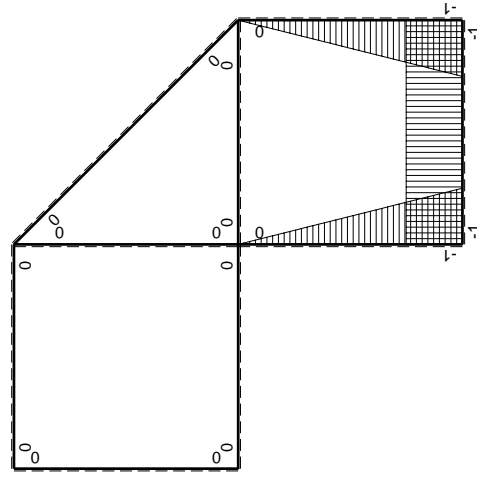


Schema di calcolo iperstatico

$(\oplus)$   $M_0$  flessione da carichi assegnati



$(\oplus)$   $M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=H_{BE}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int M_x M_x / EJ dx$
AB b	0	-5/2Fx	0	0	0	0
BA b	0	5/2Fb-5/2Fx	0	0	0	0
BC b	-x	-5/2Fb	5/2Fbx	$x^2$	$5/4Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	b-x	5/2Fb	$5/2Fb^2-5/2Fbx$	$b^2-2bx+x^2$	$3/2Fb^3/EJ$	$Xb^3/EJ$
CD b	-b	-3/2Fb	$3/2Fb^2$	$b^2$	$3/2Fb^3/EJ$	$Xb^3/EJ$
DC b	b	3/2Fb	$3/2Fb^2$	$b^2$	$3/2Fb^3/EJ$	$Xb^3/EJ$
DE b	-b+x	-3/2Fb	$3/2Fb^2-3/2Fbx$	$b^2-2bx+x^2$	$3/4Fb^3/EJ$	$1/3Xb^3/EJ$
ED b	x	3/2Fb	$3/2Fbx$	$x^2$	$3/4Fb^3/EJ$	$1/3Xb^3/EJ$
EF $\sqrt{2}b$	0	$\sqrt{2}2Fx$	0	0	0	0
FG b	0	-1/2Fx	0	0	0	0
GF b	0	$1/2Fb-1/2Fx$	0	0	0	0
GA b	0	$1/2Fb-1/2qx^2$	0	0	0	0
AG b	0	$-Fx+1/2qx^2$	0	0	0	0
FB b	0	Fb-Fx	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	$Fx-1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb+1/2qx^2$	0	0	0	0
BE	elongazione asta $N_{1, BE} \epsilon_{BE} L_{BE}$				$Fb^3/EJ$	$5/3Xb^3/EJ$
	totali				$9/2Fb^3/EJ$	$-27/10F$
	iperstatica $X=H_{BE}$					

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/2 x/b) Fb^2 1/EJ dx = [5/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (5/4 b) Fb^2 1/EJ = 5/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/2 - 5/2 x/b) Fb^2 1/EJ dx = [5/2 x - 5/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (5/2 b - 5/4 b) Fb^2 1/EJ = 5/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/2) Fb^2 1/EJ dx = [3/2 x]_0^b Fb^2 1/EJ$$

$$= (3/2 b) Fb^2 1/EJ = 3/2 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/2) Fb^2 1/EJ dx = [3/2 x]_0^b Fb^2 1/EJ$$

$$= (3/2 b) Fb^2 1/EJ = 3/2 Fb^3/EJ$$

$$L_{DE}^{xo} = \int_0^b (3/2 - 3/2 x/b) Fb^2 1/EJ dx = [3/2 x - 3/4 x^2/b]_0^b Fb^2 1/EJ$$

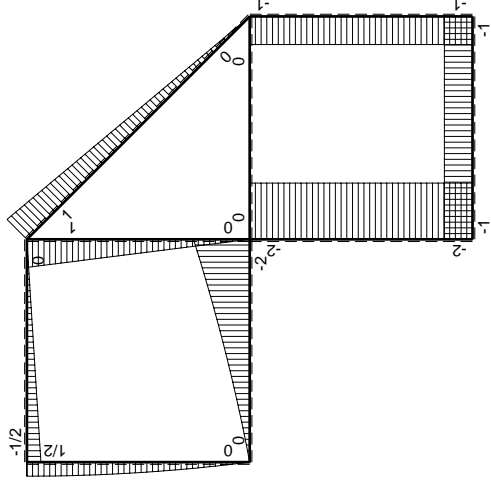
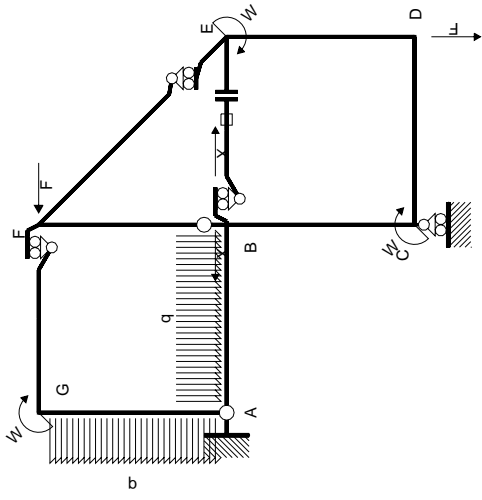
$$= (3/2 b - 3/4 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$

$$L_{ED}^{xo} = \int_0^b (3/2 x/b) Fb^2 1/EJ dx = [3/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (3/4 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$







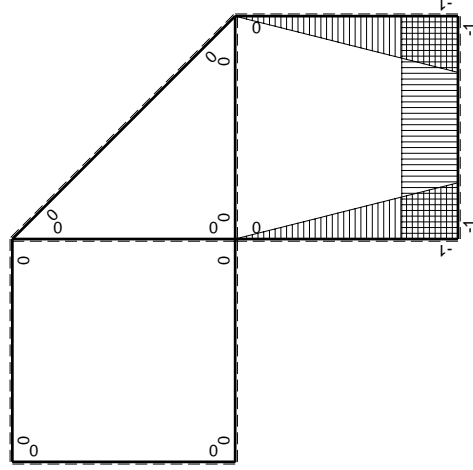
Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati

Quadro contribuiti PLV per iperstatica  $X=H_{BE}$

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int M_x M_x / EJ dx$
AB b	0	$-3/2Fx - 1/2qx^2$	0	0	0	0
BA b	0	$2Fb - 5/2Fx + 1/2qx^2$	0	0	0	0
BC b	-x	-2Fb	2Fbx	$x^2$	$Fb^3/EJ$	$1/3Xb^3/EJ$
CB b	b-x	2Fb	$2Fb^2 - 2Fbx$	$b^2 - 2bx + x^2$	$Fb^3/EJ$	$1/3Xb^3/EJ$
CD b	-b	-Fb	$Fb^2$	$b^2$	$Fb^3/EJ$	$Xb^3/EJ$
DC b	b	Fb	$Fb^2$	$b^2$	$Fb^3/EJ$	$Xb^3/EJ$
DE b	-b+x	-Fb	$Fb^2 - Fbx$	$b^2 - 2bx + x^2$	$1/2Fb^3/EJ$	$1/3Xb^3/EJ$
ED b	x	Fb	$Fbx$	$x^2$	$1/2Fb^3/EJ$	$1/3Xb^3/EJ$
EF $\sqrt{2}b$	0	$\sqrt{2}Fx$	0	0	0	0
FG b	0	-1/2Fx	0	0	0	0
GF b	0	$1/2Fb - 1/2Fx$	0	0	0	0
GA b	0	$1/2Fb - 1/2qx^2$	0	0	0	0
AG b	0	$-Fx + 1/2qx^2$	0	0	0	0
FB b	0	$Fb - Fx$	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
BE	elongazione asta $N_{1, BE}^{\epsilon} L_{BE}$				$Fb^3/EJ$	
	totali				$7/2Fb^3/EJ$	$5/3Xb^3/EJ$
	iperstatica $X=H_{BE}$				-21/10F	

Sviluppi di calcolo iperstatica



$M_x$  flessione da iperstatica  $X=1$

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (2x/b) Fb^2 1/EJ dx = [x^2/b]_0^b Fb^2 1/EJ$$

$$= (b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (2 - 2x/b) Fb^2 1/EJ dx = [2x - x^2/b]_0^b Fb^2 1/EJ$$

$$= (2b - b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1) Fb^2 1/EJ dx = [x]_0^b Fb^2 1/EJ$$

$$= (b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1) Fb^2 1/EJ dx = [x]_0^b Fb^2 1/EJ$$

$$= (b) Fb^2 1/EJ = Fb^3/EJ$$

$$L_{DE}^{xo} = \int_0^b (1 - x/b) Fb^2 1/EJ dx = [x - 1/2 x^2/b]_0^b Fb^2 1/EJ$$

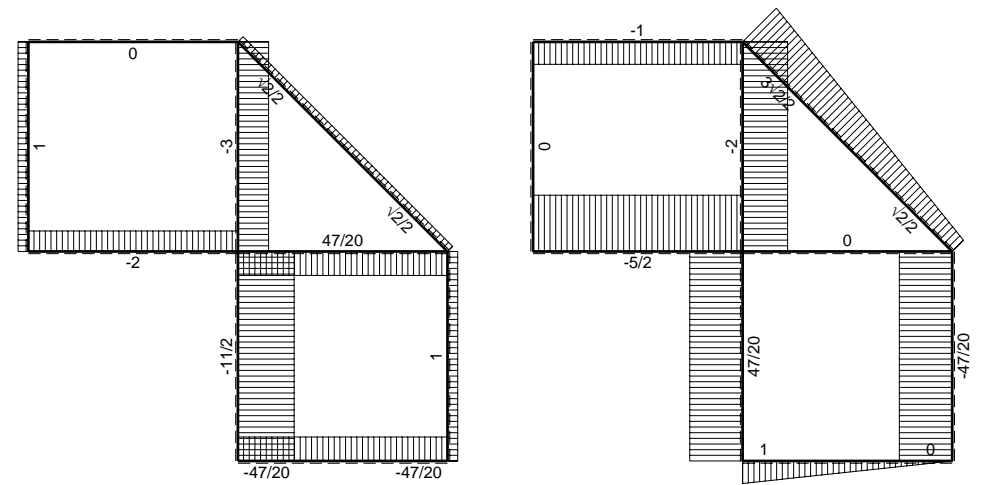
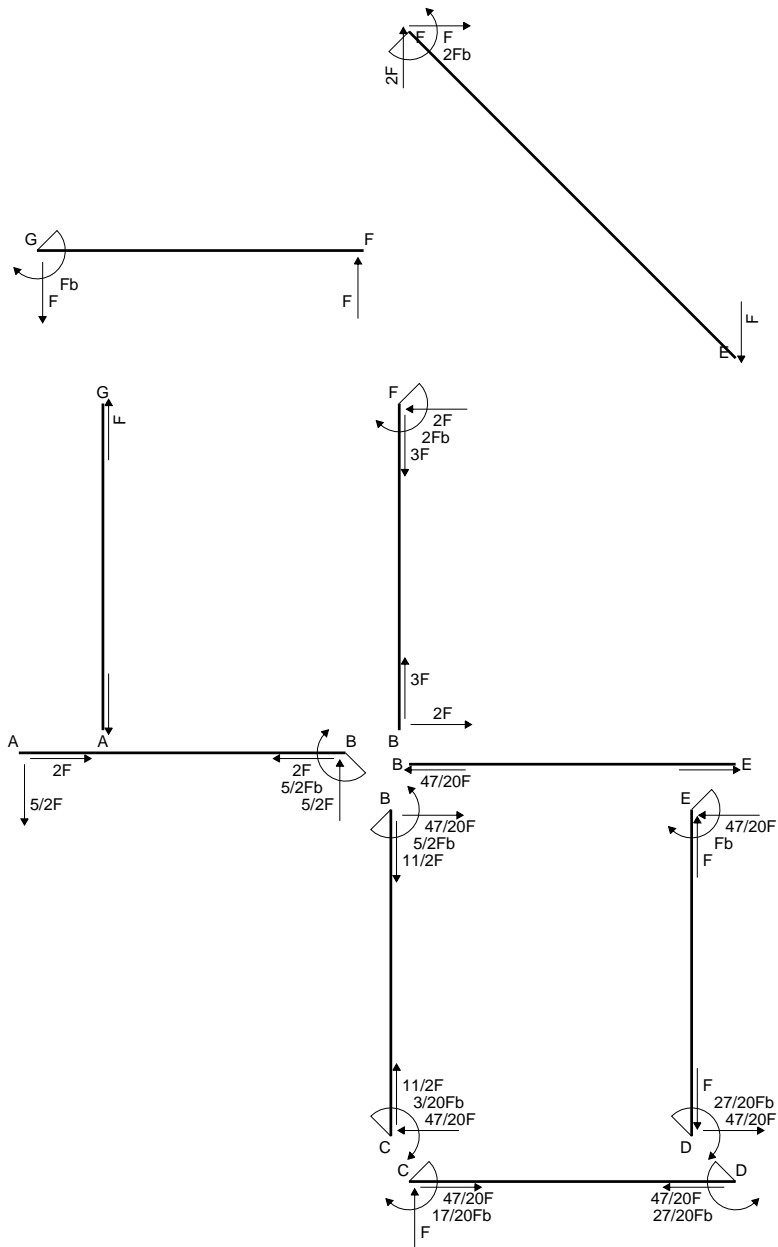
$$= (b - 1/2 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{ED}^{xo} = \int_0^b (x/b) Fb^2 1/EJ dx = [1/2 x^2/b]_0^b Fb^2 1/EJ$$

$$= (1/2 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

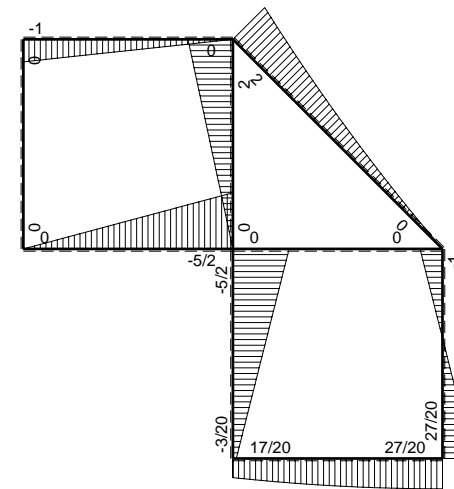




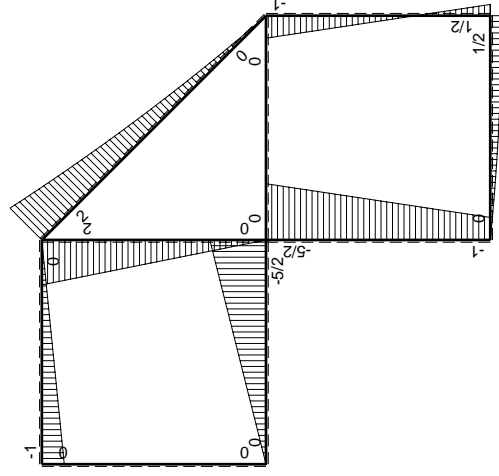
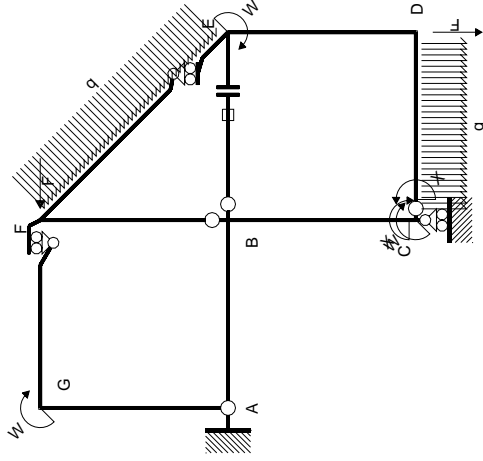


