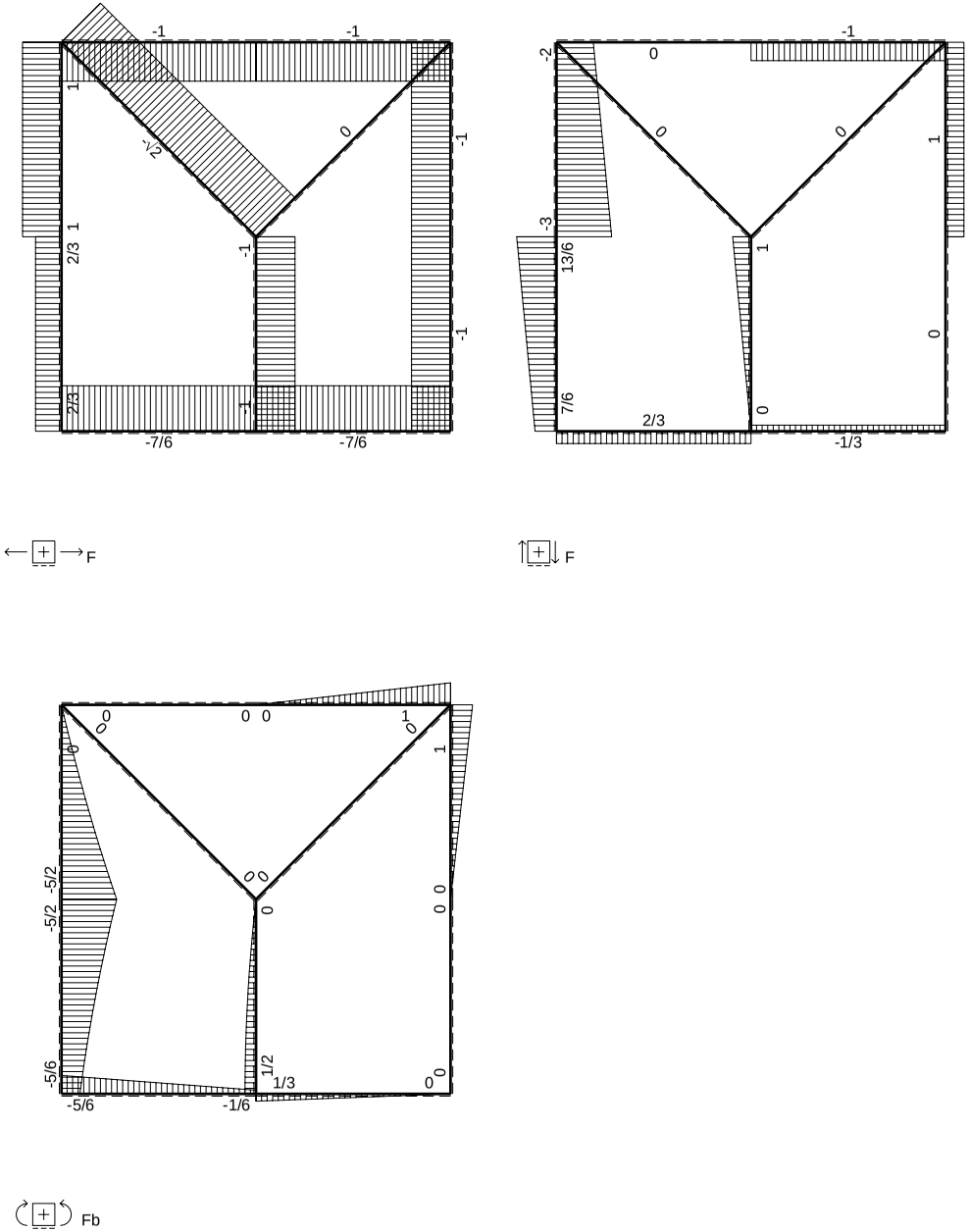
F
F
D
F
F
D
2/3F
5/6Fb
7/6F
F
1/2Fb
A
B
F
C
A
B
7/6F
5/6Fb
2/3F
7/6F
1/6Fb
2/3F
B
C
7/6F
1/3Fb
1/3F
7/6F
1/3F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½
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-1
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-1
-1
2/3
1
-7/6
-7/6
F
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-2
0
0
0
1
-3
13/6
1
0
7/6
2/3
0
-1/3
F
0 0
0
0
1
0
0
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-5/2
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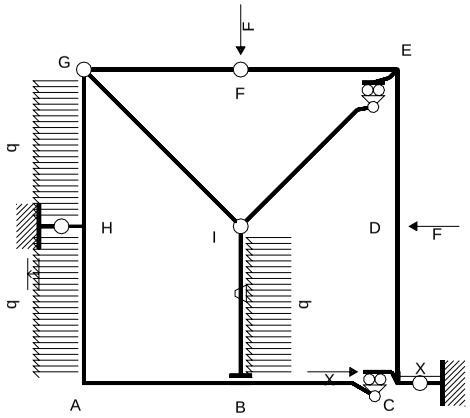
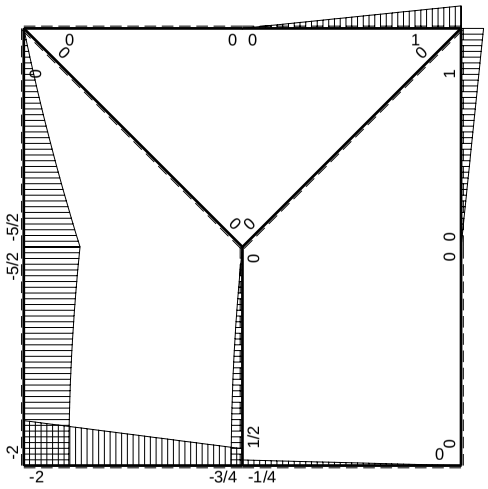
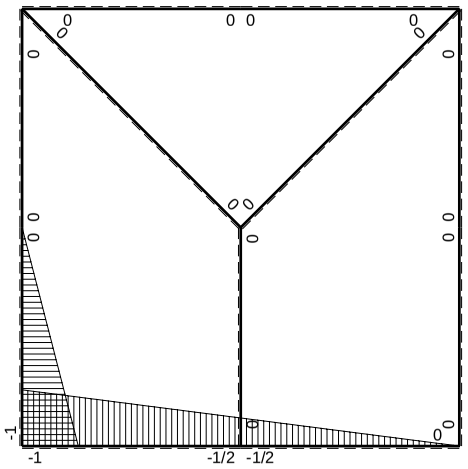
-5/6
1/2
1/3
0
-5/6
-1/6
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982951 ESΣ4.xxxx.008

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Quadro contributi PLV per iperstatica $X=H_{CB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-b+1/2x	$-2Fb+5/4Fx$	0	$2Fb^2-9/4Fbx+5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(13/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	1/2b+1/2x	$3/4Fb+5/4Fx$	0	$3/8Fb^2+Fbx+5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	-1/2b+1/2x	$-1/4Fb+1/4Fx$	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	1/2x	1/4Fx	0	$1/8Fx^2$	0	$1/4x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	-Fb/EJ	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	Fb/EJ	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	-x	$-5/2Fb+Fx-1/2qx^2$	0	$5/2Fbx-Fx^2+1/2qx^3$	0	x^2	$(25/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$2Fb+1/2qx^2$	0	$2Fb^2-2Fbx+1/2Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$7/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_{CB}$						-7/6F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982951 ESΣ4.xxxx.008
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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b \left(2 - 9/4\ x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - 9/8\ b + 5/24\ b\ \right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b \left(3/8 + x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 1/2\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b + 1/2\ b + 5/24\ b\ \right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/8 - 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/8\ b - 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b \left(1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b \left(5/2\ x/b - x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[5/4\ x^2/b - 1/3\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(5/4\ b - 1/3\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b \left(2 - 2\ x/b + 1/2\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - x^2/b + 1/6\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - b + 1/6\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 25/24\ \ Fb^3/EJ$

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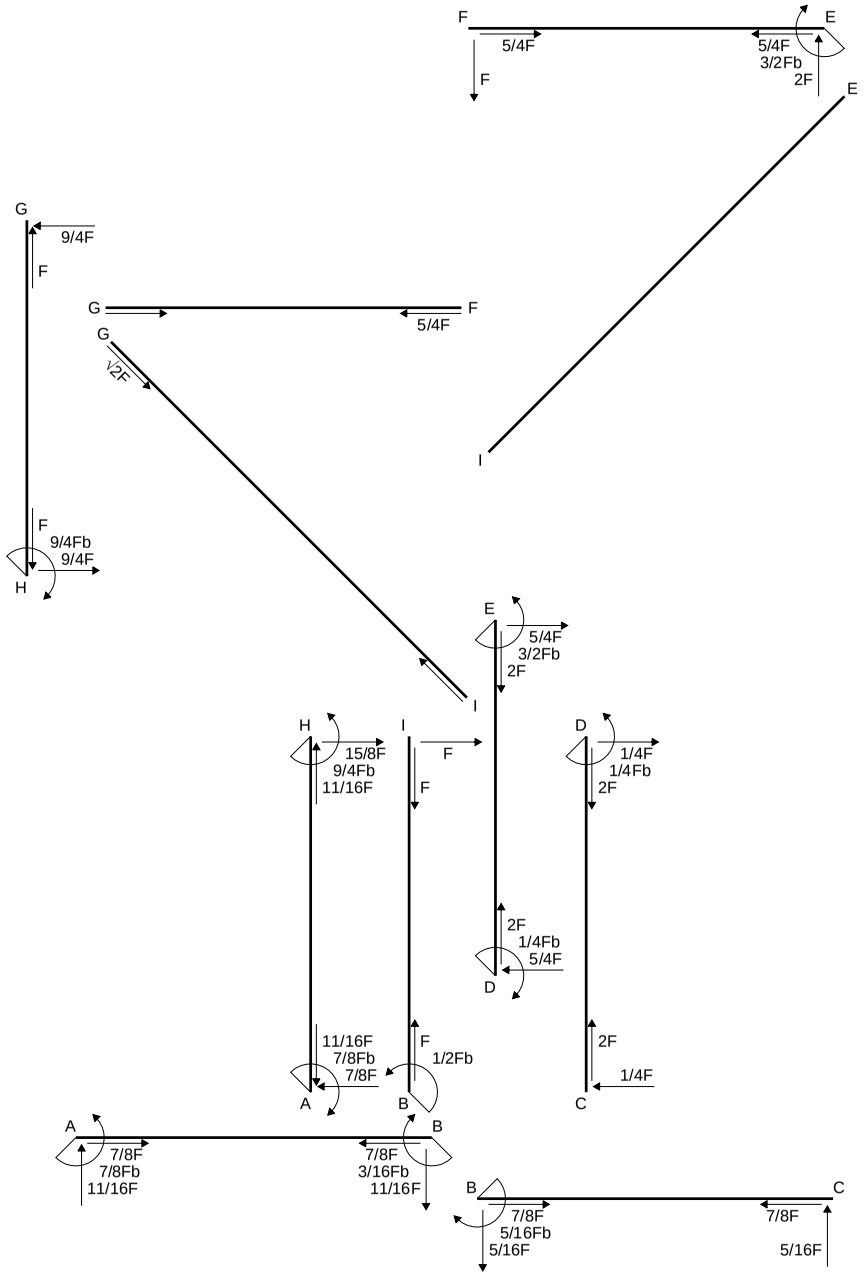
F
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G
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F
G
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5/4F
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I
F
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9/4F
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3/2Fb
2F
I
H
15/8F
9/4Fb
11/16F
I
F
F
D
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2F
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D
11/16F
7/8Fb
7/8F
F
1/2Fb
A
B
2F
1/4F
C
A
B
7/8F
7/8Fb
11/16F
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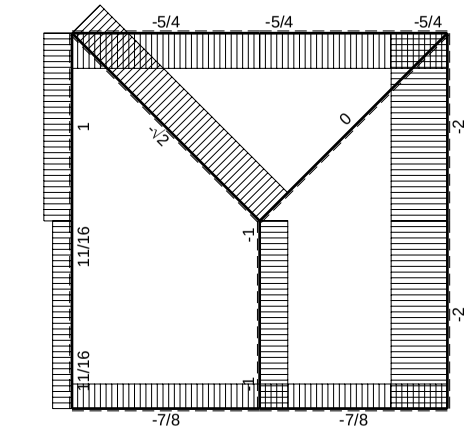
11/16F
B
C
7/8F
5/16Fb
5/16F
7/8F
5/16F


-5/4
-5/4
-5/4
1
½
0
-2
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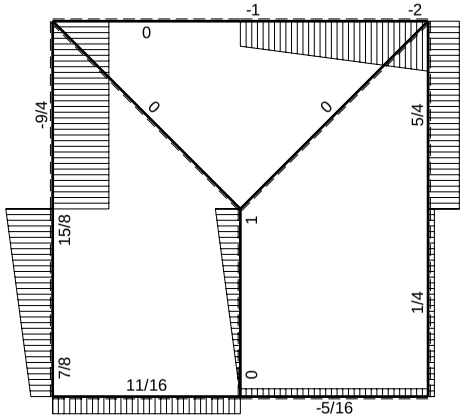
11/16
-2
11/16
-1
-7/8
-7/8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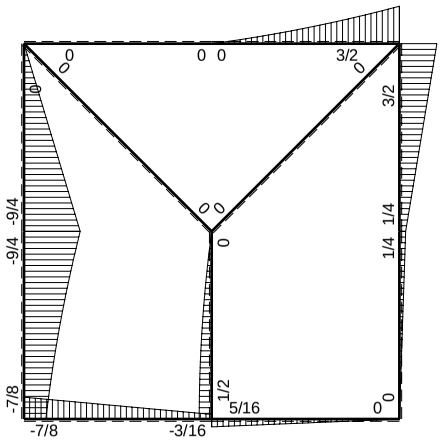
-2
0
-9/4
0
0
5/4
1
15/8
1/4
7/8
0
11/16
-5/16

←⊞→ F

↑⊞↓ F


0
0 0
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0
0
3/2
0
0
-9/4
-9/4
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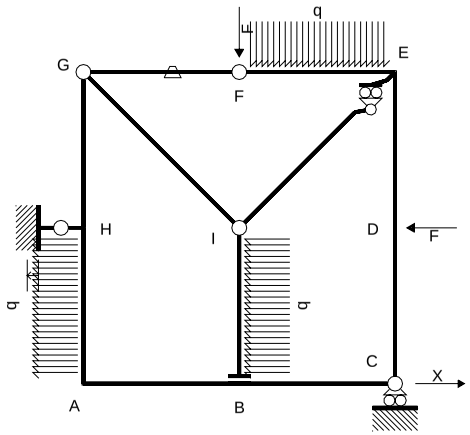
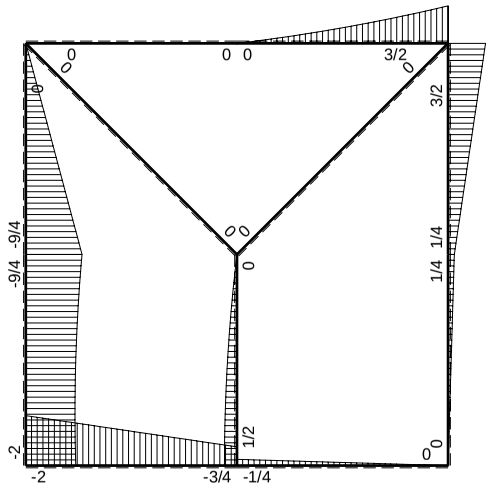
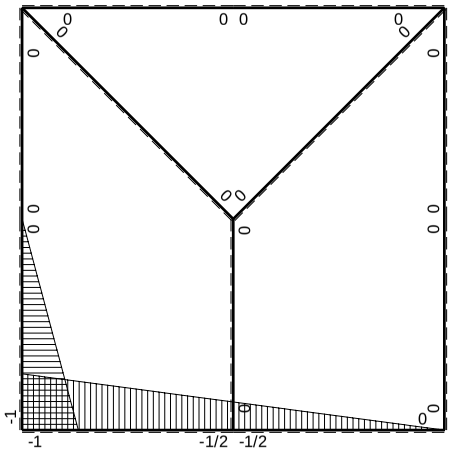
-9/4
1/4
1/4
1/4
-7/8
1/2
5/16
0
0
-7/8
-3/16

