

$q_{CD} = -q = -F/b$	$u_D = -\delta = -b^3 F/EJ$	$EJ_{BC} = EJ$
$q_{CE} = -q = -F/b$	$k_A = 4EJ/b$	$EJ_{CD} = EJ$
$\varepsilon_{BC} = -\alpha T = -b^2 F/EJ$	$k_F = 4EJ/b^3$	$EJ_{CE} = EJ$
$\theta_{AB} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{AB} = EJ$	$EJ_{EF} = EJ$

ANALISI STRUTTURE IPERSTATICHE  
PRINCIPIO DEI LAVORI VIRTUALI

Riportare sul fronte:

- 1) Declassamento Scelto
- 2) Reazioni calcolate
- 3) Diagrammi finali delle azioni interne

Sul retro:

- 4) Analisi cinematica
- 5) Diagrammi delle strutture M0 e M\*
- 6) Equazione del PLV
- 7) Reazione vincolare calcolata

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

Carichi di aste curve misurati in proiezione sugli assi x,y.

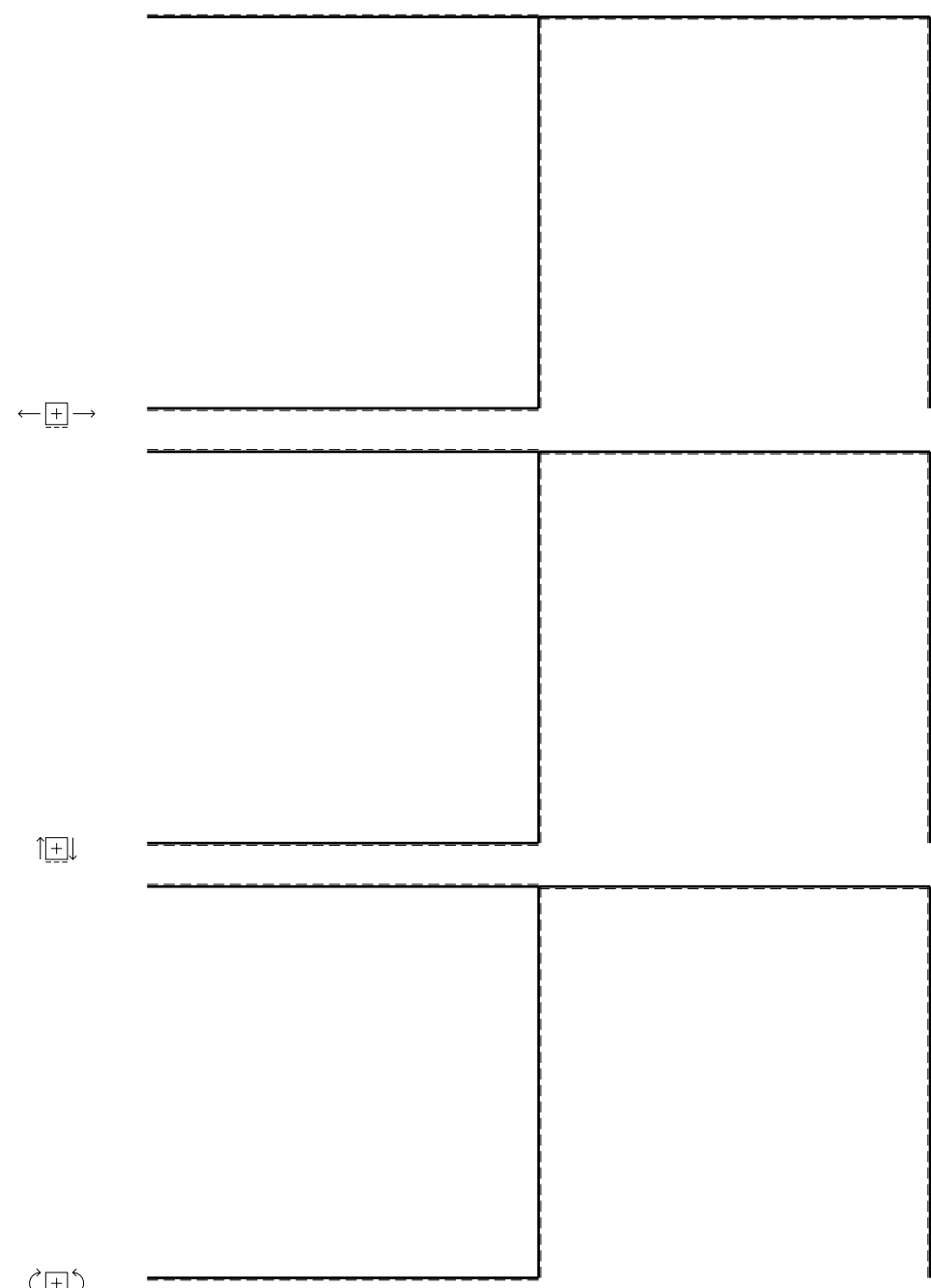
$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