← ⊕ → F

↑ ⊕ ↓ F

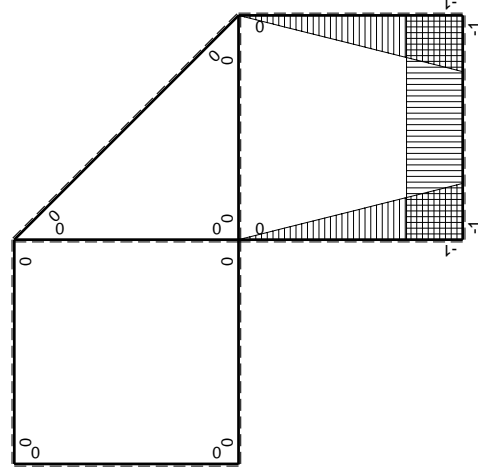


⊕ ⊖ F\_b



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contribuiti PLV per iperstatica  $X=W_{cd}$

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-5/2Fx	0	0	0	0
BA b	0	5/2Fb-5/2Fx	0	0	0	0
BC b	-x/b	-5/2Fb+3/2Fx	5/2Fx-3/2Fx <sup>2</sup> /b	x <sup>2</sup> /b <sup>2</sup>	3/4Fb <sup>2</sup> /EJ	1/3Xb/EJ
CB b	1-x/b	Fb+3/2Fx	Fb+1/2Fx-3/2Fx <sup>2</sup> /b	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	-1/3Fb <sup>2</sup> /EJ	Xb/EJ
CD b	-1	Fx-1/2qx <sup>2</sup>	-Fx+1/2Fx <sup>2</sup> /b	1	-1/3Fb <sup>2</sup> /EJ	Xb/EJ
DC b	1	-1/2Fb+1/2qx <sup>2</sup>	-1/2Fb+1/2Fx <sup>2</sup> /b	1	-1/3Fb <sup>2</sup> /EJ	Xb/EJ
DE b	-1+x/b	1/2Fb-3/2Fx	-1/2Fb+2Fx-3/2Fx <sup>2</sup> /b	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	0	1/3Xb/EJ
ED b	x/b	Fb-3/2Fx	Fx-3/2Fx <sup>2</sup> /b	x <sup>2</sup> /b <sup>2</sup>	0	1/3Xb/EJ
EF √2b	0	√2/2Fx+1/2qx <sup>2</sup>	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0	0	0
GA b	0	0	0	0	0	0
AG b	0	0	0	0	0	0
FB b	0	2Fb-2Fx	0	0	0	0
BF b	0	-2Fx	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
BE	elongazione asta $N_{1, BE} \epsilon_{BE} L_{BE}$				Fb <sup>2</sup> /EJ	
	totali				17/12Fb <sup>2</sup> /EJ	5/3Xb/EJ
	iperstatica $X=W_{cd}$				-17/20Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/2 x/b - 3/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [5/4 x^2/b - 1/2 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (5/4 b - 1/2 b) \cdot Fb \cdot 1/EJ = 3/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1 + 1/2 x/b - 3/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [x + 1/4 x^2/b - 1/2 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (b + 1/4 b - 1/2 b) \cdot Fb \cdot 1/EJ = 3/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/2 x^2/b + 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/2 b + 1/6 b) \cdot Fb \cdot 1/EJ = -1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 + 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/2 x + 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/2 b + 1/6 b) \cdot Fb \cdot 1/EJ = -1/3 Fb^2/EJ$$

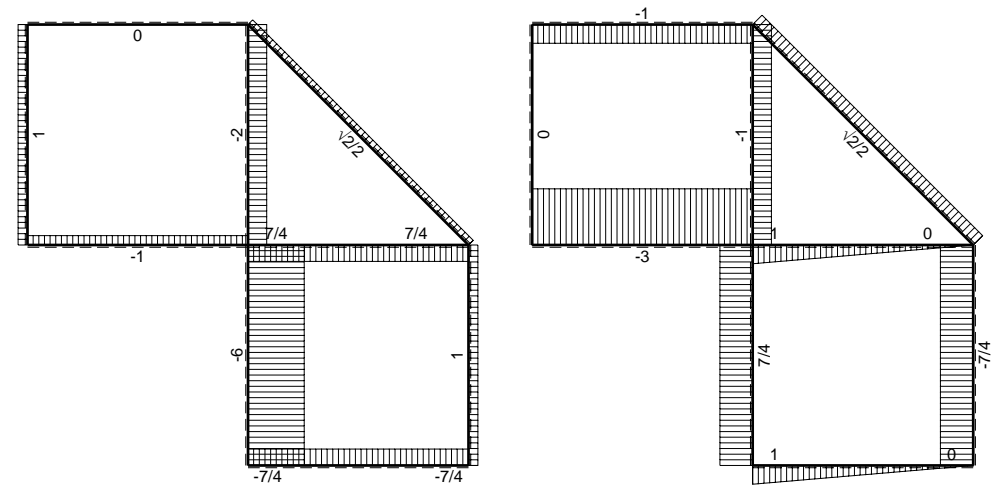
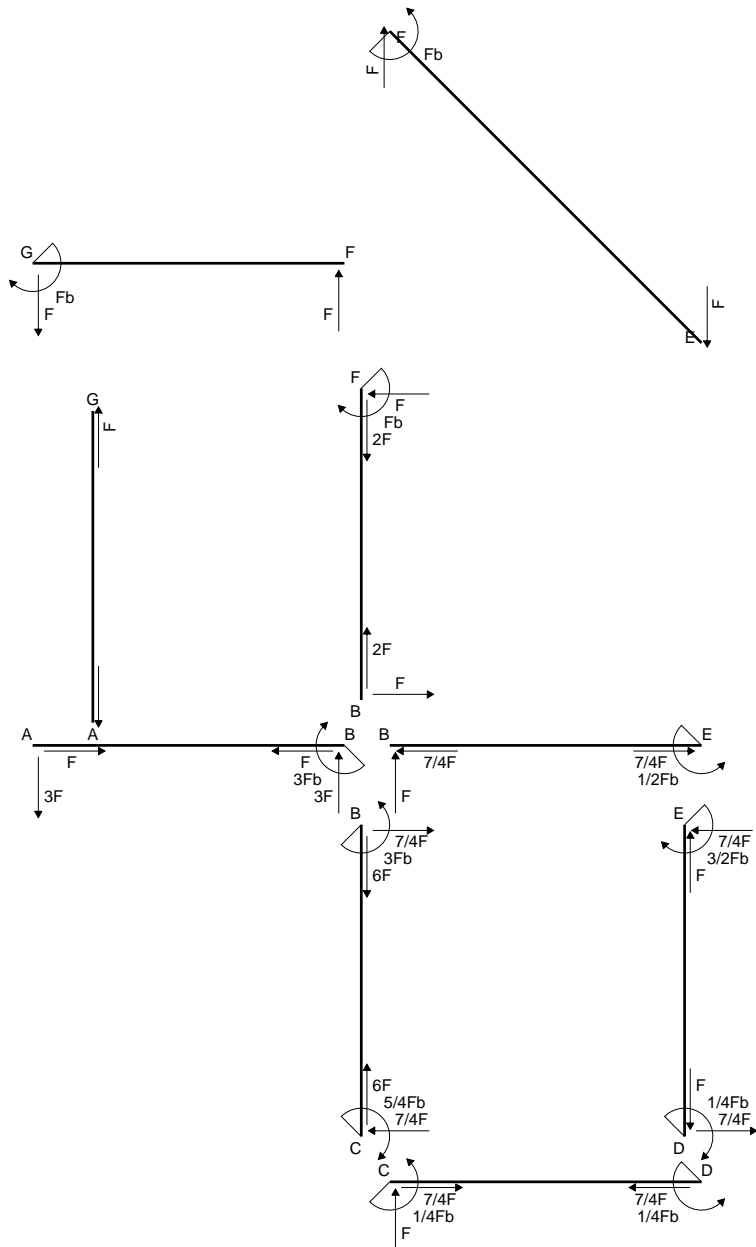
$$L_{DE}^{xo} = \int_0^b (-1/2 + 2x/b - 3/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/2 x + x^2/b - 1/2 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/2 b + b - 1/2 b) \cdot Fb \cdot 1/EJ = 0$$

$$L_{ED}^{xo} = \int_0^b (x/b - 3/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [1/2 x^2/b - 1/2 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

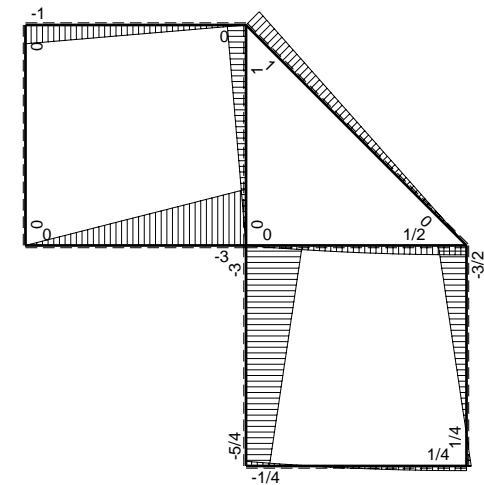
$$= (1/2 b - 1/2 b) \cdot Fb \cdot 1/EJ = 0$$



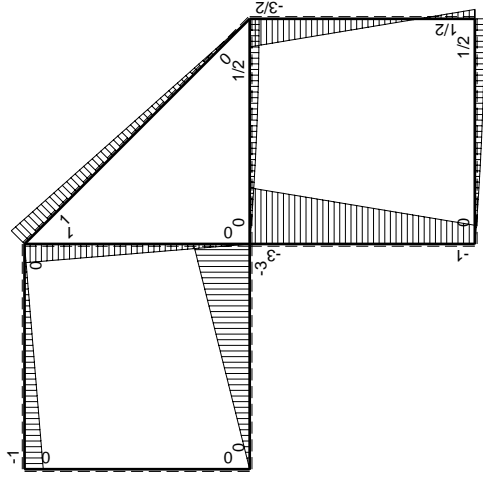
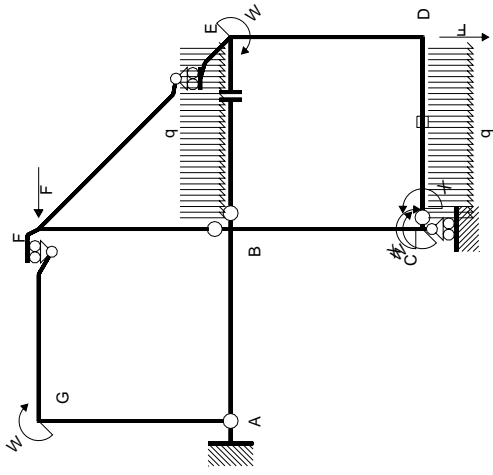


← ⊕ → F

↑ ⊕ ↓ F<sub>b</sub>

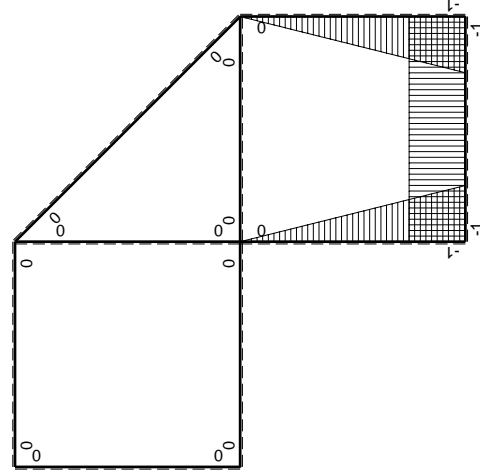


⊕ ⊖ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W<sub>cd</sub>

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-3Fx	0	0	0	0
BA b	0	3Fb-3Fx	0	0	0	0
BC b	-x/b	-3Fb+2Fx	3Fx-2Fx <sup>2</sup> /b	x <sup>2</sup> /b <sup>2</sup>	5/6Fb <sup>2</sup> /EJ	1/3Xb/EJ
CB b	1-x/b	Fb+2Fx	Fb+Fx-2Fx <sup>2</sup> /b	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	-1/3Fb <sup>2</sup> /EJ	Xb/EJ
CD b	-1	Fx-1/2qx <sup>2</sup>	-Fx+1/2Fx <sup>2</sup> /b	1		
DC b	1	-1/2Fb+1/2qx <sup>2</sup>	-1/2Fb+1/2Fx <sup>2</sup> /b	1		
DE b	-1+x/b	1/2Fb-2Fx	-1/2Fb+5/2Fx-2Fx <sup>2</sup> /b	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	1/12Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	x/b	3/2Fb-2Fx	3/2Fx-2Fx <sup>2</sup> /b	x <sup>2</sup> /b <sup>2</sup>		
EF √2b	0	√2/2Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0	0	0
GA b	0	0	0	0	0	0
AG b	0	0	0	0	0	0
FB b	0	Fb-Fx	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	Fx-1/2qx <sup>2</sup>	0	0	0	0
EB b	0	-1/2Fb+1/2qx <sup>2</sup>	0	0	0	0
CD	elongazione asta $N_{1,cd} \epsilon_{cd} l_{cd}$				-Fb <sup>2</sup> /EJ	
	totali				-5/12Fb <sup>2</sup> /EJ	5/3Xb/EJ
	iperstatica X=W <sub>cd</sub>				1/4Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3x/b - 2x^2/b^2) Fb 1/EJ dx = [3/2 x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/2 b - 2/3 b) Fb 1/EJ = 5/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1 + x/b - 2x^2/b^2) Fb 1/EJ dx = [x + 1/2 x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b + 1/2 b - 2/3 b) Fb 1/EJ = 5/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2) Fb 1/EJ dx + 1 (-1) 1 Fb^2/EJ$$

$$= [-1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + 1 (-1) 1 Fb^2/EJ$$

$$= (-1/2 b + 1/6 b) Fb 1/EJ + 1 (-1) 1 Fb^2/EJ = -4/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 + 1/2 x^2/b^2) Fb 1/EJ dx + 1 (-1) 1 Fb^2/EJ$$

$$= [-1/2 x + 1/6 x^3/b^2]_0^b Fb 1/EJ + 1 (-1) 1 Fb^2/EJ$$

$$= (-1/2 b + 1/6 b) Fb 1/EJ + 1 (-1) 1 Fb^2/EJ = -4/3 Fb^2/EJ$$

$$L_{DE}^{xo} = \int_0^b (-1/2 + 5/2 x/b - 2x^2/b^2) Fb 1/EJ dx = [-1/2 x + 5/4 x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-1/2 b + 5/4 b - 2/3 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

