

$$\begin{aligned}
 H_{DB} &= -F & K_{AE} &= 4EJ/b \\
 W_A &= -W = -Fb & E_{J_{AB}} &= EJ \\
 q_{CF} &= -q = -F/b & E_{J_{BC}} &= EJ \\
 \theta_{FA} &= -\theta = -\alpha T/b = -bF/EJ & E_{J_{DB}} &= EJ
 \end{aligned}$$

Riconsegnare questo solo foglio.

Fronte: indicazione iperstatica e valore trovato, diagrammi finali con quote frazionarie (momento dal lato fibre tese), $y_G, J_x, J_x^*, \sigma_{max}z_c, \tau_{max}$, $\sigma_{max}z_{max}$.

Retro: analisi cinematica, diagrammi M_o, M^* e N^* , equazione lavori virtuali.

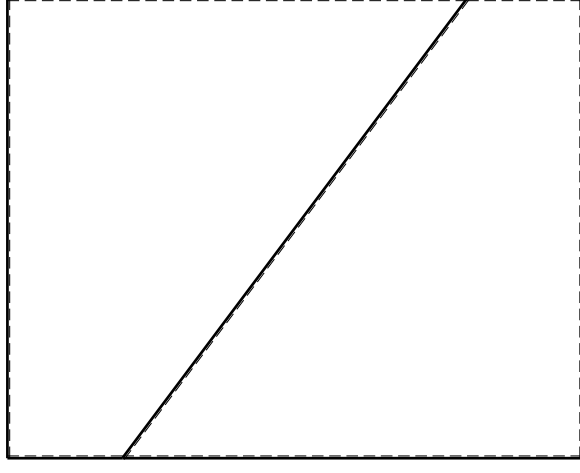
Carichi e deformazioni date hanno verso efficace in disegno. Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

$J_{yz} - x_{yz} - \theta_{yz}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta FA positiva se convessa a destra con inizio F.

