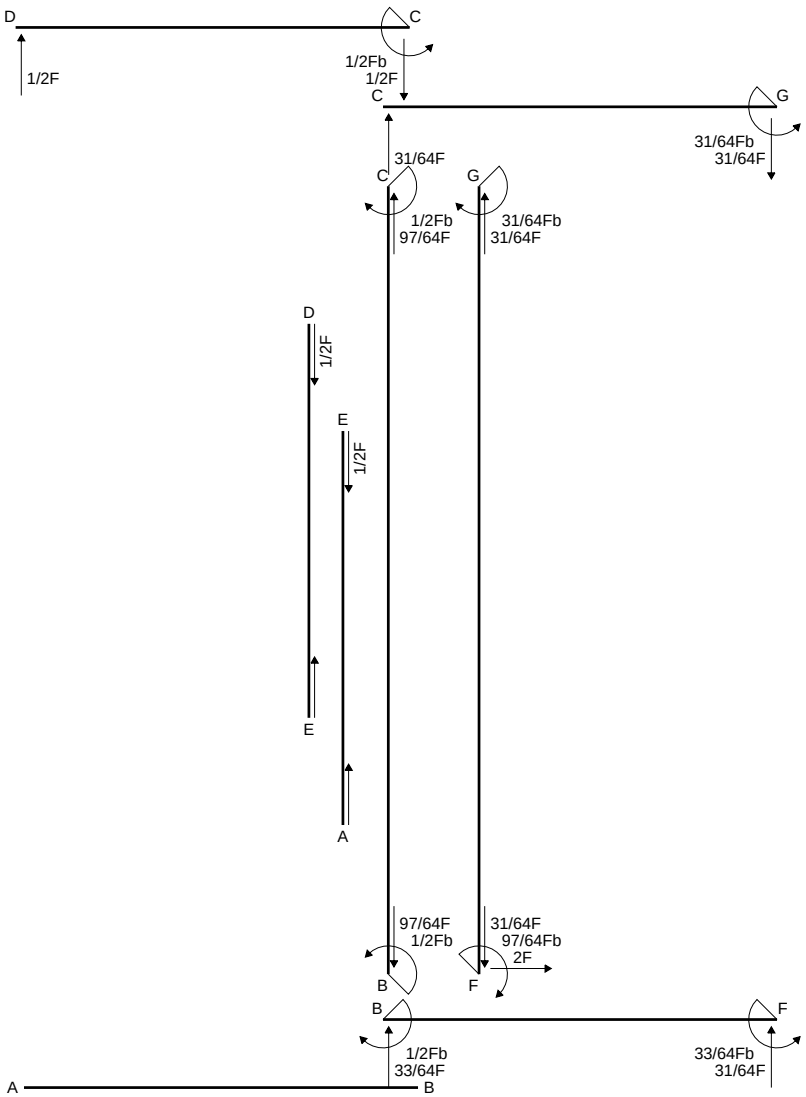
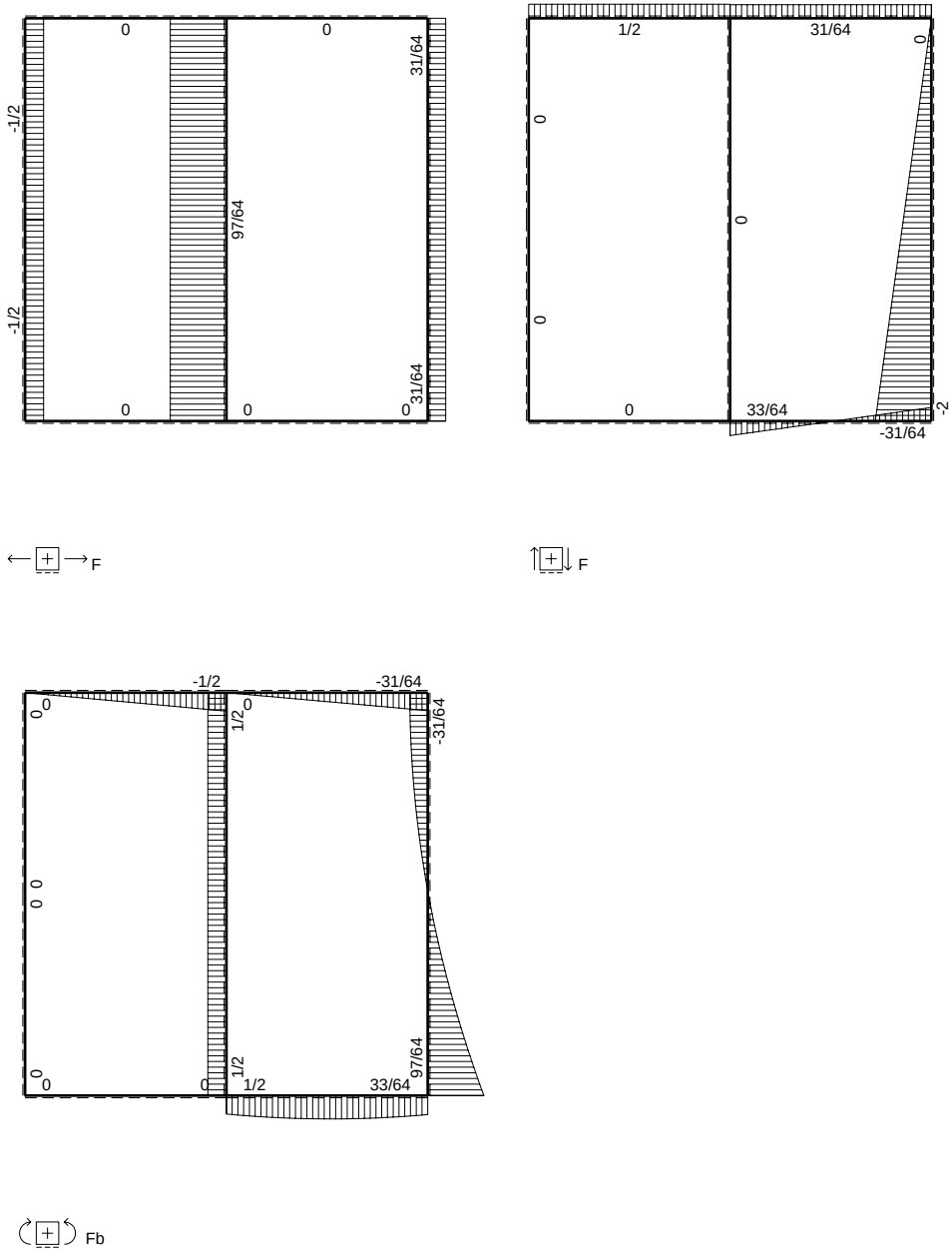

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64
F
1/2
31/64
0
0
0
0
33/64
-2
-31/64
F
-1/2
-31/64
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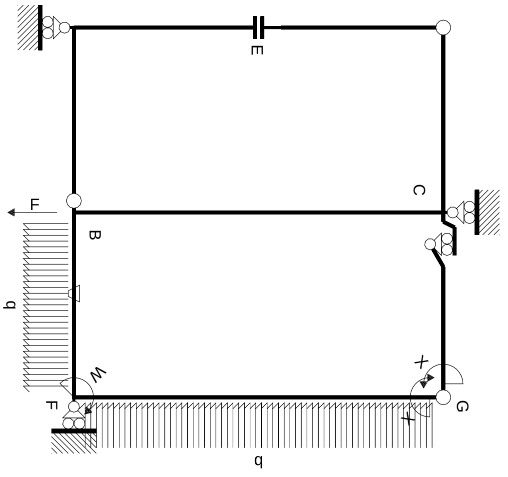
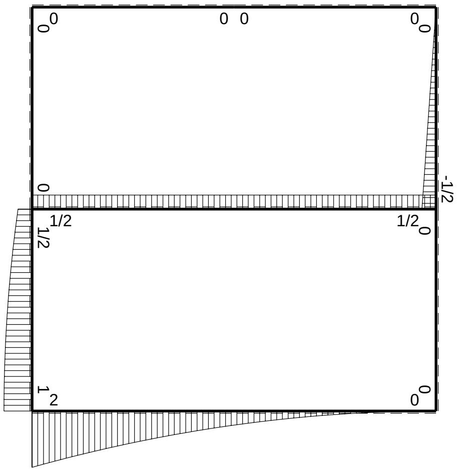
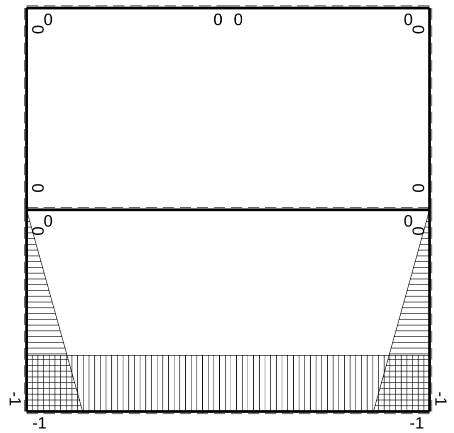
1/2
0
-31/64
0
0
0
0
1/2
1/2
33/64
97/64
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

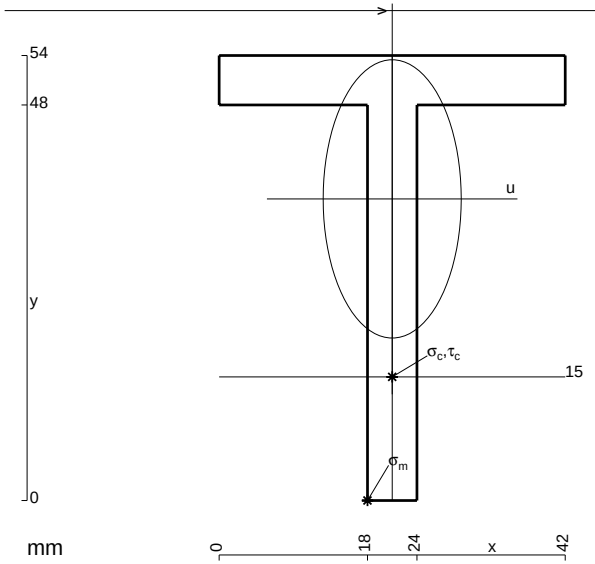
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 540. mm^2
J_u = 154030. mm^4
J_v = 37908. mm^4
y_g = 36.6 mm
T_y = 1880. N
M_x = -883600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -36.6 mm
$\sigma_m = -Mv/J_u$ = -210. N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -21.6 mm
$\sigma_c = -Mv/J_u$ = -123.9 N/mm^2
τ_c = 5.328 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 124.3 N/mm^2
S^* = 2619. mm^3

ESΣ8.001

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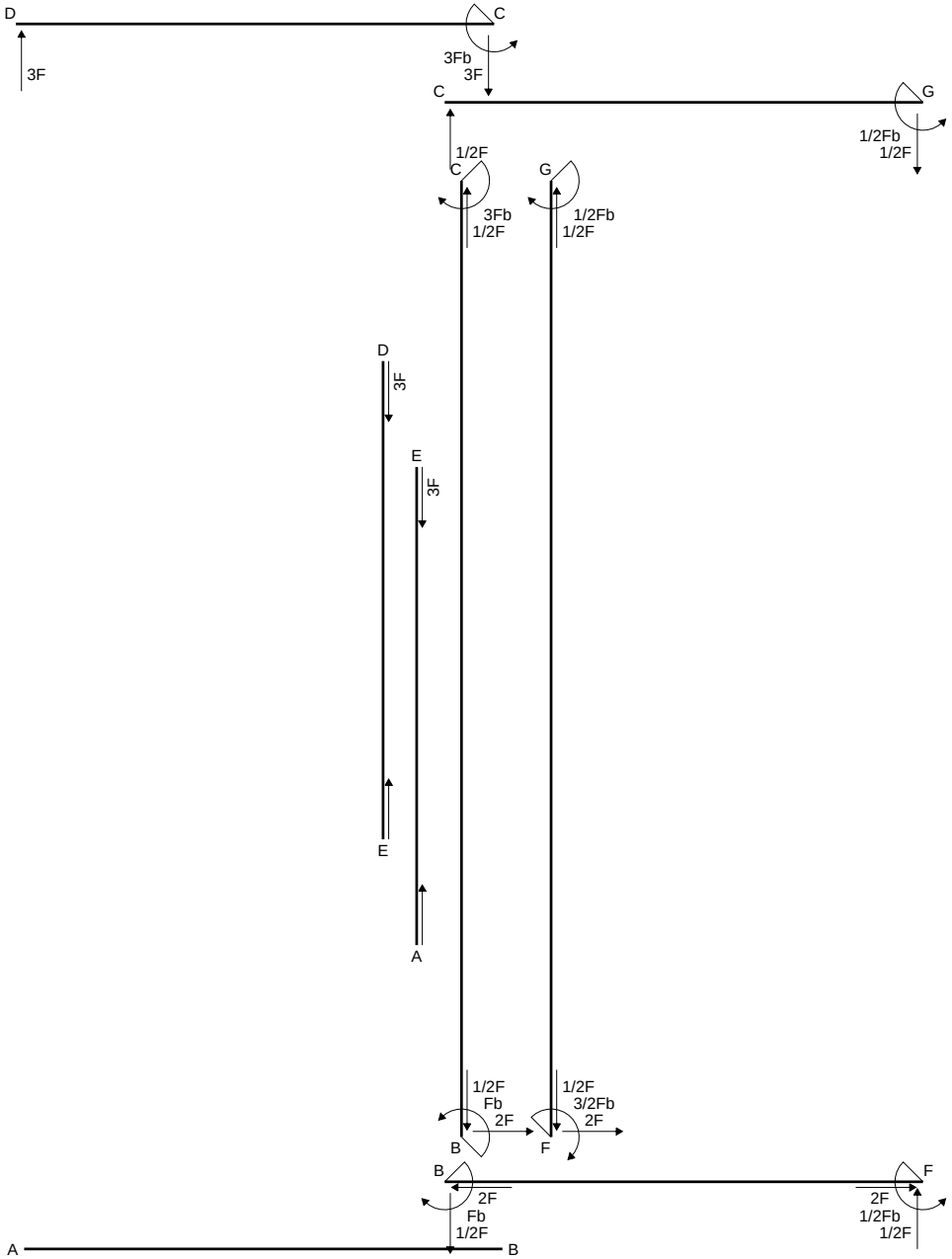
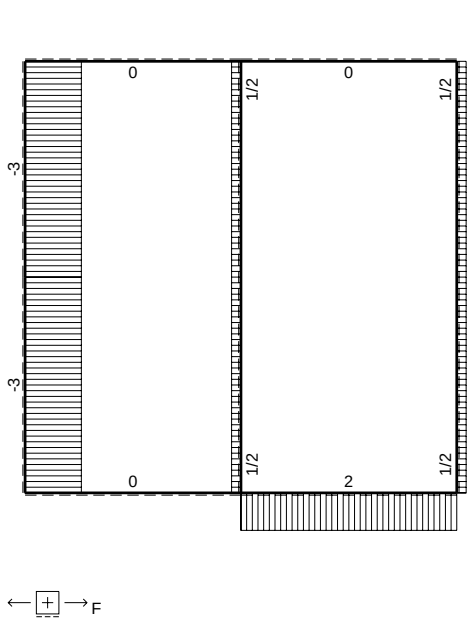
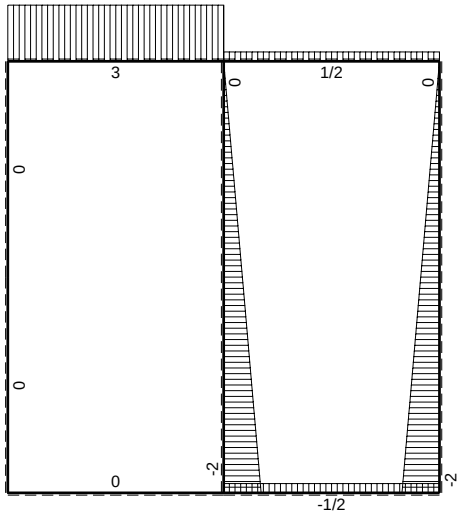
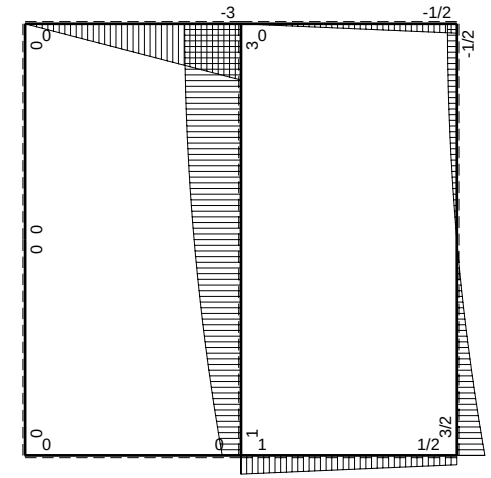

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
0
1/2
1/2
2

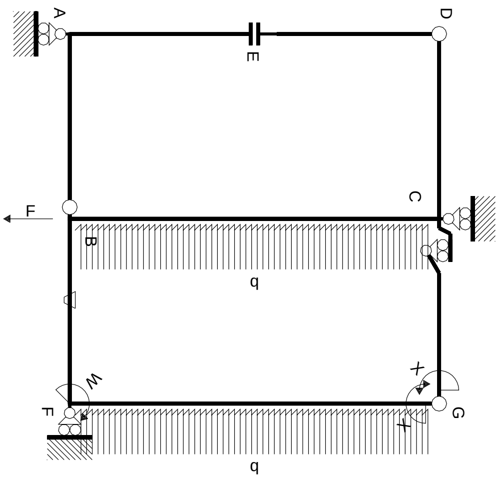
F

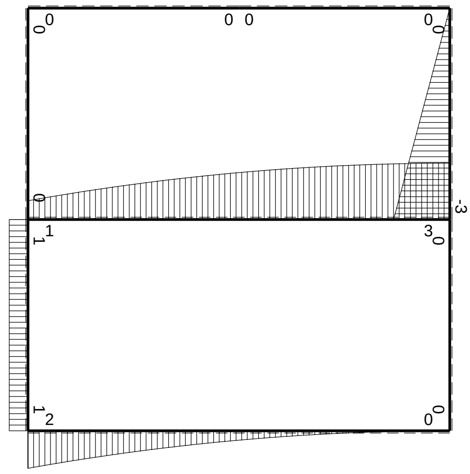

3
0
1/2
0
0
0
0
-2
-2
-1/2

F

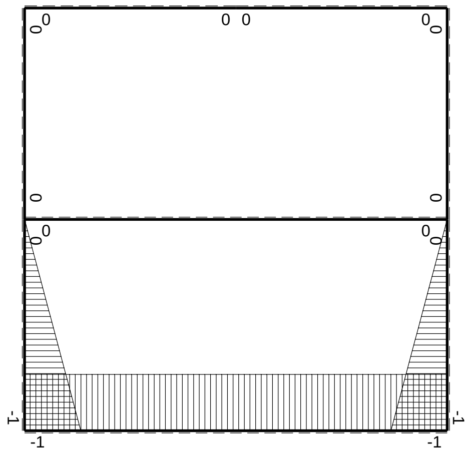

-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

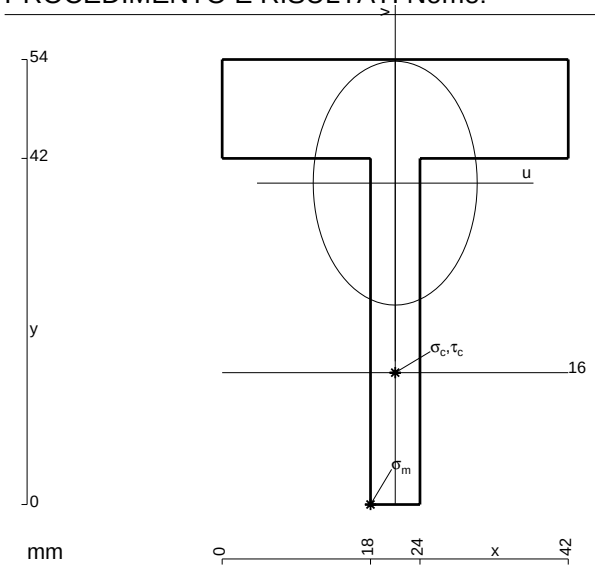
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 756. mm^2
J_u = 165564. mm^4
J_v = 74844. mm^4
y_g = 39. mm
T_y = 1830. N
M_x = -933300. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39. mm
$\sigma_m = -Mv/J_u$ = -219.8 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -23. mm
$\sigma_c = -Mv/J_u$ = -129.7 N/mm^2
τ_c = 5.482 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130. N/mm^2
S^* = 2976. mm^3

ESΣ8.002

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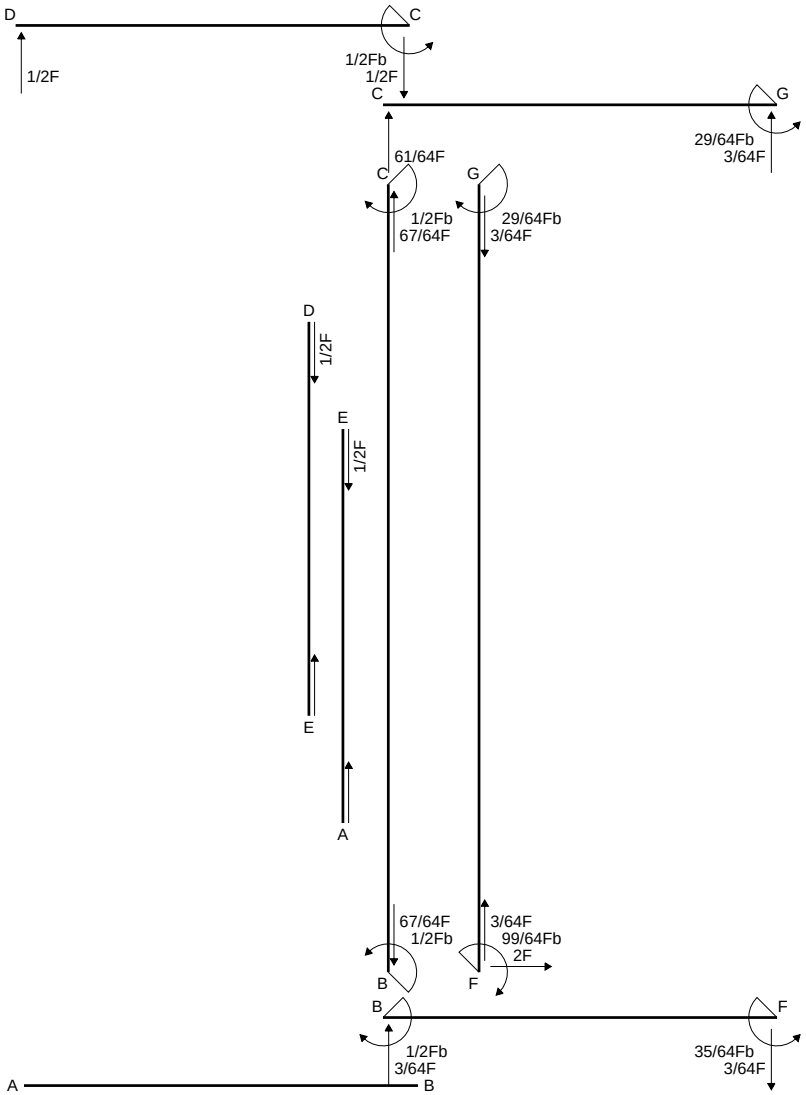
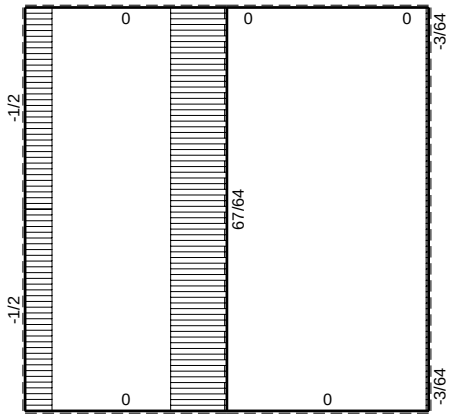

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
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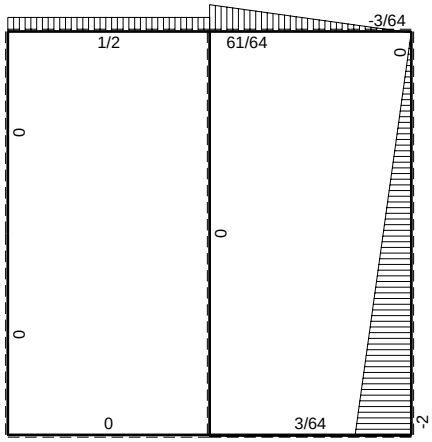
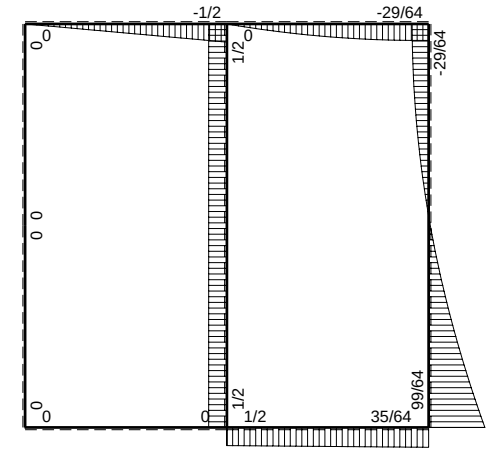
-1/2
67/64
-1/2
0
0
-3/64


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx²/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb²/EJ	1/3XLb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb²/EJ	1/3XLb/EJ
CG b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2XLb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb²/EJ	8/3XLb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

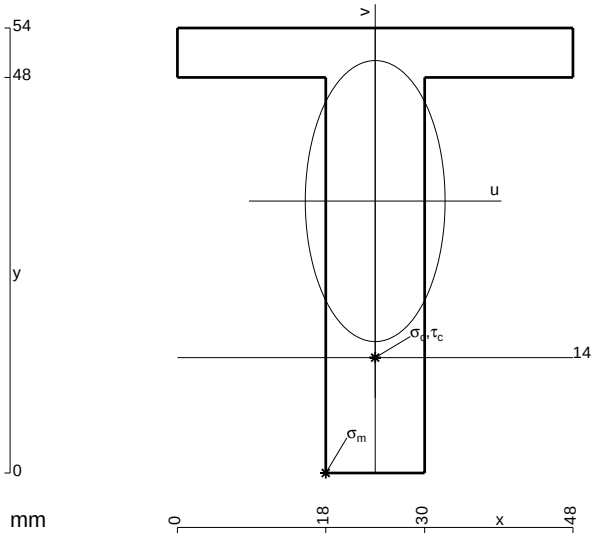
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.003



A = 864. mm^2
J_u = 251424. mm^4
J_v = 62208. mm^4
y_g = 33. mm
T_y = 3185. N
M_x = -1751750. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33. mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19. mm
$\sigma_c = -Mv/J_u$ = -132.4 N/mm^2
τ_c = 4.611 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.6 N/mm^2
S^* = 4368. mm^3

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ESΣ8.003

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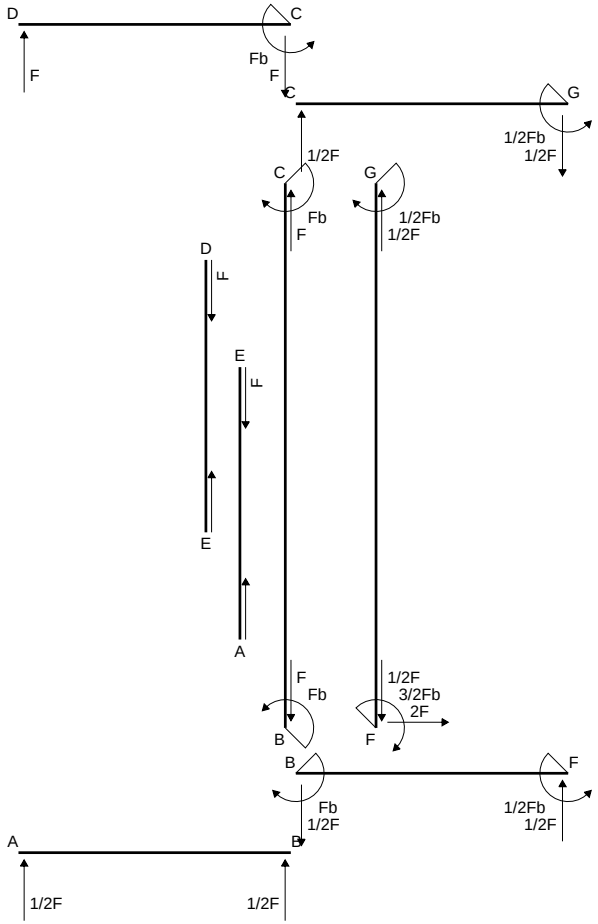
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C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
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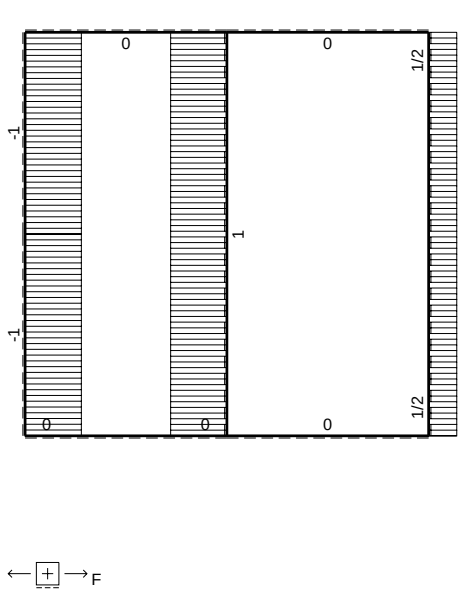
A
B
1/2F
1/2F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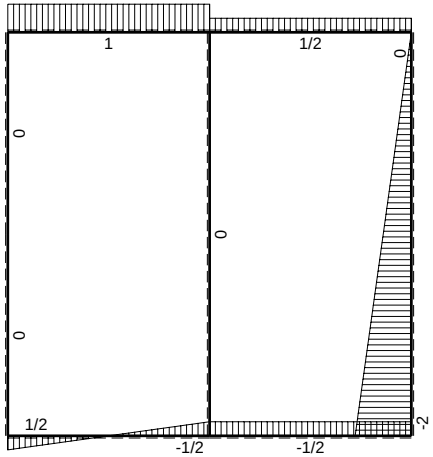
-1
0
0
0
1/2

F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1/2
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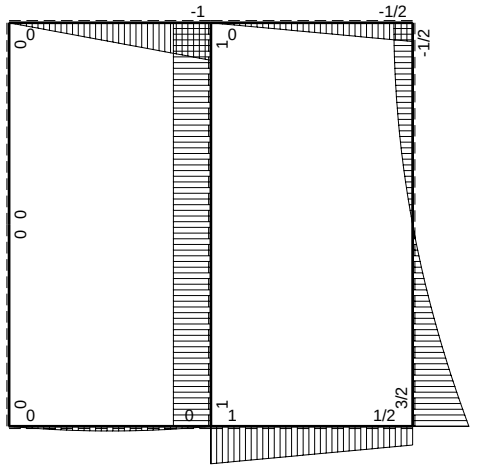
-1/2
-2

F


-1
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0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

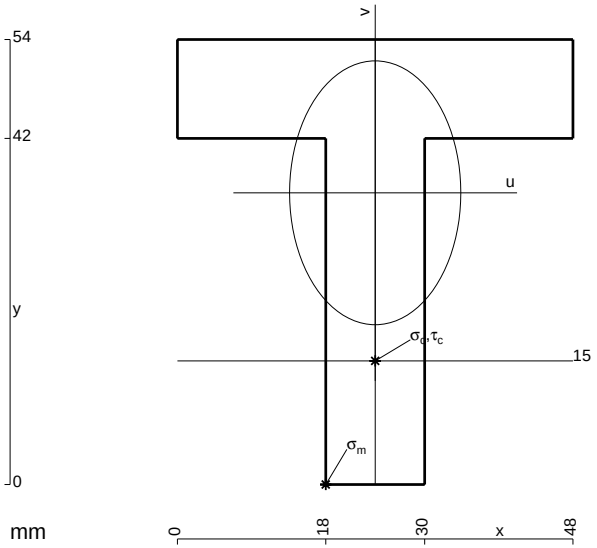
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1080. mm^2
J_u = 276955. mm^4
J_v = 116640. mm^4
y_g = 35.4 mm
T_y = 3120. N
M_x = -1872000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.4 mm
$\sigma_m = -Mv/J_u$ = -239.3 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.4 mm
$\sigma_c = -Mv/J_u$ = -137.9 N/mm^2
τ_c = 4.715 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.1 N/mm^2
S^* = 5022. mm^3

ESΣ8.004

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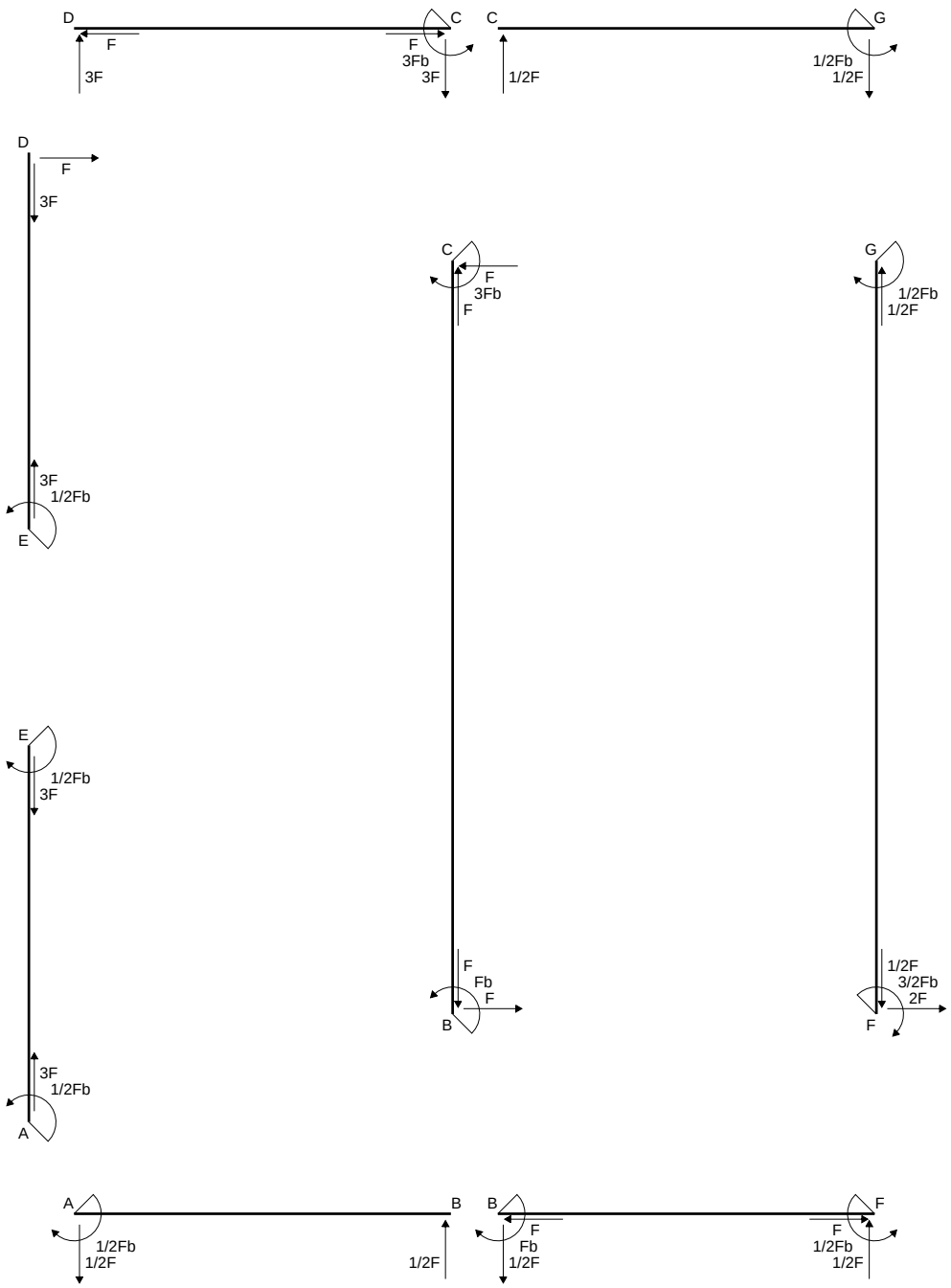
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3F
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G
1/2Fb
1/2F
D
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E
C
F
3Fb
F
G
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1/2F
E
1/2Fb
3F
F
Fb
F
B
1/2F
3/2Fb
2F
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3F
1/2Fb
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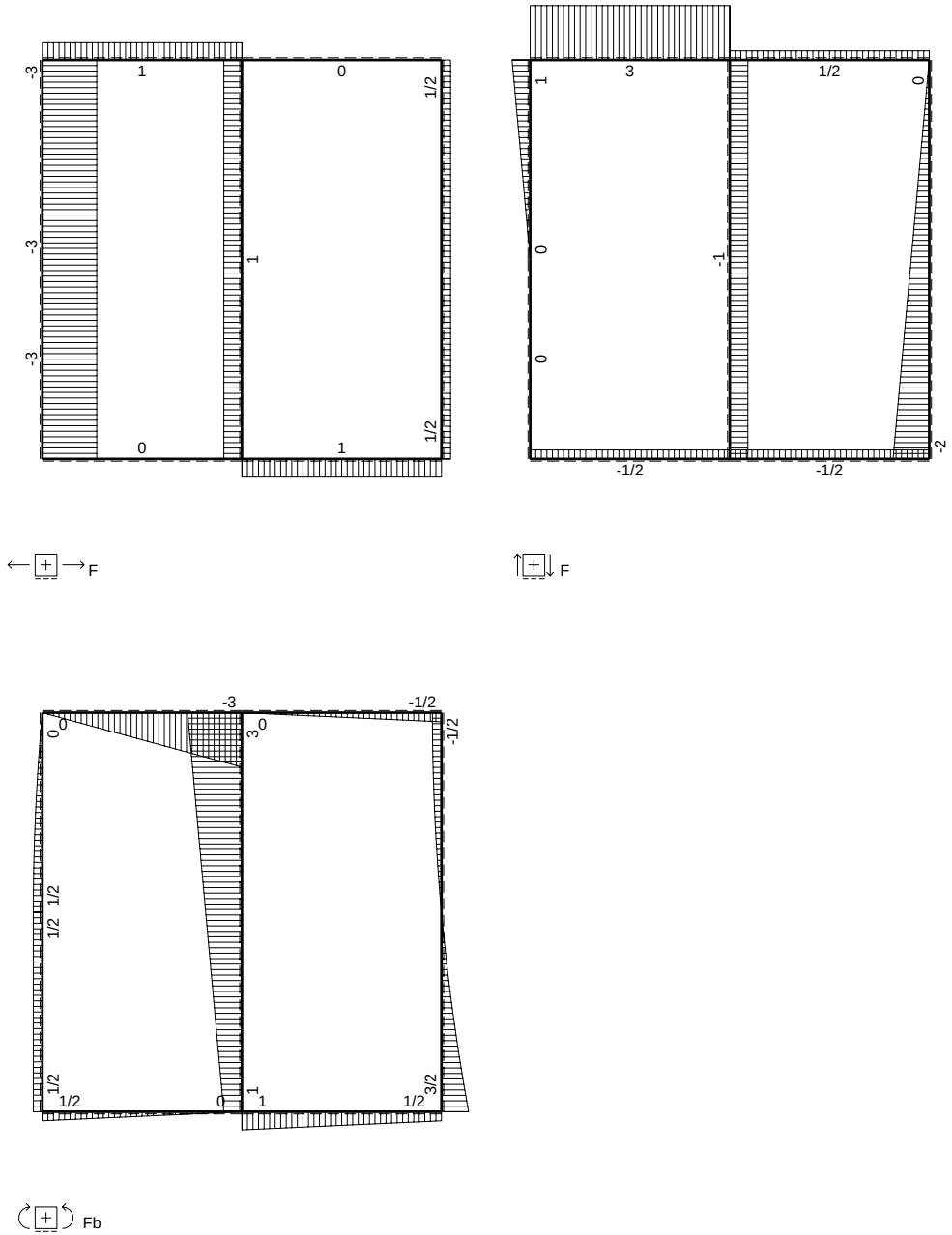
F
1/2Fb
1/2F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-1
-2
-1/2
-1/2
F
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-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

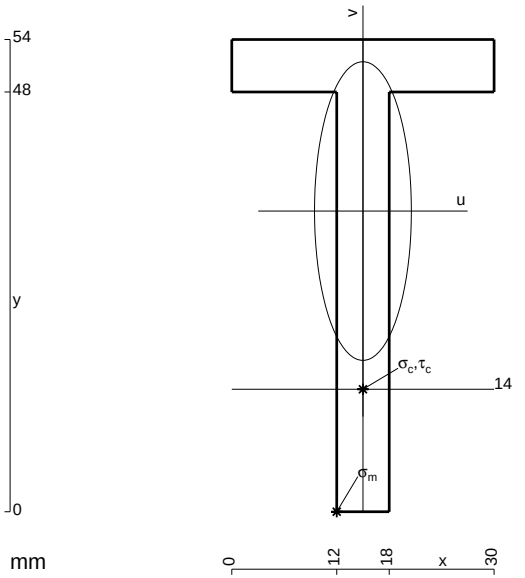
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 468. mm^2
J_u = 136587. mm^4
J_v = 14364. mm^4
y_g = 34.38 mm
N = 410. N
T_y = 1230. N
M_x = -787200. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -34.38 mm
σ_m = N/A-Mv/J_u = -197.3 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -20.38 mm
σ_c = N/A-Mv/J_u = -116.6 N/mm^2
τ_c = 3.452 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.8 N/mm^2
S^* = 2300. mm^3

ESΣ8.005

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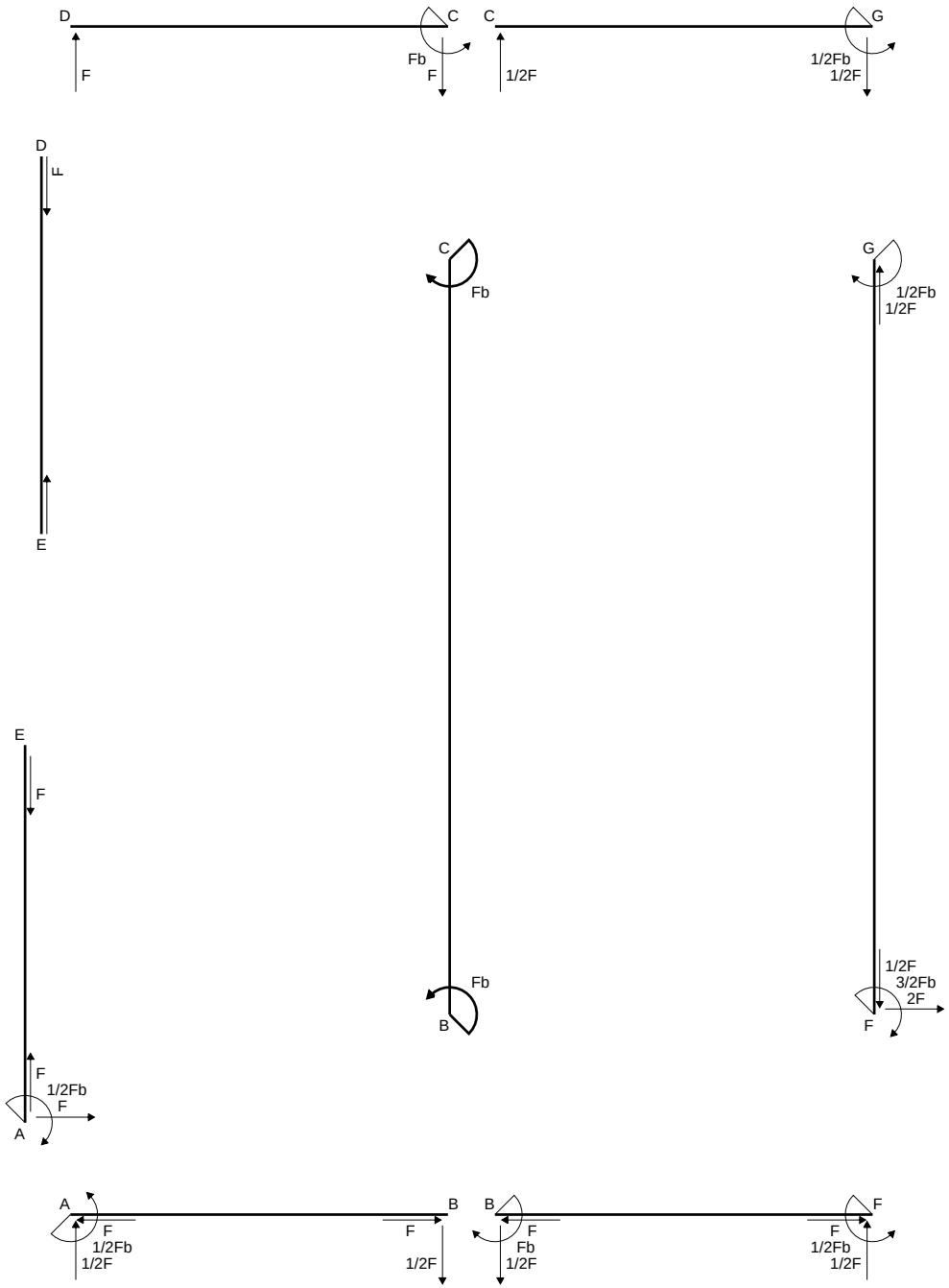
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1/2Fb
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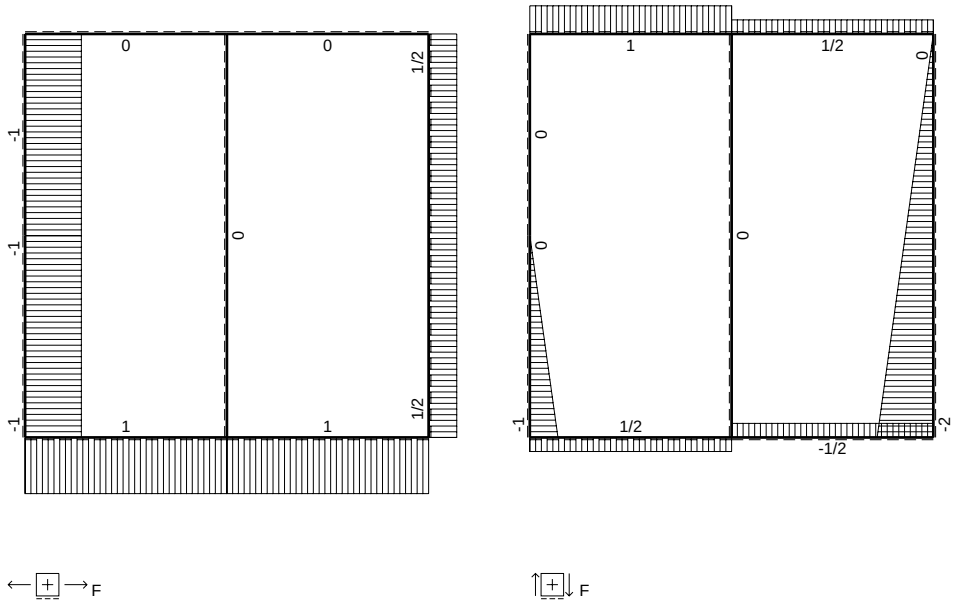
0
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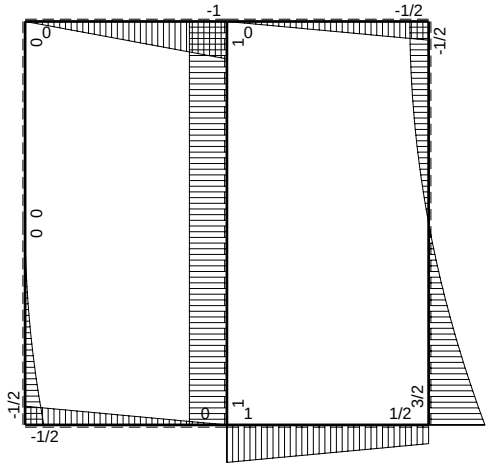
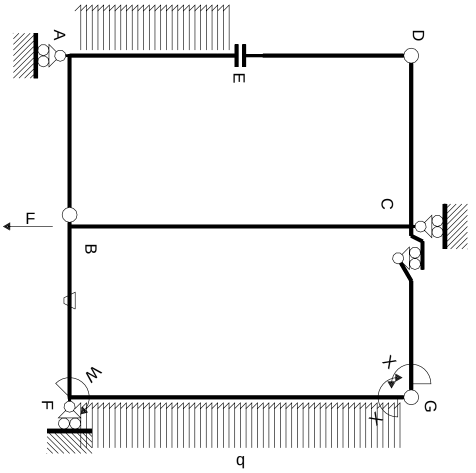
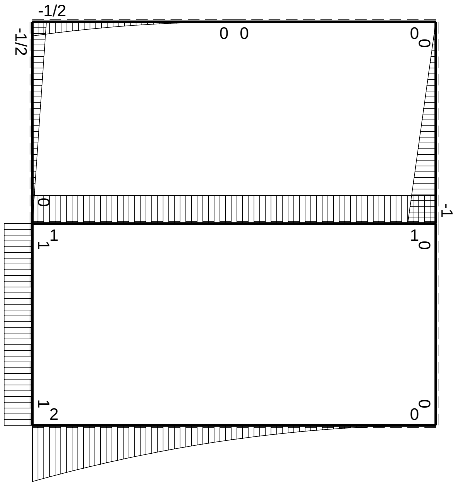
1/2
-1
-1
0
-1
1
1
1/2
1
1/2
0
0
0
0
-1
1/2
-2
-1/2
F
F

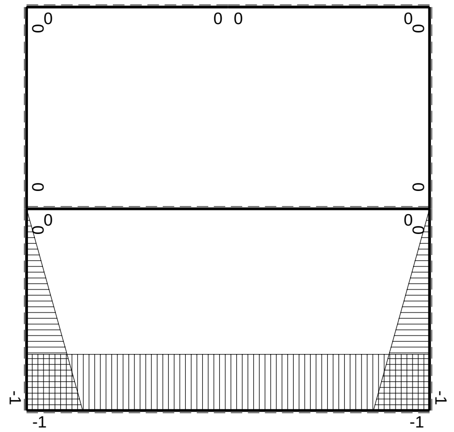

-1
-1/2
0
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

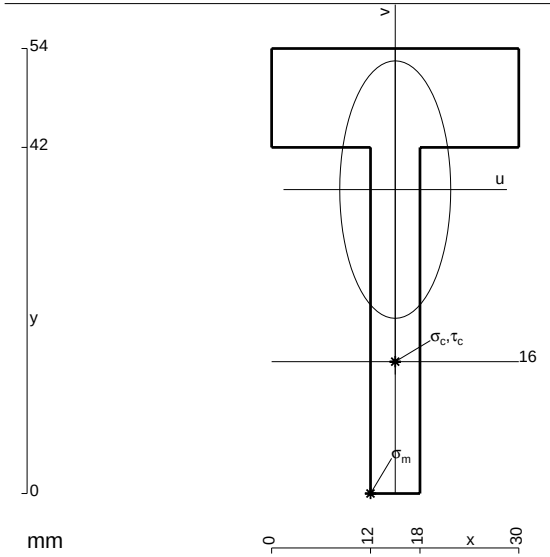
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 612. mm^2
J_u = 149427. mm^4
J_v = 27756. mm^4
y_g = 36.88 mm
T_y = 1250. N
M_x = -850000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.88 mm
$\sigma_m = -Mv/J_u$ = -209.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -20.88 mm
$\sigma_c = -Mv/J_u$ = -118.8 N/mm^2
τ_c = 3.866 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119. N/mm^2
S^* = 2773. mm^3

ESΣ8.006

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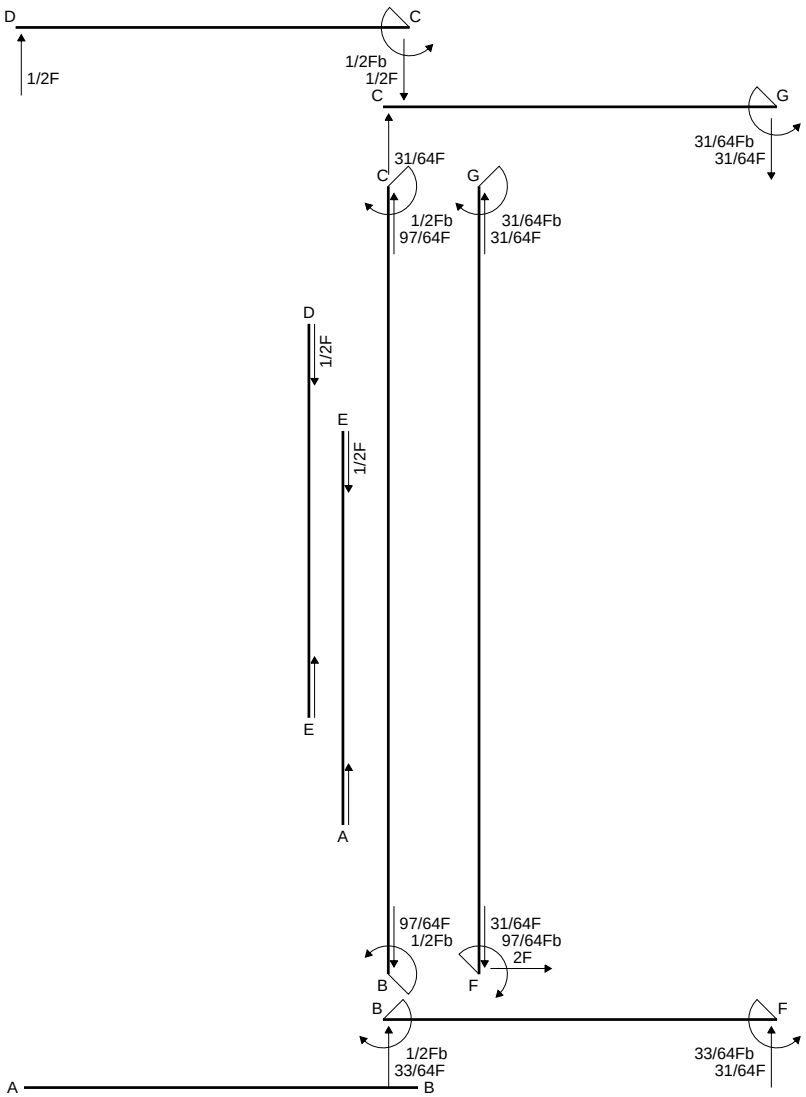
D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
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F
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F
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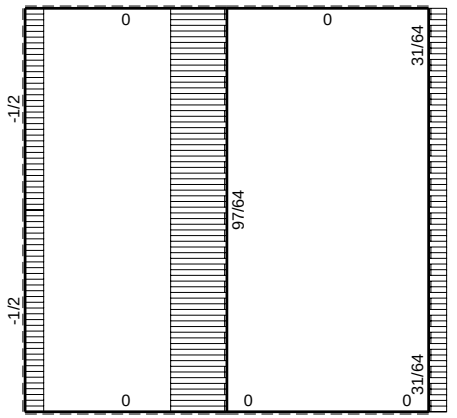
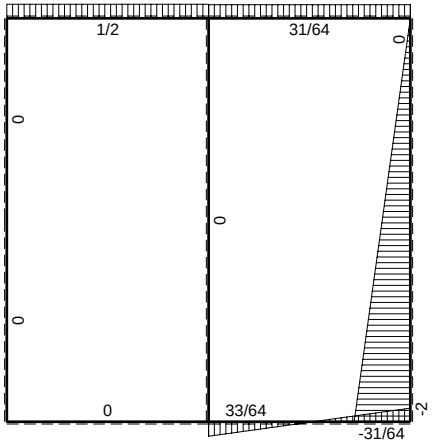
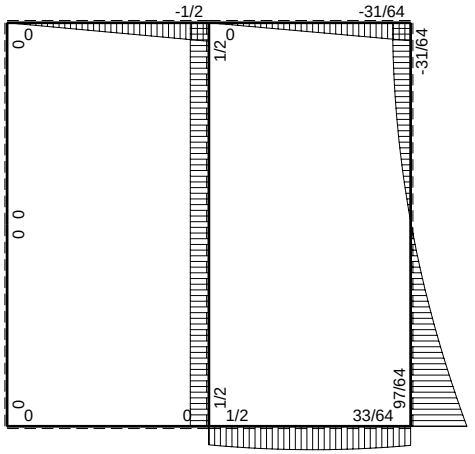
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
0
0
0
31/64

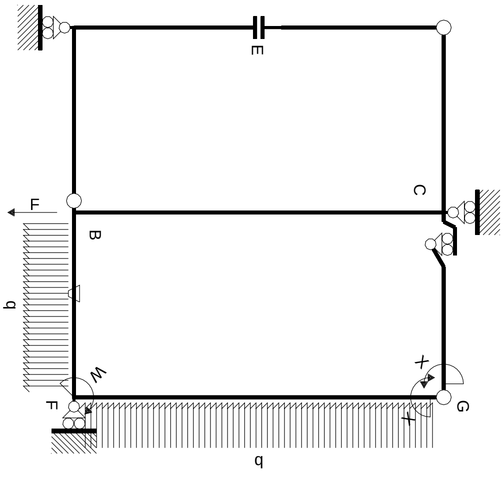

1/2
31/64
0
0
0
0
0
33/64
-2
-31/64

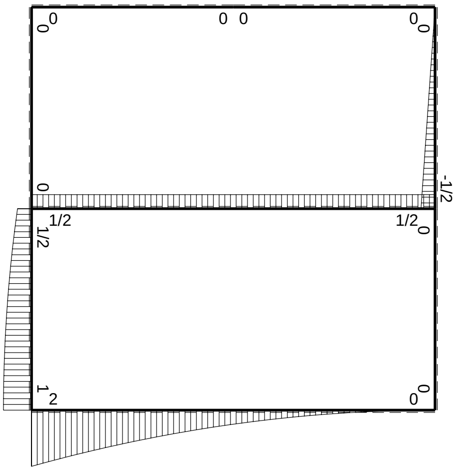
←□→ F

↑□↓ F

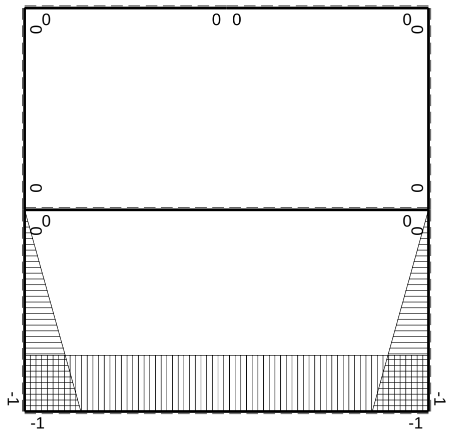

-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64

↺□↻ Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						31/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

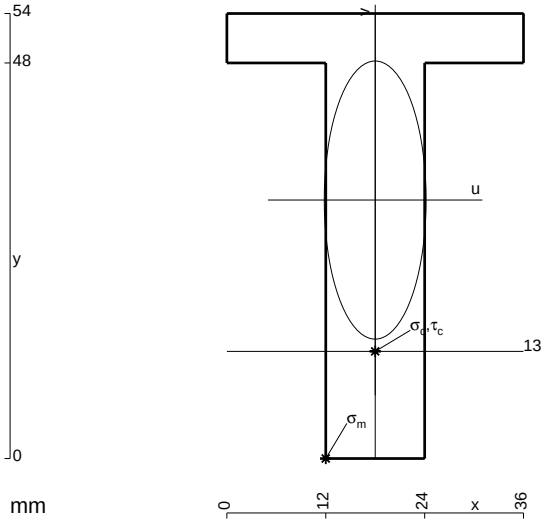
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 792. mm^2
J_u = 225759. mm^4
J_v = 30240. mm^4
y_g = 31.36 mm
T_y = 2165. N
M_x = -1580450. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.36 mm
$\sigma_m = -Mv/J_u$ = -219.6 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -18.36 mm
$\sigma_c = -Mv/J_u$ = -128.6 N/mm^2
τ_c = 3.1 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 128.7 N/mm^2
S^* = 3879. mm^3

ESΣ8.007

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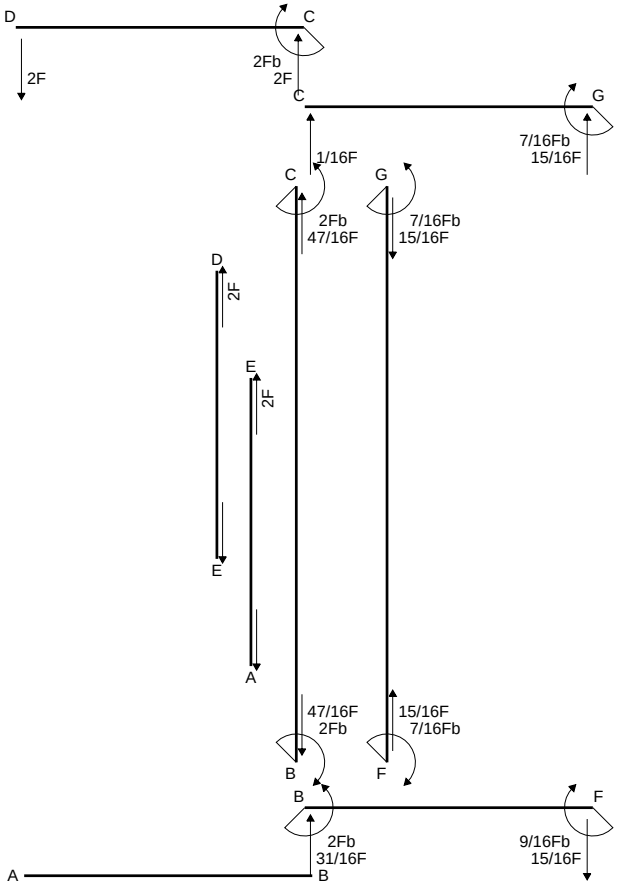
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C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
F
-2
-15/16
1/16
0
0
0
0
31/16
15/16
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3XbJ/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3XbJ/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbJ/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

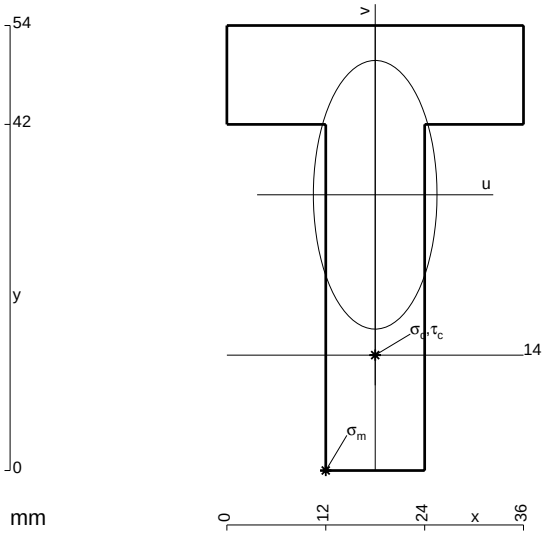
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 936. mm^2
J_u = 248849. mm^4
J_v = 52704. mm^4
y_g = 33.46 mm
T_y = -2220. N
M_x = 1709400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.46 mm
$\sigma_m = -Mv/J_u$ = 229.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.46 mm
$\sigma_c = -Mv/J_u$ = 133.7 N/mm^2
τ_c = 3.305 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 133.8 N/mm^2
S^* = 4446. mm^3

ESΣ8.008

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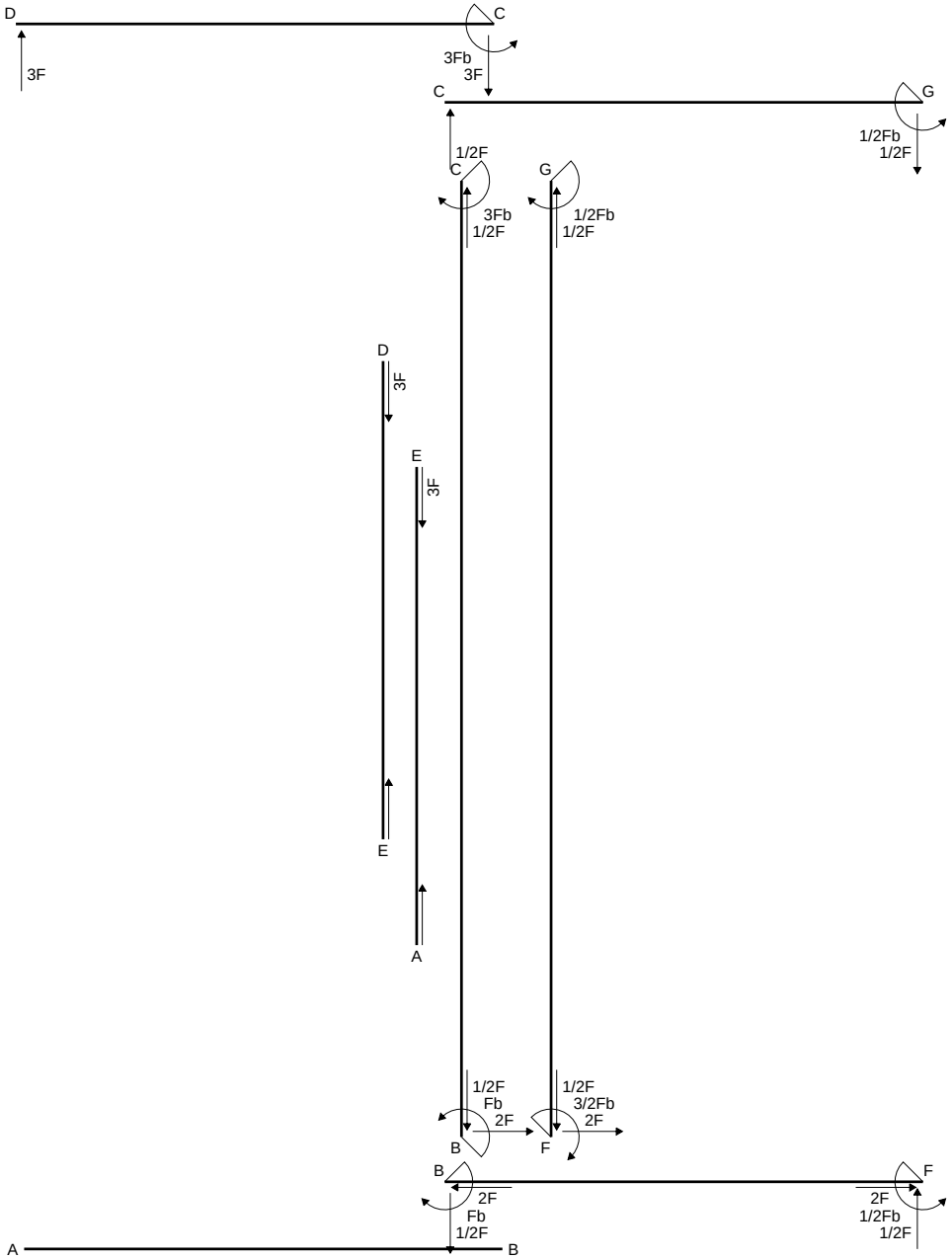
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3F
3Fb
3F
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G
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1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
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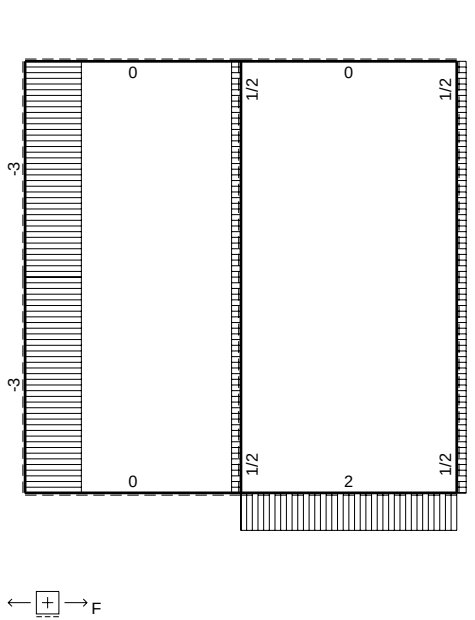
2F
1/2Fb
1/2F
A
B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-3
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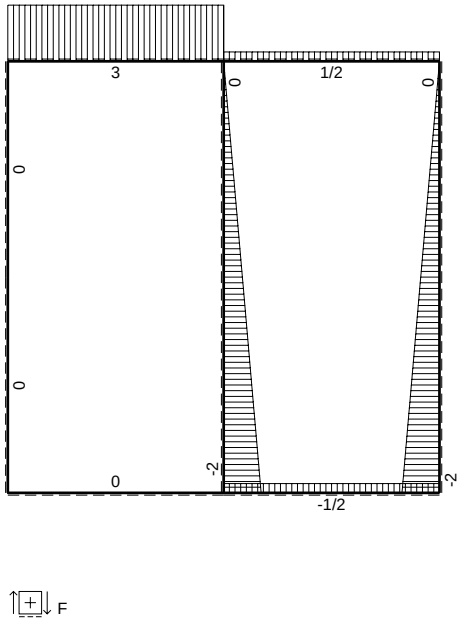
1/2
1/2
F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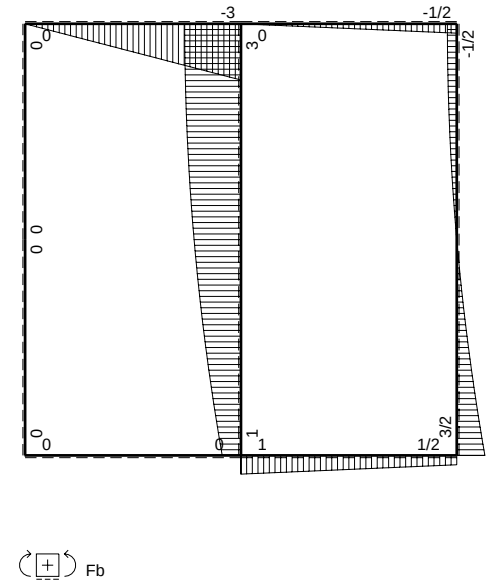
1/2
0
0
0
-2
-2
-1/2
F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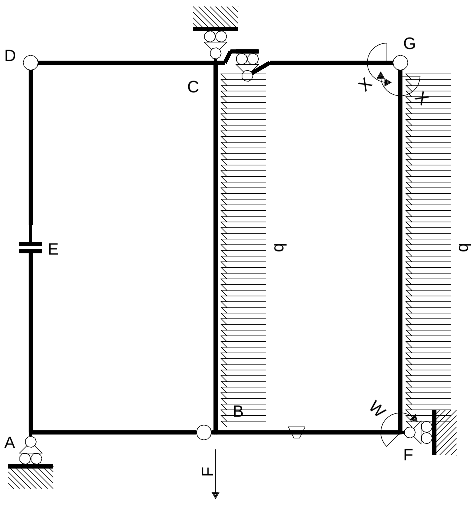
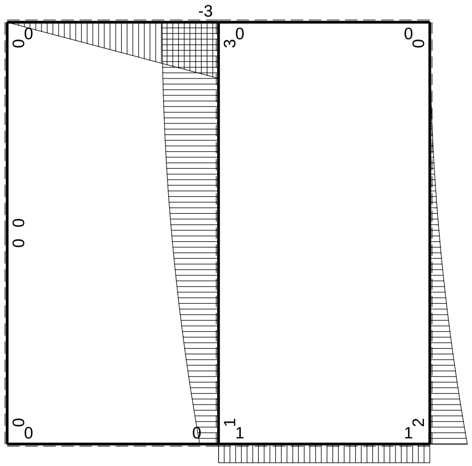
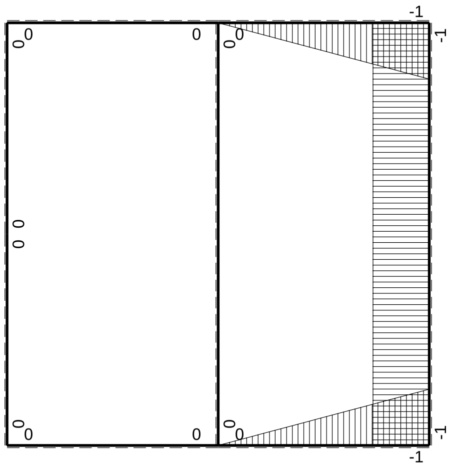
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

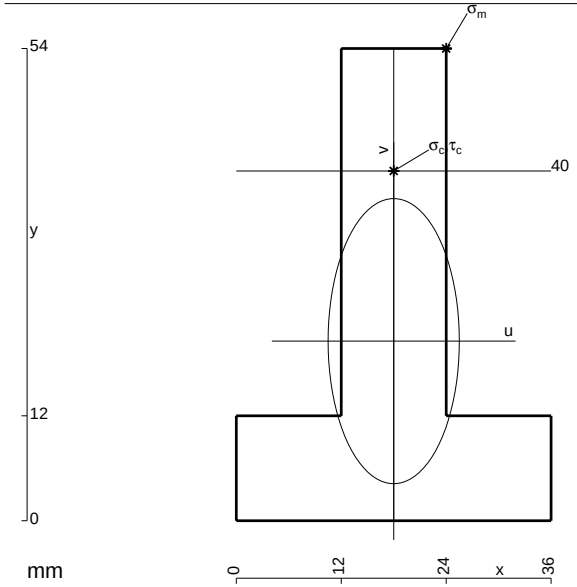
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 936. mm^2
J_u = 248849. mm^4
J_v = 52704. mm^4
y_g = 20.54 mm
T_y = 2190. N
M_x = -1773900. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.46 mm
$\sigma_m = -Mv/J_u$ = 238.5 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.46 mm
$\sigma_c = -Mv/J_u$ = 138.7 N/mm^2
τ_c = 3.26 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.8 N/mm^2
S^* = 4446. mm^3

ESΣ8.009

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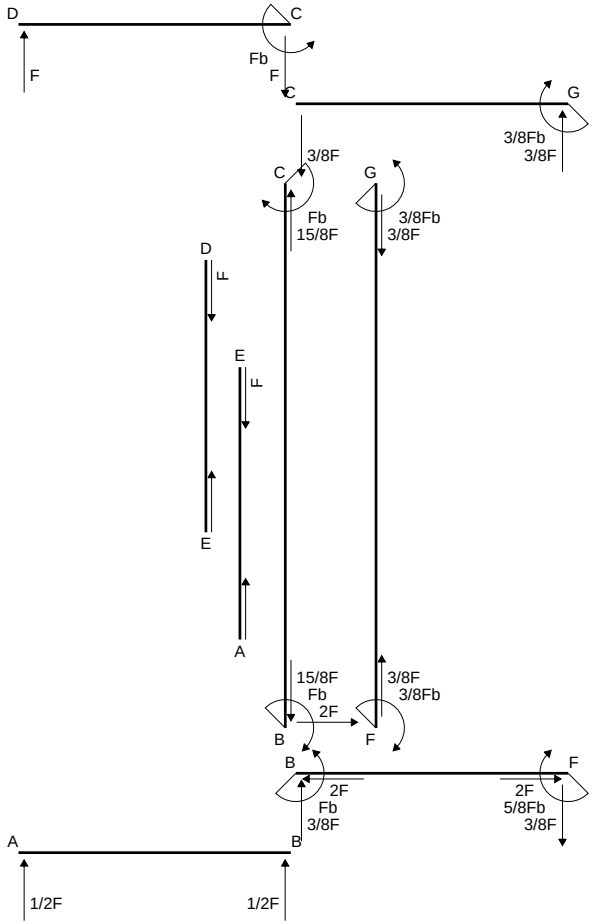
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C
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Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
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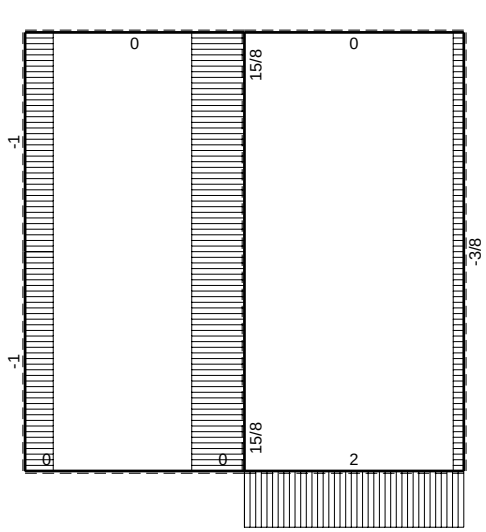
A
B
1/2F
1/2F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0
15/8
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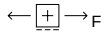
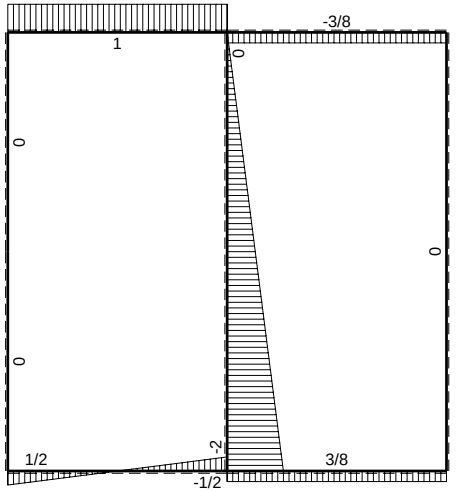
-1
-3/8
0
0
15/8
0
0
2


F


-3/8
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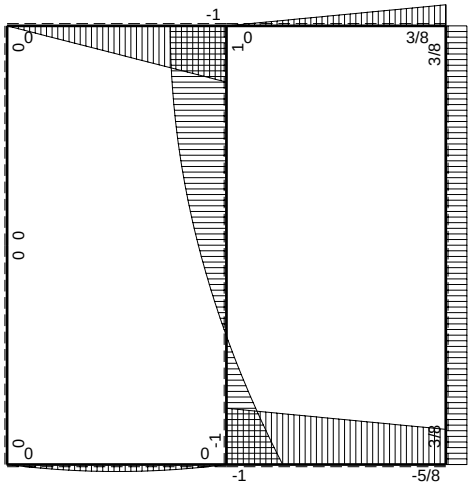
0
0
0
0
1/2
-2
3/8
-1/2


F


-1
0
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3/8
3/8
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0
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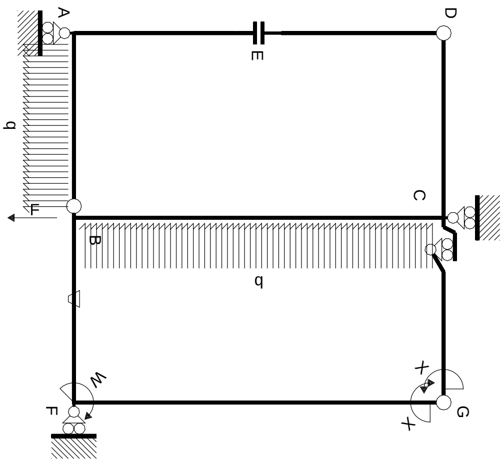
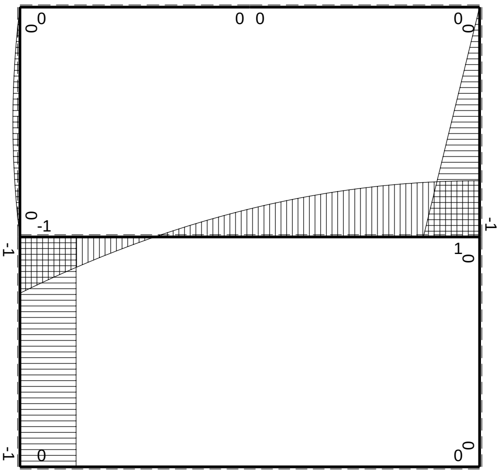
0
0
-1
0
3/8
-1
-5/8

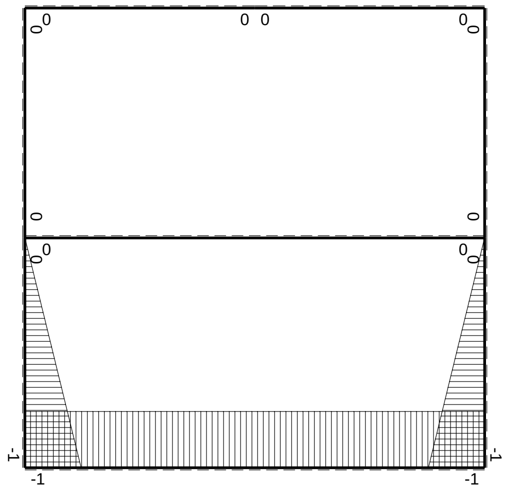

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

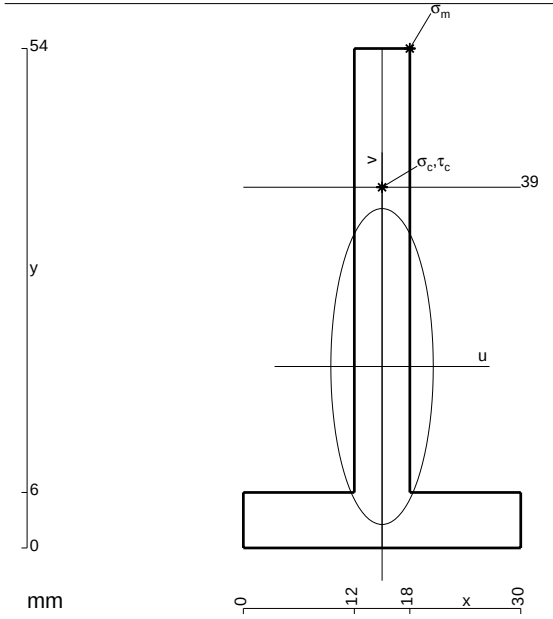
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 468. mm^2
J_u = 136587. mm^4
J_v = 14364. mm^4
y_g = 19.62 mm
N = 3338. N
T_y = -3560. N
M_x = -765400. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 34.38 mm
$\sigma_m = N/A - Mv/J_u$ = 199.8 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 19.38 mm
$\sigma_c = N/A - Mv/J_u$ = 115.8 N/mm^2
τ_c = 10.51 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.2 N/mm^2
S^* = 2420. mm^3

ESΣ8.010

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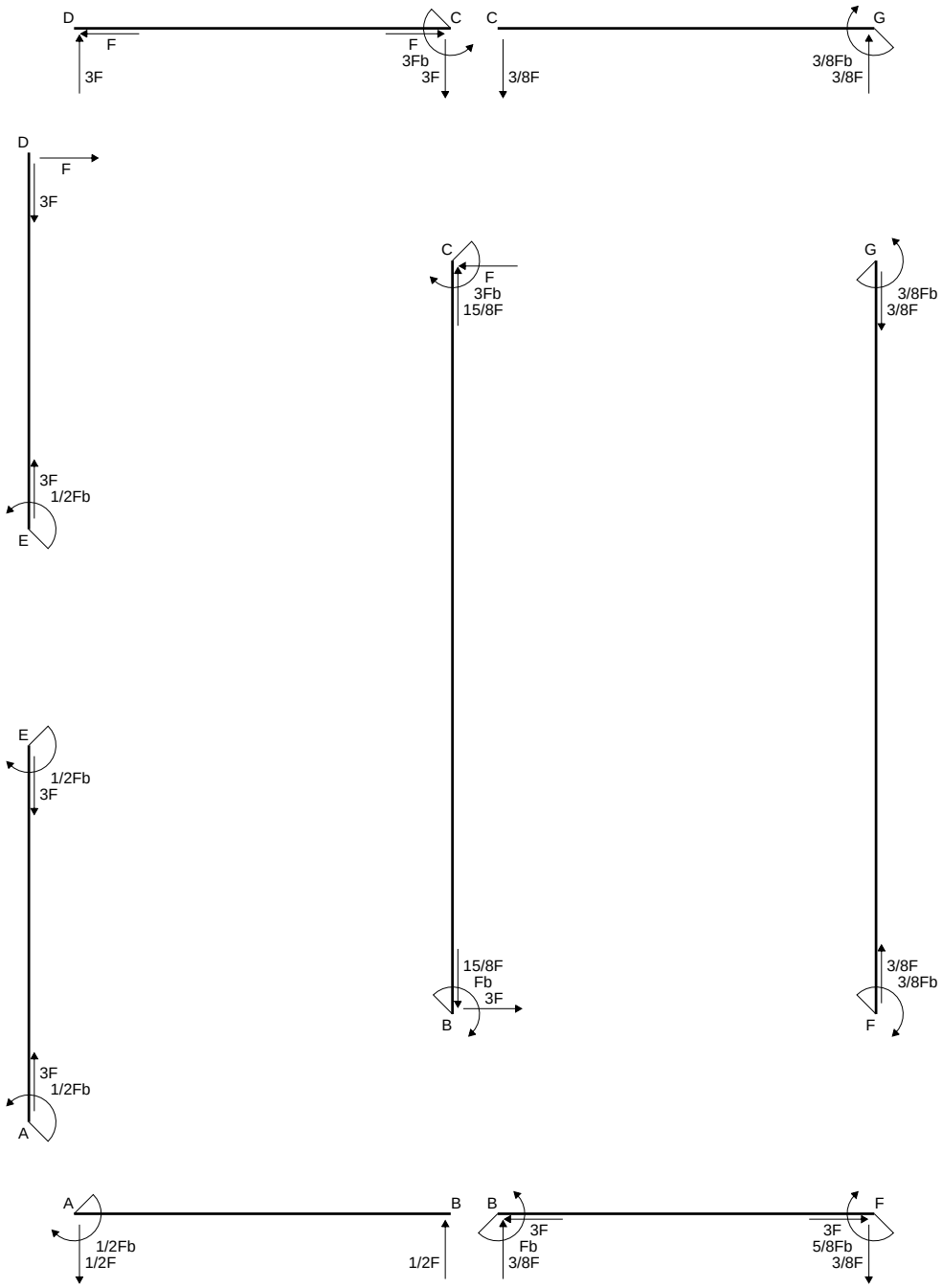
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F
3Fb
3F
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G
3/8Fb
3/8F
D
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3/8F
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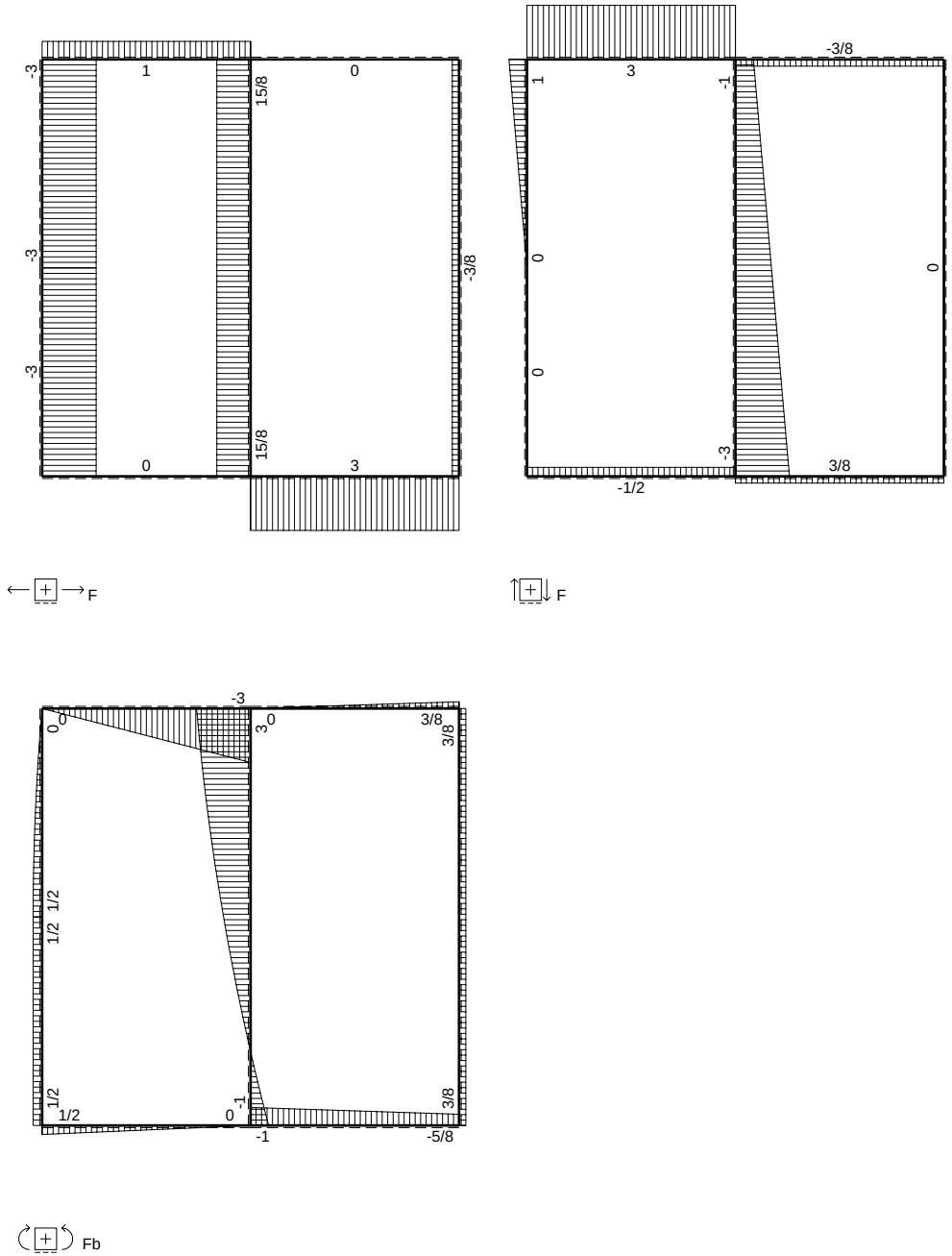
5/8Fb
3/8F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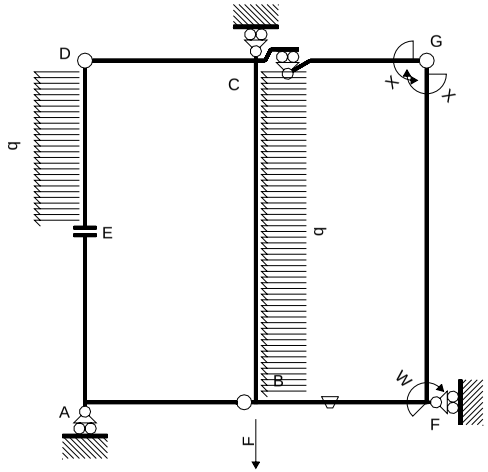
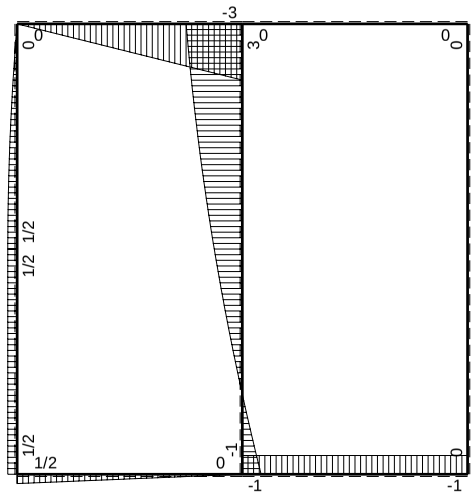
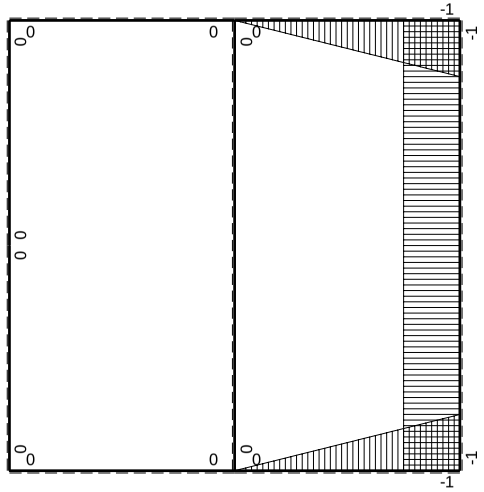
0
0
-3
3/8
-1/2
F
-3
0
0
3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.011

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

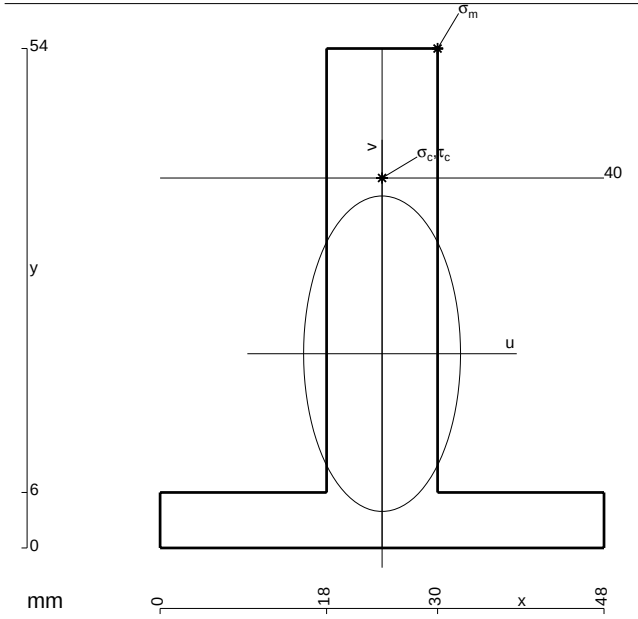
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 864. mm^2
J_u = 251424. mm^4
J_v = 62208. mm^4
y_g = 21. mm
N = 1120. N
T_y = 3360. N
M_x = -1579200. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33. mm
$\sigma_m = N/A - Mv/J_u$ = 208.6 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 19. mm
$\sigma_c = N/A - Mv/J_u$ = 120.6 N/mm^2
τ_c = 4.864 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 120.9 N/mm^2
S^* = 4368. mm^3

ESΣ8.011

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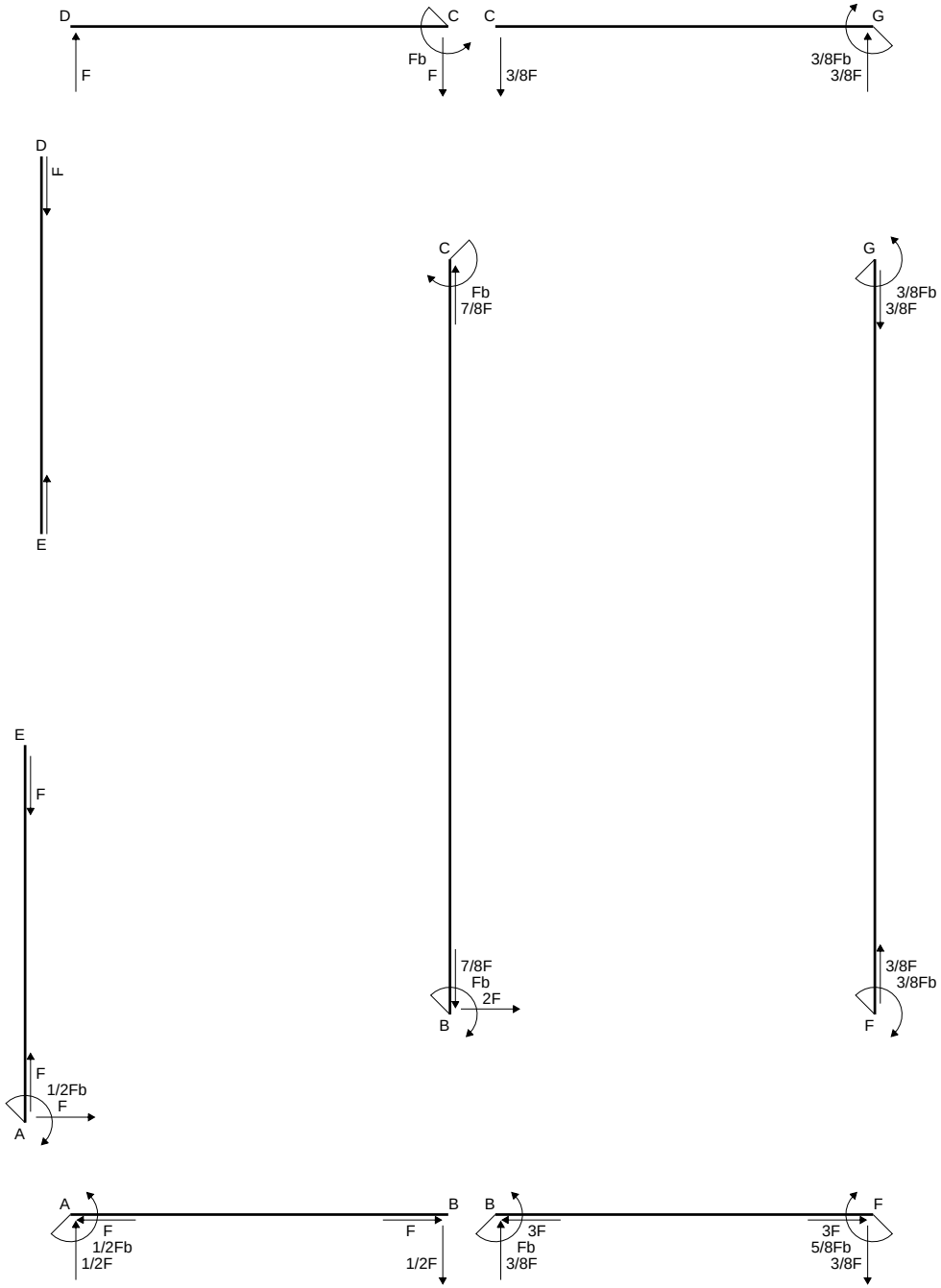
D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
E
C
Fb
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G
3/8Fb
3/8F
E
F
F
1/2Fb
F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
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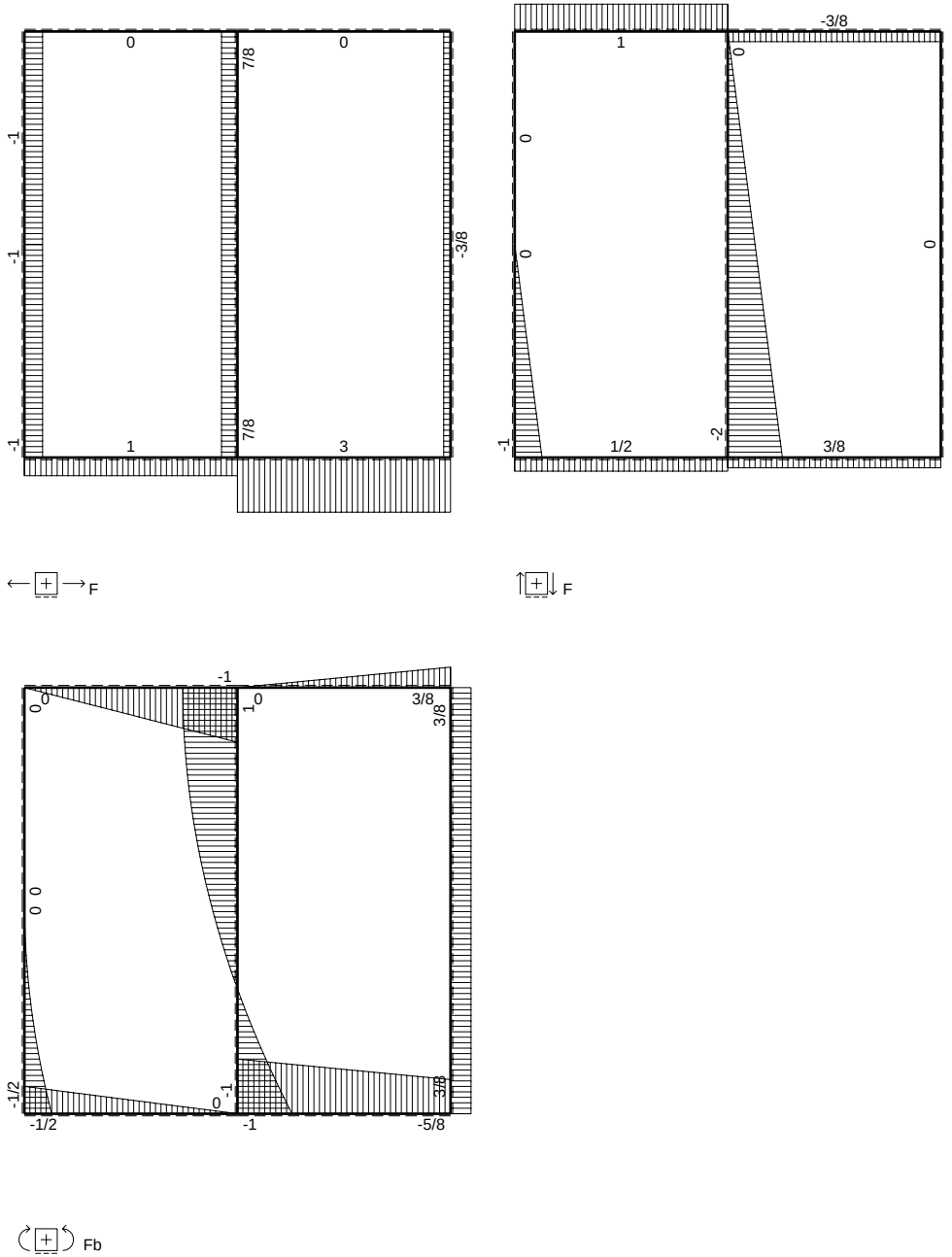
Fb
3/8F
3F
5/8Fb
3/8F


0
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7/8
-1
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3
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-3/8
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-1
1/2
-2
3/8
F
-1
0
0
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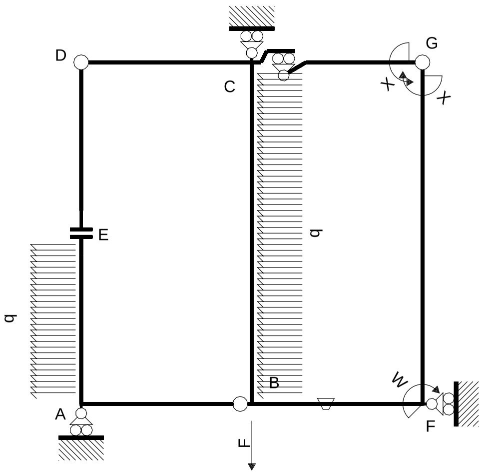
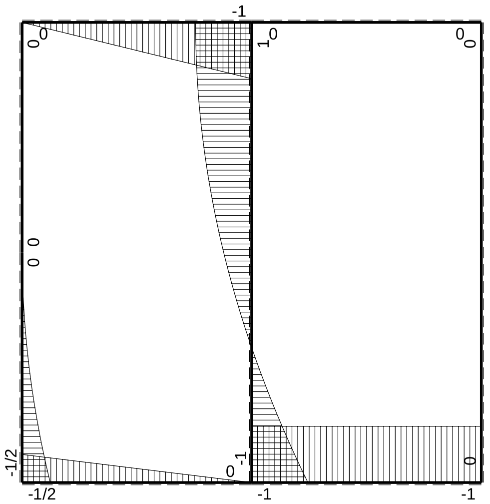
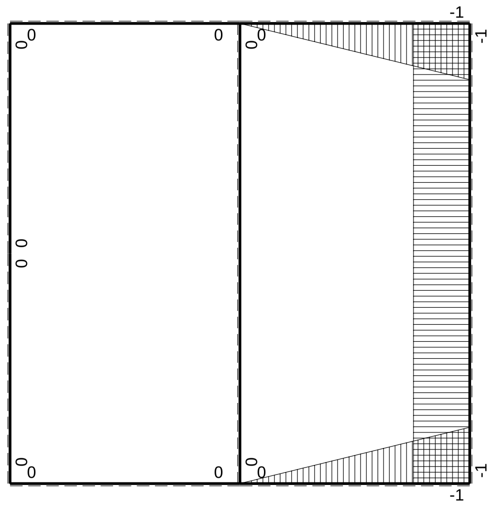
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

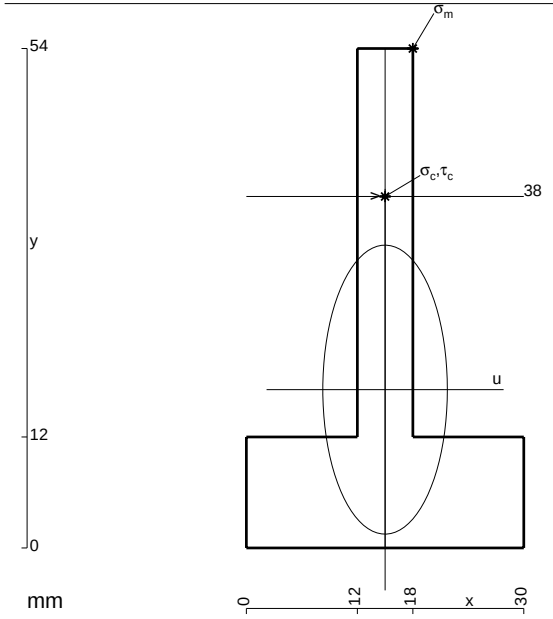
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 612. mm^2
J_u = 149428. mm^4
J_v = 27756. mm^4
y_g = 17.12 mm
N = 1505. N
T_y = -3440. N
M_x = -877200. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 36.88 mm
$\sigma_m = N/A - Mv/J_u$ = 219. N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 20.88 mm
$\sigma_c = N/A - Mv/J_u$ = 125. N/mm^2
τ_c = 10.64 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 126.4 N/mm^2
S^* = 2773. mm^3

ESΣ8.012

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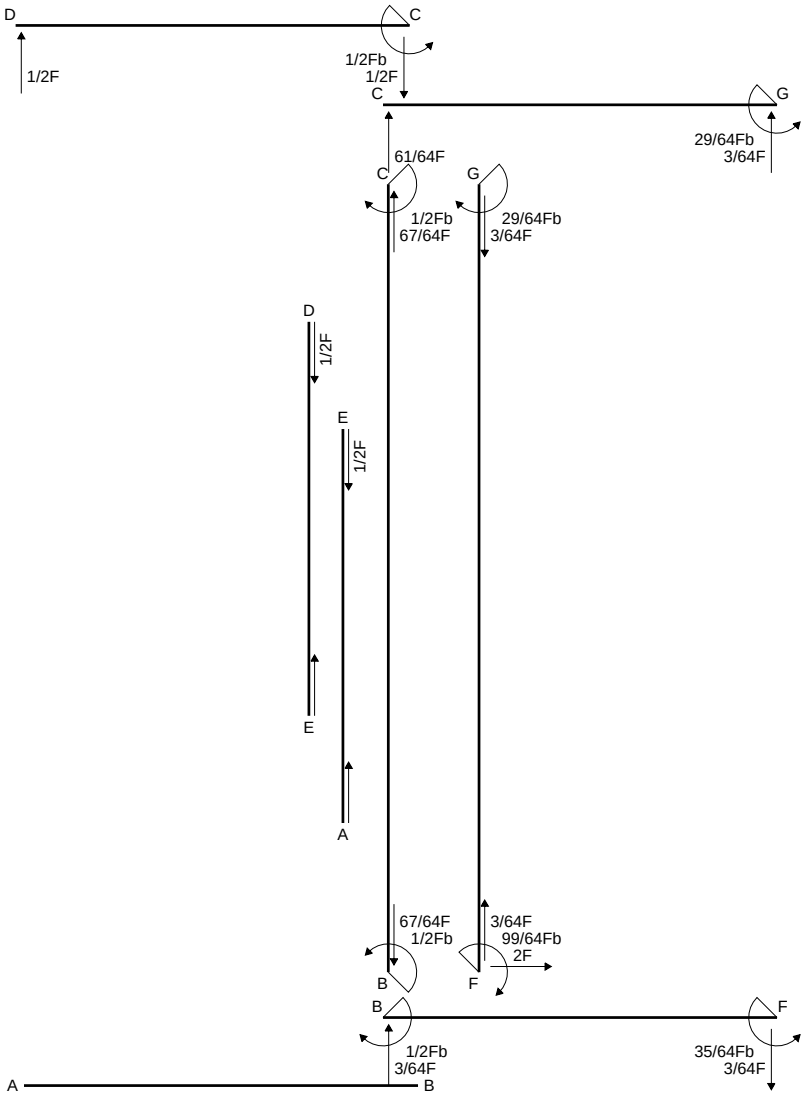
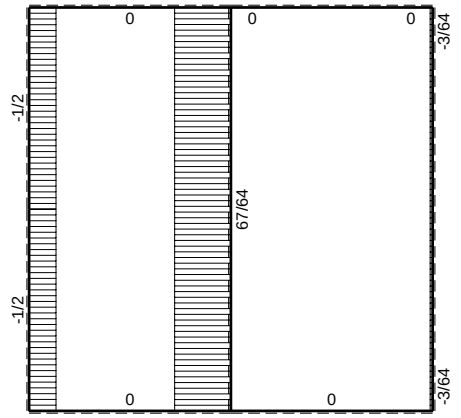

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
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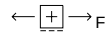
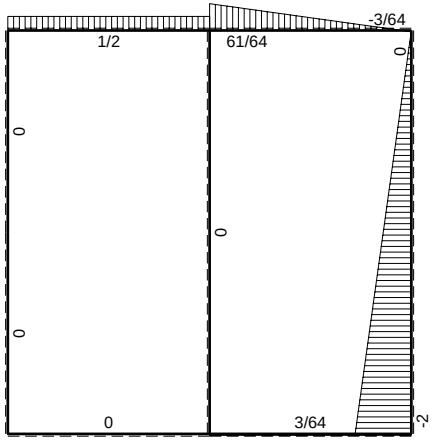
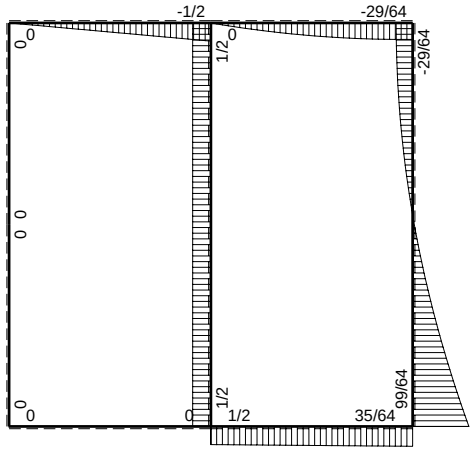
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


F


-3/64
1/2
61/64
0
0
0
0
0
0
3/64
-2


F


-1/2
-29/64
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0
-29/64
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0
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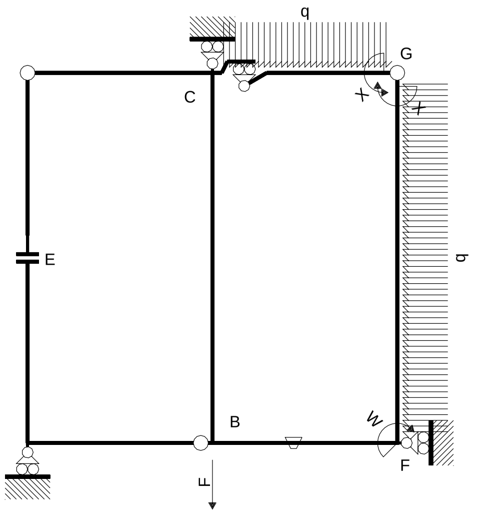
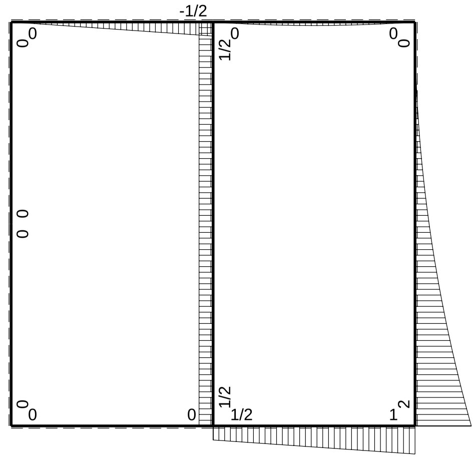
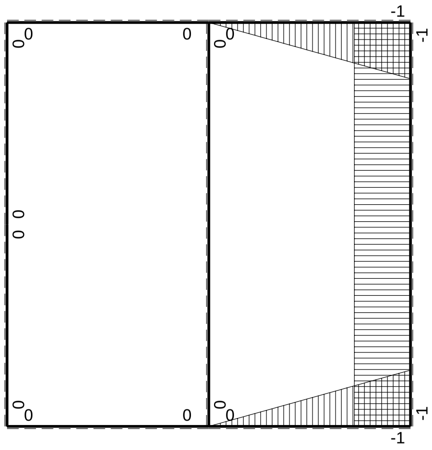
0
1/2
1/2
35/64
99/64


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int xM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$1/2Fb+1/2Fx$	$-Fb/EJ$	$-1/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(-5/12+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb+1/2Fx$	Fb/EJ	$-Fb+3/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$1/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$-1/2Fb$	0	0	0	0		
	totali						$-29/24Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$29/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

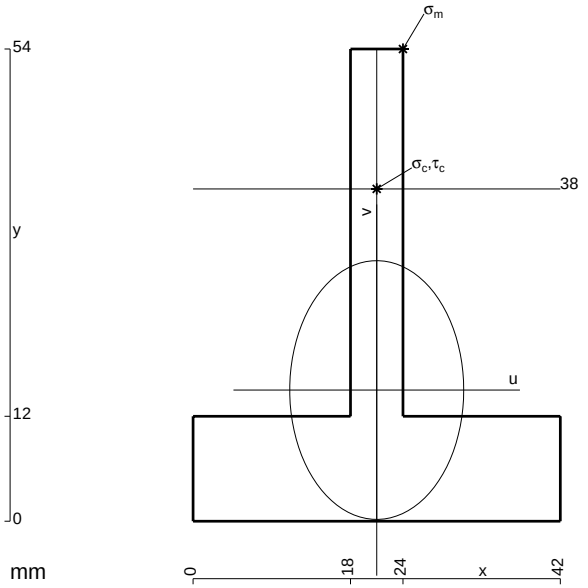
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 756. mm^2
J_u = 165564. mm^4
J_v = 74844. mm^4
y_g = 15. mm
T_y = 1775. N
M_x = -976250. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 39. mm
$\sigma_m = -Mv/J_u$ = 230. N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 23. mm
$\sigma_c = -Mv/J_u$ = 135.6 N/mm^2
τ_c = 5.318 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.9 N/mm^2
S^* = 2976. mm^3

ESΣ8.013

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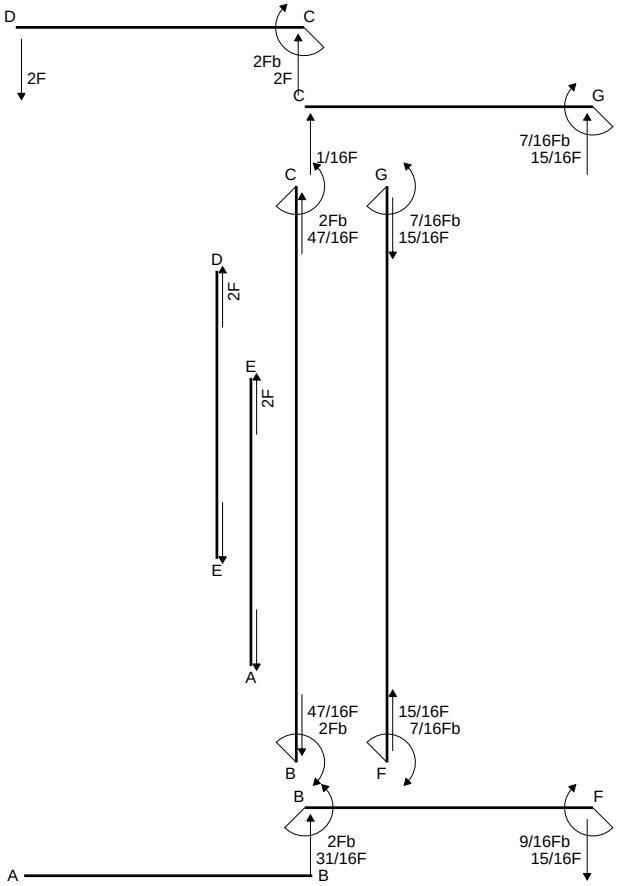
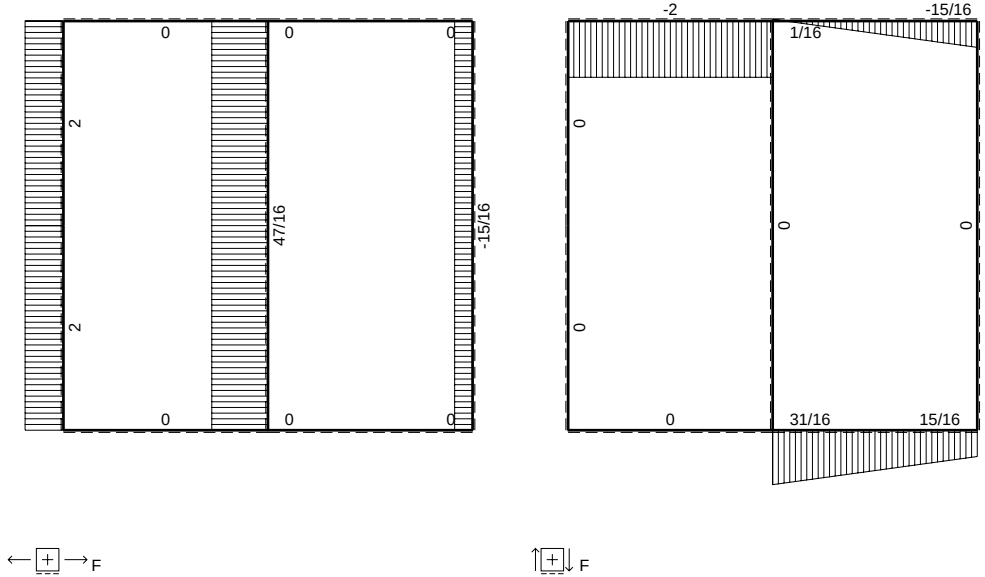

D
C
2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
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-2
-15/16
1/16
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0
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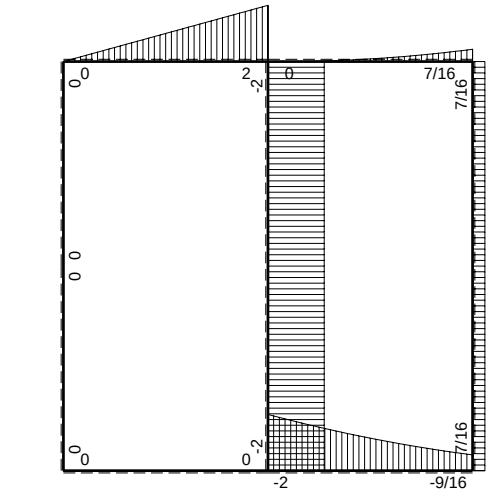
31/16
15/16
F
F


0
0
2
-2
0
7/16
7/16
0
0
0
0
-2
0
-2
7/16
-2
-9/16

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						7/6Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

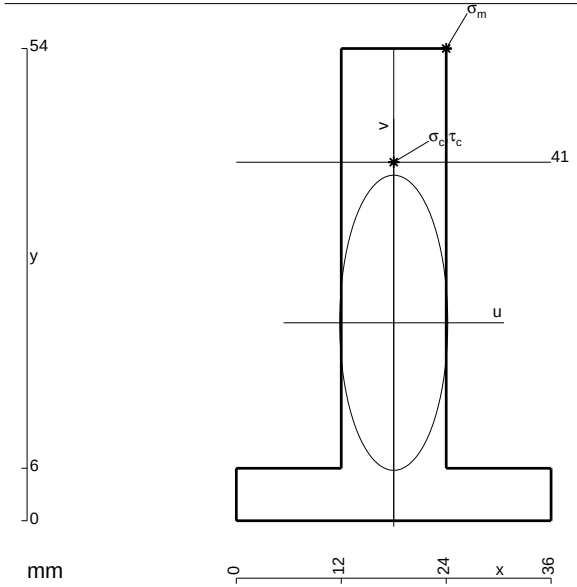
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 792. mm^2
J_u = 225759. mm^4
J_v = 30240. mm^4
y_g = 22.64 mm
T_y = -2860. N
M_x = 1716000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 31.36 mm
$\sigma_m = -Mv/J_u$ = -238.4 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 18.36 mm
$\sigma_c = -Mv/J_u$ = -139.6 N/mm^2
τ_c = 4.095 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 139.8 N/mm^2
S^* = 3879. mm^3

ESΣ8.014

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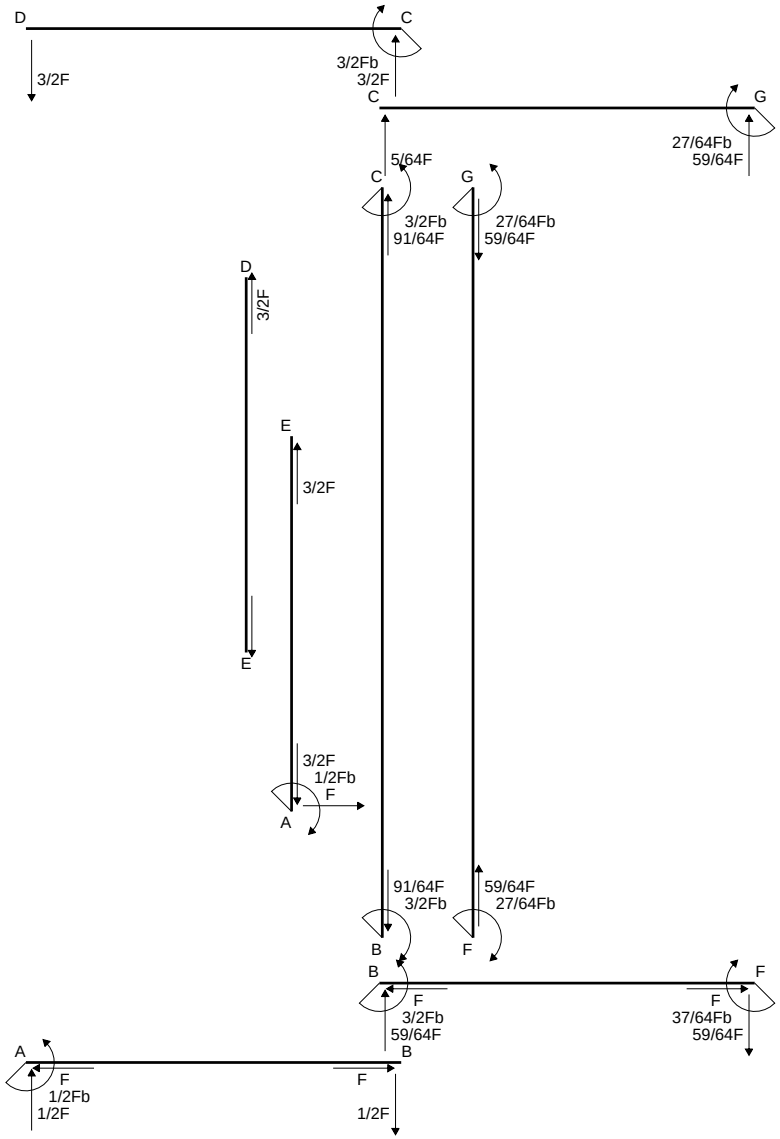
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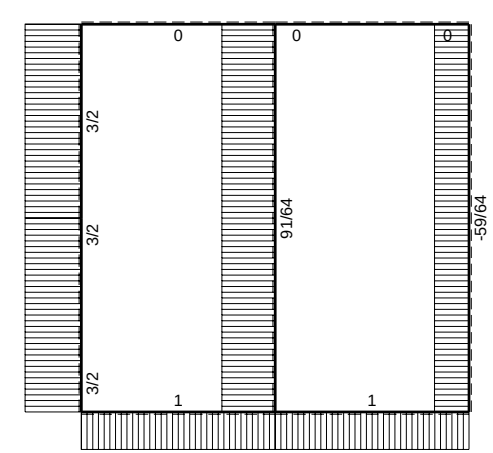
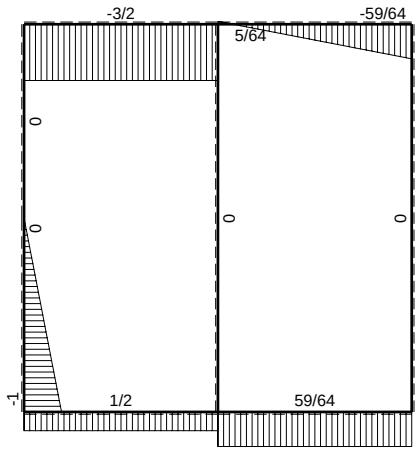
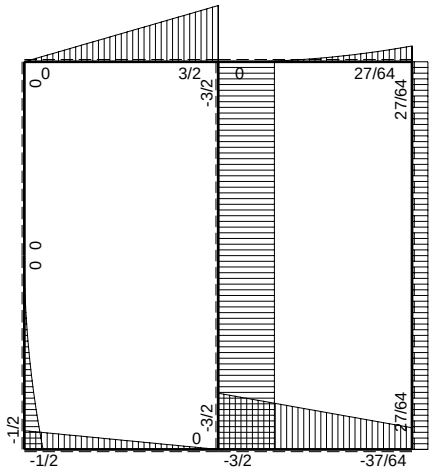
C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
3/2
91/64
-59/64
1
1


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F

F


0
0
3/2
-3/2
0
27/64
27/64
0
0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

14.11.23

M_0 flessione da carichi assegnati

PROCEDIMENTO E RISULTATI Nome: ESΣ8.015

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DC b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0	0+0	0
	0	-1/2qx²	0	0	0	0		
EA b	0	1/2Fb-Fx+1/2qx²	0	0	0	0	0+0	0
	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx²/b	Fx/EJ	x²/b²	(7/12+1/2)Fb²/EJ	1/3XbJ/EJ
BF b	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x²/b²		
FB b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²	(1/24+0)Fb²/EJ	1/3XbJ/EJ
GC b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²		
CG b	-1	0	0	0	0	1	0+0	2XbJ/EJ
FG 2b	1	0	0	0	0	1		
GF 2b	0	-3/2Fb	0	0	0	0	0+0	0
CB 2b	0	3/2Fb	0	0	0	0		
BC 2b	totali						9/8Fb²/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

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14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

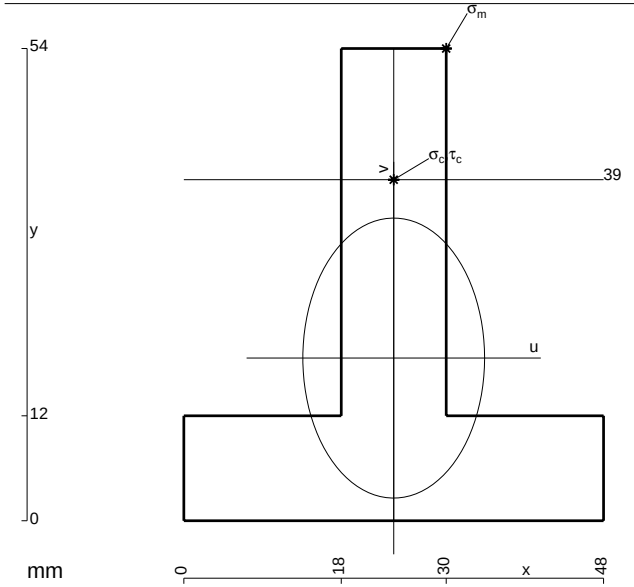
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 1080. mm^2
J_u = 276955. mm^4
J_v = 116640. mm^4
y_g = 18.6 mm
T_y = -2430. N
M_x = 1555200. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 35.4 mm
$\sigma_m = -Mv/J_u$ = -198.8 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.4 mm
$\sigma_c = -Mv/J_u$ = -114.6 N/mm^2
τ_c = 3.672 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.7 N/mm^2
S^* = 5022. mm^3

ESΣ8.015

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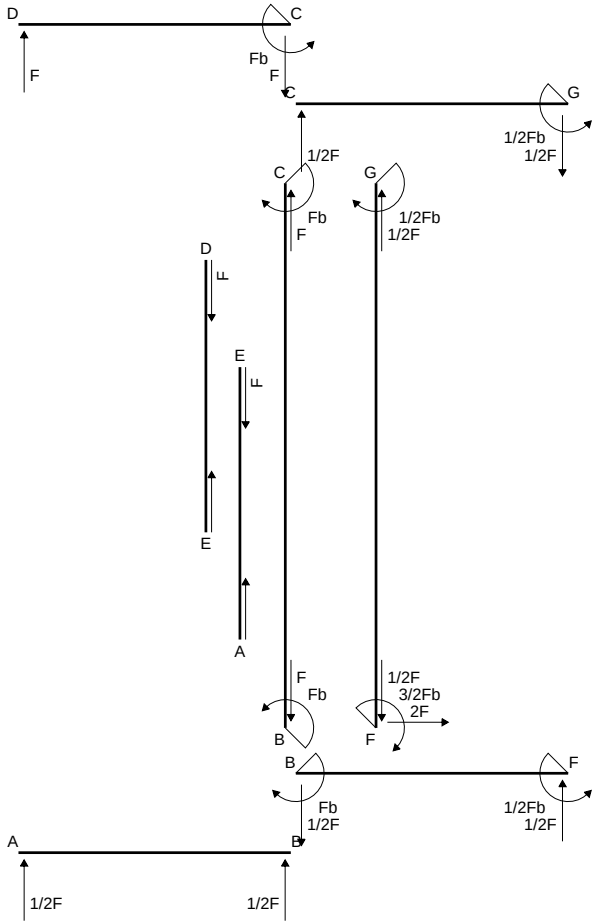
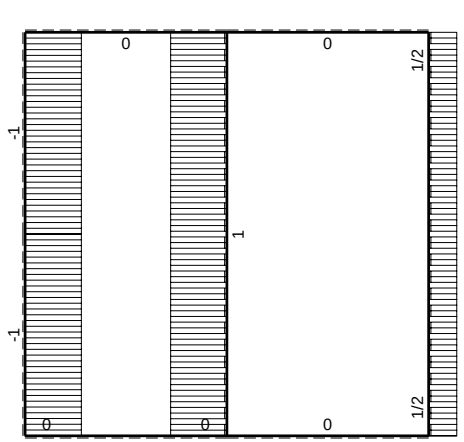
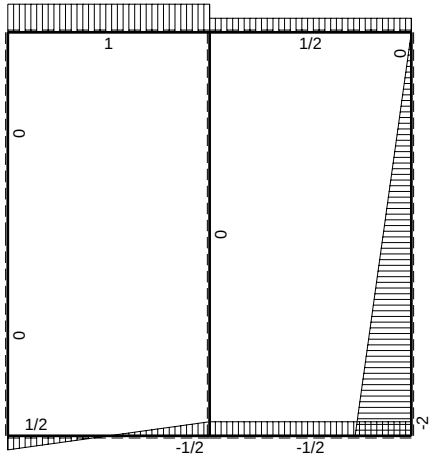

D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

F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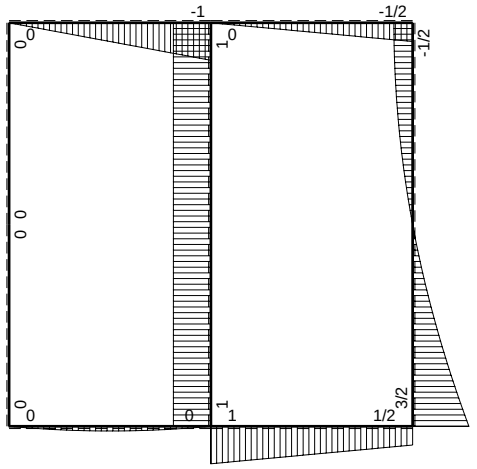
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F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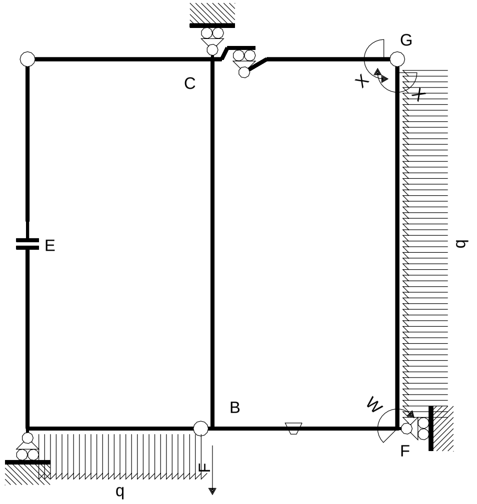
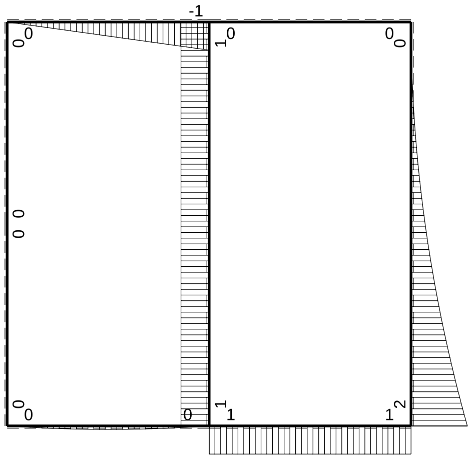
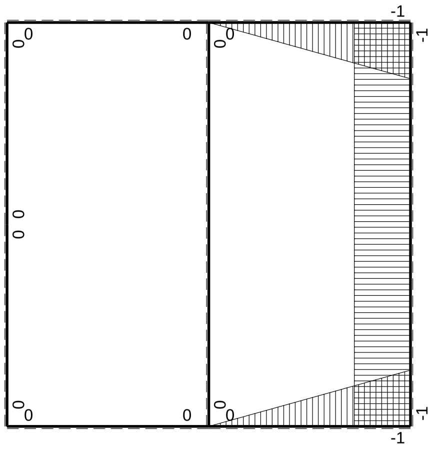
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica X=W_{GC}							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

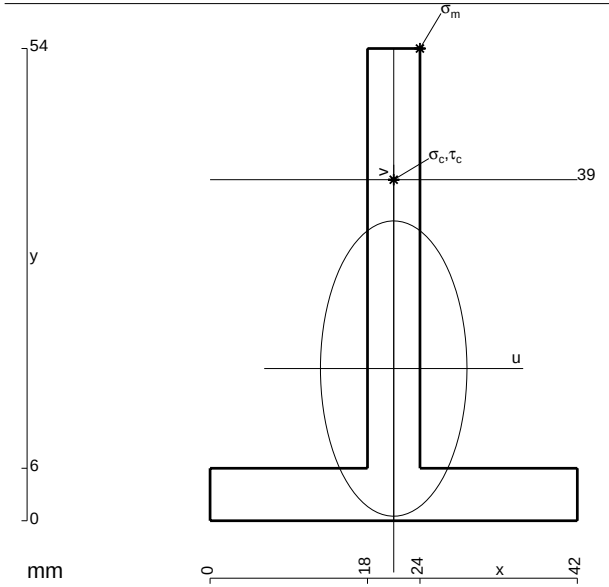
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 540. mm^2
J_u = 154030. mm^4
J_v = 37908. mm^4
y_g = 17.4 mm
T_y = 1290. N
M_x = -877200. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 36.6 mm
$\sigma_m = -Mv/J_u$ = 208.4 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 21.6 mm
$\sigma_c = -Mv/J_u$ = 123. N/mm^2
τ_c = 3.656 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.2 N/mm^2
S^* = 2619. mm^3

ESΣ8.016

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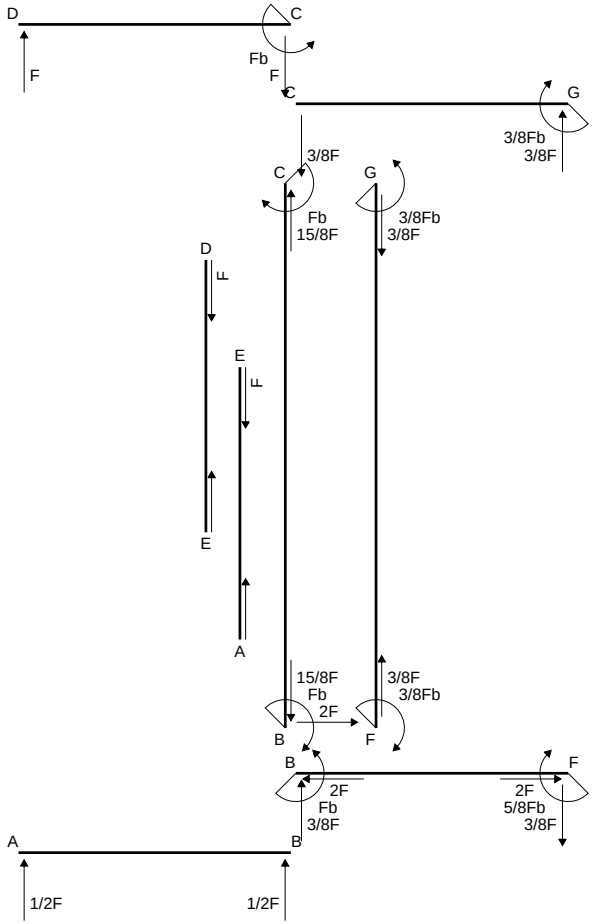
D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
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3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
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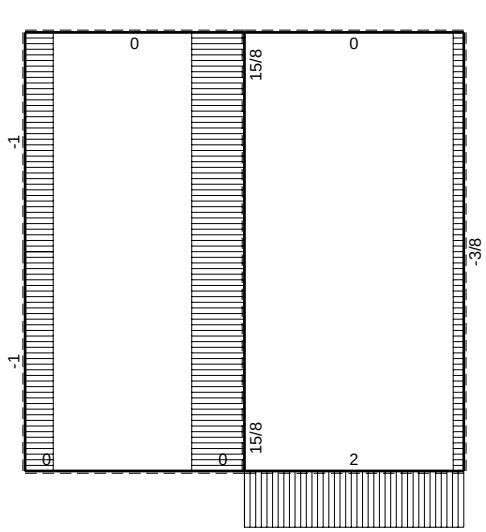
A
B
1/2F
1/2F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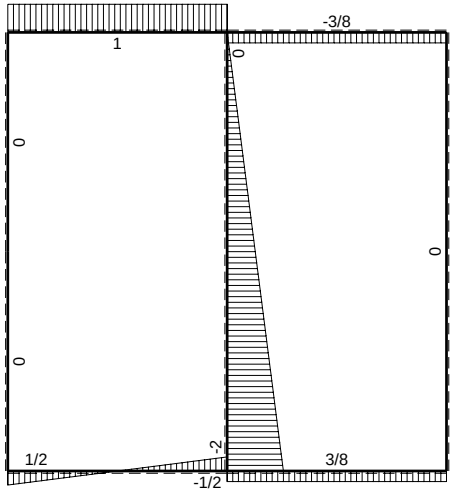
0
15/8
0
0
2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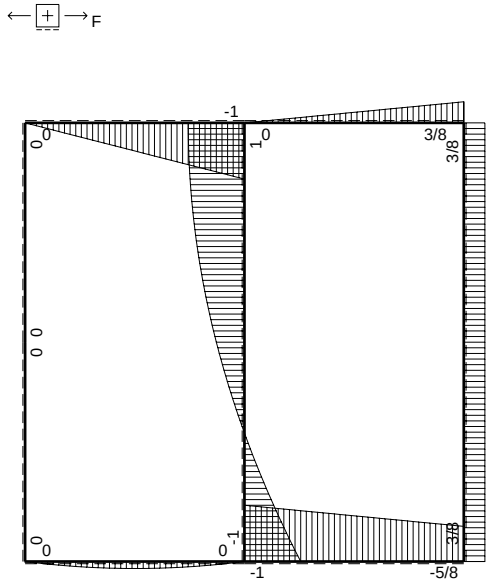
0
0
1/2
-2
3/8
-1/2


-1
0
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1
0
3/8
3/8
0 0
0
0
0
-1
-1
3/8
-1
-5/8

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

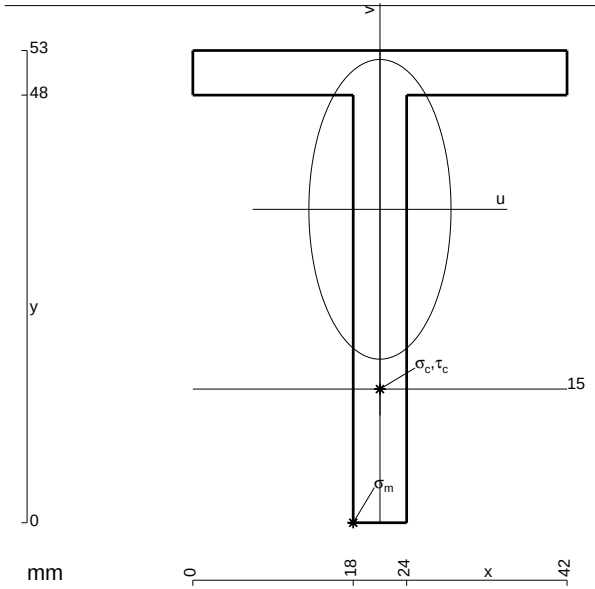
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 498. mm^2
J_u = 141019. mm^4
J_v = 31734. mm^4
y_g = 35.17 mm
N = 2363. N
T_y = -2520. N
M_x = -894600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -35.17 mm
σ_m = N/A-Mv/J_u = -218.4 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -20.17 mm
σ_c = N/A-Mv/J_u = -123.2 N/mm^2
τ_c = 7.418 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.9 N/mm^2
S^* = 2491. mm^3