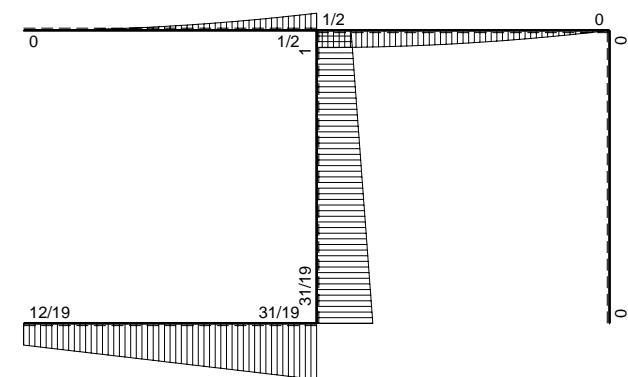
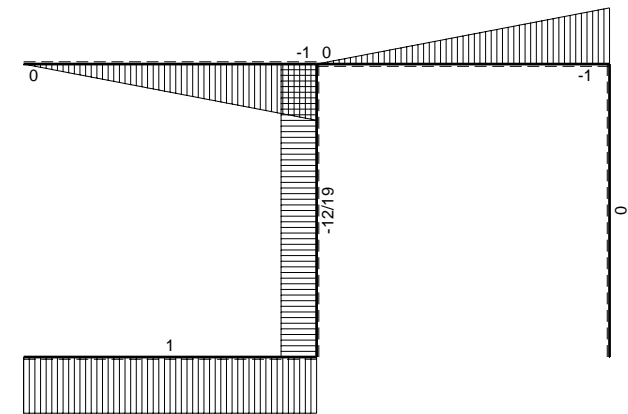
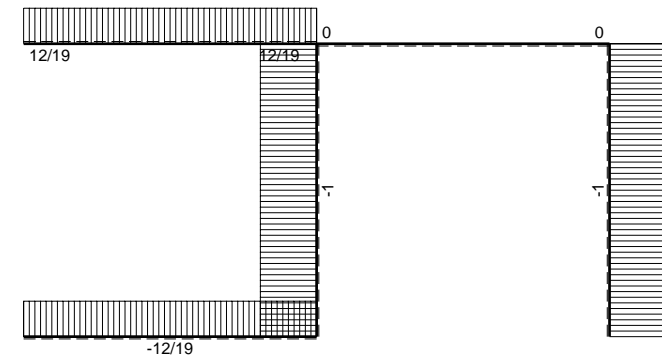
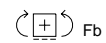
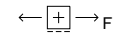
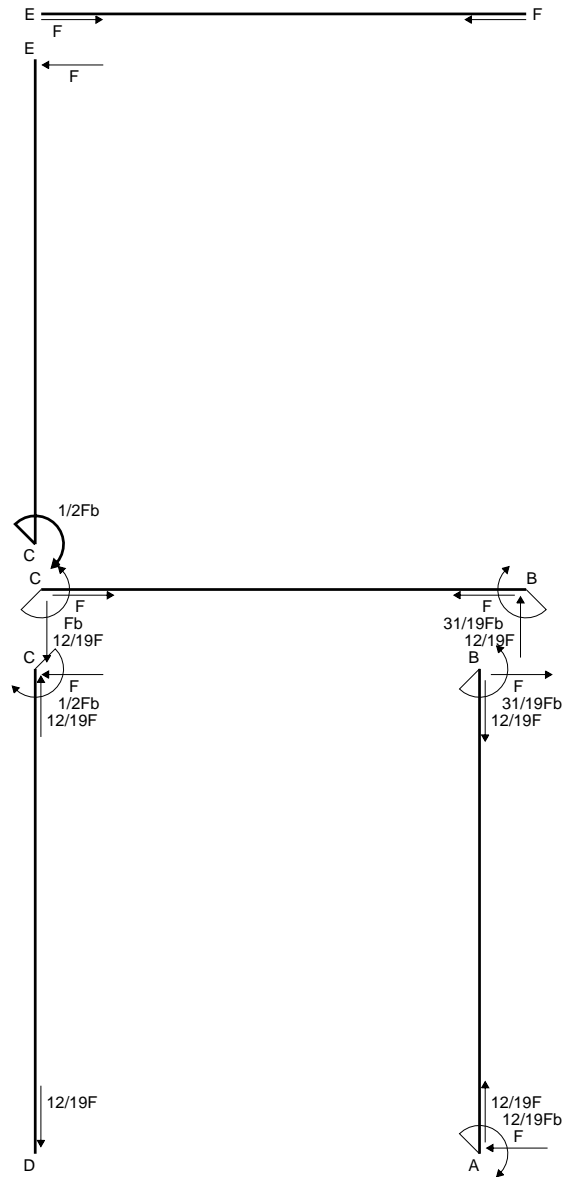
Elongazione termica specifica  $\varepsilon$  assegnata su asta BC.

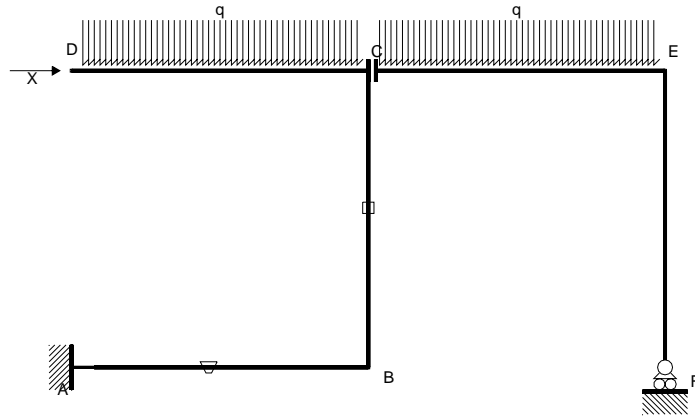
Curvatura  $\theta$  asta AB positiva se convessa a destra con inizio A.

Spostamento orizzontale assoluto u imposto al nodo D.