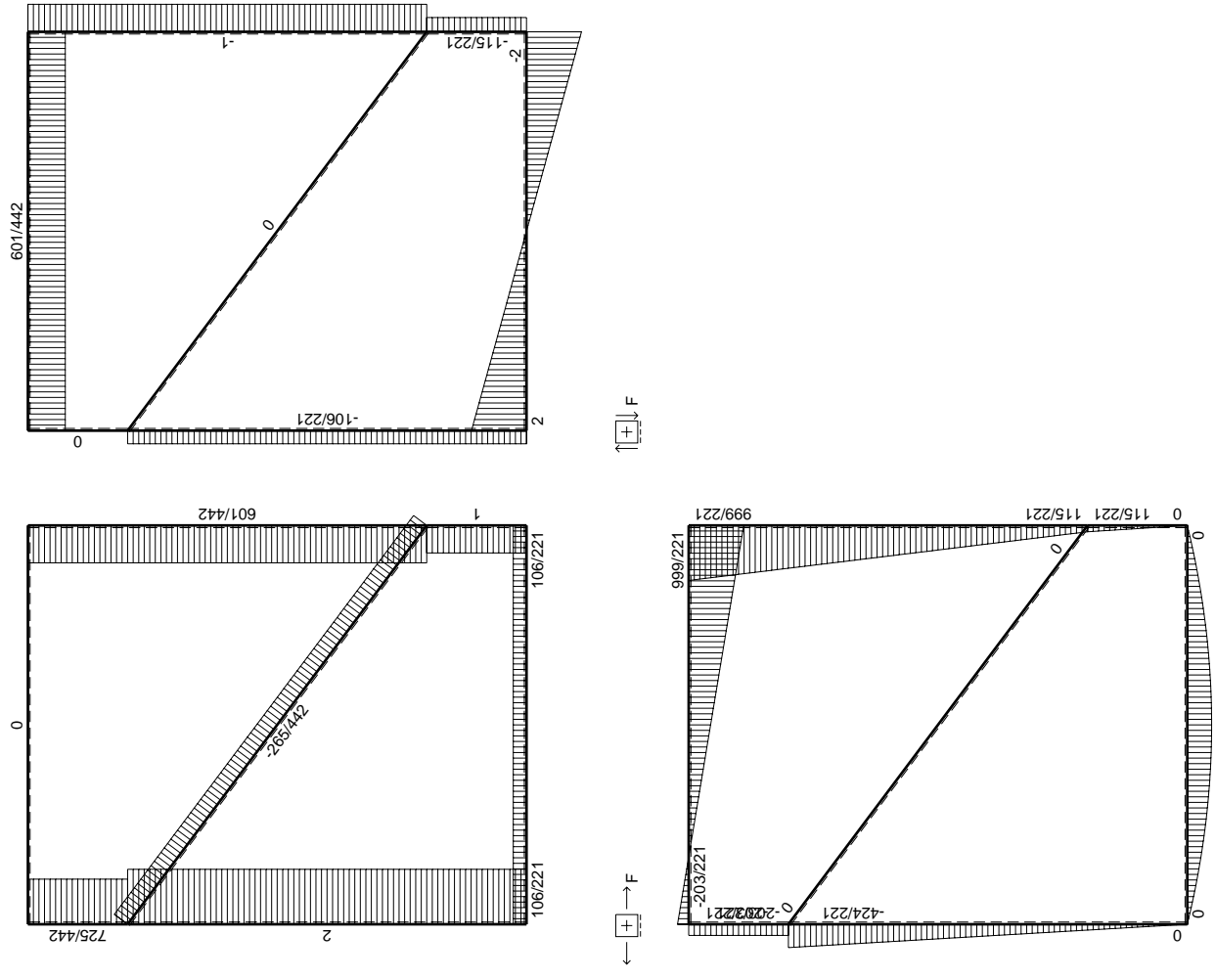
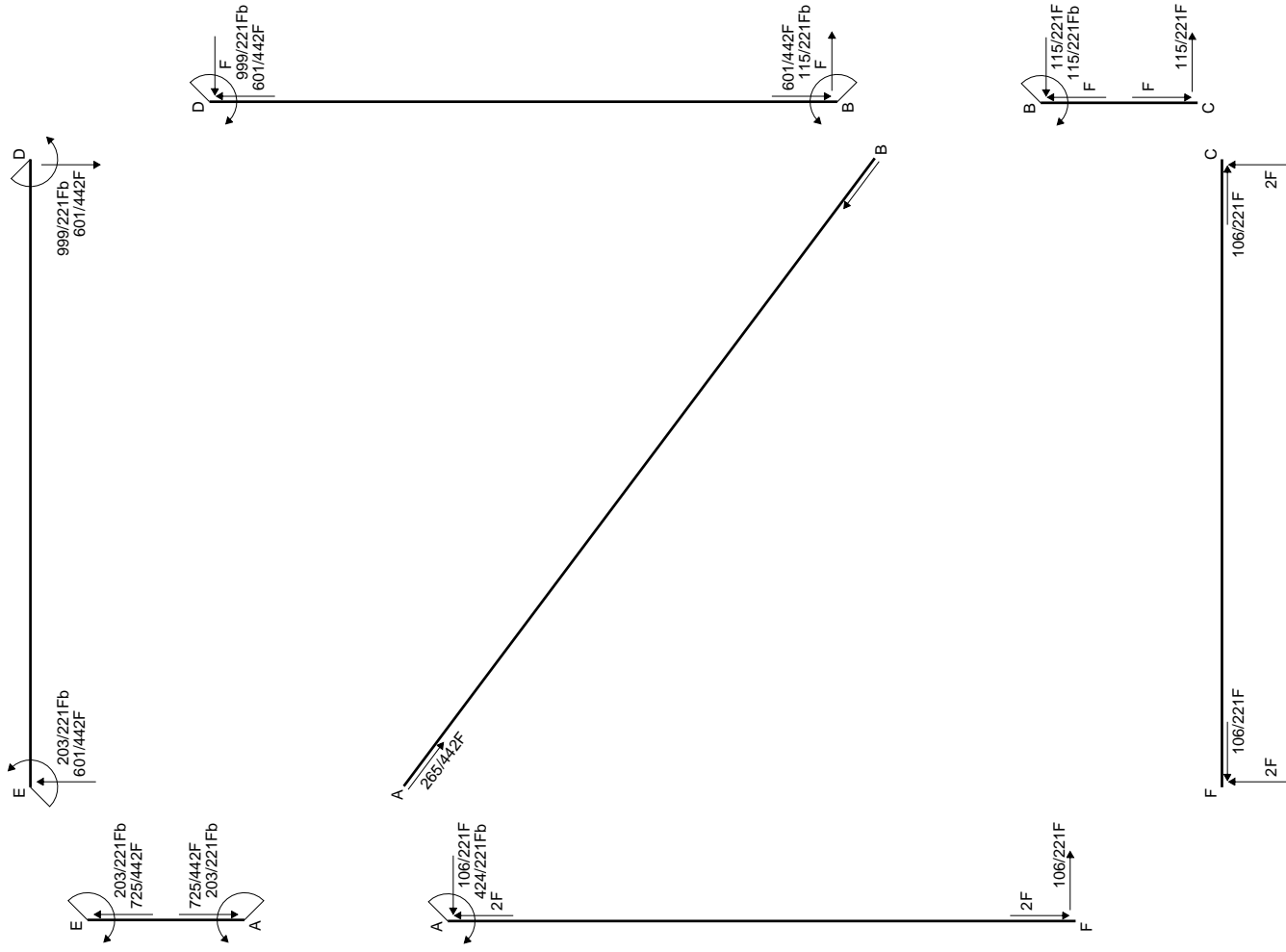
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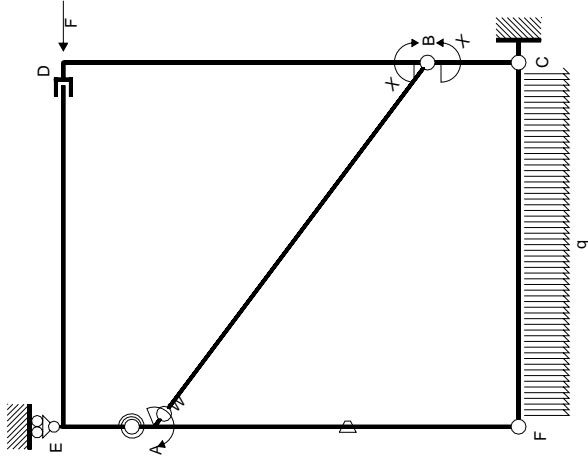
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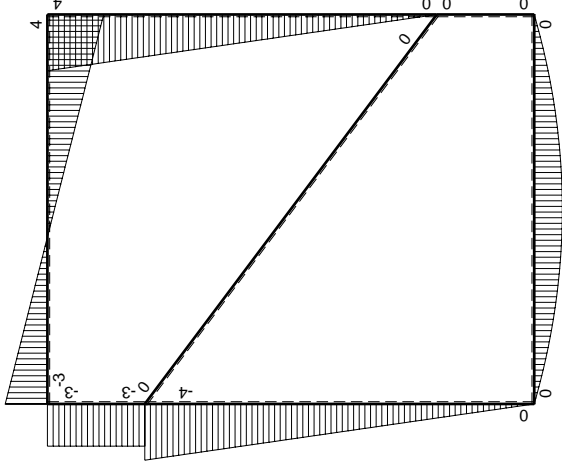
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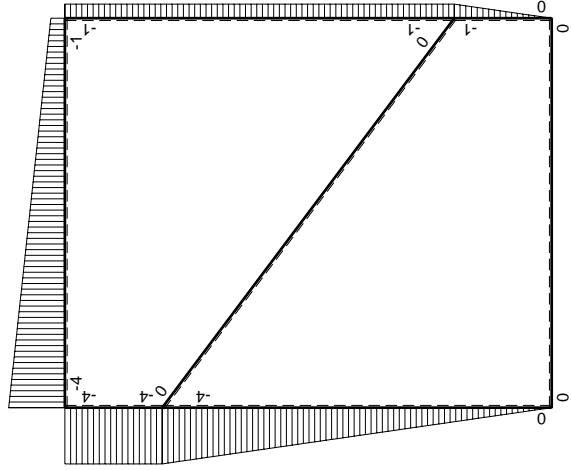
Fb