ESΣ8.017

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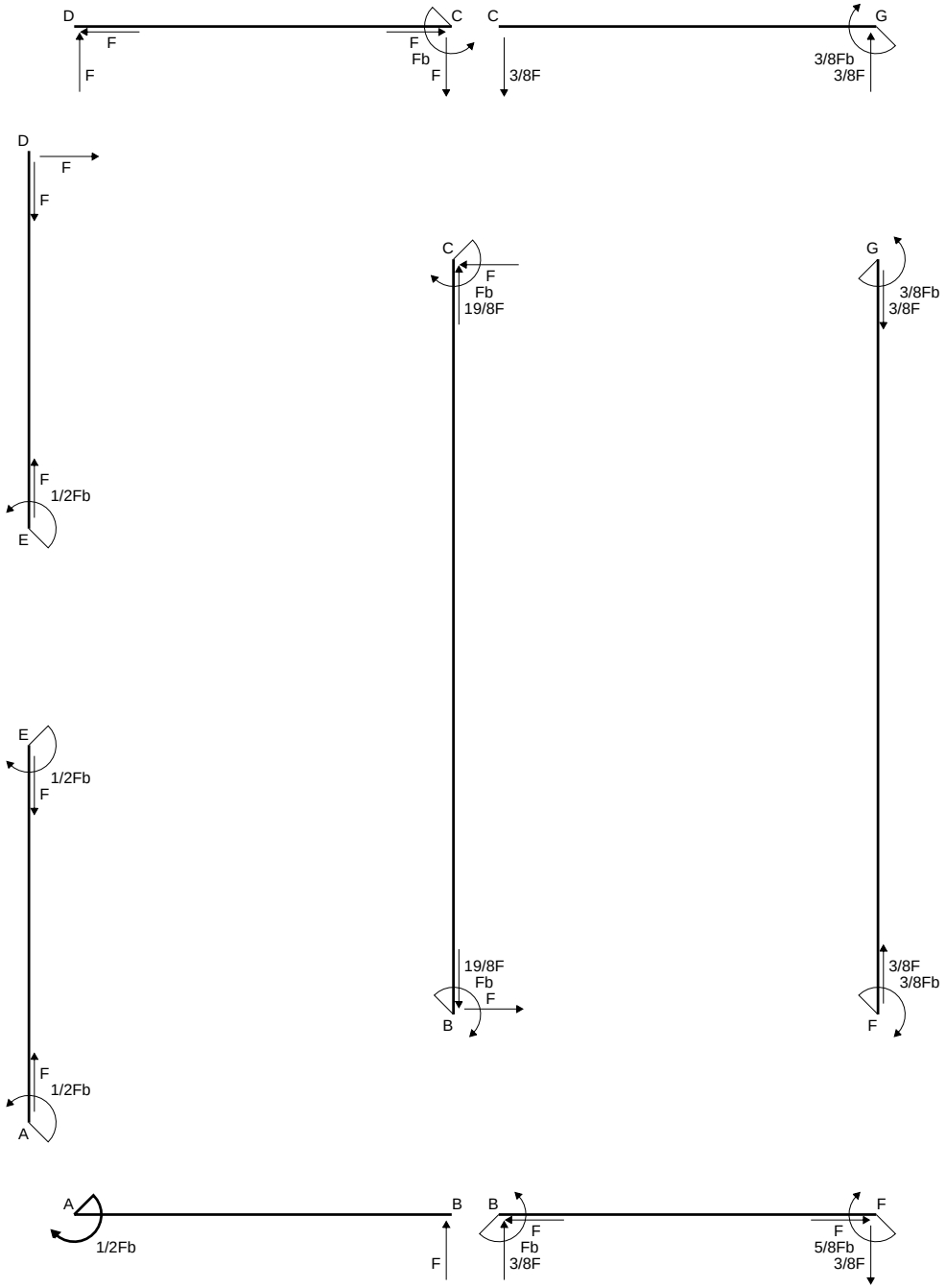
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3/8F
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Fb
19/8F
G
3/8Fb
3/8F
F
1/2Fb
E
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
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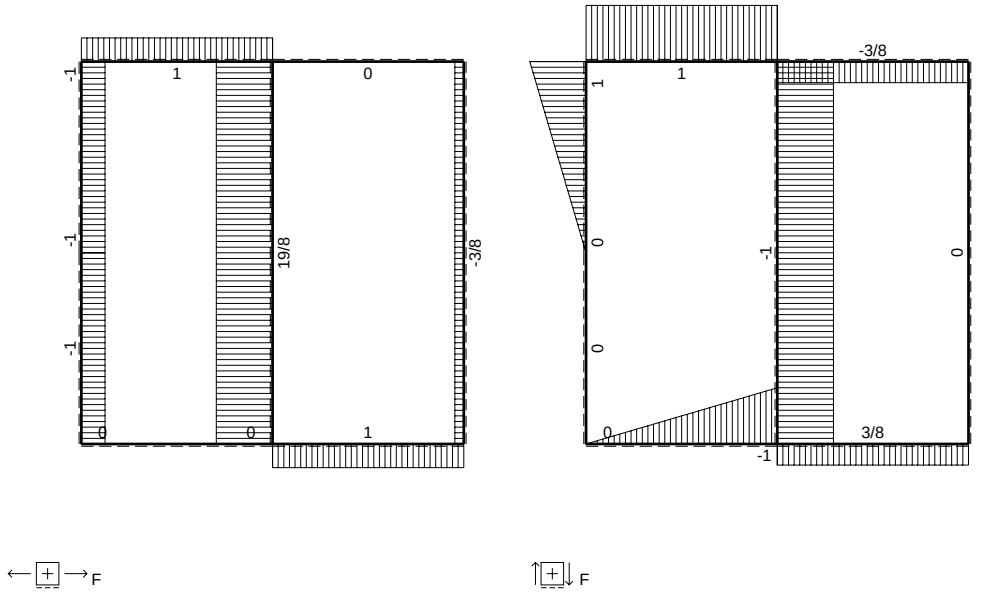
3/8F
F
5/8Fb
3/8F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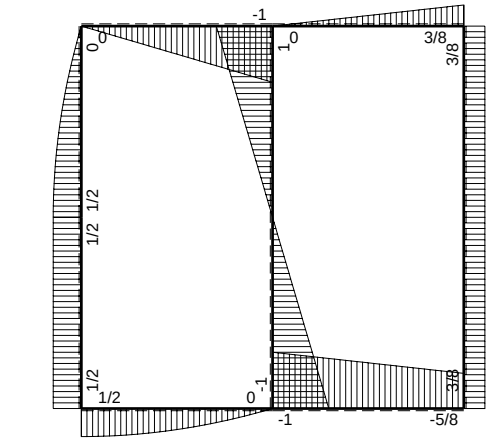
3/8
-1
F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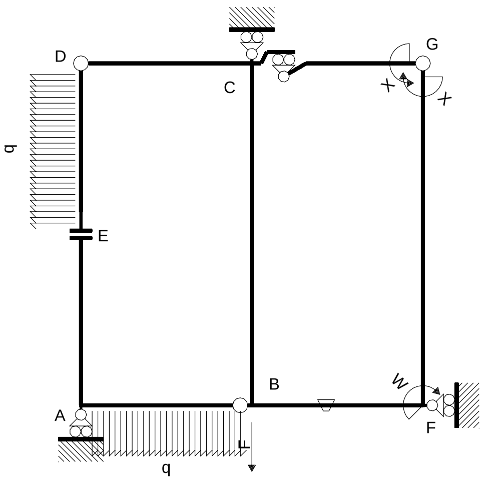
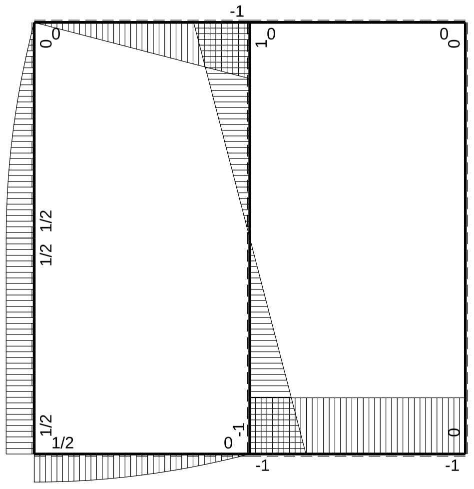
-1
-1
3/8
-1
-5/8

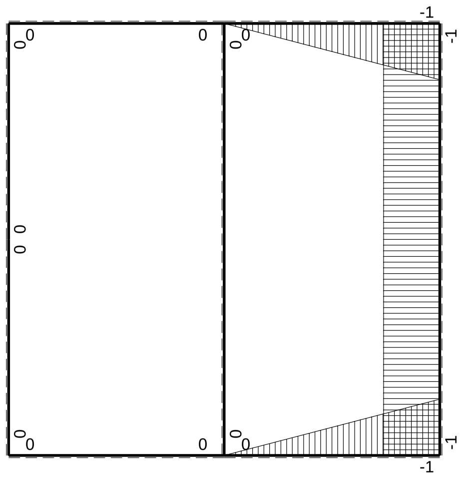

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

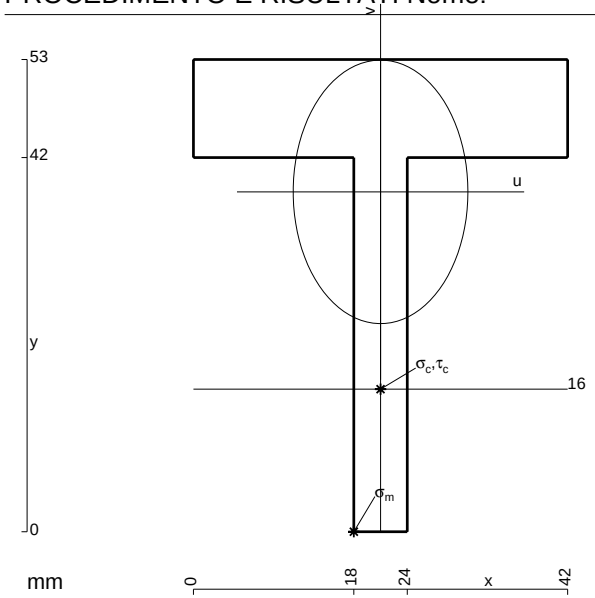
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 714. mm^2
J_u = 156210. mm^4
J_v = 68670. mm^4
y_g = 38.15 mm
N = 2921. N
T_y = -1230. N
M_x = 922500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.15 mm
$\sigma_m = N/A - Mv/J_u$ = 229.4 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.15 mm
$\sigma_c = N/A - Mv/J_u$ = 134.9 N/mm^2
τ_c = 3.798 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135. N/mm^2
S^* = 2894. mm^3

ESΣ8.018

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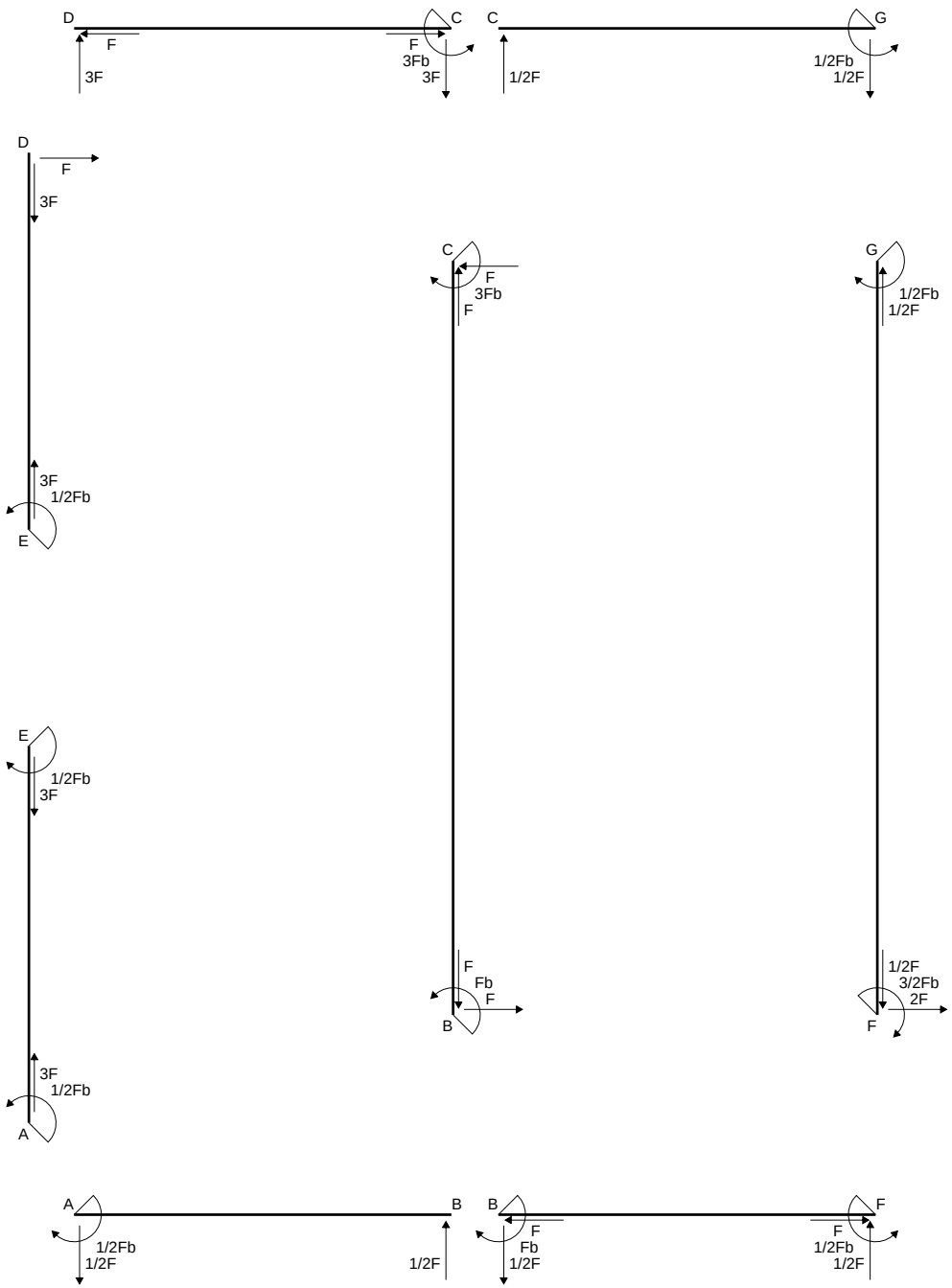
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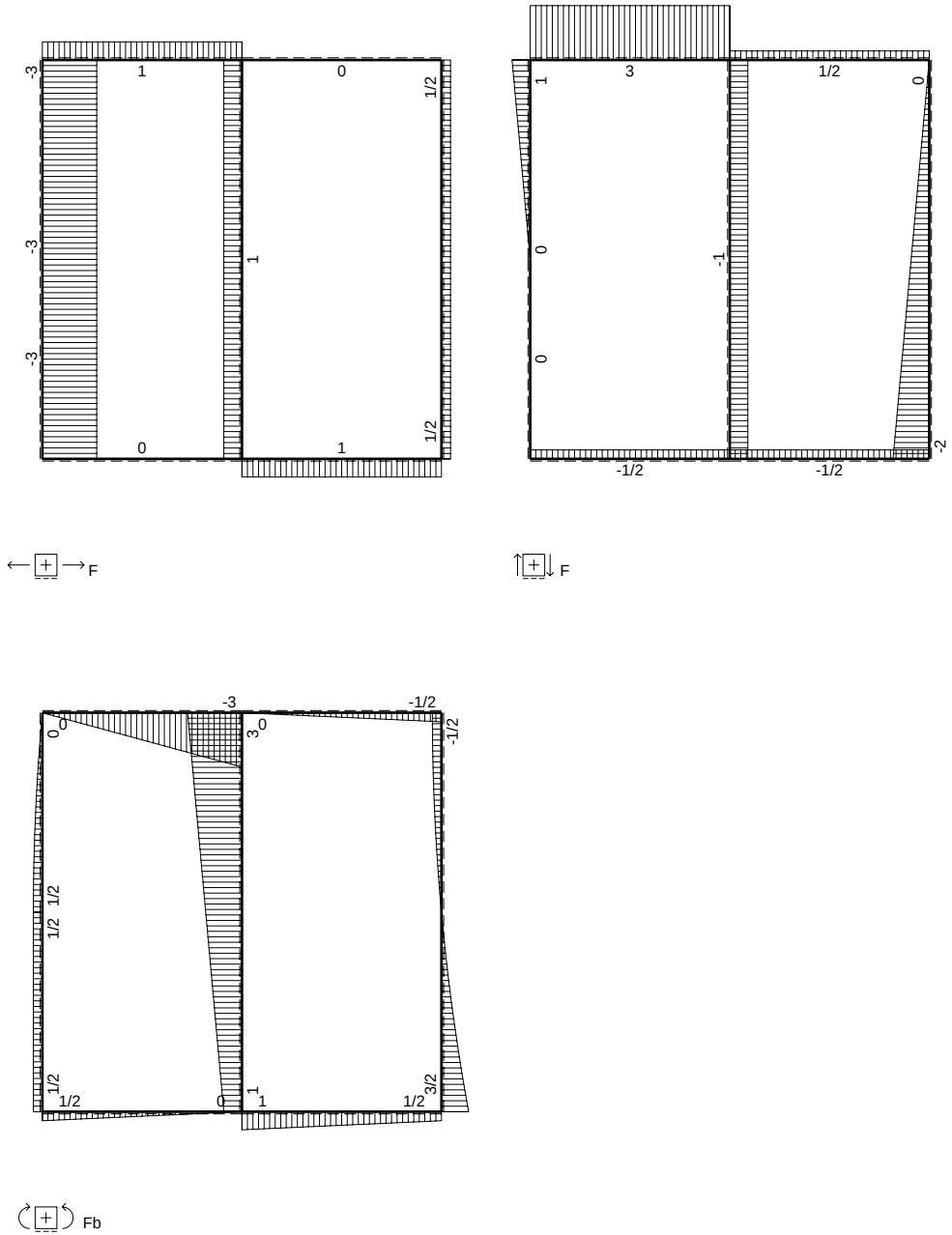
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1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

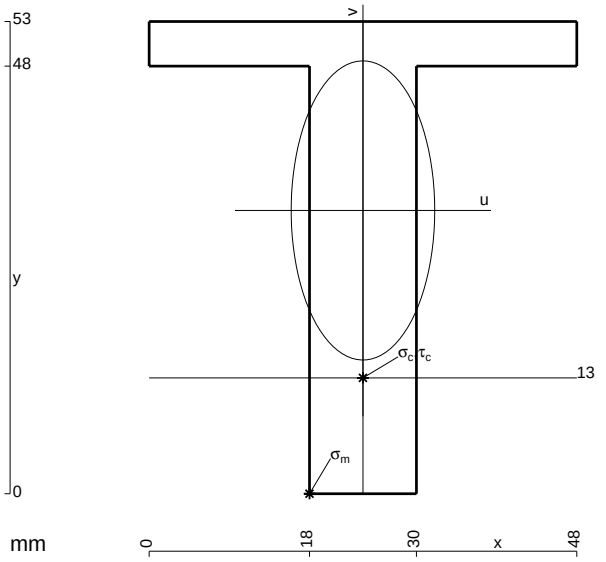
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 816. mm^2
J_u = 230061. mm^4
J_v = 52992. mm^4
y_g = 31.79 mm
N = 730. N
T_y = 2190. N
M_x = -1730100. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -31.79 mm
$\sigma_m = N/A - Mv/J_u$ = -238.2 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -18.79 mm
$\sigma_c = N/A - Mv/J_u$ = -140.4 N/mm^2
τ_c = 3.13 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 140.5 N/mm^2
S^* = 3946. mm^3

ESΣ8.019

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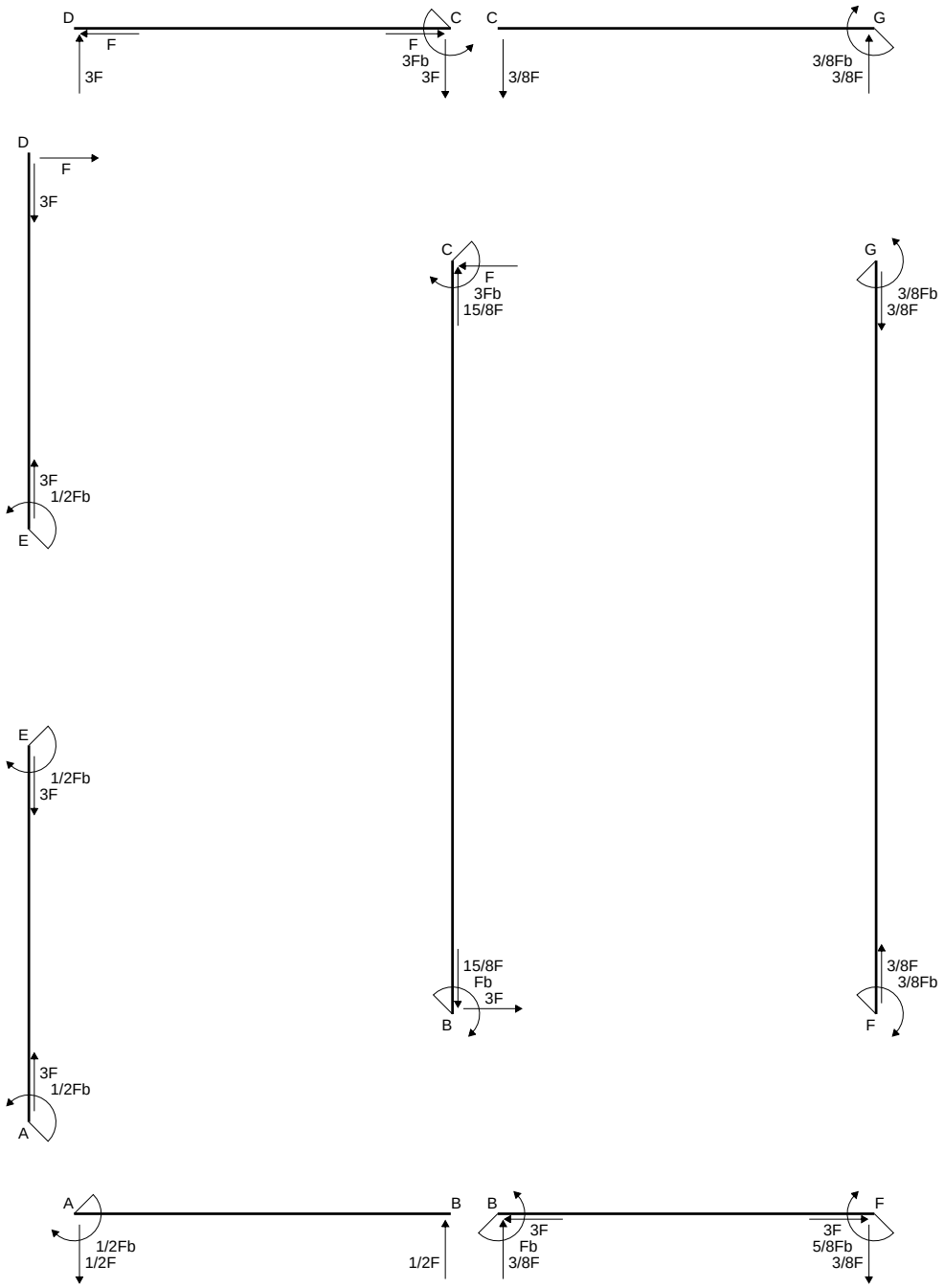
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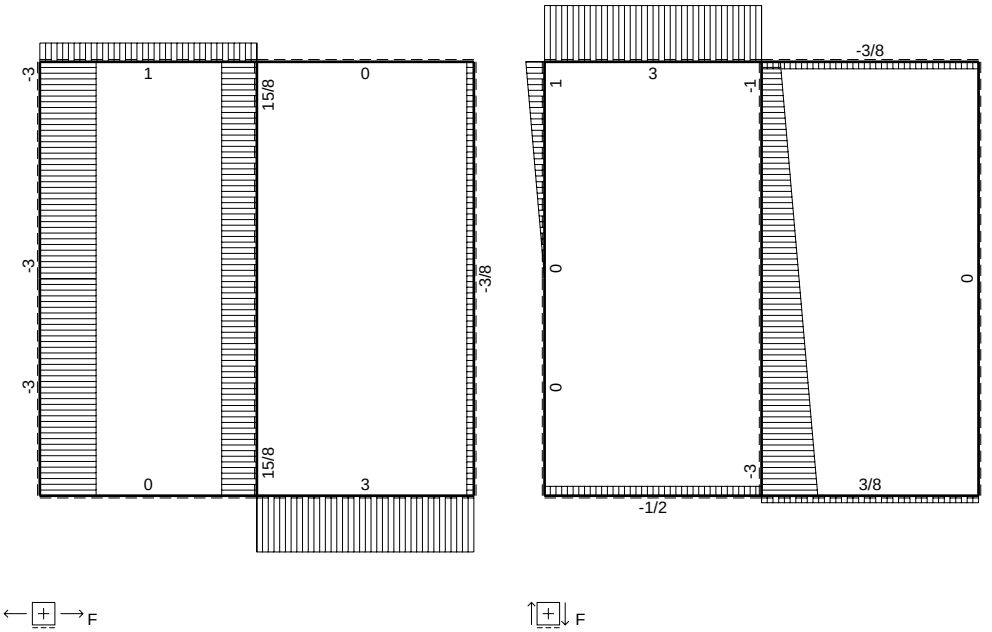
5/8Fb
3/8F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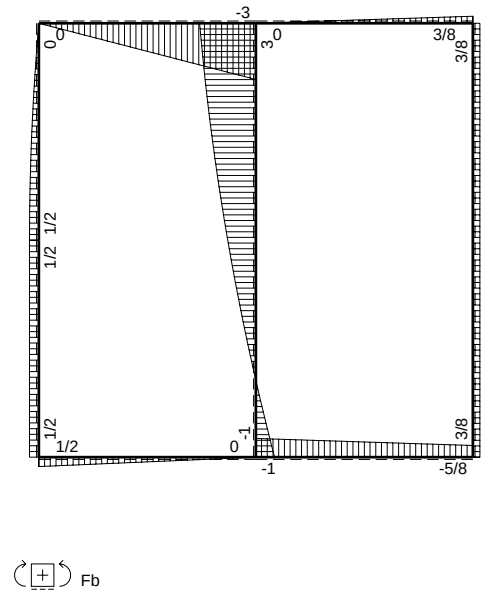
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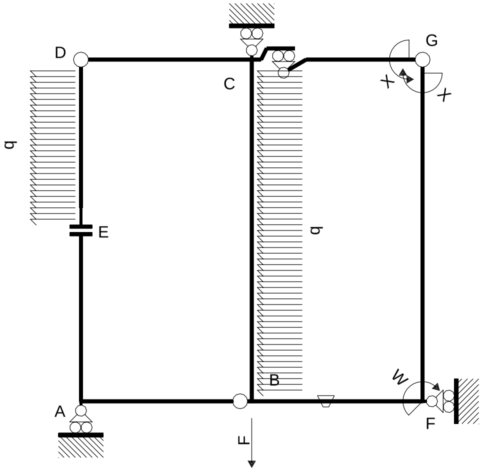
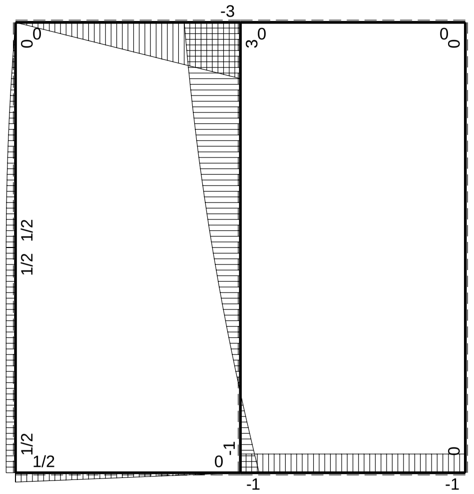
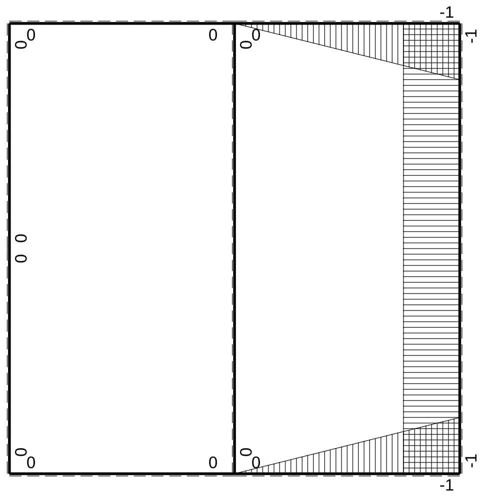
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

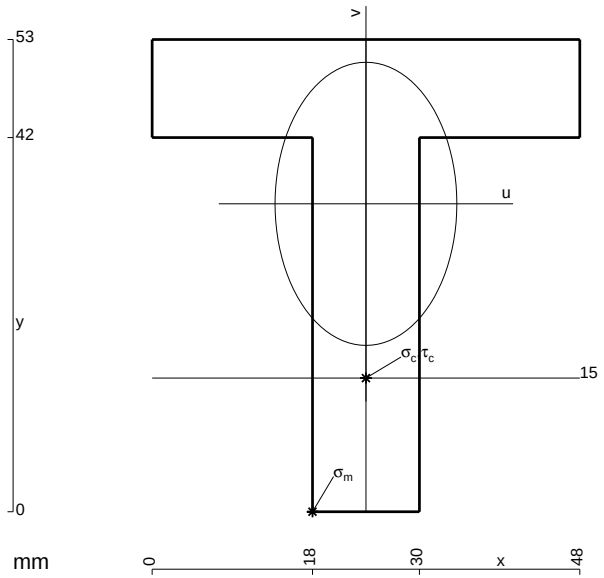
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.020



A = 1032. mm^2
J_u = 260495. mm^4
J_v = 107424. mm^4
y_g = 34.56 mm
N = 1200. N
T_y = 3600. N
M_x = -1512000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.56 mm
$\sigma_m = N/A - Mv/J_u$ = -199.4 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -19.56 mm
$\sigma_c = N/A - Mv/J_u$ = -112.4 N/mm^2
τ_c = 5.609 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 112.8 N/mm^2
S^* = 4870. mm^3

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ESΣ8.020

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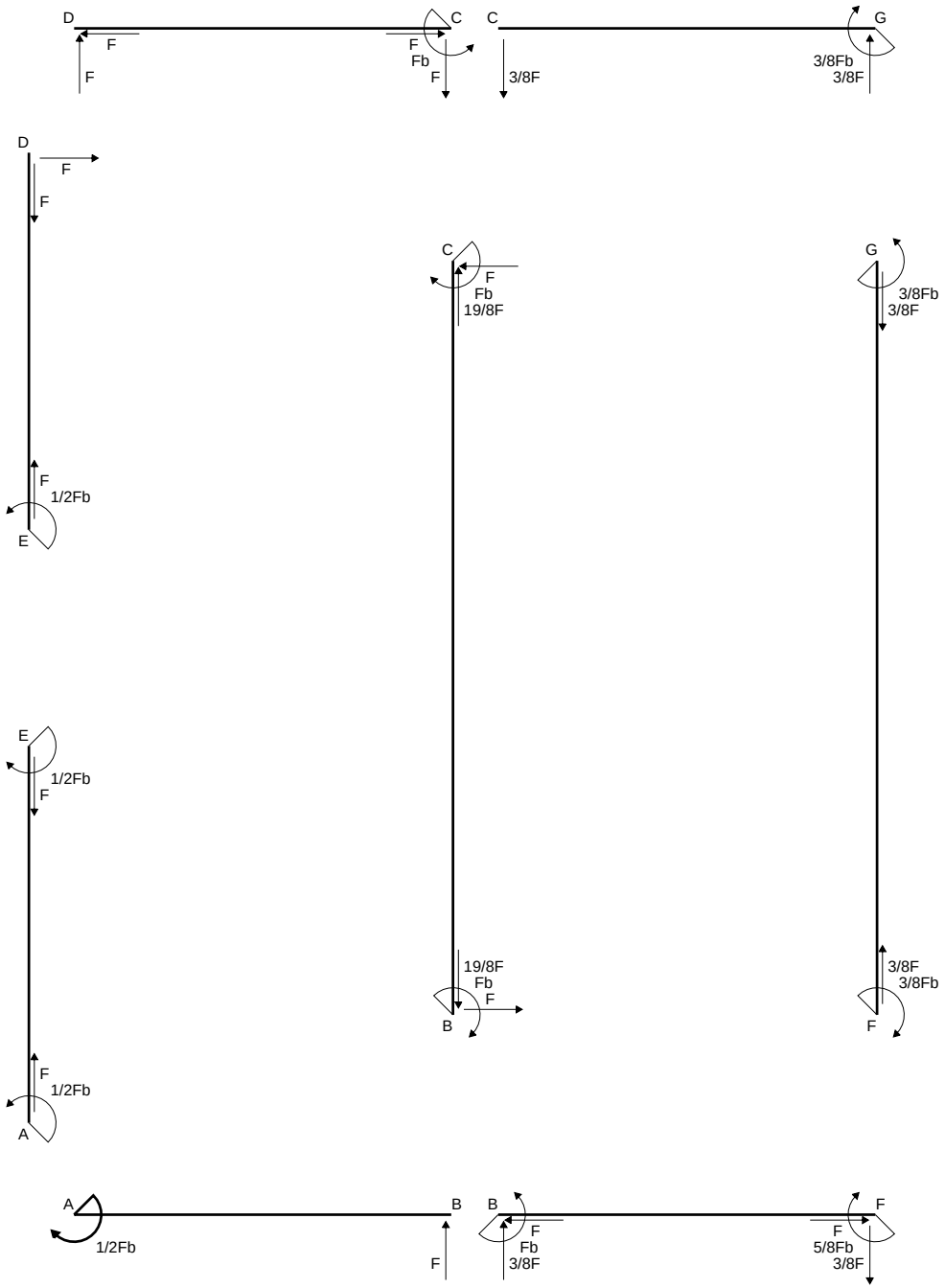
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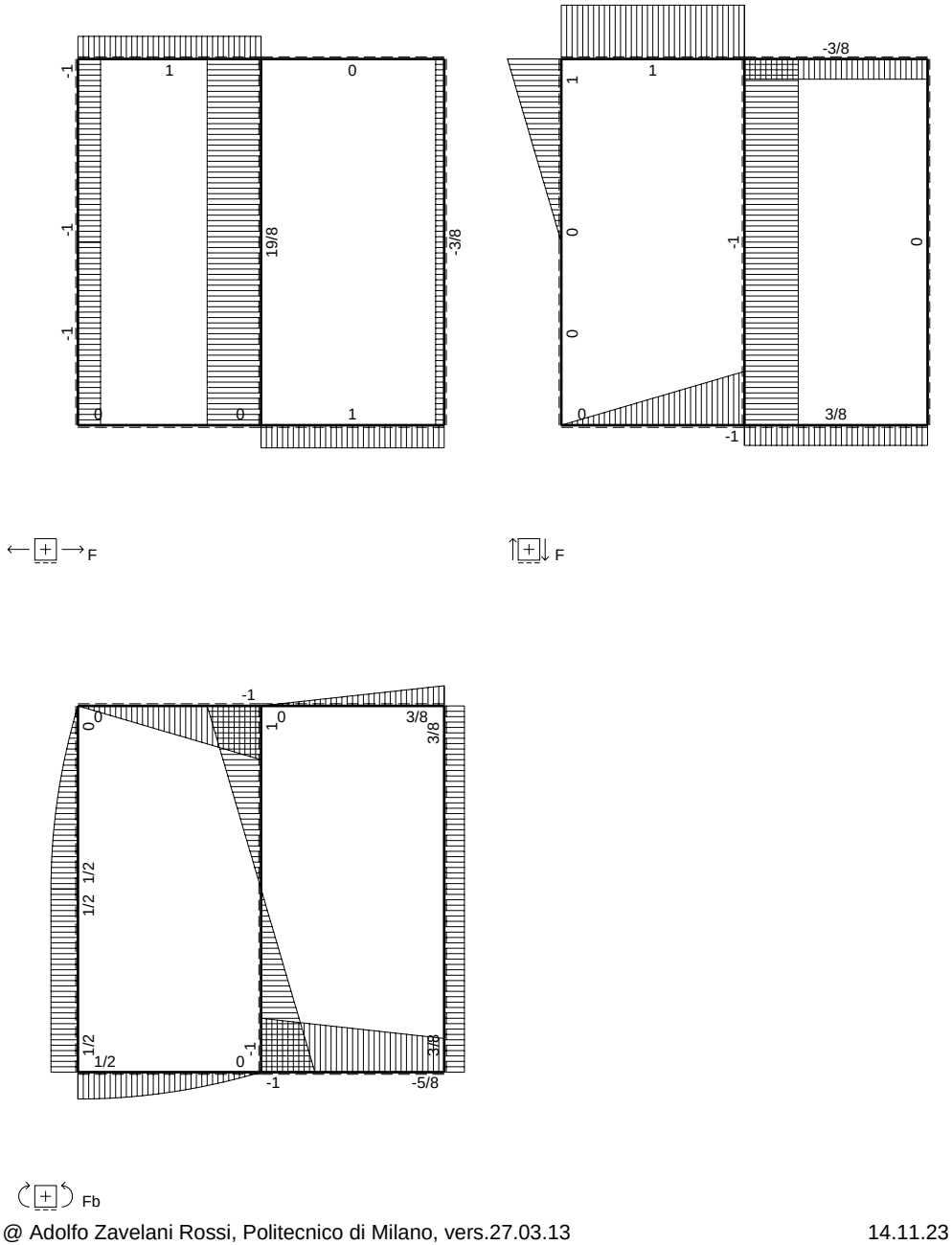
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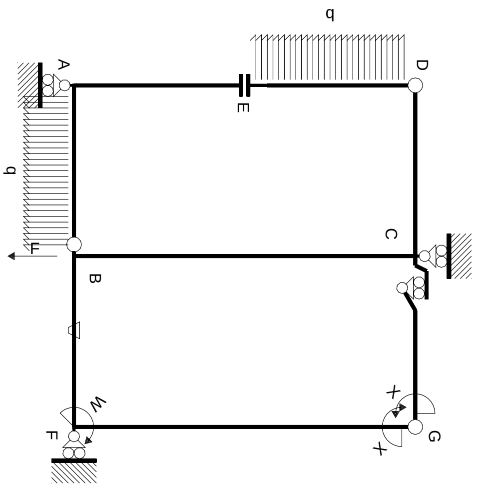
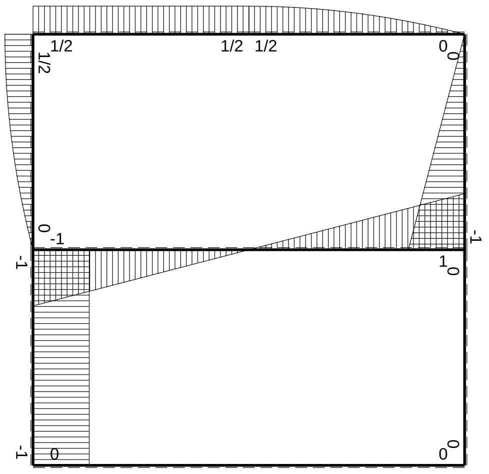
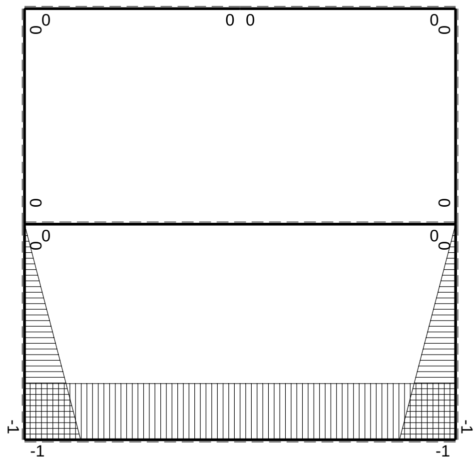
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

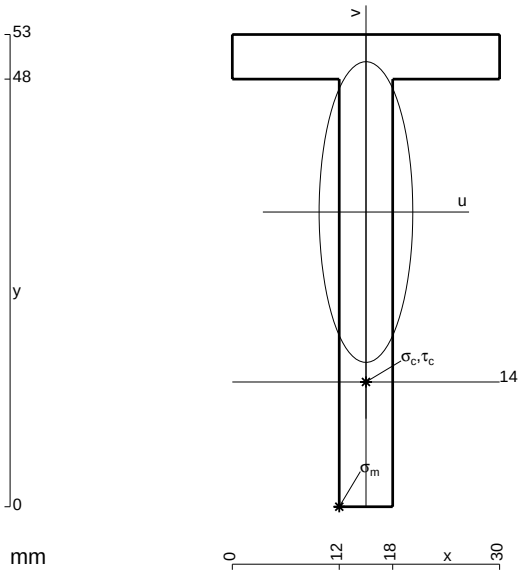
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 438. mm^2
J_u = 124871. mm^4
J_v = 12114. mm^4
y_g = 33.08 mm
N = 3919. N
T_y = -1650. N
M_x = 759000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -33.08 mm
$\sigma_m = N/A - Mv/J_u$ = 210. N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -19.08 mm
$\sigma_c = N/A - Mv/J_u$ = 124.9 N/mm^2
τ_c = 4.824 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 125.2 N/mm^2
S^* = 2190. mm^3

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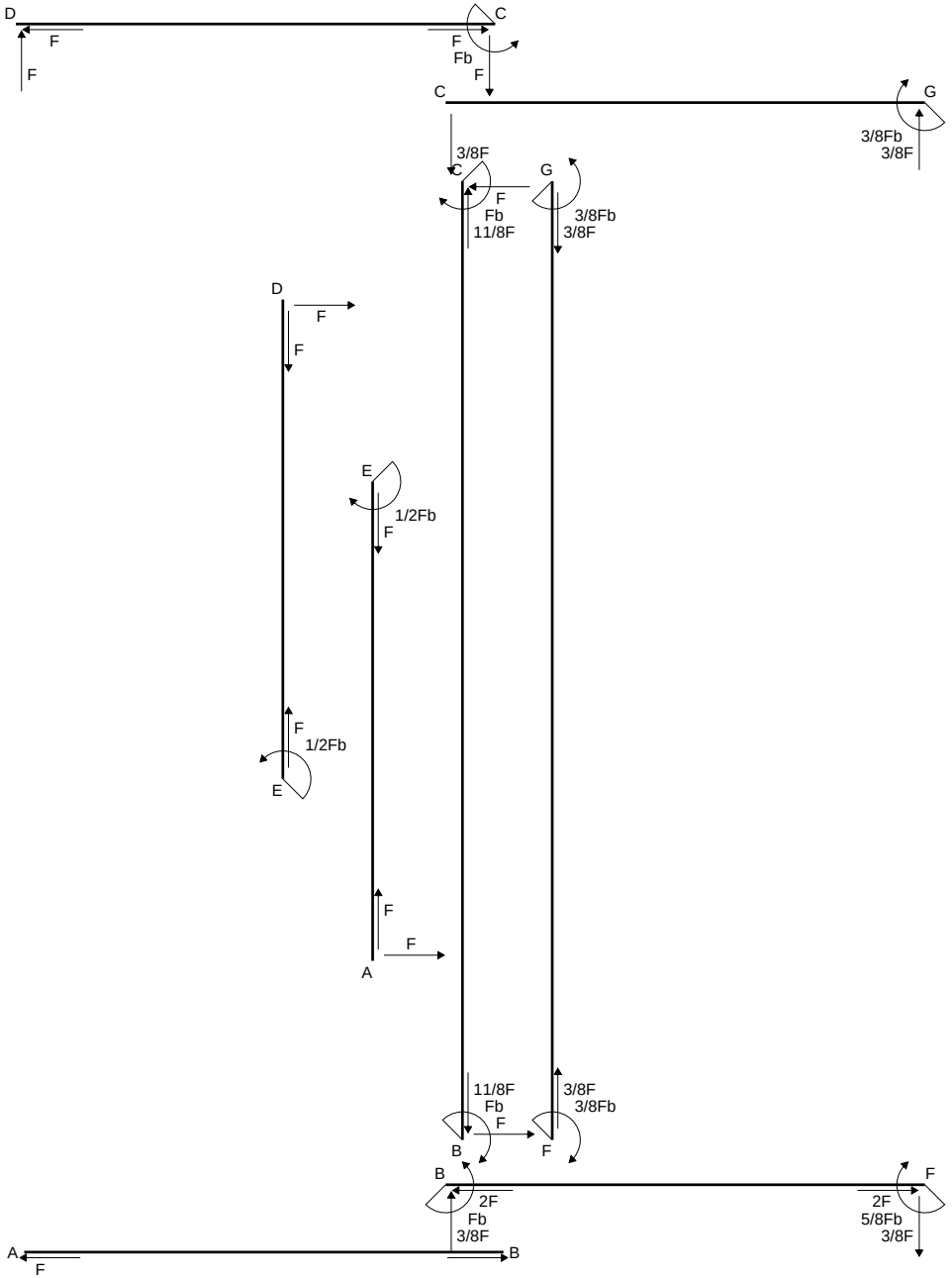
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3/8F
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B
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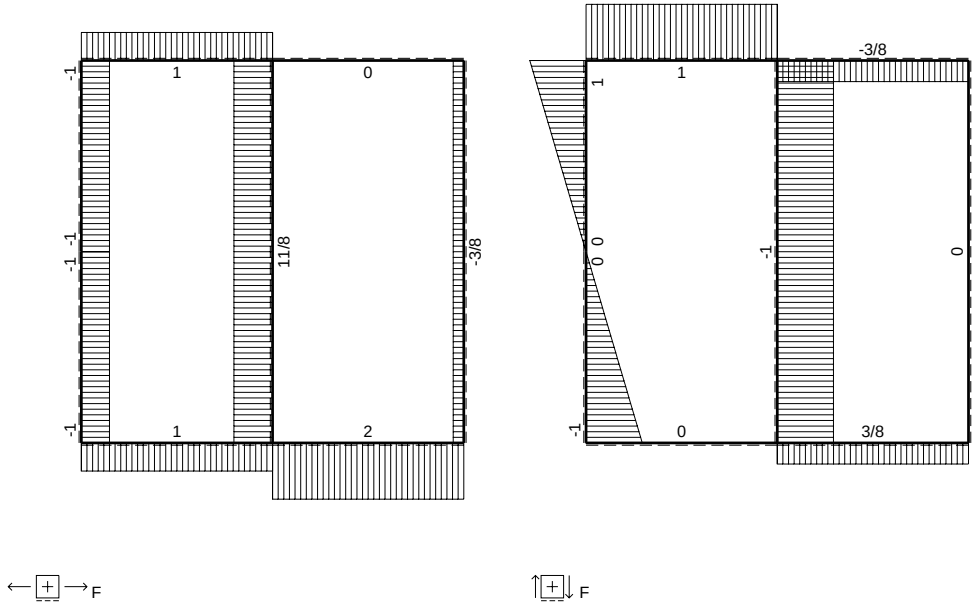
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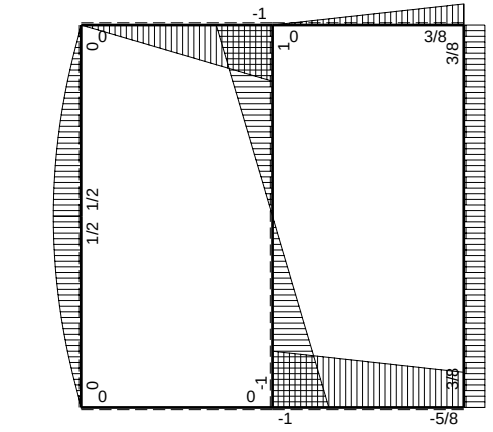
F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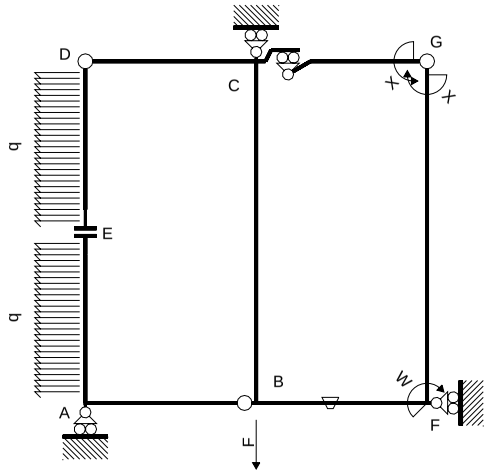
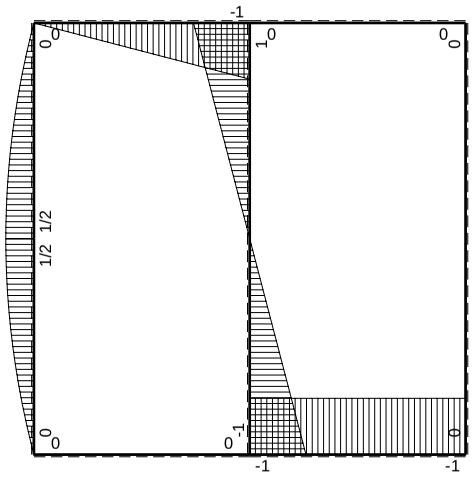
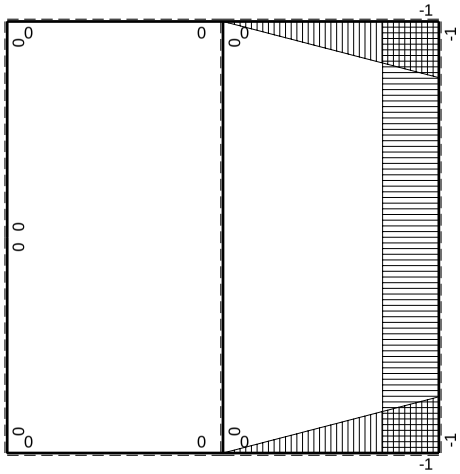
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.022

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

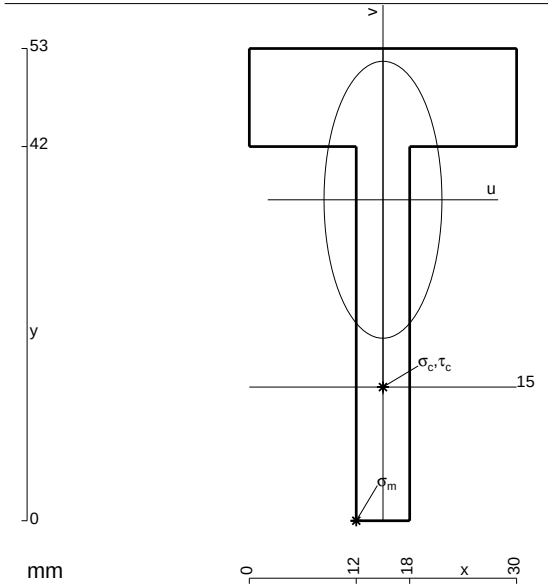
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 582. mm^2
J_u = 140714. mm^4
J_v = 25506. mm^4
y_g = 36.03 mm
N = 2310. N
T_y = -1680. N
M_x = 840000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.03 mm
$\sigma_m = N/A - Mv/J_u$ = 219. N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.03 mm
$\sigma_c = N/A - Mv/J_u$ = 129.5 N/mm^2
τ_c = 5.109 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.8 N/mm^2
S^* = 2567. mm^3

ESΣ8.022

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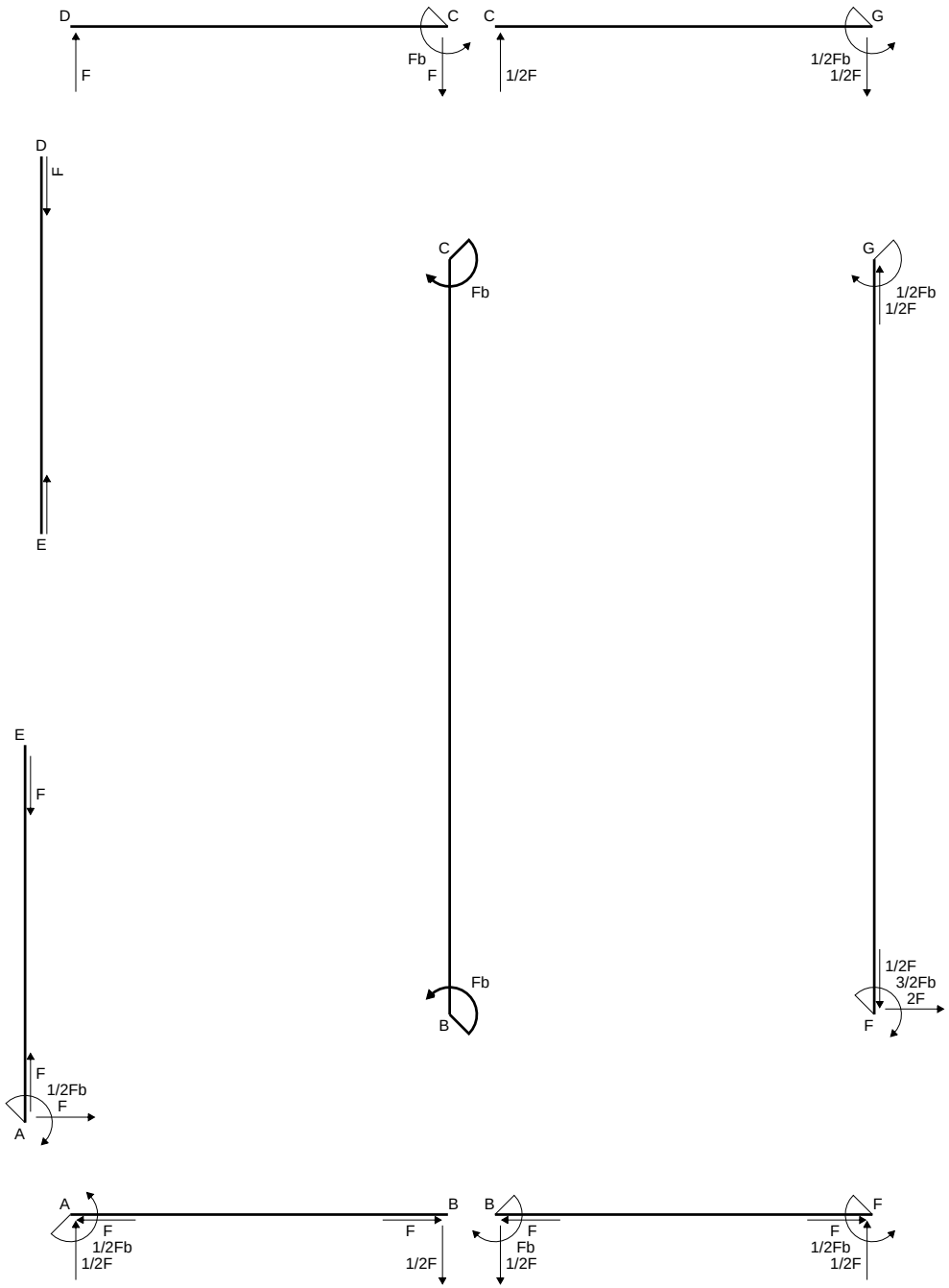
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Fb
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1/2Fb
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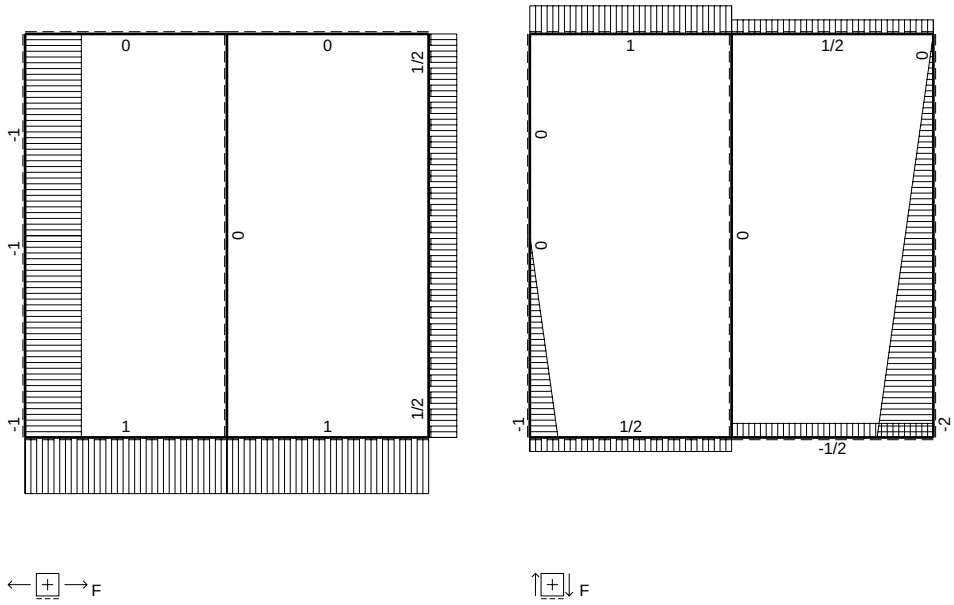
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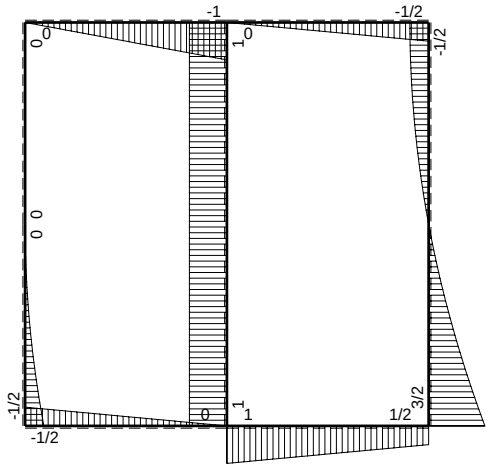
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3/2
-1/2


Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$1/2Fb$	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

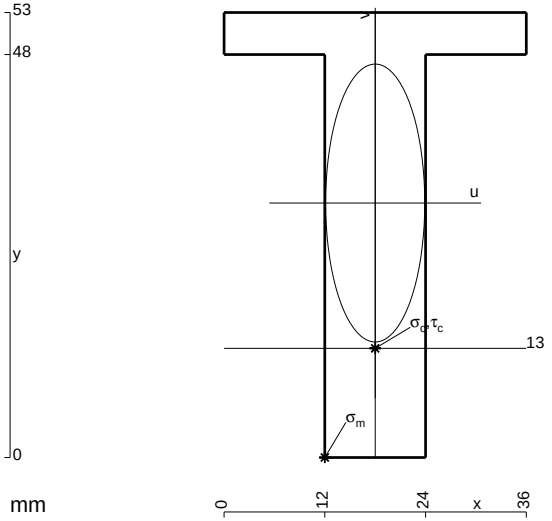
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 756. mm^2
J_u = 207276. mm^4
J_v = 26352. mm^4
y_g = 30.31 mm
T_y = 2910. N
M_x = -1571400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -30.31 mm
$\sigma_m = -Mv/J_u$ = -229.8 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -17.31 mm
$\sigma_c = -Mv/J_u$ = -131.2 N/mm^2
τ_c = 4.345 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.4 N/mm^2
S^* = 3714. mm^3

ESΣ8.023

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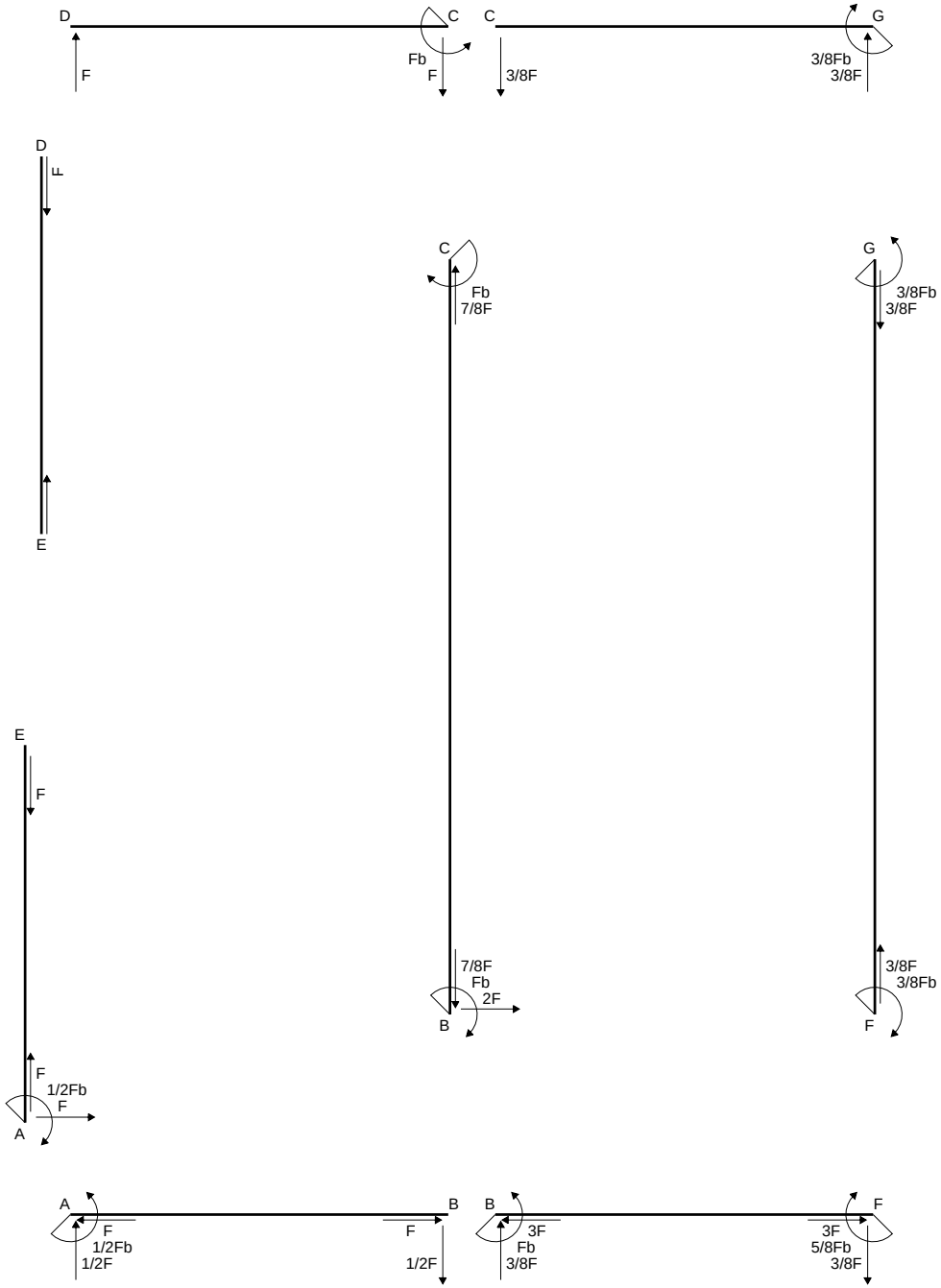
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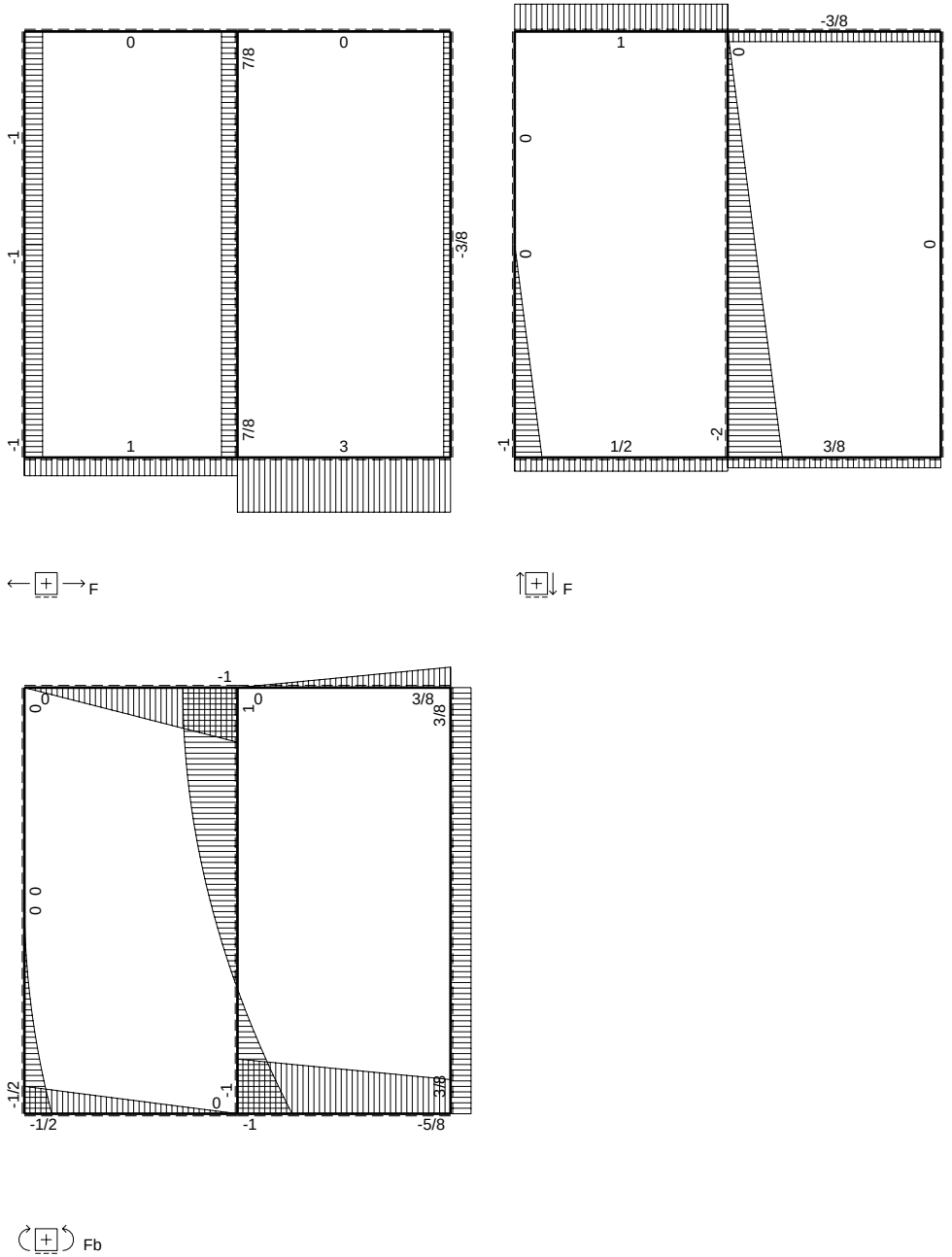
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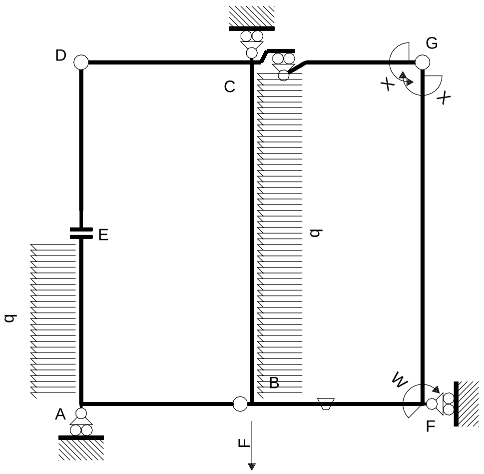
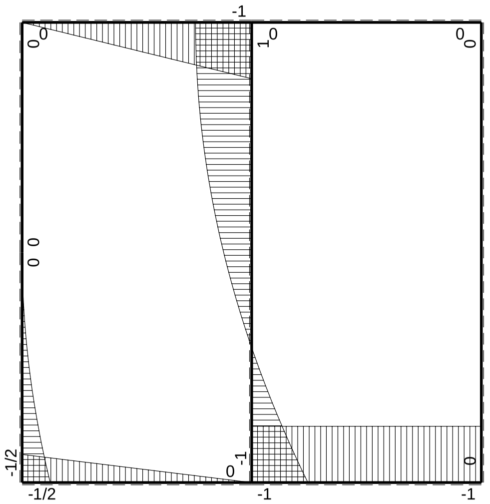
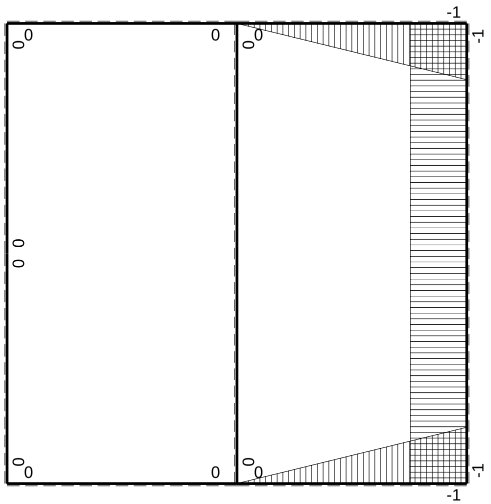
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

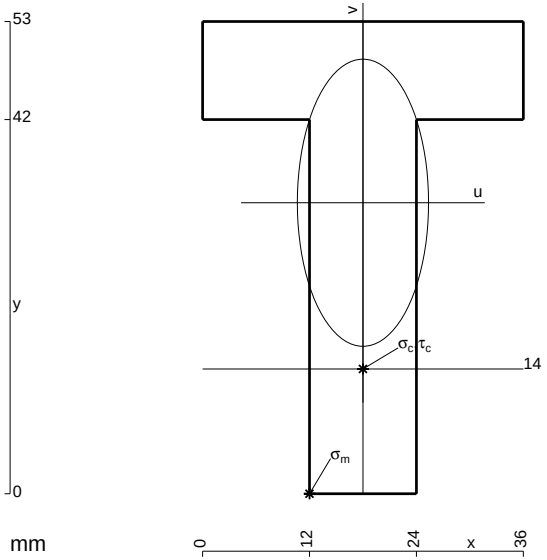
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 900. mm^2
J_u = 233812. mm^4
J_v = 48816. mm^4
y_g = 32.66 mm
N = 2616. N
T_y = -5980. N
M_x = -1734200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.66 mm
$\sigma_m = N/A - Mv/J_u$ = -239.3 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.66 mm
$\sigma_c = N/A - Mv/J_u$ = -135.5 N/mm^2
τ_c = 9.188 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.4 N/mm^2
S^* = 4311. mm^3

ESΣ8.024

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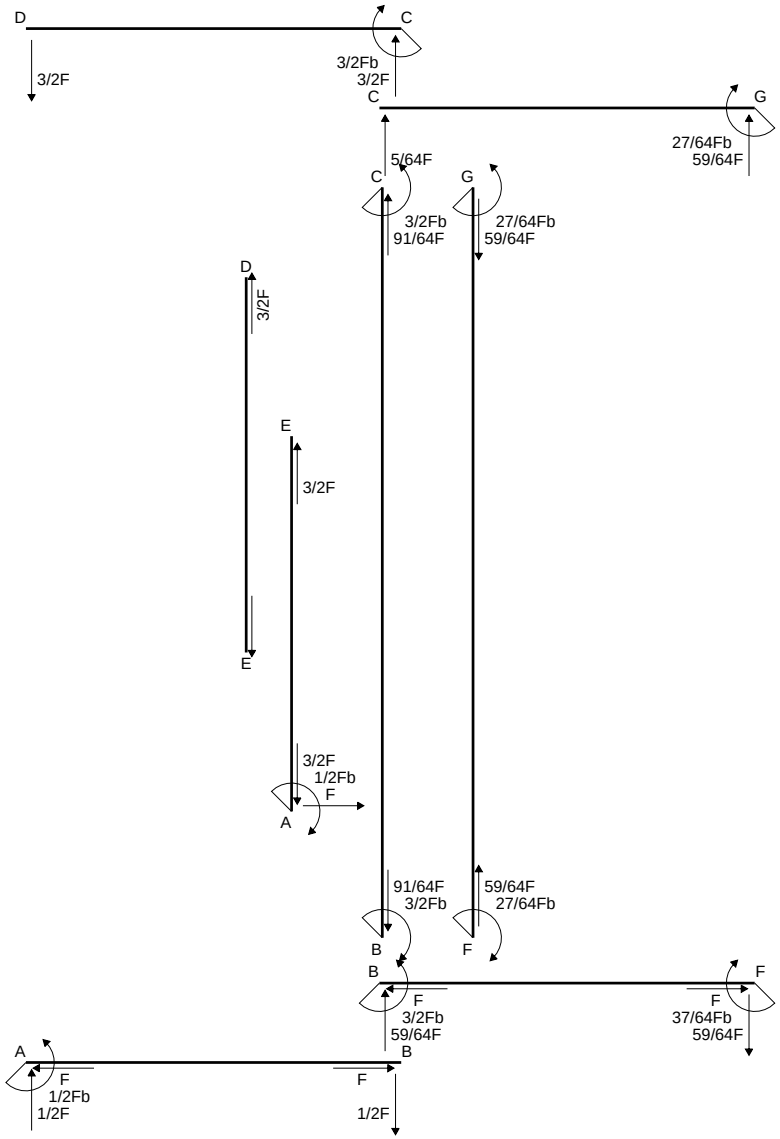
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C
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3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
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91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
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37/64Fb
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B
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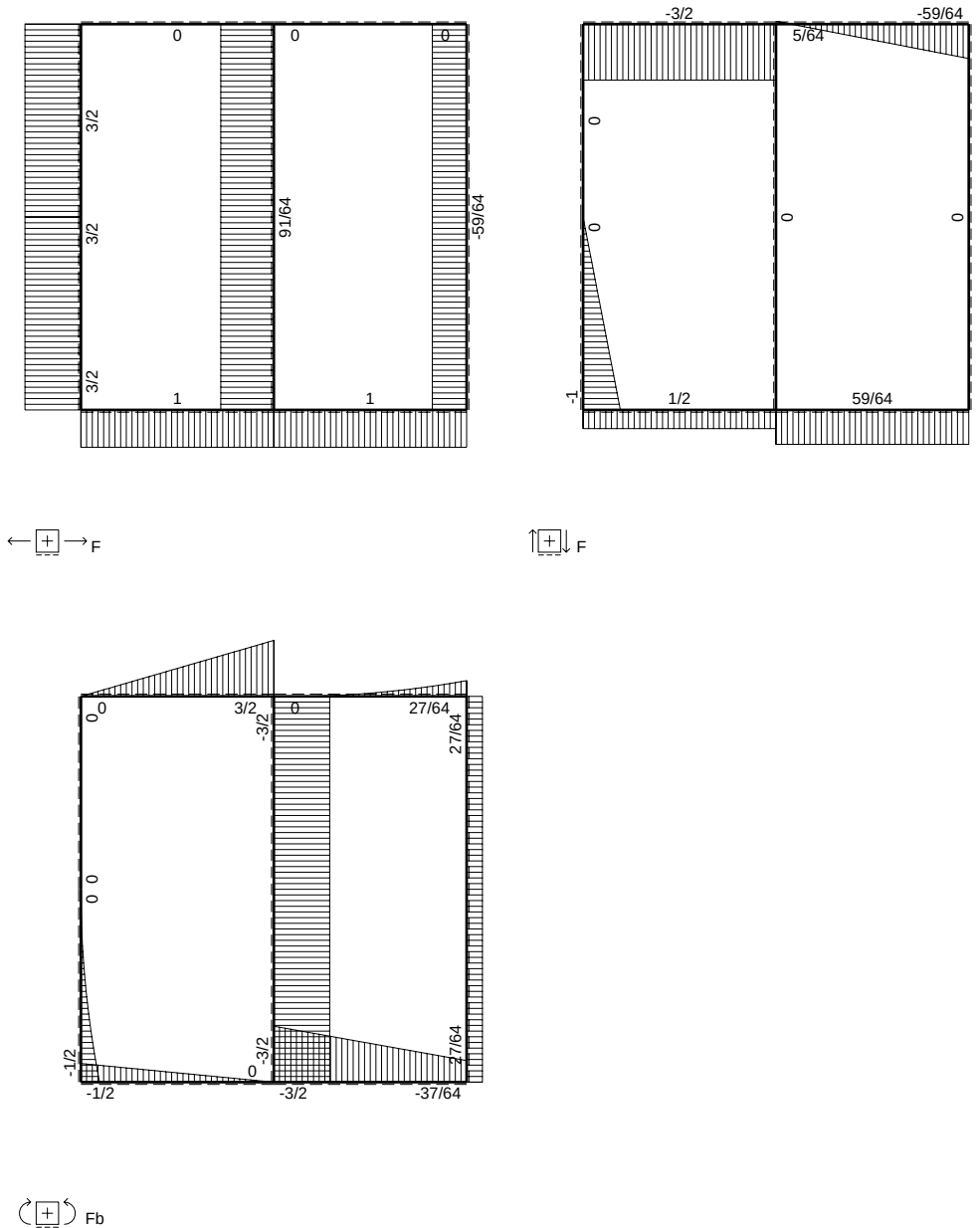
1/2Fb
1/2F
F
1/2F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91/64
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1
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-59/64
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0
0
0
-1
1/2
59/64
F
0 0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DC b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0	0+0	0
	0	-1/2qx²	0	0	0	0		
EA b	0	1/2Fb-Fx+1/2qx²	0	0	0	0	0+0	0
	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx²/b	Fx/EJ	x²/b²	(7/12+1/2)Fb²/EJ	1/3XbJ/EJ
BF b	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x²/b²		
FB b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²	(1/24+0)Fb²/EJ	1/3XbJ/EJ
GC b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x²/b²		
CG b	-1	0	0	0	0	1	0+0	2XbJ/EJ
FG 2b	1	0	0	0	0	1		
GF 2b	0	-3/2Fb	0	0	0	0	0+0	0
CB 2b	0	3/2Fb	0	0	0	0		
BC 2b	totali						9/8Fb²/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

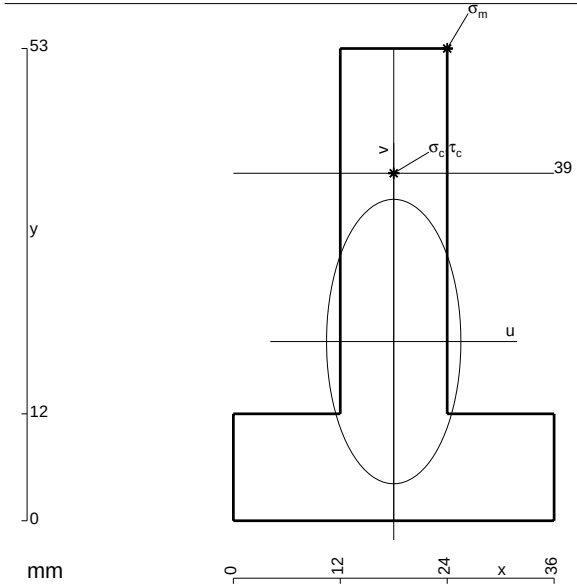
$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 924. mm^2
J_u = 235641. mm^4
J_v = 52560. mm^4
y_g = 20.11 mm
T_y = -2265. N
M_x = 1426950. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 32.89 mm
$\sigma_m = -Mv/J_u$ = -199.2 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 18.89 mm
$\sigma_c = -Mv/J_u$ = -114.4 N/mm^2
τ_c = 3.484 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.5 N/mm^2
S^* = 4349. mm^3

ESΣ8.025

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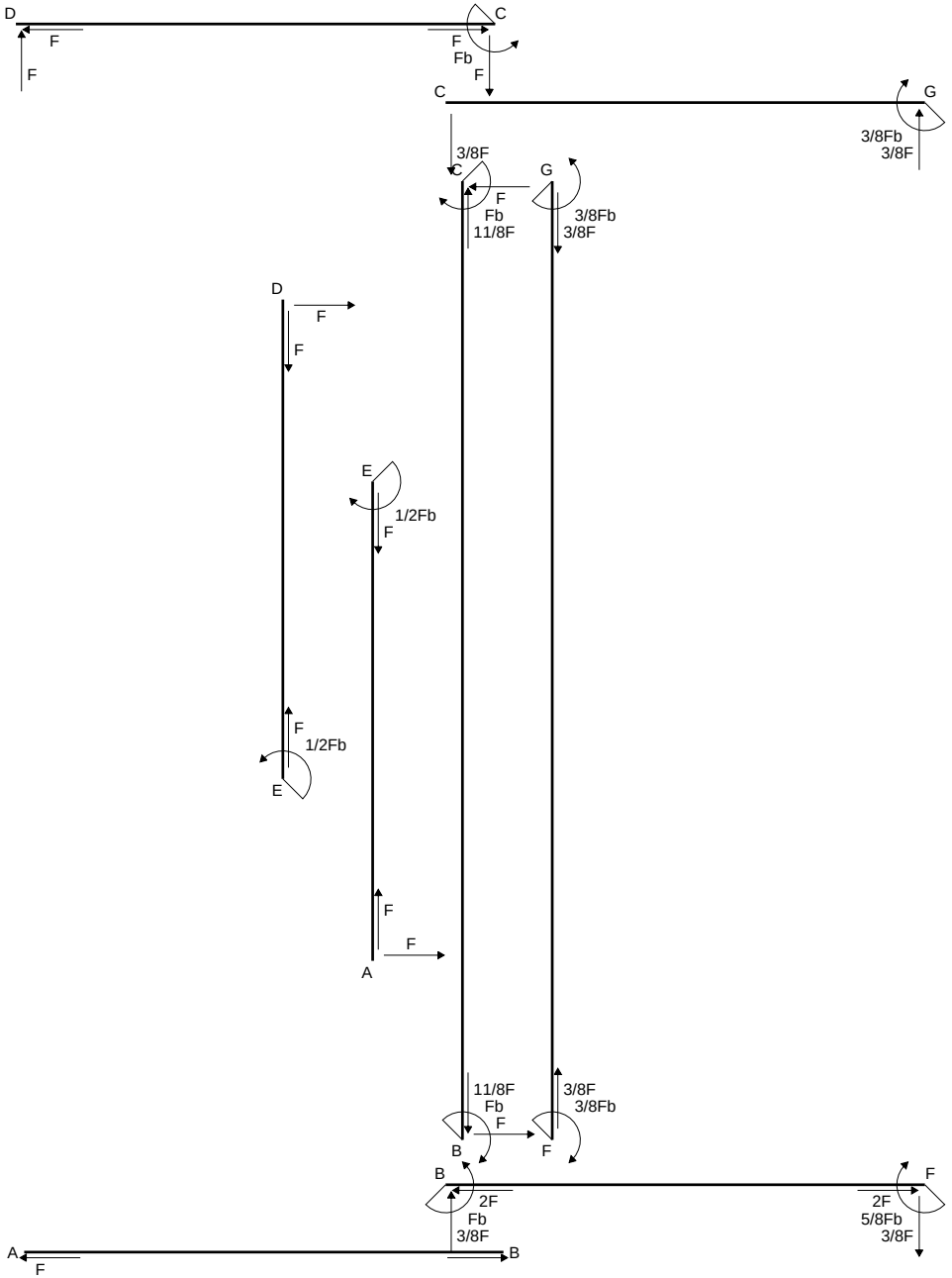
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C
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3/8F
3/8F
C
G
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Fb
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3/8F
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F
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F
3/8F
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3/8F
2F
5/8Fb
3/8F
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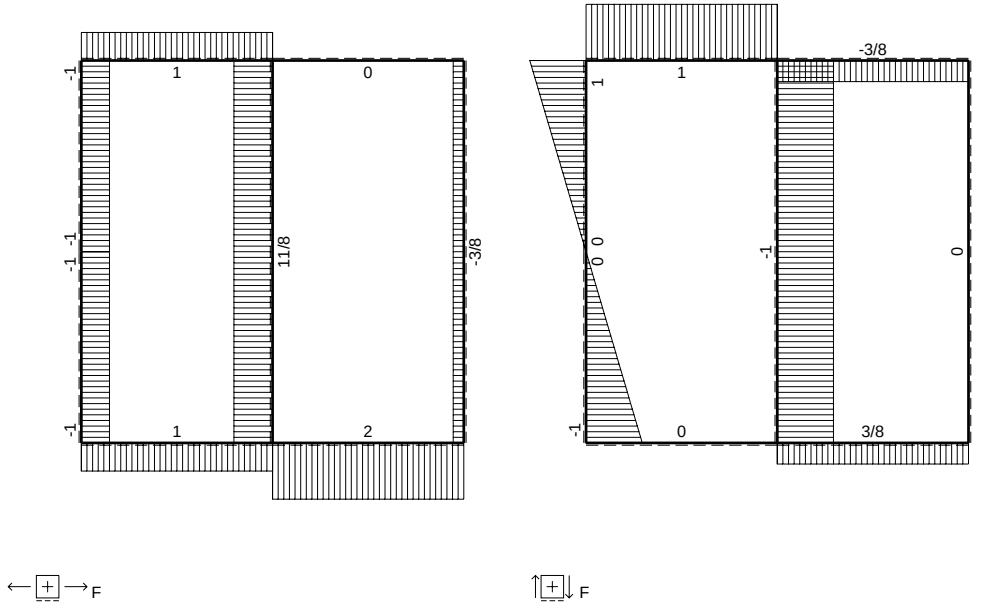
B
F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-3/8
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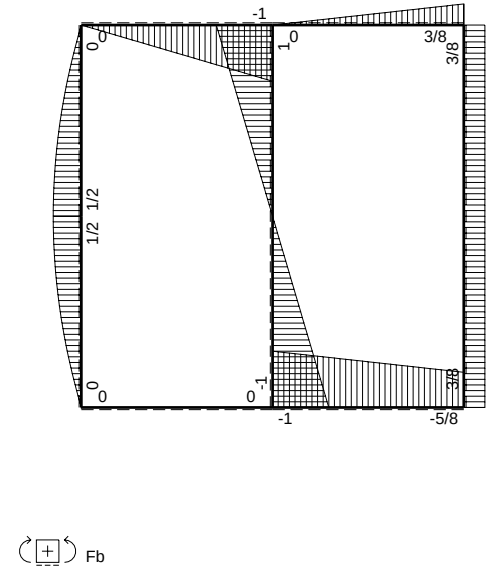
3/8
F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0
3/8
3/8
1/2
1/2
0
0
0
-1
-1
3/8
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

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PROCEDIMENTO E RISULTATI Nome: ESΣ8.026

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

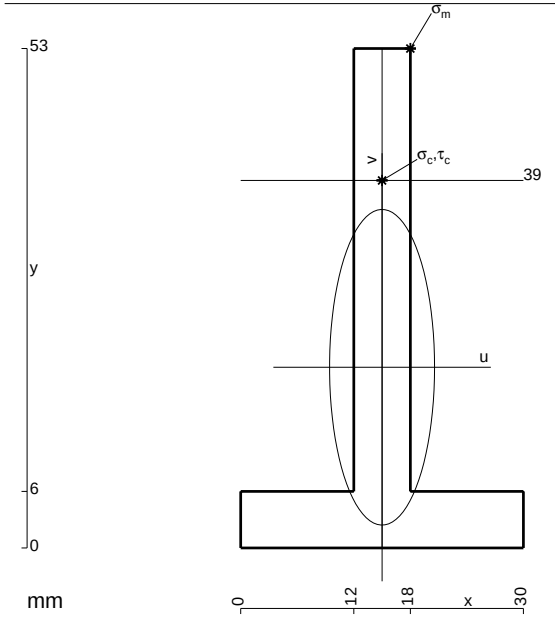
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 462. mm^2
J_u = 129608. mm^4
J_v = 14346. mm^4
y_g = 19.18 mm
N = 1623. N
T_y = -1180. N
M_x = -790600. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 33.82 mm
$\sigma_m = N/A - Mv/J_u$ = 209.8 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 19.82 mm
$\sigma_c = N/A - Mv/J_u$ = 124.4 N/mm^2
τ_c = 3.419 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.6 N/mm^2
S^* = 2253. mm^3

ESΣ8.026

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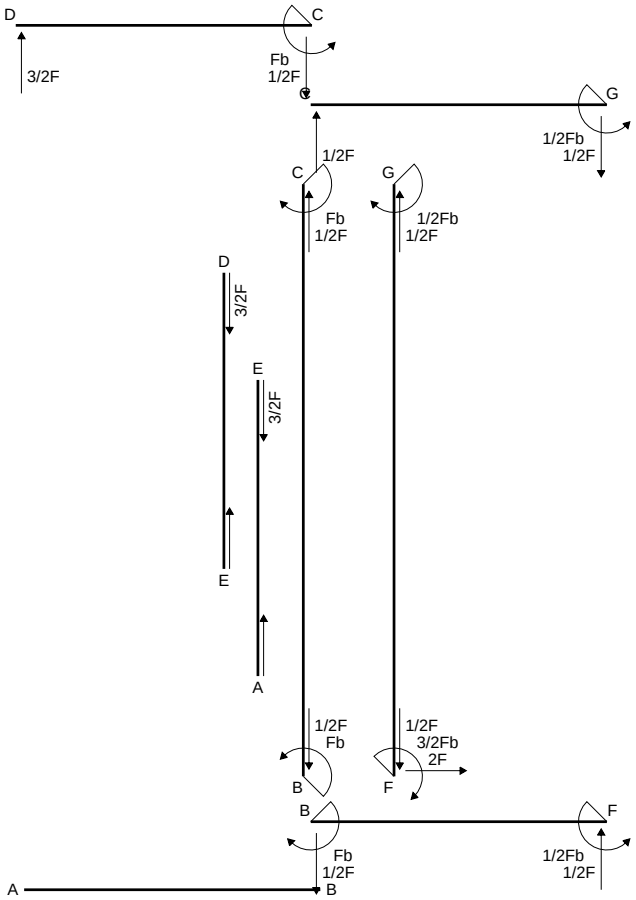
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Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
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D
3/2F
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3/2F
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1/2F
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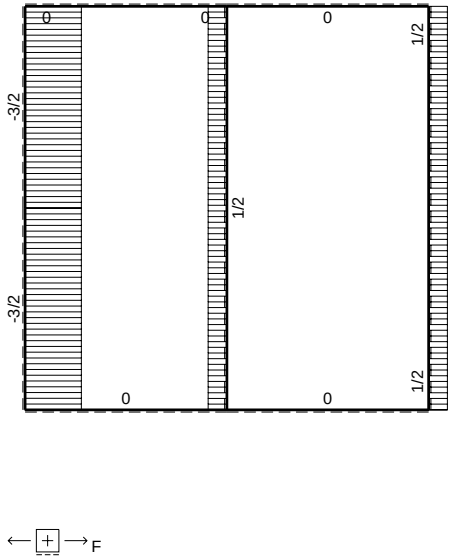
A
B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1/2
-3/2
1/2
-3/2
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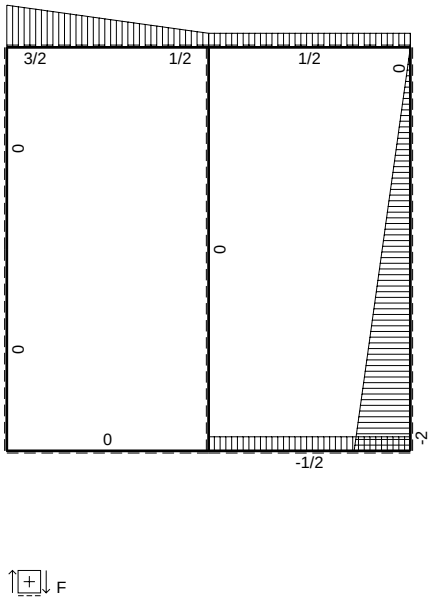
0
1/2
F


3/2
1/2
1/2
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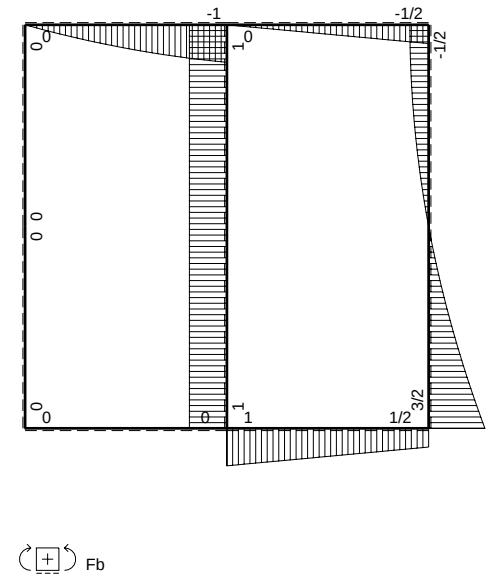
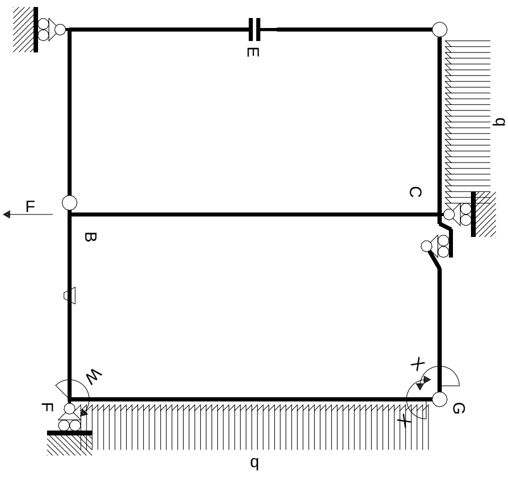
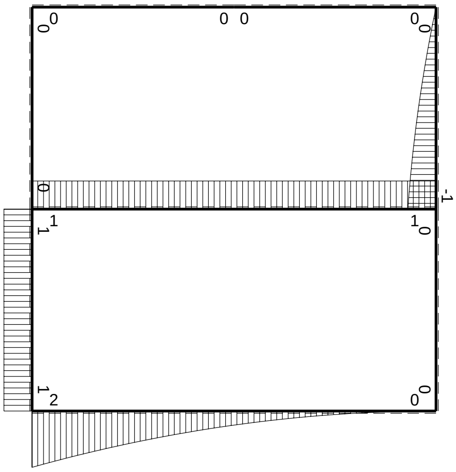
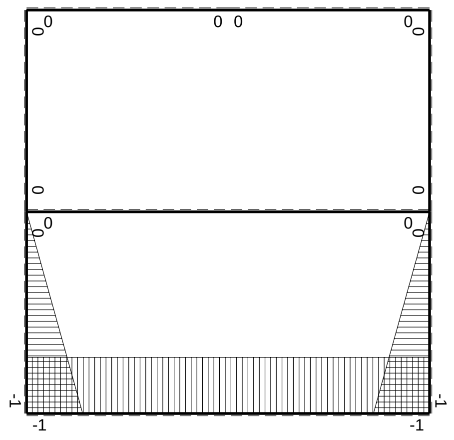
0
0
0
0
-2
-1/2
F


-1
-1/2
0
0
1
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

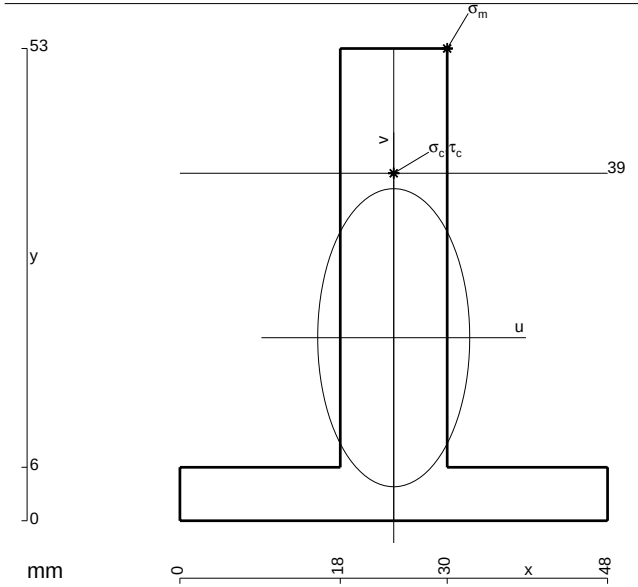
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 852. mm^2
J_u = 238569. mm^4
J_v = 62064. mm^4
y_g = 20.54 mm
T_y = 3640. N
M_x = -1615250. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 32.46 mm
$\sigma_m = -Mv/J_u$ = 219.8 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 18.46 mm
$\sigma_c = -Mv/J_u$ = 125. N/mm^2
τ_c = 5.438 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.3 N/mm^2
S^* = 4277. mm^3

ESΣ8.027

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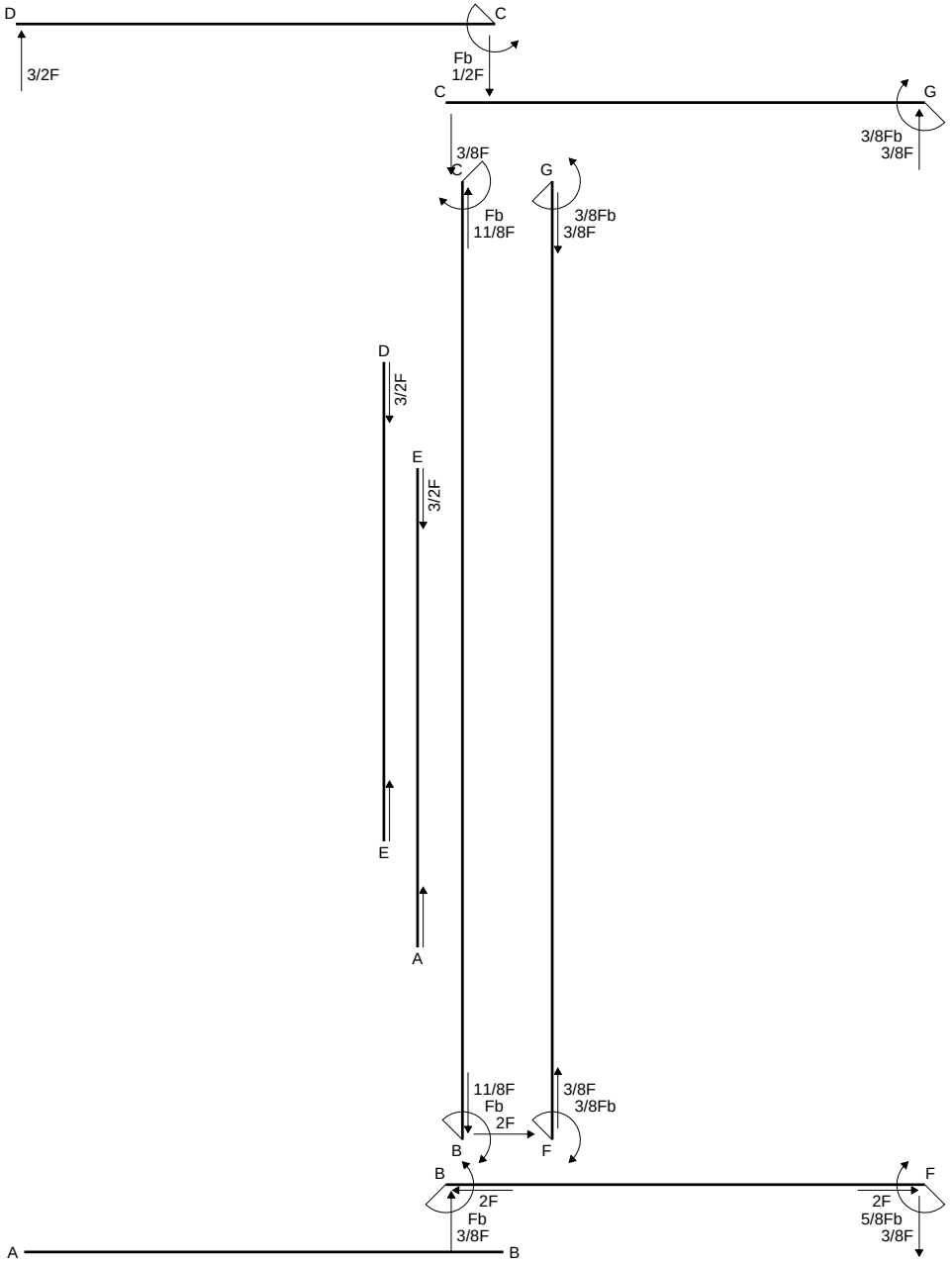
D
C
3/2F
Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
11/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
11/8F
Fb
2F
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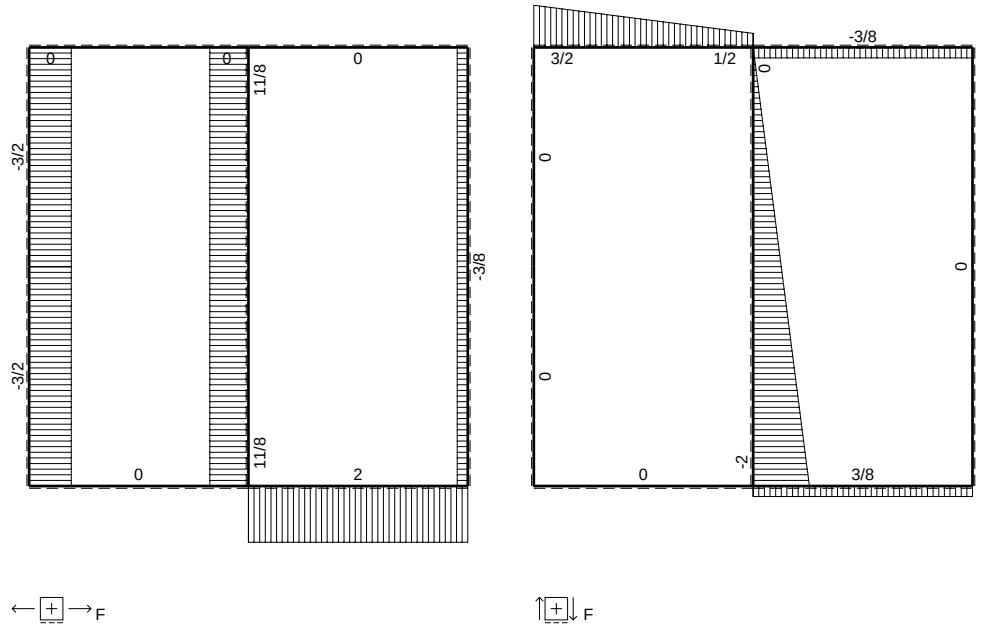
2F
5/8Fb
3/8F
A
B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0
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11/8
-3/2
-3/8
-3/2
11/8
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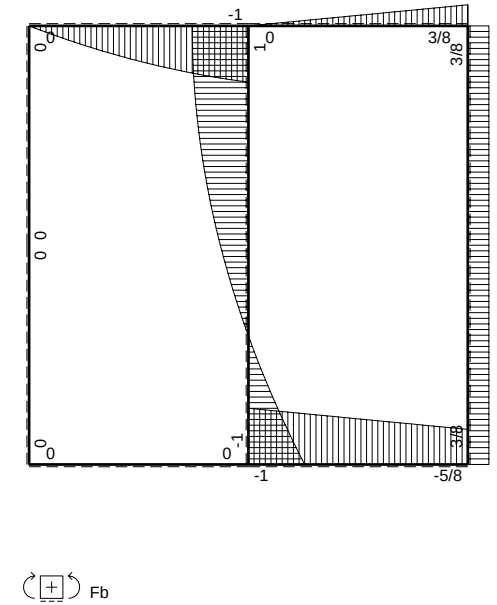
3/2
1/2
-3/8
0
0
0
0
-2
3/8
F


-1
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3/8
3/8
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0
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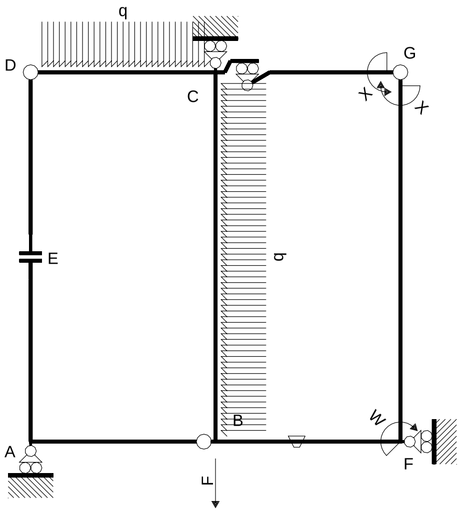
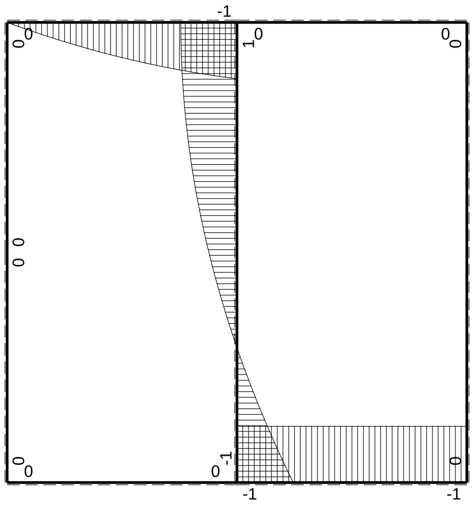
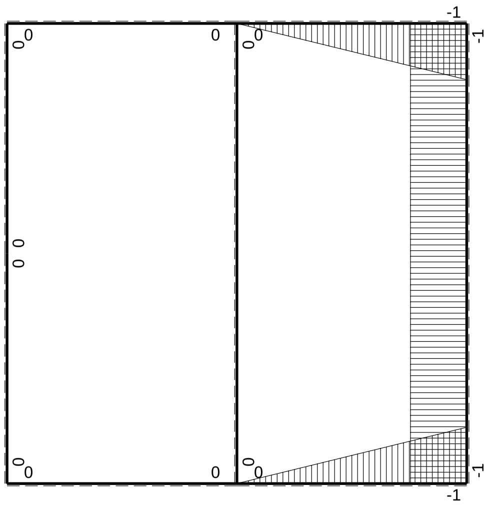
0
-1
0
-1
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

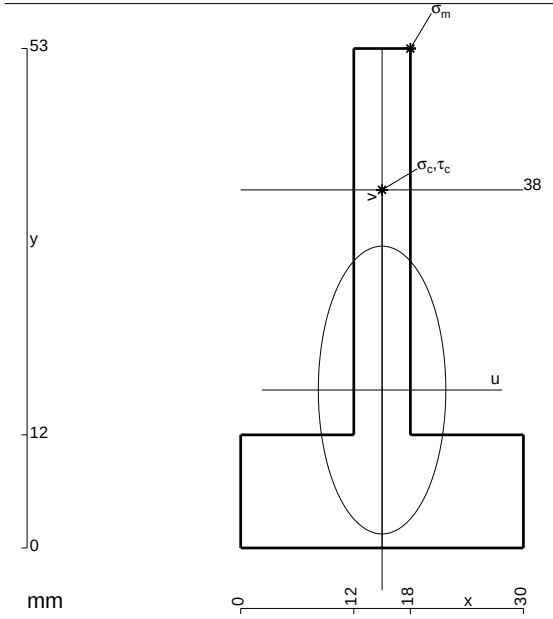
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.028



A = 606. mm^2
J_u = 141406. mm^4
J_v = 27738. mm^4
y_g = 16.76 mm
N = 1623. N
T_y = -2360. N
M_x = -885000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.24 mm
$\sigma_m = N/A - Mv/J_u$ = 229.5 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.24 mm
$\sigma_c = N/A - Mv/J_u$ = 135.6 N/mm^2
τ_c = 7.195 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.2 N/mm^2
S^* = 2587. mm^3

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ESΣ8.028

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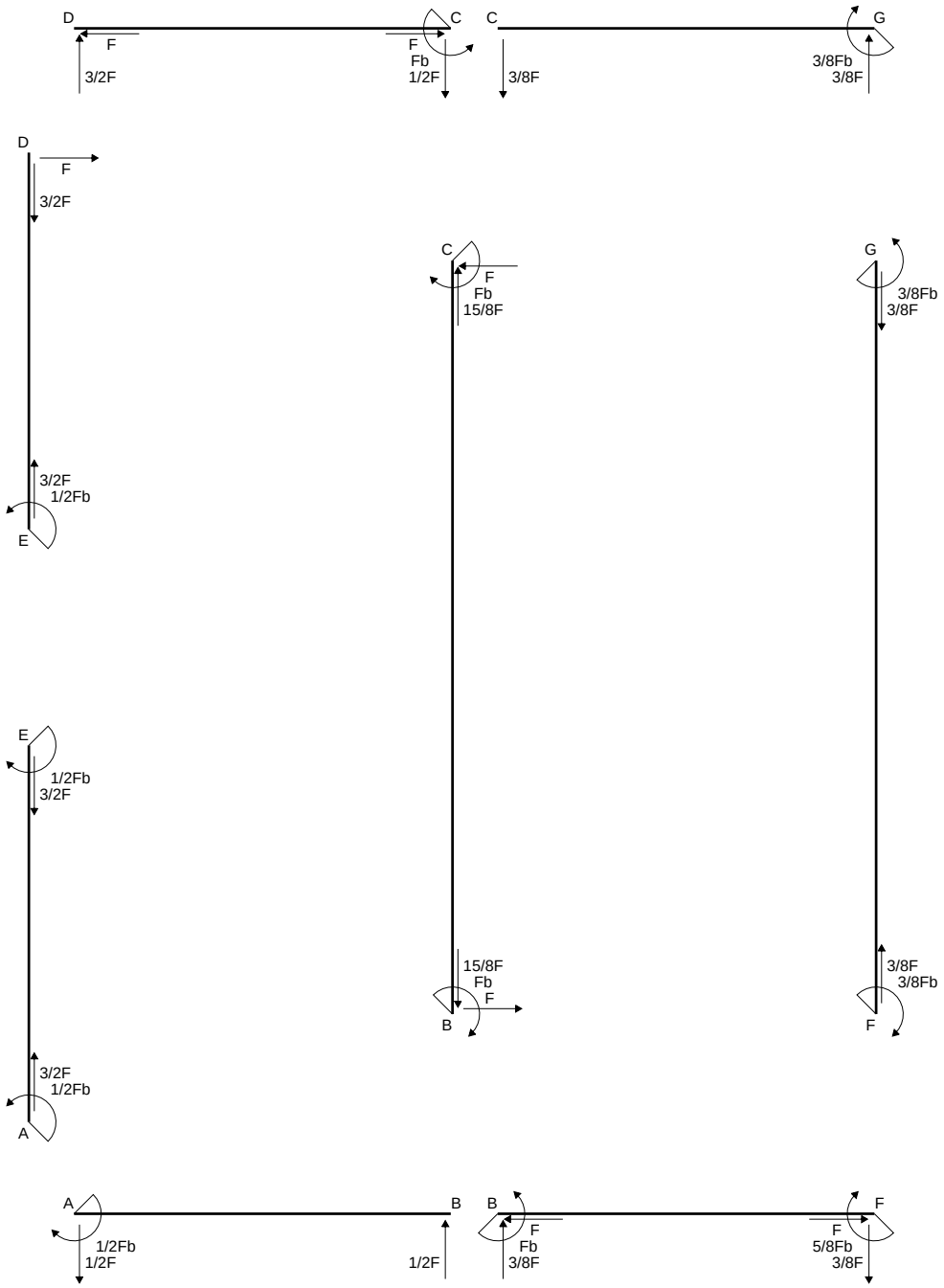
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C
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Fb
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3/8F
G
3/8Fb
3/8F
D
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C
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3/8F
3/2F
1/2Fb
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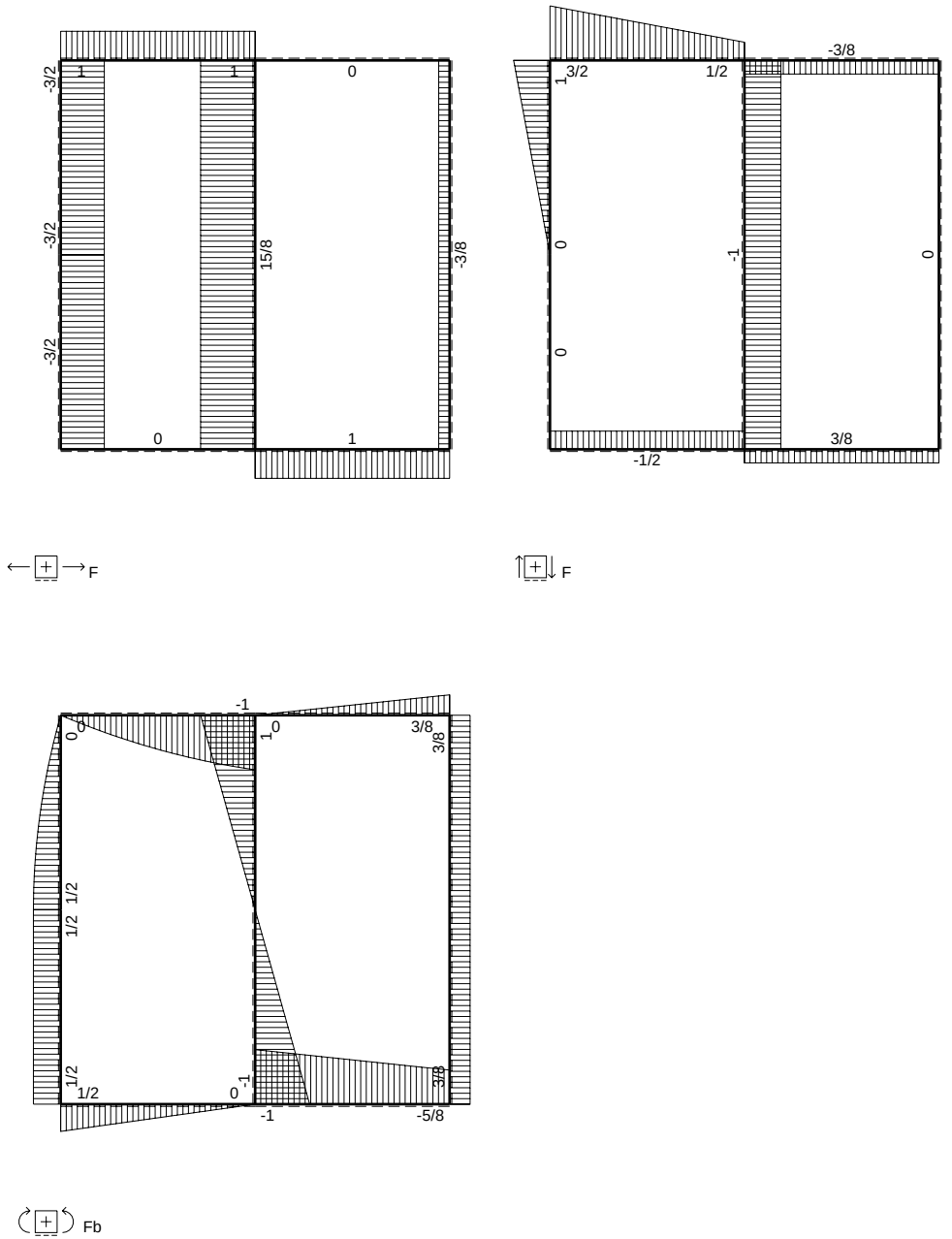
5/8Fb
3/8F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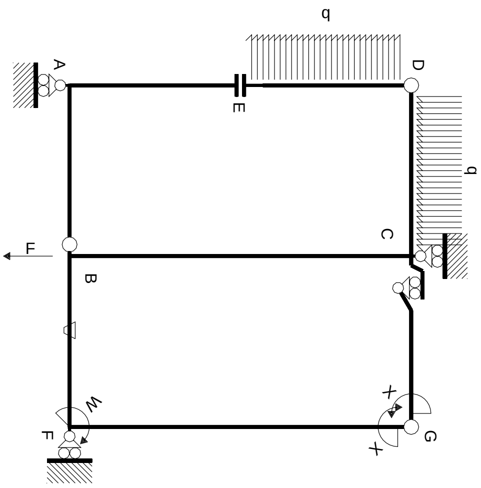
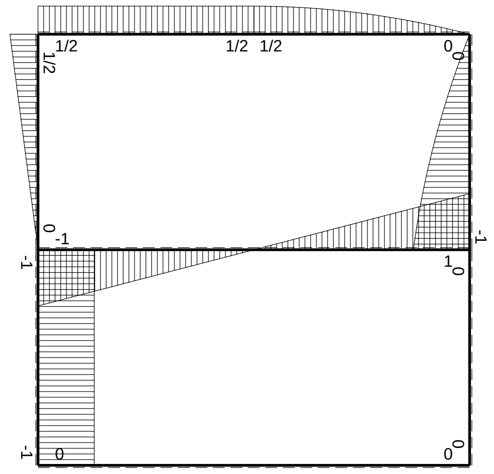
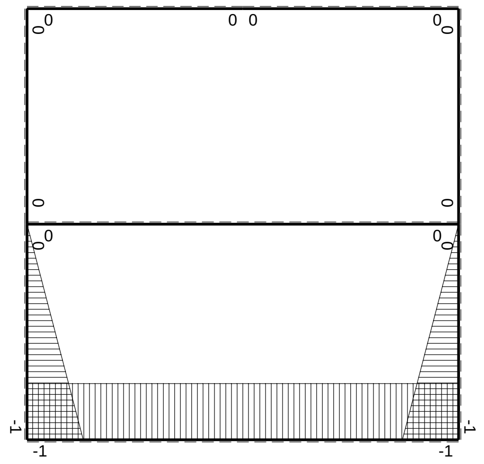
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

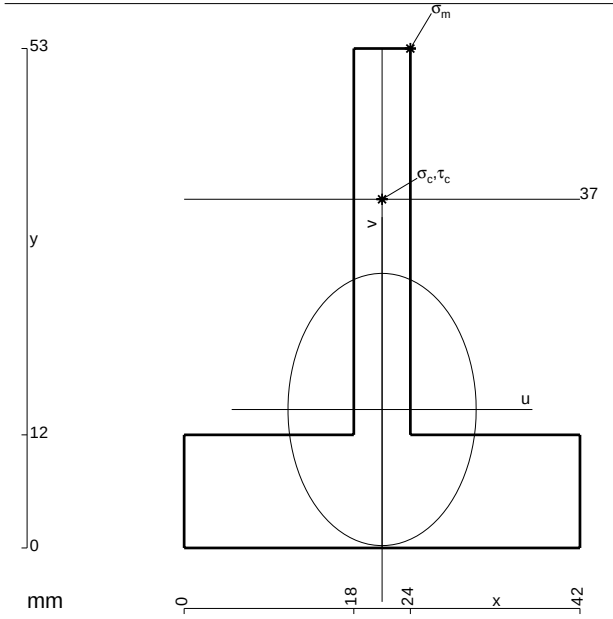
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 750. mm^2
J_u = 156599. mm^4
J_v = 74826. mm^4
y_g = 14.69 mm
N = 2288. N
T_y = -1220. N
M_x = -963800. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.31 mm
$\sigma_m = N/A - Mv/J_u$ = 238.8 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.31 mm
$\sigma_c = N/A - Mv/J_u$ = 140.3 N/mm^2
τ_c = 3.778 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 140.5 N/mm^2
S^* = 2910. mm^3

ESΣ8.029

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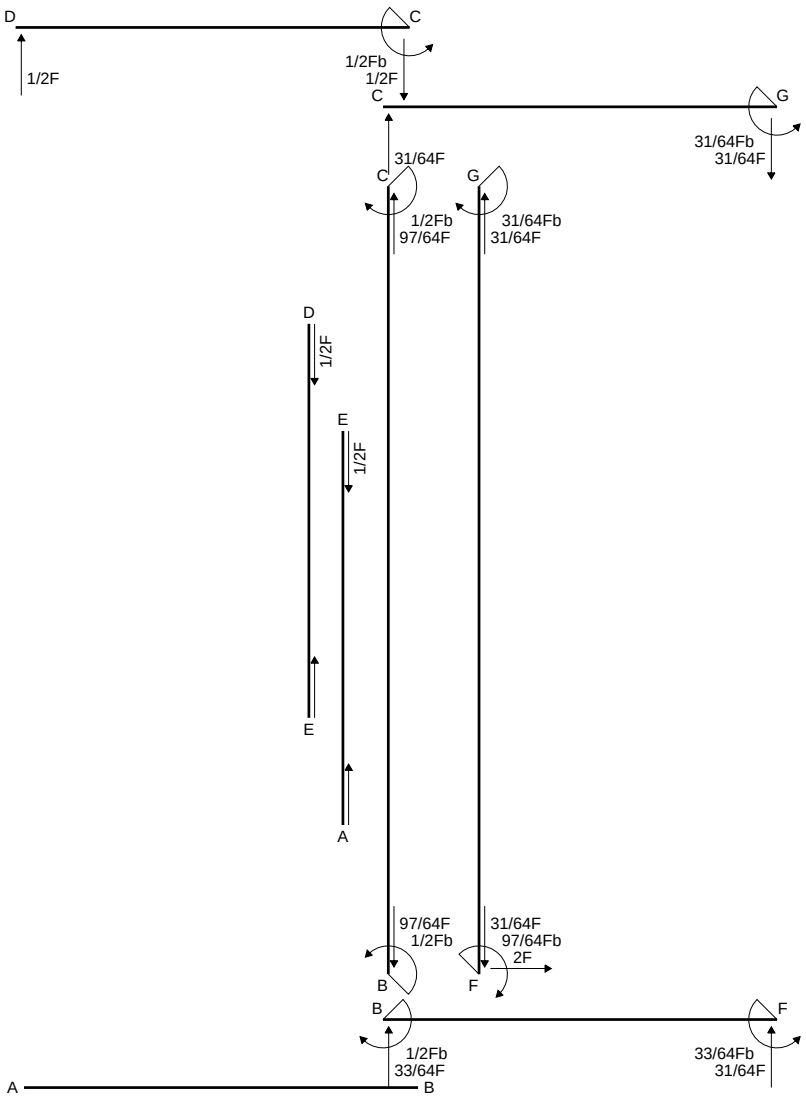
D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
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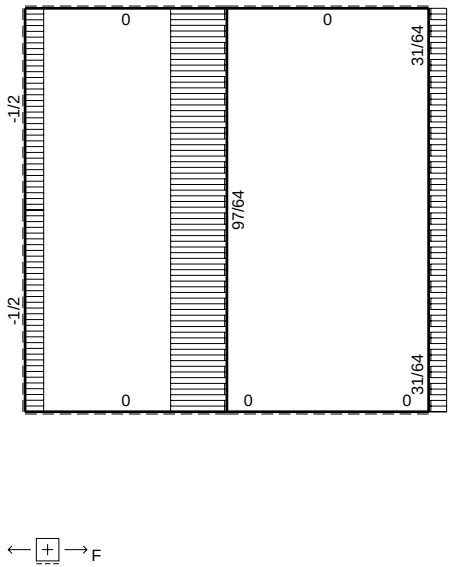
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B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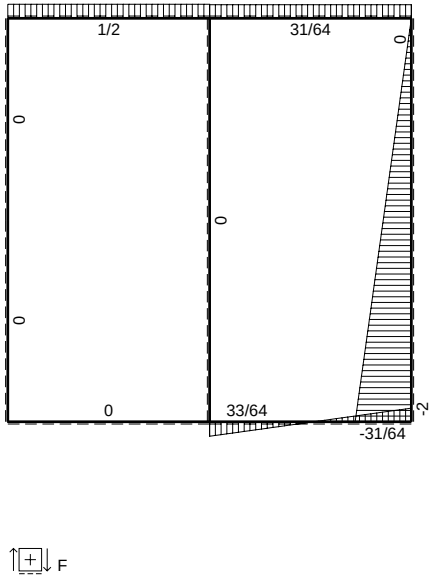
97/64
-1/2
0
0
0
31/64
F


1/2
31/64
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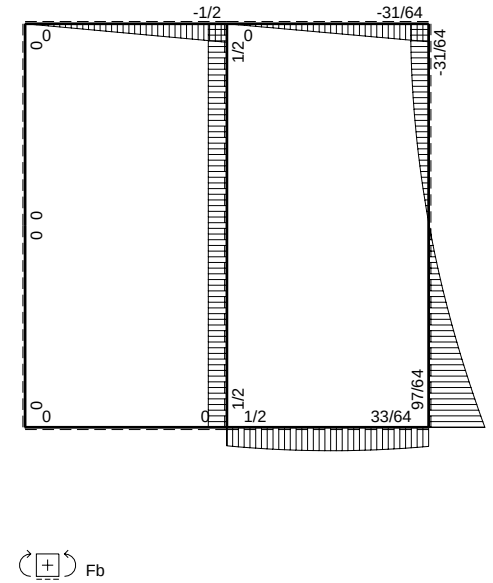
0
33/64
-2
-31/64
F


-1/2
-31/64
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0
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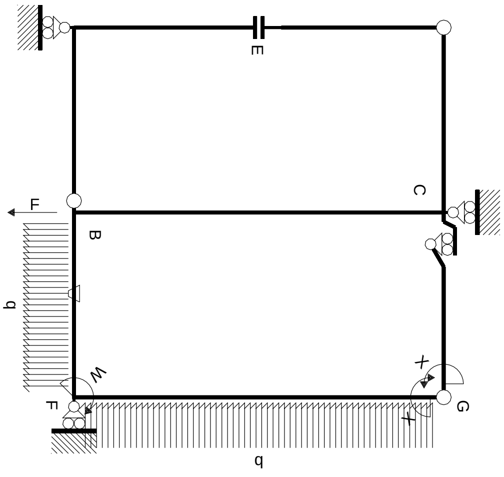
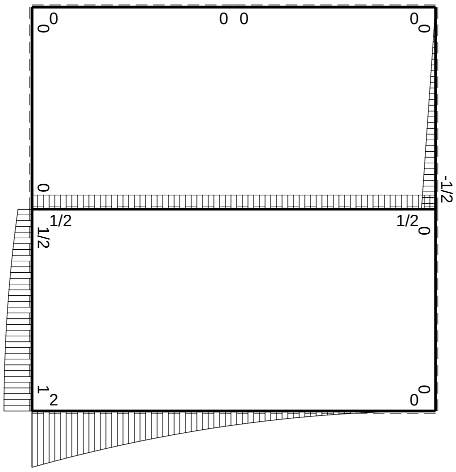
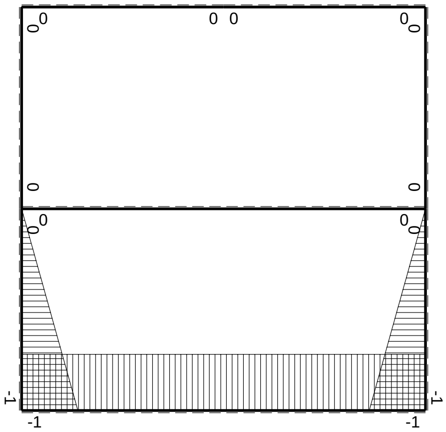
1/2
0
1/2
33/64
97/64
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

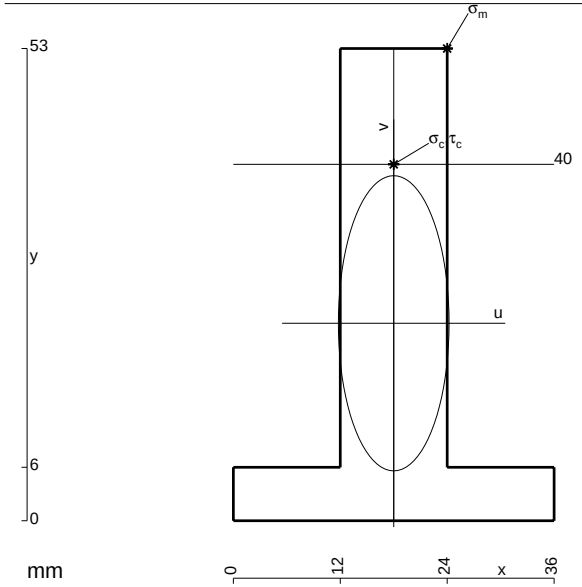
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 780. mm^2
J_u = 214152. mm^4
J_v = 30096. mm^4
y_g = 22.16 mm
T_y = 3305. N
M_x = -1388100. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 30.84 mm
$\sigma_m = -Mv/J_u$ = 199.9 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 17.84 mm
$\sigma_c = -Mv/J_u$ = 115.6 N/mm^2
τ_c = 4.883 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.9 N/mm^2
S^* = 3797. mm^3

ESΣ8.030

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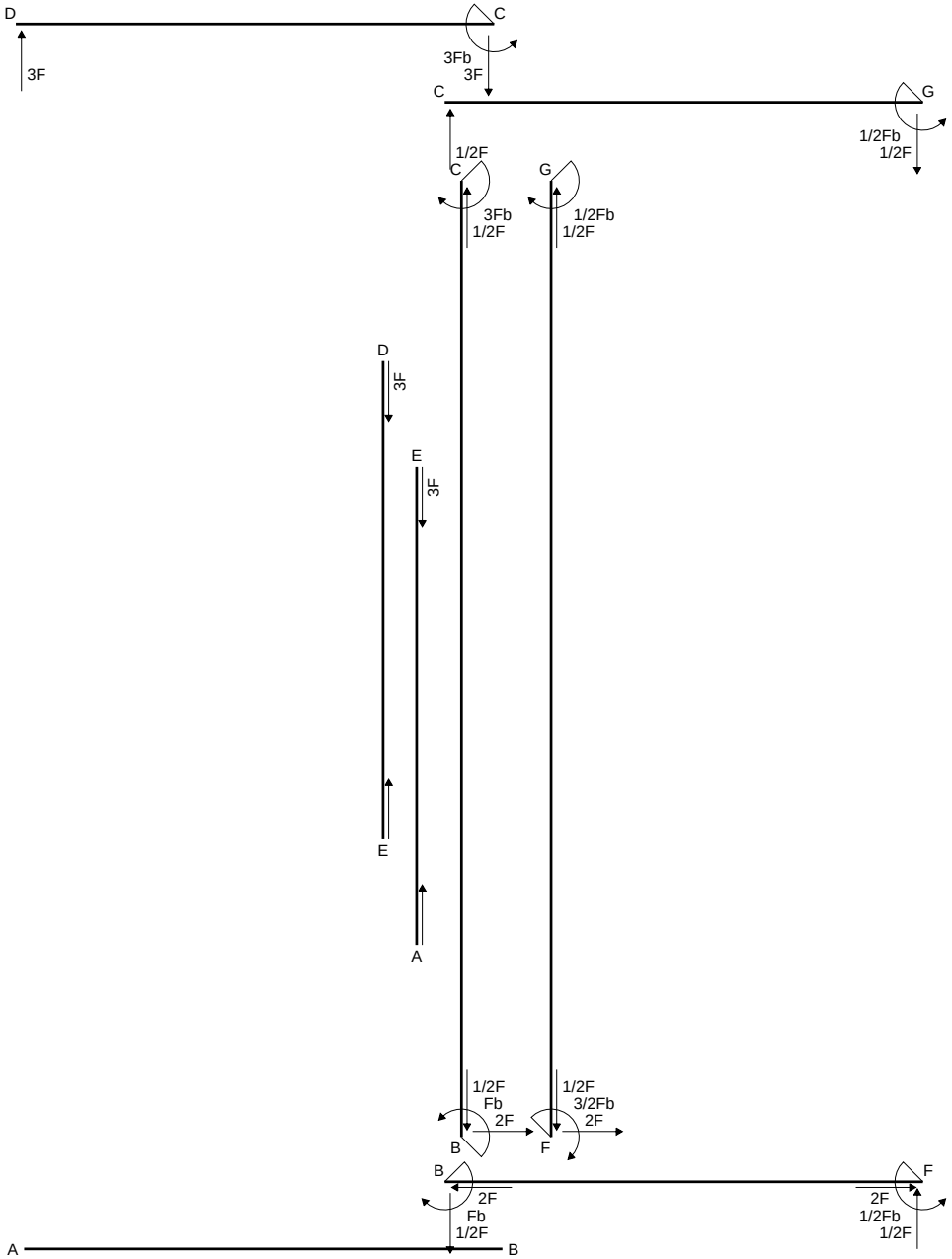
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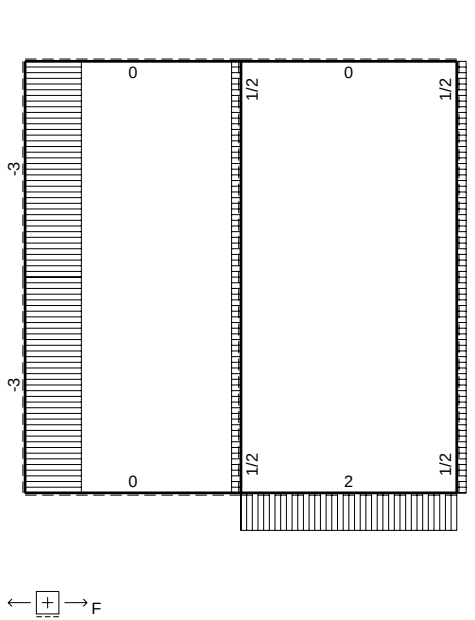
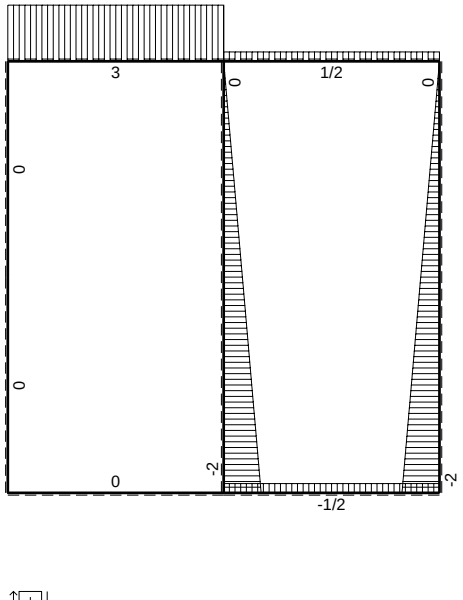
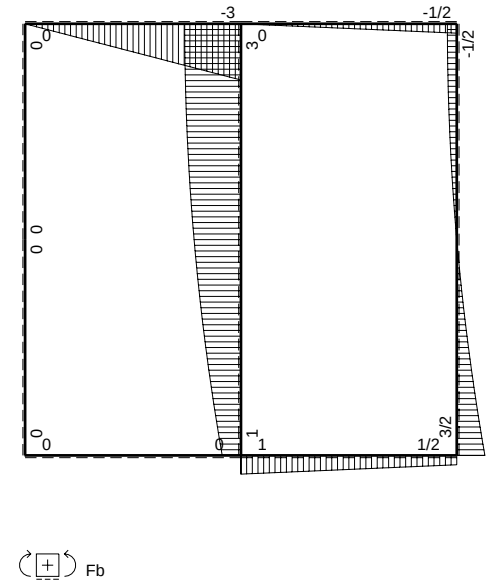
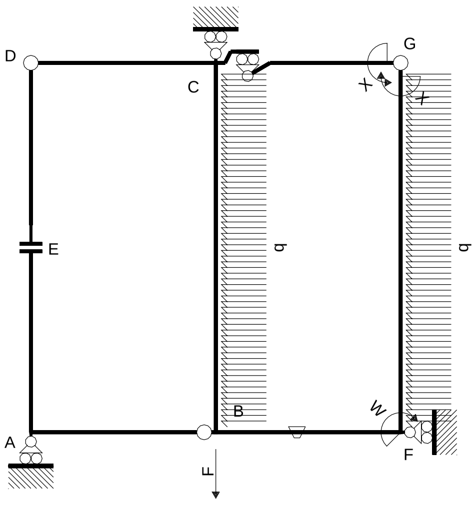
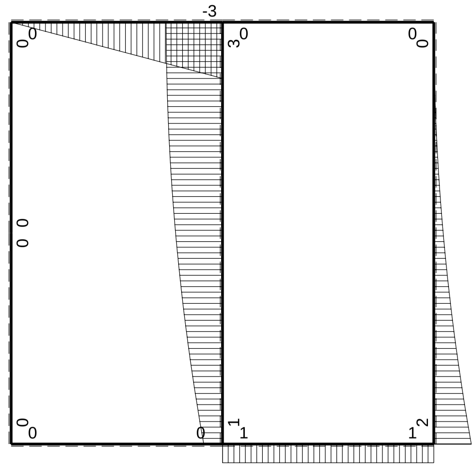
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
0
1/2
1/2
2
F

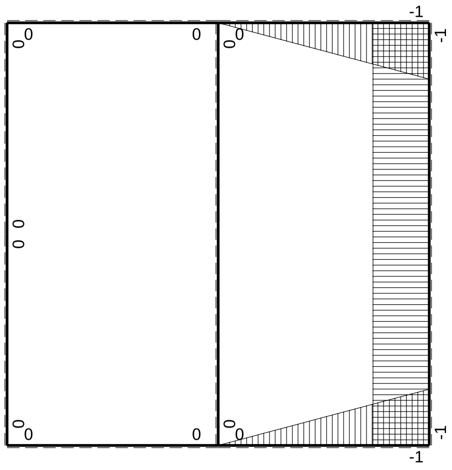

3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

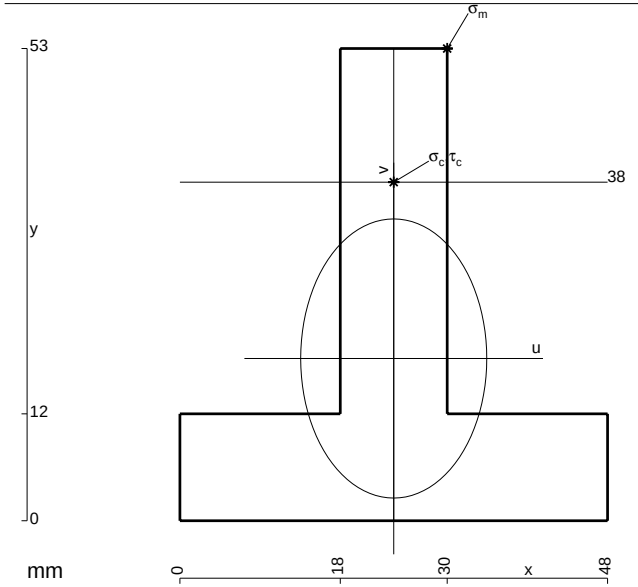
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1068. mm^2
J_u = 262174. mm^4
J_v = 116496. mm^4
y_g = 18.21 mm
T_y = 3420. N
M_x = -1573200. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.79 mm
$\sigma_m = -Mv/J_u$ = 208.8 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.79 mm
$\sigma_c = -Mv/J_u$ = 118.8 N/mm^2
τ_c = 5.34 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.1 N/mm^2
S^* = 4913. mm^3

ESΣ8.031

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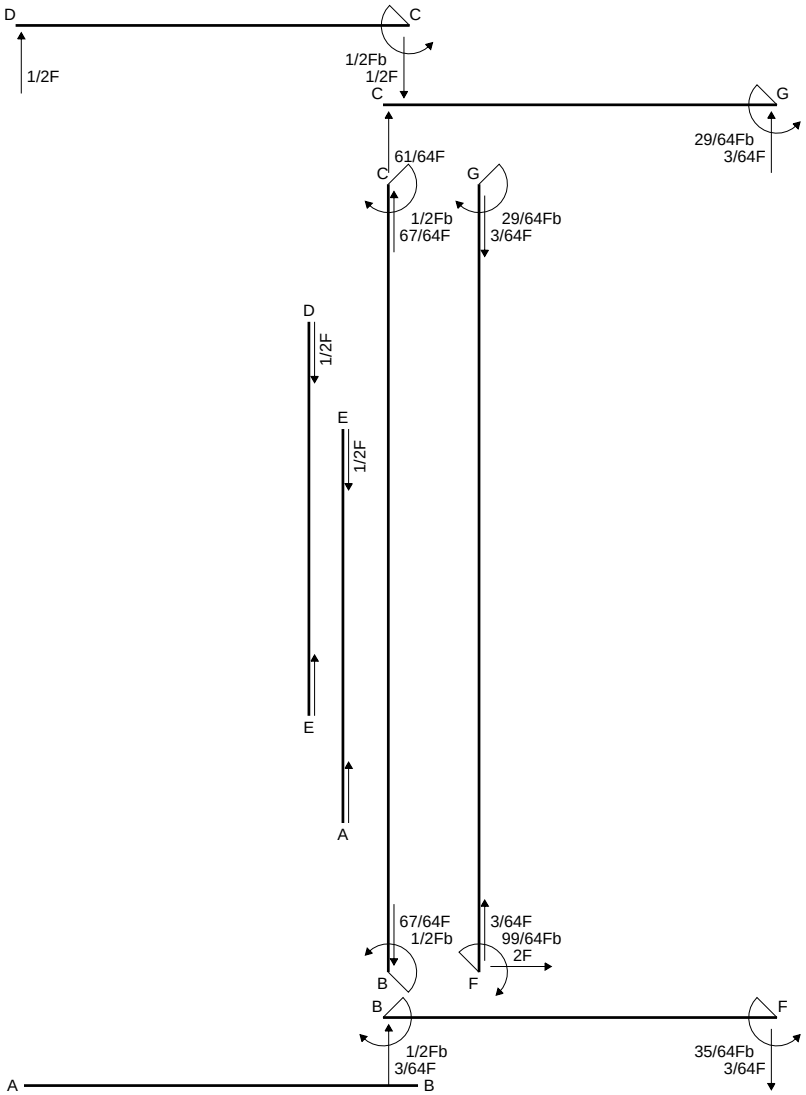
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C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
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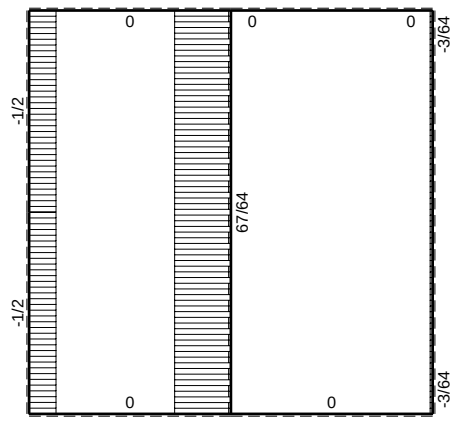
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
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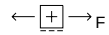
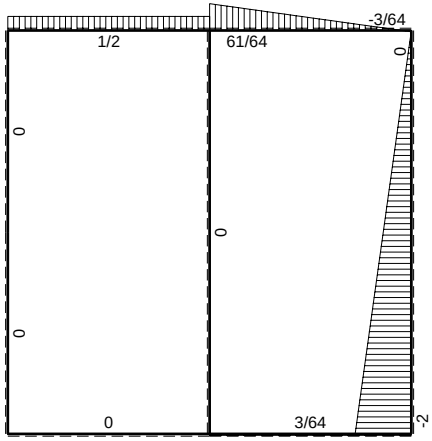
-1/2
67/64
-1/2
0
0
-3/64


F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61/64
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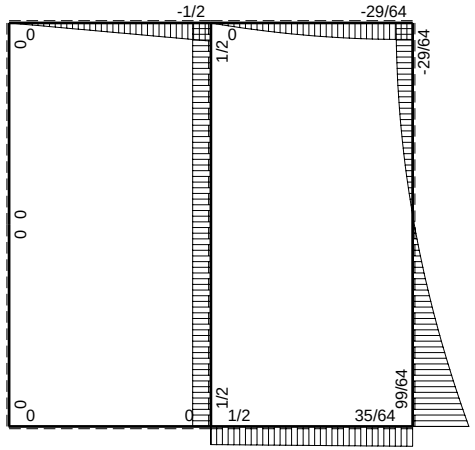
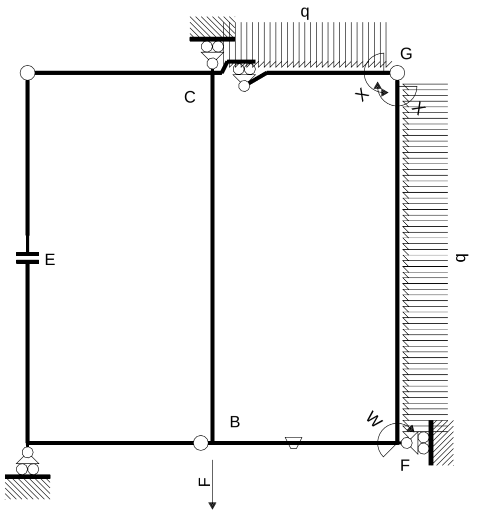
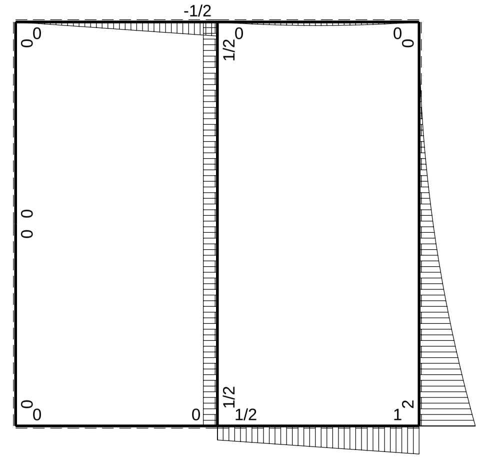
0
0
0
0
0
3/64
-2