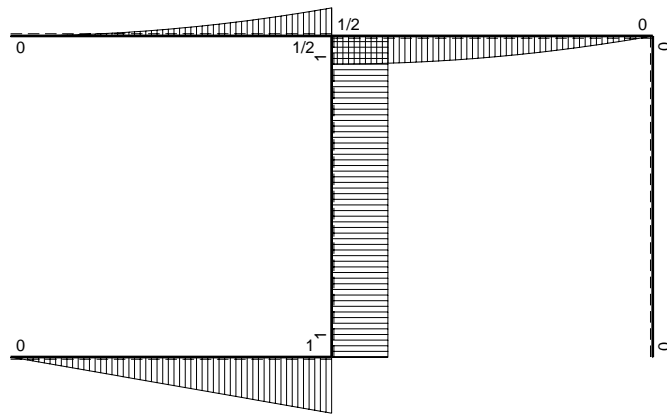
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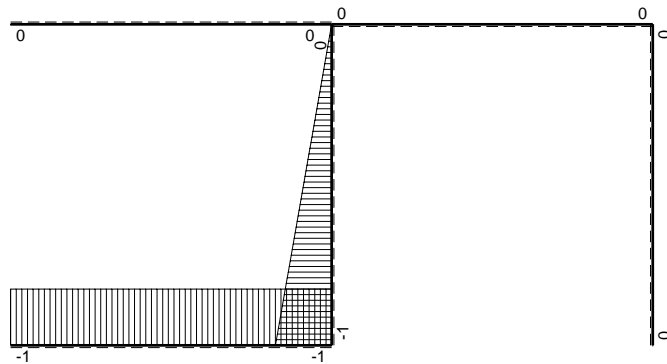




Schema di calcolo iperstatico



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=H<sub>D</sub>

→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ + \theta) dx$	$\int X M_x M_x / EJ dx$	
AB b	-b	Fx	-Fb/EJ	-Fbx	Fb <sup>2</sup> /EJ	b <sup>2</sup>	(-1/2+1)Fb <sup>3</sup> /EJ	Xb <sup>3</sup> /EJ	
BA b	b	-Fb+Fx	Fb/EJ	-Fb <sup>2</sup> +Fbx	Fb <sup>2</sup> /EJ	b <sup>2</sup>	(-1/2+1)Fb <sup>3</sup> /EJ	Xb <sup>3</sup> /EJ	
BC b	-b+x	Fb	0	-Fb <sup>2</sup> +Fbx	0	b <sup>2</sup> -2bx+x <sup>2</sup>	(-1/2+0)Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ	
CB b	x	-Fb	0	-Fbx	0	x <sup>2</sup>	(-1/2+0)Fb <sup>3</sup> /EJ	1/3Xb <sup>3</sup> /EJ	
CD b	0	1/2Fb-Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0	
DC b	0	-1/2qx <sup>2</sup>	0	0	0	0	0+0	0	
CE b	0	1/2Fb-1/2qx <sup>2</sup>	0	0	0	0	0+0	0	
EC b	0	-Fx+1/2qx <sup>2</sup>	0	0	0	0	0+0	0	
EF b	0	0	0	0	0	0	0+0	0	
FE b	0	0	0	0	0	0	0+0	0	
A	molla nodo $-W_{1A}(W_{0A}+XW_{1A})/k_A$							1/4Xb <sup>3</sup> /EJ	
D	cedimento nodo $-H_{1D}u_D$							Fb <sup>3</sup> /EJ	
	totali							Fb <sup>3</sup> /EJ	19/12Xb <sup>3</sup> /EJ
	iperstatica X=H <sub>D</sub>							-12/19F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ = (b) b^2 1/EJ = b^3/EJ$$

$$L_{BA}^{xx} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ = (b) b^2 1/EJ = b^3/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ = (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{xx} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ = (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

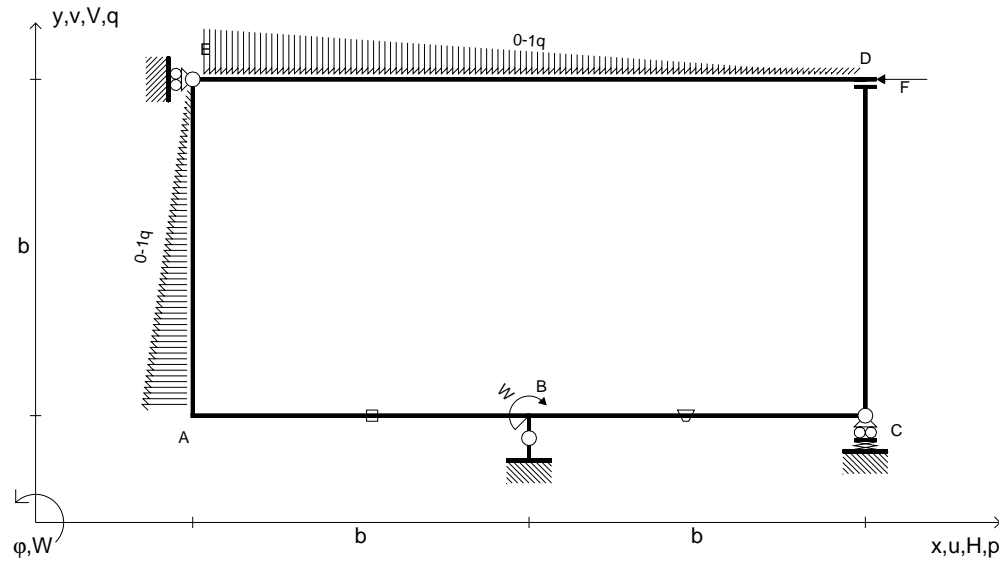
$$L_{AB}^{x0} = \int_0^b (-x/b) Fb^2 1/EJ dx + \int_0^b (1) \theta dx = [-1/2 x^2/b]_0^b Fb^2 1/EJ + [x]_0^b \theta = (-1/2 b) Fb^2 1/EJ + (b) \theta = 1/2 Fb^3/EJ$$