Schema di calcolo iperstatico



M₀ flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{BC}

←	M ^x (x)	M ₀ (x)	θ	M ^x M ₀	M ^x θ	M ^x M _x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_x M_x/EJdx$	AB 5b	BA 5b	BC b	CB b	DB 4b	BD 4b	ED 4b	DE 4b	EA b	AE b	FA 4b	AF 4b	CF 4b	FC 4b	AE	totali	iperstatica X=W _{BC}																																	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	-x/b	4/b	0	0	3Fb ² /EJ	115/3Fb ² /EJ	-115/221Fb																																
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12Fb	-12Fb	-Fb/EJ	F ² /b	0	0	0	4Fb/EJ-F ² /EJ	16-8x/b+x ² /b ²	(64/3+8)Fb ² /EJ	64/3Xb/EJ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0														
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0												
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0								
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
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	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	x/b	1/b	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Sviluppi di calcolo iperstatica

$$L_{BC}^{XX} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{CB}^{XX} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{DB}^{XX} = \int_0^{4b} (1) 1/EJ dx = [x]_0^{4b} 1/EJ$$

$$= (4b) 1/EJ = 4 b/EJ$$

$$L_{BD}^{XX} = \int_0^{4b} (1) 1/EJ dx = [x]_0^{4b} 1/EJ$$

$$= (4b) 1/EJ = 4 b/EJ$$

$$L_{ED}^{XX} = \int_0^{4b} (16 - 6x/b + 9/16 x^2/b^2) 1/EJ dx = [16x - 3x^2/b + 3/16 x^3/b^2]_0^{4b} 1/EJ$$

$$= (64b - 48b + 12b) 1/EJ = 28 b/EJ$$

$$L_{DE}^{XX} = \int_0^{4b} (1 + 3/2 x/b + 9/16 x^2/b^2) 1/EJ dx = [x + 3/4 x^2/b + 3/16 x^3/b^2]_0^{4b} 1/EJ$$