F

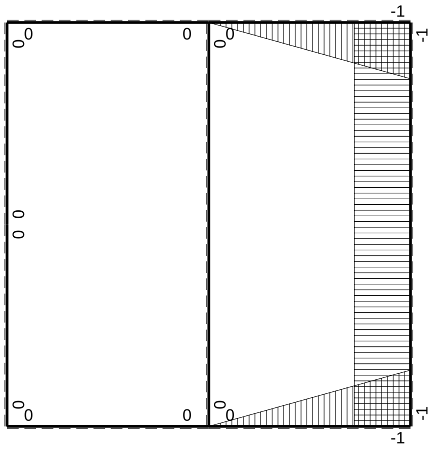

-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
0
1/2
0
0
1/2
35/64
99/64


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$1/2Fb+1/2Fx$	$-Fb/EJ$	$-1/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(-5/12+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb+1/2Fx$	Fb/EJ	$-Fb+3/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$1/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$-1/2Fb$	0	0	0	0		
	totali						$-29/24Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$29/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

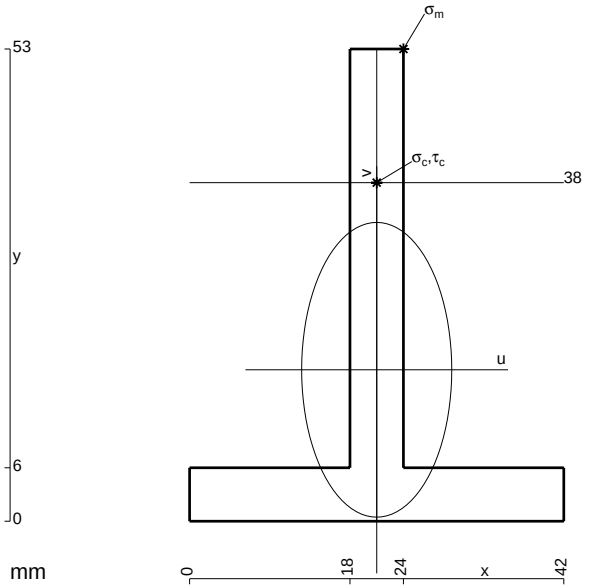
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 534. mm^2
J_u = 146122. mm^4
J_v = 37890. mm^4
y_g = 16.99 mm
T_y = 1785. N
M_x = -892500. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.01 mm
$\sigma_m = -Mv/J_u$ = 219.9 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 21.01 mm
$\sigma_c = -Mv/J_u$ = 128.3 N/mm^2
τ_c = 5.223 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.6 N/mm^2
S^* = 2566. mm^3

ESΣ8.032

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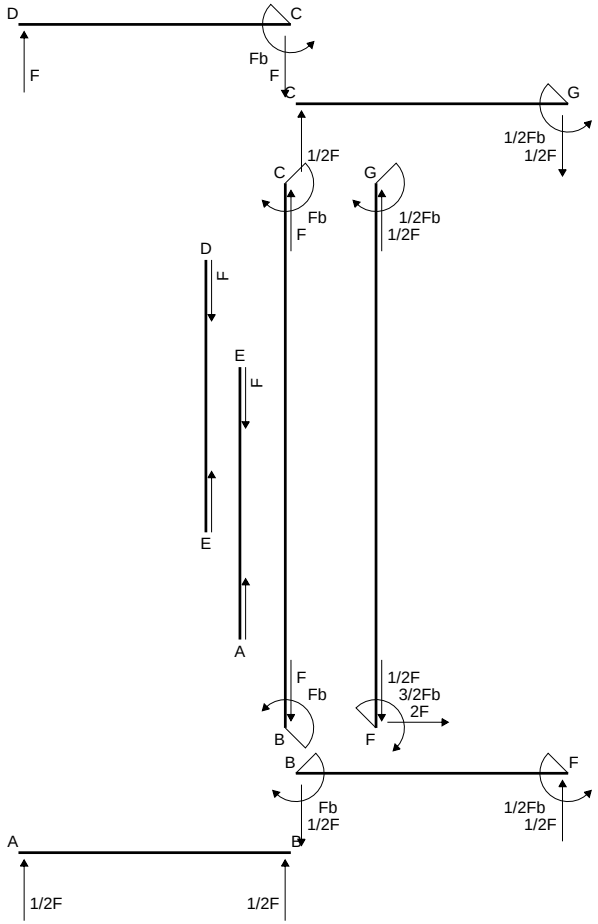
D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
1/2F
D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
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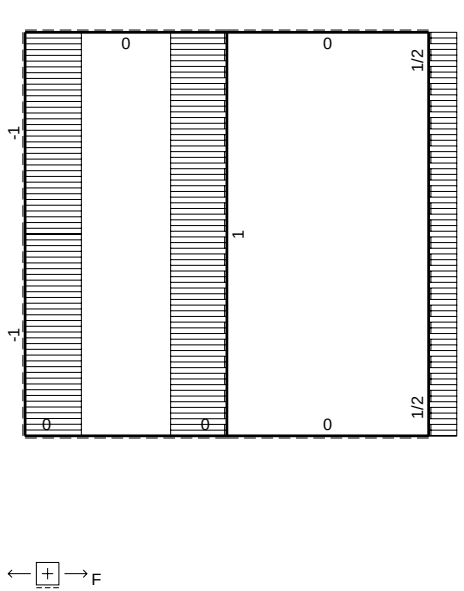
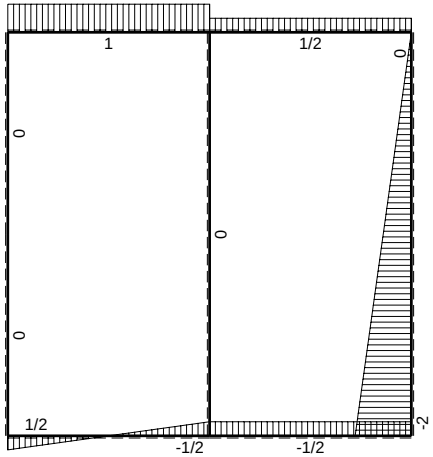
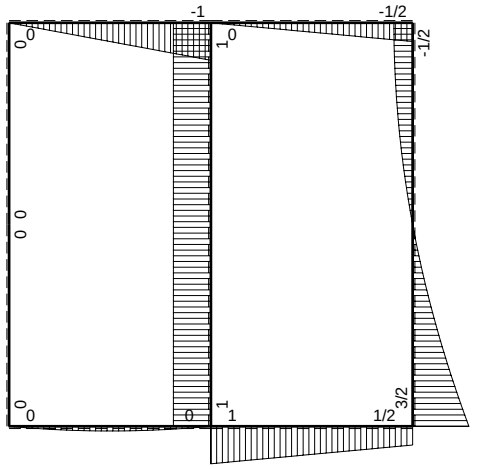
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

F


1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2

F


-1
-1/2
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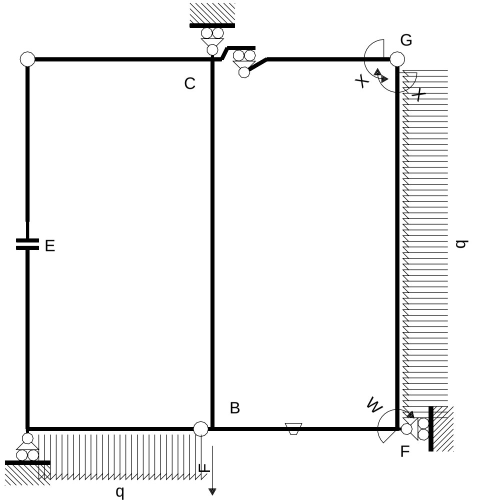
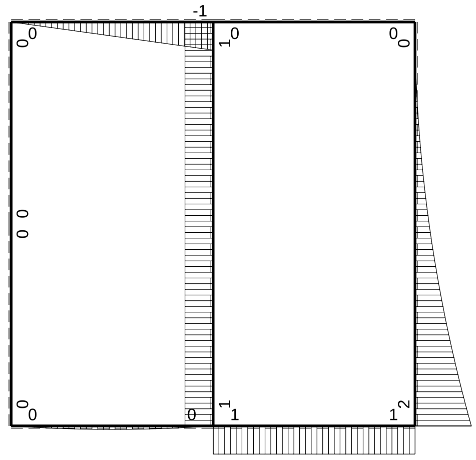
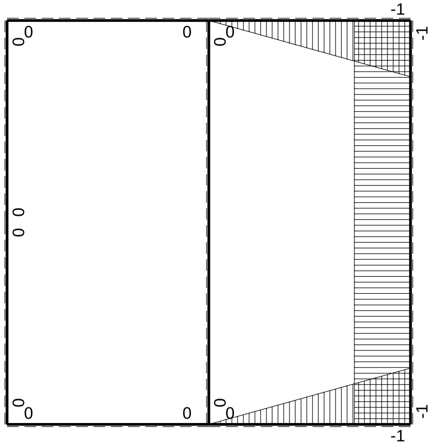
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

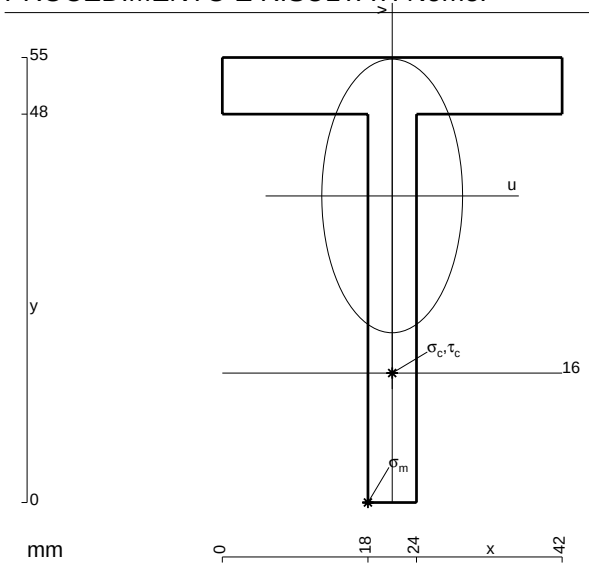
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 582. mm^2
J_u = 166519. mm^4
J_v = 44082. mm^4
y_g = 37.89 mm
T_y = 1770. N
M_x = -1008900. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.89 mm
$\sigma_m = -Mv/J_u$ = -229.6 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.89 mm
$\sigma_c = -Mv/J_u$ = -132.6 N/mm^2
τ_c = 5.084 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 132.9 N/mm^2
S^* = 2870. mm^3

ESΣ8.033

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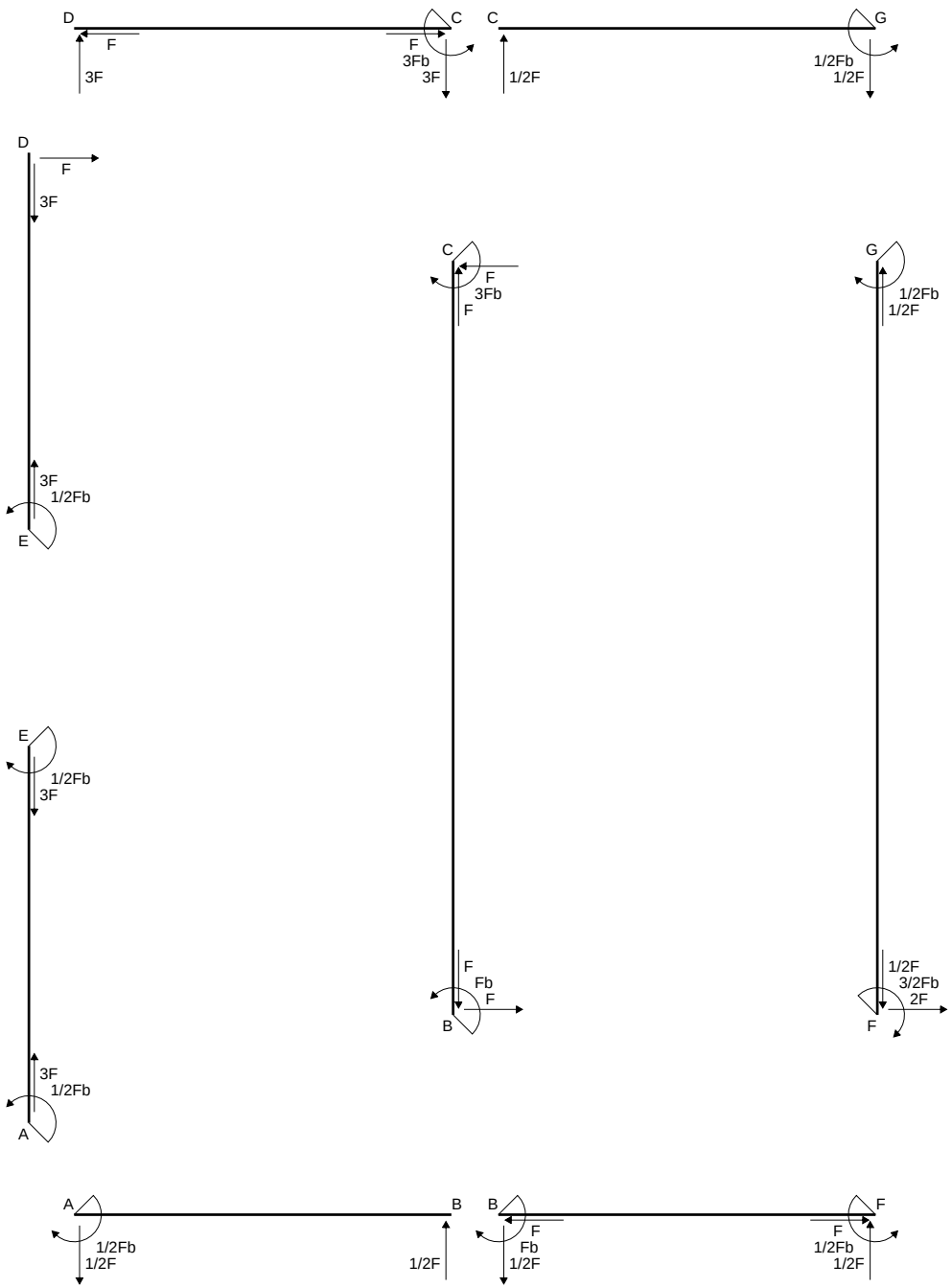
D
C
C
G
F
3F
F
3Fb
3F
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1/2Fb
1/2F
D
F
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3Fb
F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
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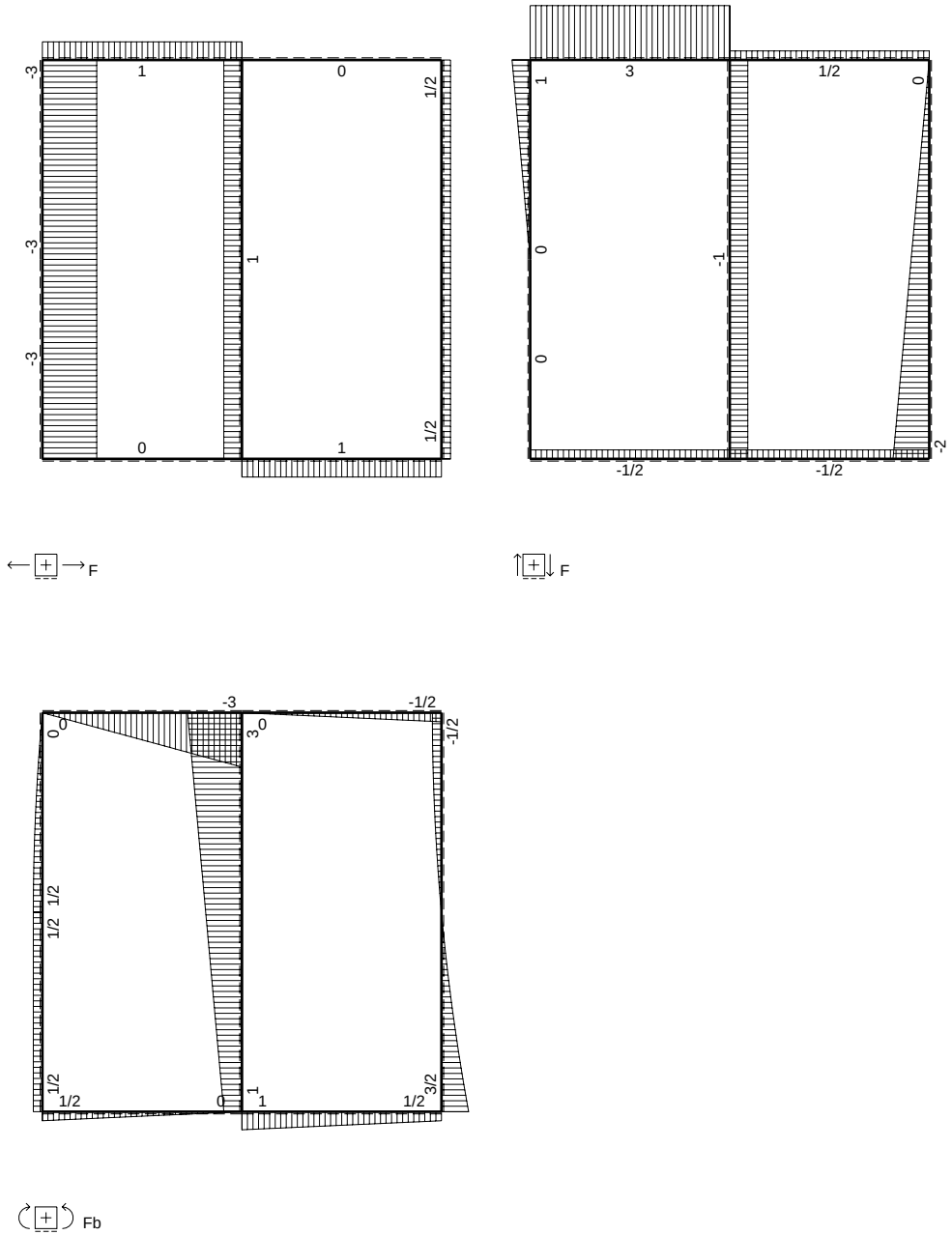
1/2F


-3
1
0
1/2
-3
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-3
0
1
1/2
F
1
3
1/2
0
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ8.034

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13

14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

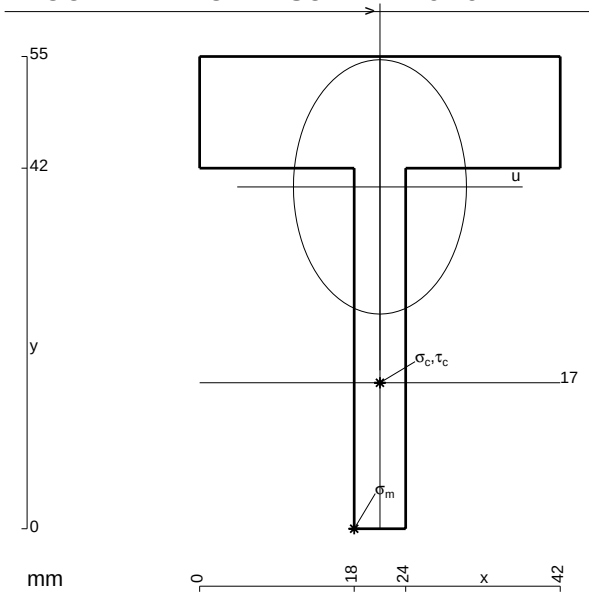
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.034



A = 798. mm^2
J_u = 175127. mm^4
J_v = 81018. mm^4
y_g = 39.82 mm
N = 570. N
T_y = 1710. N
M_x = -1043100. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.82 mm
$\sigma_m = N/A - Mv/J_u$ = -236.4 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.82 mm
$\sigma_c = N/A - Mv/J_u$ = -135.2 N/mm^2
τ_c = 5.198 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.5 N/mm^2
S^* = 3194. mm^3

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ESΣ8.034

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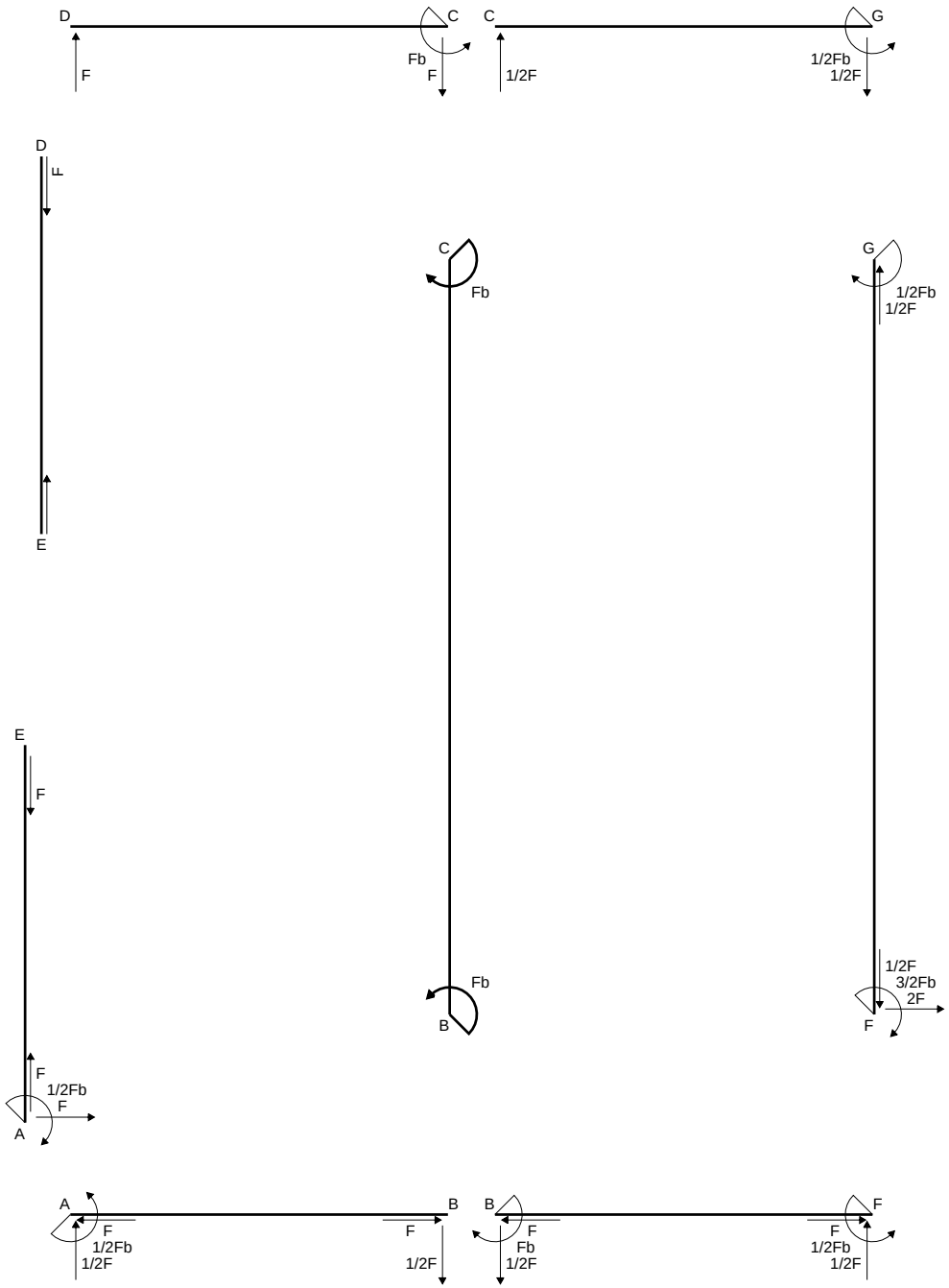
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C
G
F
Fb
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1/2Fb
1/2F
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F
E
C
Fb
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1/2F
Fb
B
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3/2Fb
2F
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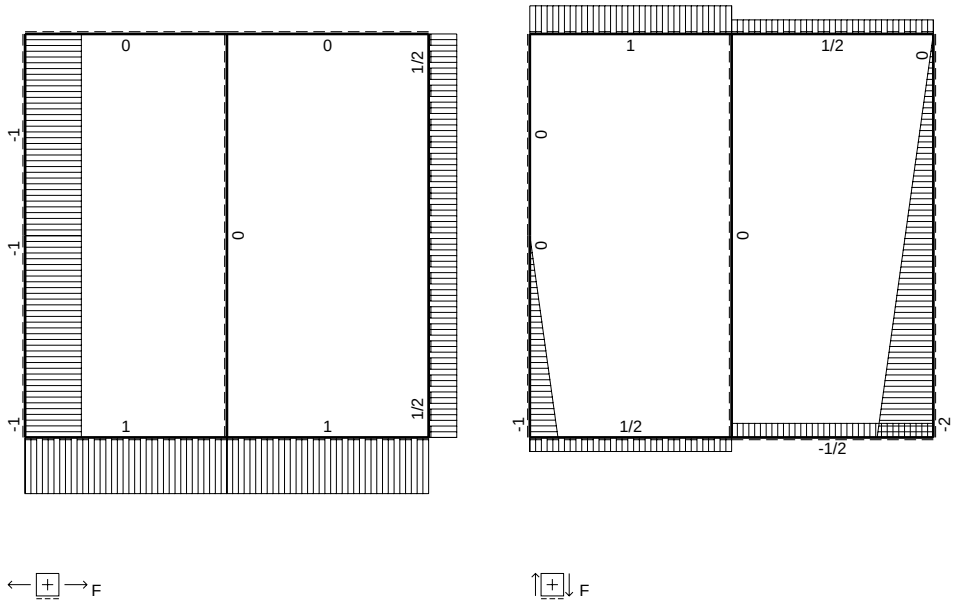
1/2Fb
1/2F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1/2
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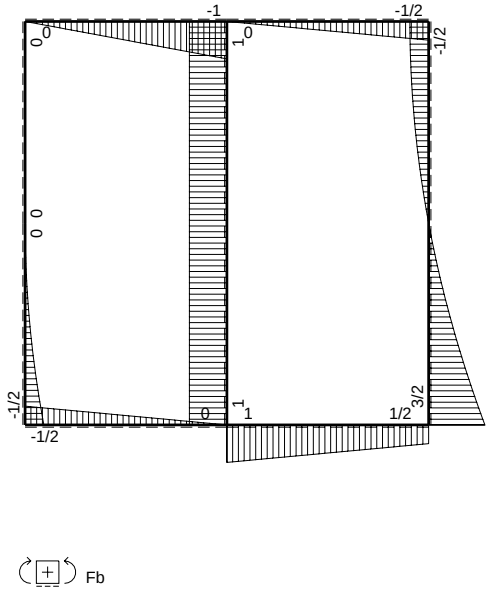
-2
-1/2
F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0 0
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0
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1
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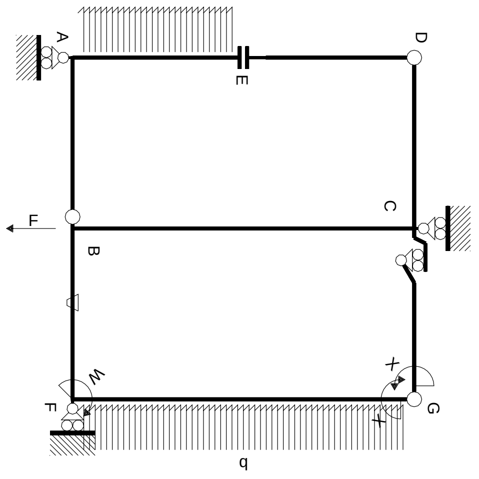
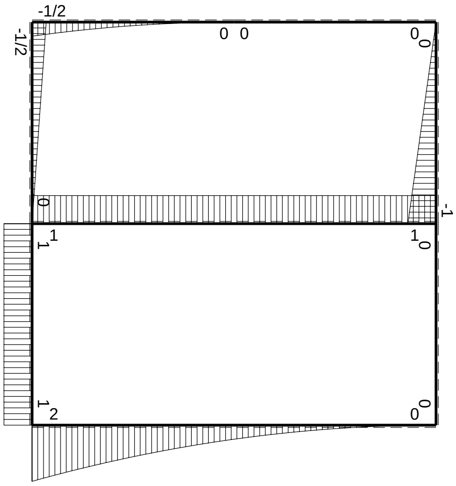
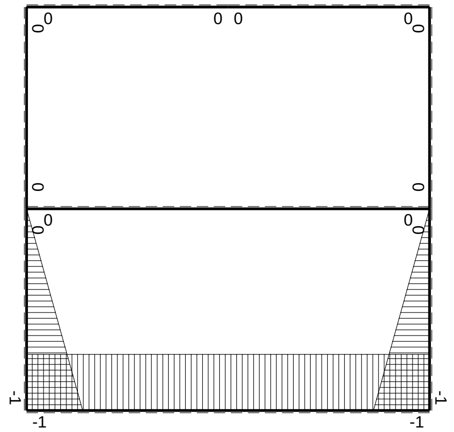
3/2
-1/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

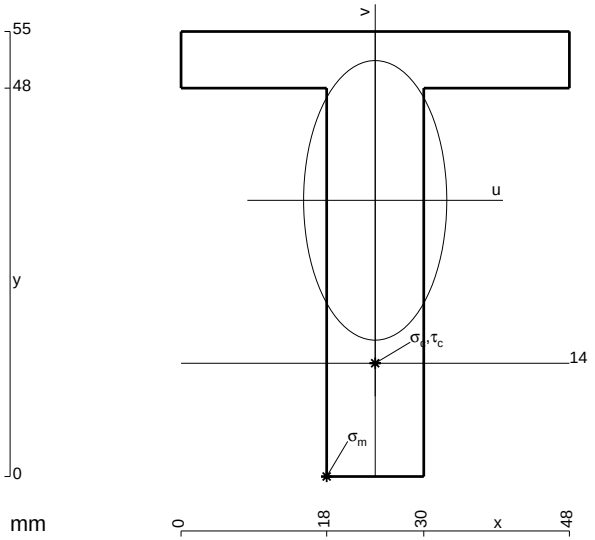
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 912. mm^2
J_u = 272448. mm^4
J_v = 71424. mm^4
y_g = 34.13 mm
T_y = 2410. N
M_x = -1590600. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.13 mm
$\sigma_m = -Mv/J_u$ = -199.3 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -20.13 mm
$\sigma_c = -Mv/J_u$ = -117.5 N/mm^2
τ_c = 3.36 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.7 N/mm^2
S^* = 4558. mm^3

ESΣ8.035

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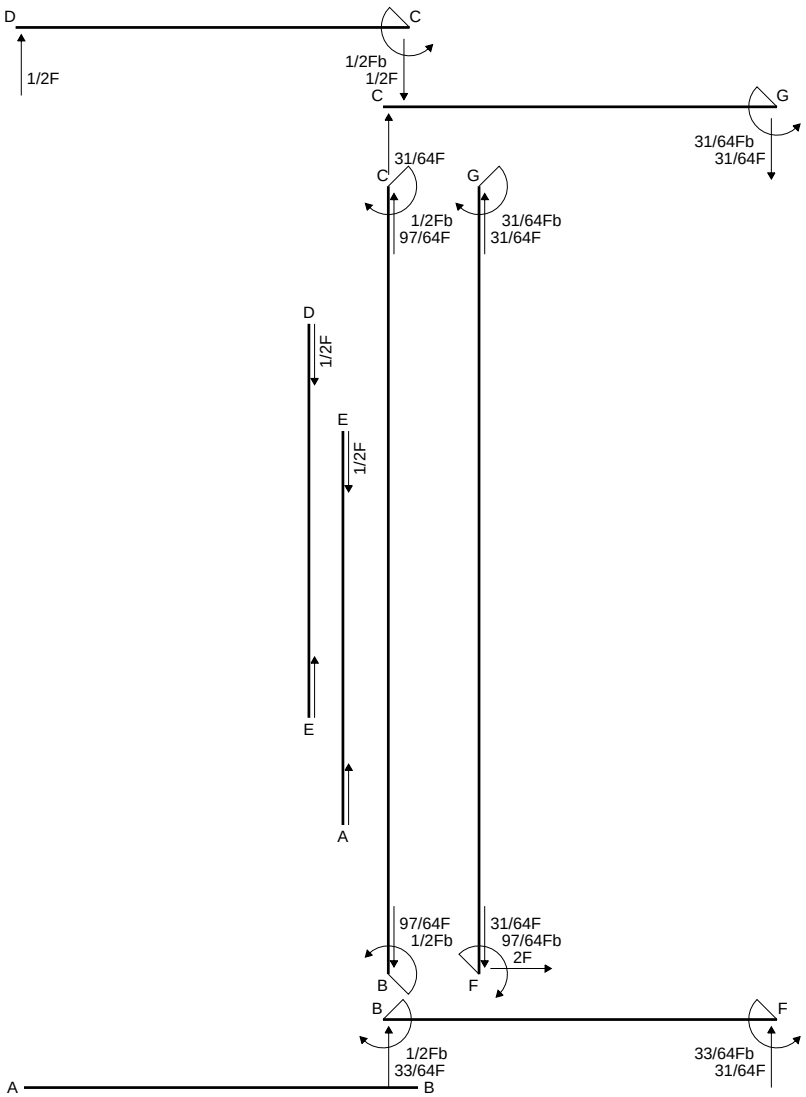
D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
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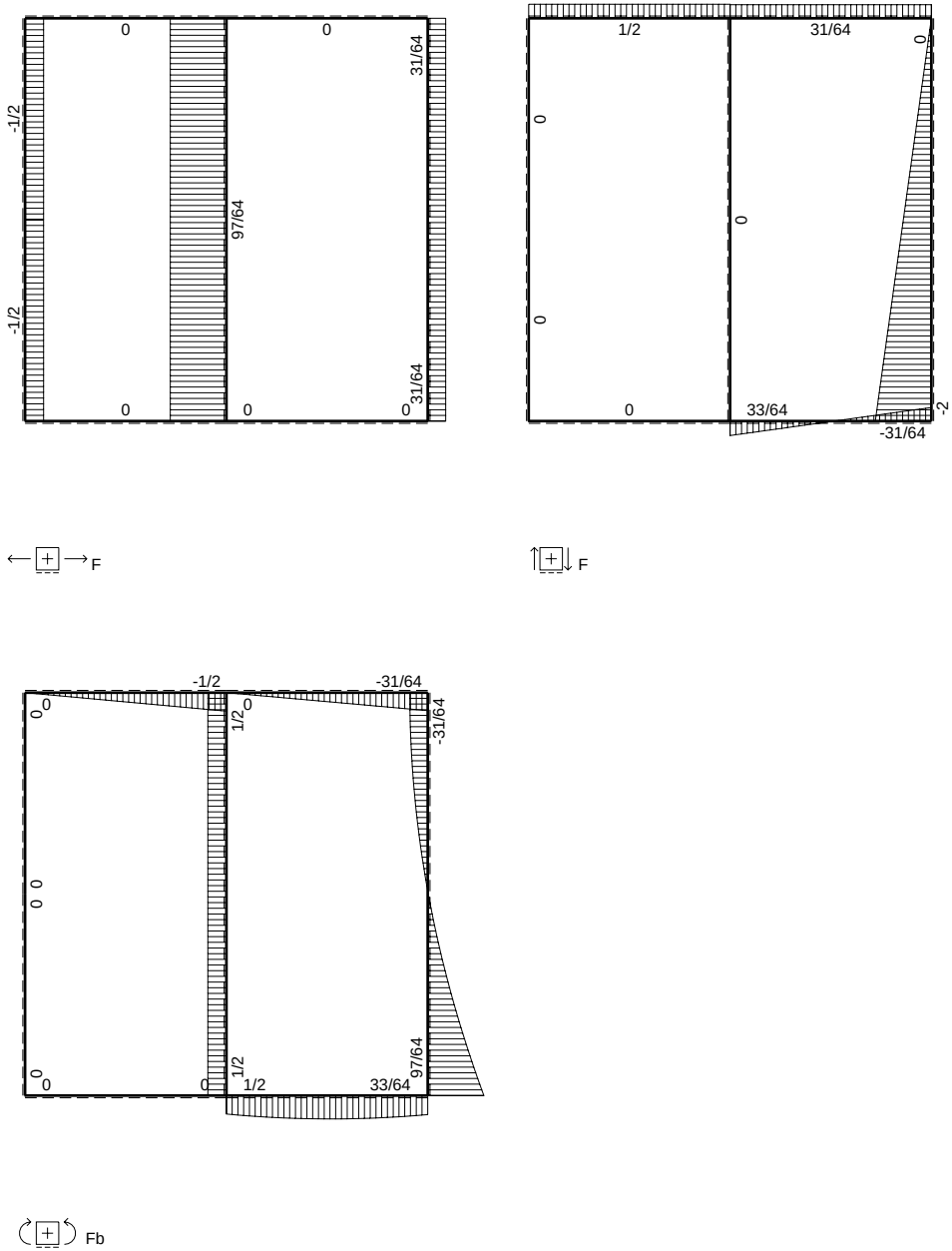
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B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31/64
F
1/2
31/64
0
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-31/64
F
-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ8.036

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

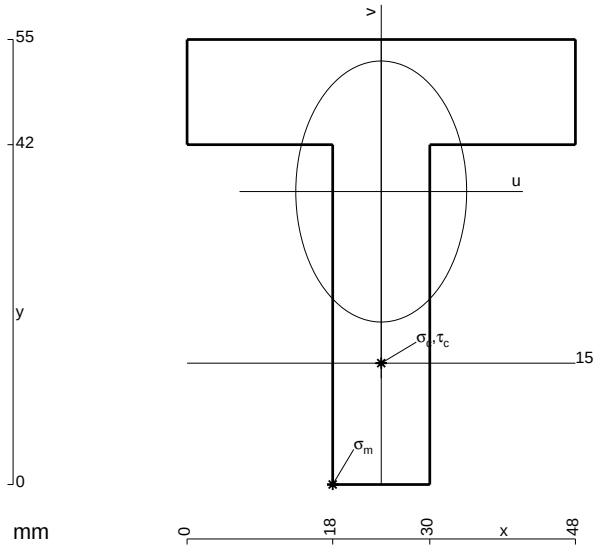
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1128. mm^2
J_u = 293725. mm^4
J_v = 125856. mm^4
y_g = 36.21 mm
T_y = 2430. N
M_x = -1701000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -36.21 mm
$\sigma_m = -Mv/J_u$ = -209.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -21.21 mm
$\sigma_c = -Mv/J_u$ = -122.8 N/mm^2
τ_c = 3.563 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123. N/mm^2
S^* = 5168. mm^3

ESΣ8.036

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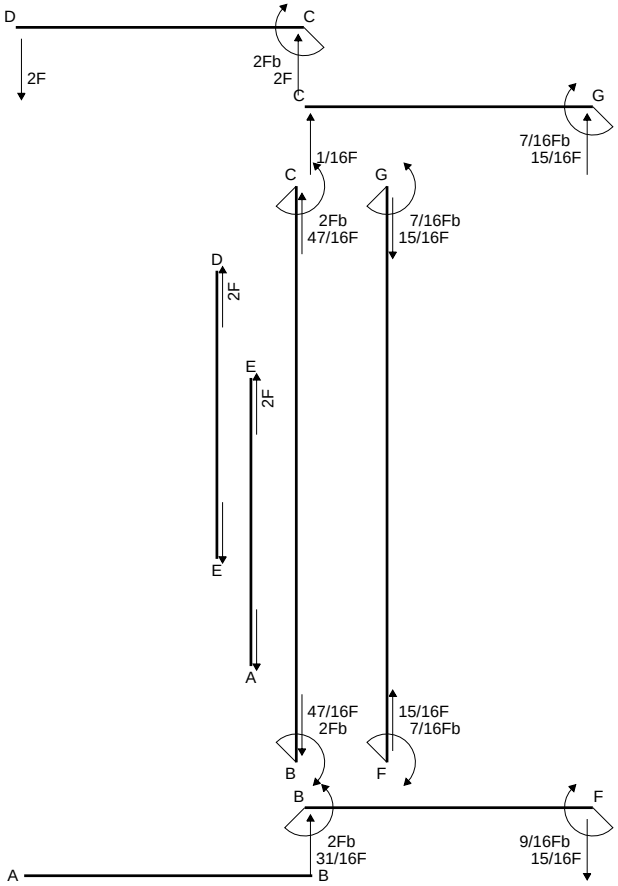
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2F
C
G
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15/16F
1/16F
C
G
2Fb
47/16F
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15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
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0
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0
31/16
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F
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7/16
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-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						7/6Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

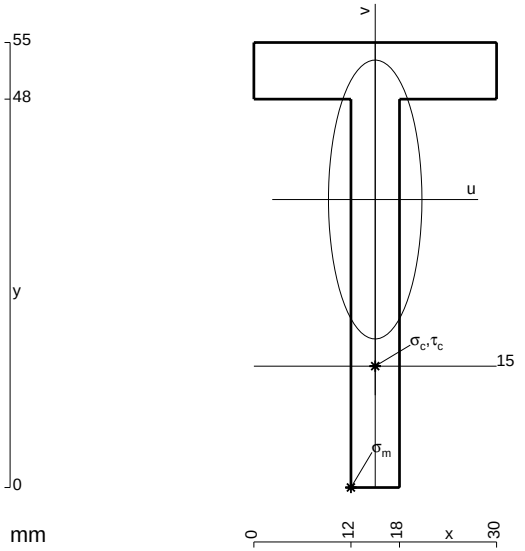
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 498. mm^2
J_u = 147997. mm^4
J_v = 16614. mm^4
y_g = 35.6 mm
T_y = -1220. N
M_x = 902800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.6 mm
$\sigma_m = -Mv/J_u$ = 217.1 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.6 mm
$\sigma_c = -Mv/J_u$ = 125.6 N/mm^2
τ_c = 3.474 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.8 N/mm^2
S^* = 2529. mm^3

ESΣ8.037

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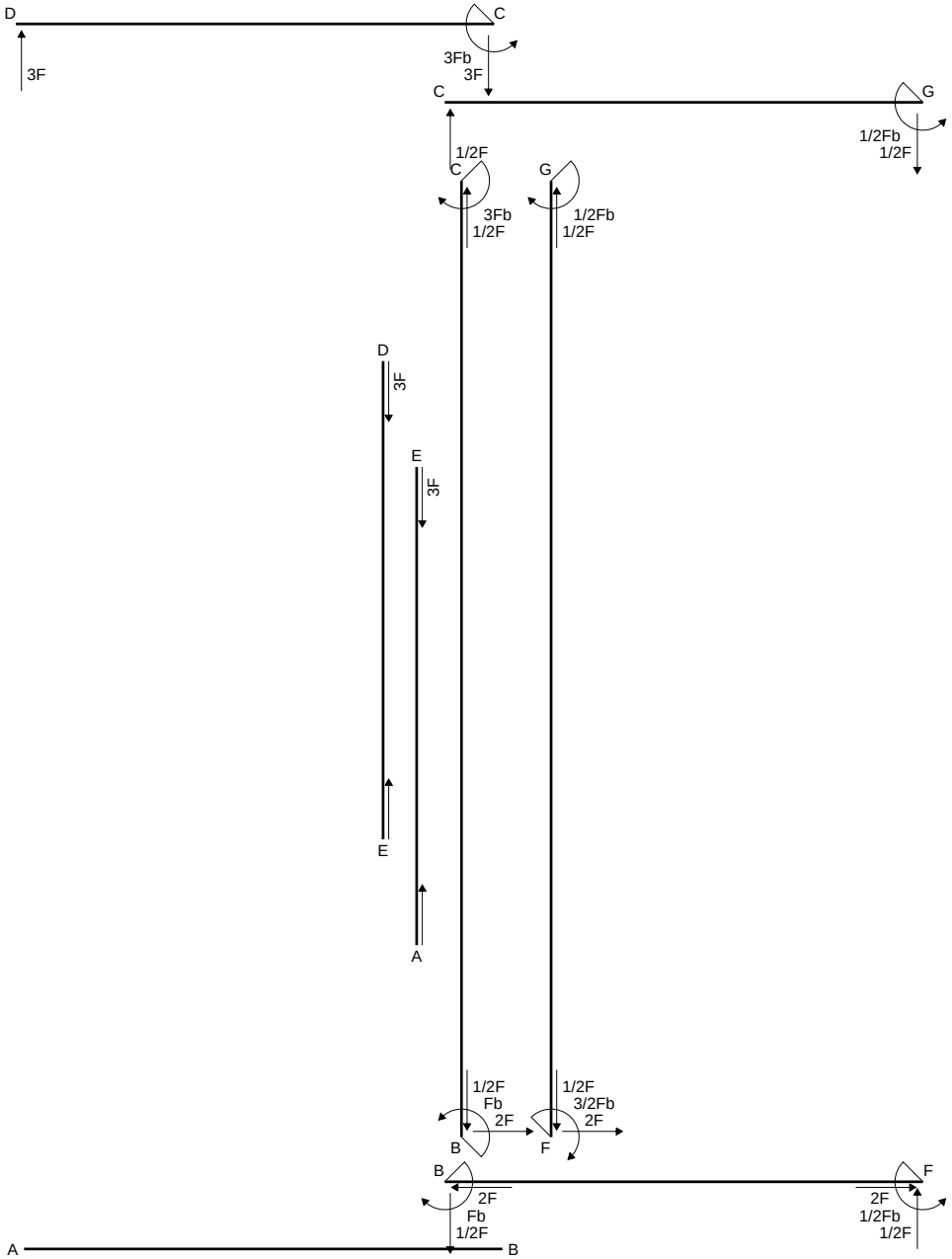
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3Fb
3F
C
G
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1/2F
1/2F
C
G
3Fb
1/2F
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1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
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F
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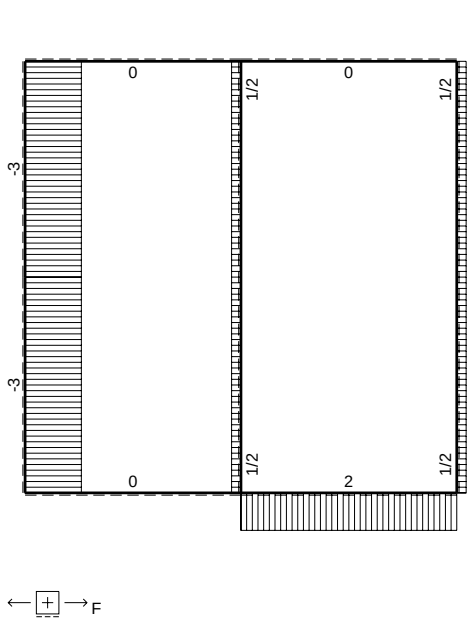
Fb
1/2F
2F
1/2Fb
1/2F
A
B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1/2
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-3
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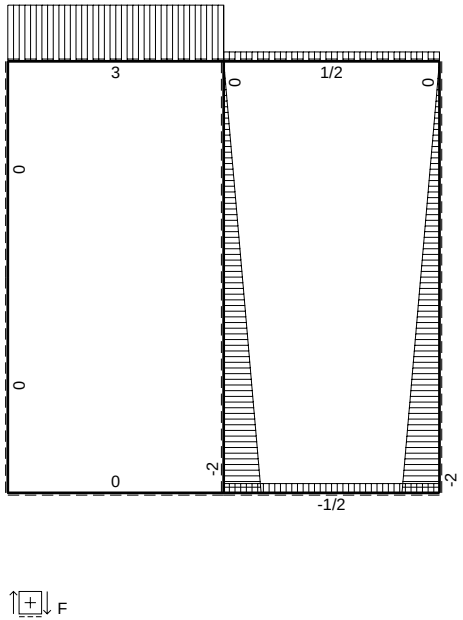
2
1/2
1/2
F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1/2
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0
-2
-2
-1/2
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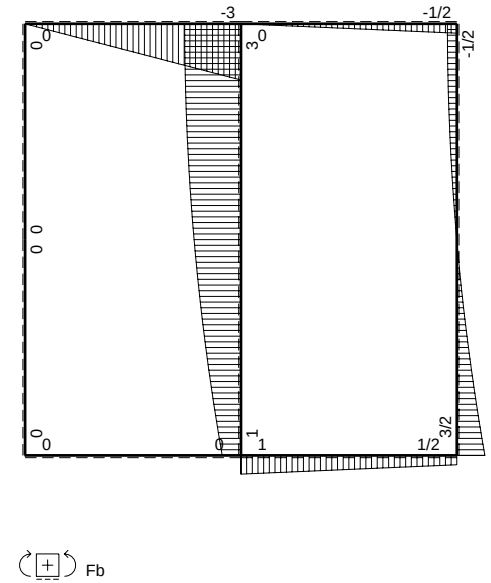
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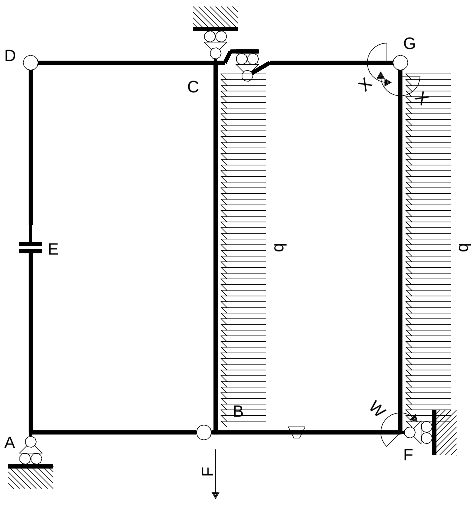
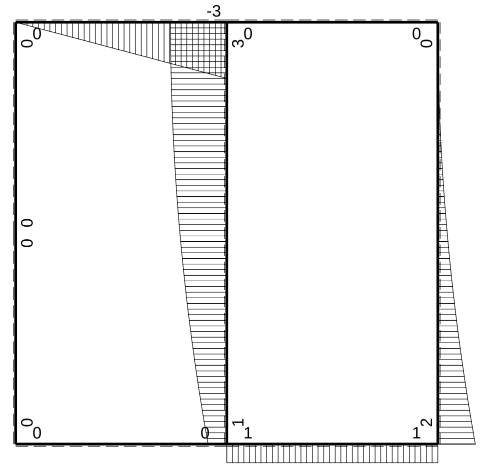
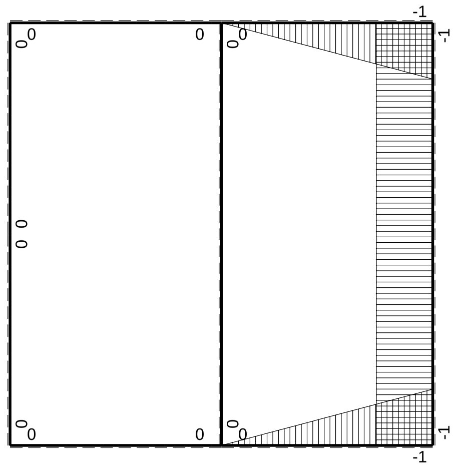
0
1
1
1/2
3/2
Fb





M$_o$ flessione da carichi assegnati



M$_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W$_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica X=W$_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

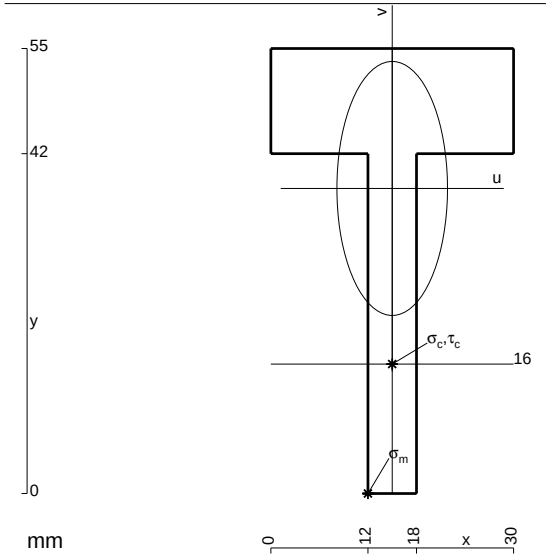
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 642. mm^2
J_u = 158306. mm^4
J_v = 30006. mm^4
y_g = 37.71 mm
T_y = 1200. N
M_x = -948000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.71 mm
$\sigma_m = -Mv/J_u$ = -225.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.71 mm
$\sigma_c = -Mv/J_u$ = -130. N/mm^2
τ_c = 3.603 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.1 N/mm^2
S^* = 2852. mm^3

ESΣ8.038

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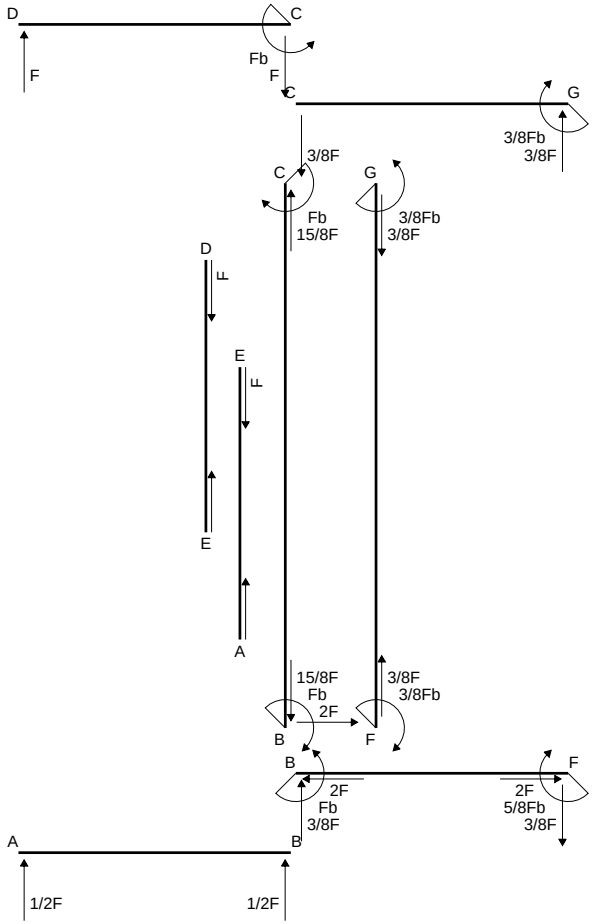
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Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
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F
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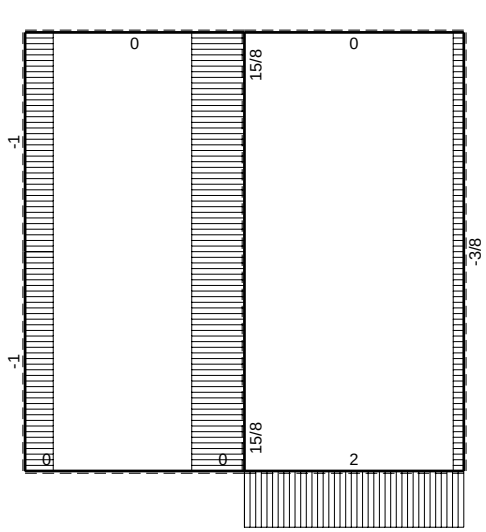
1/2F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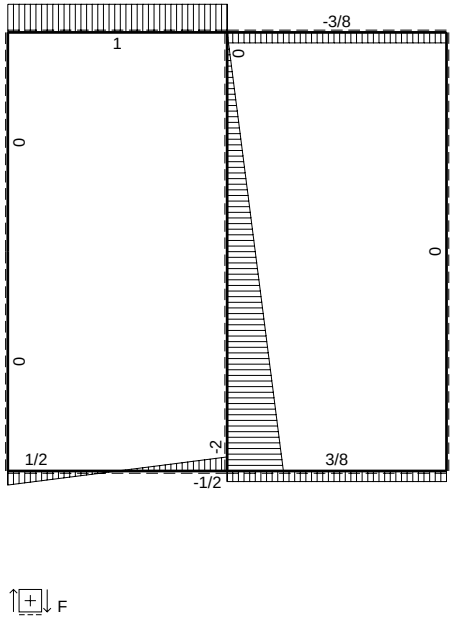
2
F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1/2
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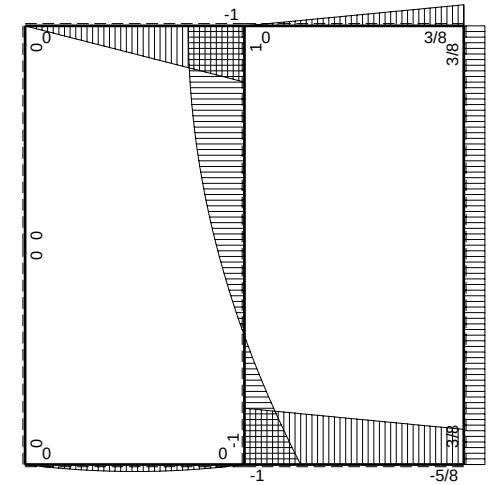
3/8
-1/2
F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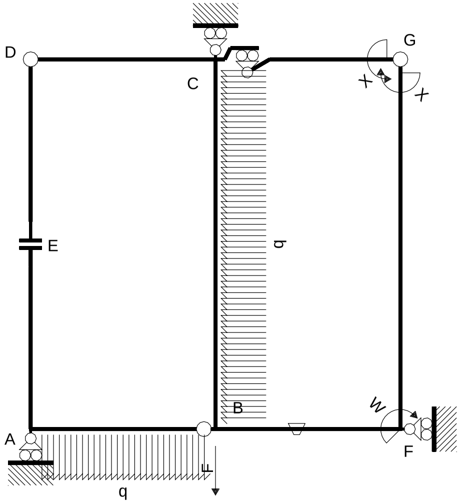
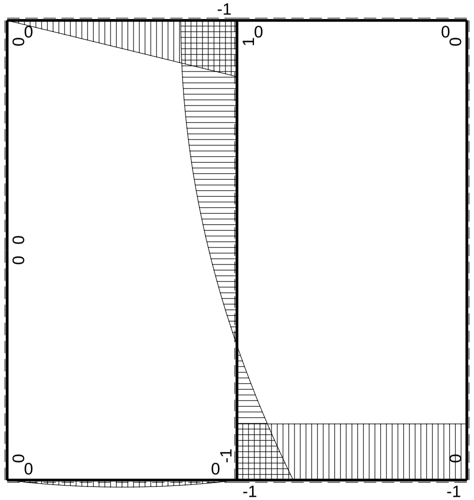
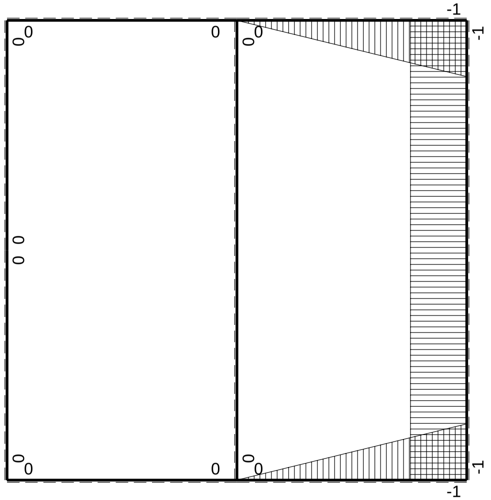
0
0
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

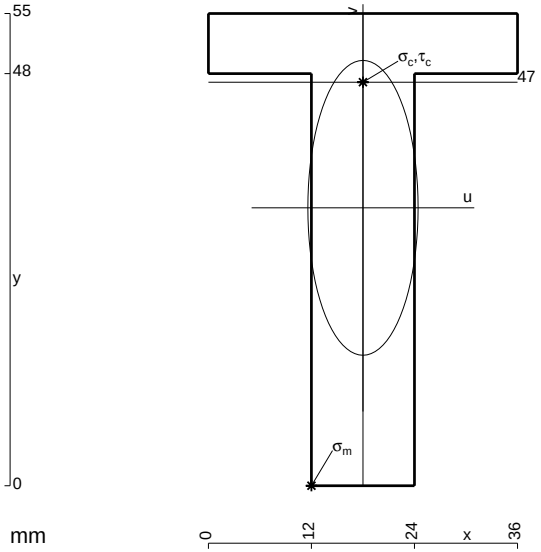
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 828. mm^2
J_u = 244195. mm^4
J_v = 34128. mm^4
y_g = 32.37 mm
N = 4163. N
T_y = -4440. N
M_x = -1842600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.37 mm
$\sigma_m = N/A - Mv/J_u$ = -239.2 N/mm^2
x_c = 18. mm
y_c = 47. mm
v_c = 14.63 mm
$\sigma_c = N/A - Mv/J_u$ = 115.4 N/mm^2
τ_c = 7.58 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.2 N/mm^2
S^* = 5002. mm^3

ESΣ8.039

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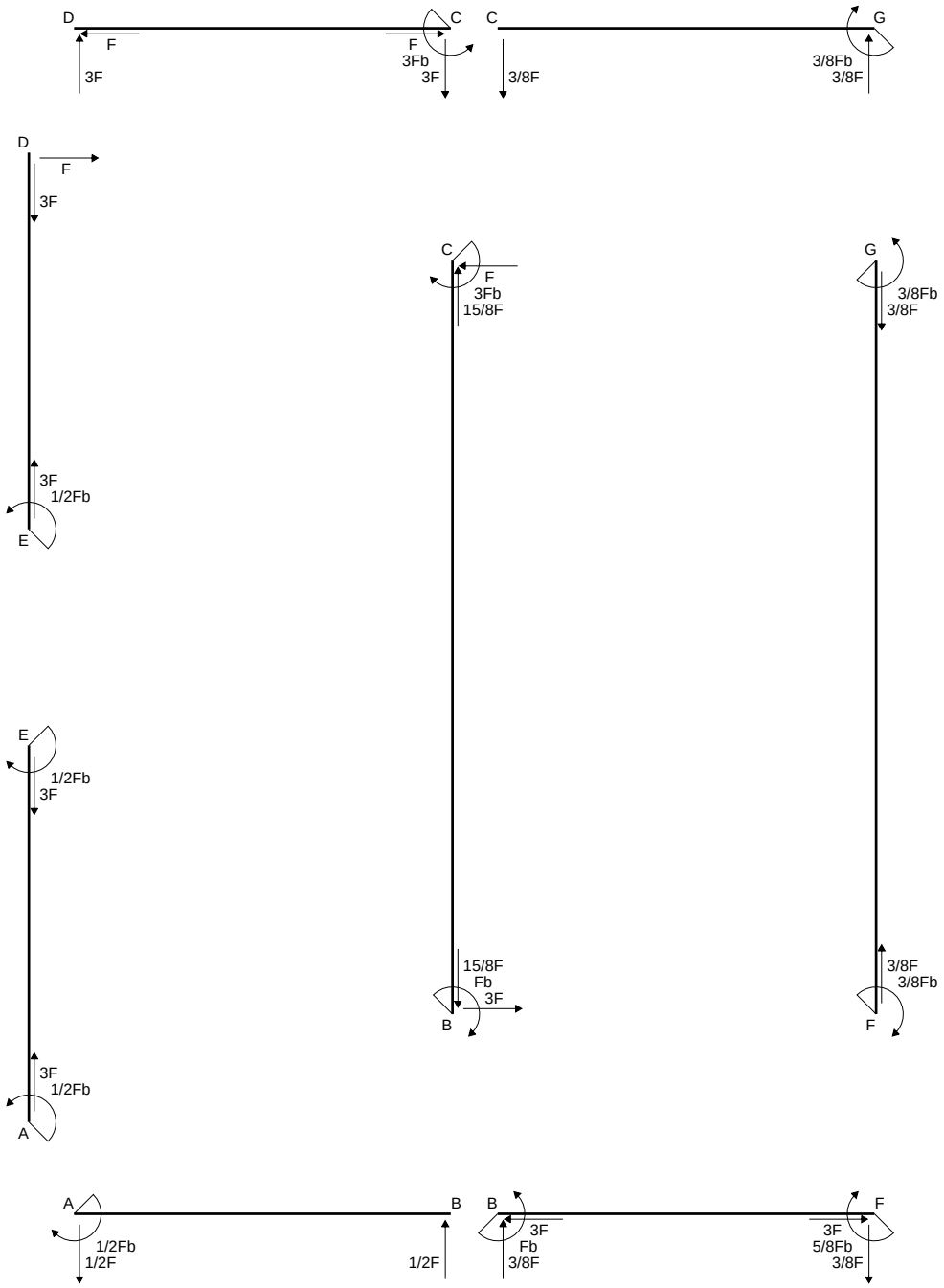
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G
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3F
F
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3F
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3/8Fb
3/8F
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F
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C
F
3Fb
15/8F
G
3/8Fb
3/8F
3F
1/2Fb
E
E
1/2Fb
3F
15/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
A
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1/2F
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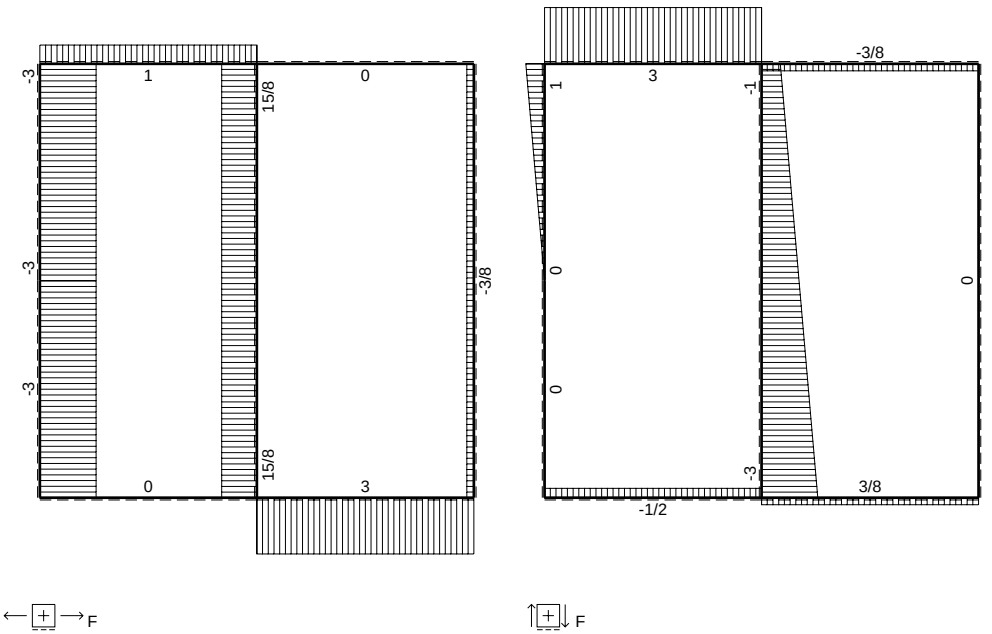
Fb
3/8F
3F
5/8Fb
3/8F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-3/8
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0
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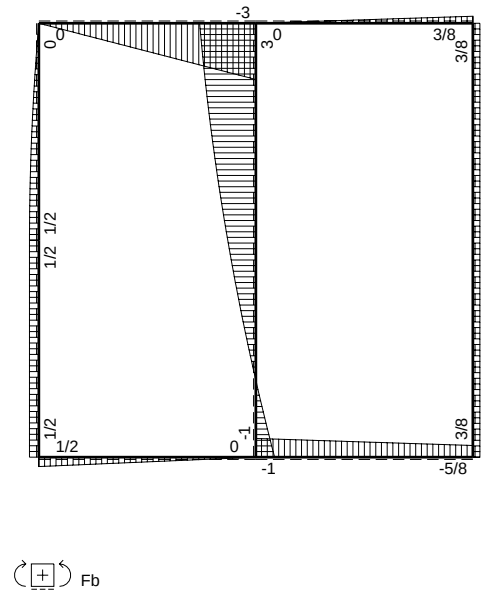
3/8
-1/2
F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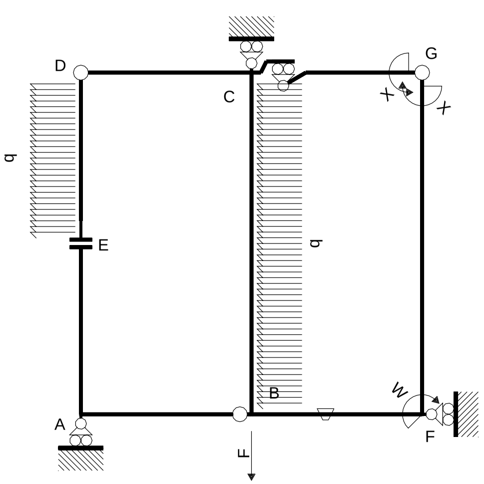
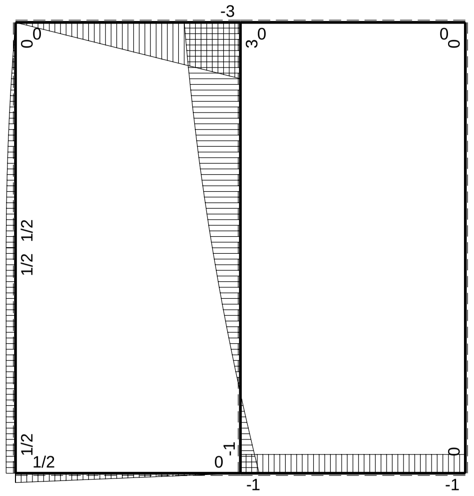
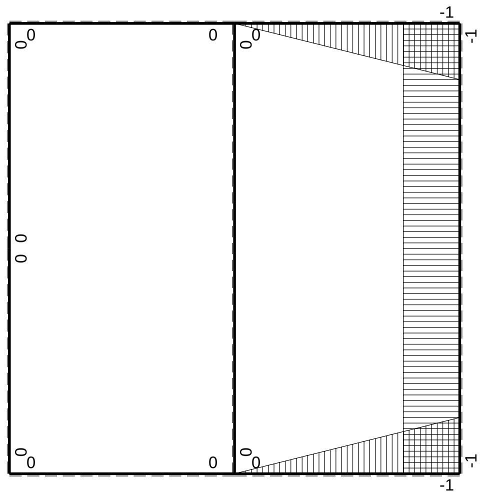
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

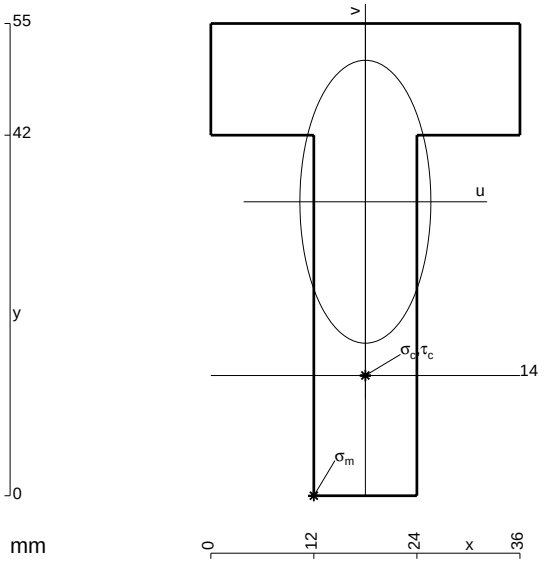
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 972. mm^2
J_u = 264196. mm^4
J_v = 56592. mm^4
y_g = 34.24 mm
N = 1170. N
T_y = 3510. N
M_x = -1544400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.24 mm
σ_m = N/A-Mv/J_u = -199. N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -20.24 mm
σ_c = N/A-Mv/J_u = -117.1 N/mm^2
τ_c = 5.067 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.4 N/mm^2
S^* = 4576. mm^3

ESΣ8.040

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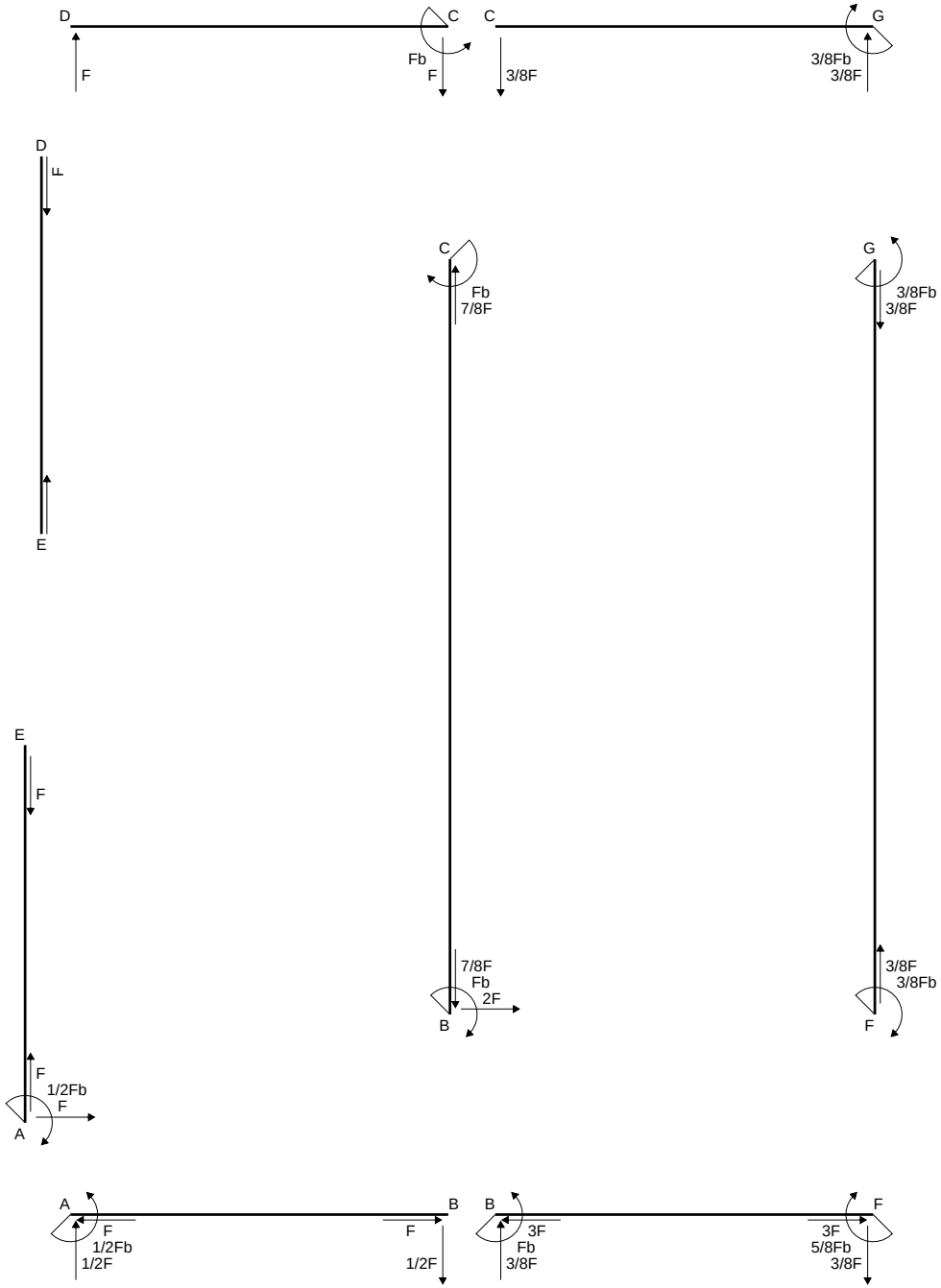
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3/8F
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7/8F
Fb
2F
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3/8Fb
F
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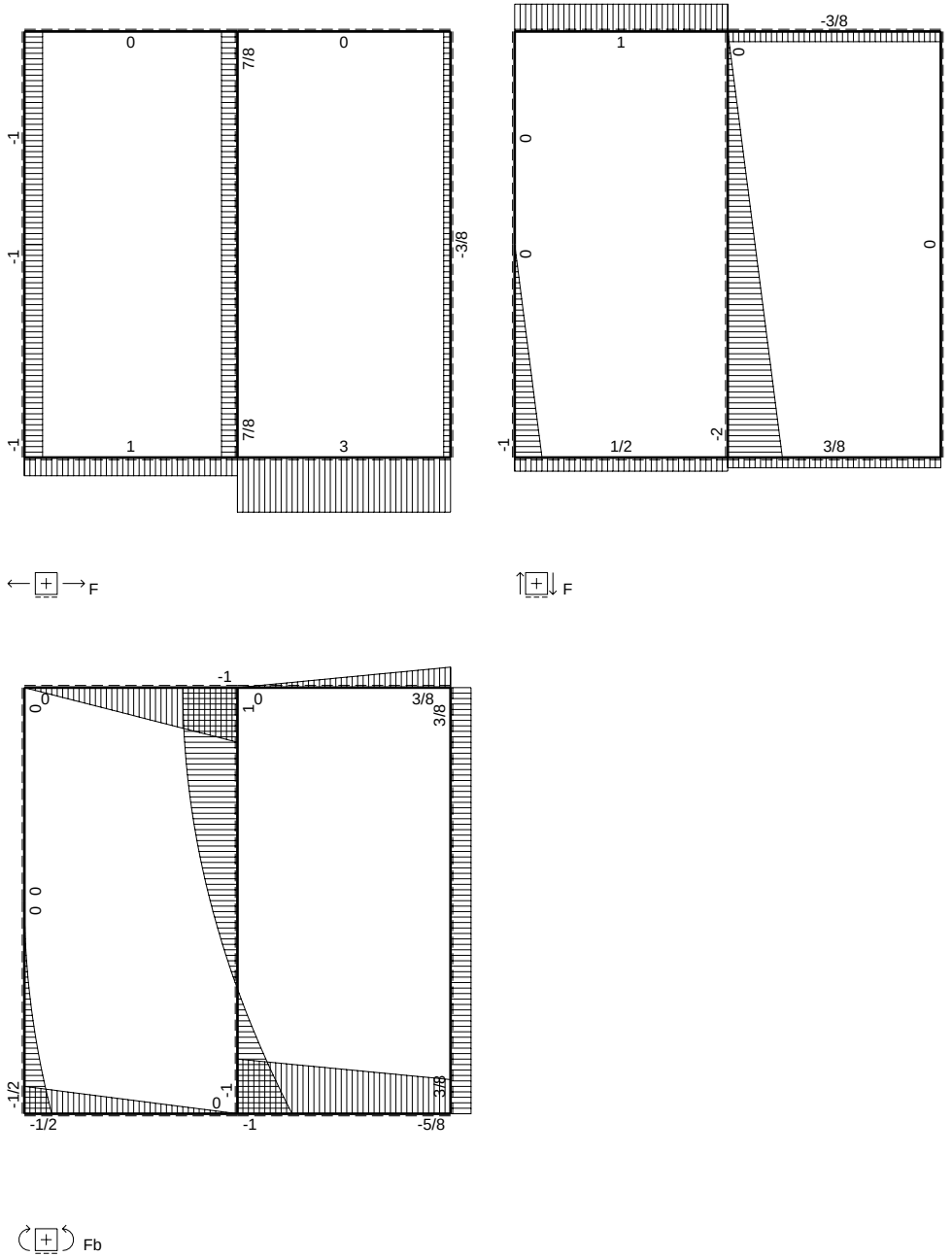
3/8F
3F
5/8Fb
3/8F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3/8
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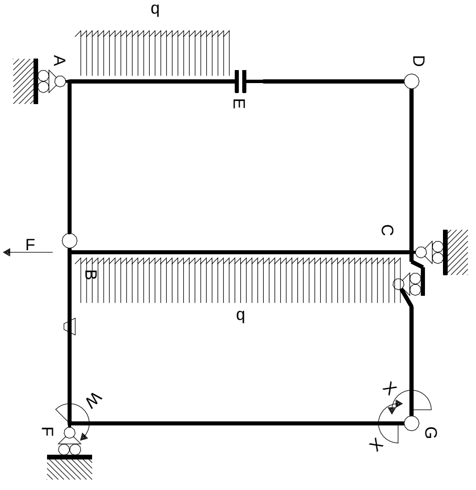
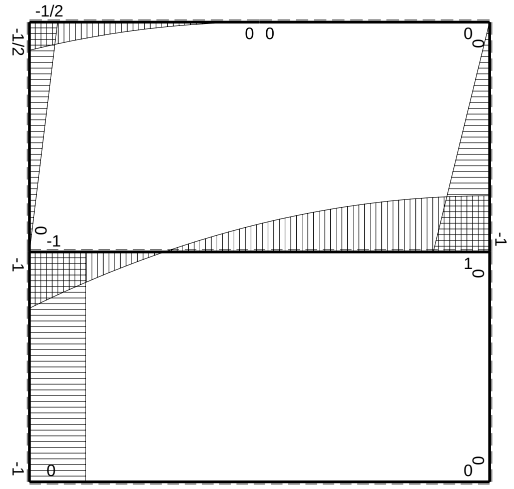
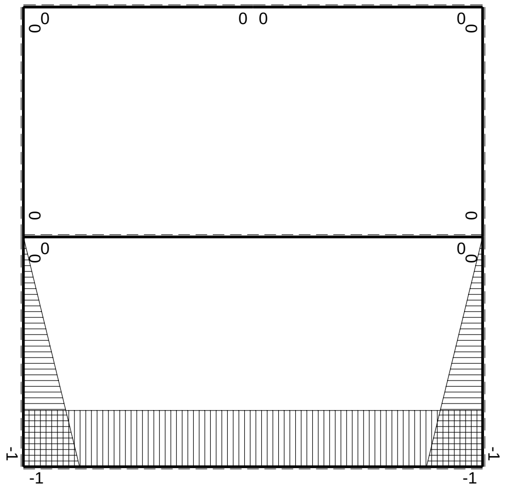
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

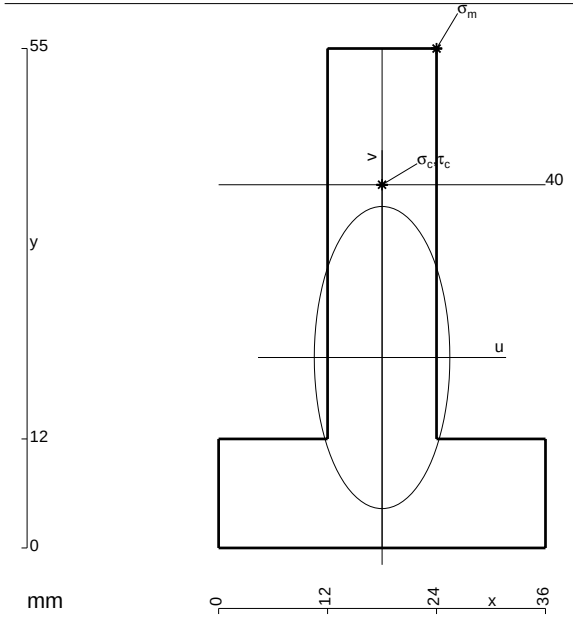
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 948. mm^2
J_u = 262515. mm^4
J_v = 52848. mm^4
y_g = 20.97 mm
N = 2905. N
T_y = -6640. N
M_x = -1593600. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 34.03 mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.03 mm
$\sigma_c = N/A - Mv/J_u$ = 118.6 N/mm^2
τ_c = 10.07 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 119.9 N/mm^2
S^* = 4776. mm^3

ESΣ8.041

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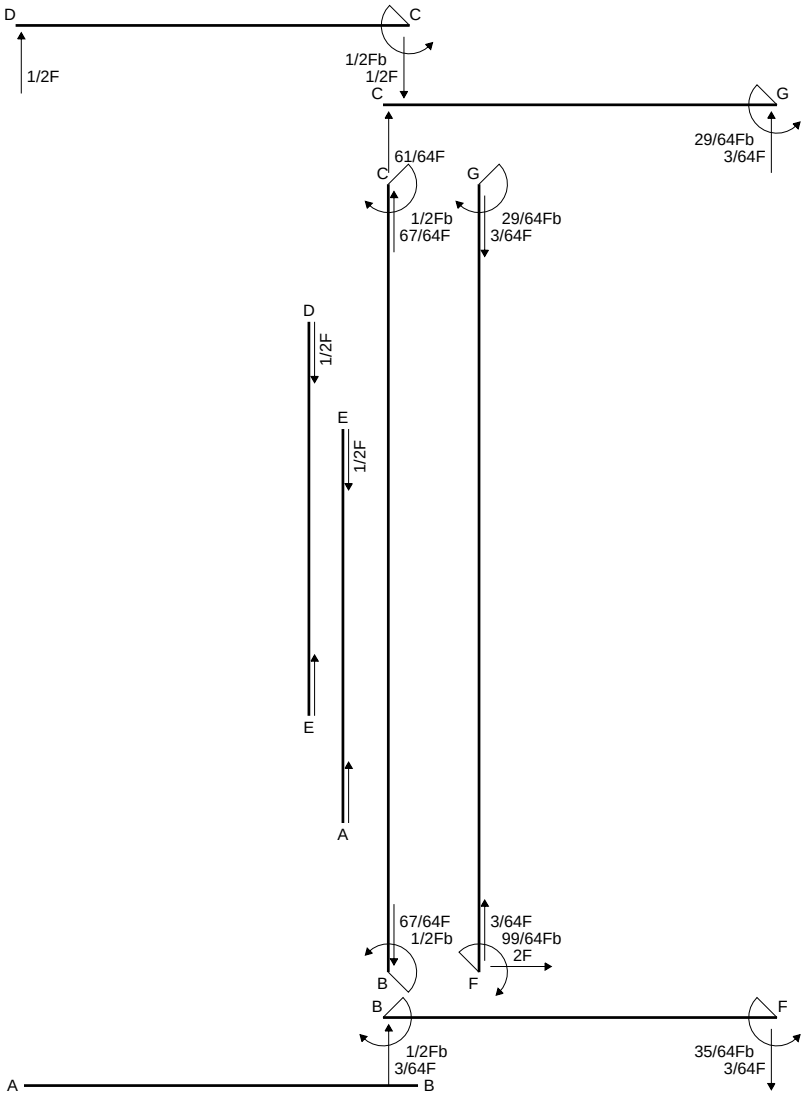
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G
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3/64F
61/64F
C
G
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67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
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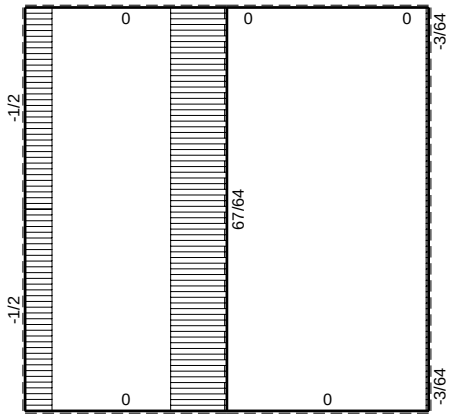
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B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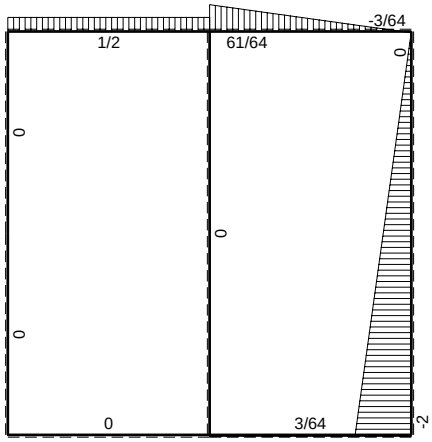
0
-3/64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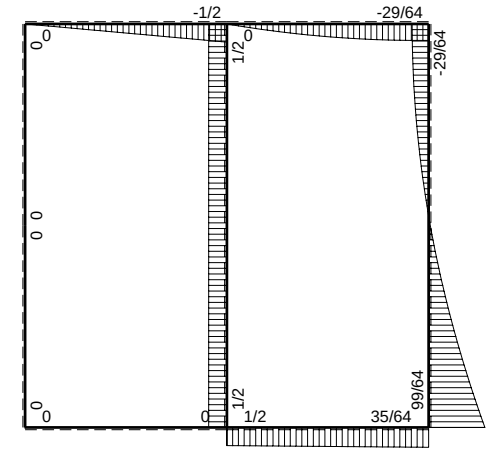
-2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-29/64
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1/2
0
0
1/2
35/64
99/64

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx²/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb²/EJ	1/3XLb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb²/EJ	1/3XLb/EJ
CG b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2XLb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
totali							-29/24Fb²/EJ	8/3XLb/EJ
iperstatica $X=W_{GC}$							29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

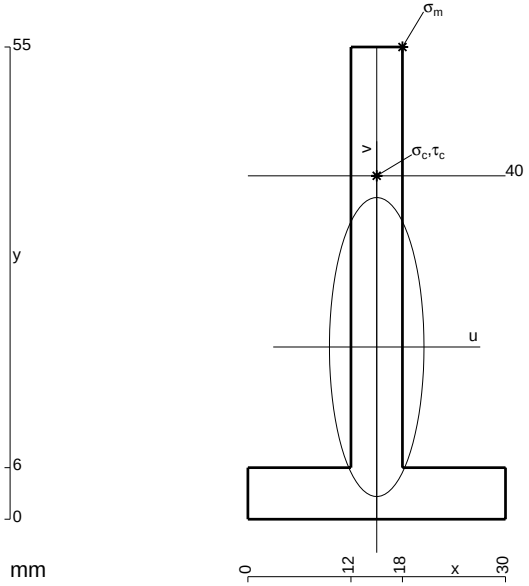
$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 474. mm^2
J_u = 143796. mm^4
J_v = 14382. mm^4
y_g = 20.06 mm
T_y = 1740. N
M_x = -904800. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 34.94 mm
$\sigma_m = -Mv/J_u$ = 219.9 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 19.94 mm
$\sigma_c = -Mv/J_u$ = 125.5 N/mm^2
τ_c = 4.981 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.8 N/mm^2
S^* = 2470. mm^3

ESΣ8.042

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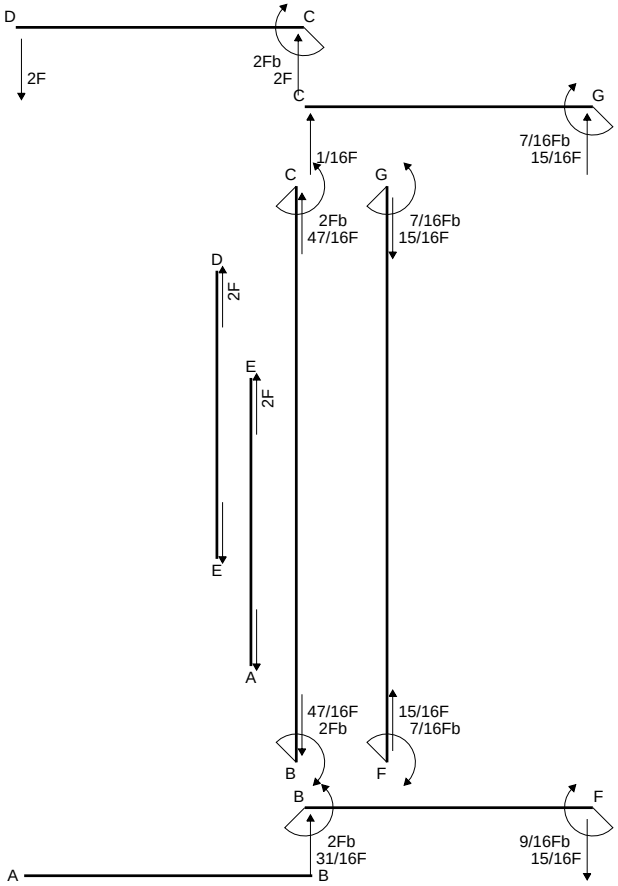
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2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
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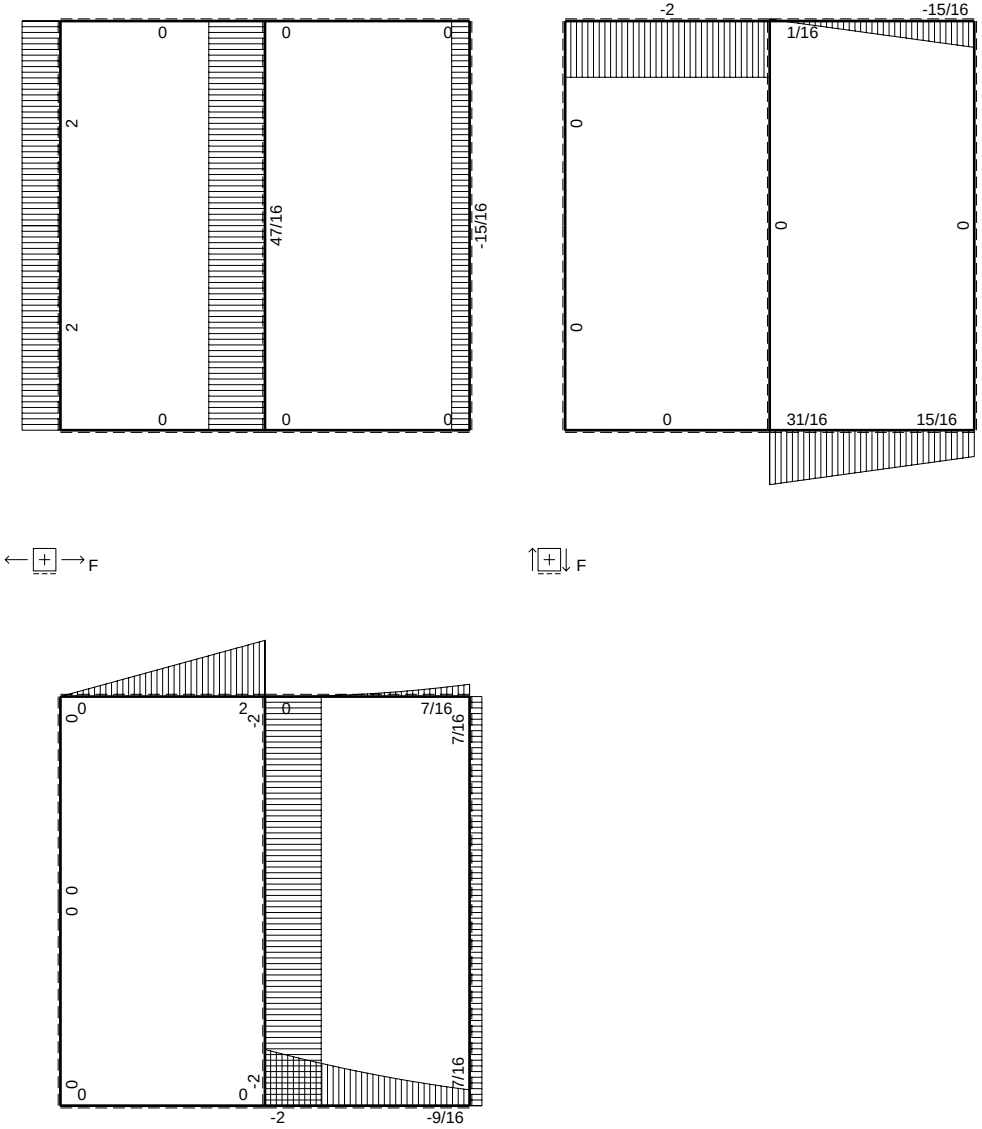
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
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2
47/16
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F
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-15/16
1/16
0
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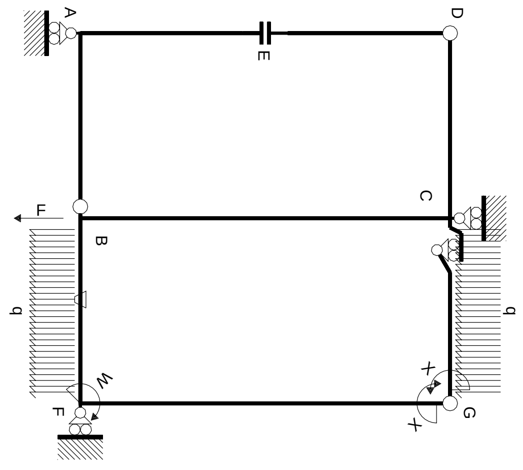
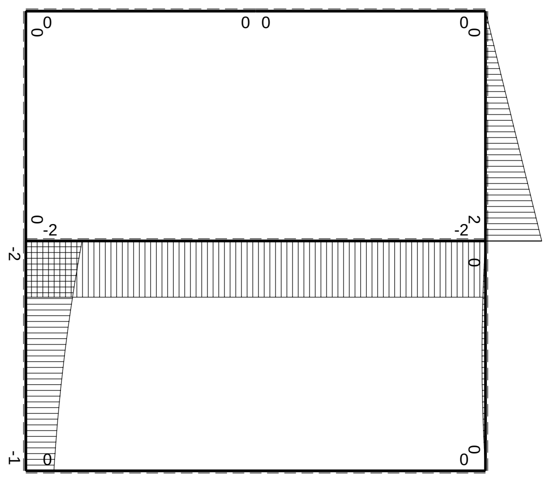
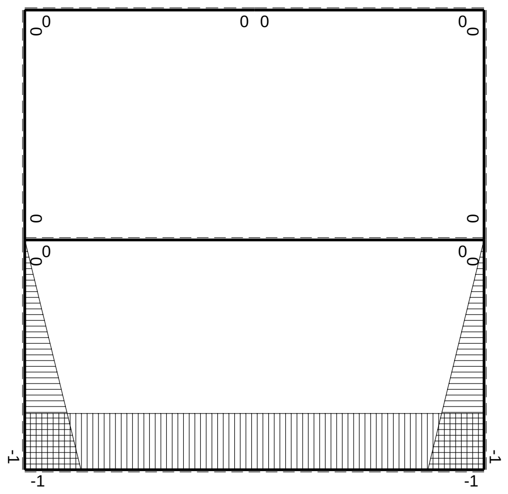
0
0
31/16
15/16
F
0
0
0
0
7/16
2
-2
0
7/16
7/16
0
0
0
-2
-2
-9/16
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

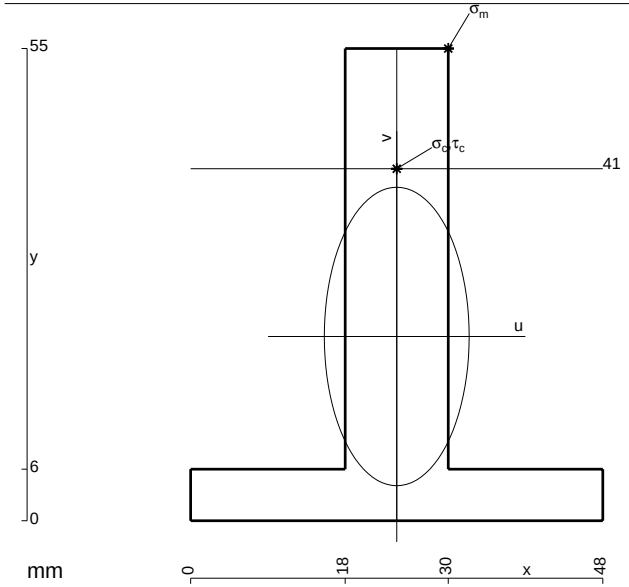
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 876. mm^2
J_u = 264708. mm^4
J_v = 62352. mm^4
y_g = 21.46 mm
T_y = -3180. N
M_x = 1812600. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 33.54 mm
$\sigma_m = -Mv/J_u$ = -229.7 N/mm^2
x_c = 24. mm
y_c = 41. mm
v_c = 19.54 mm
$\sigma_c = -Mv/J_u$ = -133.8 N/mm^2
τ_c = 4.464 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134. N/mm^2
S^* = 4459. mm^3

ESΣ8.043

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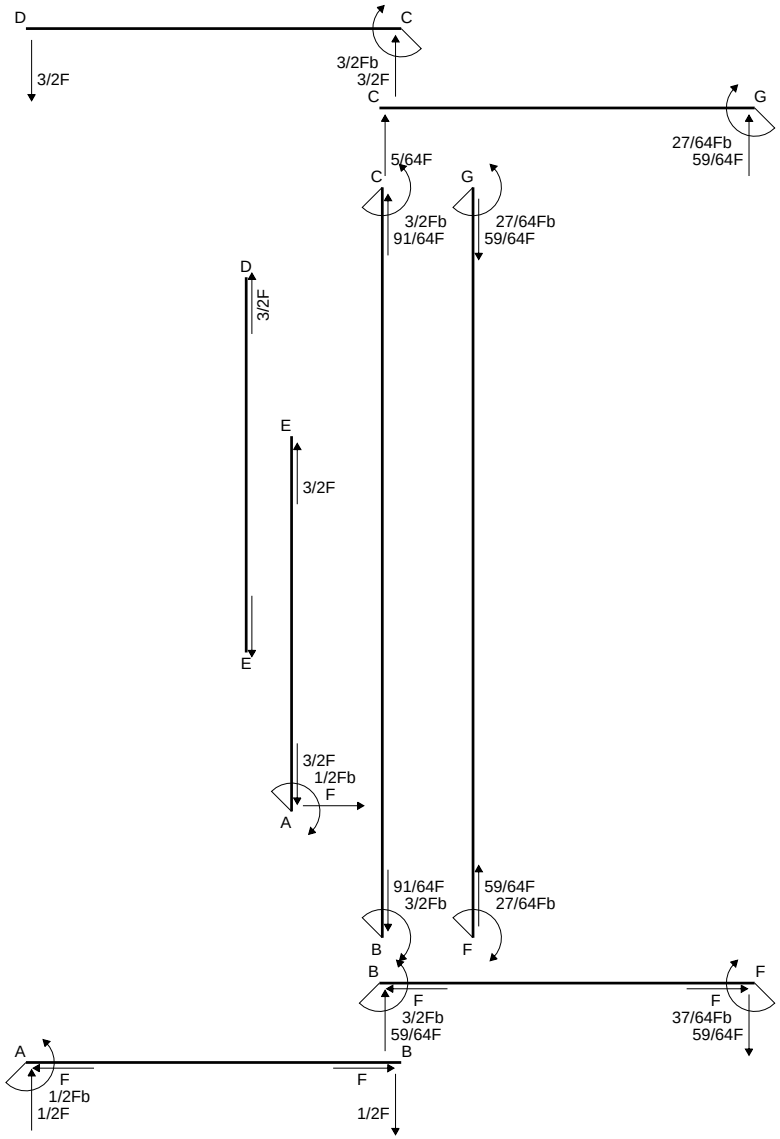
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3/2Fb
3/2F
C
G
27/64Fb
59/64F
C
5/64F
G
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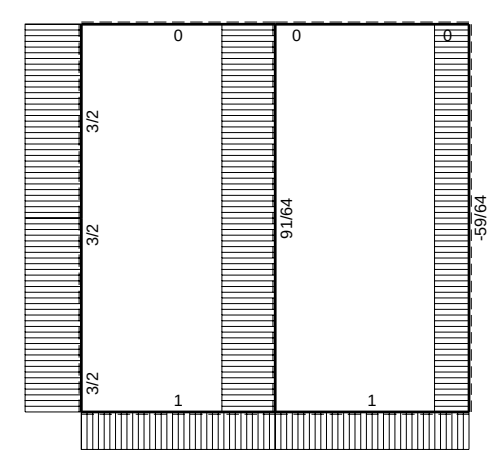
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F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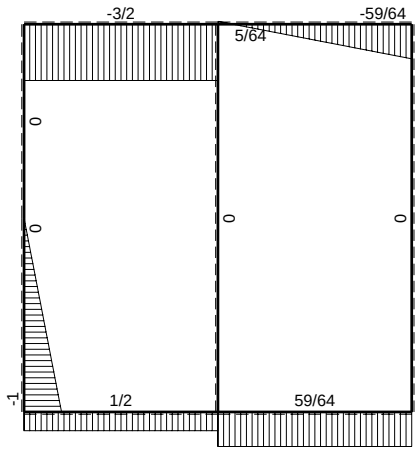
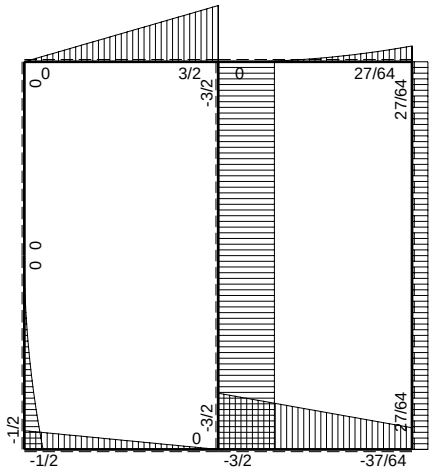
3/2
3/2
3/2
91/64
-59/64
1
1

F


-3/2
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F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3/2
-3/2
0
27/64
27/64
0
0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
BA b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
	0	-3/2Fx	0	0	0	0		
CD b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

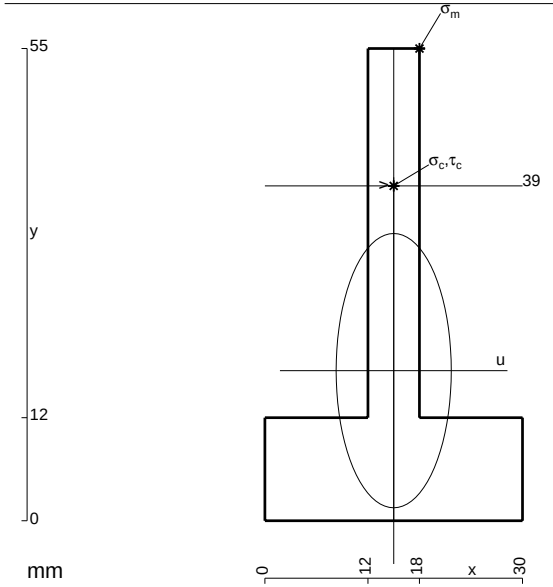
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L_{GC}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{CG}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 618. mm^2
J_u = 157731. mm^4
J_v = 27774. mm^4
y_g = 17.48 mm
T_y = -1650. N
M_x = 1006500. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.52 mm
$\sigma_m = -Mv/J_u$ = -239.4 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 21.52 mm
$\sigma_c = -Mv/J_u$ = -137.3 N/mm^2
τ_c = 4.941 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.6 N/mm^2
S^* = 2834. mm^3

ESΣ8.044

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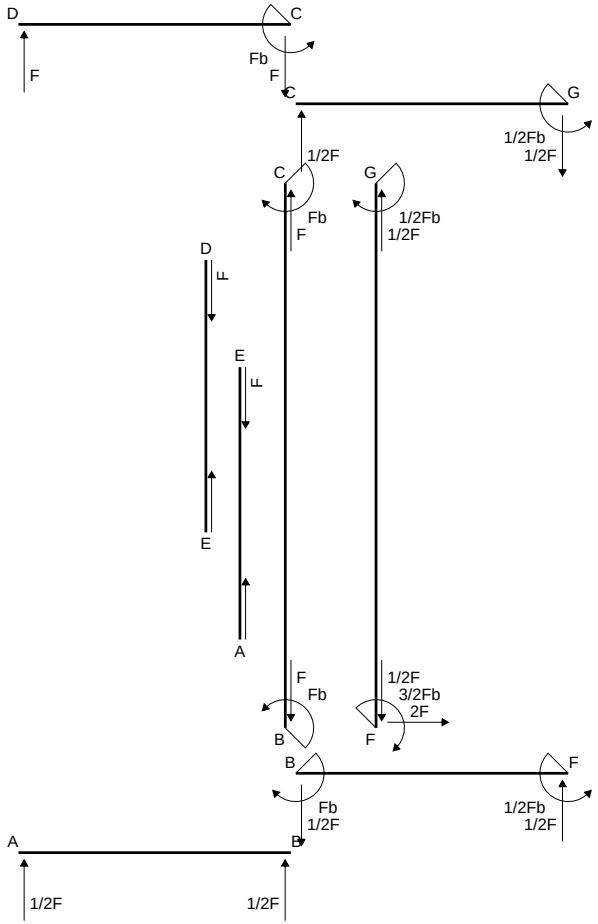
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A
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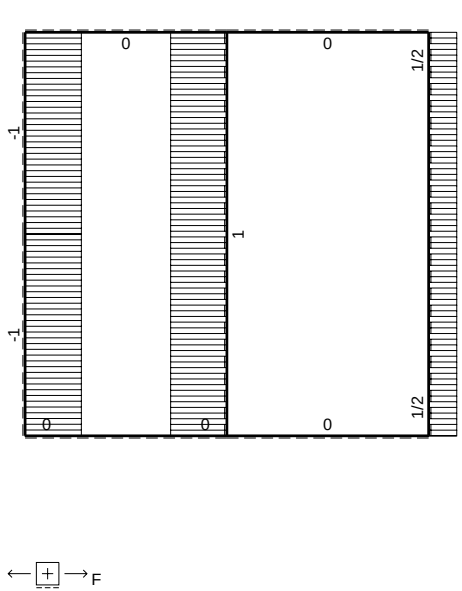
1/2F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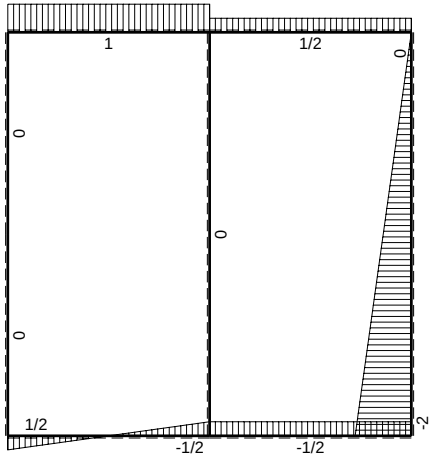
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-2

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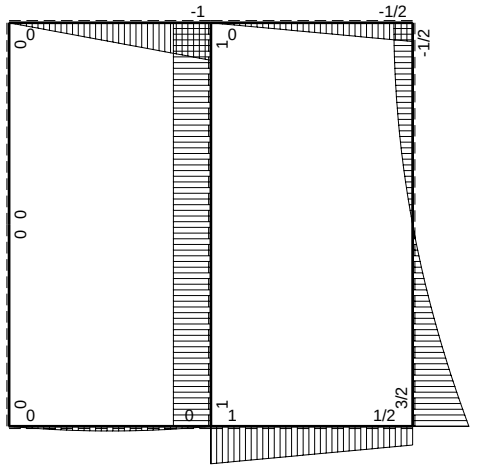
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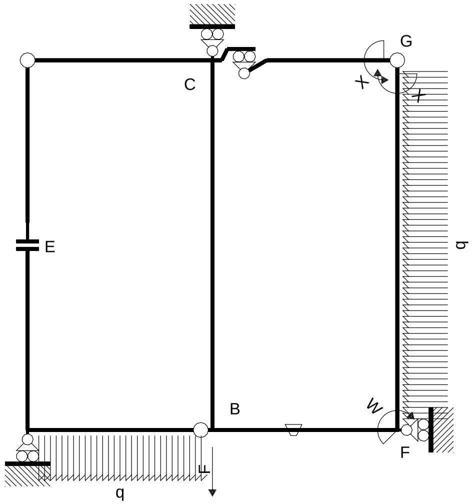
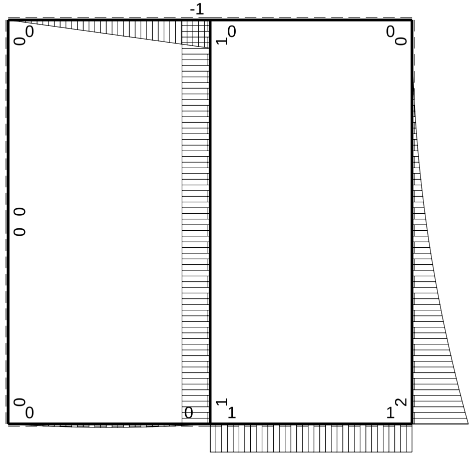
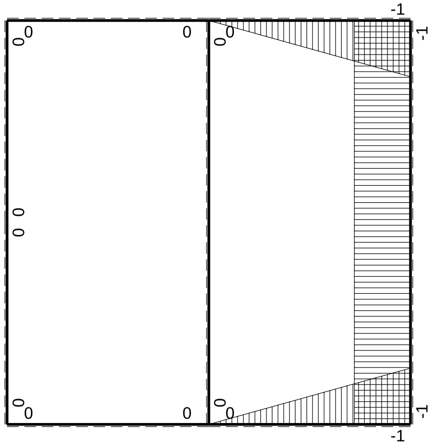
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

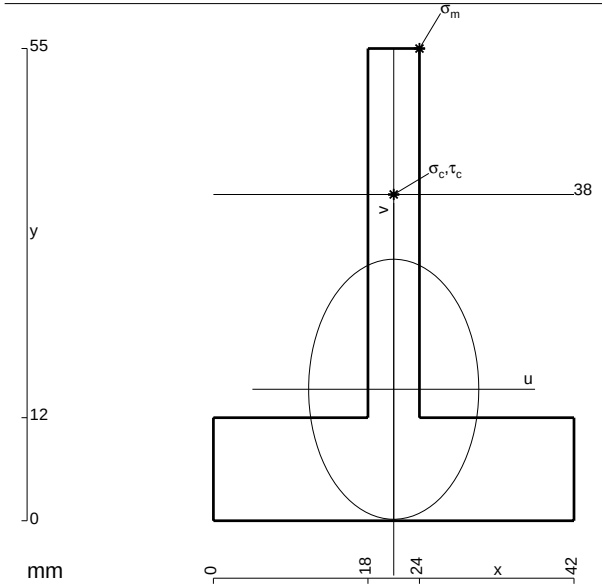
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 762. mm^2
J_u = 174852. mm^4
J_v = 74862. mm^4
y_g = 15.31 mm
T_y = 1330. N
M_x = -877800. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 39.69 mm
$\sigma_m = -Mv/J_u$ = 199.2 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 22.69 mm
$\sigma_c = -Mv/J_u$ = 113.9 N/mm^2
τ_c = 4.033 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.1 N/mm^2
S^* = 3181. mm^3

ESΣ8.045

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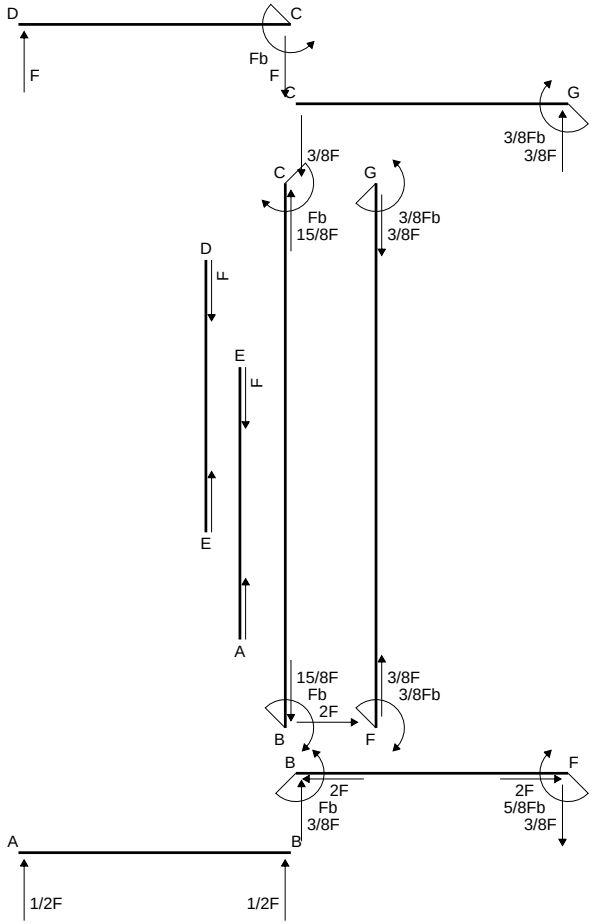
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3/8F
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B
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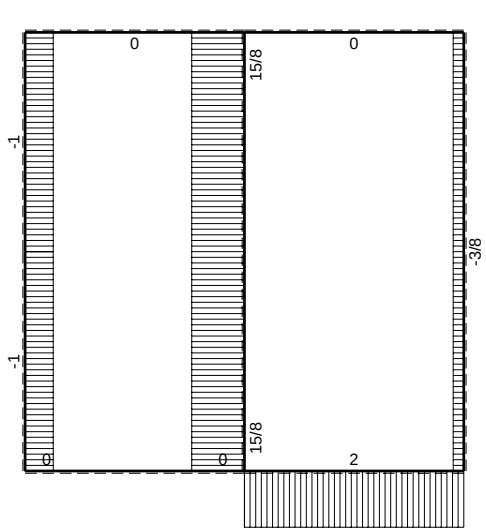
1/2F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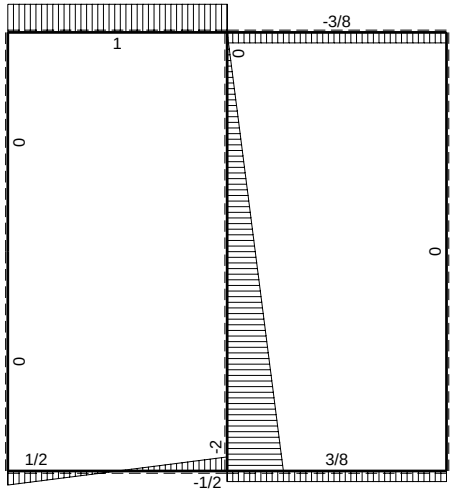
2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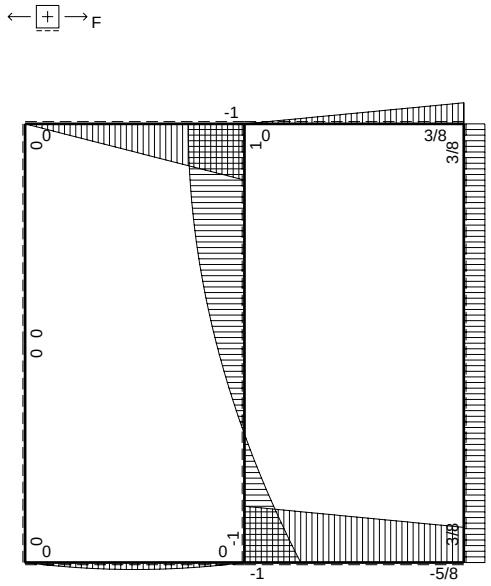
3/8
-1/2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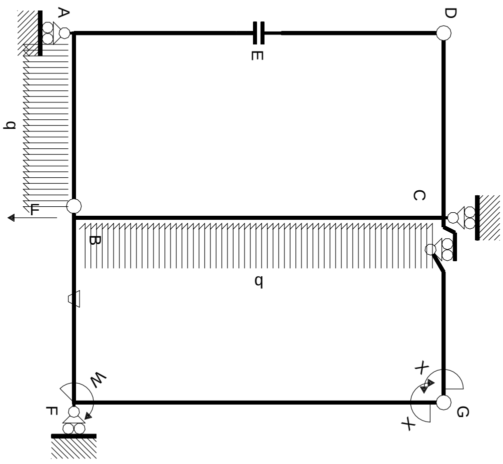
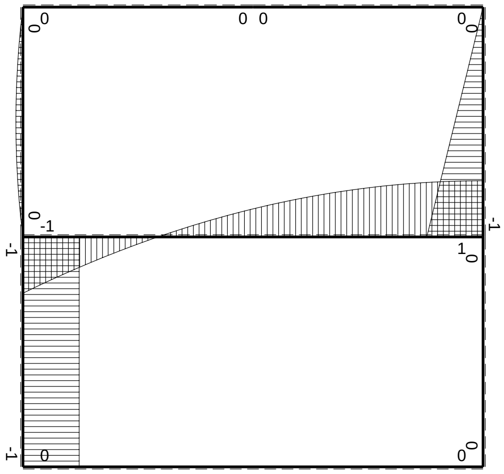
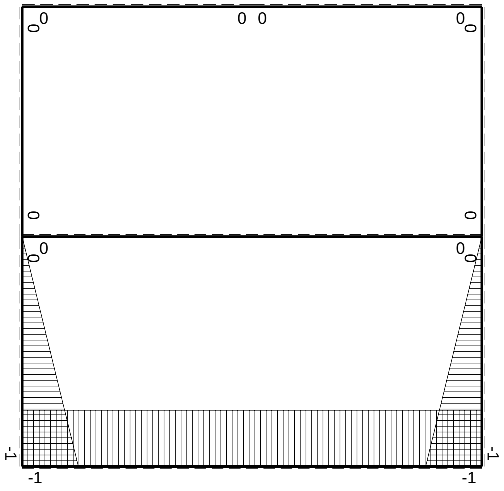
-5/8



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

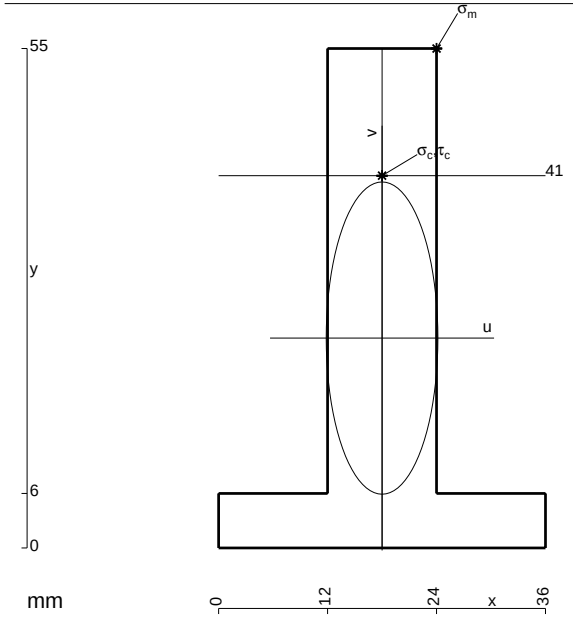
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 804. mm^2
J_u = 237762. mm^4
J_v = 30384. mm^4
y_g = 23.11 mm
N = 4088. N
T_y = -4360. N
M_x = -1526000. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 31.89 mm
$\sigma_m = N/A - Mv/J_u$ = 209.7 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 17.89 mm
$\sigma_c = N/A - Mv/J_u$ = 119.9 N/mm^2
τ_c = 6.389 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.4 N/mm^2
S^* = 4181. mm^3

ESΣ8.046

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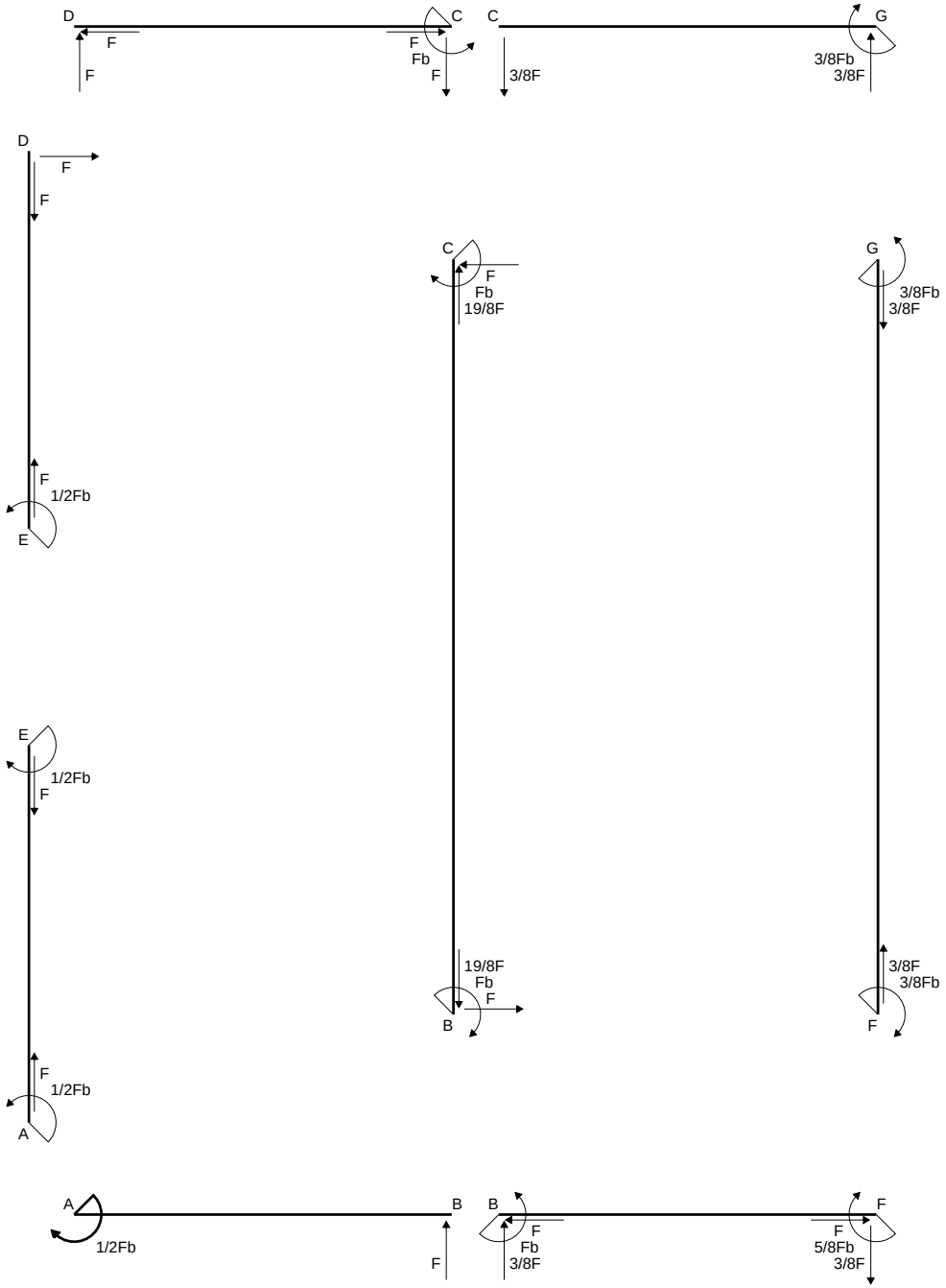
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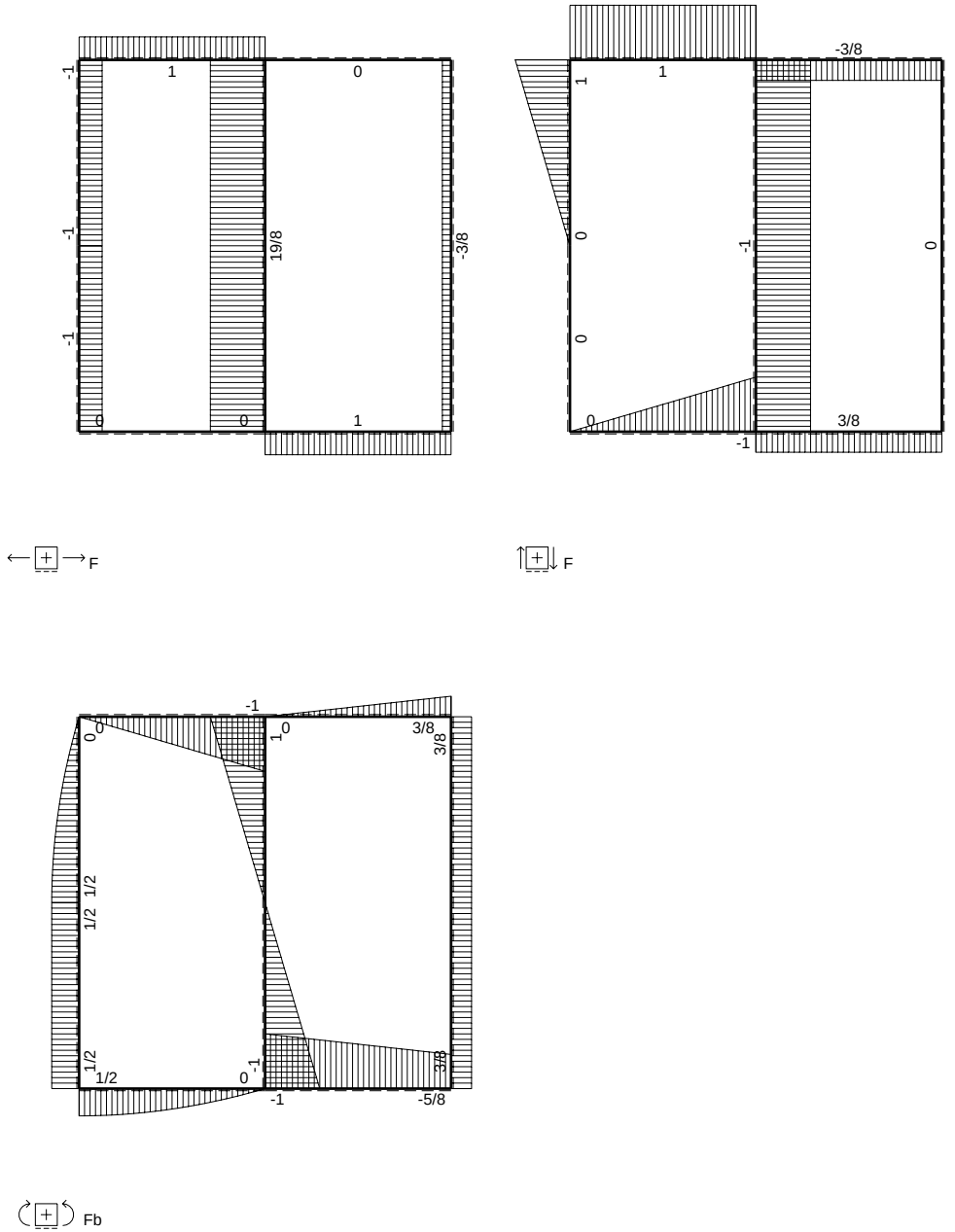
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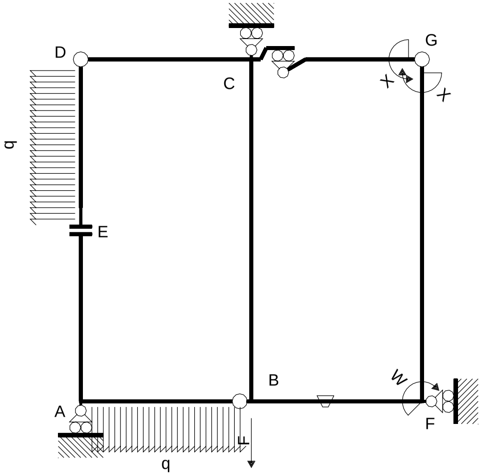
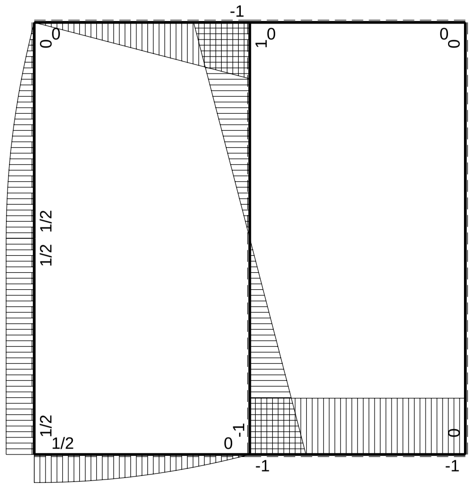
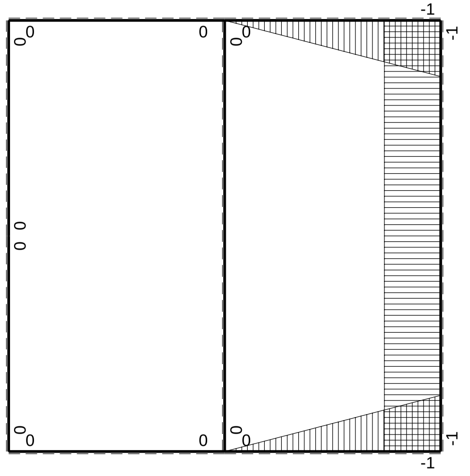
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

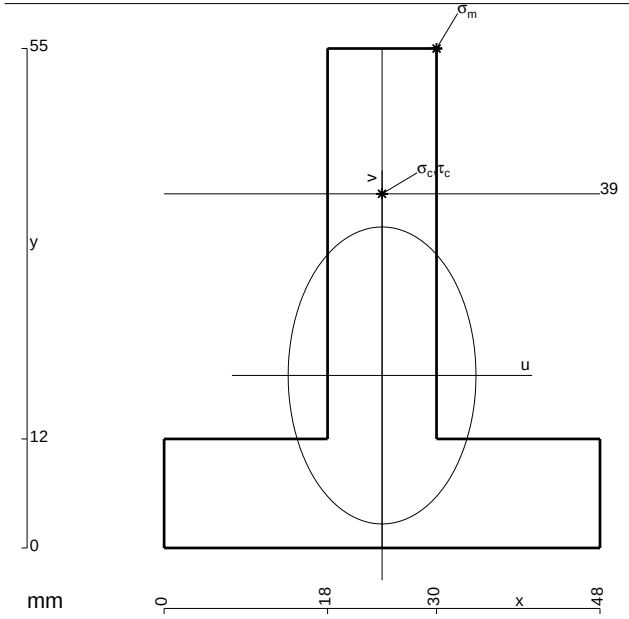
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1092. mm^2
J_u = 292252. mm^4
J_v = 116784. mm^4
y_g = 18.99 mm
N = 5581. N
T_y = -2350. N
M_x = -1739000. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 36.01 mm
$\sigma_m = N/A - Mv/J_u$ = 219.4 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.01 mm
$\sigma_c = N/A - Mv/J_u$ = 124.2 N/mm^2
τ_c = 3.603 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124.3 N/mm^2
S^* = 5377. mm^3

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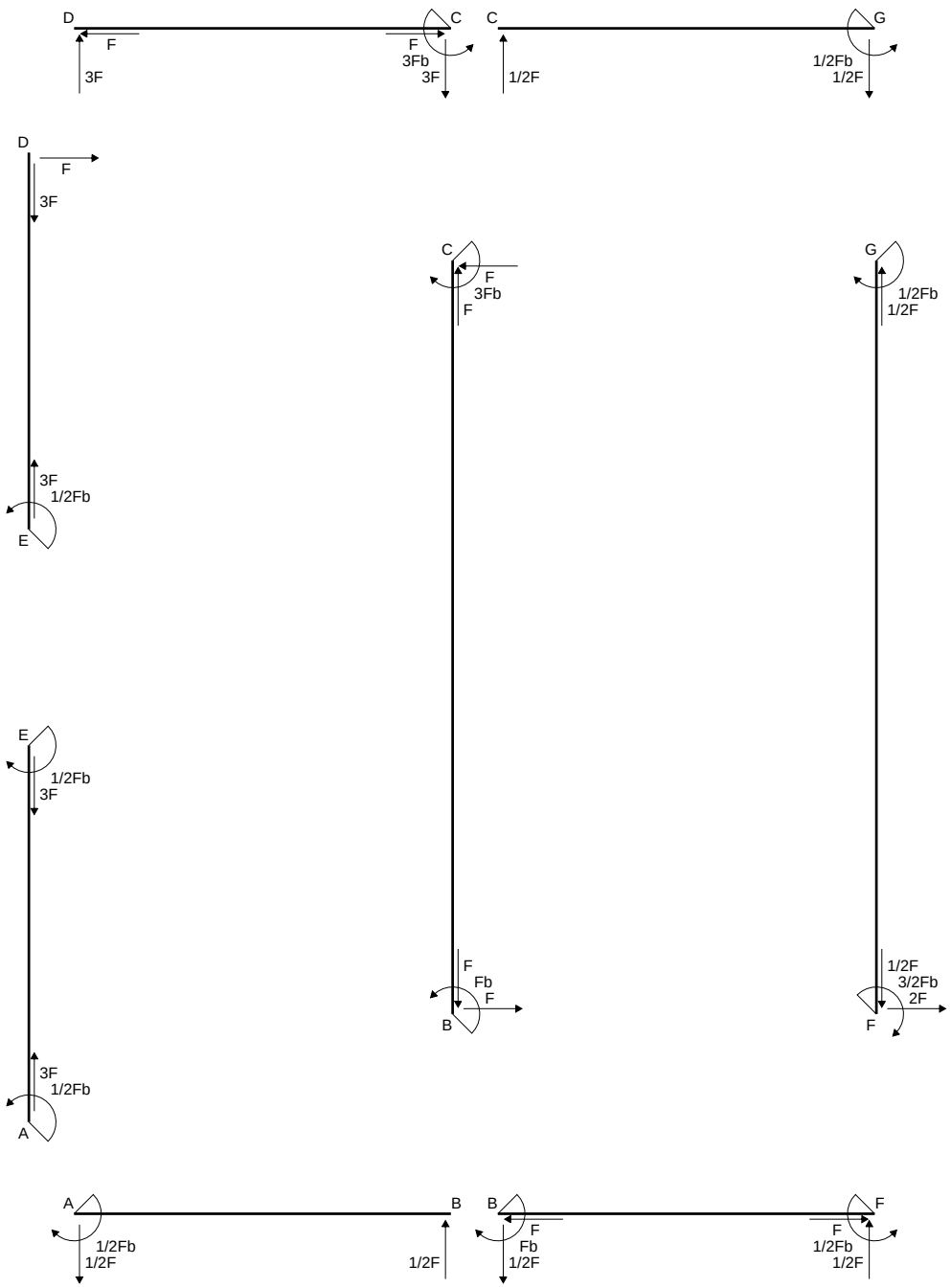
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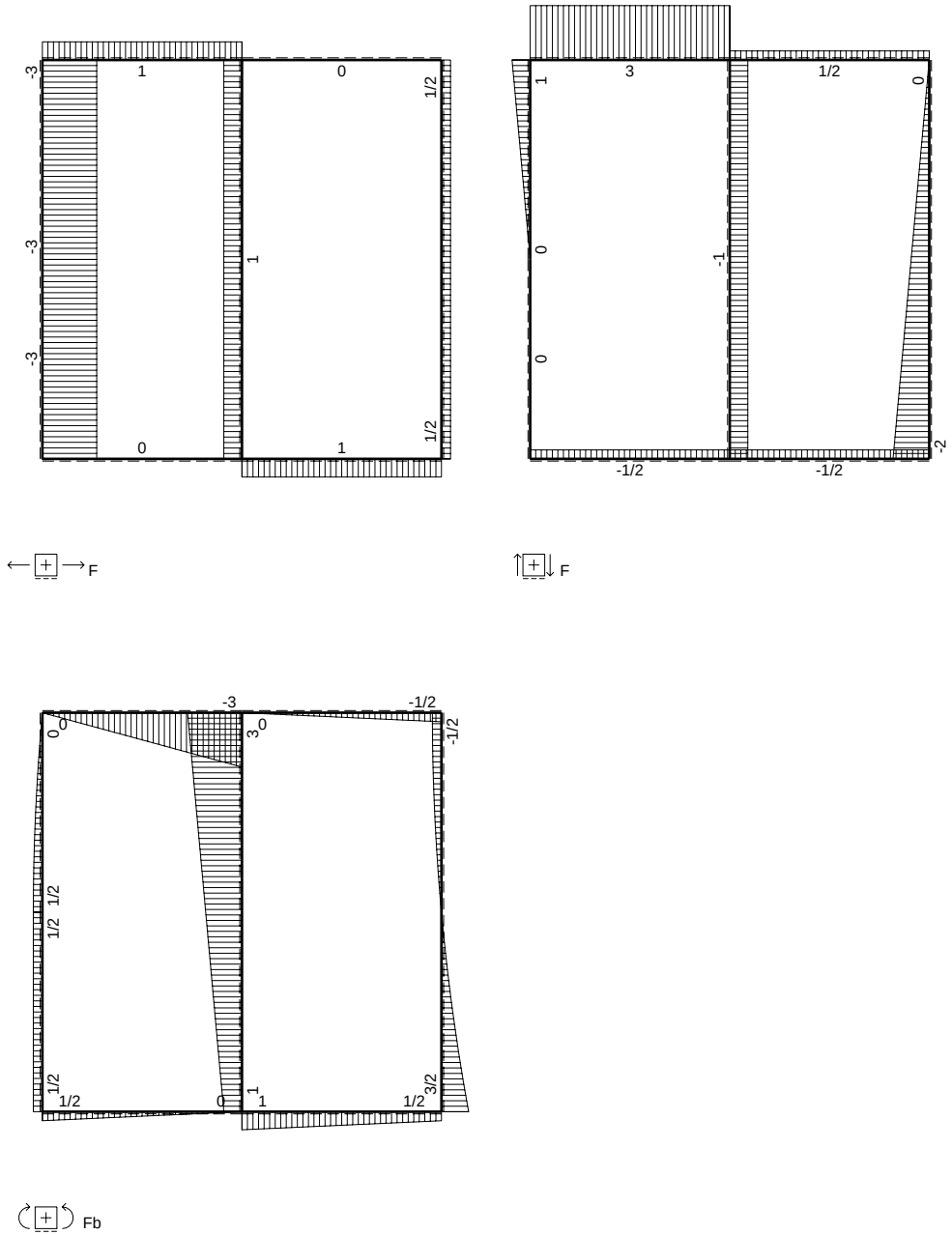
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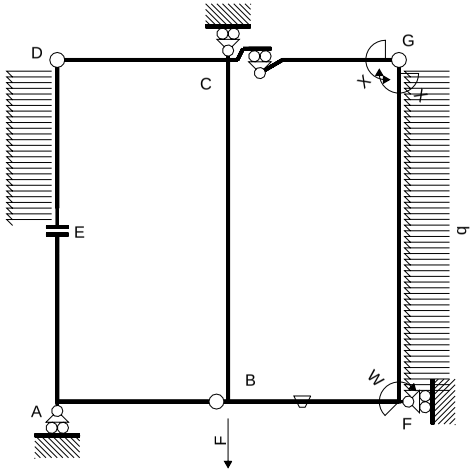
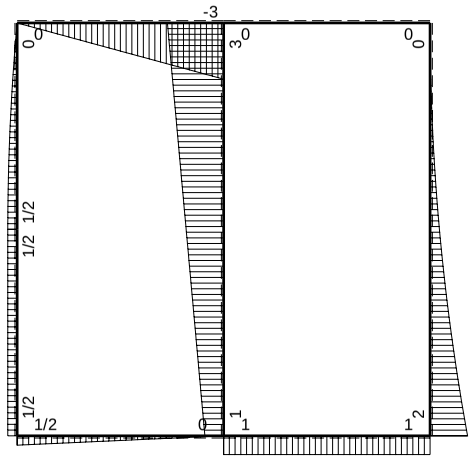
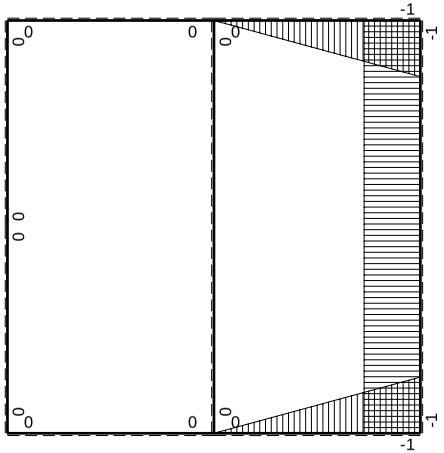