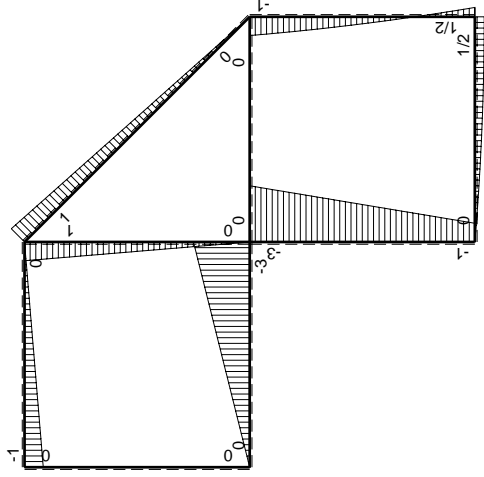
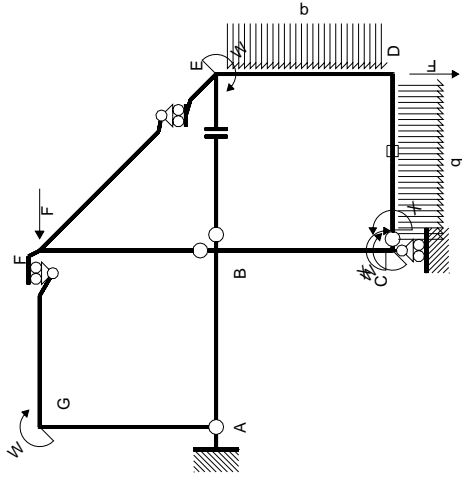
$$L_{ED}^{xo} = \int_0^b (3/2 x/b - 2x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 2/3 b) Fb 1/EJ = 1/12 Fb^2/EJ$$



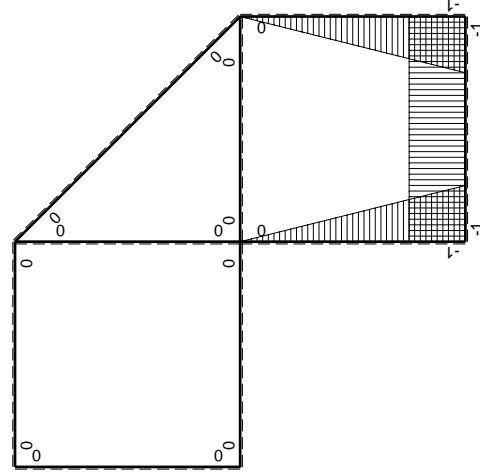






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_1$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{CD}$

→	$M_x(x)$	$M_o(x)$	$M_x M_o$	$M_x M_x$	$\int M_x M_o / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-3Fx	0	0	0	0
BA b	0	3Fb-3Fx	0	0	0	0
BC b	-x/b	-3Fb+2Fx	$3Fx-2Fx^2/b$	$x^2/b^2$	$5/6Fb^2/EJ$	$1/3Xb/EJ$
CB b	1-x/b	Fb+2Fx	$Fb+Fx-2Fx^2/b$	$1-2x/b+x^2/b^2$		
CD b	-1	$Fx-1/2qx^2$	$-Fx+1/2Fx^2/b$	1	$-1/3Fb^2/EJ$	$Xb/EJ$
DC b	1	$-1/2Fb+1/2qx^2$	$-1/2Fb+1/2Fx^2/b$	1		
DE b	-1+x/b	$1/2Fb-2Fx+1/2qx^2$	$-1/2Fb+5/2Fx-5/2Fx^2/b+1/2qx^3/b$	$1-2x/b+x^2/b^2$	$1/24Fb^2/EJ$	$1/3Xb/EJ$
ED b	x/b	$Fb-Fx-1/2qx^2$	$Fx-Fx^2/b-1/2qx^3/b$	$x^2/b^2$		
EF $\sqrt{2}b$	0	$\sqrt{2}/2Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0	0	0
GA b	0	0	0	0	0	0
AG b	0	0	0	0	0	0
FB b	0	Fb-Fx	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
CD	elongazione asta $N_{1CD} \epsilon_{CD} L_{CD}$				$-Fb^2/EJ$	
	totali				$-11/24Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{CD}$				$11/40Fb$	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3x/b - 2x^2/b^2) Fb 1/EJ dx = [3/2 x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/2 b - 2/3 b) Fb 1/EJ = 5/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1 + x/b - 2x^2/b^2) Fb 1/EJ dx = [x + 1/2 x^2/b - 2/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b + 1/2 b - 2/3 b) Fb 1/EJ = 5/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2) Fb 1/EJ dx + 1 (-1) 1 Fb^2/EJ$$

$$= [-1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + 1 (-1) 1 Fb^2/EJ$$

$$= (-1/2 b + 1/6 b) Fb 1/EJ + 1 (-1) 1 Fb^2/EJ = -4/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 + 1/2 x^2/b^2) Fb 1/EJ dx + 1 (-1) 1 Fb^2/EJ$$

$$= [-1/2 x + 1/6 x^3/b^2]_0^b Fb 1/EJ + 1 (-1) 1 Fb^2/EJ$$

$$= (-1/2 b + 1/6 b) Fb 1/EJ + 1 (-1) 1 Fb^2/EJ = -4/3 Fb^2/EJ$$

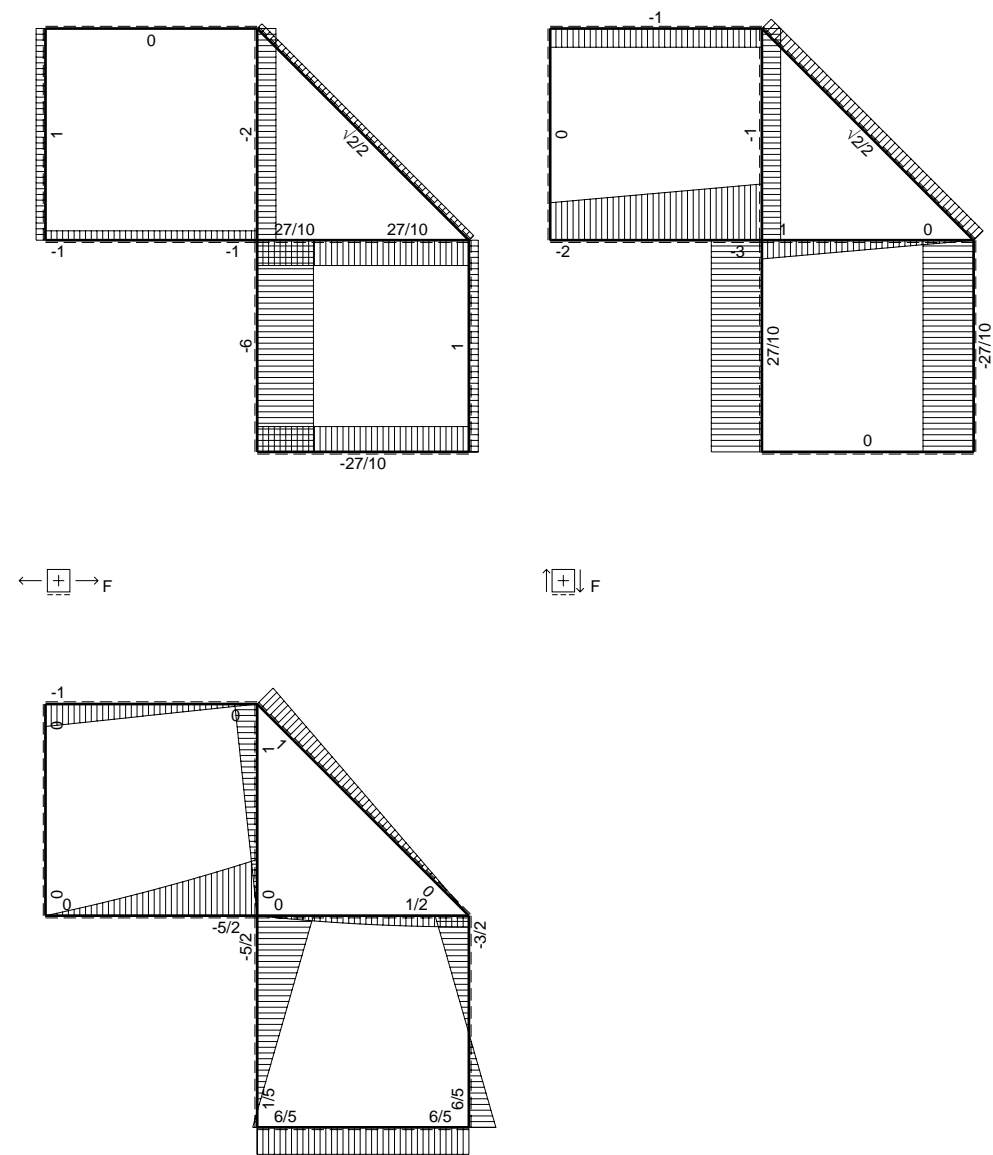
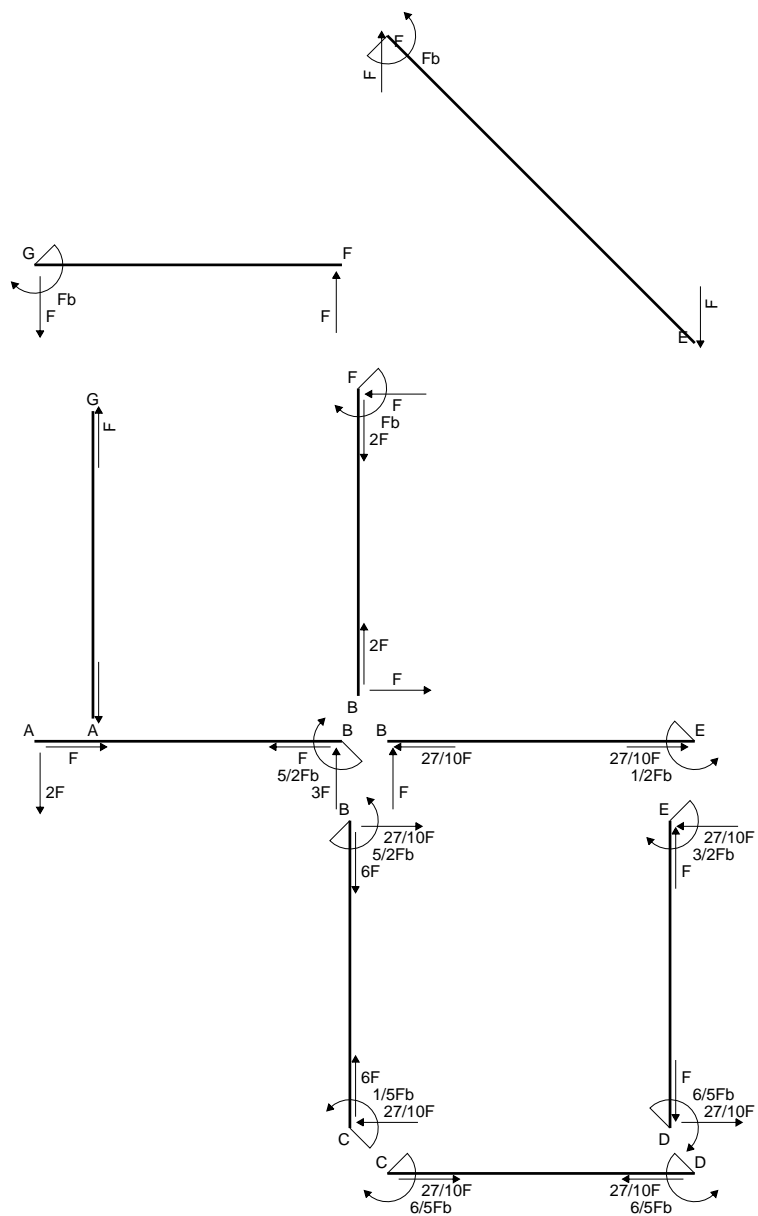
$$L_{DE}^{xo} = \int_0^b (-1/2 + 5/2 x/b - 5/2 x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx$$

$$= [-1/2 x + 5/4 x^2/b - 5/6 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