$$= (4b + 12b + 12b) 1/EJ = 28 b/EJ$$

$$L_{AE}^{XX} = \int_0^b (16) 1/EJ dx + 4 \cdot 4 \cdot 1/4 b/EJ = [16x]_0^b 1/EJ + 4 \cdot 4 \cdot 1/4 b/EJ$$

$$= (16b) 1/EJ + 4 \cdot 4 \cdot 1/4 b/EJ = 20 b/EJ$$

$$L_{EA}^{XX} = \int_0^b (16) 1/EJ dx + 4 \cdot 4 \cdot 1/4 b/EJ = [16x]_0^b 1/EJ + 4 \cdot 4 \cdot 1/4 b/EJ$$

$$= (16b) 1/EJ + 4 \cdot 4 \cdot 1/4 b/EJ = 20 b/EJ$$

$$L_{FA}^{XX} = \int_0^{4b} (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^{4b} 1/EJ$$

$$= (64/3 b) 1/EJ = 64/3 b/EJ$$

$$L_{AF}^{XX} = \int_0^{4b} (16 - 8x/b + x^2/b^2) 1/EJ dx = [16x - 4x^2/b + 1/3 x^3/b^2]_0^{4b} 1/EJ$$

$$= (64b - 64b + 64/3 b) 1/EJ = 64/3 b/EJ$$

$$L_{DB}^{Xo} = \int_0^{4b} (-4 + x/b) Fb 1/EJ dx = [-4x + 1/2 x^2/b]_0^{4b} Fb 1/EJ$$

$$= (-16b + 8b) Fb 1/EJ = -8 Fb^2/EJ$$

$$L_{BD}^{Xo} = \int_0^{4b} (-x/b) Fb 1/EJ dx = [-1/2 x^2/b]_0^{4b} Fb 1/EJ$$

$$= (-8b) Fb 1/EJ = -8 Fb^2/EJ$$

$$L_{ED}^{Xo} = \int_0^{4b} (12 - 37/4 x/b + 21/16 x^2/b^2) Fb 1/EJ dx = [12x - 37/8 x^2/b + 7/16 x^3/b^2]_0^{4b} Fb 1/EJ$$

$$= (48b - 74b + 28b) Fb 1/EJ = 2 Fb^2/EJ$$

$$L_{DE}^{Xo} = \int_0^{4b} (-4 - 5/4 x/b + 21/16 x^2/b^2) Fb 1/EJ dx = [-4x - 5/8 x^2/b + 7/16 x^3/b^2]_0^{4b} Fb 1/EJ$$

$$= (-16b - 10b + 28b) Fb 1/EJ = 2 Fb^2/EJ$$

$$L_{AE}^{Xo} = \int_0^b (12) Fb 1/EJ dx + 4 \cdot 3 \cdot 1/4 Fb^2/EJ = [12x]_0^b Fb 1/EJ + 4 \cdot 3 \cdot 1/4 Fb^2/EJ$$

$$= (12b) Fb 1/EJ + 4 \cdot 3 \cdot 1/4 Fb^2/EJ = 15 Fb^2/EJ$$

$$L_{EA}^{Xo} = \int_0^b (12) Fb 1/EJ dx + 4 \cdot 3 \cdot 1/4 Fb^2/EJ = [12x]_0^b Fb 1/EJ + 4 \cdot 3 \cdot 1/4 Fb^2/EJ$$