$$L_{BA}^{x0} = \int_0^b (-1 + x/b) Fb^2 1/EJ dx + \int_0^b (-1) \theta dx = [-x + 1/2 x^2/b]_0^b Fb^2 1/EJ + [-x]_0^b \theta = (-b + 1/2 b) Fb^2 1/EJ + (-b) \theta = 1/2 Fb^3/EJ$$

$$L_{BC}^{x0} = \int_0^b (-1 + x/b) Fb^2 1/EJ dx = [-x + 1/2 x^2/b]_0^b Fb^2 1/EJ$$

$$\begin{aligned} &= (-b + 1/2 b) Fb^2 1/EJ = -1/2 Fb^3/EJ \\ L_{CB}^{x_0} &= \int_0^b (-x/b) Fb^2 1/EJ dx = [-1/2 x^2/b]_0^b Fb^2 1/EJ \\ &= (-1/2 b) Fb^2 1/EJ = -1/2 Fb^3/EJ \end{aligned}$$

---



$H_{DE} = -F$	$\theta_{BC} = -\theta = -\alpha T/b = -bF/EJ$	$EJ_{CD} = EJ$
$W_B = -W = -Fb$	$V_E = -\delta = -b^3 F/EJ$	$EJ_{DE} = EJ$
$P_{AE} = -q = -F/b$	$k_C = 4EJ/b^3$	$EJ_{EA} = EJ$
$q_{ED} = -q = -F/b$	$EJ_{AB} = EJ$	
$\epsilon_{AB} = -\alpha T = -b^2 F/EJ$	$EJ_{BC} = EJ$	

ANALISI STRUTTURE IPERSTATICHE  
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Carichi di aste curve misurati in proiezione sugli assi x,y.

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Elongazione termica specifica  $\epsilon$  assegnata su asta AB.

Curvatura  $\theta$  asta BC positiva se convessa a destra con inizio B.

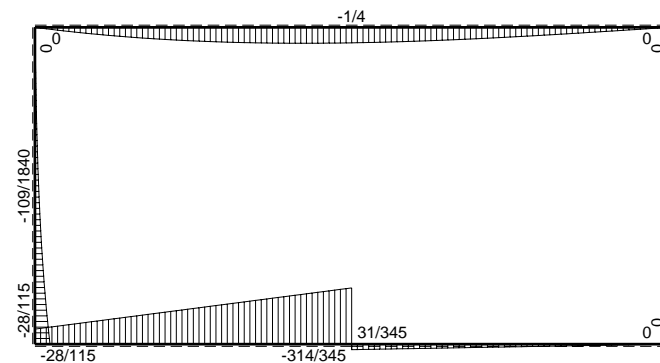
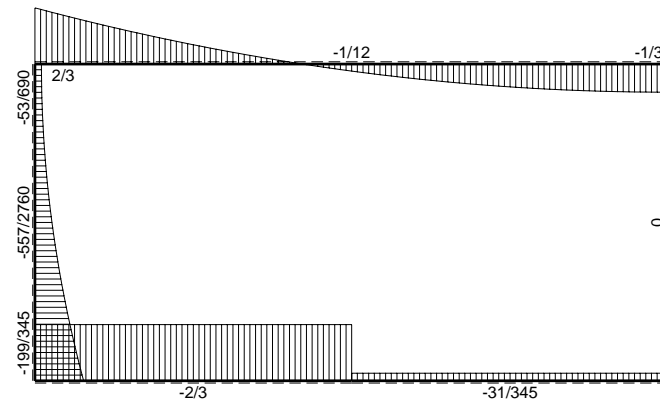
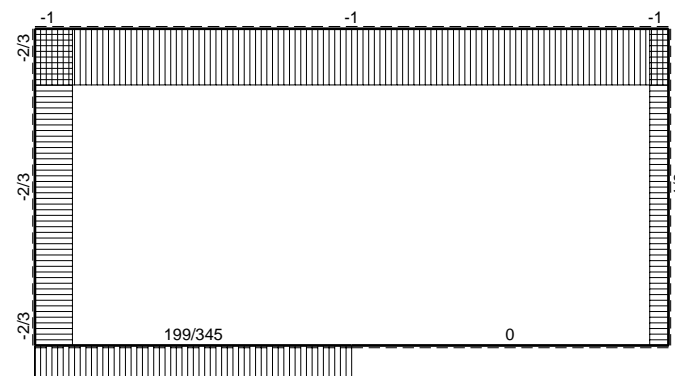
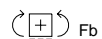
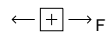
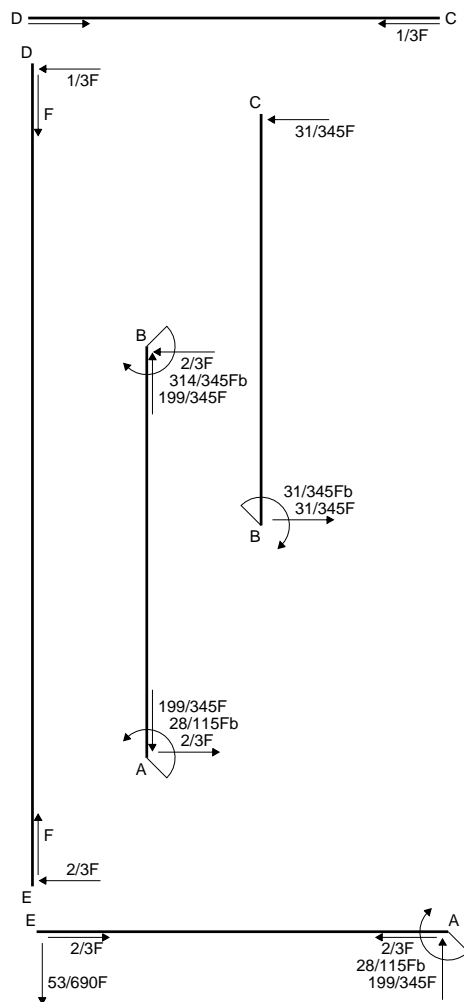
Spostamento verticale assoluto v imposto al nodo E.

