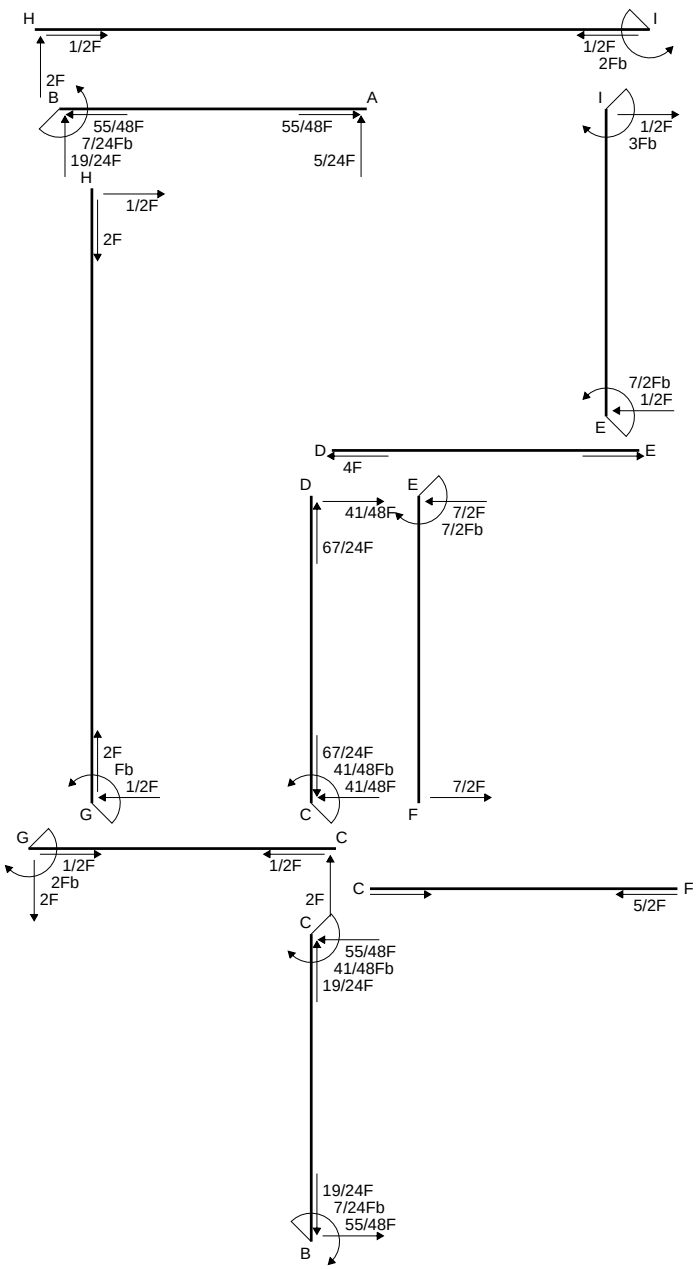

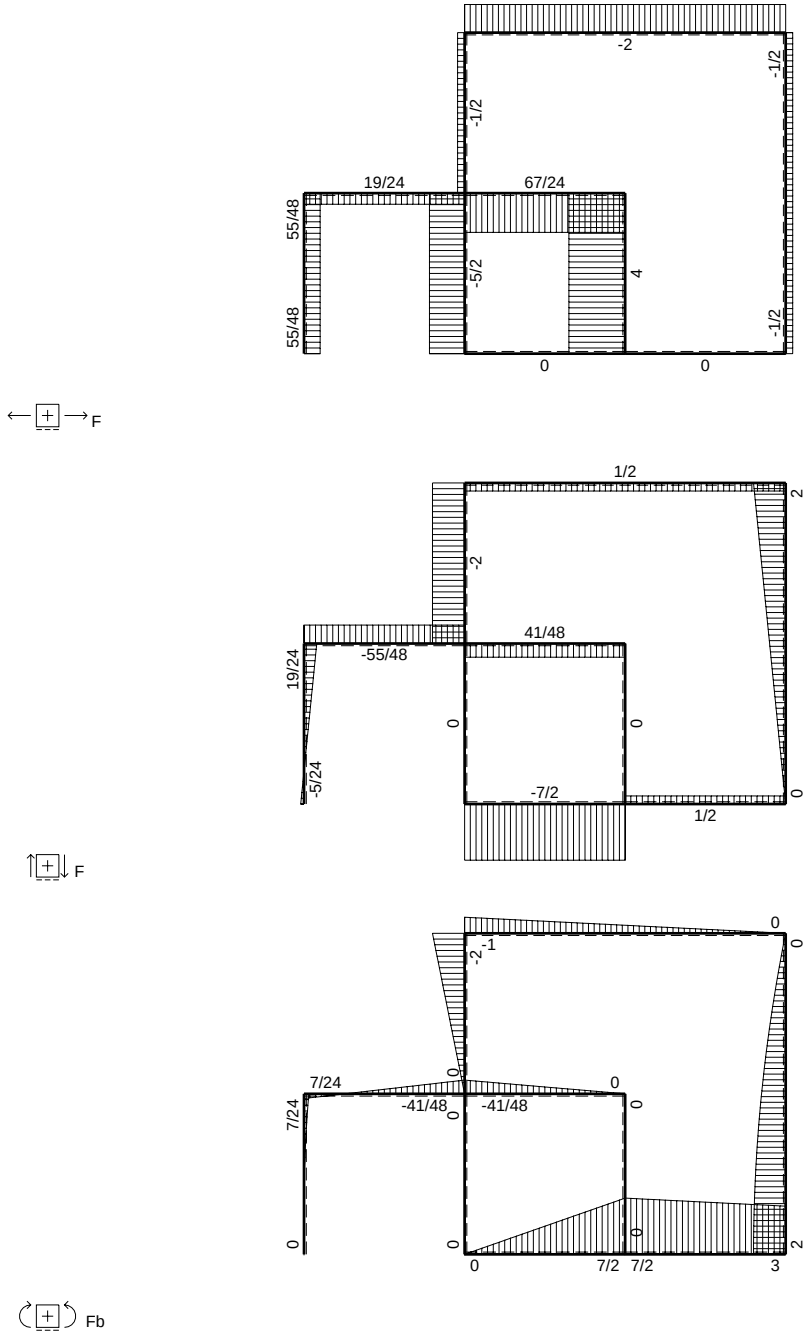

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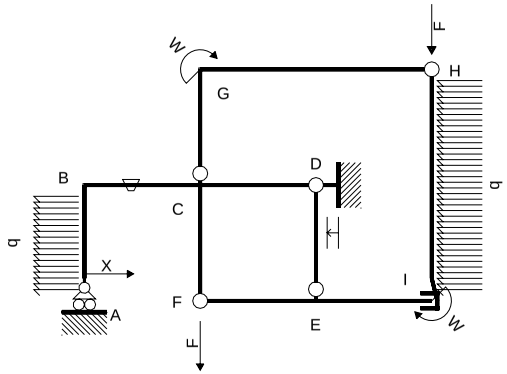
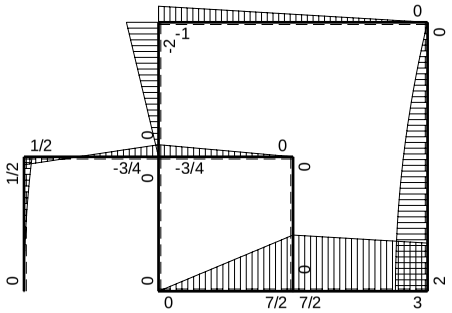
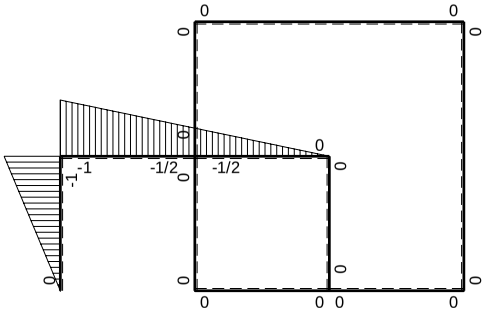


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-5/4Fx$	$-Fb/EJ$	$-1/2Fb^2+3/2Fbx-5/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/24+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-5/4Fx$	Fb/EJ	$3/8Fb^2-1/4Fbx-5/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-5/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_{AB}$						5/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +3/2\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [-1/2\, x +3/4\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [\, x -1/4\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -5/24\, b)\, Fb^2\, 1/EJ + (\, b -1/4\, b)\, \theta = 19/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -1/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [3/8\, x -1/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b -5/24\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 19/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

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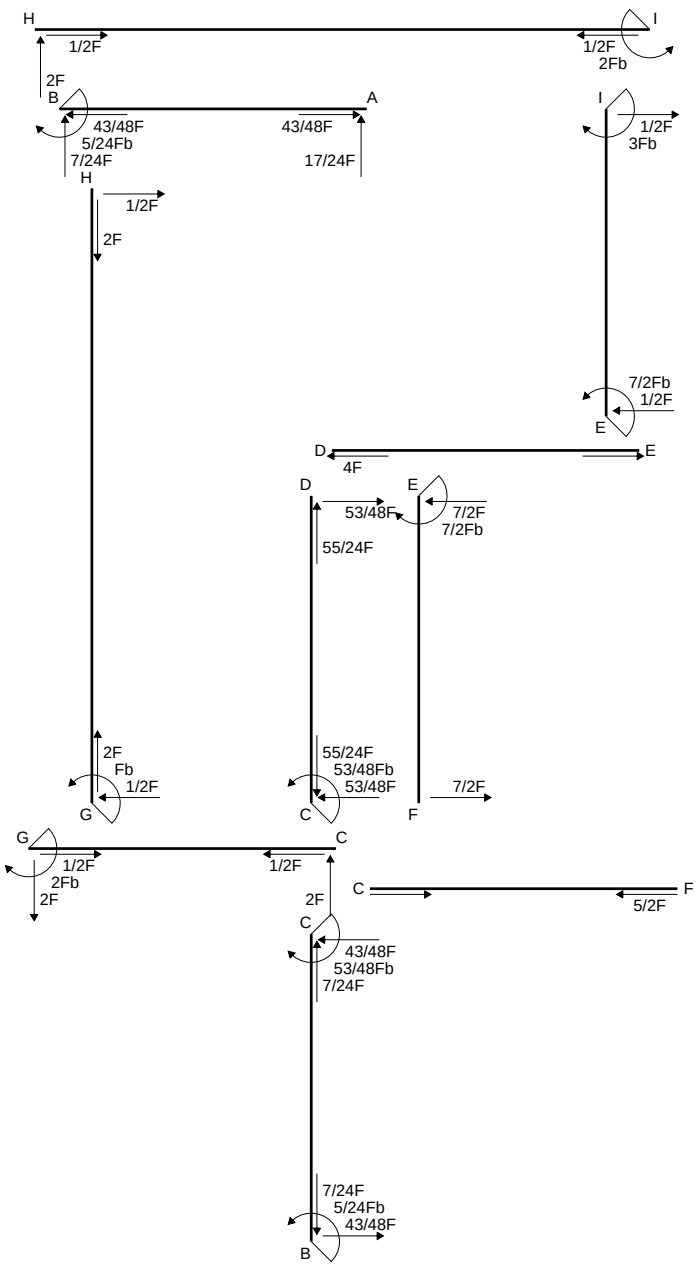
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2F
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5/24Fb
7/24F
17/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
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Fb
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2Fb
2F
C
F
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2F
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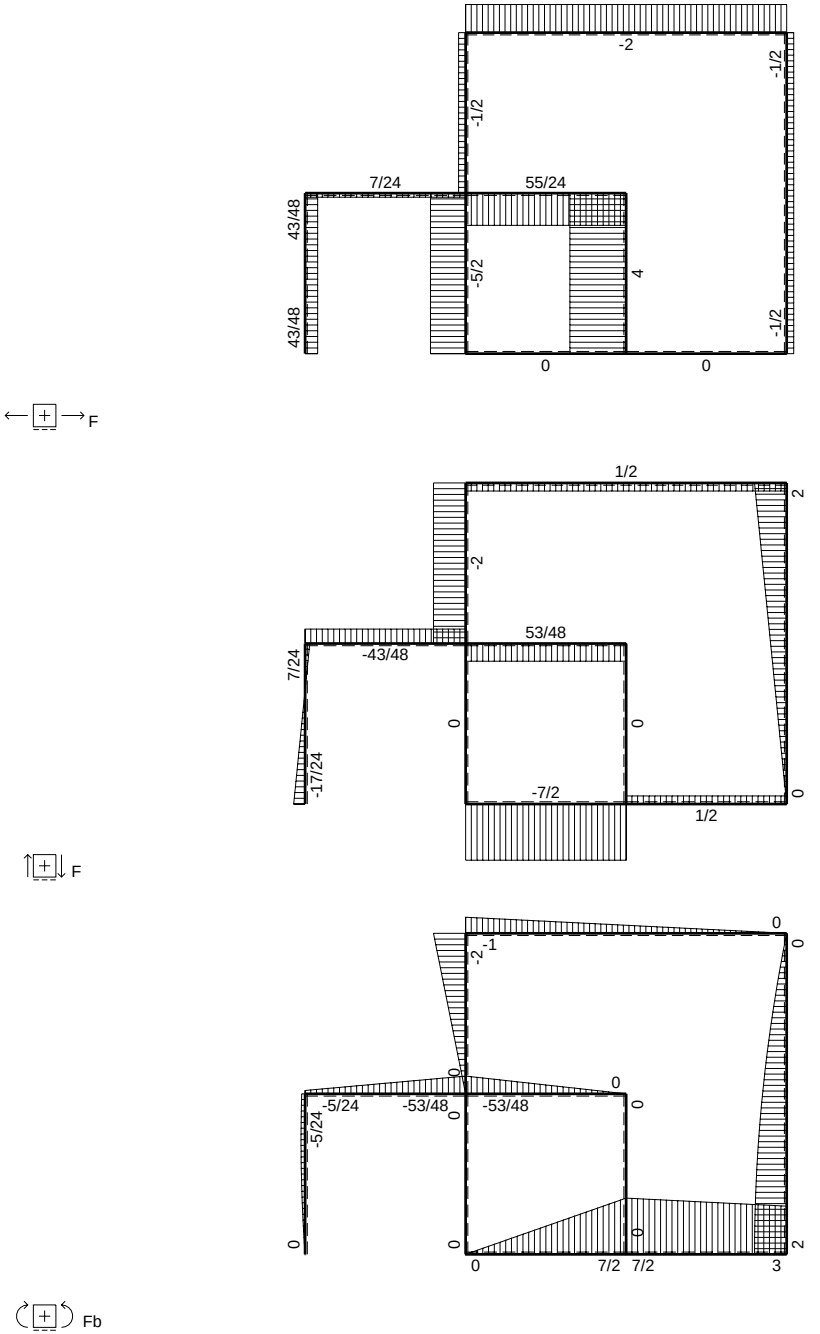
53/48Fb
7/24F
7/24F
5/24Fb
43/48F
B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-1/2
7/24
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-5/2
4
43/48
-1/2
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0
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1/2
2
-2
53/48
-43/48
7/24
0
0
-17/24
-7/2
0
1/2
F
0
0
-2
-1
0
0
-5/24
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0
-5/24
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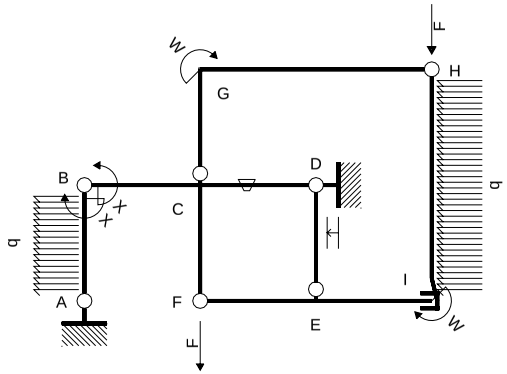
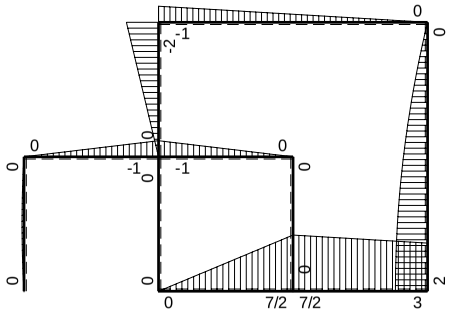
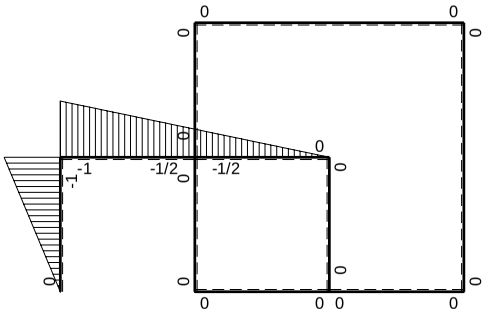
0
0
0
7/2
7/2
3
2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981883 ESΣ1.xxxx.003

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PROCEDIMENTO E RISULTATI 981883 ESΣ1.xxxx.003

Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$-x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	$1-x/b$	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	$-Fx$	0	$Fx-1/2Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(1/3+0)Fb^2/EJ$	$7/12Xb/EJ$
CB b	$1/2+1/2x/b$	$Fb-Fx$	0	$1/2Fb-1/2Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	$-Fb+Fx$	$-Fb/EJ$	$1/2Fb-Fx+1/2Fx^2/b$	$1/2Fb/EJ-1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(1/6+1/4)Fb^2/EJ$	$1/12Xb/EJ$
DC b	$1/2x/b$	Fx	Fb/EJ	$1/2Fx^2/b$	$1/2Fx/EJ$	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$-5/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						$5/24Fb$	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1/2 -1/2\ x/b)\ \theta\ dx$

$= [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b -1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2\ x/b)\ \theta\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/4\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b)\ Fb\ 1/EJ\ + (-1/4\ b)\ \theta\ = 5/12\ Fb^2/EJ$

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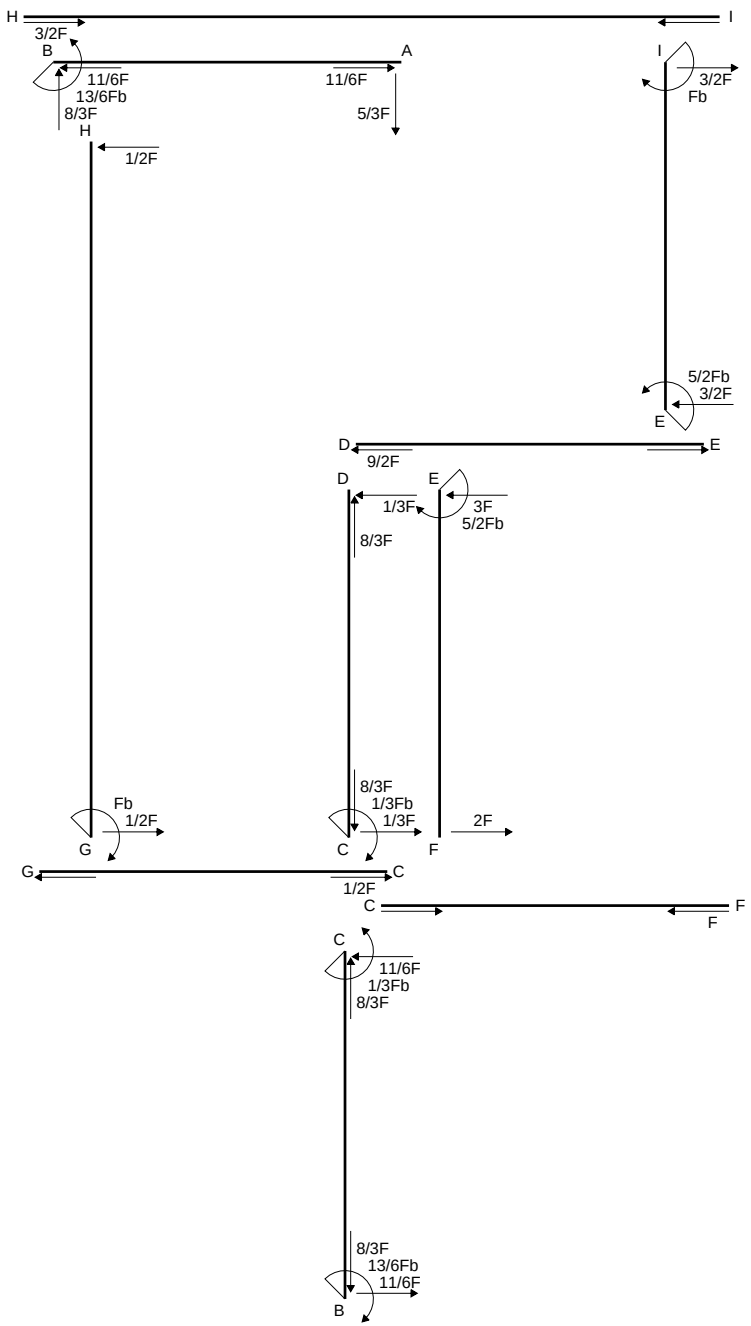
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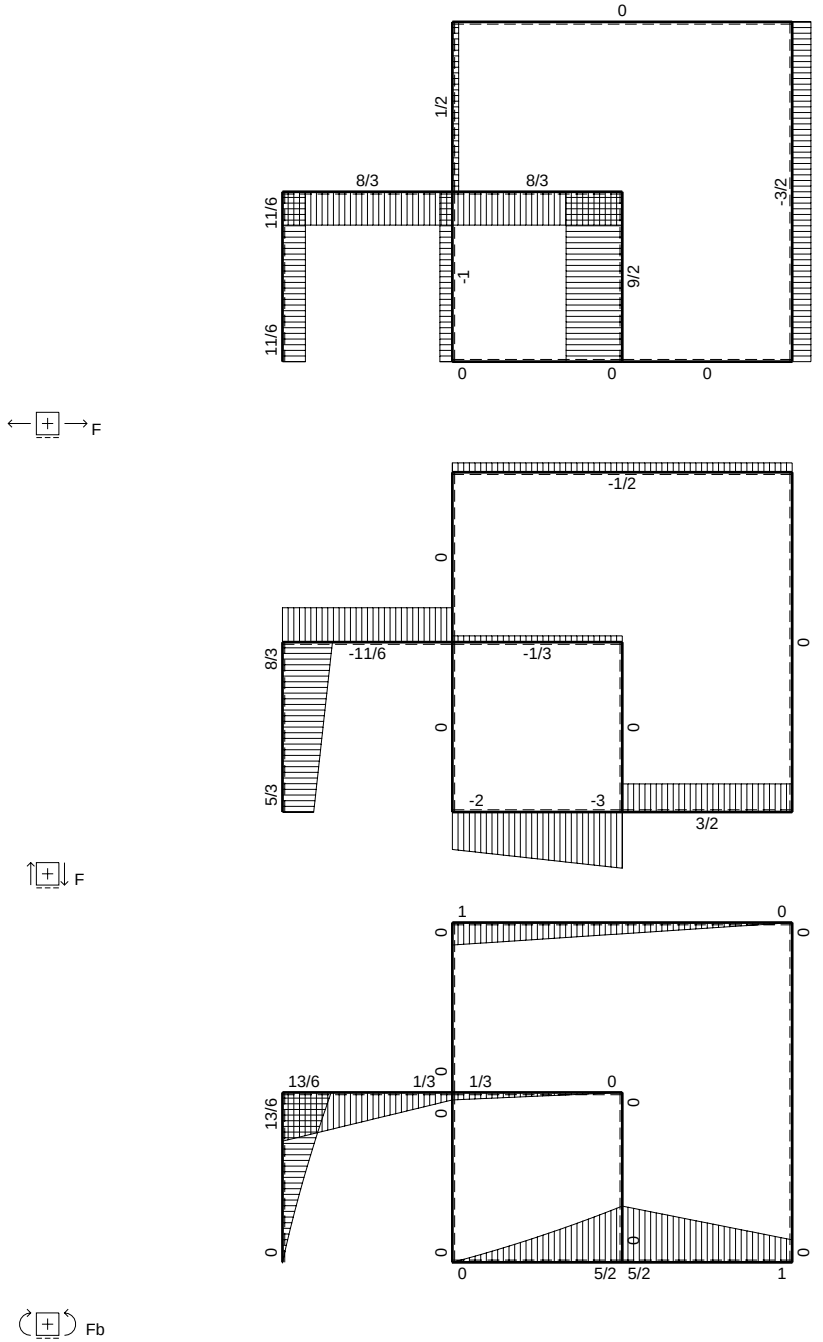
11/6F
B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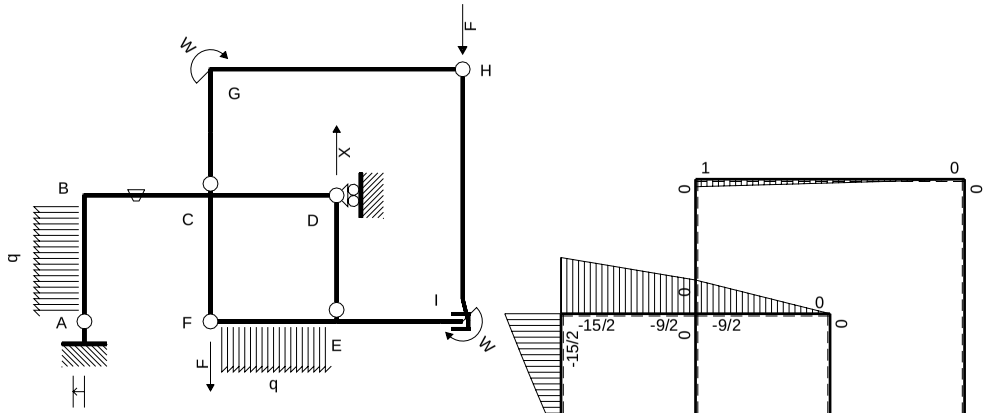
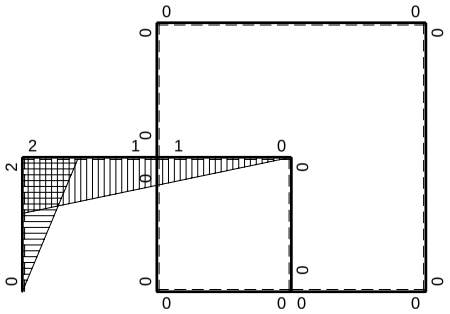
5/2
1
Fb



Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 957564 ESΣ1.xxxx.010

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Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2x	$-8Fx+1/2qx^2$	0	$-16Fx^2+qx^3$	0	$4x^2$	$(-61/12+0)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	$-2b+2x$	$15/2Fb-7Fx-1/2qx^2$	0	$-15Fb^2+29Fbx-13Fx^2-qx^3$	0	$4b^2-8bx+4x^2$		
BC b	$2b-x$	$-15/2Fb+3Fx$	$-Fb/EJ$	$-15Fb^2+27/2Fbx-3Fx^2$	$-2Fb^2/EJ+Fxb/EJ$	$4b^2-4bx+x^2$	$(-37/4-3/2)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	$-b-x$	$9/2Fb+3Fx$	Fb/EJ	$-9/2Fb^2-15/2Fbx-3Fx^2$	$-Fb^2/EJ-Fxb/EJ$	$b^2+2bx+x^2$		
CD b	$b-x$	$-9/2Fb+9/2Fx$	0	$-9/2Fb^2+9Fbx-9/2Fx^2$	0	$b^2-2bx+x^2$	$(-3/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	$-x$	$9/2Fx$	0	$-9/2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$-2Fb^3/EJ$	
	totali						$-58/3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_D$						29/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b \left(4\, x^2/b^2\right) b^2\, 1/EJ\, dx = \left[4/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$

$= \left(4/3\, b\right) b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b \left(4 - 8\, x/b + 4\, x^2/b^2\right) b^2\, 1/EJ\, dx = \left[4\, x - 4\, x^2/b + 4/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$

$= \left(4\, b - 4\, b + 4/3\, b\right) b^2\, 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b \left(4 - 4\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[4\, x - 2\, x^2/b + 1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$

$= \left(4\, b - 2\, b + 1/3\, b\right) b^2\, 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b \left(1 + 2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[x + x^2/b + 1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$

$= \left(b + b + 1/3\, b\right) b^2\, 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b \left(1 - 2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[x - x^2/b + 1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$

$= \left(b - b + 1/3\, b\right) b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b \left(x^2/b^2\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$

$= \left(1/3\, b\right) b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b \left(-16\, x^2/b^2 + x^3/b^3\right) Fb^2\, 1/EJ\, dx = \left[-16/3\, x^3/b^2 + 1/4\, x^4/b^3\right]_o^b Fb^2\, 1/EJ$

$= \left(-16/3\, b + 1/4\, b\right) Fb^2\, 1/EJ = -61/12\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b \left(-15 + 29\, x/b - 13\, x^2/b^2 - x^3/b^3\right) Fb^2\, 1/EJ\, dx$

$= \left[-15\, x + 29/2\, x^2/b - 13/3\, x^3/b^2 - 1/4\, x^4/b^3\right]_o^b Fb^2\, 1/EJ$

$= \left(-15\, b + 29/2\, b - 13/3\, b - 1/4\, b\right) Fb^2\, 1/EJ = -61/12\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b \left(-15 + 27/2\, x/b - 3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + \int_o^b \left(-2 + x/b\right) \theta\ dx$

$= \left[-15\, x + 27/4\, x^2/b - x^3/b^2\right]_o^b Fb^2\, 1/EJ + \left[-2\, x + 1/2\, x^2/b\right]_o^b \theta$

$= \left(-15\, b + 27/4\, b - b\right) Fb^2\, 1/EJ + \left(-2\, b + 1/2\, b\right) \theta = -43/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b \left(-9/2 - 15/2\, x/b - 3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + \int_o^b \left(1 + x/b\right) \theta\ dx$

$= \left[-9/2\, x - 15/4\, x^2/b - x^3/b^2\right]_o^b Fb^2\, 1/EJ + \left[x + 1/2\, x^2/b\right]_o^b \theta$

$= \left(-9/2\, b - 15/4\, b - b\right) Fb^2\, 1/EJ + \left(b + 1/2\, b\right) \theta = -43/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b \left(-9/2 + 9\, x/b - 9/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-9/2\, x + 9/2\, x^2/b - 3/2\, x^3/b^2\right]_o^b Fb^2\, 1/EJ$

$= \left(-9/2\, b + 9/2\, b - 3/2\, b\right) Fb^2\, 1/EJ = -3/2\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b \left(-9/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[-3/2\, x^3/b^2\right]_o^b Fb^2\, 1/EJ$

$= \left(-3/2\, b\right) Fb^2\, 1/EJ = -3/2\ Fb^3/EJ$

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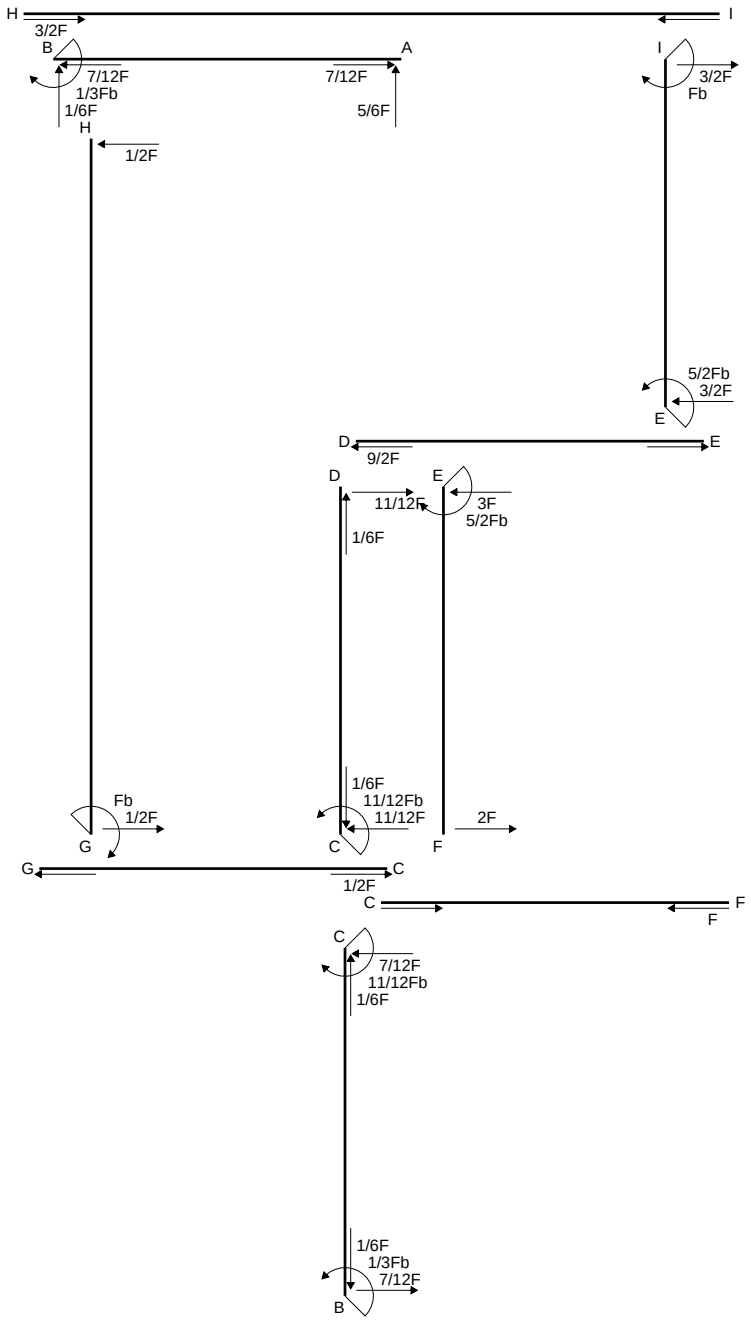
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F
G
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1/3Fb
7/12F
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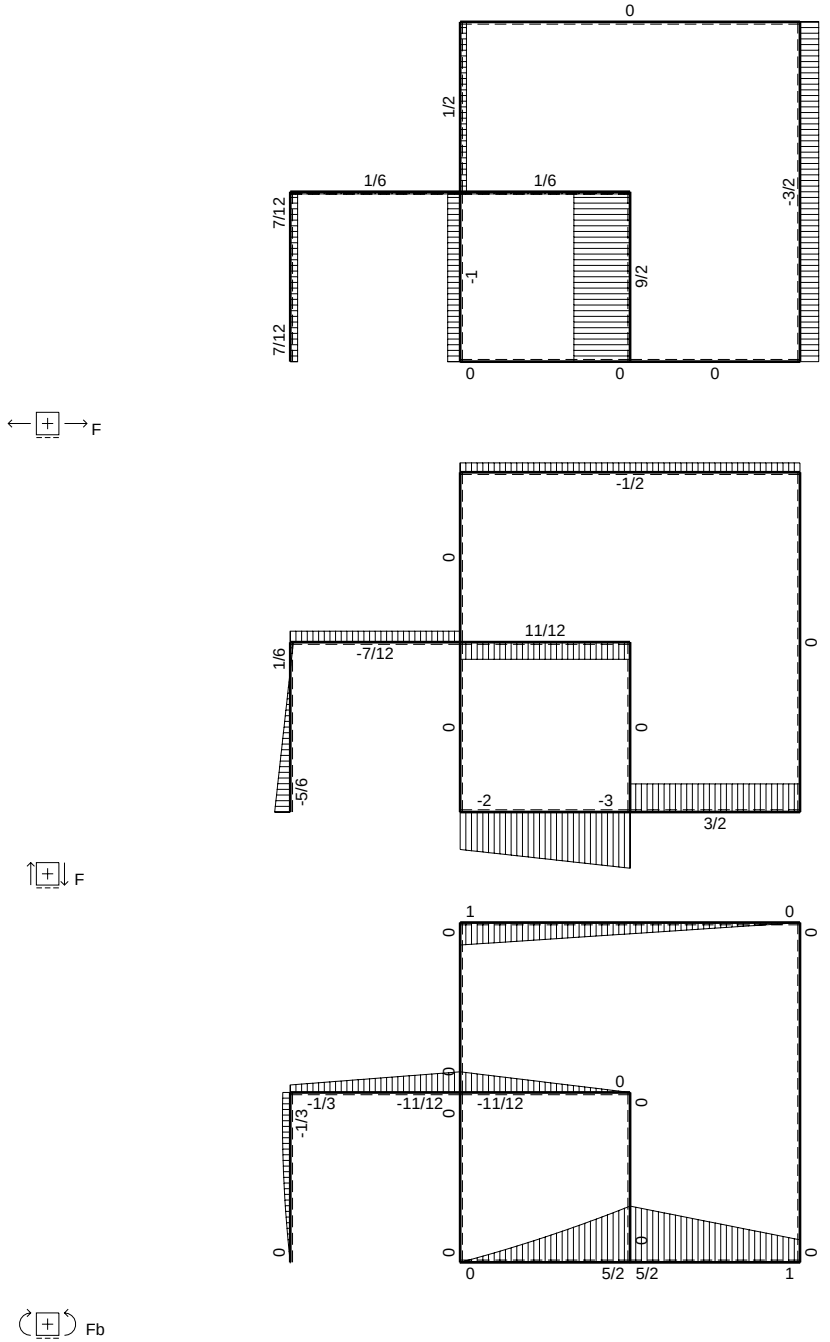
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1/6
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0
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-5/6
-2
-3
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0
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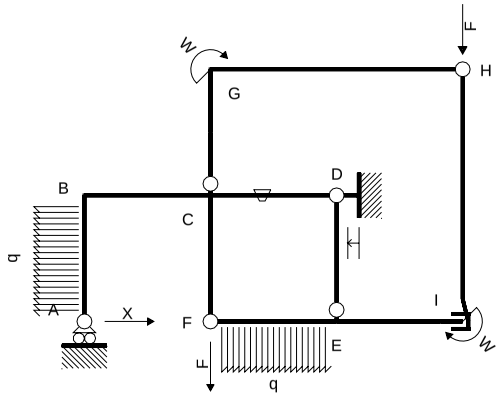
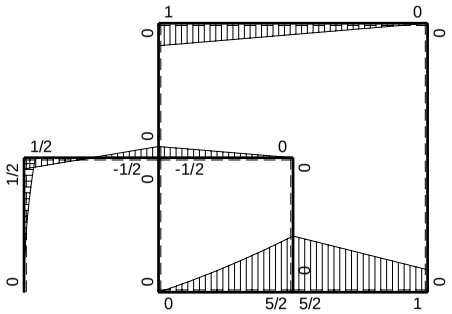
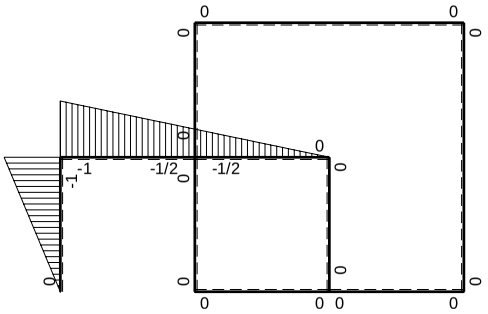