⟲⊞⟳ Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982314 ESΣ4.xxxx.015

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PROCEDIMENTO E RISULTATI 982314 ESΣ4.xxxx.015

Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-2Fb+5/4Fx$	0	$2Fb^2-9/4Fbx+5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(13/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$3/4Fb+5/4Fx$	0	$3/8Fb^2+Fbx+5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-1/4Fb+1/4Fx$	0	$1/8Fb^2-1/4Fbx+1/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$1/4Fx$	0	$1/8Fx^2$	0	$1/4x^2$		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	$-Fb/EJ$	0	0	0	0+0	0
GF b	0	0	Fb/EJ	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-9/4Fb+3/4Fx-1/2qx^2$	0	$9/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(1+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	$b-x$	$2Fb-1/4Fx+1/2qx^2$	0	$2Fb^2-9/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$9/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						-9/8F	

Sviluppi di calcolo iperstatica

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$$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$$

$$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$$

$$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$$

$$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$$

$$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$$

$$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$$
$$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$$

$$L_{AB}^{Xo} = \int_o^b \left(2 - 9/4\ x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(2\ b - 9/8\ b + 5/24\ b\ \right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$$

$$L_{BA}^{Xo} = \int_o^b \left(3/8 + x/b + 5/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 1/2\ x^2/b + 5/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(3/8\ b + 1/2\ b + 5/24\ b\ \right) Fb^2\ 1/EJ\ = 13/12\ \ Fb^3/EJ$$

$$L_{BC}^{Xo} = \int_o^b \left(1/8 - 1/4\ x/b + 1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/8\ x - 1/8\ x^2/b + 1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/8\ b - 1/8\ b + 1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$$

$$L_{CB}^{Xo} = \int_o^b \left(1/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(1/24\ b\ \right) Fb^2\ 1/EJ\ = 1/24\ \ Fb^3/EJ$$

$$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 3/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[9/8\ x^2/b - 1/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(9/8\ b - 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = \ \ Fb^3/EJ$$

$$L_{AH}^{Xo} = \int_o^b \left(2 - 9/4\ x/b + 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$$
$$= \left(2\ b - 9/8\ b + 1/4\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = \ \ Fb^3/EJ$$

ESΣ4.xxxx.015

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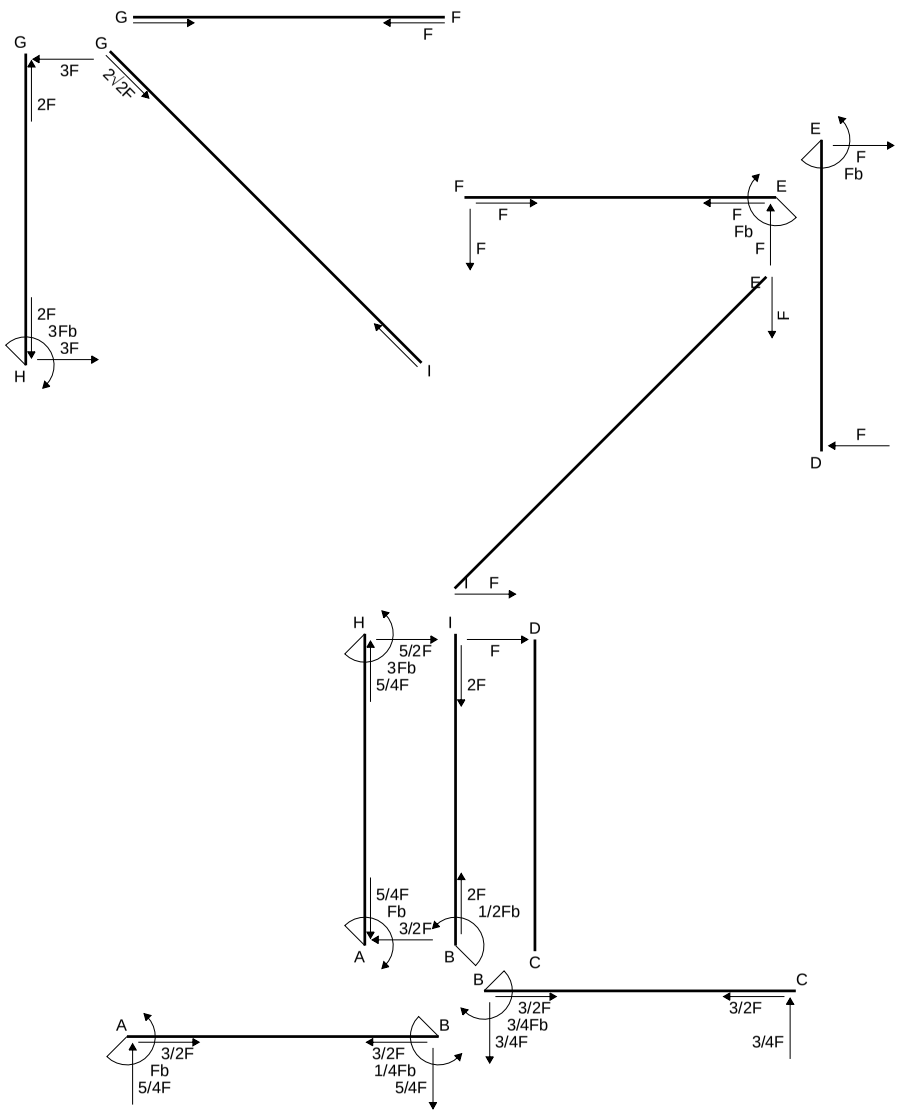
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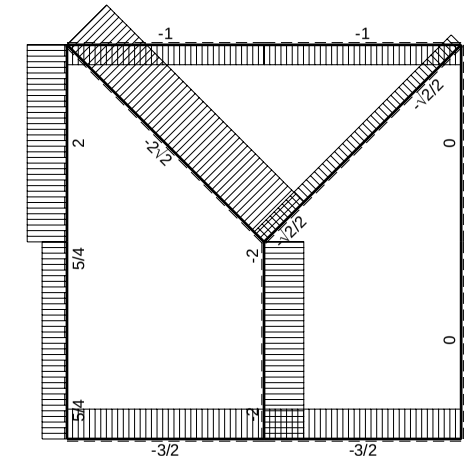
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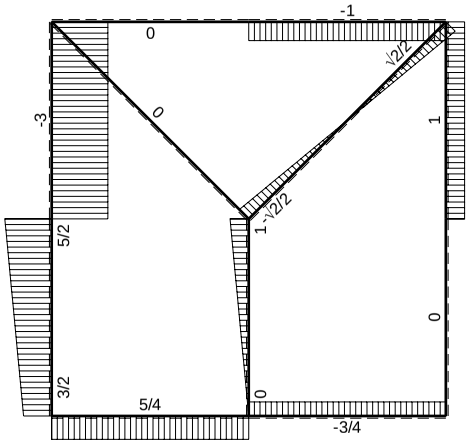
-3/2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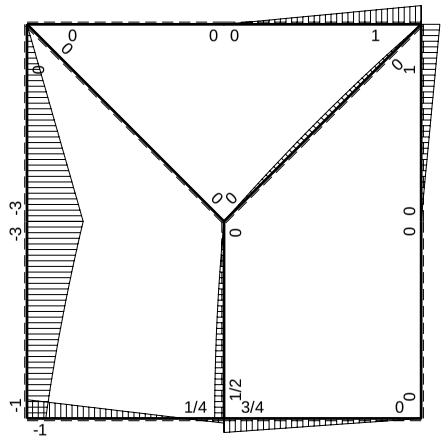
↑□↓ F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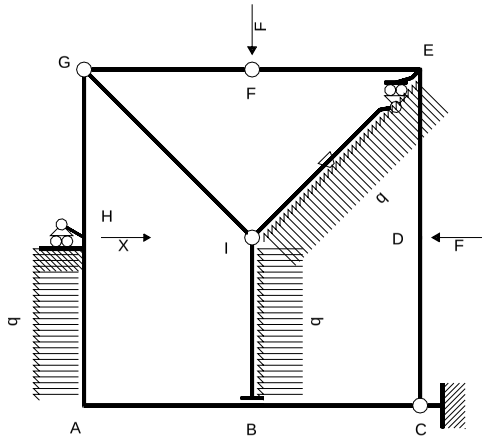
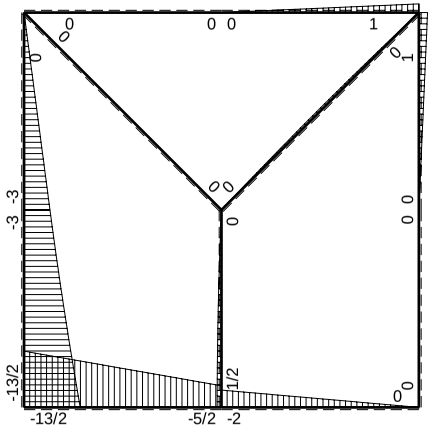
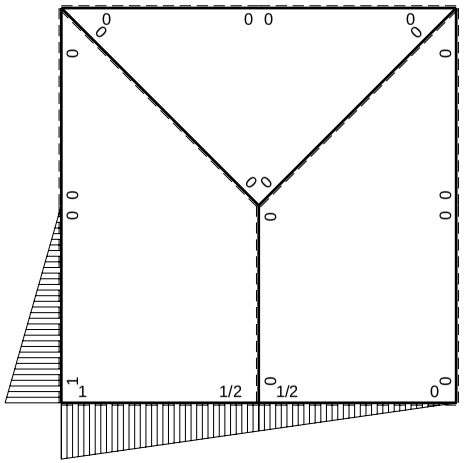
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 985260 ESΣ4.xxxx.035

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Quadro contributi PLV per iperstatica $X=H_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	b-1/2x	-13/2Fb+4Fx	0	$-13/2Fb^2+29/4Fbx-2Fx^2$	0	$b^2-bx+1/4x^2$	$(-85/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	-1/2b-1/2x	5/2Fb+4Fx	0	$-5/4Fb^2-13/4Fbx-2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	1/2b-1/2x	-2Fb+2Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-1/3+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	-1/2x	2Fx	0	$-Fx^2$	0	$1/4x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3Fx	0	0	0	0	0+0	0
HG b	0	3Fb-3Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	$-\sqrt{2}/2Fx+1/2qx^2$	-Fb/EJ	0	0	0	0	0
HA b	x	$-3Fb-3Fx-1/2qx^2$	0	$-3Fbx-3Fx^2-1/2qx^3$	0	x^2	$(-21/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	-b+x	$13/2Fb-4Fx+1/2qx^2$	0	$-13/2Fb^2+21/2Fbx-9/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						Fb^3/EJ	
	totali						$-11/2Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_H$						11/2F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 985260 ESΣ4.xxxx.035
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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b \left(-13/2 + 29/4\ x/b - 2\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-13/2\ x + 29/8\ x^2/b - 2/3\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-13/2\ b + 29/8\ b - 2/3\ b\ \right) Fb^2\ 1/EJ\ = -85/24\ \ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b \left(-5/4 - 13/4\ x/b - 2\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-5/4\ x - 13/8\ x^2/b - 2/3\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5/4\ b - 13/8\ b - 2/3\ b\ \right) Fb^2\ 1/EJ\ = -85/24\ \ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b \left(-1 + 2\ x/b - x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-\ x + x^2/b - 1/3\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-\ b + b - 1/3\ b\ \right) Fb^2\ 1/EJ\ = -1/3\ \ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b \left(-\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-1/3\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-1/3\ b\ \right) Fb^2\ 1/EJ\ = -1/3\ \ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b \left(-3\ x/b - 3\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[-3/2\ x^2/b - x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-3/2\ b - b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = -21/8\ \ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b \left(-13/2 + 21/2\ x/b - 9/2\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx$

$= \left[-13/2\ x + 21/4\ x^2/b - 3/2\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-13/2\ b + 21/4\ b - 3/2\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = -21/8\ \ Fb^3/EJ$