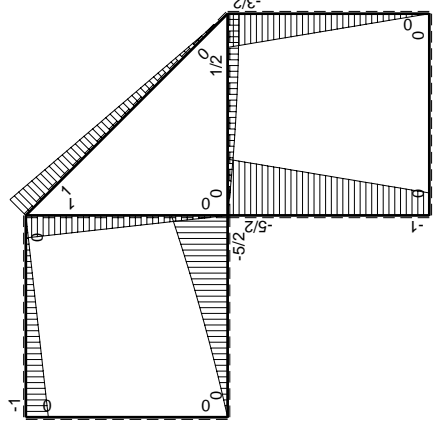
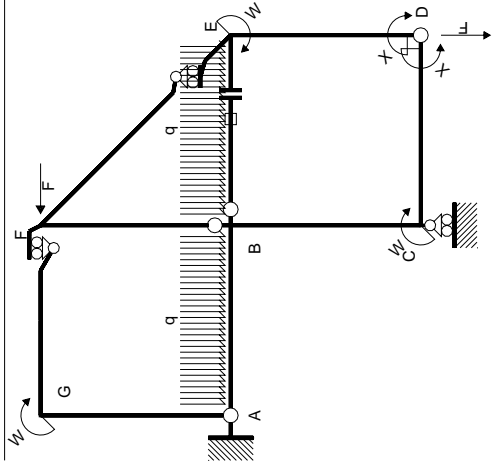
$$= (-1/2 b + 5/4 b - 5/6 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{ED}^{xo} = \int_0^b (x/b - x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

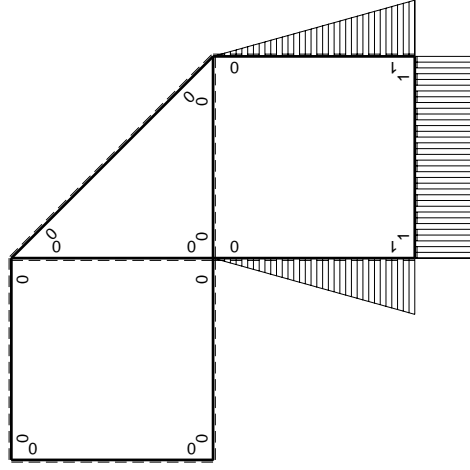
$$= (1/2 b - 1/3 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$



$\curvearrowright (+) F_b$



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contribuiti PLV per iperstatica  $X=W_{DC}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int M_x M_x / EJ dx$
AB b	0	$-2Fx - 1/2qx^2$	0	0	0	0
BA b	0	$5/2Fb - 3Fx + 1/2qx^2$	0	0	0	0
BC b	$x/b$	$-5/2Fb + 3/2Fx$	$-5/2Fx + 3/2Fx^2/b$	$x^2/b^2$	$-3/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	$-1+x/b$	$Fb + 3/2Fx$	$-Fb - 1/2Fx + 3/2Fx^2/b$	$1 - 2x/b + x^2/b^2$	0	$Xb/EJ$
CD b	1	0	0	1	0	0
DC b	-1	0	0	1	0	0
DE b	$1-x/b$	$-3/2Fx$	$-3/2Fx + 3/2Fx^2/b$	$1 - 2x/b + x^2/b^2$	$-1/4Fb^2/EJ$	$1/3Xb/EJ$
ED b	$-x/b$	$3/2Fb - 3/2Fx$	$-3/2Fx + 3/2Fx^2/b$	$x^2/b^2$	0	0
EF $\sqrt{2}b$	0	$\sqrt{2}/2Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	$Fb - Fx$	0	0	0	0
GA b	0	0	0	0	0	0
AG b	0	0	0	0	0	0
FB b	0	$Fb - Fx$	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	$Fx - 1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb + 1/2qx^2$	0	0	0	0
BE	elongazione asta $N_{1, BE}^E - L_{BE}^{E-BE}$					
	totali					
	iperstatica $X=W_{DC}$					
					$-Fb^2/EJ$	$5/3Xb/EJ$
					$-2Fb^2/EJ$	$6/5Fb$

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (-5/2 x/b + 3/2 x^2/b^2) Fb 1/EJ dx = [-5/4 x^2/b + 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-5/4 b + 1/2 b) Fb 1/EJ = -3/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (-1 - 1/2 x/b + 3/2 x^2/b^2) Fb 1/EJ dx = [-x - 1/4 x^2/b + 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-b - 1/4 b + 1/2 b) Fb 1/EJ = -3/4 Fb^2/EJ$$

$$L_{DE}^{xo} = \int_0^b (-3/2 x/b + 3/2 x^2/b^2) Fb 1/EJ dx = [-3/4 x^2/b + 1/2 x^3/b^2]_0^b Fb 1/EJ$$

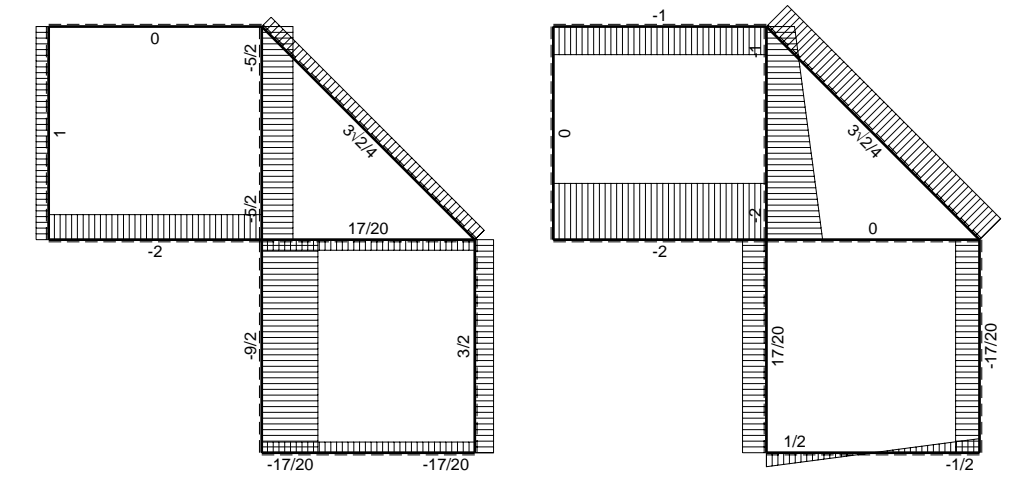
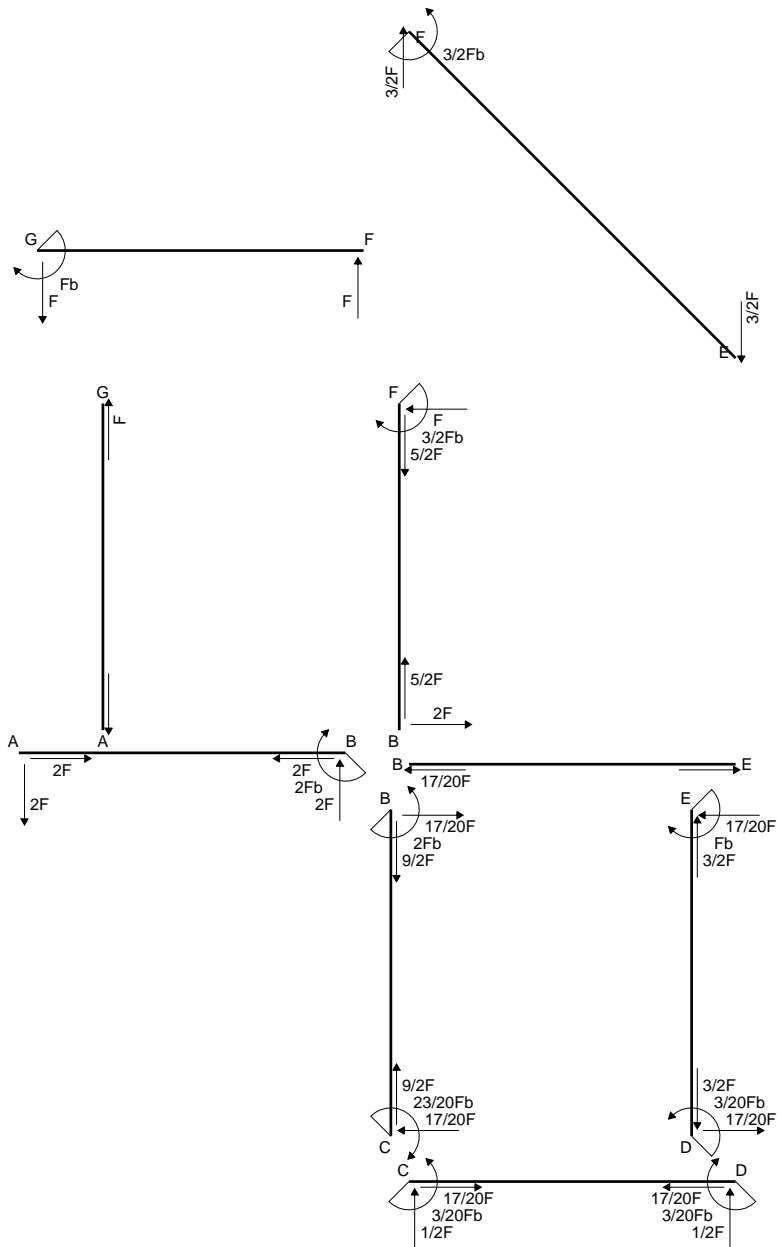
$$= (-3/4 b + 1/2 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

$$L_{ED}^{xo} = \int_0^b (-3/2 x/b + 3/2 x^2/b^2) Fb 1/EJ dx = [-3/4 x^2/b + 1/2 x^3/b^2]_0^b Fb 1/EJ$$

$$= (-3/4 b + 1/2 b) Fb 1/EJ = -1/4 Fb^2/EJ$$

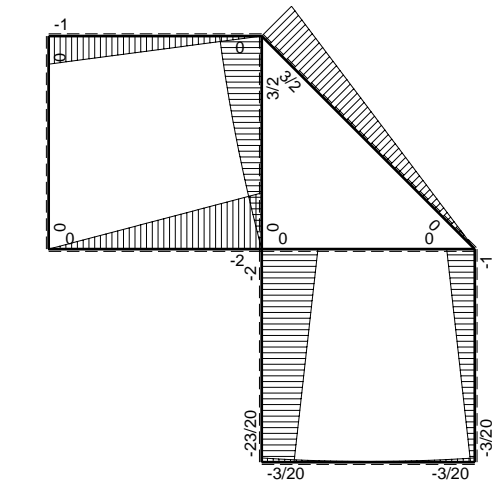




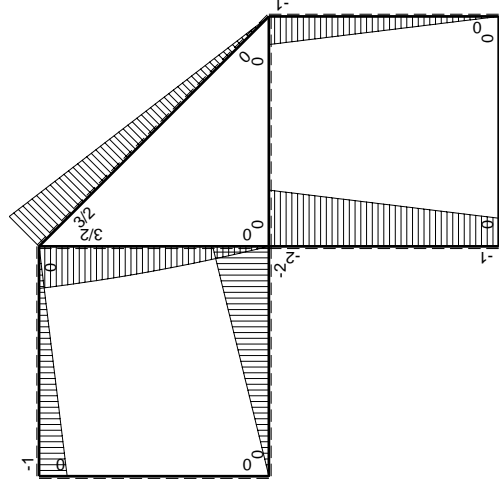
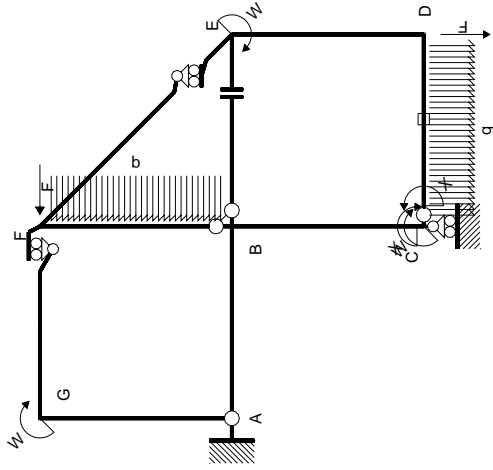


← ⊕ → F

↑ ⊕ ↓ F

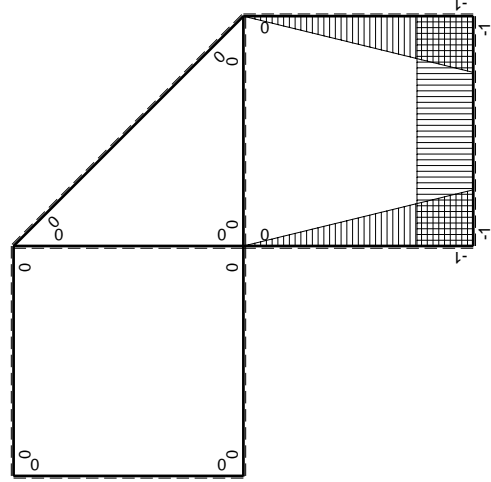


⊕ ⊖ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contribuiti PLV per iperstatica  $X=W_{cd}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-2Fx	0	0	0	0
BA b	0	2Fb-2Fx	0	0	0	0
BC b	-x/b	-2Fb+Fx	2Fx-Fx <sup>2</sup> /b	x <sup>2</sup> /b <sup>2</sup>	2/3Fb <sup>2</sup> /EJ	1/3Xb/EJ
CB b	1-x/b	Fb+Fx	Fb-Fx <sup>2</sup> /b	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	2/3Fb <sup>2</sup> /EJ	1/3Xb/EJ
CD b	-1	1/2Fx-1/2qx <sup>2</sup>	-1/2Fx+1/2Fx <sup>2</sup> /b	1	-1/12Fb <sup>2</sup> /EJ	Xb/EJ
DC b	1	-1/2Fx+1/2qx <sup>2</sup>	-1/2Fx+1/2Fx <sup>2</sup> /b	1	-1/12Fb <sup>2</sup> /EJ	Xb/EJ
DE b	-1+x/b	-Fx	Fx-Fx <sup>2</sup> /b	1-2x/b+x <sup>2</sup> /b <sup>2</sup>	1/6Fb <sup>2</sup> /EJ	1/3Xb/EJ
ED b	x/b	Fb-Fx	Fx-Fx <sup>2</sup> /b	x <sup>2</sup> /b <sup>2</sup>	1/6Fb <sup>2</sup> /EJ	1/3Xb/EJ
EF $\sqrt{2}b$	0	3 $\sqrt{2}$ /4Fx	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	Fb-Fx	0	0	0	0
GA b	0	0	0	0	0	0
AG b	0	0	0	0	0	0
FB b	0	3/2Fb-Fx-1/2qx <sup>2</sup>	0	0	0	0
BF b	0	-2Fx+1/2qx <sup>2</sup>	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
CD	elongazione asta $N_{1,cd} \epsilon_{cd} L_{cd}$			0	-Fb <sup>2</sup> /EJ	
	totali				-1/4Fb <sup>2</sup> /EJ	5/3Xb/EJ
	iperstatica $X=W_{cd}$				3/20Fb	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (2x/b - x^2/b^2) Fb 1/EJ dx = [x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b - 1/3 b) Fb 1/EJ = 2/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1 - x^2/b^2) Fb 1/EJ dx = [x - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (b - 1/3 b) Fb 1/EJ = 2/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-1/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx + 1 (-1) 1 Fb^2/EJ$$