M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.048

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

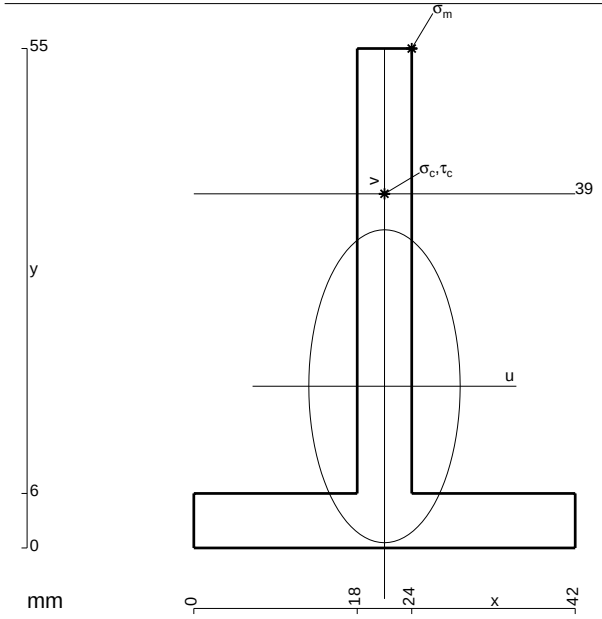
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 546. mm^2
J_u = 162198. mm^4
J_v = 37926. mm^4
y_g = 17.81 mm
N = 420. N
T_y = 1260. N
M_x = -995400. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.19 mm
$\sigma_m = N/A - Mv/J_u$ = 229. N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 21.19 mm
$\sigma_c = N/A - Mv/J_u$ = 130.8 N/mm^2
τ_c = 3.628 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131. N/mm^2
S^* = 2802. mm^3

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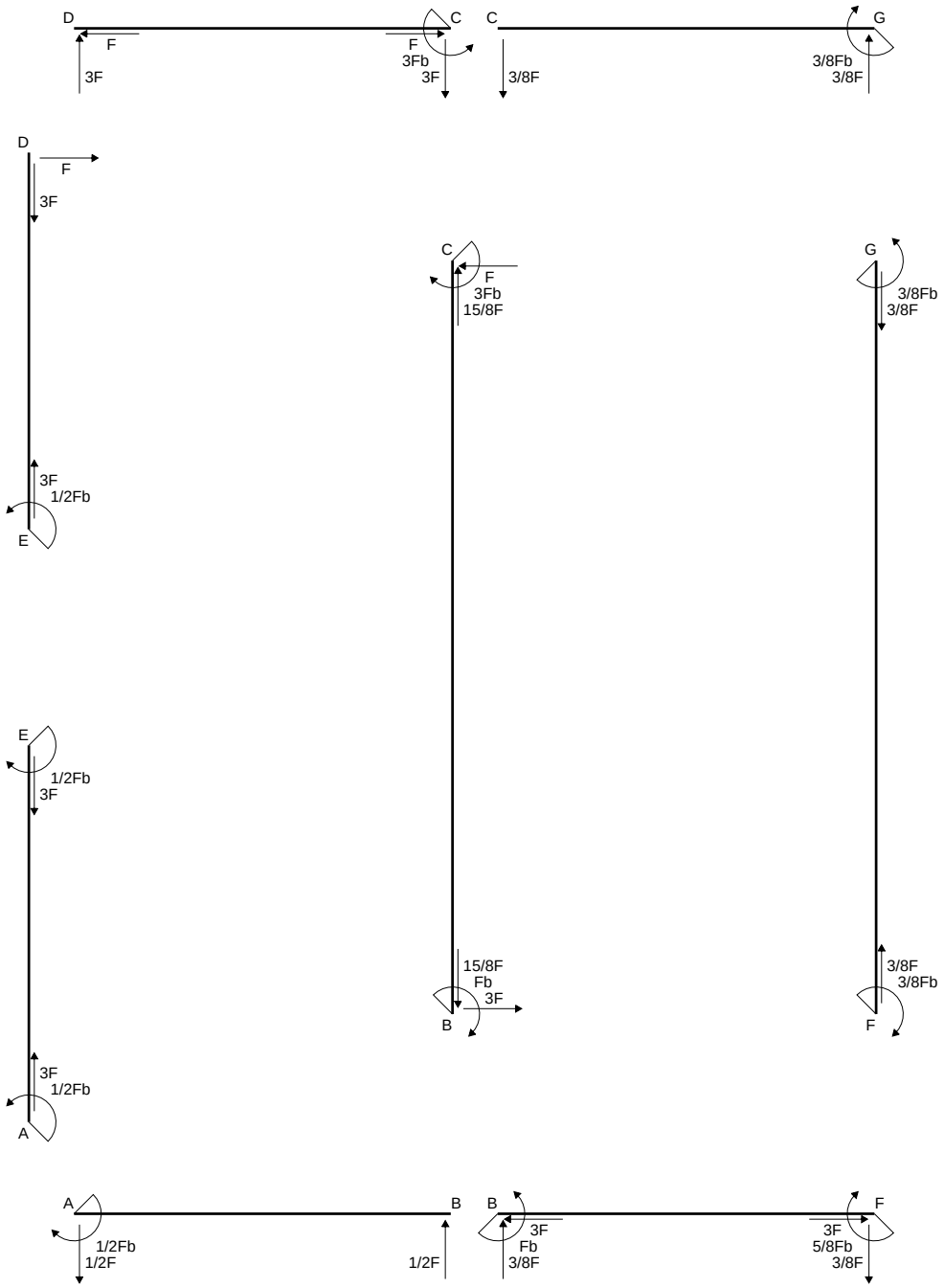
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3F
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A
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1/2F
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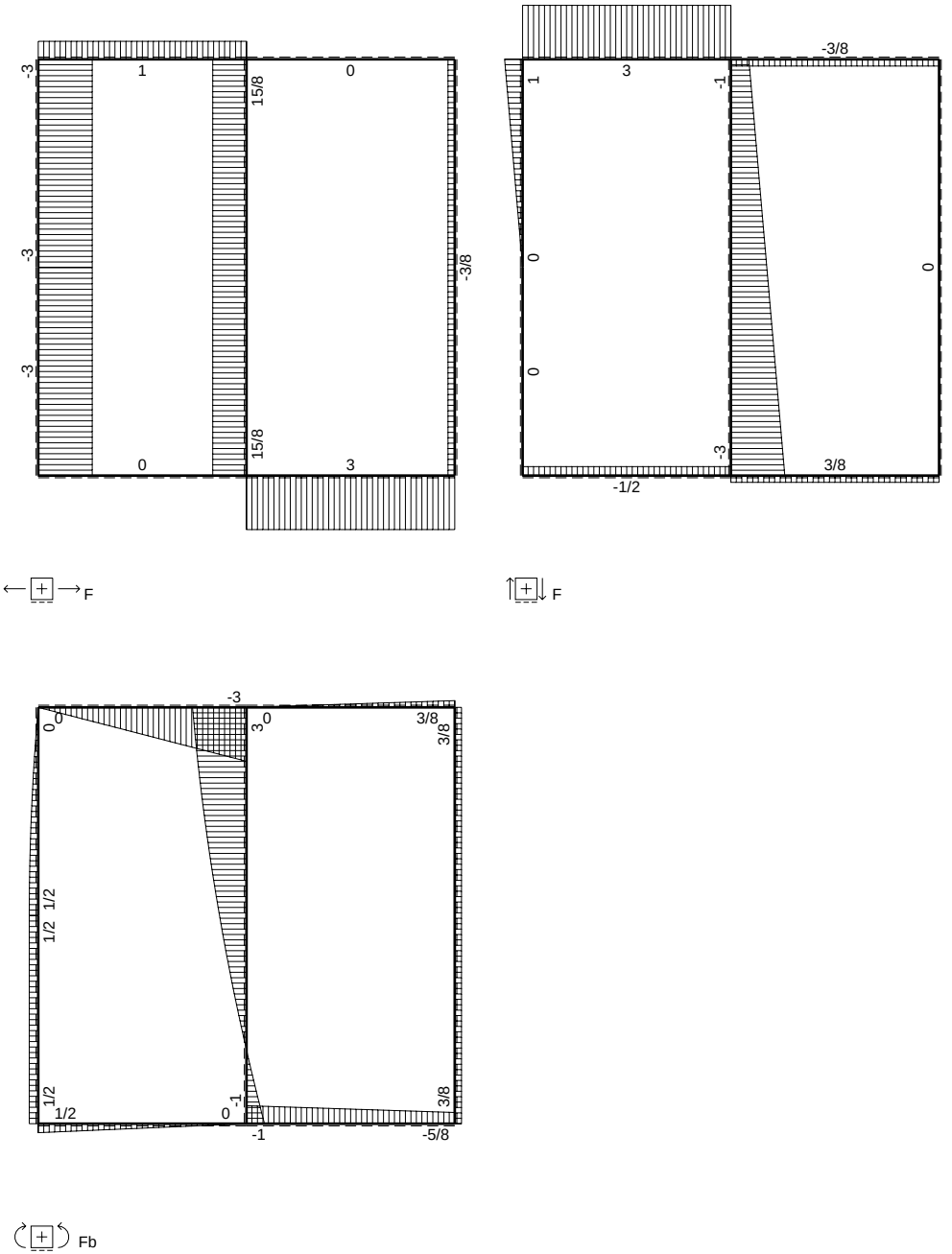
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1/2
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-1
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-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
	totali					iperstatica $X=W_{GC}$	Fb^2/EJ	8/3Xb/EJ
							-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

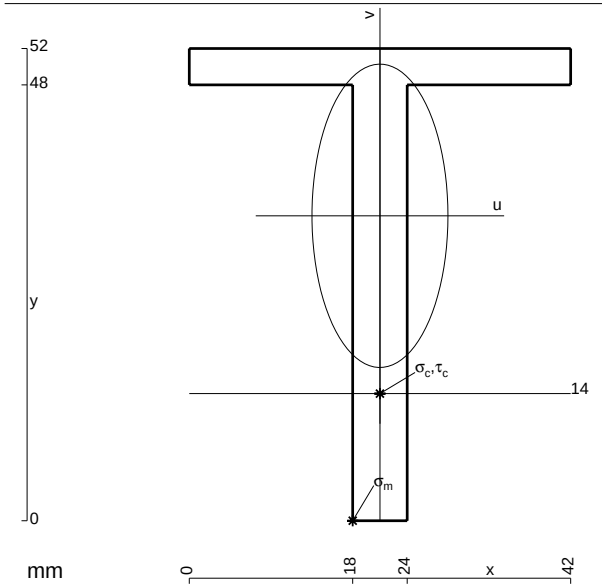
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 456. mm^2
J_u = 127247. mm^4
J_v = 25560. mm^4
y_g = 33.58 mm
N = 390. N
T_y = 1170. N
M_x = -900900. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -33.58 mm
$\sigma_m = N/A - Mv/J_u$ = -236.9 N/mm^2
x_c = 21. mm
y_c = 14. mm
v_c = -19.58 mm
$\sigma_c = N/A - Mv/J_u$ = -137.8 N/mm^2
τ_c = 3.421 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.9 N/mm^2
S^* = 2233. mm^3

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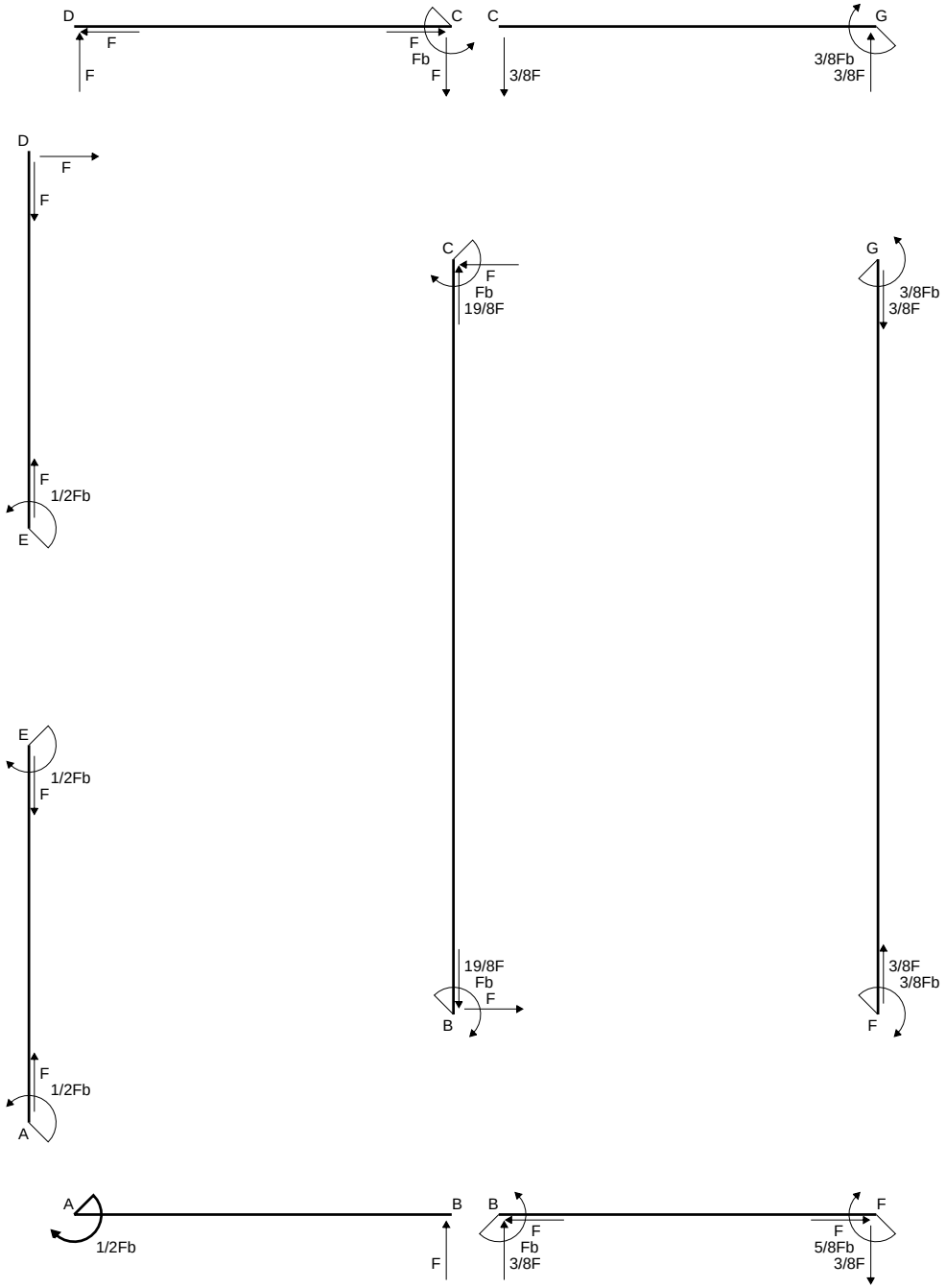
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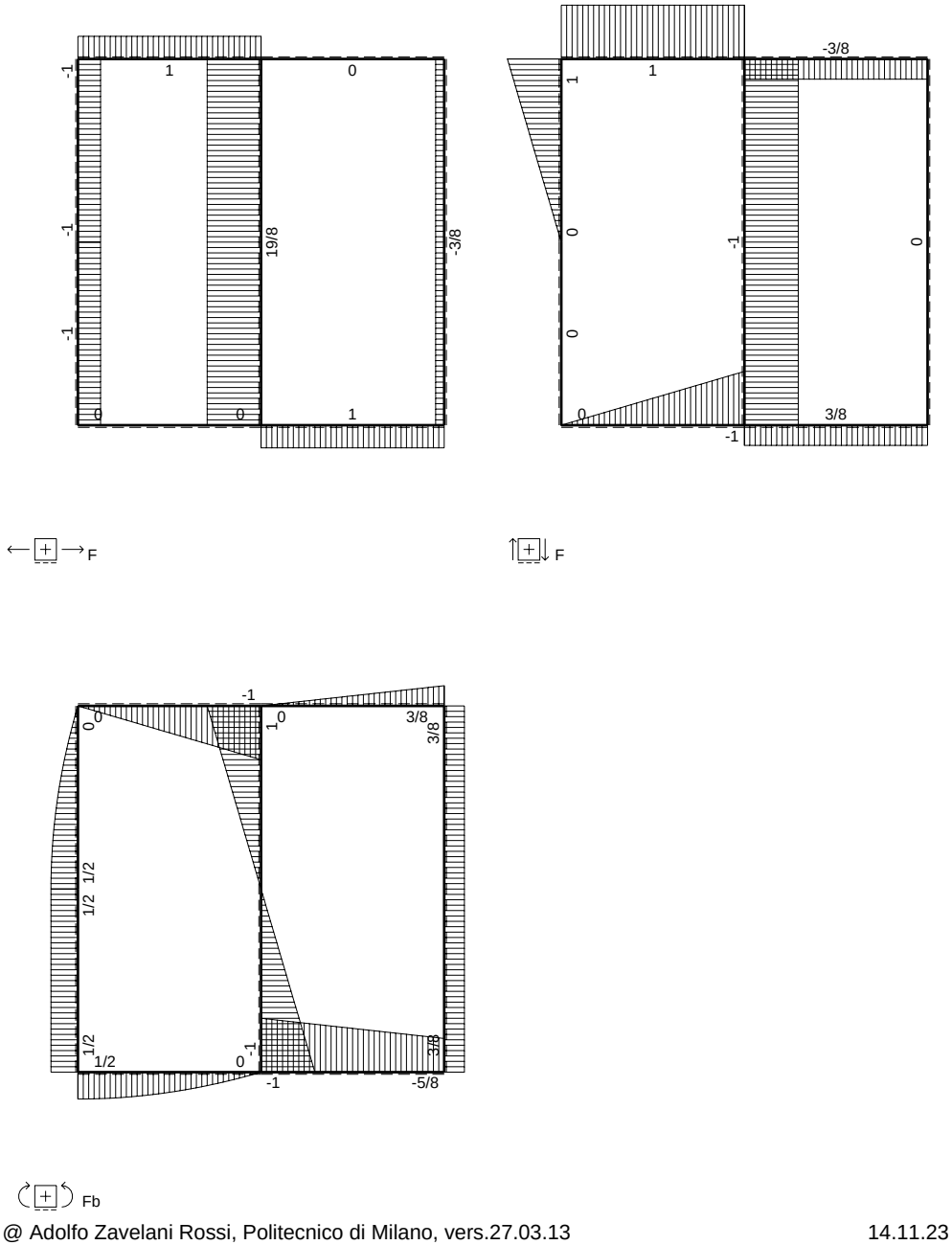
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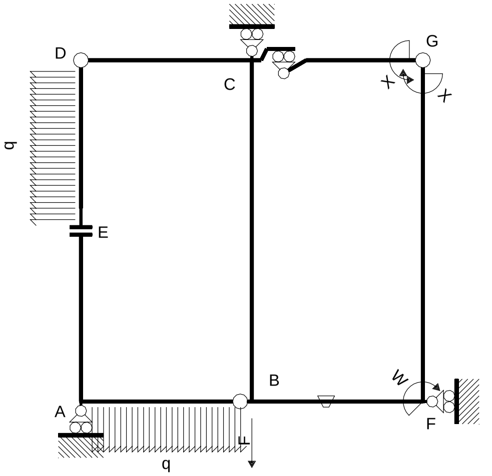
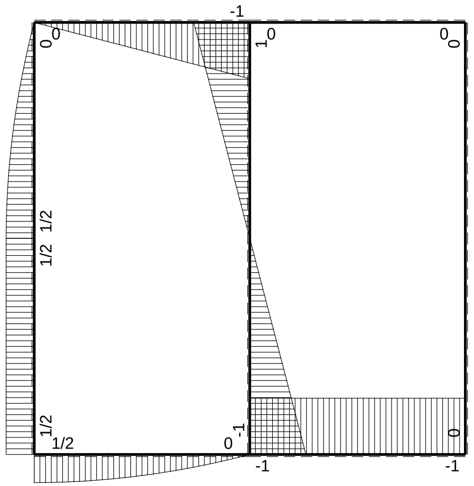
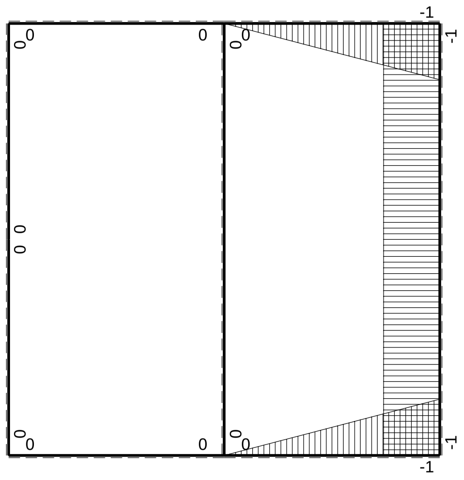
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

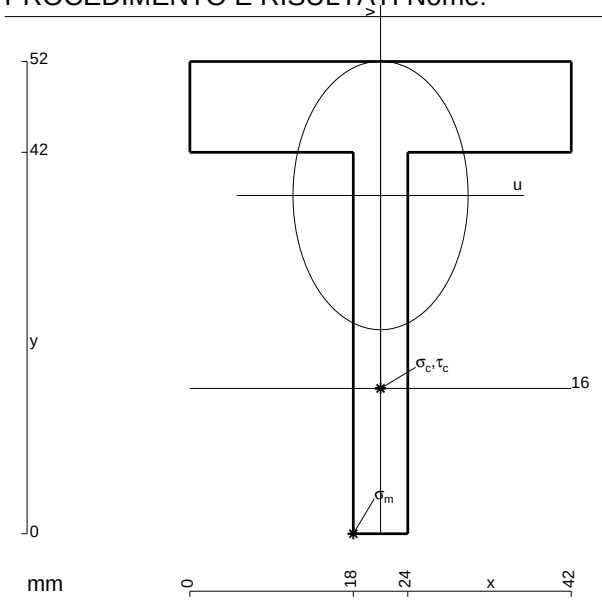
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 672. mm^2
J_u = 147014. mm^4
J_v = 62496. mm^4
y_g = 37.25 mm
N = 4418. N
T_y = -1860. N
M_x = 762600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.25 mm
$\sigma_m = N/A - Mv/J_u$ = 199.8 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.25 mm
$\sigma_c = N/A - Mv/J_u$ = 116.8 N/mm^2
τ_c = 5.921 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.3 N/mm^2
S^* = 2808. mm^3

ESΣ8.050

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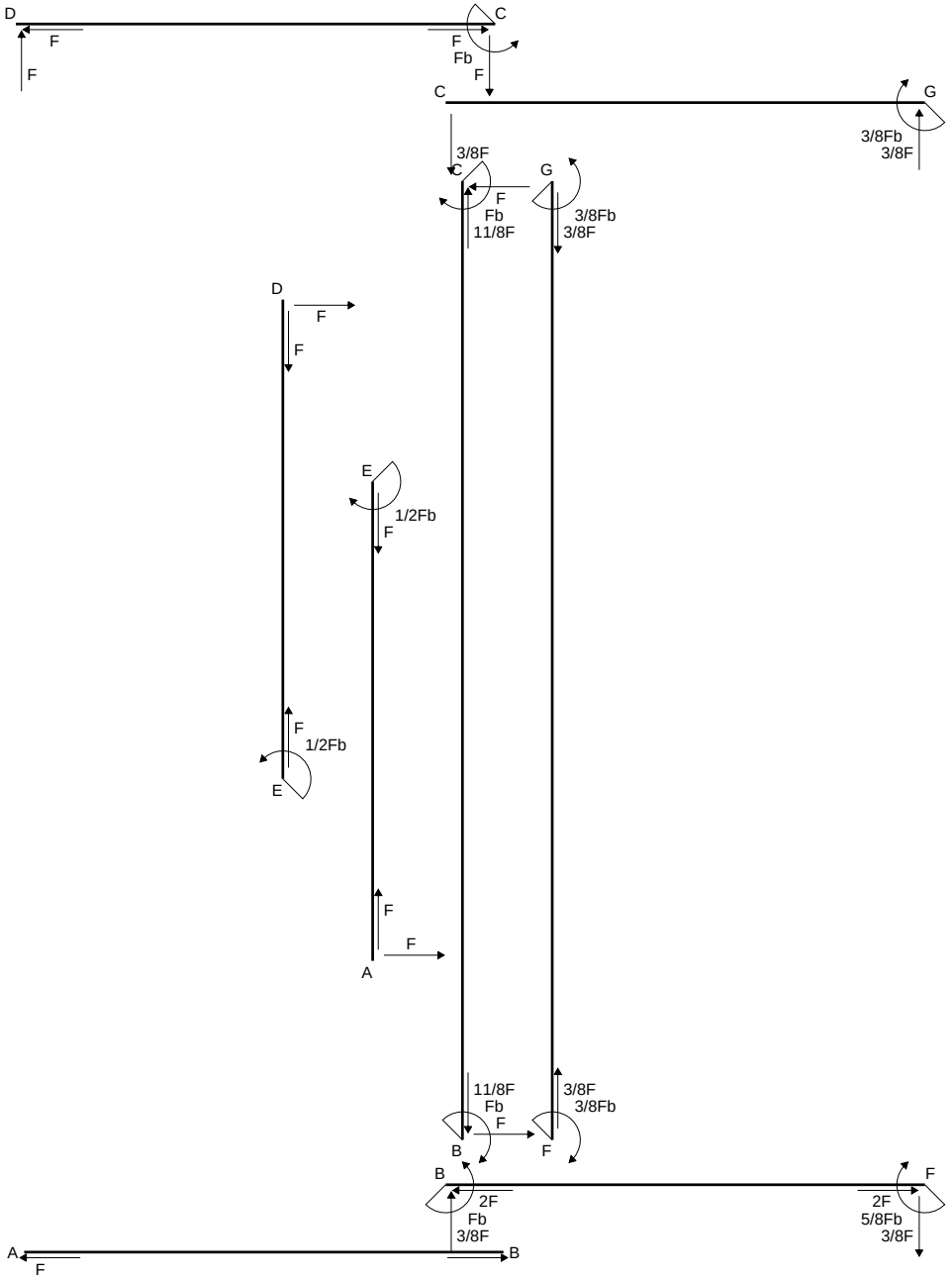
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Fb
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3/8F
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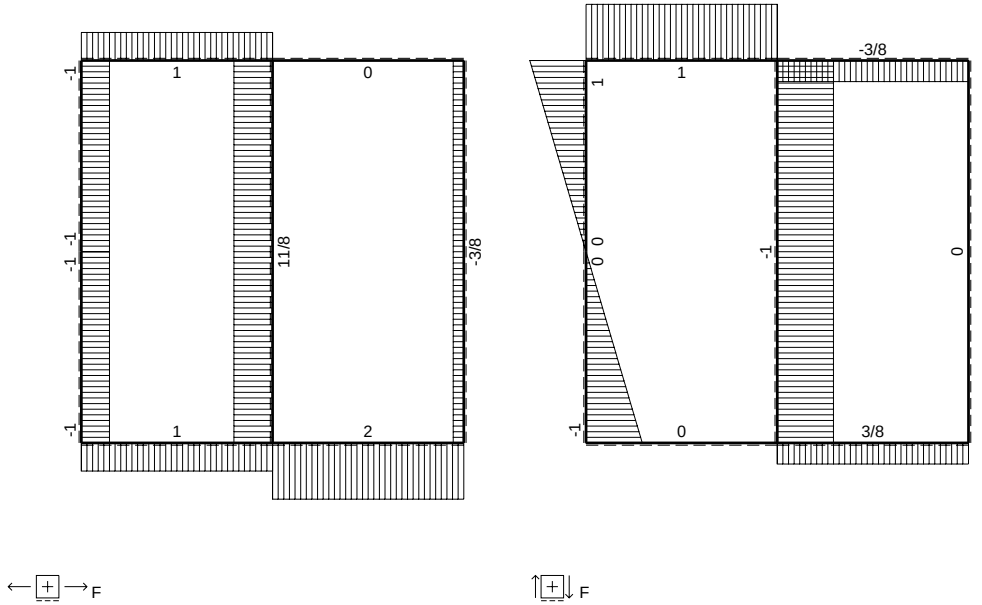
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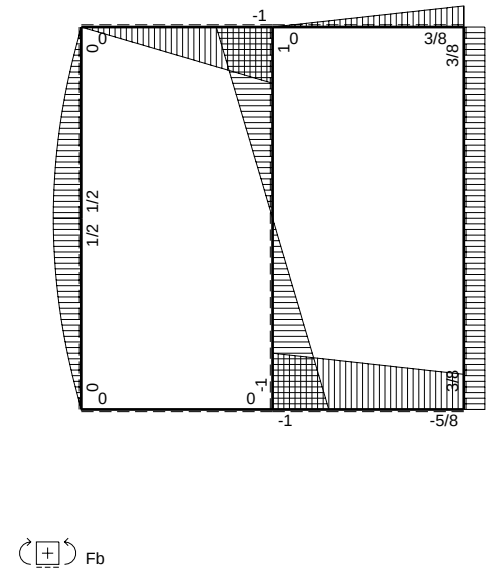
3/8
F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-1
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3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$							-3/8Fb

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

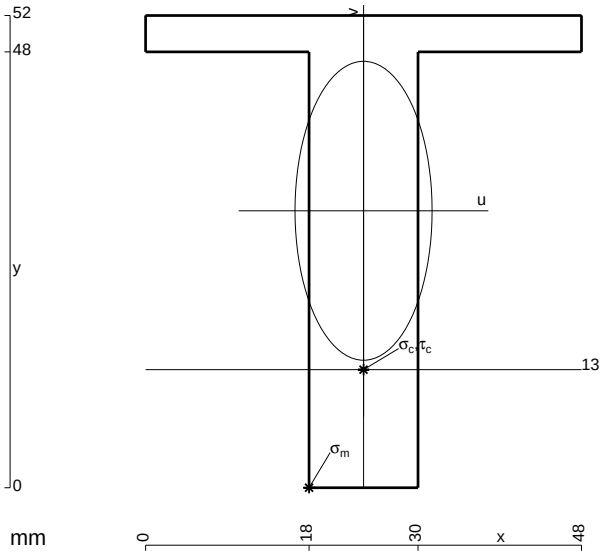
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 768. mm^2
J_u = 208192. mm^4
J_v = 43776. mm^4
y_g = 30.5 mm
N = 4263. N
T_y = -3100. N
M_x = 1395000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -30.5 mm
$\sigma_m = N/A - Mv/J_u$ = 209.9 N/mm^2
x_c = 24. mm
y_c = 13. mm
v_c = -17.5 mm
$\sigma_c = N/A - Mv/J_u$ = 122.8 N/mm^2
τ_c = 4.646 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.1 N/mm^2
S^* = 3744. mm^3

ESΣ8.051

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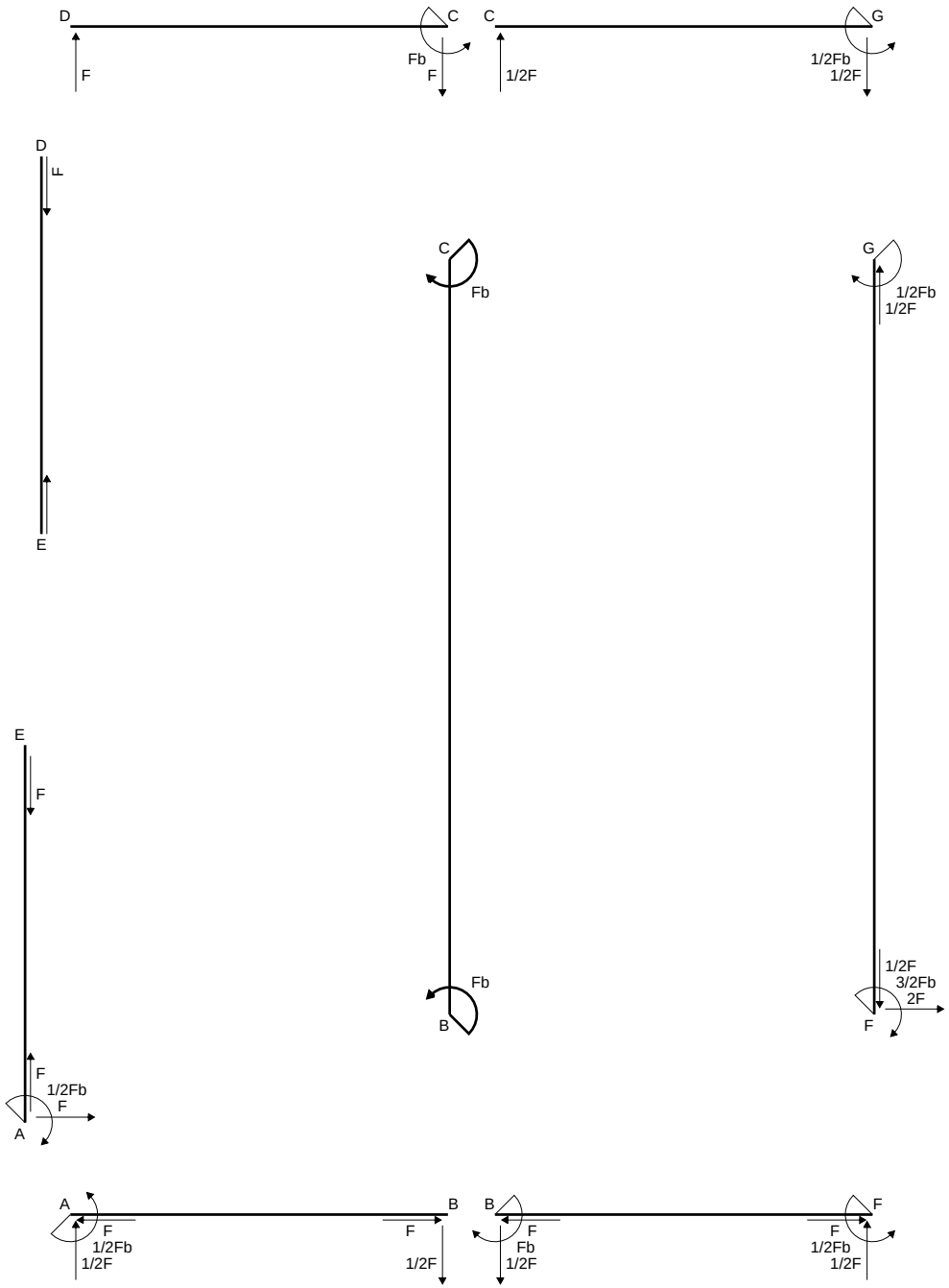
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Fb
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1/2Fb
1/2F
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F
E
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Fb
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1/2F
Fb
B
1/2F
3/2Fb
2F
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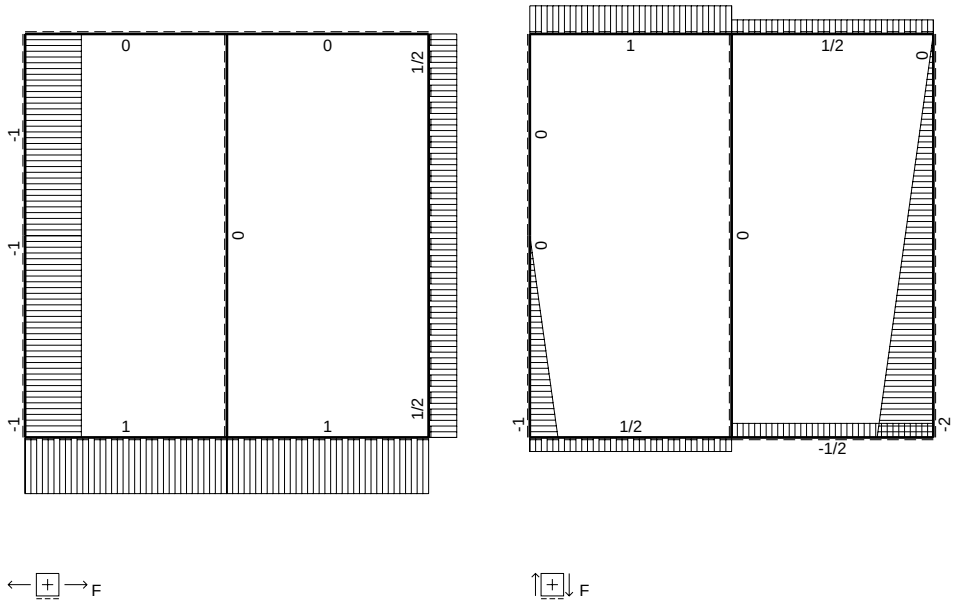
1/2Fb
1/2F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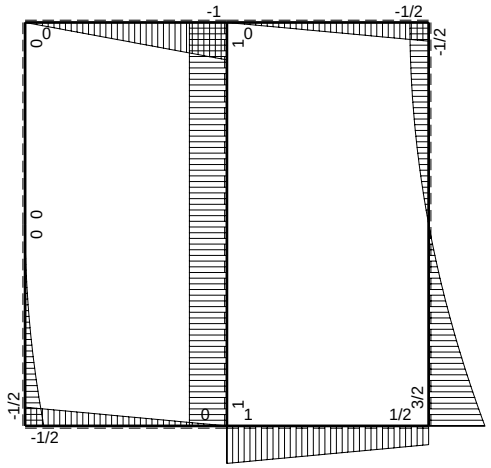
-2
-1/2
F
F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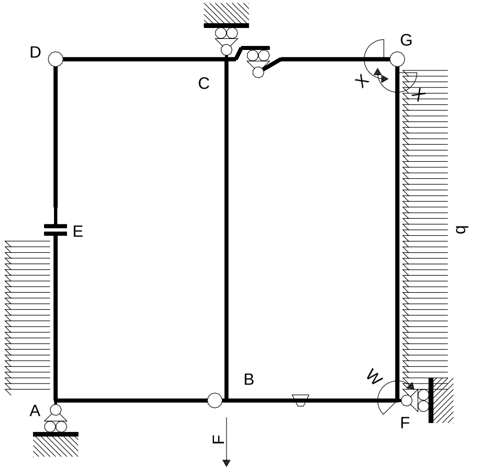
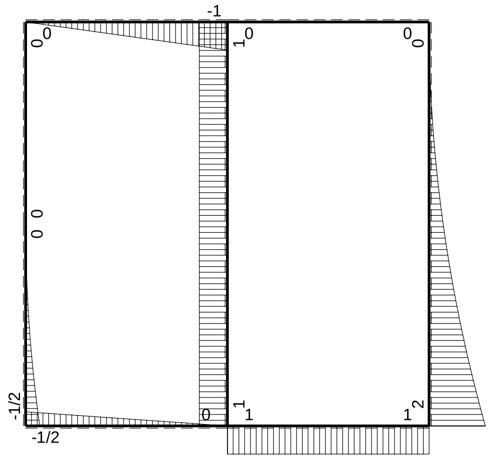
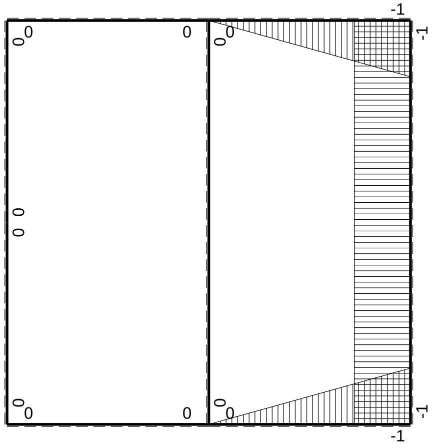
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int xM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$1/2Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

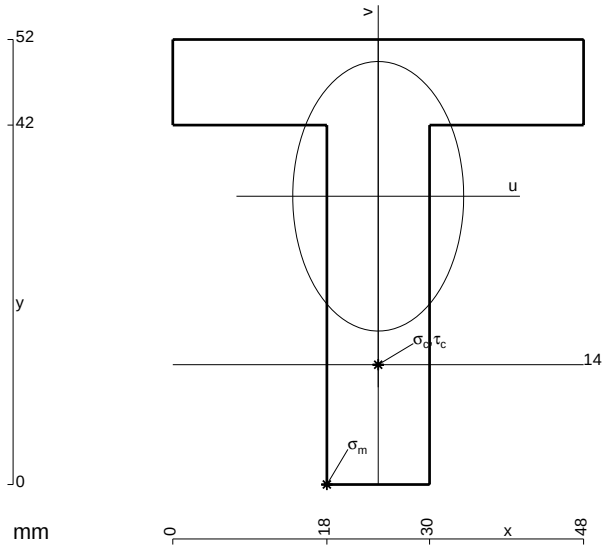
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 984. mm^2
J_u = 244285. mm^4
J_v = 98208. mm^4
y_g = 33.68 mm
T_y = 3250. N
M_x = -1592500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.68 mm
$\sigma_m = -Mv/J_u$ = -219.6 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.68 mm
$\sigma_c = -Mv/J_u$ = -128.3 N/mm^2
τ_c = 4.97 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.6 N/mm^2
S^* = 4483. mm^3

ESΣ8.052

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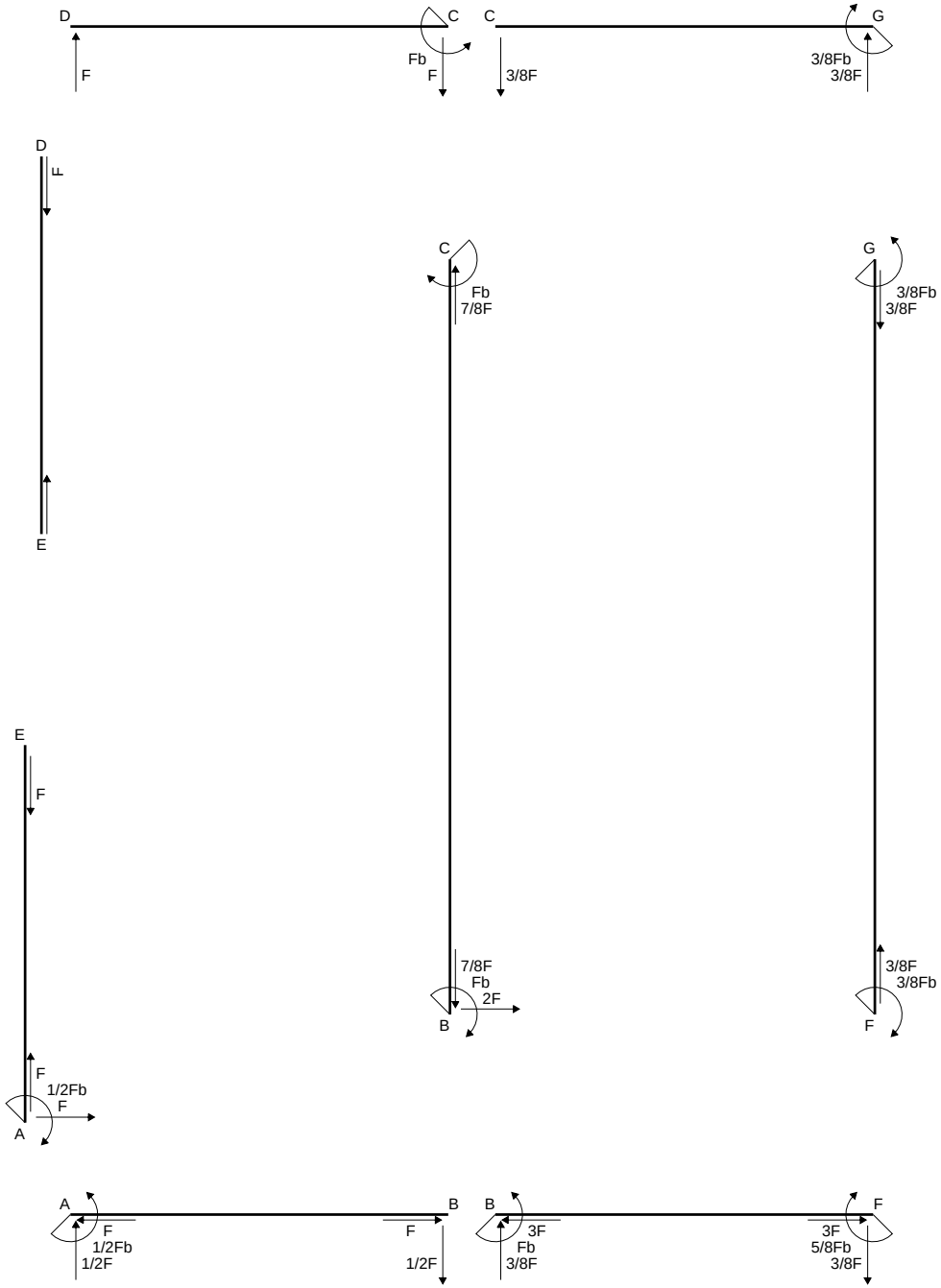
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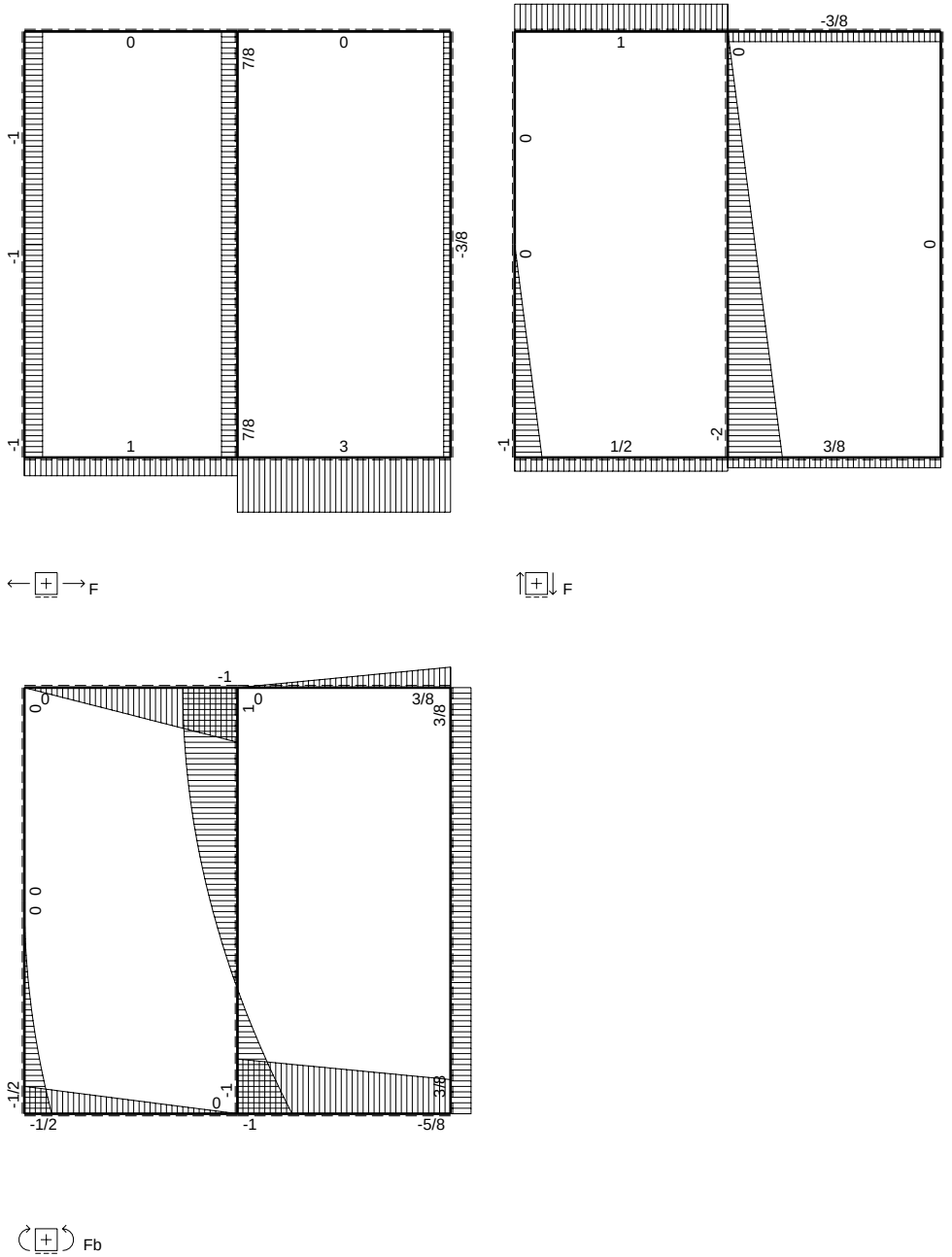
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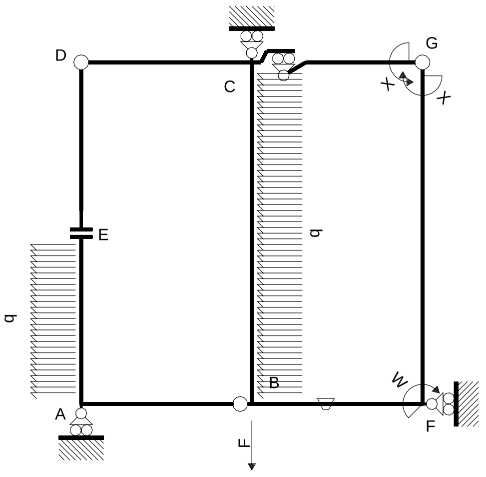
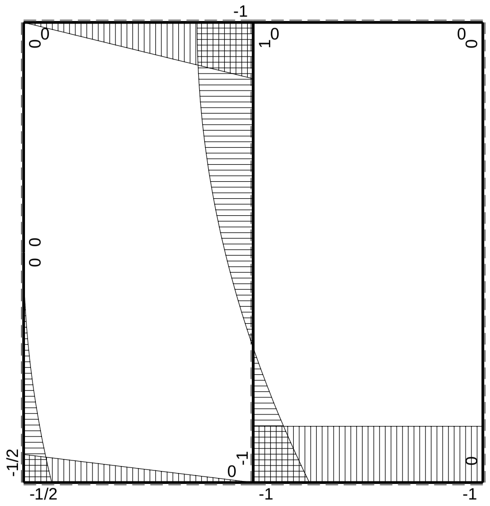
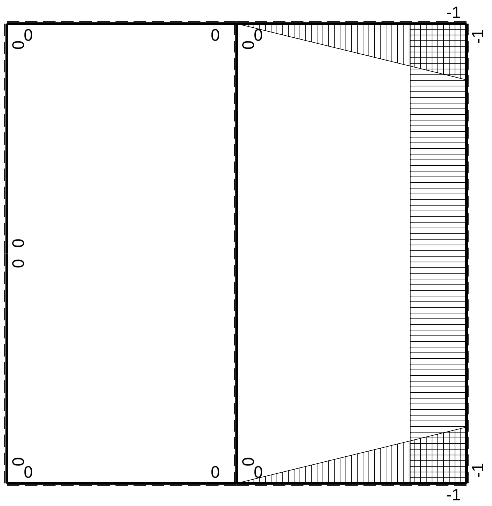


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

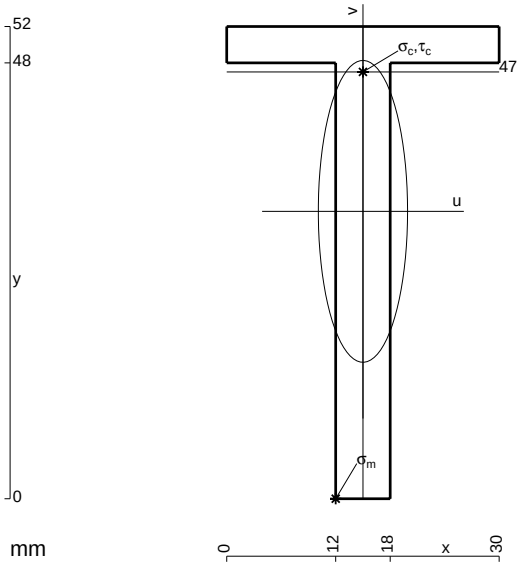
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 408. mm^2
J_u = 112717. mm^4
J_v = 9864. mm^4
y_g = 31.65 mm
N = 1365. N
T_y = -3120. N
M_x = -826800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -31.65 mm
$\sigma_m = N/A - Mv/J_u$ = -228.8 N/mm^2
x_c = 15. mm
y_c = 47. mm
v_c = 15.35 mm
$\sigma_c = N/A - Mv/J_u$ = 116. N/mm^2
τ_c = 10.6 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.4 N/mm^2
S^* = 2297. mm^3

ESΣ8.053

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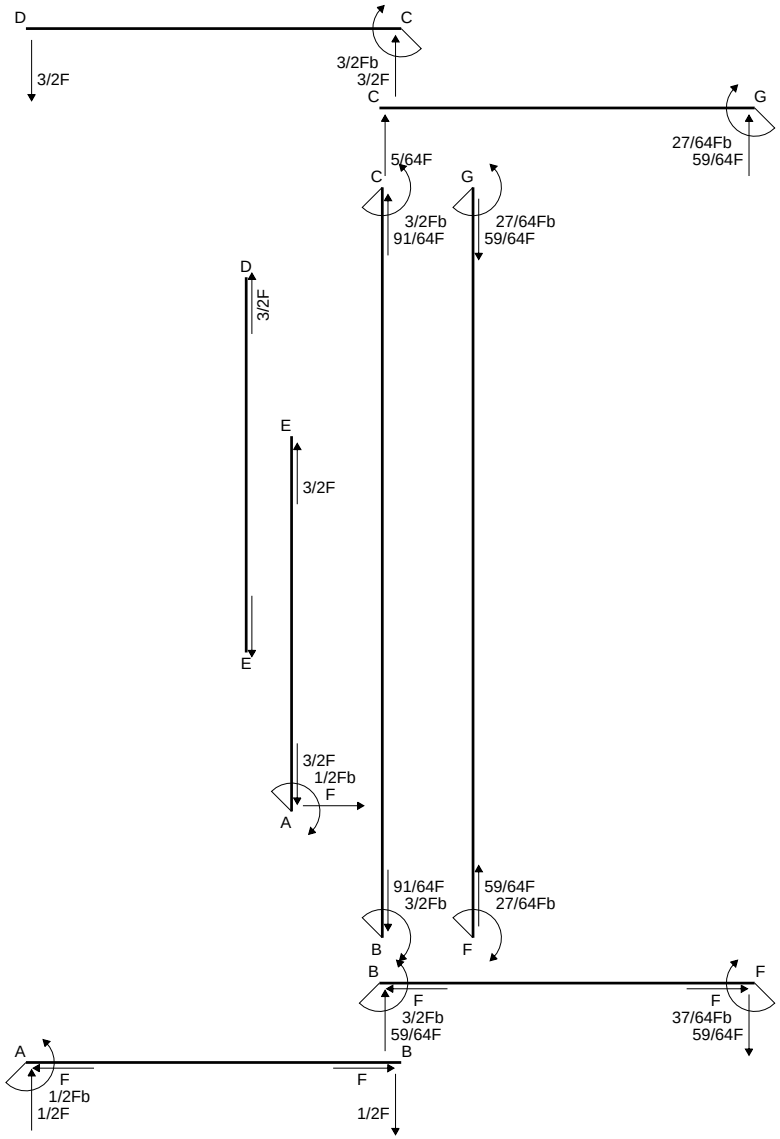
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G
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59/64F
C
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27/64Fb
59/64F
D
3/2F
E
3/2F
3/2F
1/2Fb
F
A
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59/64F
27/64Fb
B
F
B
F
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1/2F
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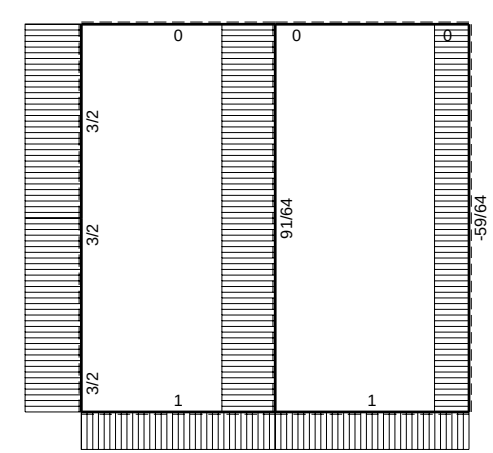
1/2F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1

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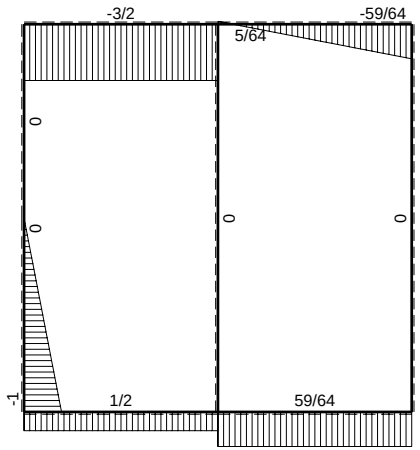
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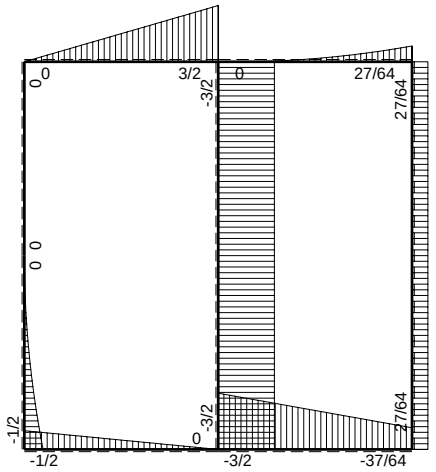
1/2
59/64

F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-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

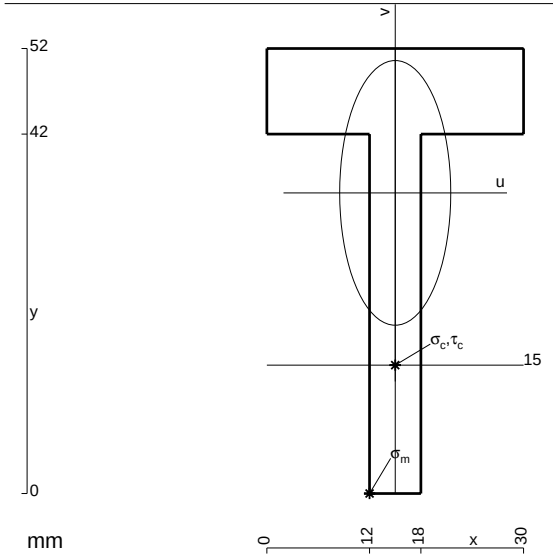
$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 552. mm^2
J_u = 132127. mm^4
J_v = 23256. mm^4
y_g = 35.13 mm
T_y = -1575. N
M_x = 897750. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.13 mm
$\sigma_m = -Mv/J_u$ = 238.7 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.13 mm
$\sigma_c = -Mv/J_u$ = 136.8 N/mm^2
τ_c = 4.94 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137. N/mm^2
S^* = 2487. mm^3

ESΣ8.054

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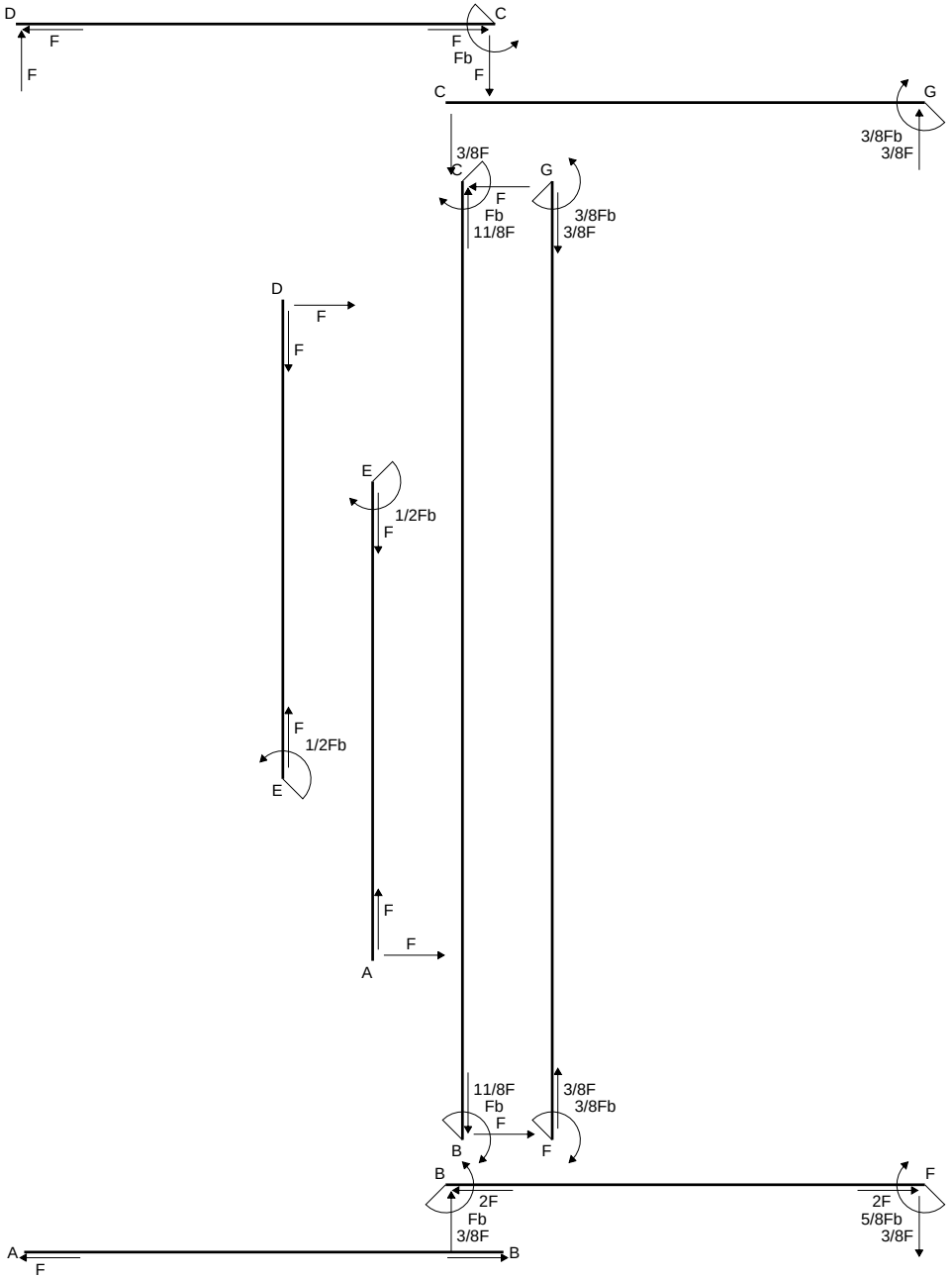
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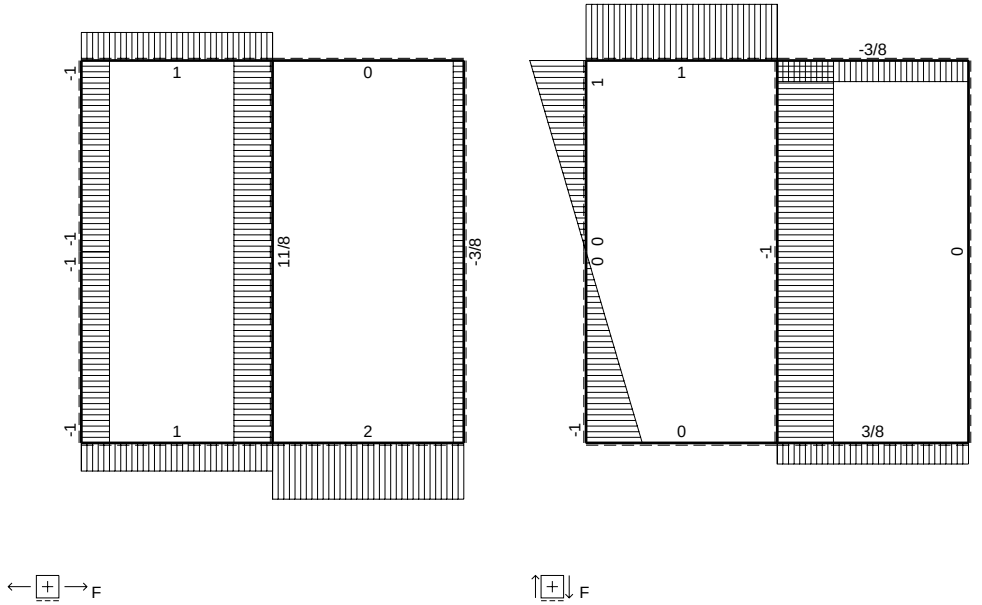
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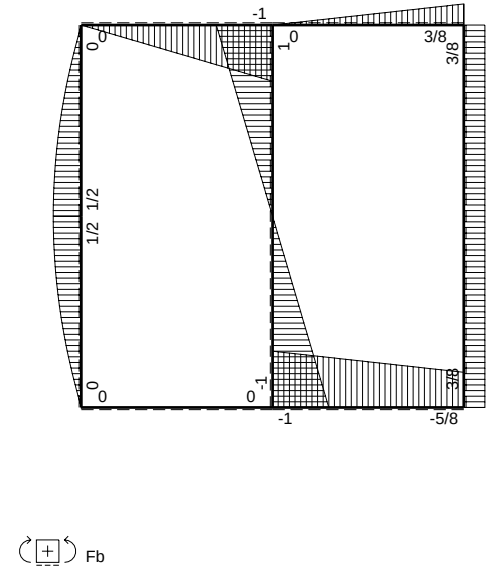
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1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$							-3/8Fb

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

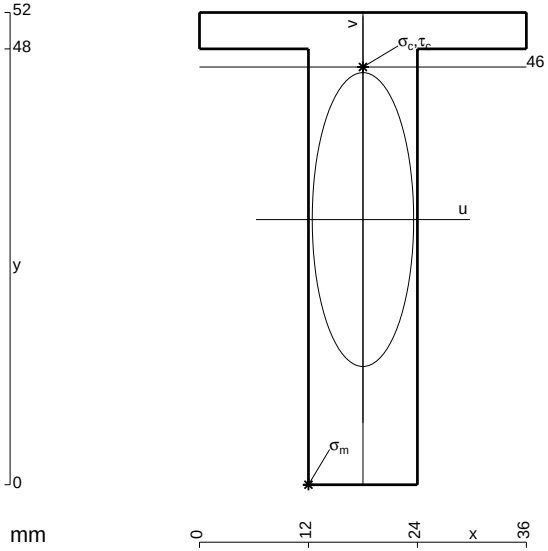
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 720. mm^2
J_u = 188659. mm^4
J_v = 22464. mm^4
y_g = 29.2 mm
N = 2970. N
T_y = -2160. N
M_x = -1317600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -29.2 mm
$\sigma_m = N/A - Mv/J_u$ = -199.8 N/mm^2
x_c = 18. mm
y_c = 46. mm
v_c = 16.8 mm
$\sigma_c = N/A - Mv/J_u$ = 121.5 N/mm^2
τ_c = 3.265 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.6 N/mm^2
S^* = 3422. mm^3

ESΣ8.055

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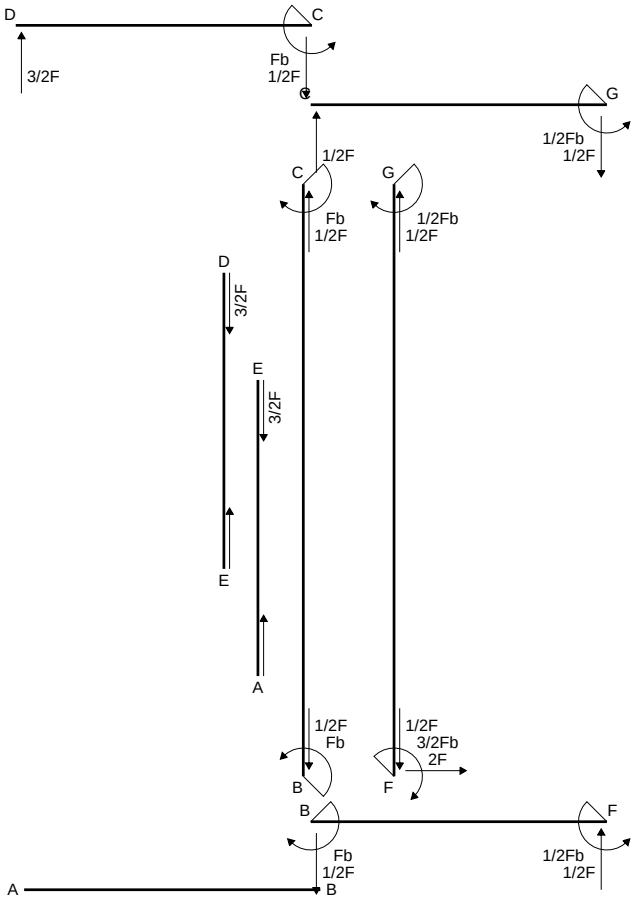
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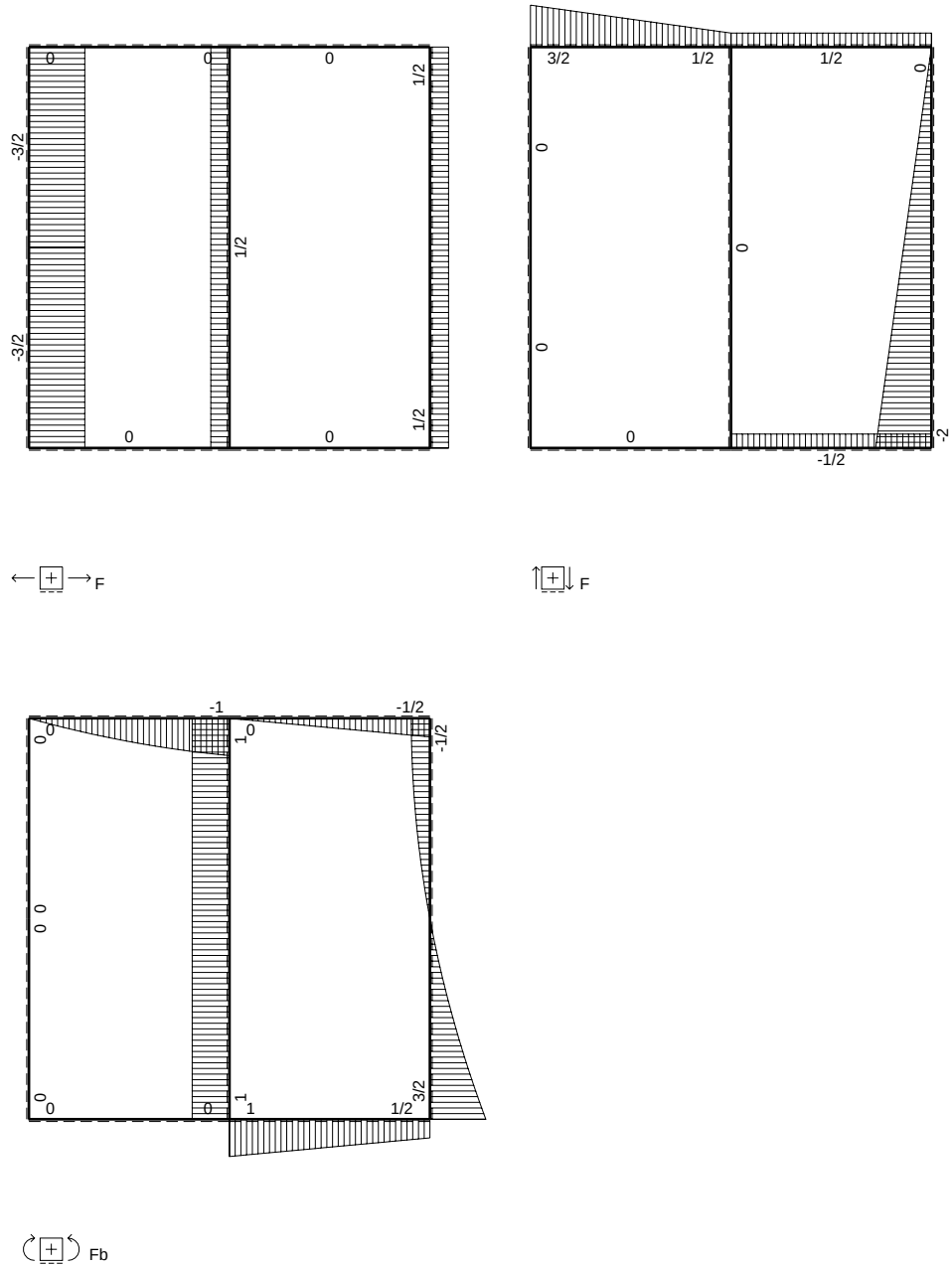
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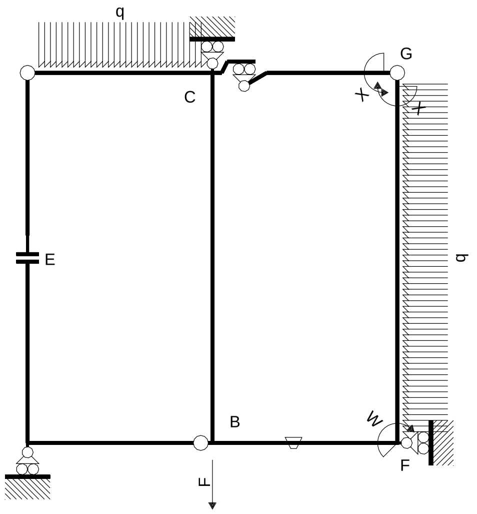
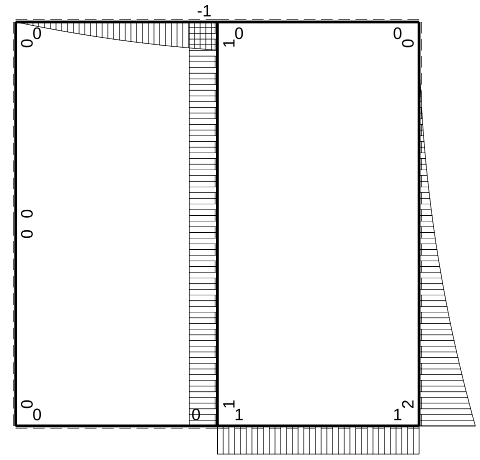
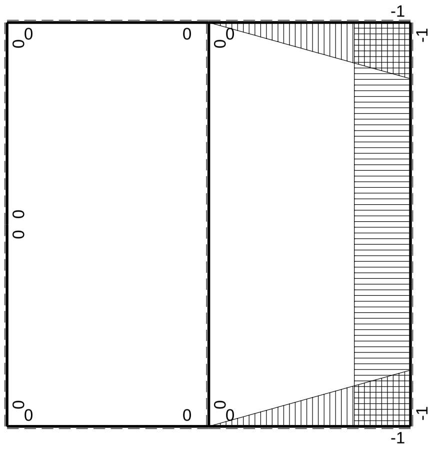




M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$1/2Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

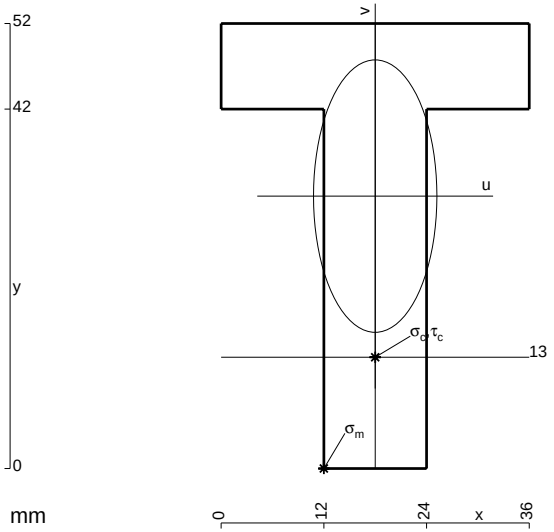
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 864. mm^2
J_u = 219048. mm^4
J_v = 44928. mm^4
y_g = 31.83 mm
T_y = 3550. N
M_x = -1442190. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.83 mm
$\sigma_m = -Mv/J_u$ = -209.6 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -18.83 mm
$\sigma_c = -Mv/J_u$ = -124. N/mm^2
τ_c = 5.337 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 124.3 N/mm^2
S^* = 3952. mm^3

ESΣ8.056

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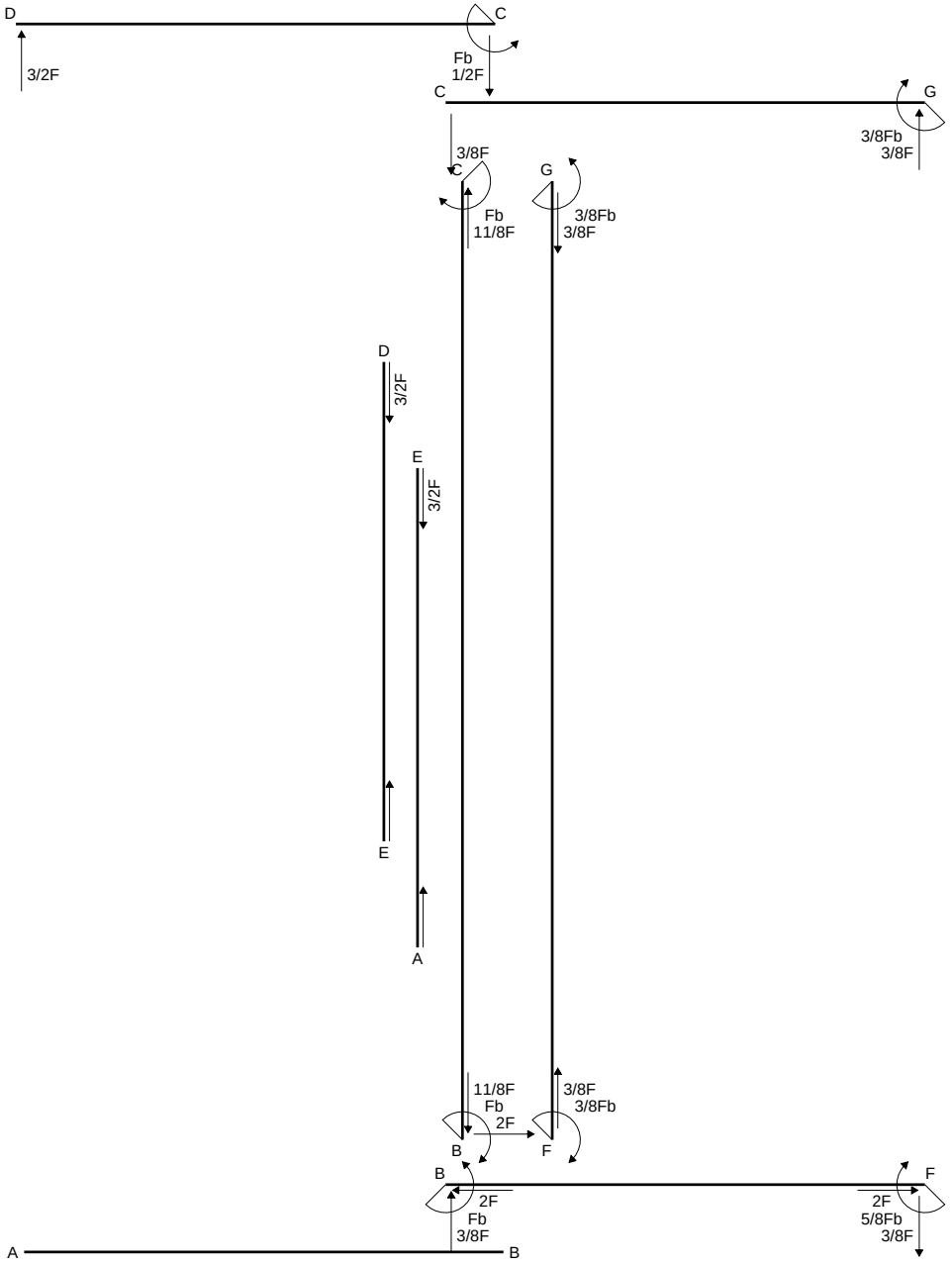
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E
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E
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2F
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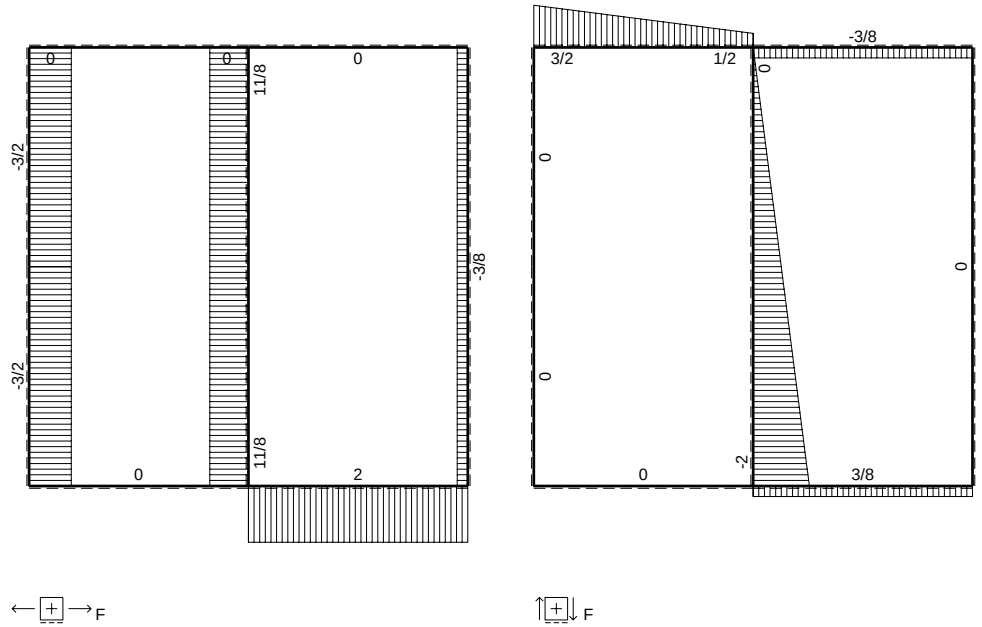
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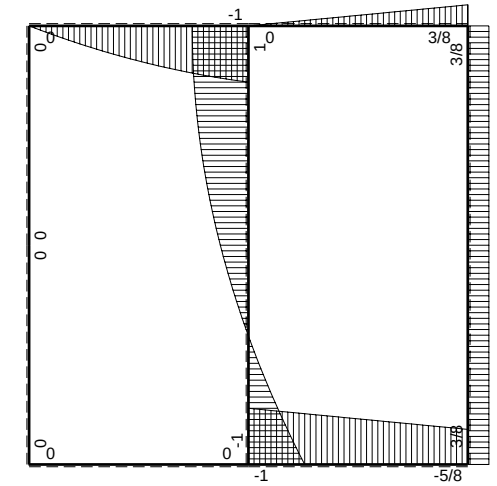
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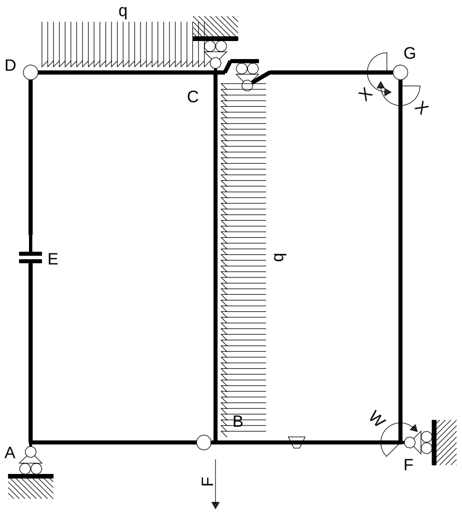
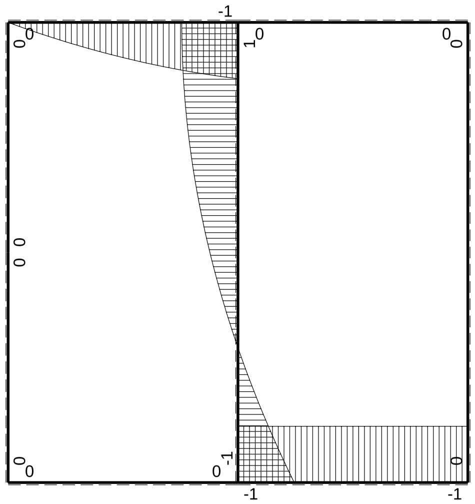
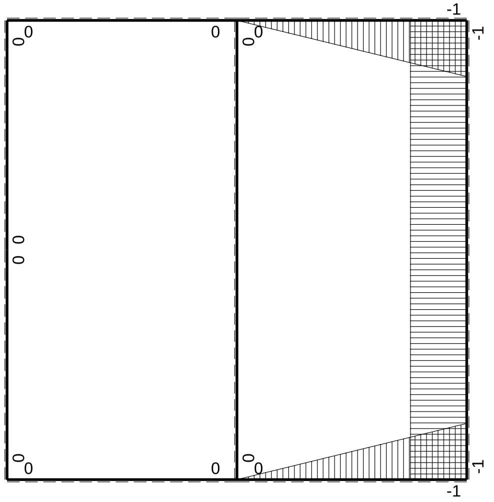
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

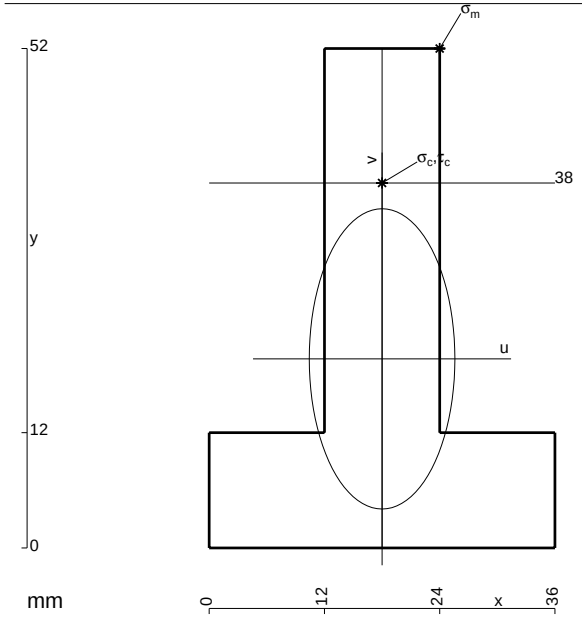
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 912. mm^2
J_u = 222885. mm^4
J_v = 52416. mm^4
y_g = 19.68 mm
N = 2970. N
T_y = -4320. N
M_x = -1490400. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 32.32 mm
$\sigma_m = N/A - Mv/J_u$ = 219.3 N/mm^2
x_c = 18. mm
y_c = 38. mm
v_c = 18.32 mm
$\sigma_c = N/A - Mv/J_u$ = 125.7 N/mm^2
τ_c = 6.869 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.3 N/mm^2
S^* = 4253. mm^3

ESΣ8.057

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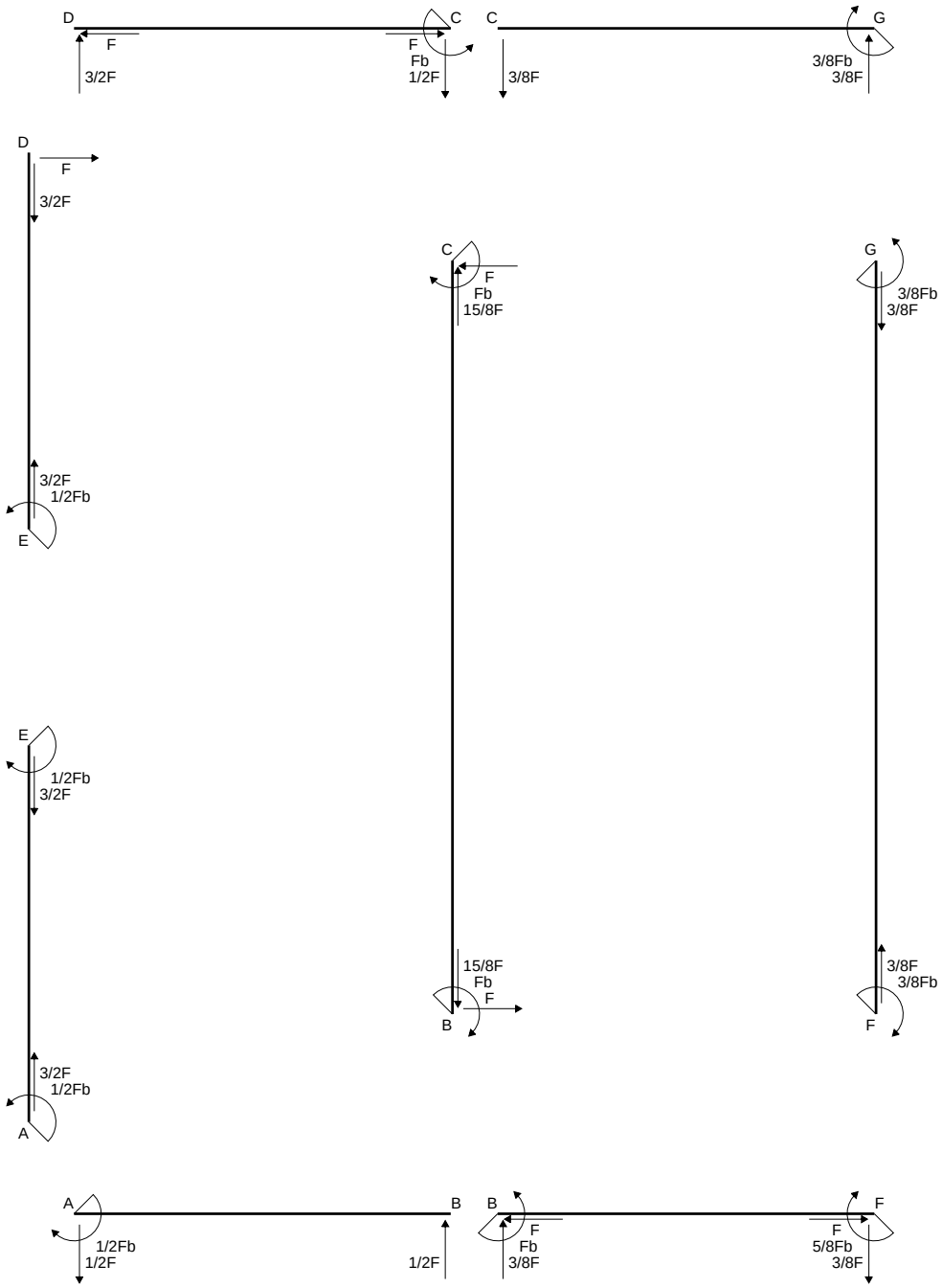
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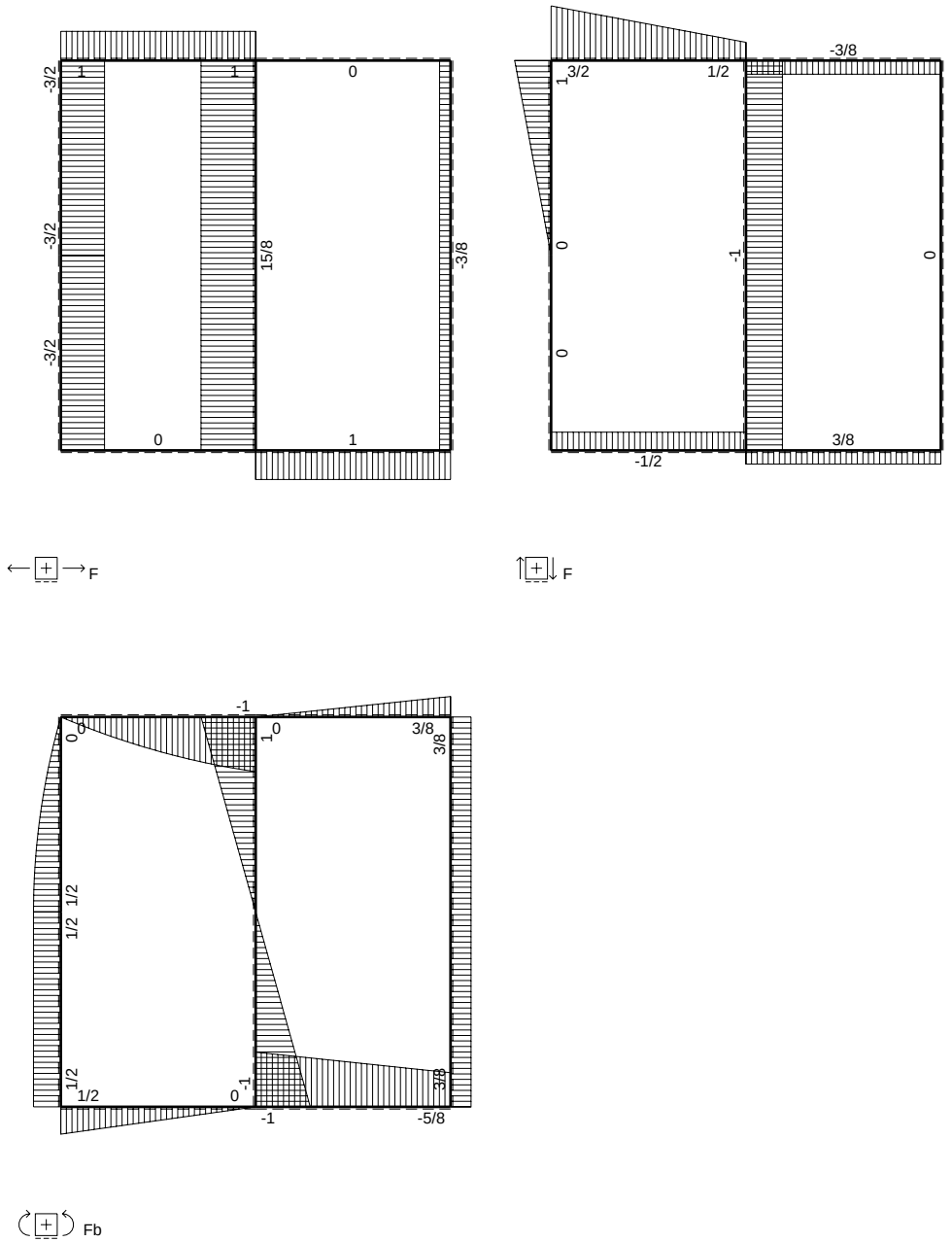
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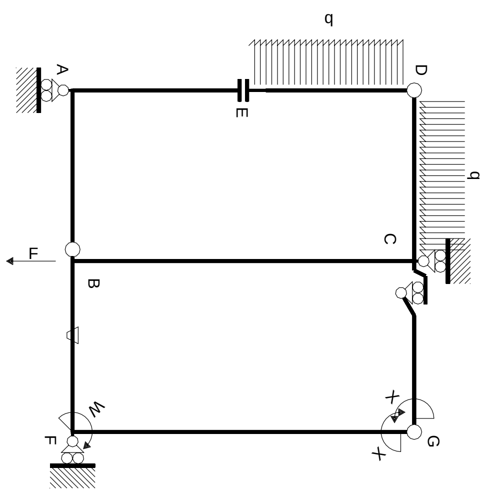
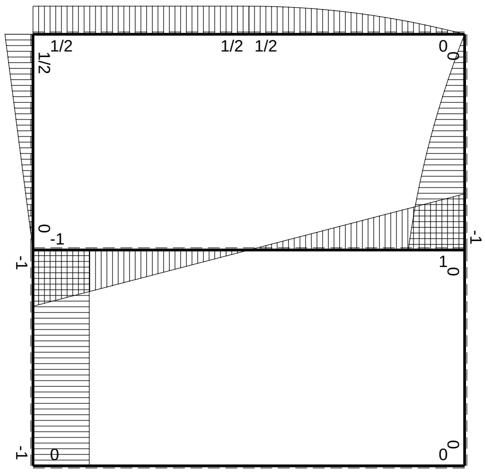
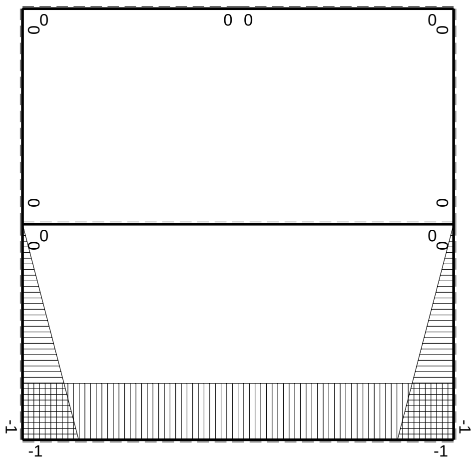


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

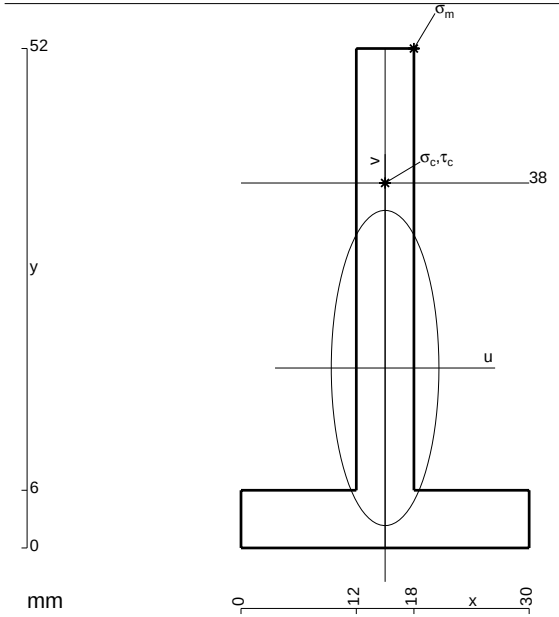
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 456. mm^2
J_u = 122856. mm^4
J_v = 14328. mm^4
y_g = 18.74 mm
N = 2119. N
T_y = -1130. N
M_x = -824900. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 33.26 mm
$\sigma_m = N/A - Mv/J_u$ = 228. N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 19.26 mm
$\sigma_c = N/A - Mv/J_u$ = 134. N/mm^2
τ_c = 3.382 N/mm^2
$\sigma_0 = \sqrt{\sigma^2 + 3\tau^2}$ = 134.1 N/mm^2
S^* = 2206. mm^3

ESΣ8.058

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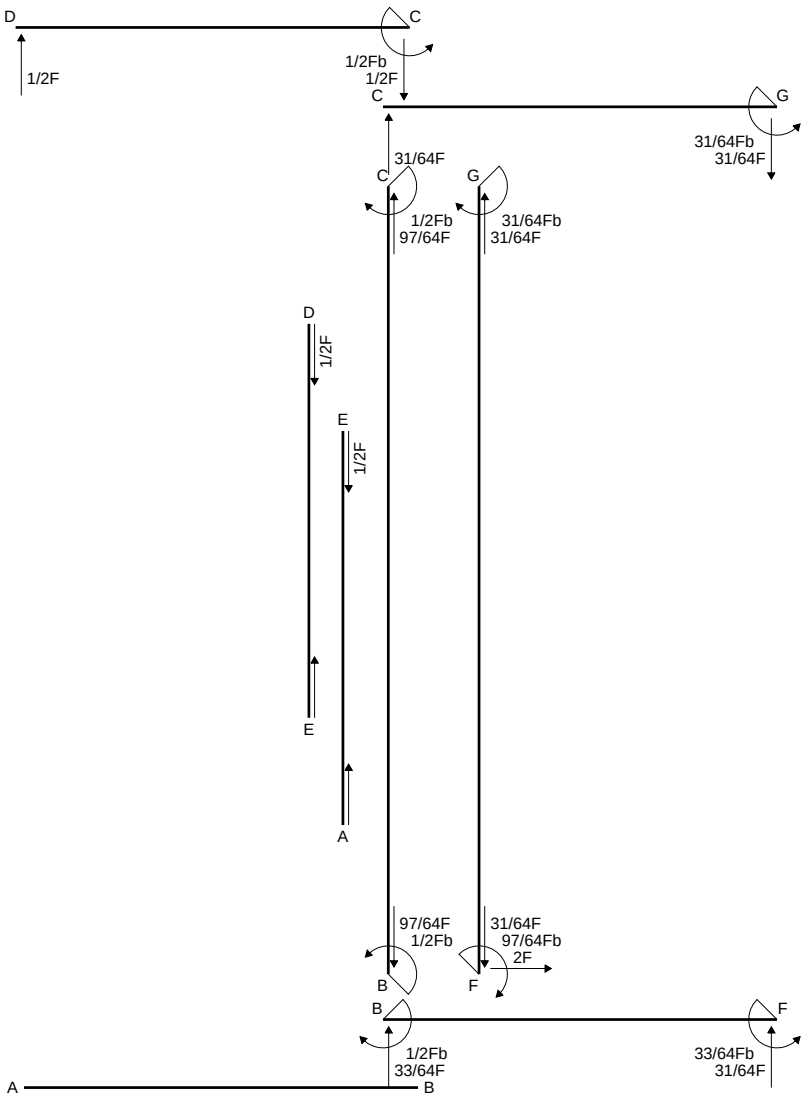
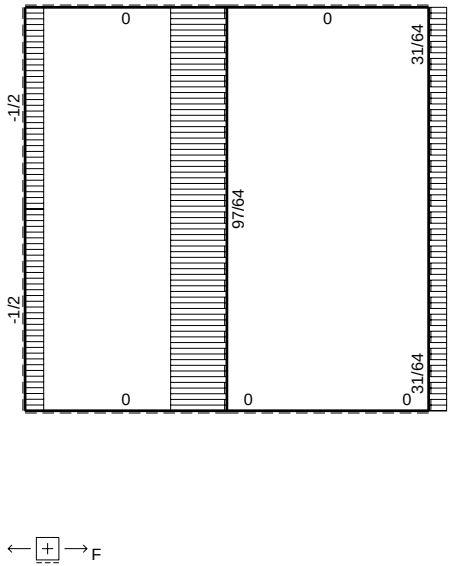

D
C
1/2F
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
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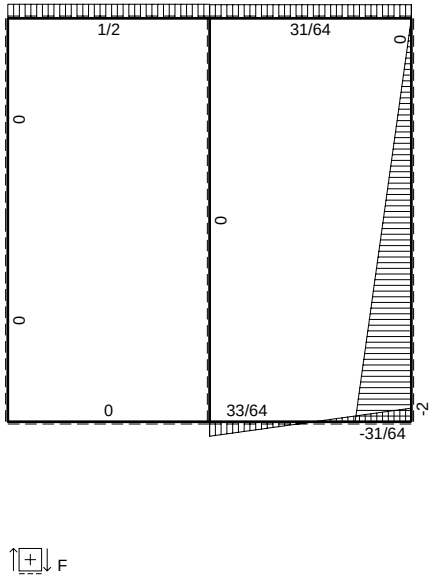
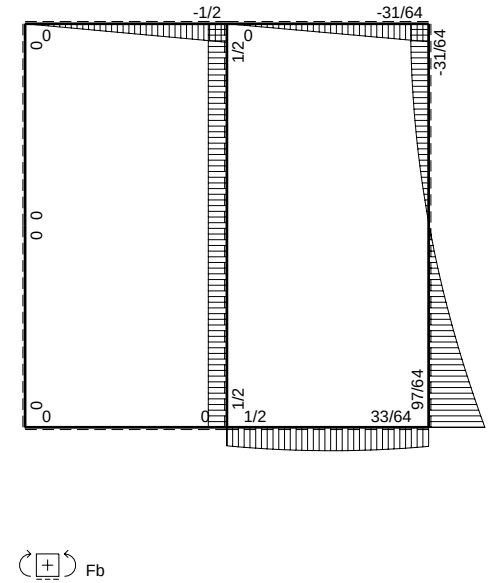
0
0
0
31/64
F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64
F


-1/2
-31/64
0
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	FX/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3XbJ/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-FX/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbJ/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2XbJ/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

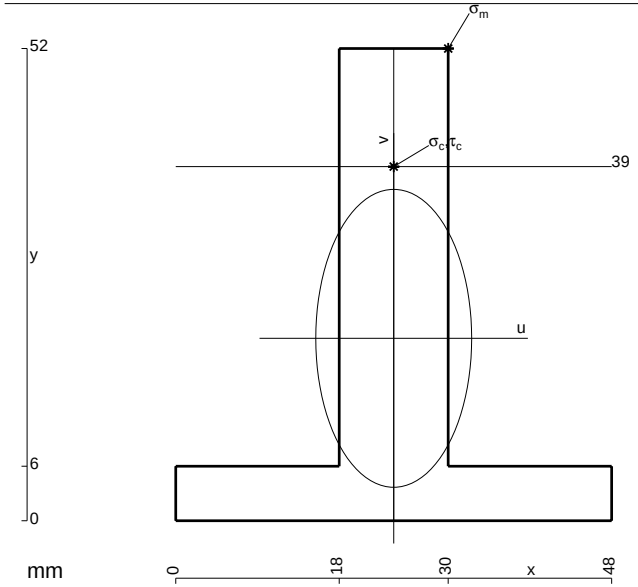
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 840. mm^2
J_u = 226138. mm^4
J_v = 61920. mm^4
y_g = 20.09 mm
T_y = 2205. N
M_x = -1697850. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 31.91 mm
$\sigma_m = -Mv/J_u$ = 239.6 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 18.91 mm
$\sigma_c = -Mv/J_u$ = 142. N/mm^2
τ_c = 3.221 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 142.1 N/mm^2
S^* = 3965. mm^3

ESΣ8.059

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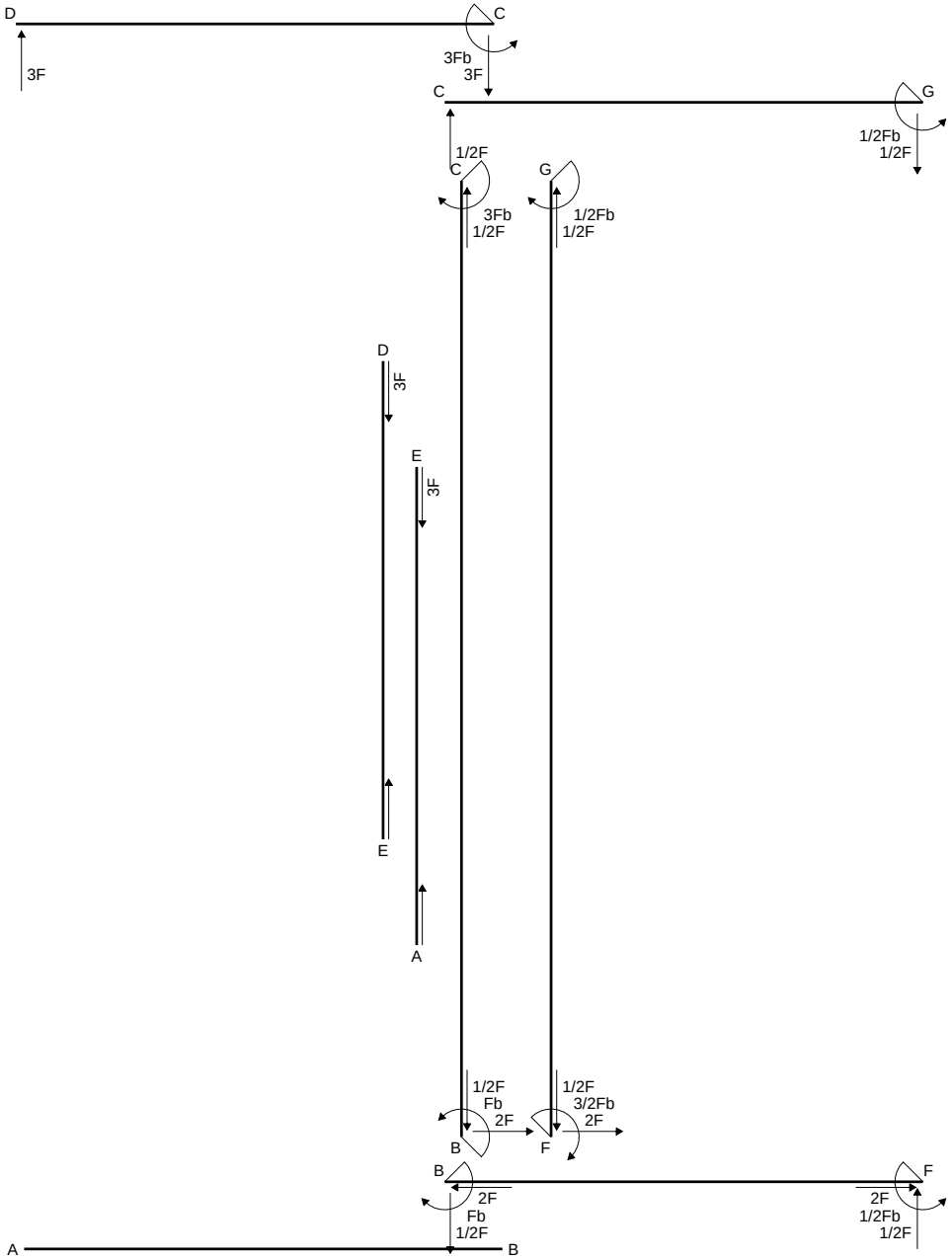
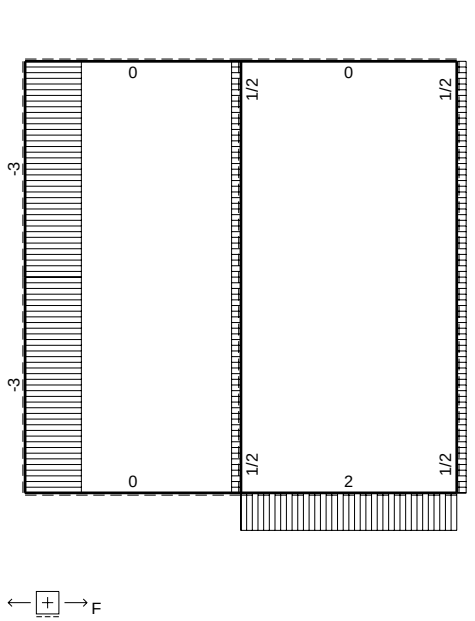
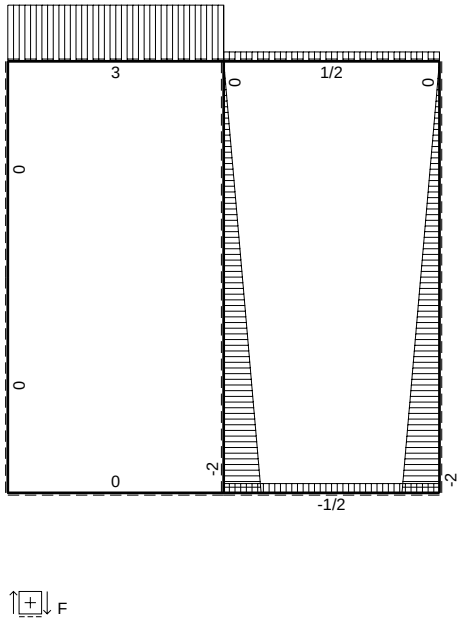
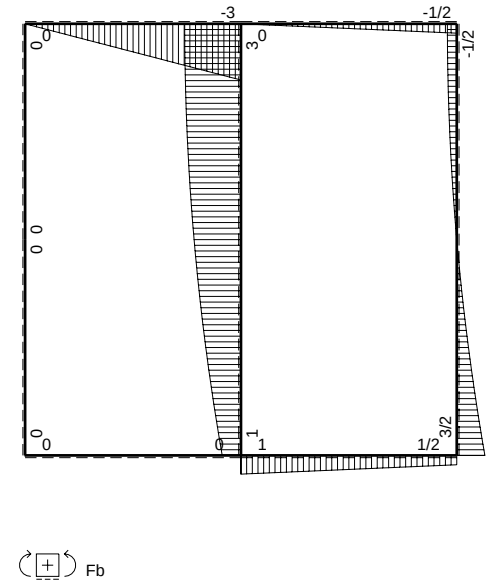
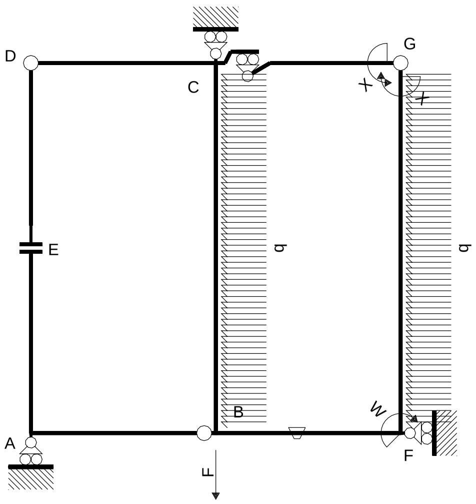
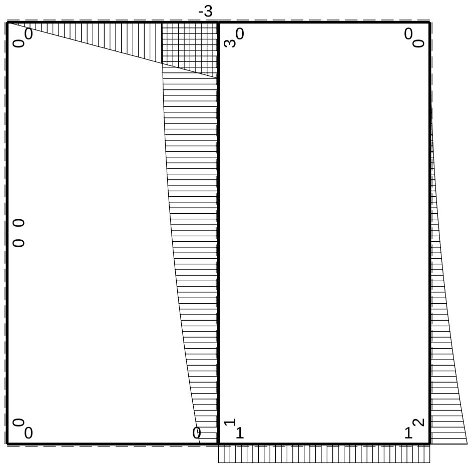

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
0
1/2
1/2
-3
-3
0
2
0
1/2
1/2
F

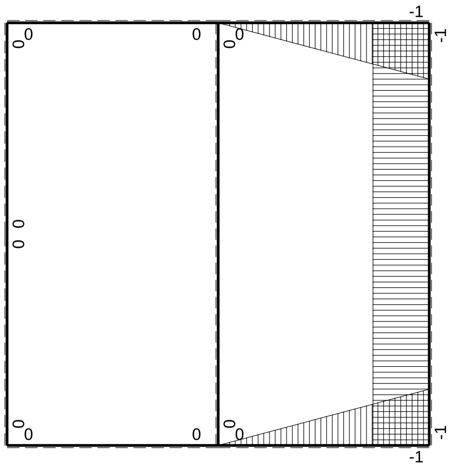

3
0
1/2
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica X=W_{GC}							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

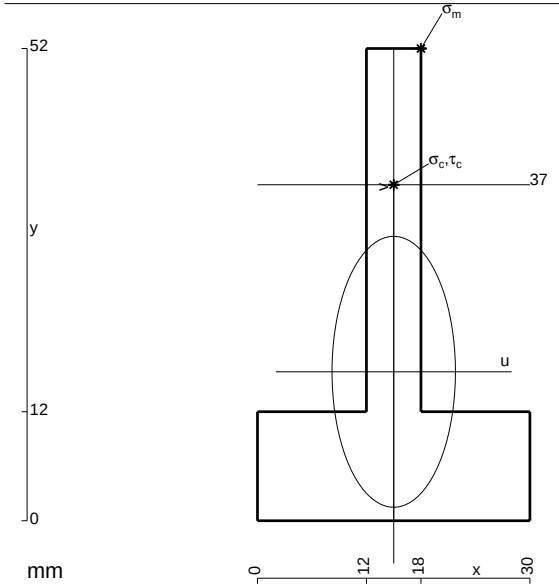
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 600. mm^2
J_u = 133664. mm^4
J_v = 27720. mm^4
y_g = 16.4 mm
T_y = 1830. N
M_x = -750300. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.6 mm
$\sigma_m = -Mv/J_u$ = 199.8 N/mm^2
x_c = 15. mm
y_c = 37. mm
v_c = 20.6 mm
$\sigma_c = -Mv/J_u$ = 115.6 N/mm^2
τ_c = 5.771 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.1 N/mm^2
S^* = 2529. mm^3

ESΣ8.060

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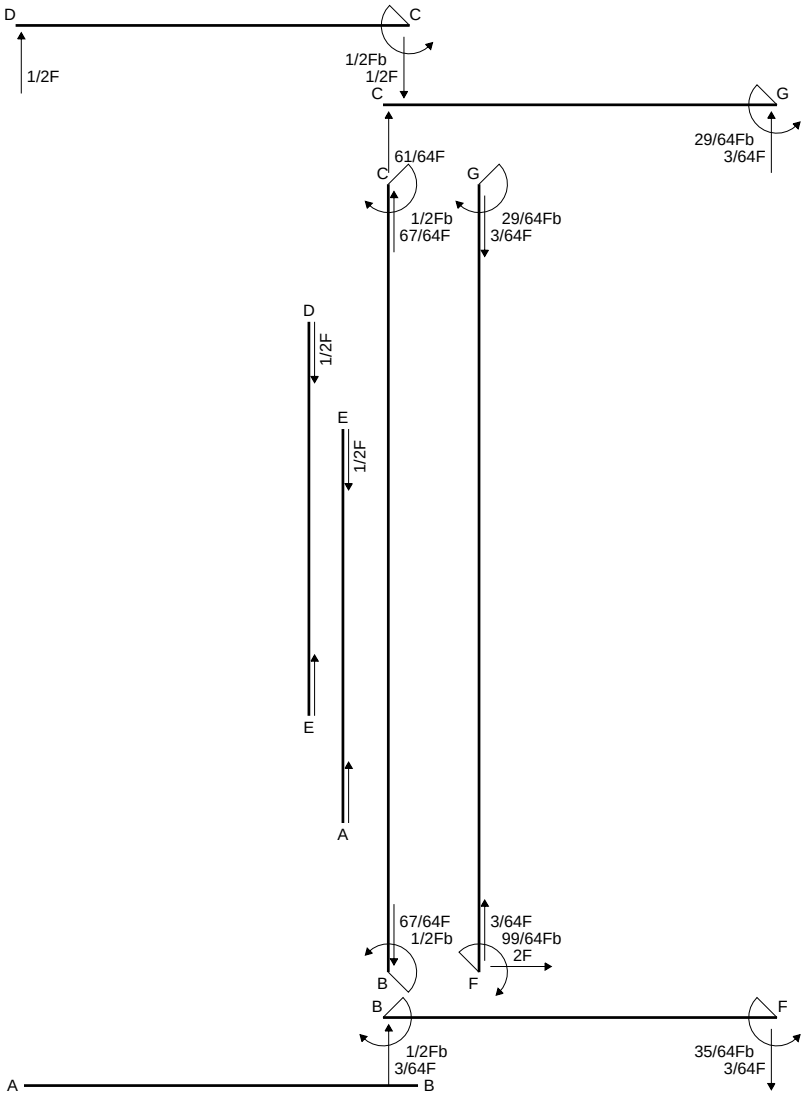
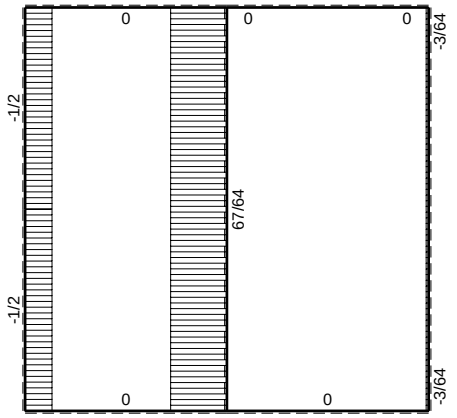

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B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67/64
-1/2
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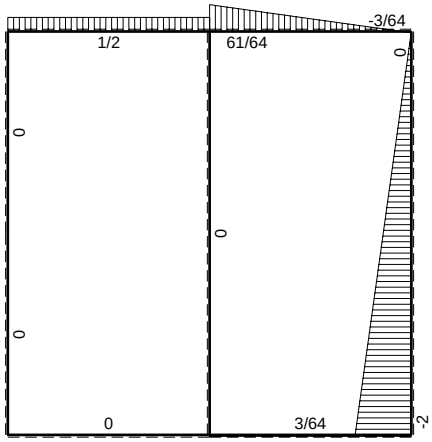
0
-3/64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1/2
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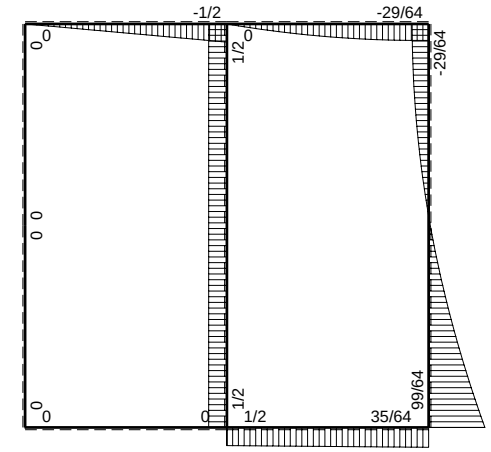
0
0
0
3/64
-2


-1/2
-29/64
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0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx²/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb²/EJ	1/3XbJ/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x²/b²		
GC b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x²/b²	(1/24+0)Fb²/EJ	1/3XbJ/EJ
CG b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2XbJ/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
totali							-29/24Fb²/EJ	8/3XbJ/EJ
iperstatica $X=W_{GC}$							29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

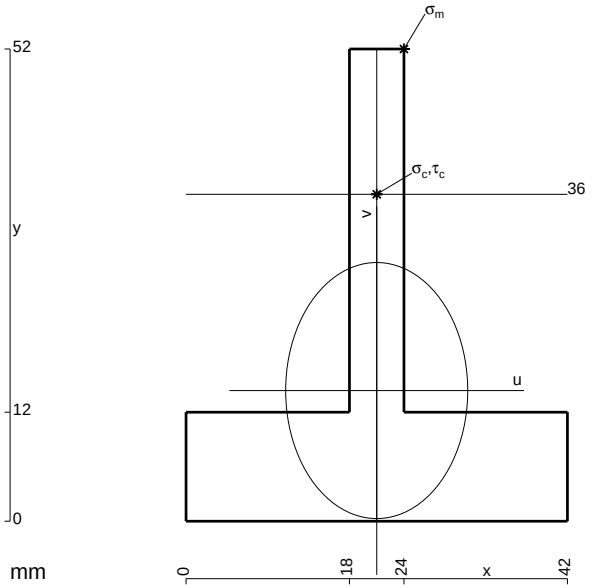
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.061



A = 744. mm^2
J_u = 147953. mm^4
J_v = 74808. mm^4
y_g = 14.39 mm
T_y = 1835. N
M_x = -825750. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 37.61 mm
$\sigma_m = -Mv/J_u$ = 209.9 N/mm^2
x_c = 21. mm
y_c = 36. mm
v_c = 21.61 mm
$\sigma_c = -Mv/J_u$ = 120.6 N/mm^2
τ_c = 5.876 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.1 N/mm^2
S^* = 2843. mm^3

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ESΣ8.061

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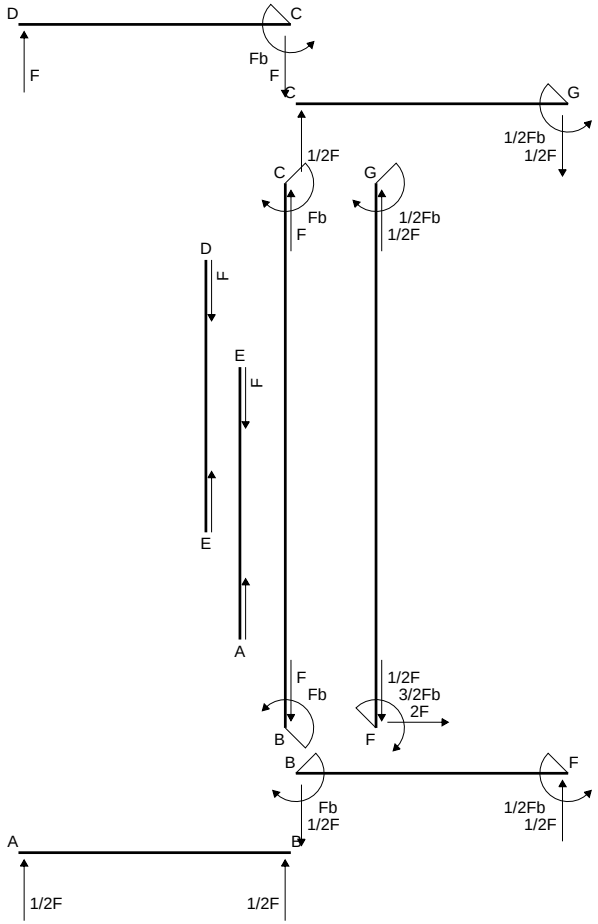
D
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F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
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D
F
E
F
E
A
F
Fb
1/2F
3/2Fb
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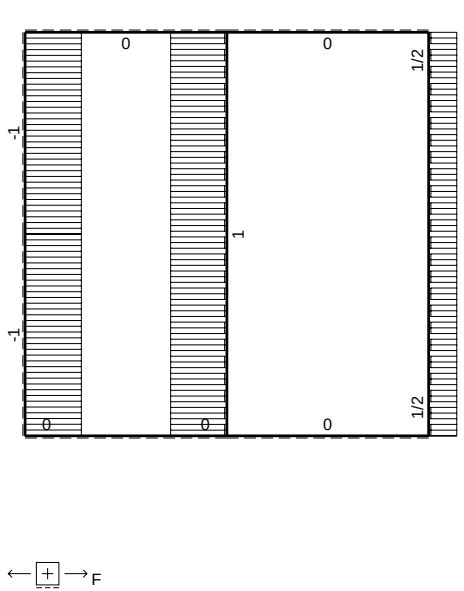
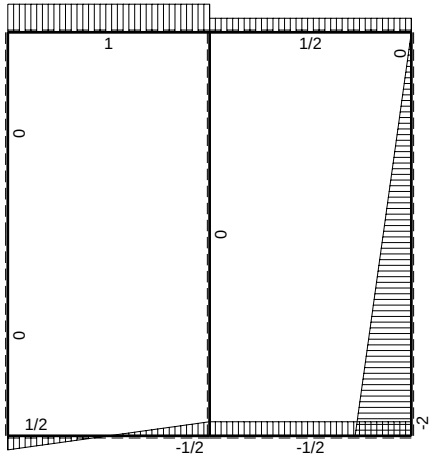
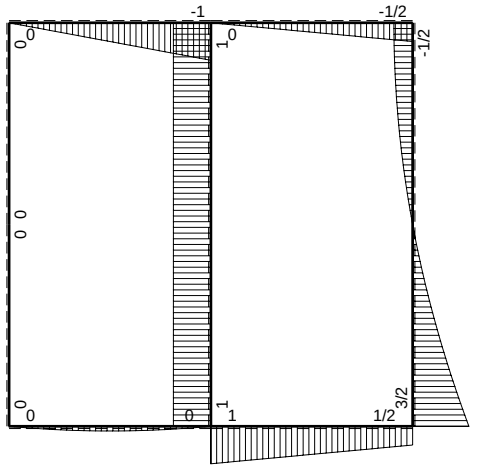
A
B
1/2F
1/2F


0
0
1/2
-1
1
-1
0
0
0
1/2

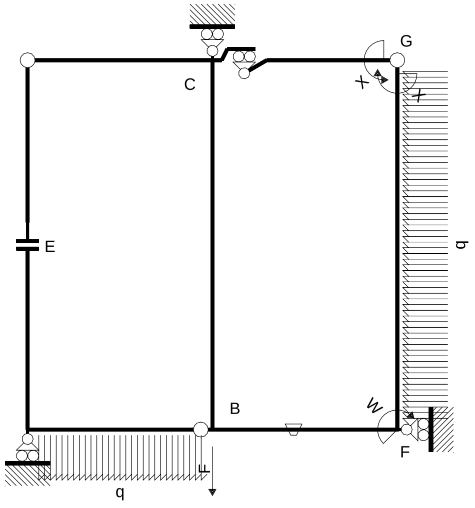
F

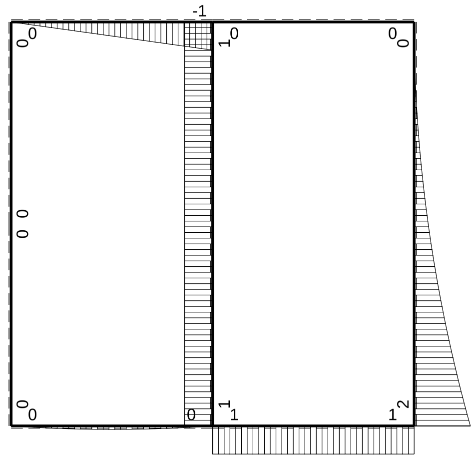

1
1/2
0
0
0
0
1/2
-2
-1/2
-1/2

F

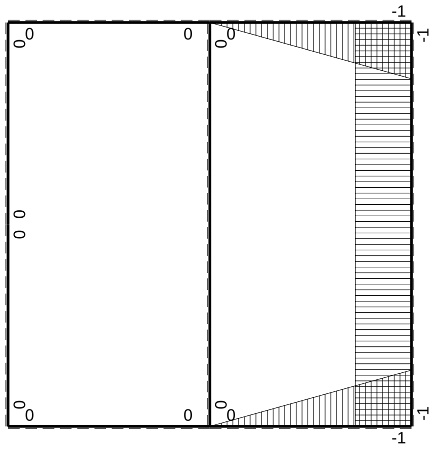

-1
-1/2
0
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

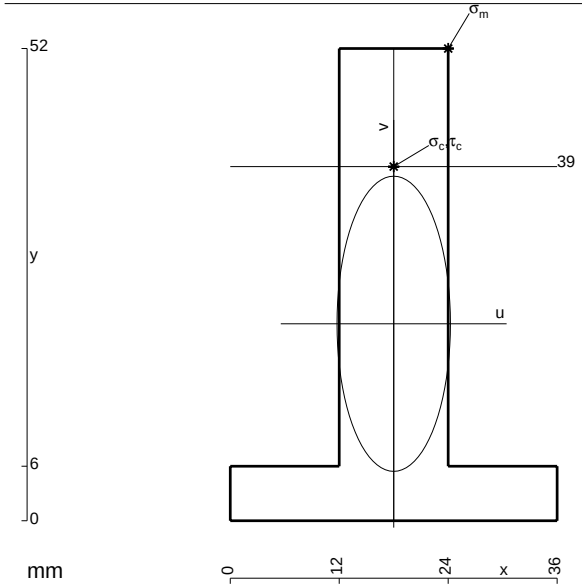
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 768. mm^2
J_u = 202933. mm^4
J_v = 29952. mm^4
y_g = 21.69 mm
T_y = 3000. N
M_x = -1470000. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 30.31 mm
$\sigma_m = -Mv/J_u$ = 219.6 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 17.31 mm
$\sigma_c = -Mv/J_u$ = 125.4 N/mm^2
τ_c = 4.576 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.7 N/mm^2
S^* = 3715. mm^3

ESΣ8.062

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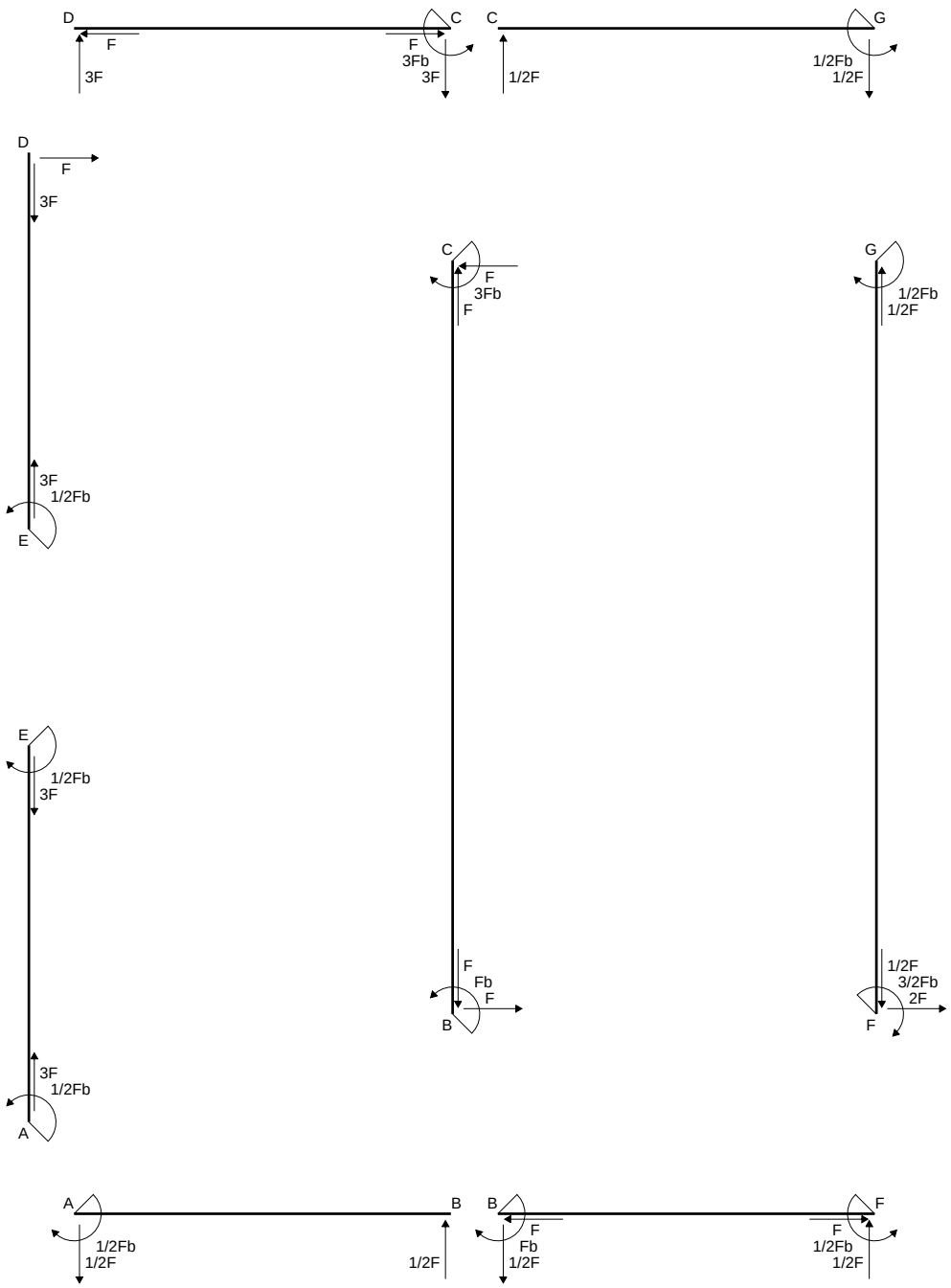
D
C
C
G
F
3F
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3Fb
3F
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1/2Fb
1/2F
D
F
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G
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1/2Fb
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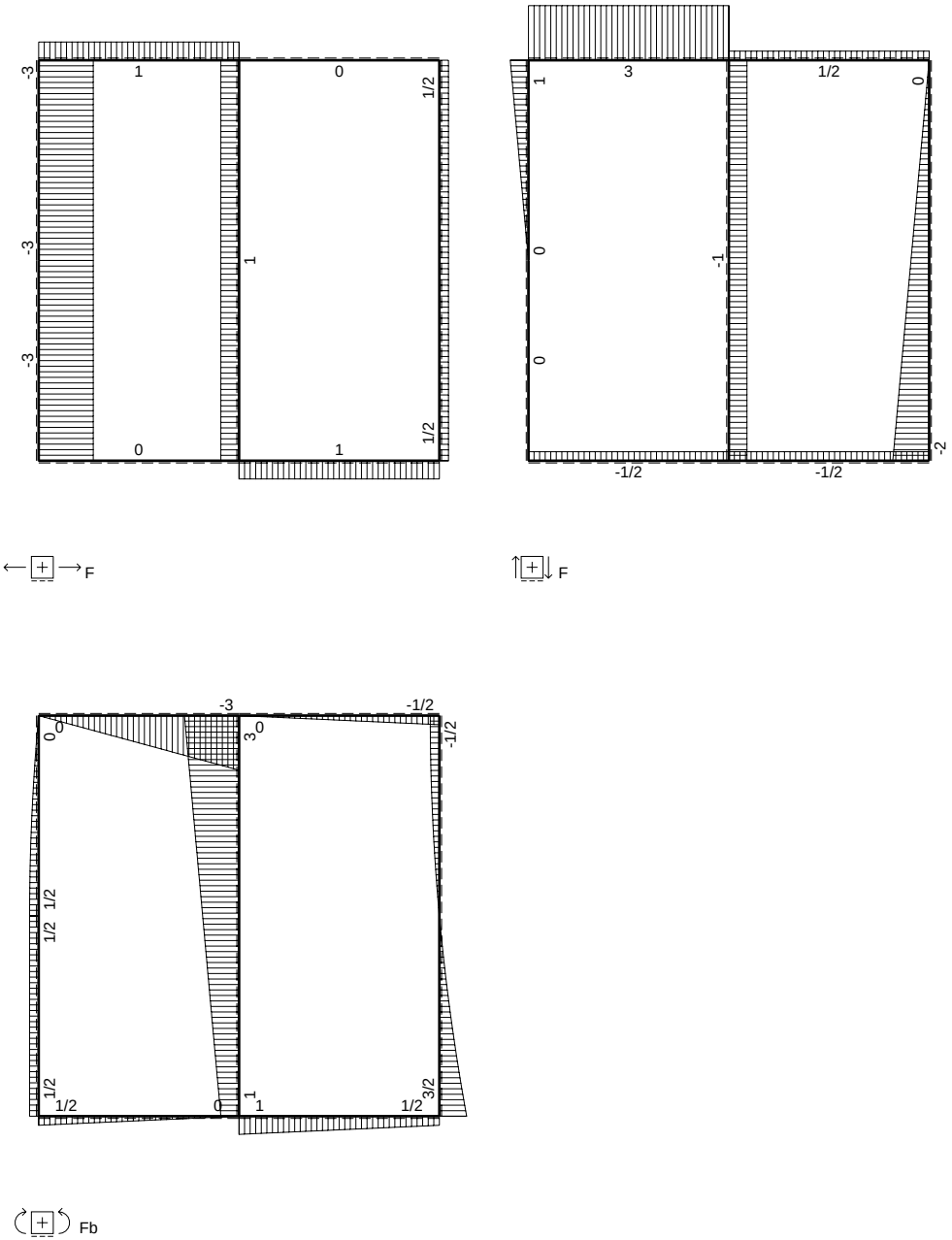
1/2Fb
1/2F


-3
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-1
-2
-1/2
-1/2
F
-3
-1/2
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1/2
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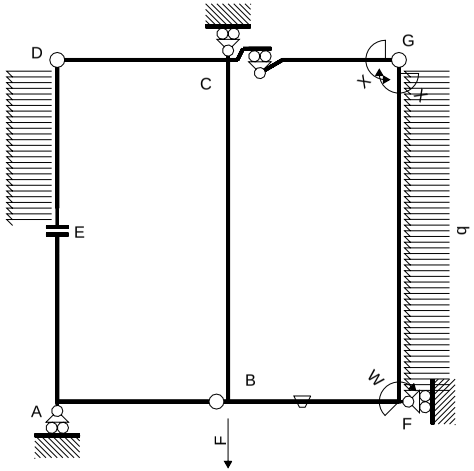
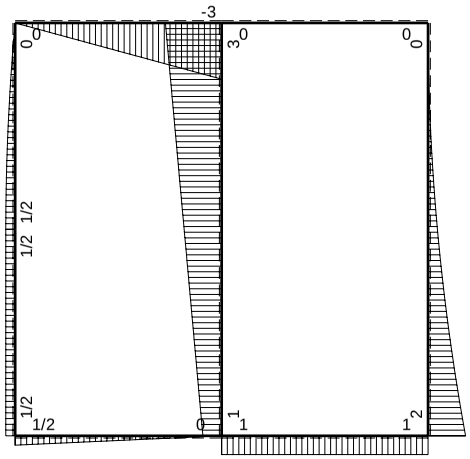
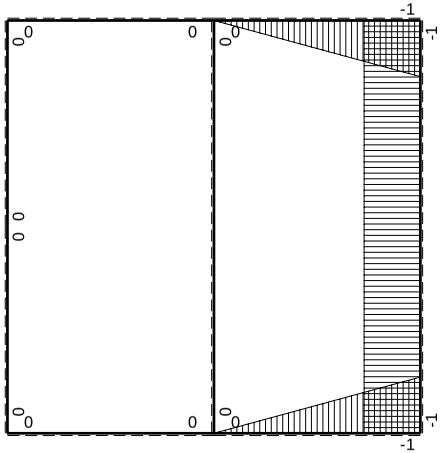
1/2
1/2
0
1
1
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.063

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

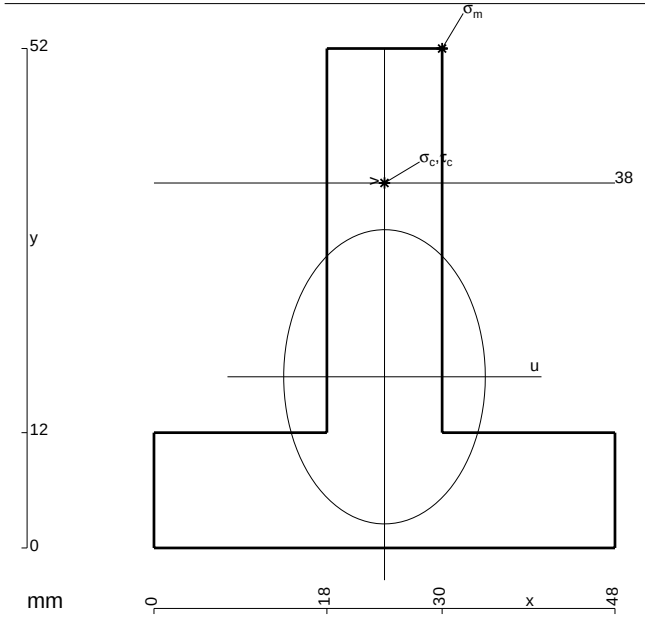
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1056. mm^2
J_u = 247901. mm^4
J_v = 116352. mm^4
y_g = 17.82 mm
N = 1040. N
T_y = 3120. N
M_x = -1653600. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 34.18 mm
$\sigma_m = N/A - Mv/J_u$ = 229. N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 20.18 mm
$\sigma_c = N/A - Mv/J_u$ = 135.6 N/mm^2
τ_c = 4.789 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135.9 N/mm^2
S^* = 4567. mm^3