$$= (12b) Fb 1/EJ + 4 \cdot 3 \cdot 1/4 Fb^2/EJ = 15 Fb^2/EJ$$

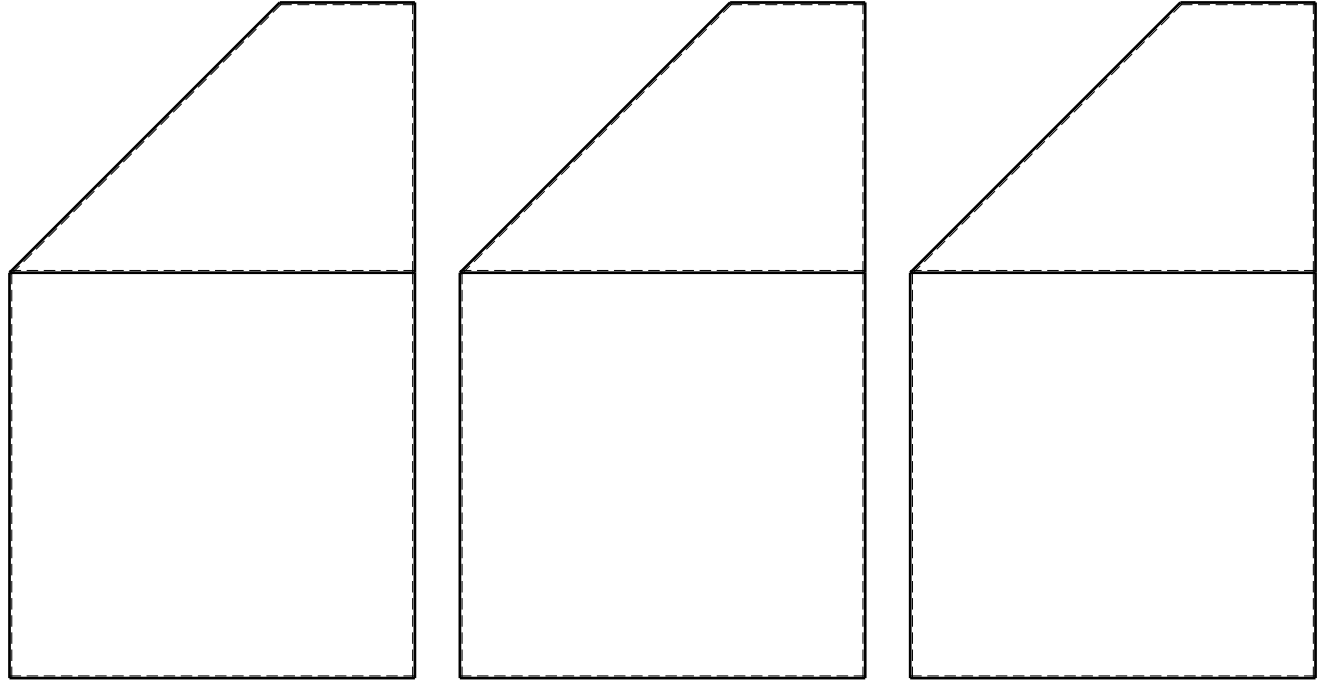
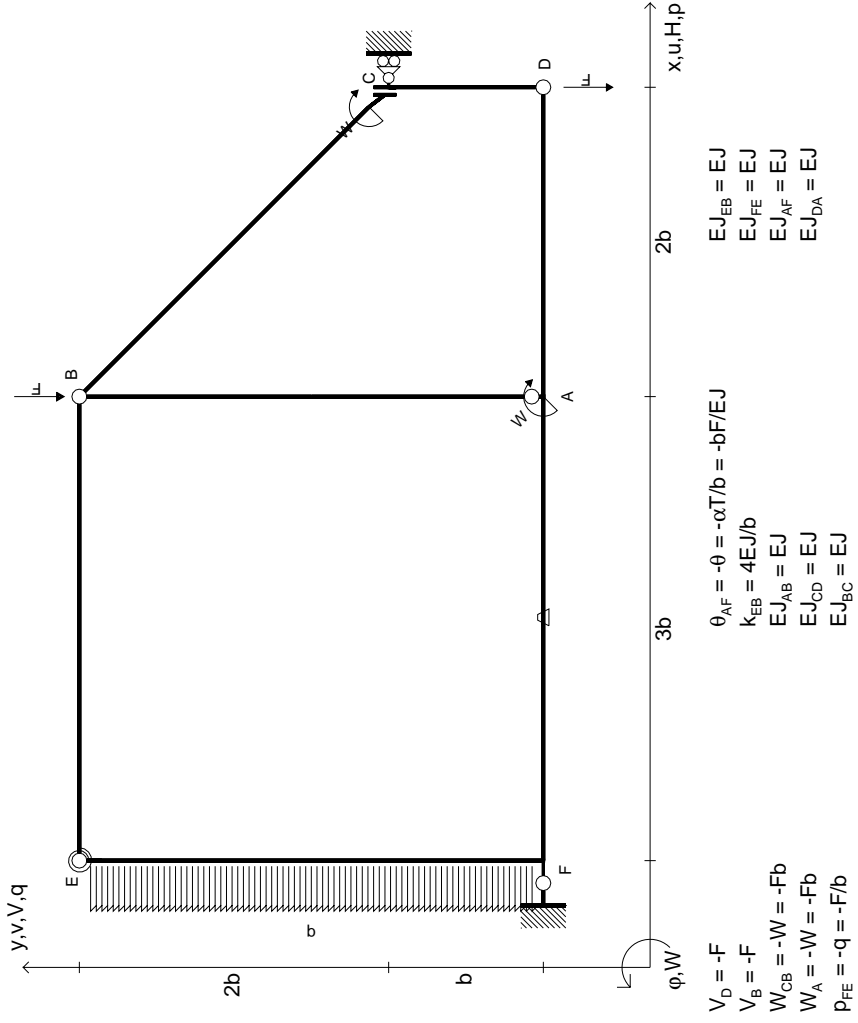
$$L_{FA}^{Xo} = \int_0^{4b} (x^2/b^2) Fb 1/EJ dx + \int_0^{4b} (x/b) \theta dx = [1/3 x^3/b^2]_0^{4b} Fb 1/EJ + [1/2 x^2/b]_0^{4b} \theta$$