$$= [-1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + 1 (-1) 1 Fb^2/EJ$$

$$= (-1/4 b + 1/6 b) Fb 1/EJ + 1 (-1) 1 Fb^2/EJ = -13/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 x/b + 1/2 x^2/b^2) Fb 1/EJ dx + 1 (-1) 1 Fb^2/EJ$$

$$= [-1/4 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + 1 (-1) 1 Fb^2/EJ$$

$$= (-1/4 b + 1/6 b) Fb 1/EJ + 1 (-1) 1 Fb^2/EJ = -13/12 Fb^2/EJ$$

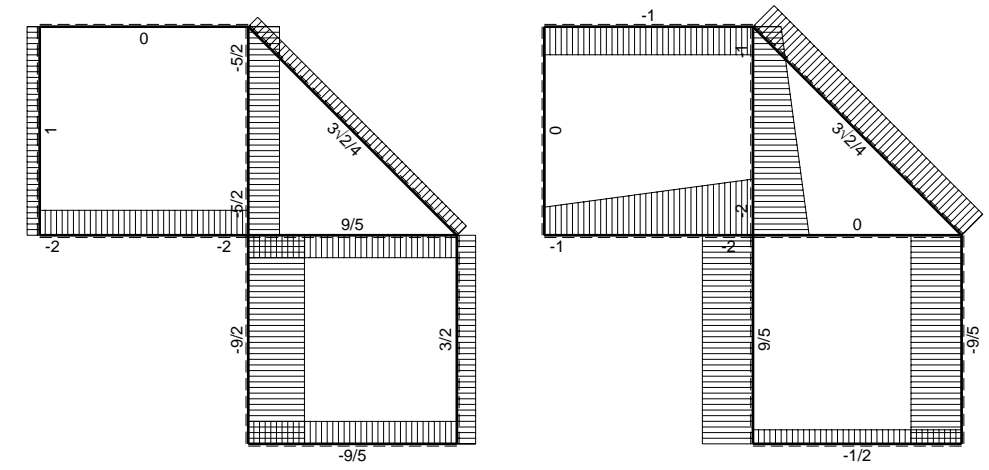
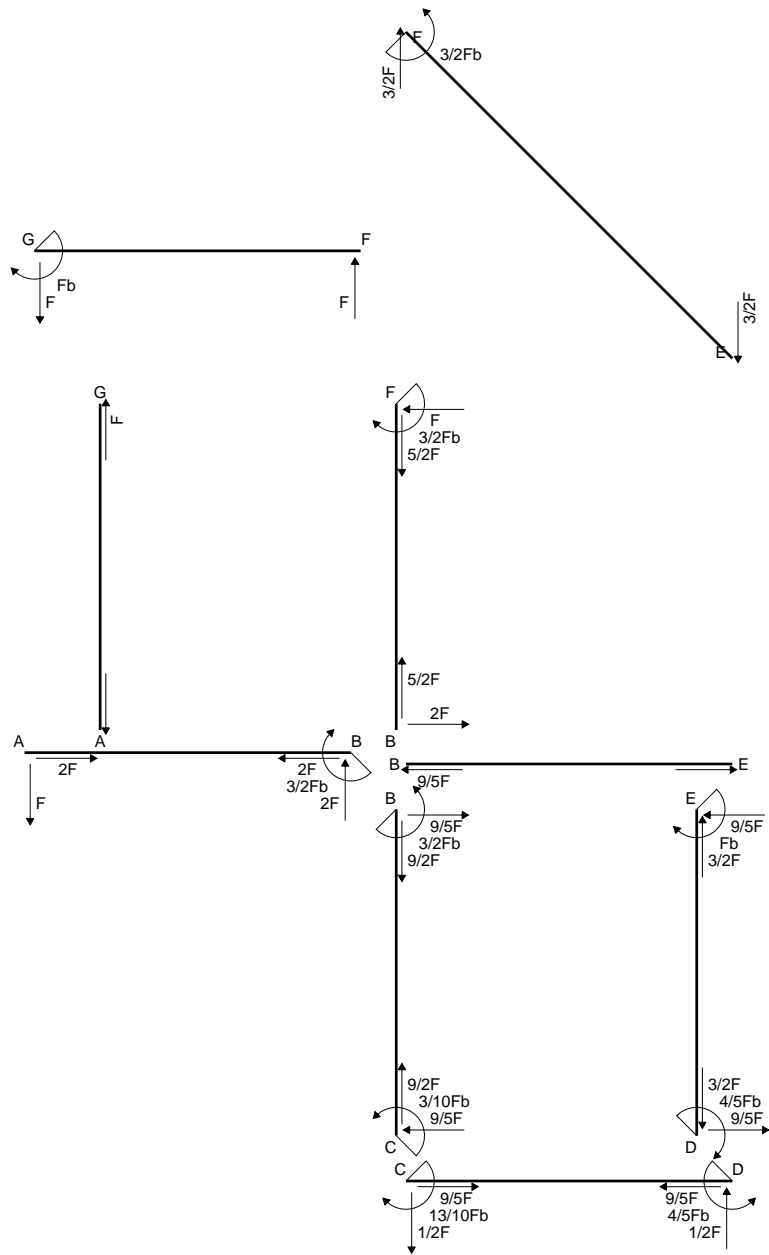
$$L_{DE}^{xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{ED}^{xo} = \int_0^b (x/b - x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/3 x^3/b^2]_0^b Fb 1/EJ$$

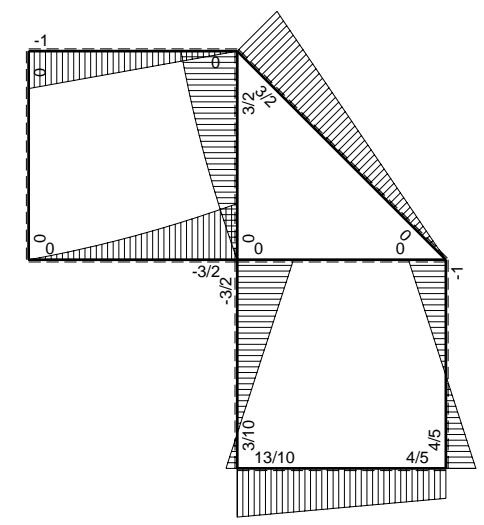
$$= (1/2 b - 1/3 b) Fb 1/EJ = 1/6 Fb^2/EJ$$



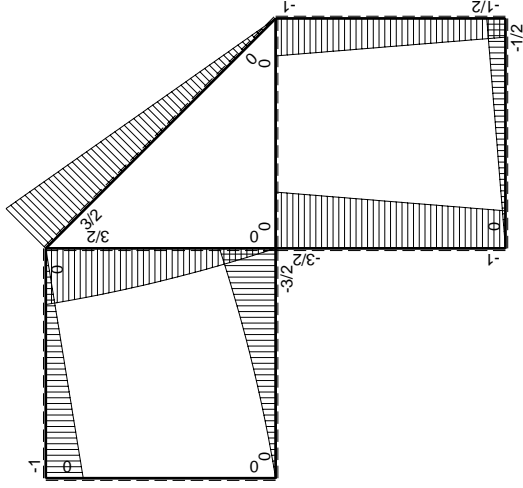
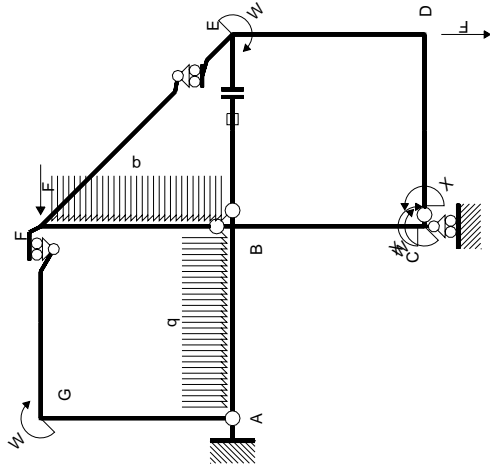


← ⊕ → F

↑ ⊕ ↓ F

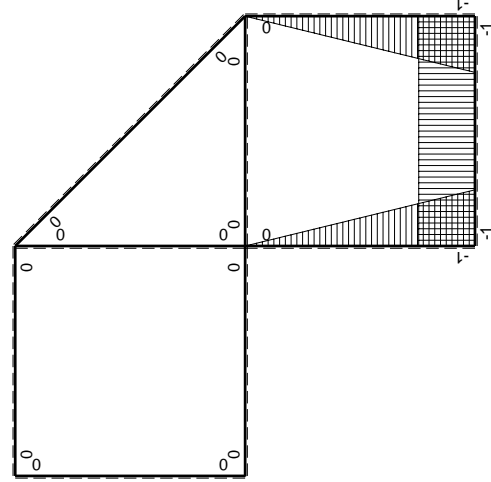


⊕ ⊖ F\_b



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contribuiti PLV per iperstatica X=W<sub>cd</sub>

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / E J dx$	$\int X M_x M_x / E J dx$
AB b	0	$-Fx - 1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb - 2Fx + 1/2qx^2$	0	0	0	0
BC b	$-x/b$	$-3/2Fb + 1/2Fx$	$3/2Fx - 1/2Fx^2 / b$	$x^2 / b^2$	$7/12Fb^2 / EJ$	$1/3Xb / EJ$
CB b	$1-x/b$	$Fb + 1/2Fx$	$Fb - 1/2Fx - 1/2Fx^2 / b$	$1 - 2x/b + x^2 / b^2$	$1/4Fb^2 / EJ$	$Xb / EJ$
CD b	-1	$-1/2Fx$	$1/2Fx$	1	$1/4Fb^2 / EJ$	$Xb / EJ$
DC b	1	$1/2Fb - 1/2Fx$	$1/2Fb - 1/2Fx$	1	$1/3Fb^2 / EJ$	$1/3Xb / EJ$
DE b	$-1+x/b$	$-1/2Fb - 1/2Fx$	$1/2Fb - 1/2Fx^2 / b$	$1 - 2x/b + x^2 / b^2$	$1/3Fb^2 / EJ$	$1/3Xb / EJ$
ED b	$x/b$	$Fb - 1/2Fx$	$Fx - 1/2Fx^2 / b$	$x^2 / b^2$	0	0
EF $\sqrt{2}b$	0	$3\sqrt{2}/4Fx$	0	0	0	0
FG b	0	-Fx	0	0	0	0
GF b	0	$Fb - Fx$	0	0	0	0
GA b	0	0	0	0	0	0
AG b	0	0	0	0	0	0
FB b	0	$3/2Fb - Fx - 1/2qx^2$	0	0	0	0
BF b	0	$-2Fx + 1/2qx^2$	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
BE	elongazione asta $N_{1, BE}^{\epsilon_{BE-BE}}$				$Fb^2 / EJ$	
	totali				$13/6Fb^2 / EJ$	$5/3Xb / EJ$
	iperstatica X=W <sub>cd</sub>				$-13/10Fb$	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [3/4 x^2/b - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (3/4 b - 1/6 b) \cdot Fb \cdot 1/EJ = 7/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1 - 1/2 x/b - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [x - 1/4 x^2/b - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (b - 1/4 b - 1/6 b) \cdot Fb \cdot 1/EJ = 7/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 x/b) \cdot Fb \cdot 1/EJ \, dx = [1/4 x^2/b]_0^b \cdot Fb \cdot 1/EJ$$

$$= (1/4 b) \cdot Fb \cdot 1/EJ = 1/4 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 - 1/2 x/b) \cdot Fb \cdot 1/EJ \, dx = [1/2 x - 1/4 x^2/b]_0^b \cdot Fb \cdot 1/EJ$$

$$= (1/2 b - 1/4 b) \cdot Fb \cdot 1/EJ = 1/4 Fb^2/EJ$$

$$L_{DE}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [1/2 x - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

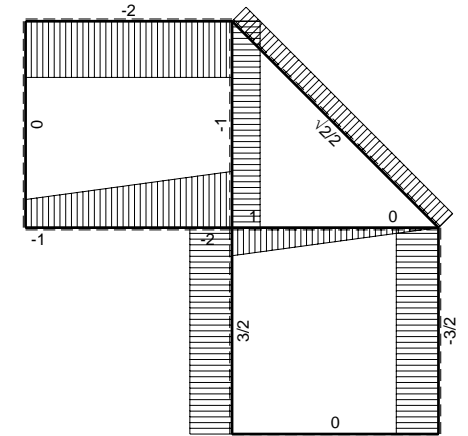
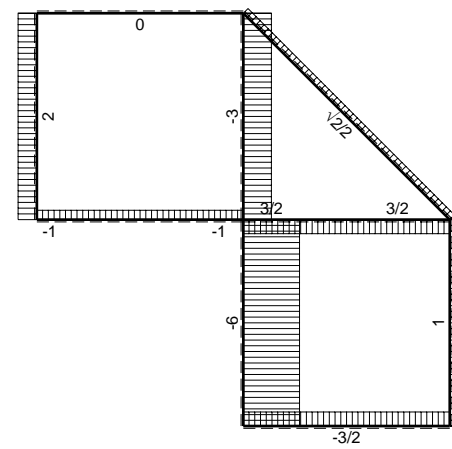
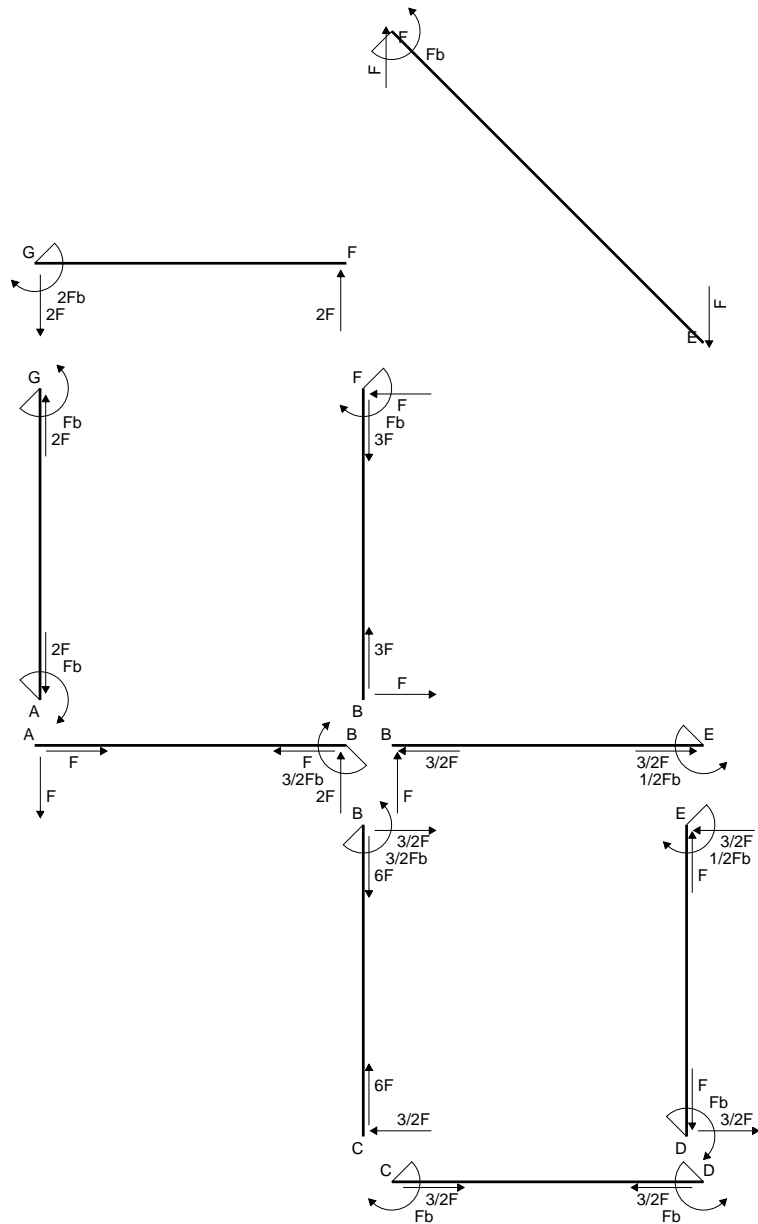
$$= (1/2 b - 1/6 b) \cdot Fb \cdot 1/EJ = 1/3 Fb^2/EJ$$