ESΣ8.063

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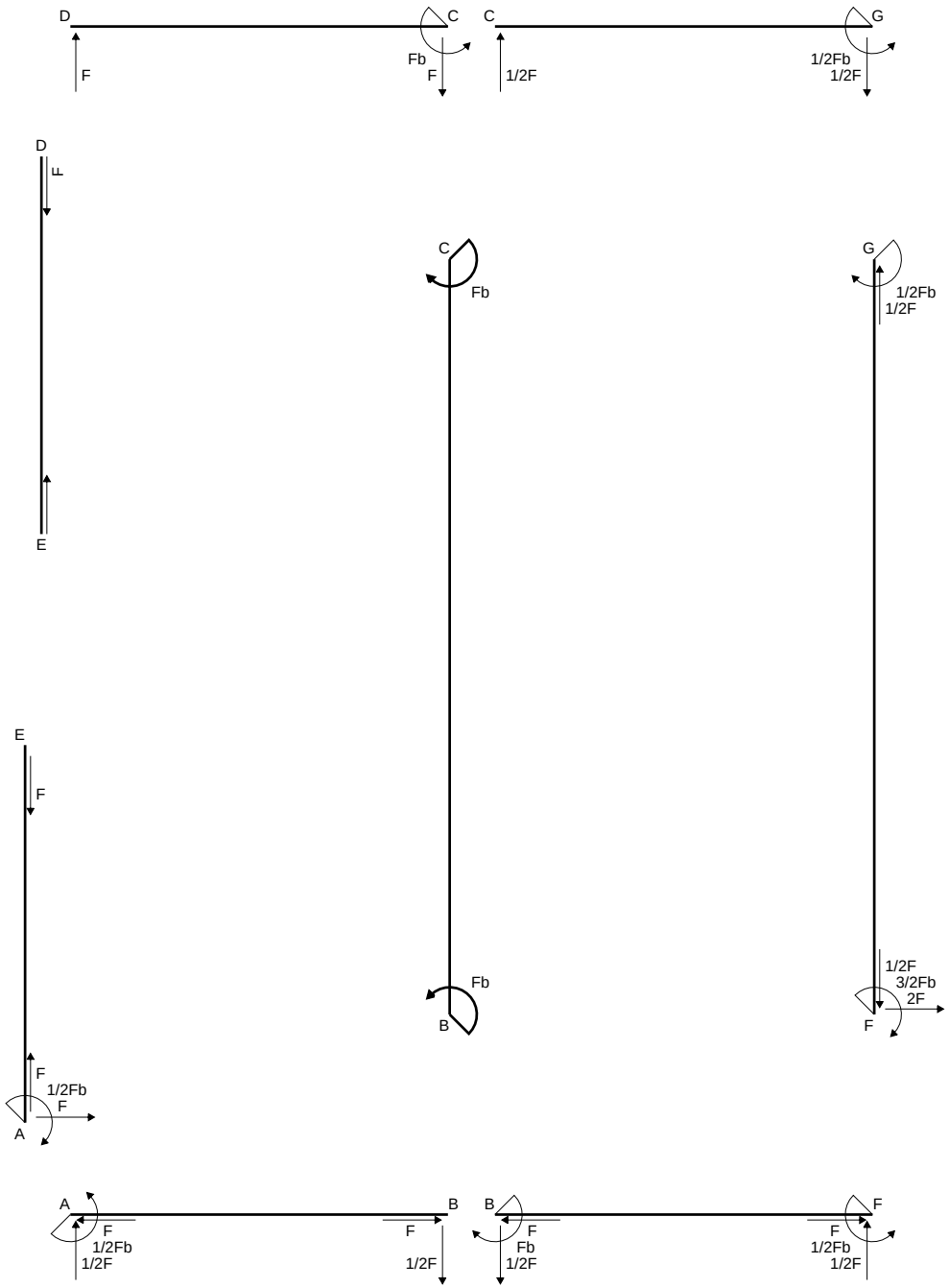
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C
C
G
F
Fb
F
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1/2Fb
1/2F
D
F
E
C
Fb
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1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
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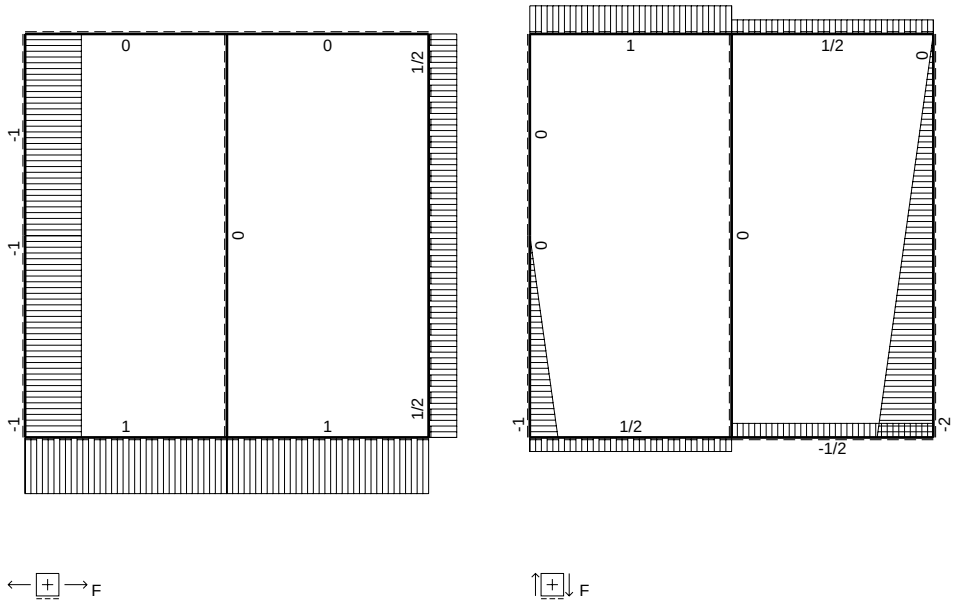
1/2Fb
1/2F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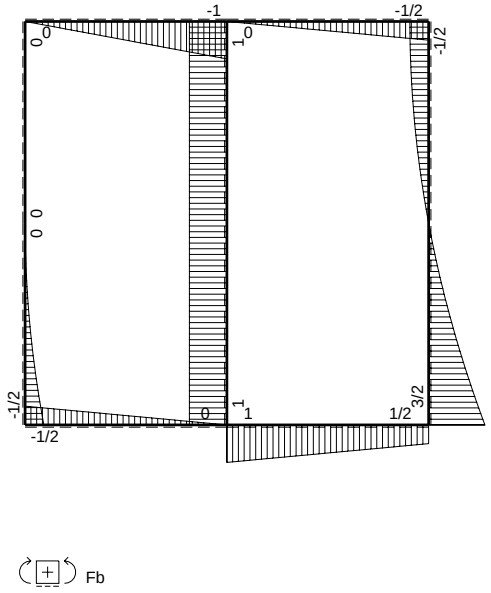
-1/2
F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-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

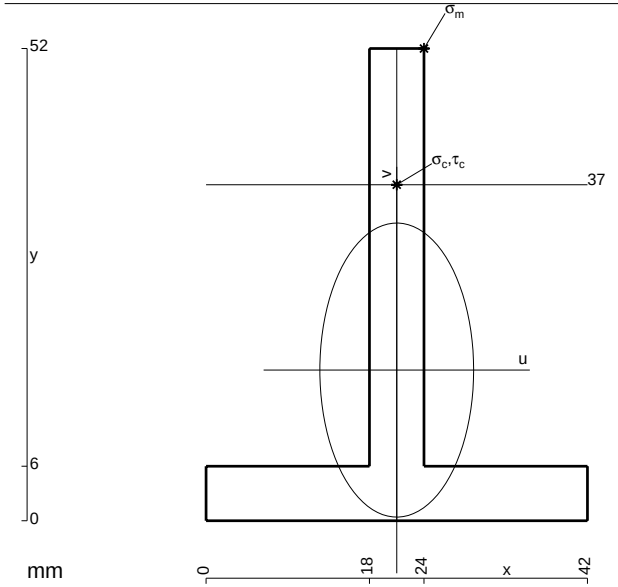
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 528. mm^2
J_u = 138472. mm^4
J_v = 37872. mm^4
y_g = 16.59 mm
T_y = 1640. N
M_x = -934800. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.41 mm
$\sigma_m = -Mv/J_u$ = 239. N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 20.41 mm
$\sigma_c = -Mv/J_u$ = 137.8 N/mm^2
τ_c = 4.958 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138. N/mm^2
S^* = 2512. mm^3

ESΣ8.064

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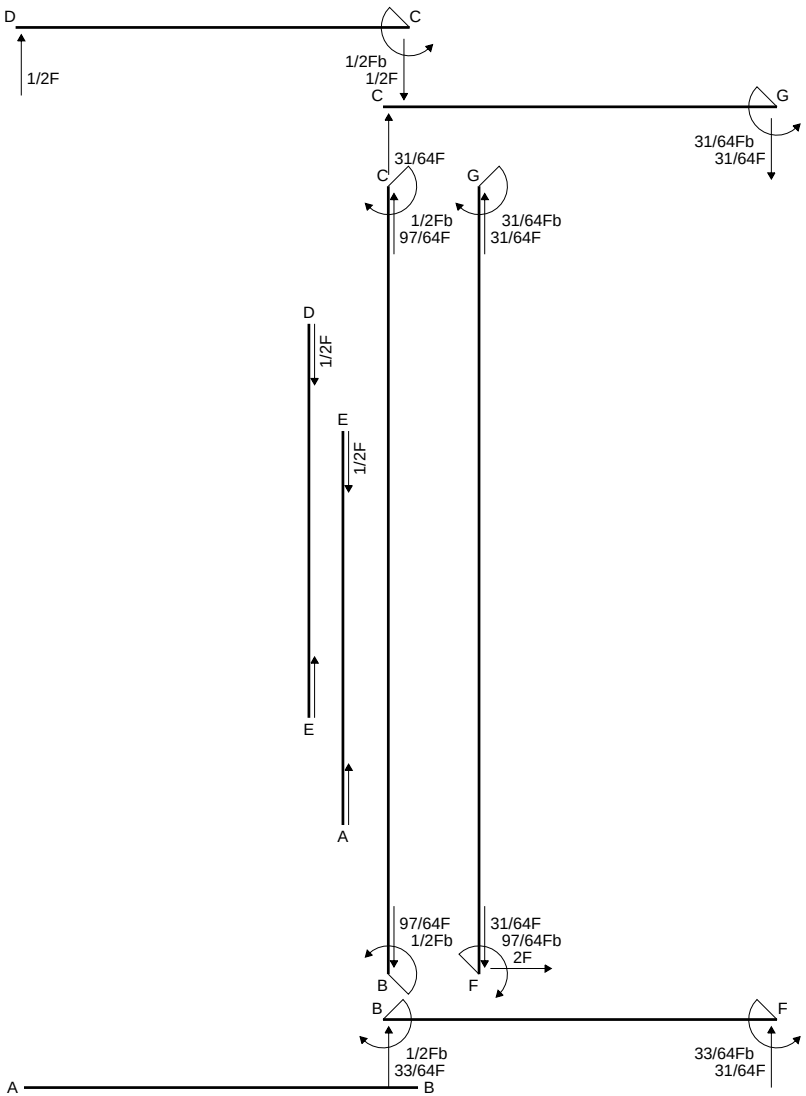
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1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
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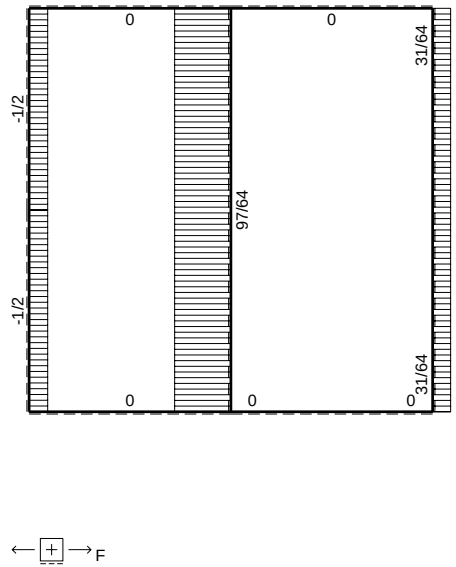
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
97/64
-1/2
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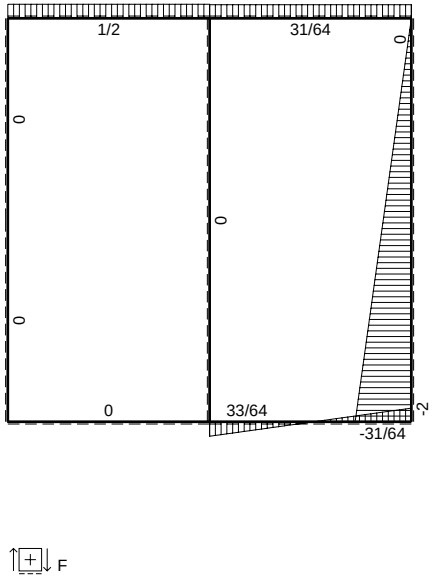
0
0
31/64
F


1/2
31/64
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0
0
33/64
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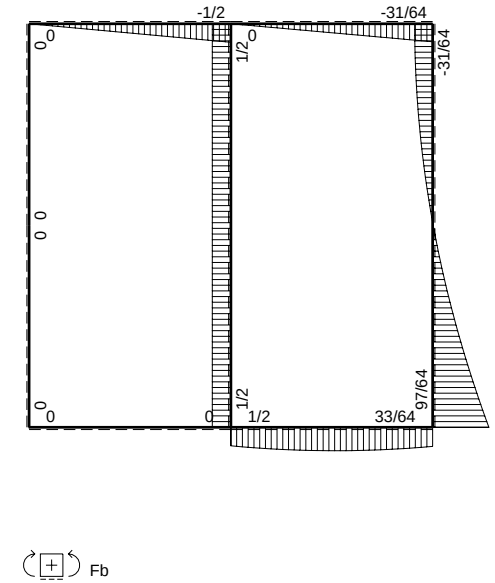
-31/64
F


-1/2
-31/64
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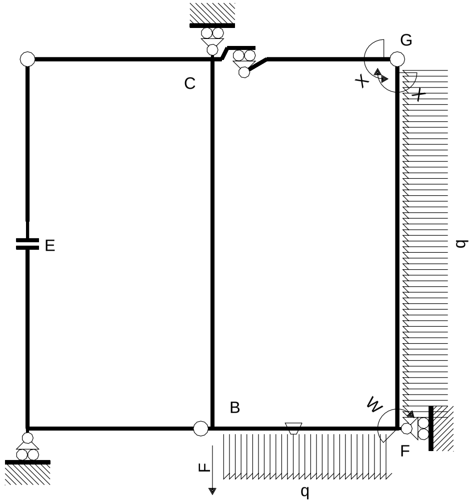
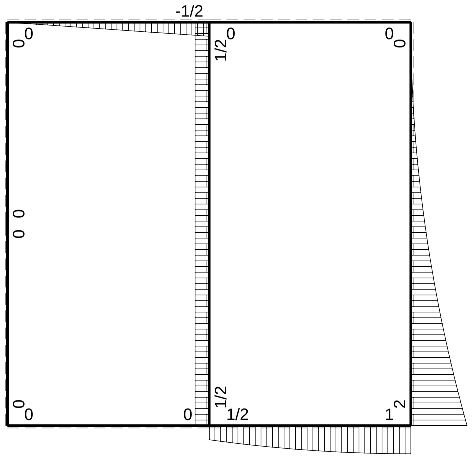
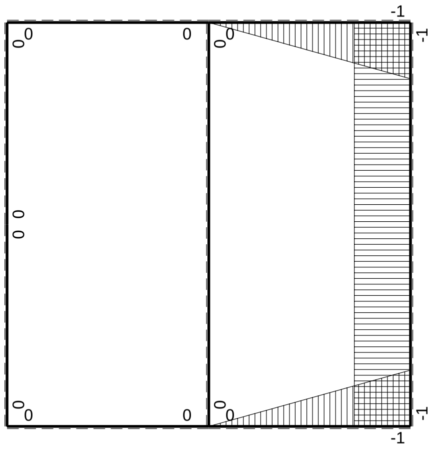
1/2
0
1/2
33/64
97/64
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

↓	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$1/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$-1/2Fb$	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$31/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

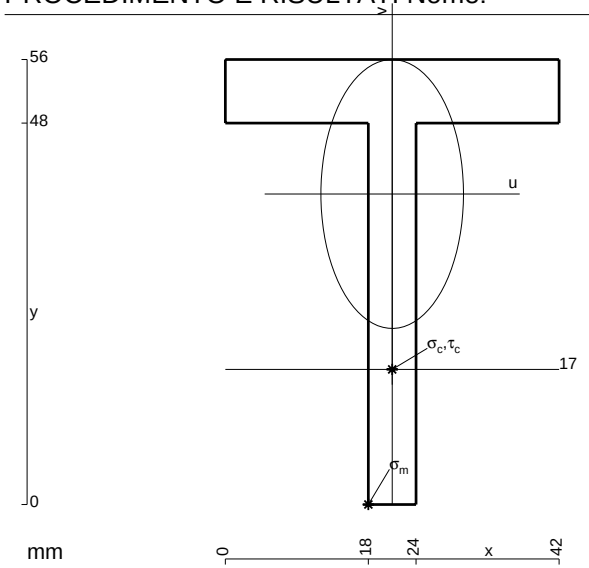
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.065



A = 624. mm^2
J_u = 178668. mm^4
J_v = 50256. mm^4
y_g = 39.08 mm
T_y = 1385. N
M_x = -914100. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.08 mm
$\sigma_m = -Mv/J_u$ = -199.9 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.08 mm
$\sigma_c = -Mv/J_u$ = -112.9 N/mm^2
τ_c = 4.029 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 113.2 N/mm^2
S^* = 3119. mm^3

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ESΣ8.065

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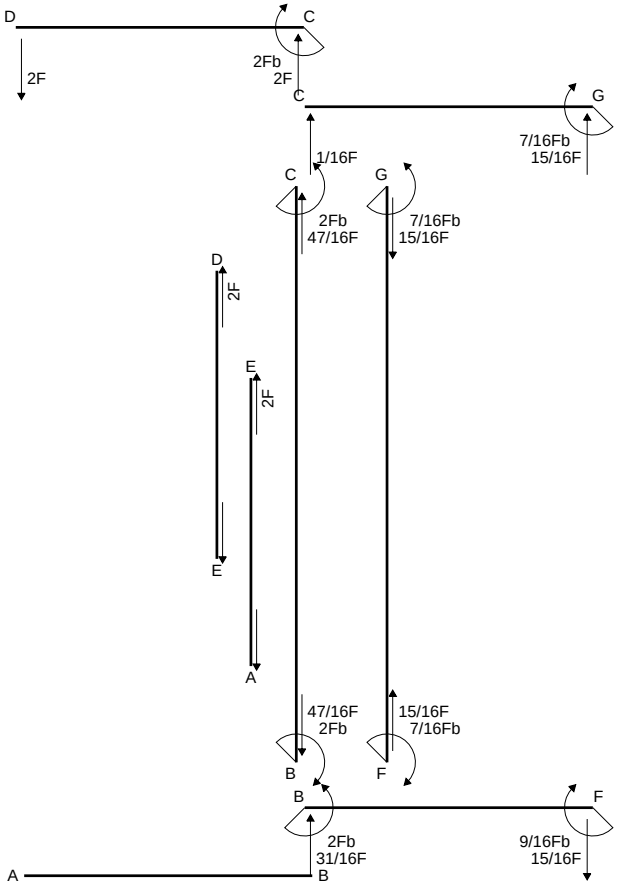
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2F
C
G
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15/16F
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G
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47/16F
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15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
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B
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31/16F
9/16Fb
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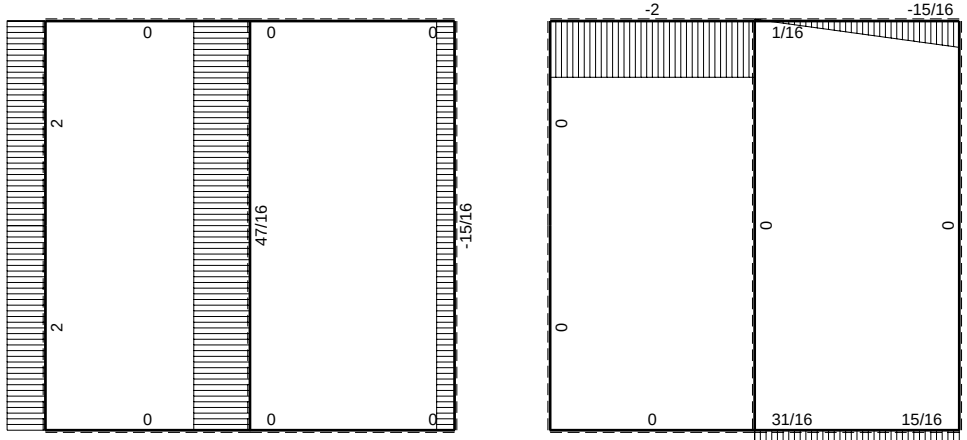
B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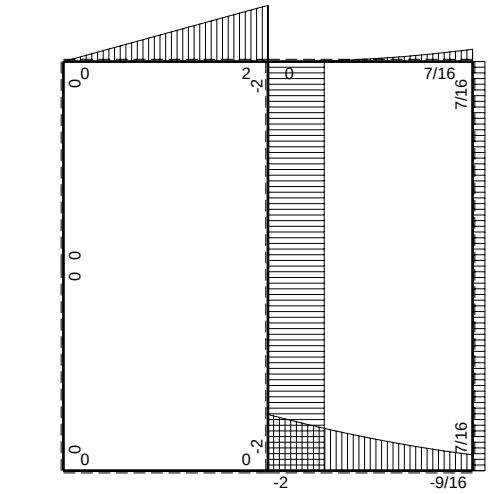
0
31/16
15/16

F F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-2
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-2
-9/16

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.066

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

14.11.23

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

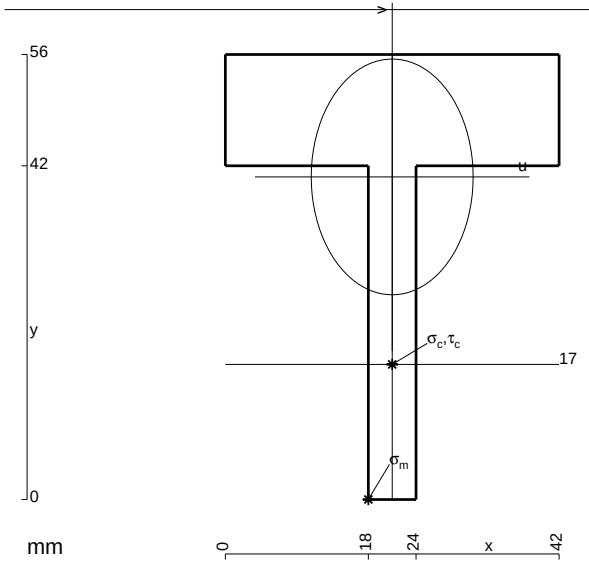
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.066



A = 840. mm^2
J_u = 184946. mm^4
J_v = 87192. mm^4
y_g = 40.6 mm
T_y = -1360. N
M_x = 952000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -40.6 mm
$\sigma_m = -Mv/J_u$ = 209. N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -23.6 mm
$\sigma_c = -Mv/J_u$ = 121.5 N/mm^2
τ_c = 4.013 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 121.7 N/mm^2
S^* = 3274. mm^3

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ESΣ8.066

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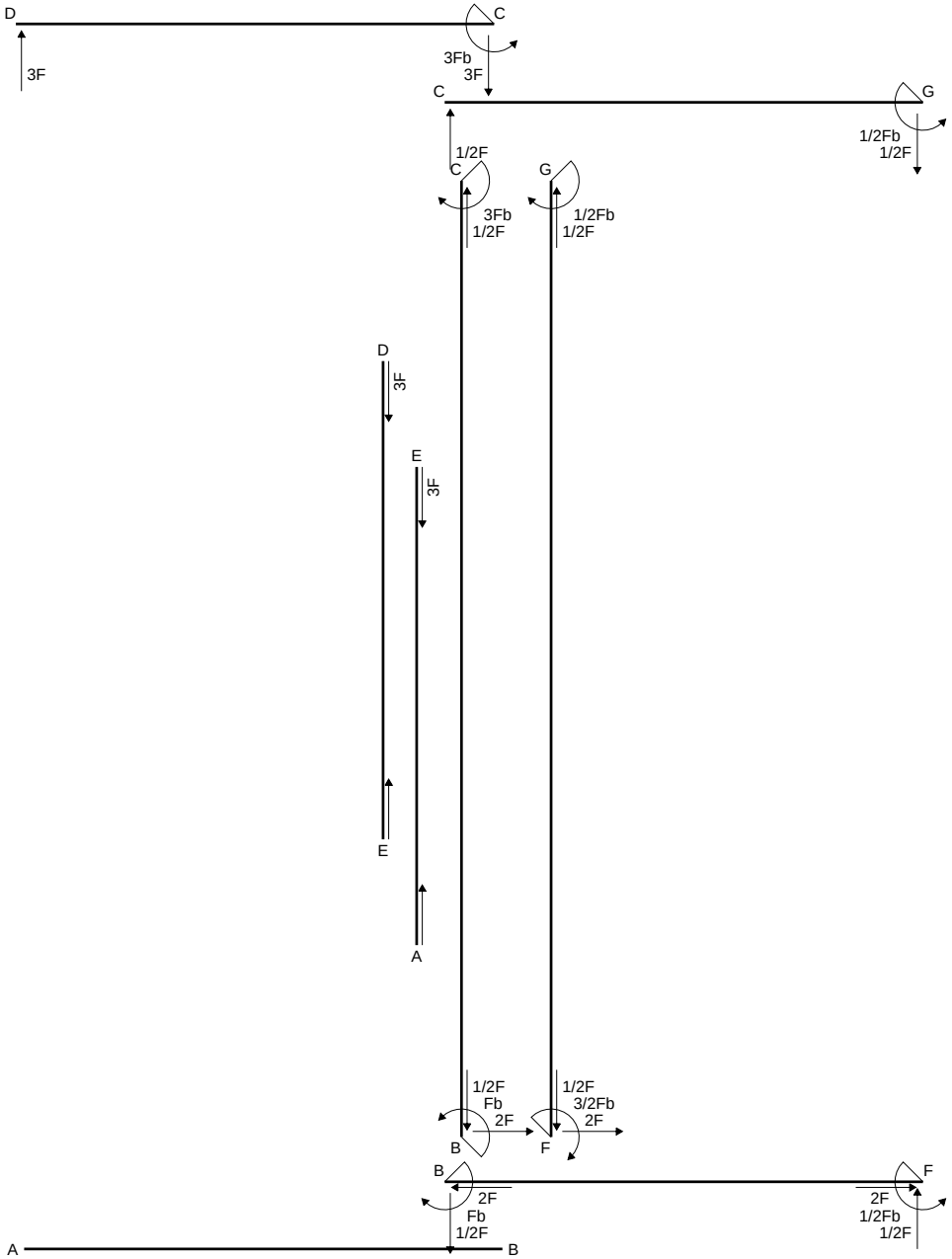
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3F
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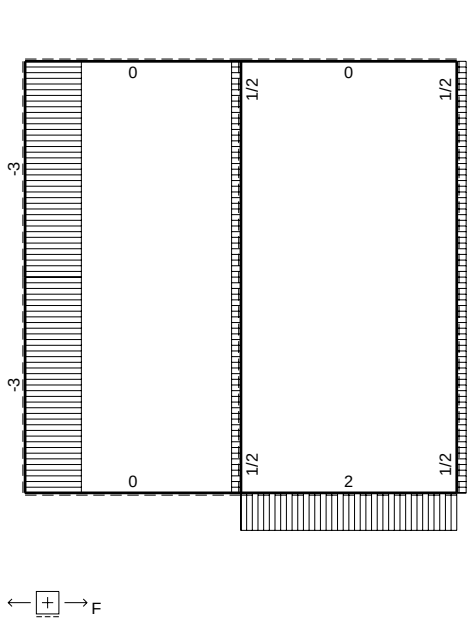
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B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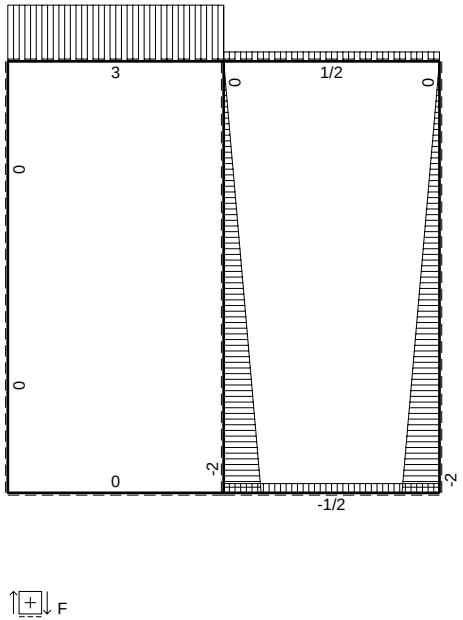
1/2
-3
-3
0
2
1/2
1/2
F


3
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0
0
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-2
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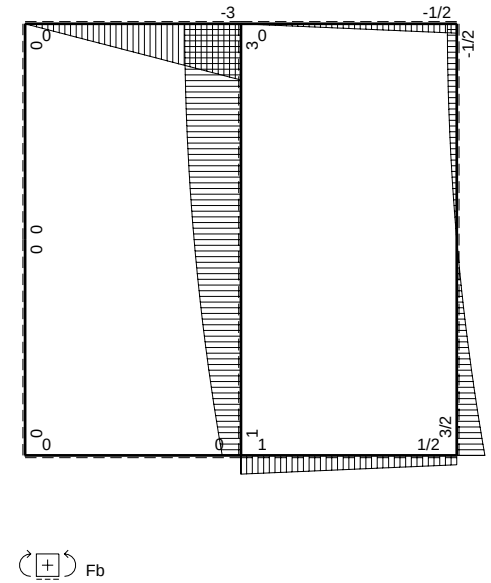
-1/2
F


-3
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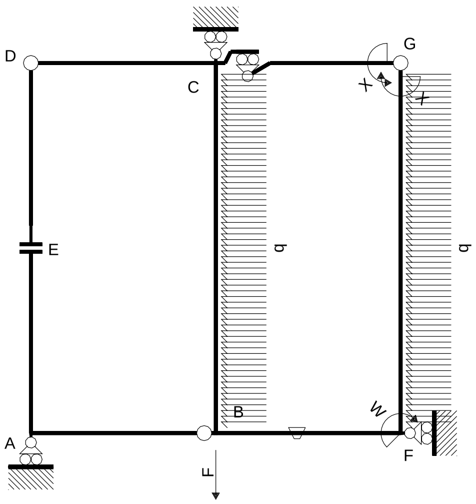
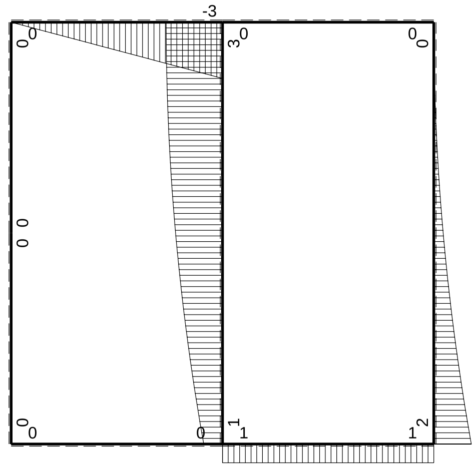
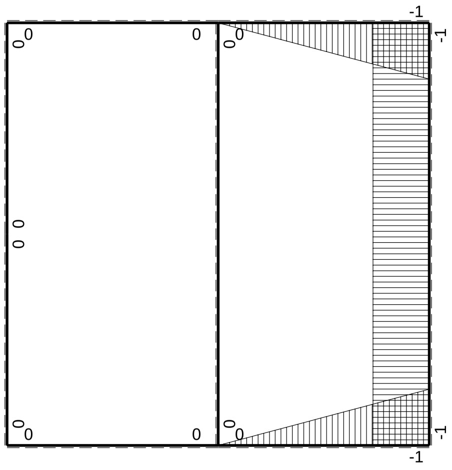
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

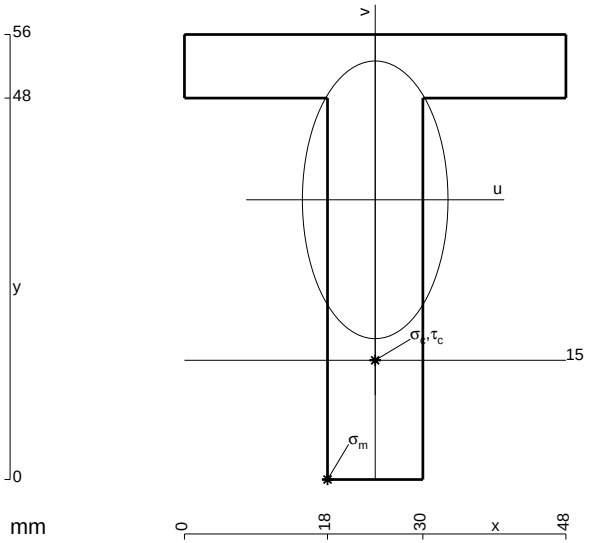
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 960. mm^2
J_u = 293274. mm^4
J_v = 80640. mm^4
y_g = 35.2 mm
T_y = 2460. N
M_x = -1820400. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.2 mm
$\sigma_m = -Mv/J_u$ = -218.5 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.2 mm
$\sigma_c = -Mv/J_u$ = -125.4 N/mm^2
τ_c = 3.485 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 125.5 N/mm^2
S^* = 4986. mm^3

ESΣ8.067

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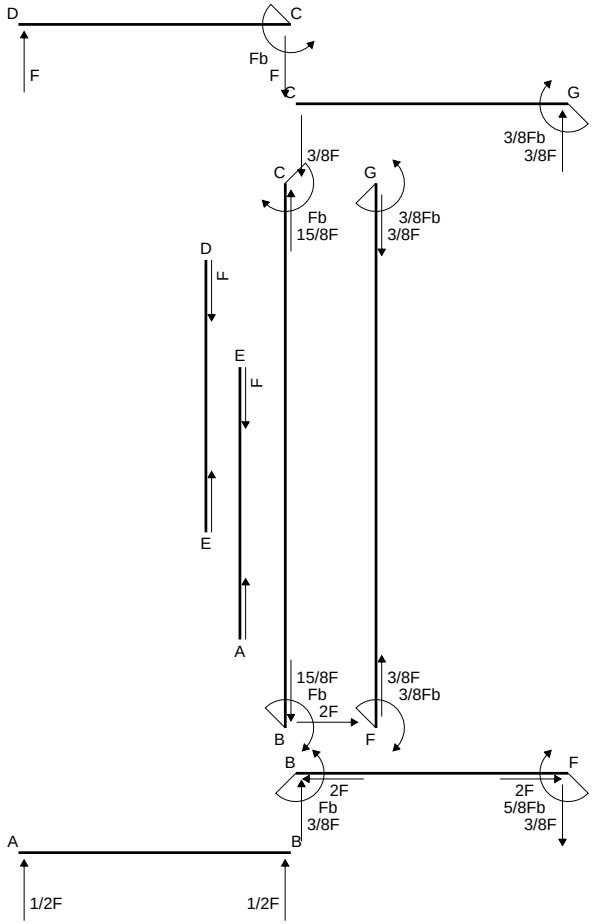
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15/8F
3/8Fb
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D
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E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
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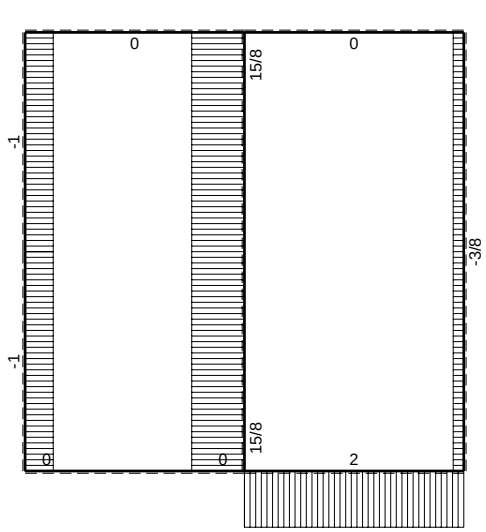
B
1/2F
1/2F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-3/8
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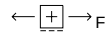
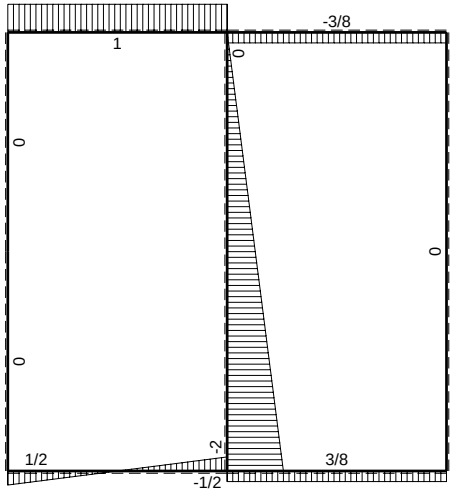
2


F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1/2
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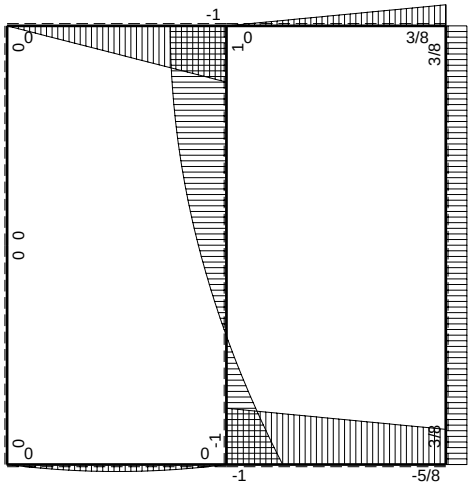
3/8
-1/2


F


-1
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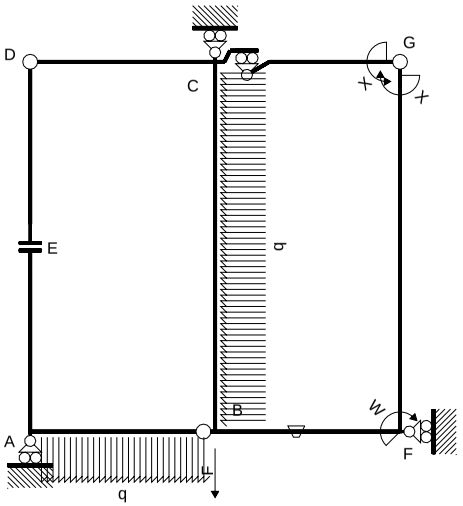
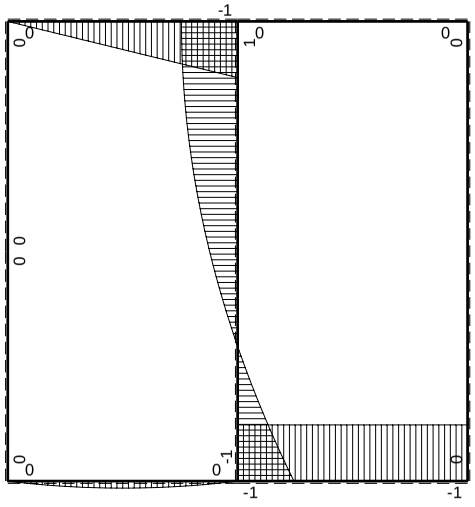
-1
-5/8

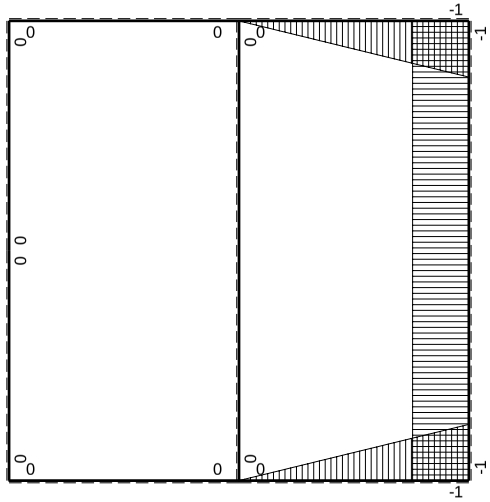

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.068

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

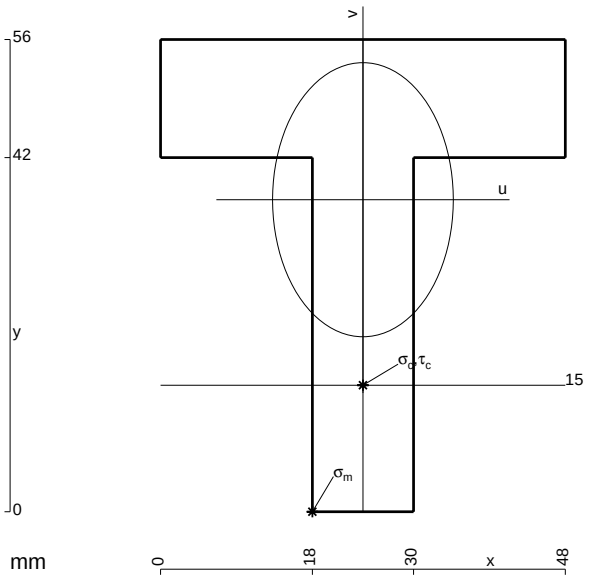
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1176. mm^2
J_u = 310856. mm^4
J_v = 135072. mm^4
y_g = 37. mm
N = 4650. N
T_y = -4960. N
M_x = -1959200. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -37. mm
$\sigma_m = N/A - Mv/J_u$ = -229.2 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -22. mm
$\sigma_c = N/A - Mv/J_u$ = -134.7 N/mm^2
τ_c = 7.061 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135.3 N/mm^2
S^* = 5310. mm^3

ESΣ8.068

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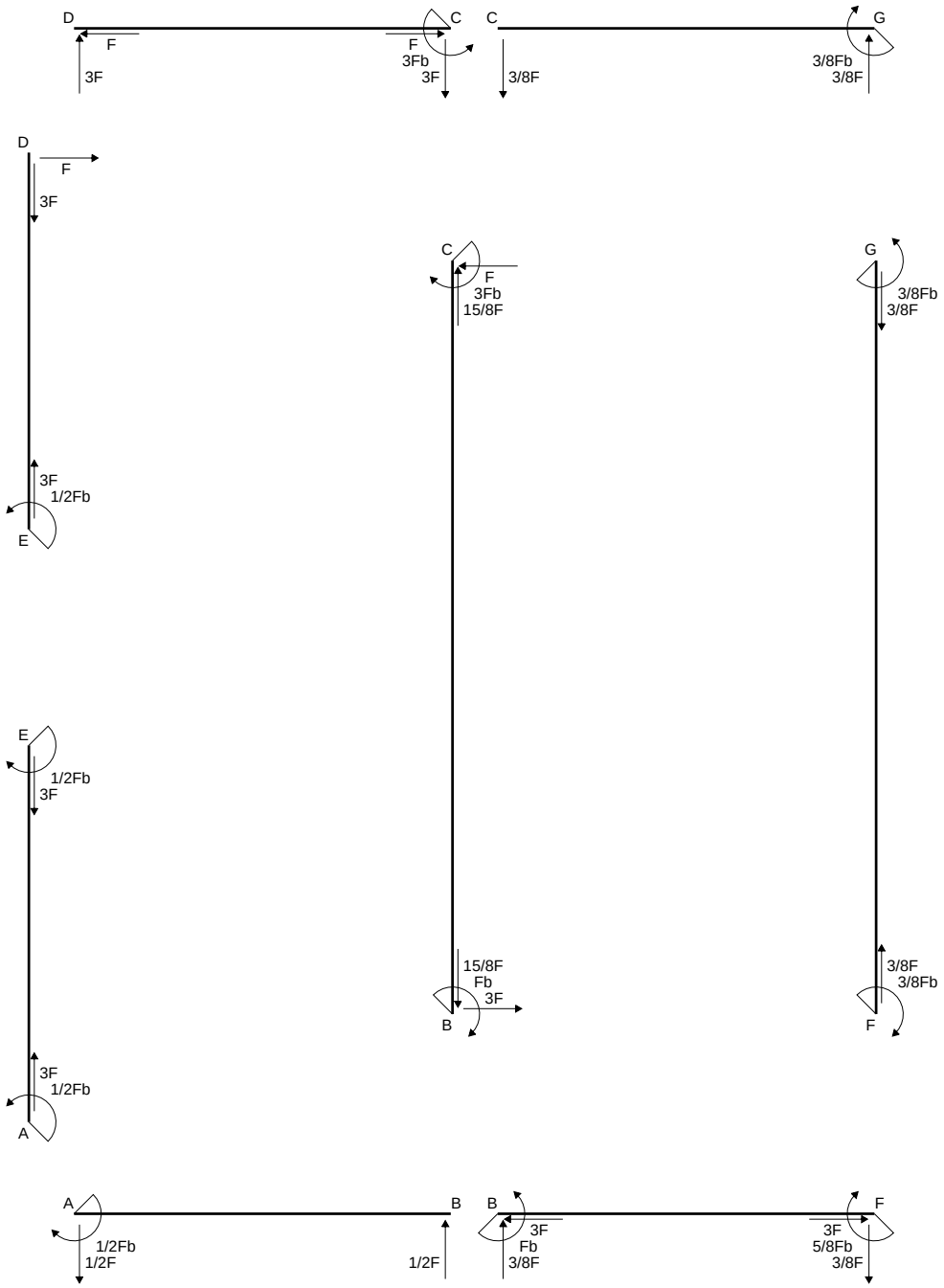
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G
3/8Fb
3/8F
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15/8F
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3F
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3/8F
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3F
1/2Fb
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3/8F
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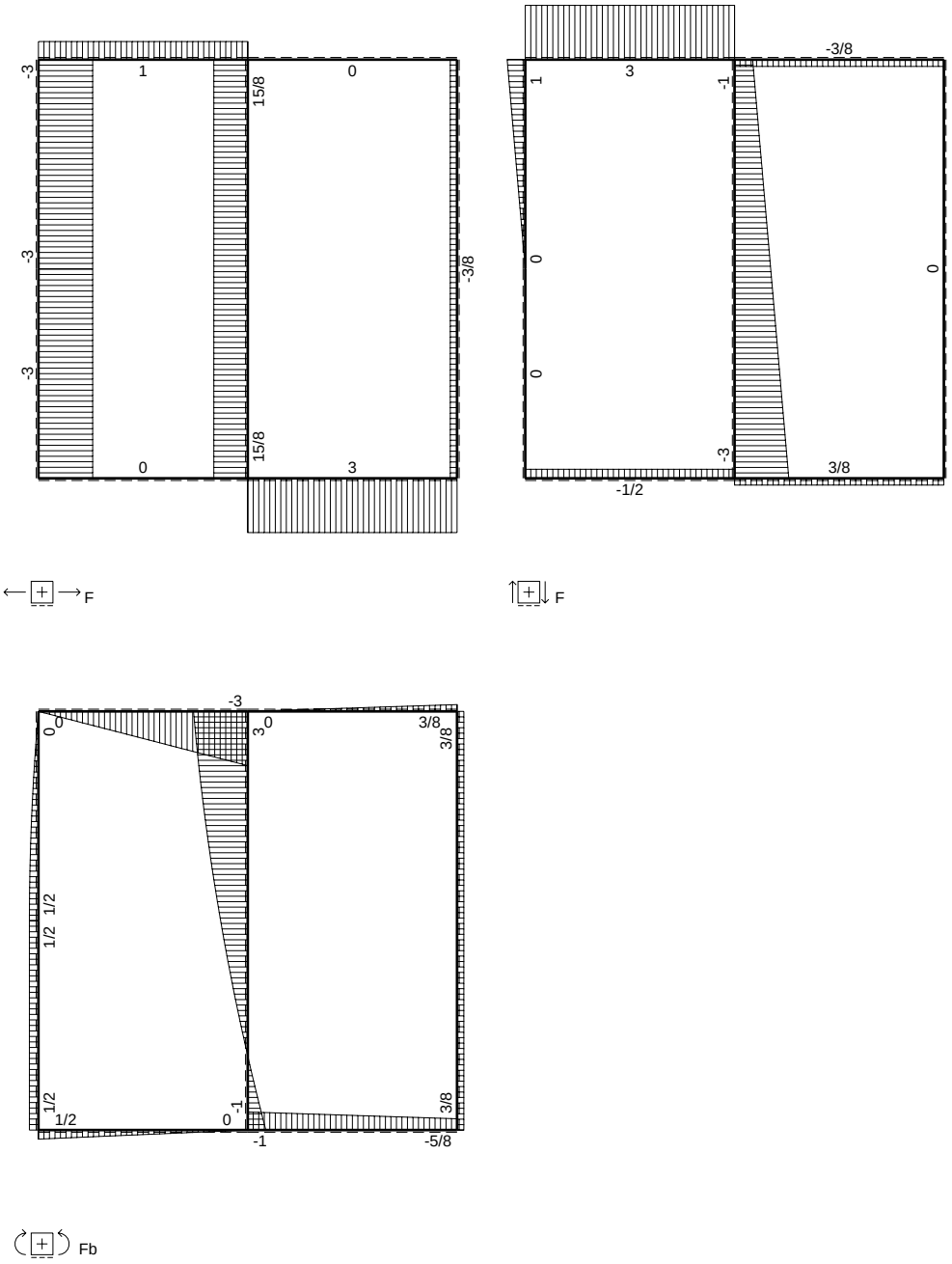
3F
5/8Fb
3/8F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-3/8
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-1/2
F
-3
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0
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1/2
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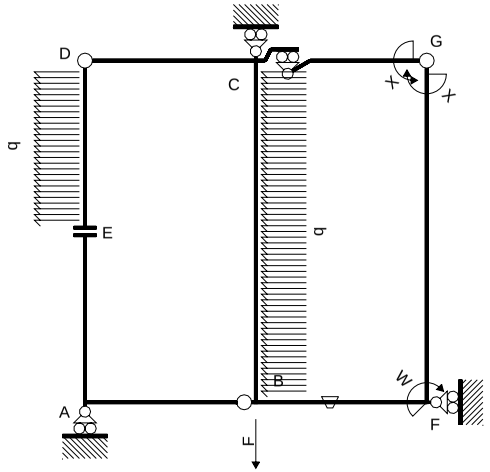
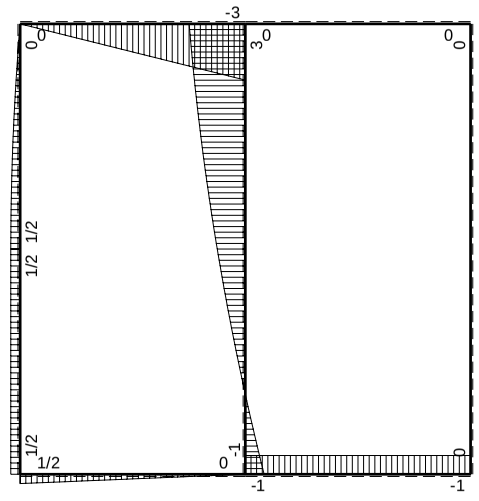
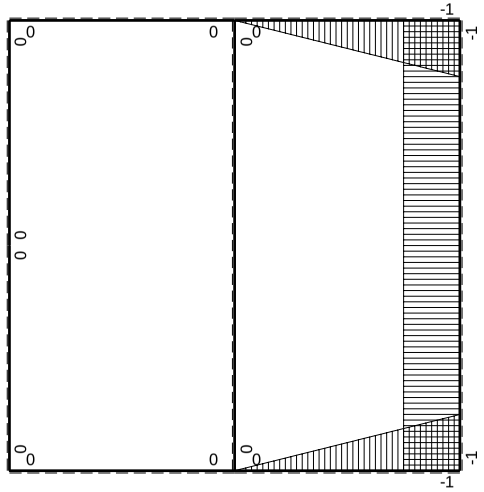
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
				totali			Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.069

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

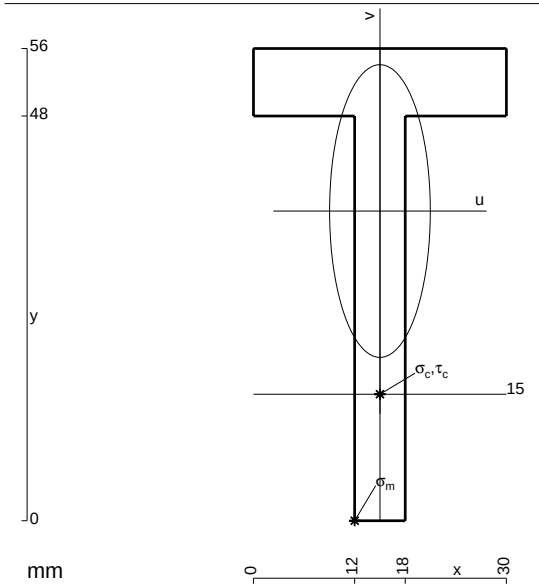
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 528. mm^2
J_u = 159209. mm^4
J_v = 18864. mm^4
y_g = 36.73 mm
N = 410. N
T_y = 1230. N
M_x = -1020900. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.73 mm
$\sigma_m = N/A - Mv/J_u$ = -234.7 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.73 mm
$\sigma_c = N/A - Mv/J_u$ = -138.5 N/mm^2
τ_c = 3.387 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.7 N/mm^2
S^* = 2630. mm^3

ESΣ8.069

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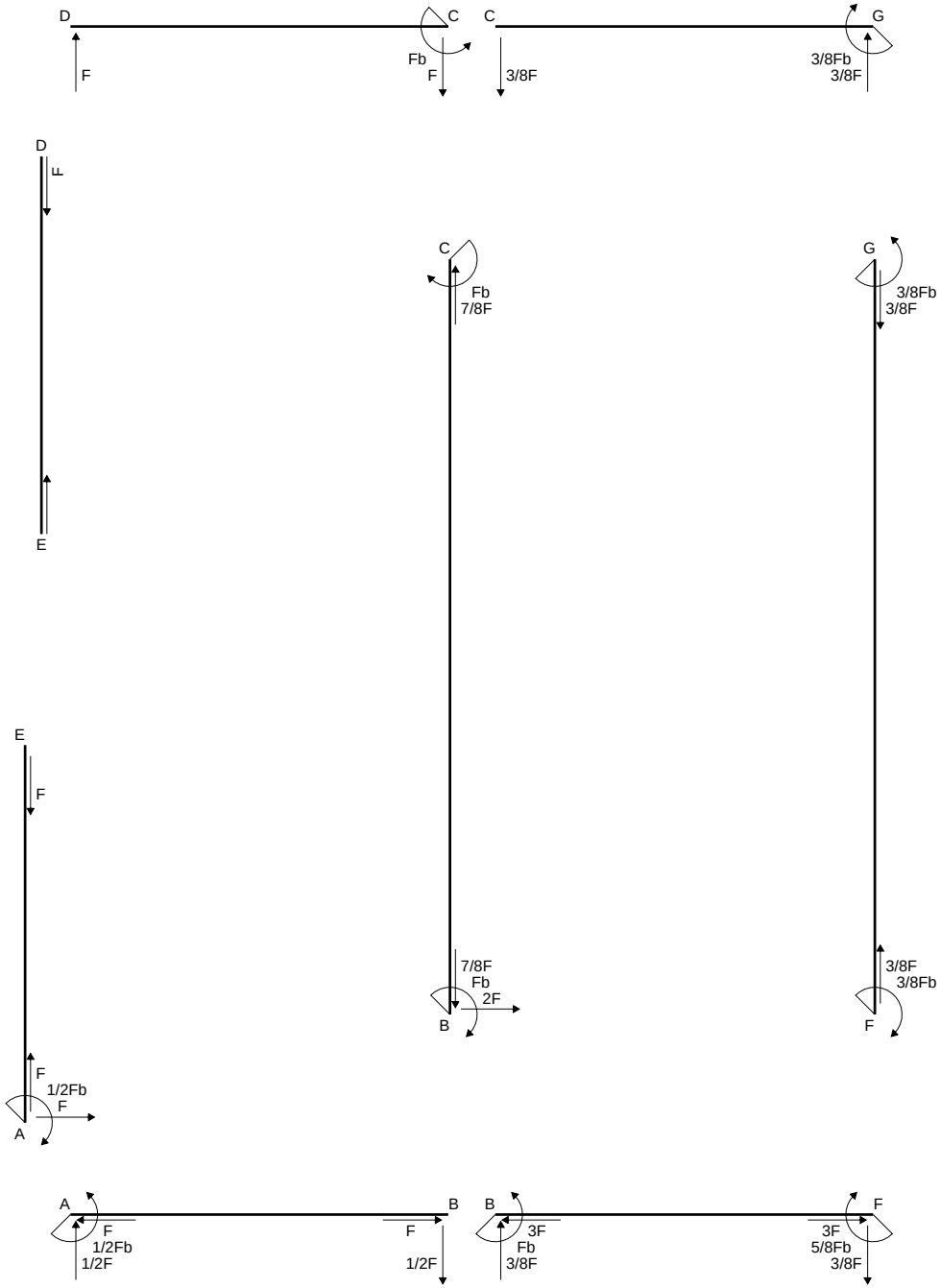
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C
G
F
Fb
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3/8F
3/8Fb
3/8F
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F
E
C
Fb
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G
3/8Fb
3/8F
E
F
F
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F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
B
F
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1/2F
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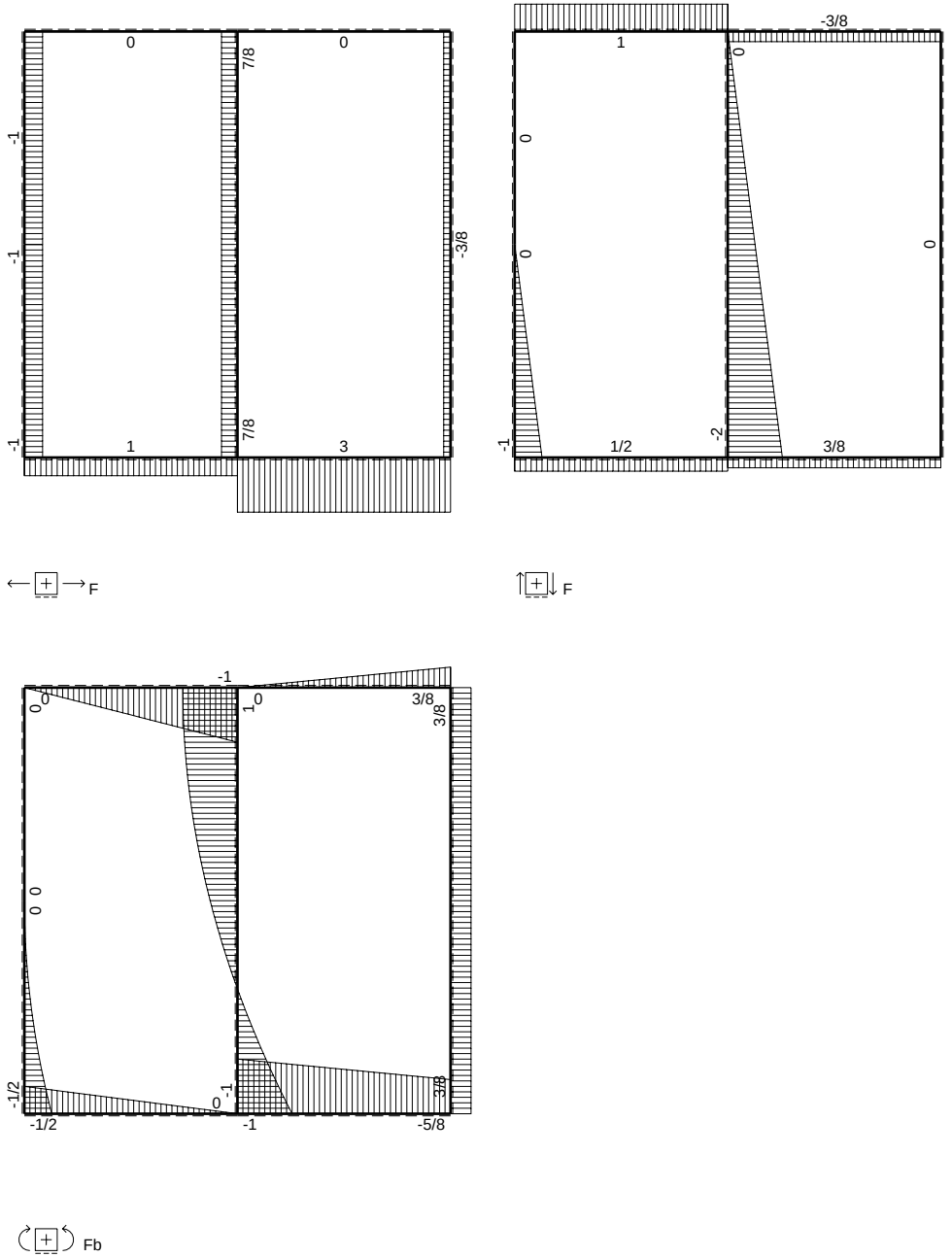
Fb
3/8F
3F
5/8Fb
3/8F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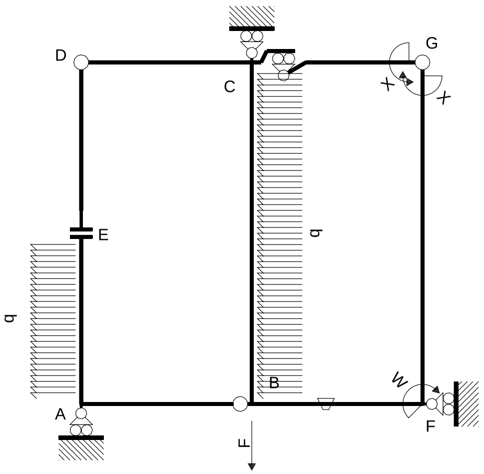
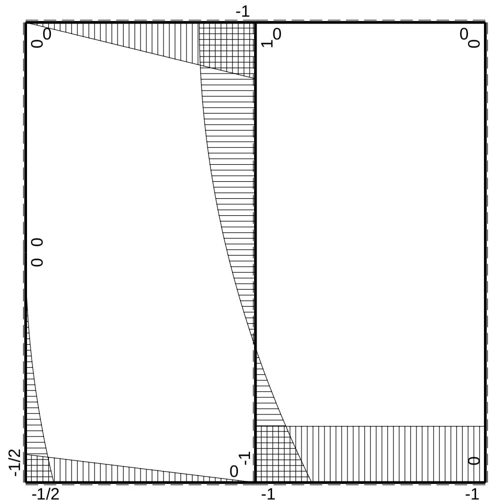
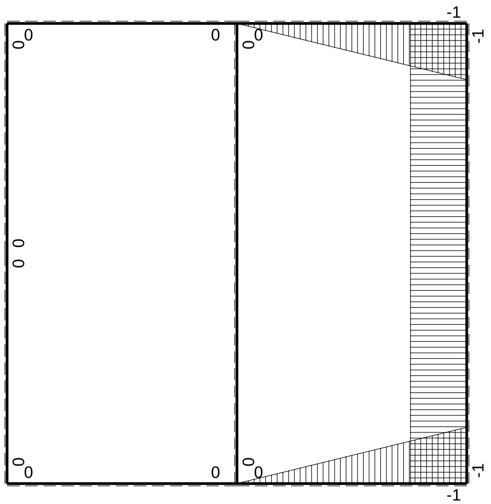
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

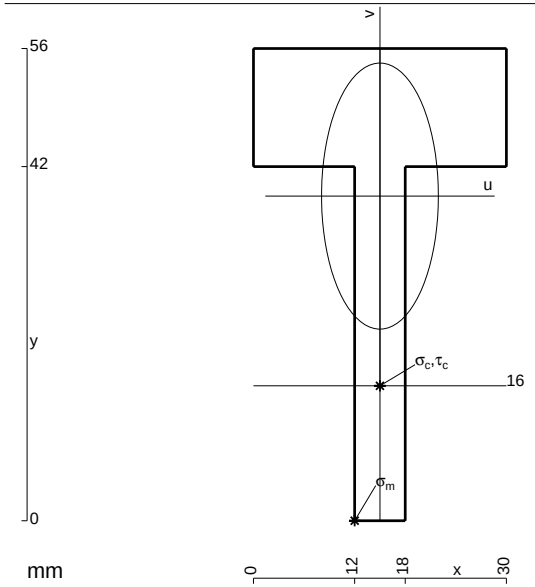
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 672. mm^2
J_u = 167384. mm^4
J_v = 32256. mm^4
y_g = 38.5 mm
N = 1750. N
T_y = -4000. N
M_x = -880000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -38.5 mm
$\sigma_m = N/A - Mv/J_u$ = -199.8 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -22.5 mm
$\sigma_c = N/A - Mv/J_u$ = -115.7 N/mm^2
τ_c = 11.66 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.4 N/mm^2
S^* = 2928. mm^3

ESΣ8.070

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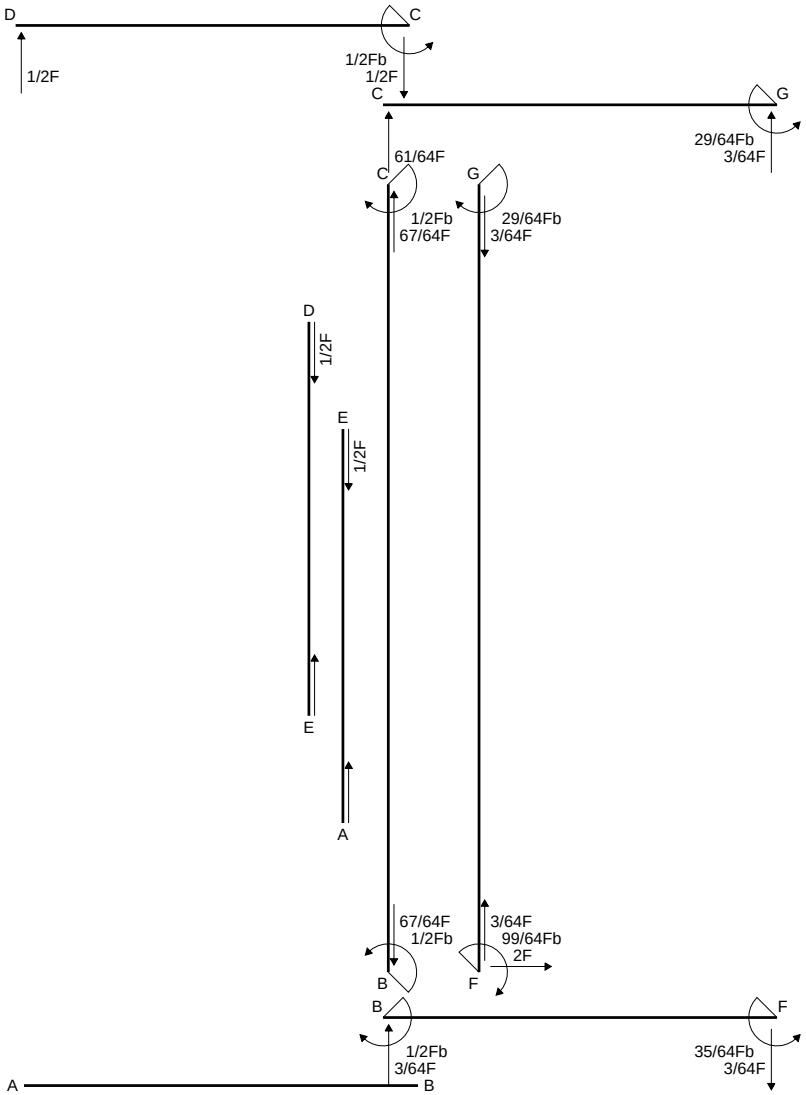
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61/64F
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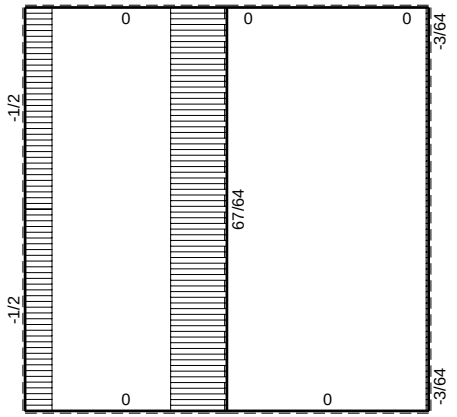
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B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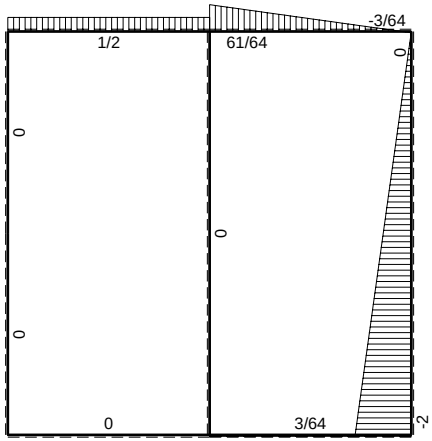
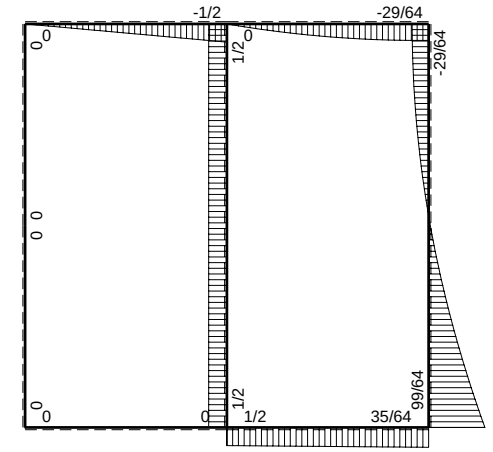
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx²/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb²/EJ	1/3XLb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb²/EJ	1/3XLb/EJ
CG b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2XLb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb²/EJ	8/3XLb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

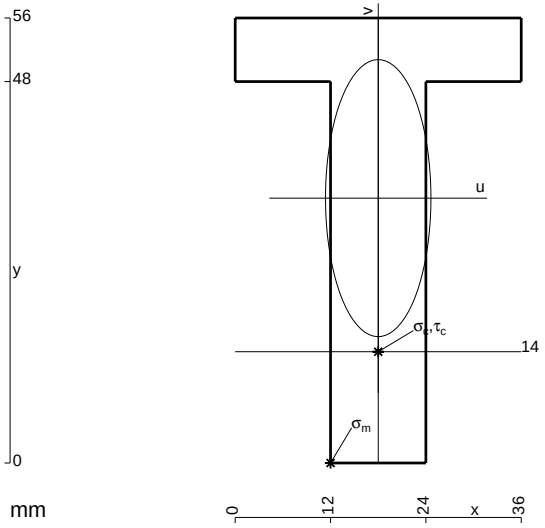
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.071



A = 864. mm^2
J_u = 262656. mm^4
J_v = 38016. mm^4
y_g = 33.33 mm
T_y = 3445. N
M_x = -1653600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.33 mm
$\sigma_m = -Mv/J_u$ = -209.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.33 mm
$\sigma_c = -Mv/J_u$ = -121.7 N/mm^2
τ_c = 4.835 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122. N/mm^2
S^* = 4424. mm^3

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ESΣ8.071

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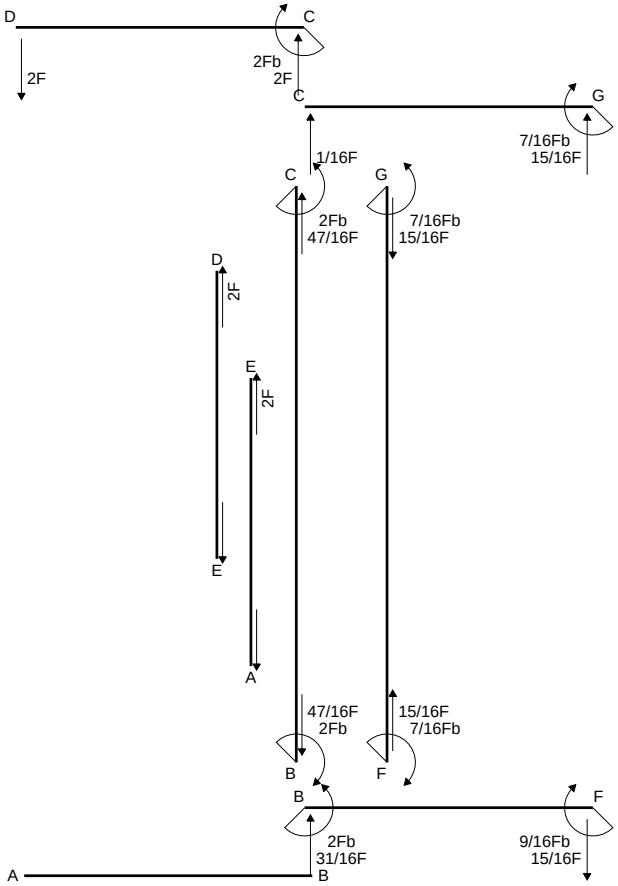
D
C
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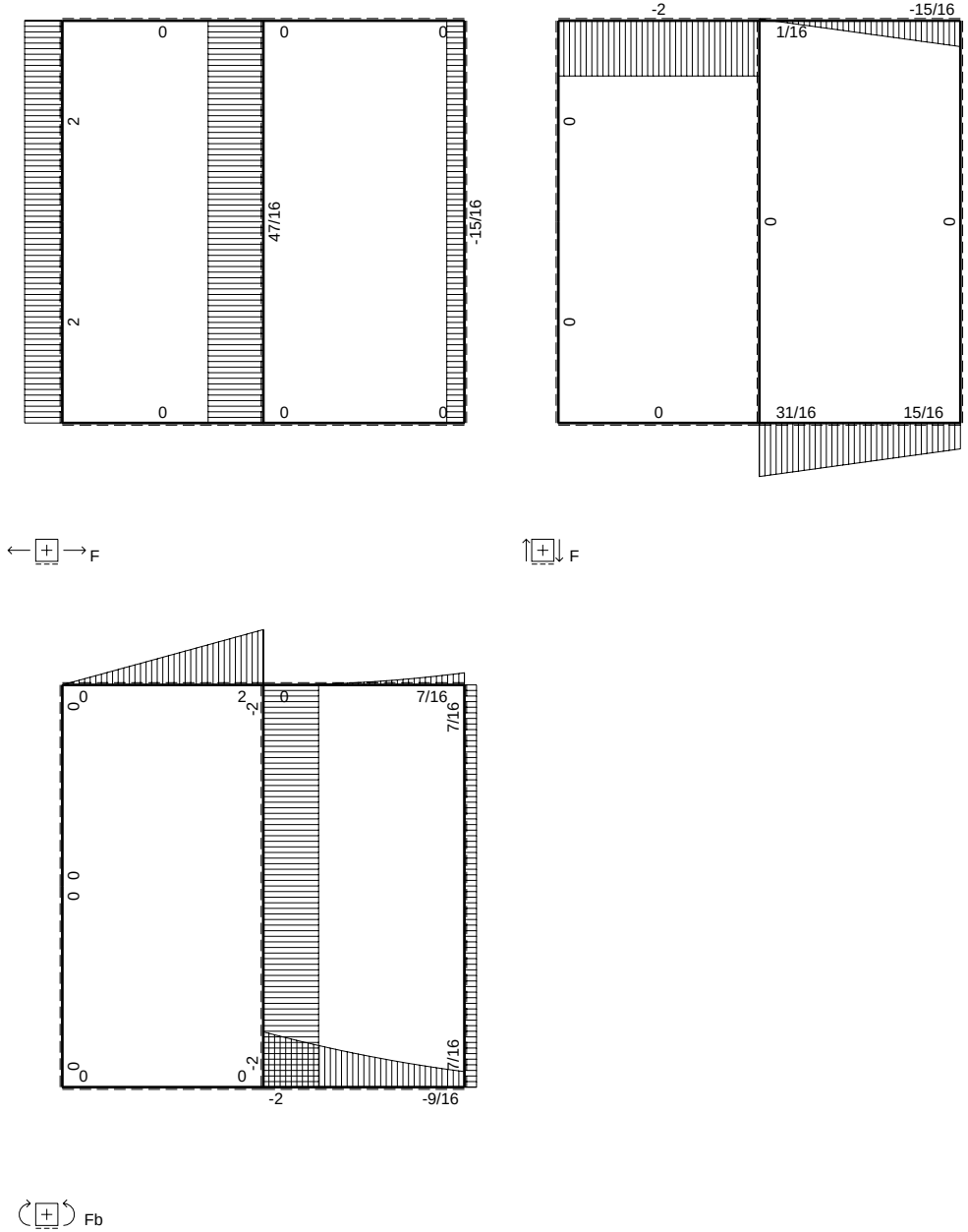
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
0
0
2
47/16
-15/16
2
0
0
0
-2
-15/16
1/16
0
0
0
0
0
31/16
15/16
F
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$\rightarrow$	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b		0	0	0	0	0	0	0+0	0
BA b		0	0	0	0	0	0		
CD b		0	2Fb-2Fx	0	0	0	0	0+0	0
DC b		0	-2Fx	0	0	0	0		
DE b		0	0	0	0	0	0	0+0	0
ED b		0	0	0	0	0	0		
EA b		0	0	0	0	0	0	0+0	0
AE b		0	0	0	0	0	0		
BF b		-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3XLb/EJ
FB b		1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b		-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3XLb/EJ
CG b		x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b		-1	0	0	0	0	1	0+0	2XLb/EJ
GF 2b		1	0	0	0	0	1		
CB 2b		0	-2Fb	0	0	0	0	0+0	0
BC 2b		0	2Fb	0	0	0	0		
		totali						$7/6Fb^2/EJ$	8/3XLb/EJ
		iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

Schema di calcolo iperstatico

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

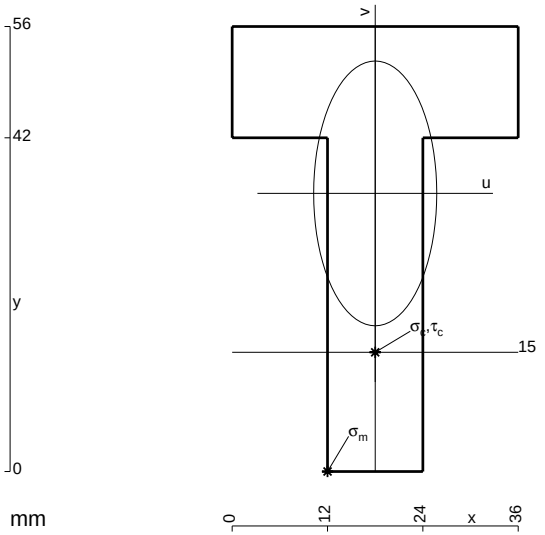
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 1008. mm^2
J_u = 279888. mm^4
J_v = 60480. mm^4
y_g = 35. mm
T_y = -3380. N
M_x = 1757600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -35. mm
$\sigma_m = -Mv/J_u$ = 219.8 N/mm^2
x_c = 18. mm
y_c = 15. mm
v_c = -20. mm
$\sigma_c = -Mv/J_u$ = 125.6 N/mm^2
τ_c = 4.981 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 125.9 N/mm^2
S^* = 4950. mm^3

ESΣ8.072

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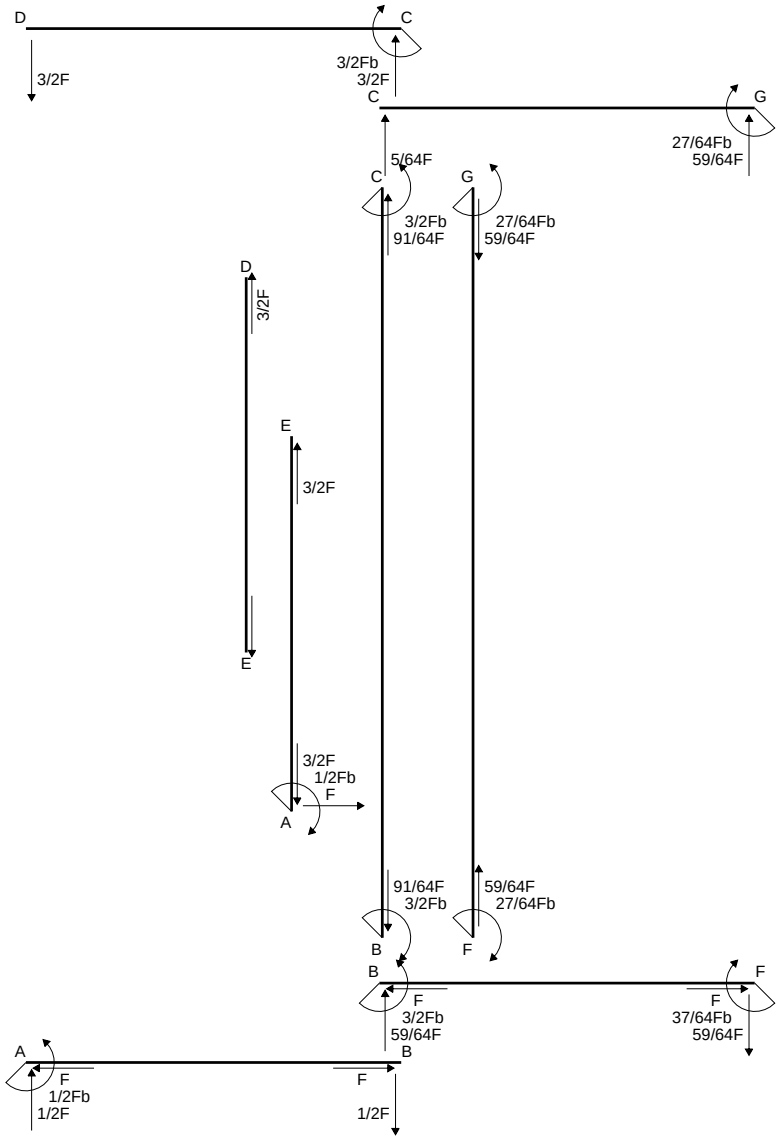
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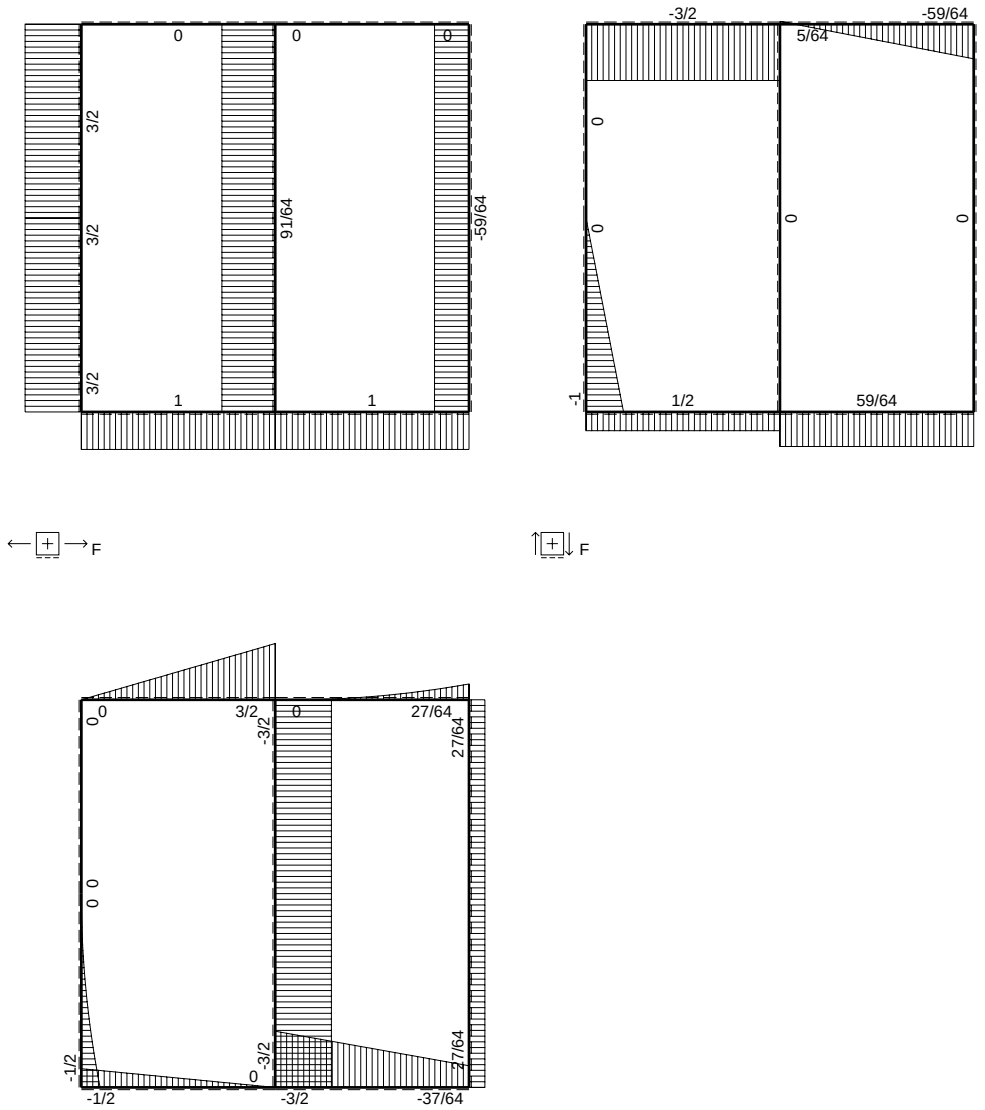
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
3/2Fb
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F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3/2
3/2
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91/64
-59/64
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1
F
-3/2
-59/64
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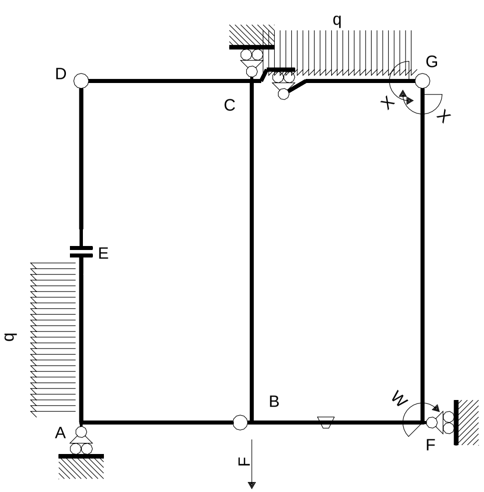
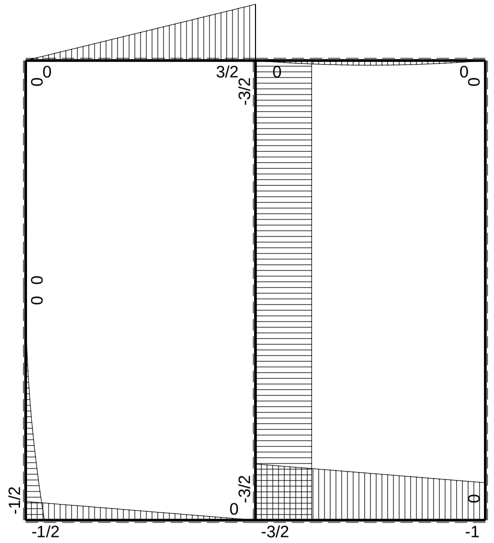
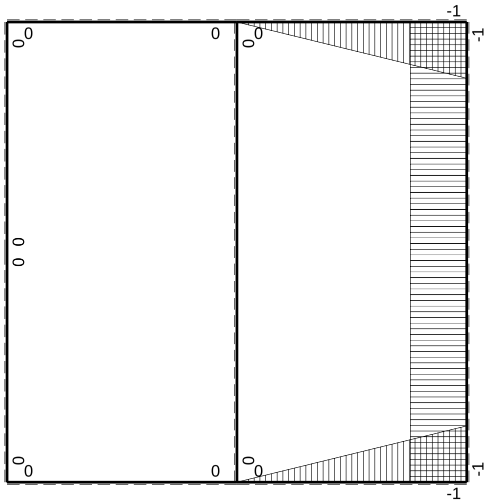
0
0
0
0
-1
1/2
59/64
F
0 0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-3/2
27/64
-1/2
-1/2
-3/2
-37/64
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0
DC b	0	$-3/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	$-3/2Fb+1/2Fx$	$-Fb/EJ$	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$Fb+1/2Fx$	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$-3/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$3/2Fb$	0	0	0	0		
	totali						$9/8Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$-27/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

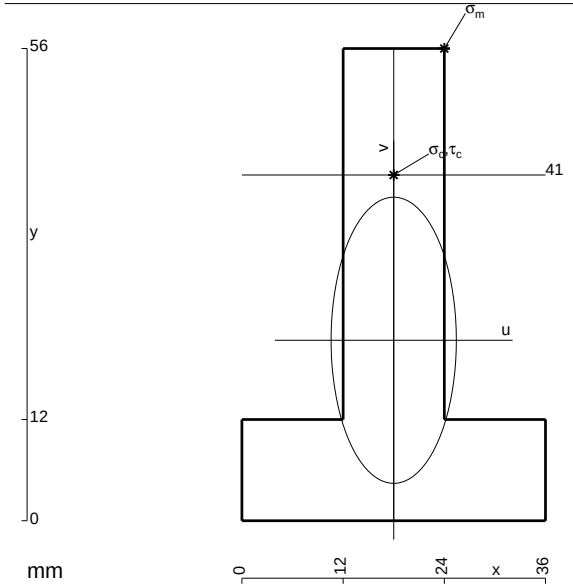
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 960. mm^2
J_u = 276646. mm^4
J_v = 52992. mm^4
y_g = 21.4 mm
T_y = -3225. N
M_x = 1838250. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 34.6 mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.6 mm
$\sigma_c = -Mv/J_u$ = -130.2 N/mm^2
τ_c = 4.739 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.5 N/mm^2
S^* = 4878. mm^3

ESΣ8.073

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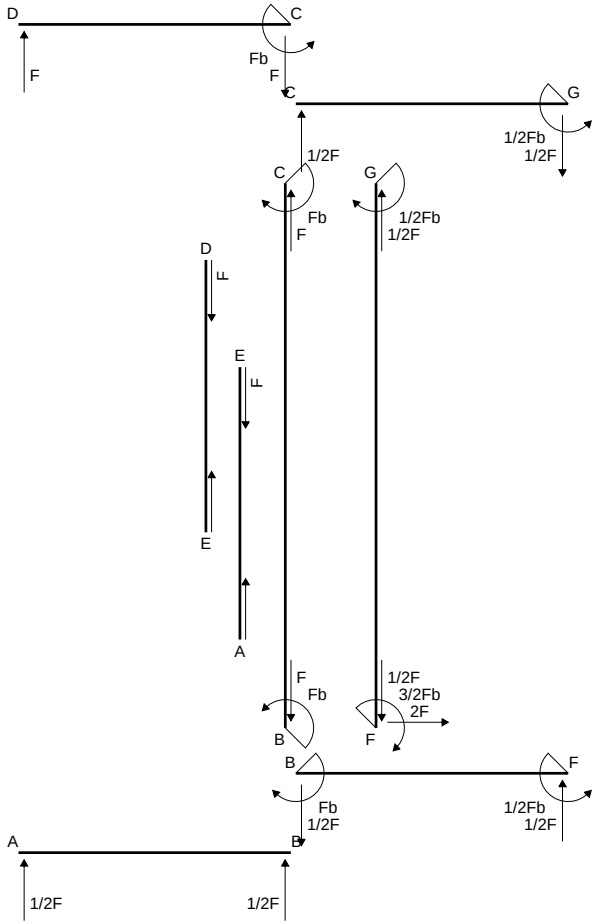
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G
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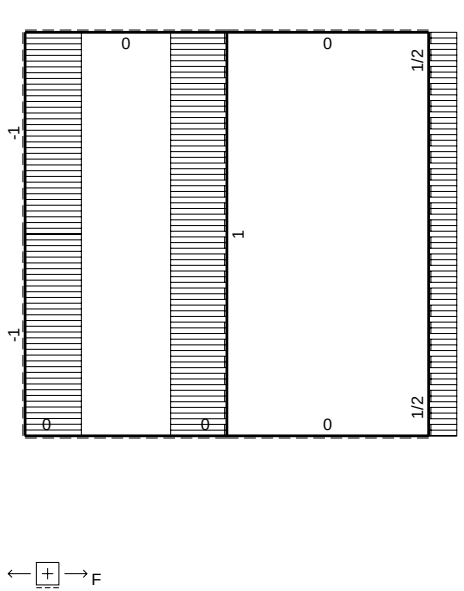
A
B
1/2F
1/2F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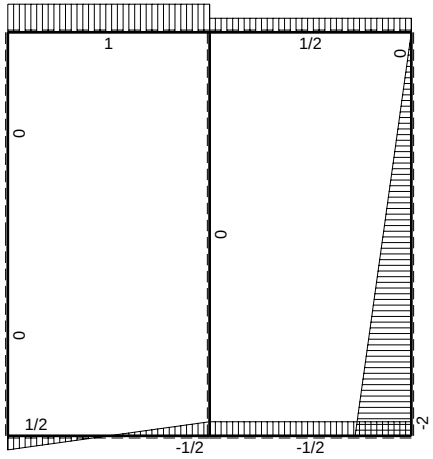
1/2

F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1/2
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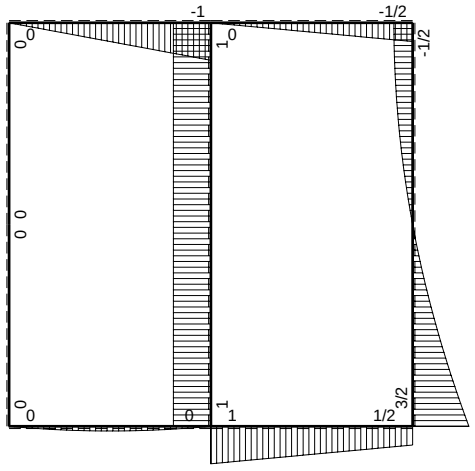
-1/2
-1/2

F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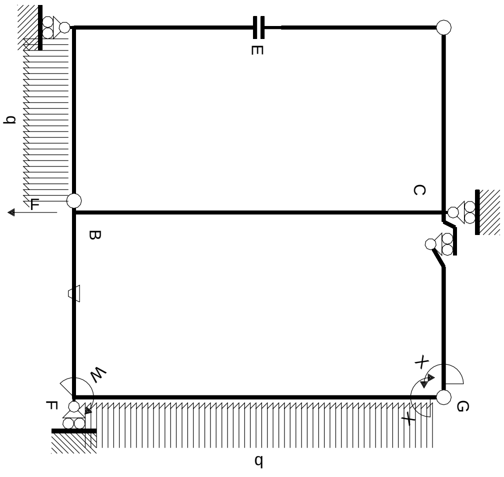
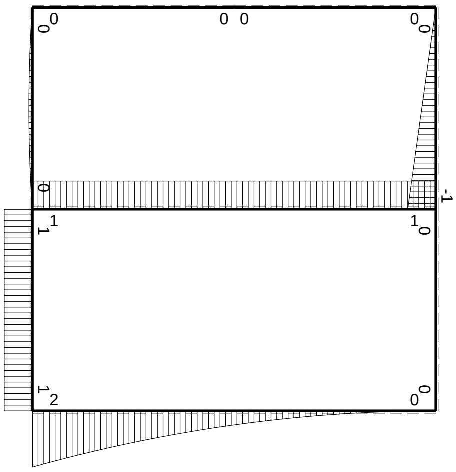
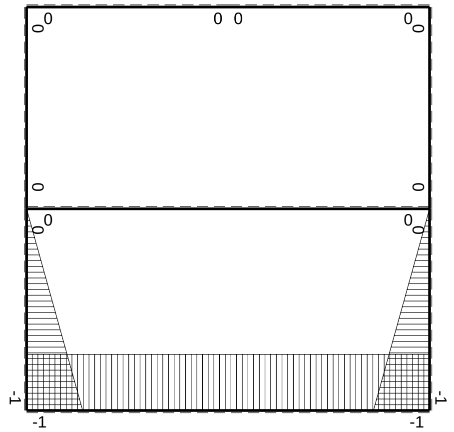
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

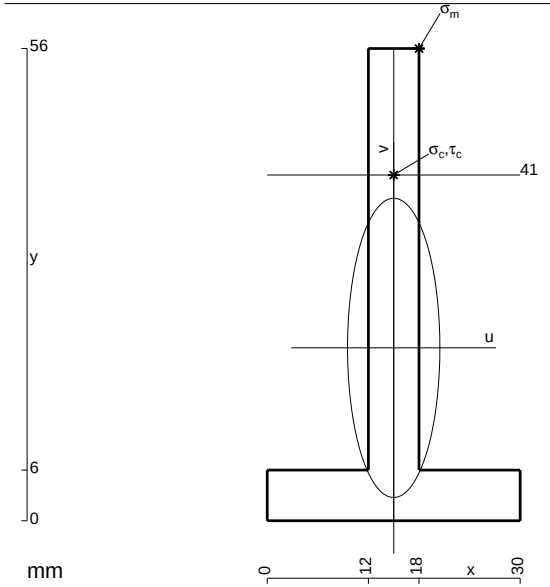
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.074



A = 480. mm^2
J_u = 151240. mm^4
J_v = 14400. mm^4
y_g = 20.5 mm
T_y = 1670. N
M_x = -1018700. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 35.5 mm
$\sigma_m = -Mv/J_u$ = 239.1 N/mm^2
x_c = 15. mm
y_c = 41. mm
v_c = 20.5 mm
$\sigma_c = -Mv/J_u$ = 138.1 N/mm^2
τ_c = 4.638 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.3 N/mm^2
S^* = 2520. mm^3

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ESΣ8.074

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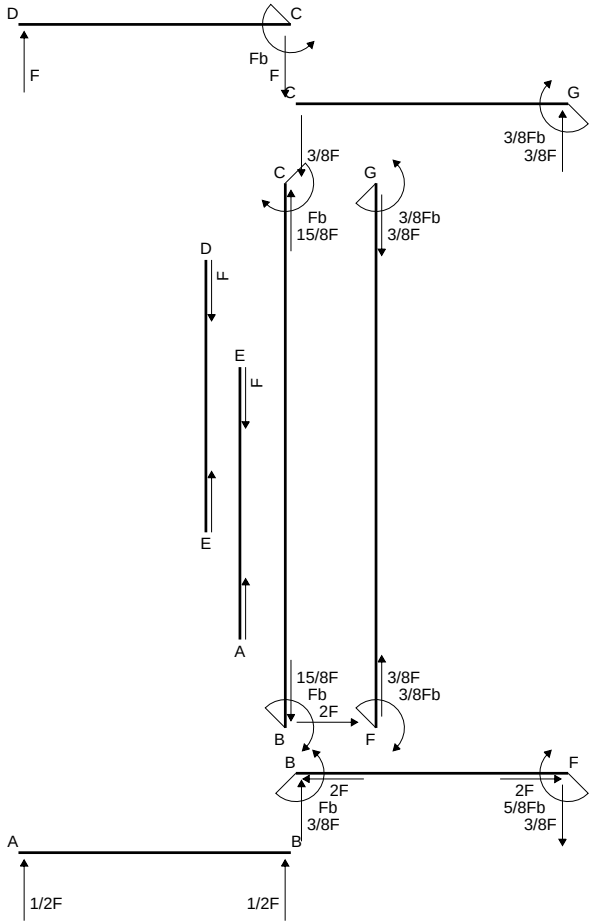
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C
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3/8F
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G
Fb
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3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
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2F
Fb
3/8F
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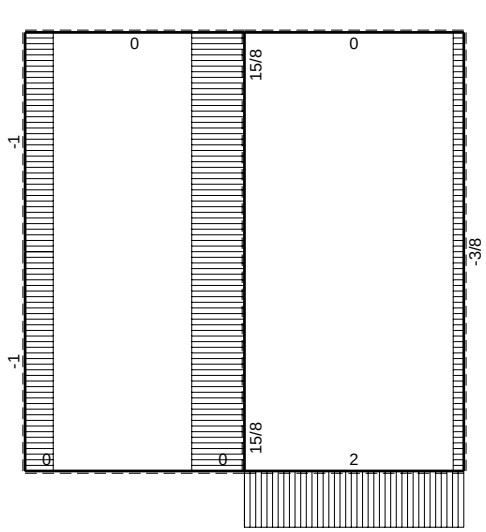
A
B
1/2F
1/2F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15/8
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-1
-3/8
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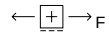
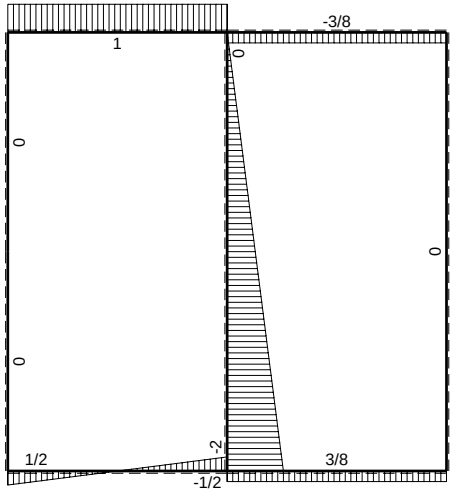
0
15/8
0
0
2


F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1/2
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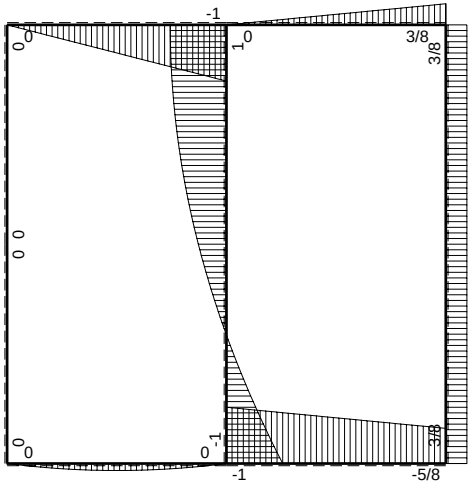
3/8
-1/2


F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3/8
3/8
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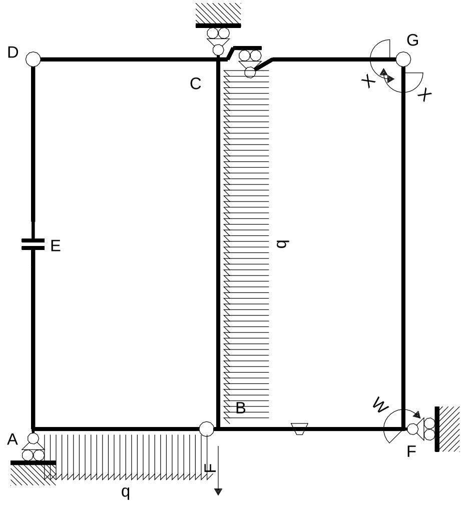
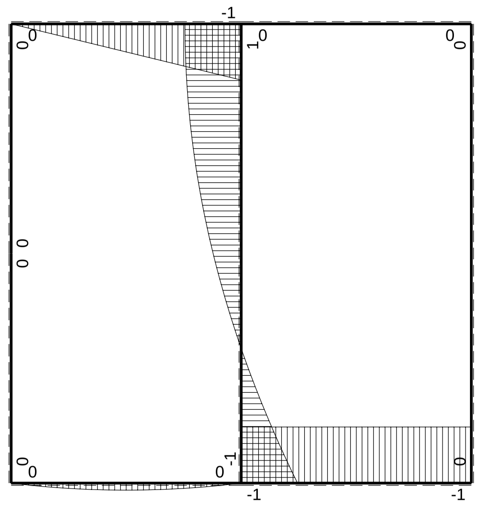
-1
0
-1
3/8
-1
-5/8

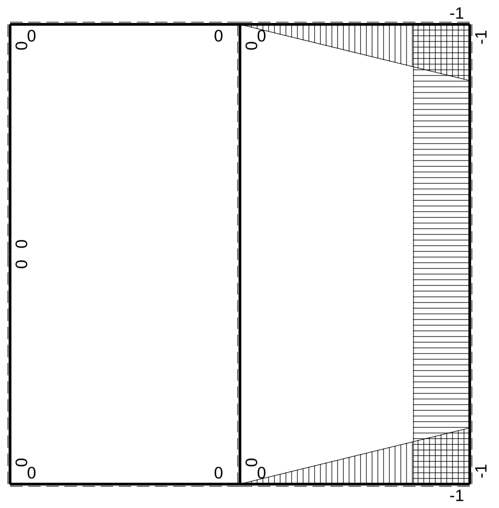

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

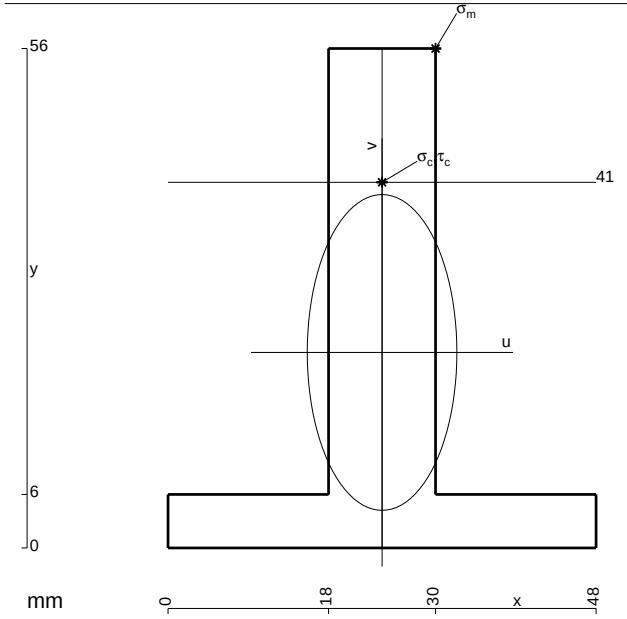
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 888. mm^2
J_u = 278426. mm^4
J_v = 62496. mm^4
y_g = 21.92 mm
N = 4519. N
T_y = -4820. N
M_x = -1590600. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 34.08 mm
$\sigma_m = N/A - Mv/J_u$ = 199.8 N/mm^2
x_c = 24. mm
y_c = 41. mm
v_c = 19.08 mm
$\sigma_c = N/A - Mv/J_u$ = 114.1 N/mm^2
τ_c = 6.902 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.7 N/mm^2
S^* = 4785. mm^3

ESΣ8.075

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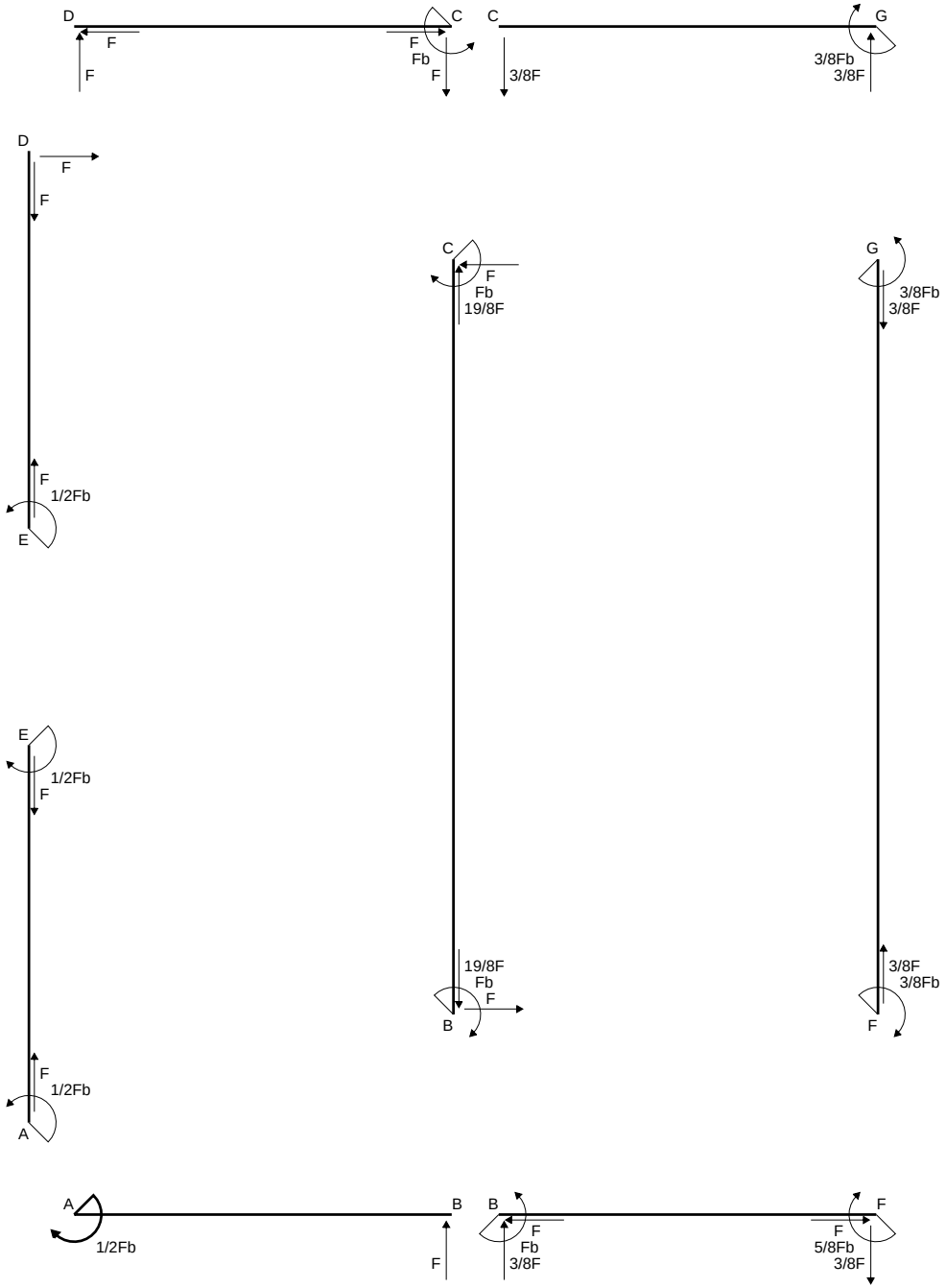
D
C
C
G
F
F
F
Fb
F
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3/8Fb
3/8F
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C
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19/8F
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19/8F
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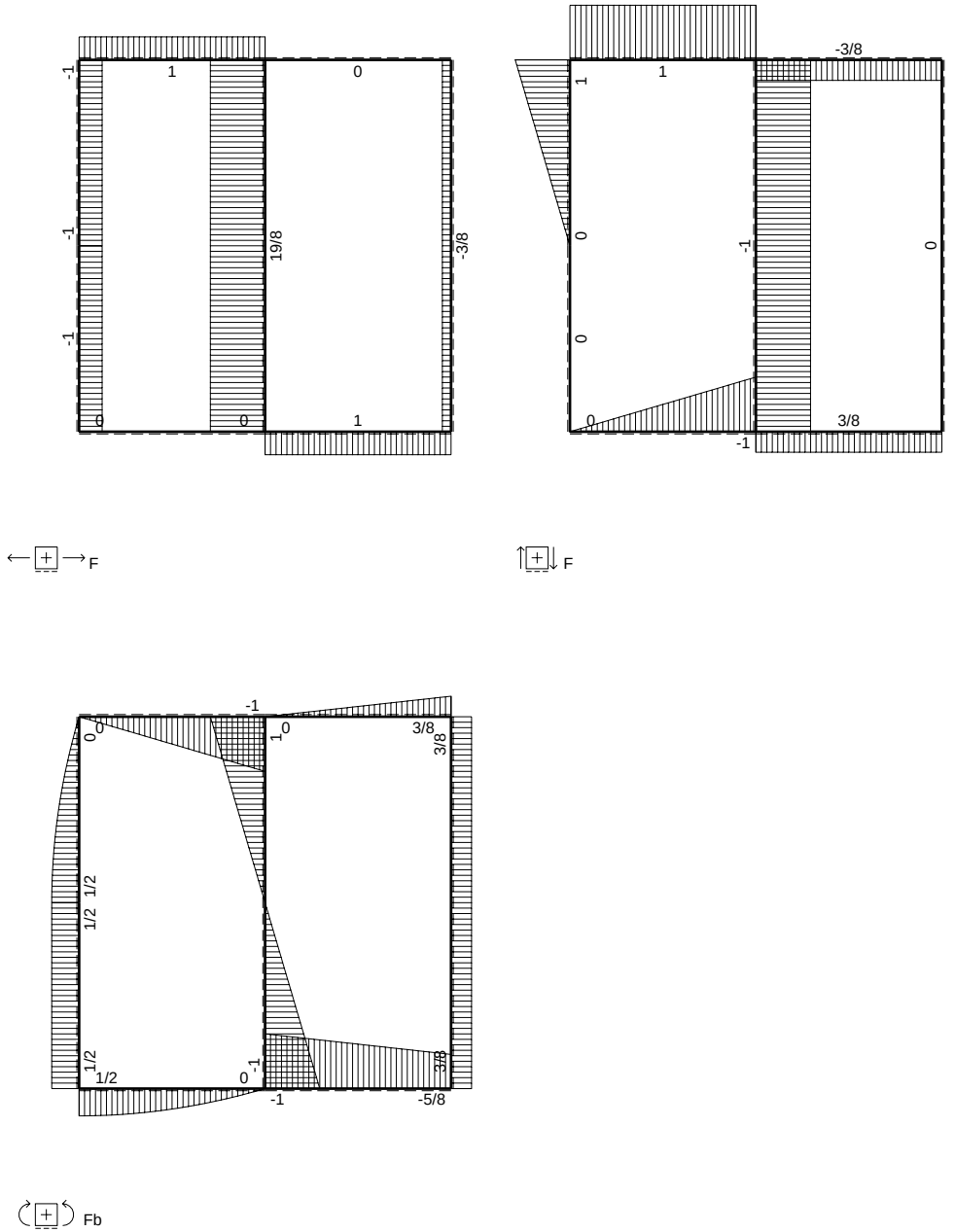
F
5/8Fb
3/8F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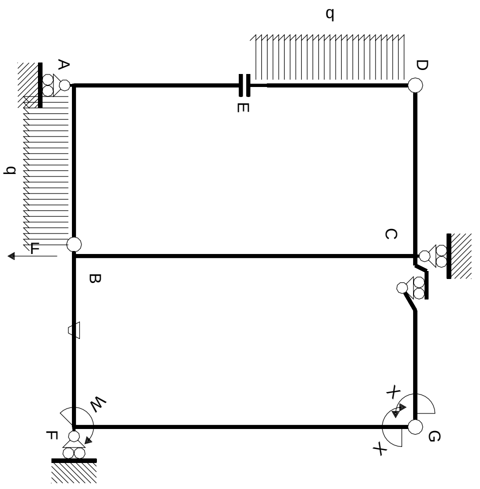
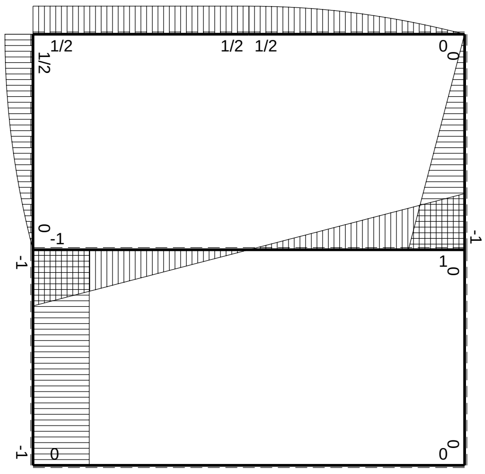
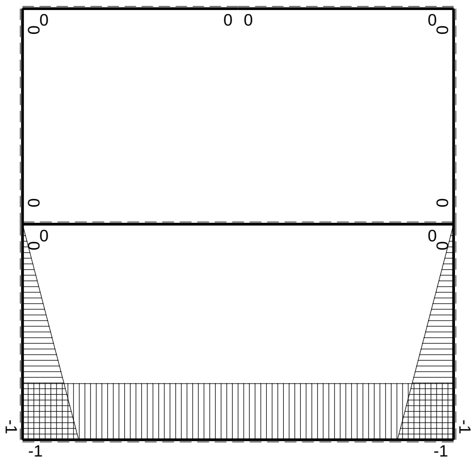
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

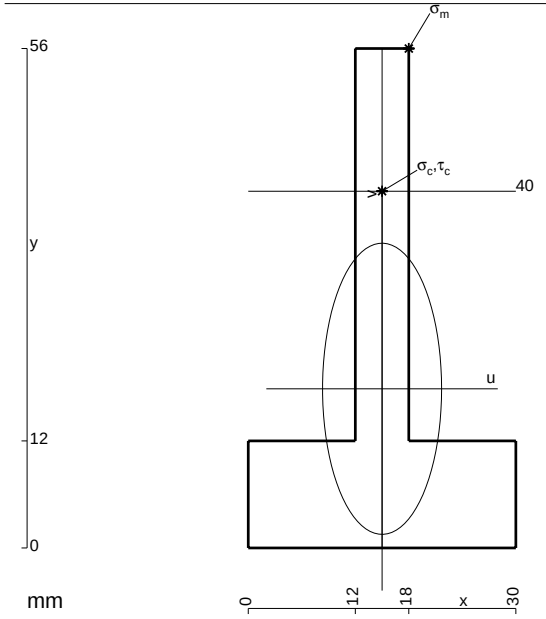
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 624. mm^2
J_u = 166321. mm^4
J_v = 27792. mm^4
y_g = 17.85 mm
N = 3016. N
T_y = -1270. N
M_x = -889000. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 38.15 mm
$\sigma_m = N/A - Mv/J_u$ = 208.8 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 22.15 mm
$\sigma_c = N/A - Mv/J_u$ = 123.2 N/mm^2
τ_c = 3.684 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.4 N/mm^2
S^* = 2895. mm^3

ESΣ8.076

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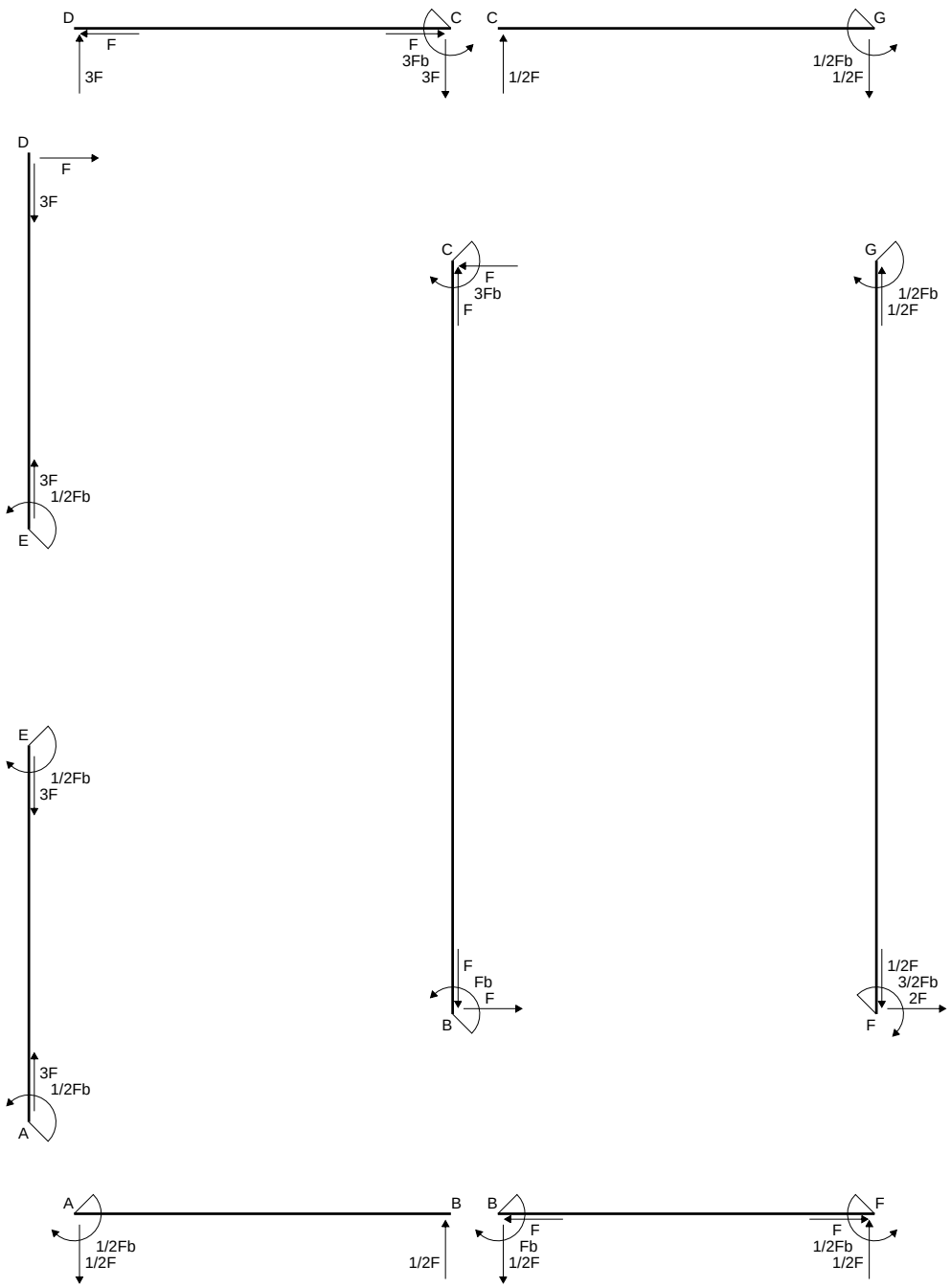
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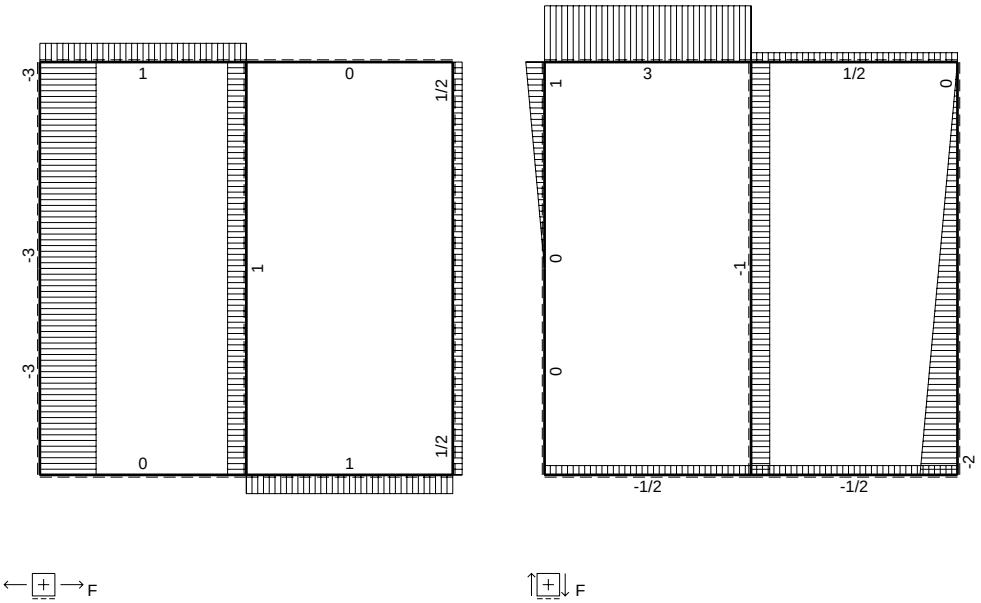
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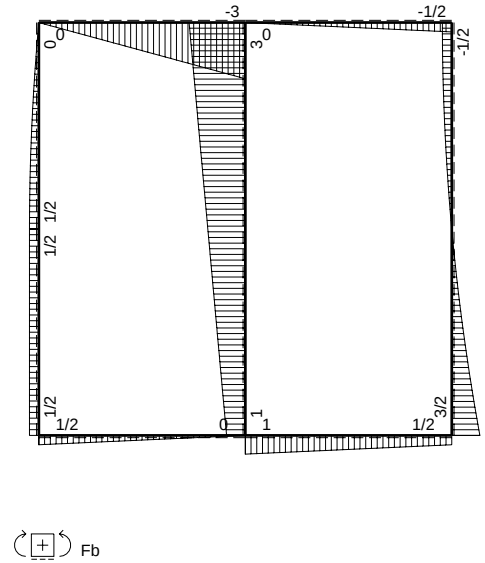
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Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

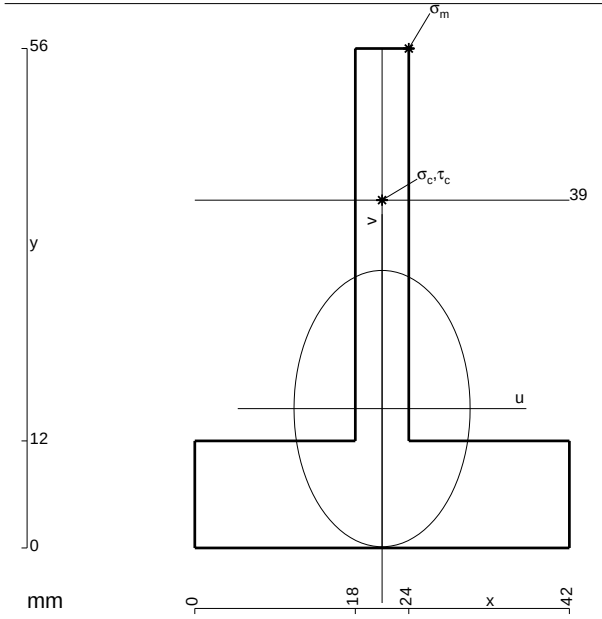
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 768. mm^2
J_u = 184468. mm^4
J_v = 74880. mm^4
y_g = 15.63 mm
N = 450. N
T_y = 1350. N
M_x = -999000. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 40.38 mm
$\sigma_m = N/A - Mv/J_u$ = 219.2 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 23.38 mm
$\sigma_c = N/A - Mv/J_u$ = 127.2 N/mm^2
τ_c = 3.966 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 127.4 N/mm^2
S^* = 3251. mm^3

ESΣ8.077

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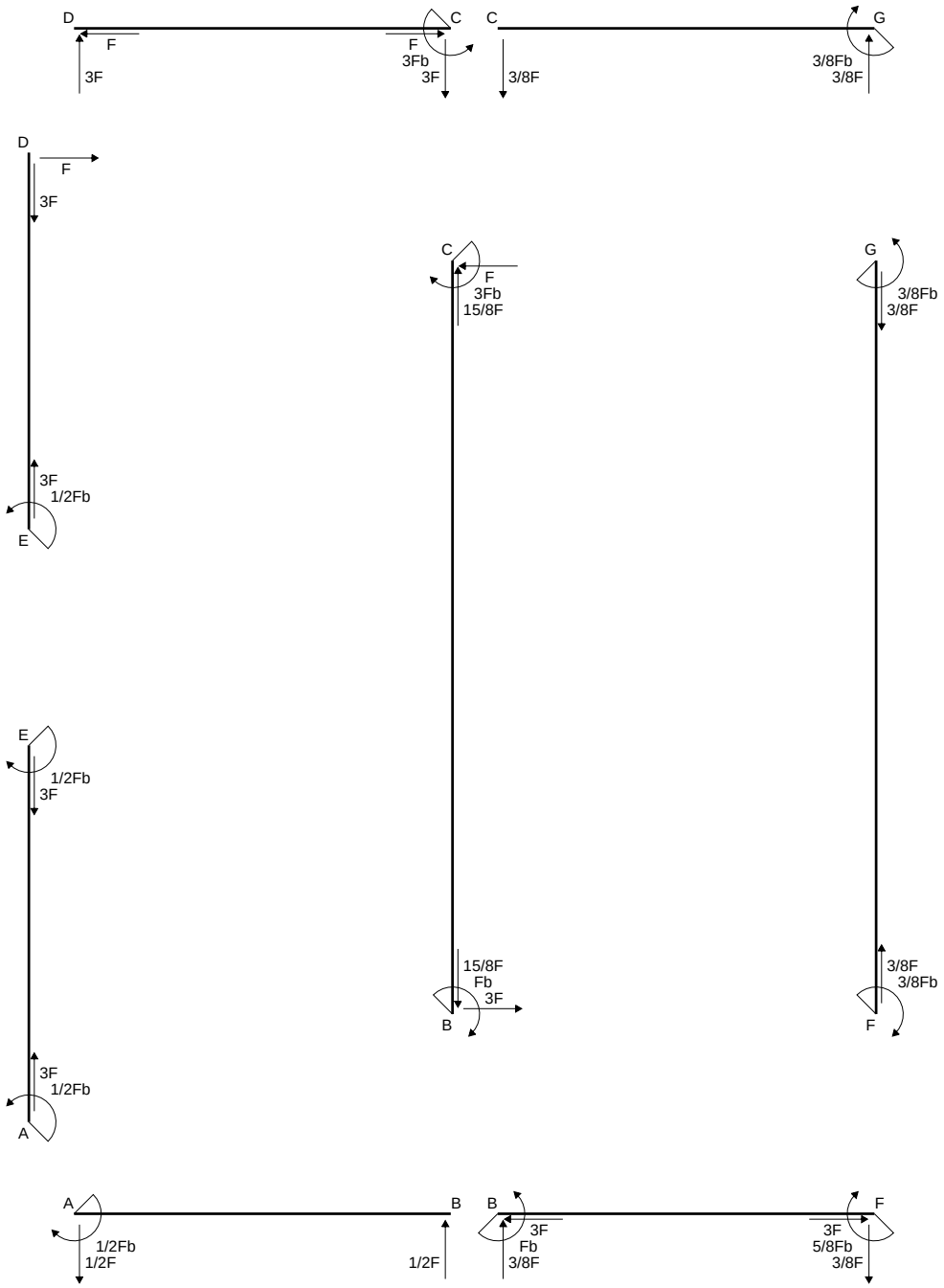
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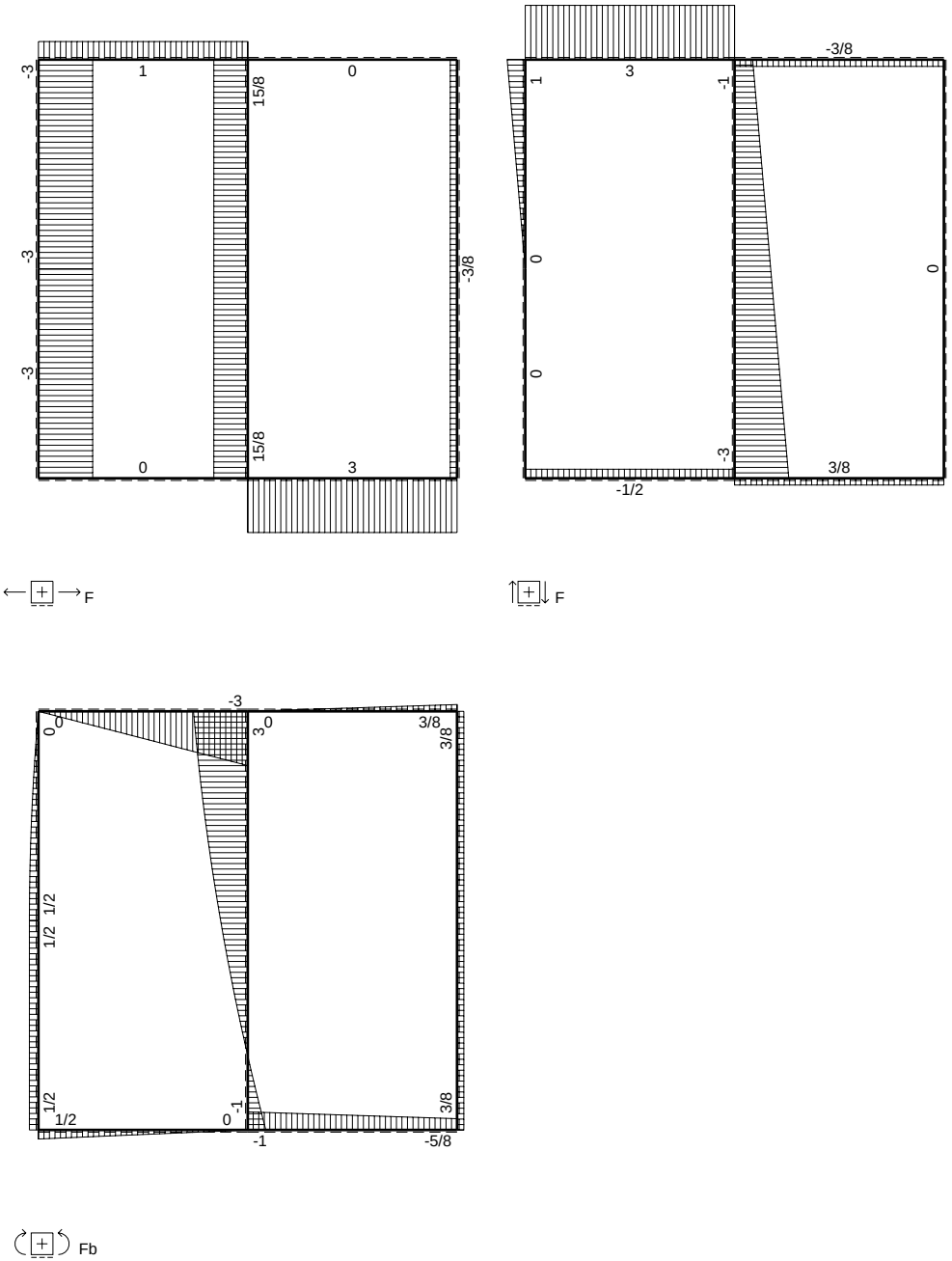
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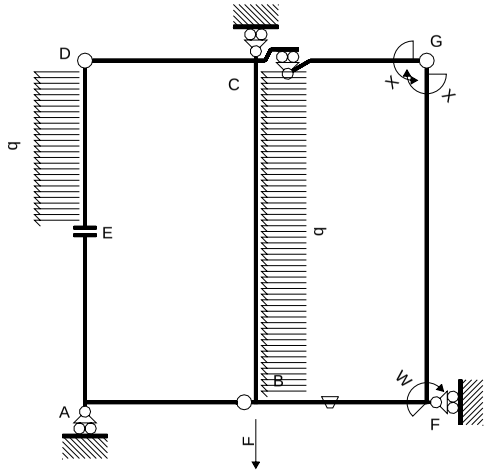
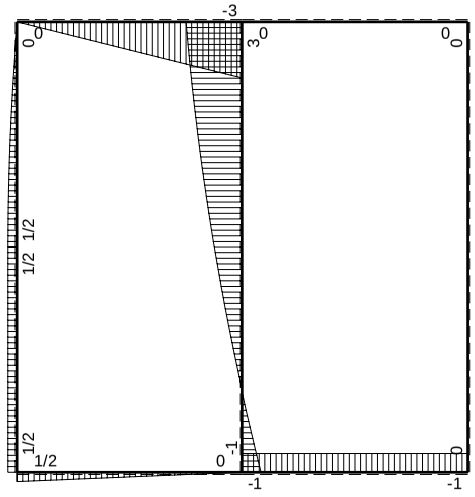
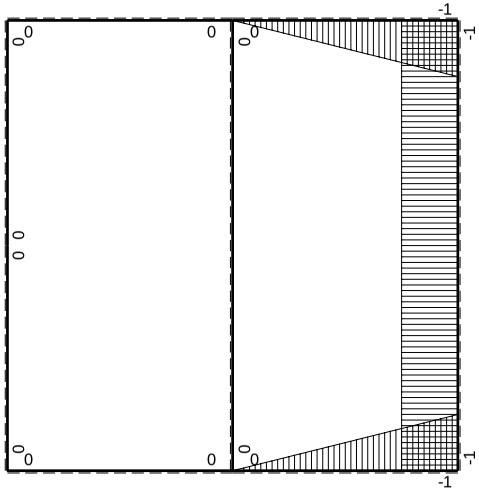
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.078

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

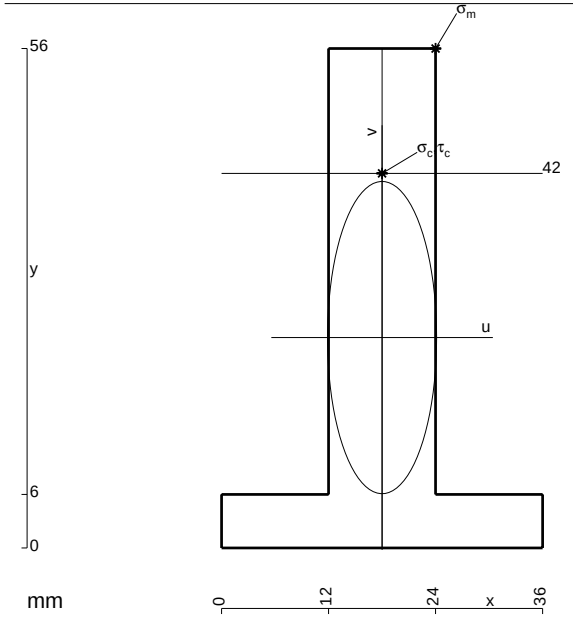
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.078



A = 816. mm^2
J_u = 250166. mm^4
J_v = 30528. mm^4
y_g = 23.59 mm
N = 740. N
T_y = 2220. N
M_x = -1753800. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 32.41 mm
$\sigma_m = N/A - Mv/J_u$ = 228.1 N/mm^2
x_c = 18. mm
y_c = 42. mm
v_c = 18.41 mm
$\sigma_c = N/A - Mv/J_u$ = 130. N/mm^2
τ_c = 3.157 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 130.1 N/mm^2
S^* = 4269. mm^3

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ESΣ8.078

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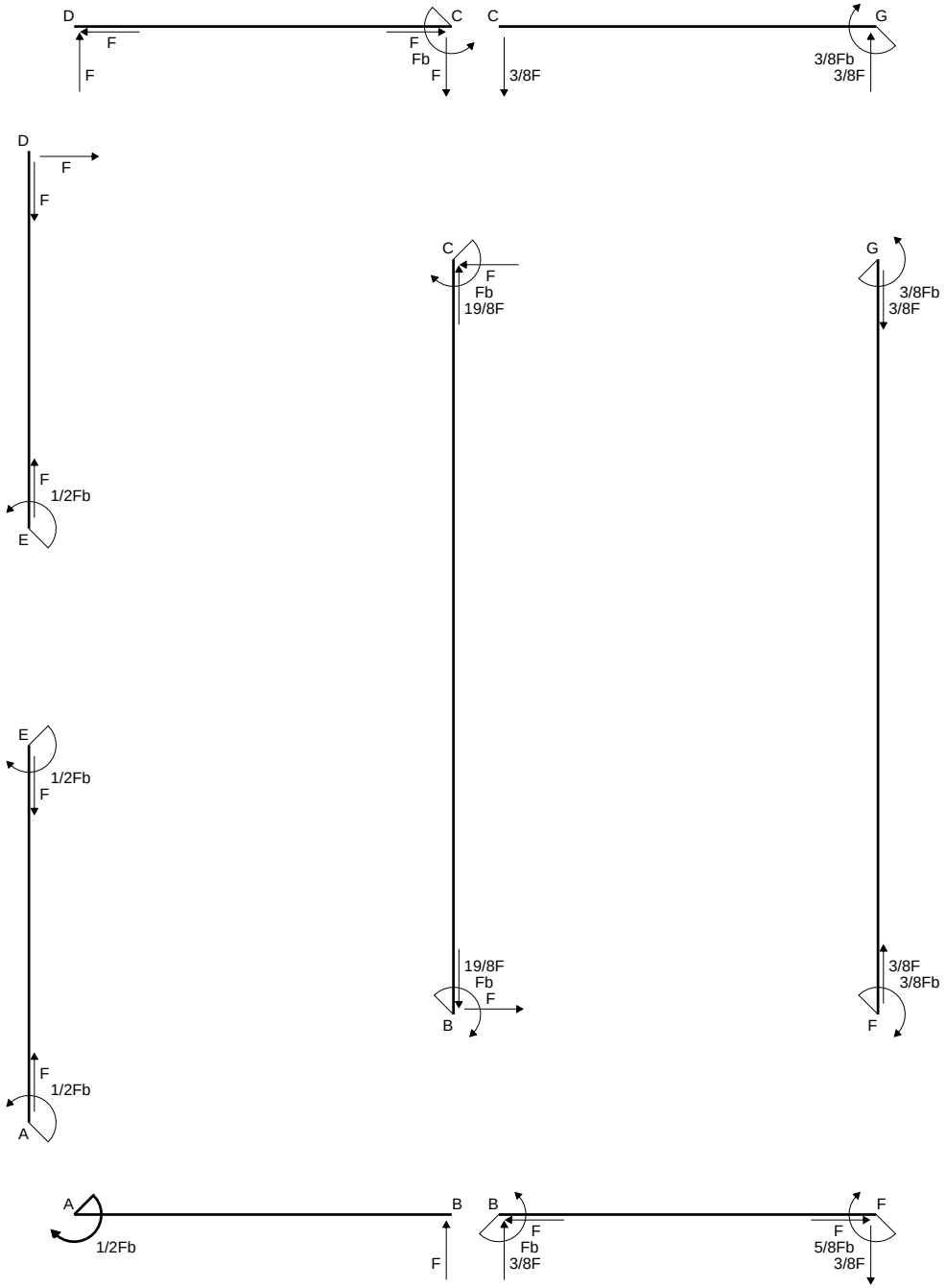
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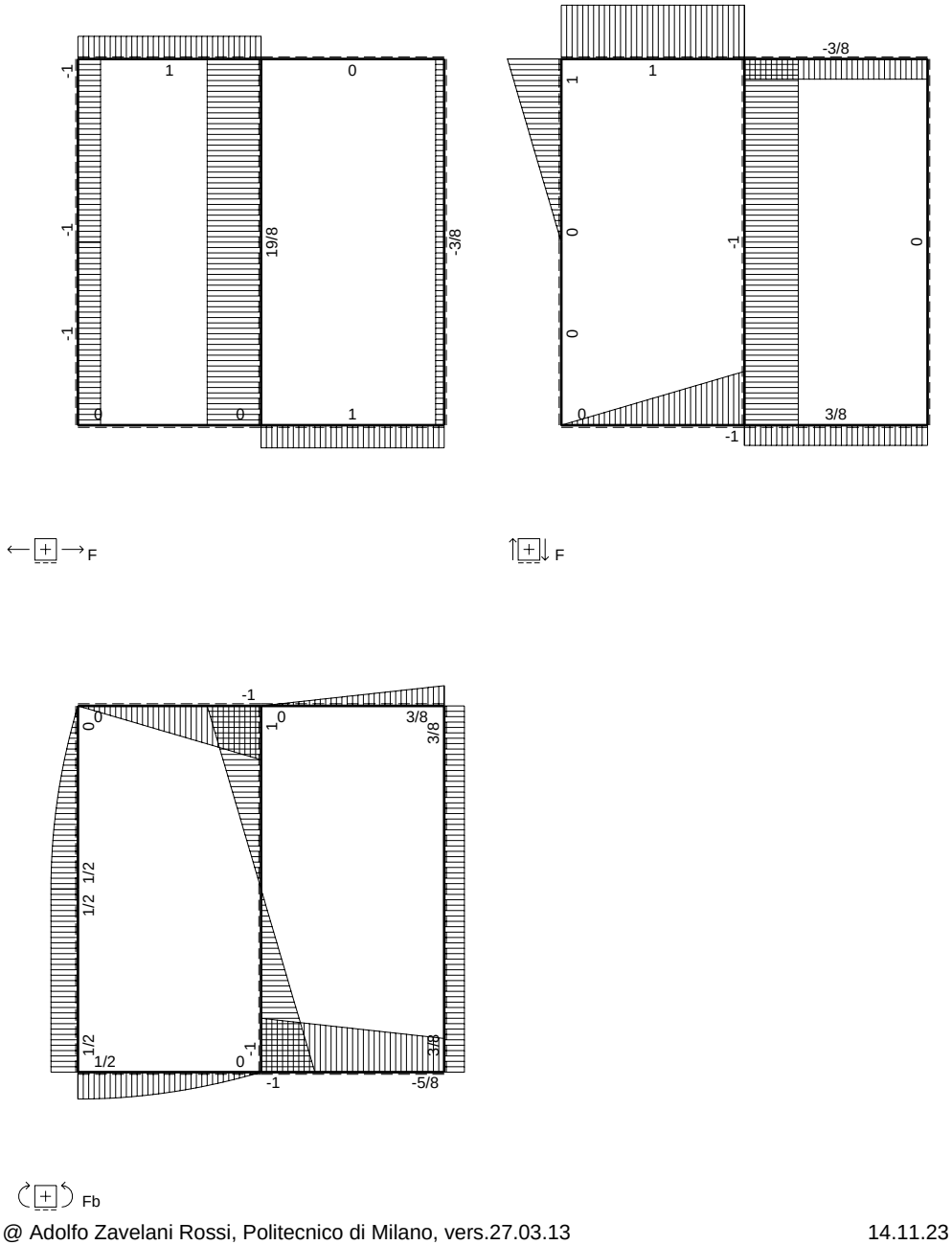
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Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali					iperstatica $X=W_{GC}$	Fb^2/EJ	8/3Xb/EJ
							-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