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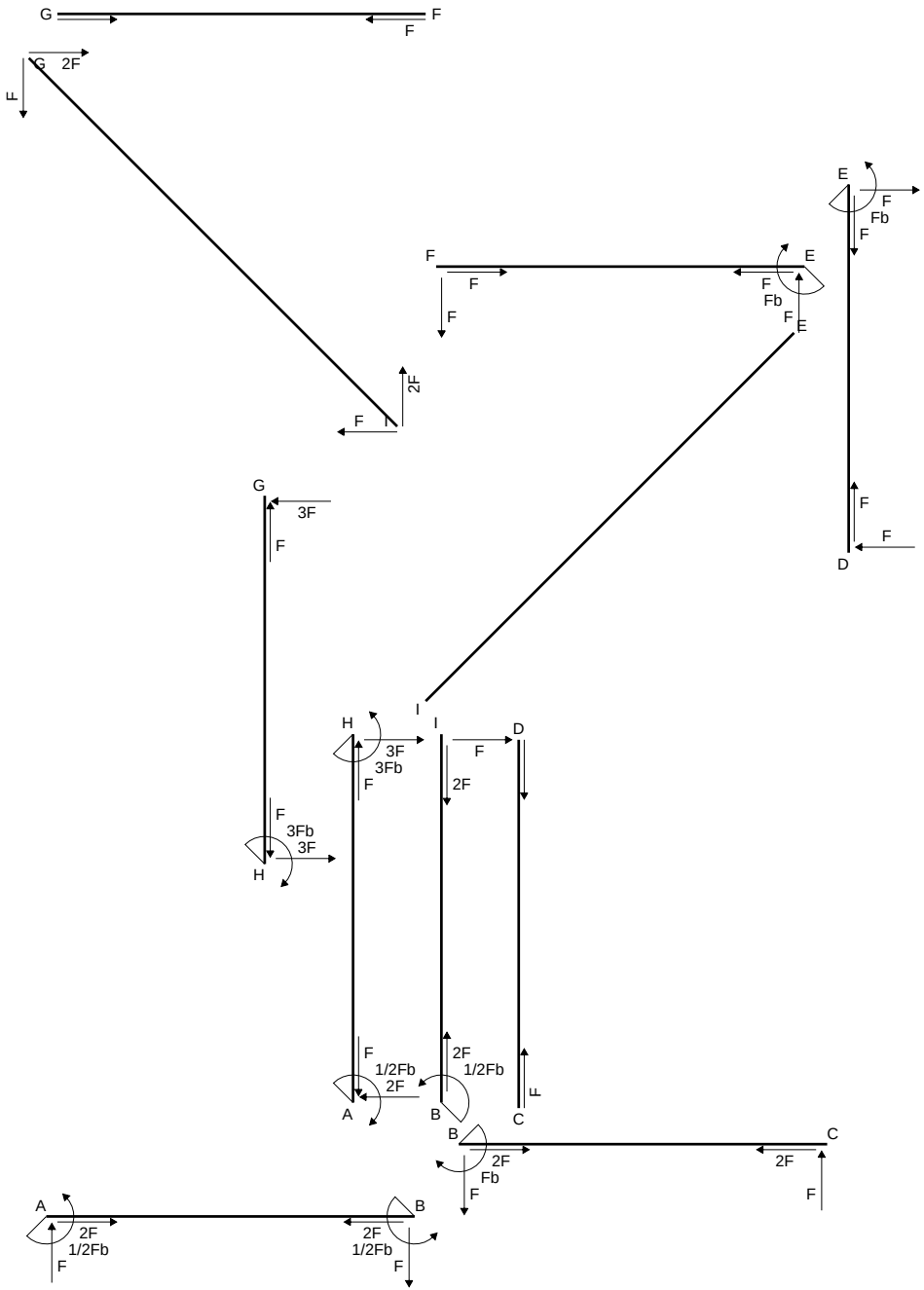
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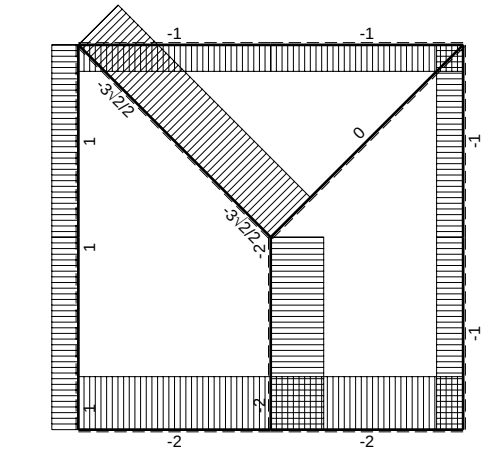
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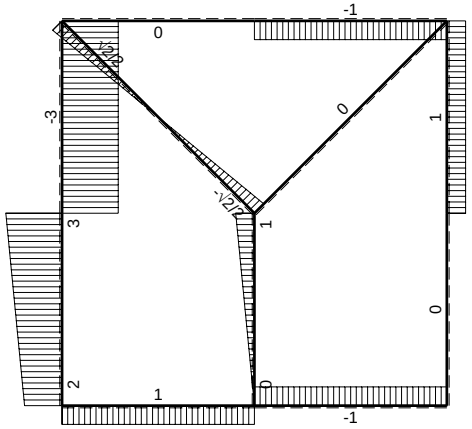
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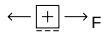
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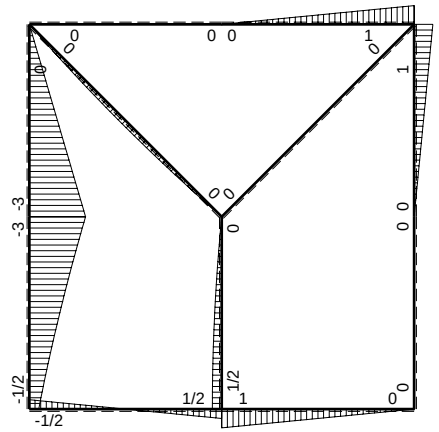
F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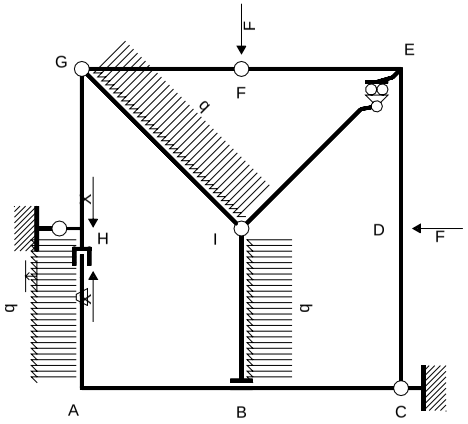
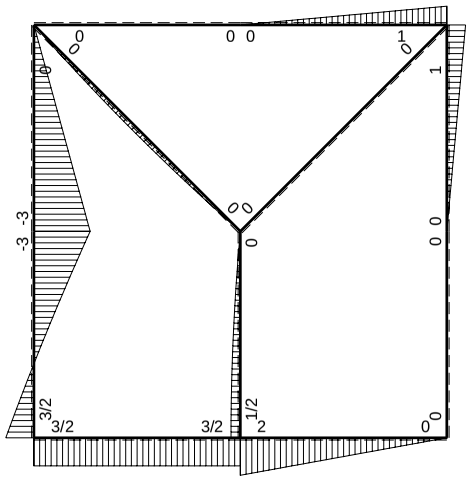
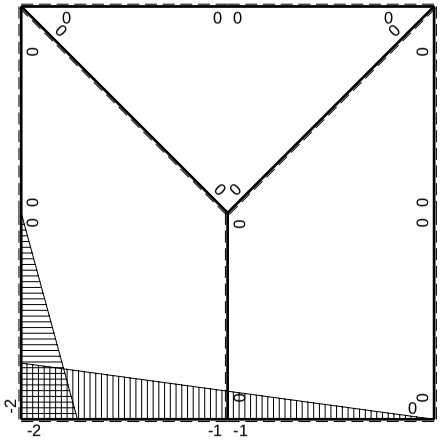
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981822 ESΣ4.xxxx.051

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Quadro contributi PLV per iperstatica $X=V_{HA}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	$3/2Fb$	0	$-3Fb^2+3/2Fbx$	0	$4b^2-4bx+x^2$	$(-9/4+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	$-3/2Fb$	0	$-3/2Fb^2-3/2Fbx$	0	$b^2+2bx+x^2$		
BC b	$-b+x$	$2Fb-2Fx$	0	$-2Fb^2+4Fbx-2Fx^2$	0	$b^2-2bx+x^2$	$(-2/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	x	$-2Fx$	0	$-2Fx^2$	0	x^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	$-Fb+Fx$	0	0	0	0		
EF b	0	$Fb-Fx$	0	0	0	0	0+0	0
FE b	0	$-Fx$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-3Fx$	0	0	0	0	0+0	0
HG b	0	$3Fb-3Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}b$	0	0	0	0	0	0	0	0
HA b	$-2x$	$-3Fb+5Fx-1/2qx^2$	$-Fb/EJ$	$6Fbx-10Fx^2+qx^3$	$2Fxb/EJ$	$4x^2$	$(-1/12+1)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$-3/2Fb+4Fx+1/2qx^2$	Fb/EJ	$-3Fb^2+11Fbx-7Fx^2-qx^3$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$-4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_{HA}$						F	

PROCEDIMENTO E RISULTATI 981822 ESΣ4.xxxx.051

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-3 + 3/2\,x/b)\, Fb^2\, 1/EJ\, dx = [-3\,x + 3/4\,x^2/b]_o^b\, Fb^2\, 1/EJ$

$= (-3\,b + 3/4\,b)\, Fb^2\, 1/EJ = -9/4\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-3/2 - 3/2\,x/b)\, Fb^2\, 1/EJ\, dx = [-3/2\,x - 3/4\,x^2/b]_o^b\, Fb^2\, 1/EJ$

$= (-3/2\,b - 3/4\,b)\, Fb^2\, 1/EJ = -9/4\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-2 + 4\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2\,x + 2\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2\,b + 2\,b - 2/3\,b)\, Fb^2\, 1/EJ = -2/3\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-2/3\,b)\, Fb^2\, 1/EJ = -2/3\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (6\,x/b - 10\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (2\,x/b)\,\theta\, dx$

$= [3\,x^2/b - 10/3\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ + [x^2/b]_o^b\,\theta$

$= (3\,b - 10/3\,b + 1/4\,b)\, Fb^2\, 1/EJ + (b)\,\theta = 11/12\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (-3 + 11\,x/b - 7\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-2 + 2\,x/b)\,\theta\, dx$

$= [-3\,x + 11/2\,x^2/b - 7/3\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ + [-2\,x + x^2/b]_o^b\,\theta$

$= (-3\,b + 11/2\,b - 7/3\,b - 1/4\,b)\, Fb^2\, 1/EJ + (-2\,b + b)\,\theta = 11/12\; Fb^3/EJ$

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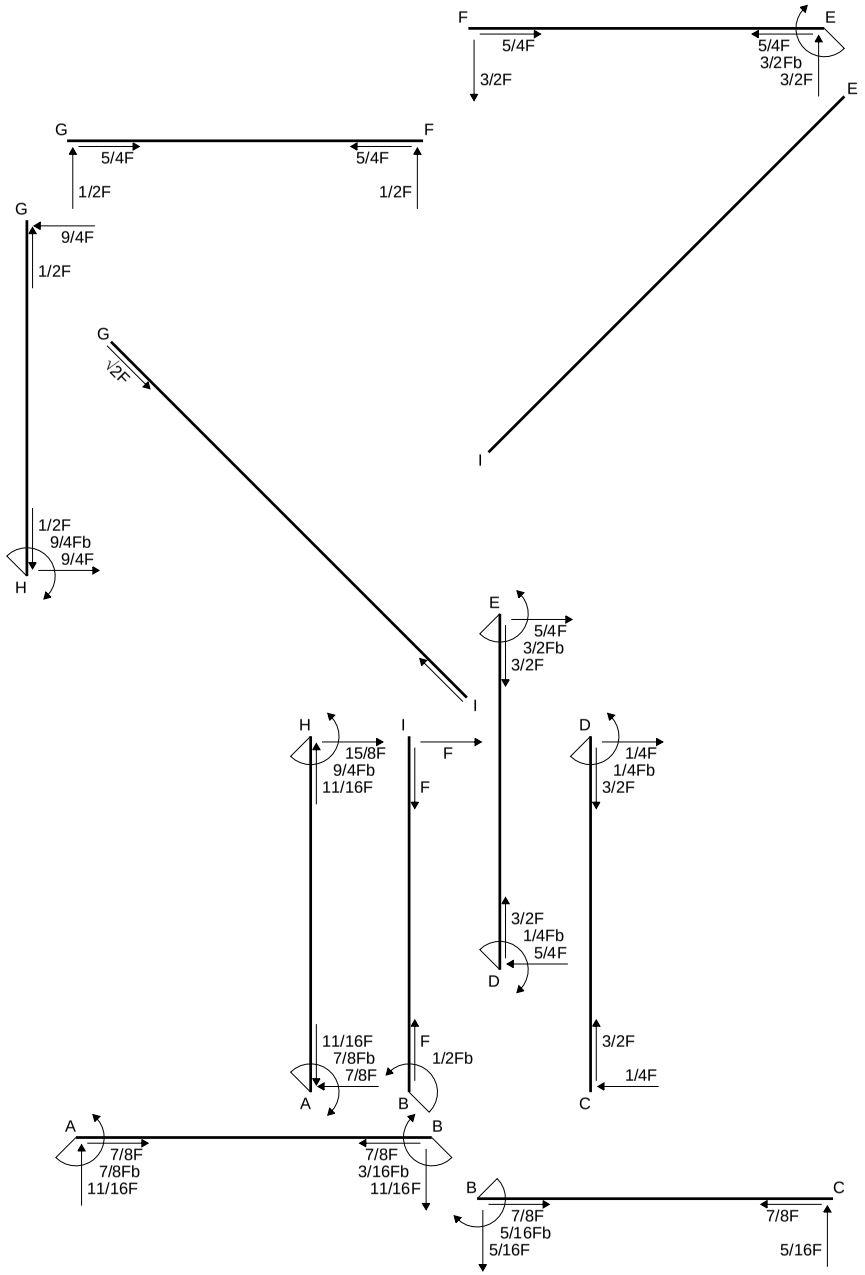
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G
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1/2F
G
½F
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1/2F
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H
E
5/4F
3/2Fb
3/2F
I
H
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D
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F
1/4F
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7/8Fb
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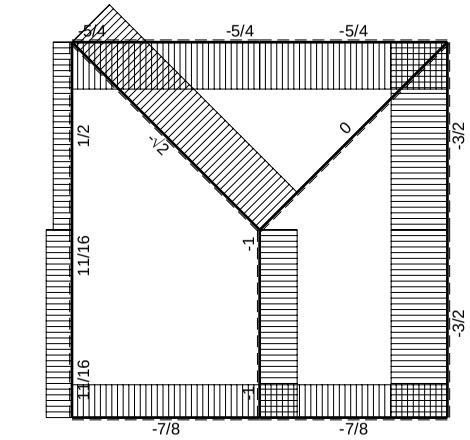
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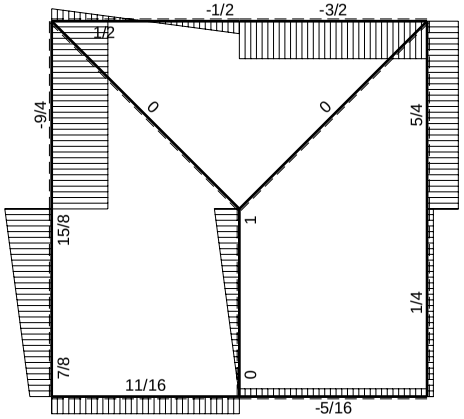
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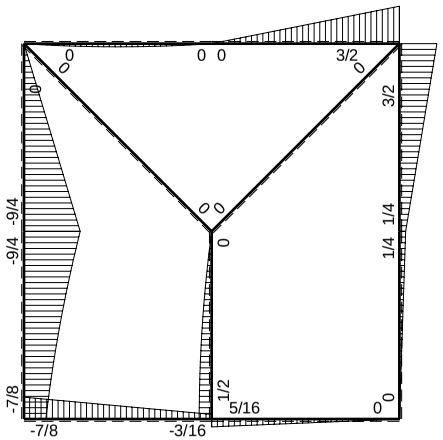
11/16
0
-5/16

←□→ F

↑□↓ F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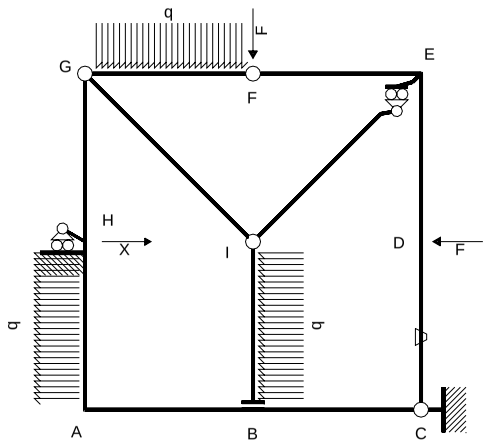
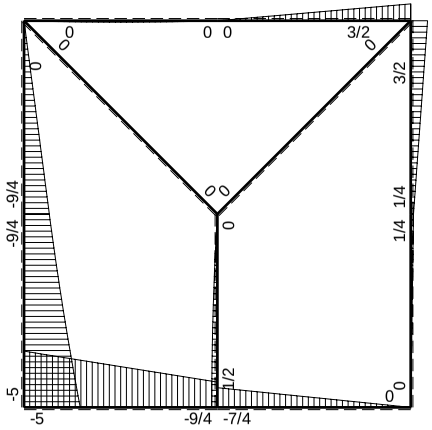
0
-7/8
-3/16