$$L_{ED}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (1/2 b - 1/6 b) \cdot Fb \cdot 1/EJ = 1/3 Fb^2/EJ$$

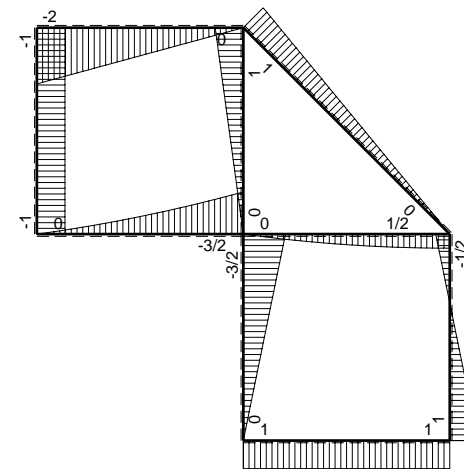




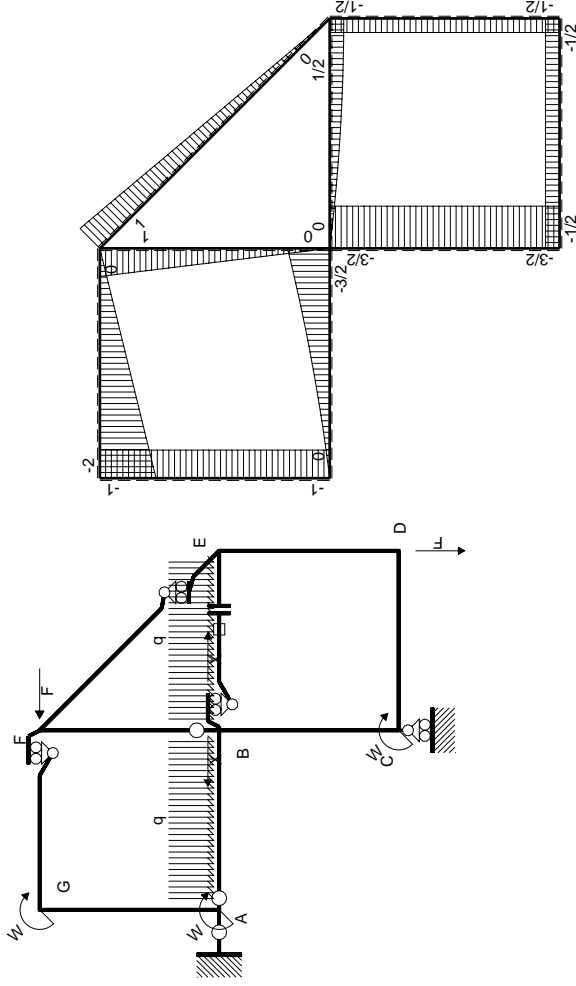


← (+) → F

↑ (+) ↓ F

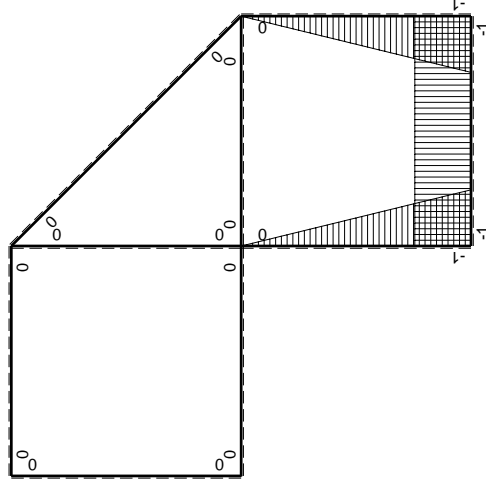


⊙ (+) ⊙ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>BE</sub>

→	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	$-Fx - 1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb - 2Fx + 1/2qx^2$	0	0	0	0
BC b	-x	-3/2Fb	3/2Fbx	$x^2$	3/4Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
CB b	b-x	3/2Fb	3/2Fb <sup>2</sup> -3/2Fbx	$b^2-2bx+x^2$	1/2Fb <sup>3</sup> /EJ	Xb <sup>3</sup> /EJ
CD b	-b	-1/2Fb	1/2Fb <sup>2</sup>	$b^2$	1/4Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
DC b	b	1/2Fb	1/2Fb <sup>2</sup>	$b^2$	1/4Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
DE b	-b+x	-1/2Fb	1/2Fb <sup>2</sup> -1/2Fbx	$b^2-2bx+x^2$	0	0
ED b	x	1/2Fb	1/2Fbx	$x^2$	0	0
EF √2b	0	√2/2Fx	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0	0	0
GA b	0	-Fb	0	0	0	0
AG b	0	Fb	0	0	0	0
FB b	0	Fb-Fx	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	$Fx - 1/2qx^2$	0	0	0	0
EB b	0	$-1/2Fb + 1/2qx^2$	0	0	0	0
BE	elongazione asta $N_{1, BE} \epsilon_{BE} L_{BE}$				Fb <sup>3</sup> /EJ	
	totali				5/2Fb <sup>3</sup> /EJ	5/3Xb <sup>3</sup> /EJ
	iperstatica X=H <sub>BE</sub>				-3/2F	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b) Fb^2 1/EJ dx = [3/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (3/4 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/2 - 3/2 x/b) Fb^2 1/EJ dx = [3/2 x - 3/4 x^2/b]_0^b Fb^2 1/EJ$$

$$= (3/2 b - 3/4 b) Fb^2 1/EJ = 3/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2) Fb^2 1/EJ dx = [1/2 x]_0^b Fb^2 1/EJ$$

$$= (1/2 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2) Fb^2 1/EJ dx = [1/2 x]_0^b Fb^2 1/EJ$$

$$= (1/2 b) Fb^2 1/EJ = 1/2 Fb^3/EJ$$

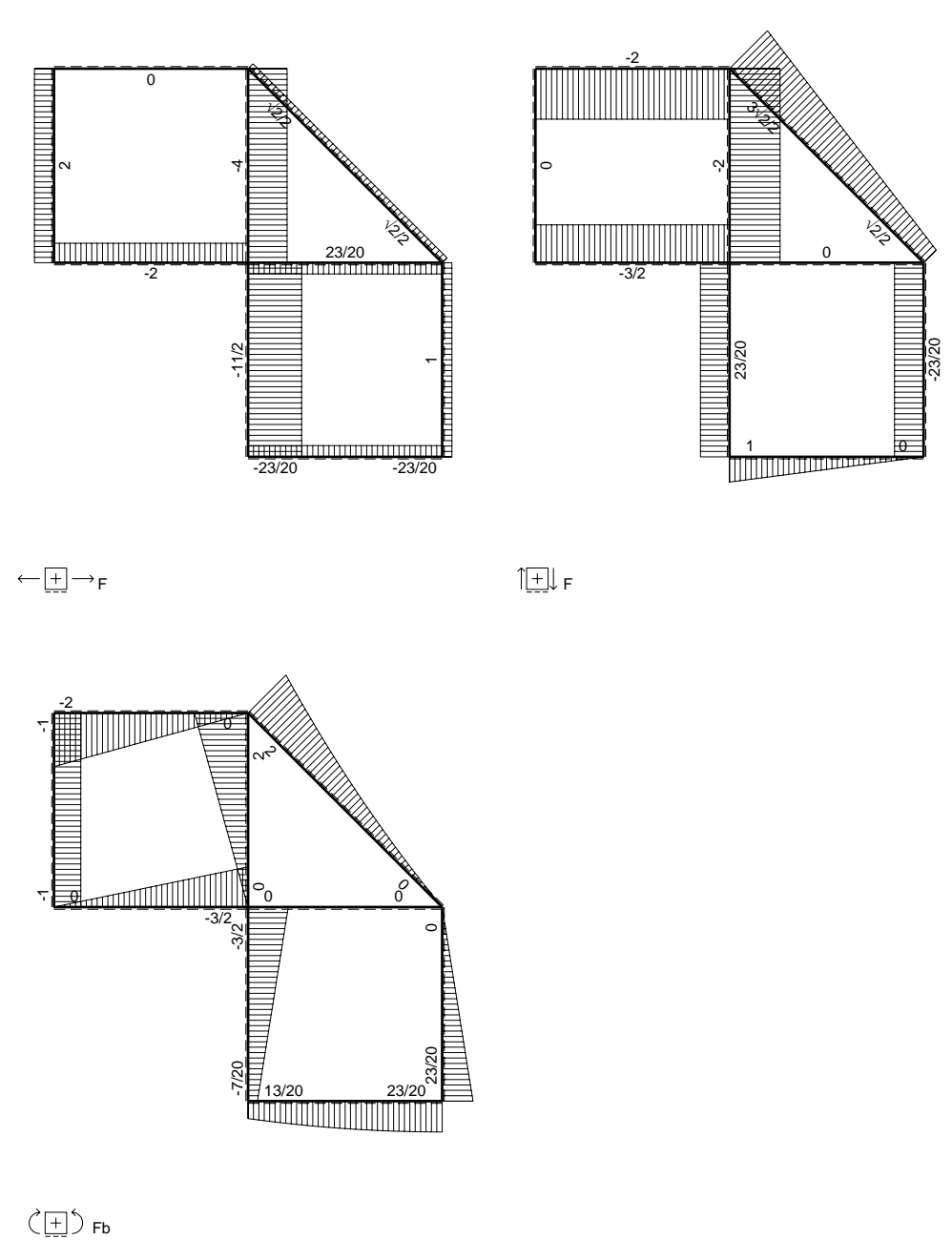
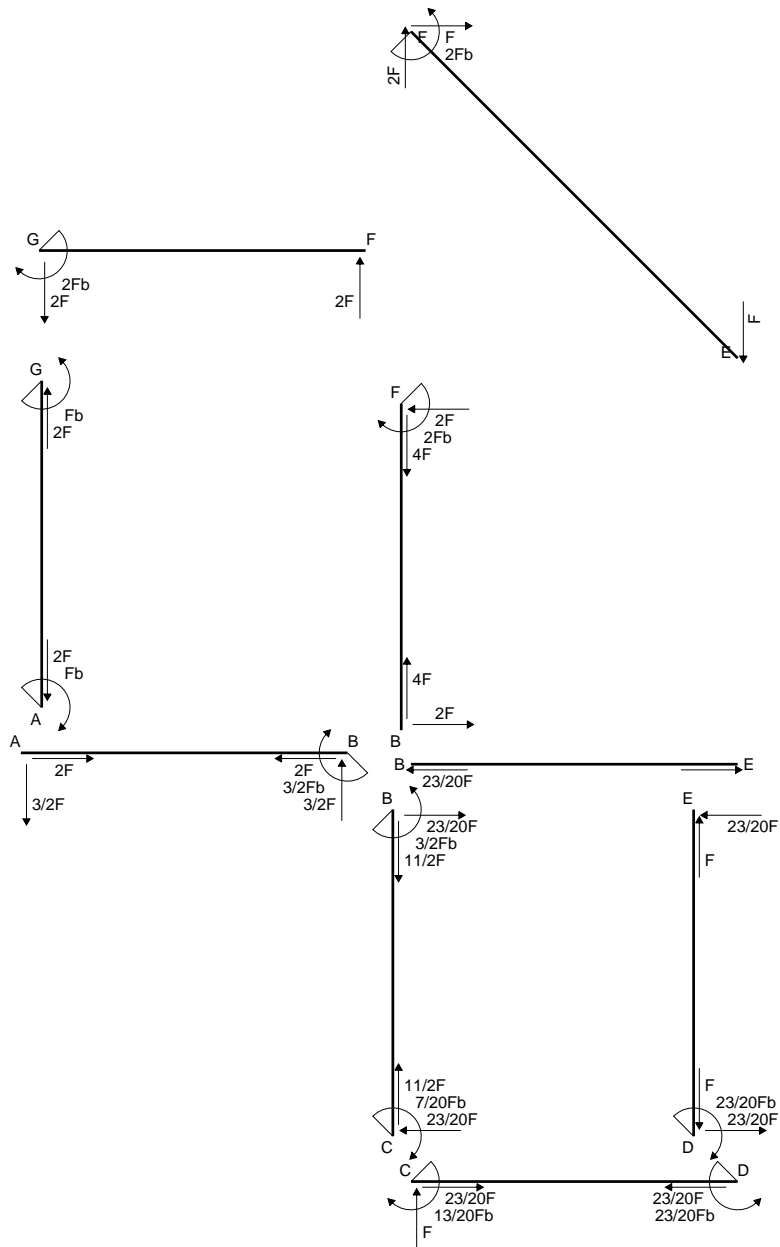
$$L_{DE}^{xo} = \int_0^b (1/2 - 1/2 x/b) Fb^2 1/EJ dx = [1/2 x - 1/4 x^2/b]_0^b Fb^2 1/EJ$$