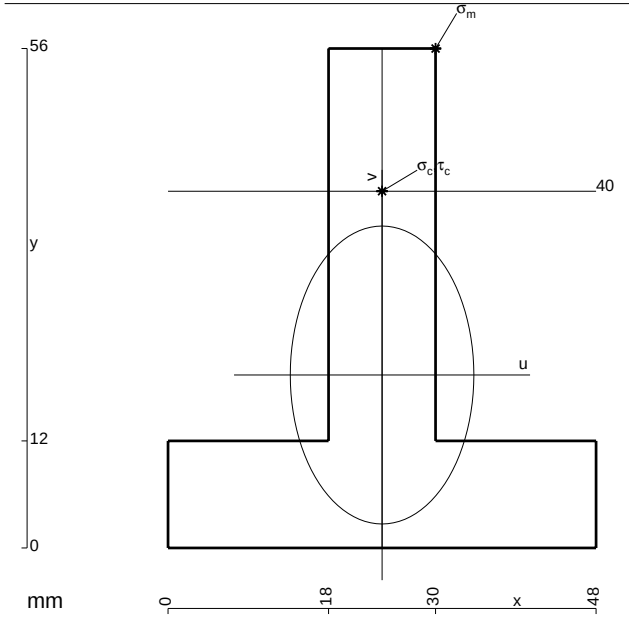
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1104. mm^2
J_u = 308071. mm^4
J_v = 116928. mm^4
y_g = 19.39 mm
N = 5653. N
T_y = -2380. N
M_x = -1975400. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 36.61 mm
$\sigma_m = N/A - Mv/J_u$ = 239.9 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 20.61 mm
$\sigma_c = N/A - Mv/J_u$ = 137.3 N/mm^2
τ_c = 3.536 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 137.4 N/mm^2
S^* = 5493. mm^3

ESΣ8.079

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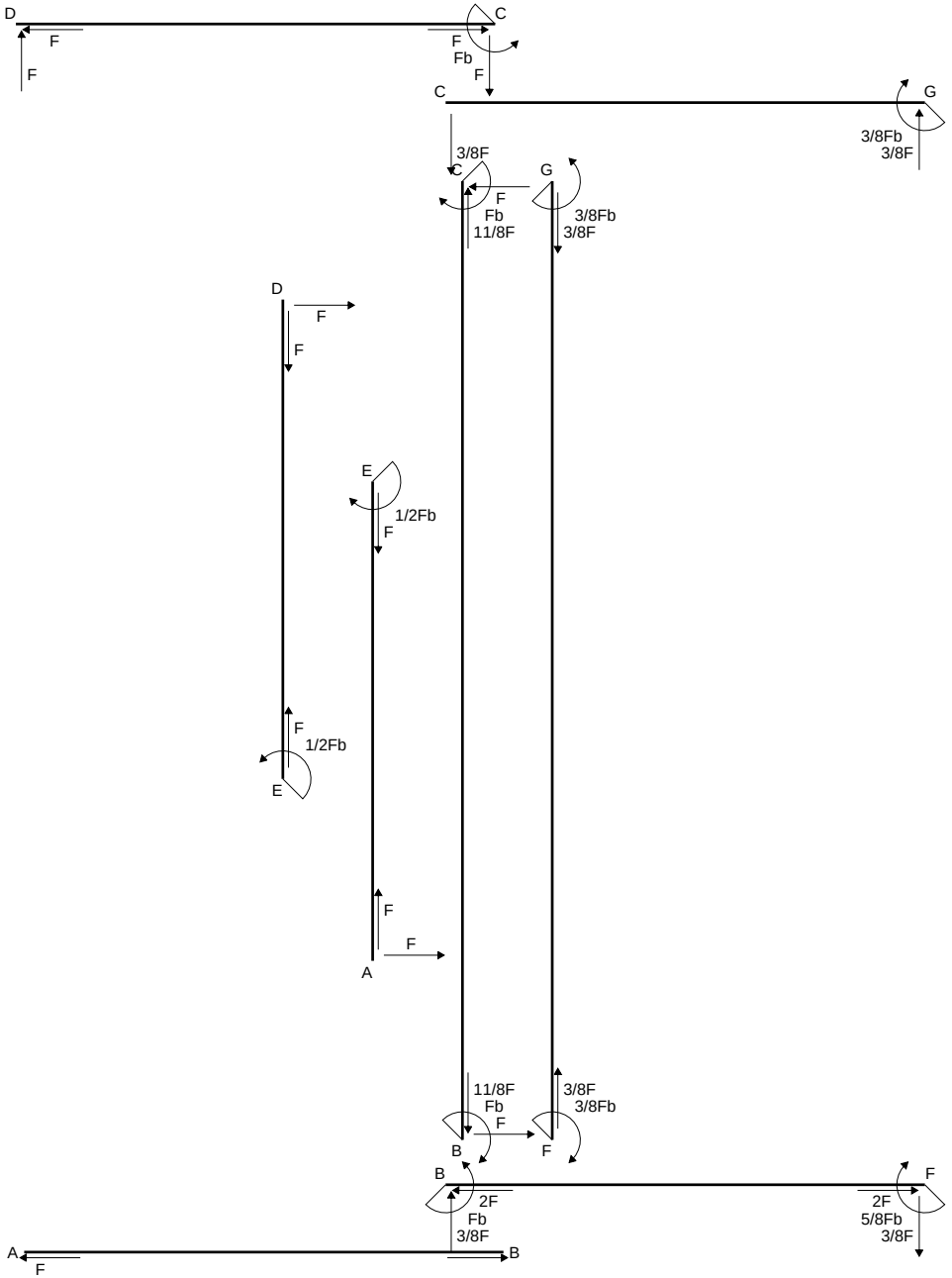
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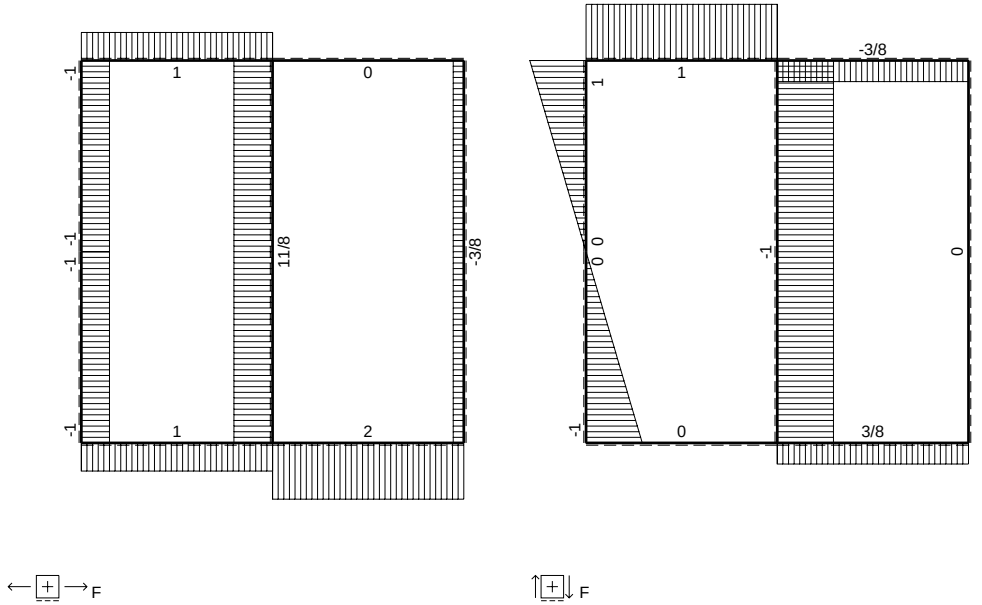
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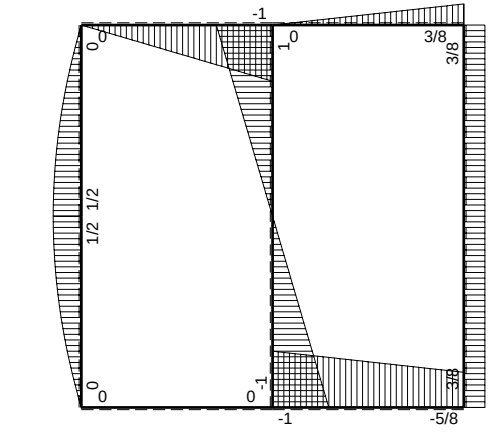
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Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

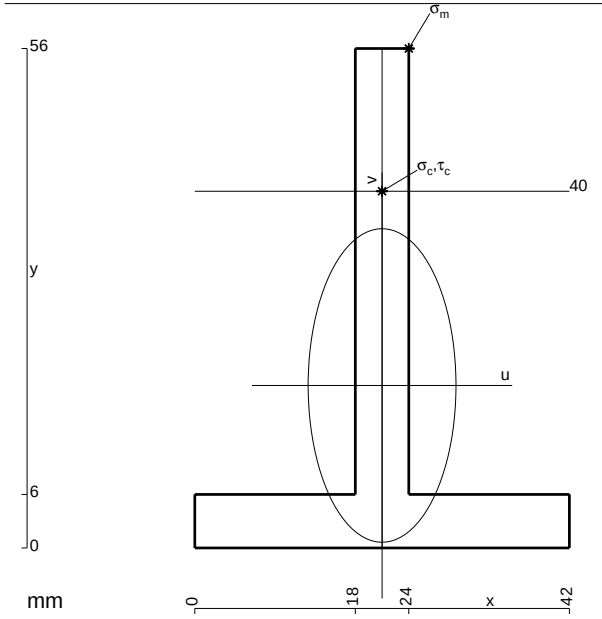
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 552. mm^2
J_u = 170630. mm^4
J_v = 37944. mm^4
y_g = 18.22 mm
N = 2750. N
T_y = -2000. N
M_x = -880000. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 37.78 mm
$\sigma_m = N/A - Mv/J_u$ = 199.8 N/mm^2
x_c = 21. mm
y_c = 40. mm
v_c = 21.78 mm
$\sigma_c = N/A - Mv/J_u$ = 117.3 N/mm^2
τ_c = 5.585 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.7 N/mm^2
S^* = 2859. mm^3

ESΣ8.080

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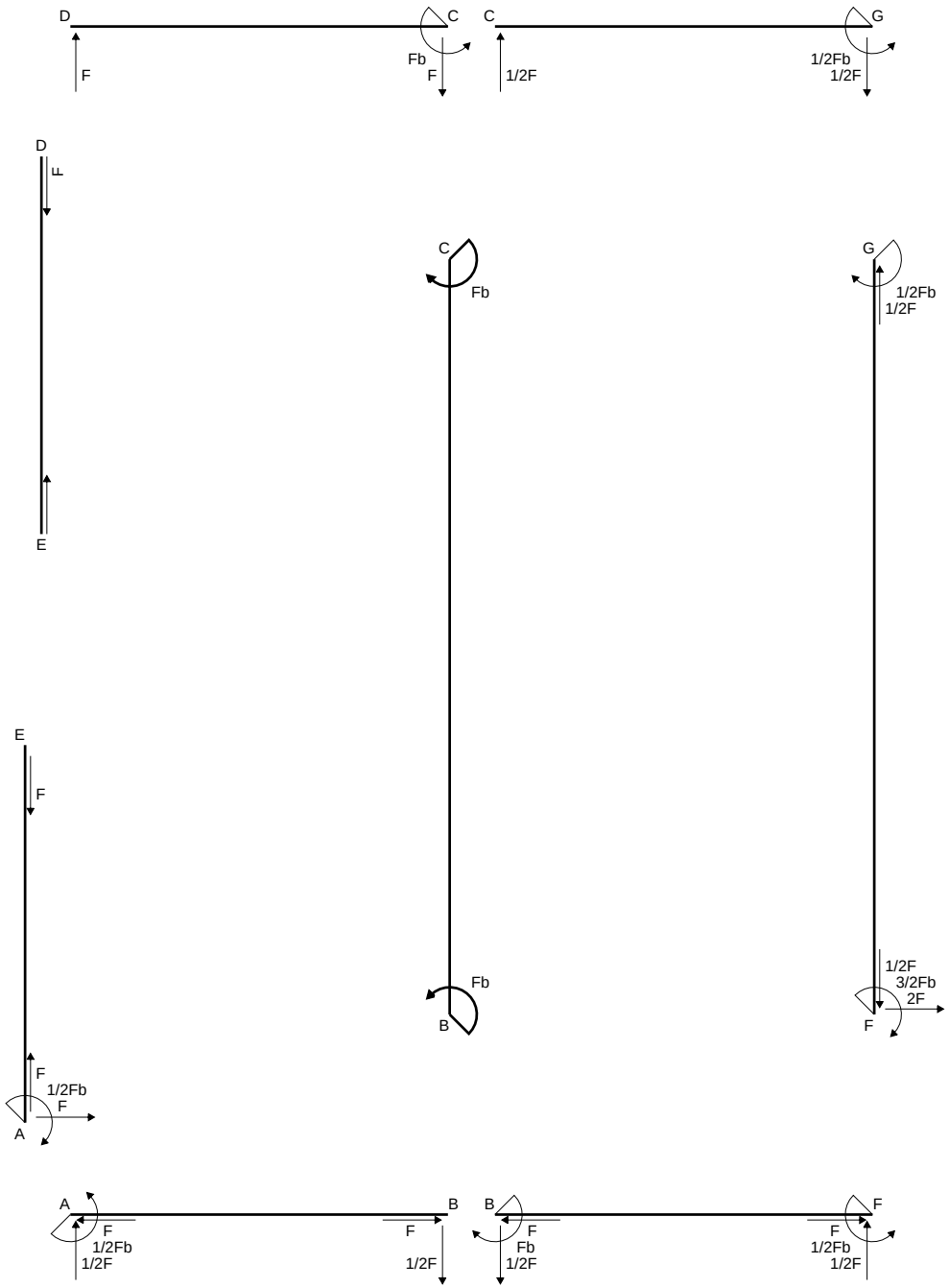
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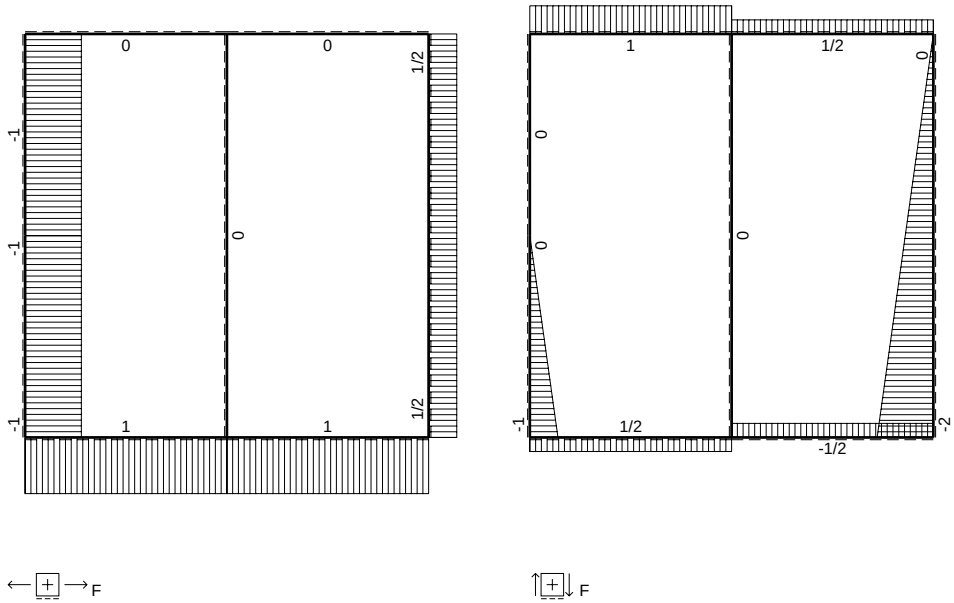
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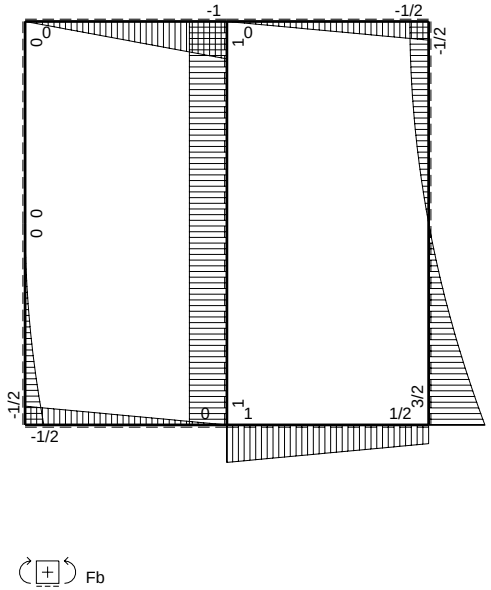
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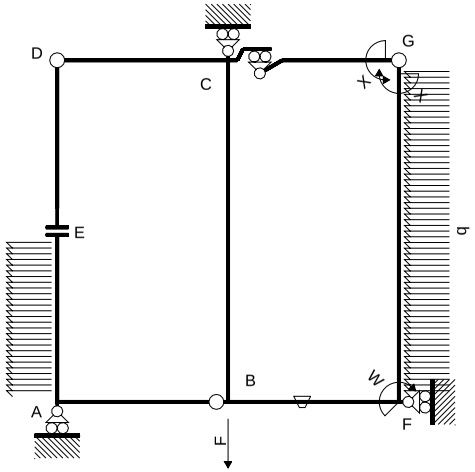
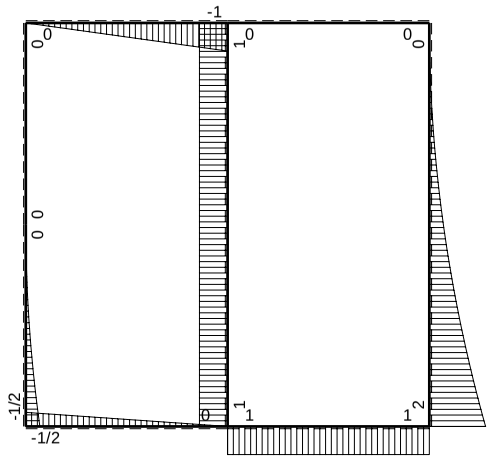
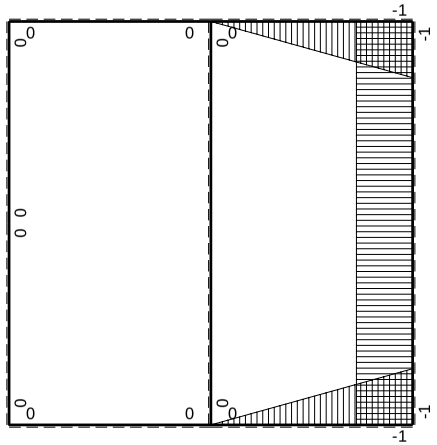




M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
		totali					$-4/3Fb^2/EJ$	8/3Xb/EJ
		iperstatica $X=W_{GC}$					1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.081

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

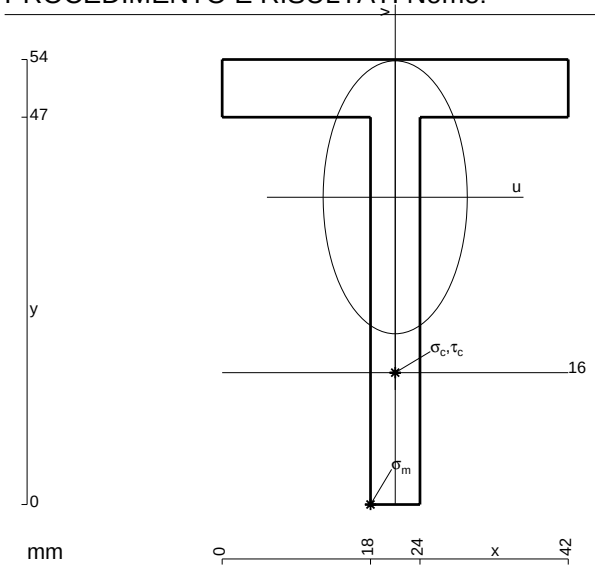
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 576. mm^2
J_u = 158042. mm^4
J_v = 44064. mm^4
y_g = 37.28 mm
T_y = 1890. N
M_x = -888300. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.28 mm
$\sigma_m = -Mv/J_u$ = -209.5 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.28 mm
$\sigma_c = -Mv/J_u$ = -119.6 N/mm^2
τ_c = 5.603 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120. N/mm^2
S^* = 2811. mm^3

ESΣ8.081

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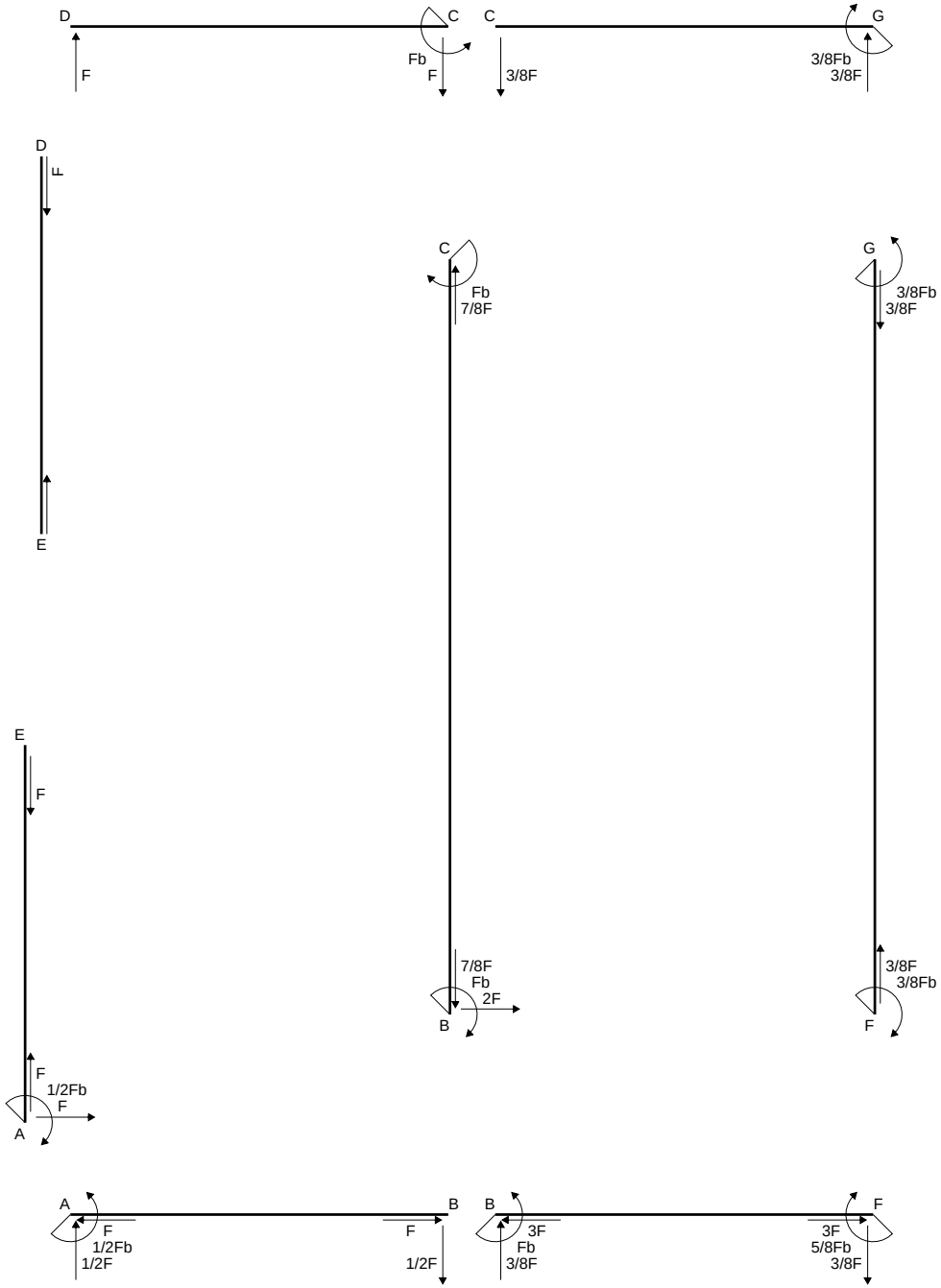
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C
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3/8F
3/8Fb
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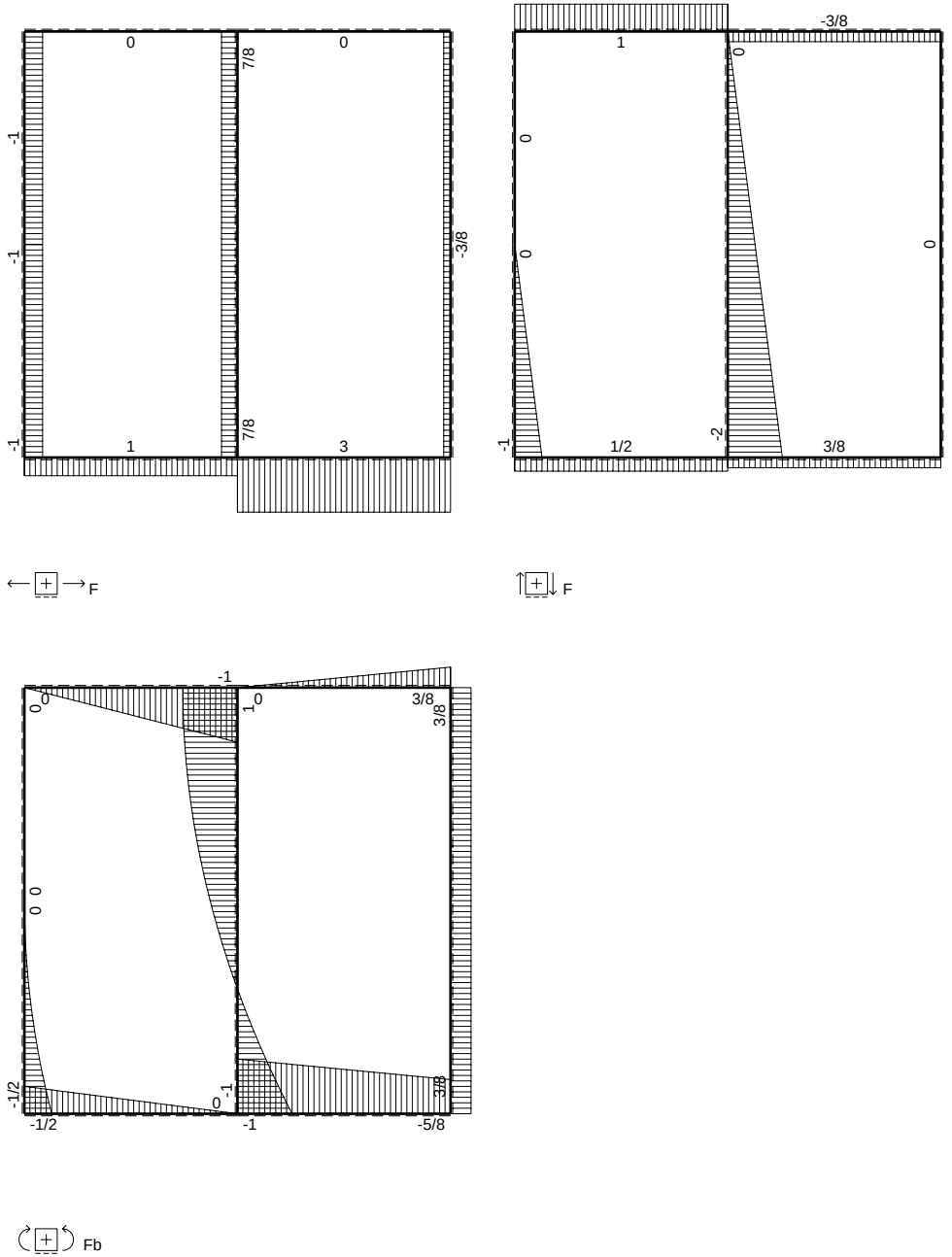
3F
5/8Fb
3/8F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-1
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-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Schema di calcolo iperstatico

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PROCEDIMENTO E RISULTATI Nome: ESΣ8.082

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

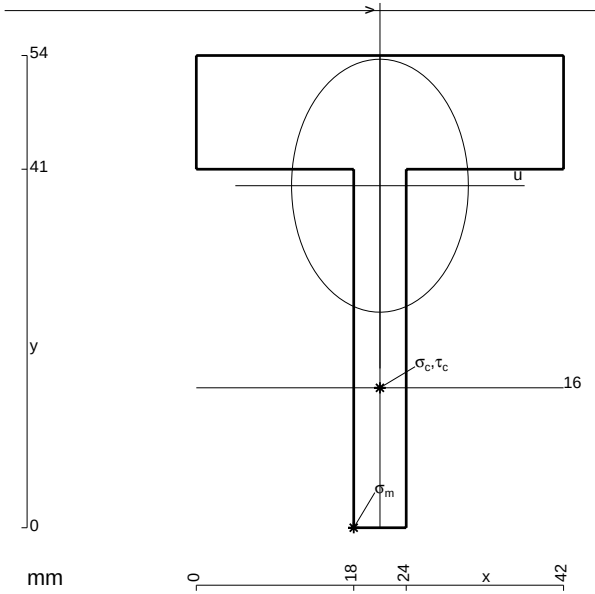
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 792. mm^2
J_u = 165782. mm^4
J_v = 81000. mm^4
y_g = 39.11 mm
N = 1610. N
T_y = -3680. N
M_x = -938400. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.11 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -23.11 mm
$\sigma_c = N/A - Mv/J_u$ = -128.8 N/mm^2
τ_c = 11.05 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.2 N/mm^2
S^* = 2987. mm^3

ESΣ8.082

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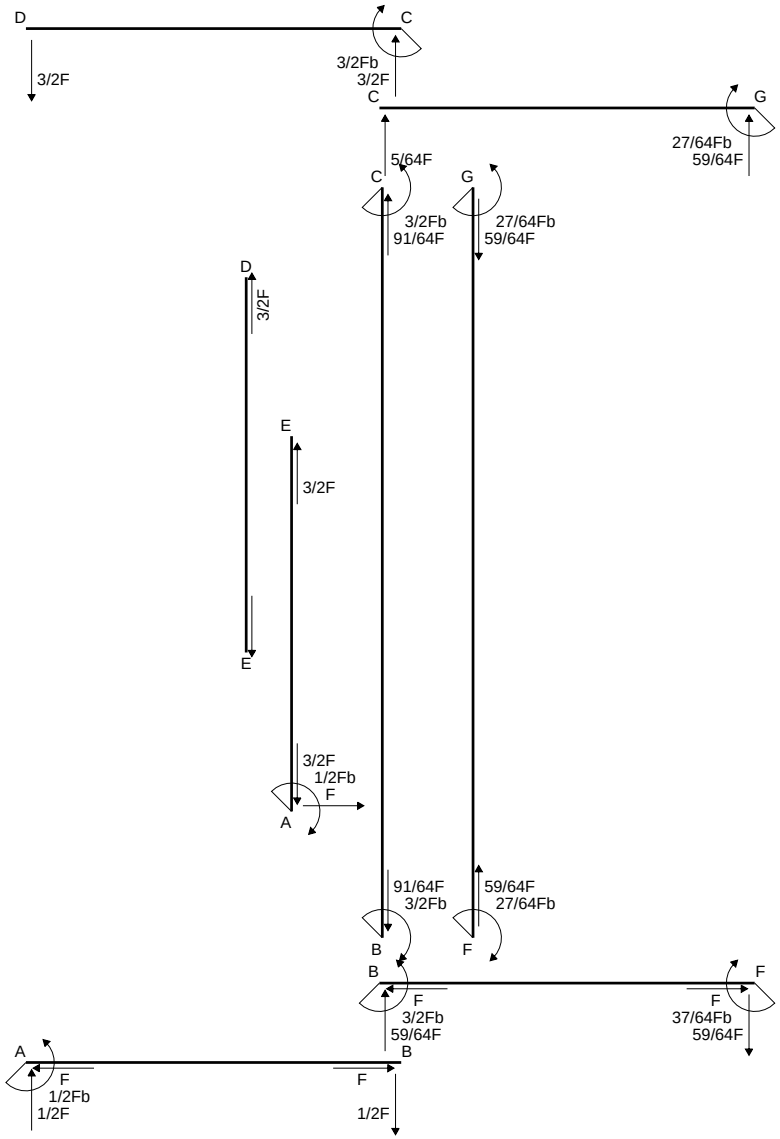
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3/2F
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G
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59/64F
C
5/64F
G
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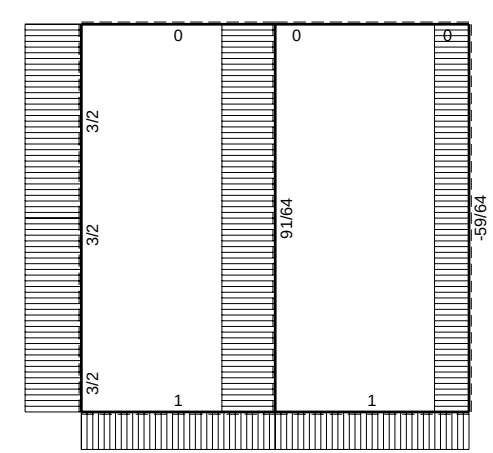
27/64Fb
59/64F
D
3/2F
E
3/2F
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F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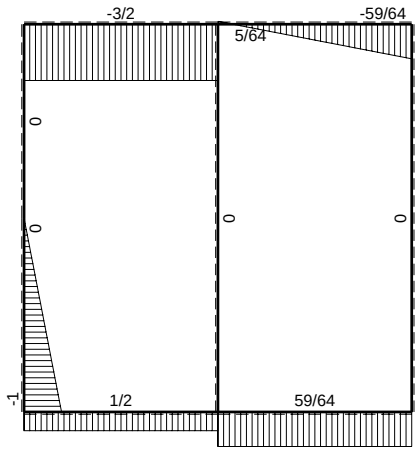
3/2
3/2
3/2
91/64
-59/64
1
1

F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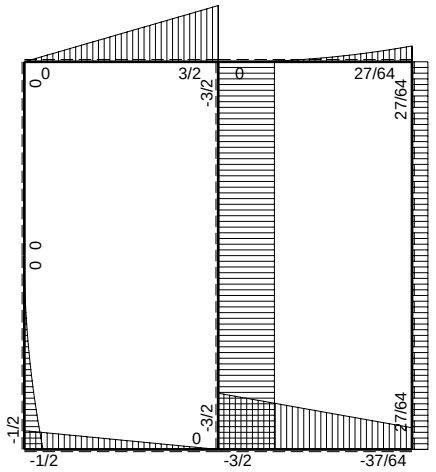
-59/64
5/64
0
0
0
0
-1
1/2
59/64

F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3/2
-3/2
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27/64
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-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	-1/2qx^2	0	0	0	0	0+0	0
AE b	0	1/2Fb-Fx+1/2qx^2	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	3/2Fx-1/2Fx^2/b	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3XbJ/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	Fb-1/2Fx-1/2Fx^2/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3XbJ/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbJ/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						9/8Fb^2/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

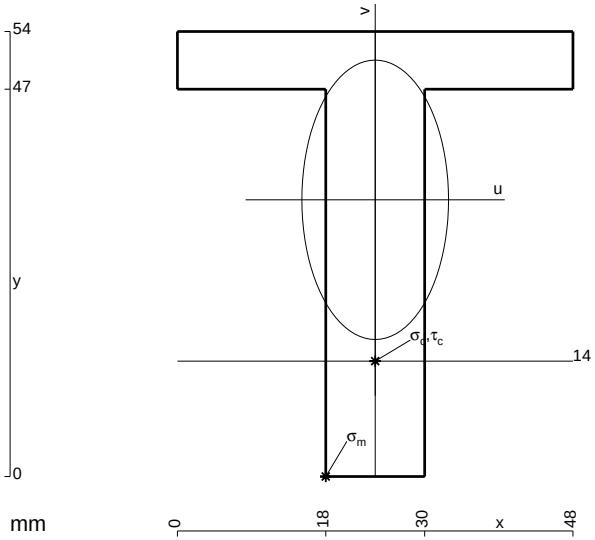
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 900. mm^2
J_u = 258693. mm^4
J_v = 71280. mm^4
y_g = 33.58 mm
T_y = -3210. N
M_x = 1765500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.58 mm
$\sigma_m = -Mv/J_u$ = 229.2 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.58 mm
$\sigma_c = -Mv/J_u$ = 133.6 N/mm^2
τ_c = 4.617 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 133.9 N/mm^2
S^* = 4465. mm^3

ESΣ8.083

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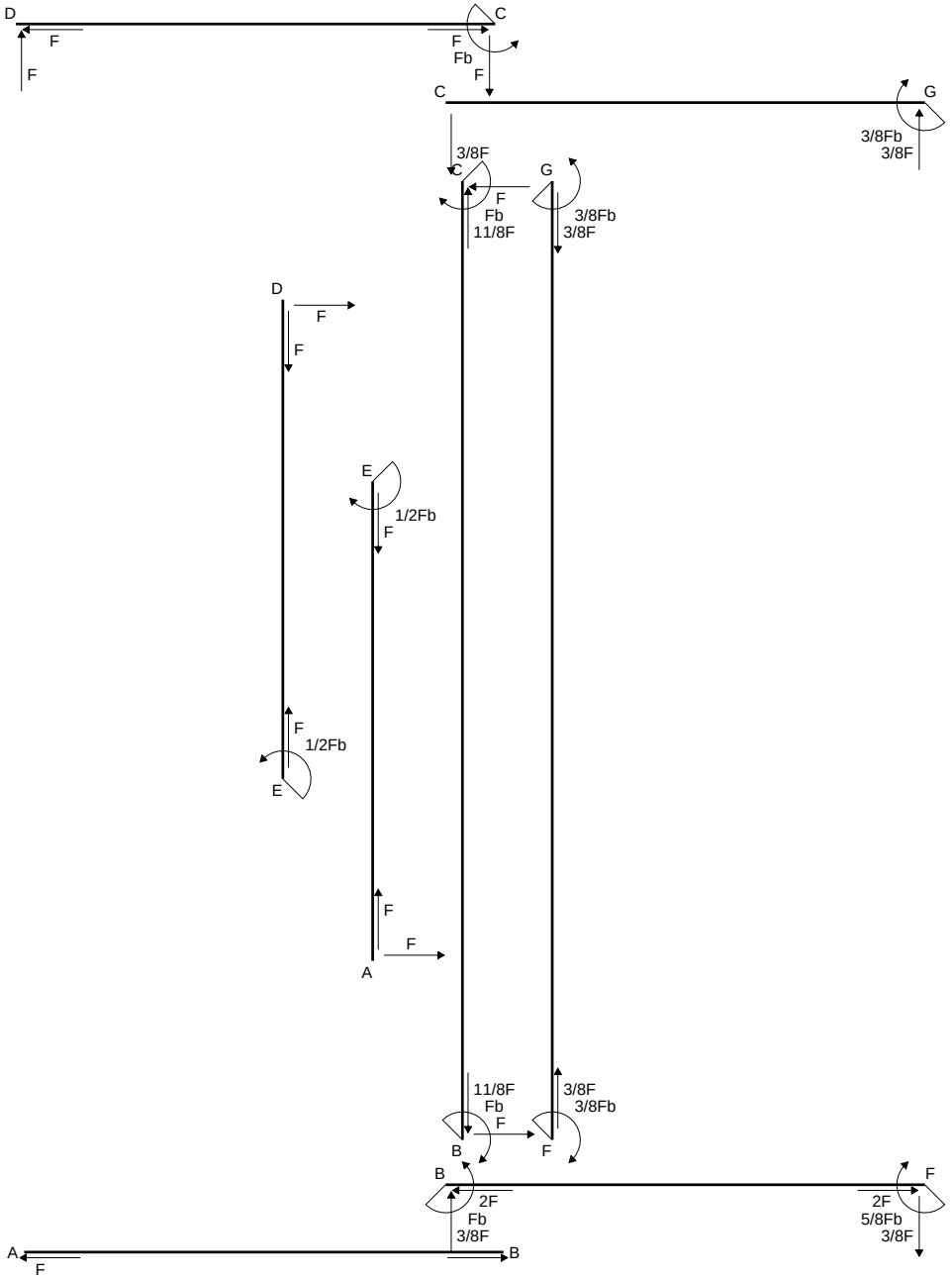
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B
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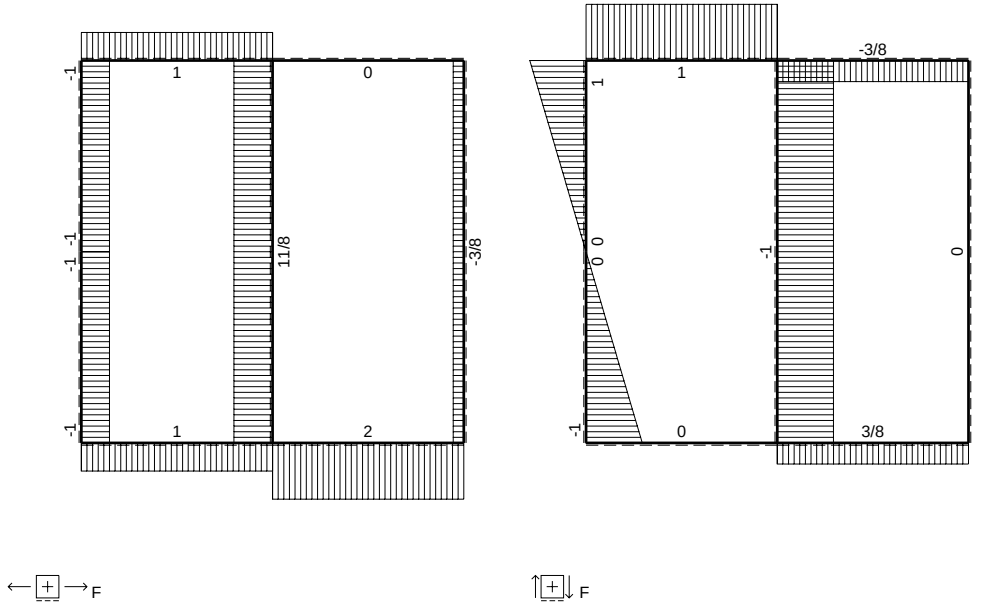
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-3/8
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-1
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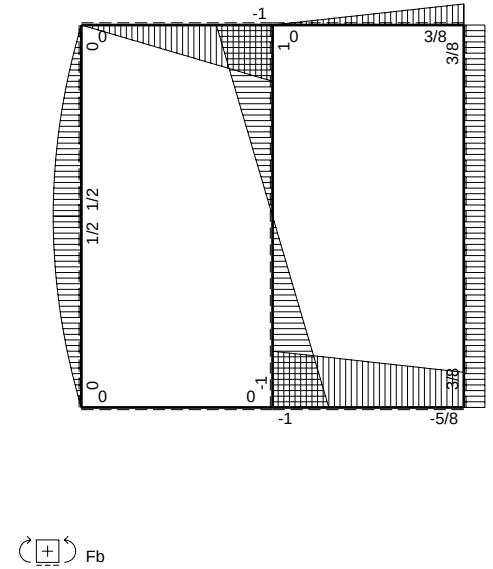
-1
0
3/8
F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3/8
1/2
1/2
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0
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-1
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3XbEJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbEJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$							-3/8Fb

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

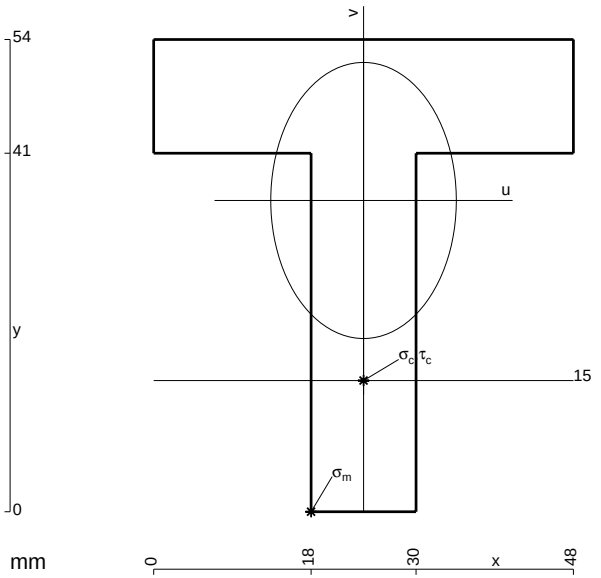
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.084



A = 1116. mm^2
J_u = 278254. mm^4
J_v = 125712. mm^4
y_g = 35.6 mm
N = 4221. N
T_y = -3070. N
M_x = 1842000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.6 mm
$\sigma_m = N/A - Mv/J_u$ = 239.4 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.6 mm
$\sigma_c = N/A - Mv/J_u$ = 140.1 N/mm^2
τ_c = 4.65 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.4 N/mm^2
S^* = 5057. mm^3

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ESΣ8.084

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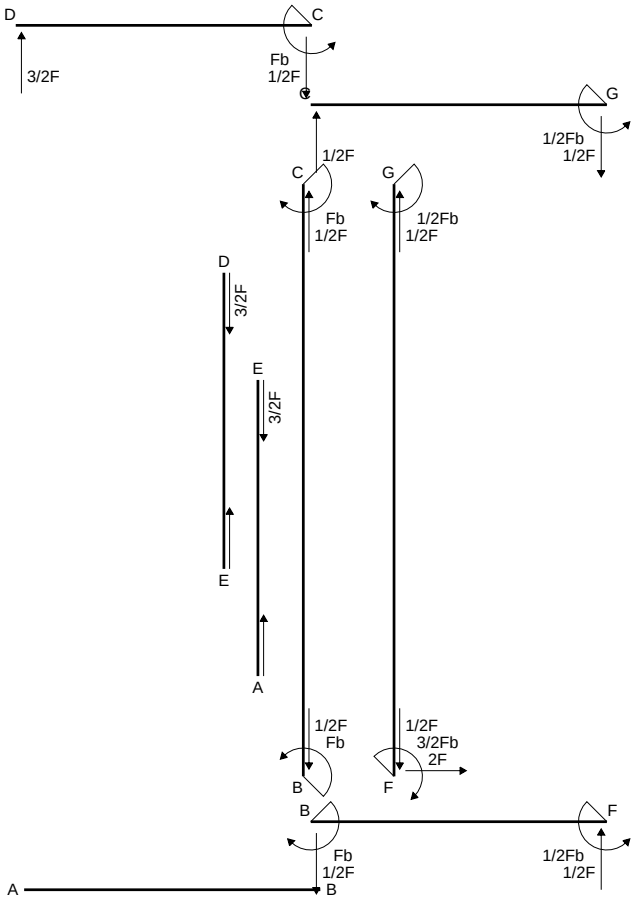
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Fb
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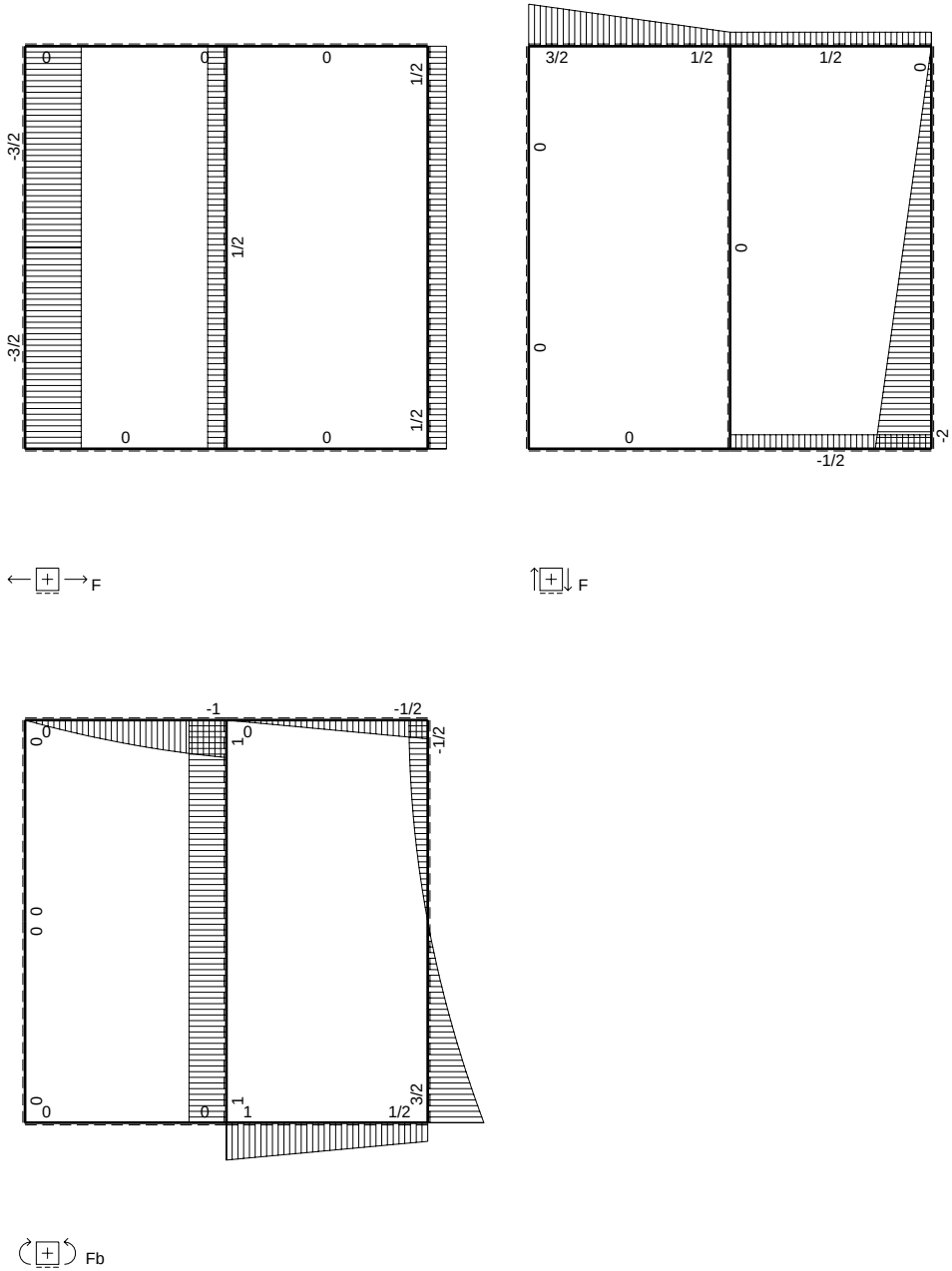
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0
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0
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1/2
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Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

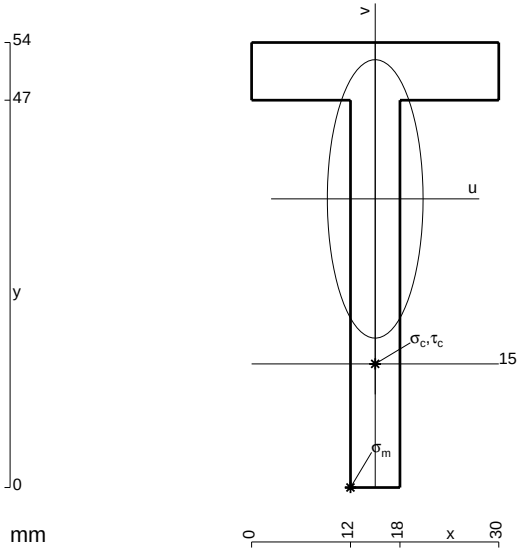
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 492. mm^2
J_u = 140516. mm^4
J_v = 16596. mm^4
y_g = 35.02 mm
T_y = 2000. N
M_x = -800000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.02 mm
$\sigma_m = -Mv/J_u$ = -199.4 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.02 mm
$\sigma_c = -Mv/J_u$ = -114. N/mm^2
τ_c = 5.876 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.5 N/mm^2
S^* = 2477. mm^3

ESΣ8.085

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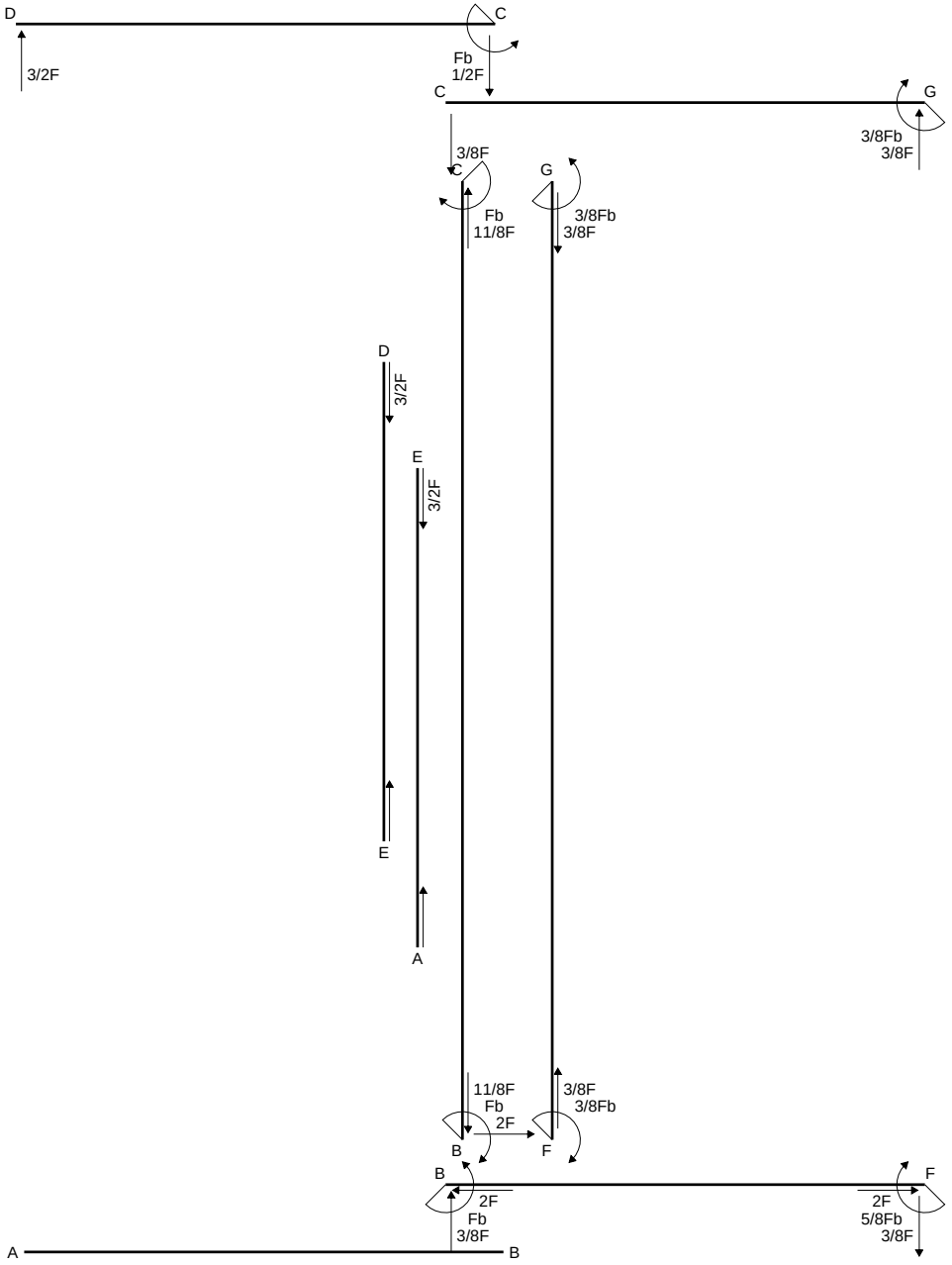
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2F
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A
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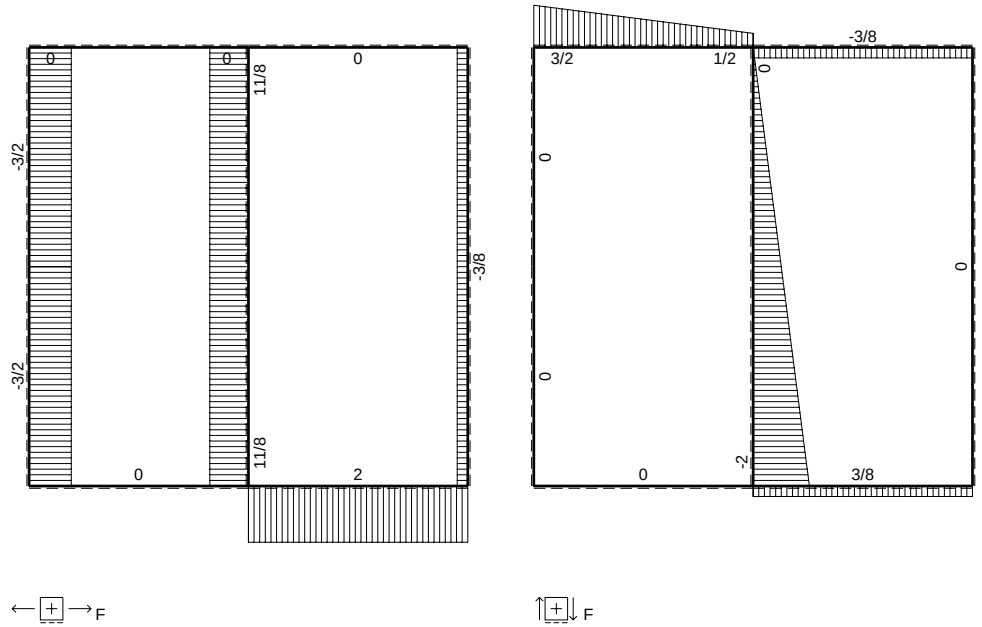
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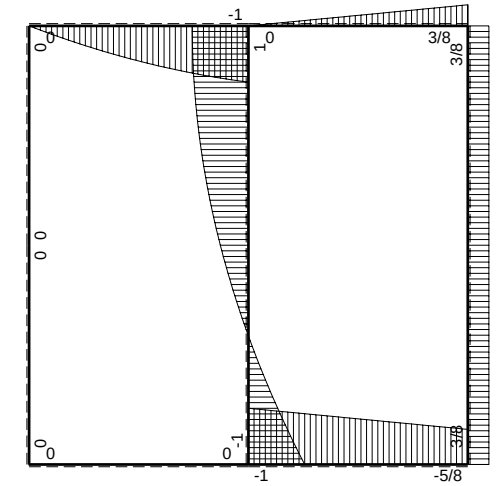
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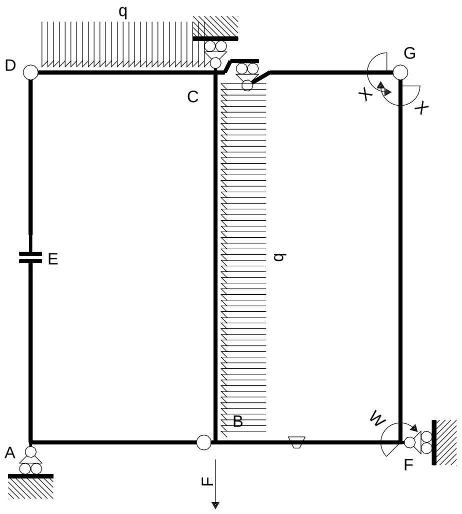
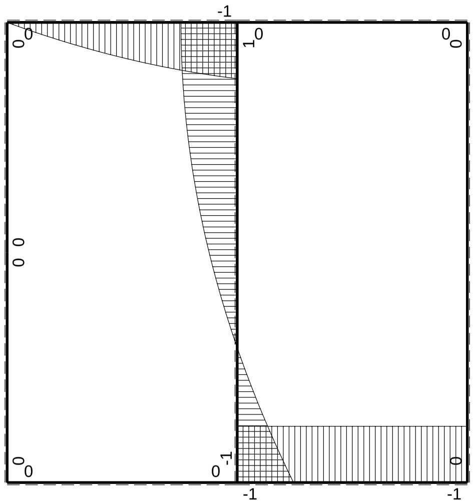
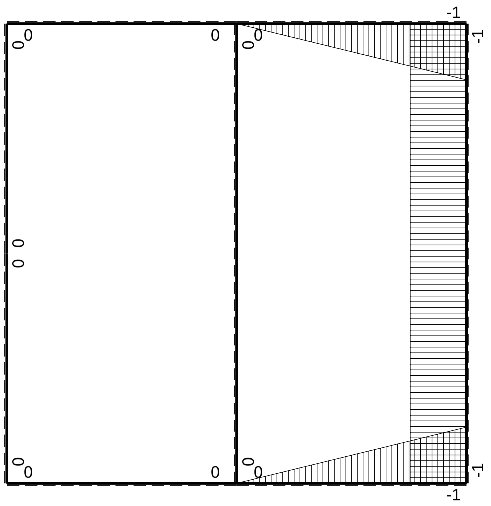
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

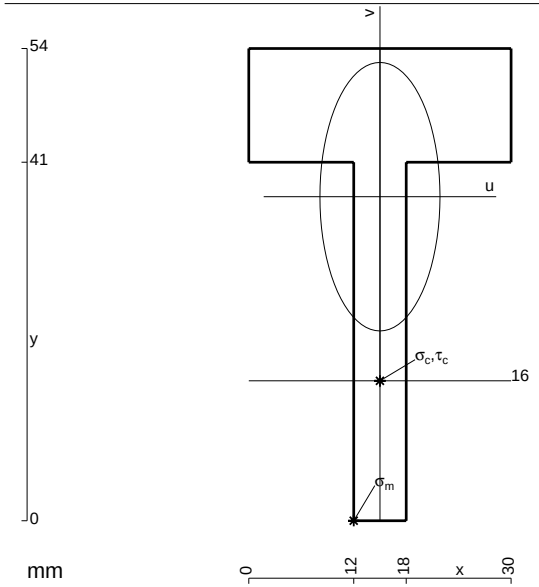
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 636. mm^2
J_u = 149922. mm^4
J_v = 29988. mm^4
y_g = 37.06 mm
N = 1733. N
T_y = -2520. N
M_x = -856800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.06 mm
$\sigma_m = N/A - Mv/J_u$ = -209.1 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.06 mm
$\sigma_c = N/A - Mv/J_u$ = -117.6 N/mm^2
τ_c = 7.814 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.4 N/mm^2
S^* = 2789. mm^3

ESΣ8.086

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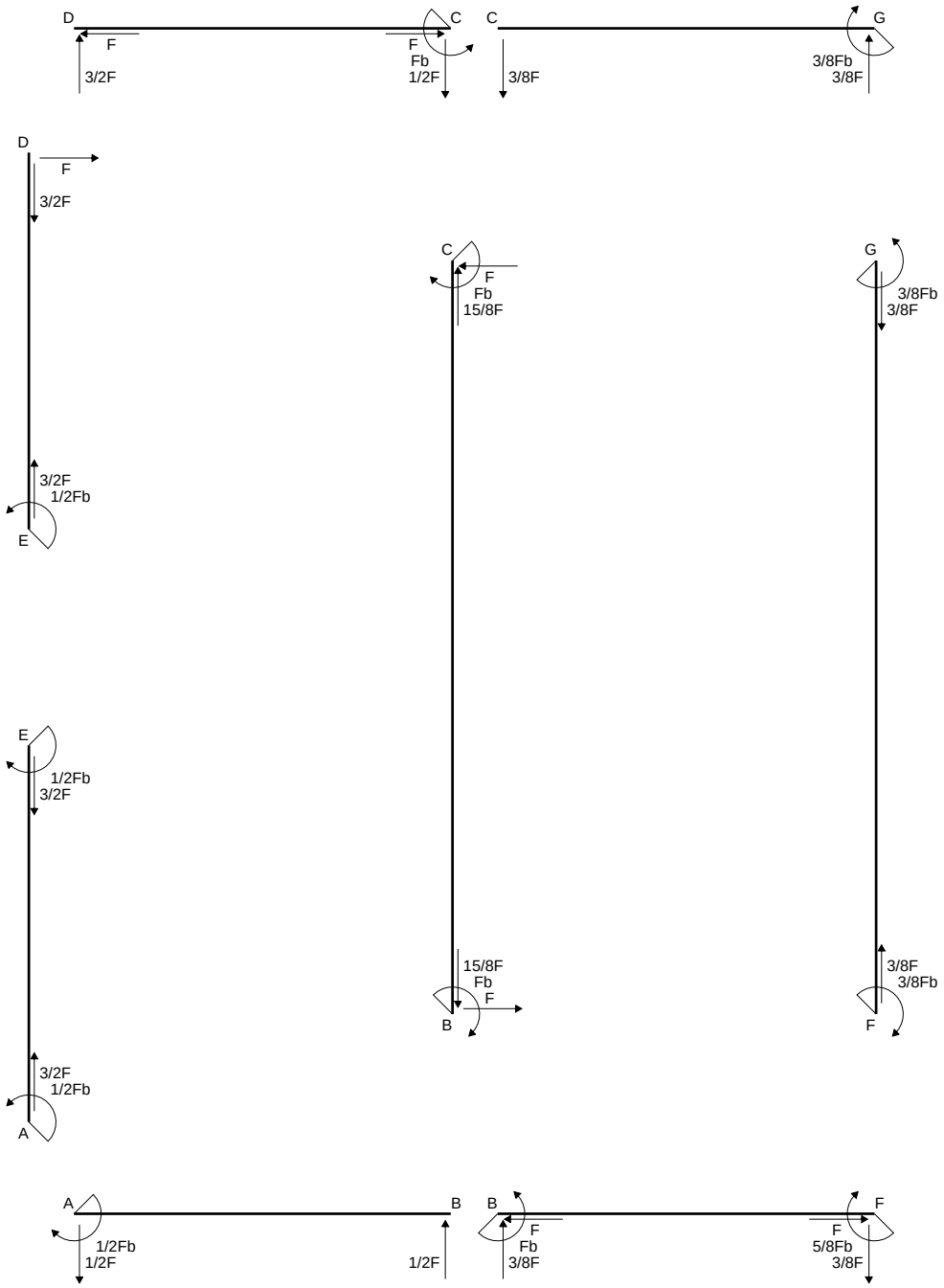
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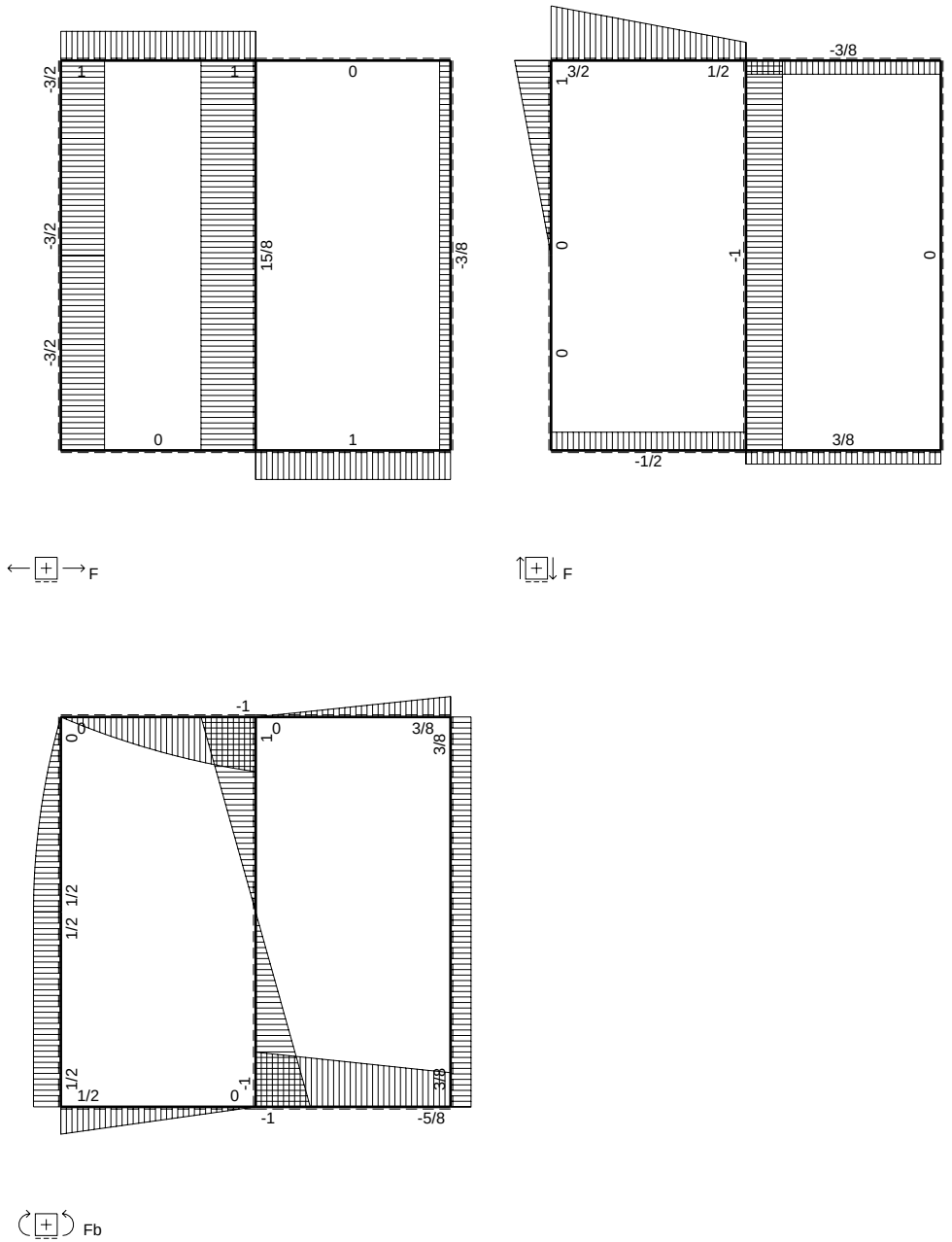
3/8F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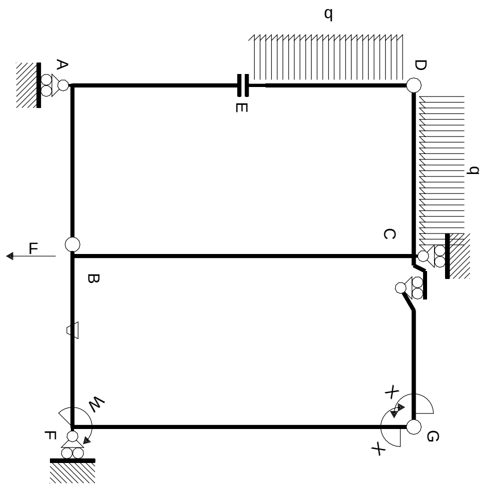
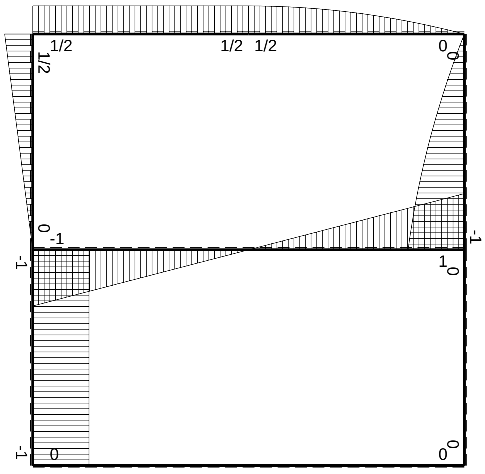
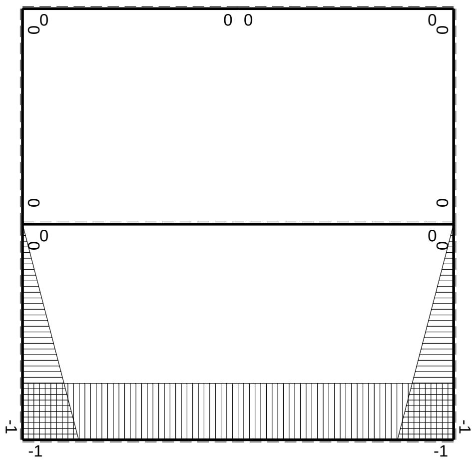
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

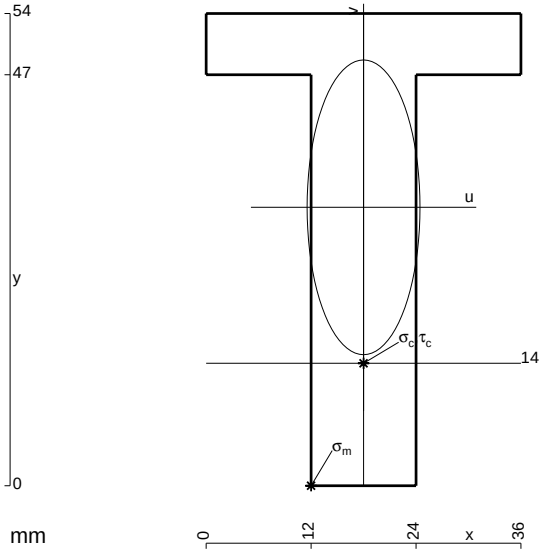
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 816. mm^2
J_u = 231827. mm^4
J_v = 33984. mm^4
y_g = 31.84 mm
N = 4013. N
T_y = -2140. N
M_x = 1562200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.84 mm
$\sigma_m = N/A - Mv/J_u$ = 219.5 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -17.84 mm
$\sigma_c = N/A - Mv/J_u$ = 125.1 N/mm^2
τ_c = 3.21 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.2 N/mm^2
S^* = 4173. mm^3

ESΣ8.087

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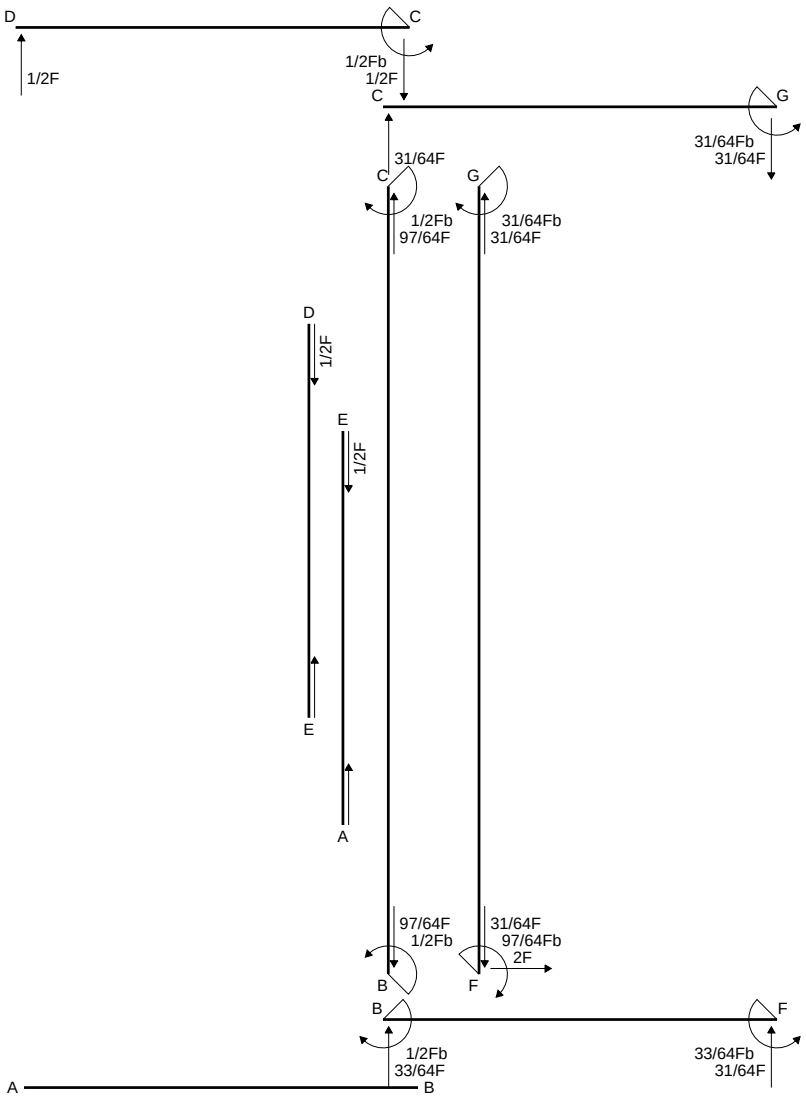
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1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
97/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
97/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
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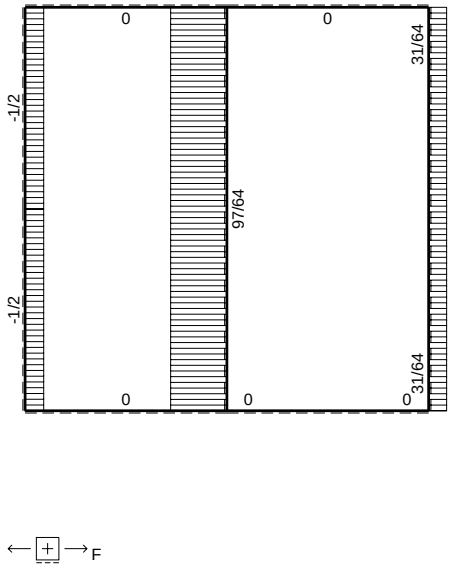
33/64F
33/64Fb
31/64F
A
B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-1/2
0
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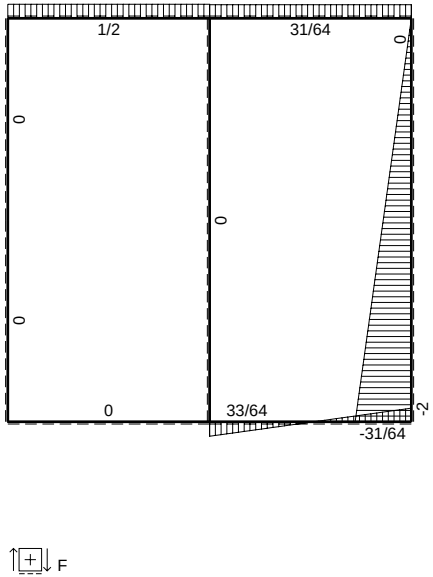
31/64
F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31/64
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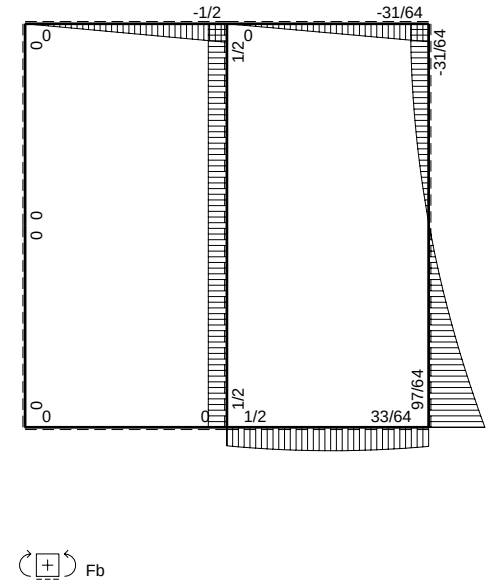
33/64
-2
-31/64
F


-1/2
-31/64
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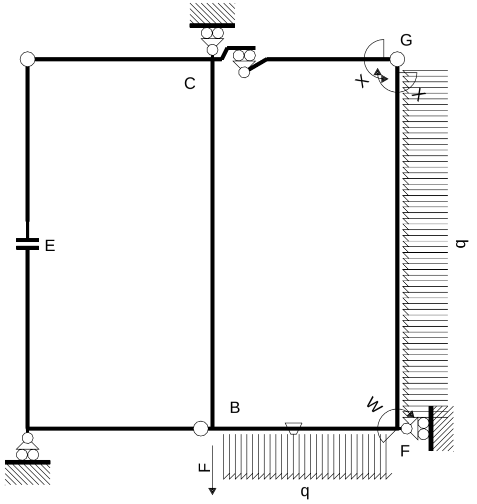
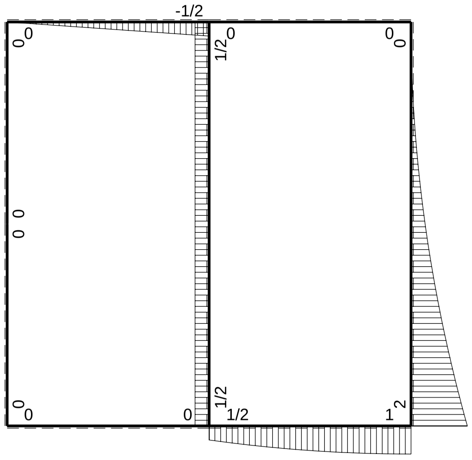
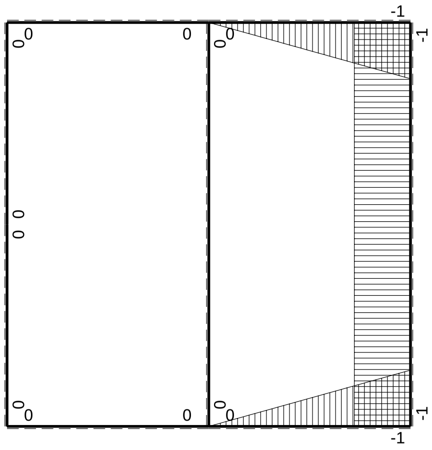
0
1/2
33/64
97/64
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

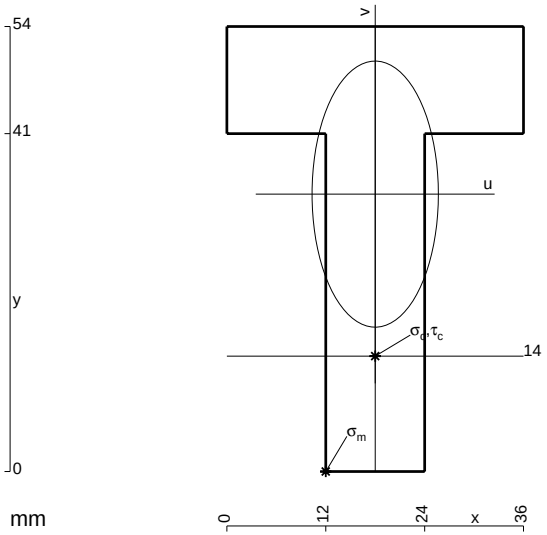
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 960. mm^2
J_u = 250363. mm^4
J_v = 56448. mm^4
y_g = 33.66 mm
T_y = 2220. N
M_x = -1709400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.66 mm
$\sigma_m = -Mv/J_u$ = -229.8 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.66 mm
$\sigma_c = -Mv/J_u$ = -134.2 N/mm^2
τ_c = 3.31 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 134.4 N/mm^2
S^* = 4479. mm^3

ESΣ8.088

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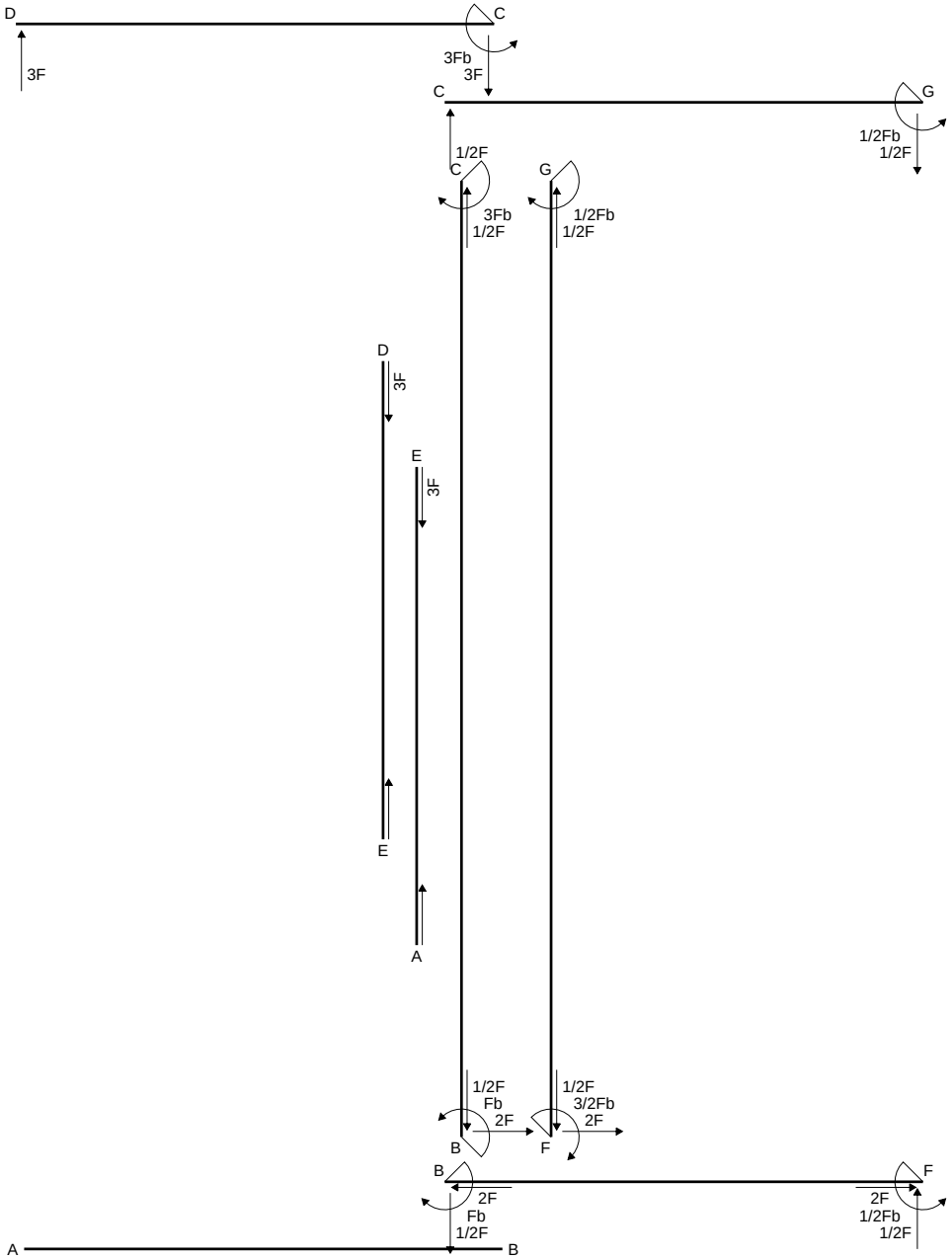
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3Fb
3F
C
G
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1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
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B
F
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F
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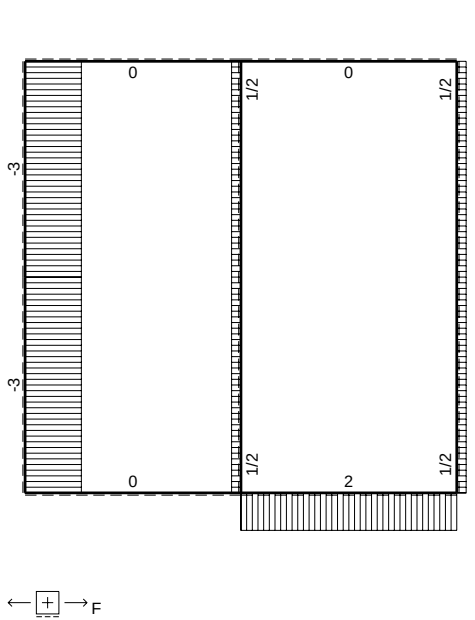
Fb
1/2F
2F
1/2Fb
1/2F
A
B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-3
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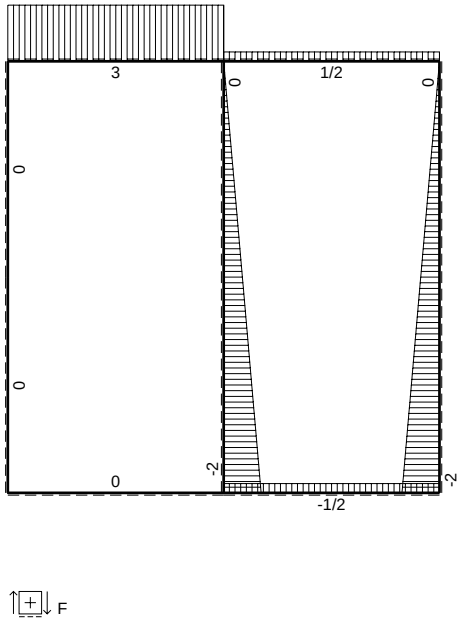
2
1/2
1/2
F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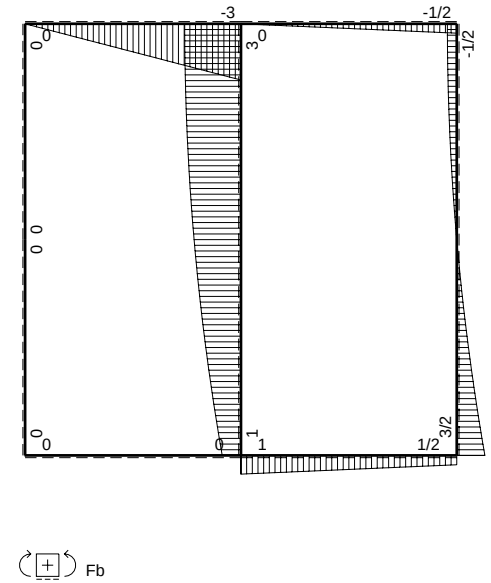
-1/2
F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-1/2
0
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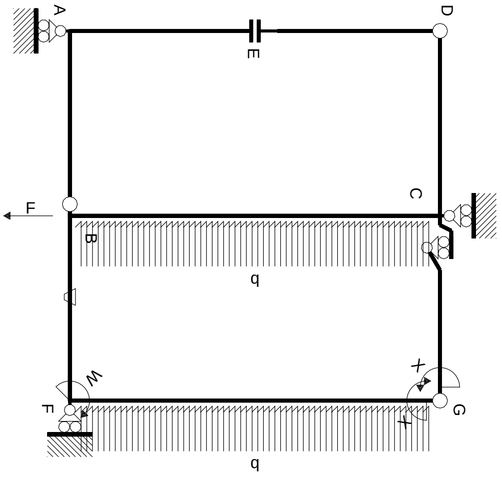
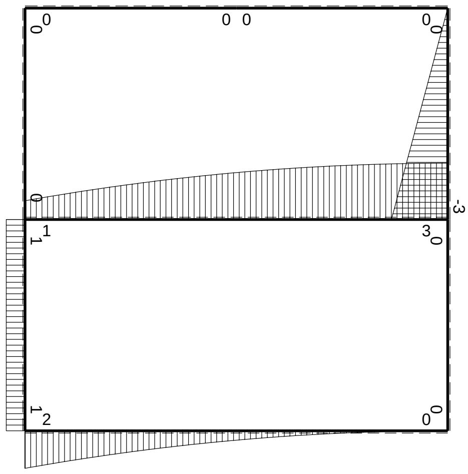
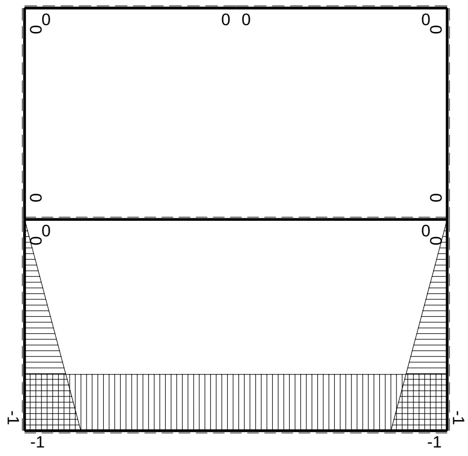
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

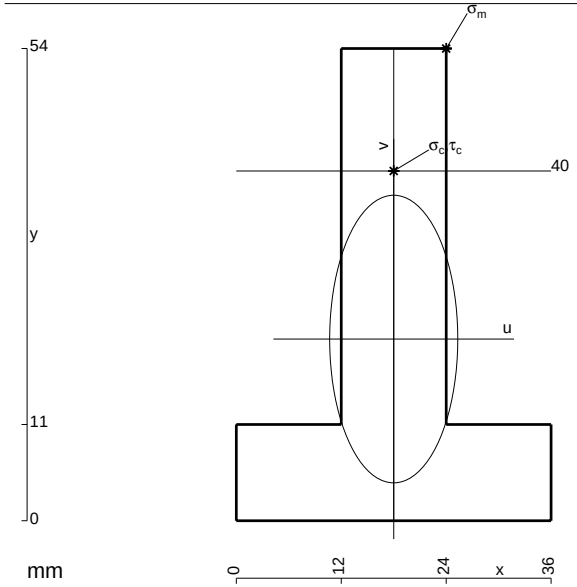
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 912. mm^2
J_u = 246834. mm^4
J_v = 48960. mm^4
y_g = 20.78 mm
T_y = 2190. N
M_x = -1773900. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.22 mm
$\sigma_m = -Mv/J_u$ = 238.8 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.22 mm
$\sigma_c = -Mv/J_u$ = 138.2 N/mm^2
τ_c = 3.257 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.3 N/mm^2
S^* = 4406. mm^3

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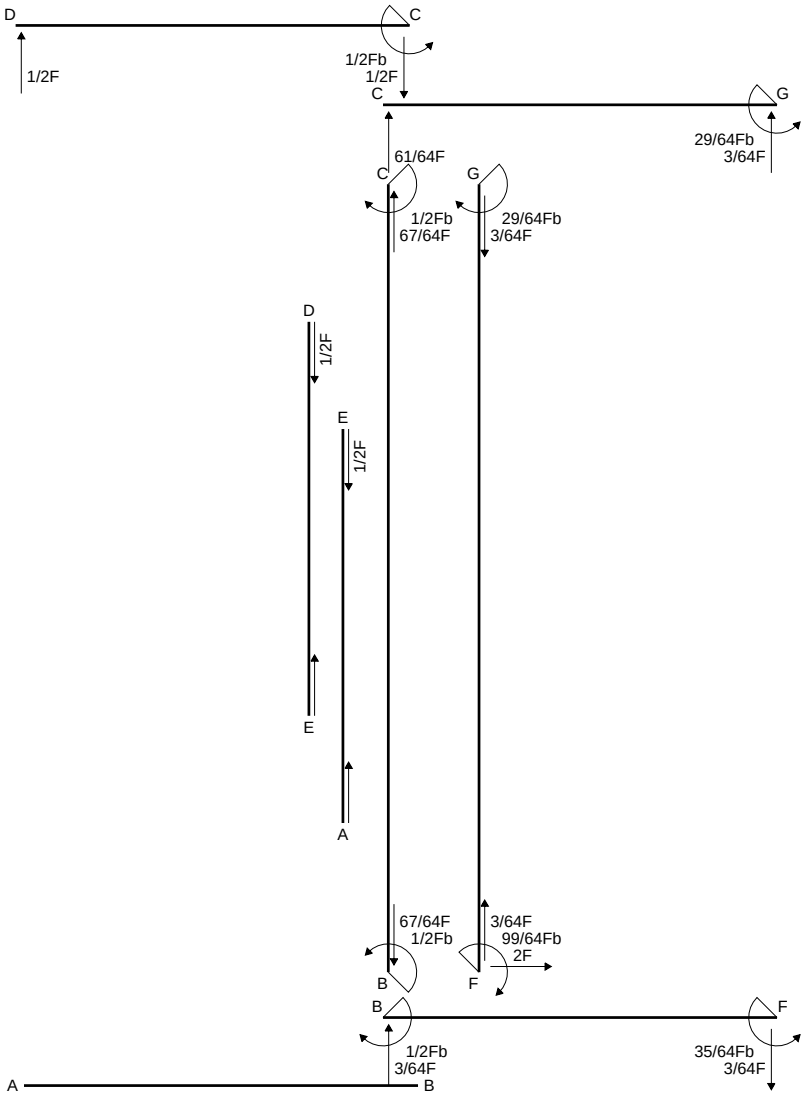
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C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
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F
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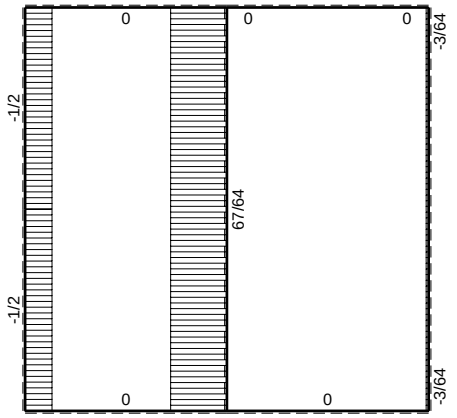
3/64F
35/64Fb
3/64F
A
B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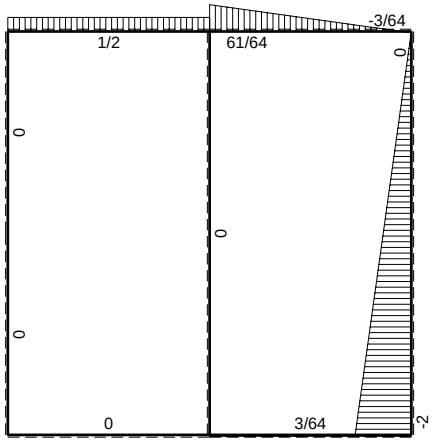
-1/2
67/64
-1/2
0
0
-3/64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1/2
61/64
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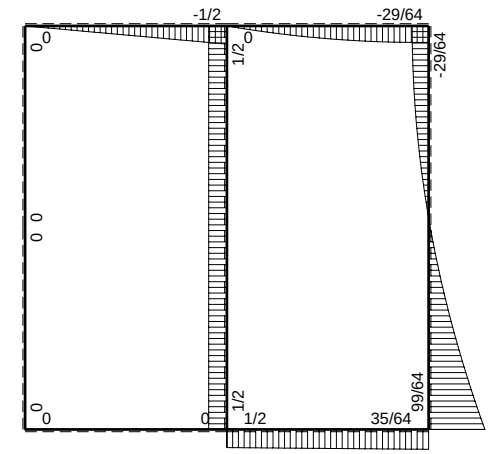
0
0
3/64
-2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-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx²/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb²/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
totali							-29/24Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

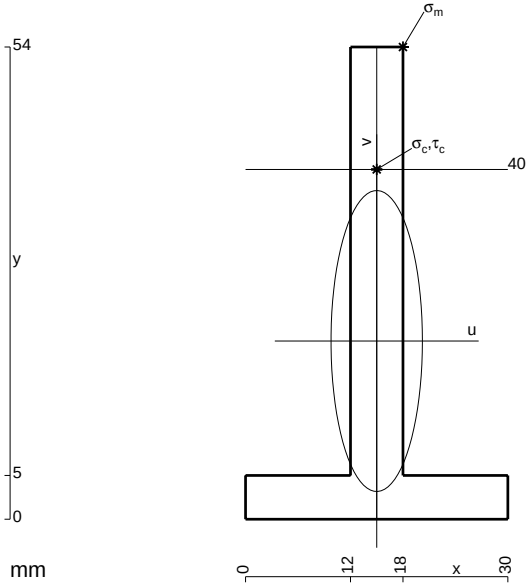
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.090



A = 444. mm^2
J_u = 131544. mm^4
J_v = 12132. mm^4
y_g = 20.38 mm
T_y = 1815. N
M_x = -780450. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 33.62 mm
$\sigma_m = -Mv/J_u$ = 199.5 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 19.62 mm
$\sigma_c = -Mv/J_u$ = 116.4 N/mm^2
τ_c = 5.142 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.8 N/mm^2
S^* = 2236. mm^3

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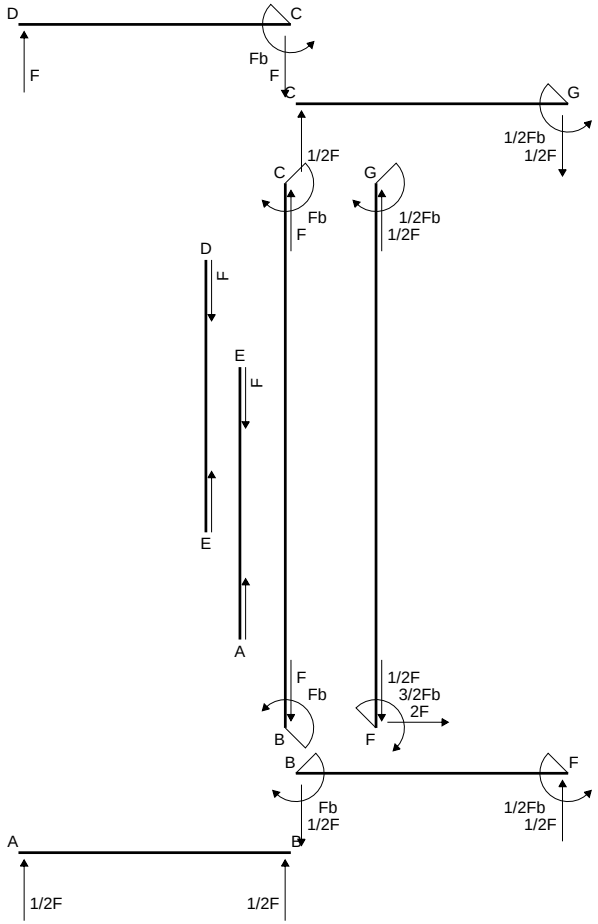
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C
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Fb
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C
G
1/2Fb
1/2F
1/2F
C
G
Fb
F
1/2Fb
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D
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E
F
E
A
F
Fb
1/2F
3/2Fb
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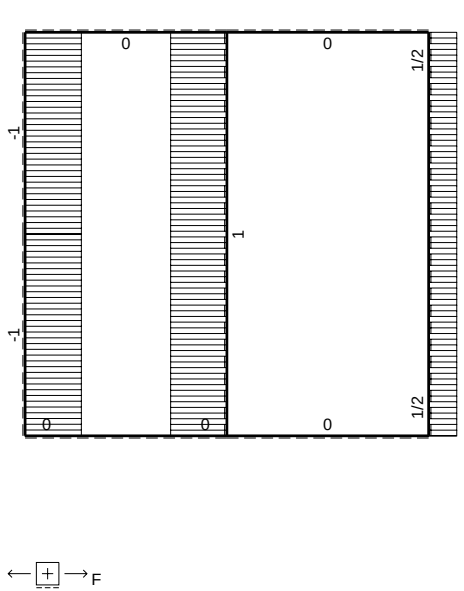
A
B
1/2F
1/2F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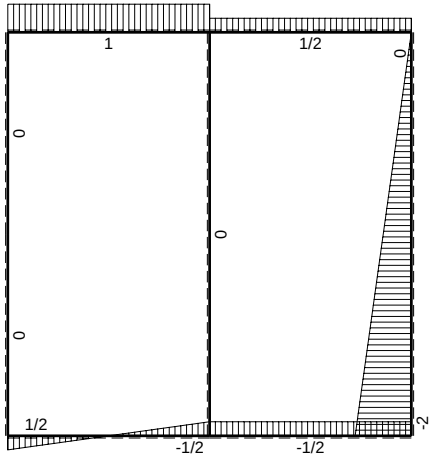
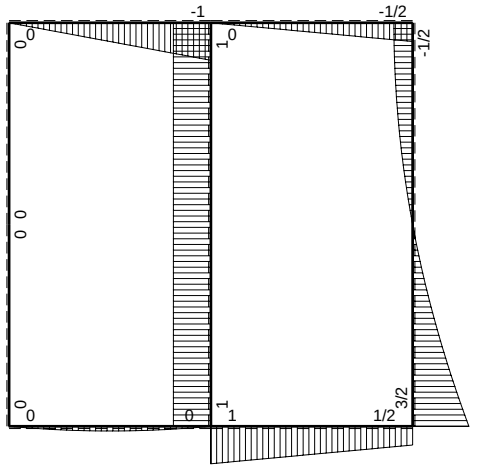
0
1/2
-1
1
-1
0
0
0
1/2

F


1
1/2
0
0
0
0
1/2
-2
-1/2
-1/2

F


-1
-1/2
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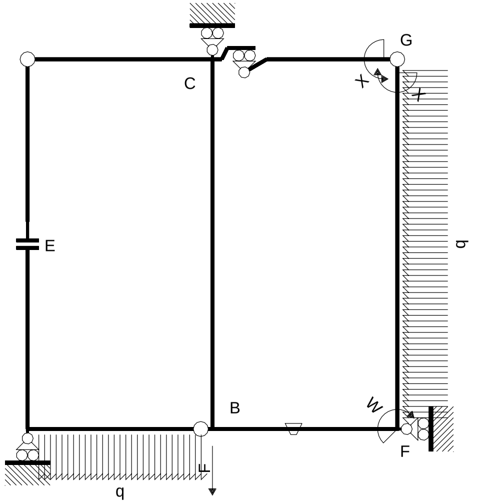
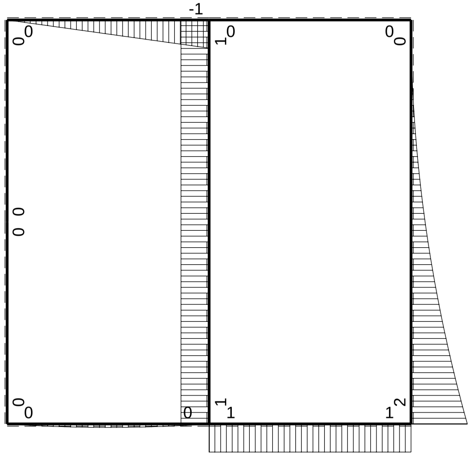
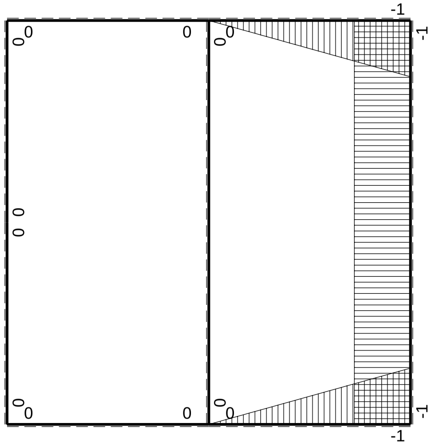
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica X=W_{GC}							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

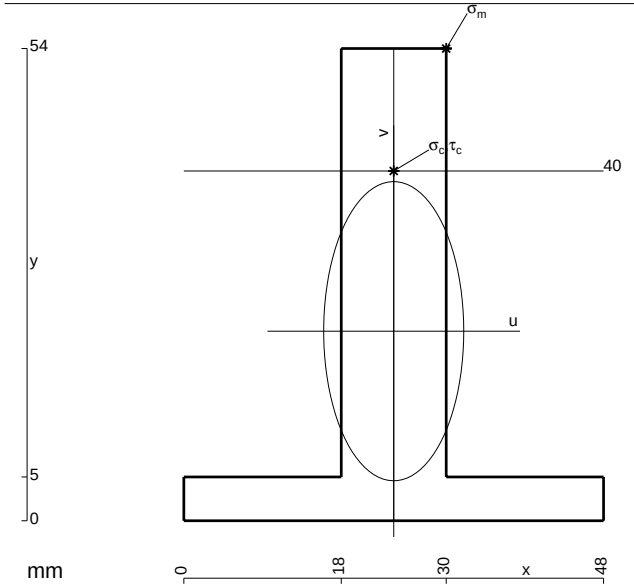
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 828. mm^2
J_u = 242396. mm^4
J_v = 53136. mm^4
y_g = 21.67 mm
T_y = 3350. N
M_x = -1574500. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 32.33 mm
$\sigma_m = -Mv/J_u$ = 210. N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 18.33 mm
$\sigma_c = -Mv/J_u$ = 119. N/mm^2
τ_c = 4.9 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.3 N/mm^2
S^* = 4255. mm^3

ESΣ8.091

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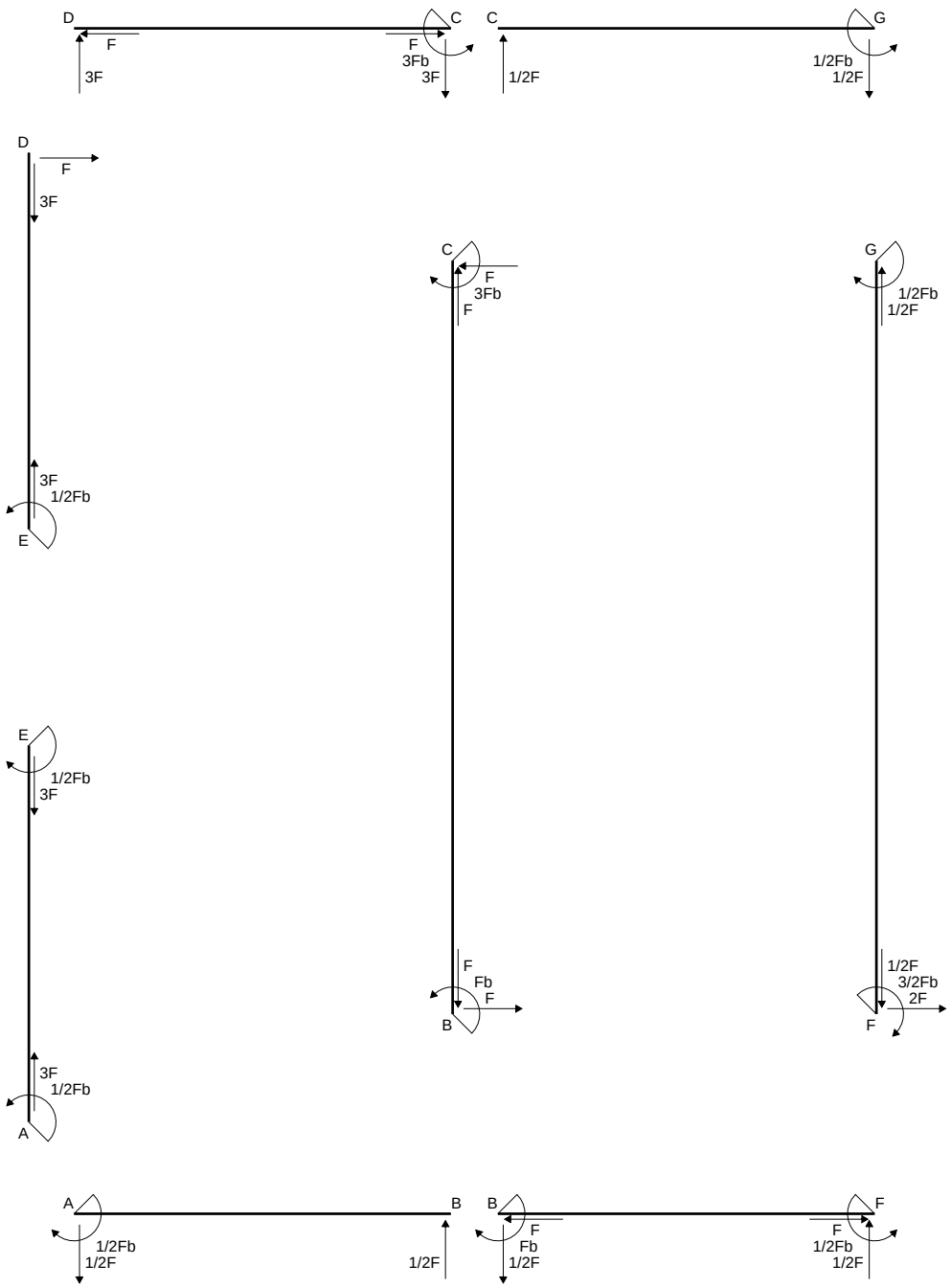
D
C
C
G
F
3F
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3Fb
3F
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1/2Fb
1/2F
D
F
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F
G
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F
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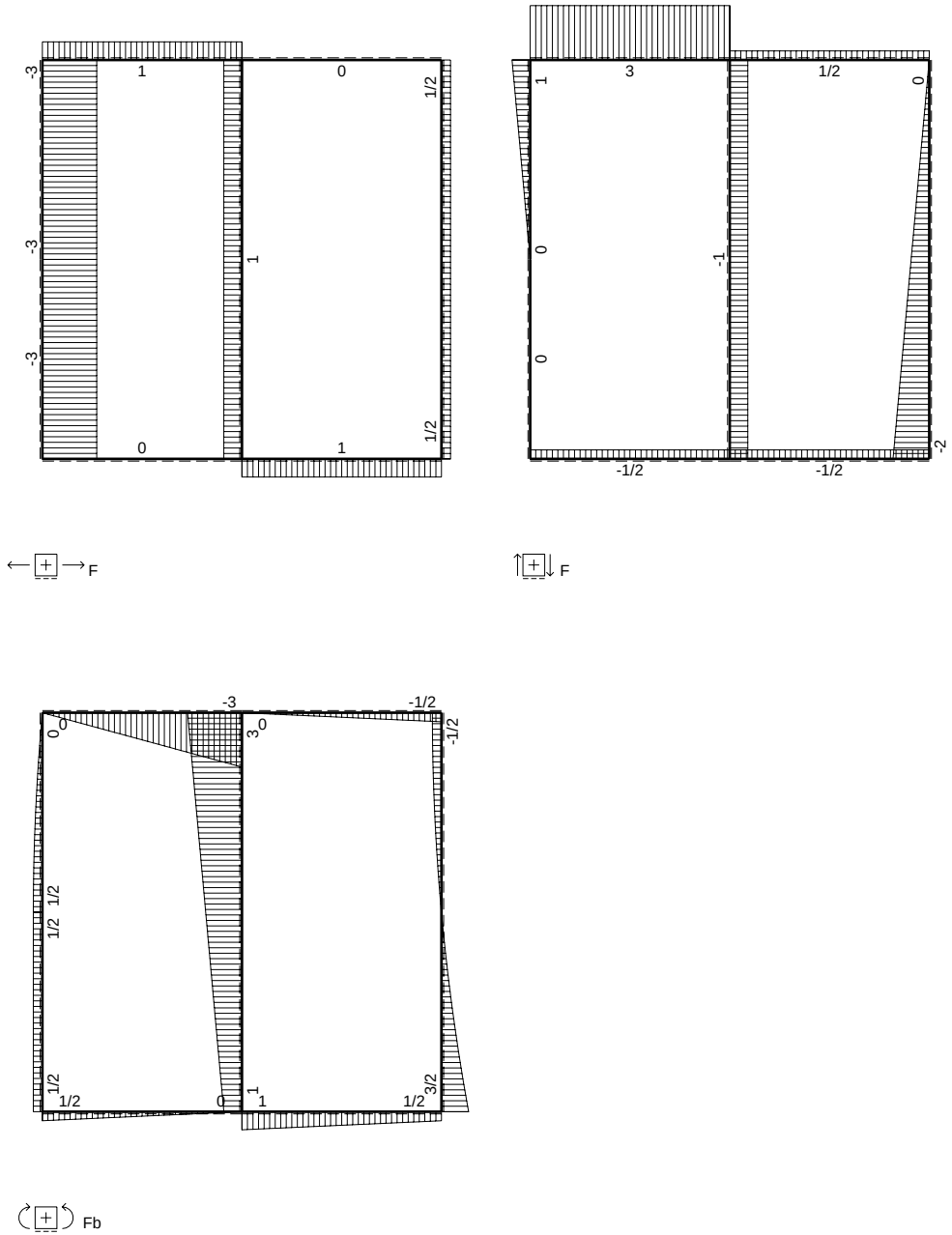
-3
1
0
1/2
-3
1
-3
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1
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F
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3
1/2
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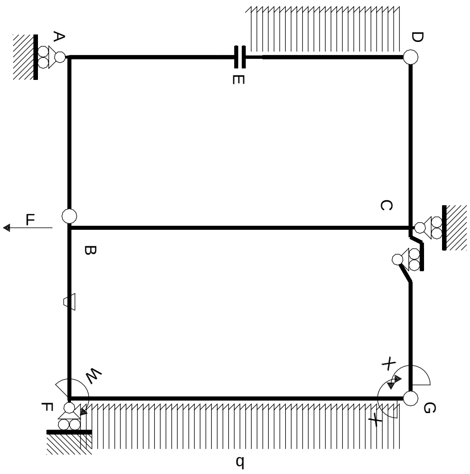
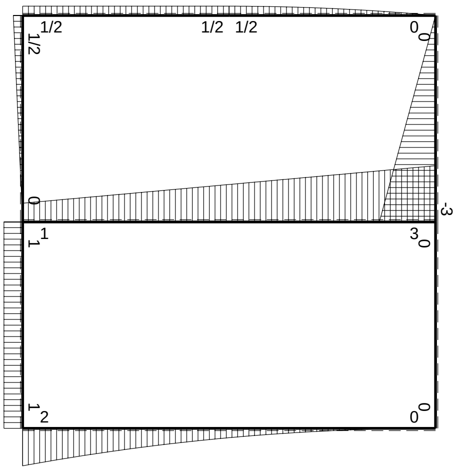
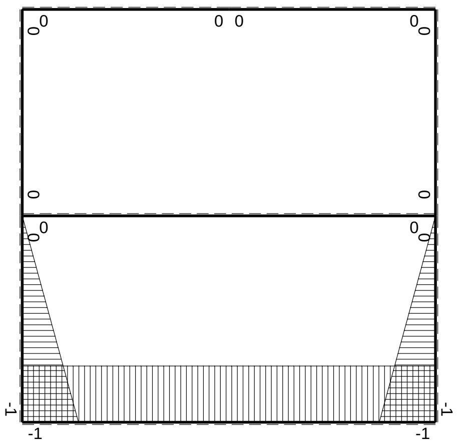
1
0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

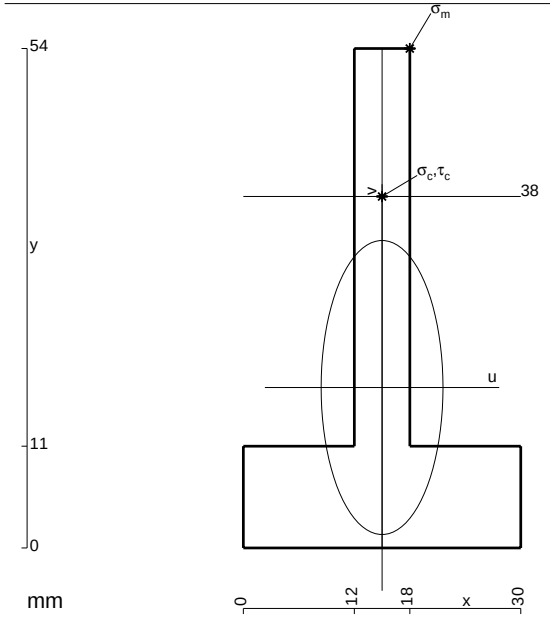
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 588. mm^2
J_u = 148637. mm^4
J_v = 25524. mm^4
y_g = 17.35 mm
N = 580. N
T_y = 1740. N
M_x = -887400. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 36.65 mm
$\sigma_m = N/A - Mv/J_u$ = 219.8 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 20.65 mm
$\sigma_c = N/A - Mv/J_u$ = 124.3 N/mm^2
τ_c = 5.367 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 124.6 N/mm^2
S^* = 2751. mm^3

ESΣ8.092

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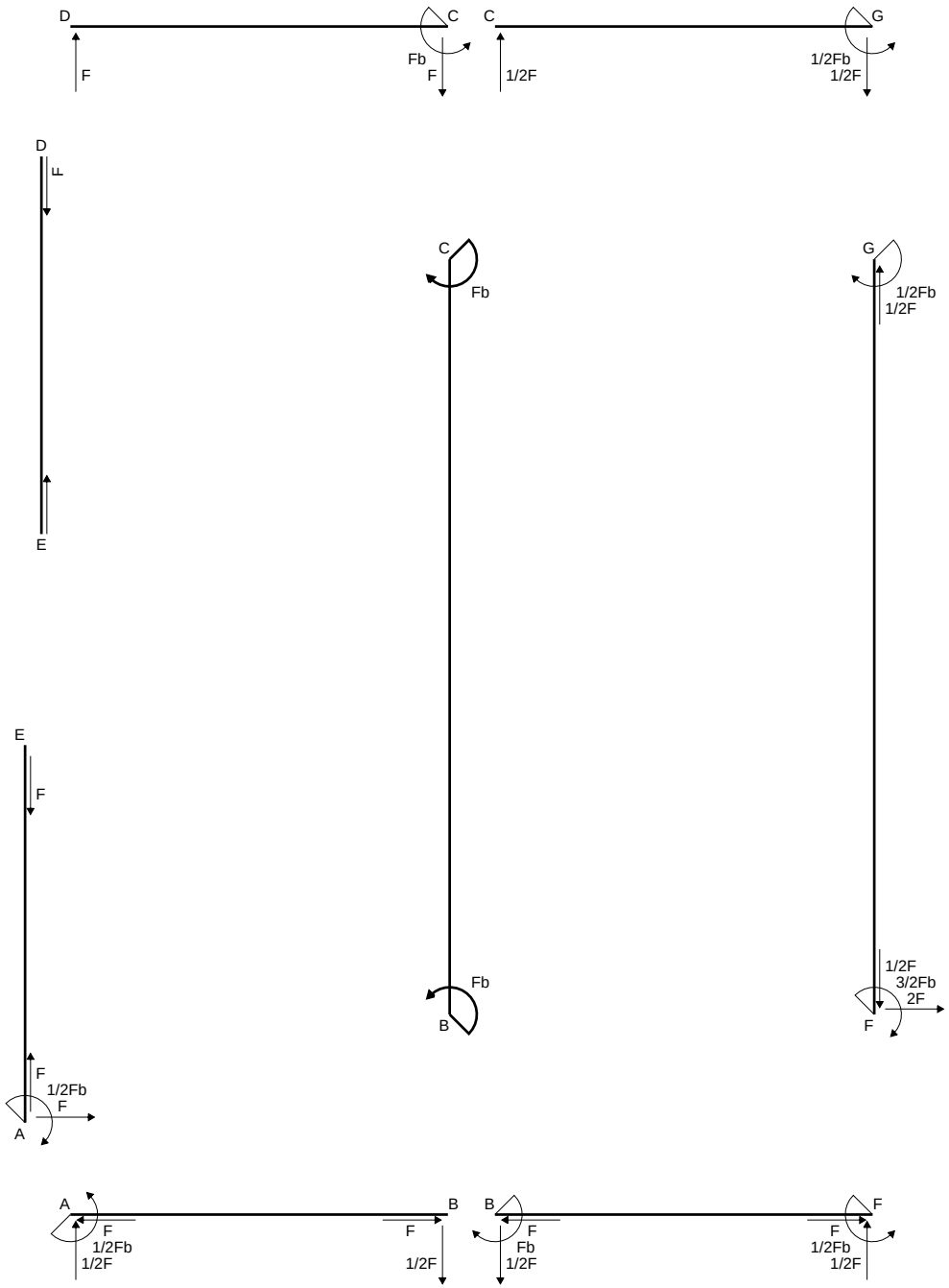
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F
Fb
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1/2Fb
1/2F
D
F
E
C
Fb
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1/2F
Fb
B
1/2F
3/2Fb
2F
F
E
F
F
1/2Fb
F
A
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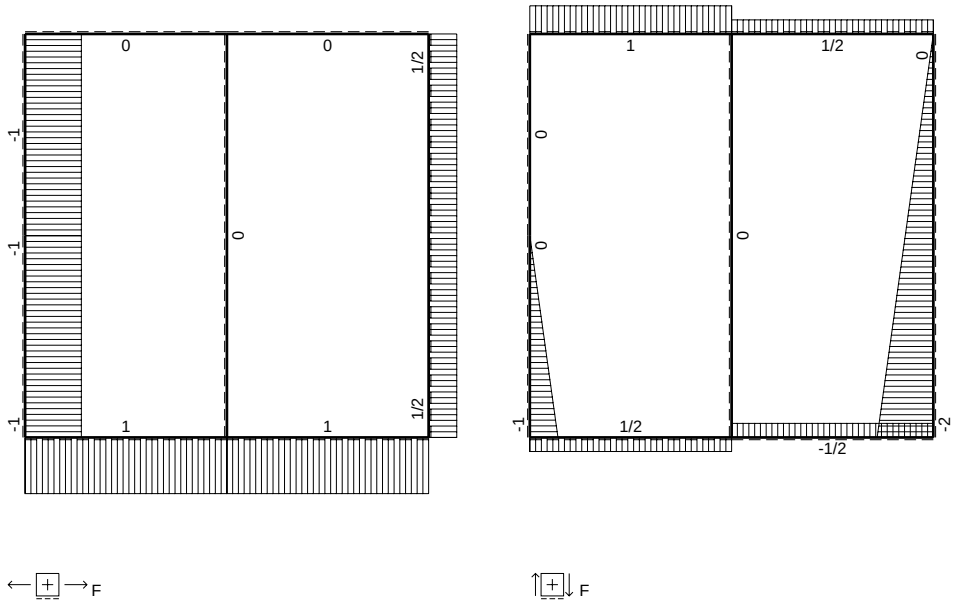
1/2Fb
1/2F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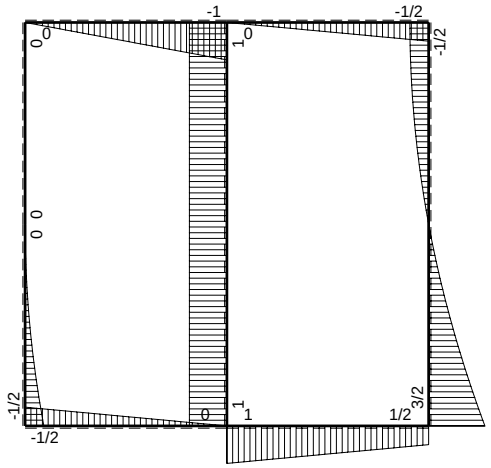
F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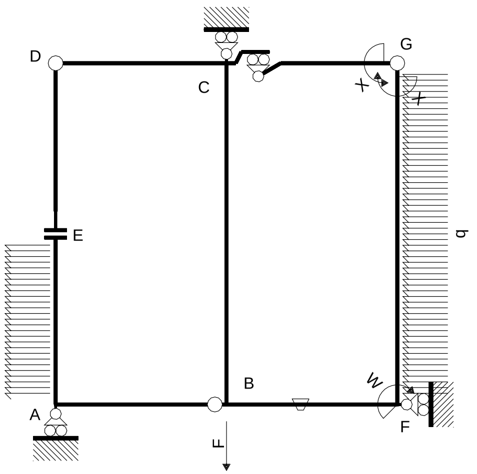
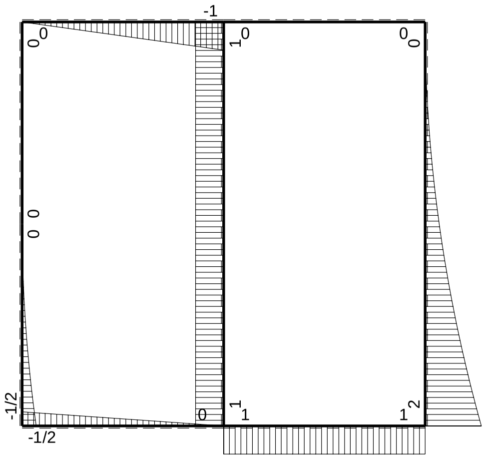
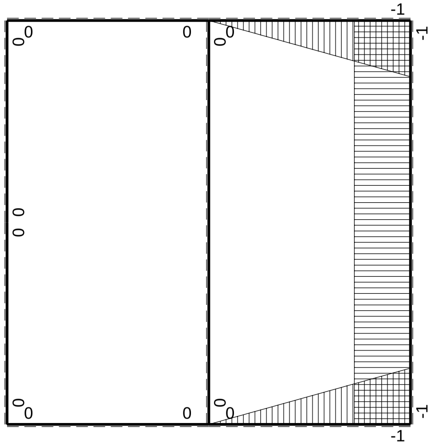
-1/2


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

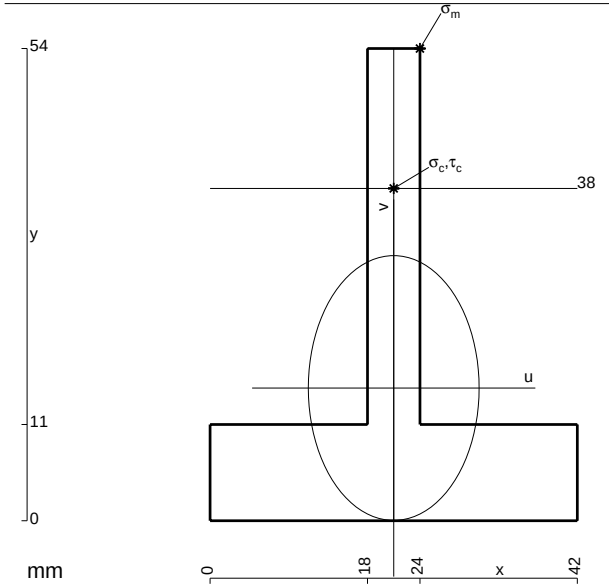
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 720. mm^2
J_u = 165098. mm^4
J_v = 68688. mm^4
y_g = 15.18 mm
T_y = 1770. N
M_x = -973500. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 38.83 mm
$\sigma_m = -Mv/J_u$ = 228.9 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 22.83 mm
$\sigma_c = -Mv/J_u$ = 134.6 N/mm^2
τ_c = 5.288 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.9 N/mm^2
S^* = 2959. mm^3

ESΣ8.093

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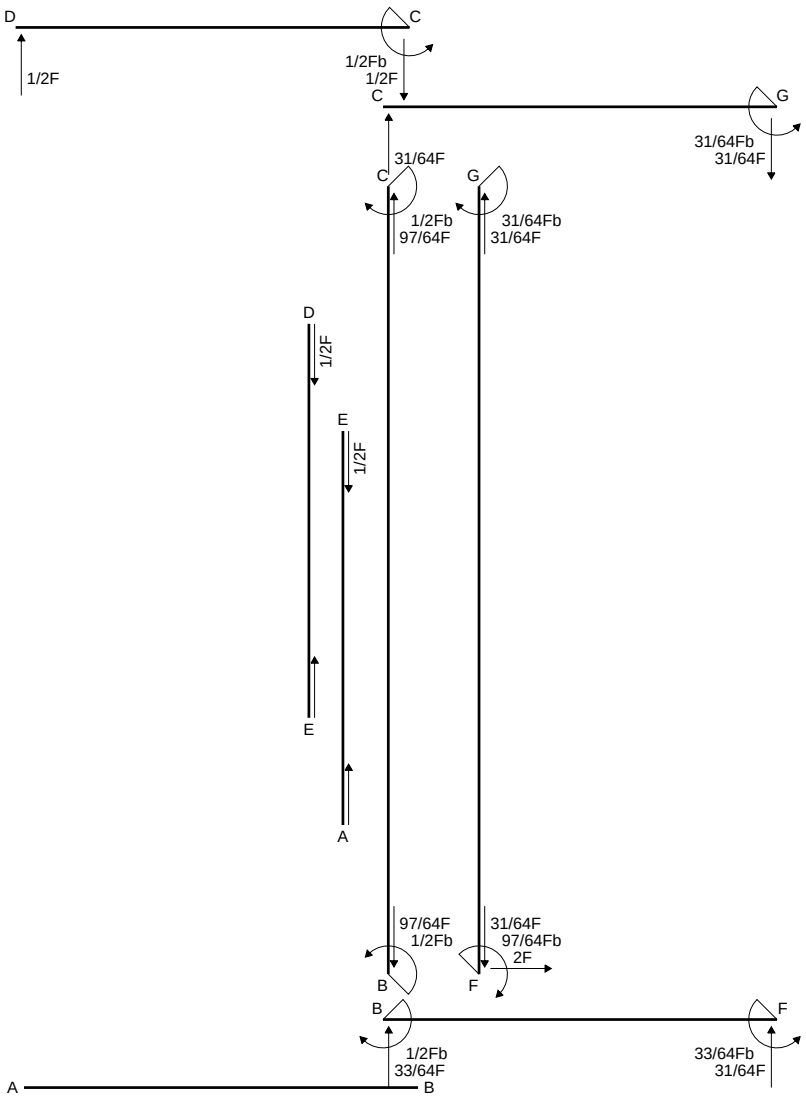
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1/2Fb
1/2F
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G
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C
31/64F
G
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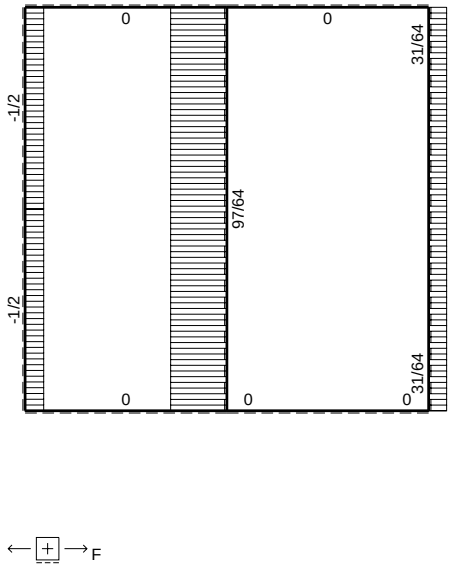
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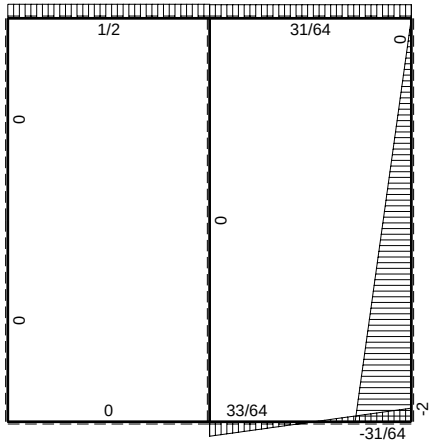
31/64

F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-31/64

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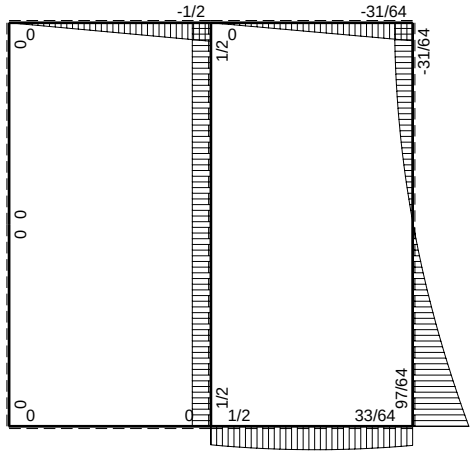
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1/2
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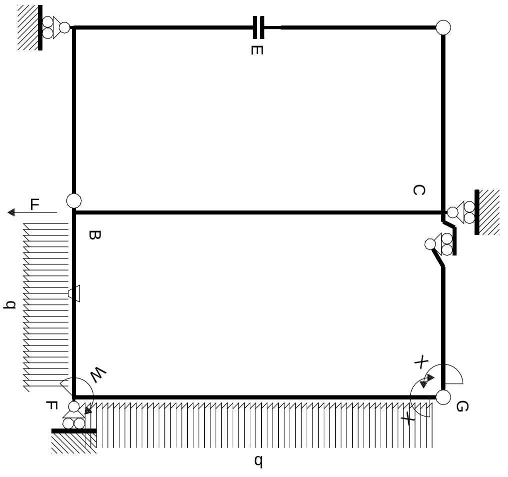
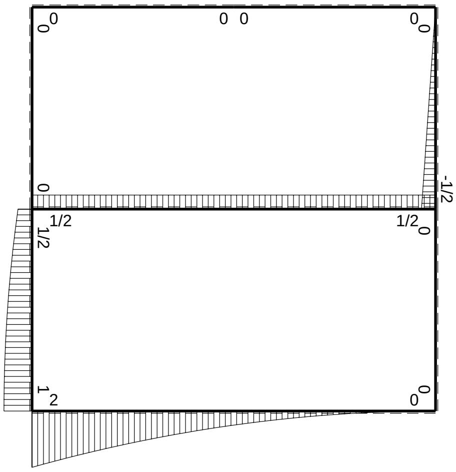
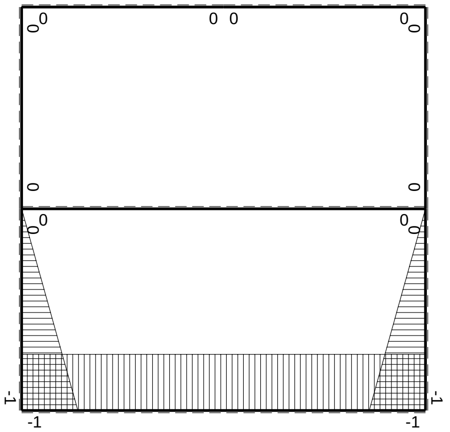
1/2
33/64
97/64

Fb





M$_o$ flessione da carichi assegnati



M$_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W$_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W$_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

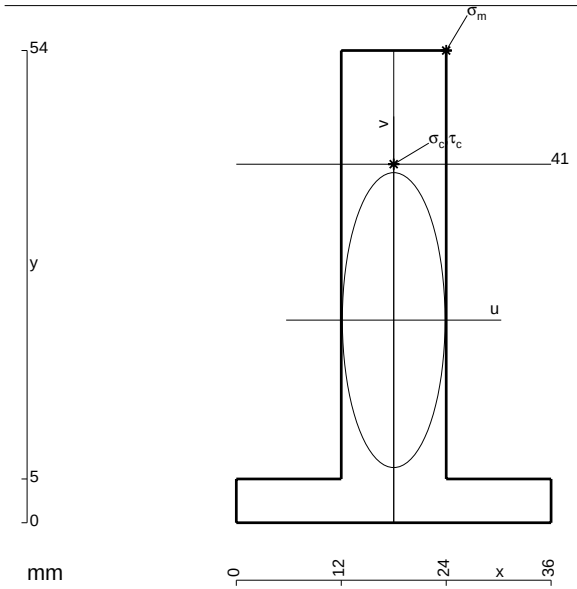
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 768. mm^2
J_u = 218489. mm^4
J_v = 26496. mm^4
y_g = 23.17 mm
T_y = 2830. N
M_x = -1698000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 30.83 mm
$\sigma_m = -Mv/J_u$ = 239.6 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 17.83 mm
$\sigma_c = -Mv/J_u$ = 138.6 N/mm^2
τ_c = 4.096 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 138.7 N/mm^2
S^* = 3795. mm^3