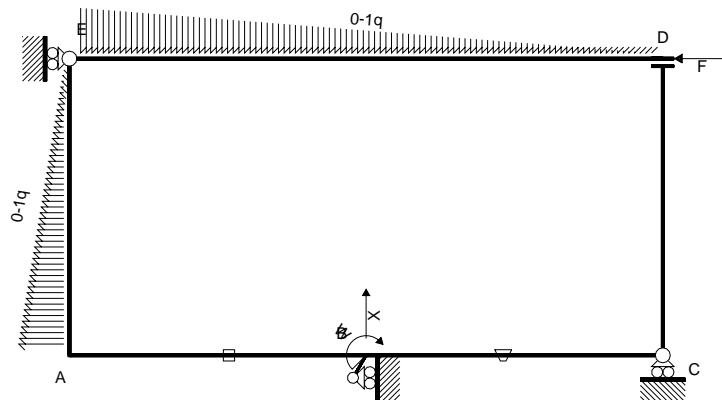
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$\leftarrow \boxed{+} \rightarrow$

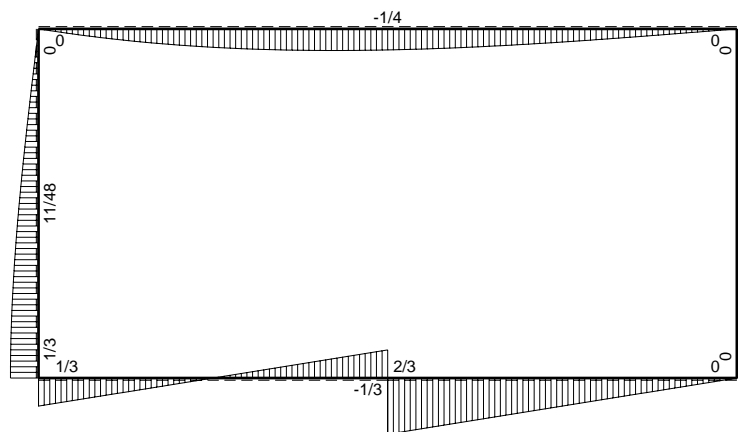
$\uparrow \boxed{+} \downarrow$

$\left( \boxed{+} \right)$

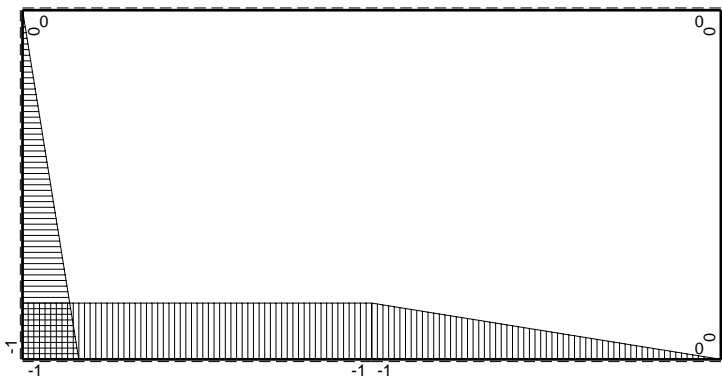




Schema di calcolo iperstatico



$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=V_B$ 

→	$M_x(x)$	$M_o(x)$	$\theta$	$M_x M_o$	$M_x \theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int X M_x M_x/EJ dx$	
AB b	-b	$1/3Fb-2/3Fx$	0	$-1/3Fb^2+2/3Fbx$	0	$b^2$	0+0	$Xb^3/EJ$	
BA b	b	$1/3Fb-2/3Fx$	0	$1/3Fb^2-2/3Fbx$	0	$b^2$			
BC b	-b+x	$2/3Fb-2/3Fx$	-Fb/EJ	$-2/3Fb^2+4/3Fbx-2/3Fx^2$	$Fb^2/EJ-Fxb/EJ$	$b^2-2bx+x^2$	$(-2/9+1/2)Fb^3/EJ$	$1/3Xb^3/EJ$	
CB b	x	$-2/3Fx$	Fb/EJ	$-2/3Fx^2$	$Fxb/EJ$	$x^2$			
CD b	0	0	0	0	0	0	0+0	0	
DC b	0	0	0	0	0	0			
DE 2b	0	$-1/3Fx+1/12qx^3/b$	0	0	0	0	0+0	0	
ED 2b	0	$2/3Fx-1/2qx^2+1/12qx^3/b$	0	0	0	0			
EA b	-x	$1/2Fx-1/6qx^3/b$	0	$-1/2Fx^2+1/6qx^4/b$	0	$x^2$	$(-2/15+0)Fb^3/EJ$	$1/3Xb^3/EJ$	
AE b	b-x	$-1/3Fb+1/2qx^2-1/6qx^3/b$	0	$-1/3Fb^2+1/3Fbx+1/2Fx^2-2/3qx^3+1/6qx^4/b$	0	$b^2-2bx+x^2$			
AB	elongazione asta $N_{1AB} \epsilon_{AB} L_{AB}$							$-Fb^3/EJ$	
C	molla nodo $-V_{1C}(V_{oC}+XV_{1C})/k_C$							$-1/4Fb^3/EJ$	$1/4Xb^3/EJ$
	totali							$-199/180Fb^3/EJ$	$23/12Xb^3/EJ$
	iperstatica $X=V_B$							199/345F	

Sviluppi di calcolo iperstatica

$$L_{AB}^{XX} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{BA}^{XX} = \int_0^b (1) b^2 1/EJ dx = [x]_0^b b^2 1/EJ$$

$$= (b) b^2 1/EJ = b^3/EJ$$