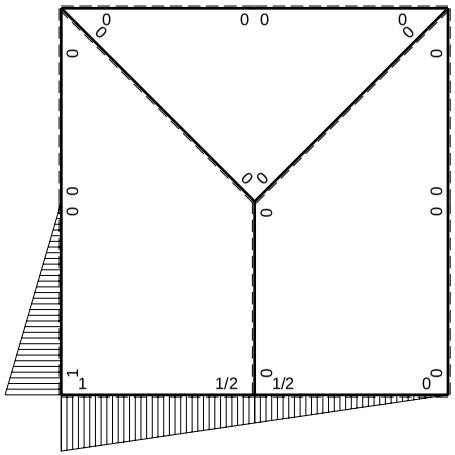
↻□↺ Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	b-1/2x	-5Fb+11/4Fx	0	$-5Fb^2+21/4Fbx-11/8Fx^2$	0	$b^2-bx+1/4x^2$	$(-17/6+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	-1/2b-1/2x	9/4Fb+11/4Fx	0	$-9/8Fb^2-5/2Fbx-11/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	1/2b-1/2x	-7/4Fb+7/4Fx	0	$-7/8Fb^2+7/4Fbx-7/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-7/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	-1/2x	7/4Fx	0	$-7/8Fx^2$	0	$1/4x^2$		
CD b	0	1/4Fx	-Fb/EJ	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	Fb/EJ	0	0	0		
DE b	0	1/4Fb+5/4Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	0	0	0	0		
EF b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
FE b	0	-3/2Fx	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	x	$-9/4Fb-9/4Fx-1/2qx^2$	0	$-9/4Fbx-9/4Fx^2-1/2qx^3$	0	x^2	$(-2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	-b+x	$5Fb-13/4Fx+1/2qx^2$	0	$-5Fb^2+33/4Fbx-15/4Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						Fb^3/EJ	
	totali						$-33/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_H$						33/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-5 + 21/4\ x/b - 11/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-5\ x + 21/8\ x^2/b - 11/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5\ b + 21/8\ b - 11/24\ b\ \right) Fb^2\ 1/EJ\ = -17/6\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-9/8 - 5/2\ x/b - 11/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-9/8\ x - 5/4\ x^2/b - 11/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-9/8\ b - 5/4\ b - 11/24\ b\ \right) Fb^2\ 1/EJ\ = -17/6\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-7/8 + 7/4\ x/b - 7/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-7/8\ x + 7/8\ x^2/b - 7/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-7/8\ b + 7/8\ b - 7/24\ b\ \right) Fb^2\ 1/EJ\ = -7/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-7/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-7/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-7/24\ b\ \right) Fb^2\ 1/EJ\ = -7/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(-9/4\ x/b - 9/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[-9/8\ x^2/b - 3/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-9/8\ b - 3/4\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = -2\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(-5 + 33/4\ x/b - 15/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx$

$= \left[-5\ x + 33/8\ x^2/b - 5/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5\ b + 33/8\ b - 5/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = -2\ \ Fb^3/EJ$

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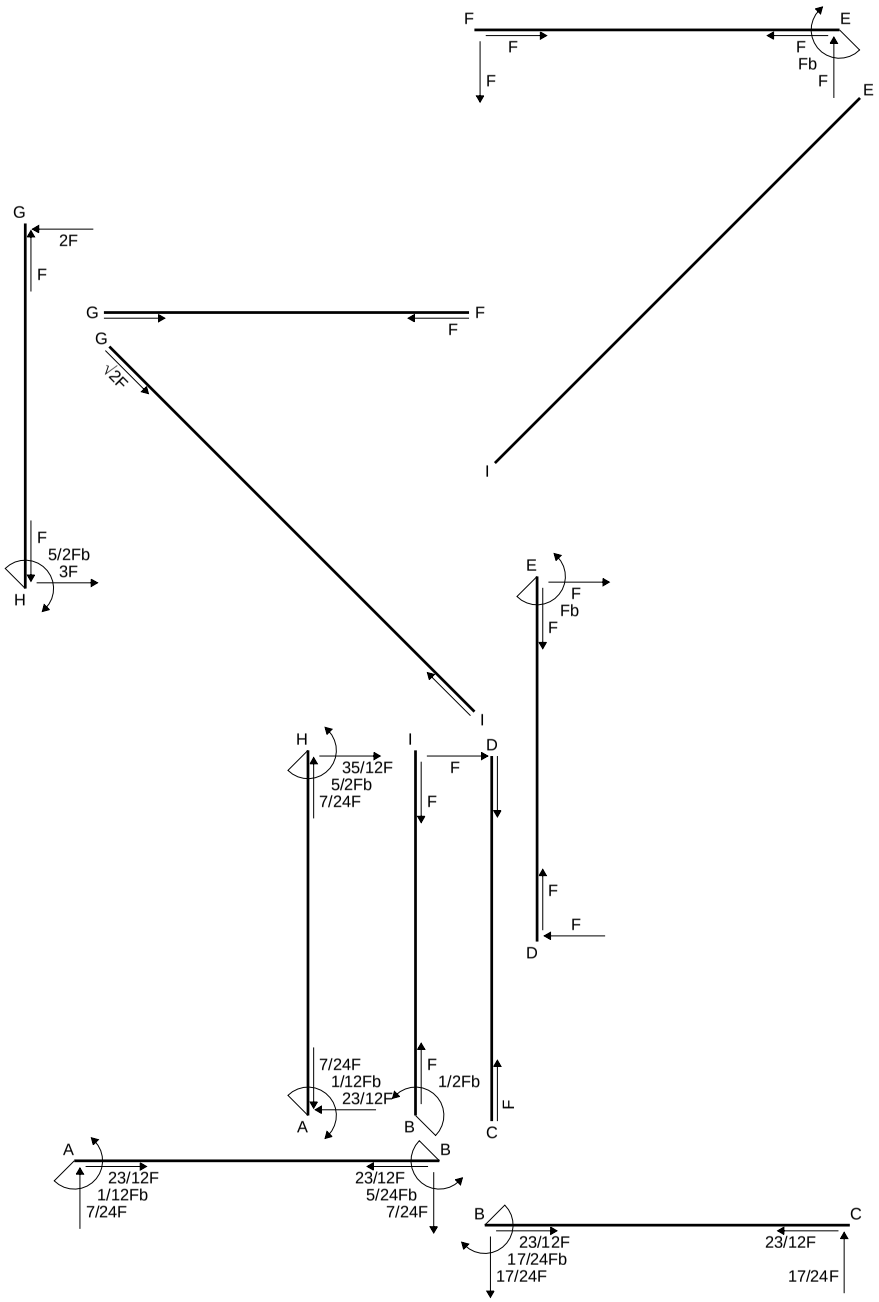
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F
D
F
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D
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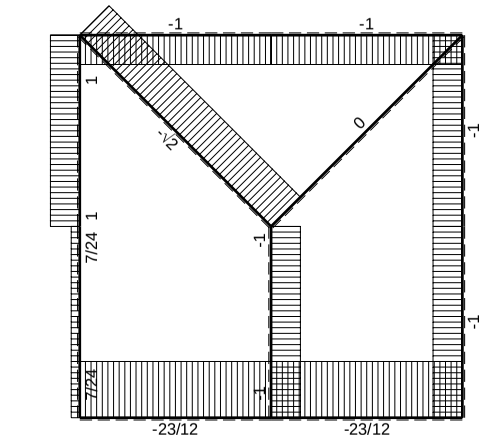
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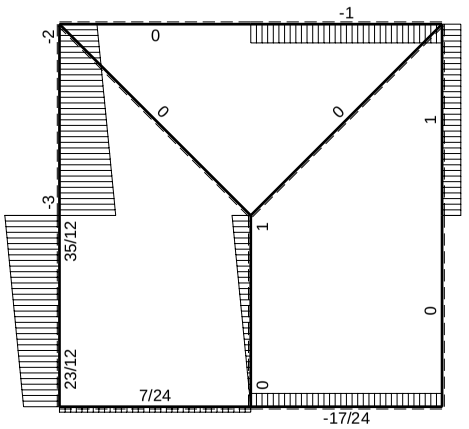
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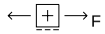
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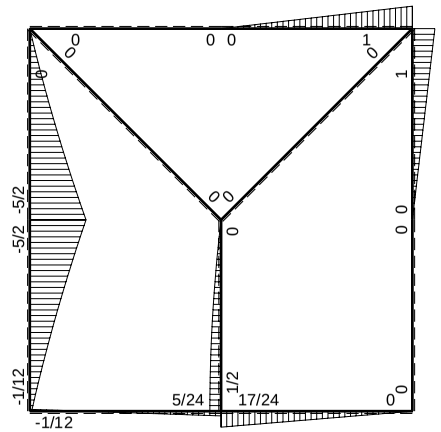
F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17/24
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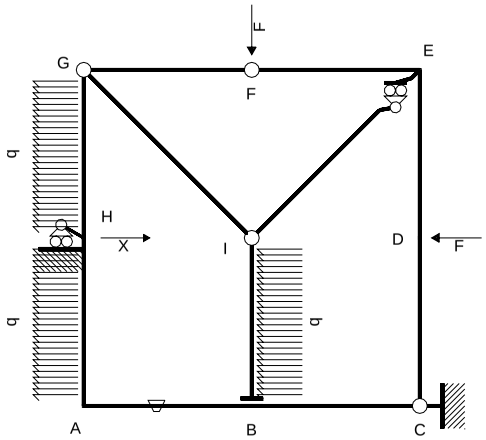
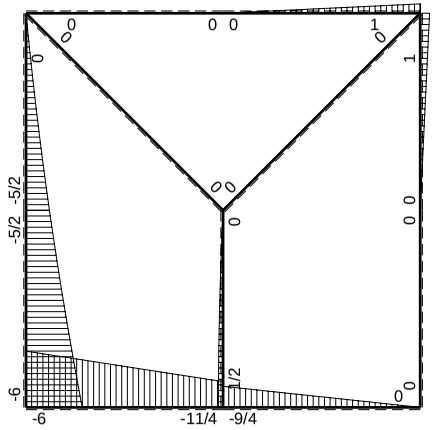
-1/12

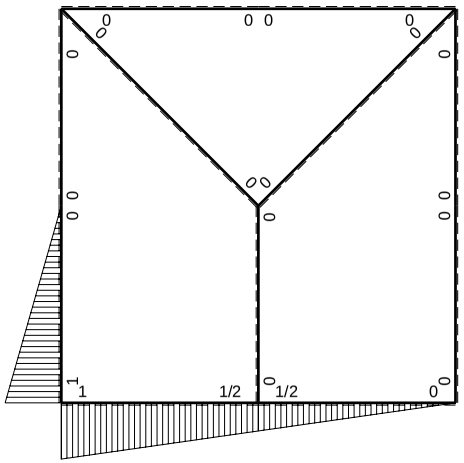

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981482 ESΣ4.xxxx.067

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Quadro contributi PLV per iperstatica $X=H_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	b-1/2x	-6Fb+13/4Fx	-Fb/EJ	$-6Fb^2+25/4Fbx-13/8Fx^2$	$-Fb^2/EJ+1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(-41/12-3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	-1/2b-1/2x	11/4Fb+13/4Fx	Fb/EJ	$-11/8Fb^2-3Fbx-13/8Fx^2$	$-1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
BC b	1/2b-1/2x	-9/4Fb+9/4Fx	0	$-9/8Fb^2+9/4Fbx-9/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-3/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	-1/2x	9/4Fx	0	$-9/8Fx^2$	0	$1/4x^2$		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	x	$-5/2Fb-3Fx-1/2qx^2$	0	$-5/2Fbx-3Fx^2-1/2qx^3$	0	x^2	$(-19/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	-b+x	$6Fb-4Fx+1/2qx^2$	0	$-6Fb^2+10Fbx-9/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						Fb^3/EJ	
	totali						$-71/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_H$						71/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-6 + 25/4\ x/b - 13/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(-1 + 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[-6\ x + 25/8\ x^2/b - 13/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[-\ x + 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-6\ b + 25/8\ b - 13/24\ b\ \right) Fb^2\ 1/EJ\ + \left(-\ b + 1/4\ b\ \right) \theta\ \ = -25/6\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-11/8 - 3\ x/b - 13/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(1/2 + 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[-11/8\ x - 3/2\ x^2/b - 13/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[1/2\ x + 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-11/8\ b - 3/2\ b - 13/24\ b\ \right) Fb^2\ 1/EJ\ + \left(1/2\ b + 1/4\ b\ \right) \theta\ \ = -25/6\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-9/8 + 9/4\ x/b - 9/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-9/8\ x + 9/8\ x^2/b - 3/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-9/8\ b + 9/8\ b - 3/8\ b\ \right) Fb^2\ 1/EJ\ = -3/8\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-9/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-3/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-3/8\ b\ \right) Fb^2\ 1/EJ\ = -3/8\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(-5/2\ x/b - 3\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[-5/4\ x^2/b - \ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-5/4\ b - \ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = -19/8\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(-6 + 10\ x/b - 9/2\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[-6\ x + 5\ x^2/b - 3/2\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-6\ b + 5\ b - 3/2\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = -19/8\ \ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 981482 ESΣ4.xxxx.067