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-Fx$	0	$-1/2Fb^2+5/4Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(-1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$1/2Fb-Fx$	0	$1/4Fb^2-1/4Fbx-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	-Fb/EJ	$1/4Fb^2-1/2Fbx+1/4Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/12+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	Fb/EJ	$1/4Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-5/6Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						5/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +5/4\, x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +5/8\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +5/8\, b -1/6\, b)\, Fb^2\, 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/4 -1/4\, x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/8\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/8\, b -1/6\, b)\, Fb^2\, 1/EJ = -1/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/12\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 1/3\ Fb^3/EJ$

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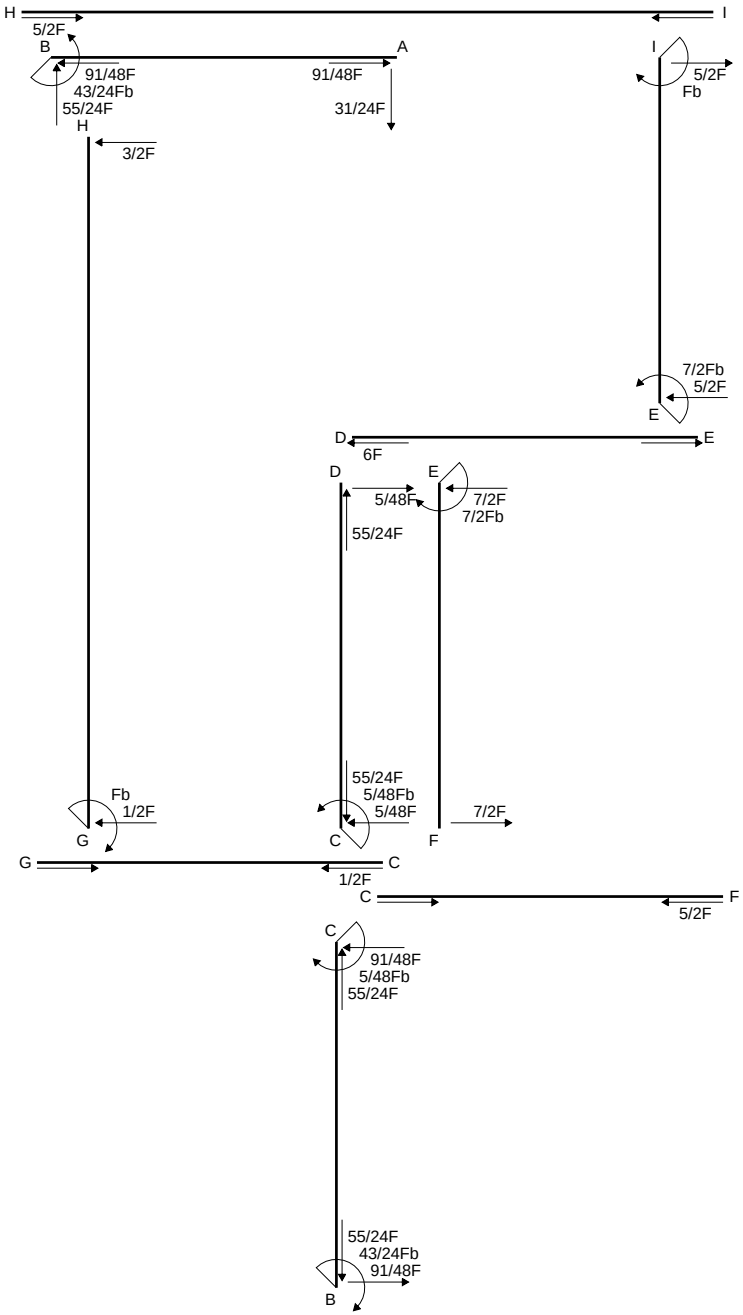
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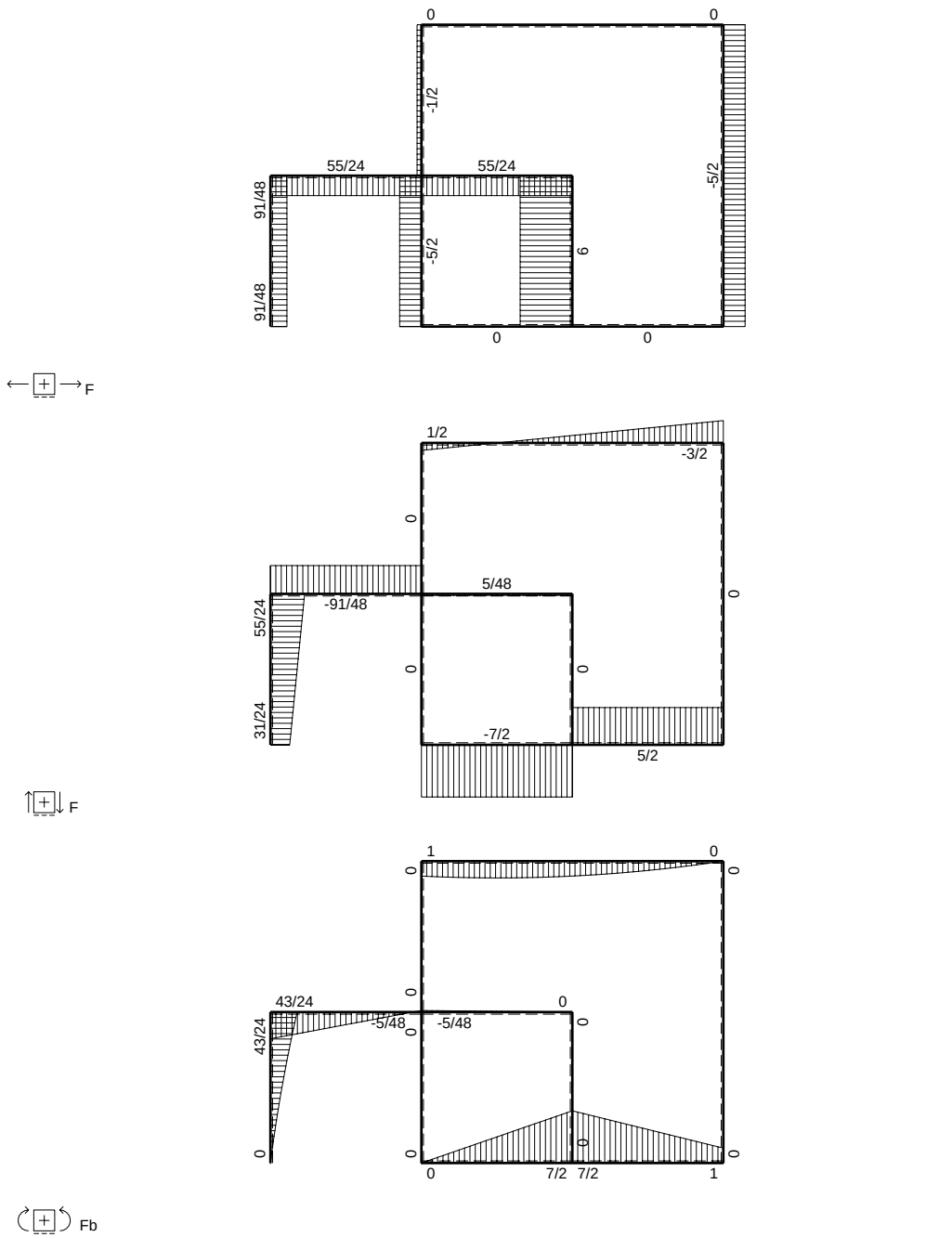
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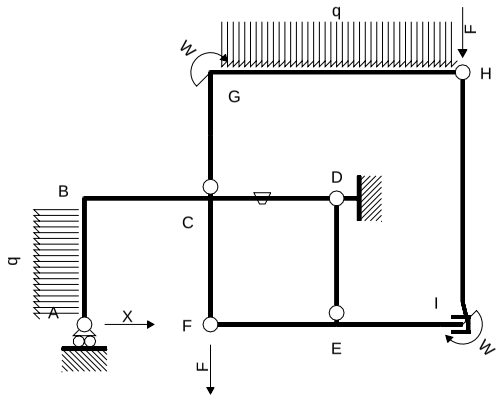
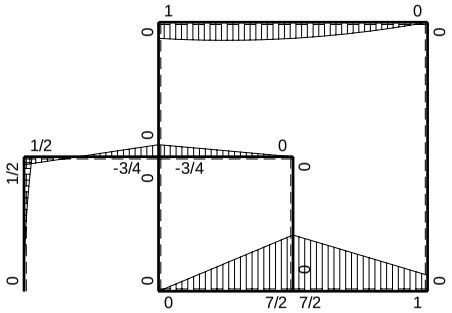
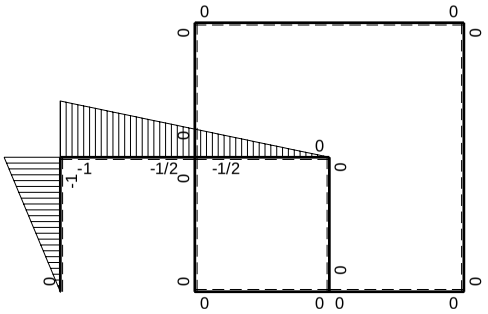
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982642 ESΣ1.xxxx.025

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	0	$-1/2qx^3$	0	x^2	$(-1/8+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	0	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-5/4Fx$	0	$-1/2Fb^2+3/2Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/24+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-5/4Fx$	0	$3/8Fb^2-1/4Fbx-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	-Fb/EJ	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+5/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$31/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-31/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ = -1/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +3/2\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +3/4\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +3/4\, b -5/24\, b)\, Fb^2\, 1/EJ = 1/24\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -1/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 1/24\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\ dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

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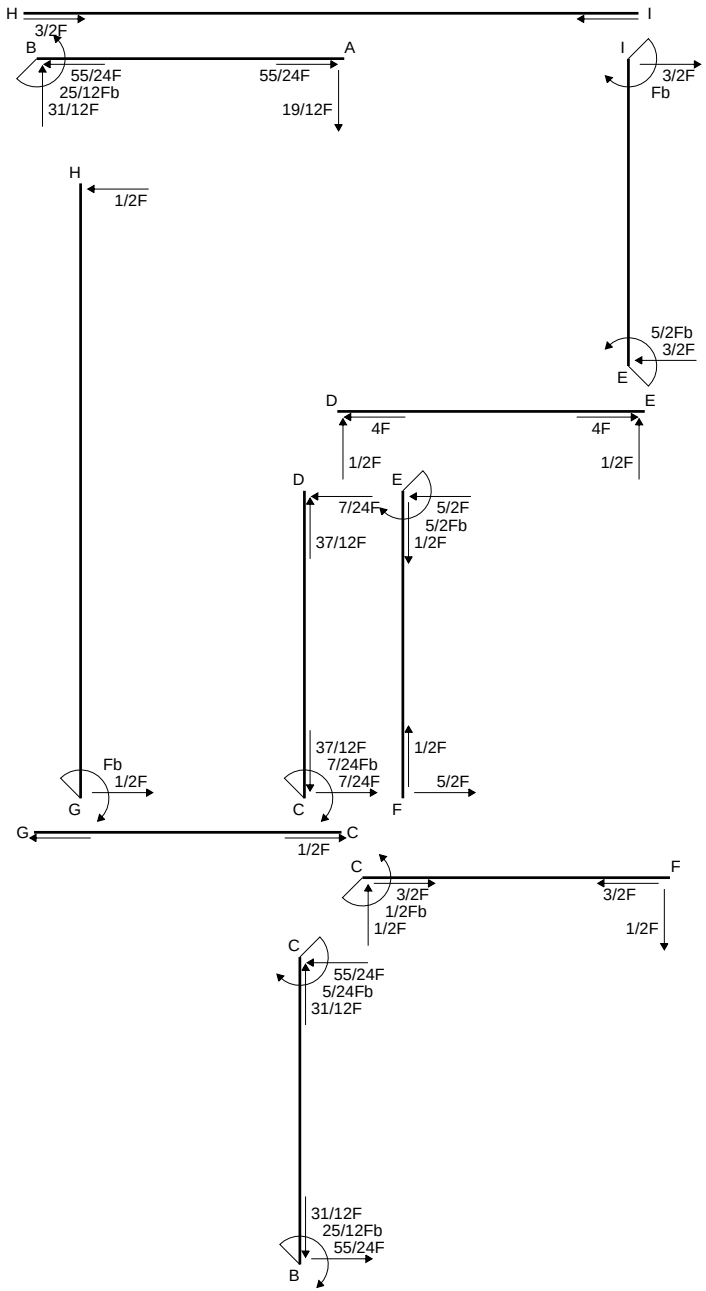
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19/12F
3/2F
Fb
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4F
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C
F
3/2F
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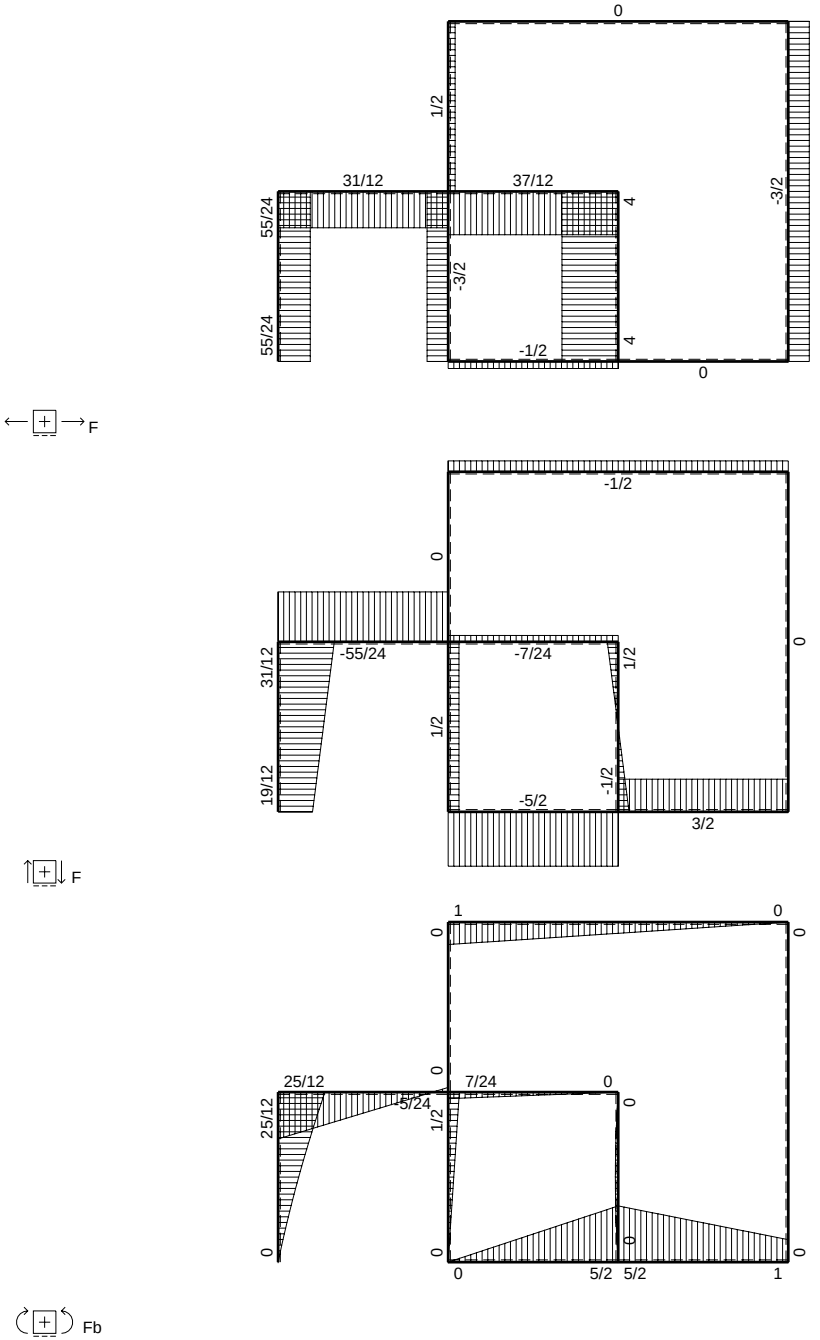
1/2F
3/2F
1/2F
C
55/24F
5/24Fb
31/12F
31/12F
25/12Fb
55/24F
B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-1/2
4
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F
-1/2
0
0
31/12
-55/24
-7/24
1/2
1/2
19/12
-1/2
-5/2
3/2
F
1
0
0
0
0
0
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0
0
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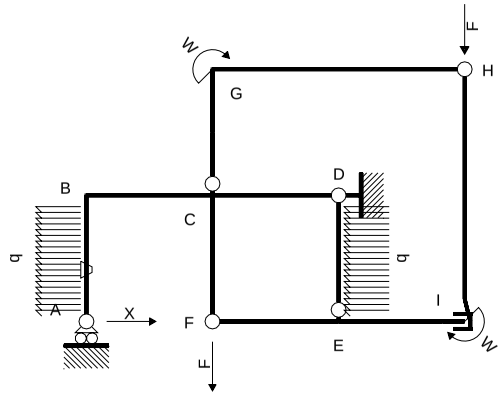
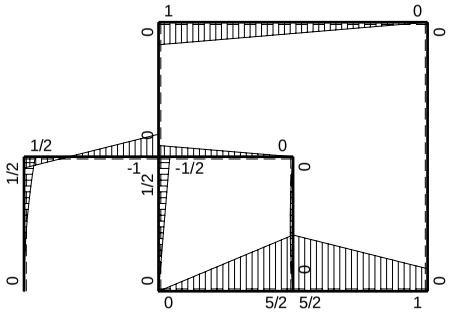
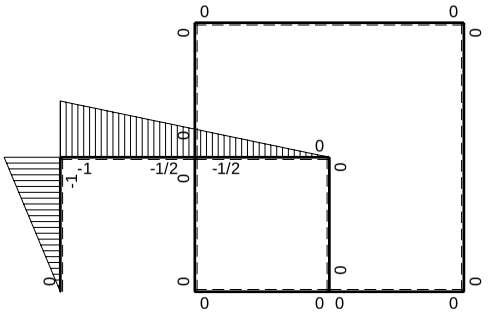
5/2
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 985260 ESΣ1.xxxx.035

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PROCEDIMENTO E RISULTATI 985260 ESΣ1.xxxx.035

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	-Fb/EJ	$-1/2qx^3$	Fxb/EJ	x^2	$(-1/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-3/2Fx$	0	$-1/2Fb^2+7/4Fbx-3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$Fb-3/2Fx$	0	$1/2Fb^2-1/4Fbx-3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	$5/2Fb-5/2Fx$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx$	0	0	0	0		
FC b	0	$1/2Fx$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+1/2Fx$	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$19/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-19/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (x/b)\, \theta\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [1/2\, x^2/b]_o^b\, \theta$

$= (-1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b)\, \theta\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [- x +1/2\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ + (- b +1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +7/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +7/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +7/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

ESΣ1.xxxx.035

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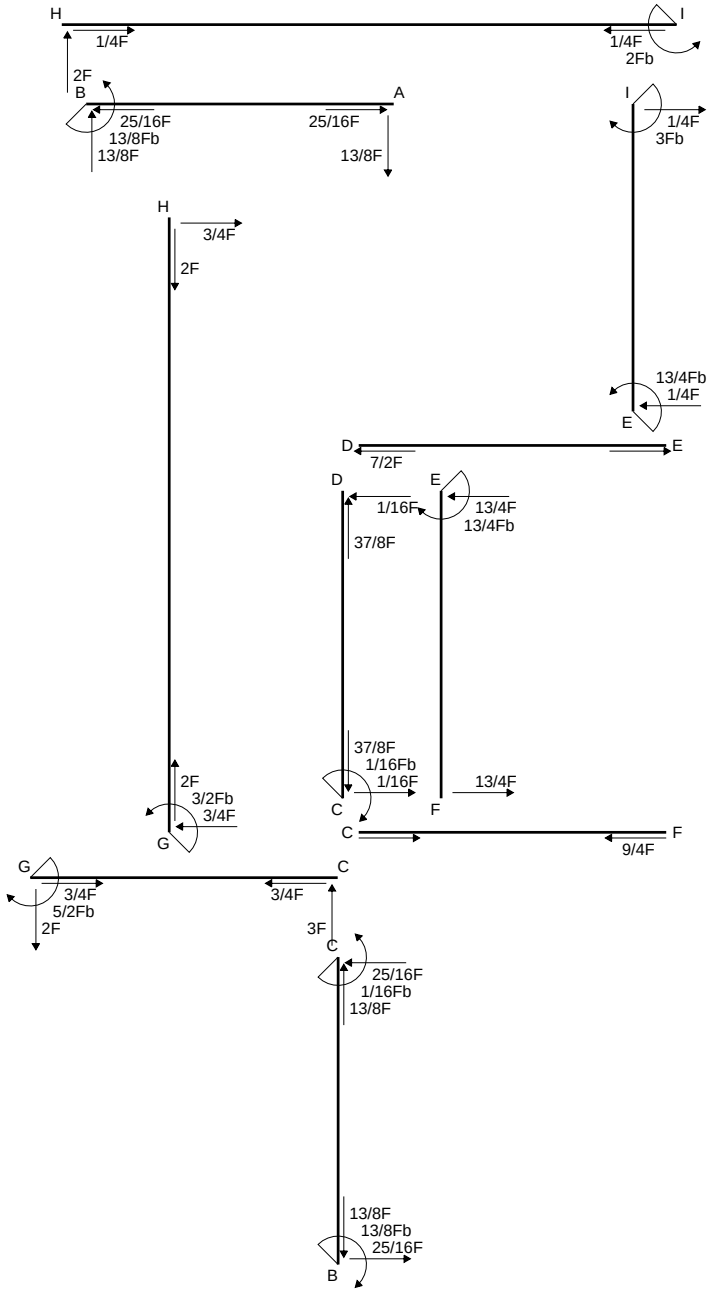
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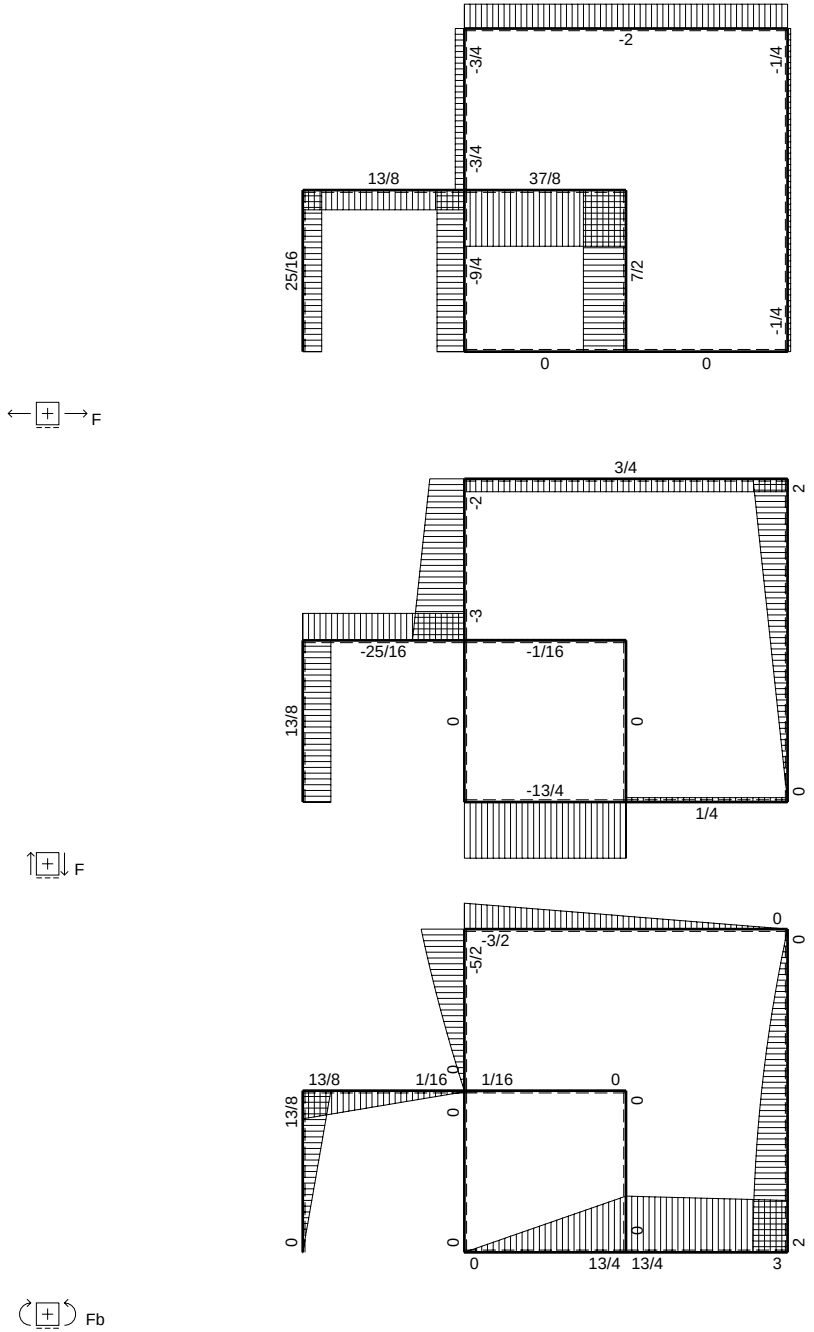
B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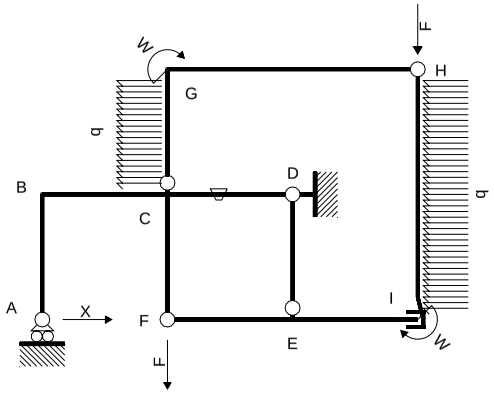
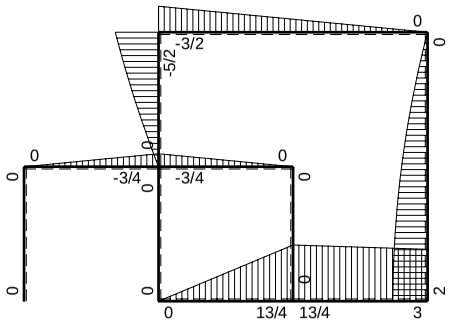
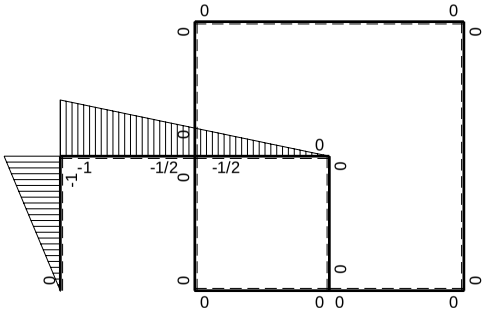


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 980874 ESΣ1.xxxx.046

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PROCEDIMENTO E RISULTATI 980874 ESΣ1.xxxx.046

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	$-Fb/EJ$	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$13/4Fb-13/4Fx$	0	0	0	0	0+0	0
FE b	0	$-13/4Fx$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-3Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$5/2Fb-2Fx-1/2qx^2$	0	0	0	0		
GH 2b	0	$-3/2Fb+3/4Fx$	0	0	0	0	0+0	0
HG 2b	0	$3/4Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/4Fx$	0	0	0	0	0+0	0
EI b	0	$-13/4Fb+1/4Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$13/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-13/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\ dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

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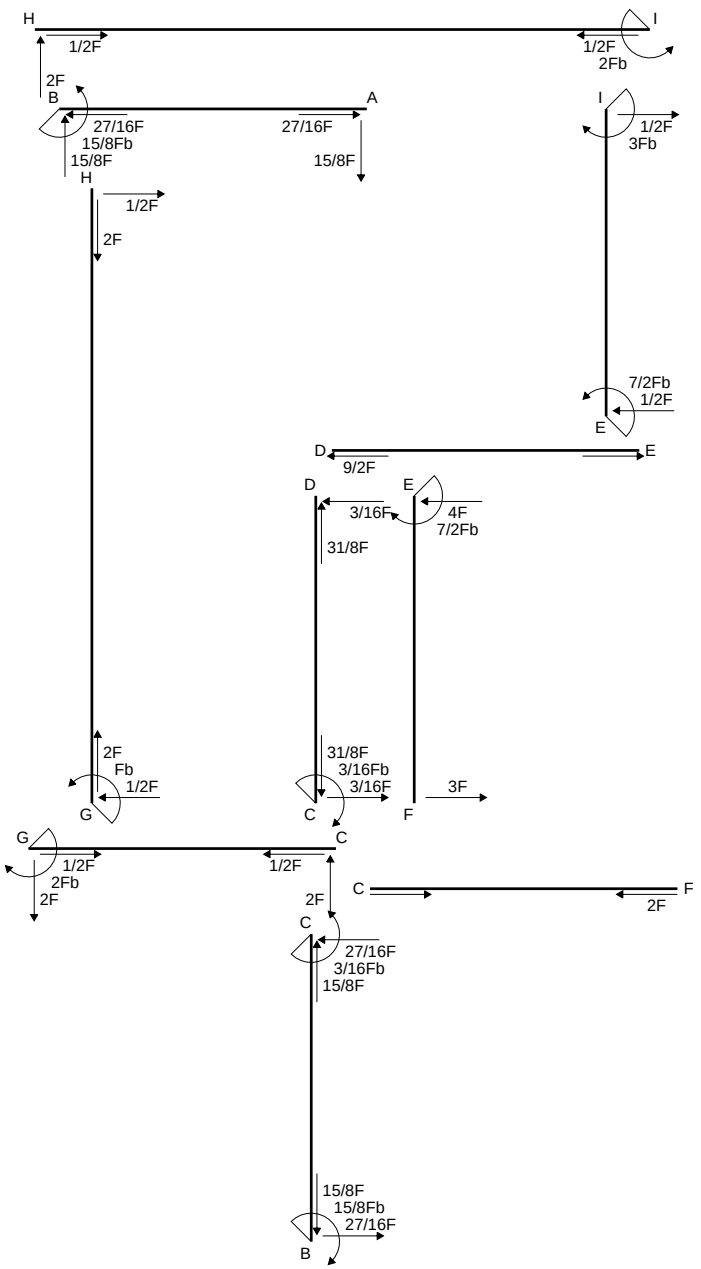