$$L_{BC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{CB}^{XX} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{EA}^{XX} = \int_0^b (x^2/b^2) b^2 1/EJ dx = [1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$

$$L_{AE}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) b^2 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b b^2 1/EJ$$

$$= (b - b + 1/3 b) b^2 1/EJ = 1/3 b^3/EJ$$



$$\begin{aligned}
 L_{AB}^{x_0} &= \int_0^b (-1/3 + 2/3 x/b) Fb^2 1/EJ dx + 1 \quad (-1) \quad 1 \quad Fb^3/EJ \\
 &= [-1/3 x + 1/3 x^2/b]_0^b Fb^2 1/EJ + 1 \quad (-1) \quad 1 \quad Fb^3/EJ \\
 &= (-1/3 b + 1/3 b) Fb^2 1/EJ + 1 \quad (-1) \quad 1 \quad Fb^3/EJ = - Fb^3/EJ
 \end{aligned}$$

$$\begin{aligned}
 L_{BA}^{x_0} &= \int_0^b (1/3 - 2/3 x/b) Fb^2 1/EJ dx + 1 \quad (-1) \quad 1 \quad Fb^3/EJ \\
 &= [1/3 x - 1/3 x^2/b]_0^b Fb^2 1/EJ + 1 \quad (-1) \quad 1 \quad Fb^3/EJ \\
 &= (1/3 b - 1/3 b) Fb^2 1/EJ + 1 \quad (-1) \quad 1 \quad Fb^3/EJ = - Fb^3/EJ
 \end{aligned}$$

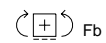
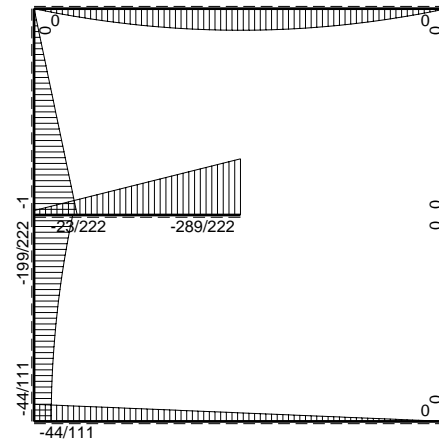
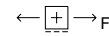
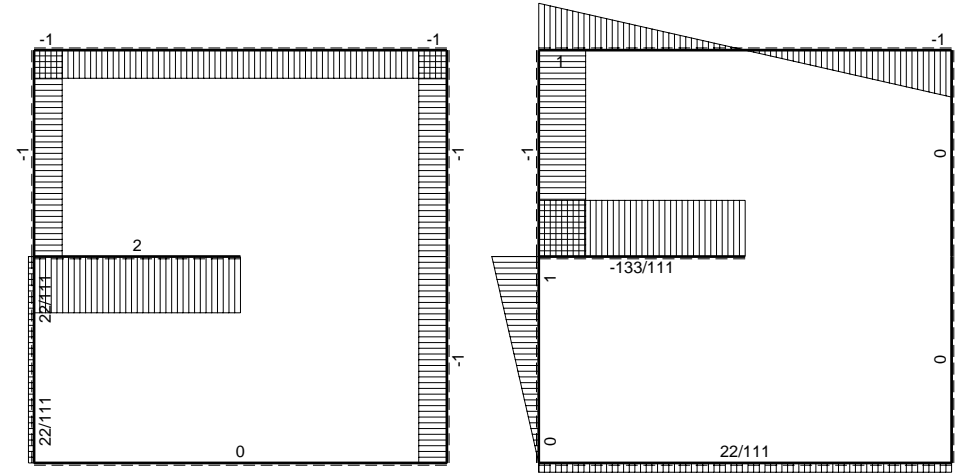
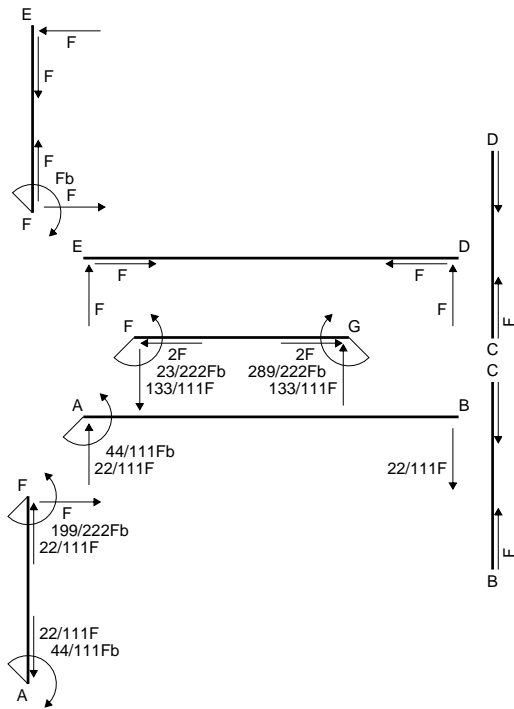
$$\begin{aligned}
 L_{BC}^{x_0} &= \int_0^b (-2/3 + 4/3 x/b - 2/3 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (1 - x/b) \theta dx \\
 &= [-2/3 x + 2/3 x^2/b - 2/9 x^3/b^2]_0^b Fb^2 1/EJ + [x - 1/2 x^2/b]_0^b \theta \\
 &= (-2/3 b + 2/3 b - 2/9 b) Fb^2 1/EJ + (b - 1/2 b) \theta = 5/18 Fb^3/EJ
 \end{aligned}$$