ESΣ8.094

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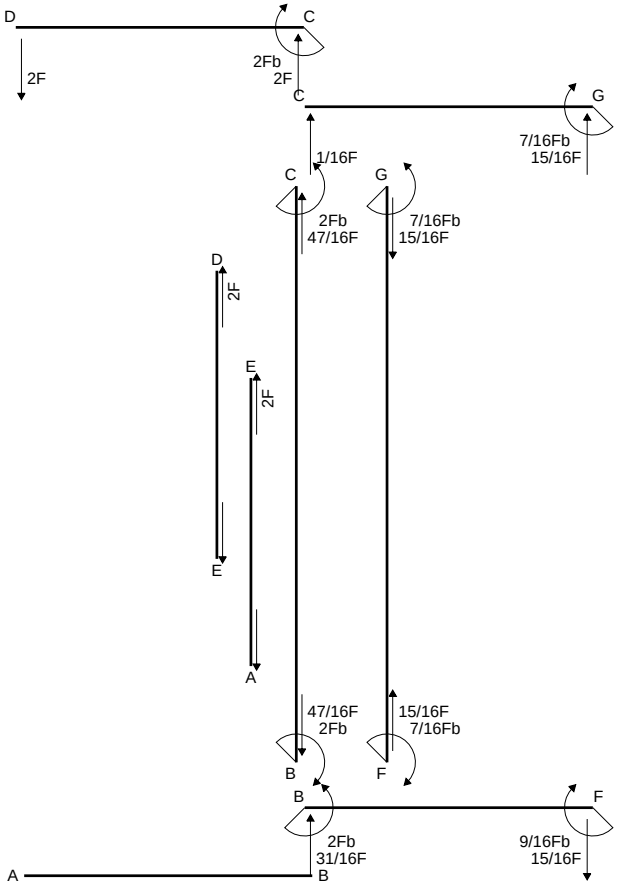
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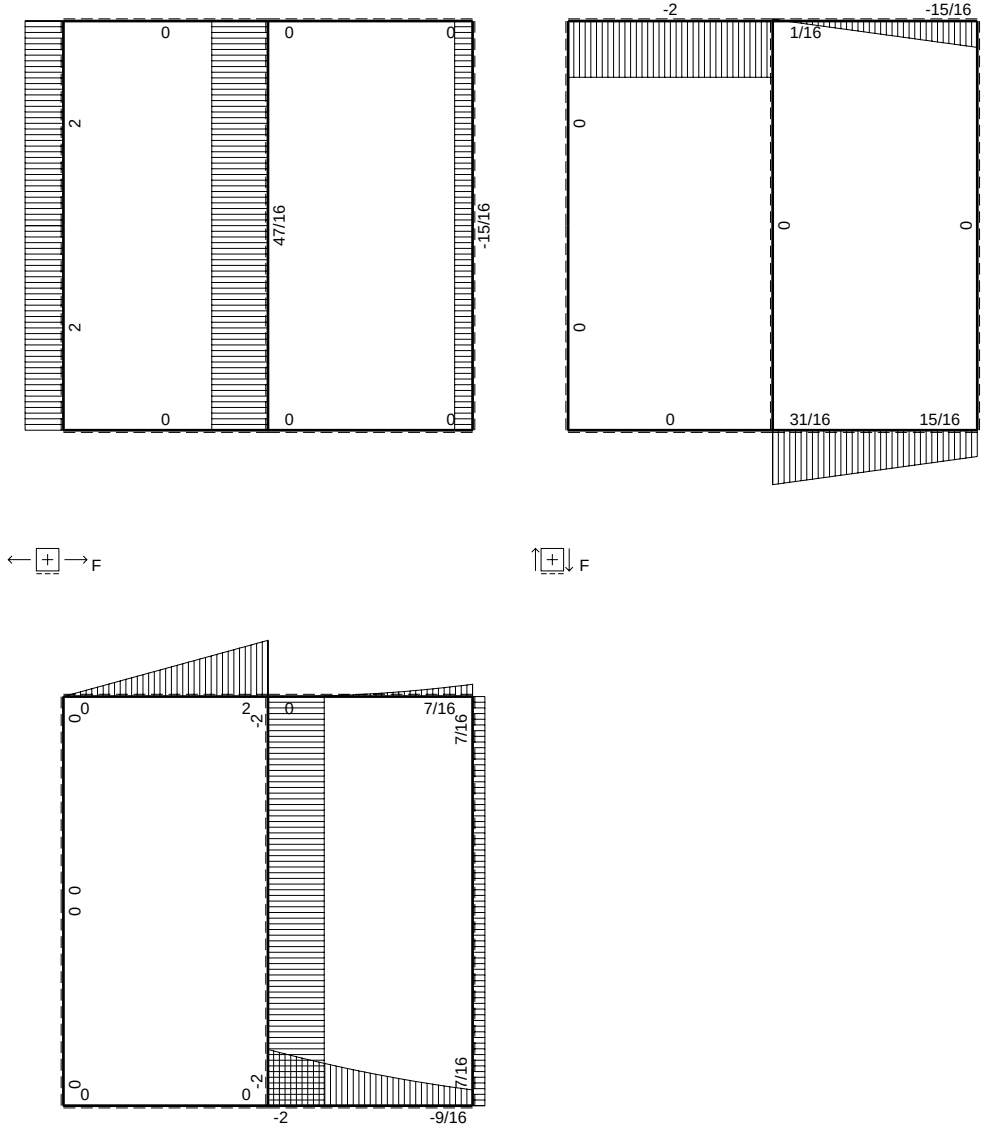
C
2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
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2
47/16
-15/16
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0
0
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-2
-15/16
1/16
0
0
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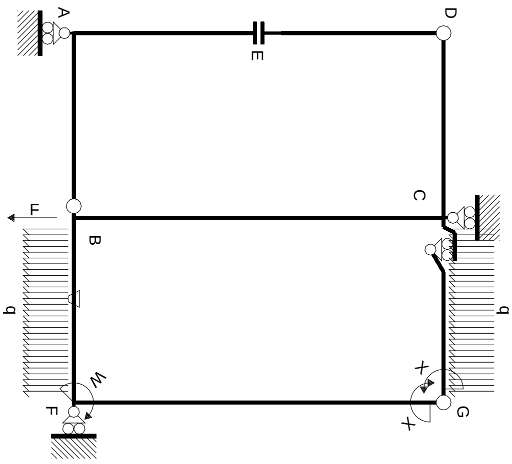
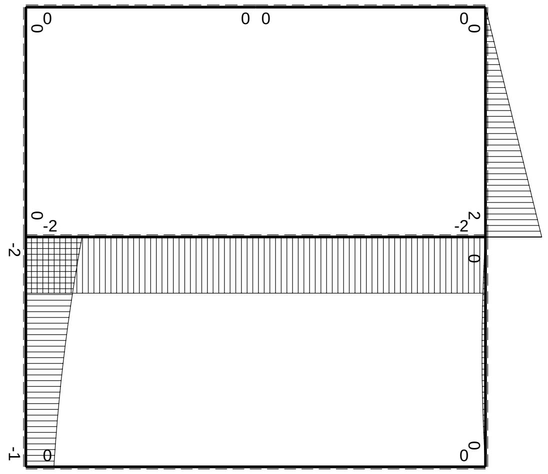
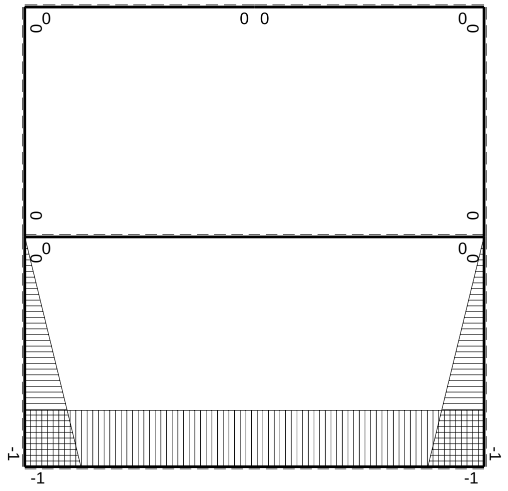
0
0
31/16
15/16
F
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

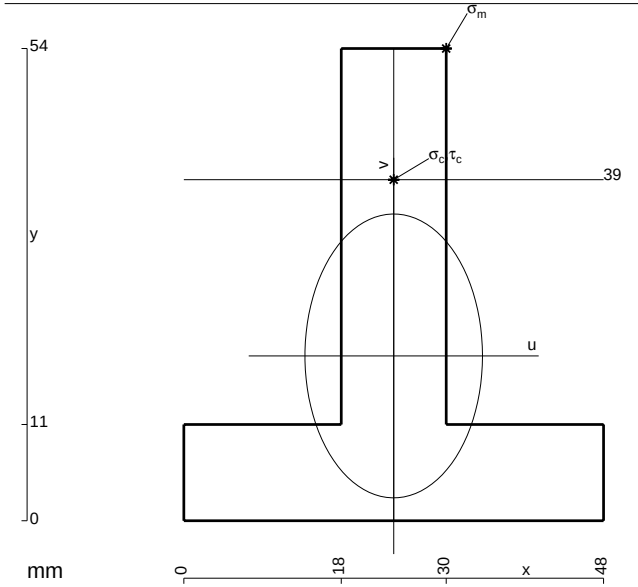
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.095



A = 1044. mm^2
J_u = 275075. mm^4
J_v = 107568. mm^4
y_g = 18.84 mm
T_y = -2440. N
M_x = 1561600. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 35.16 mm
$\sigma_m = -Mv/J_u$ = -199.6 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.16 mm
$\sigma_c = -Mv/J_u$ = -114.4 N/mm^2
τ_c = 3.68 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.6 N/mm^2
S^* = 4978. mm^3

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ESΣ8.095

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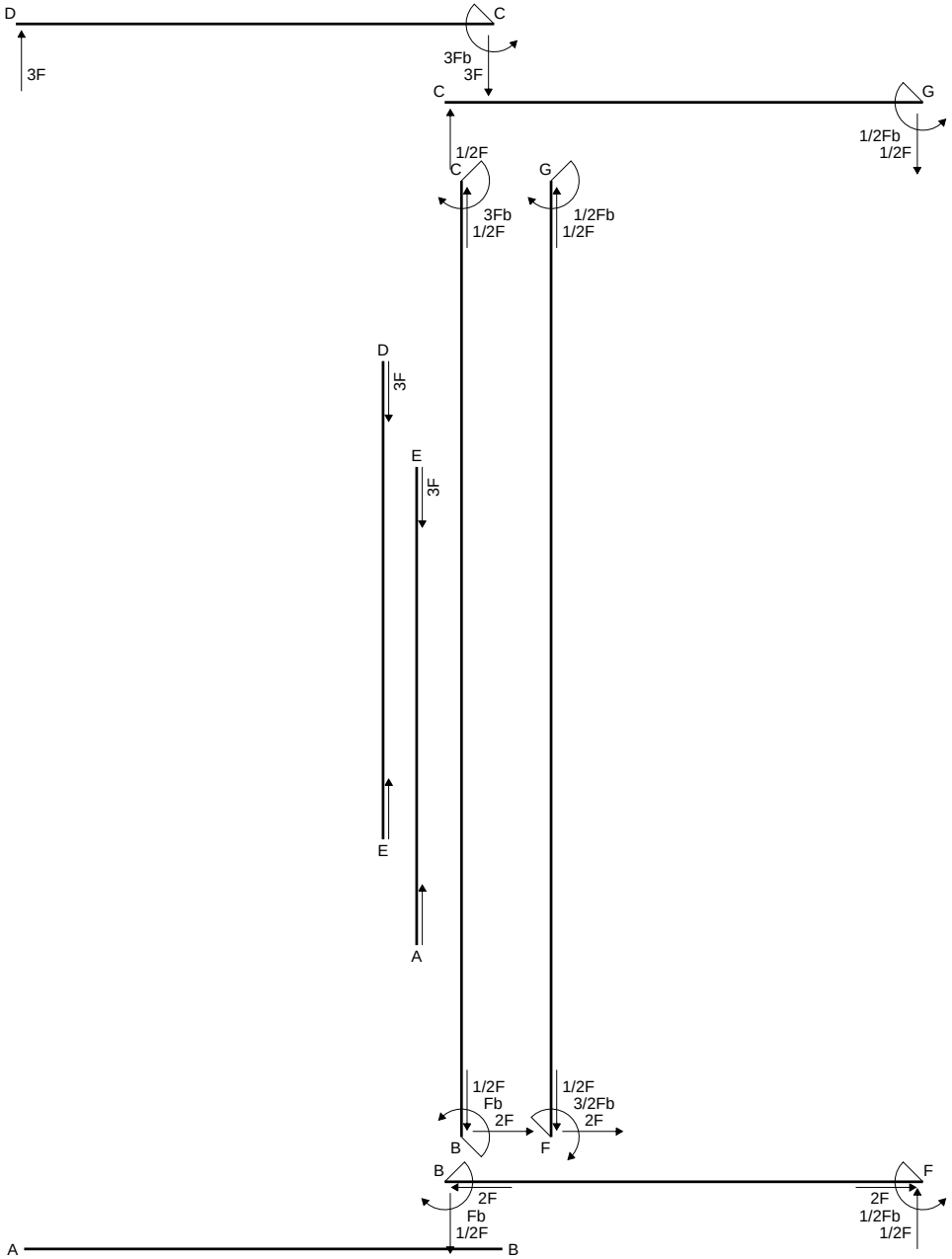
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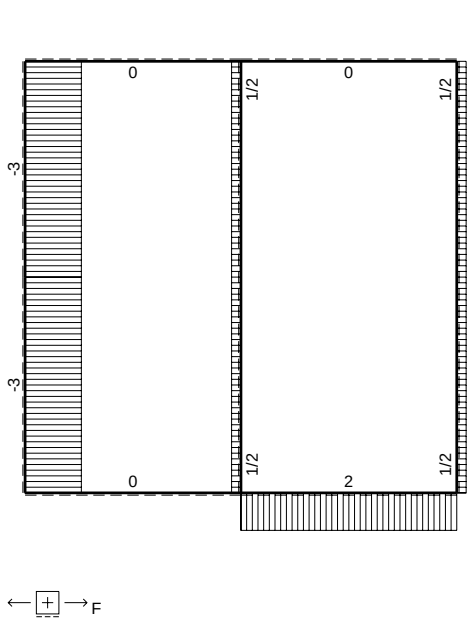
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
1/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
1/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B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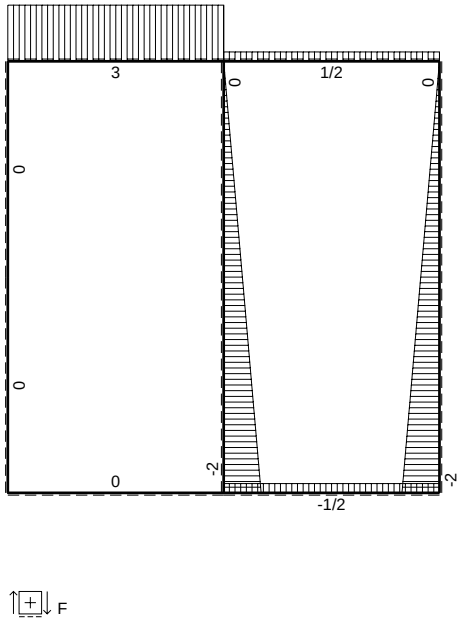
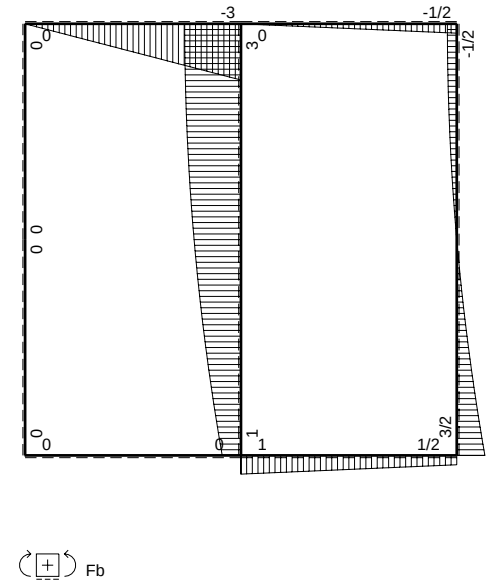
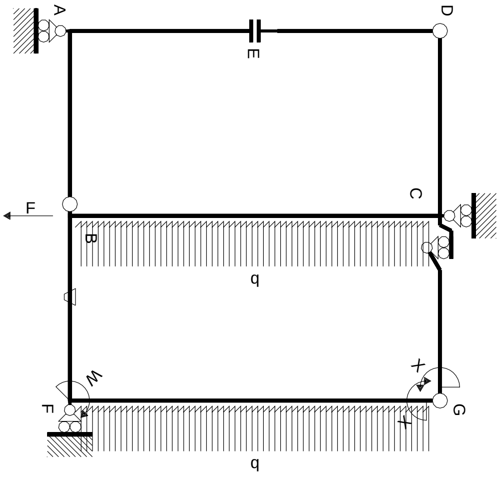
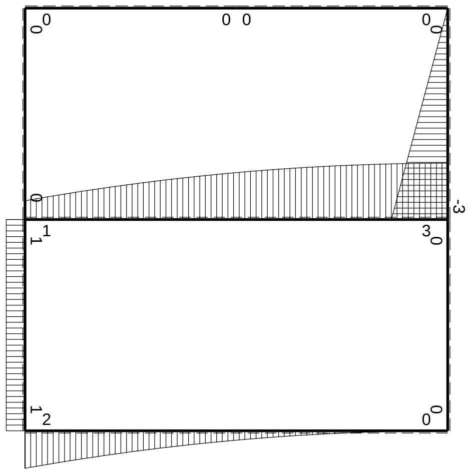
0
1/2
1/2
-3
-3
0
0
1/2
1/2
2
F

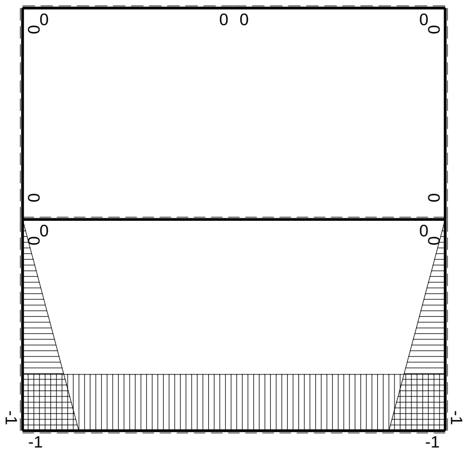

3
0
1/2
0
0
0
0
-2
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

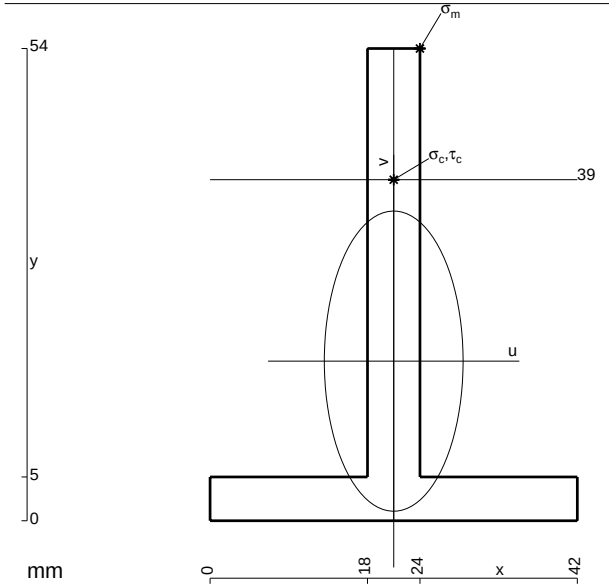
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 504. mm^2
J_u = 148565. mm^4
J_v = 31752. mm^4
y_g = 18.25 mm
T_y = 1260. N
M_x = -856800. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 35.75 mm
$\sigma_m = -Mv/J_u$ = 206.2 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 20.75 mm
$\sigma_c = -Mv/J_u$ = 119.7 N/mm^2
τ_c = 3.594 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.8 N/mm^2
S^* = 2543. mm^3

ESΣ8.096

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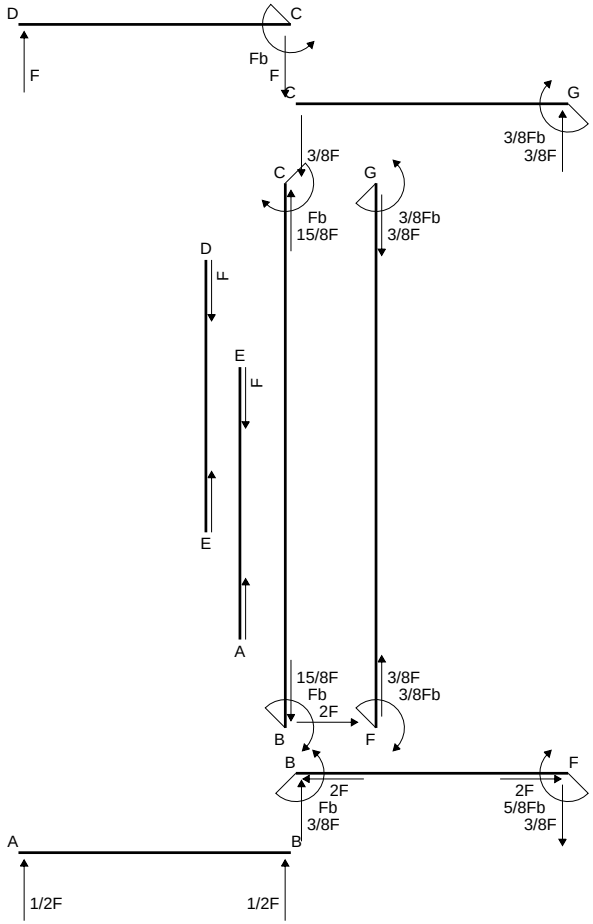
D
C
F
Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
15/8F
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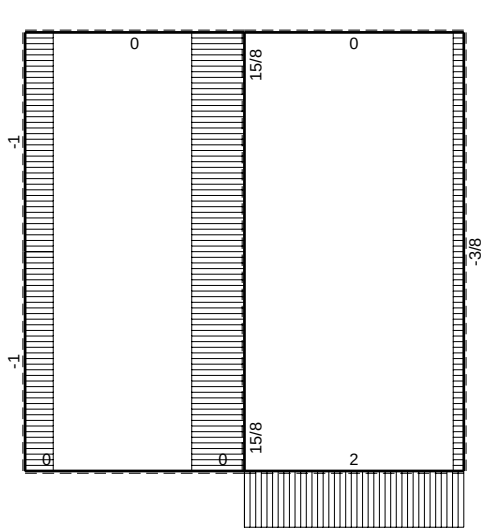
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
B
F
B
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
1/2F
1/2F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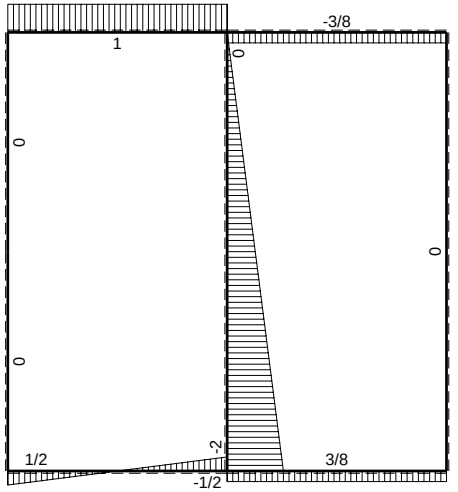
15/8
-1
-3/8
-1
15/8
0
0
2

F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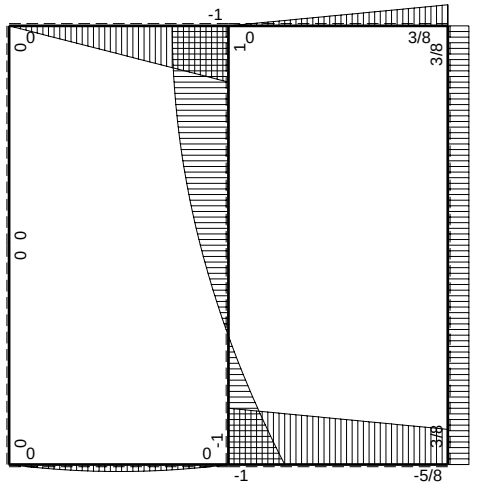
0
0
0
0
0
1/2
-2
3/8
-1/2

F


-1
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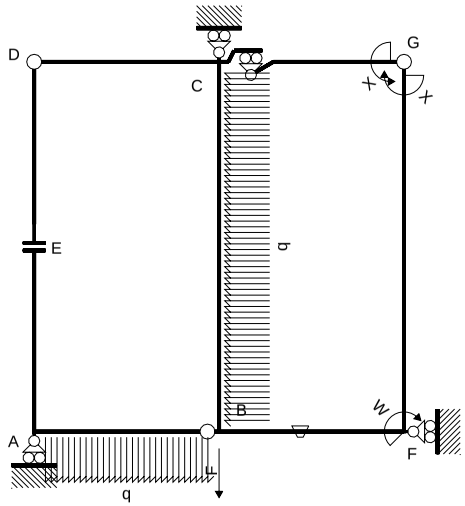
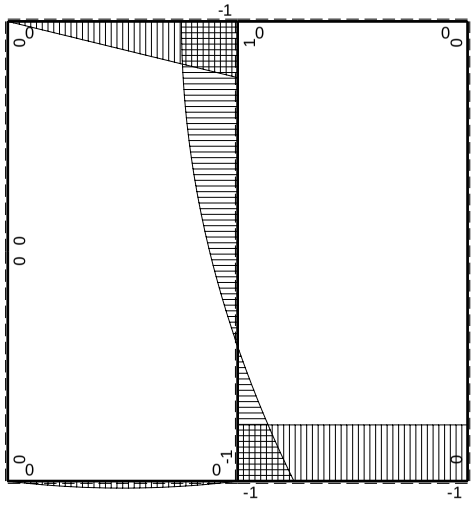
0
1
0
3/8
3/8
0 0
0
0
0
-1
3/8
-1
-5/8

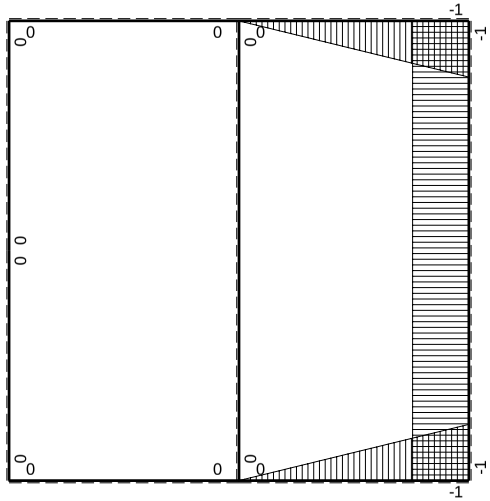
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.097

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

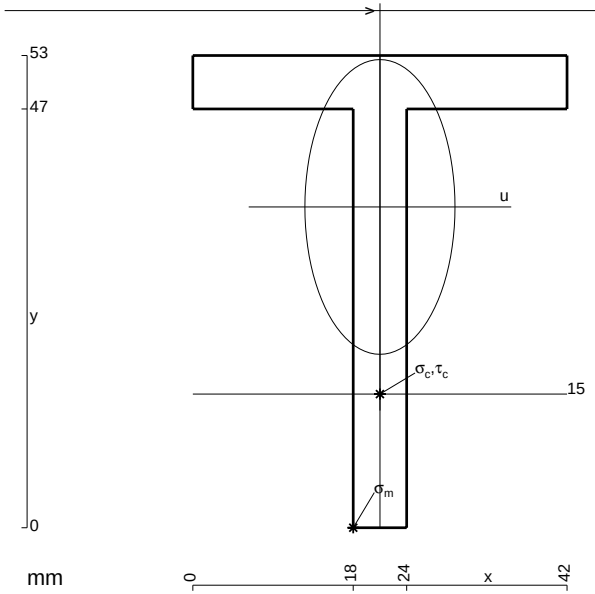
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 534. mm^2
J_u = 146122. mm^4
J_v = 37890. mm^4
y_g = 36.01 mm
N = 2400. N
T_y = -2560. N
M_x = -908800. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -36.01 mm
$\sigma_m = N/A - Mv/J_u$ = -219.4 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -21.01 mm
$\sigma_c = N/A - Mv/J_u$ = -126.1 N/mm^2
τ_c = 7.491 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.8 N/mm^2
S^* = 2566. mm^3

ESΣ8.097

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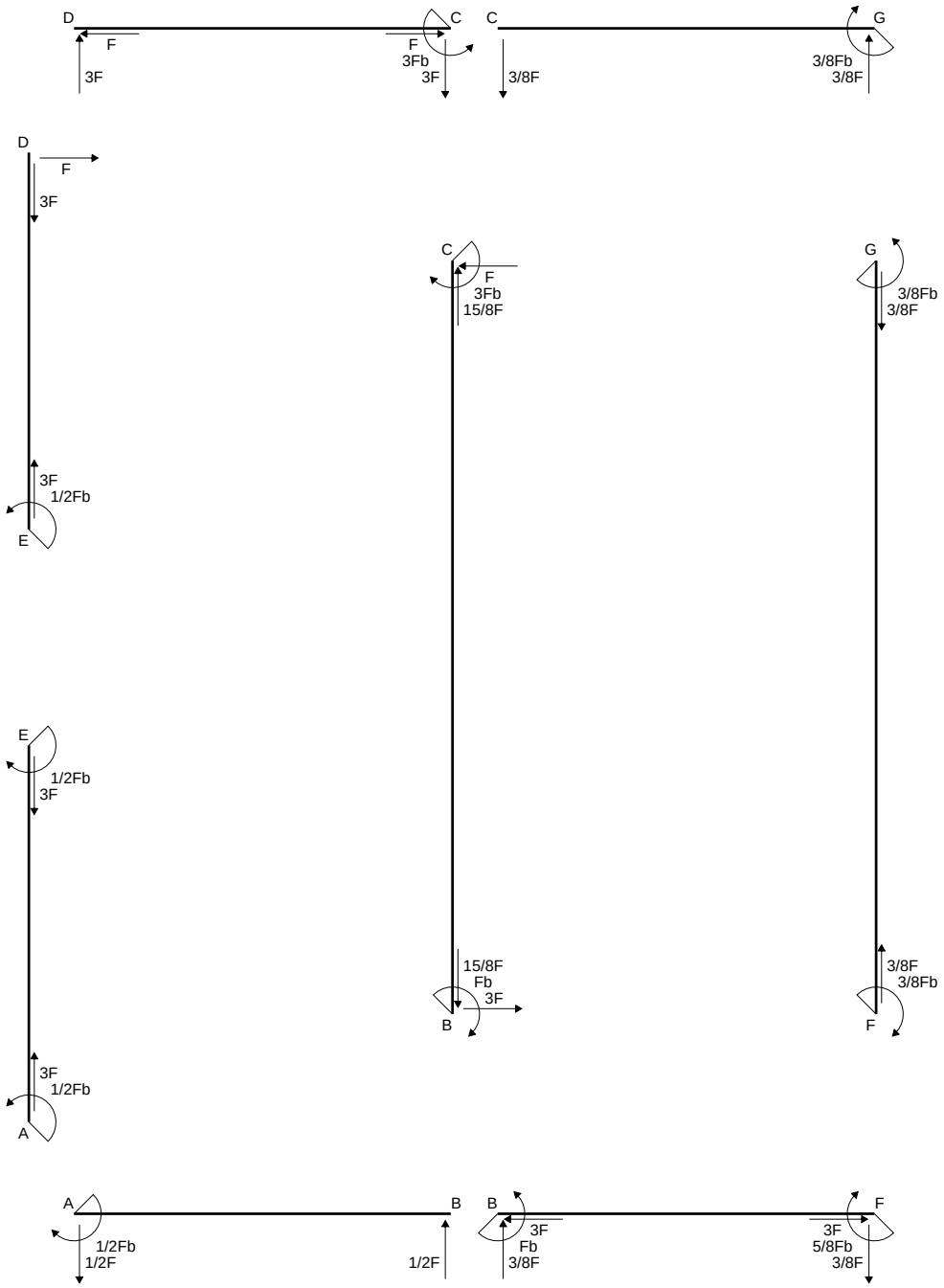
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C
F
3Fb
3F
C
3/8F
G
3/8Fb
3/8F
D
F
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C
F
3Fb
15/8F
G
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3/8F
E
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3F
15/8F
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3F
B
3/8F
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F
3F
1/2Fb
A
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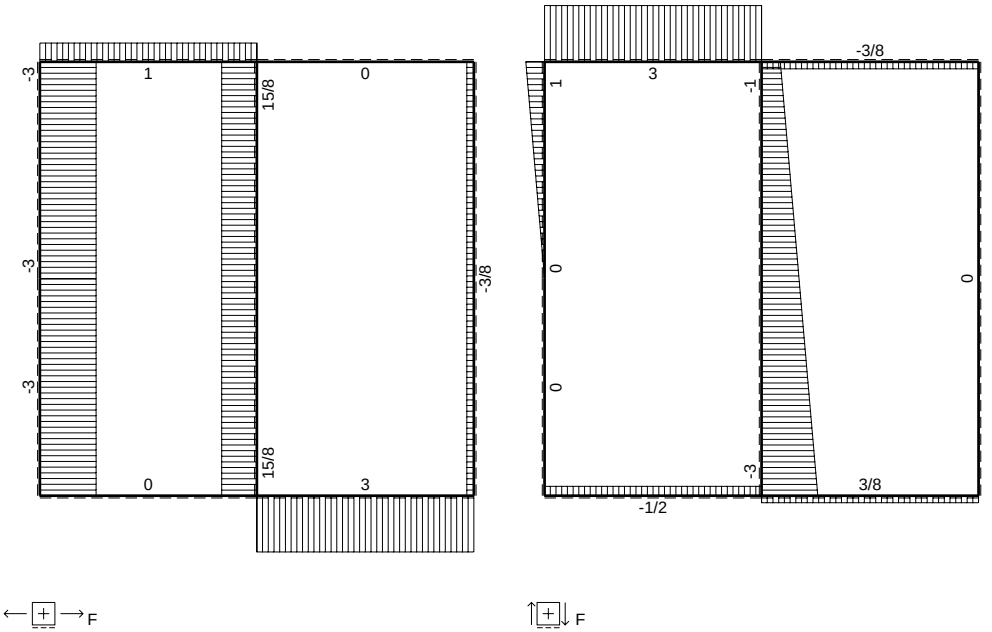
Fb
3/8F
F
3F
5/8Fb
3/8F


-3
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F
-3/8
1
3
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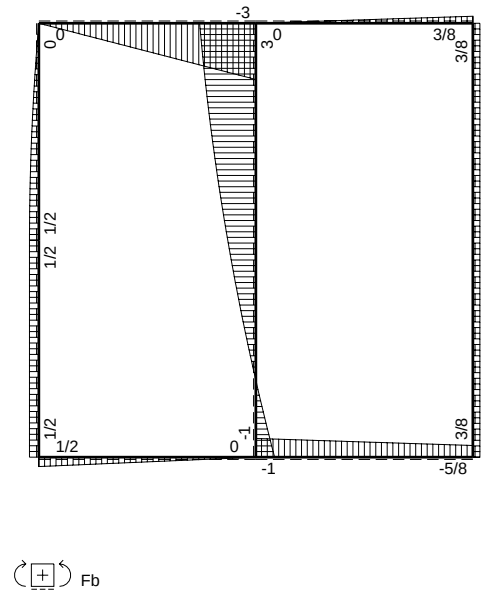
0
0
-3
3/8
-1/2
F


-3
0
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3
0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
	totali					iperstatica $X=W_{GC}$	Fb^2/EJ	8/3Xb/EJ
							-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

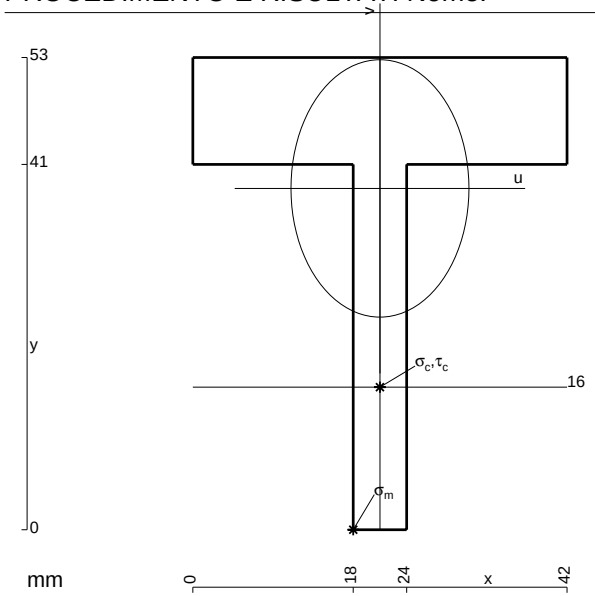
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 750. mm^2
J_u = 156599. mm^4
J_v = 74826. mm^4
y_g = 38.31 mm
N = 410. N
T_y = 1230. N
M_x = -922500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.31 mm
$\sigma_m = N/A - Mv/J_u$ = -225.1 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.31 mm
$\sigma_c = N/A - Mv/J_u$ = -130.9 N/mm^2
τ_c = 3.809 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131. N/mm^2
S^* = 2910. mm^3

ESΣ8.098

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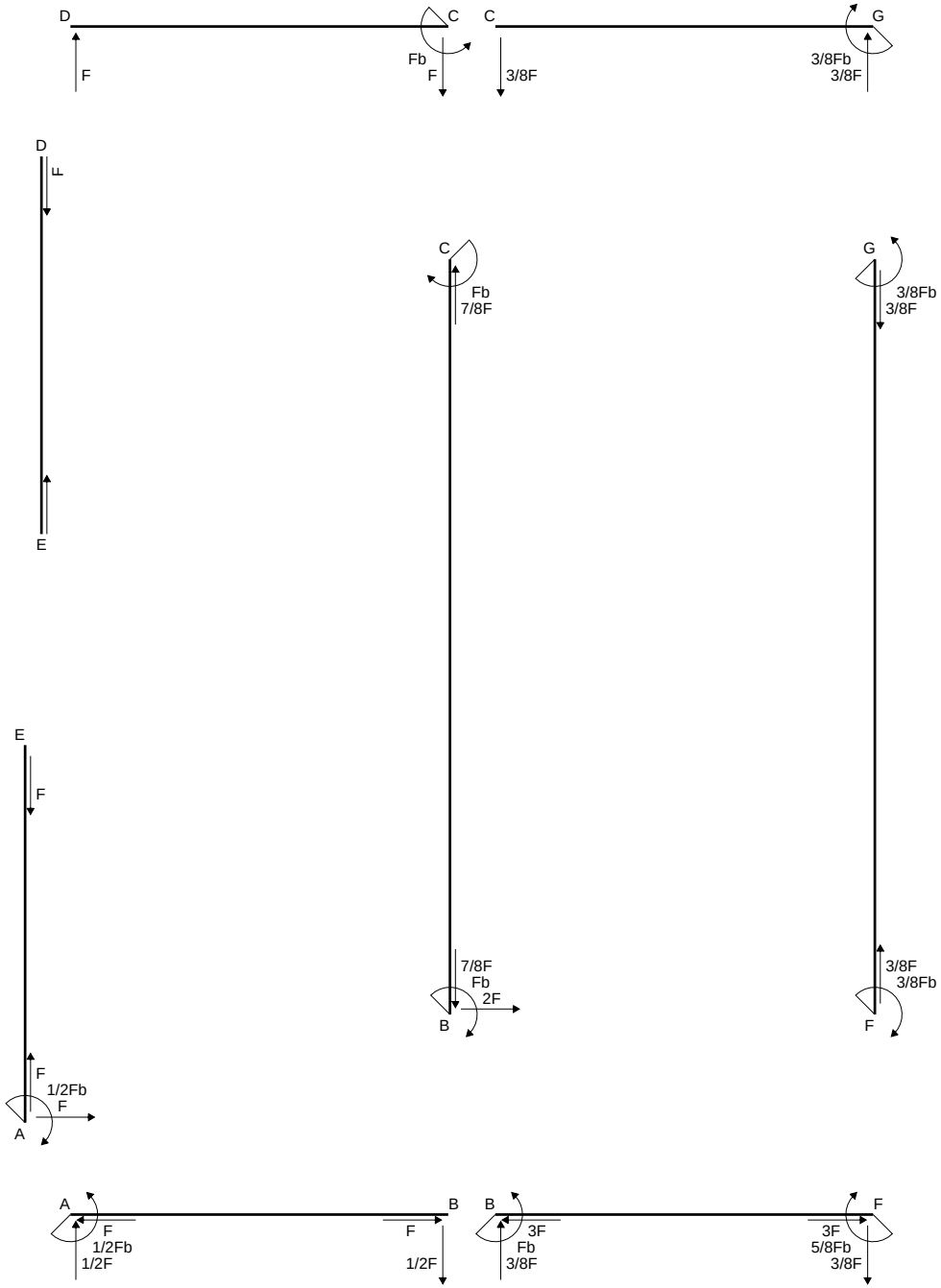
D
C
C
G
F
Fb
F
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3/8Fb
3/8F
D
F
E
C
Fb
7/8F
G
3/8Fb
3/8F
E
F
F
1/2Fb
F
A
7/8F
Fb
2F
B
3/8F
3/8Fb
F
A
B
B
F
F
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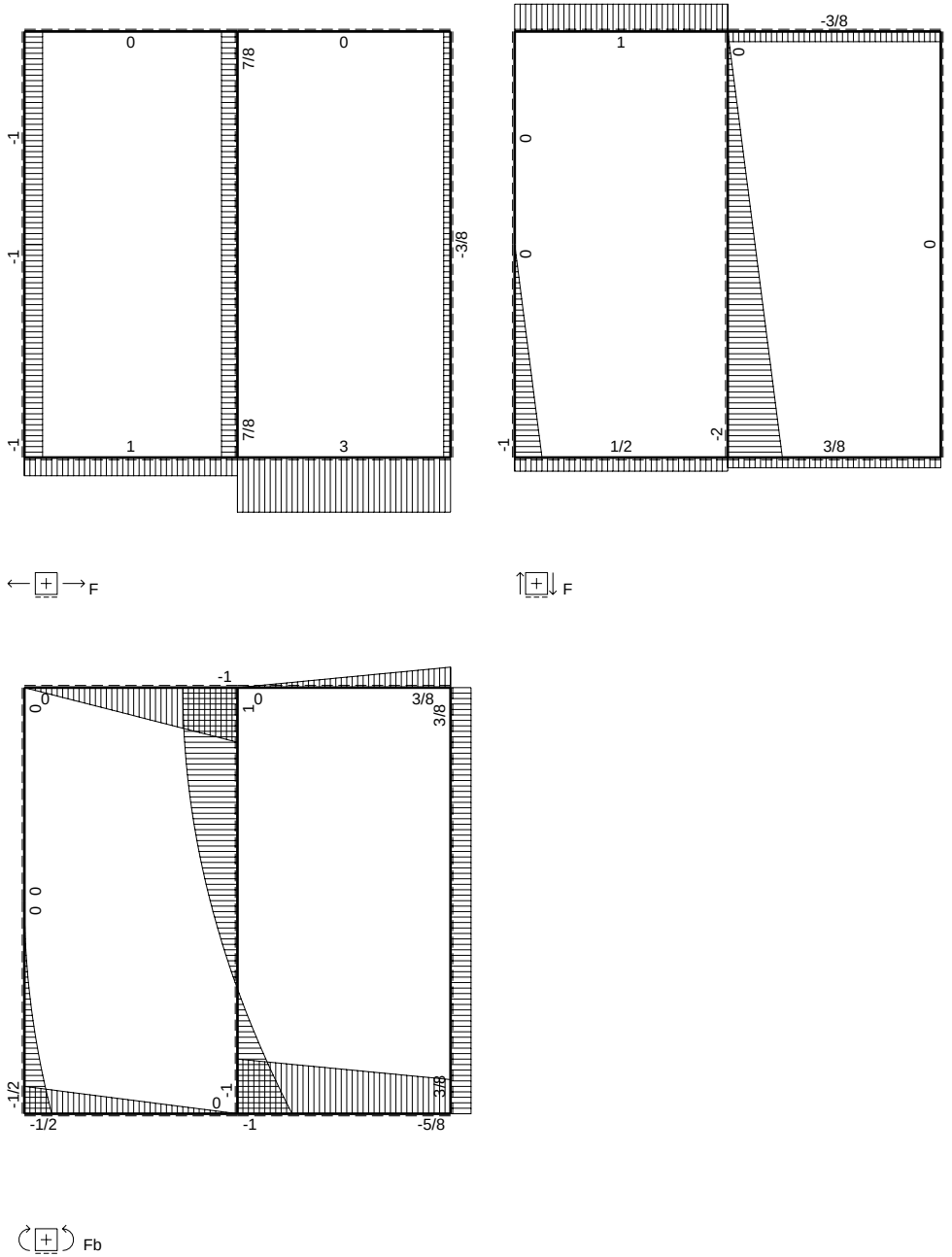
1/2F
F
1/2F
3F
Fb
3/8F
3F
5/8Fb
3/8F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7/8
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-1
-3/8
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1
7/8
3
F
-3/8
1
0
0
0
-1
1/2
-2
3/8
F
-1
0
0
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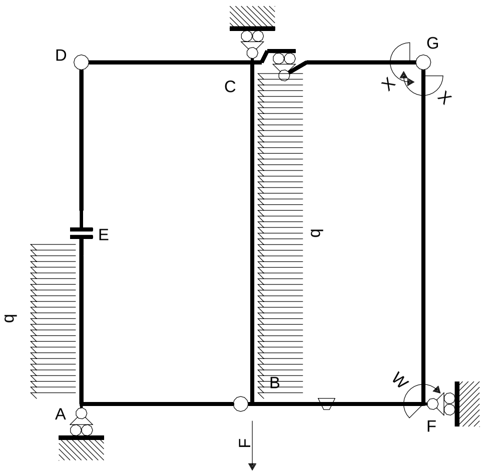
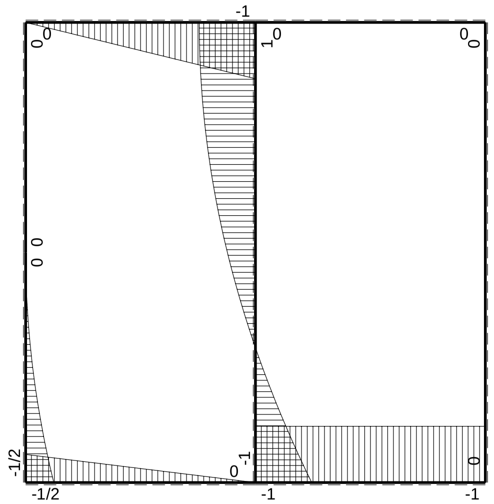
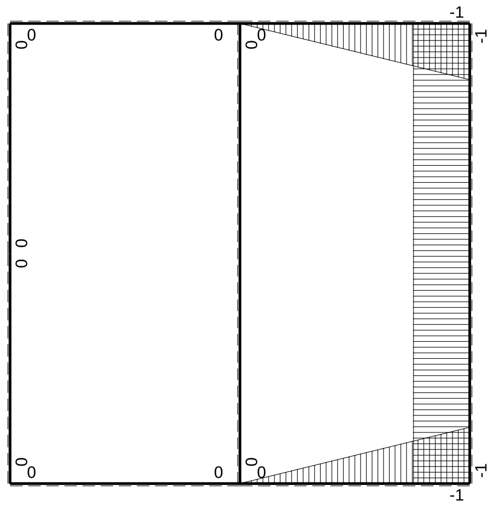
0
3/8
3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 - 2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b + 1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b + 1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

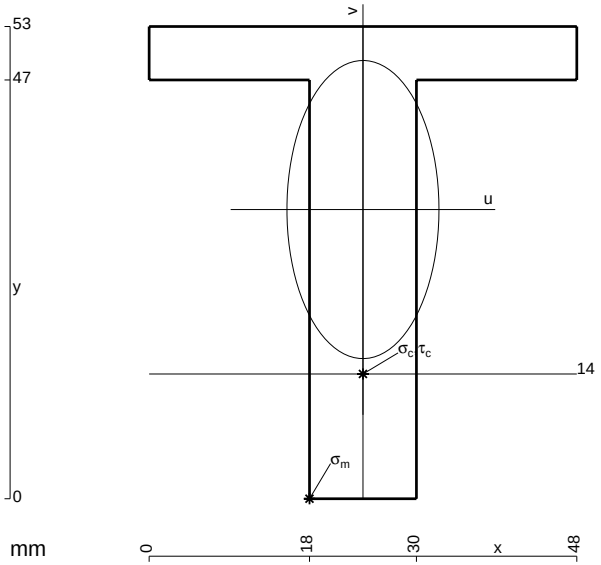
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x - 1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x + 1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b - 1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b + 1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 852. mm^2
J_u = 238570. mm^4
J_v = 62064. mm^4
y_g = 32.46 mm
N = 1969. N
T_y = -4500. N
M_x = -1777500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -32.46 mm
$\sigma_m = N/A - Mv/J_u$ = -239.5 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -18.46 mm
$\sigma_c = N/A - Mv/J_u$ = -135.2 N/mm^2
τ_c = 6.723 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 135.7 N/mm^2
S^* = 4277. mm^3

ESΣ8.099

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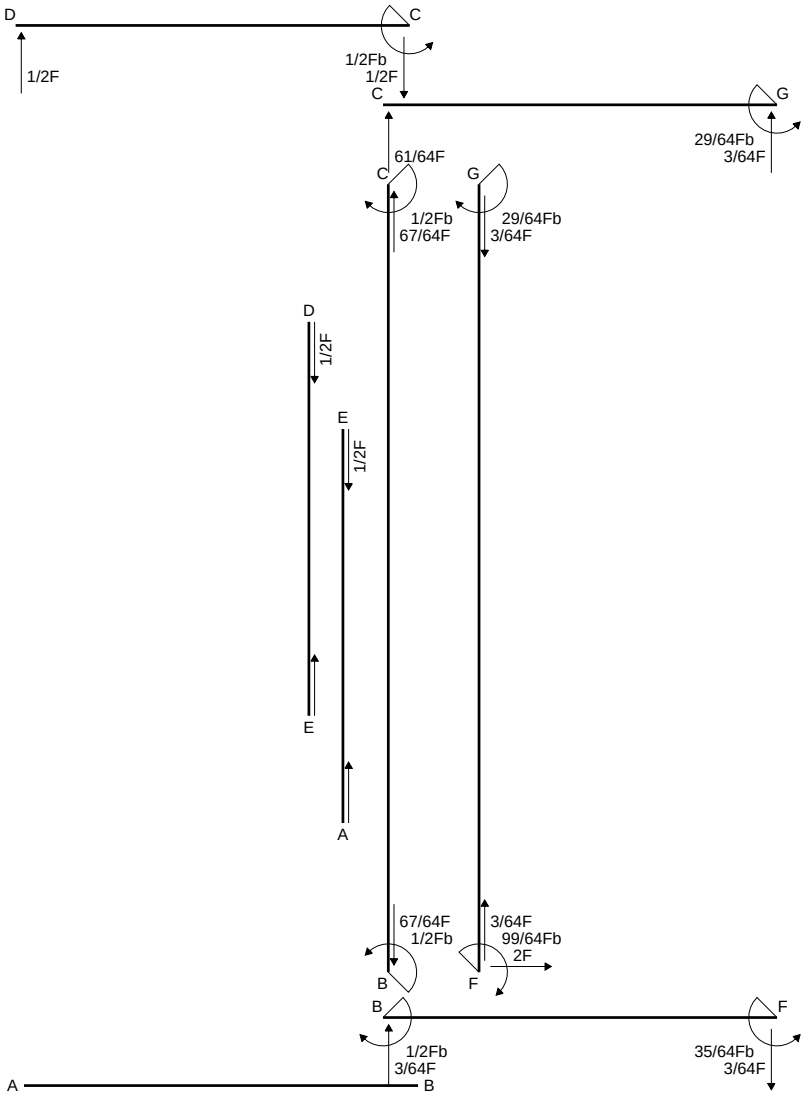
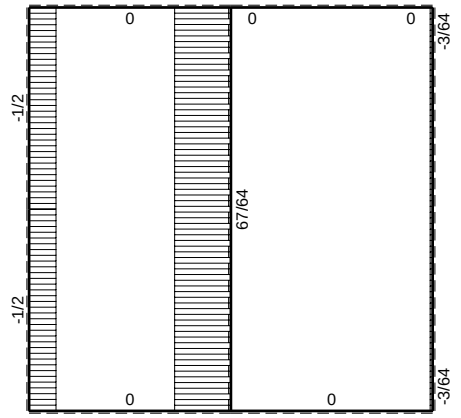
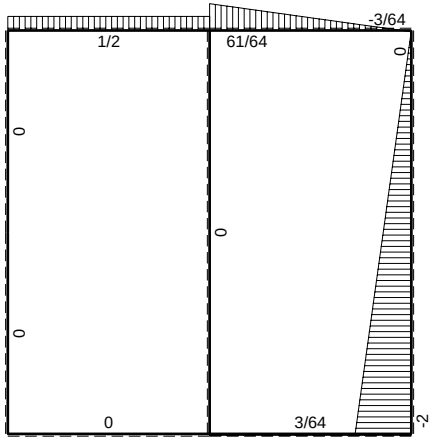
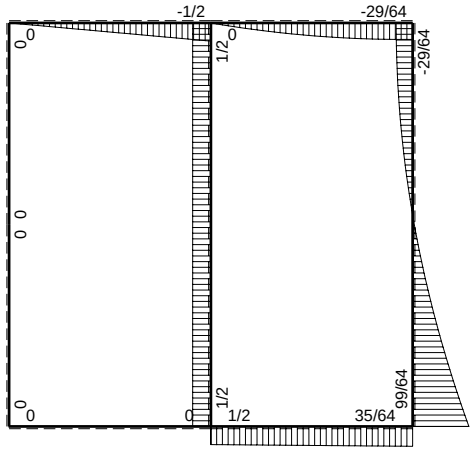

D
C
1/2F
1/2Fb
1/2F
C
G
29/64Fb
3/64F
61/64F
C
G
1/2Fb
67/64F
29/64Fb
3/64F
D
1/2F
E
1/2F
E
A
67/64F
1/2Fb
3/64F
99/64Fb
2F
B
F
B
F
1/2Fb
3/64F
35/64Fb
3/64F
A
B


0
0
0
-3/64
-1/2
67/64
-1/2
0
0
-3/64


F


-3/64
1/2
61/64
0
0
0
0
0
3/64
-2


F


-1/2
-29/64
0
0
1/2
0
-29/64
0
0
0
0
1/2
0
0
1/2
35/64
99/64


Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	1/2Fb+1/2Fx	-Fb/EJ	-1/2Fx-1/2Fx²/b	Fx/EJ	x^2/b^2	(-5/12+1/2)Fb²/EJ	1/3XLb/EJ
FB b	1-x/b	-Fb+1/2Fx	Fb/EJ	-Fb+3/2Fx-1/2Fx²/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx²	0	1/2Fx-Fx²/b+1/2qx³/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb²/EJ	1/3XLb/EJ
CG b	x/b	1/2Fx-1/2qx²	0	1/2Fx²/b-1/2qx³/b	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2XLb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-29/24Fb²/EJ	8/3XLb/EJ
	iperstatica $X=W_{GC}$						29/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 +3/2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +3/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

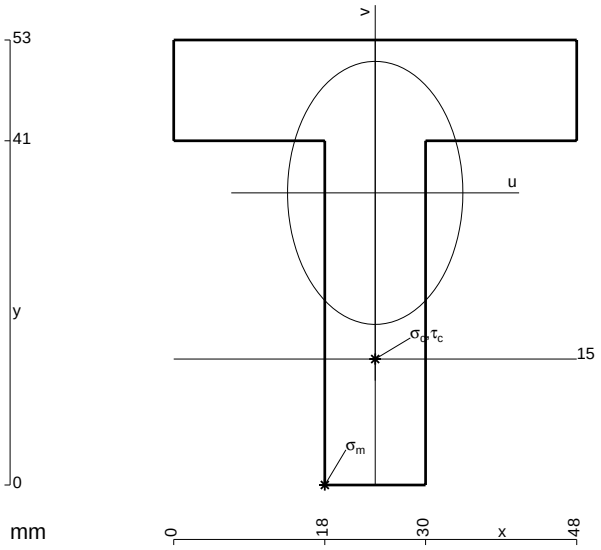
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.100



A = 1068. mm^2
J_u = 262174. mm^4
J_v = 116496. mm^4
y_g = 34.79 mm
T_y = 3585. N
M_x = -1505700. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.79 mm
$\sigma_m = -Mv/J_u$ = -199.8 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -19.79 mm
$\sigma_c = -Mv/J_u$ = -113.7 N/mm^2
τ_c = 5.598 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.1 N/mm^2
S^* = 4913. mm^3

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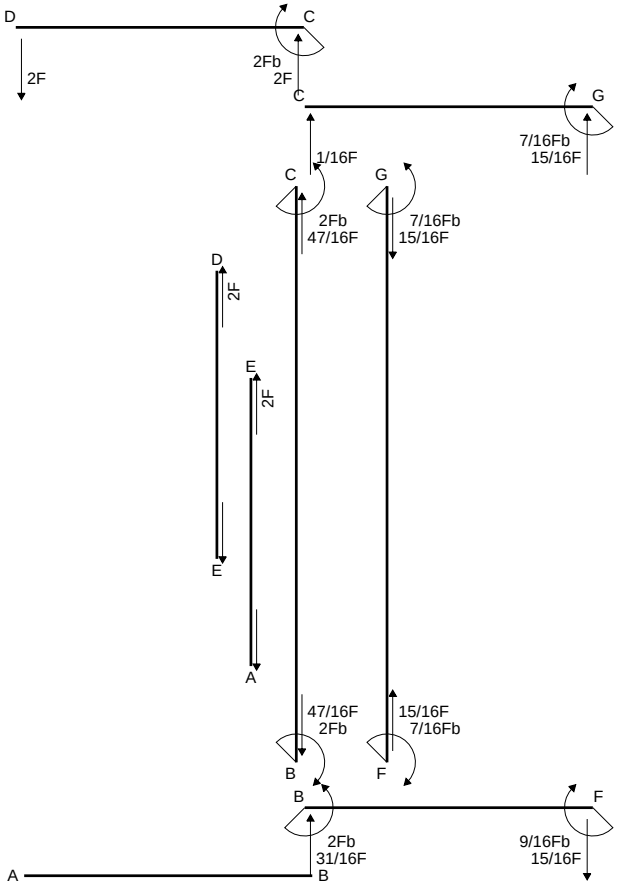
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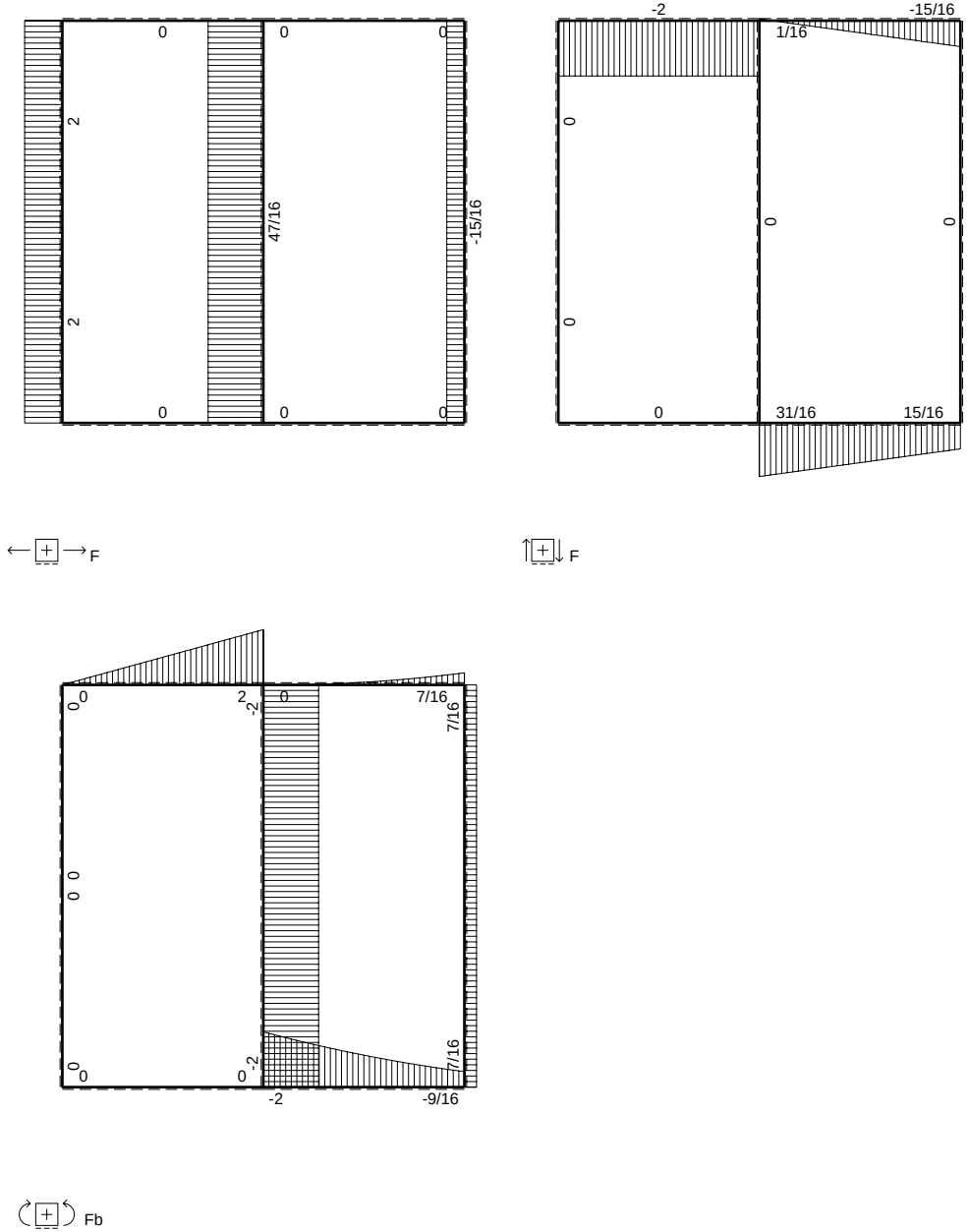
C
2F
2Fb
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
47/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
47/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B


0
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0
2
47/16
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-2
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1/16
0
0
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0
0
31/16
15/16
F
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

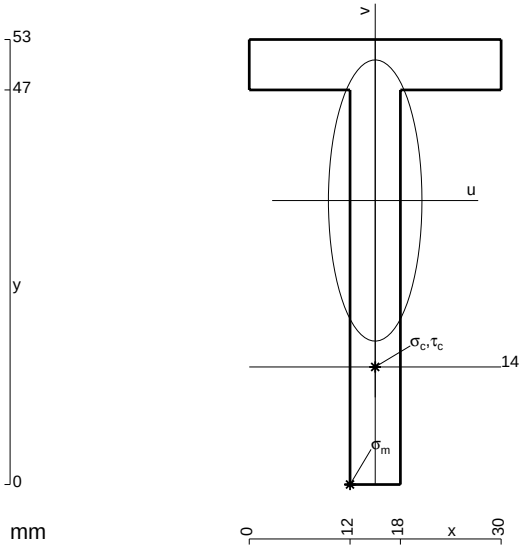
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 462. mm^2
J_u = 129608. mm^4
J_v = 14346. mm^4
y_g = 33.82 mm
T_y = -1740. N
M_x = 800400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -33.82 mm
$\sigma_m = -Mv/J_u$ = 208.9 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -19.82 mm
$\sigma_c = -Mv/J_u$ = 122.4 N/mm^2
τ_c = 5.042 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 122.7 N/mm^2
S^* = 2253. mm^3

ESΣ8.101

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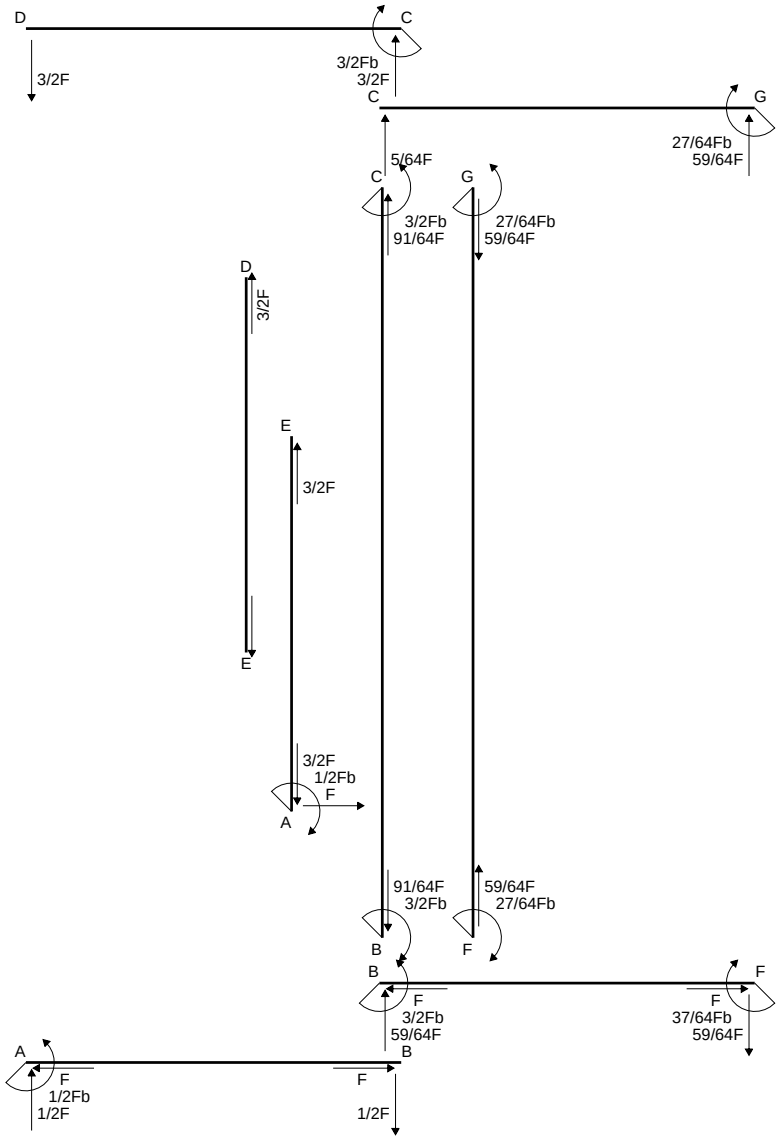
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C
3/2F
3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
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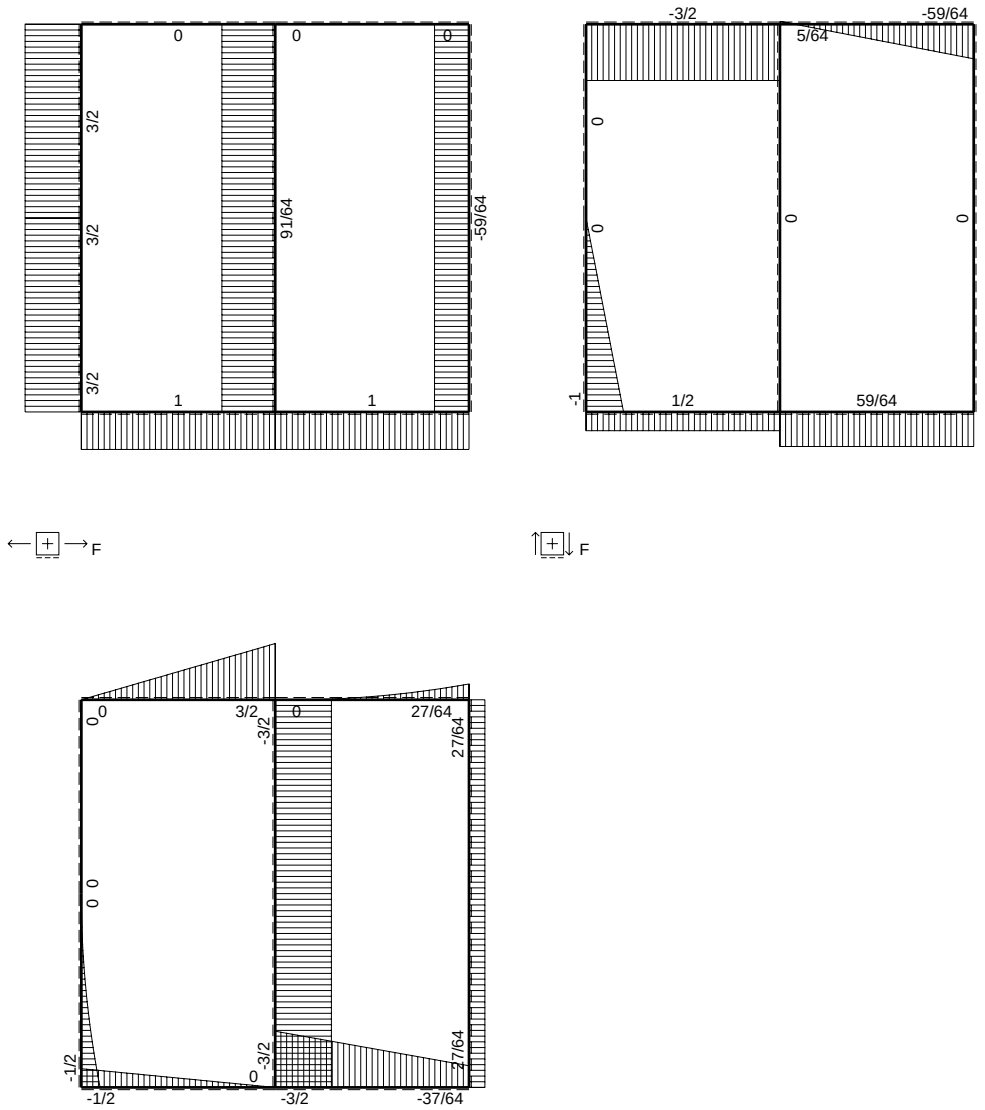
91/64F
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F


0
0
0
3/2
3/2
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91/64
-59/64
1
1
F
-3/2
-59/64
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0
0
-1
1/2
59/64
F
0 0
3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
-3/2
27/64
-1/2
-3/2
-37/64
Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

14.11.23

M_0 flessione da carichi assegnati

PROCEDIMENTO E RISULTATI Nome: ESΣ8.102

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

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14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

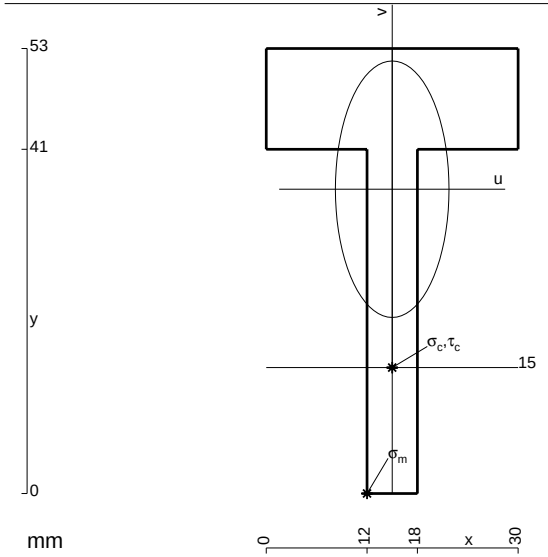
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 606. mm^2
J_u = 141406. mm^4
J_v = 27738. mm^4
y_g = 36.24 mm
T_y = -1710. N
M_x = 855000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.24 mm
$\sigma_m = -Mv/J_u$ = 219.1 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.24 mm
$\sigma_c = -Mv/J_u$ = 128.4 N/mm^2
τ_c = 5.214 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 128.8 N/mm^2
S^* = 2587. mm^3

ESΣ8.102

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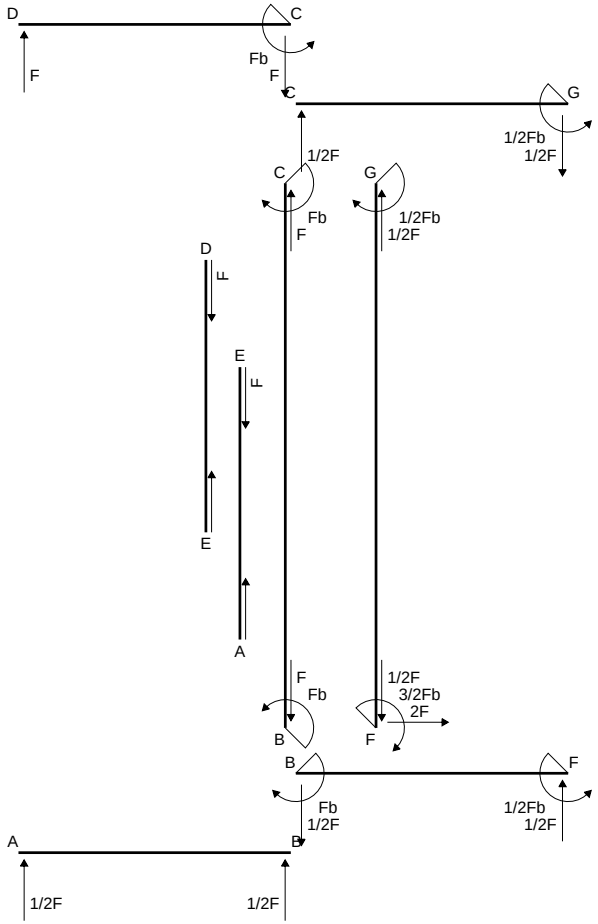
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F
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G
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1/2F
1/2F
C
G
Fb
F
1/2Fb
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D
F
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F
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A
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Fb
1/2F
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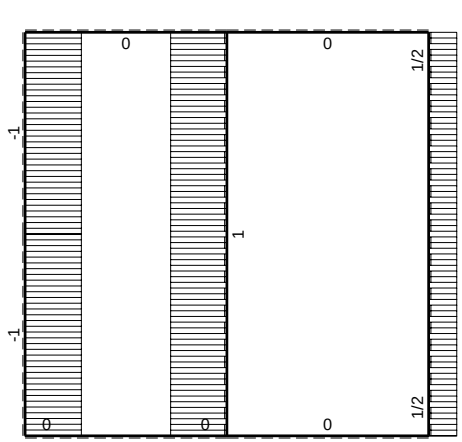
A
B
1/2F
1/2F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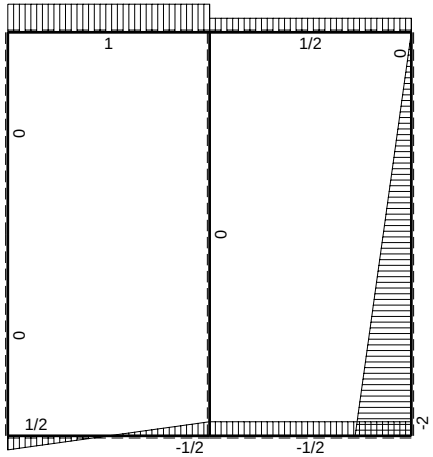
0
0
0
1/2


F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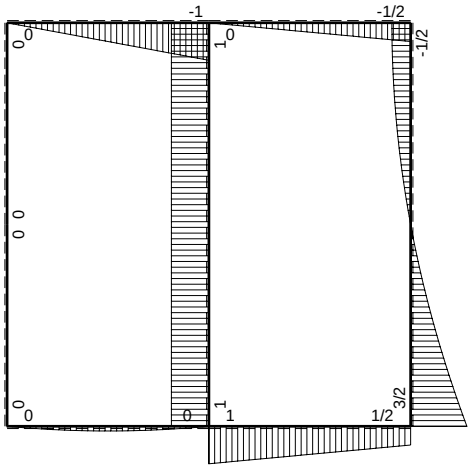
1/2
-2
-1/2
-1/2


F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1
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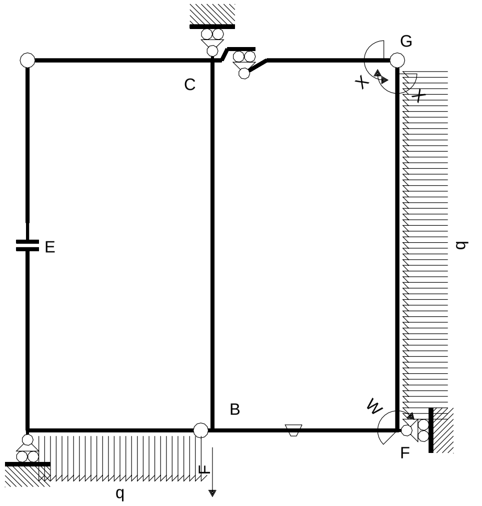
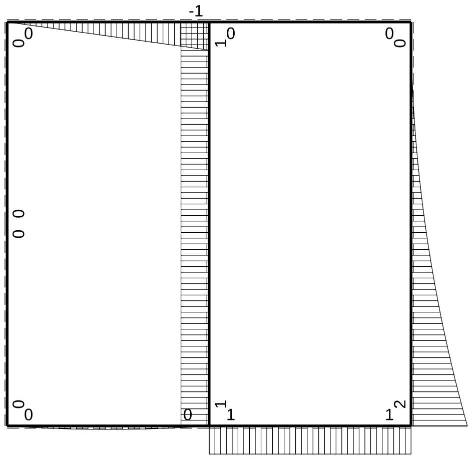
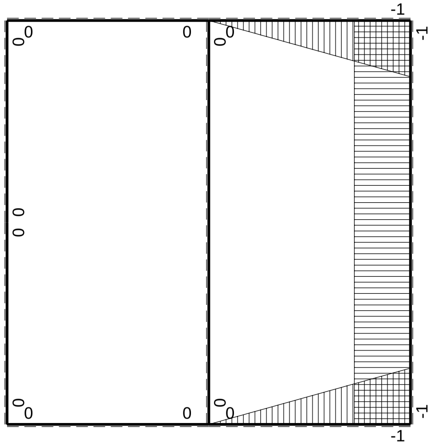
0
0
0
0
1
1
1/2
3/2


Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

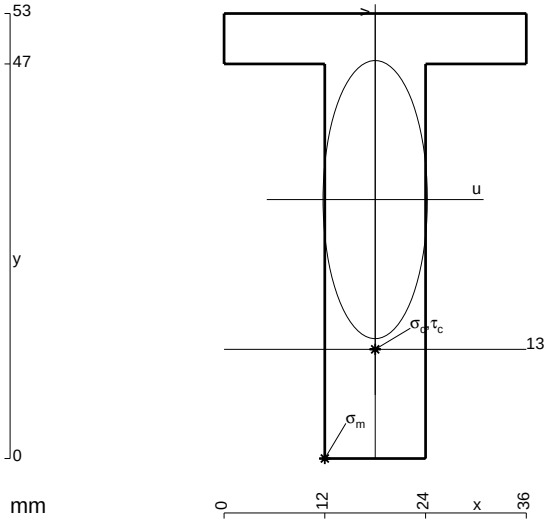
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 780. mm^2
J_u = 214152. mm^4
J_v = 30096. mm^4
y_g = 30.84 mm
T_y = 2950. N
M_x = -1593000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -30.84 mm
$\sigma_m = -Mv/J_u$ = -229.4 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -17.84 mm
$\sigma_c = -Mv/J_u$ = -132.7 N/mm^2
τ_c = 4.359 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.9 N/mm^2
S^* = 3797. mm^3

ESΣ8.103

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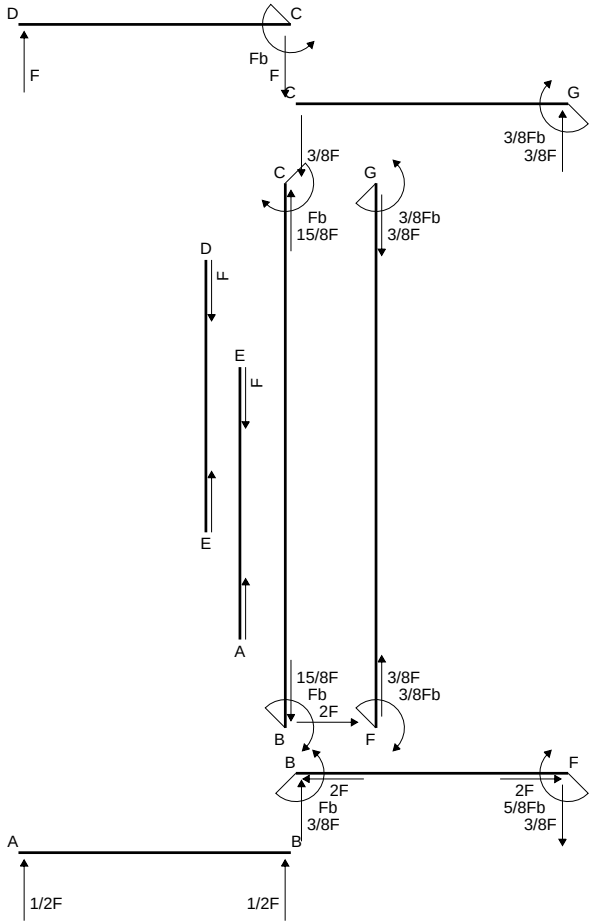
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Fb
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3/8F
3/8F
C
G
Fb
15/8F
3/8Fb
3/8F
D
F
E
F
E
A
15/8F
Fb
2F
3/8F
3/8Fb
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F
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Fb
3/8F
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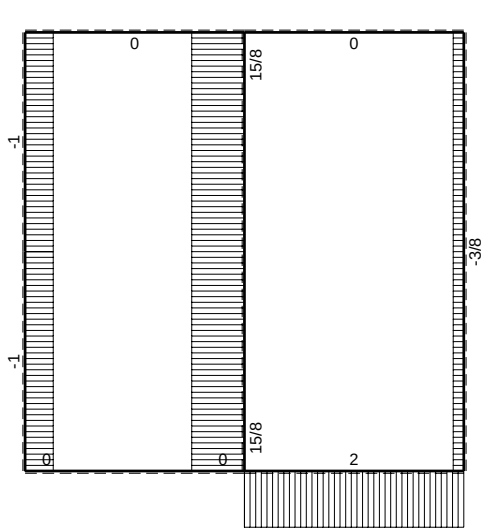
B
1/2F
1/2F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15/8
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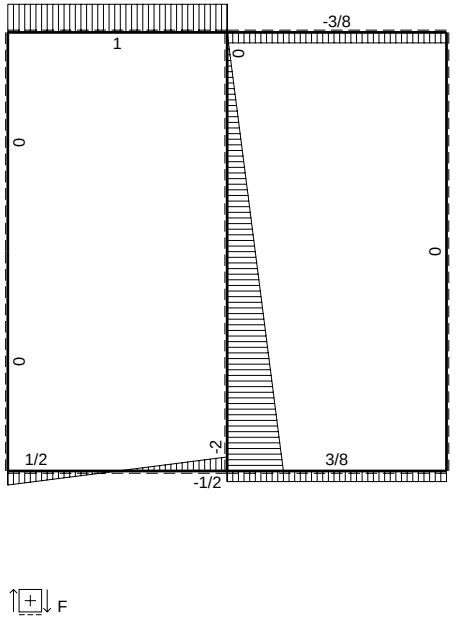
F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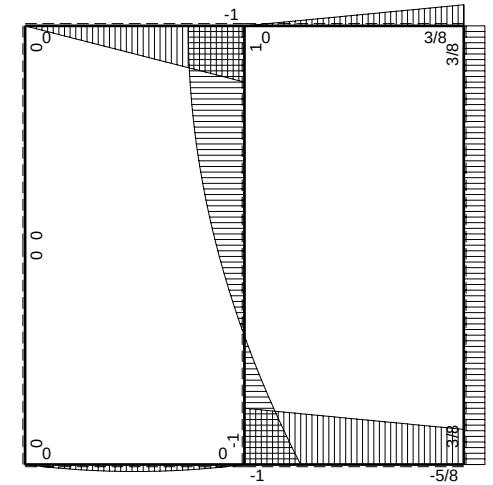
3/8
-1/2
F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0
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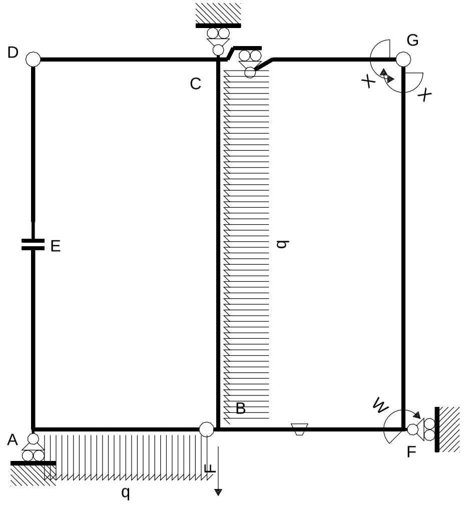
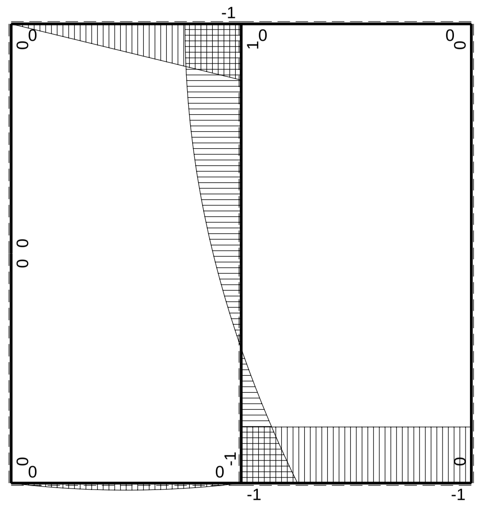
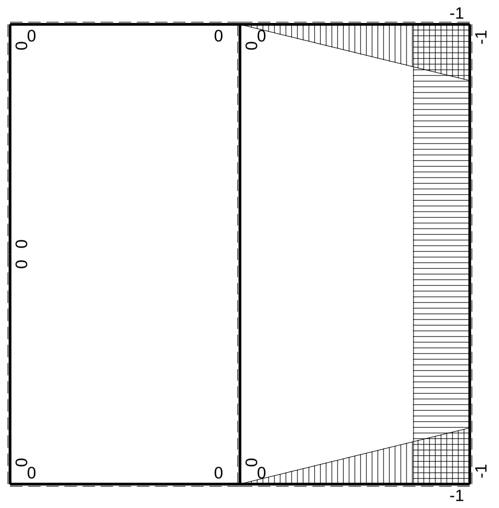
-1
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

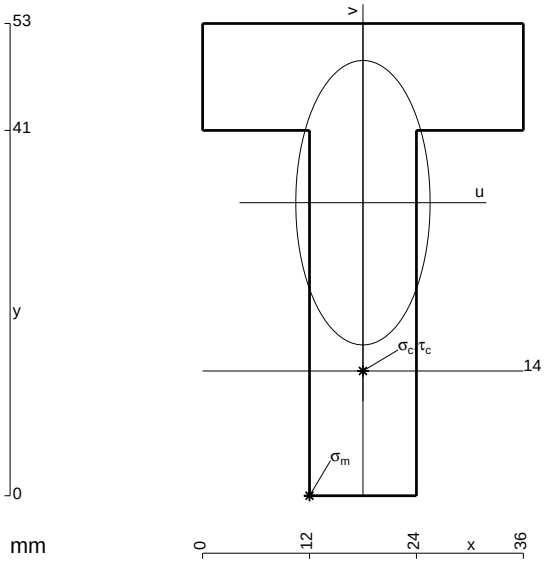
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 924. mm^2
J_u = 235641. mm^4
J_v = 52560. mm^4
y_g = 32.89 mm
N = 5700. N
T_y = -6080. N
M_x = -1763200. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.89 mm
$\sigma_m = N/A - Mv/J_u$ = -239.9 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.89 mm
$\sigma_c = N/A - Mv/J_u$ = -135.2 N/mm^2
τ_c = 9.352 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.1 N/mm^2
S^* = 4349. mm^3

ESΣ8.104

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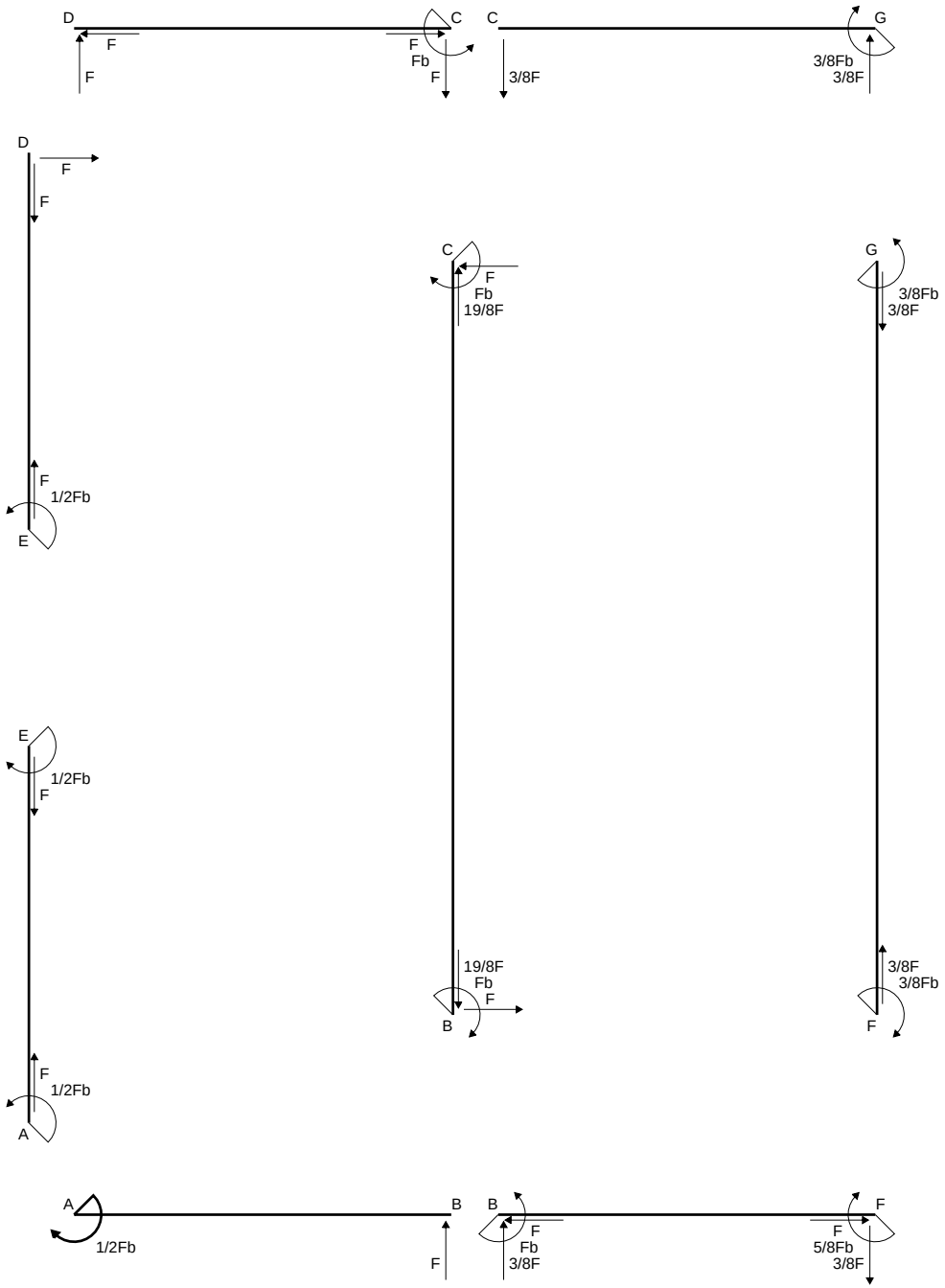
D
C
C
G
F
F
F
Fb
F
3/8F
3/8Fb
3/8F
D
F
F
C
F
Fb
19/8F
G
3/8Fb
3/8F
F
1/2Fb
E
E
1/2Fb
F
19/8F
Fb
F
B
3/8F
3/8Fb
F
F
1/2Fb
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B
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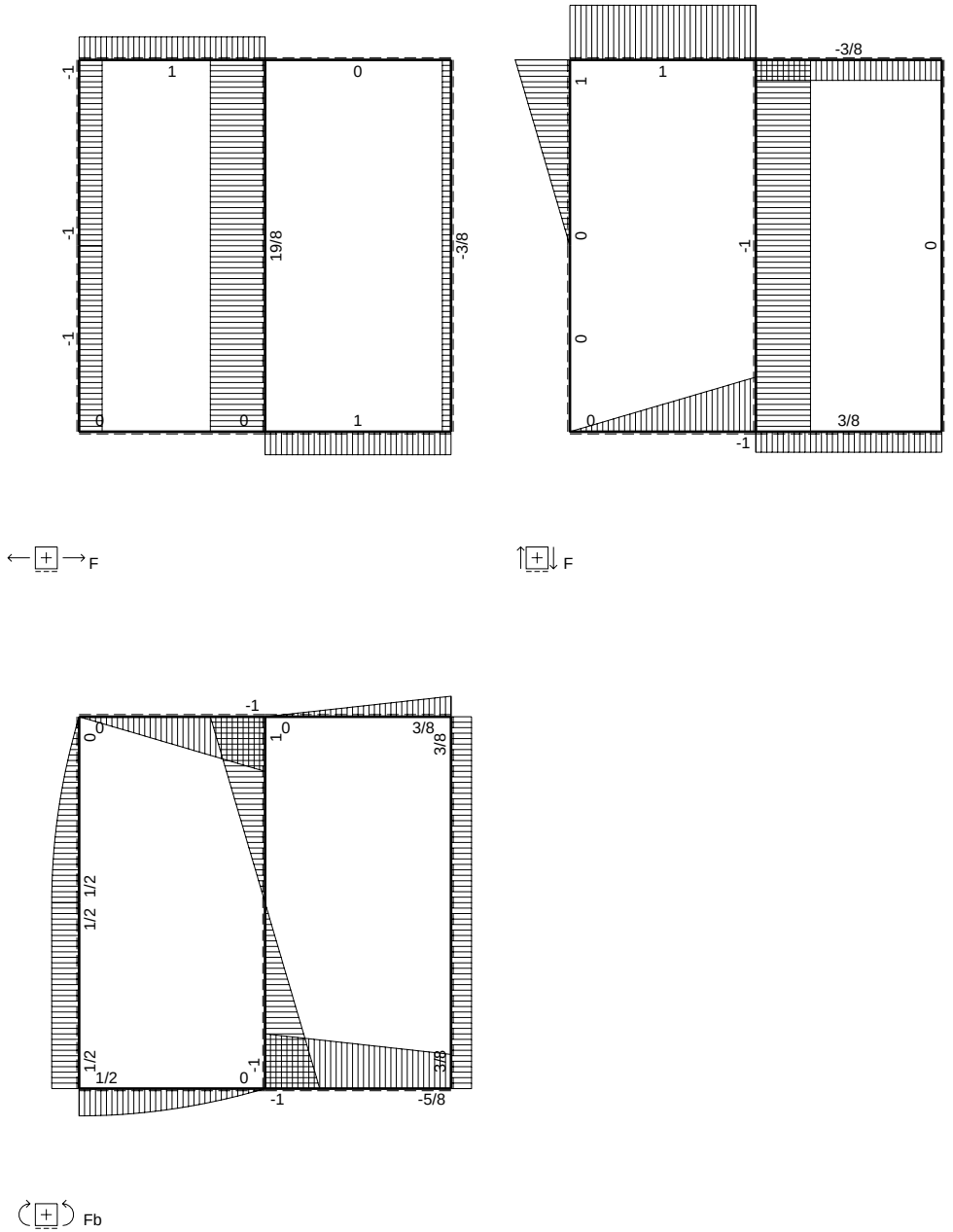
3/8F
F
5/8Fb
3/8F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0
0
1
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3/8
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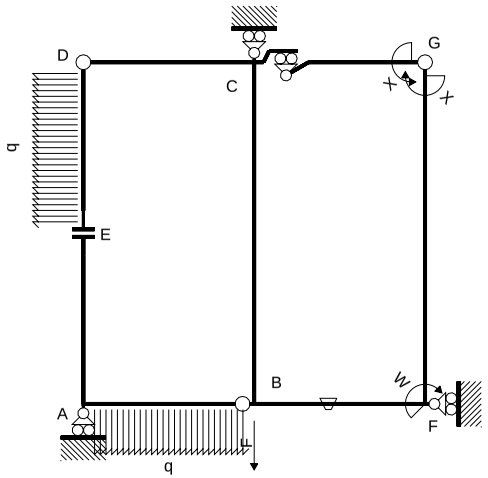
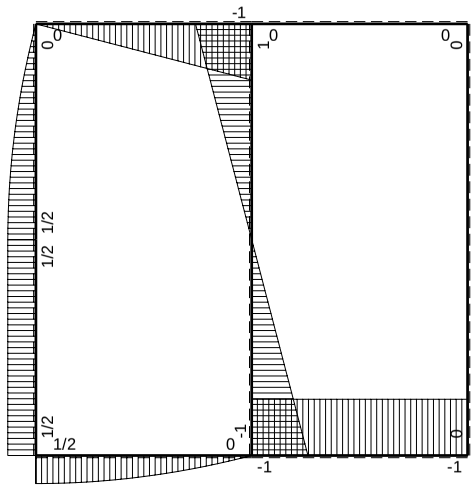
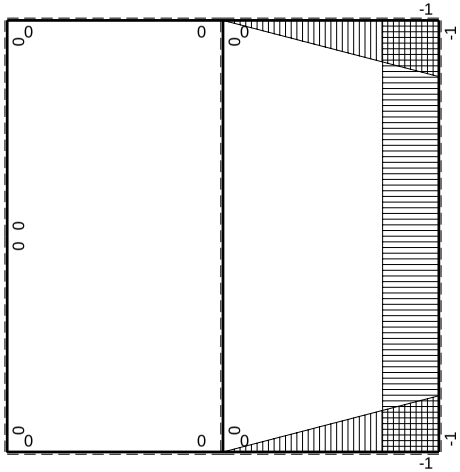
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.105

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

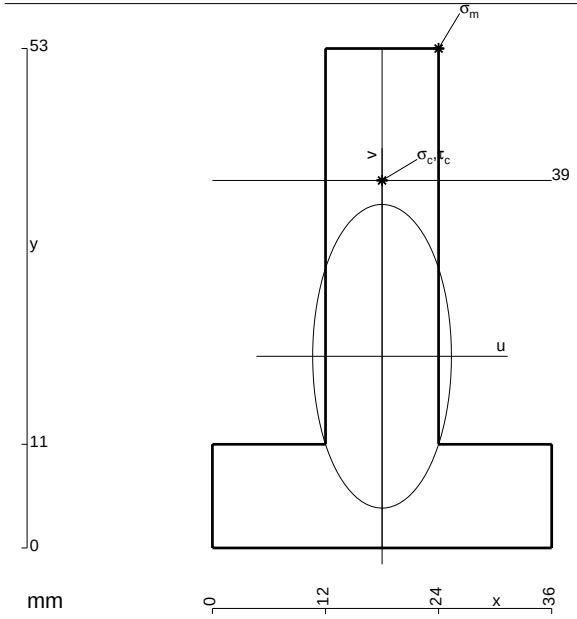
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 900. mm^2
J_u = 233812. mm^4
J_v = 48816. mm^4
y_g = 20.34 mm
N = 5225. N
T_y = -2200. N
M_x = -1386000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 32.66 mm
$\sigma_m = N/A - Mv/J_u$ = 199.4 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 18.66 mm
$\sigma_c = N/A - Mv/J_u$ = 116.4 N/mm^2
τ_c = 3.38 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 116.6 N/mm^2
S^* = 4311. mm^3

ESΣ8.105

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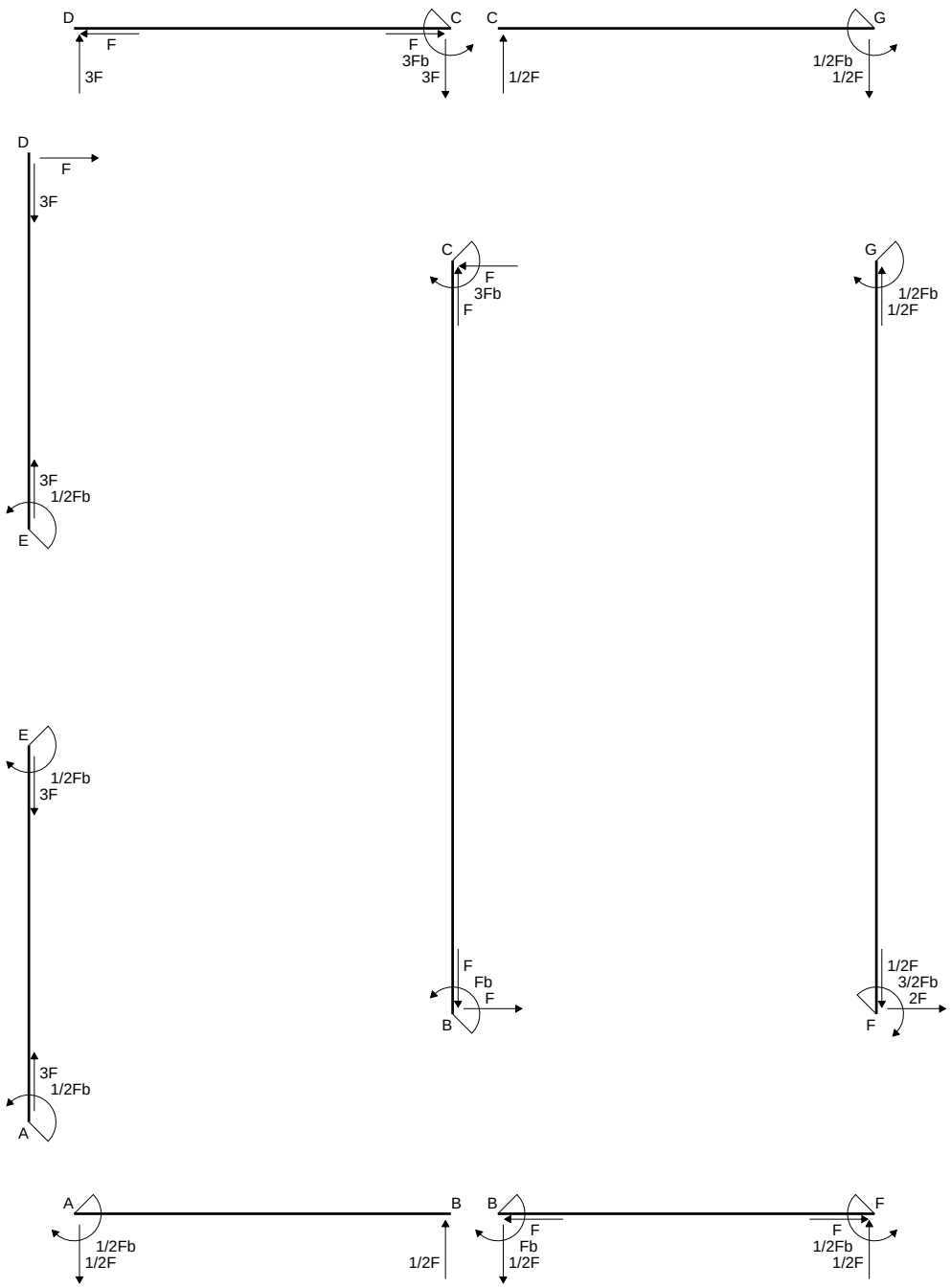
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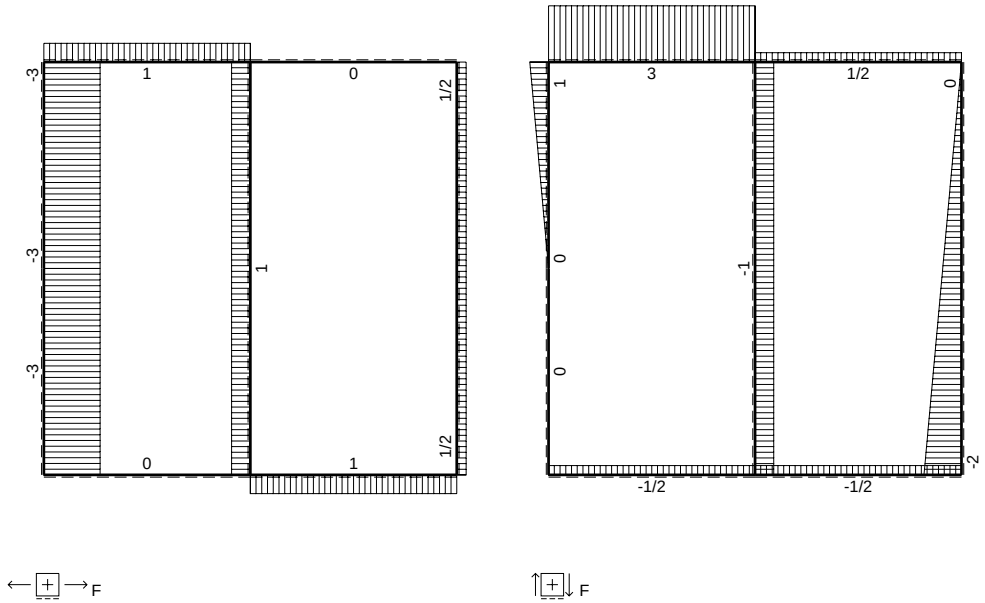
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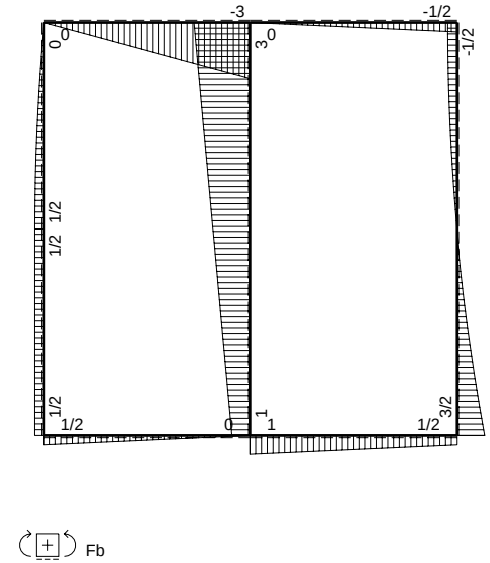
-1/2
F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1/2
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1/2
0
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1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbJ/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbJ/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbJ/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3XbJ/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

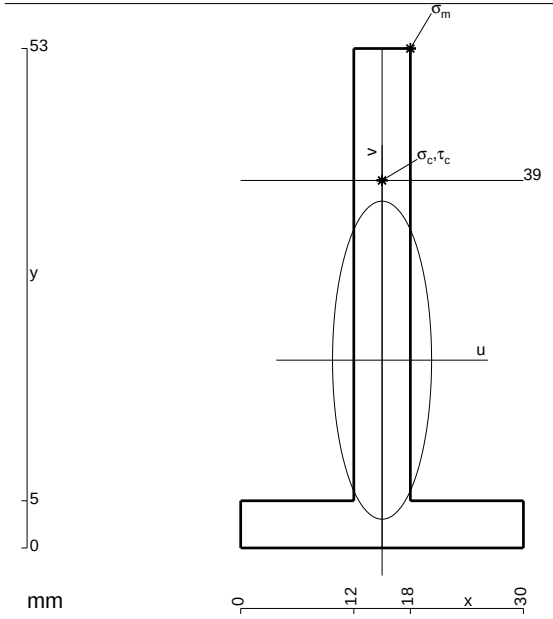
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 438. mm^2
J_u = 124872. mm^4
J_v = 12114. mm^4
y_g = 19.92 mm
N = 390. N
T_y = 1170. N
M_x = -783900. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 33.08 mm
$\sigma_m = N/A - Mv/J_u$ = 208.5 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 19.08 mm
$\sigma_c = N/A - Mv/J_u$ = 120.6 N/mm^2
τ_c = 3.42 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.8 N/mm^2
S^* = 2190. mm^3

ESΣ8.106

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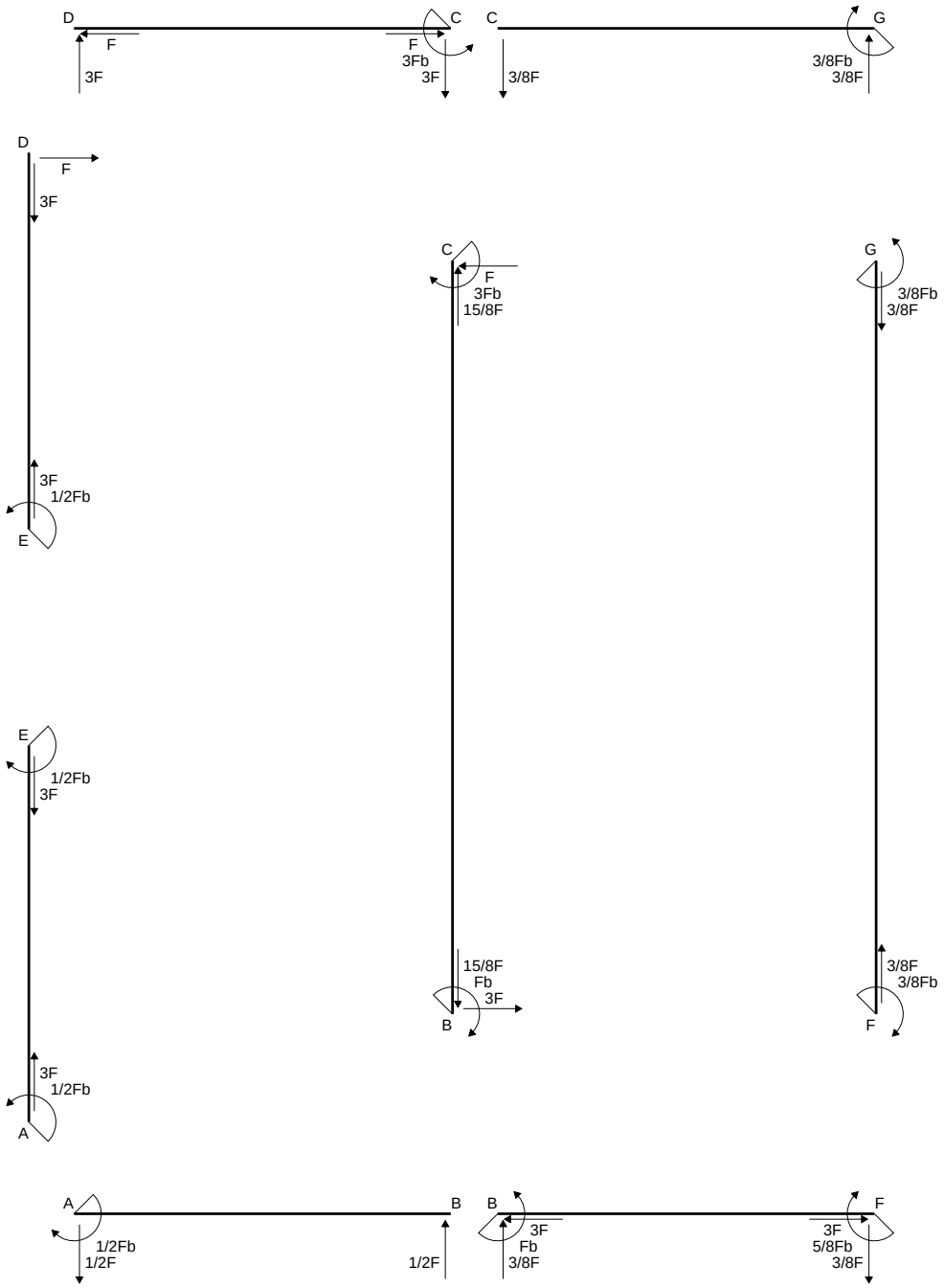
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3F
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3/8Fb
3/8F
D
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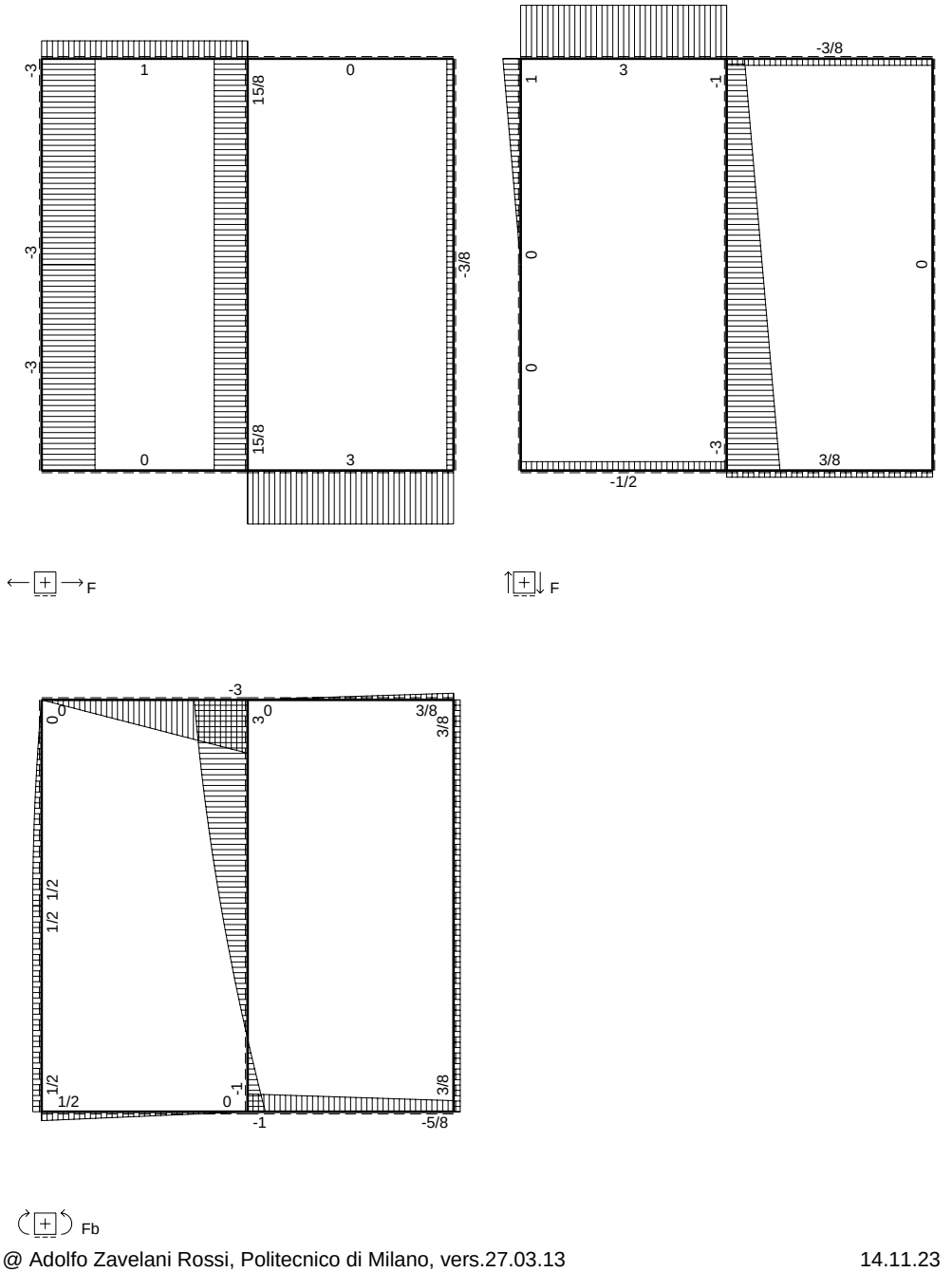
Fb
3/8F
3F
5/8Fb
3/8F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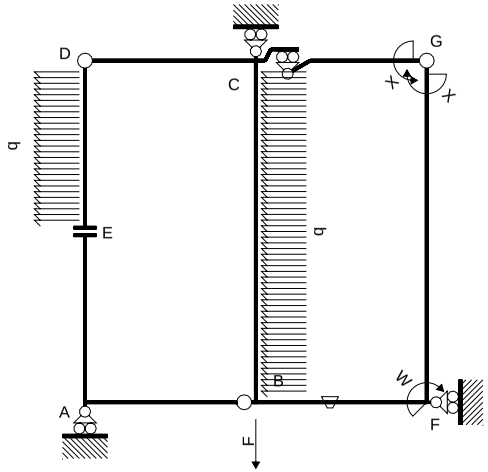
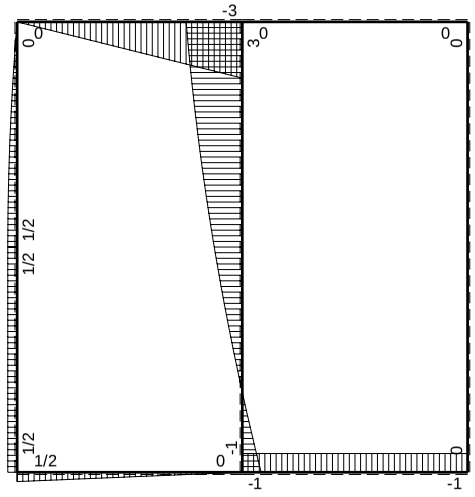
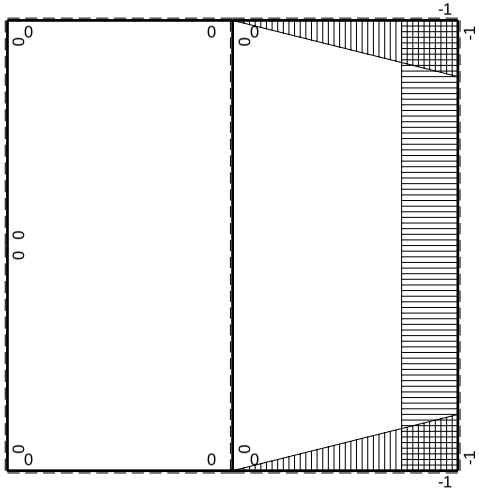
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
				totali			Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.107

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.27.03.13 14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

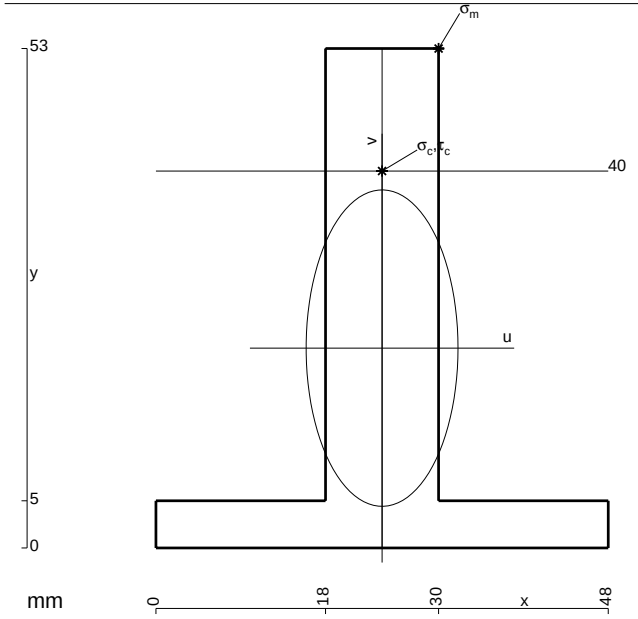
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.107



A = 816. mm^2
J_u = 230061. mm^4
J_v = 52992. mm^4
y_g = 21.21 mm
N = 740. N
T_y = 2220. N
M_x = -1576200. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 31.79 mm
$\sigma_m = N/A - Mv/J_u$ = 218.7 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 18.79 mm
$\sigma_c = N/A - Mv/J_u$ = 129.7 N/mm^2
τ_c = 3.173 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.8 N/mm^2
S^* = 3946. mm^3

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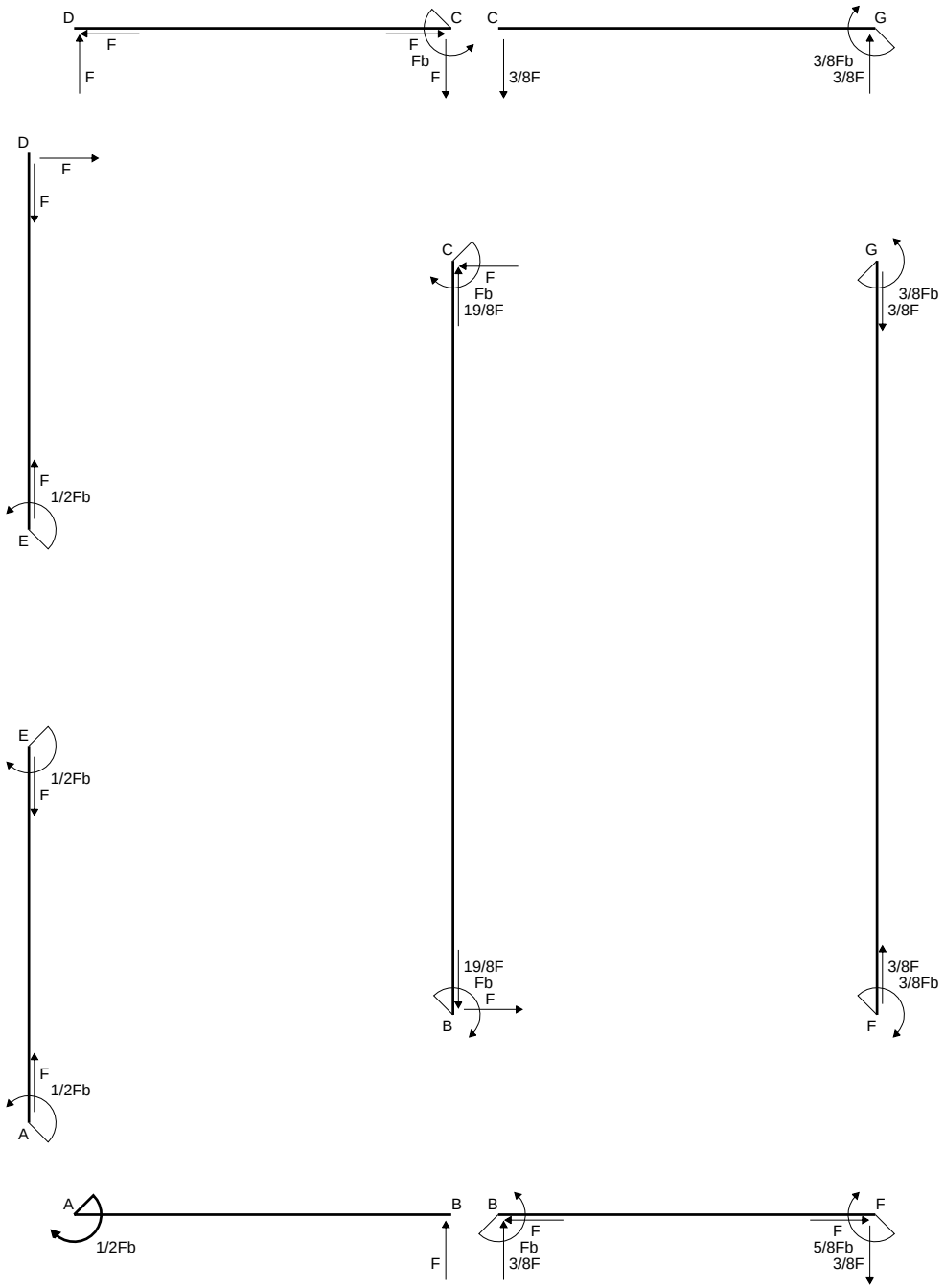
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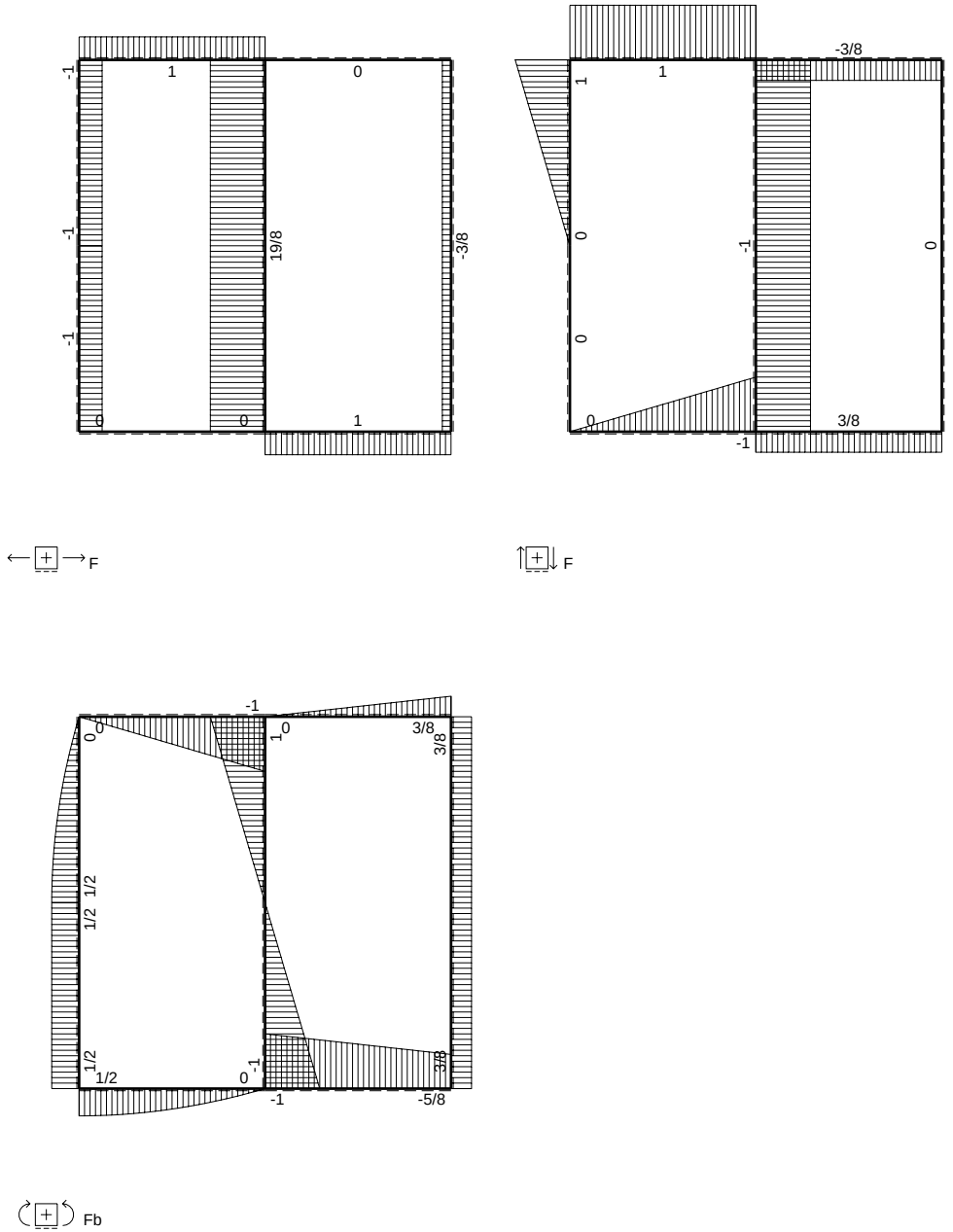
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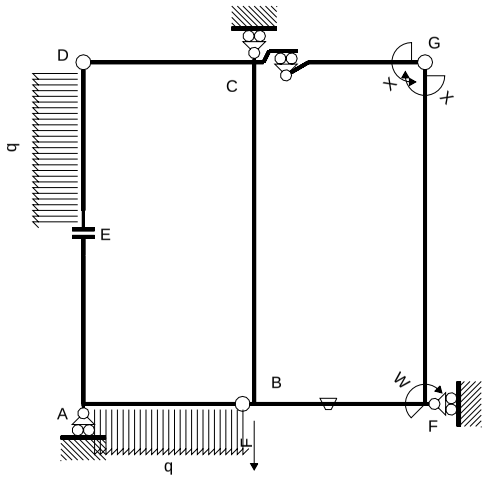
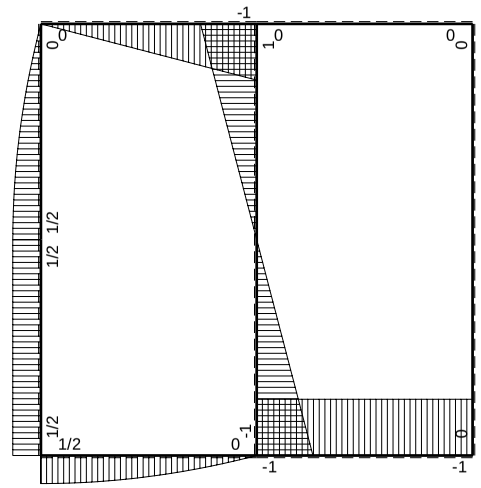
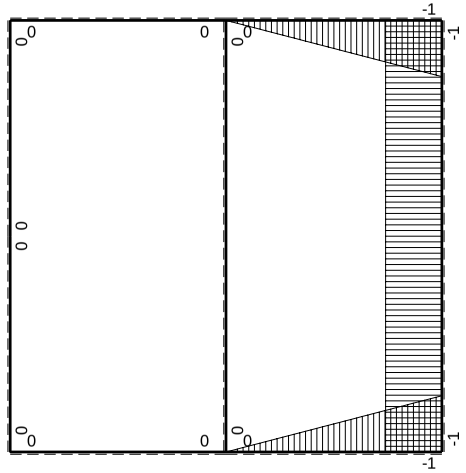


Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.108

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

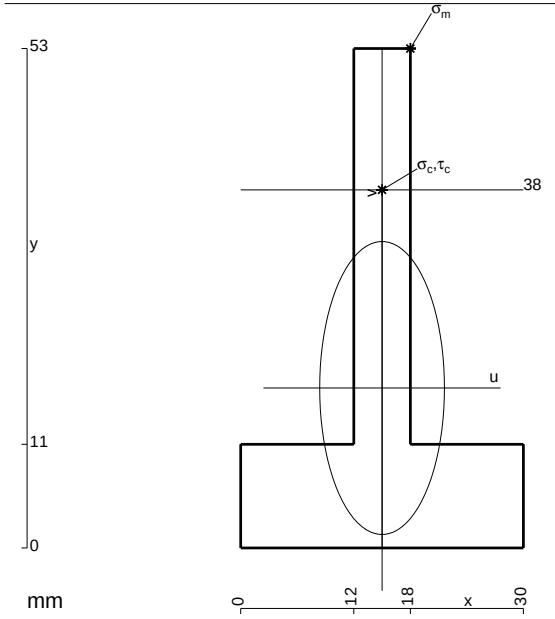
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 582. mm^2
J_u = 140714. mm^4
J_v = 25506. mm^4
y_g = 16.97 mm
N = 2779. N
T_y = -1170. N
M_x = -877500. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.03 mm
$\sigma_m = N/A - Mv/J_u$ = 229.4 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.03 mm
$\sigma_c = N/A - Mv/J_u$ = 135.9 N/mm^2
τ_c = 3.558 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136. N/mm^2
S^* = 2567. mm^3

ESΣ8.108

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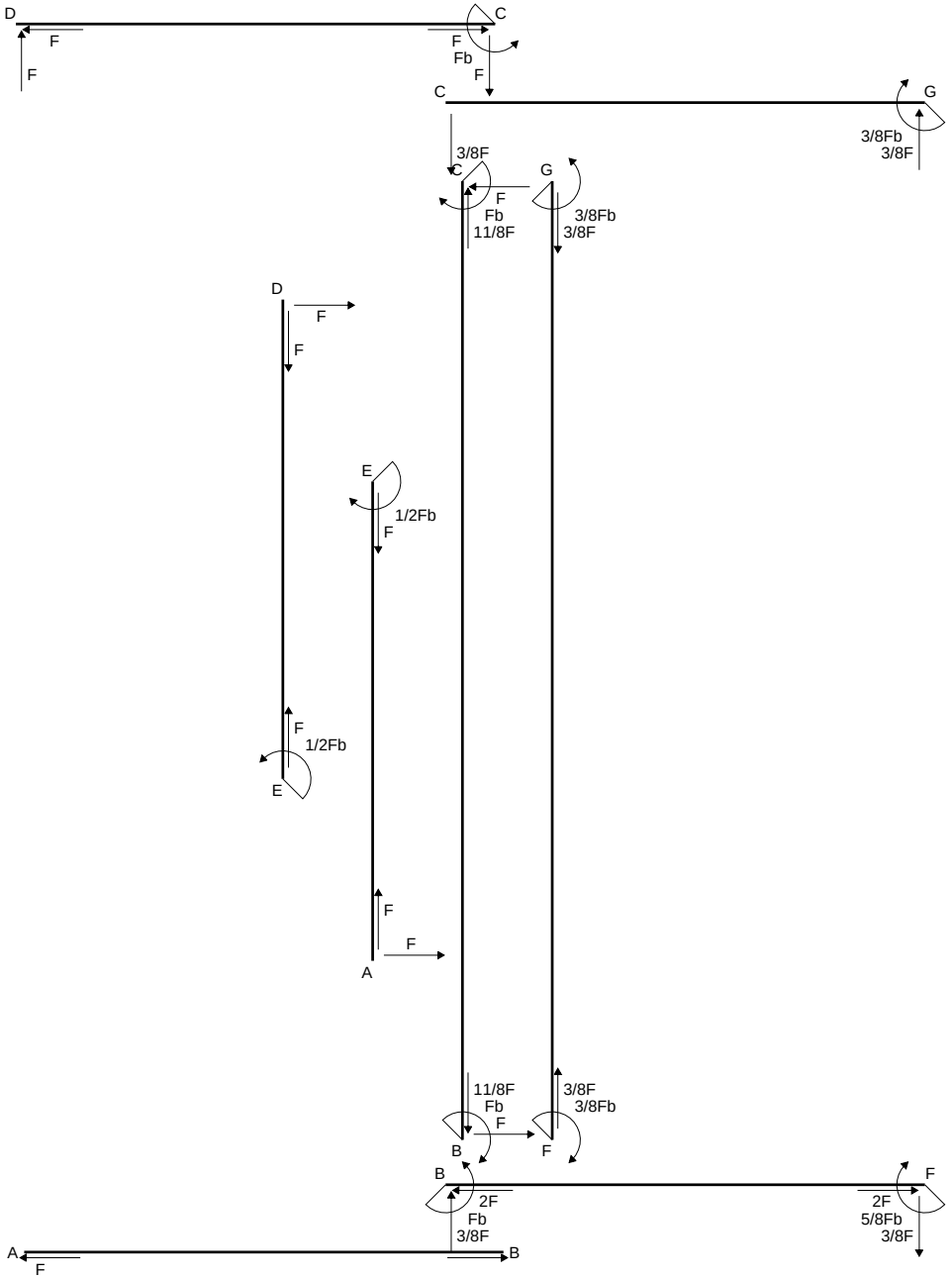
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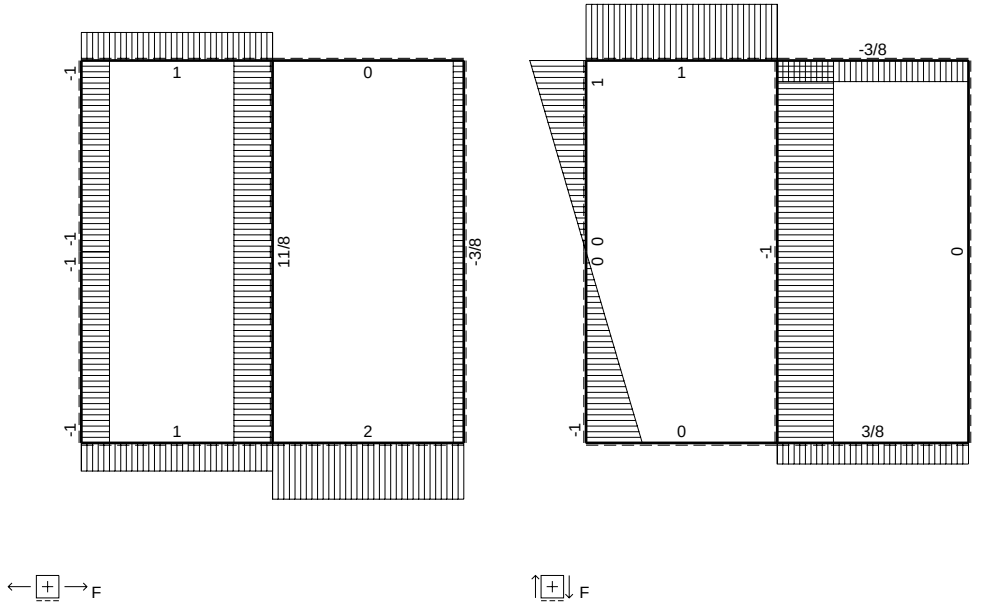
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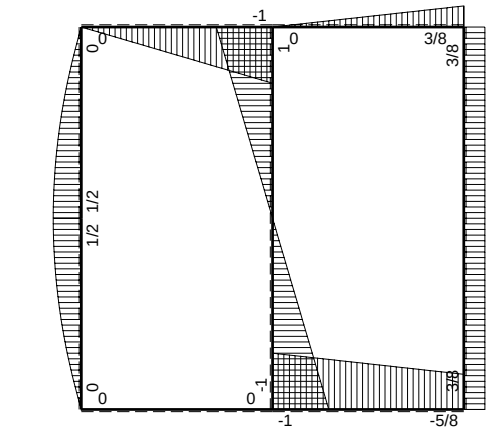
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Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3XbEJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2XbEJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$							-3/8Fb

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

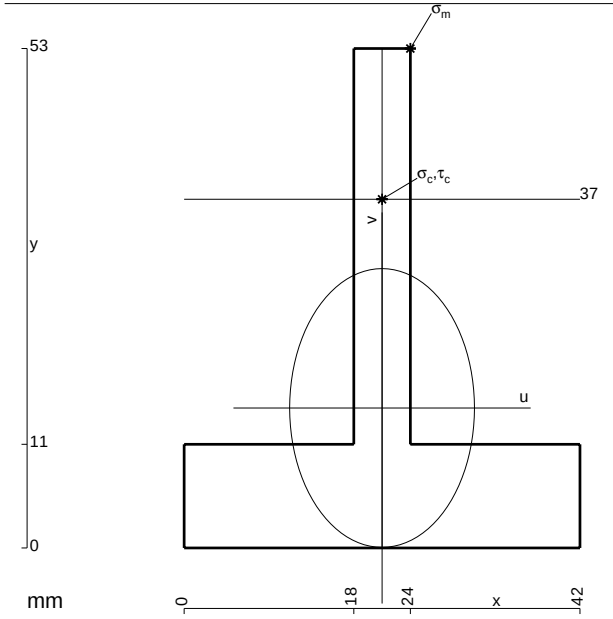
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.109



A = 714. mm^2
J_u = 156211. mm^4
J_v = 68670. mm^4
y_g = 14.85 mm
N = 1691. N
T_y = -1230. N
M_x = -971700. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.15 mm
$\sigma_m = N/A - Mv/J_u$ = 239.7 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.15 mm
$\sigma_c = N/A - Mv/J_u$ = 140.1 N/mm^2
τ_c = 3.798 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.3 N/mm^2
S^* = 2894. mm^3

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ESΣ8.109

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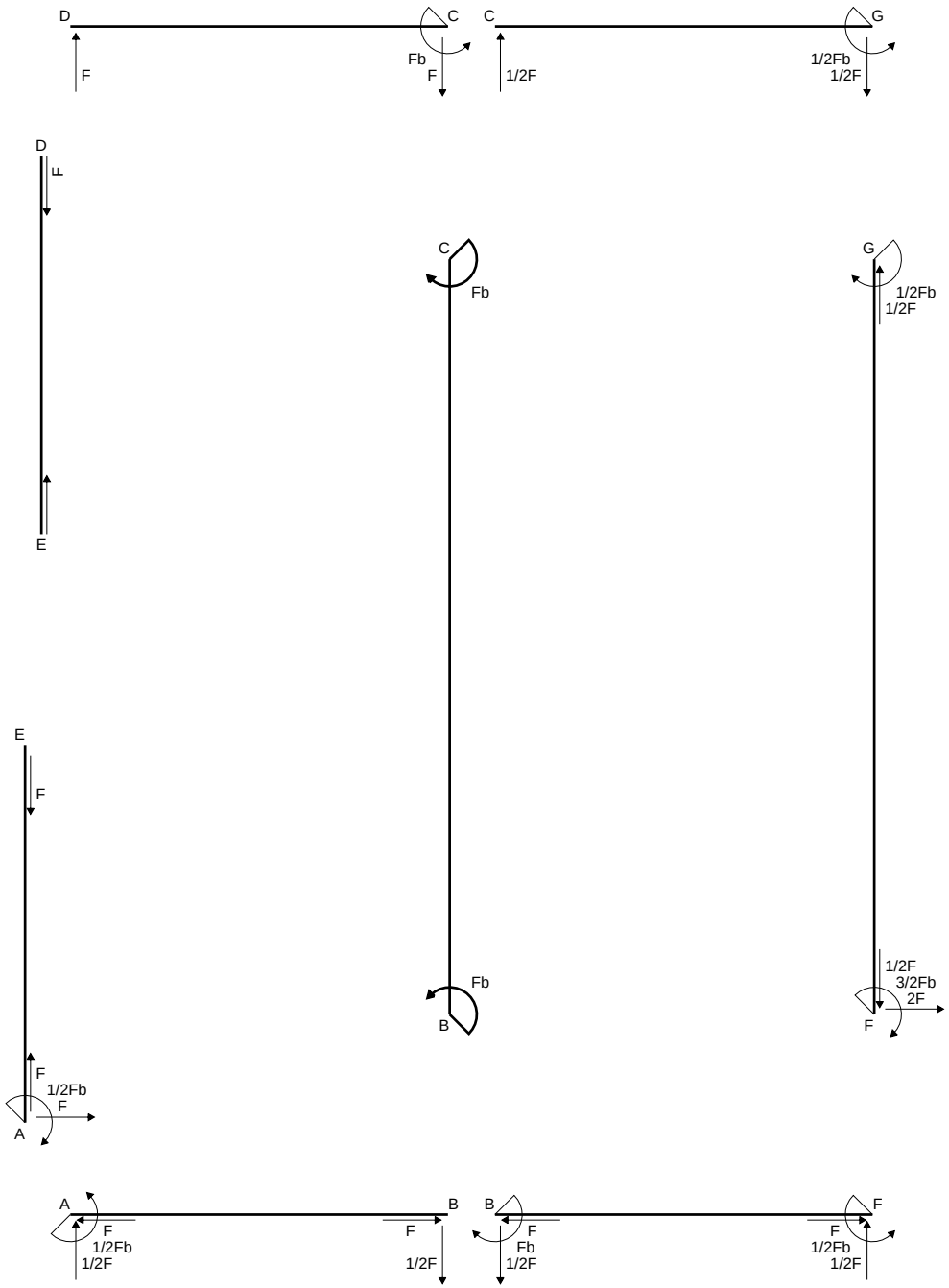
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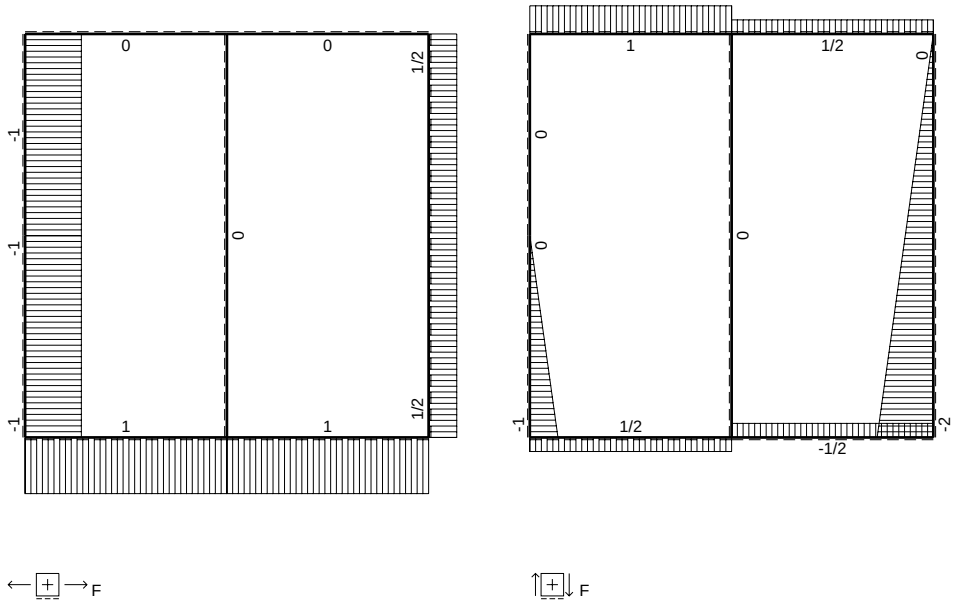
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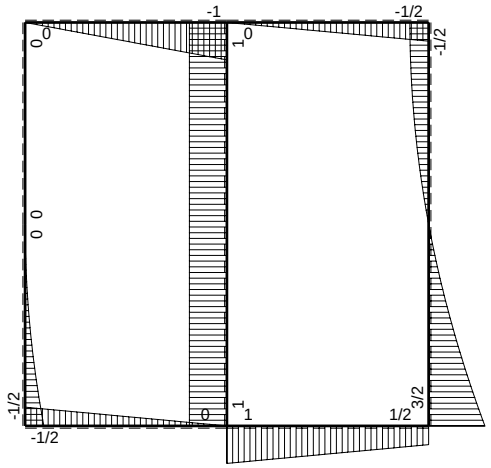
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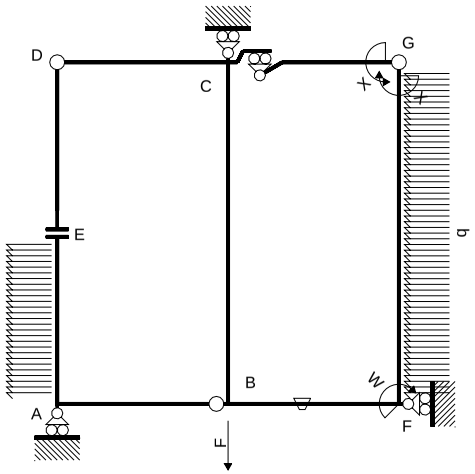
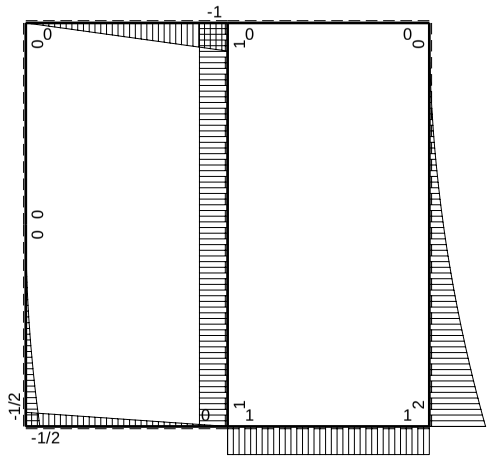
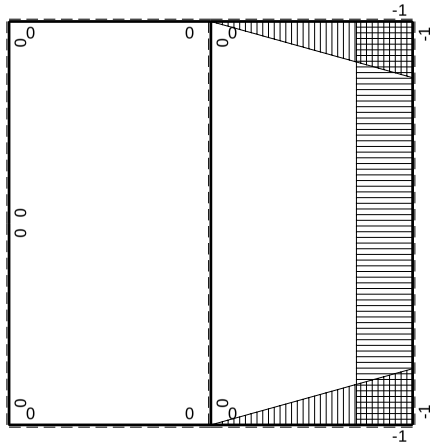