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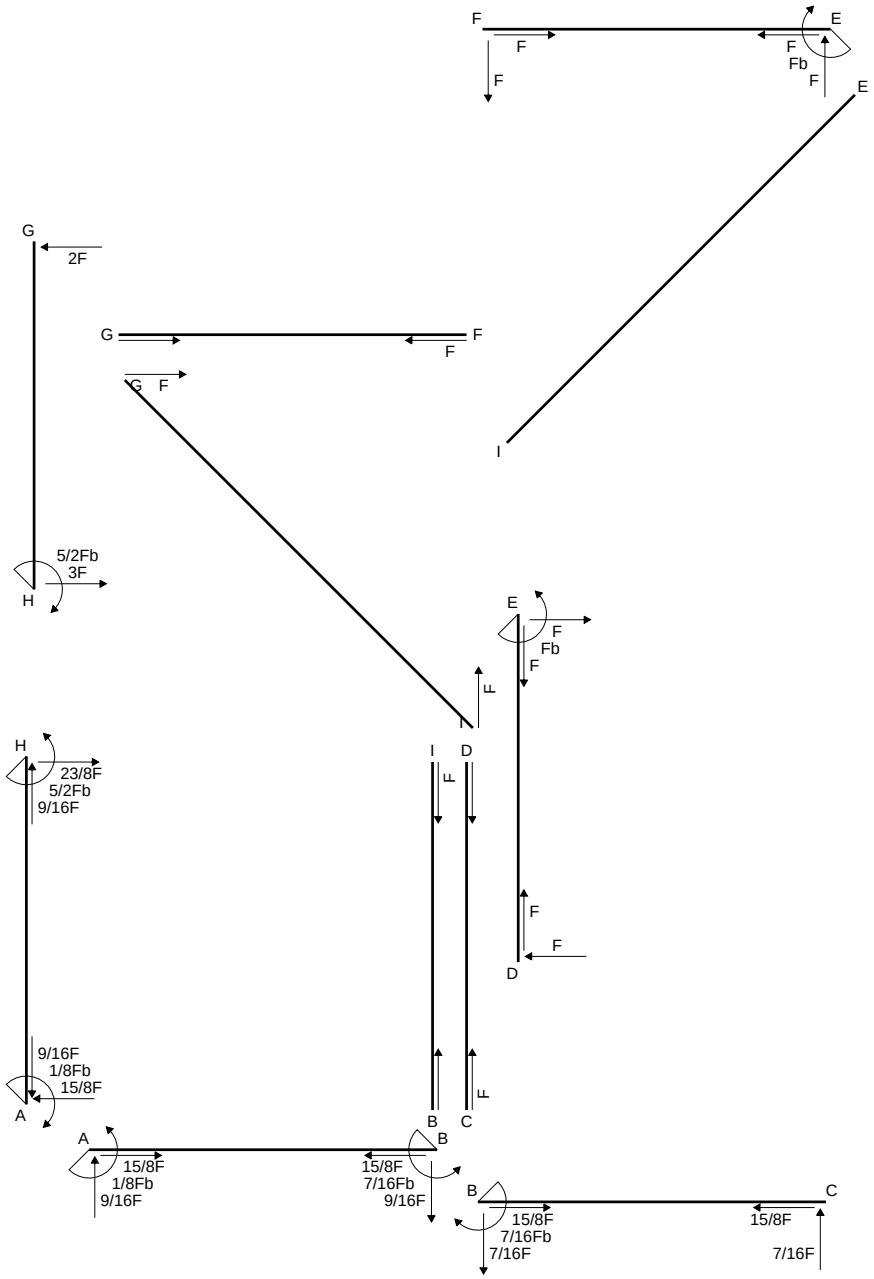
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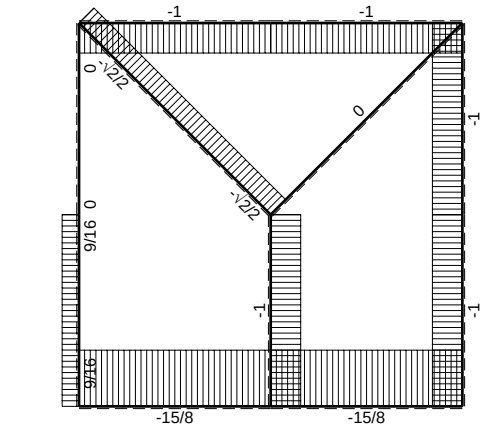
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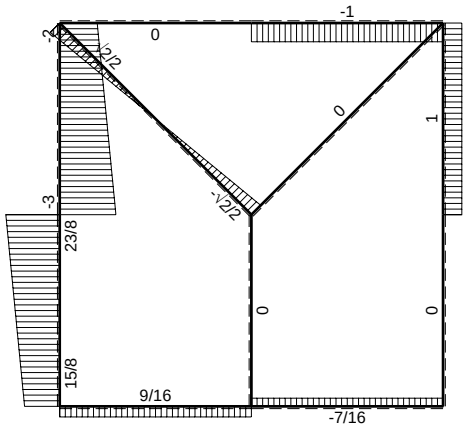
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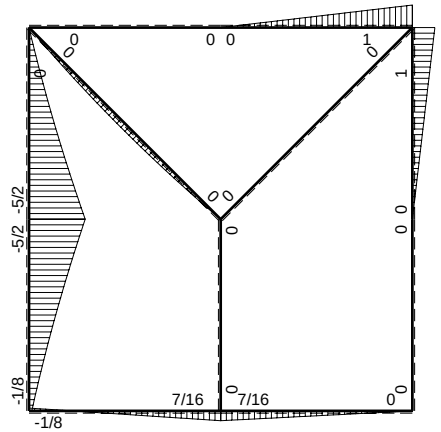
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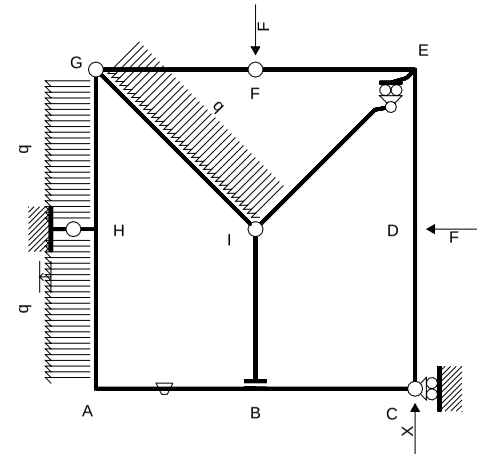
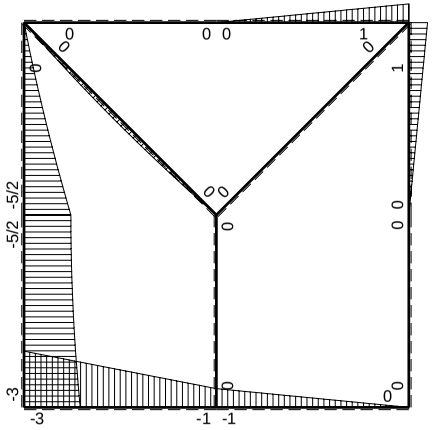
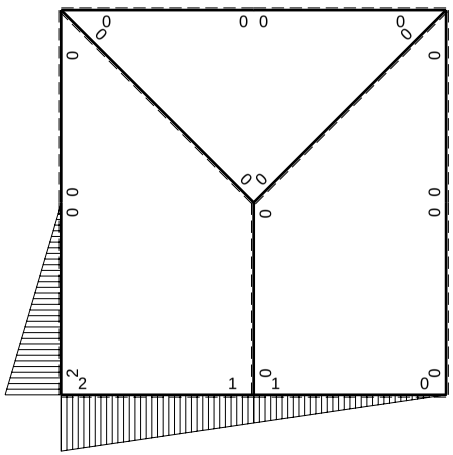
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983213 ESΣ4.xxxx.111

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Quadro contributi PLV per iperstatica $X=V_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2b-x	-3Fb+2Fx	-Fb/EJ	$-6Fb^2+7Fbx-2Fx^2$	$-2Fb^2/EJ+Fxb/EJ$	$4b^2-4bx+x^2$	$(-19/6-3/2)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	-b-x	Fb+2Fx	Fb/EJ	$-Fb^2-3Fbx-2Fx^2$	$-Fb^2/EJ-Fxb/EJ$	$b^2+2bx+x^2$		
BC b	b-x	-Fb+Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	-x	Fx	0	$-Fx^2$	0	x^2		
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0		
DE b	0	Fx	0	0	0	0	0+0	0
ED b	0	-Fb+Fx	0	0	0	0		
EF b	0	Fb-Fx	0	0	0	0	0+0	0
FE b	0	-Fx	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	2x	$-5/2Fb-1/2qx^2$	0	$-5Fbx-qx^3$	0	$4x^2$	$(-11/4+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	-2b+2x	$3Fb-Fx+1/2qx^2$	0	$-6Fb^2+8Fbx-3Fx^2+qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$2Fb^3/EJ$	
	totali						$-23/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_C$						23/16F	

PROCEDIMENTO E RISULTATI 983213 ESΣ4.xxxx.111
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$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-6 + 7\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-2 + x/b)\,\theta\, dx$

$= [-6\,x + 7/2\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-2\,x + 1/2\,x^2/b]_o^b\,\theta$

$= (-6\,b + 7/2\,b - 2/3\,b)\, Fb^2\, 1/EJ + (-2\,b + 1/2\,b)\,\theta = -14/3\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 - 3\,x/b - 2\,x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 + x/b)\,\theta\, dx$

$= [-x - 3/2\,x^2/b - 2/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x + 1/2\,x^2/b]_o^b\,\theta$

$= (-b - 3/2\,b - 2/3\,b)\, Fb^2\, 1/EJ + (b + 1/2\,b)\,\theta = -14/3\; Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1 + 2\,x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-x + x^2/b - 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-b + b - 1/3\,b)\, Fb^2\, 1/EJ = -1/3\; Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/3\,b)\, Fb^2\, 1/EJ = -1/3\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (-5\,x/b - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-5/2\,x^2/b - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-5/2\,b - 1/4\,b)\, Fb^2\, 1/EJ = -11/4\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (-6 + 8\,x/b - 3\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-6\,x + 4\,x^2/b - x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-6\,b + 4\,b - b + 1/4\,b)\, Fb^2\, 1/EJ = -11/4\; Fb^3/EJ$

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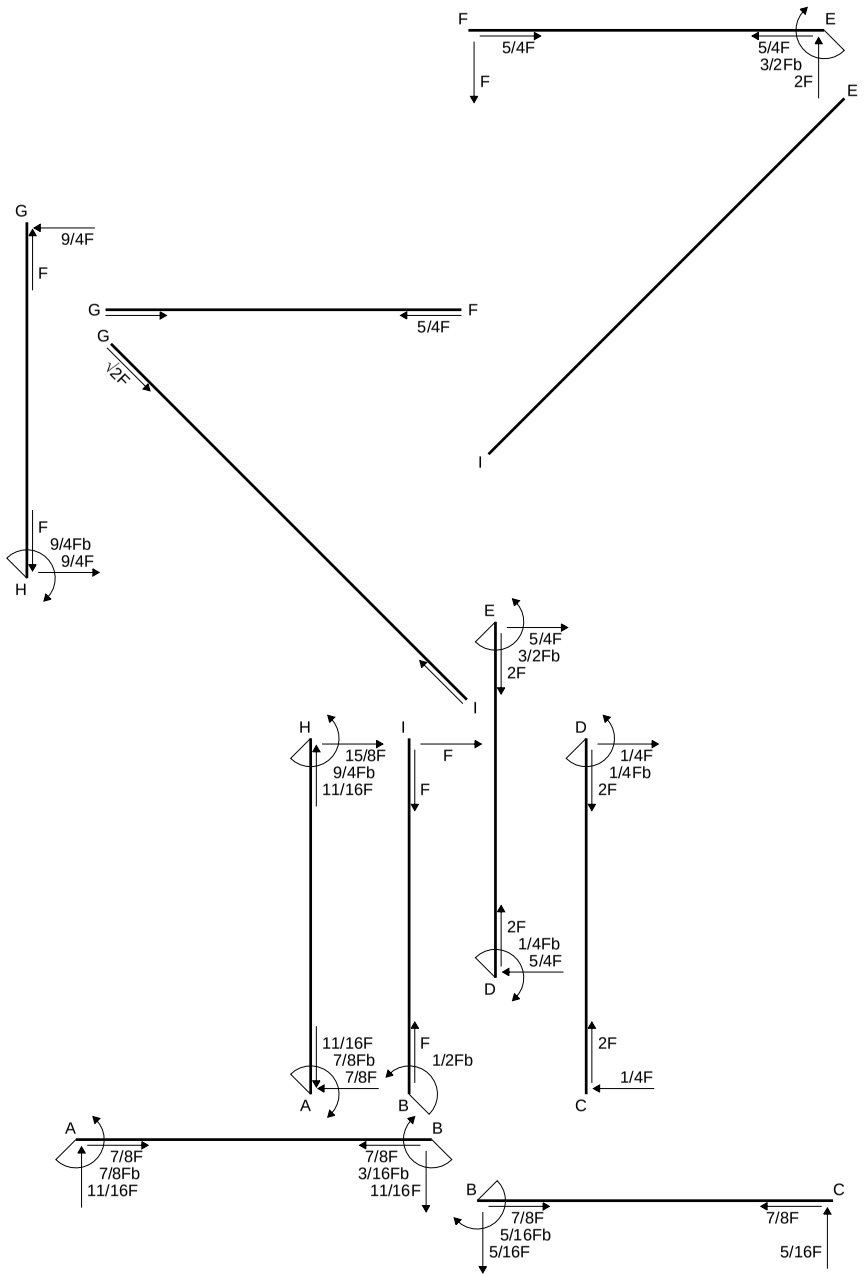

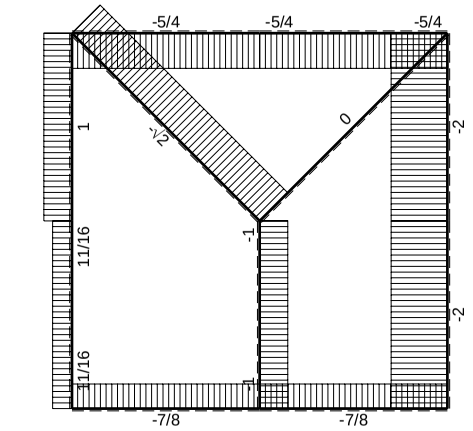

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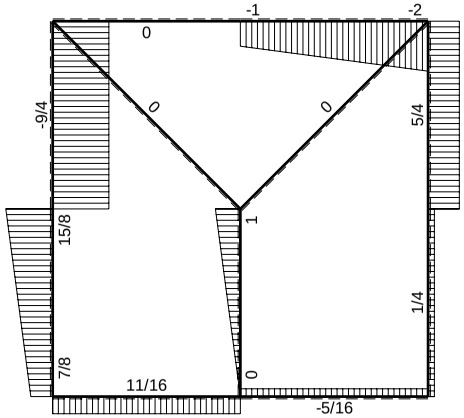

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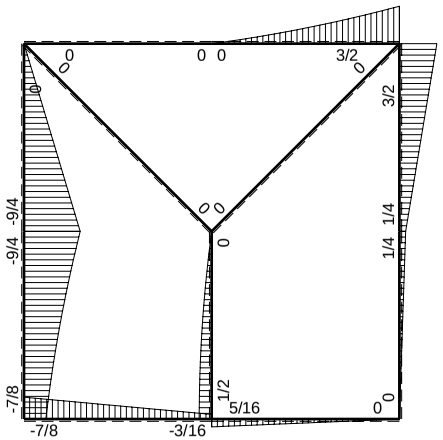

Fb

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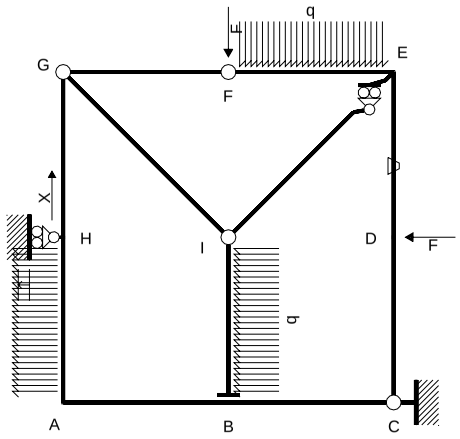
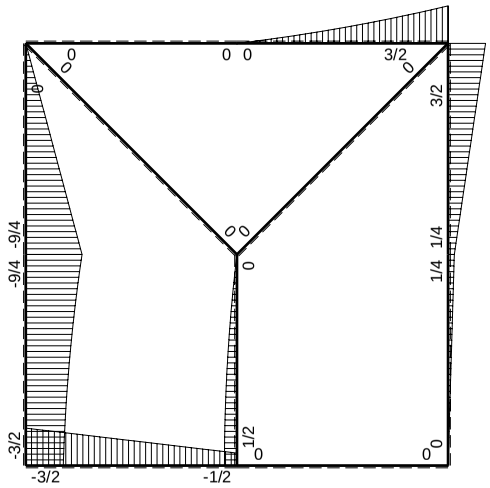
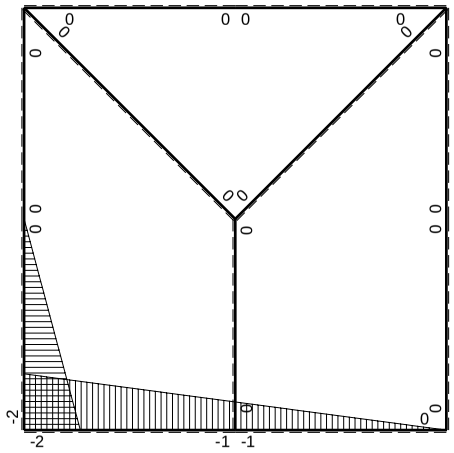




M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983331 ESΣ4.xxxx.136

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PROCEDIMENTO E RISULTATI 983331 ESΣ4.xxxx.136