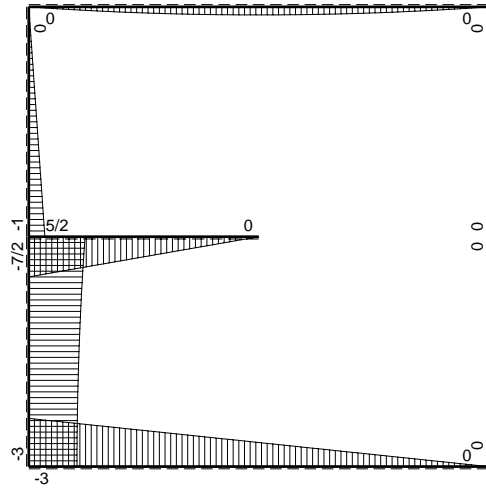
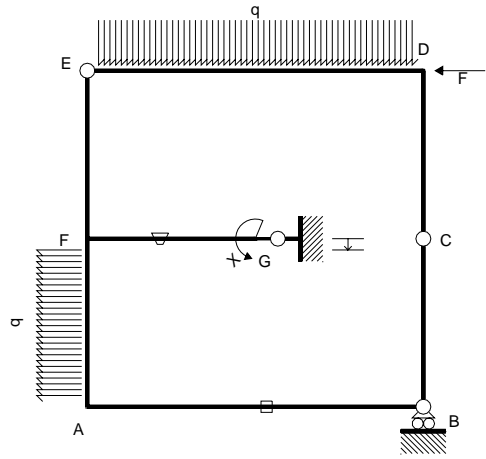
$$\begin{aligned}
 L_{CB}^{x_0} &= \int_0^b (-2/3 x^2/b^2) Fb^2 1/EJ dx + \int_0^b (-x/b) \theta dx = [-2/9 x^3/b^2]_0^b Fb^2 1/EJ + [-1/2 x^2/b]_0^b \theta \\
 &= (-2/9 b) Fb^2 1/EJ + (-1/2 b) \theta = 5/18 Fb^3/EJ
 \end{aligned}$$

$$\begin{aligned}
 L_{EA}^{x_0} &= \int_0^b (-1/2 x^2/b^2 + 1/6 x^4/b^4) Fb^2 1/EJ dx = [-1/6 x^3/b^2 + 1/30 x^5/b^4]_0^b Fb^2 1/EJ \\
 &= (-1/6 b + 1/30 b) Fb^2 1/EJ = -2/15 Fb^3/EJ
 \end{aligned}$$

$$\begin{aligned}
 L_{AE}^{x_0} &= \int_0^b (-1/3 + 1/3 x/b + 1/2 x^2/b^2 - 2/3 x^3/b^3 + 1/6 x^4/b^4) Fb^2 1/EJ dx \\
 &= [-1/3 x + 1/6 x^2/b + 1/6 x^3/b^2 - 1/6 x^4/b^3 + 1/30 x^5/b^4]_0^b Fb^2 1/EJ \\
 &= (-1/3 b + 1/6 b + 1/6 b - 1/6 b + 1/30 b) Fb^2 1/EJ = -2/15 Fb^3/EJ
 \end{aligned}$$

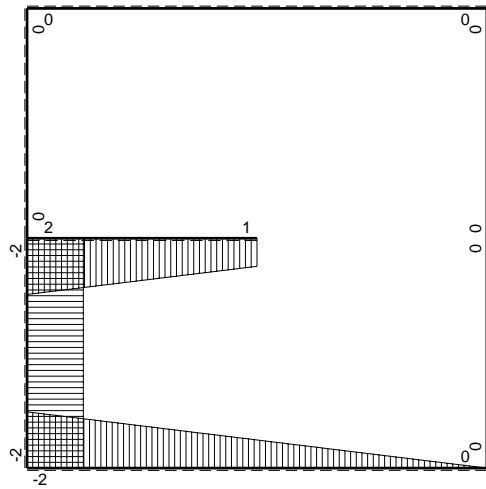






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contribuiti PLV per iperstatica X=W<sub>G</sub>