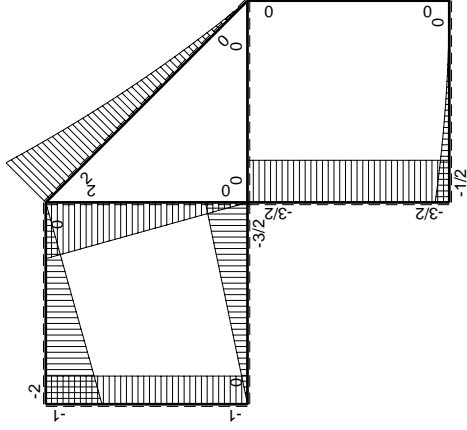
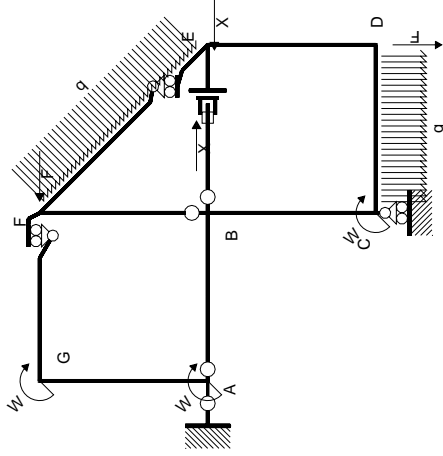
$$= (1/2 b - 1/4 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$

$$L_{ED}^{xo} = \int_0^b (1/2 x/b) Fb^2 1/EJ dx = [1/4 x^2/b]_0^b Fb^2 1/EJ$$

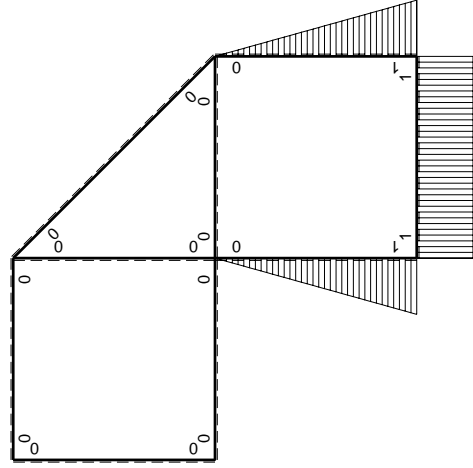
$$= (1/4 b) Fb^2 1/EJ = 1/4 Fb^3/EJ$$







$(\oplus)$   $M_0$  flessione da carichi assegnati



$(\oplus)$   $M_x$  flessione da iperstatica  $X=1$

Quadro contribuiti PLV per iperstatica  $X=H_{EB}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int X M_x M_x / EJ dx$
AB b	0	-3/2Fx	0	0	0	0
BA b	0	3/2Fb-3/2Fx	0	0	0	0
BC b	x	-3/2Fb	-3/2Fbx	$x^2$	-3/4Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ
CB b	-b+x	3/2Fb	-3/2Fb <sup>2</sup> +3/2Fbx	$b^2-2bx+x^2$	-1/6Fb <sup>3</sup> /EJ	Xb <sup>3</sup> /EJ
CD b	b	-1/2Fb+Fx-1/2qx <sup>2</sup>	-1/2Fb <sup>2</sup> +Fbx-1/2Fx <sup>2</sup>	$b^2$	-1/6Fb <sup>3</sup> /EJ	Xb <sup>3</sup> /EJ
DC b	-b	1/2qx <sup>2</sup>	-1/2Fx <sup>2</sup>	$b^2$	-1/6Fb <sup>3</sup> /EJ	Xb <sup>3</sup> /EJ
DE b	b-x	0	0	$b^2-2bx+x^2$	0	1/3Xb <sup>3</sup> /EJ
ED b	-x	0	0	$x^2$	0	0
EF $\sqrt{2}b$	0	$\sqrt{2}2Fx+1/2qx^2$	0	0	0	0
FG b	0	-2Fx	0	0	0	0
GF b	0	2Fb-2Fx	0	0	0	0
GA b	0	-Fb	0	0	0	0
AG b	0	Fb	0	0	0	0
FB b	0	2Fb-2Fx	0	0	0	0
BF b	0	-2Fx	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
BE	elongazione asta $N_{1, BE} \epsilon_{BE} L_{BE}$				-Fb <sup>3</sup> /EJ	
	totali				-23/12Fb <sup>3</sup> /EJ	5/3Xb <sup>3</sup> /EJ
	iperstatica $X=H_{EB}$				23/20F	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[ \frac{1}{3} x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = \left[ x - x^2/b + \frac{1}{3} x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) b^2 1/EJ dx = \left[ x \right]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) b^2 1/EJ dx = \left[ x \right]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = \left[ x - x^2/b + \frac{1}{3} x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = \left[ \frac{1}{3} x^3/b^2 \right]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{BC}^{xo} = \int_0^b (-3/2 x/b) Fb^2 1/EJ dx = \left[ -3/4 x^2/b \right]_0^b Fb^2 1/EJ$$

$$= (-3/4 b) Fb^2 1/EJ = -3/4 Fb^3/EJ$$

$$L_{CB}^{xo} = \int_0^b (-3/2 + 3/2 x/b) Fb^2 1/EJ dx = \left[ -3/2 x + 3/4 x^2/b \right]_0^b Fb^2 1/EJ$$

$$= (-3/2 b + 3/4 b) Fb^2 1/EJ = -3/4 Fb^3/EJ$$

$$L_{CD}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb^2 1/EJ dx = \left[ -1/2 x + 1/2 x^2/b - 1/6 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

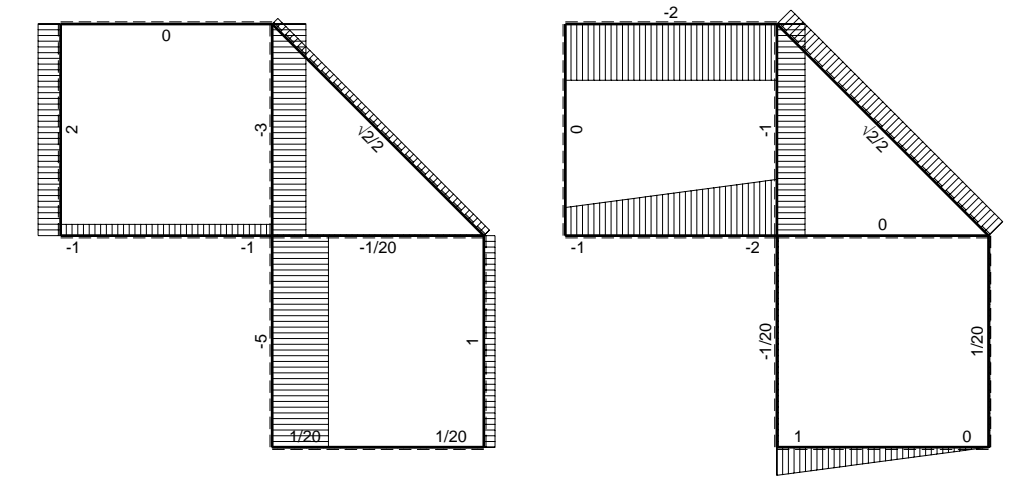
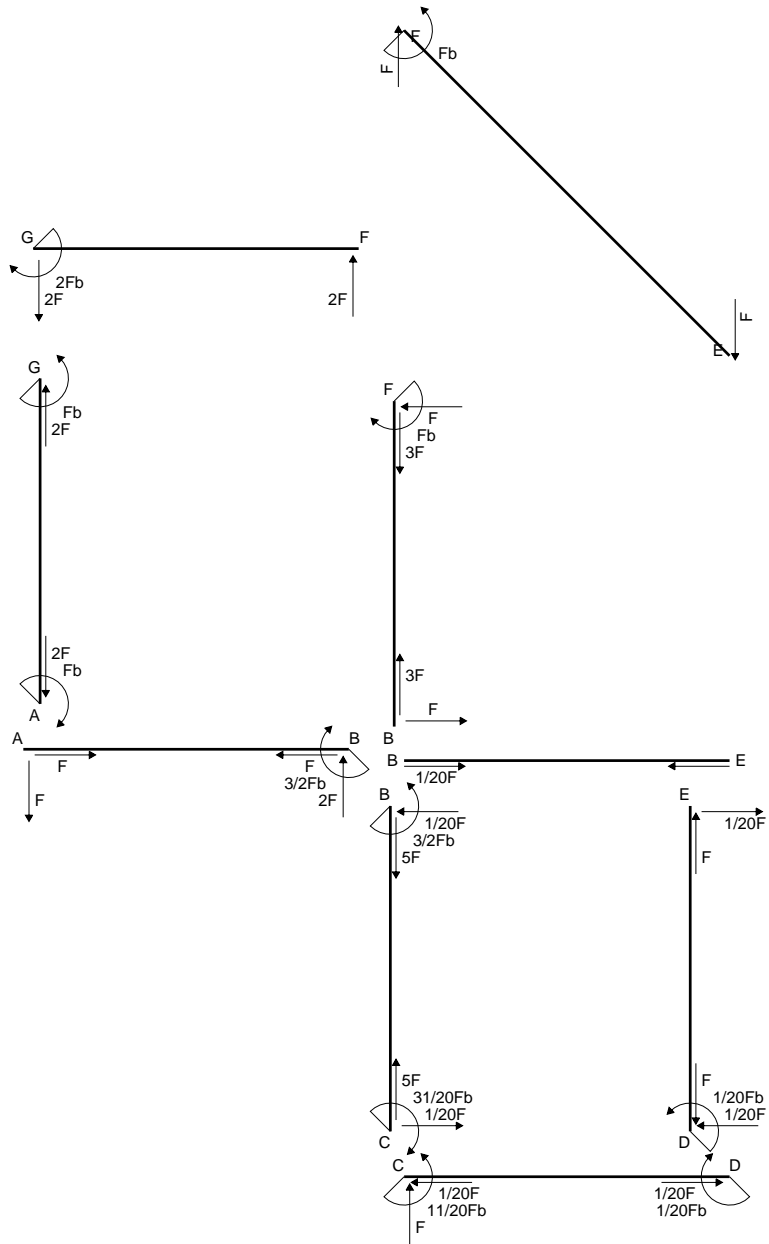
$$= (-1/2 b + 1/2 b - 1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 x^2/b^2) Fb^2 1/EJ dx = \left[ -1/6 x^3/b^2 \right]_0^b Fb^2 1/EJ$$

$$= (-1/6 b) Fb^2 1/EJ = -1/6 Fb^3/EJ$$

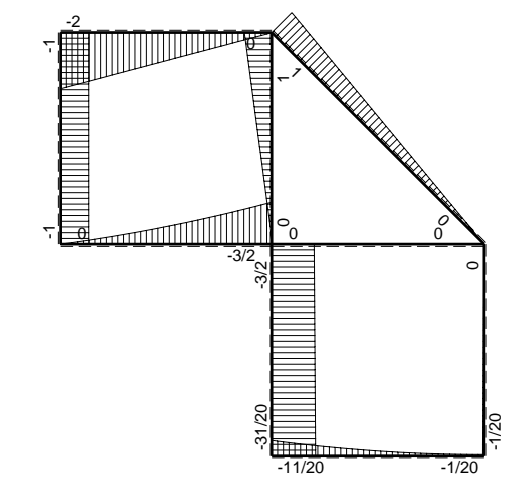




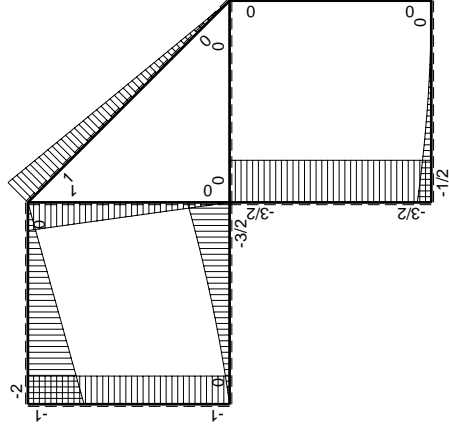
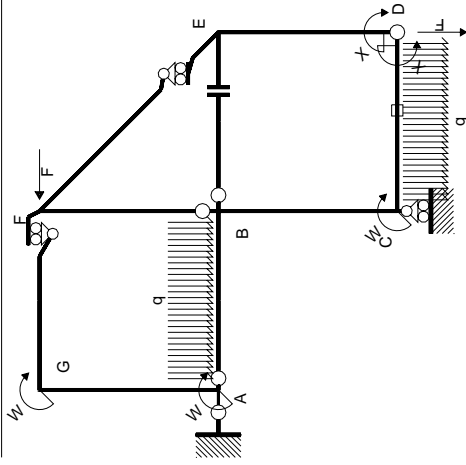