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2b+x	-3/2Fb+Fx	0	$3Fb^2-7/2Fbx+Fx^2$	0	$4b^2-4bx+x^2$	$(19/12+0)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	b+x	1/2Fb+Fx	0	$1/2Fb^2+3/2Fbx+Fx^2$	0	$b^2+2bx+x^2$		
BC b	-b+x	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CB b	x	0	0	0	0	x^2		
CD b	0	1/4Fx	0	0	0	0	0+0	0
DC b	0	-1/4Fb+1/4Fx	0	0	0	0		
DE b	0	1/4Fb+5/4Fx	-Fb/EJ	0	0	0	0+0	0
ED b	0	-3/2Fb+5/4Fx	Fb/EJ	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-9/4Fx	0	0	0	0	0+0	0
HG b	0	9/4Fb-9/4Fx	0	0	0	0		
GI √2b	0	0	0	0	0	0	0	0
IB b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
BI b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
IE √2b	0	0	0	0	0	0	0	0
HA b	-2x	$-9/4Fb+5/4Fx-1/2qx^2$	0	$9/2Fbx-5/2Fx^2+qx^3$	0	$4x^2$	$(5/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	2b-2x	$3/2Fb+1/4Fx+1/2qx^2$	0	$3Fb^2-5/2Fbx+1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$5/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						-5/16F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (3 - 7/2\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3\,x - 7/4\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3\,b - 7/4\,b + 1/3\,b)\, Fb^2\, 1/EJ = 19/12\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2 + 3/2\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\,x + 3/4\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\,b + 3/4\,b + 1/3\,b)\, Fb^2\, 1/EJ = 19/12\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (9/2\,x/b - 5/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [9/4\,x^2/b - 5/6\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (9/4\,b - 5/6\,b + 1/4\,b)\, Fb^2\, 1/EJ = 5/3\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (3 - 5/2\,x/b + 1/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [3\,x - 5/4\,x^2/b + 1/6\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (3\,b - 5/4\,b + 1/6\,b - 1/4\,b)\, Fb^2\, 1/EJ = 5/3\; Fb^3/EJ$

ESΣ4.xxxx.136

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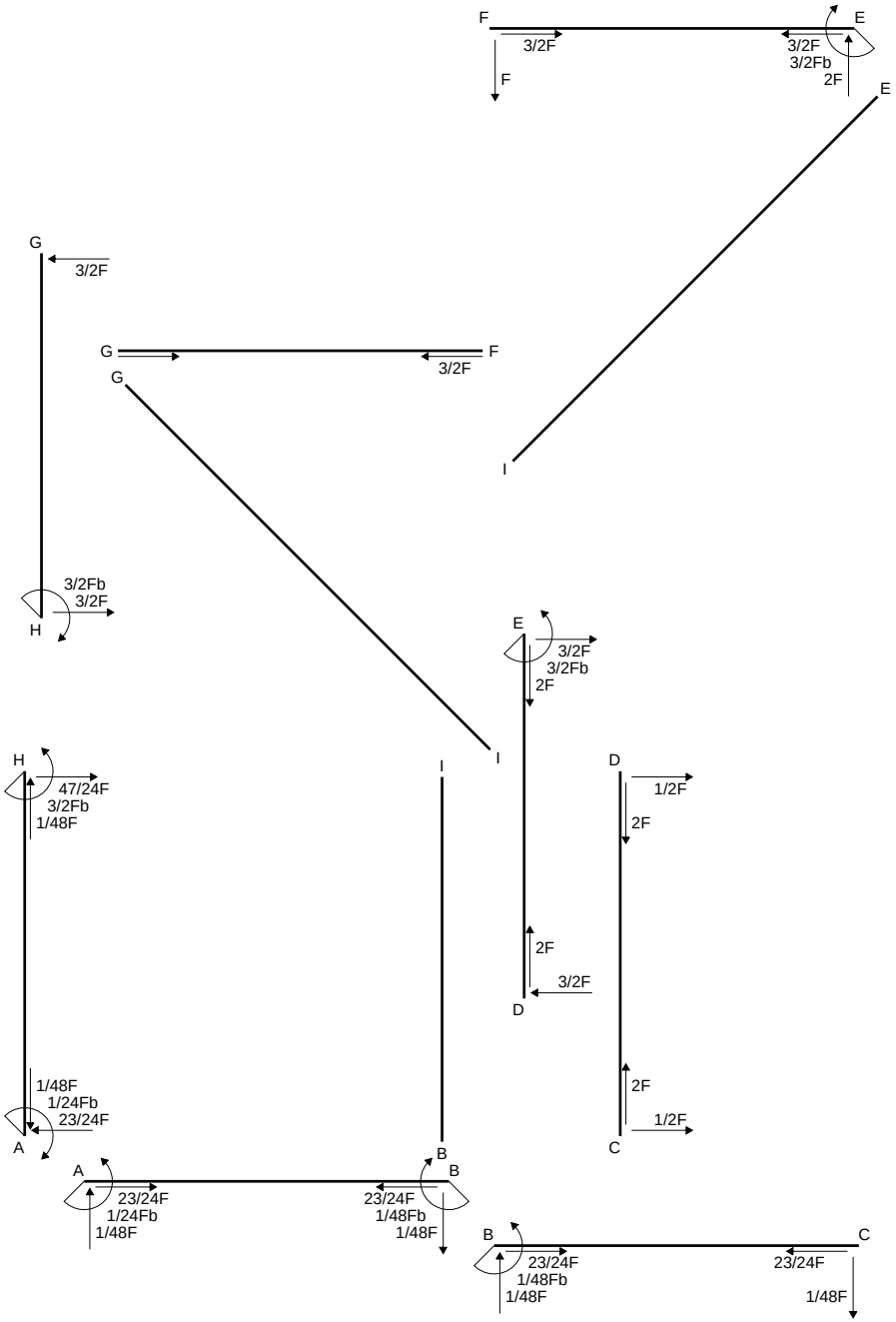
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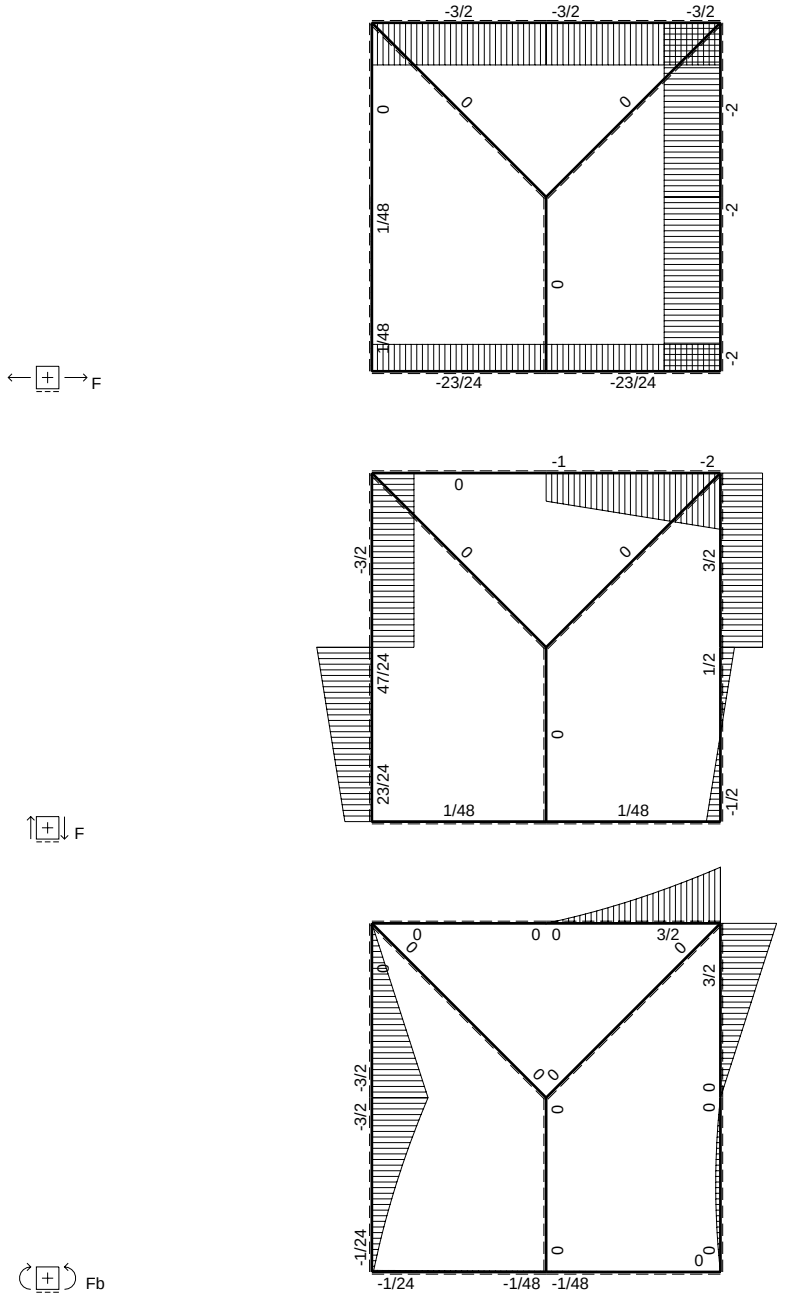
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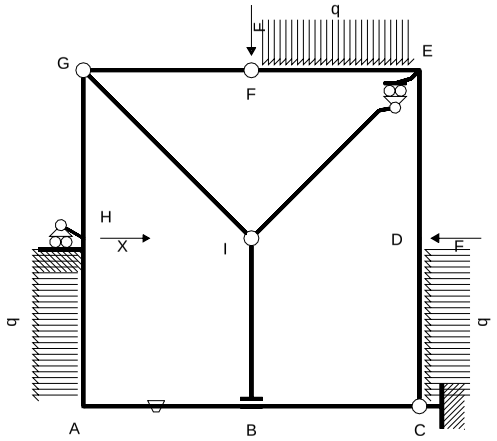
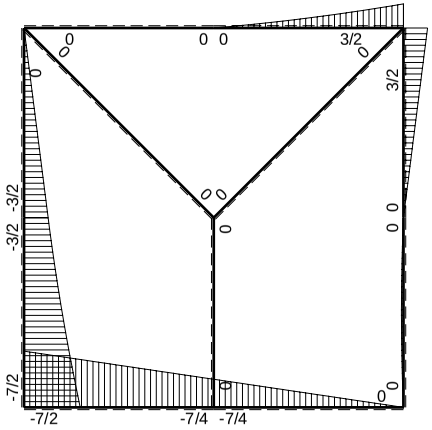
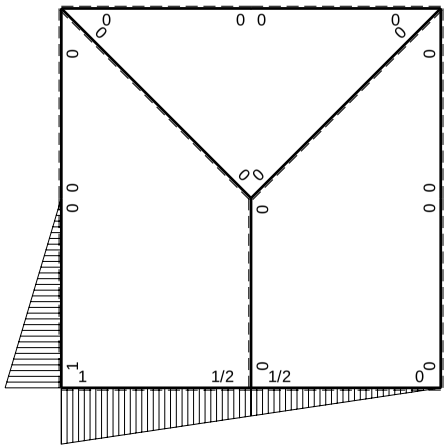
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 980904 ESΣ4.xxxx.153

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Quadro contributi PLV per iperstatica $X=H_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	b-1/2x	-7/2Fb+7/4Fx	-Fb/EJ	$-7/2Fb^2+7/2Fbx-7/8Fx^2$	$-Fb^2/EJ+1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(-49/24-3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	-1/2b-1/2x	7/4Fb+7/4Fx	Fb/EJ	$-7/8Fb^2-7/4Fbx-7/8Fx^2$	$-1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
BC b	1/2b-1/2x	-7/4Fb+7/4Fx	0	$-7/8Fb^2+7/4Fbx-7/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(-7/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	-1/2x	7/4Fx	0	$-7/8Fx^2$	0	$1/4x^2$		
CD b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	3/2Fx	0	0	0	0	0+0	0
ED b	0	-3/2Fb+3/2Fx	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	-3/2Fx	0	0	0	0	0+0	0
HG b	0	3/2Fb-3/2Fx	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	x	$-3/2Fb-3/2Fx-1/2qx^2$	0	$-3/2Fbx-3/2Fx^2-1/2qx^3$	0	x^2	$(-11/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	-b+x	$7/2Fb-5/2Fx+1/2qx^2$	0	$-7/2Fb^2+6Fbx-3Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						Fb^3/EJ	
	totali						$-83/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_H$						83/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-7/2 + 7/2\ x/b - 7/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(-1 + 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[-7/2\ x + 7/4\ x^2/b - 7/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[-\ x + 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-7/2\ b + 7/4\ b - 7/24\ b\ \right) Fb^2\ 1/EJ\ + \left(-\ b + 1/4\ b\ \right) \theta\ \ = -67/24\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-7/8 - 7/4\ x/b - 7/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(1/2 + 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[-7/8\ x - 7/8\ x^2/b - 7/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[1/2\ x + 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(-7/8\ b - 7/8\ b - 7/24\ b\ \right) Fb^2\ 1/EJ\ + \left(1/2\ b + 1/4\ b\ \right) \theta\ \ = -67/24\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-7/8 + 7/4\ x/b - 7/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-7/8\ x + 7/8\ x^2/b - 7/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-7/8\ b + 7/8\ b - 7/24\ b\ \right) Fb^2\ 1/EJ\ = -7/24\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-7/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[-7/24\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-7/24\ b\ \right) Fb^2\ 1/EJ\ = -7/24\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(-3/2\ x/b - 3/2\ x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[-3/4\ x^2/b - 1/2\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-3/4\ b - 1/2\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = -11/8\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(-7/2 + 6\ x/b - 3\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[-7/2\ x + 3\ x^2/b - x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(-7/2\ b + 3\ b - b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = -11/8\ \ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 980904 ESΣ4.xxxx.153

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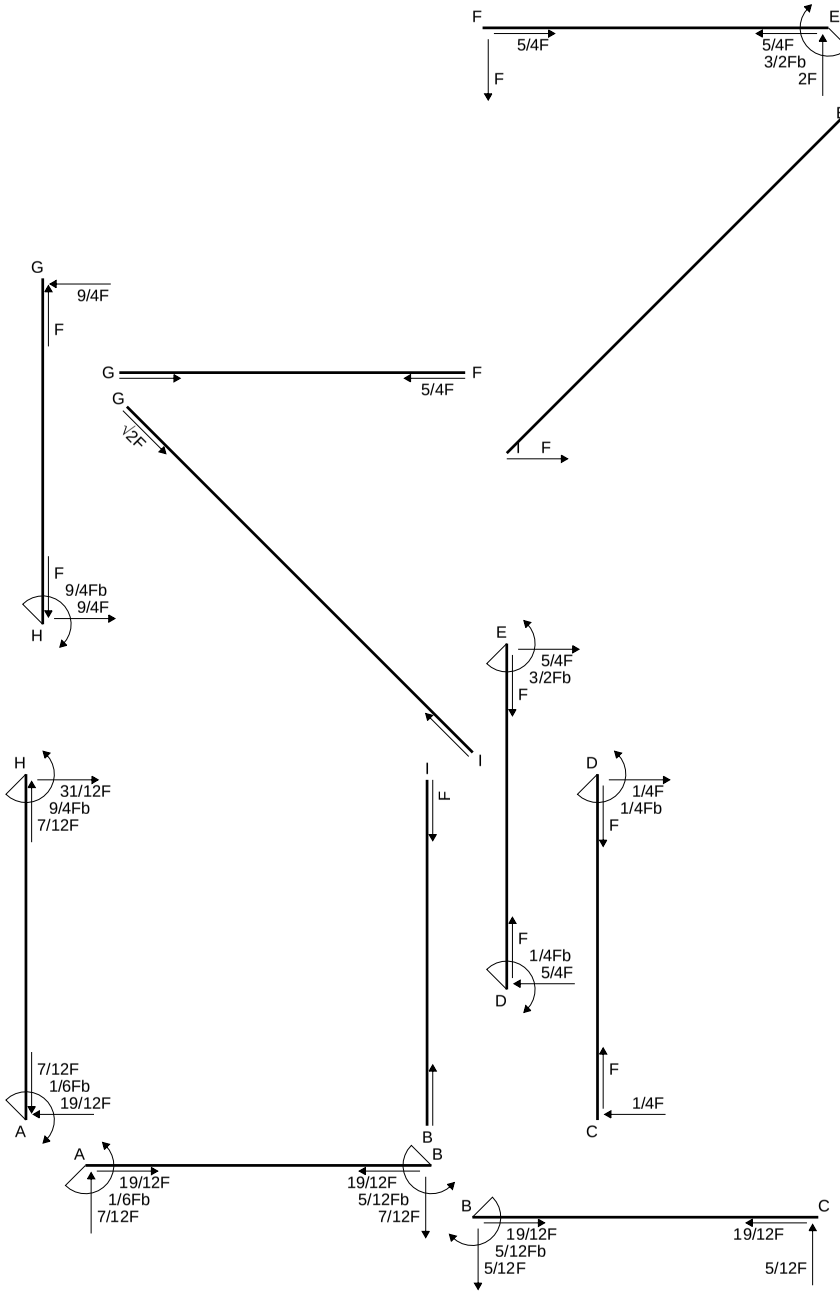
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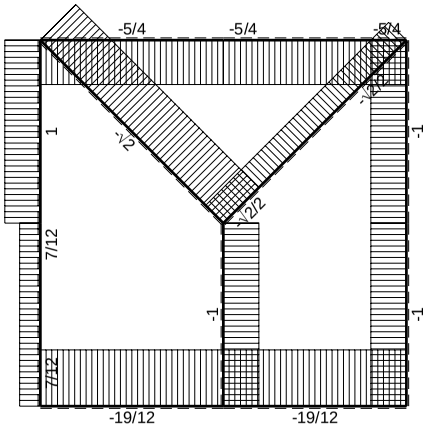
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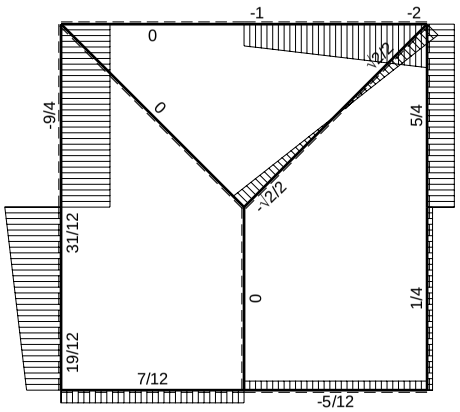
-19/12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F