$$= (64/3 b) Fb 1/EJ + (8b) \theta = 88/3 Fb^2/EJ$$

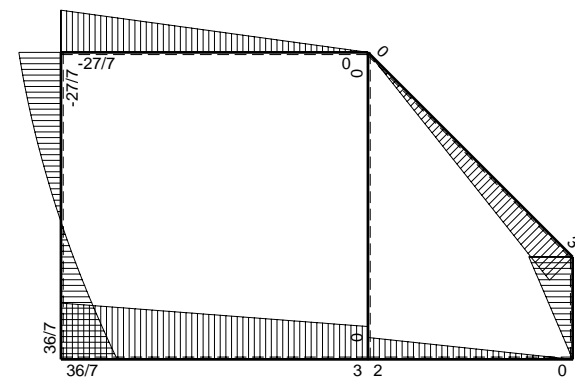
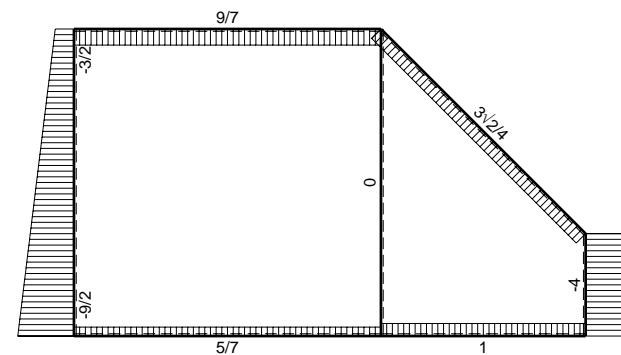
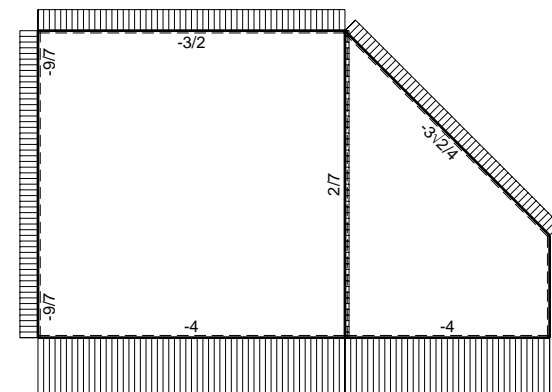
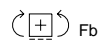
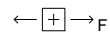
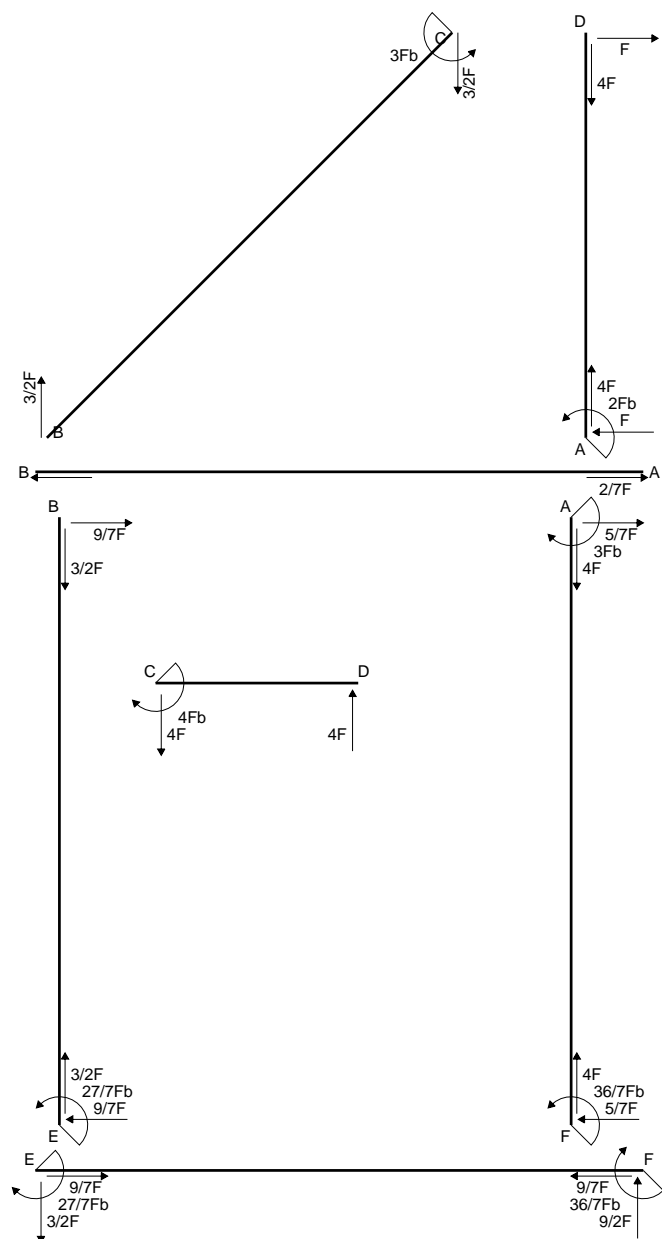
$$L_{AF}^{Xo} = \int_0^{4b} (16 - 8x/b + x^2/b^2) Fb 1/EJ dx + \int_0^{4b} (-4 + x/b) \theta dx$$