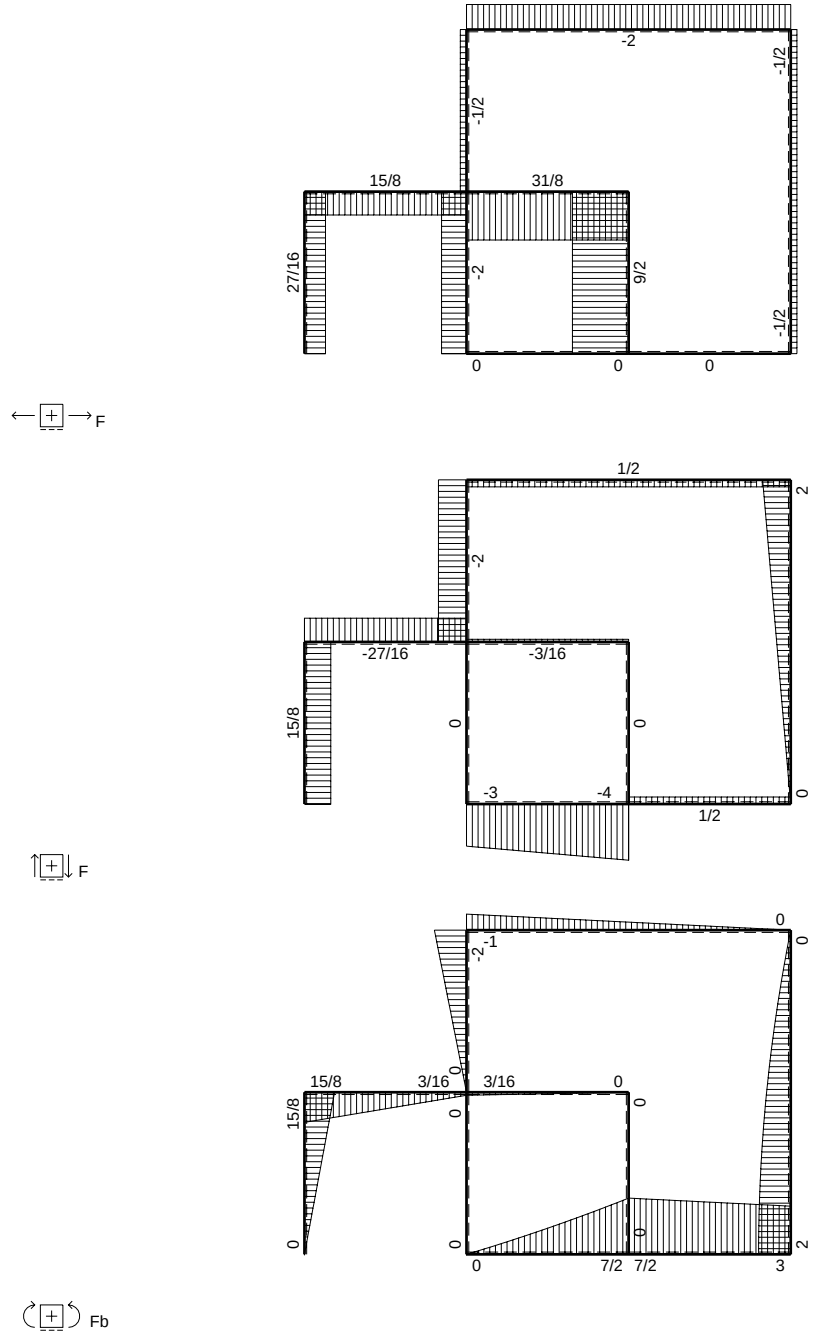

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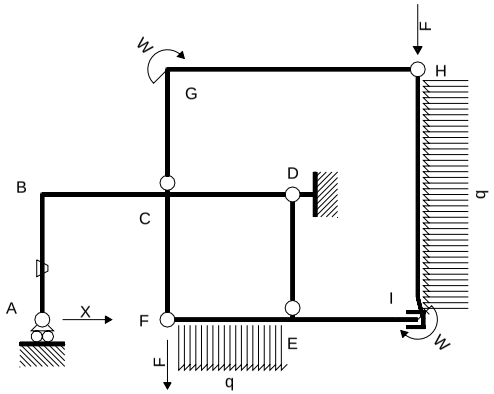
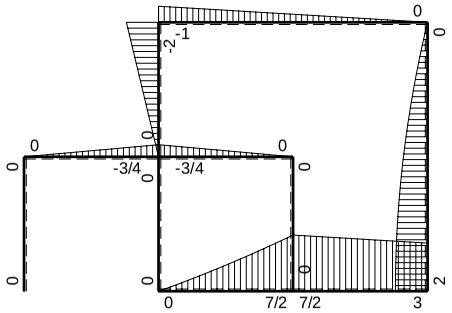
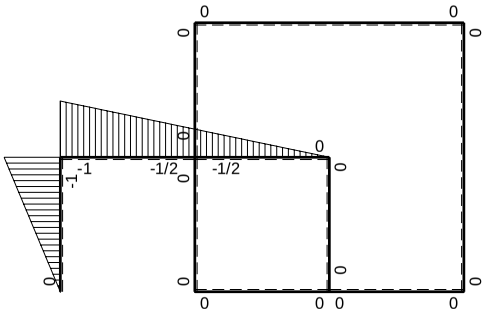


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983864 ESΣ1.xxxx.050

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PROCEDIMENTO E RISULTATI 983864 ESΣ1.xxxx.050

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/4Fx	0	3/4Fbx-3/8Fx^2	0	b^2-bx+1/4x^2	(1/4+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	3/8Fb^2-3/8Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	3/8Fb^2-3/4Fbx+3/8Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/8+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	3/4Fx	0	3/8Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-4Fx+1/2qx^2	0	0	0	0	0+0	0
FE b	0	-3Fx-1/2qx^2	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	2Fx-1/2qx^2	0	0	0	0	0+0	0
IH 2b	0	-2Fb+1/2qx^2	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						15/8Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-15/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ1.xxxx.050

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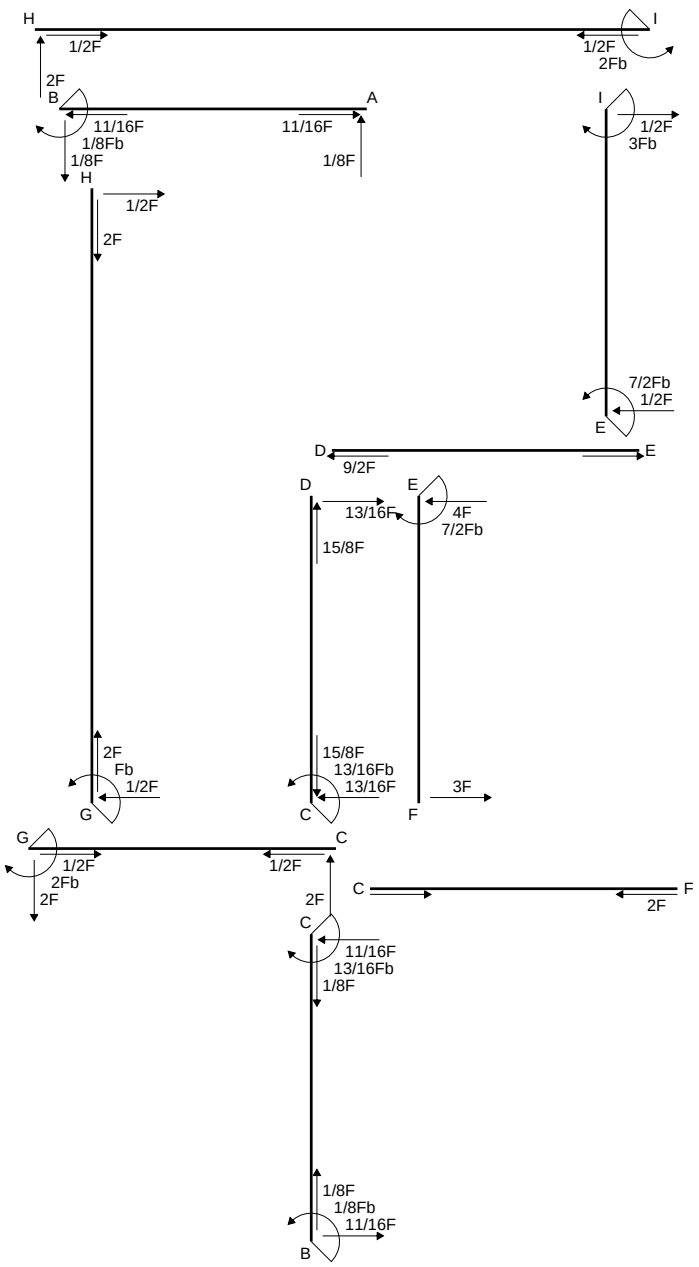

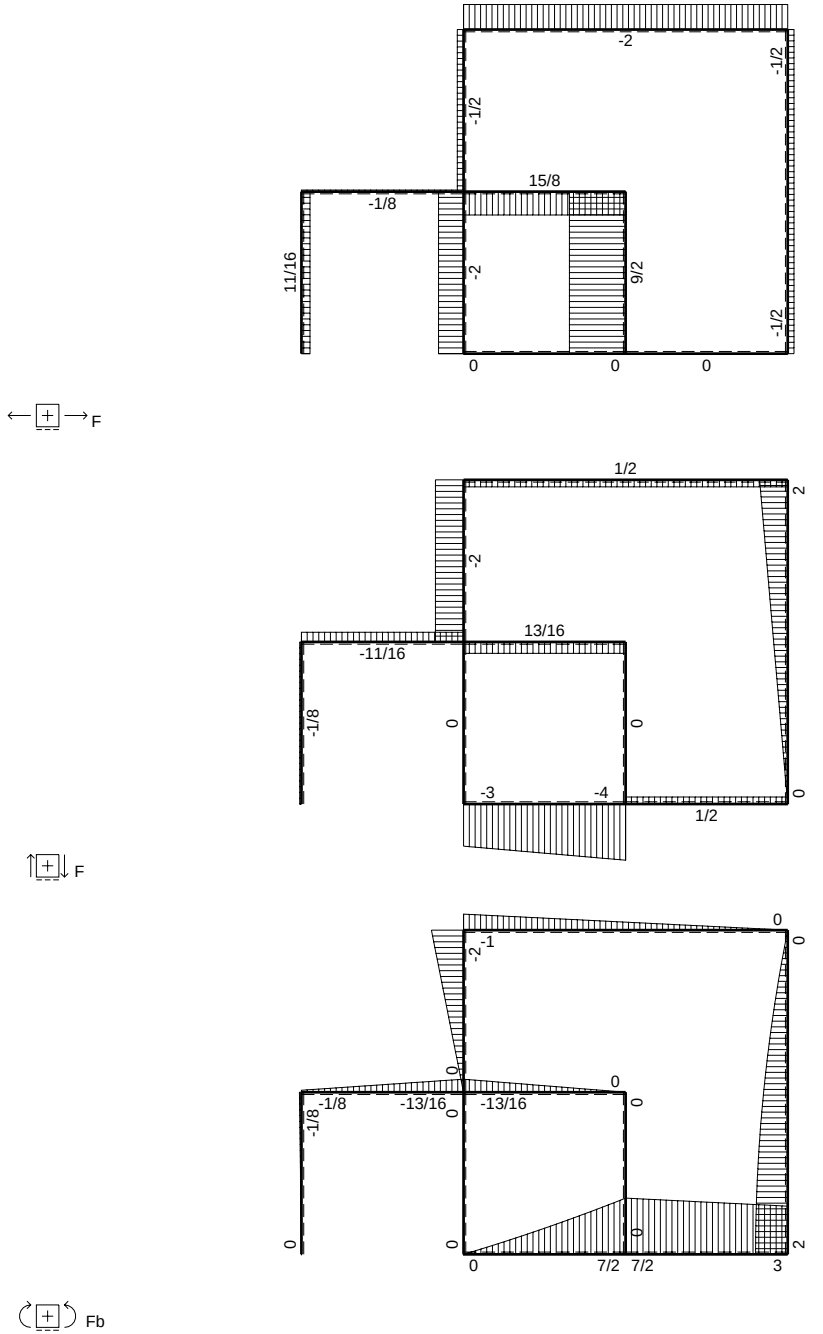

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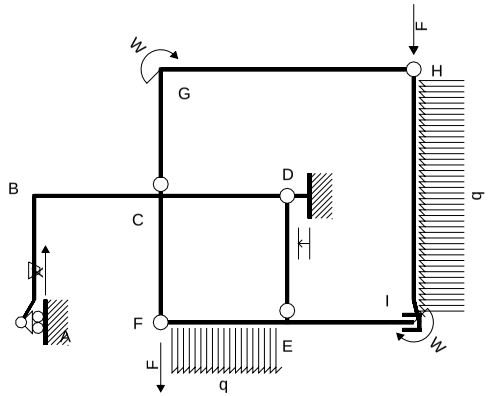
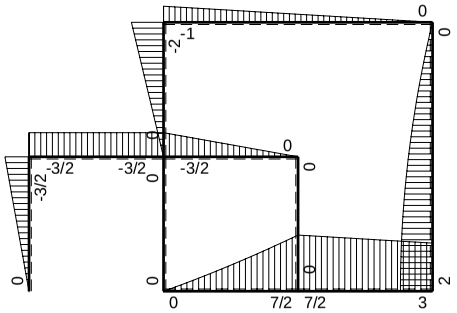
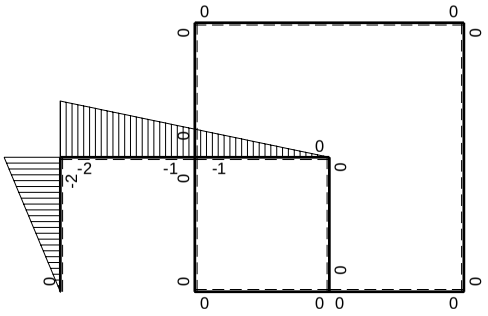


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981822 ESΣ1.xxxx.051

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Quadro contributi PLV per iperstatica $X=V_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-2x	-3/2Fx	-Fb/EJ	$3Fx^2$	2Fxb/EJ	$4x^2$	$(1+1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	2b-2x	3/2Fb-3/2Fx	Fb/EJ	$3Fb^2-6Fbx+3Fx^2$	$2Fb^2/EJ-2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	-2b+x	-3/2Fb	0	$3Fb^2-3/2Fbx$	0	$4b^2-4bx+x^2$	$(9/4+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	b+x	3/2Fb	0	$3/2Fb^2+3/2Fbx$	0	$b^2+2bx+x^2$		
CD b	-b+x	-3/2Fb+3/2Fx	0	$3/2Fb^2-3Fbx+3/2Fx^2$	0	$b^2-2bx+x^2$	$(1/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	x	3/2Fx	0	$3/2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-2Fb^3/EJ$	
	totali						$11/4Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_{AB}$						-11/16F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 981822 ESΣ1.xxxx.051
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$$L^{XX}_{AB} = \int_o^b \left(4\, x^2/b^2\right) b^2\, 1/EJ\, dx = \left[4/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$$

$$= \left(4/3\, b\right) b^2\, 1/EJ = 4/3\ b^3/EJ$$

$$L^{XX}_{BA} = \int_o^b \left(4 - 8\, x/b + 4\, x^2/b^2\right) b^2\, 1/EJ\, dx = \left[4\, x - 4\, x^2/b + 4/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$$

$$= \left(4\, b - 4\, b + 4/3\, b\right) b^2\, 1/EJ = 4/3\ b^3/EJ$$

$$L^{XX}_{BC} = \int_o^b \left(4 - 4\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[4\, x - 2\, x^2/b + 1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$$

$$= \left(4\, b - 2\, b + 1/3\, b\right) b^2\, 1/EJ = 7/3\ b^3/EJ$$

$$L^{XX}_{CB} = \int_o^b \left(1 + 2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[x + x^2/b + 1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$$

$$= \left(b + b + 1/3\, b\right) b^2\, 1/EJ = 7/3\ b^3/EJ$$

$$L^{XX}_{CD} = \int_o^b \left(1 - 2\, x/b + x^2/b^2\right) b^2\, 1/EJ\, dx = \left[x - x^2/b + 1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$$

$$= \left(b - b + 1/3\, b\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{XX}_{DC} = \int_o^b \left(x^2/b^2\right) b^2\, 1/EJ\, dx = \left[1/3\, x^3/b^2\right]_o^b b^2\, 1/EJ$$

$$= \left(1/3\, b\right) b^2\, 1/EJ = 1/3\ b^3/EJ$$

$$L^{Xo}_{AB} = \int_o^b \left(3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + \int_o^b \left(2\, x/b\right) \theta\ dx = \left[x^3/b^2\right]_o^b Fb^2\, 1/EJ + \left[x^2/b\right]_o^b \theta$$

$$= \left(b\right) Fb^2\, 1/EJ + \left(b\right) \theta = 2\ Fb^3/EJ$$

$$L^{Xo}_{BA} = \int_o^b \left(3 - 6\, x/b + 3\, x^2/b^2\right) Fb^2\, 1/EJ\, dx + \int_o^b \left(-2 + 2\, x/b\right) \theta\ dx$$

$$= \left[3\, x - 3\, x^2/b + x^3/b^2\right]_o^b Fb^2\, 1/EJ + \left[-2\, x + x^2/b\right]_o^b \theta$$

$$= \left(3\, b - 3\, b + b\right) Fb^2\, 1/EJ + \left(-2\, b + b\right) \theta = 2\ Fb^3/EJ$$

$$L^{Xo}_{BC} = \int_o^b \left(3 - 3/2\, x/b\right) Fb^2\, 1/EJ\, dx = \left[3\, x - 3/4\, x^2/b\right]_o^b Fb^2\, 1/EJ$$

$$= \left(3\, b - 3/4\, b\right) Fb^2\, 1/EJ = 9/4\ Fb^3/EJ$$

$$L^{Xo}_{CB} = \int_o^b \left(3/2 + 3/2\, x/b\right) Fb^2\, 1/EJ\, dx = \left[3/2\, x + 3/4\, x^2/b\right]_o^b Fb^2\, 1/EJ$$

$$= \left(3/2\, b + 3/4\, b\right) Fb^2\, 1/EJ = 9/4\ Fb^3/EJ$$

$$L^{Xo}_{CD} = \int_o^b \left(3/2 - 3\, x/b + 3/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[3/2\, x - 3/2\, x^2/b + 1/2\, x^3/b^2\right]_o^b Fb^2\, 1/EJ$$

$$= \left(3/2\, b - 3/2\, b + 1/2\, b\right) Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$$

$$L^{Xo}_{DC} = \int_o^b \left(3/2\, x^2/b^2\right) Fb^2\, 1/EJ\, dx = \left[1/2\, x^3/b^2\right]_o^b Fb^2\, 1/EJ$$

$$= \left(1/2\, b\right) Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$$

ESΣ1.xxxx.051

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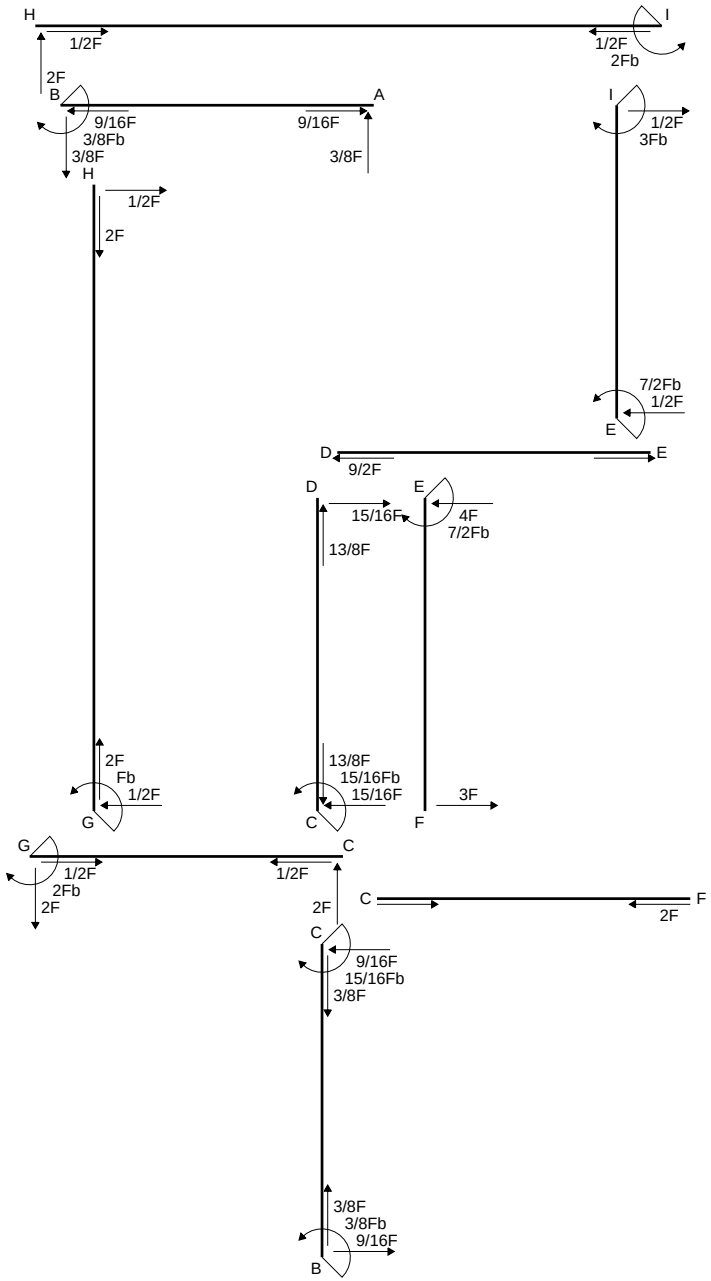
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3Fb
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E
D
E
9/2F
D
E
15/16F
4F
7/2Fb
13/8F
2F
Fb
1/2F
13/8F
15/16Fb
15/16F
3F
G
C
F
G
C
1/2F
2Fb
2F
1/2F
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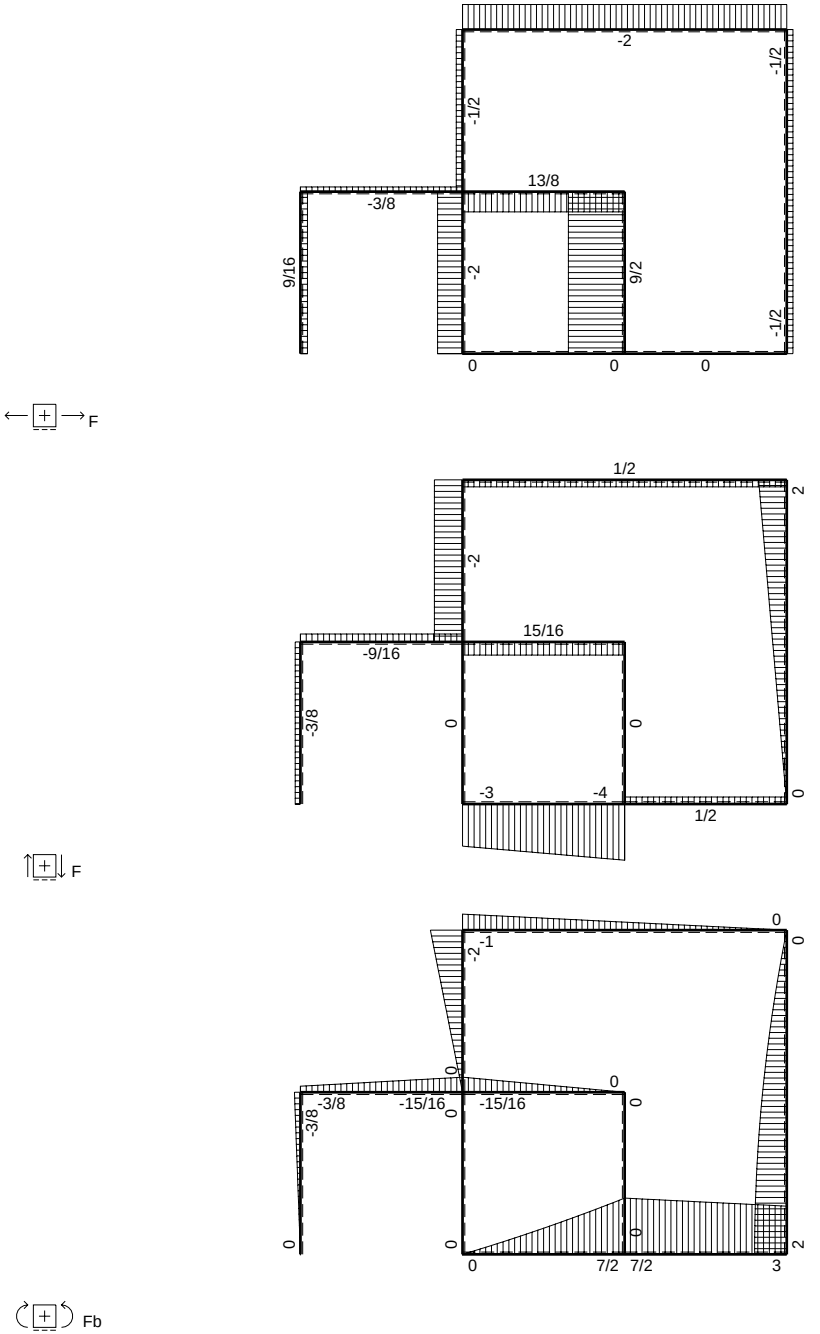
9/16F
B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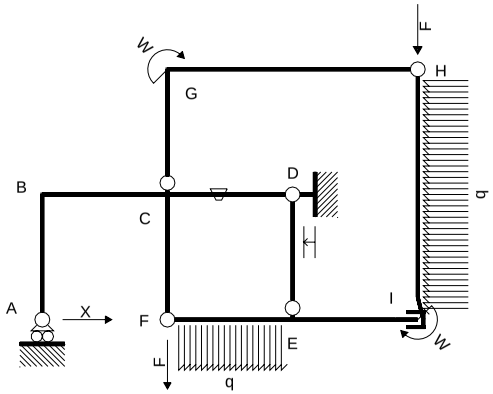
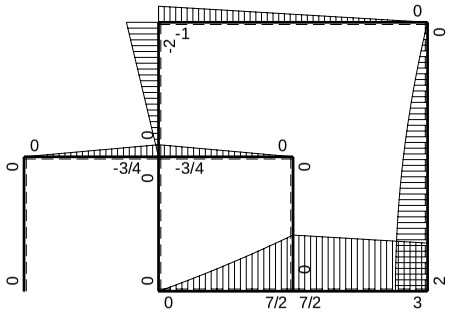
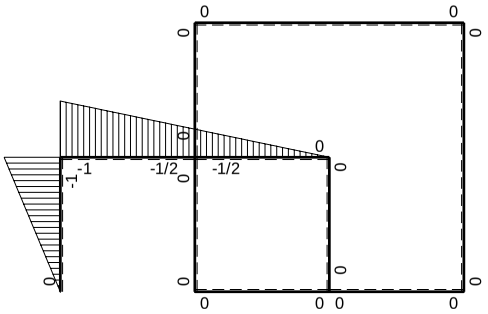
0
7/2
7/2
3
2
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 910738 ESΣ1.xxxx.053

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-3/4Fx$	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$3/4Fb-3/4Fx$	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	$-Fb/EJ$	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-3/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						3/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 910738 ESΣ1.xxxx.053

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\ dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

ESΣ1.xxxx.053

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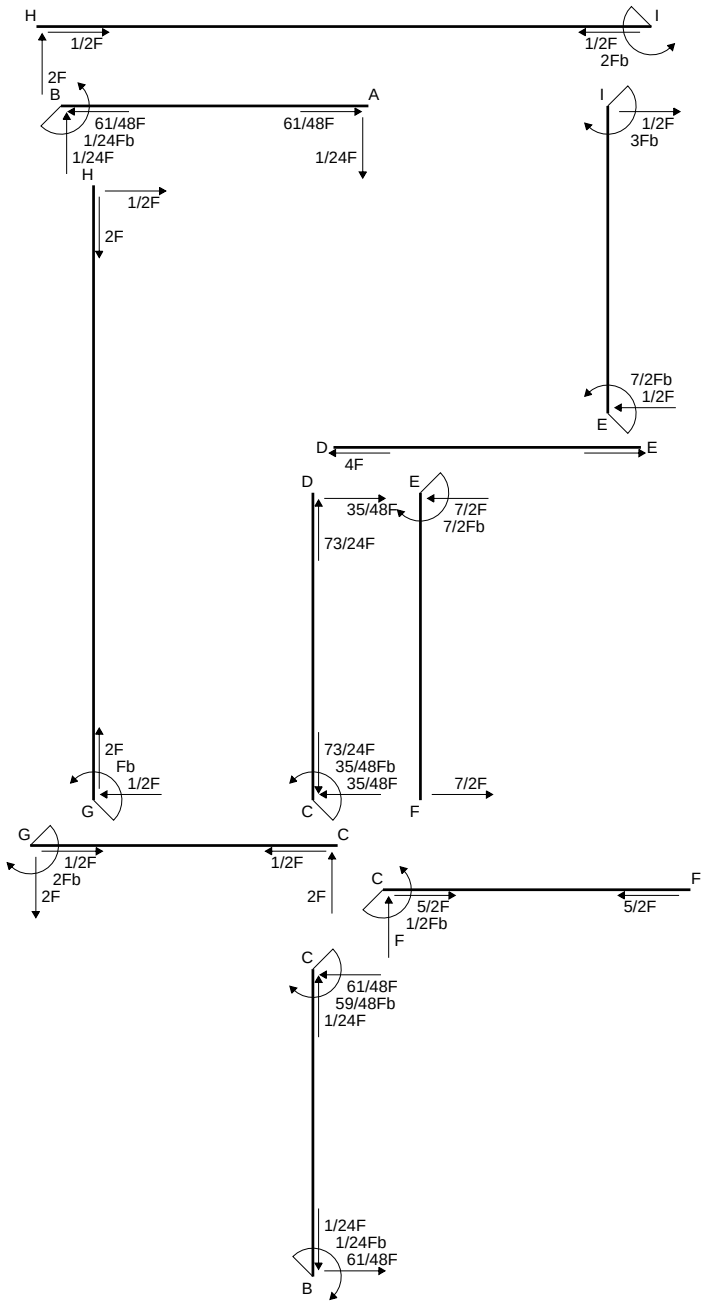
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C
F
5/2F
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5/2F
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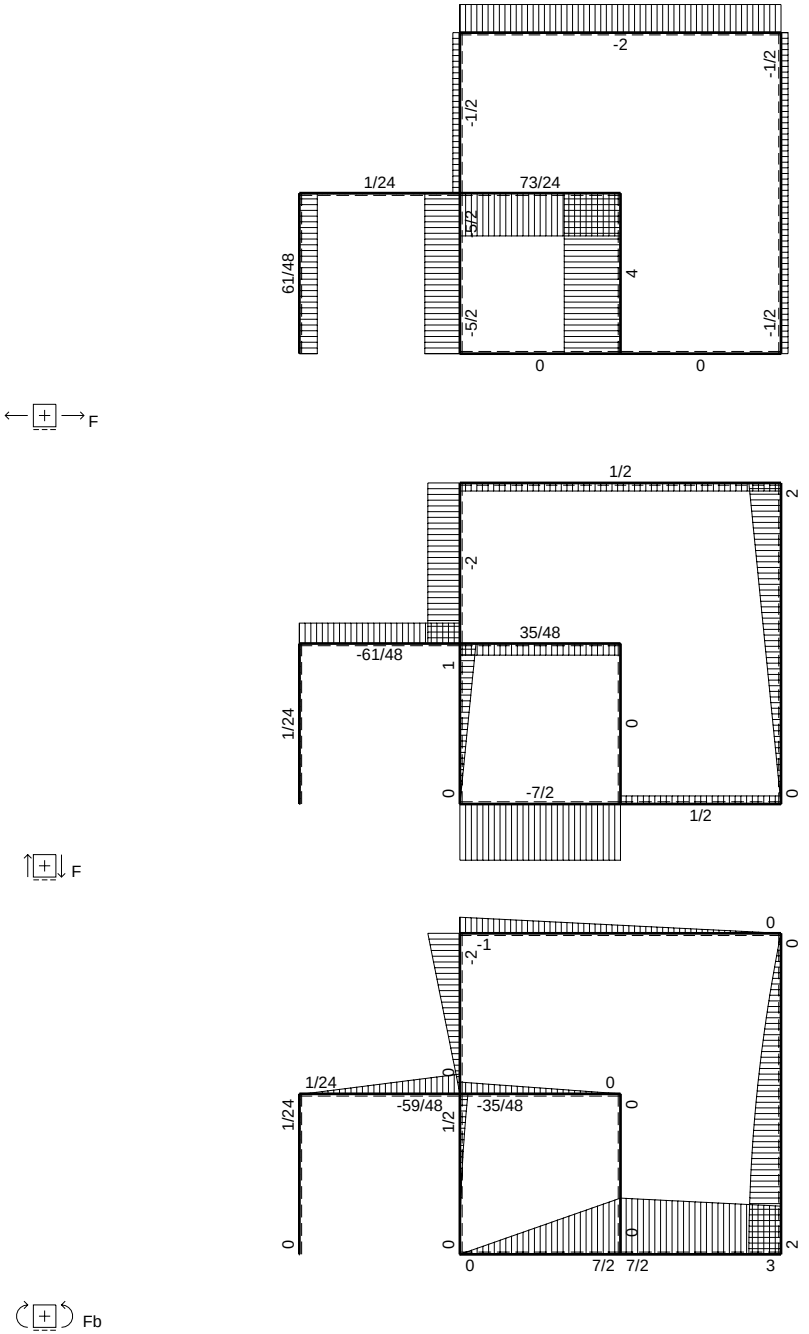
59/48Fb
1/24F
1/24F
1/24Fb
61/48F
B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-1/2
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0
F
1/2
2
-2
35/48
-61/48
1
1/24
0
0
-7/2
0
1/2
Fb
0
0
-2
-1
1/24
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-35/48
1/2
1/24
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0
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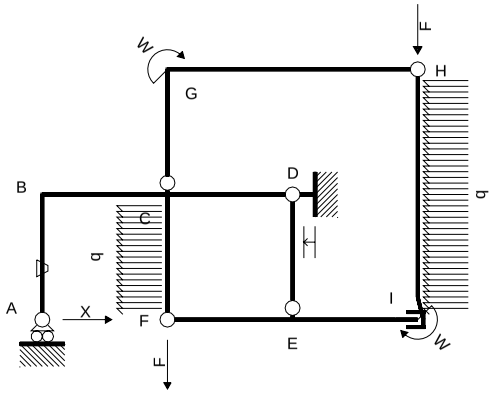
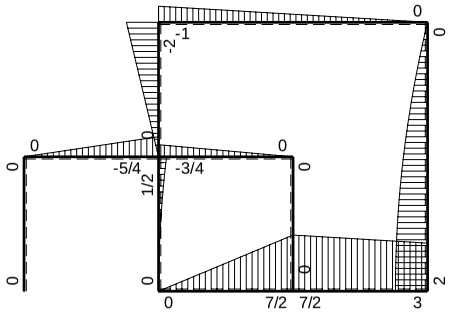
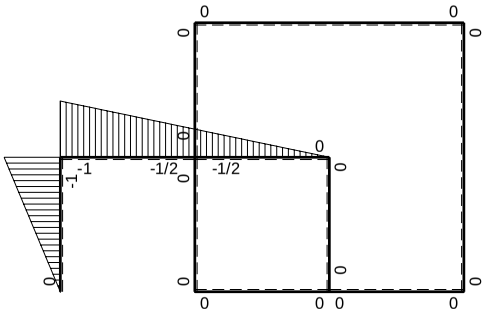
7/2
7/2
3
2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982777 ESΣ1.xxxx.066

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PROCEDIMENTO E RISULTATI 982777 ESΣ1.xxxx.066

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-5/4Fx$	0	$5/4Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(5/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$5/4Fb-5/4Fx$	0	$5/8Fb^2-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ1.xxxx.066