→	$M_x(x)$	$M_0(x)$	$\theta$	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x (M_0/EJ+\theta) dx$	$\int X M_x M_x / EJ dx$
AB 2b	$-2+x/b$	$-3Fb+3/2Fx$	0	$6Fb-6Fx+3/2Fx^2/b$	0	$4-4x/b+x^2/b^2$	$(4+0)Fb^2/EJ$	$8/3Xb/EJ$
BA 2b	$x/b$	$3/2Fx$	0	$3/2Fx^2/b$	0	$x^2/b^2$	0	0
BC b	0	0	0	0	0	0	0+0	0
CB b	0	0	0	0	0	0	0+0	0
CD b	0	0	0	0	0	0	0+0	0
DC b	0	0	0	0	0	0	0+0	0
DE 2b	0	$-Fx+1/2qx^2$	0	0	0	0	0+0	0
ED 2b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
EF b	0	$-Fx$	0	0	0	0	0+0	0
FE b	0	$Fb-Fx$	0	0	0	0	0+0	0
FG b	$2-x/b$	$5/2Fb-5/2Fx$	$-Fb/EJ$	$5Fb-15/2Fx+5/2Fx^2/b$	$-2Fb/EJ+Fx/EJ$	$4-4x/b+x^2/b^2$	$(25/12-3/2)Fb^2/EJ$	$7/3Xb/EJ$
GF b	$-1-x/b$	$-5/2Fx$	$Fb/EJ$	$5/2Fx+5/2Fx^2/b$	$-Fb/EJ-Fx/EJ$	$1+2x/b+x^2/b^2$	$(19/3+0)Fb^2/EJ$	$4Xb/EJ$
FA b	-2	$-7/2Fb+Fx-1/2qx^2$	0	$7Fb-2Fx+Fx^2/b$	0	4	0	0
AF b	2	$3Fb+1/2qx^2$	0	$6Fb+Fx^2/b$	0	4	0	0
B	molla nodo $-V_{1B}(V_{0B}+XV_{1B})/k_B$						$1/8Fb^2/EJ$	$1/4Xb/EJ$
G	cedimento nodo $-V_{1G}U_G$						$Fb^2/EJ$	
	totali						$289/24Fb^2/EJ$	$37/4Xb/EJ$
	iperstatica X=W <sub>G</sub>						$-289/222Fb$	

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^{2b} (4 - 4x/b + x^2/b^2) 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^{2b} 1/EJ$$

$$= (8b - 8b + 8/3 b) 1/EJ = 8/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^{2b} (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^{2b} 1/EJ$$

$$= (8/3 b) 1/EJ = 8/3 b/EJ$$

$$L_{FG}^{xx} = \int_0^b (4 - 4x/b + x^2/b^2) 1/EJ dx = [4x - 2x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (4b - 2b + 1/3 b) 1/EJ = 7/3 b/EJ$$

$$L_{GF}^{xx} = \int_0^b (1 + 2x/b + x^2/b^2) 1/EJ dx = [x + x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b + b + 1/3 b) 1/EJ = 7/3 b/EJ$$

$$L_{FA}^{xx} = \int_0^b (4) 1/EJ dx = [4x]_0^b 1/EJ$$

$$= (4b) 1/EJ = 4 b/EJ$$

$$L_{AF}^{xx} = \int_0^b (4) 1/EJ dx = [4x]_0^b 1/EJ$$

$$= (4b) 1/EJ = 4 b/EJ$$

$$L_{AB}^{xo} = \int_0^{2b} (6 - 6x/b + 3/2 x^2/b^2) Fb 1/EJ dx = [6x - 3x^2/b + 1/2 x^3/b^2]_0^{2b} Fb 1/EJ$$

$$= (12b - 12b + 4b) Fb 1/EJ = 4 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^{2b} (3/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^3/b^2]_0^{2b} Fb 1/EJ$$

$$= (4b) Fb 1/EJ = 4 Fb^2/EJ$$

$$L_{FG}^{xo} = \int_0^b (5 - 15/2 x/b + 5/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-2 + x/b) \theta dx$$

$$= [5x - 15/4 x^2/b + 5/6 x^3/b^2]_0^b Fb 1/EJ + [-2x + 1/2 x^2/b]_0^b \theta$$

$$= (5b - 15/4 b + 5/6 b) Fb 1/EJ + (-2b + 1/2 b) \theta = 7/12 Fb^2/EJ$$

$$L_{GF}^{xo} = \int_0^b (5/2 x/b + 5/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 + x/b) \theta dx$$

$$= [5/4 x^2/b + 5/6 x^3/b^2]_0^b Fb 1/EJ + [x + 1/2 x^2/b]_0^b \theta$$

$$= (5/4 b + 5/6 b) Fb 1/EJ + (b + 1/2 b) \theta = 7/12 Fb^2/EJ$$

$$L_{FA}^{xo} = \int_0^b (7 - 2x/b + x^2/b^2) Fb 1/EJ dx = [7x - x^2/b + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (7b - b + 1/3 b) Fb 1/EJ = 19/3 Fb^2/EJ$$

$$L_{AF}^{xo} = \int_0^b (6 + x^2/b^2) Fb 1/EJ dx = [6x + 1/3 x^3/b^2]_0^b Fb 1/EJ$$

$$= (6b + 1/3 b) Fb 1/EJ = 19/3 Fb^2/EJ$$