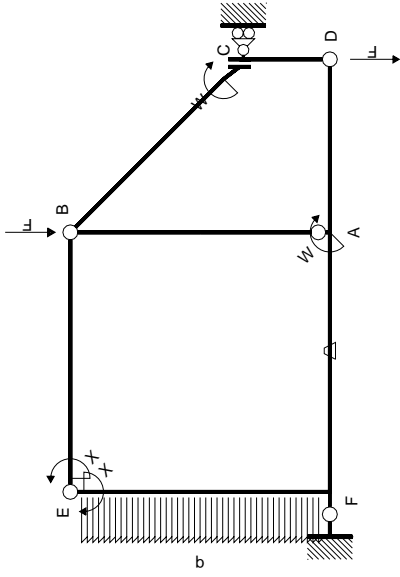
$$= [16x - 4x^2/b + 1/3 x^3/b^2]_0^{4b} Fb 1/EJ + [-4x + 1/2 x^2/b]_0^{4b} \theta$$

$$= (64b - 64b + 64/3 b) Fb 1/EJ + (-16b + 8b) \theta = 88/3 Fb^2/EJ$$

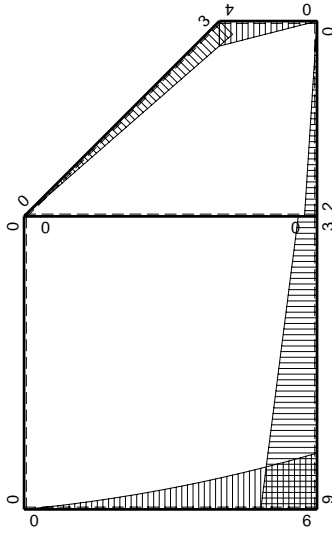


Consegnare SOLO questo foglio con quote frazionarie intere sui diagrammi. Indicare l'iperstatica scelta e i risultati richiesti per la sezione.
 Sul retro: analisi cinematica, diagrammi strutture principale e di servizio, equazione PLV con funzioni esplicite dei momenti.
 Carichi e deformazioni date hanno verso efficace in disegno. Calcolare reazioni vincolari della struttura e delle aste. Tracciare i diagrammi quotati delle azioni interne nelle aste. $J_{YZ} - X_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y. Curvatura θ asta AF positiva se convessa a destra con inizio A.
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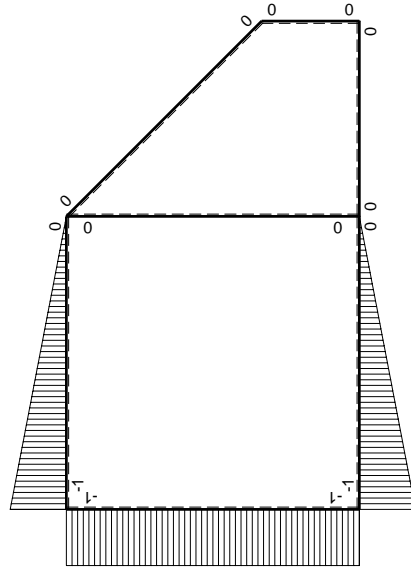




Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{EB}

→	M ₀ (x)	M _x (x)	θ	M ₀ ^x	M _θ	M _x ^x	$\int M_x(M_0/EJ+\theta)dx$	$\int X M_x M_0/EJ dx$		
AB 3b	0	0	0	0	0	0	0	0		
BA 3b	0	0	0	0	0	0	0	0		
CD b	0	4Fb-4Fx	0	0	0	0	0	0		
DC b	0	-4Fx	0	0	0	0	0	0		
BC 2\2b	0	3\2\4Fx	0	0	0	0	0	0		
EB 3b	-1+1\3x/b	0	0	0	0	0	1-2\3x/b+1\9x ² /b ²	0+0		
BE 3b	1\3x/b	0	0	0	0	0	1\9x ² /b ²	0+0		
FE 3b	-1	9Fb-9\2Fx+1\2qx ²	0	-9Fb+9\2Fx-1\2Fx ² /b	0	0	1	(-45\4+0)F ² /EJ		
EF 3b	1	-3\2Fx-1\2qx ²	0	-3\2Fx-1\2Fx ² /b	0	0	1	3Xb/EJ		
AF 3b	-1\3x/b	3Fb+2Fx	-Fb/EJ	-Fx-2\3Fx ² /b	1\3Fx/EJ	1\9x ² /b ²	(-2\12+3\2)Fb ² /EJ	Xb/EJ		
FA 3b	1-1\3x/b	-9Fb+2Fx	Fb/EJ	-9Fb+5Fx-2\3Fx ² /b	Fb/EJ-1\3Fx/EJ	1-2\3x/b+1\9x ² /b ²	(-2\12+3\2)Fb ² /EJ	Xb/EJ		
DA 2b	0	Fx	0	0	0	0	0	0		
AD 2b	0	-2Fb+Fx	0	0	0	0	0	0		
EB	molla asta - W _{1EB} (W _{0EB} +XW _{1EB})/k _{EB}								1\4Xb/EJ	
totali									-81\4Fb ² /EJ	21\4Xb/EJ
									27\7Fb	