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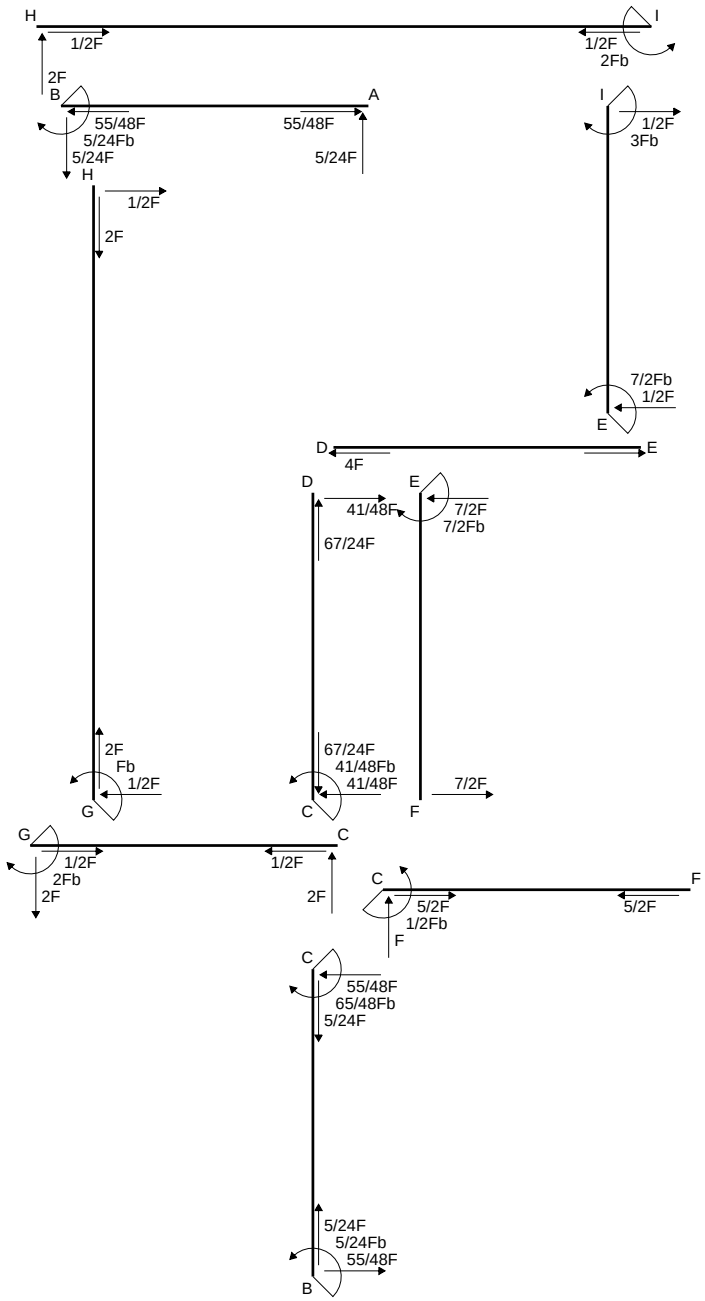
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1/2F
2Fb
2F
B
A
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55/48F
5/24Fb
5/24F
5/24F
I
1/2F
3Fb
H
1/2F
2F
7/2Fb
1/2F
E
D
E
4F
D
E
41/48F
7/2F
7/2Fb
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Fb
1/2F
67/24F
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41/48F
7/2F
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2F
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5/2F
1/2Fb
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5/24F
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55/48F
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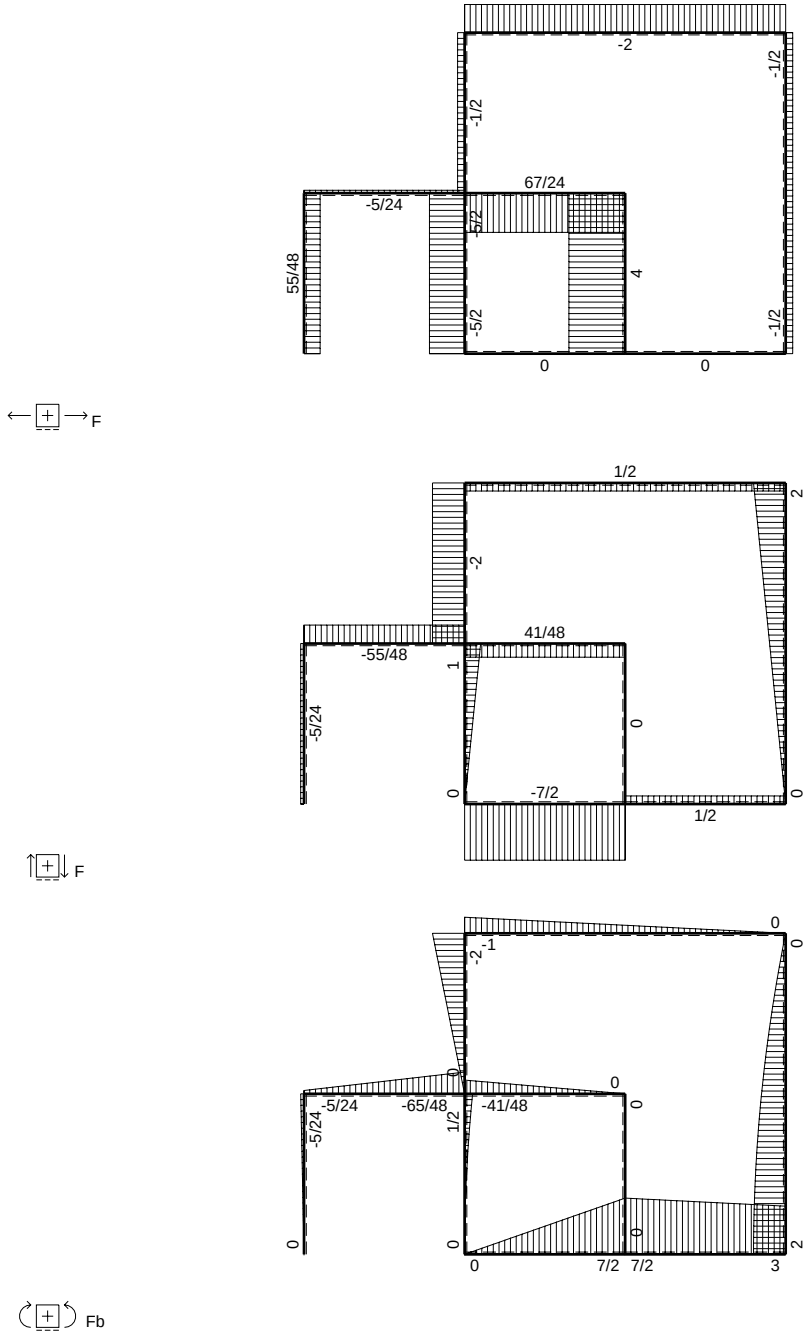
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-1/2
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-5/24
-5/2
55/48
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-5/2
-1/2
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0
F
1/2
2
-2
41/48
-55/48
1
-5/24
0
0
-7/2
0
1/2
Fb
0
0
-1
-2
0
0
-5/24
-5/24
-65/48
1/2
-41/48
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0
0
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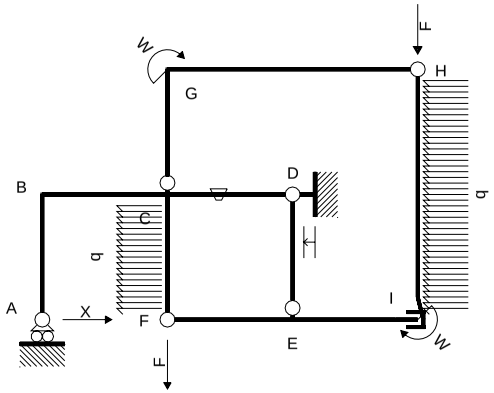
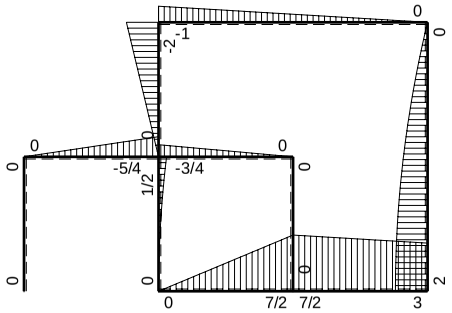
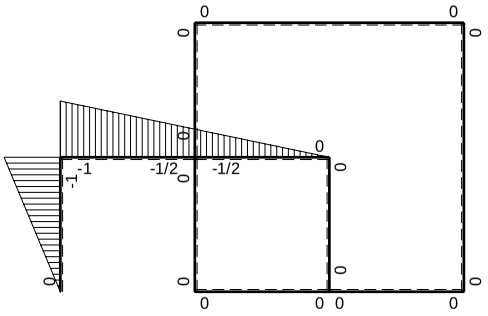
7/2
7/2
3
2



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$-5/4Fx$	0	$5/4Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(5/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$5/4Fb-5/4Fx$	0	$5/8Fb^2-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-3/4Fb+3/4Fx$	$-Fb/EJ$	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$3/4Fx$	Fb/EJ	$3/8Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-7/2Fx$	0	0	0	0	0+0	0
FE b	0	$-7/2Fx$	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	$-2Fx$	0	0	0	0	0+0	0
GC b	0	$2Fb-2Fx$	0	0	0	0		
GH 2b	0	$-Fb+1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$1/2Fx$	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	$3Fb+1/2Fx$	0	0	0	0	0+0	0
EI b	0	$-7/2Fb+1/2Fx$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-5/24Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						5/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

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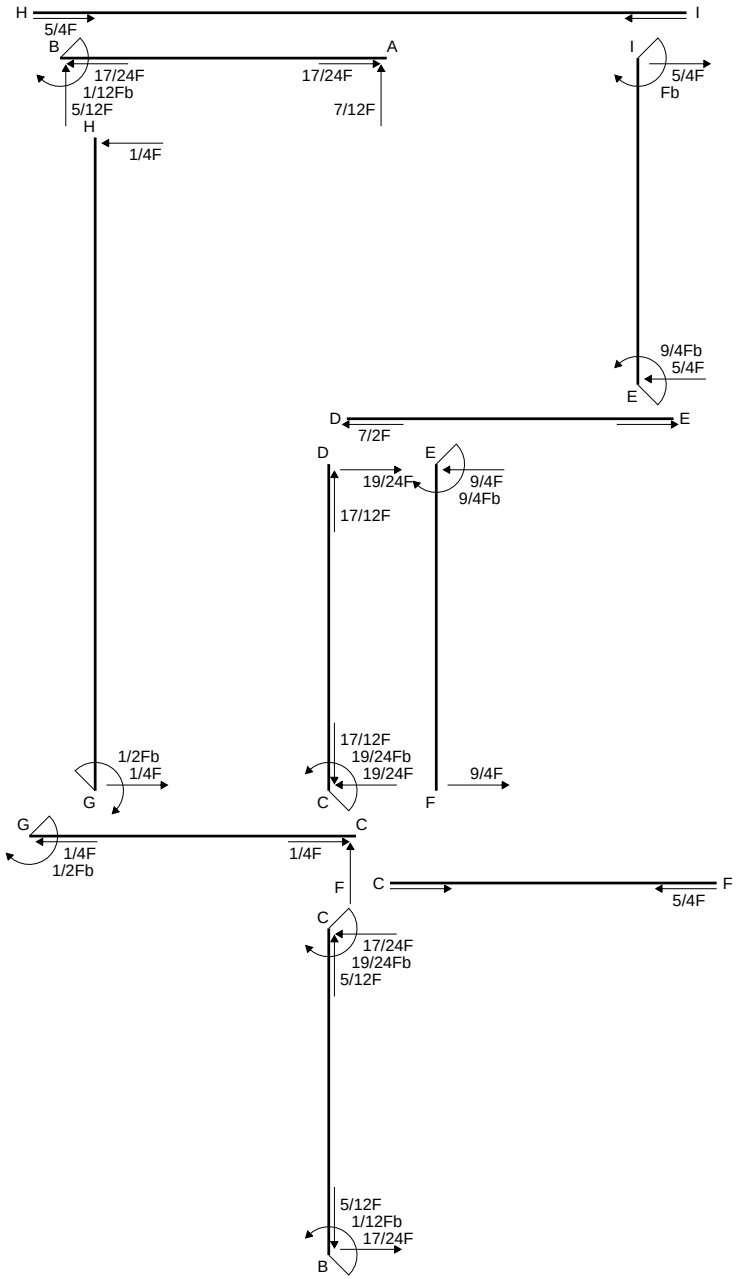
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1/2Fb
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17/24F
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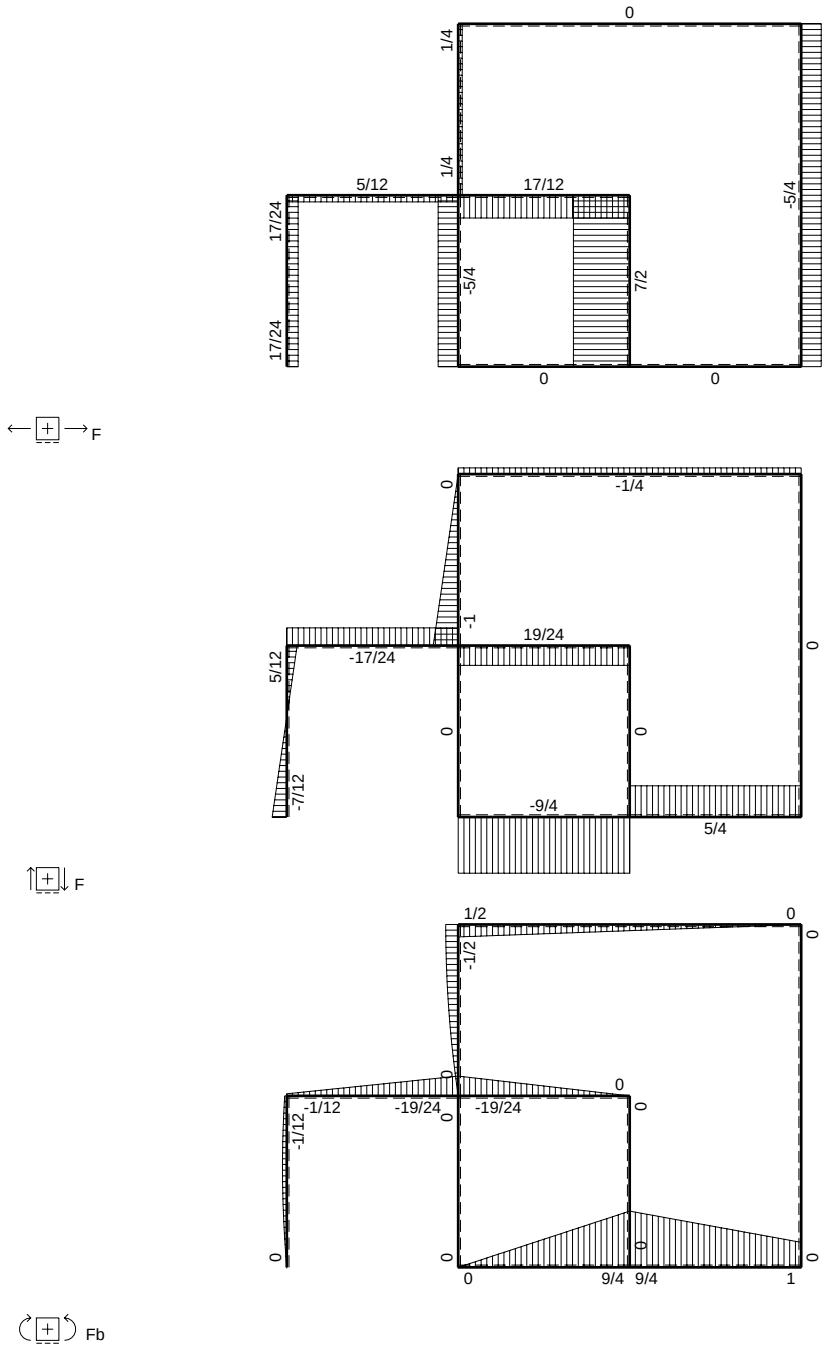
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17/24
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0
-1/4
-1
19/24
5/12
-17/24
0
0
0
-7/12
-9/4
5/4
F
1/2
0
0
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0
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9/4
9/4
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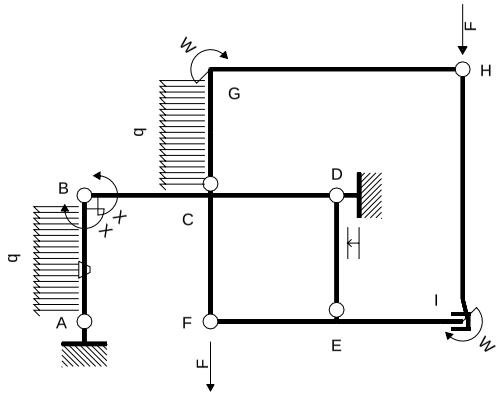
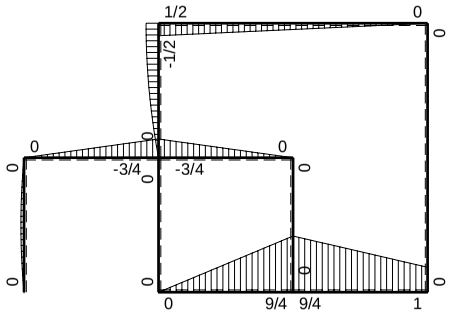
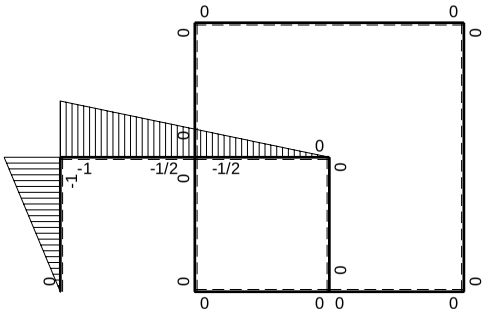
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982219 ESΣ1.xxxx.074

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	$-1/2Fx+1/2qx^2$	-Fb/EJ	$1/2Fx^2/b-1/2qx^3/b$	Fx/EJ	x^2/b^2	$(1/24+1/2)Fb^2/EJ$	1/3Xb/EJ
BA b	1-x/b	$1/2Fx-1/2qx^2$	Fb/EJ	$1/2Fx-Fx^2/b+1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	-3/4Fx	0	$3/4Fx-3/8Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(1/4+0)Fb^2/EJ$	7/12Xb/EJ
CB b	$1/2+1/2x/b$	3/4Fb-3/4Fx	0	$3/8Fb-3/8Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	$-3/4Fb+3/4Fx$	0	$3/8Fb-3/4Fx+3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+0)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	3/4Fx	0	$3/8Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^2/EJ$	
	totali						$-1/12Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						1/12Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 13/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/4\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -3/8\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b\ )\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

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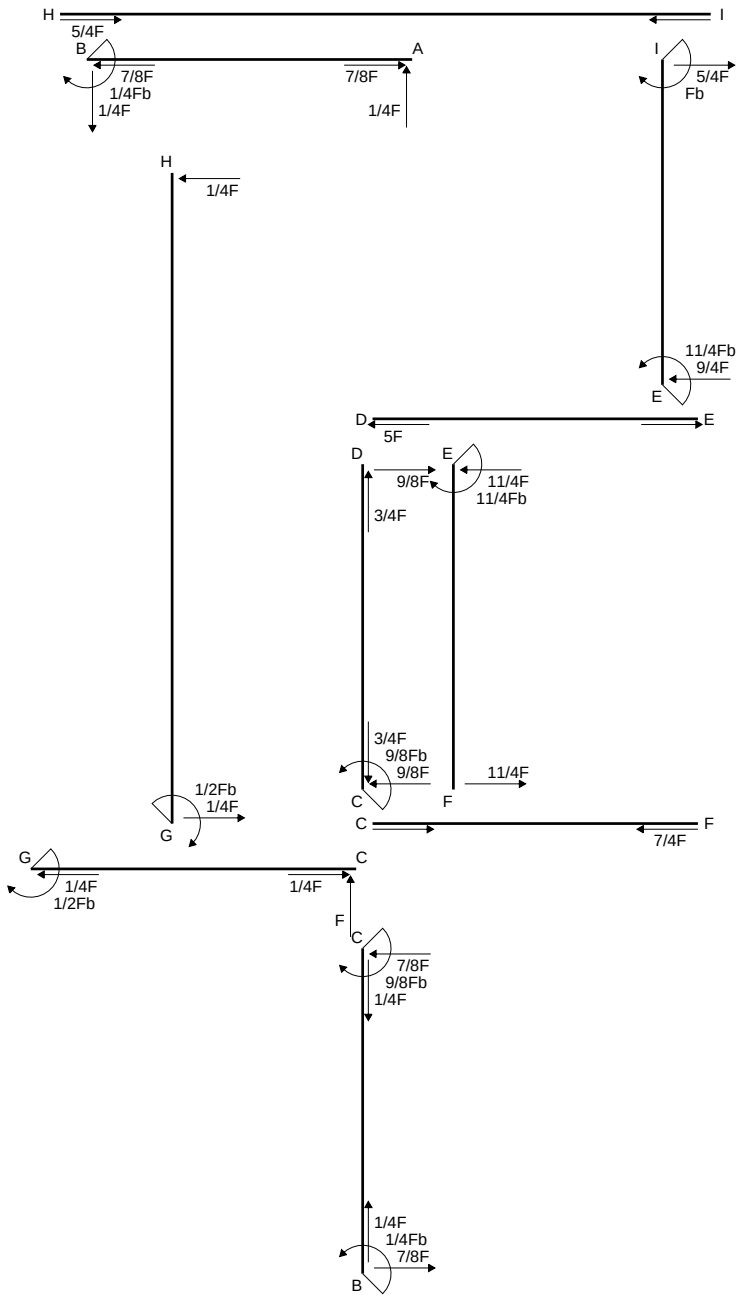
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7/8F
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7/8F
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11/4Fb
9/4F
E
D
E
5F
D
E
9/8F
11/4F
11/4Fb
3/4F
3/4F
9/8Fb
9/8F
11/4F
C
F
1/2Fb
1/4F
G
C
F
7/4F
G
C
1/4F
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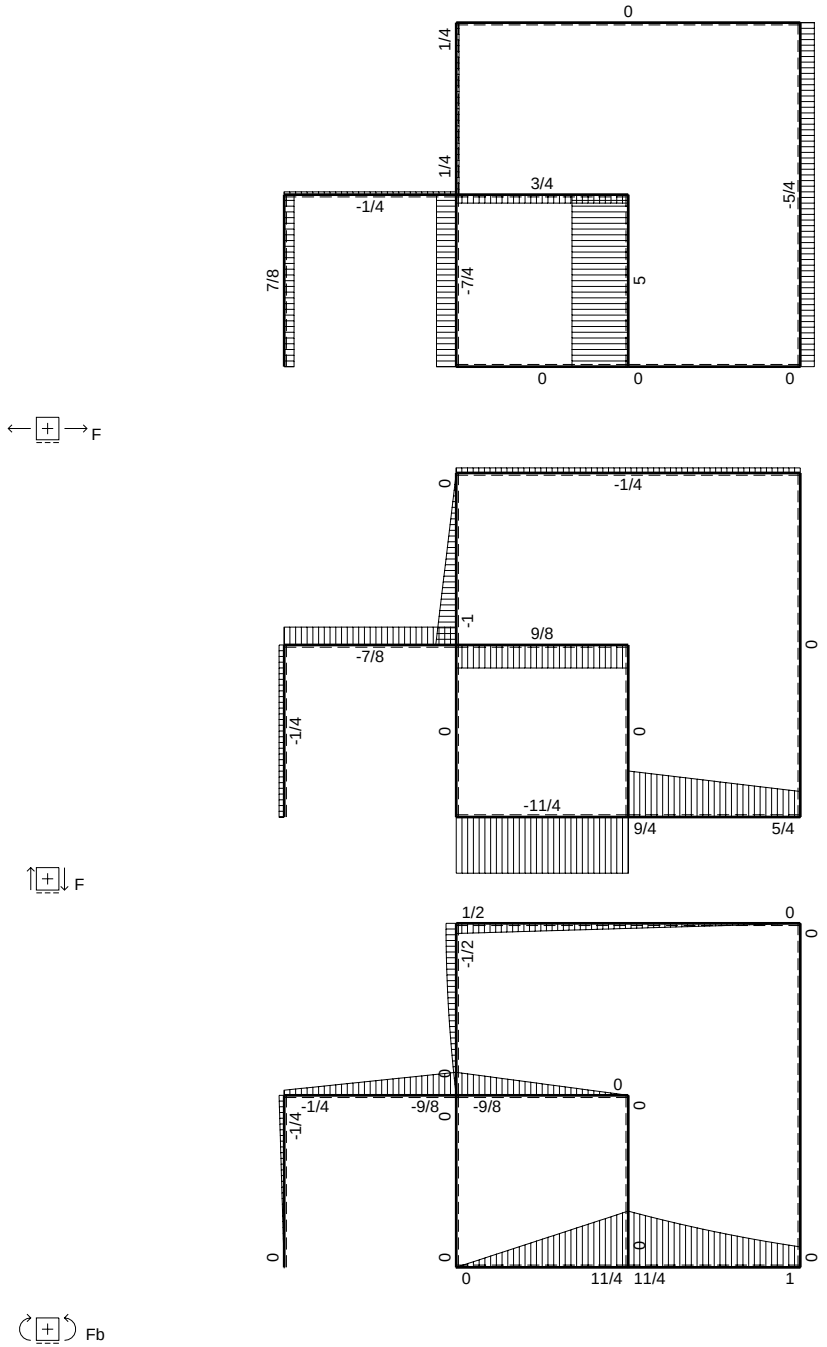
1/4F
1/4Fb
7/8F
B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0
-1/4
-1
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-7/8
-1/4
-11/4
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11/4
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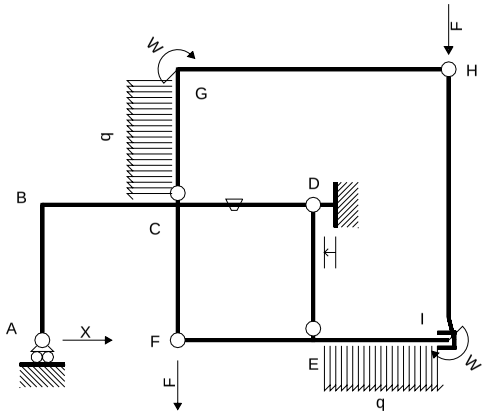
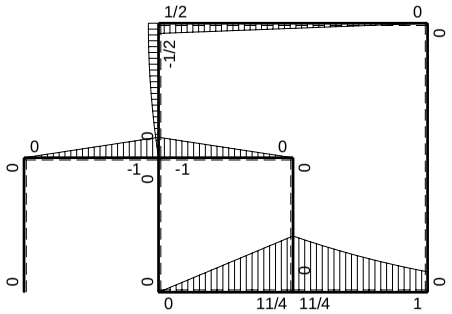
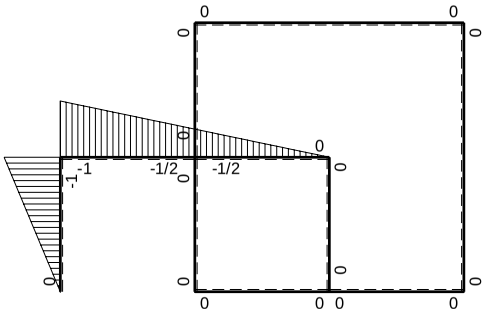


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982885 ESΣ1.xxxx.093

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-Fb+Fx$	$-Fb/EJ$	$1/2Fb^2-Fbx+1/2Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/6+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	Fx	Fb/EJ	$1/2Fx^2$	$1/2Fxb/EJ$	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	11/4Fb-11/4Fx	0	0	0	0	0+0	0
FE b	0	-11/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$-1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						1/4F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 982885 ESΣ1.xxxx.093

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/6\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

ESΣ1.xxxx.093

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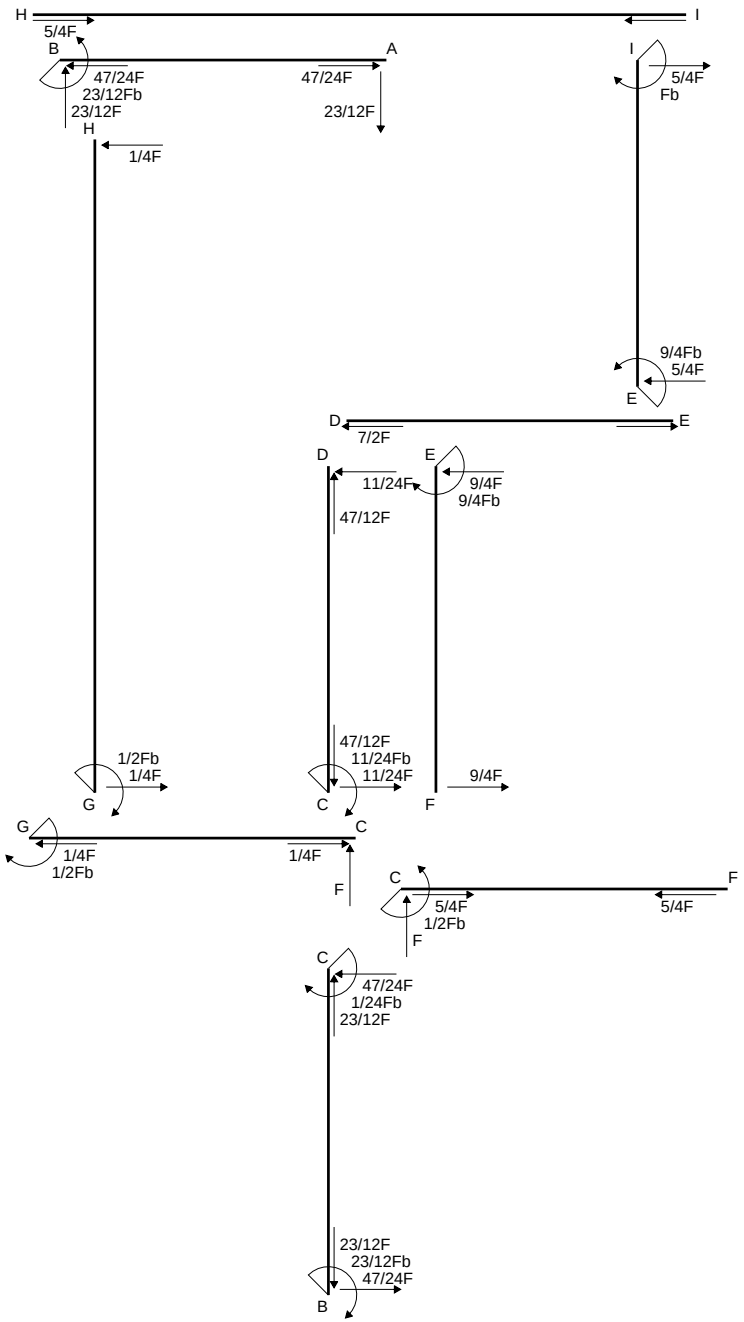
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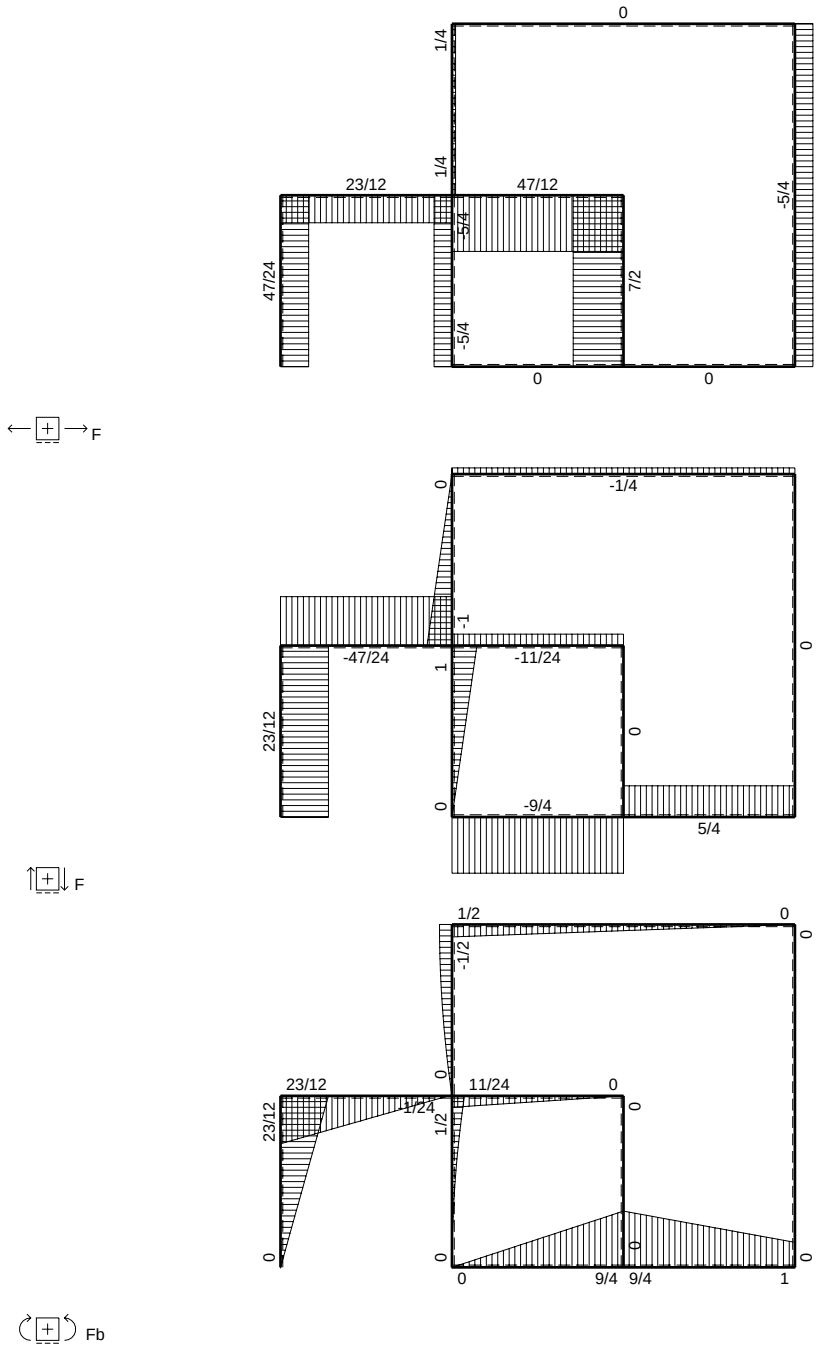
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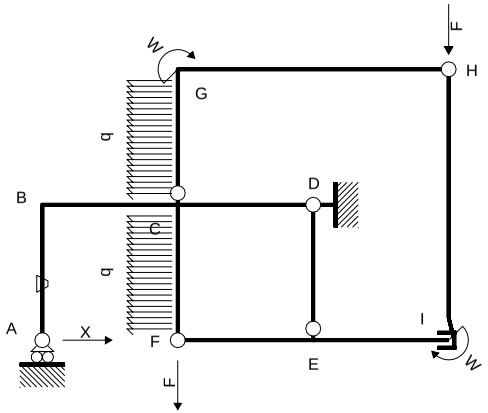
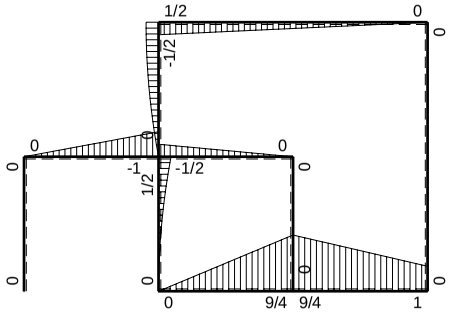
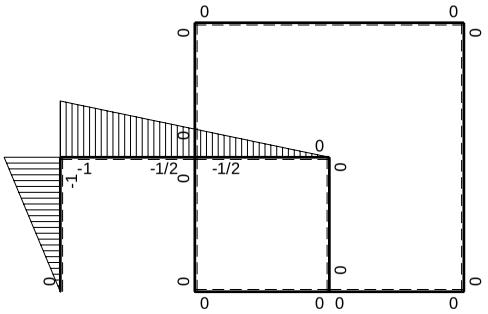