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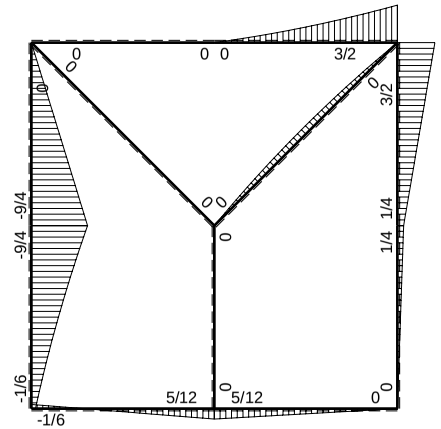
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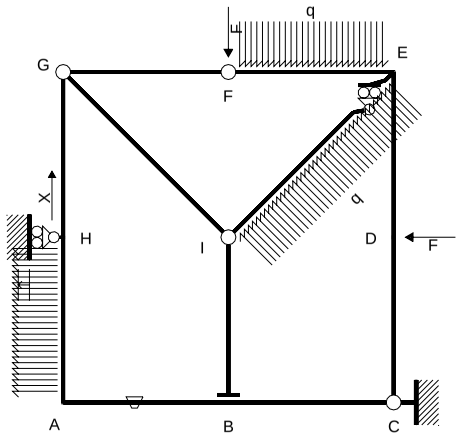
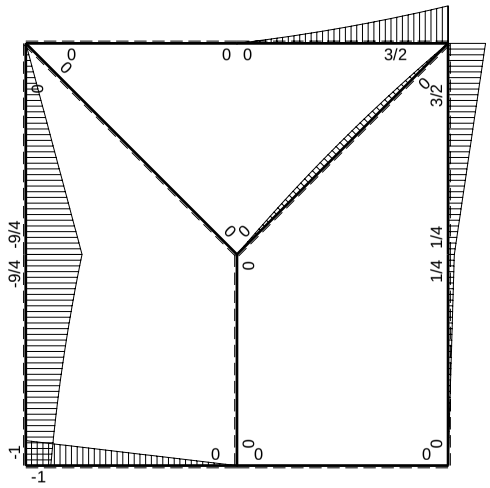
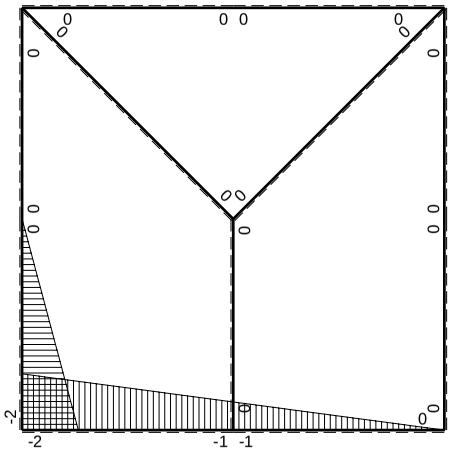




M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 986207 ESΣ4.xxxx.166

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PROCEDIMENTO E RISULTATI 986207 ESΣ4.xxxx.166

Quadro contributi PLV per iperstatica $X=V_H$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-2b+x$	$-Fb+Fx$	$-Fb/EJ$	$2Fb^2-3Fbx+Fx^2$	$2Fb^2/EJ-Fxb/EJ$	$4b^2-4bx+x^2$	$(5/6+3/2)Fb^3/EJ$	$7/3Xb^3/EJ$
BA b	$b+x$	Fx	Fb/EJ	$Fbx+Fx^2$	$Fb^2/EJ+Fxb/EJ$	$b^2+2bx+x^2$		
BC b	$-b+x$	0	0	0	0	$b^2-2bx+x^2$	0+0	$1/3Xb^3/EJ$
CB b	x	0	0	0	0	x^2		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	0	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}b$	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}b$	0	$-\sqrt{2}/2Fx+1/2qx^2$	0	0	0	0	0	0
HA b	$-2x$	$-9/4Fb+7/4Fx-1/2qx^2$	0	$9/2Fbx-7/2Fx^2+qx^3$	0	$4x^2$	$(4/3+0)Fb^3/EJ$	$4/3Xb^3/EJ$
AH b	$2b-2x$	$Fb+3/4Fx+1/2qx^2$	0	$2Fb^2-1/2Fbx-1/2Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-2Fb^3/EJ$	
	totali						$5/3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_H$						$-5/12F$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4 - 4\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 2\,x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 2\,b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 + 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x + x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b + b + 1/3\,b)\, b^2\, 1/EJ = 7/3\; b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - 2\,x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b + 1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b + 1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{CB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\,b)\, b^2\, 1/EJ = 1/3\; b^3/EJ$

$L^{XX}_{HA} = \int_o^b (4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{XX}_{AH} = \int_o^b (4 - 8\,x/b + 4\,x^2/b^2)\, b^2\, 1/EJ\, dx = [4\,x - 4\,x^2/b + 4/3\,x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (4\,b - 4\,b + 4/3\,b)\, b^2\, 1/EJ = 4/3\; b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (2 - 3\,x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (2 - x/b)\,\theta\; dx$

$= [2\,x - 3/2\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ + [2\,x - 1/2\,x^2/b]_o^b\,\theta$

$= (2\,b - 3/2\,b + 1/3\,b)\, Fb^2\, 1/EJ + (2\,b - 1/2\,b)\,\theta = 7/3\; Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (x/b + x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 - x/b)\,\theta\; dx$

$= [1/2\,x^2/b + 1/3\,x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-x - 1/2\,x^2/b]_o^b\,\theta$

$= (1/2\,b + 1/3\,b)\, Fb^2\, 1/EJ + (-b - 1/2\,b)\,\theta = 7/3\; Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b (9/2\,x/b - 7/2\,x^2/b^2 + x^3/b^3)\, Fb^2\, 1/EJ\, dx = [9/4\,x^2/b - 7/6\,x^3/b^2 + 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (9/4\,b - 7/6\,b + 1/4\,b)\, Fb^2\, 1/EJ = 4/3\; Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b (2 - 1/2\,x/b - 1/2\,x^2/b^2 - x^3/b^3)\, Fb^2\, 1/EJ\, dx = [2\,x - 1/4\,x^2/b - 1/6\,x^3/b^2 - 1/4\,x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (2\,b - 1/4\,b - 1/6\,b - 1/4\,b)\, Fb^2\, 1/EJ = 4/3\; Fb^3/EJ$

ESΣ4.xxxx.166

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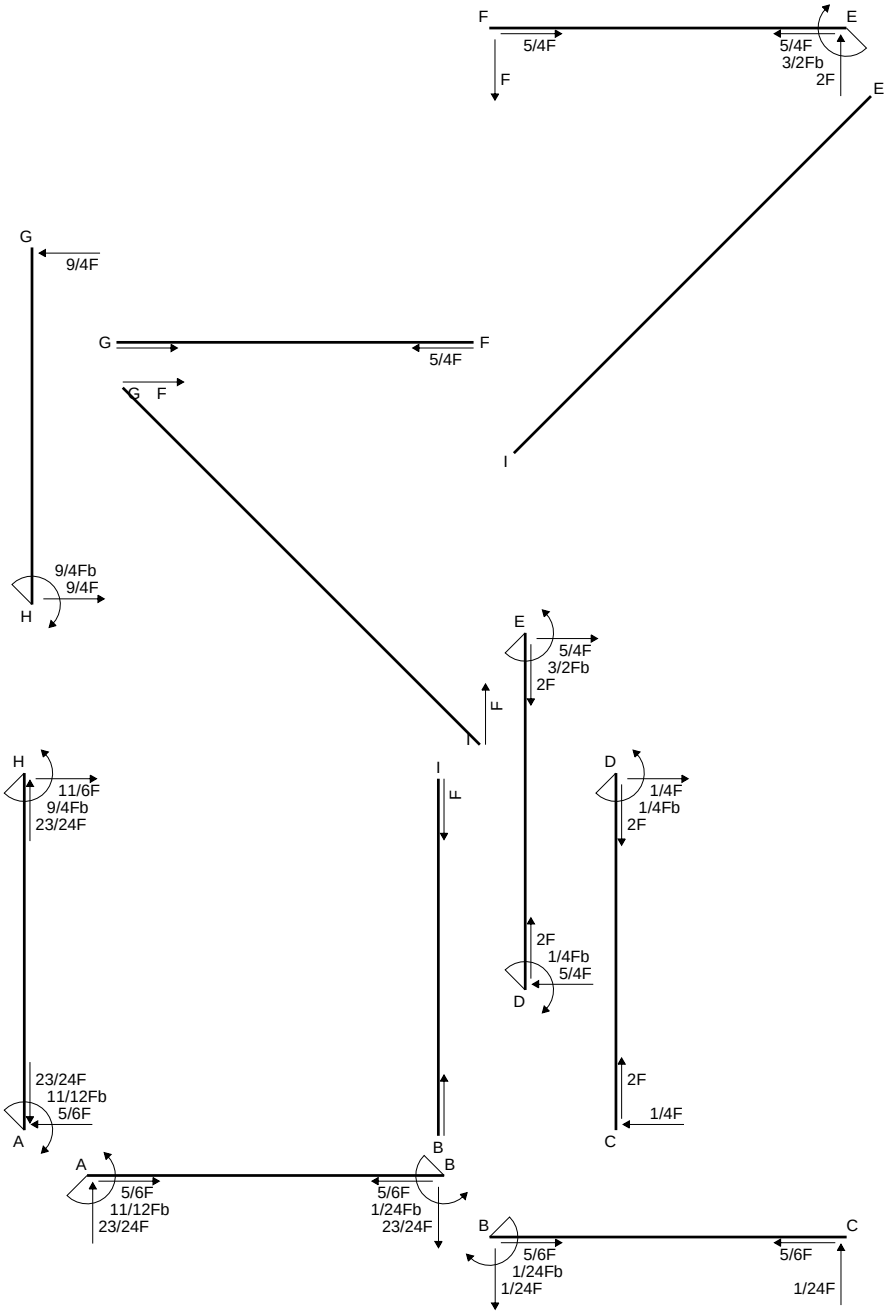
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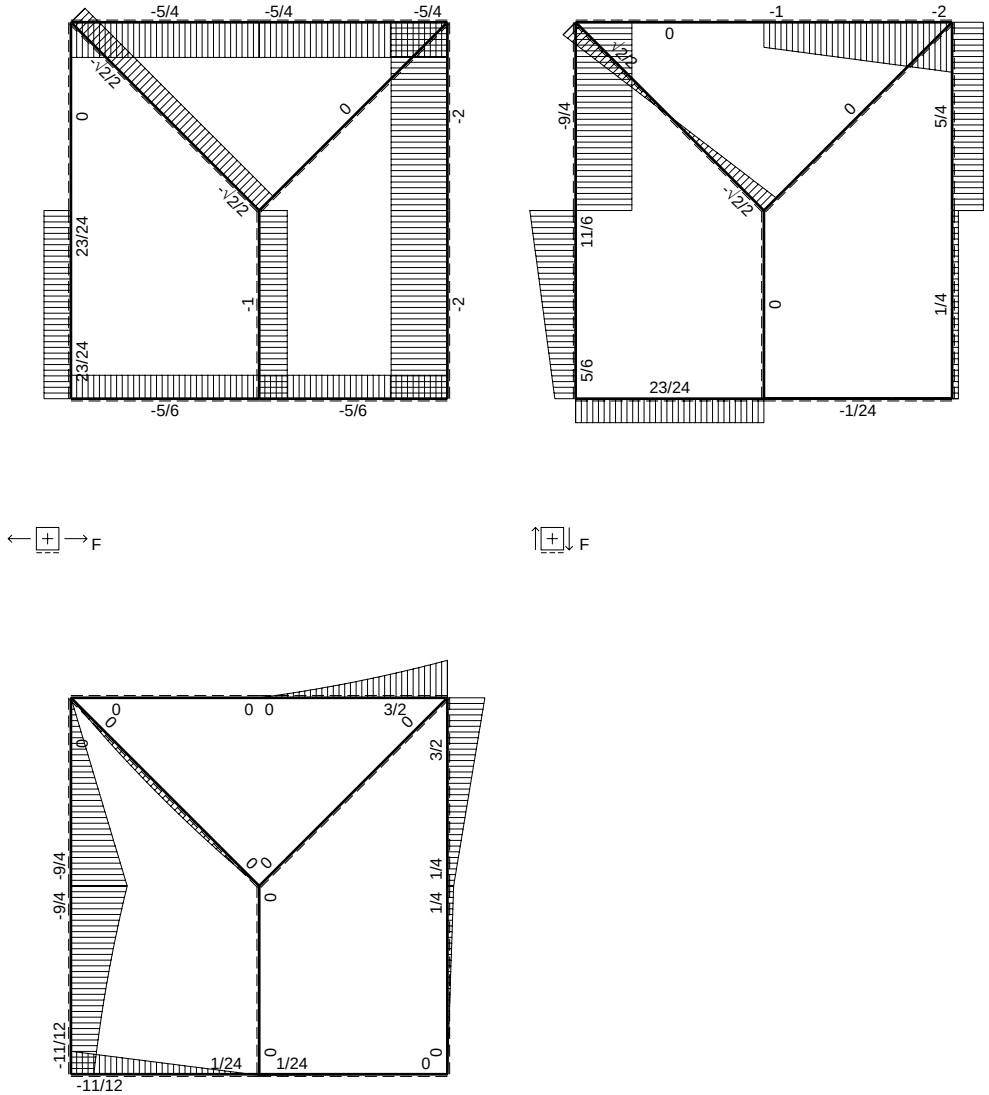
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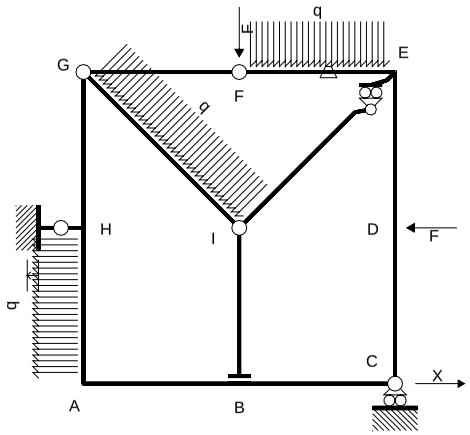
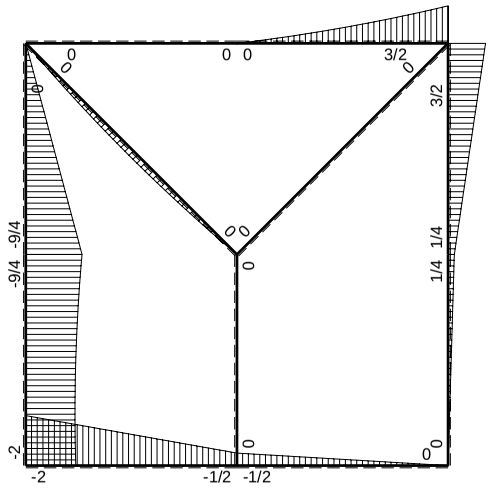
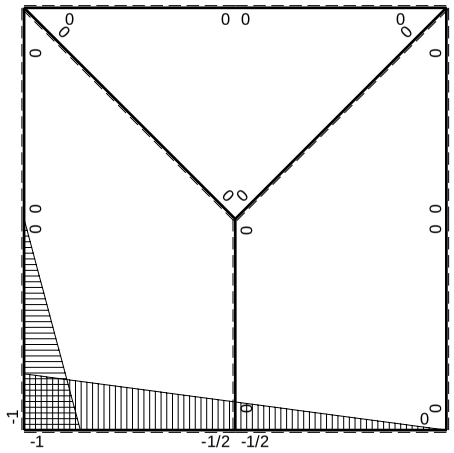
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-2Fb+3/2Fx$	0	$2Fb^2-5/2Fbx+3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$1/2Fb+3/2Fx$	0	$1/4Fb^2+Fbx+3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
CD b	0	$1/4Fx$	0	0	0	0	0+0	0
DC b	0	$-1/4Fb+1/4Fx$	0	0	0	0		
DE b	0	$1/4Fb+5/4Fx$	0	0	0	0	0+0	0
ED b	0	$-3/2Fb+5/4Fx$	0	0	0	0		
EF b	0	$3/2Fb-2Fx+1/2qx^2$	$-Fb/EJ$	0	0	0	0+0	0
FE b	0	$-Fx-1/2qx^2$	Fb/EJ	0	0	0		
FG b	0	0	0	0	0	0	0+0	0
GF b	0	0	0	0	0	0		
GH b	0	$-9/4Fx$	0	0	0	0	0+0	0
HG b	0	$9/4Fb-9/4Fx$	0	0	0	0		
GI $\sqrt{2}$b	0	$\sqrt{2}/2Fx-1/2qx^2$	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-9/4Fb+3/4Fx-1/2qx^2$	0	$9/4Fbx-3/4Fx^2+1/2qx^3$	0	x^2	$(1+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$2Fb-1/4Fx+1/2qx^2$	0	$2Fb^2-9/4Fbx+3/4Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$13/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						$-13/12F$	