Sviluppi di calcolo iperstatica

$$L_{EB}^{xx} = \int_0^{3b} (1 - 2/3 x/b + 1/9 x^2/b^2) 1/EJ dx + 1 \cdot 1 \cdot 1/4 b/EJ$$

$$= \left[x - 1/3 x^2/b + 1/27 x^3/b^2 \right]_0^{3b} 1/EJ + 1 \cdot 1 \cdot 1/4 b/EJ$$

$$= (3b - 3b + b) 1/EJ + 1 \cdot 1 \cdot 1/4 b/EJ = 5/4 b/EJ$$

$$L_{BE}^{xx} = \int_0^{3b} (1/9 x^2/b^2) 1/EJ dx + 1 \cdot 1 \cdot 1/4 b/EJ = \left[1/27 x^3/b^2 \right]_0^{3b} 1/EJ + 1 \cdot 1 \cdot 1/4 b/EJ$$

$$= (b) 1/EJ + 1 \cdot 1 \cdot 1/4 b/EJ = 5/4 b/EJ$$

$$L_{FE}^{xx} = \int_0^{3b} (1) 1/EJ dx = \left[x \right]_0^{3b} 1/EJ$$

$$= (3b) 1/EJ = 3 b/EJ$$

$$L_{EF}^{xx} = \int_0^{3b} (1) 1/EJ dx = \left[x \right]_0^{3b} 1/EJ$$

$$= (3b) 1/EJ = 3 b/EJ$$

$$L_{AF}^{xx} = \int_0^{3b} (1/9 x^2/b^2) 1/EJ dx = \left[1/27 x^3/b^2 \right]_0^{3b} 1/EJ$$

$$= (b) 1/EJ = b/EJ$$

$$L_{FA}^{xx} = \int_0^{3b} (1 - 2/3 x/b + 1/9 x^2/b^2) 1/EJ dx = \left[x - 1/3 x^2/b + 1/27 x^3/b^2 \right]_0^{3b} 1/EJ$$

$$= (3b - 3b + b) 1/EJ = b/EJ$$

$$L_{FE}^{xo} = \int_0^{3b} (-9 + 9/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx = \left[-9x + 9/4 x^2/b - 1/6 x^3/b^2 \right]_0^{3b} Fb 1/EJ$$

$$= (-27b + 81/4 b - 9/2 b) Fb 1/EJ = -45/4 Fb^2/EJ$$

$$L_{EF}^{xo} = \int_0^{3b} (-3/2 x/b - 1/2 x^2/b^2) Fb 1/EJ dx = \left[-3/4 x^2/b - 1/6 x^3/b^2 \right]_0^{3b} Fb 1/EJ$$

$$= (-27/4 b - 9/2 b) Fb 1/EJ = -45/4 Fb^2/EJ$$

$$L_{AF}^{xo} = \int_0^{3b} (-x/b - 2/3 x^2/b^2) Fb 1/EJ dx + \int_0^{3b} (1/3 x/b) \theta dx$$

$$= \left[-1/2 x^2/b - 2/9 x^3/b^2 \right]_0^{3b} Fb 1/EJ + \left[1/6 x^2/b \right]_0^{3b} \theta$$

$$= (-9/2 b - 6b) Fb 1/EJ + (3/2 b) \theta = -9 Fb^2/EJ$$

$$L_{FA}^{xo} = \int_0^{3b} (-9 + 5x/b - 2/3 x^2/b^2) Fb 1/EJ dx + \int_0^{3b} (-1 + 1/3 x/b) \theta dx$$

$$= \left[-9x + 5/2 x^2/b - 2/9 x^3/b^2 \right]_0^{3b} Fb 1/EJ + \left[-x + 1/6 x^2/b \right]_0^{3b} \theta$$

$$= (-27b + 45/2 b - 6b) Fb 1/EJ + (-3b + 3/2 b) \theta = -9 Fb^2/EJ$$