← ⊕ → F

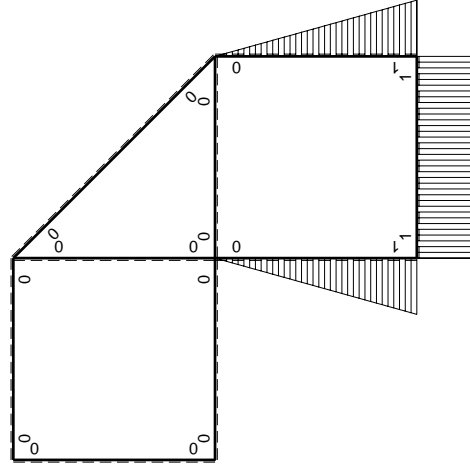
↑ ⊕ ↓ F



⊕ ⊖ Fb



$(\oplus)$   $M_0$  flessione da carichi assegnati



$(\oplus)$   $M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{DC}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int M_x M_x / EJ dx$
AB b	0	$-Fx - 1/2qx^2$	0	0	0	0
BA b	0	$3/2Fb - 2Fx + 1/2qx^2$	0	0	0	0
BC b	$x/b$	$-3/2Fb$	$-3/2Fx$	$x^2/b^2$	$-3/4Fb^2/EJ$	$1/3Xb/EJ$
CB b	$-1+x/b$	$3/2Fb$	$-3/2Fb + 3/2Fx$	$1 - 2x/b + x^2/b^2$	$-1/6Fb^2/EJ$	$Xb/EJ$
CD b	1	$-1/2Fb + Fx - 1/2qx^2$	$-1/2Fb + Fx - 1/2Fx^2/b$	1	$-1/6Fb^2/EJ$	$Xb/EJ$
DC b	-1	$1/2qx^2$	$-1/2Fx^2/b$	1	$-1/6Fb^2/EJ$	$Xb/EJ$
DE b	$1-x/b$	0	0	$1 - 2x/b + x^2/b^2$	0	$1/3Xb/EJ$
ED b	$-x/b$	0	0	$x^2/b^2$	0	0
EF $\sqrt{2}b$	0	$\sqrt{2}/2Fx$	0	0	0	0
FG b	0	$-2Fx$	0	0	0	0
GF b	0	$2Fb - 2Fx$	0	0	0	0
GA b	0	$-Fb$	0	0	0	0
AG b	0	$Fb$	0	0	0	0
FB b	0	$Fb - Fx$	0	0	0	0
BF b	0	$-Fx$	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
CD	elongazione asta $N_{1,cd} \epsilon_{cd} L_{cd}$					
	totali					
	iperstatica $X=W_{DC}$					
					$Fb^2/EJ$	$5/3Xb/EJ$
					$-1/20Fb$	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) 1/EJ dx = [x]_0^b 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (-3/2 x/b) Fb 1/EJ dx = [-3/4 x^2/b]_0^b Fb 1/EJ$$

$$= (-3/4 b) Fb 1/EJ = -3/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (-3/2 + 3/2 x/b) Fb 1/EJ dx = [-3/2 x + 3/4 x^2/b]_0^b Fb 1/EJ$$

$$= (-3/2 b + 3/4 b) Fb 1/EJ = -3/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) 1 Fb^2/EJ$$

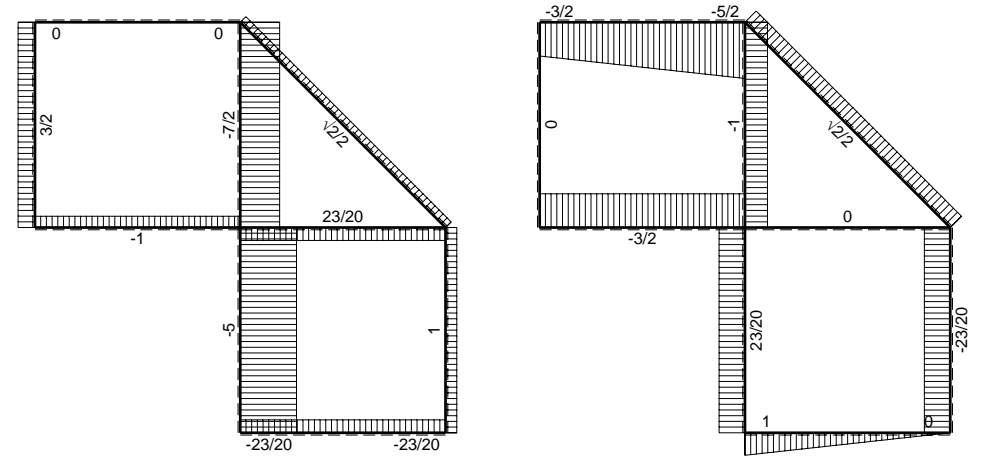
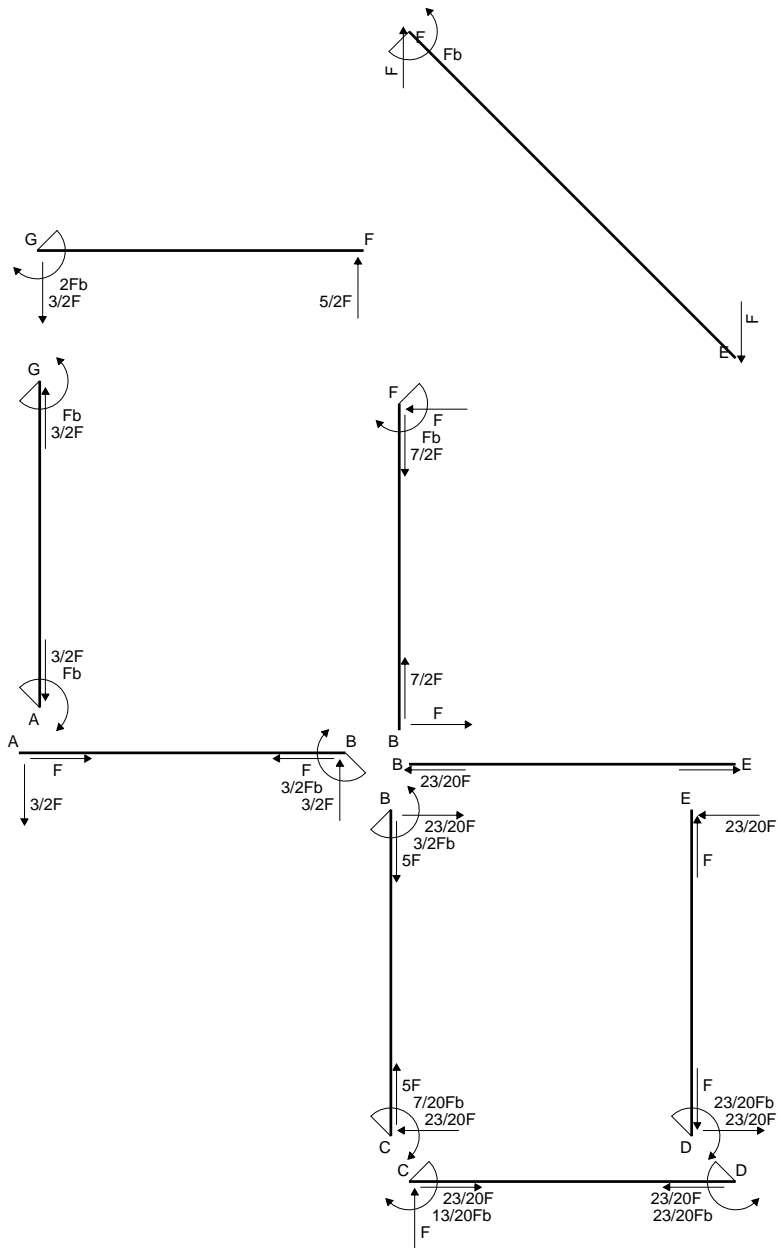
$$= [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) 1 Fb^2/EJ$$

$$= (-1/2 b + 1/2 b - 1/6 b) Fb 1/EJ - 1 (-1) 1 Fb^2/EJ = 5/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 x^2/b^2) Fb 1/EJ dx - 1 (-1) 1 Fb^2/EJ = [-1/6 x^3/b^2]_0^b Fb 1/EJ - 1 (-1) 1 Fb^2/EJ$$

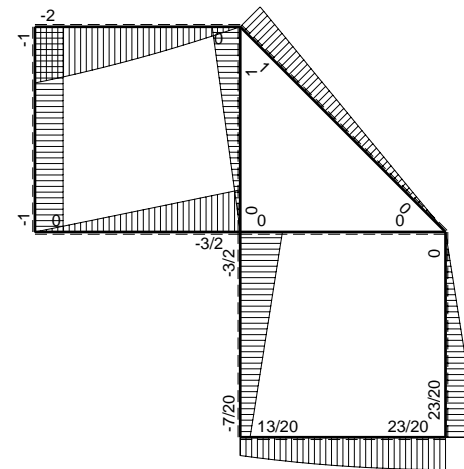
$$= (-1/6 b) Fb 1/EJ - 1 (-1) 1 Fb^2/EJ = 5/6 Fb^2/EJ$$



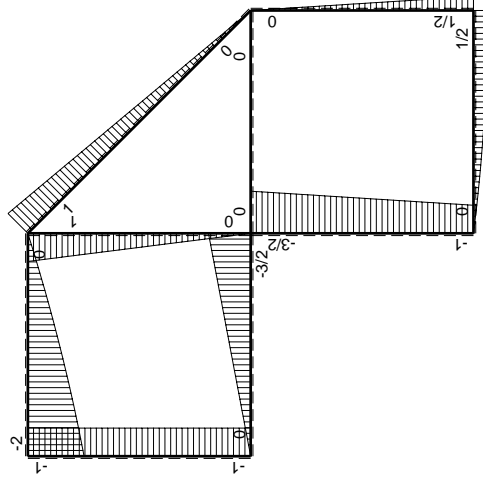
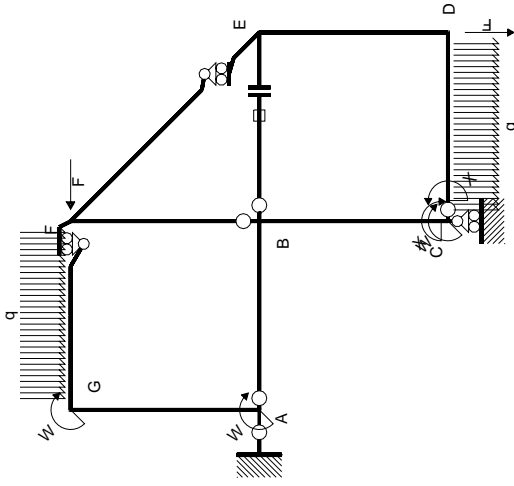


← ⊕ → F

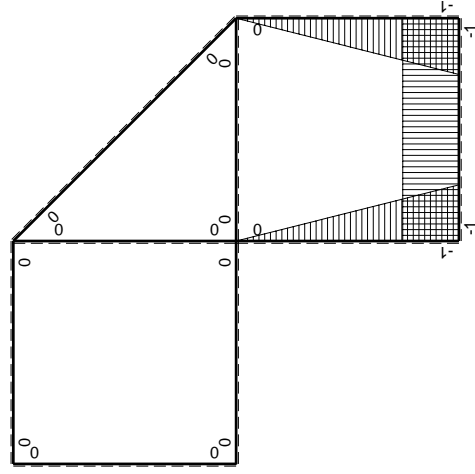
↑ ⊕ ↓ F



⊕ ⊖ F<sub>b</sub>



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contribuiti PLV per iperstatica  $X=W_{cd}$

$\rightarrow$	$M_x(x)$	$M_0(x)$	$M_x M_0$	$M_x M_x$	$\int M_x M_0 / EJ dx$	$\int M_x M_x / EJ dx$
AB b	0	$-3/2Fx$	0	0	0	0
BA b	0	$3/2Fb-3/2Fx$	0	0	0	0
BC b	$-x/b$	$-3/2Fb+1/2Fx$	$3/2Fx-1/2Fx^2/b$	$x^2/b^2$	$7/12Fb^2/EJ$	$1/3Xb/EJ$
CB b	$1-x/b$	$Fb+1/2Fx$	$Fb-1/2Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$-1/3Fb^2/EJ$	$Xb/EJ$
CD b	-1	$Fx-1/2qx^2$	$-Fx+1/2Fx^2/b$	1	$-1/3Fb^2/EJ$	$Xb/EJ$
DC b	1	$-1/2Fb+1/2qx^2$	$-1/2Fb+1/2Fx^2/b$	1	$-1/3Fb^2/EJ$	$Xb/EJ$
DE b	$-1+x/b$	$1/2Fb-1/2Fx$	$-1/2Fb+Fx-1/2Fx^2/b$	$1-2x/b+x^2/b^2$	$-1/6Fb^2/EJ$	$1/3Xb/EJ$
ED b	$x/b$	$-1/2Fx$	$-1/2Fx^2/b$	$x^2/b^2$	$-1/6Fb^2/EJ$	$1/3Xb/EJ$
EF $\sqrt{2}b$	0	$\sqrt{2}/2Fx$	0	0	0	0
FG b	0	$-5/2Fx+1/2qx^2$	0	0	0	0
GF b	0	$2Fb-3/2Fx-1/2qx^2$	0	0	0	0
GA b	0	-Fb	0	0	0	0
AG b	0	Fb	0	0	0	0
FB b	0	$Fb-Fx$	0	0	0	0
BF b	0	-Fx	0	0	0	0
BE b	0	0	0	0	0	0
EB b	0	0	0	0	0	0
BE	elongazione asta $N_{1, BE}^{\epsilon_{BE}} L_{BE}$				$Fb^2/EJ$	
	totali				$13/12Fb^2/EJ$	$5/3Xb/EJ$
	iperstatica $X=W_{cd}$				$-13/20Fb$	

Sviluppi di calcolo iperstatica

$$L_{BC}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1) \cdot 1/EJ \, dx = [x]_0^b \cdot 1/EJ$$

$$= (b) \cdot 1/EJ = b/EJ$$

$$L_{DE}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) \cdot 1/EJ \, dx = [x - x^2/b + 1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (b - b + 1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{ED}^{xx} = \int_0^b (x^2/b^2) \cdot 1/EJ \, dx = [1/3 x^3/b^2]_0^b \cdot 1/EJ$$

$$= (1/3 b) \cdot 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [3/4 x^2/b - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (3/4 b - 1/6 b) \cdot Fb \cdot 1/EJ = 7/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1 - 1/2 x/b - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [x - 1/4 x^2/b - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (b - 1/4 b - 1/6 b) \cdot Fb \cdot 1/EJ = 7/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (-x/b + 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/2 x^2/b + 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/2 b + 1/6 b) \cdot Fb \cdot 1/EJ = -1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (-1/2 + 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/2 x + 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/2 b + 1/6 b) \cdot Fb \cdot 1/EJ = -1/3 Fb^2/EJ$$

$$L_{DE}^{xo} = \int_0^b (-1/2 + x/b - 1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/2 x + 1/2 x^2/b - 1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/2 b + 1/2 b - 1/6 b) \cdot Fb \cdot 1/EJ = -1/6 Fb^2/EJ$$

$$L_{ED}^{xo} = \int_0^b (-1/2 x^2/b^2) \cdot Fb \cdot 1/EJ \, dx = [-1/6 x^3/b^2]_0^b \cdot Fb \cdot 1/EJ$$

$$= (-1/6 b) \cdot Fb \cdot 1/EJ = -1/6 Fb^2/EJ$$