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982732 ESΣ1.xxxx.102

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PROCEDIMENTO E RISULTATI 982732 ESΣ1.xxxx.102

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	-Fx	0	$Fbx-1/2Fx^2$	0	$b^2-bx+1/4x^2$	$(1/3+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	Fb-Fx	0	$1/2Fb^2-1/2Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-1/2Fb+1/2Fx$	0	$1/4Fb^2-1/2Fbx+1/4Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/12+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	$1/2Fx$	0	$1/4Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	$1/2qx^2$	0	0	0	0	0+0	0
CF b	0	$-1/2Fb+Fx-1/2qx^2$	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$23/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-23/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ = 1/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/12\, b)\, Fb^2\, 1/EJ = 1/12\ Fb^3/EJ$

ESΣ1.xxxx.102

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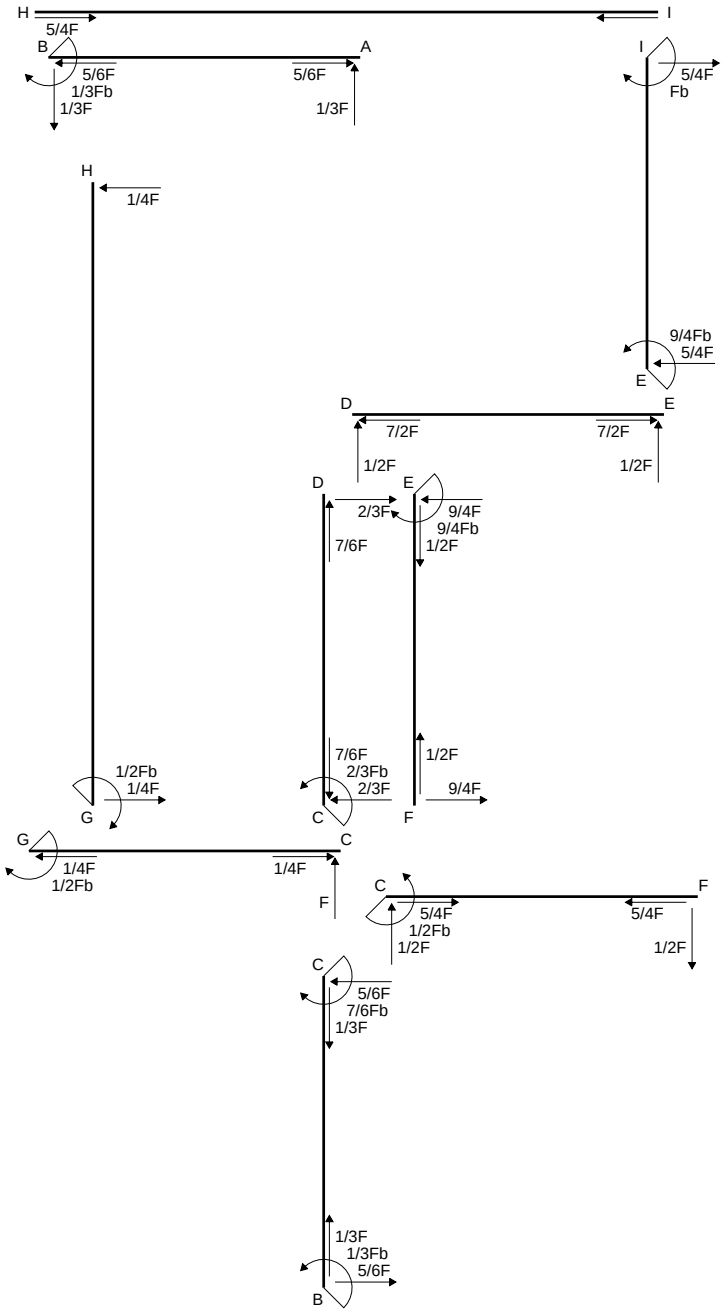
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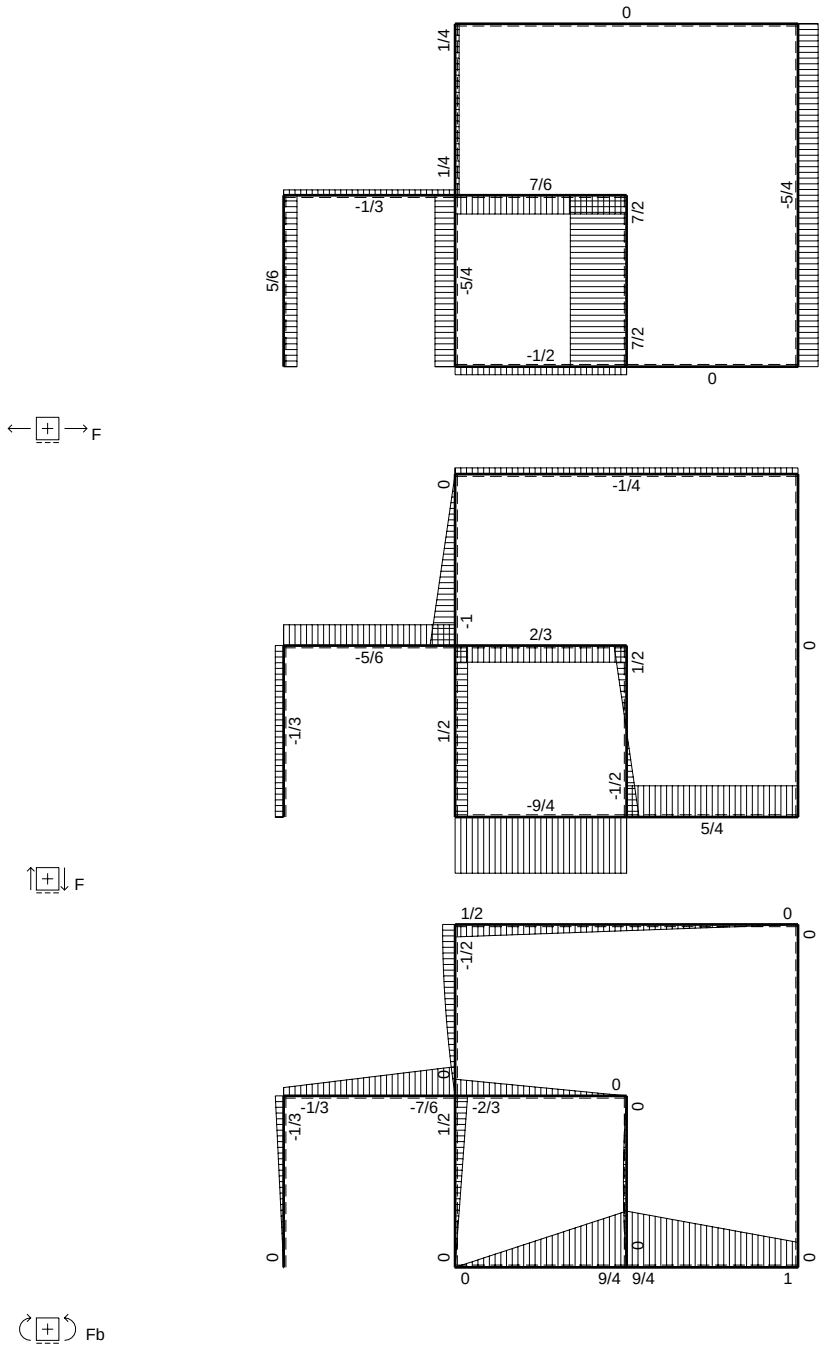
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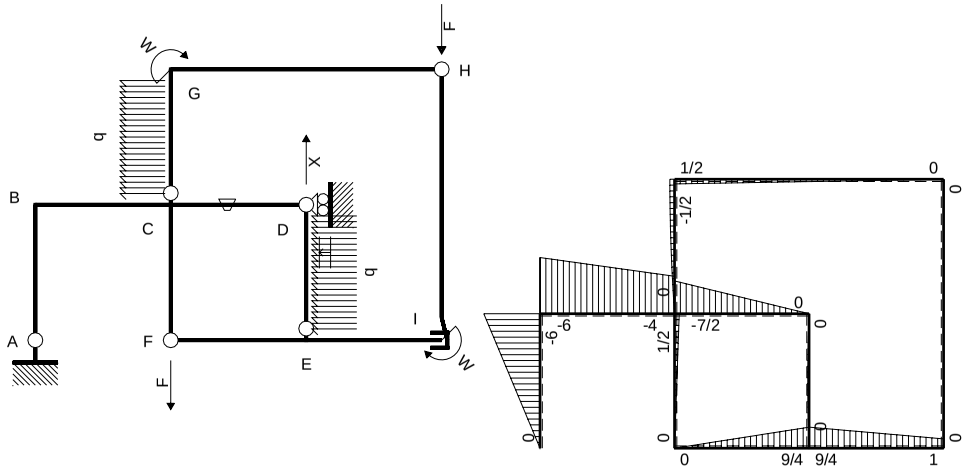
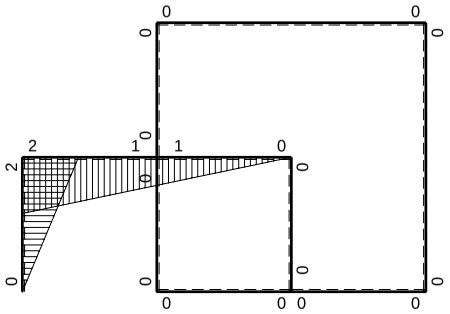


Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983213 ESΣ1.xxxx.111

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PROCEDIMENTO E RISULTATI 983213 ESΣ1.xxxx.111

Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	2x	-6Fx	0	$-12Fx^2$	0	$4x^2$	$(-4+0)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	-2b+2x	6Fb-6Fx	0	$-12Fb^2+24Fbx-12Fx^2$	0	$4b^2-8bx+4x^2$		
BC b	2b-x	-6Fb+2Fx	0	$-12Fb^2+10Fbx-2Fx^2$	0	$4b^2-4bx+x^2$	$(-23/3+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	-b-x	4Fb+2Fx	0	$-4Fb^2-6Fbx-2Fx^2$	0	$b^2+2bx+x^2$		
CD b	b-x	-7/2Fb+7/2Fx	-Fb/EJ	$-7/2Fb^2+7Fbx-7/2Fx^2$	$-Fb^2/EJ+Fxb/EJ$	$b^2-2bx+x^2$	$(-7/6-1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	-x	7/2Fx	Fb/EJ	$-7/2Fx^2$	-Fxb/EJ	x^2		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$2Fb^3/EJ$	
	totali						$-34/3Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_D$						17/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-12\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-4\ b)\ Fb^2\ 1/EJ = -4\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-12 + 24\ x/b - 12\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-12\ x + 12\ x^2/b - 4\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-12\ b + 12\ b - 4\ b)\ Fb^2\ 1/EJ = -4\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-12 + 10\ x/b - 2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-12\ x + 5\ x^2/b - 2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-12\ b + 5\ b - 2/3\ b)\ Fb^2\ 1/EJ = -23/3\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-4 - 6\ x/b - 2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-4\ x - 3\ x^2/b - 2/3\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-4\ b - 3\ b - 2/3\ b)\ Fb^2\ 1/EJ = -23/3\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-7/2 + 7\ x/b - 7/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [-7/2\ x + 7/2\ x^2/b - 7/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [-x + 1/2\ x^2/b]_o^b\ \theta$

$= (-7/2\ b + 7/2\ b - 7/6\ b)\ Fb^2\ 1/EJ + (-b + 1/2\ b)\ \theta = -5/3\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-7/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx = [-7/6\ x^3/b^2]_o^b\ Fb^2\ 1/EJ + [1/2\ x^2/b]_o^b\ \theta$

$= (-7/6\ b)\ Fb^2\ 1/EJ + (1/2\ b)\ \theta = -5/3\ Fb^3/EJ$

ESΣ1.xxxx.111

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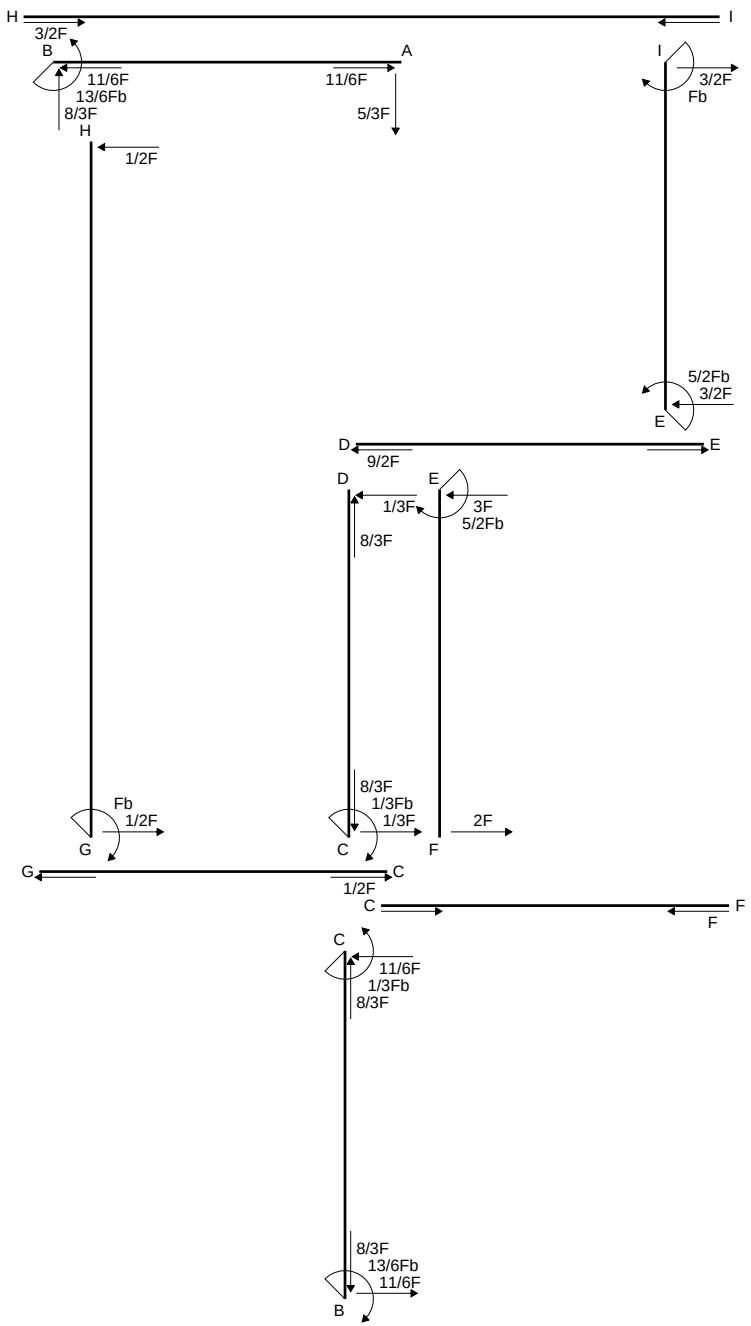
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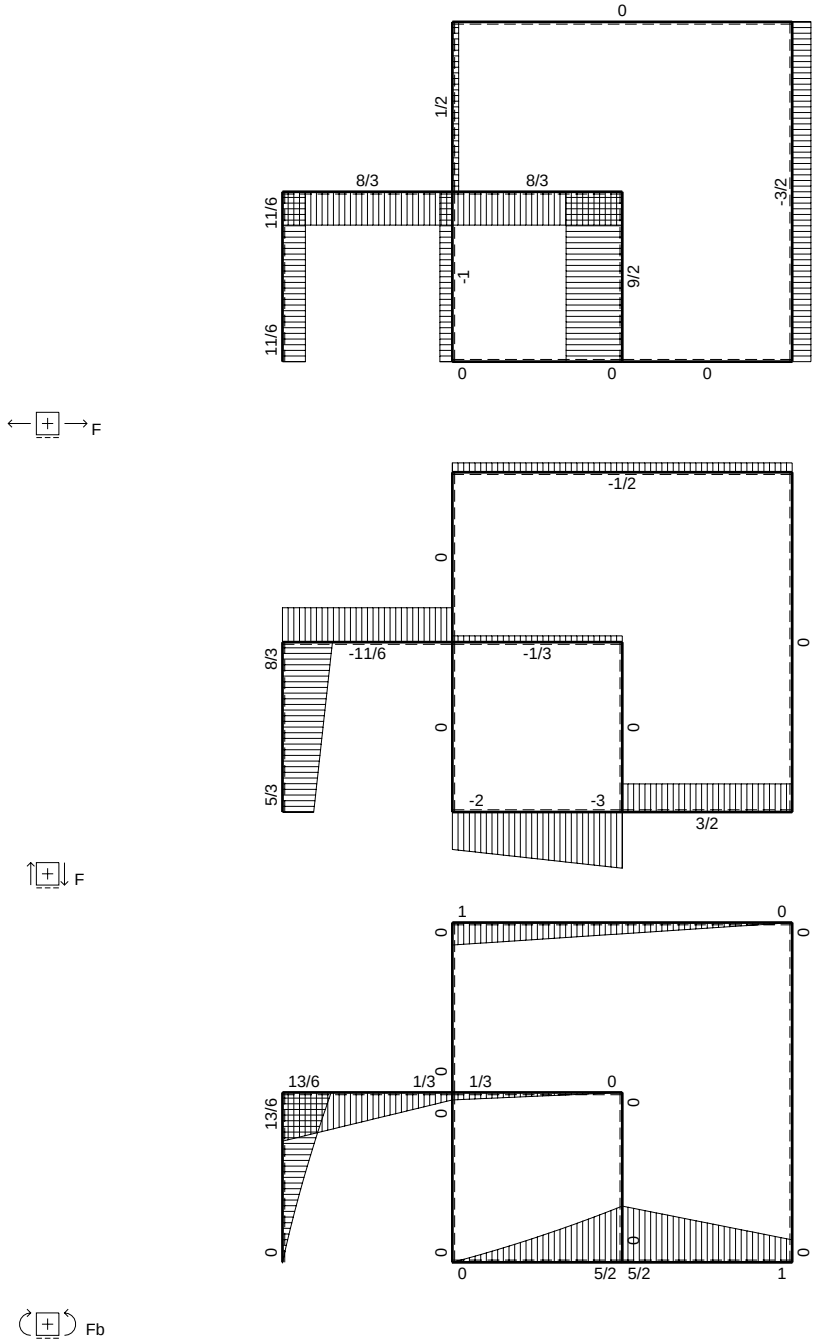
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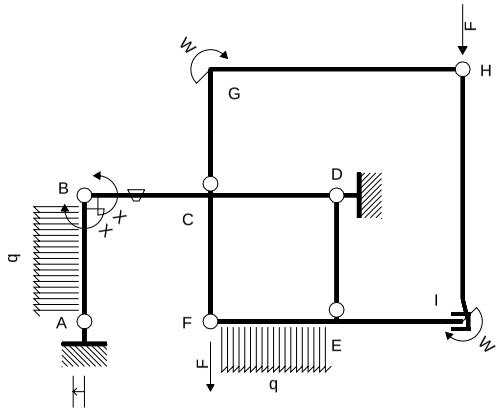
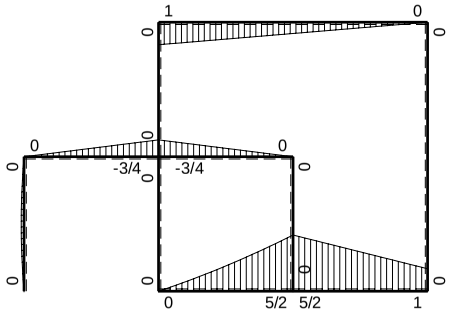
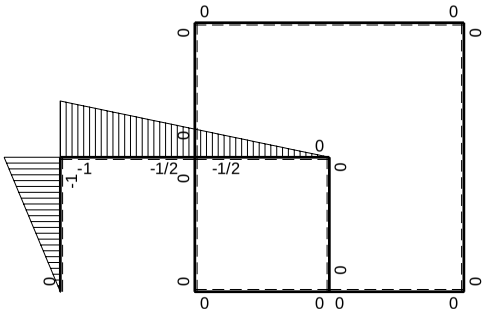


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 958558 ESΣ1.xxxx.112

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
BA b	1-x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
BC b	-1+1/2x/b	-3/4Fx	-Fb/EJ	$3/4Fx-3/8Fx^2/b$	Fb/EJ-1/2Fx/EJ	$1-x/b+1/4x^2/b^2$	$(1/4+3/4)Fb^2/EJ$	7/12Xb/EJ
CB b	1/2+1/2x/b	3/4Fb-3/4Fx	Fb/EJ	$3/8Fb-3/8Fx^2/b$	1/2Fb/EJ+1/2Fx/EJ	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	-1/2+1/2x/b	-3/4Fb+3/4Fx	0	$3/8Fb-3/4Fx+3/8Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	$(1/8+0)Fb^2/EJ$	1/12Xb/EJ
DC b	1/2x/b	3/4Fx	0	$3/8Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx	0	0	0	0	0+0	0
EI b	0	-5/2Fb+3/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^2/EJ	
	totali						$13/6Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BC}$						-13/6Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 958558 ESΣ1.xxxx.112

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\ x/b -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (1 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x^2/b -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 -1/2\ x/b)\ \theta\ dx$

$= [3/8\ x -1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x -1/4\ x^2/b\ ]_o^b\ \theta$

$= (3/8\ b -1/8\ b)\ Fb\ 1/EJ\ + (-1/2\ b -1/4\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\ x/b +3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [3/8\ x -3/8\ x^2/b +1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (3/8\ b -3/8\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\ x^2/b^2)\ Fb\ 1/EJ\ dx = [1/8\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/8\ b)\ Fb\ 1/EJ\ = 1/8\ Fb^2/EJ$

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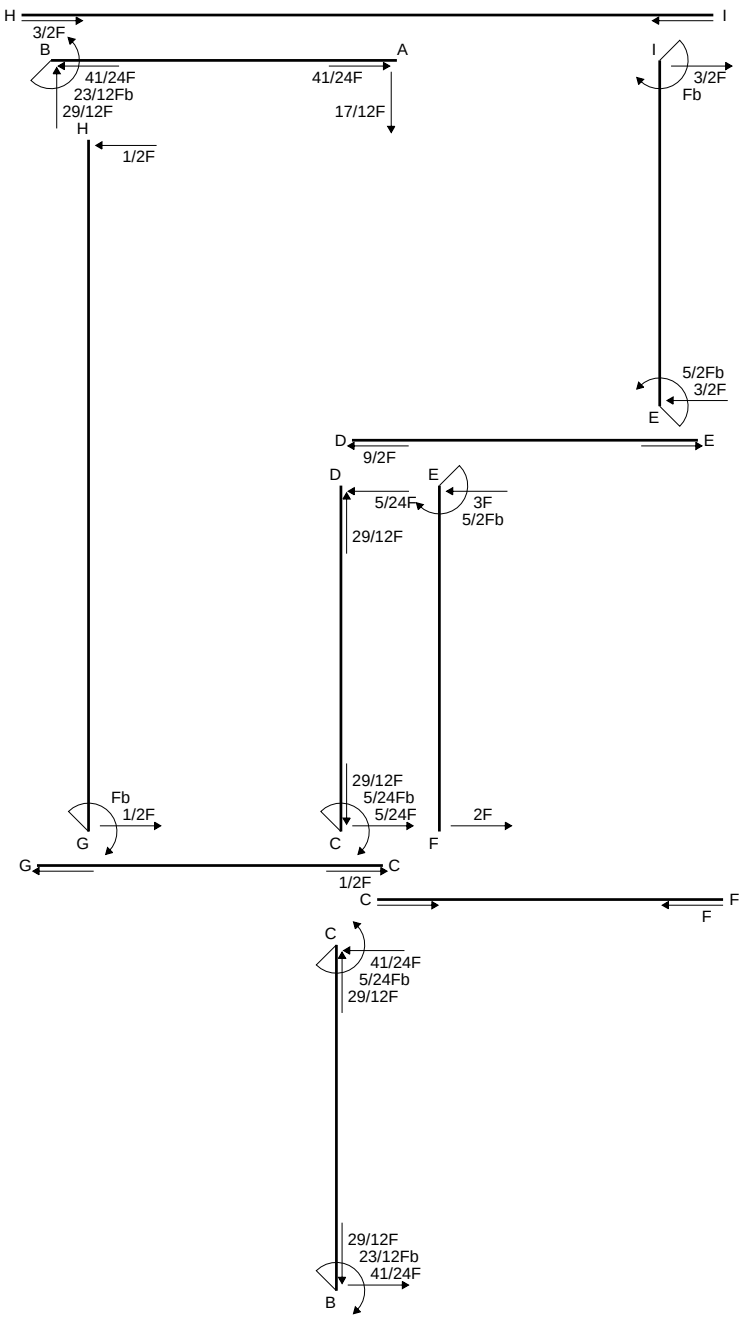
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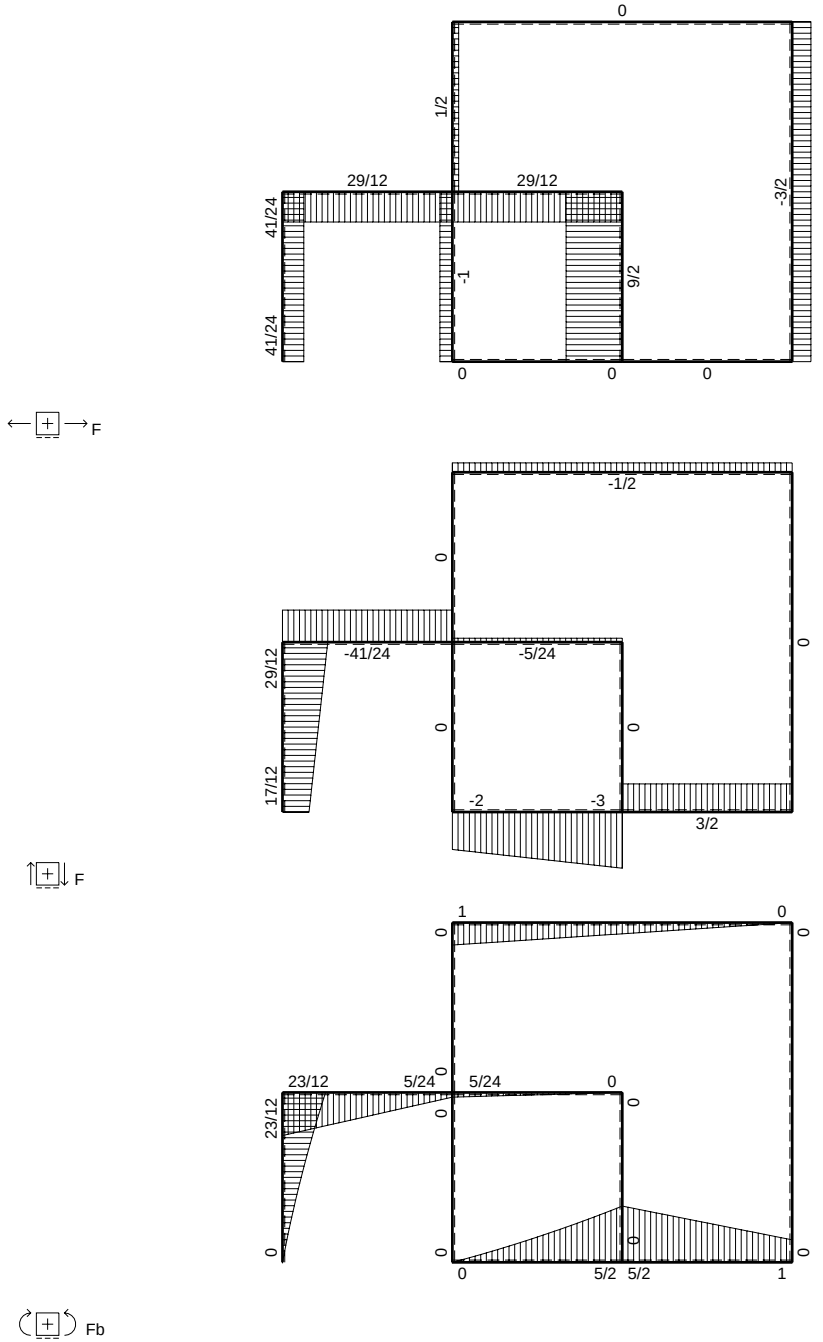
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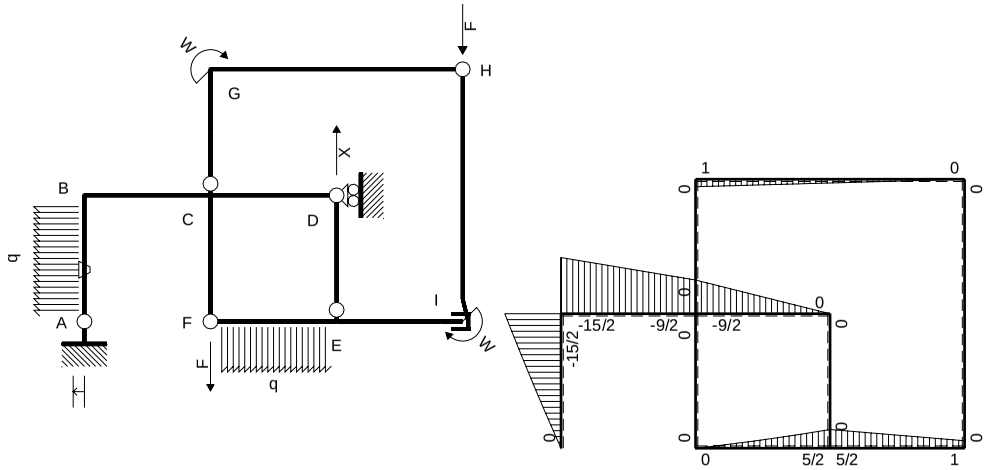
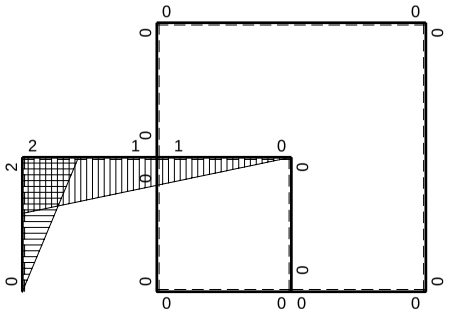


Schema di calcolo iperstatico

M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981376 ESΣ1.xxxx.114

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PROCEDIMENTO E RISULTATI 981376 ESΣ1.xxxx.114

Quadro contributi PLV per iperstatica $X=V_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	$2x$	$-8Fx+1/2qx^2$	$-Fb/EJ$	$-16Fx^2+qx^3$	$-2Fxb/EJ$	$4x^2$	$(-61/12-1)Fb^3/EJ$	$4/3Xb^3/EJ$
BA b	$-2b+2x$	$15/2Fb-7Fx-1/2qx^2$	Fb/EJ	$-15Fb^2+29Fbx-13Fx^2-qx^3$	$-2Fb^2/EJ+2Fxb/EJ$	$4b^2-8bx+4x^2$		
BC b	$2b-x$	$-15/2Fb+3Fx$	0	$-15Fb^2+27/2Fbx-3Fx^2$	0	$4b^2-4bx+x^2$	$(-37/4+0)Fb^3/EJ$	$7/3Xb^3/EJ$
CB b	$-b-x$	$9/2Fb+3Fx$	0	$-9/2Fb^2-15/2Fbx-3Fx^2$	0	$b^2+2bx+x^2$		
CD b	$b-x$	$-9/2Fb+9/2Fx$	0	$-9/2Fb^2+9Fbx-9/2Fx^2$	0	$b^2-2bx+x^2$	$(-3/2+0)Fb^3/EJ$	$1/3Xb^3/EJ$
DC b	$-x$	$9/2Fx$	0	$-9/2Fx^2$	0	x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$5/2Fb-3Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb-1/2Fx$	0	0	0	0	0+0	0
HG 2b	0	$-1/2Fx$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx$	0	0	0	0	0+0	0
EI b	0	$-5/2Fb+3/2Fx$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$-2Fb^3/EJ$	
	totali						$-113/6Fb^3/EJ$	$4Xb^3/EJ$
	iperstatica $X=V_D$						113/24F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (4 - 8\ x/b + 4\ x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 4\ x^2/b + 4/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 4\ b + 4/3\ b)\ b^2\ 1/EJ = 4/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (4 - 4\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [4\ x - 2\ x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (4\ b - 2\ b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1 + 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x + x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b + b + 1/3\ b)\ b^2\ 1/EJ = 7/3\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ b^2\ 1/EJ\ dx = [x - x^2/b + 1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (b - b + 1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (x^2/b^2)\ b^2\ 1/EJ\ dx = [1/3\ x^3/b^2]_o^b\ b^2\ 1/EJ$

$= (1/3\ b)\ b^2\ 1/EJ = 1/3\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-16\ x^2/b^2 + x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (-2\ x/b)\ \theta\ dx$

$= [-16/3\ x^3/b^2 + 1/4\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [-x^2/b]_o^b\ \theta$

$= (-16/3\ b + 1/4\ b)\ Fb^2\ 1/EJ + (-b)\ \theta = -73/12\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-15 + 29\ x/b - 13\ x^2/b^2 - x^3/b^3)\ Fb^2\ 1/EJ\ dx + \int_o^b (2 - 2\ x/b)\ \theta\ dx$

$= [-15\ x + 29/2\ x^2/b - 13/3\ x^3/b^2 - 1/4\ x^4/b^3]_o^b\ Fb^2\ 1/EJ + [2\ x - x^2/b]_o^b\ \theta$

$= (-15\ b + 29/2\ b - 13/3\ b - 1/4\ b)\ Fb^2\ 1/EJ + (2\ b - b)\ \theta = -73/12\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-15 + 27/2\ x/b - 3\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-15\ x + 27/4\ x^2/b - x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-15\ b + 27/4\ b - b)\ Fb^2\ 1/EJ = -37/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-9/2 - 15/2\ x/b - 3\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-9/2\ x - 15/4\ x^2/b - x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-9/2\ b - 15/4\ b - b)\ Fb^2\ 1/EJ = -37/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-9/2 + 9\ x/b - 9/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-9/2\ x + 9/2\ x^2/b - 3/2\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-9/2\ b + 9/2\ b - 3/2\ b)\ Fb^2\ 1/EJ = -3/2\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-9/2\ x^2/b^2)\ Fb^2\ 1/EJ\ dx = [-3/2\ x^3/b^2]_o^b\ Fb^2\ 1/EJ$