Sviluppi di calcolo iperstatica

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$L_{AB}^{XX} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BA}^{XX} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L_{BC}^{XX} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{CB}^{XX} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L_{HA}^{XX} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AH}^{XX} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L_{AB}^{Xo} = \int_o^b \left(2 - 5/2\ x/b + 3/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 5/4\ x^2/b + 1/4\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - 5/4\ b + 1/4\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L_{BA}^{Xo} = \int_o^b \left(1/4 + x/b + 3/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/4\ x + 1/2\ x^2/b + 1/4\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/4\ b + 1/2\ b + 1/4\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L_{BC}^{Xo} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L_{CB}^{Xo} = \int_o^b \left(1/4\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(1/12\ b\ \right) Fb^2\ 1/EJ\ = 1/12\ \ Fb^3/EJ$

$L_{HA}^{Xo} = \int_o^b \left(9/4\ x/b - 3/4\ x^2/b^2 + 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[9/8\ x^2/b - 1/4\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(9/8\ b - 1/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

$L_{AH}^{Xo} = \int_o^b \left(2 - 9/4\ x/b + 3/4\ x^2/b^2 - 1/2\ x^3/b^3\right) Fb^2\ 1/EJ\ dx = \left[2\ x - 9/8\ x^2/b + 1/4\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(2\ b - 9/8\ b + 1/4\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = \ Fb^3/EJ$

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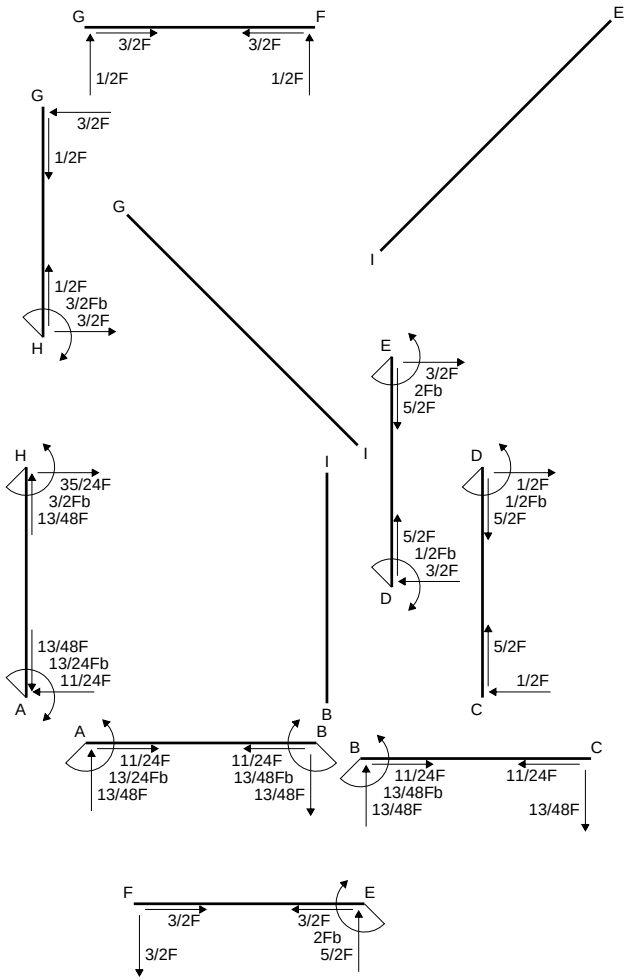

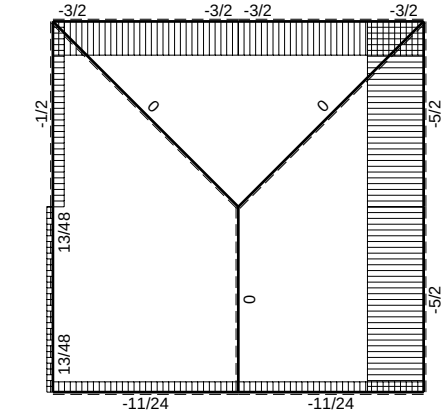

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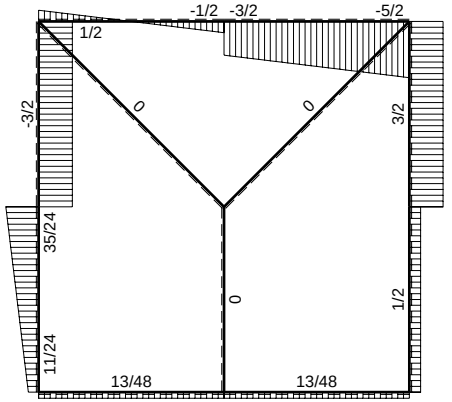

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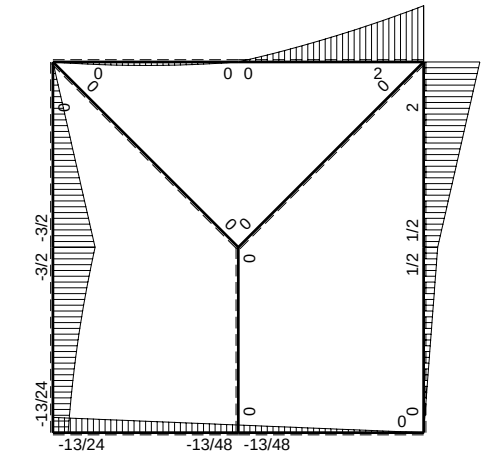

Fb

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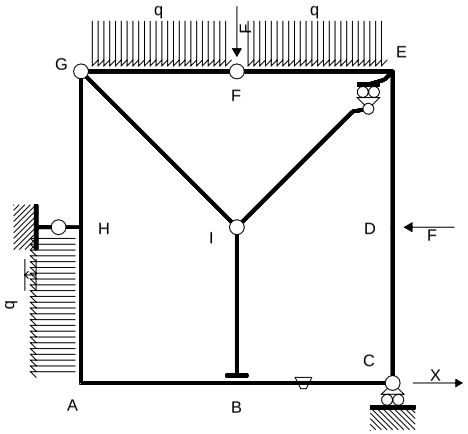
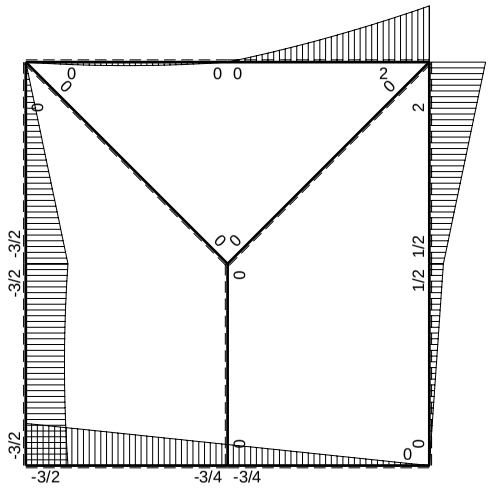
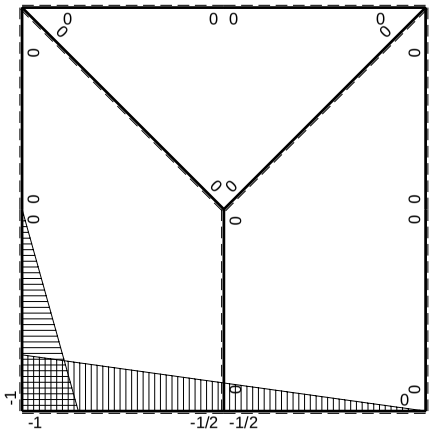




M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_C$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-b+1/2x$	$-3/2Fb+3/4Fx$	0	$3/2Fb^2-3/2Fbx+3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(7/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
BA b	$1/2b+1/2x$	$3/4Fb+3/4Fx$	0	$3/8Fb^2+3/4Fbx+3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
BC b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	$-Fb/EJ$	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
CB b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
CD b	0	$1/2Fx$	0	0	0	0	0+0	0
DC b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
DE b	0	$1/2Fb+3/2Fx$	0	0	0	0	0+0	0
ED b	0	$-2Fb+3/2Fx$	0	0	0	0		
EF b	0	$2Fb-5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3/2Fx-1/2qx^2$	0	0	0	0		
FG b	0	$-1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
GF b	0	$1/2Fx-1/2qx^2$	0	0	0	0		
GH b	0	$-3/2Fx$	0	0	0	0	0+0	0
HG b	0	$3/2Fb-3/2Fx$	0	0	0	0		
GI $\sqrt{2}$b	0	0	0	0	0	0	0	0
IB b	0	0	0	0	0	0	0+0	0
BI b	0	0	0	0	0	0		
IE $\sqrt{2}$b	0	0	0	0	0	0	0	0
HA b	$-x$	$-3/2Fb+1/2Fx-1/2qx^2$	0	$3/2Fbx-1/2Fx^2+1/2qx^3$	0	x^2	$(17/24+0)Fb^3/EJ$	$1/3Xb^3/EJ$
AH b	b-x	$3/2Fb-1/2Fx+1/2qx^2$	0	$3/2Fb^2-2Fbx+Fx^2-1/2qx^3$	0	$b^2-2bx+x^2$		
H	cedimento nodo $-H_{1H}u_H$						$-Fb^3/EJ$	
	totali						$23/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_C$						$-23/24F$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(1 - x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - 1/2\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - 1/2\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(1/4 + 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x + 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b + 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 7/12\ \ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(1/4 - 1/2\ x/b + 1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/4\ x - 1/4\ x^2/b + 1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/4\ b - 1/4\ b + 1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1/4\ x^2/b^2\right) b^2\ 1/EJ\ dx = \left[1/12\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/12\ b\ \right) b^2\ 1/EJ\ = 1/12\ \ b^3/EJ$

$L^{XX}_{HA} = \int_o^b \left(\ x^2/b^2\ \right) b^2\ 1/EJ\ dx = \left[1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{XX}_{AH} = \int_o^b \left(1 - 2\ x/b + x^2/b^2\right) b^2\ 1/EJ\ dx = \left[\ x - x^2/b + 1/3\ x^3/b^2\ \right]_o^b b^2\ 1/EJ$

$= \left(\ b - b + 1/3\ b\ \right) b^2\ 1/EJ\ = 1/3\ \ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(3/2 - 3/2\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - 3/4\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - 3/4\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(3/8 + 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx = \left[3/8\ x + 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/8\ b + 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 7/8\ \ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(3/8 - 3/4\ x/b + 3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(1/2 - 1/2\ x/b\ \right) \theta\ \ dx$

$= \left[3/8\ x - 3/8\ x^2/b + 1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[1/2\ x - 1/4\ x^2/b\ \right]_o^b \theta$

$= \left(3/8\ b - 3/8\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ + \left(1/2\ b - 1/4\ b\ \right) \theta\ \ = 3/8\ \ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(3/8\ x^2/b^2\right) Fb^2\ 1/EJ\ dx + \int_o^b \left(-1/2\ x/b\ \right) \theta\ \ dx = \left[1/8\ x^3/b^2\ \right]_o^b Fb^2\ 1/EJ\ + \left[-1/4\ x^2/b\ \right]_o^b \theta$

$= \left(1/8\ b\ \right) Fb^2\ 1/EJ\ + \left(-1/4\ b\ \right) \theta\ \ = 3/8\ \ Fb^3/EJ$

$L^{Xo}_{HA} = \int_o^b \left(3/2\ x/b - 1/2\ x^2/b^2 + 1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[3/4\ x^2/b - 1/6\ x^3/b^2 + 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/4\ b - 1/6\ b + 1/8\ b\ \right) Fb^2\ 1/EJ\ = 17/24\ \ Fb^3/EJ$

$L^{Xo}_{AH} = \int_o^b \left(3/2 - 2\ x/b + x^2/b^2 - 1/2\ x^3/b^3\ \right) Fb^2\ 1/EJ\ dx = \left[3/2\ x - x^2/b + 1/3\ x^3/b^2 - 1/8\ x^4/b^3\ \right]_o^b Fb^2\ 1/EJ$

$= \left(3/2\ b - b + 1/3\ b - 1/8\ b\ \right) Fb^2\ 1/EJ\ = 17/24\ \ Fb^3/EJ$

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