$= (-3/2\ b)\ Fb^2\ 1/EJ = -3/2\ Fb^3/EJ$

PROCEDIMENTO E RISULTATI 981376 ESΣ1.xxxx.114

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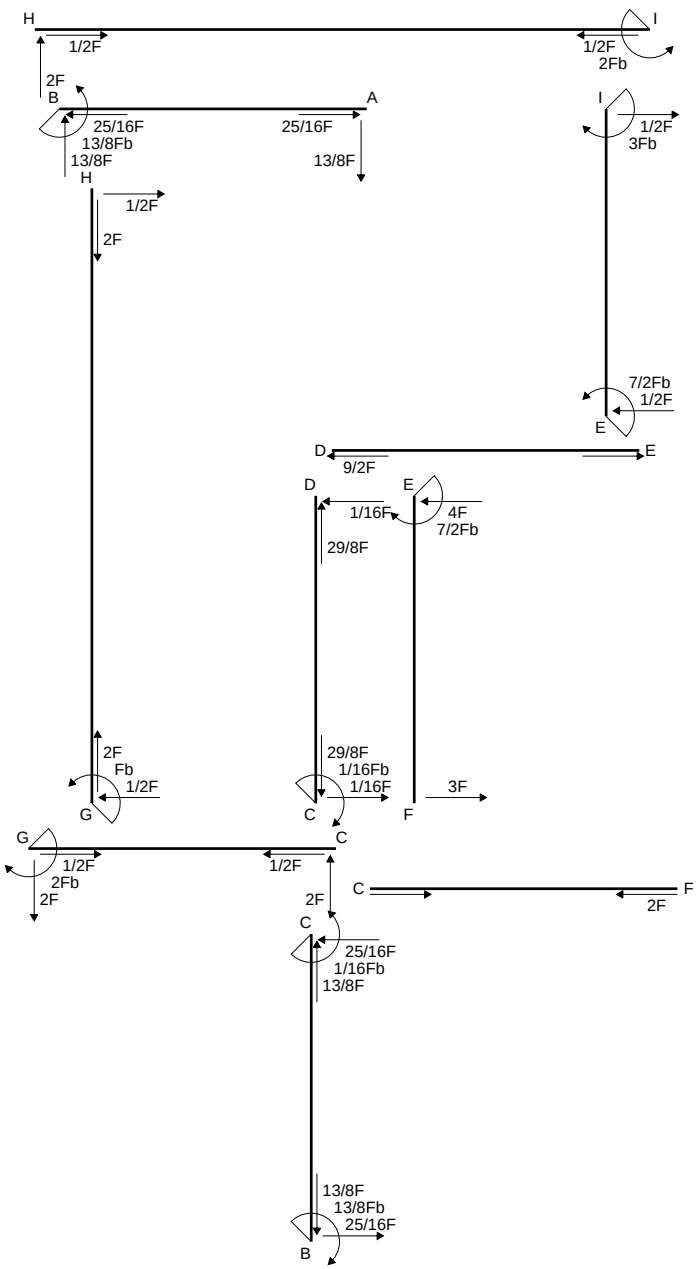

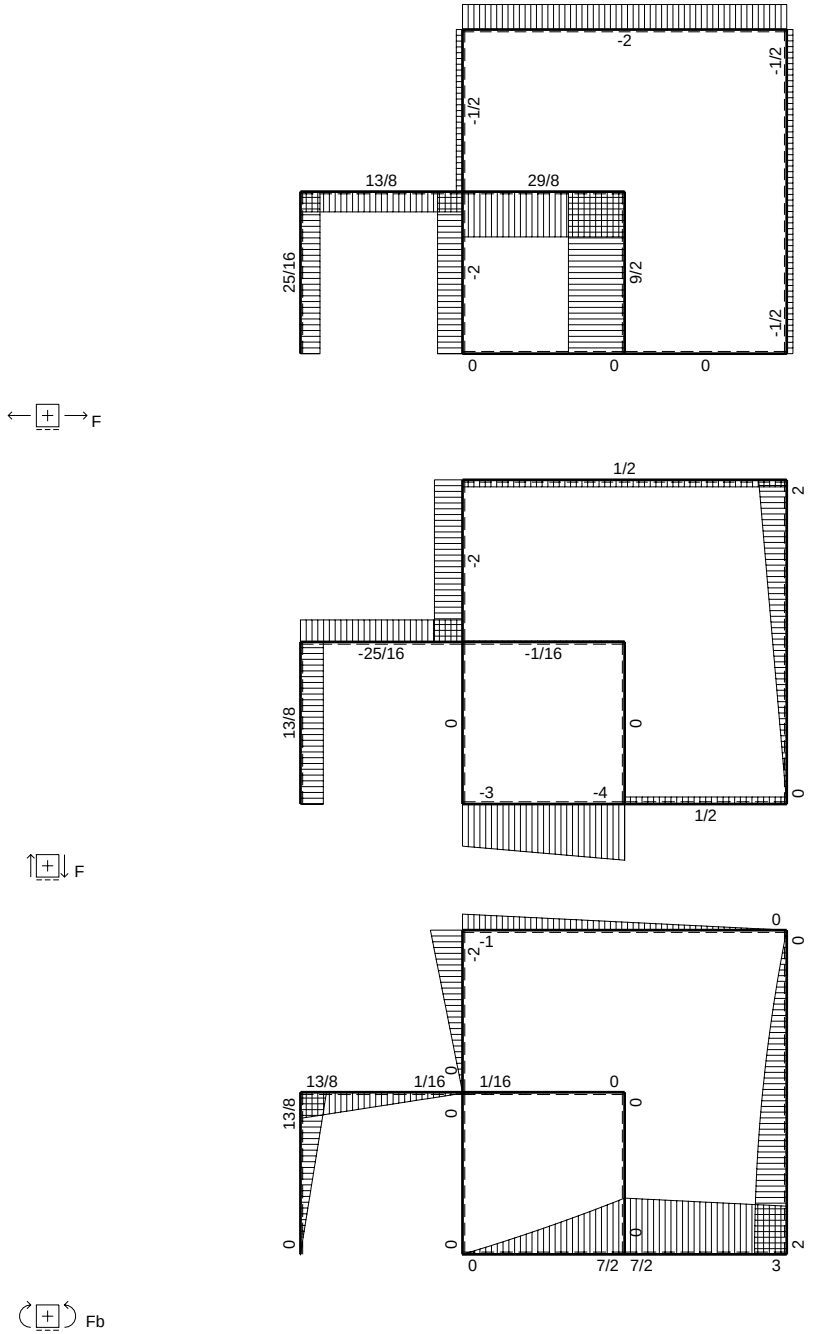

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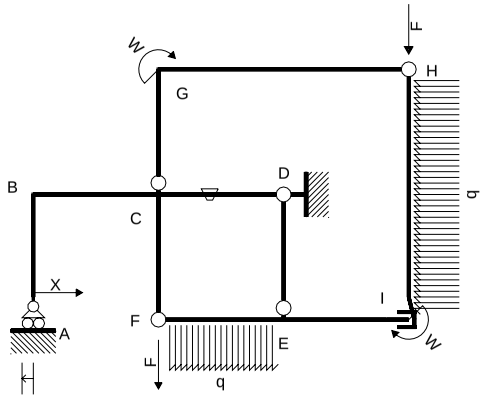
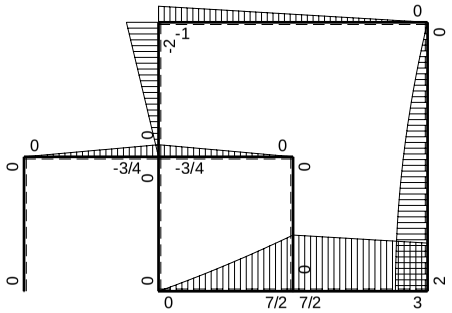
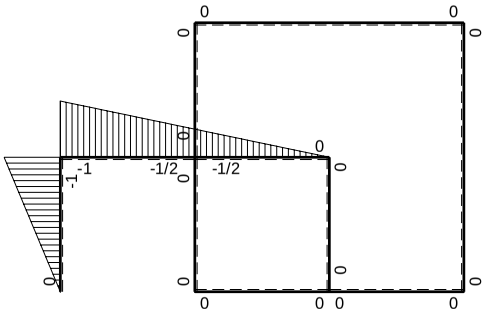


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 912079 ESΣ1.xxxx.122

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Quadro contributi PLV per iperstatica $X=H_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/4Fx	0	$3/4Fbx-3/8Fx^2$	0	$b^2-bx+1/4x^2$	$(1/4+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	3/4Fb-3/4Fx	0	$3/8Fb^2-3/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	-Fb/EJ	$3/8Fb^2-3/4Fbx+3/8Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/8+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	3/4Fx	Fb/EJ	$3/8Fx^2$	1/2Fxb/EJ	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	-2Fx	0	0	0	0	0+0	0
GC b	0	2Fb-2Fx	0	0	0	0		
GH 2b	0	-Fb+1/2Fx	0	0	0	0	0+0	0
HG 2b	0	1/2Fx	0	0	0	0		
HI 2b	0	$2Fx-1/2qx^2$	0	0	0	0	0+0	0
IH 2b	0	$-2Fb+1/2qx^2$	0	0	0	0		
IE b	0	3Fb+1/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+1/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$13/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_{AB}$						-13/8F	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI 912079 ESΣ1.xxxx.122

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ = 1/4\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\ dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/8\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 3/8\ Fb^3/EJ$

ESΣ1.xxxx.122

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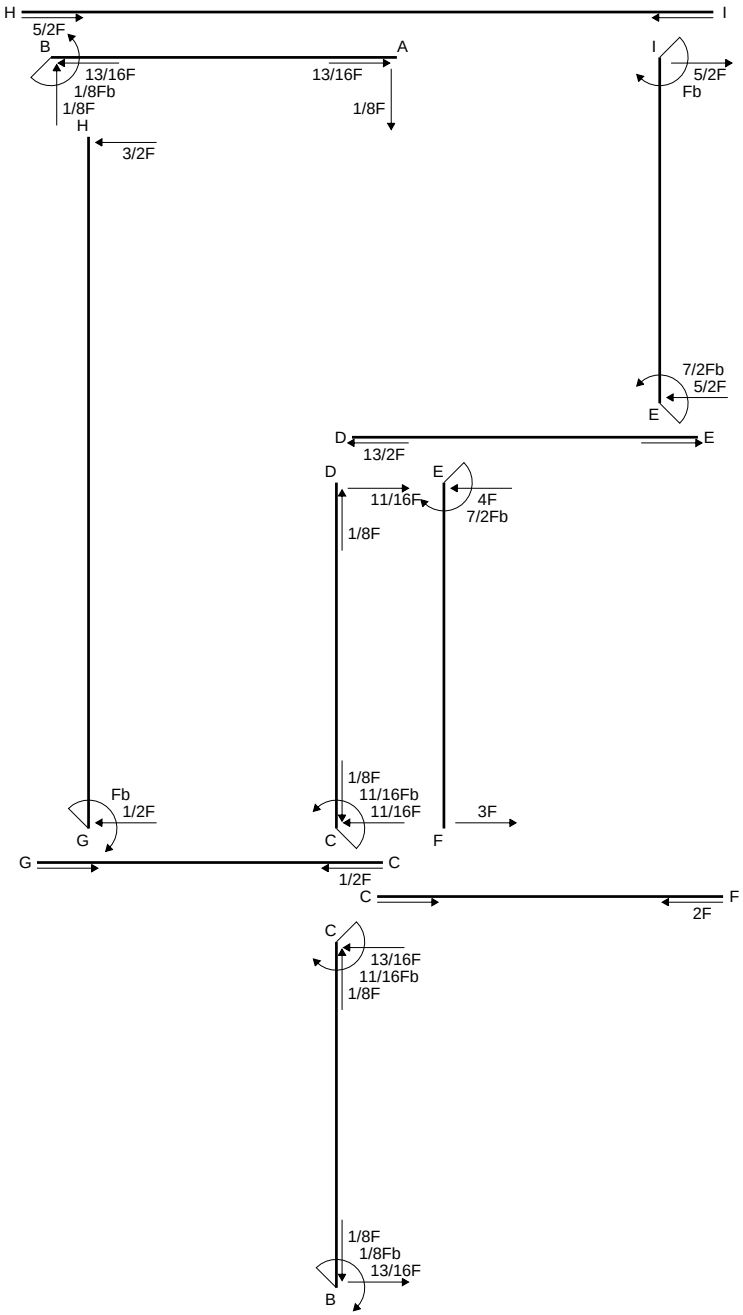
H
I
5/2F
B
A
13/16F
1/8Fb
1/8F
13/16F
1/8F
I
5/2F
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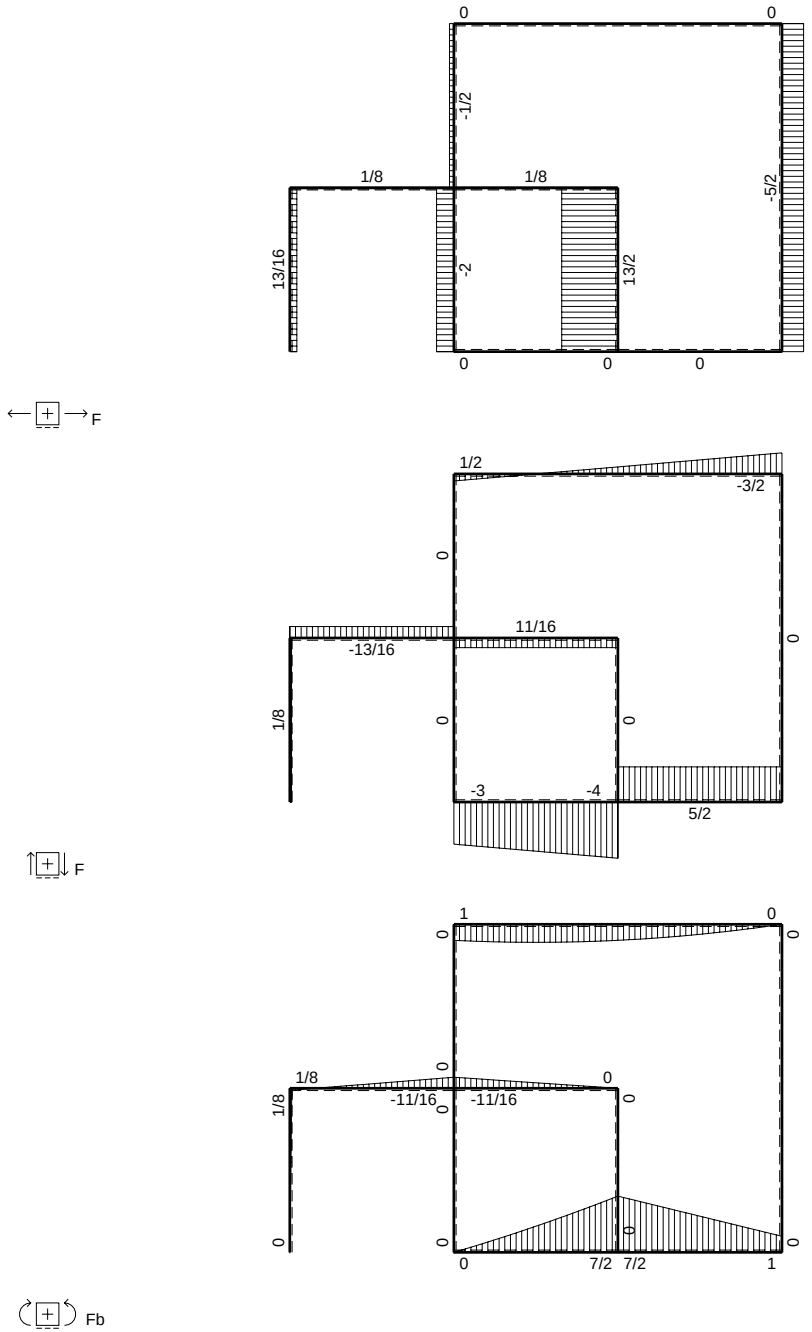
H
3/2F
7/2Fb
5/2F
E
D
E
13/2F
D
E
11/16F
4F
7/2Fb
1/8F
1/8F
11/16Fb
11/16F
Fb
1/2F
G
C
F
3F
G
C
1/2F
C
F
2F
C
13/16F
11/16Fb
1/8F
1/8F
1/8Fb
13/16F
B


0
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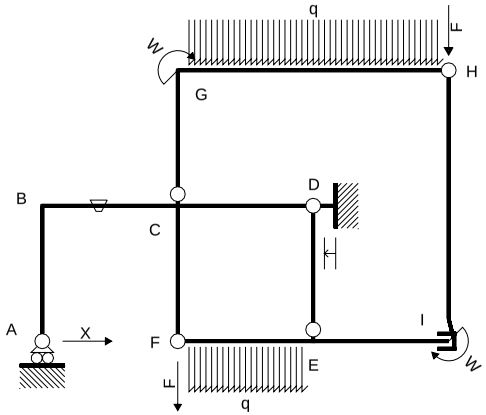
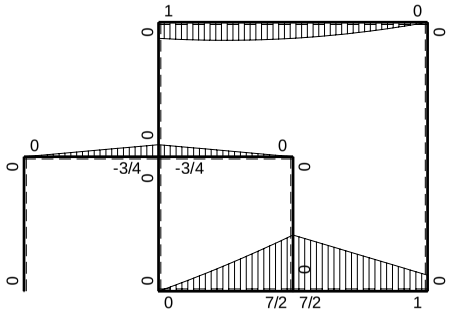
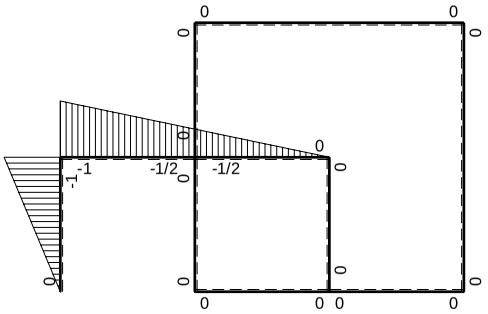
1/8
1/8
-5/2
13/16
-2
13/2
0
0
0
F
1/2
-3/2
0
11/16
-13/16
0
1/8
0
0
-3
-4
5/2
F
1
0
0
0
0
1/8
0
-11/16
-11/16
0
1/8
0
0
0
0
0
7/2
7/2
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 983331 ESΣ1.xxxx.136

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PROCEDIMENTO E RISULTATI 983331 ESΣ1.xxxx.136

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/4Fx	-Fb/EJ	$3/4Fbx-3/8Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/4+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	3/4Fb-3/4Fx	Fb/EJ	$3/8Fb^2-3/8Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-3/4Fb+3/4Fx	0	$3/8Fb^2-3/4Fbx+3/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	3/4Fx	0	$3/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$7/2Fb-4Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-3Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+5/2Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/4\, x/b -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x^2/b -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/8 -3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\ dx$

$= [3/8\, x -1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (3/8\, b -1/8\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (3/8 -3/4\, x/b +3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/8\, x -3/8\, x^2/b +1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/8\, b -3/8\, b +1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (3/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/8\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/8\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

ESΣ1.xxxx.136

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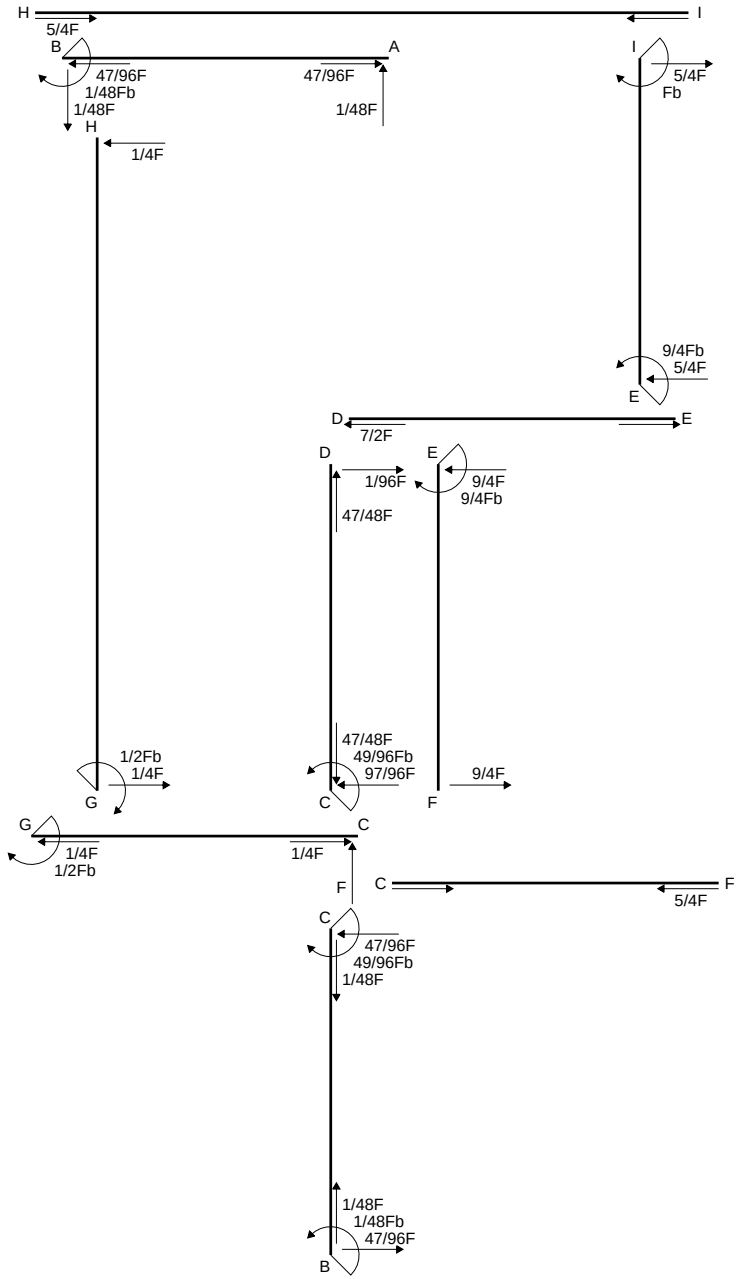
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B
A
47/96F
1/48Fb
1/48F
47/96F
1/48F
I
5/4F
Fb
H
1/4F
9/4Fb
5/4F
E
D
E
7/2F
D
E
1/96F
9/4F
9/4Fb
47/48F
1/2Fb
1/4F
G
47/48F
49/96Fb
97/96F
9/4F
C
F
G
C
1/4F
1/2Fb
1/4F
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5/4F
C
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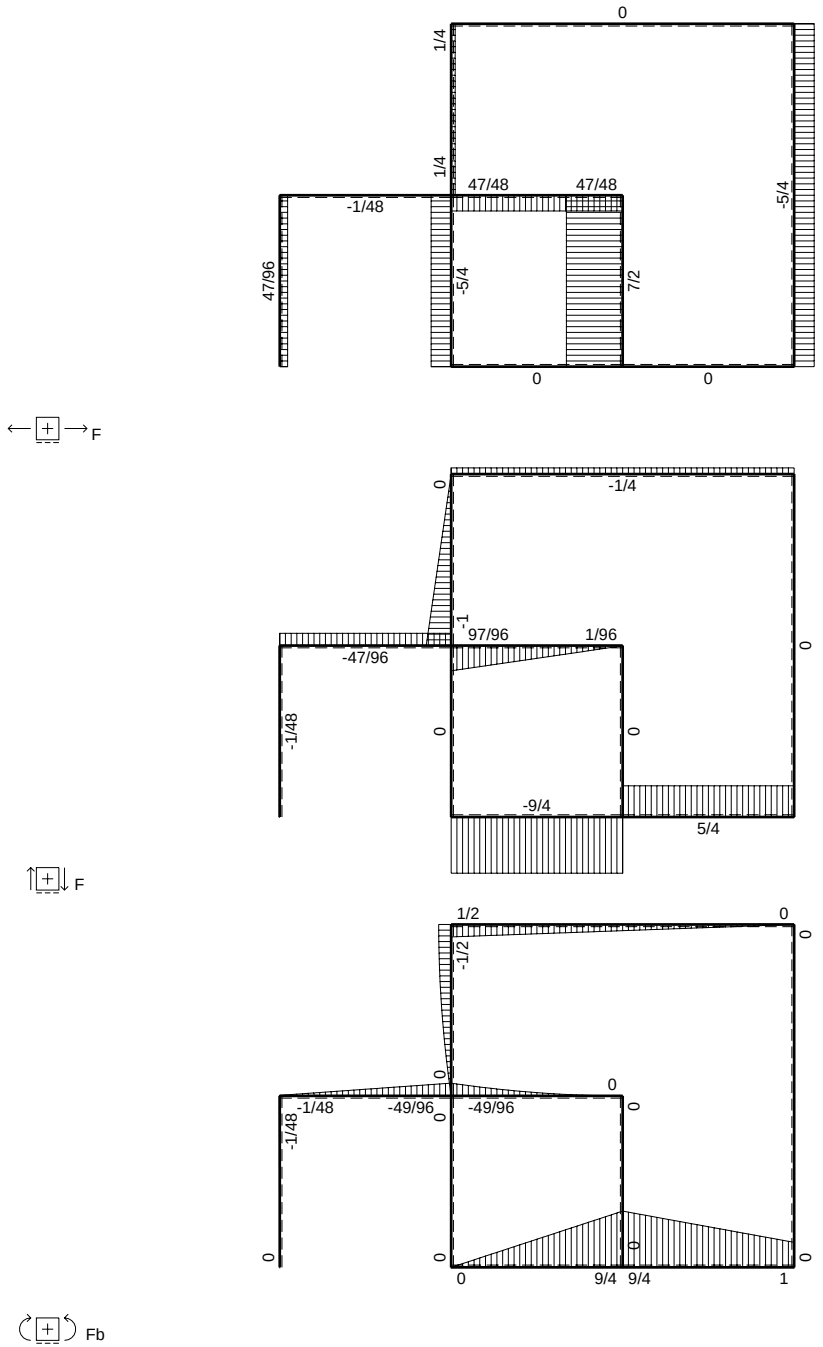
49/96Fb
1/48F
1/48F
1/48Fb
47/96F
B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47/48
-5/4
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47/96
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0
-1/4
-1
97/96
1/96
-47/96
-1/48
0
0
0
-9/4
5/4
F
1/2
0
0
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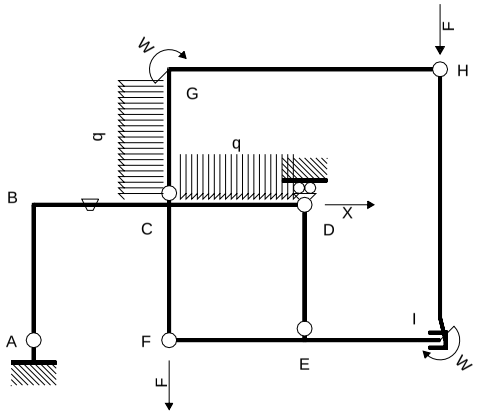
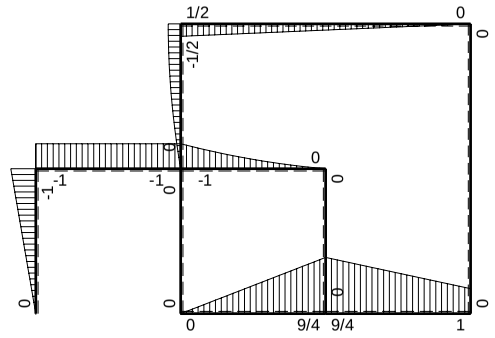
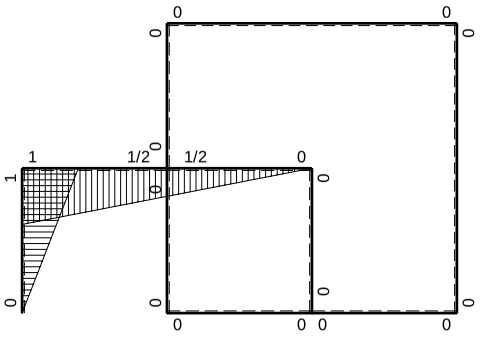
0
0
0
0
9/4
9/4
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 980904 ESΣ1.xxxx.153

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Quadro contributi PLV per iperstatica $X=H_D$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	x	-Fx	0	$-Fx^2$	0	x^2	$(-1/3+0)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	-b+x	Fb-Fx	0	$-Fb^2+2Fbx-Fx^2$	0	$b^2-2bx+x^2$		
BC b	b-1/2x	-Fb	-Fb/EJ	$-Fb^2+1/2Fbx$	$-Fb^2/EJ+1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(-3/4-3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	-1/2b-1/2x	Fb	Fb/EJ	$-1/2Fb^2-1/2Fbx$	$-1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	1/2b-1/2x	$-Fb+3/2Fx-1/2qx^2$	0	$-1/2Fb^2+5/4Fbx-Fx^2+1/4qx^3$	0	$1/4b^2-1/2bx+1/4x^2$	$(-7/48+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	-1/2x	$1/2Fx+1/2qx^2$	0	$-1/4Fx^2-1/4qx^3$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	9/4Fb-9/4Fx	0	0	0	0	0+0	0
FE b	0	-9/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/4Fx	0	0	0	0	0+0	0
EI b	0	-9/4Fb+5/4Fx	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						Fb^3/EJ	
	totali						$-47/48Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_D$						47/48F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (- x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 +2\, x/b - x^2/b^2)\, Fb^2\, 1/EJ\, dx = [- x + x^2/b -1/3\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (- b + b -1/3\, b)\, Fb^2\, 1/EJ = -1/3\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1 +1/2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 +1/2\, x/b)\, \theta\, dx$

$= [- x +1/4\, x^2/b]_o^b\, Fb^2\, 1/EJ + [- x +1/4\, x^2/b]_o^b\, \theta$

$= (- b +1/4\, b)\, Fb^2\, 1/EJ + (- b +1/4\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (-1/2 -1/2\, x/b)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 +1/2\, x/b)\, \theta\, dx$

$= [-1/2\, x -1/4\, x^2/b]_o^b\, Fb^2\, 1/EJ + [1/2\, x +1/4\, x^2/b]_o^b\, \theta$

$= (-1/2\, b -1/4\, b)\, Fb^2\, 1/EJ + (1/2\, b +1/4\, b)\, \theta = -3/2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (-1/2 +5/4\, x/b - x^2/b^2 +1/4\, x^3/b^3)\, Fb^2\, 1/EJ\, dx$

$= [-1/2\, x +5/8\, x^2/b -1/3\, x^3/b^2 +1/16\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +5/8\, b -1/3\, b +1/16\, b)\, Fb^2\, 1/EJ = -7/48\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (-1/4\, x^2/b^2 -1/4\, x^3/b^3)\, Fb^2\, 1/EJ\, dx = [-1/12\, x^3/b^2 -1/16\, x^4/b^3]_o^b\, Fb^2\, 1/EJ$

$= (-1/12\, b -1/16\, b)\, Fb^2\, 1/EJ = -7/48\ Fb^3/EJ$

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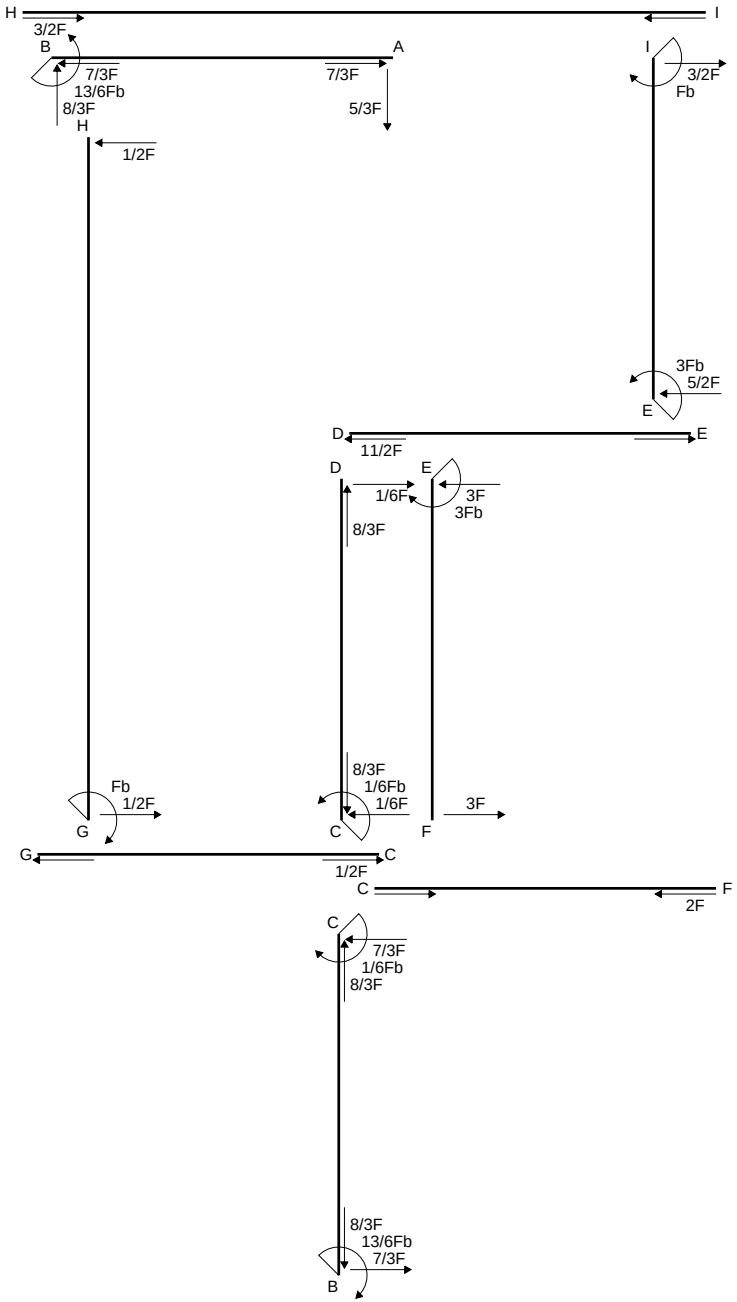
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3/2F
B
A
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8/3F
7/3F
5/3F
3/2F
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3Fb
5/2F
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11/2F
D
E
1/6F
3F
3Fb
8/3F
8/3F
1/6Fb
1/6F
3F
Fb
1/2F
G
C
F
G
C
1/2F
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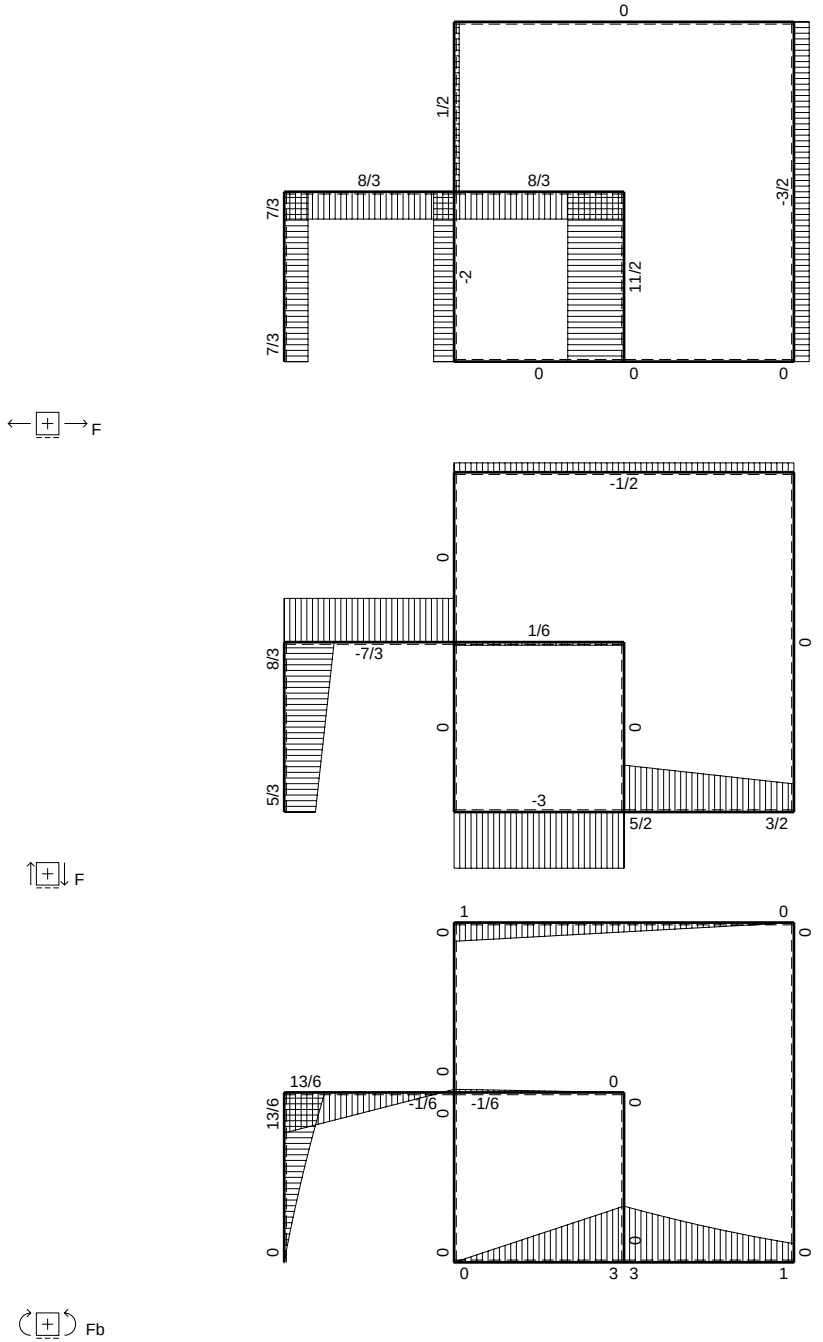
2F
C
7/3F
1/6Fb
8/3F
8/3F
13/6Fb
7/3F
B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11/2
7/3
0
0
0
F
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8/3
-7/3
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0
5/3
-3
5/2
3/2
F
1
0
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13/6
0
13/6
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0
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0
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3
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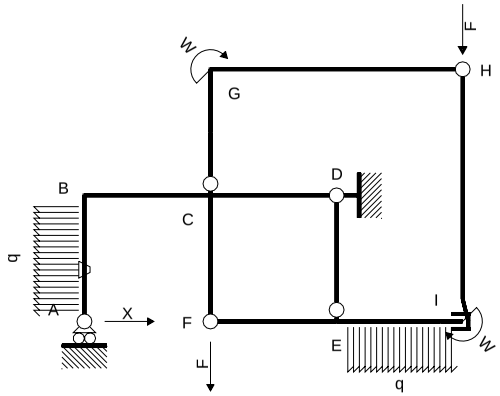
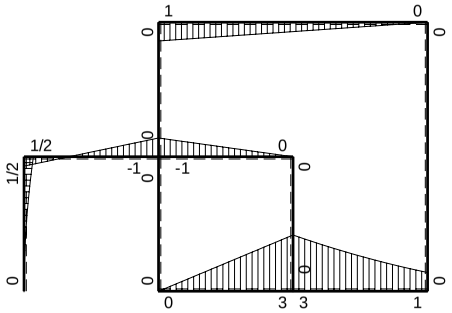
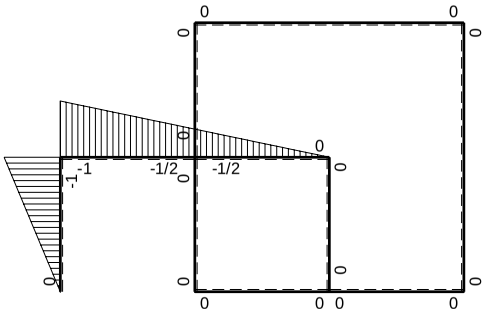
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982789 ESΣ1.xxxx.157

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PROCEDIMENTO E RISULTATI 982789 ESΣ1.xxxx.157

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	$1/2qx^2$	-Fb/EJ	$-1/2qx^3$	Fxb/EJ	x^2	$(-1/8+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	$-1/2Fb+Fx-1/2qx^2$	Fb/EJ	$-1/2Fb^2+3/2Fbx-3/2Fx^2+1/2qx^3$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	$-b+1/2x$	$1/2Fb-3/2Fx$	0	$-1/2Fb^2+7/4Fbx-3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/8+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	$1/2b+1/2x$	$Fb-3/2Fx$	0	$1/2Fb^2-1/4Fbx-3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	$-1/2b+1/2x$	$-Fb+Fx$	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	$1/2x$	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$5/3Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-5/3F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (x/b)\, \theta\, dx = [-1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [1/2\, x^2/b]_o^b\, \theta$

$= (-1/8\, b)\, Fb^2\, 1/EJ + (1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2 +3/2\, x/b -3/2\, x^2/b^2 +1/2\, x^3/b^3)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1 + x/b)\, \theta\, dx$

$= [-1/2\, x +3/4\, x^2/b -1/2\, x^3/b^2 +1/8\, x^4/b^3]_o^b\, Fb^2\, 1/EJ + [- x +1/2\, x^2/b]_o^b\, \theta$

$= (-1/2\, b +3/4\, b -1/2\, b +1/8\, b)\, Fb^2\, 1/EJ + (- b +1/2\, b)\, \theta = 3/8\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (-1/2 +7/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [-1/2\, x +7/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (-1/2\, b +7/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/4\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/8\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/8\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/8\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

ESΣ1.xxxx.157

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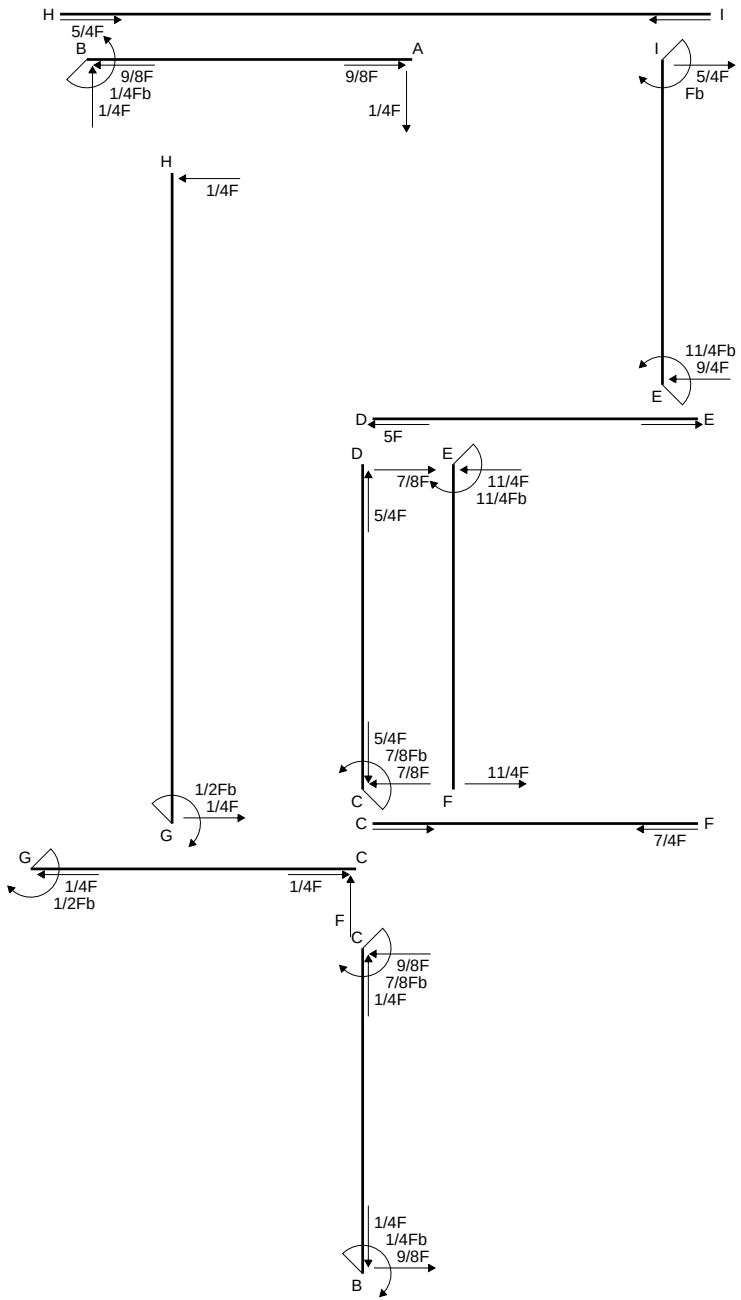
H
I
5/4F
B
A
9/8F
1/4Fb
1/4F
9/8F
1/4F
I
5/4F
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H
1/4F
11/4Fb
9/4F
E
D
E
5F
D
E
7/8F
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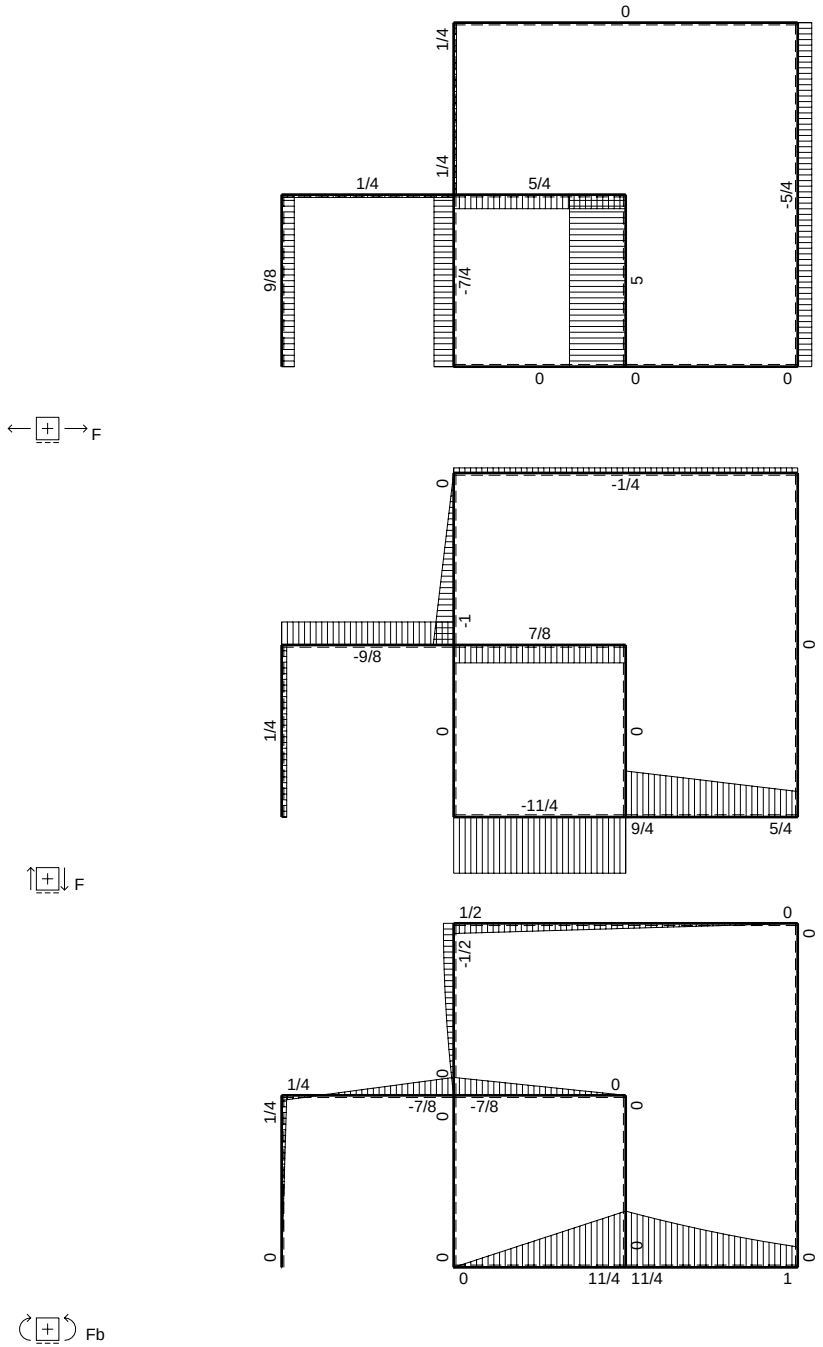
1/2Fb
1/4F
F
C
9/8F
7/8Fb
1/4F
1/4F
1/4Fb
9/8F
B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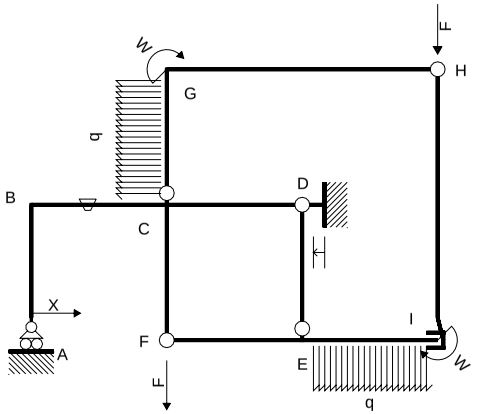
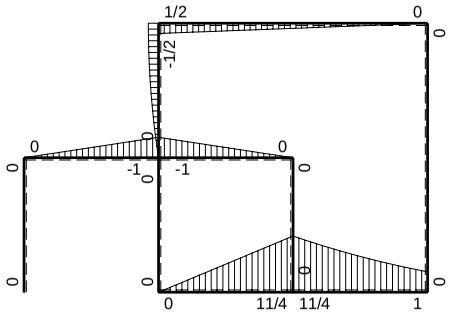
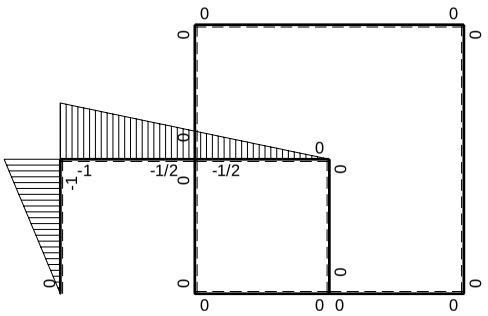
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 981975 ESΣ1.xxxx.168

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Quadro contributi PLV per iperstatica $X=H_{AB}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-Fx	-Fb/EJ	$Fbx-1/2Fx^2$	$Fb^2/EJ-1/2Fxb/EJ$	$b^2-bx+1/4x^2$	$(1/3+3/4)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	Fb-Fx	Fb/EJ	$1/2Fb^2-1/2Fx^2$	$1/2Fb^2/EJ+1/2Fxb/EJ$	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-Fb+Fx	0	$1/2Fb^2-Fbx+1/2Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(1/6+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	Fx	0	$1/2Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	11/4Fb-11/4Fx	0	0	0	0	0+0	0
FE b	0	-11/4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
GC b	0	$1/2Fb-1/2qx^2$	0	0	0	0		
GH 2b	0	1/2Fb-1/4Fx	0	0	0	0	0+0	0
HG 2b	0	-1/4Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/4Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-11/4Fb+9/4Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/4Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_{AB}$						-1/4F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (x/b -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x^2/b -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/6\, b)\, Fb^2\, 1/EJ + (-1/2\, b -1/4\, b)\, \theta = 13/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

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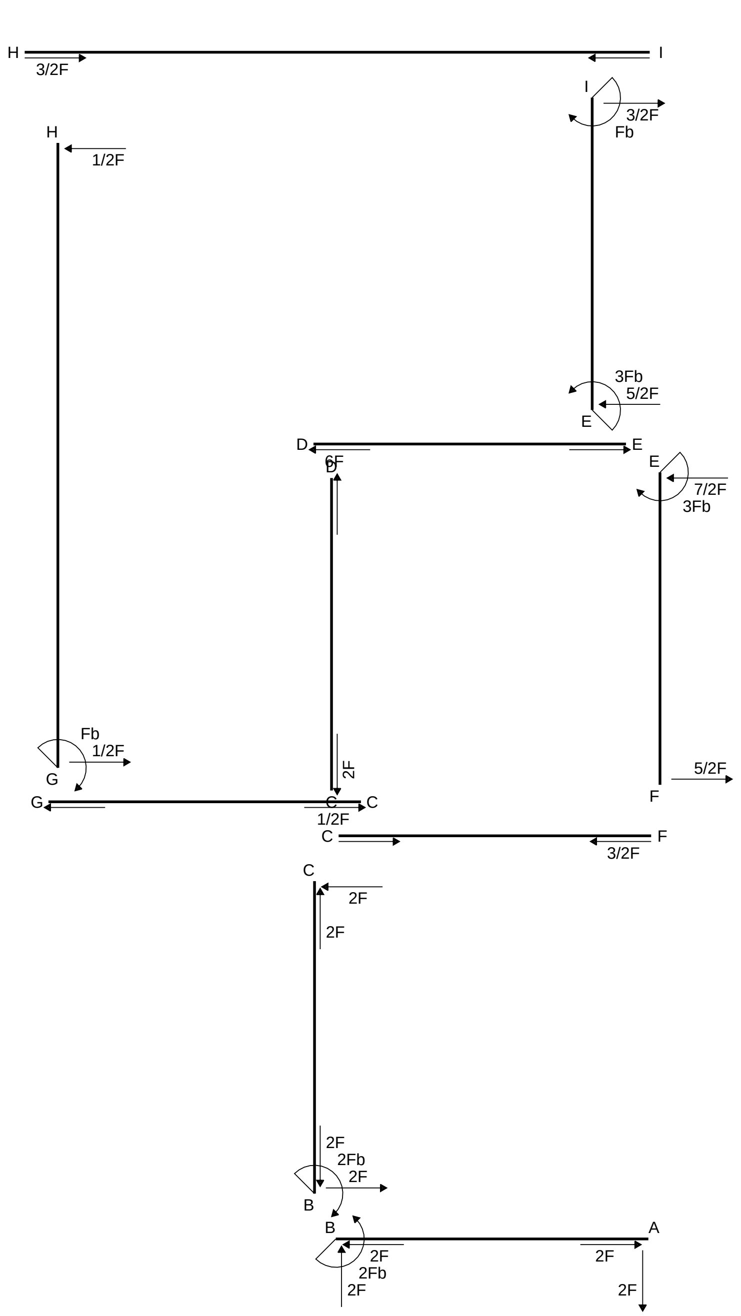
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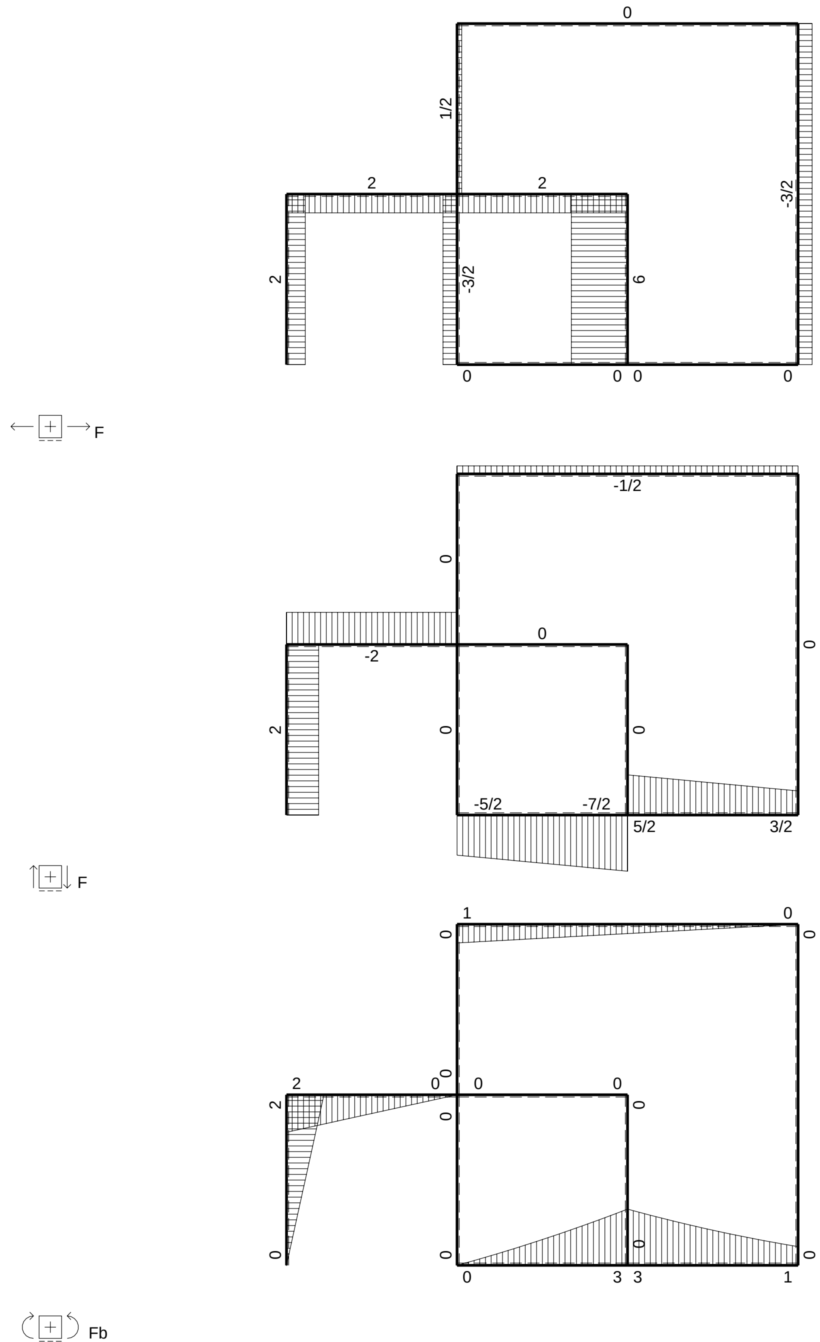
2Fb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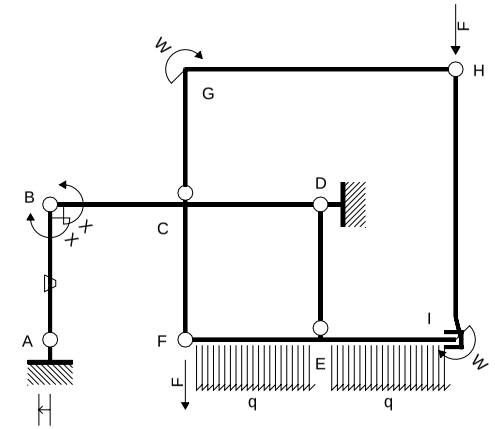
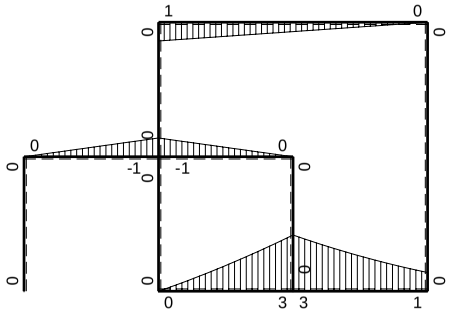
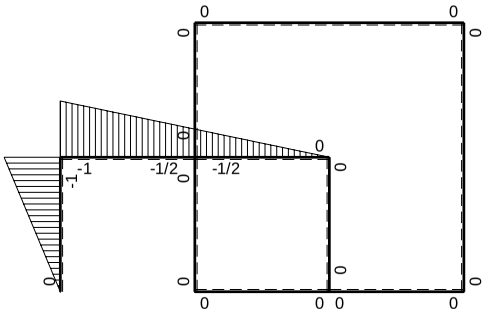
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 956425 ESΣ1.xxxx.174

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Quadro contributi PLV per iperstatica $X=W_{BC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x/b	0	-Fb/EJ	0	Fx/EJ	x^2/b^2	(0+1/2)Fb^2/EJ	1/3Xb/EJ
BA b	1-x/b	0	Fb/EJ	0	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
BC b	$-1+1/2x/b$	-Fx	0	$Fx-1/2Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	(1/3+0)Fb^2/EJ	7/12Xb/EJ
CB b	$1/2+1/2x/b$	Fb-Fx	0	$1/2Fb-1/2Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	$-1/2+1/2x/b$	-Fb+Fx	0	$1/2Fb-Fx+1/2Fx^2/b$	0	$1/4-1/2x/b+1/4x^2/b^2$	(1/6+0)Fb^2/EJ	1/12Xb/EJ
DC b	1/2x/b	Fx	0	$1/2Fx^2/b$	0	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	$3Fb-7/2Fx+1/2qx^2$	0	0	0	0	0+0	0
FE b	0	$-5/2Fx-1/2qx^2$	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^2/EJ	
	totali						$2Fb^2$/EJ	Xb/EJ
	iperstatica $X=W_{BC}$						-2Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b)\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2)\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2)\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b)\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b\ )\ \theta\ dx = [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (- b +1/2\ b)\ \theta\ = 1/2\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (1/2 -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/6\ b)\ Fb\ 1/EJ\ = 1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/2\ x -1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/2\ b -1/2\ b +1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b)\ Fb\ 1/EJ\ = 1/6\ Fb^2/EJ$

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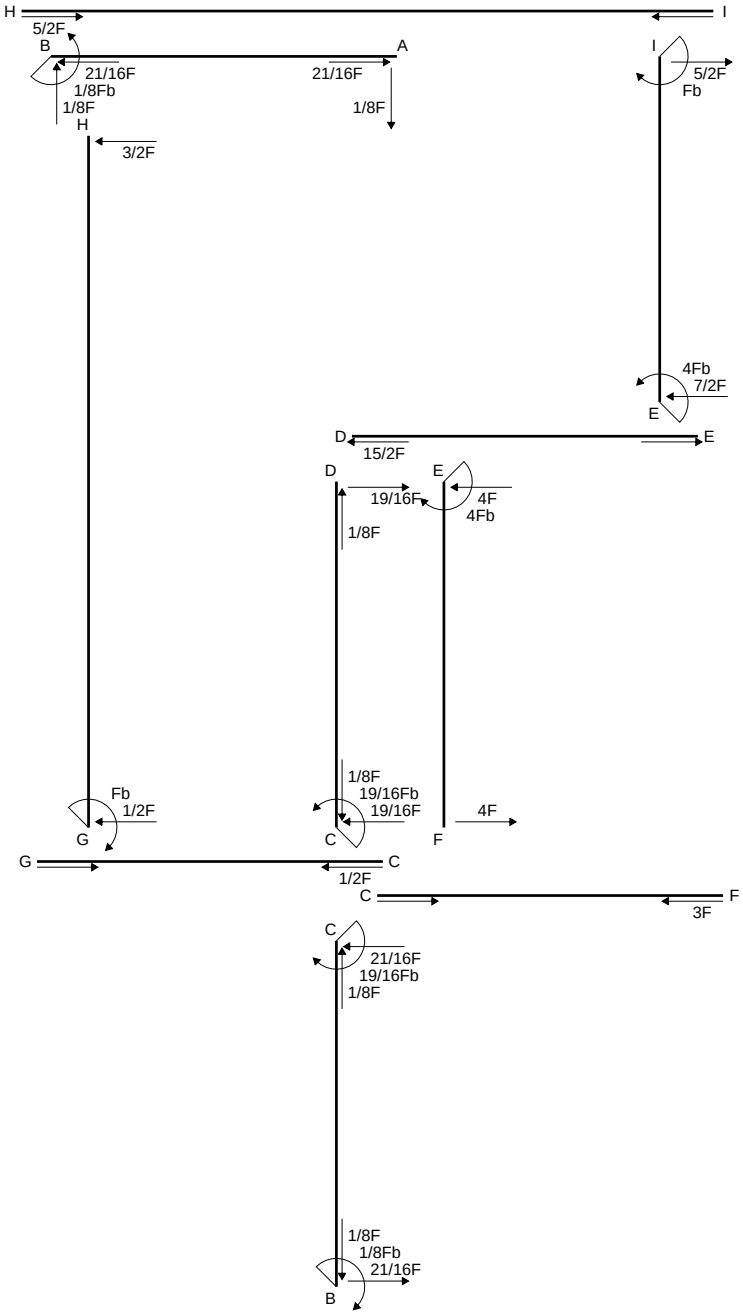
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21/16F
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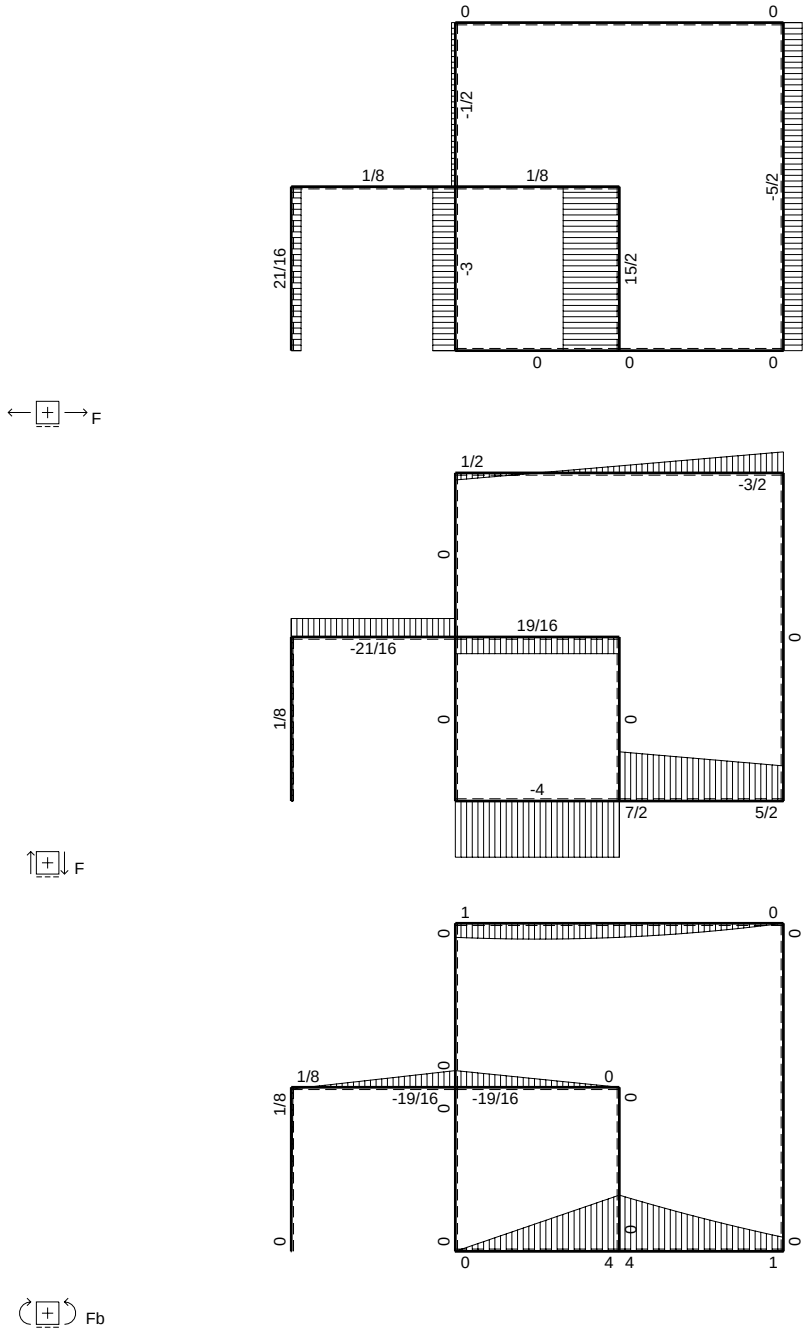
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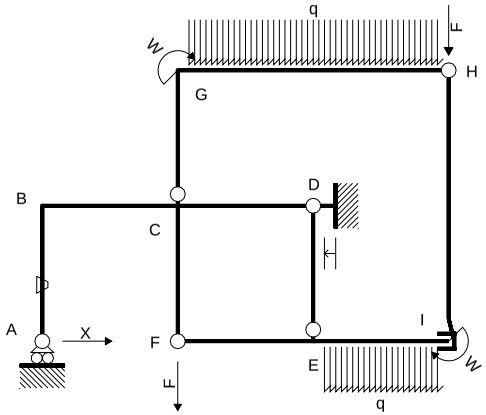
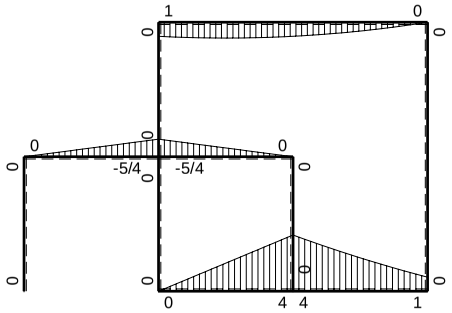
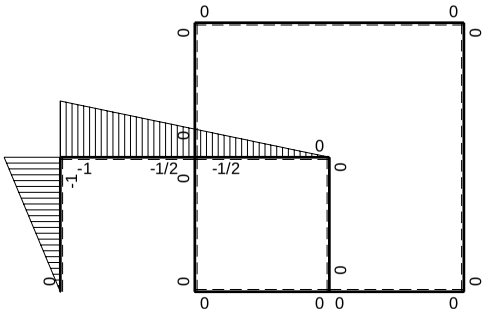


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 982644 ESΣ1.xxxx.180

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	$(0+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$
BA b	b-x	0	Fb/EJ	0	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$		
BC b	-b+1/2x	-5/4Fx	0	$5/4Fbx-5/8Fx^2$	0	$b^2-bx+1/4x^2$	$(5/12+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	5/4Fb-5/4Fx	0	$5/8Fb^2-5/8Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-5/4Fb+5/4Fx	0	$5/8Fb^2-5/4Fbx+5/8Fx^2$	0	$1/4b^2-1/2bx+1/4x^2$	$(5/24+0)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	5/4Fx	0	$5/8Fx^2$	0	$1/4x^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	4Fb-4Fx	0	0	0	0	0+0	0
FE b	0	-4Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+5/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-4Fb+7/2Fx-1/2qx^2$	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						$-Fb^3/EJ$	
	totali						$1/8Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-1/8F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (5/4\, x/b -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x^2/b -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (5/8 -5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/24\, b)\, Fb^2\, 1/EJ = 5/12\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (5/8 -5/4\, x/b +5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/8\, x -5/8\, x^2/b +5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/8\, b -5/8\, b +5/24\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (5/8\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [5/24\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (5/24\, b)\, Fb^2\, 1/EJ = 5/24\ Fb^3/EJ$

ESΣ1.xxxx.180

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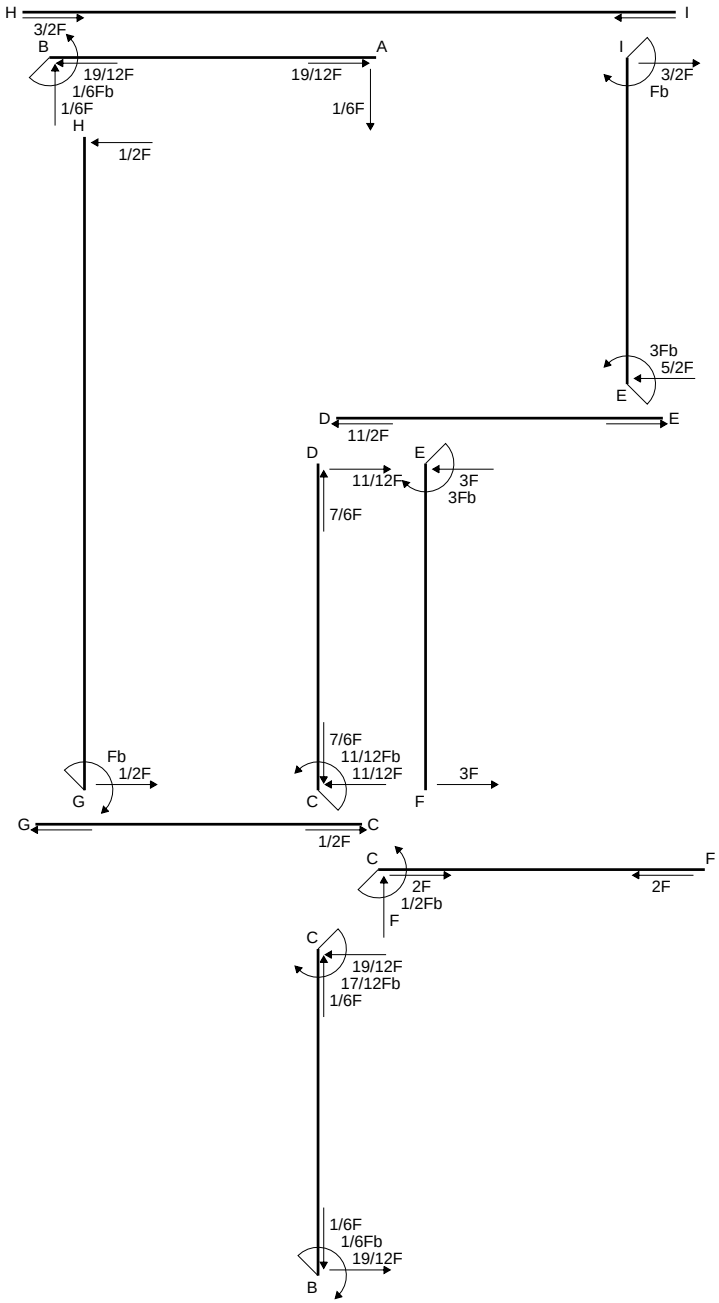
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1/6F
I
3/2F
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3Fb
7/6F
7/6F
11/12Fb
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C
F
2F
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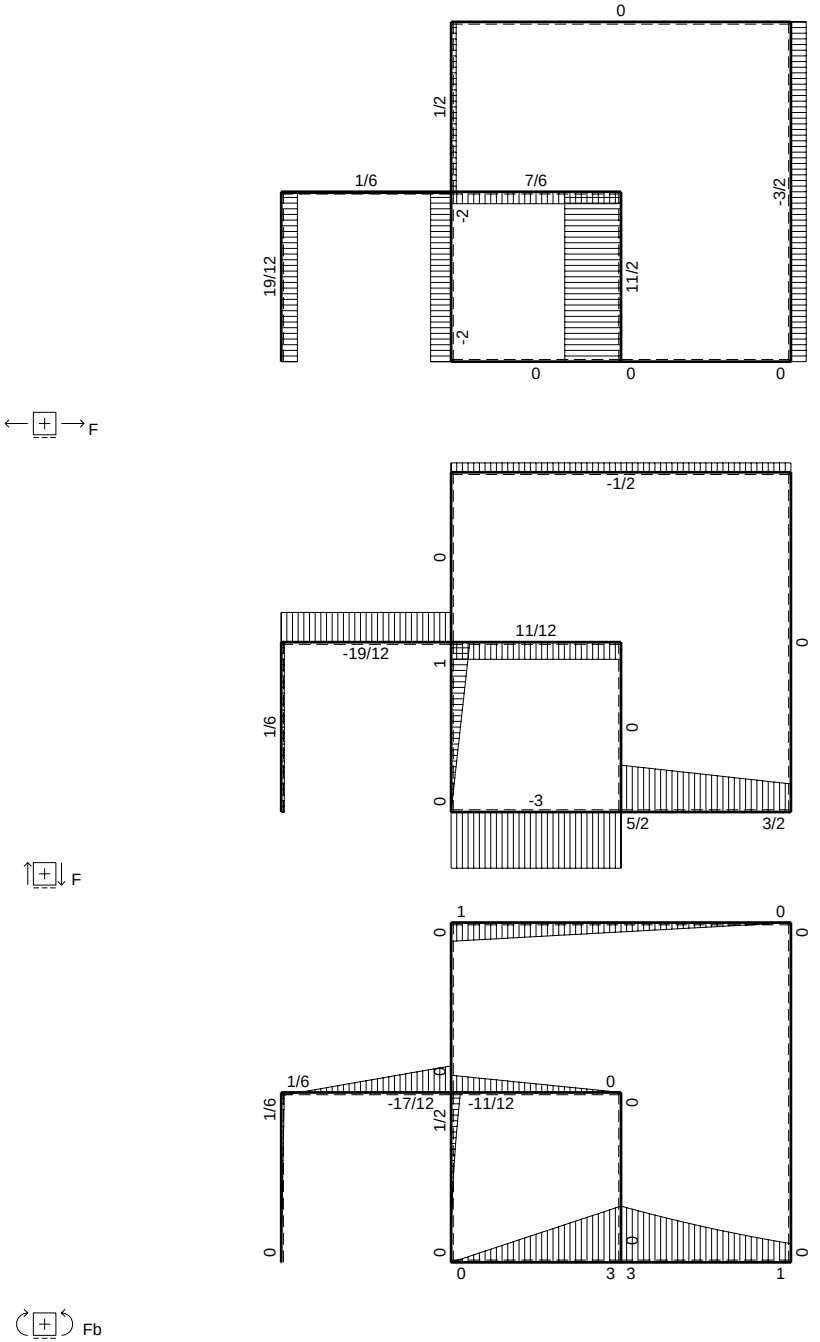
19/12F
B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-2
19/12
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-2
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F
1
0
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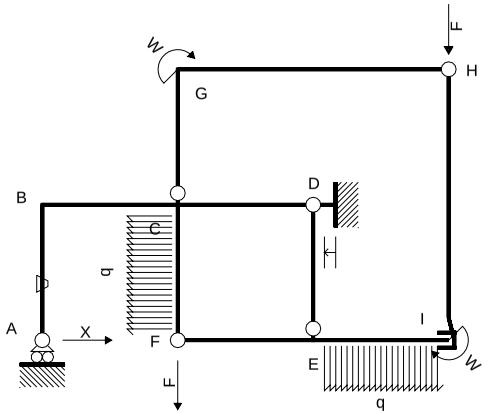
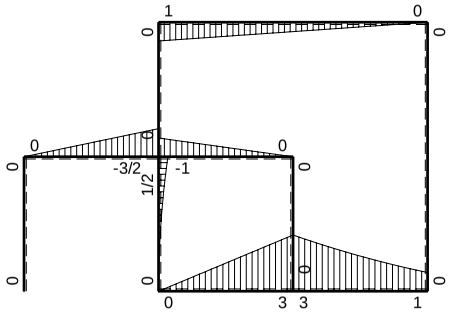
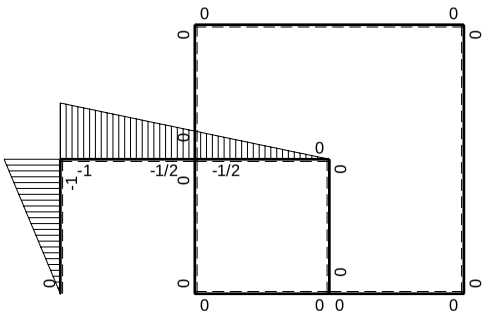
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 987434 ESΣ1.xxxx.186

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Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	-Fb/EJ	0	Fxb/EJ	x^2	(0+1/2)Fb^3/EJ	1/3Xb^3/EJ
BA b	b-x	0	Fb/EJ	0	Fb^2/EJ-Fxb/EJ	b^2-2bx+x^2		
BC b	-b+1/2x	-3/2Fx	0	3/2Fbx-3/4Fx^2	0	b^2-bx+1/4x^2	(1/2+0)Fb^3/EJ	7/12Xb^3/EJ
CB b	1/2b+1/2x	3/2Fb-3/2Fx	0	3/4Fb^2-3/4Fx^2	0	1/4b^2+1/2bx+1/4x^2		
CD b	-1/2b+1/2x	-Fb+Fx	0	1/2Fb^2-Fbx+1/2Fx^2	0	1/4b^2-1/2bx+1/4x^2	(1/6+0)Fb^3/EJ	1/12Xb^3/EJ
DC b	1/2x	Fx	0	1/2Fx^2	0	1/4x^2		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	1/2qx^2	0	0	0	0	0+0	0
CF b	0	-1/2Fb+Fx-1/2qx^2	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+3/2Fx+1/2qx^2	0	0	0	0	0+0	0
EI b	0	-3Fb+5/2Fx-1/2qx^2	0	0	0	0		
D	cedimento nodo $-H_{1D}u_D$						-Fb^3/EJ	
	totali						1/6Fb^3/EJ	Xb^3/EJ
	iperstatica $X=H_A$						-1/6F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{AB} = \int_o^b (x/b)\, \theta\, dx = [1/2\, x^2/b]_o^b\, \theta$

$= (1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BA} = \int_o^b (-1 + x/b)\, \theta\, dx = [- x +1/2\, x^2/b]_o^b\, \theta$

$= (- b +1/2\, b)\, \theta = 1/2\ Fb^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (1/6\, b)\, Fb^2\, 1/EJ = 1/6\ Fb^3/EJ$

ESΣ1.xxxx.186

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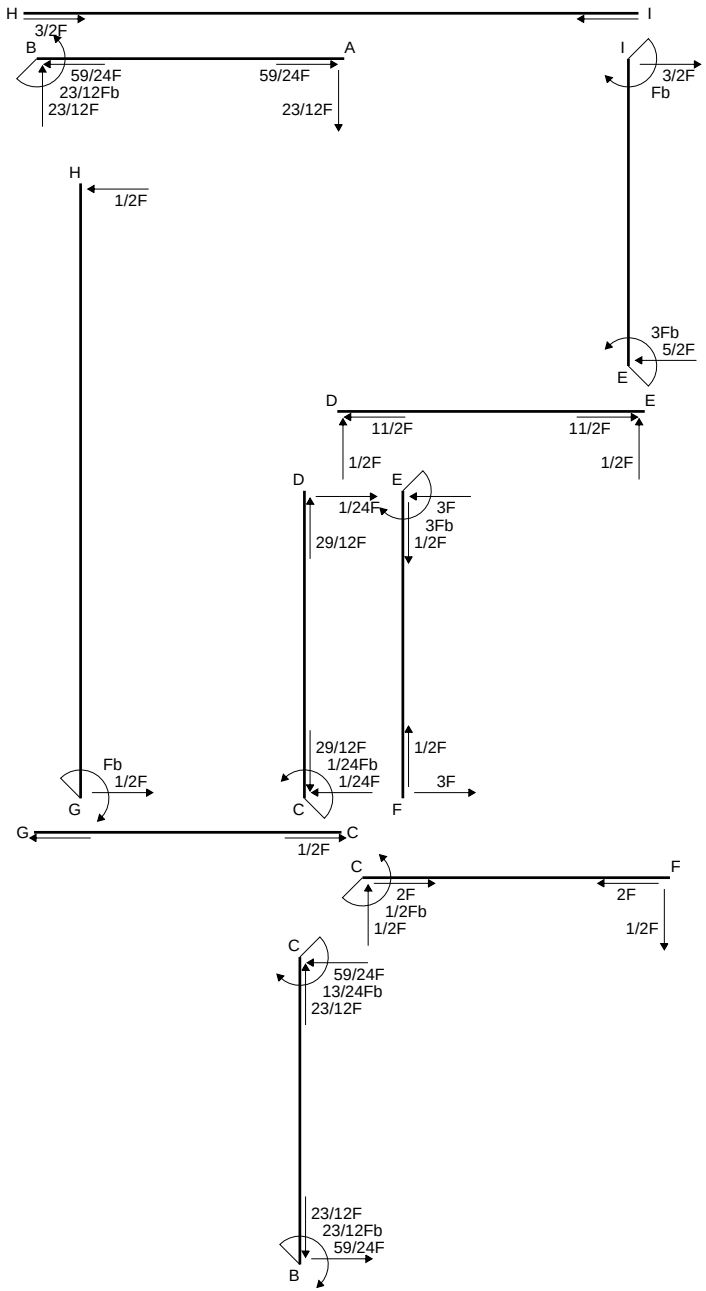
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1/2Fb
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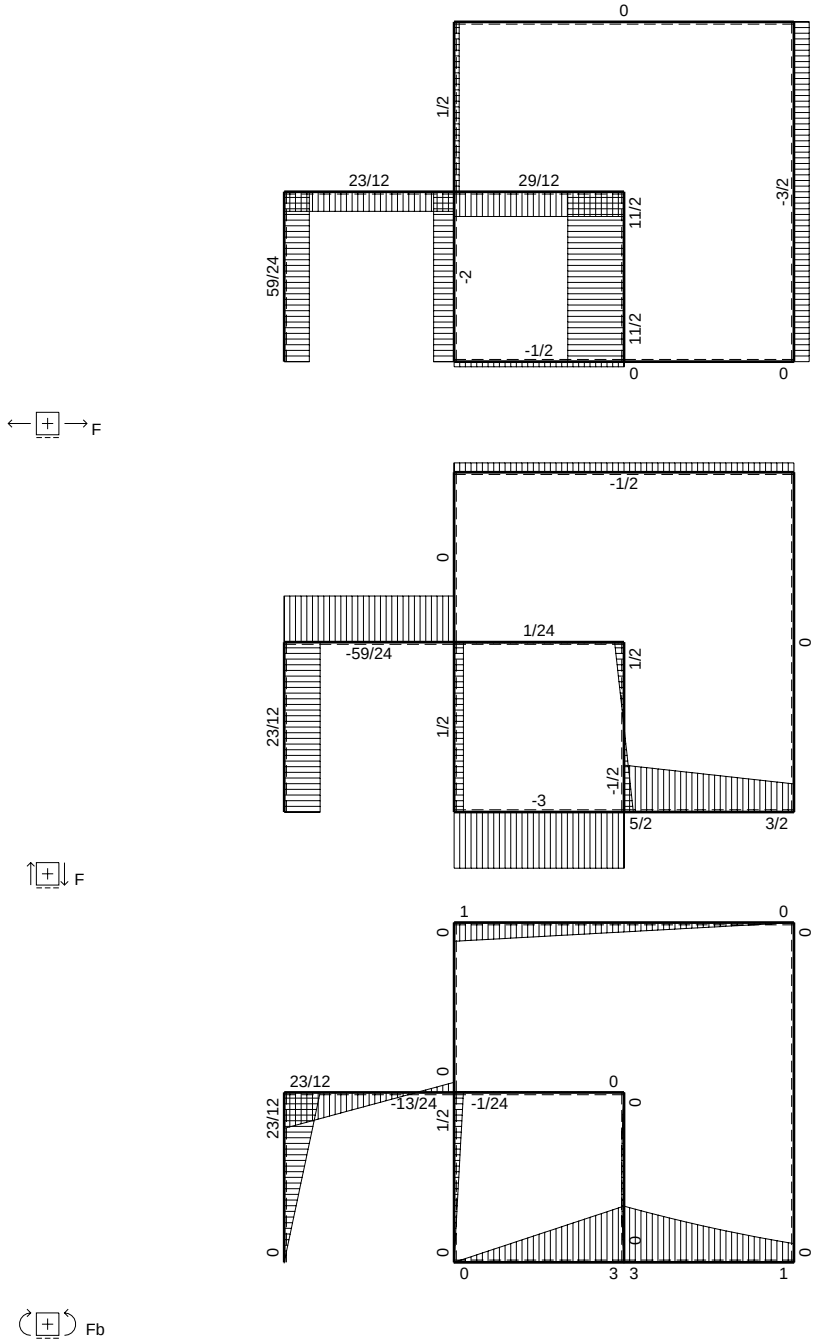
23/12Fb
59/24F
B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3/2
F
1
0
0
0
0
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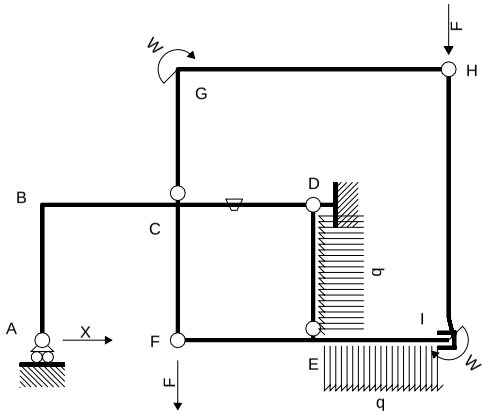
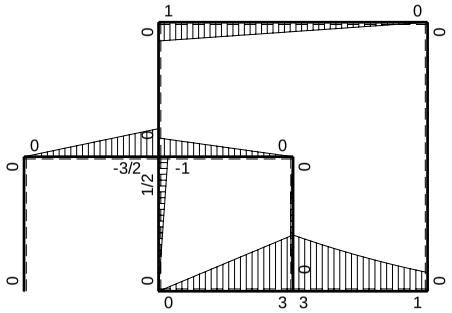
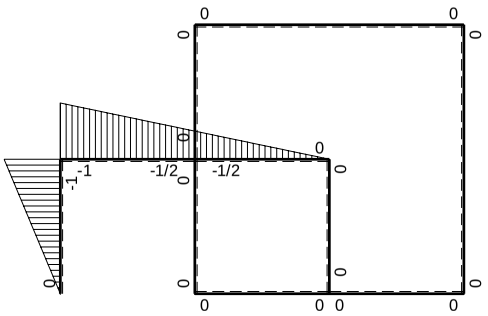
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI 957662 ESΣ1.xxxx.193

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PROCEDIMENTO E RISULTATI 957662 ESΣ1.xxxx.193

Quadro contributi PLV per iperstatica $X=H_A$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	-x	0	0	0	0	x^2	0+0	$1/3Xb^3/EJ$
BA b	b-x	0	0	0	0	$b^2-2bx+x^2$		
BC b	-b+1/2x	-3/2Fx	0	$3/2Fbx-3/4Fx^2$	0	$b^2-bx+1/4x^2$	$(1/2+0)Fb^3/EJ$	$7/12Xb^3/EJ$
CB b	1/2b+1/2x	3/2Fb-3/2Fx	0	$3/4Fb^2-3/4Fx^2$	0	$1/4b^2+1/2bx+1/4x^2$		
CD b	-1/2b+1/2x	-Fb+Fx	-Fb/EJ	$1/2Fb^2-Fbx+1/2Fx^2$	$1/2Fb^2/EJ-1/2Fxb/EJ$	$1/4b^2-1/2bx+1/4x^2$	$(1/6+1/4)Fb^3/EJ$	$1/12Xb^3/EJ$
DC b	1/2x	Fx	Fb/EJ	$1/2Fx^2$	1/2Fxb/EJ	$1/4x^2$		
DE b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
EF b	0	3Fb-3Fx	0	0	0	0	0+0	0
FE b	0	-3Fx	0	0	0	0		
FC b	0	1/2Fx	0	0	0	0	0+0	0
CF b	0	-1/2Fb+1/2Fx	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	Fb-1/2Fx	0	0	0	0	0+0	0
HG 2b	0	-1/2Fx	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	$Fb+3/2Fx+1/2qx^2$	0	0	0	0	0+0	0
EI b	0	$-3Fb+5/2Fx-1/2qx^2$	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						Fb^3/EJ	
	totali						$23/12Fb^3/EJ$	Xb^3/EJ
	iperstatica $X=H_A$						-23/12F	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\, b^2\, 1/EJ\, dx = [1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\, x/b + x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x - x^2/b +1/3\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b - b +1/3\, b)\, b^2\, 1/EJ = 1/3\ b^3/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [\, x -1/2\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (\, b -1/2\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x +1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b +1/4\, b +1/12\, b)\, b^2\, 1/EJ = 7/12\ b^3/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\, x/b +1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/4\, x -1/4\, x^2/b +1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/4\, b -1/4\, b +1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\, x^2/b^2)\, b^2\, 1/EJ\, dx = [1/12\, x^3/b^2]_o^b\, b^2\, 1/EJ$

$= (1/12\, b)\, b^2\, 1/EJ = 1/12\ b^3/EJ$

$L^{Xo}_{BC} = \int_o^b (3/2\, x/b -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x^2/b -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CB} = \int_o^b (3/4 -3/4\, x^2/b^2)\, Fb^2\, 1/EJ\, dx = [3/4\, x -1/4\, x^3/b^2]_o^b\, Fb^2\, 1/EJ$

$= (3/4\, b -1/4\, b)\, Fb^2\, 1/EJ = 1/2\ Fb^3/EJ$

$L^{Xo}_{CD} = \int_o^b (1/2 - x/b +1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (1/2 -1/2\, x/b)\, \theta\, dx$

$= [1/2\, x -1/2\, x^2/b +1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [1/2\, x -1/4\, x^2/b]_o^b\, \theta$

$= (1/2\, b -1/2\, b +1/6\, b)\, Fb^2\, 1/EJ + (1/2\, b -1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

$L^{Xo}_{DC} = \int_o^b (1/2\, x^2/b^2)\, Fb^2\, 1/EJ\, dx + \int_o^b (-1/2\, x/b)\, \theta\, dx = [1/6\, x^3/b^2]_o^b\, Fb^2\, 1/EJ + [-1/4\, x^2/b]_o^b\, \theta$

$= (1/6\, b)\, Fb^2\, 1/EJ + (-1/4\, b)\, \theta = 5/12\ Fb^3/EJ$

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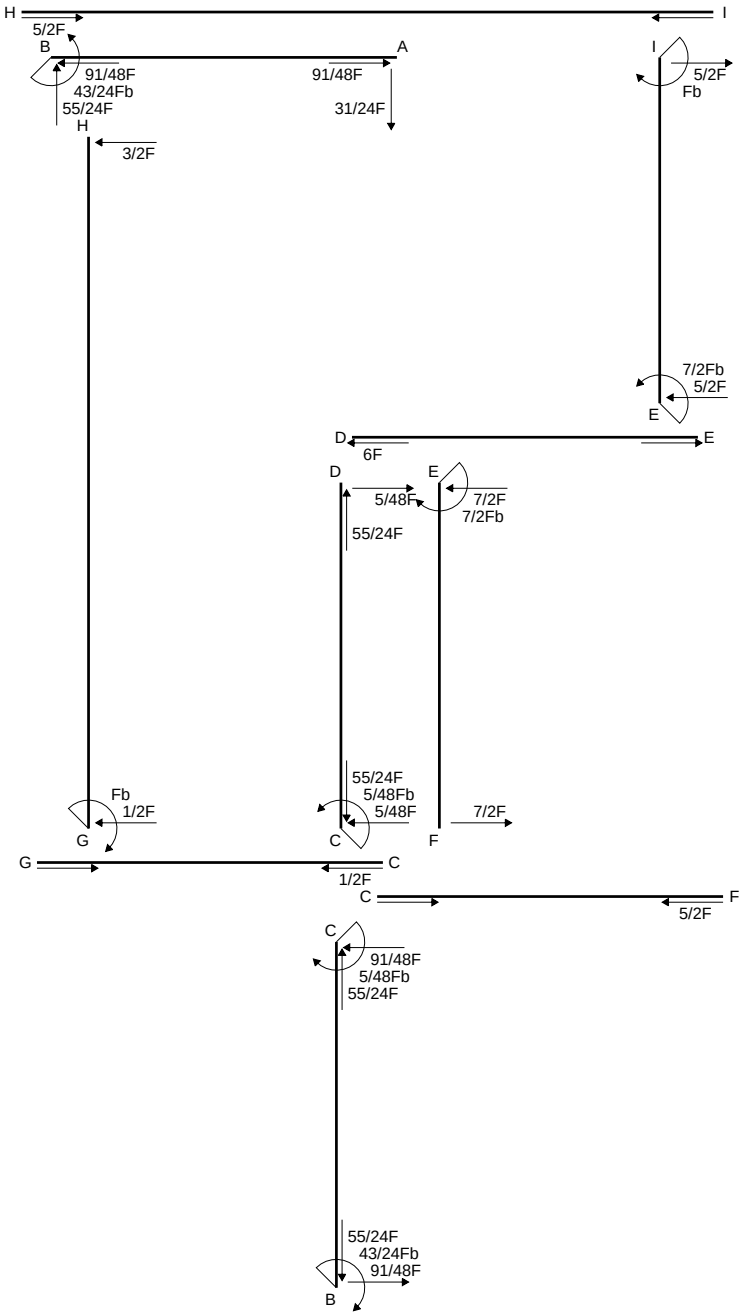
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B
A
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43/24Fb
55/24F
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I
5/2F
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3/2F
7/2Fb
5/2F
E
D
E
6F
D
E
5/48F
7/2F
7/2Fb
55/24F
55/24F
5/48Fb
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Fb
1/2F
7/2F
G
C
F
G
C
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C
91/48F
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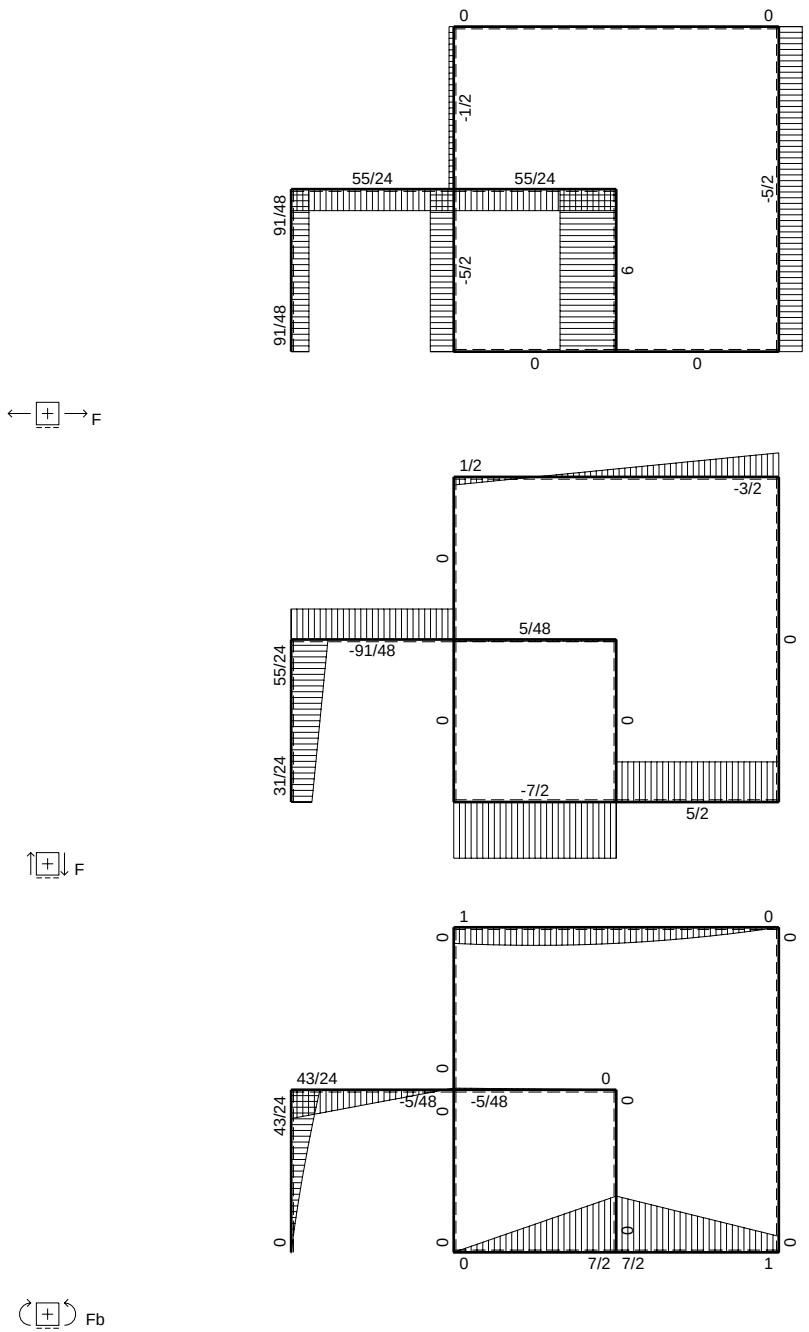
55/24F
55/24F
43/24Fb
91/48F
B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5/2
F
1
0
0
0
0
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-5/48
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0
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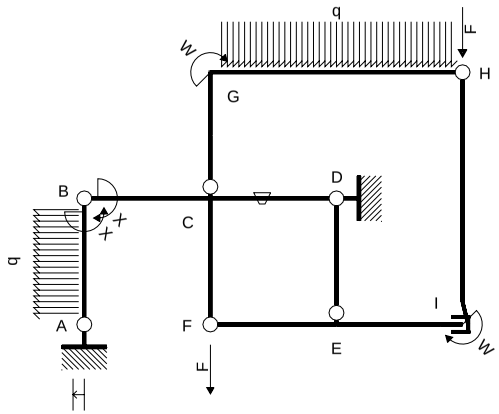
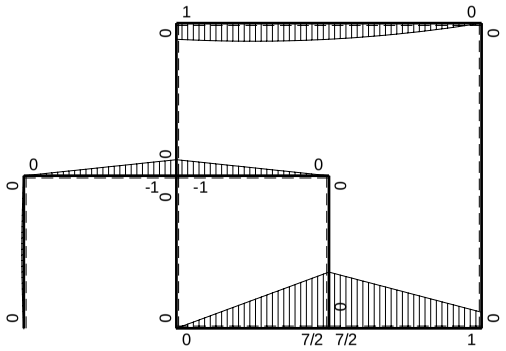
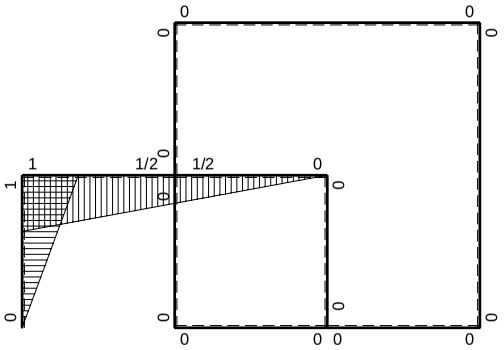
0
0
7/2
7/2
1
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

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Quadro contributi PLV per iperstatica $X=W_{BA}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	x/b	$-1/2Fx+1/2qx^2$	0	$-1/2Fx^2/b+1/2qx^3/b$	0	x^2/b^2	$(-1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
BA b	-1+x/b	$1/2Fx-1/2qx^2$	0	$-1/2Fx+Fx^2/b-1/2qx^3/b$	0	$1-2x/b+x^2/b^2$		
BC b	1-1/2x/b	-Fx	0	$-Fx+1/2Fx^2/b$	0	$1-x/b+1/4x^2/b^2$	$(-1/3+0)Fb^2/EJ$	$7/12Xb/EJ$
CB b	-1/2-1/2x/b	Fb-Fx	0	$-1/2Fb+1/2Fx^2/b$	0	$1/4+1/2x/b+1/4x^2/b^2$		
CD b	1/2-1/2x/b	-Fb+Fx	-Fb/EJ	$-1/2Fb+Fx-1/2Fx^2/b$	$-1/2Fb/EJ+1/2Fx/EJ$	$1/4-1/2x/b+1/4x^2/b^2$	$(-1/6-1/4)Fb^2/EJ$	$1/12Xb/EJ$
DC b	-1/2x/b	Fx	Fb/EJ	$-1/2Fx^2/b$	$-1/2Fx/EJ$	$1/4x^2/b^2$		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EF b	0	7/2Fb-7/2Fx	0	0	0	0	0+0	0
FE b	0	-7/2Fx	0	0	0	0		
FC b	0	0	0	0	0	0	0+0	0
CF b	0	0	0	0	0	0		
CG b	0	0	0	0	0	0	0+0	0
GC b	0	0	0	0	0	0		
GH 2b	0	$Fb+1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
HG 2b	0	$-3/2Fx+1/2qx^2$	0	0	0	0		
HI 2b	0	0	0	0	0	0	0+0	0
IH 2b	0	0	0	0	0	0		
IE b	0	Fb+5/2Fx	0	0	0	0	0+0	0
EI b	0	-7/2Fb+5/2Fx	0	0	0	0		
A	cedimento nodo $-H_{1A}u_A$						$-Fb^2/EJ$	
	totali						$-43/24Fb^2/EJ$	Xb/EJ
	iperstatica $X=W_{BA}$						43/24Fb	

Sviluppi di calcolo iperstatica

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$L^{XX}_{AB} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BA} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{BC} = \int_o^b (1 - x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [\ x -1/2\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b -1/2\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CB} = \int_o^b (1/4 +1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x +1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b +1/4\ b +1/12\ b\ )\ 1/EJ\ = 7/12\ b/EJ$

$L^{XX}_{CD} = \int_o^b (1/4 -1/2\ x/b +1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/4\ x -1/4\ x^2/b +1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/4\ b -1/4\ b +1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{XX}_{DC} = \int_o^b (1/4\ x^2/b^2\ )\ 1/EJ\ dx = [1/12\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/12\ b\ )\ 1/EJ\ = 1/12\ b/EJ$

$L^{Xo}_{AB} = \int_o^b (-1/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/6\ b +1/8\ b\ )\ Fb\ 1/EJ\ = -1/24\ Fb^2/EJ$

$L^{Xo}_{BA} = \int_o^b (-1/2\ x/b + x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [-1/4\ x^2/b +1/3\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (-1/4\ b +1/3\ b -1/8\ b\ )\ Fb\ 1/EJ\ = -1/24\ Fb^2/EJ$

$L^{Xo}_{BC} = \int_o^b (-\ x/b +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x^2/b +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{CB} = \int_o^b (-1/2 +1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/2\ x +1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ$

$= (-1/2\ b +1/6\ b\ )\ Fb\ 1/EJ\ = -1/3\ Fb^2/EJ$

$L^{Xo}_{CD} = \int_o^b (-1/2 + x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1/2 +1/2\ x/b\ )\ \theta\ dx$

$= [-1/2\ x +1/2\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-1/2\ x +1/4\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b +1/2\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-1/2\ b +1/4\ b\ )\ \theta\ = -5/12\ Fb^2/EJ$

$L^{Xo}_{DC} = \int_o^b (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (1/2\ x/b\ )\ \theta\ dx = [-1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/4\ x^2/b\ ]_o^b\ \theta$

$= (-1/6\ b\ )\ Fb\ 1/EJ\ + (1/4\ b\ )\ \theta\ = -5/12\ Fb^2/EJ$

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