M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.110

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

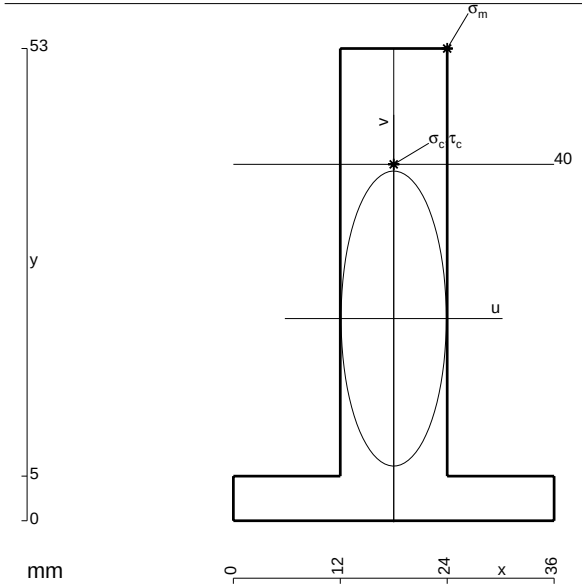
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 756. mm^2
J_u = 207276. mm^4
J_v = 26352. mm^4
y_g = 22.69 mm
T_y = 3250. N
M_x = -1365000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 30.31 mm
$\sigma_m = -Mv/J_u$ = 199.6 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 17.31 mm
$\sigma_c = -Mv/J_u$ = 114. N/mm^2
τ_c = 4.853 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.3 N/mm^2
S^* = 3714. mm^3

ESΣ8.110

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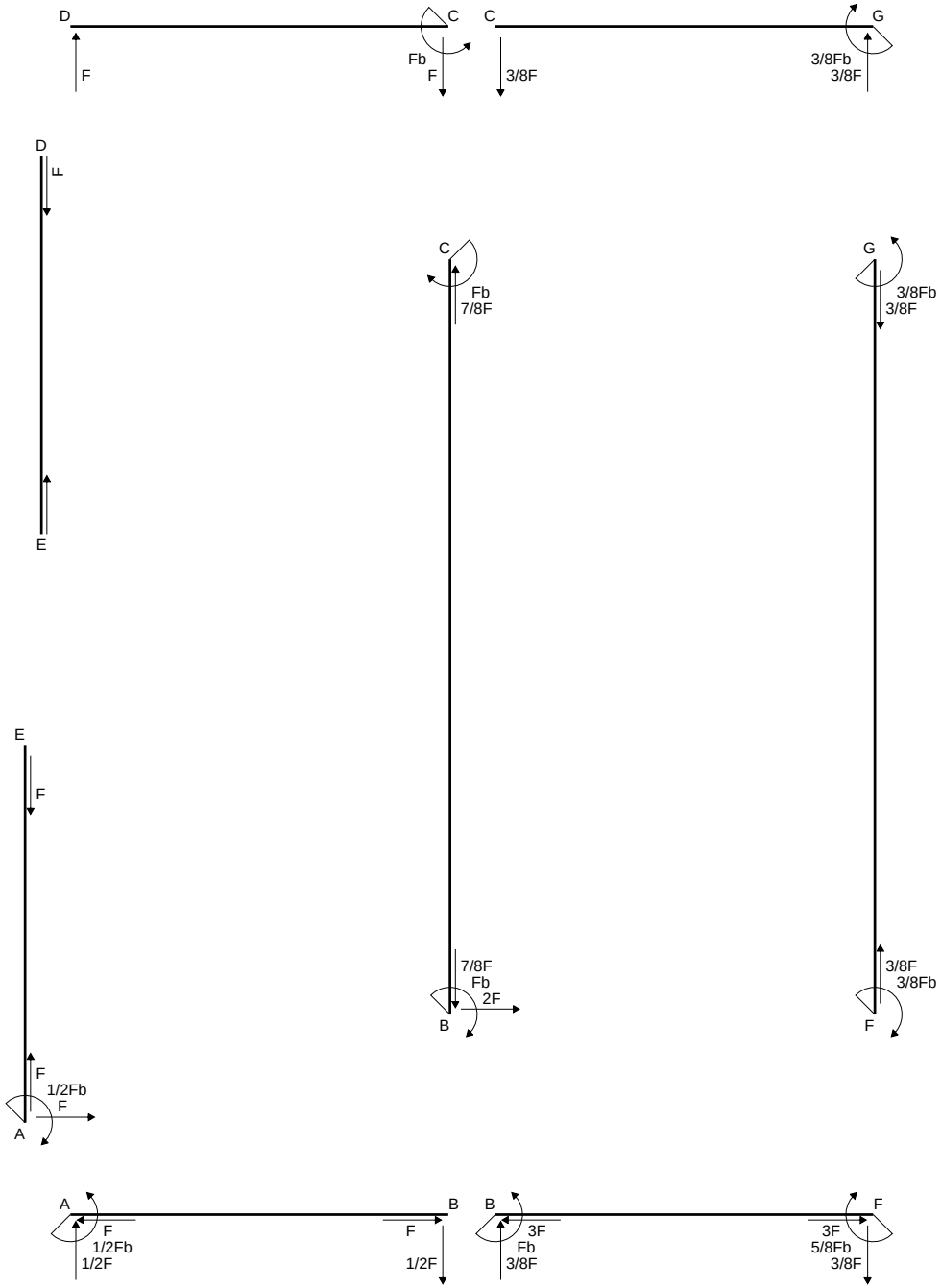
D
C
C
G
F
Fb
F
3/8F
3/8Fb
3/8F
D
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E
C
Fb
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G
3/8Fb
3/8F
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1/2Fb
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A
7/8F
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3/8Fb
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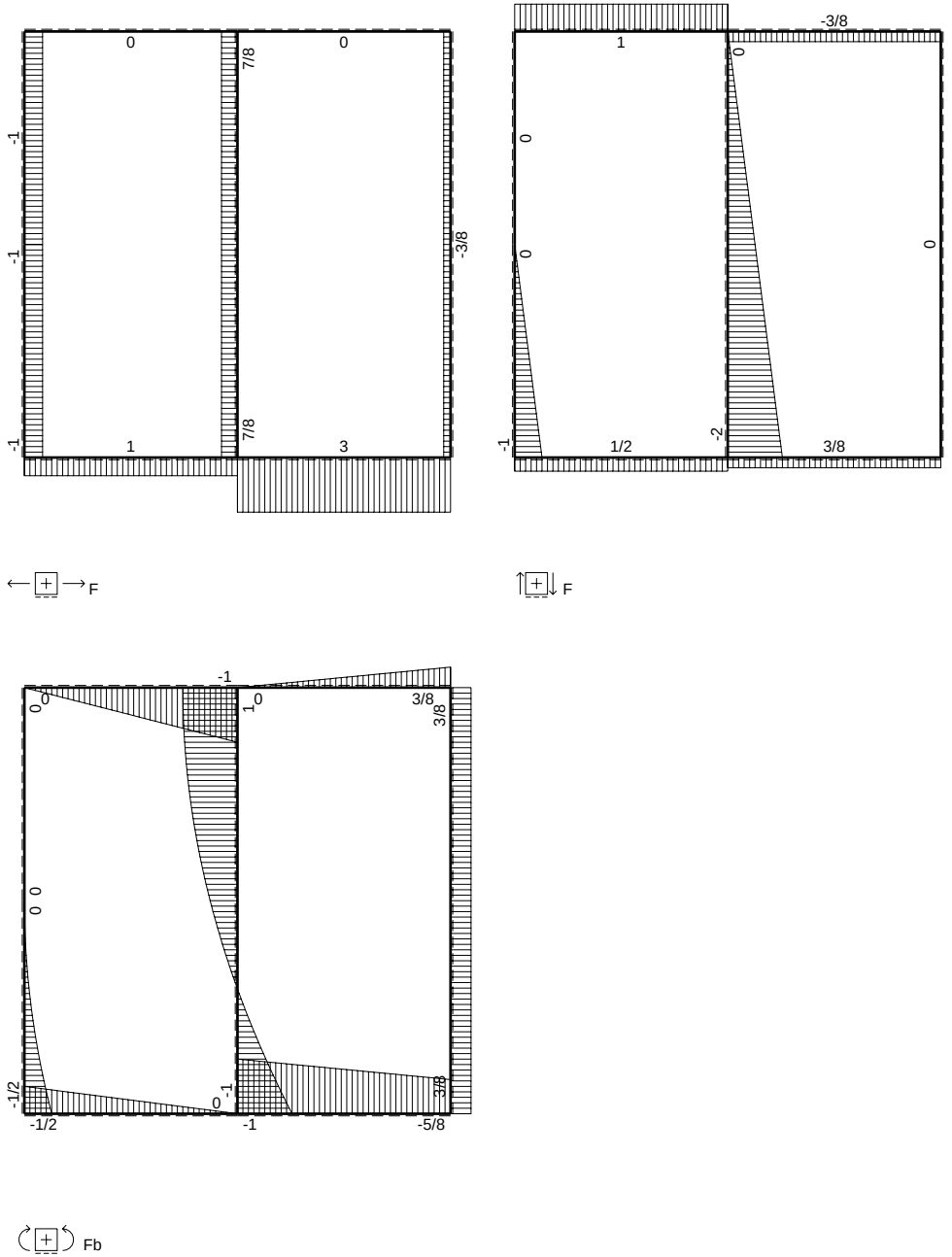
3/8F
3F
5/8Fb
3/8F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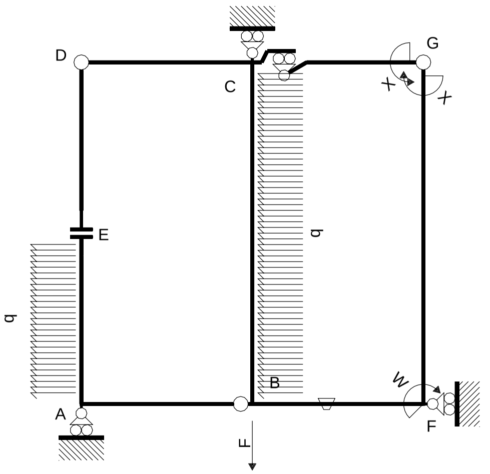
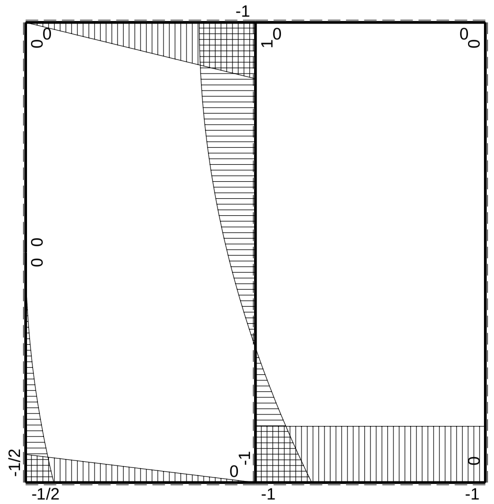
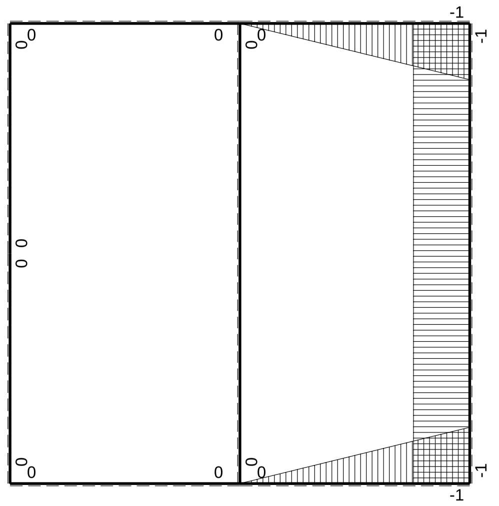
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

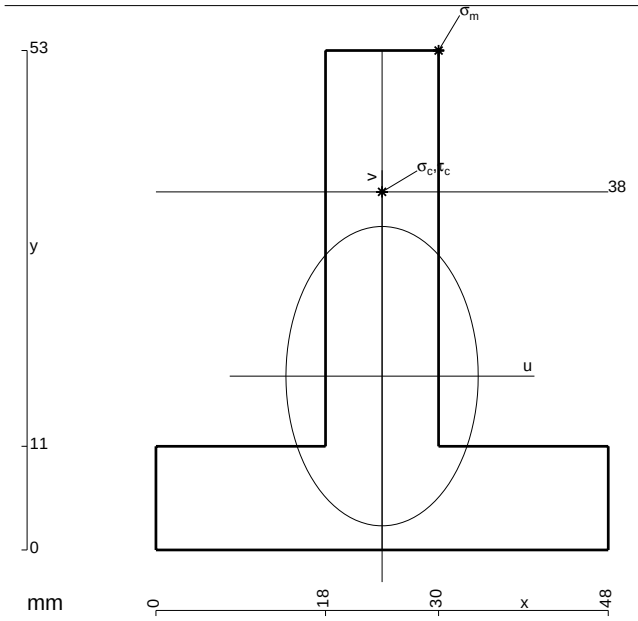
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1032. mm^2
J_u = 260495. mm^4
J_v = 107424. mm^4
y_g = 18.44 mm
N = 2966. N
T_y = -6780. N
M_x = -1559400. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.56 mm
$\sigma_m = N/A - Mv/J_u$ = 209.8 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.56 mm
$\sigma_c = N/A - Mv/J_u$ = 120. N/mm^2
τ_c = 10.56 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.3 N/mm^2
S^* = 4870. mm^3

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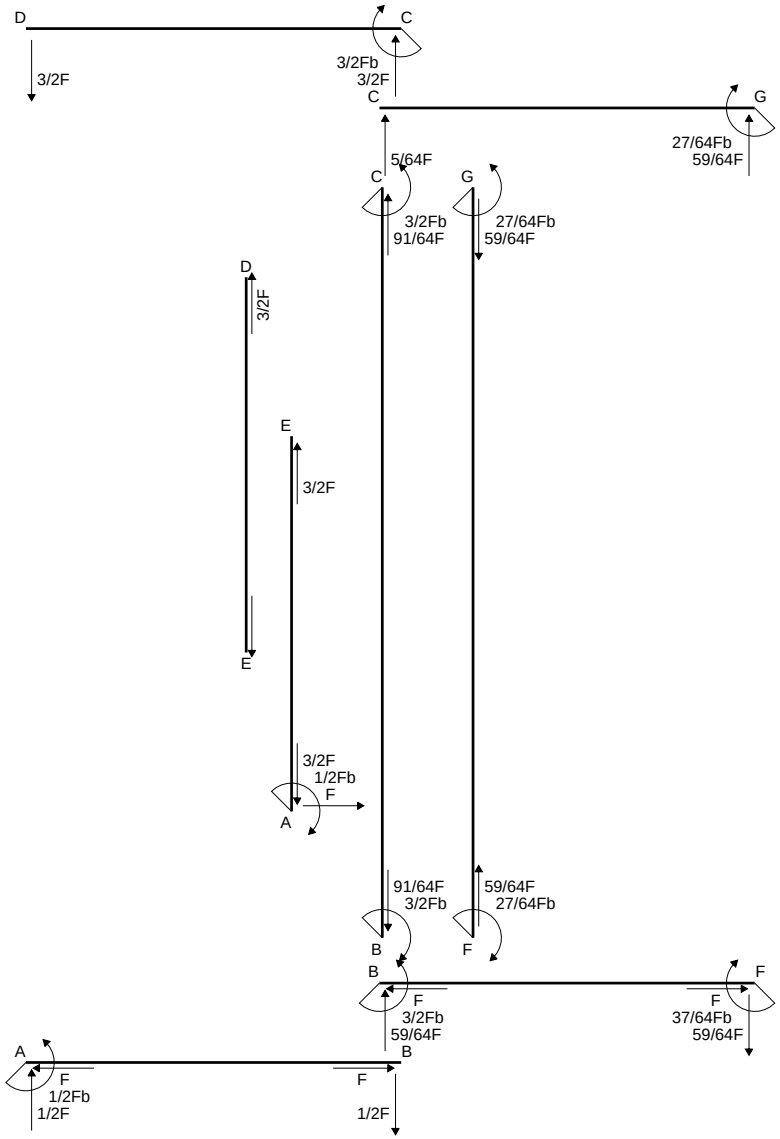
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3/2Fb
3/2F
C
G
27/64Fb
59/64F
5/64F
C
G
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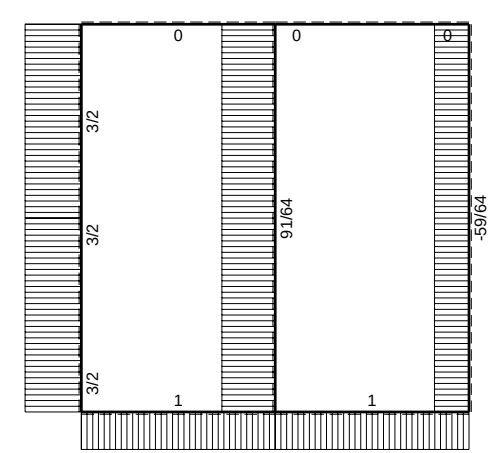
27/64Fb
59/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
91/64F
3/2Fb
59/64F
27/64Fb
B
F
B
F
F
3/2Fb
59/64F
F
37/64Fb
59/64F
A
B
F
1/2Fb
1/2F
F
1/2F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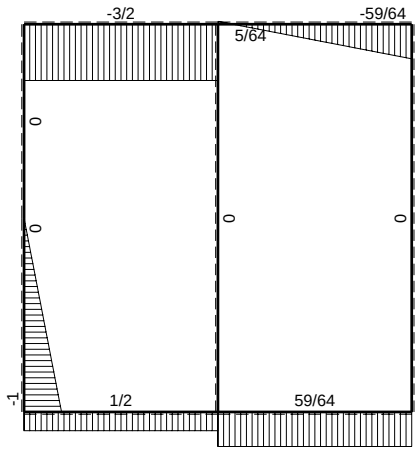
3/2
3/2
3/2
91/64
-59/64
1
1

F


-3/2
-59/64
5/64
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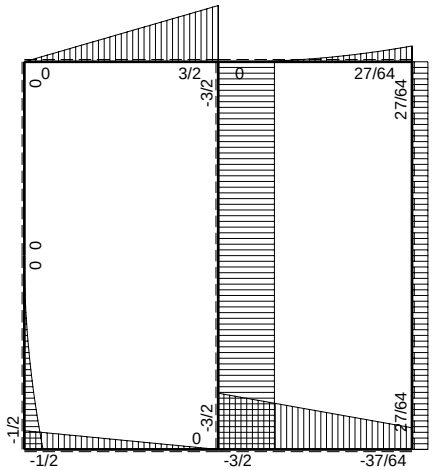
0
0
0
-1
1/2
59/64

F


0
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3/2
-3/2
0
27/64
27/64
0 0
-1/2
-3/2
0
27/64
-1/2
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	$-3/2Fb+1/2Fx$	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

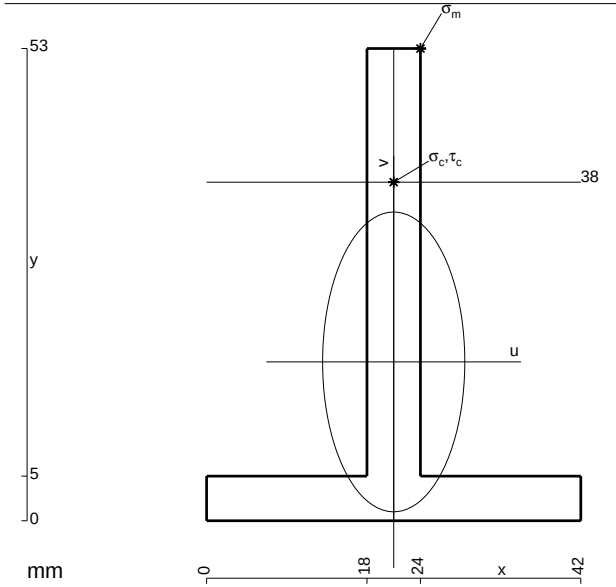
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 498. mm^2
J_u = 141019. mm^4
J_v = 31734. mm^4
y_g = 17.83 mm
T_y = -1755. N
M_x = 877500. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 35.17 mm
$\sigma_m = -Mv/J_u$ = -218.9 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 20.17 mm
$\sigma_c = -Mv/J_u$ = -125.5 N/mm^2
τ_c = 5.166 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125.9 N/mm^2
S^* = 2491. mm^3

ESΣ8.112

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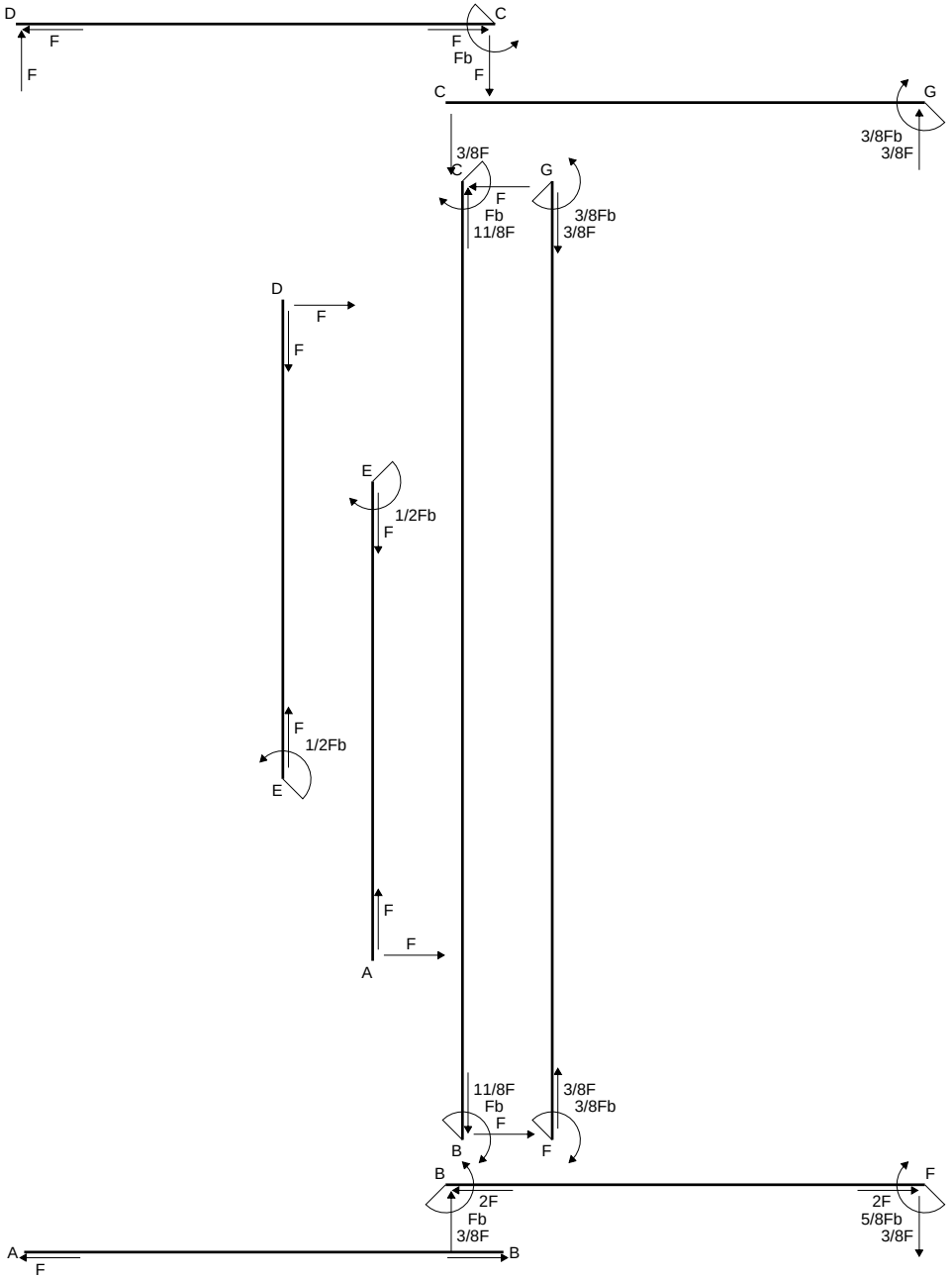
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E
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E
F
F
A
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Fb
F
3/8F
3/8Fb
F
B
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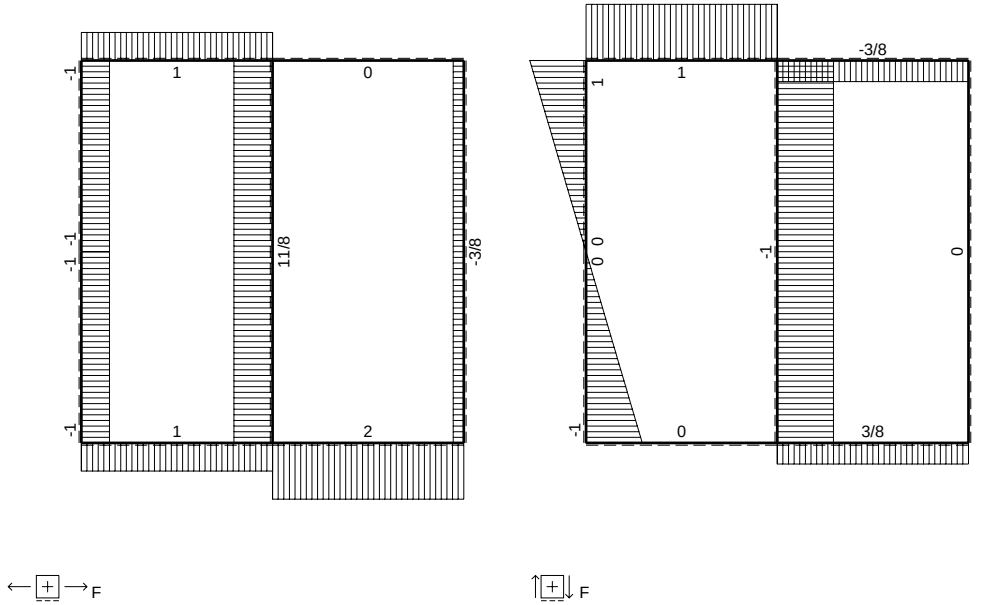
F
2F
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F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1
2
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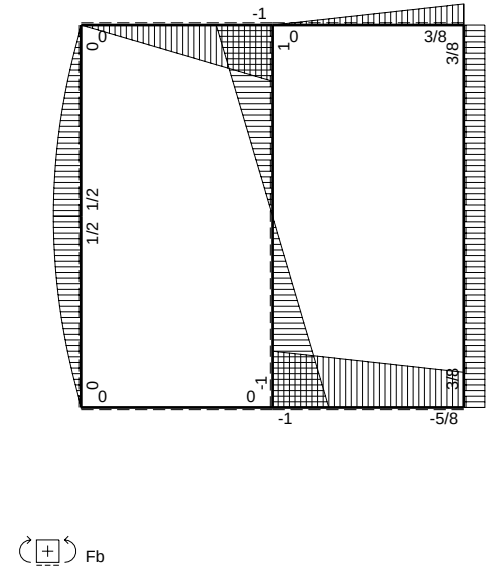
-3/8
1
1
0 0
-1
0
-1
0
3/8
F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0
3/8
3/8
1/2 1/2
0
0
0
-1
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

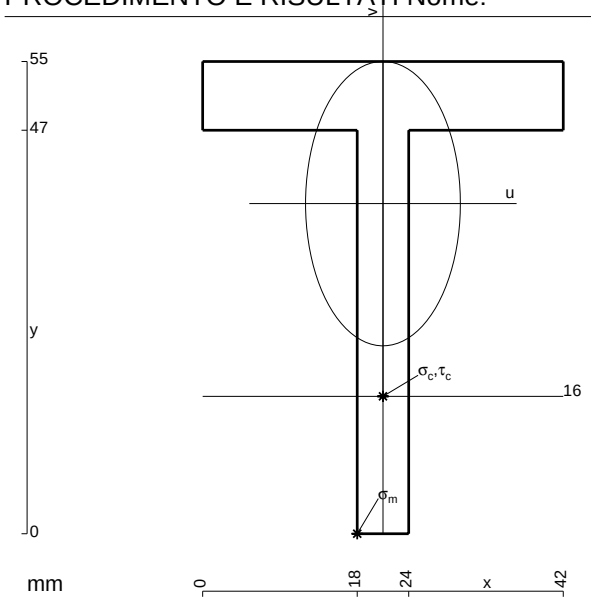
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.113



A = 618. mm^2
J_u = 169652. mm^4
J_v = 50238. mm^4
y_g = 38.45 mm
N = 2406. N
T_y = -1750. N
M_x = 997500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.45 mm
$\sigma_m = N/A - Mv/J_u$ = 230. N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.45 mm
$\sigma_c = N/A - Mv/J_u$ = 135.9 N/mm^2
τ_c = 5.026 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.2 N/mm^2
S^* = 2923. mm^3

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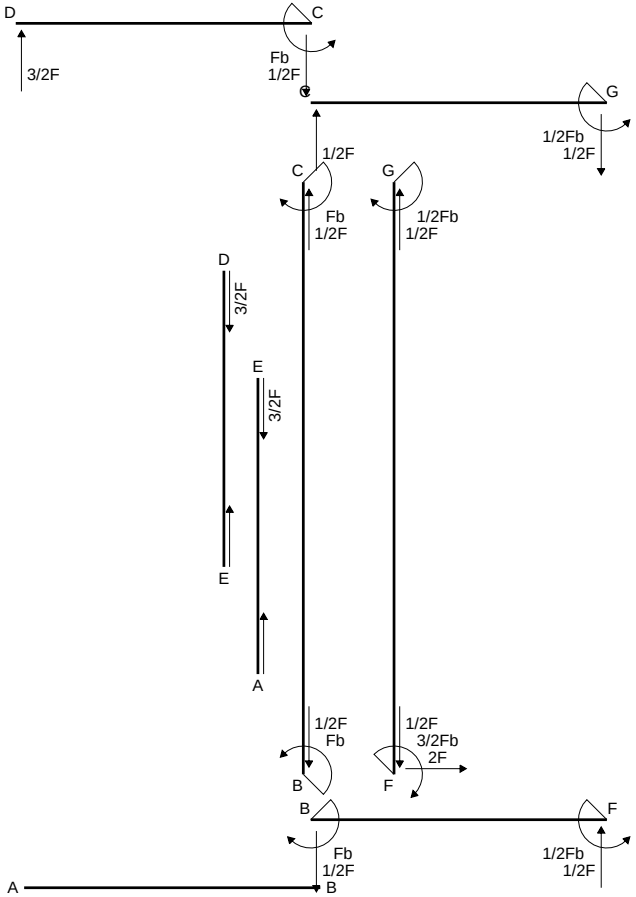
D
C
3/2F
Fb
1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
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1/2F
D
3/2F
E
3/2F
E
A
1/2F
Fb
1/2F
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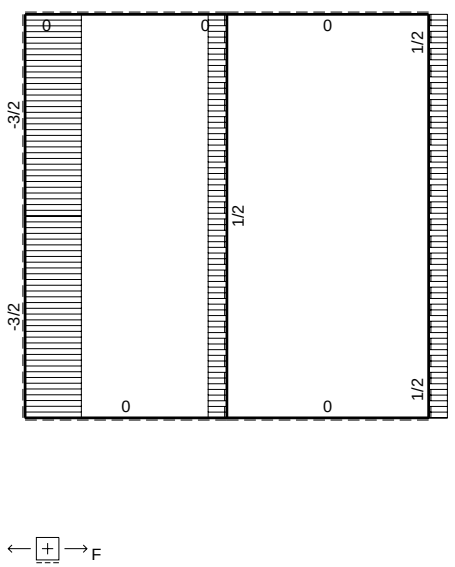
2F
B
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B


0
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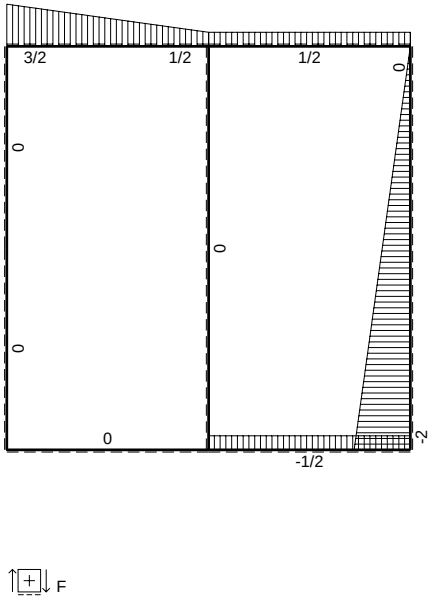
1/2
-3/2
1/2
-3/2
0
0
1/2
F


3/2
1/2
1/2
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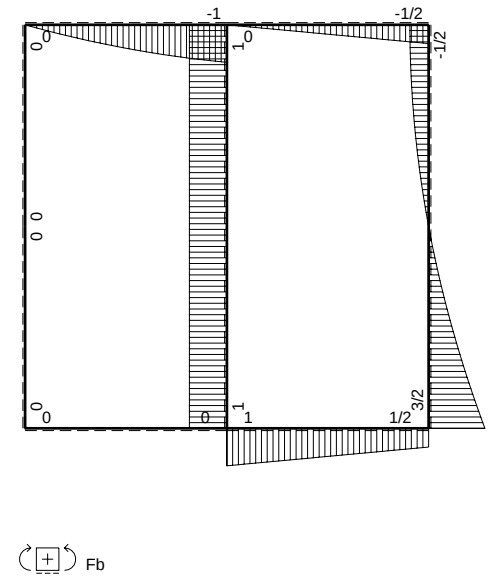
0
0
0
-2
-1/2
F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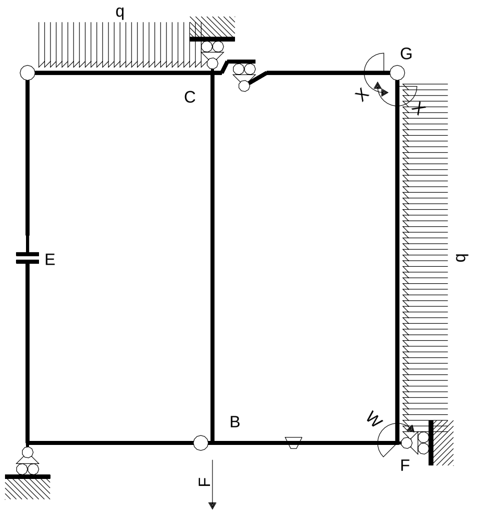
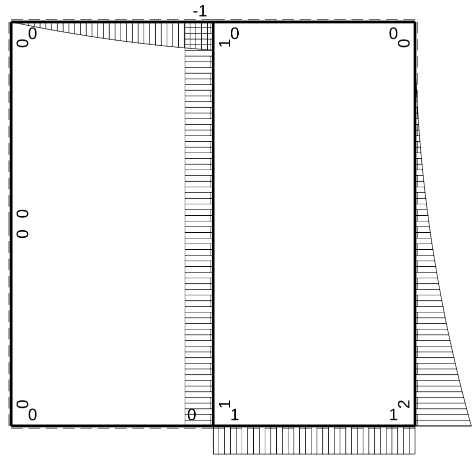
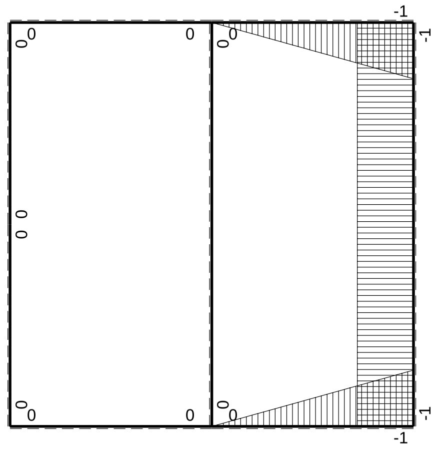
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

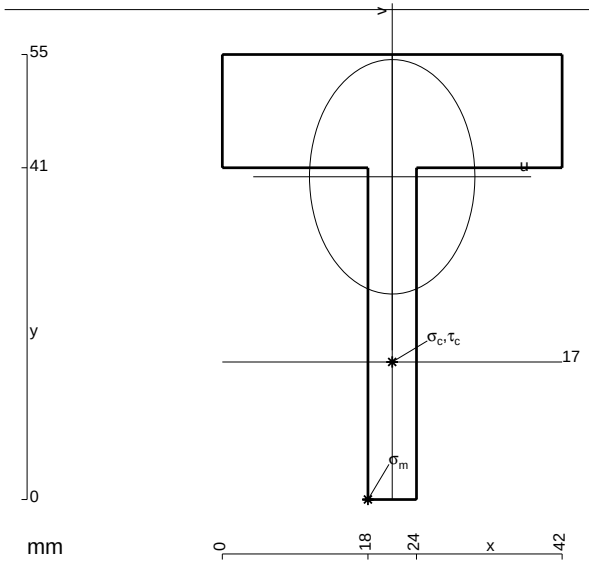
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 834. mm^2
J_u = 175228. mm^4
J_v = 87174. mm^4
y_g = 39.89 mm
T_y = 2760. N
M_x = -1052250. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.89 mm
$\sigma_m = -Mv/J_u$ = -239.5 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.89 mm
$\sigma_c = -Mv/J_u$ = -137.4 N/mm^2
τ_c = 8.405 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 138.2 N/mm^2
S^* = 3202. mm^3

ESΣ8.114

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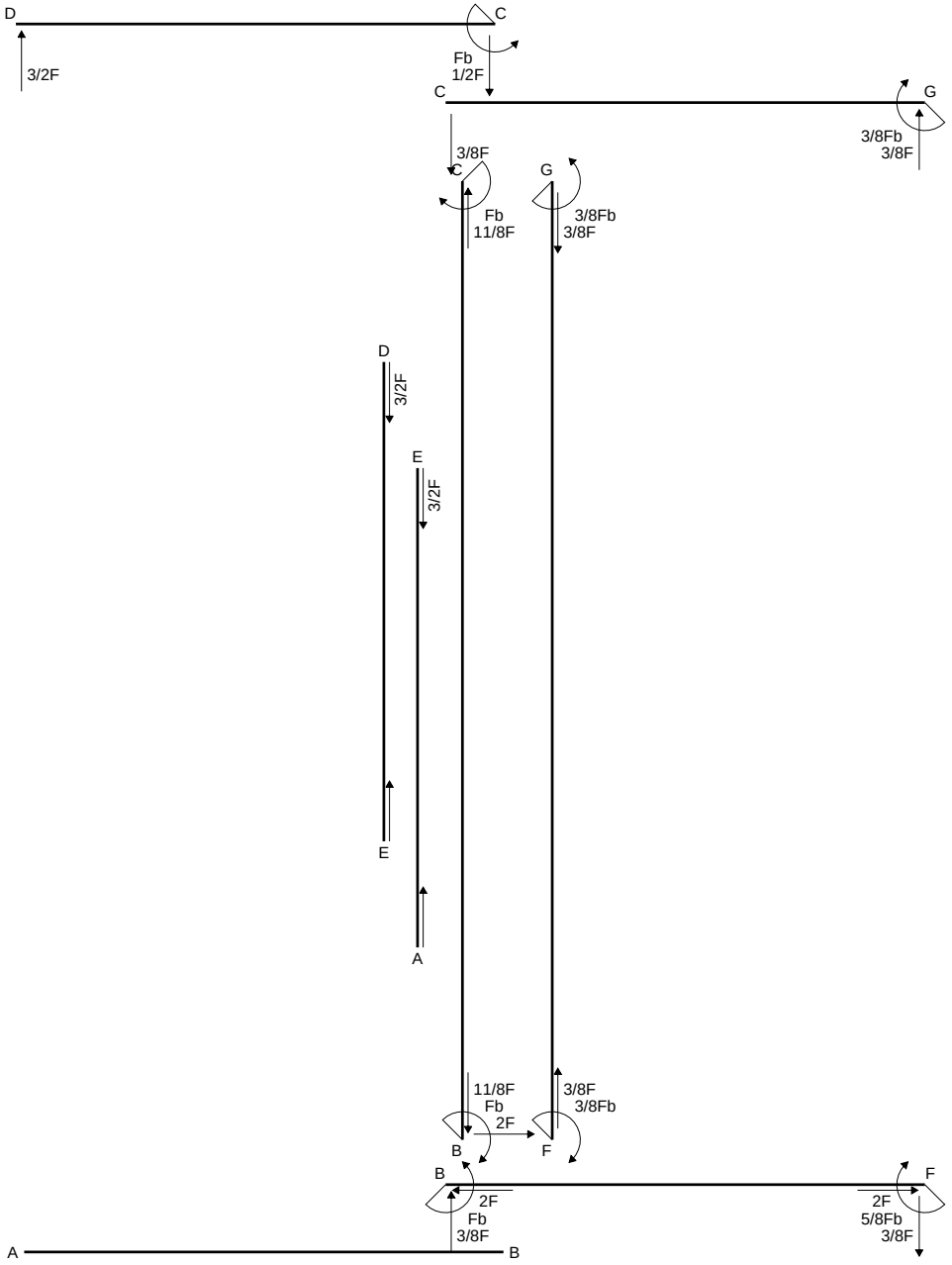
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C
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Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
11/8F
3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
11/8F
Fb
2F
3/8F
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F
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Fb
3/8F
2F
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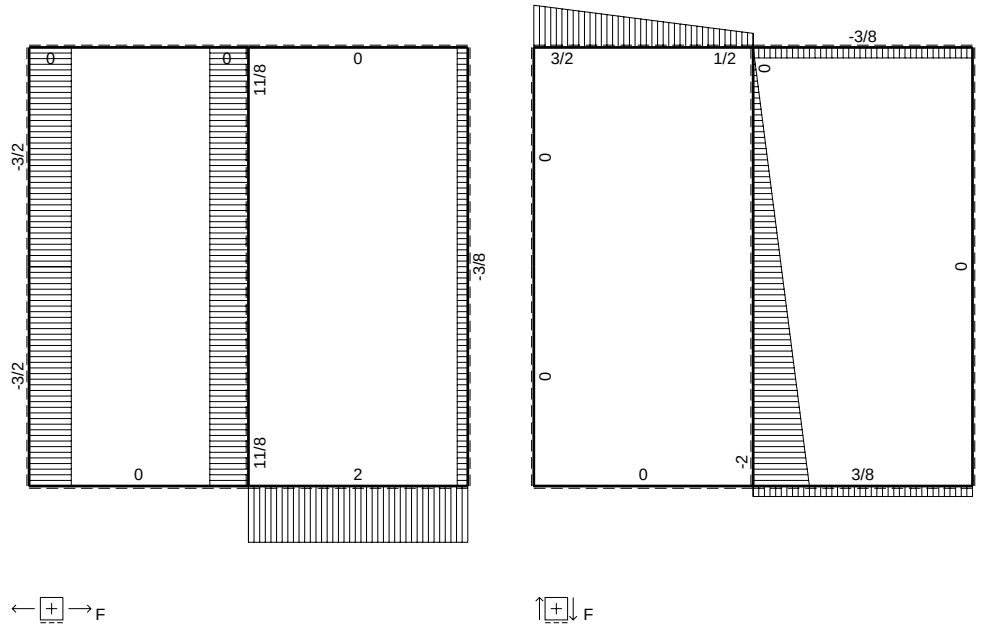
3/8F
A
B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0
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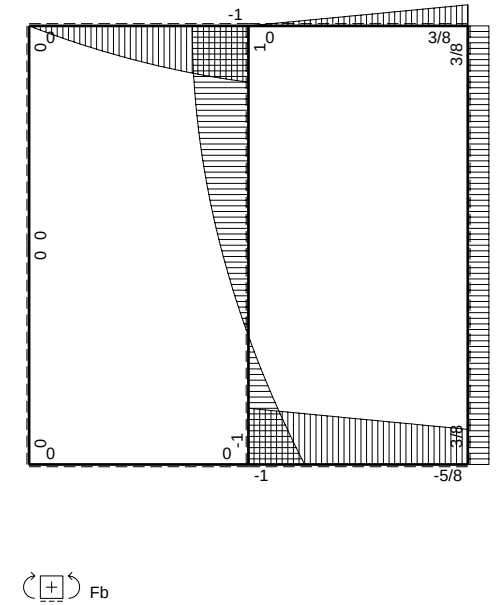
3/8
F


-1
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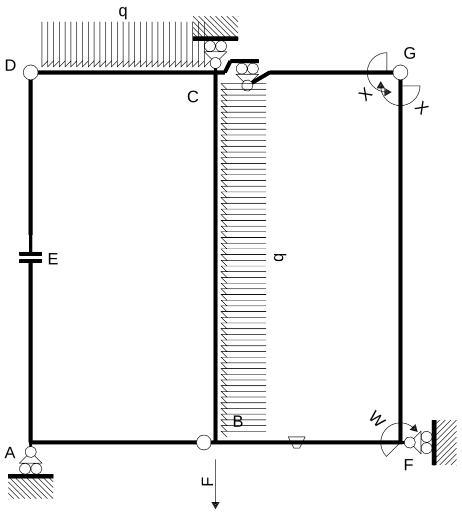
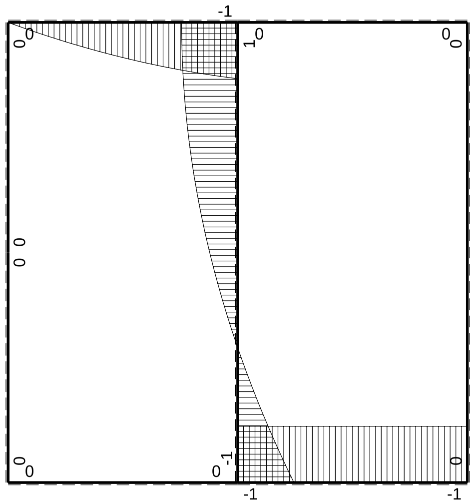
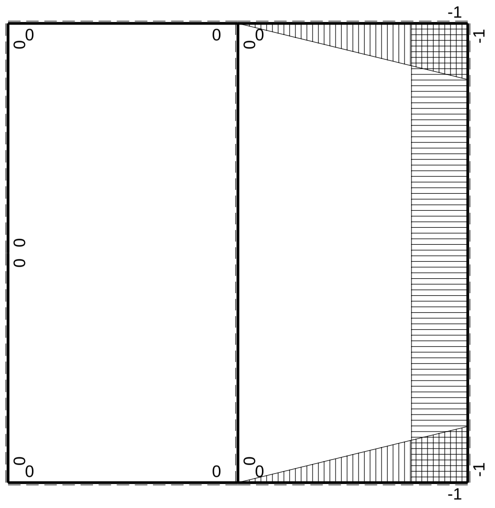
0
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

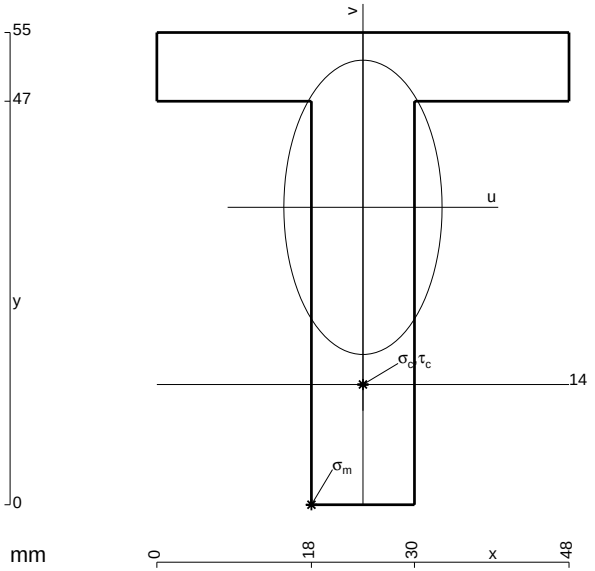
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.115



A = 948. mm^2
J_u = 278641. mm^4
J_v = 80496. mm^4
y_g = 34.64 mm
N = 3410. N
T_y = -4960. N
M_x = -1636800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.64 mm
$\sigma_m = N/A - Mv/J_u$ = -199.9 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -20.64 mm
$\sigma_c = N/A - Mv/J_u$ = -117.6 N/mm^2
τ_c = 6.888 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 118.2 N/mm^2
S^* = 4643. mm^3

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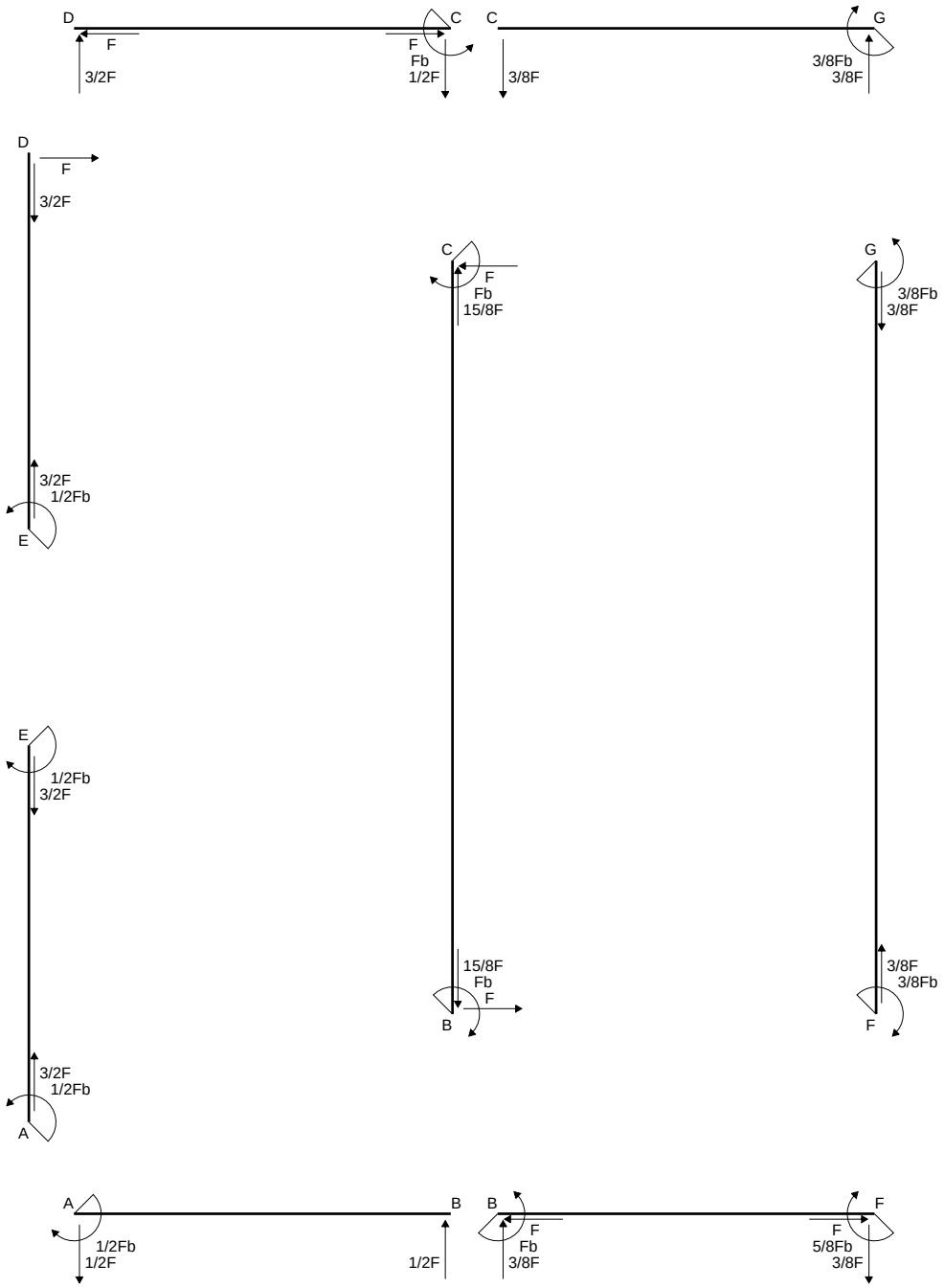
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C
F
Fb
1/2F
C
3/8F
G
3/8Fb
3/8F
D
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3/2F
1/2Fb
E
C
F
Fb
15/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
15/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
A
1/2Fb
1/2F
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5/8Fb
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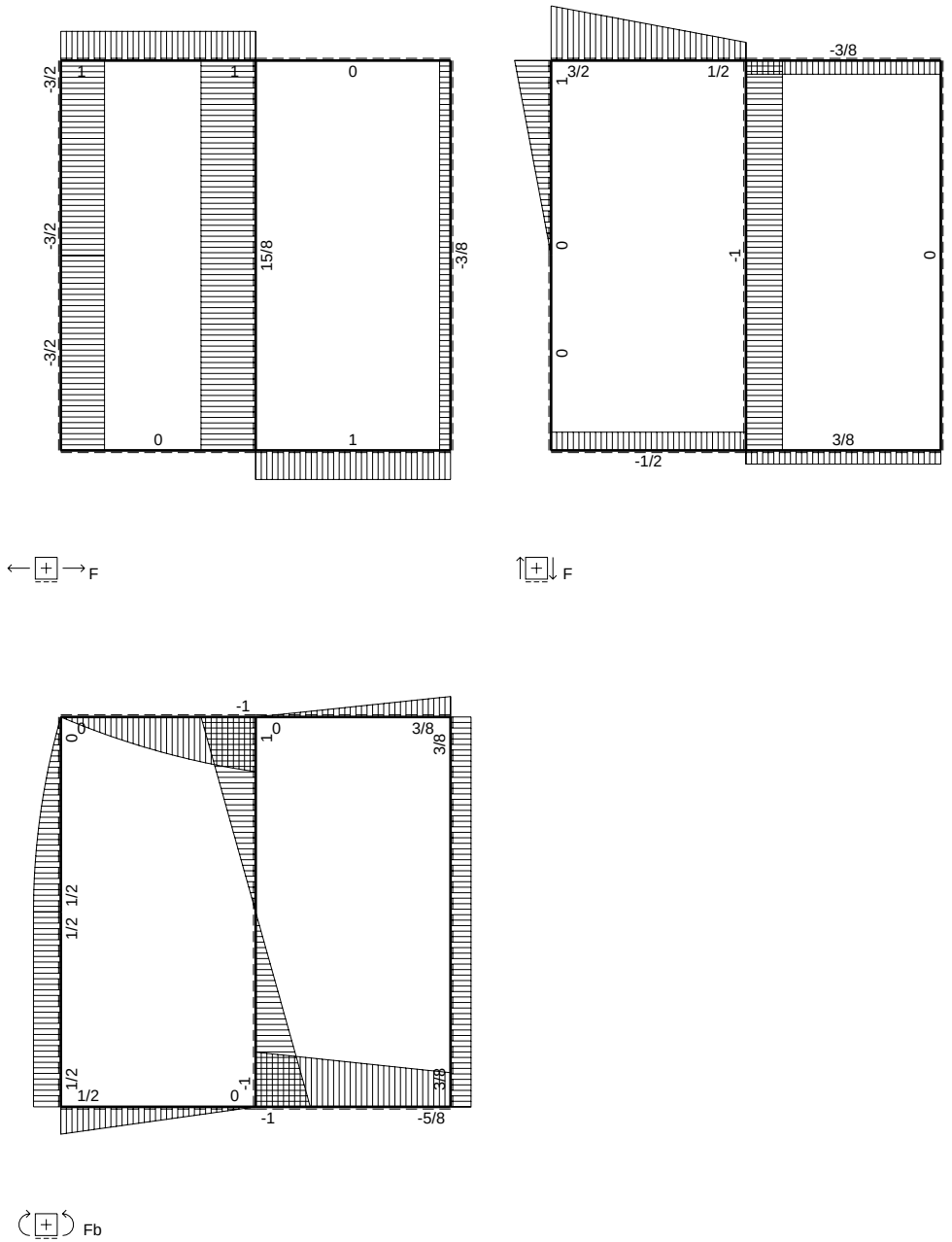
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-1
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0
1
0
3/8
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1/2
1/2
1/2
0
-1
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
→								
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-Fb+1/2Fx+1/2qx^2	0	0	0	0	0+0	0
DC b	0	3/2Fx-1/2qx^2	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

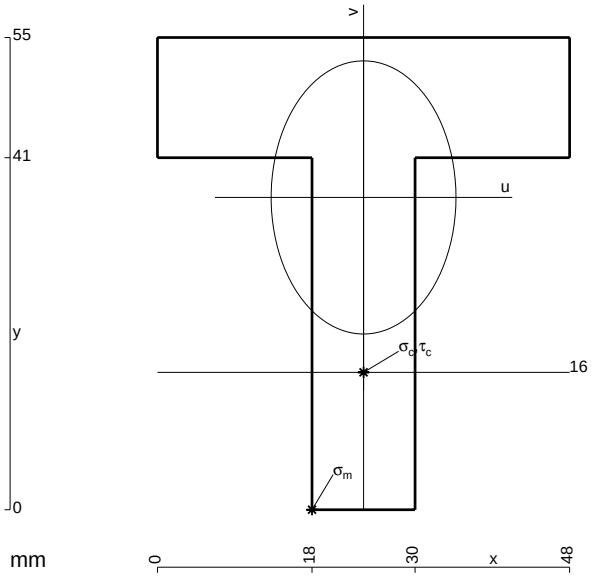
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.116



A = 1164. mm^2
J_u = 294703. mm^4
J_v = 134928. mm^4
y_g = 36.38 mm
N = 4463. N
T_y = -2380. N
M_x = 1666000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -36.38 mm
σ_m = N/A-Mv/J_u = 209.5 N/mm^2
x_c = 24. mm
y_c = 16. mm
v_c = -20.38 mm
σ_c = N/A-Mv/J_u = 119. N/mm^2
τ_c = 3.667 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.2 N/mm^2
S^* = 5448. mm^3

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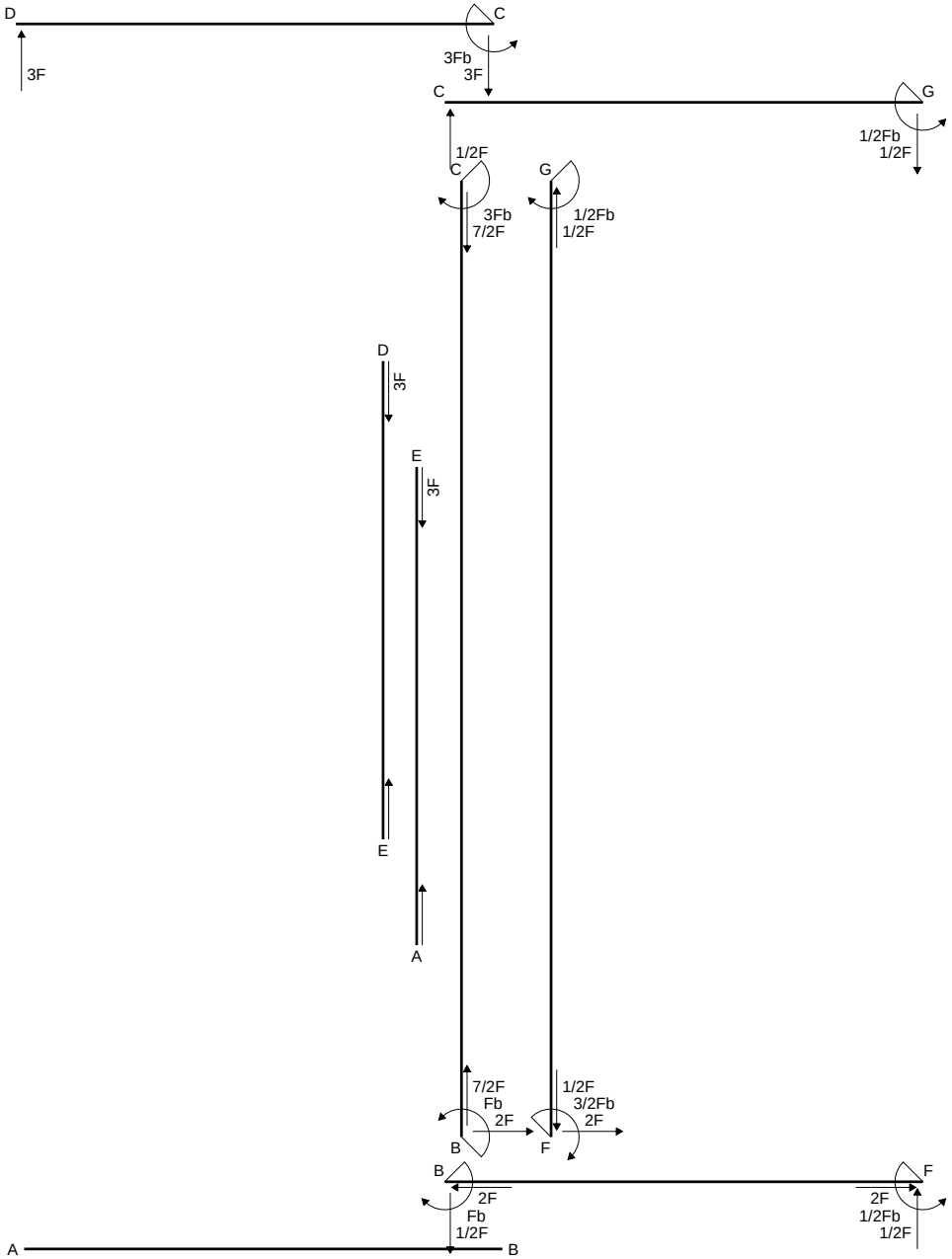
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3F
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G
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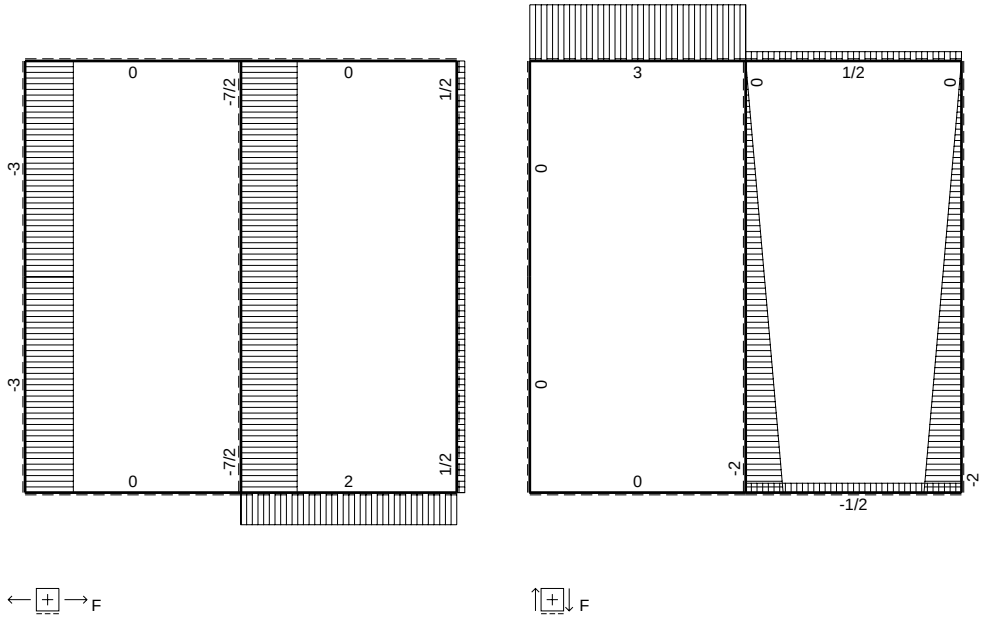
C
G
3Fb
7/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
7/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B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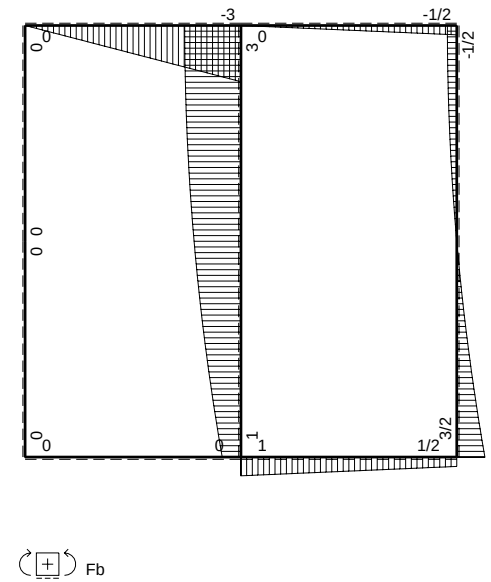
-1/2
F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-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	3Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx²	0	0	0	0		
	totali						-4/3Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

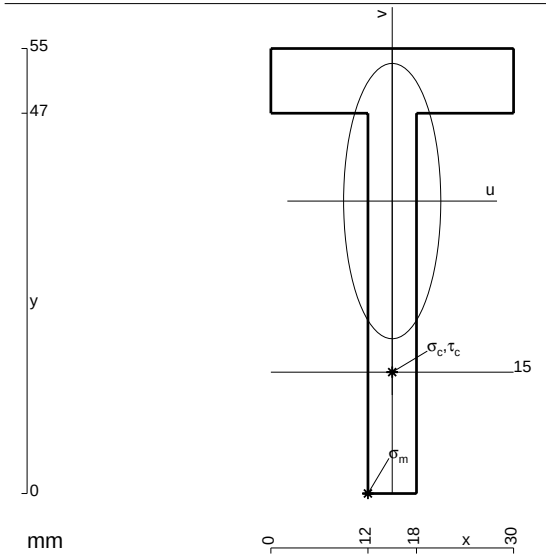
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 522. mm^2
J_u = 151243. mm^4
J_v = 18846. mm^4
y_g = 36.14 mm
T_y = 1230. N
M_x = -910200. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.14 mm
$\sigma_m = -Mv/J_u$ = -217.5 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.14 mm
$\sigma_c = -Mv/J_u$ = -127.2 N/mm^2
τ_c = 3.494 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 127.4 N/mm^2
S^* = 2578. mm^3

ESΣ8.117

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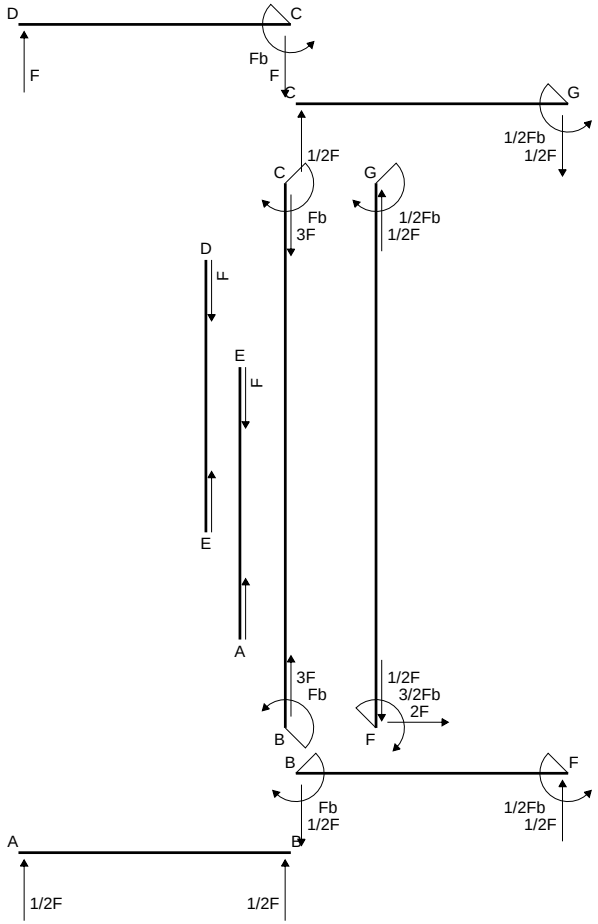
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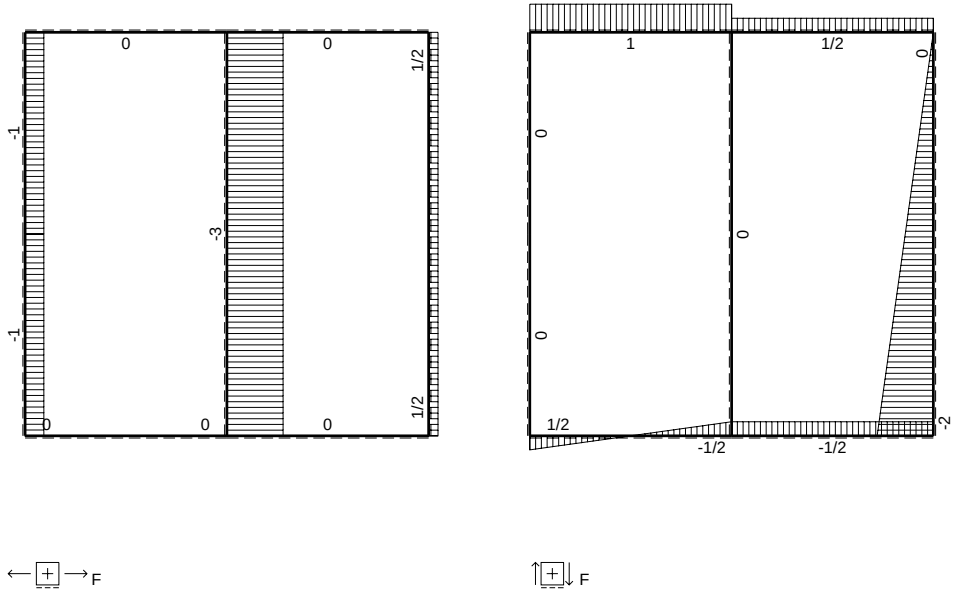
1/2F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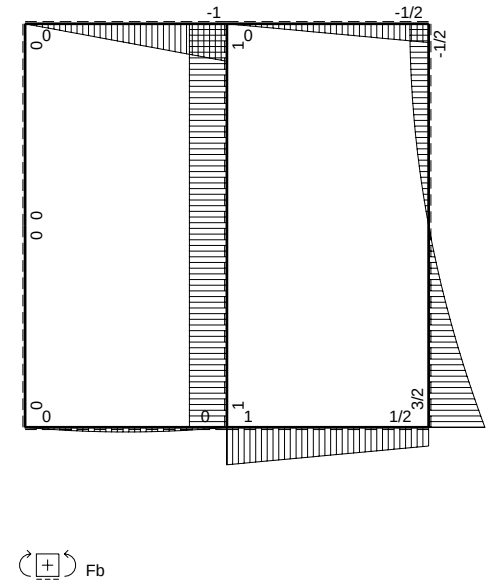
-1/2
F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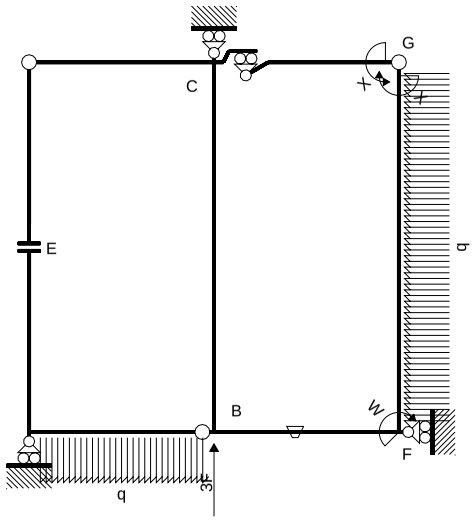
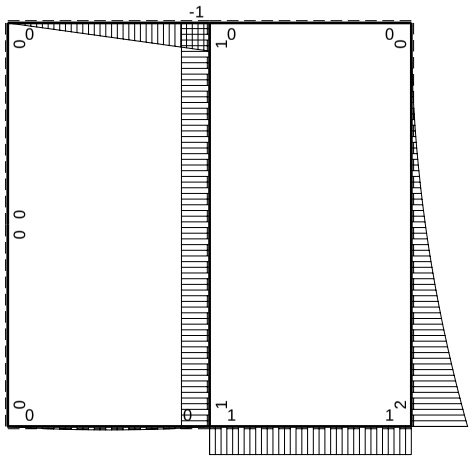
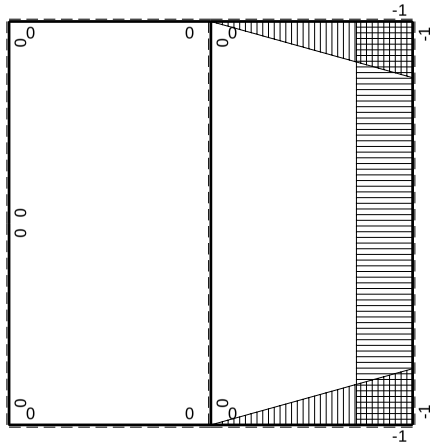
1/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
		totali					$-4/3Fb^2/EJ$	8/3Xb/EJ
		iperstatica $X=W_{GC}$					1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.118

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

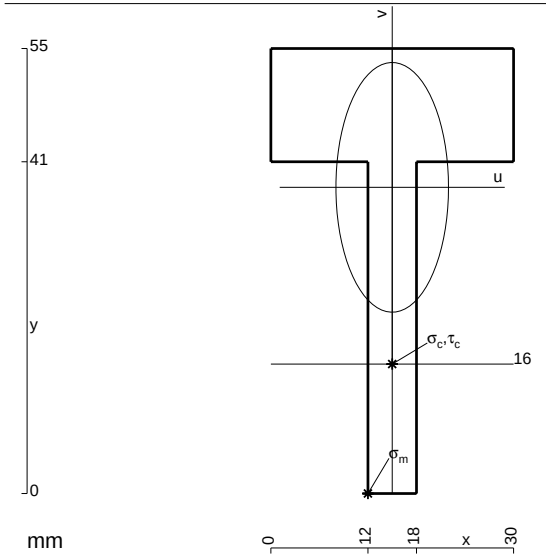
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.118



A = 666. mm^2
J_u = 158641. mm^4
J_v = 32238. mm^4
y_g = 37.84 mm
T_y = 1220. N
M_x = -963800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.84 mm
$\sigma_m = -Mv/J_u$ = -229.9 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.84 mm
$\sigma_c = -Mv/J_u$ = -132.7 N/mm^2
τ_c = 3.672 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.9 N/mm^2
S^* = 2865. mm^3

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ESΣ8.118

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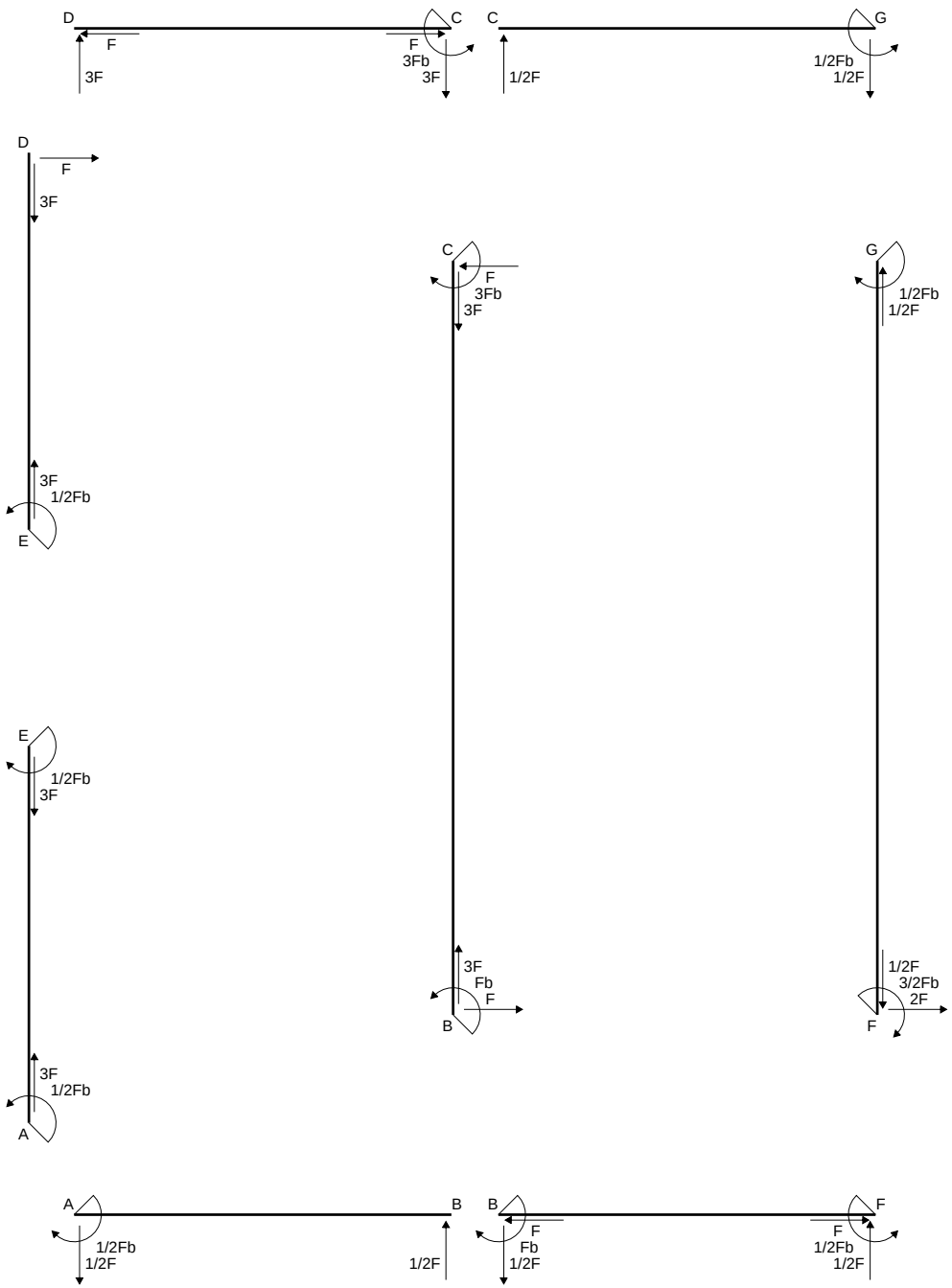
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G
F
3F
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3Fb
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F
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F
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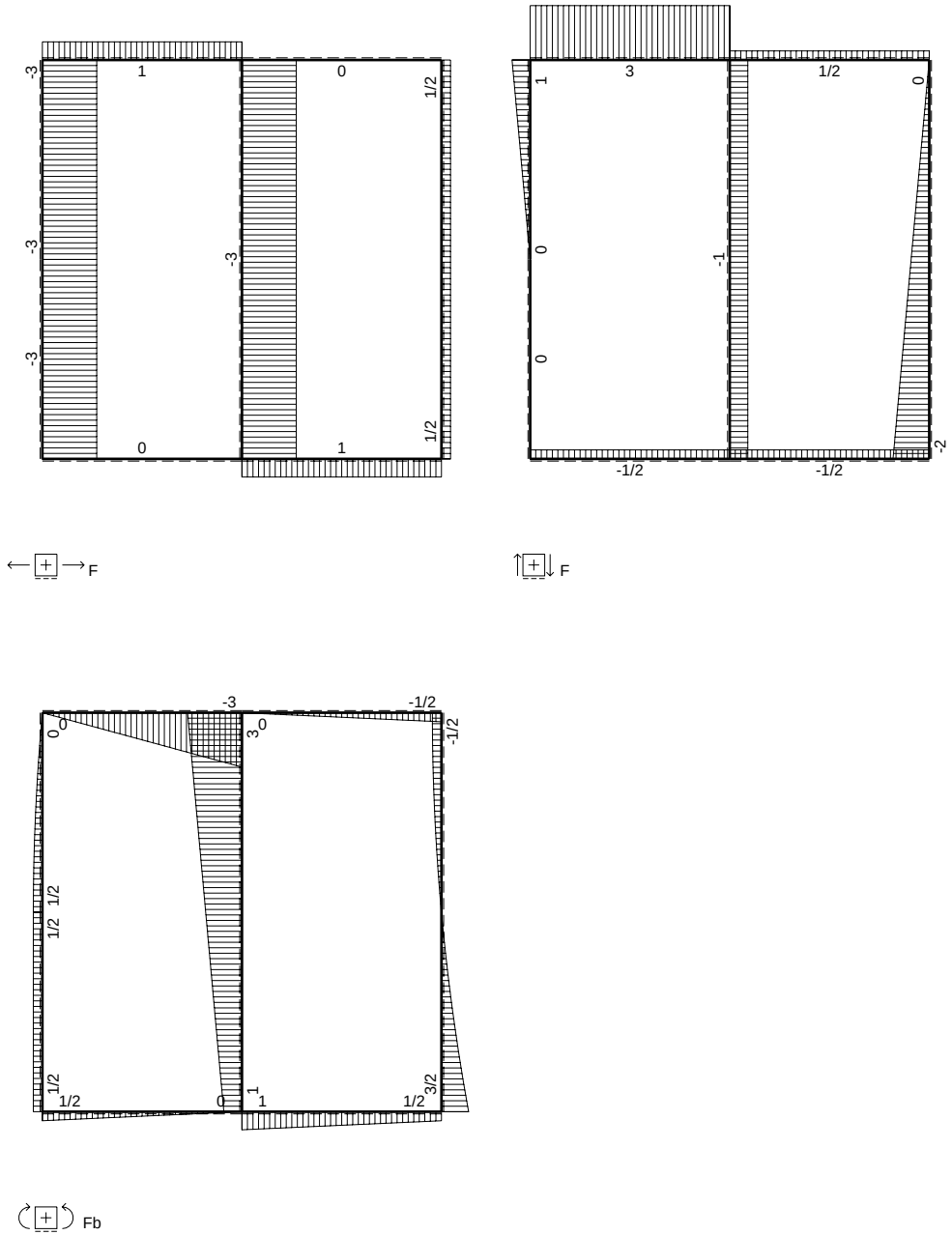
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F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx²	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx²	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x²/b²	(-1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x²/b²		
GC b	-1+x/b	0	0	0	0	1-2x/b+x²/b²	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x²/b²		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
	totali							-4/3Fb²/EJ 8/3Xb/EJ
	iperstatica $X=W_{GC}$							1/2Fb

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

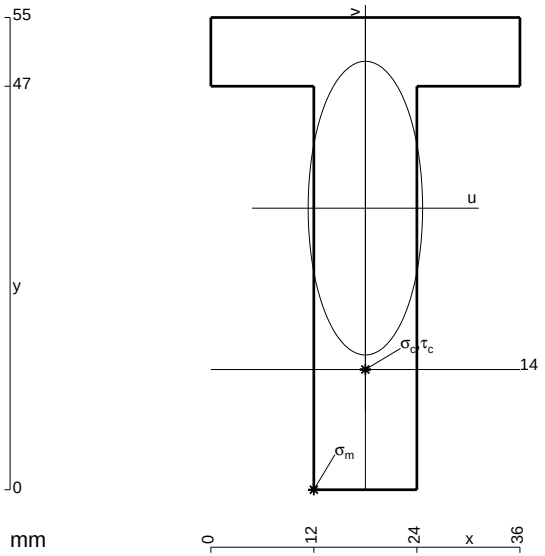
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 852. mm^2
J_u = 249536. mm^4
J_v = 37872. mm^4
y_g = 32.8 mm
N = 730. N
T_y = 2190. N
M_x = -1817700. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.8 mm
$\sigma_m = N/A - Mv/J_u$ = -238. N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.8 mm
$\sigma_c = N/A - Mv/J_u$ = -136.1 N/mm^2
τ_c = 3.169 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 136.2 N/mm^2
S^* = 4334. mm^3

ESΣ8.119

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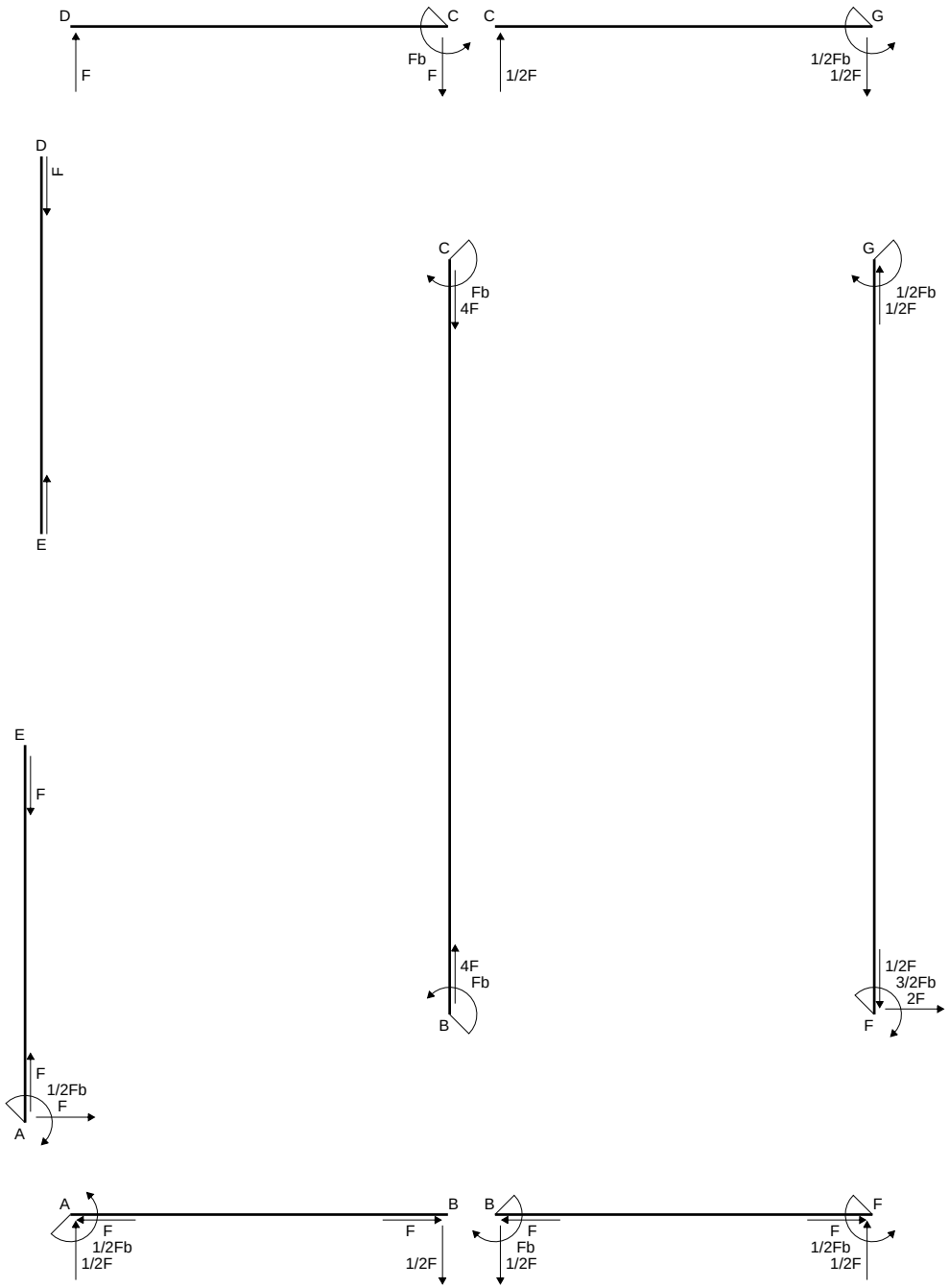
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Fb
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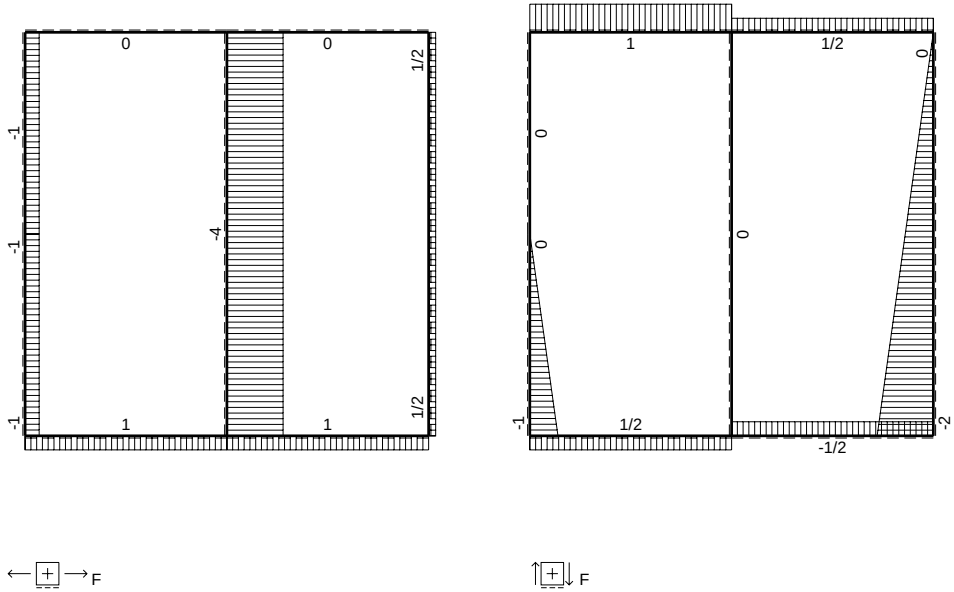
F
1/2Fb
1/2F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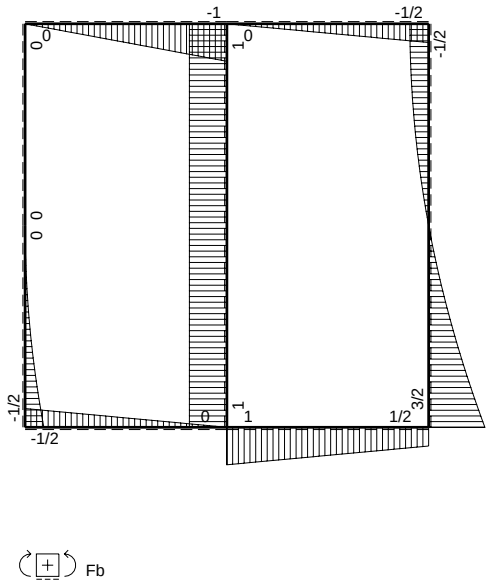
-1/2
F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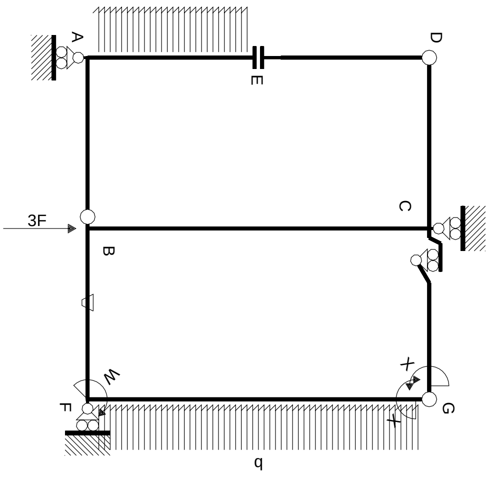
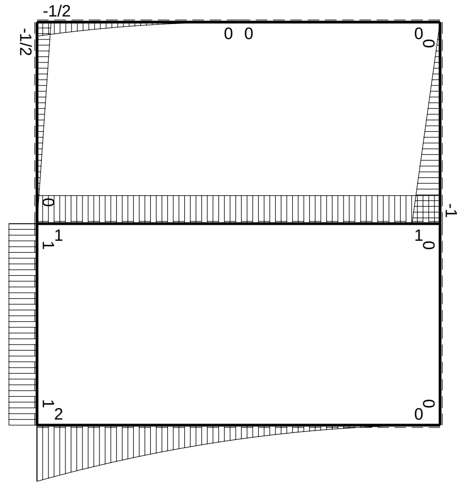
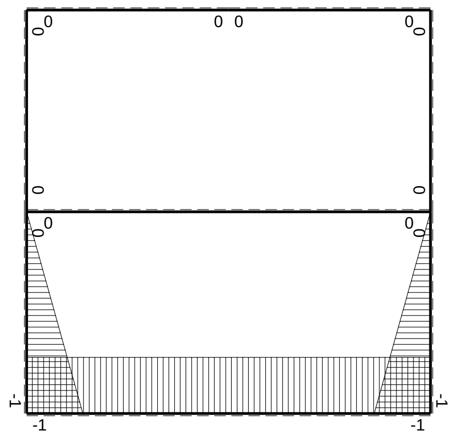
1/2
3/2
-1/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

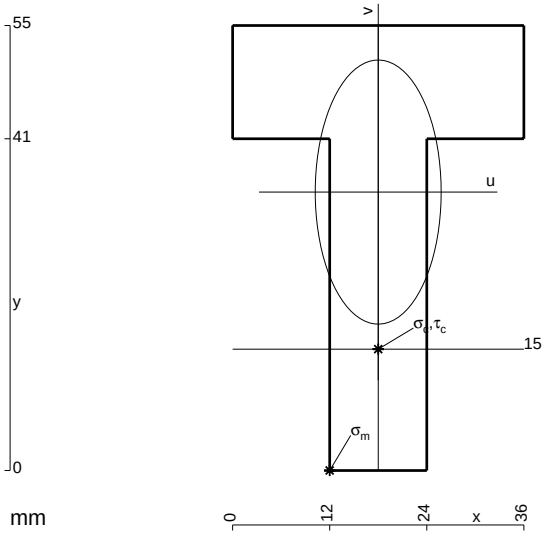
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 996. mm^2
J_u = 265432. mm^4
J_v = 60336. mm^4
y_g = 34.42 mm
T_y = 3500. N
M_x = -1540000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -34.42 mm
$\sigma_m = -Mv/J_u$ = -199.7 N/mm^2
x_c = 18. mm
y_c = 15. mm
v_c = -19.42 mm
$\sigma_c = -Mv/J_u$ = -112.6 N/mm^2
τ_c = 5.324 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113. N/mm^2
S^* = 4845. mm^3

ESΣ8.120

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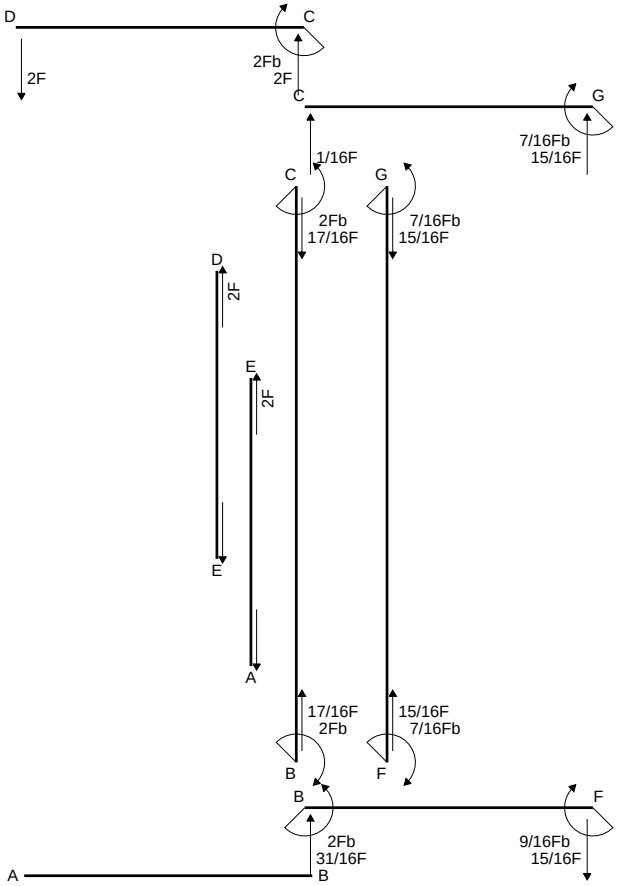
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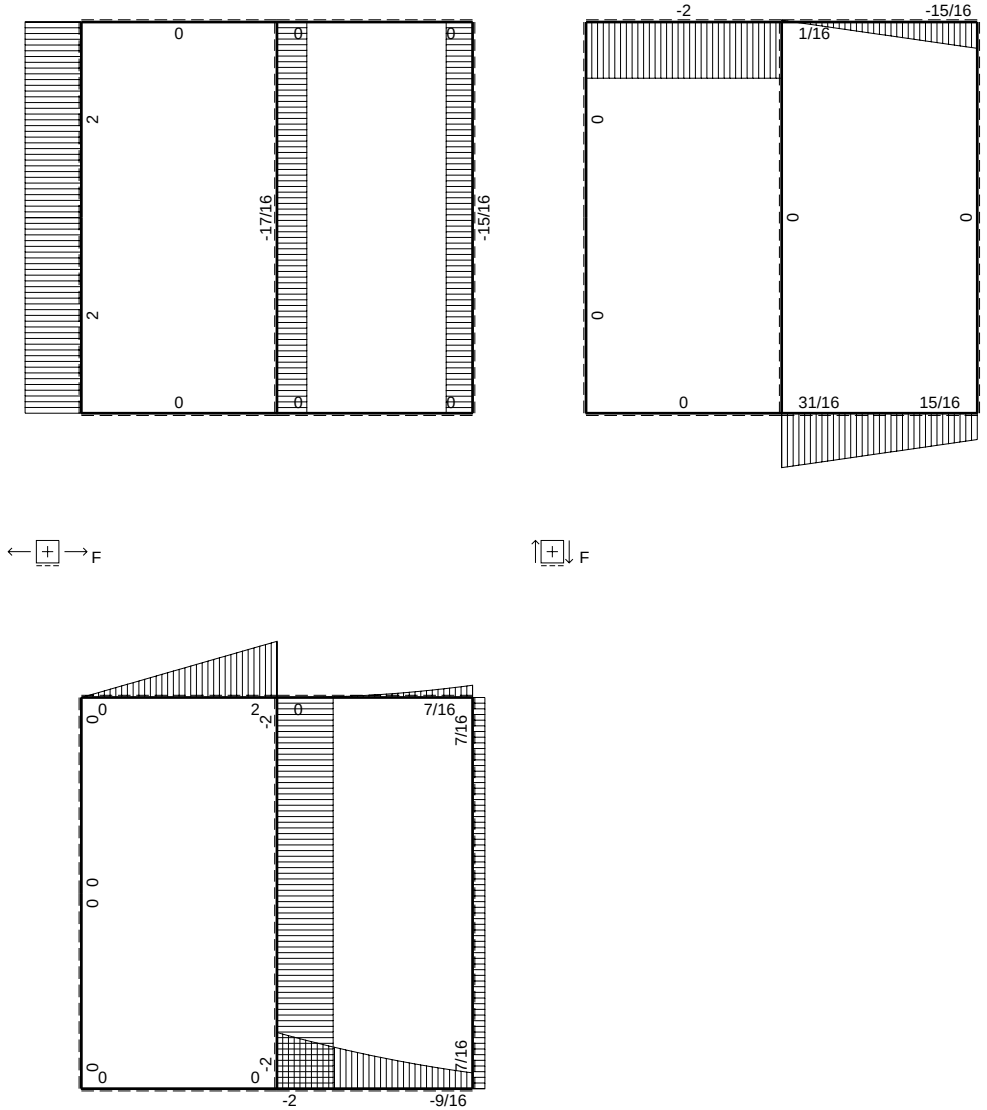
B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31/16
15/16
F
F
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

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14.11.23

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
	0	-2Fx	0	0	0	0		
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
	1	0	0	0	0	1		
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
	0	2Fb	0	0	0	0		
BC 2b	0	2Fb	0	0	0	0		
	totali						7/6Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

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14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

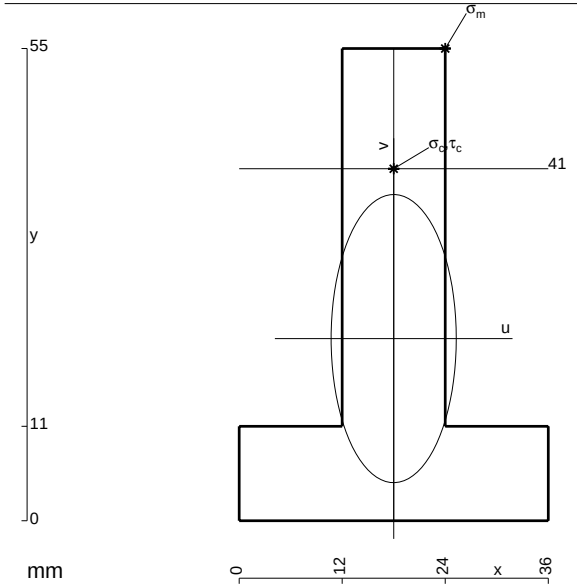
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 924. mm^2
J_u = 260306. mm^4
J_v = 49104. mm^4
y_g = 21.21 mm
T_y = -3360. N
M_x = 1612800. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 33.79 mm
$\sigma_m = -Mv/J_u$ = -209.3 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.79 mm
$\sigma_c = -Mv/J_u$ = -122.6 N/mm^2
τ_c = 4.84 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.9 N/mm^2
S^* = 4500. mm^3

ESΣ8.121

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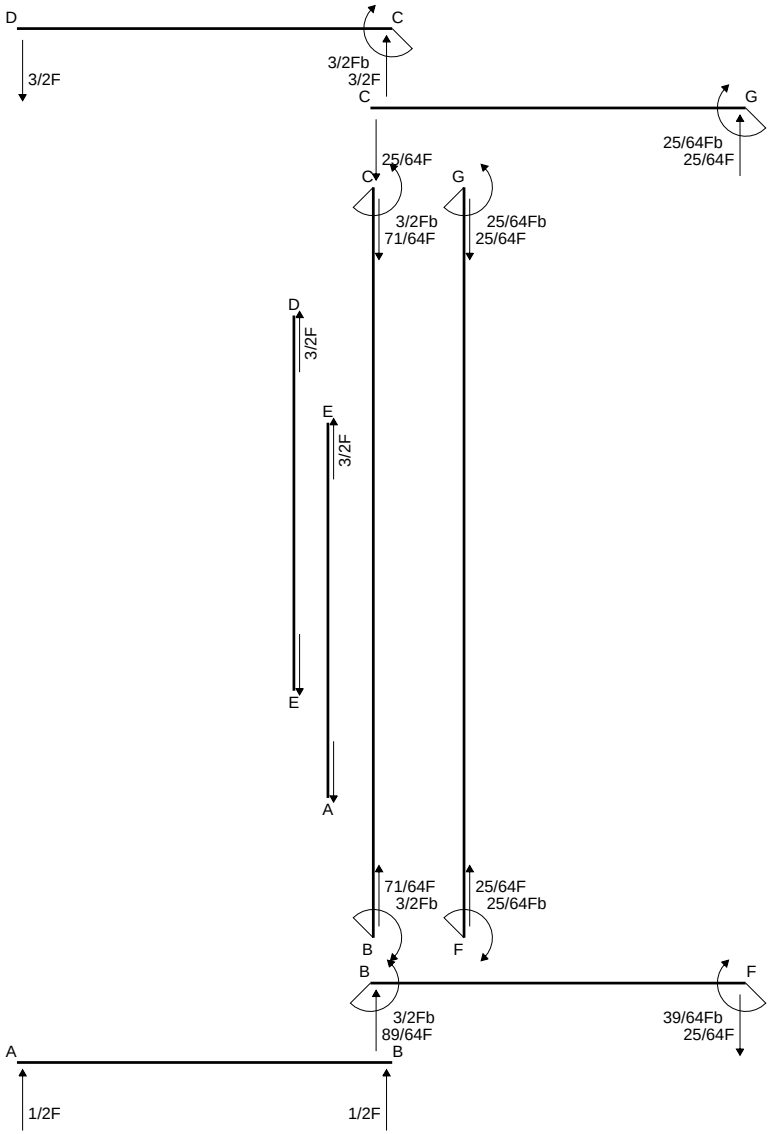
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3/2Fb
3/2F
C
G
25/64F
25/64Fb
25/64F
C
G
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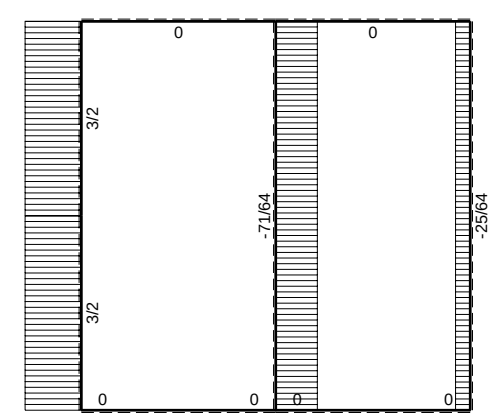
71/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
A
71/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
3/2Fb
89/64F
39/64Fb
25/64F
B
A
1/2F
1/2F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-25/64
3/2
0
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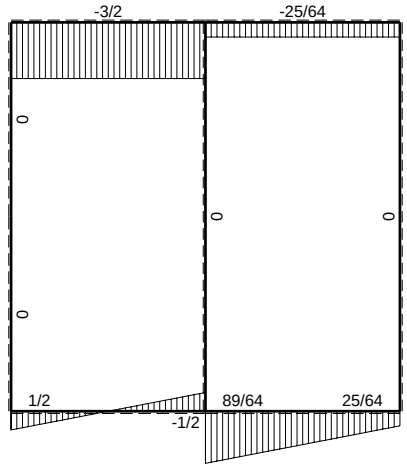
0
0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-25/64
0
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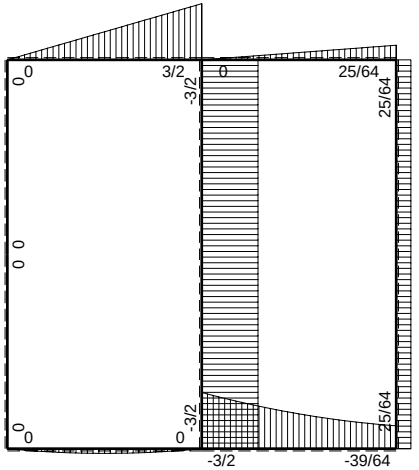
0
0
1/2
89/64
25/64
-1/2

F

F


0
0
3/2
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0
25/64
25/64
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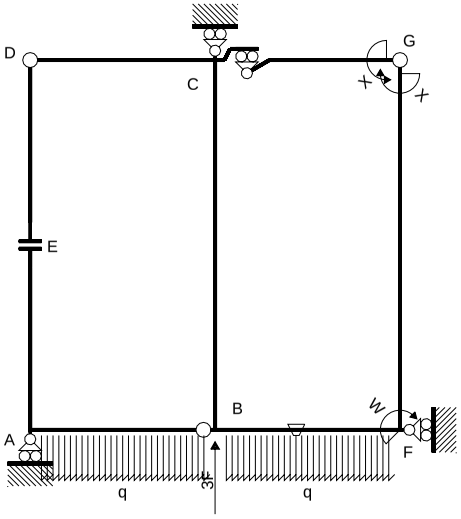
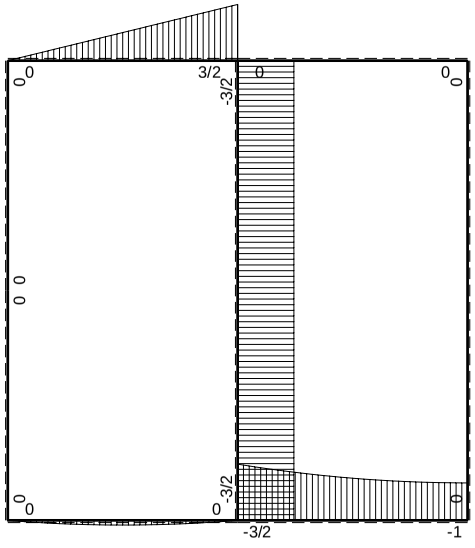
0
0
0
-3/2
25/64
-3/2
-39/64

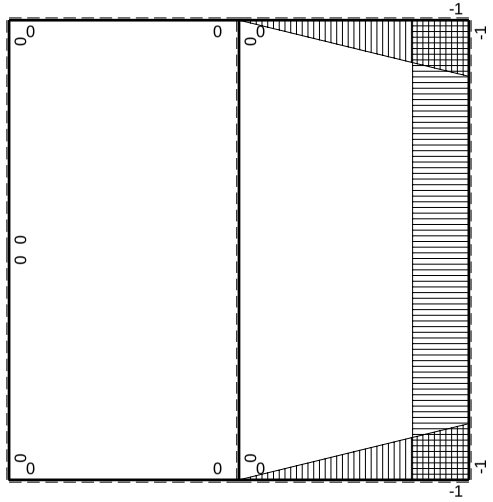
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	$M_x M_o$	$M_x\theta$	$M_x M_x$	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
BA b	0	-1/2Fx+1/2qx^2	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-3/2Fb+Fx-1/2qx^2	-Fb/EJ	3/2Fx-Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(13/24+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2qx^2	Fb/EJ	Fb-Fx+1/2Fx^2/b-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
				totali			25/24Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-25/64Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.122

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

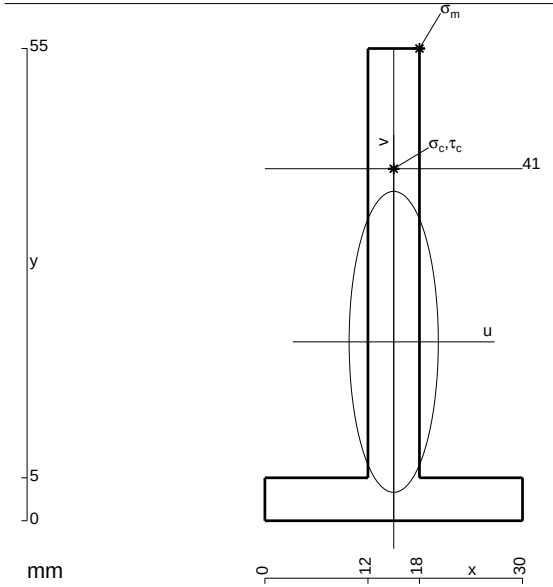
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 450. mm^2
J_u = 138437. mm^4
J_v = 12150. mm^4
y_g = 20.83 mm
T_y = -1710. N
M_x = 889200. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 34.17 mm
$\sigma_m = -Mv/J_u$ = -219.5 N/mm^2
x_c = 15. mm
y_c = 41. mm
v_c = 20.17 mm
$\sigma_c = -Mv/J_u$ = -129.5 N/mm^2
τ_c = 4.698 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129.8 N/mm^2
S^* = 2282. mm^3

ESΣ8.122

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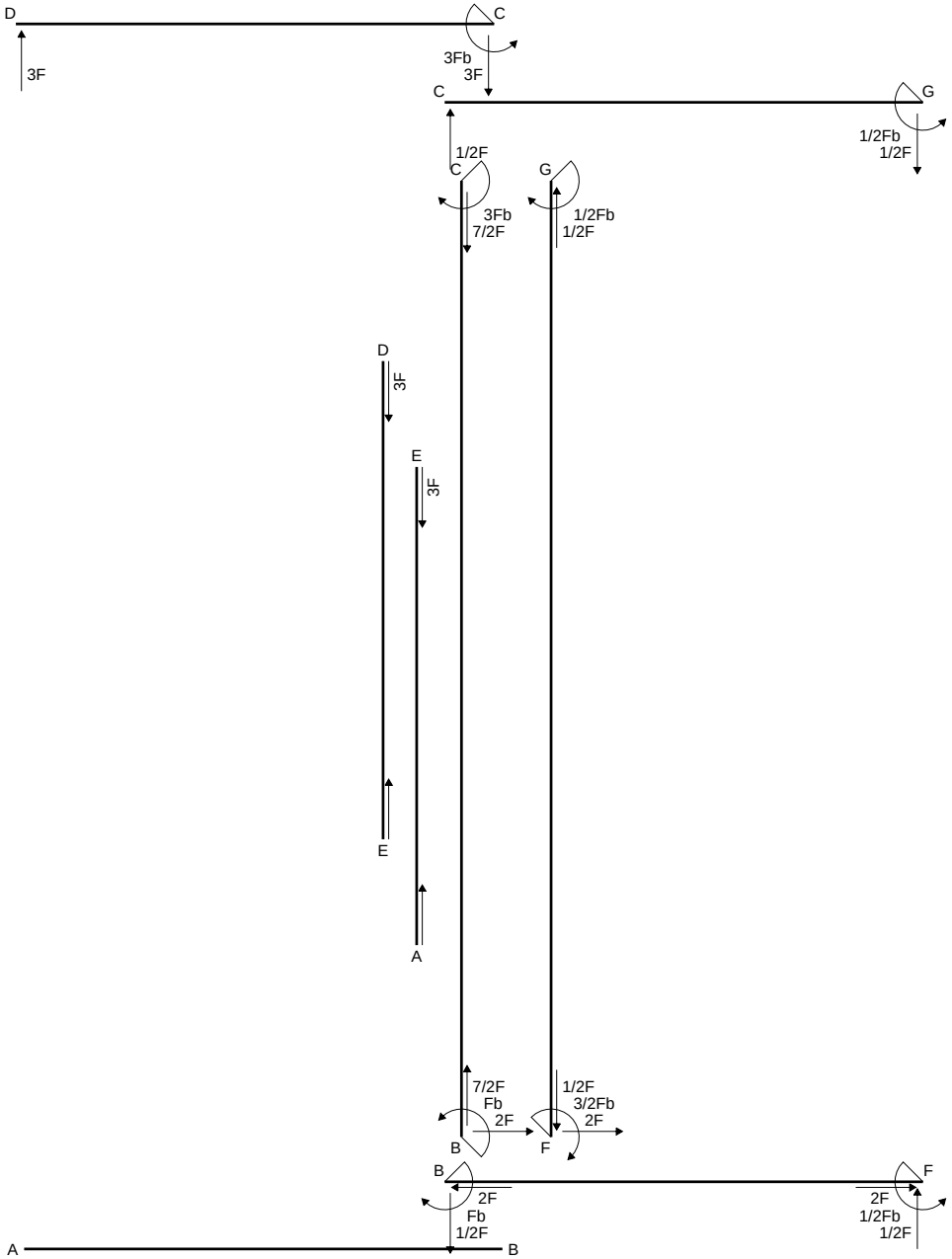
D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
7/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
7/2F
Fb
2F
1/2F
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1/2F
2F
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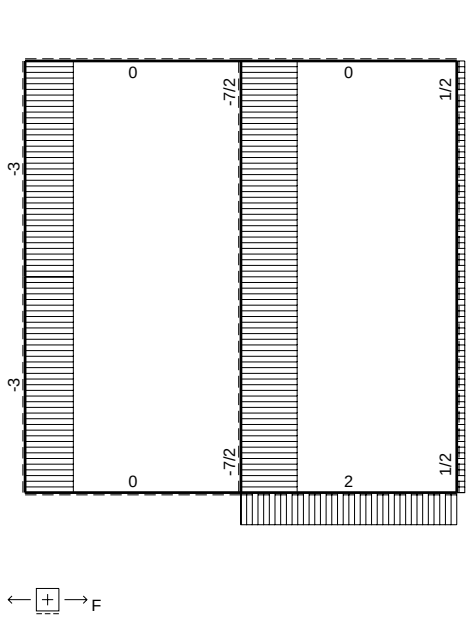
1/2F
A
B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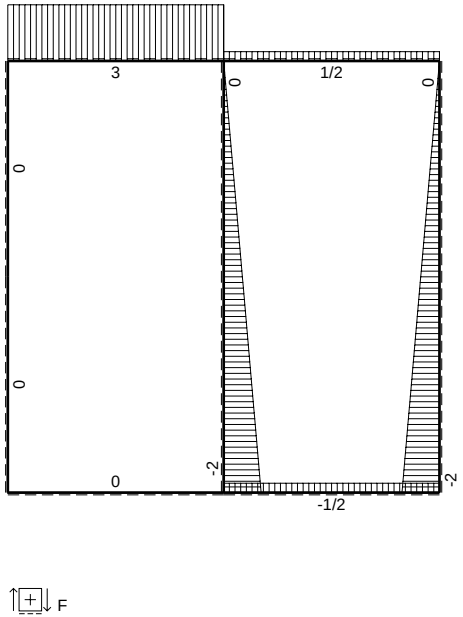
0
1/2
-3
-3
0
-7/2
2
1/2
F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0
0
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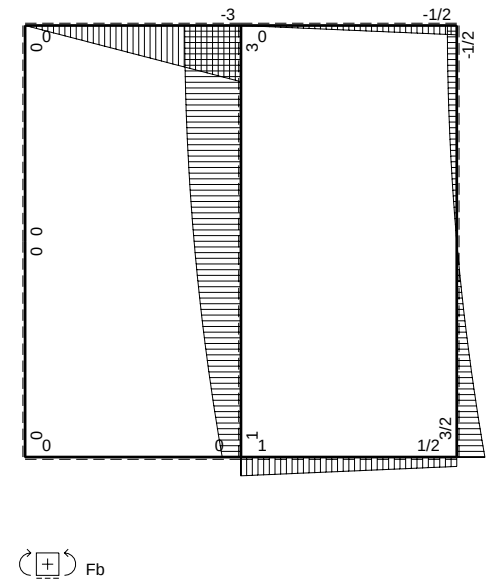
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
1
1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx²	0	-2Fb+2Fx-1/2Fx²/b	0	1	(-4/3+0)Fb²/EJ	2Xb/EJ
GF 2b	1	-1/2qx²	0	-1/2Fx²/b	0	1		
CB 2b	0	3Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx²	0	0	0	0		
totali							-4/3Fb²/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

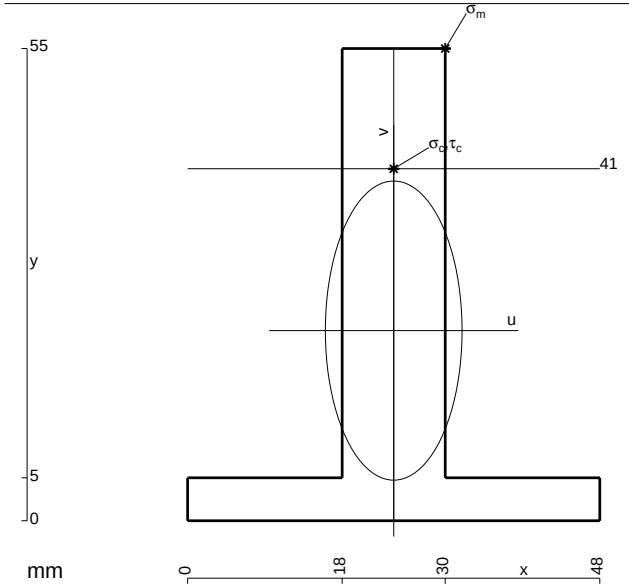
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 840. mm^2
J_u = 255143. mm^4
J_v = 53280. mm^4
y_g = 22.14 mm
T_y = 3120. N
M_x = -1778400. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 32.86 mm
$\sigma_m = -Mv/J_u$ = 229. N/mm^2
x_c = 24. mm
y_c = 41. mm
v_c = 18.86 mm
$\sigma_c = -Mv/J_u$ = 131.4 N/mm^2
τ_c = 4.427 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.7 N/mm^2
S^* = 4344. mm^3

ESΣ8.123

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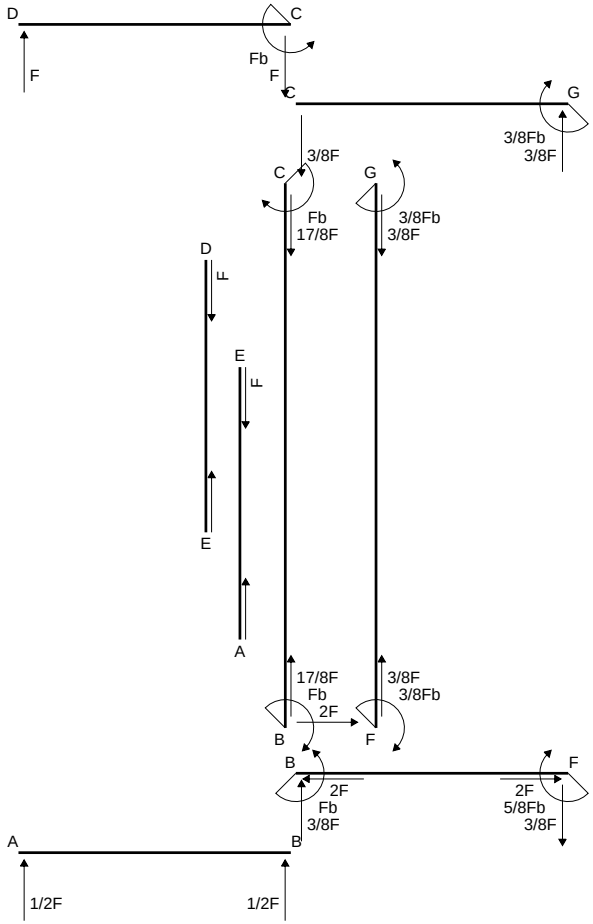
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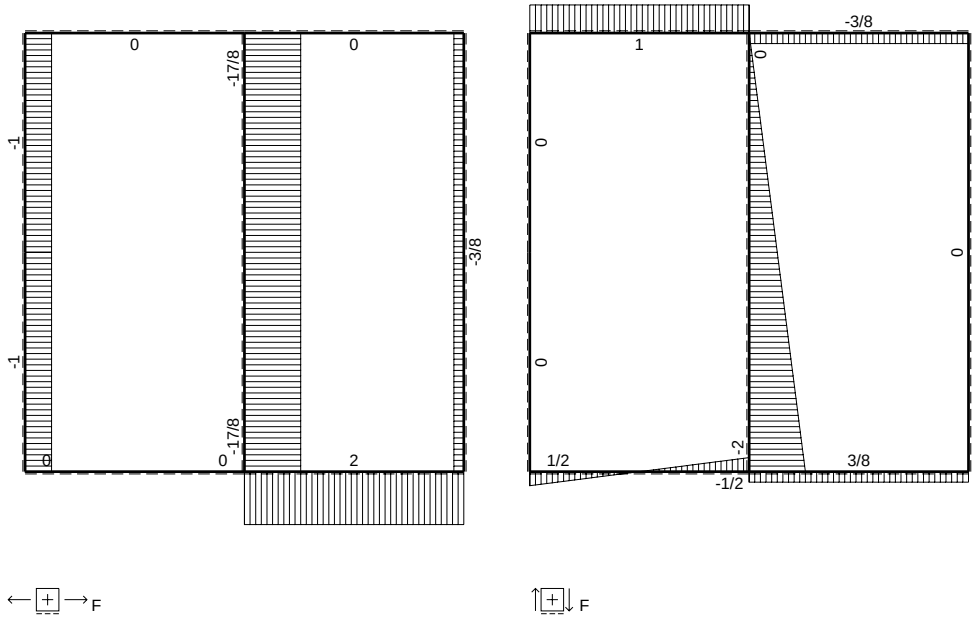
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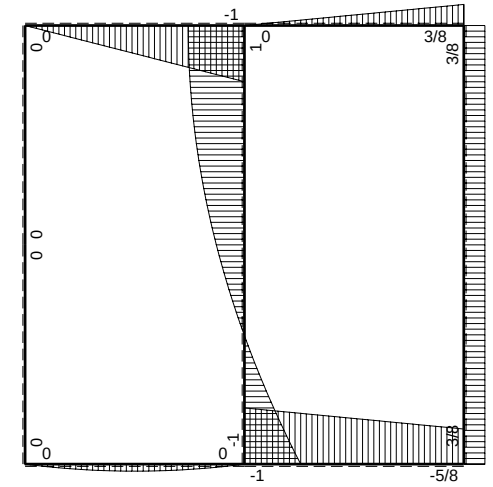
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-1
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Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$							$-3/8Fb$

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

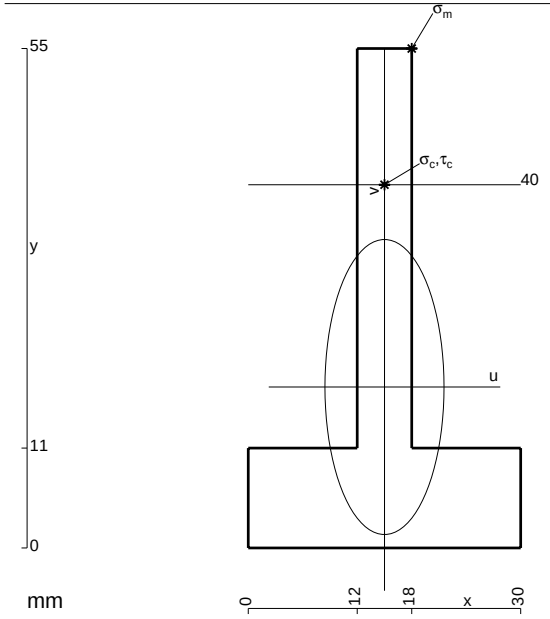
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 594. mm^2
J_u = 156836. mm^4
J_v = 25542. mm^4
y_g = 17.72 mm
N = -3591. N
T_y = -3380. N
M_x = -1030900. Nmm
x_m = 18. mm
y_m = 55. mm
u_m = 3. mm
v_m = 37.28 mm
$\sigma_m = N/A - Mv/J_u$ = 239. N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 22.28 mm
$\sigma_c = N/A - Mv/J_u$ = 140.4 N/mm^2
τ_c = 9.626 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 141.4 N/mm^2
S^* = 2680. mm^3

ESΣ8.124

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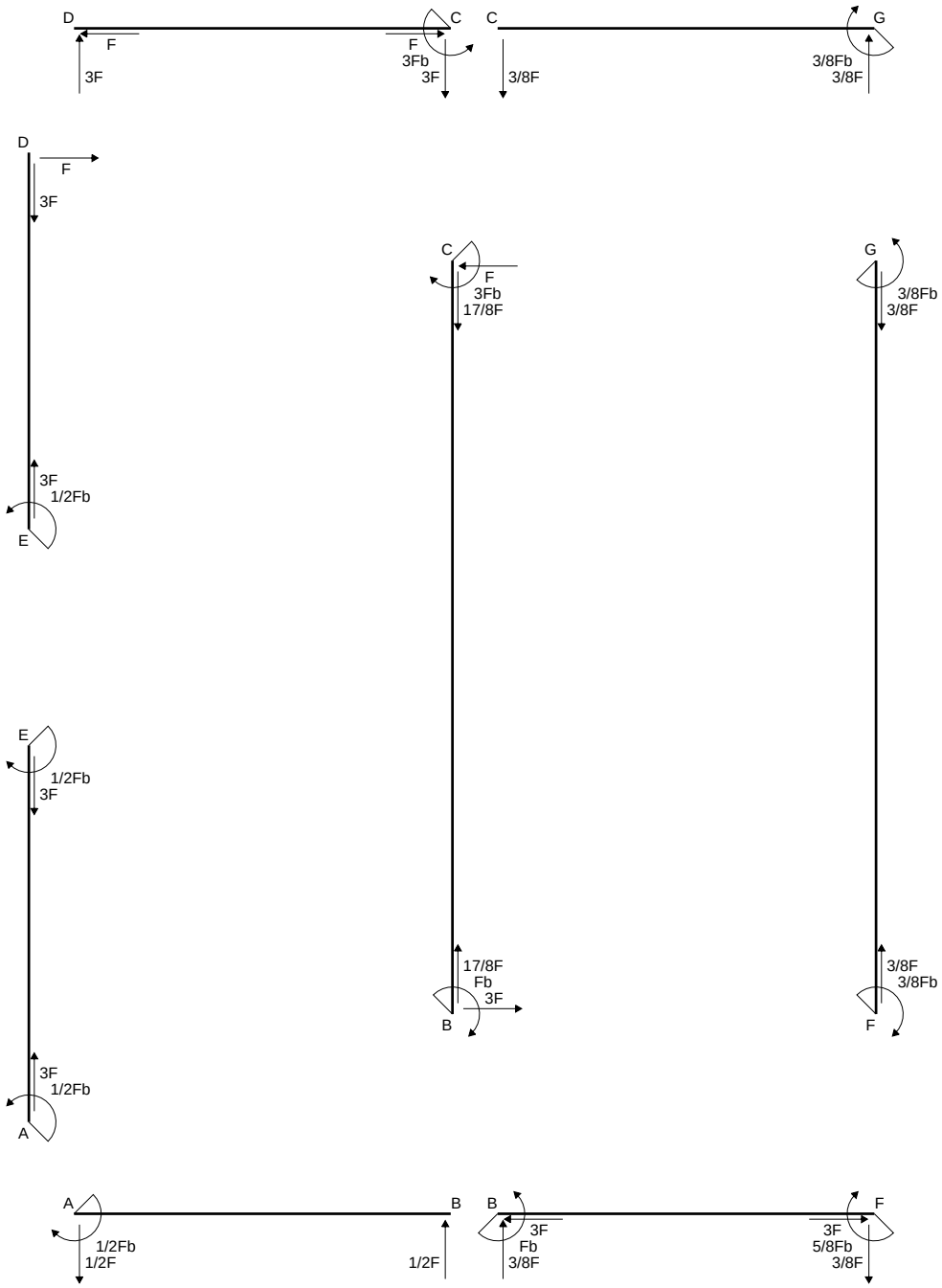
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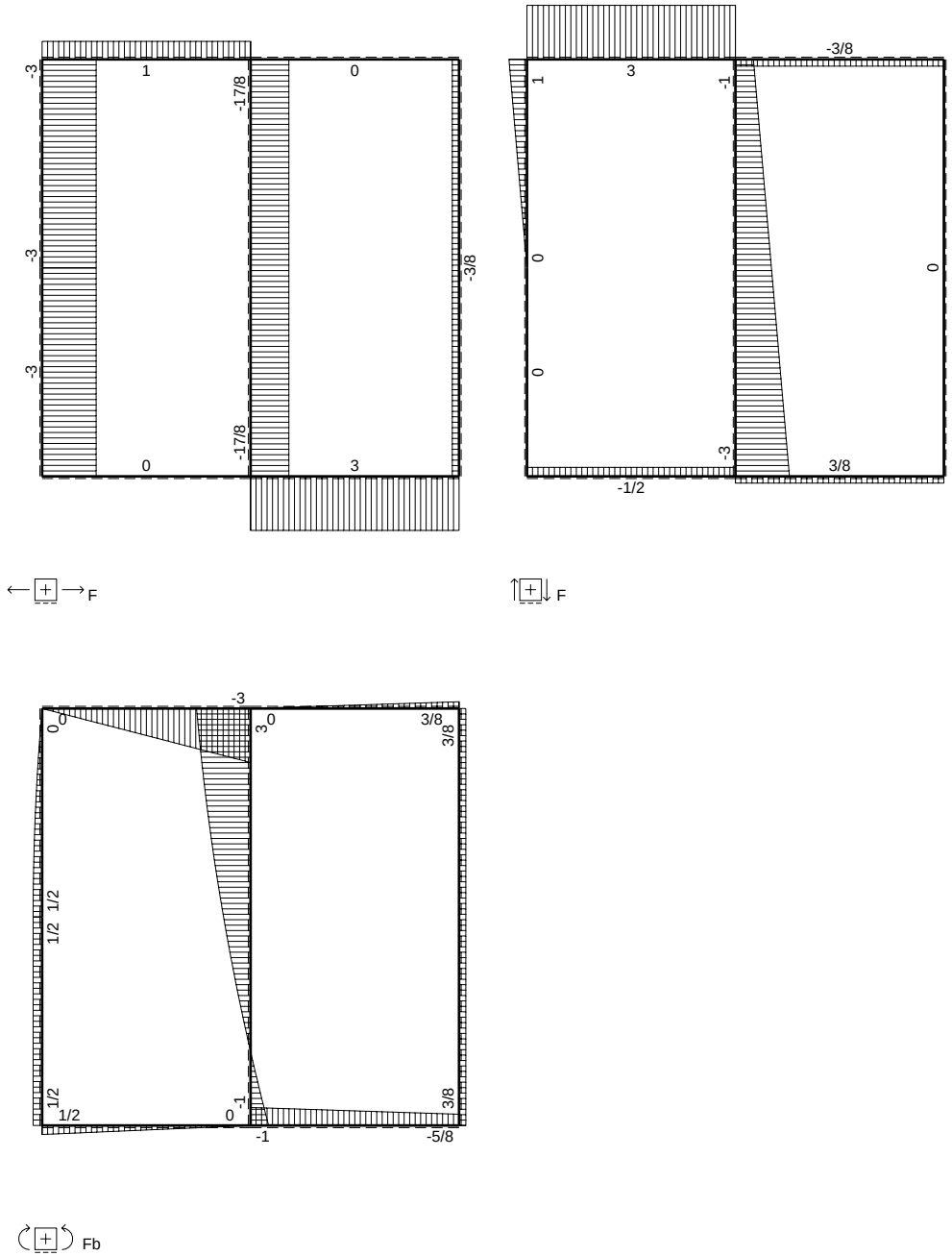
5/8Fb
3/8F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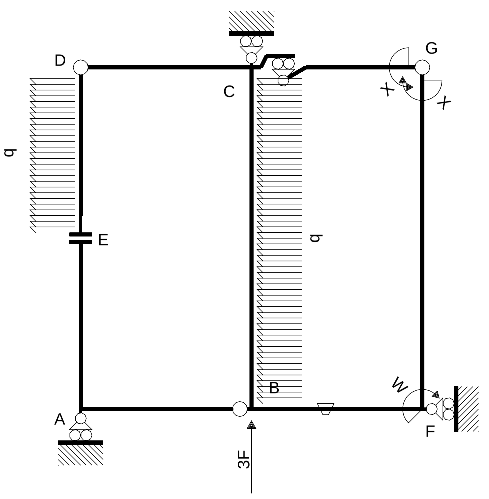
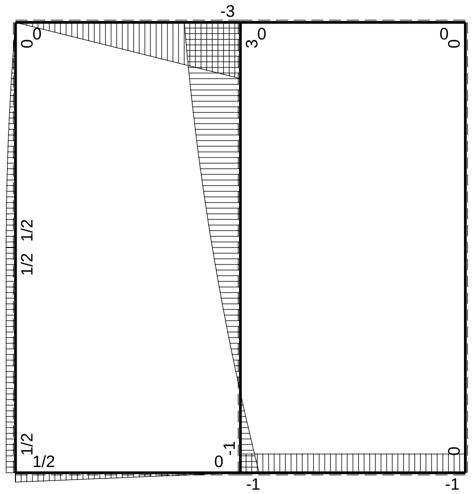
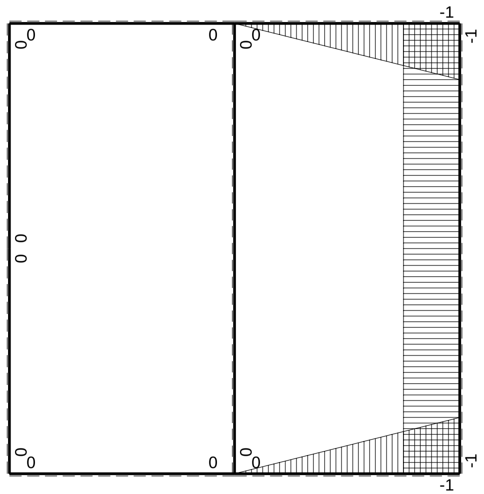
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

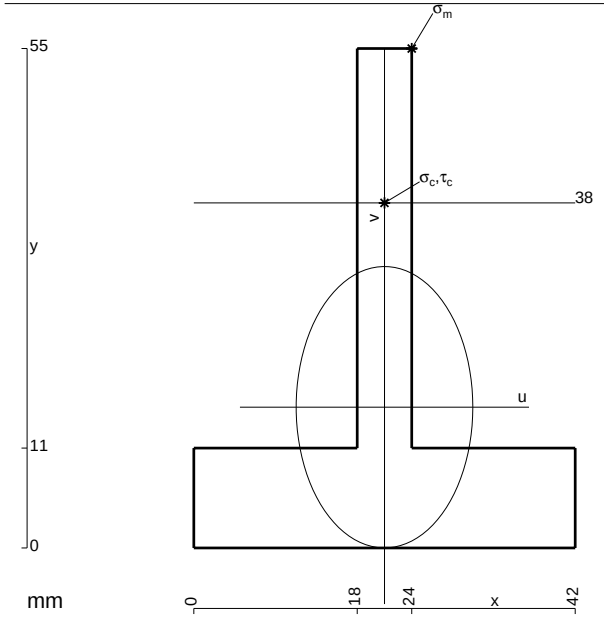
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 726. mm^2
J_u = 174301. mm^4
J_v = 68706. mm^4
y_g = 15.5 mm
N = 440. N
T_y = 1320. N
M_x = -871200. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 39.5 mm
$\sigma_m = N/A - Mv/J_u$ = 198. N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 22.5 mm
$\sigma_c = N/A - Mv/J_u$ = 113.1 N/mm^2
τ_c = 3.991 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.3 N/mm^2
S^* = 3162. mm^3

ESΣ8.125

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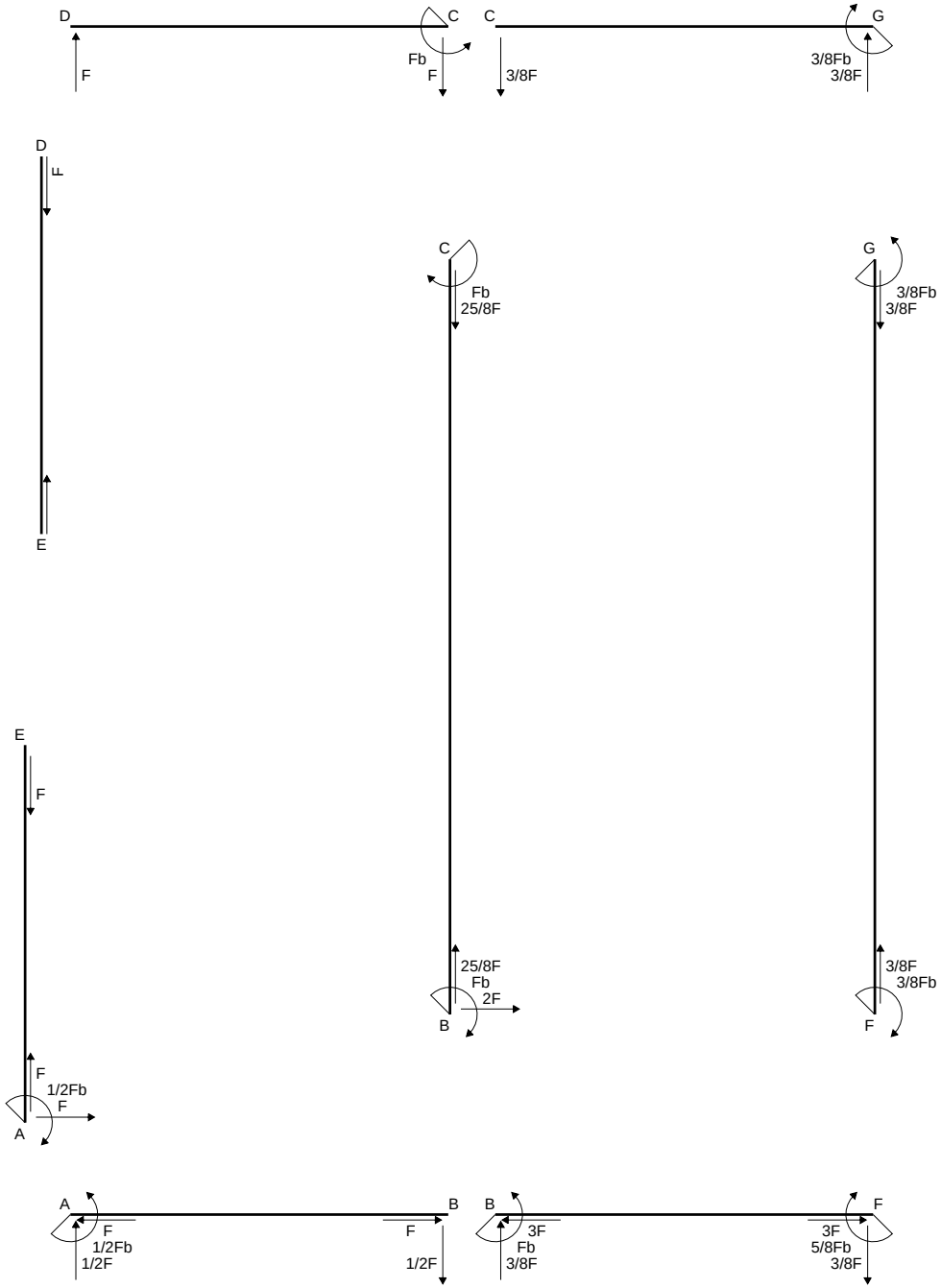
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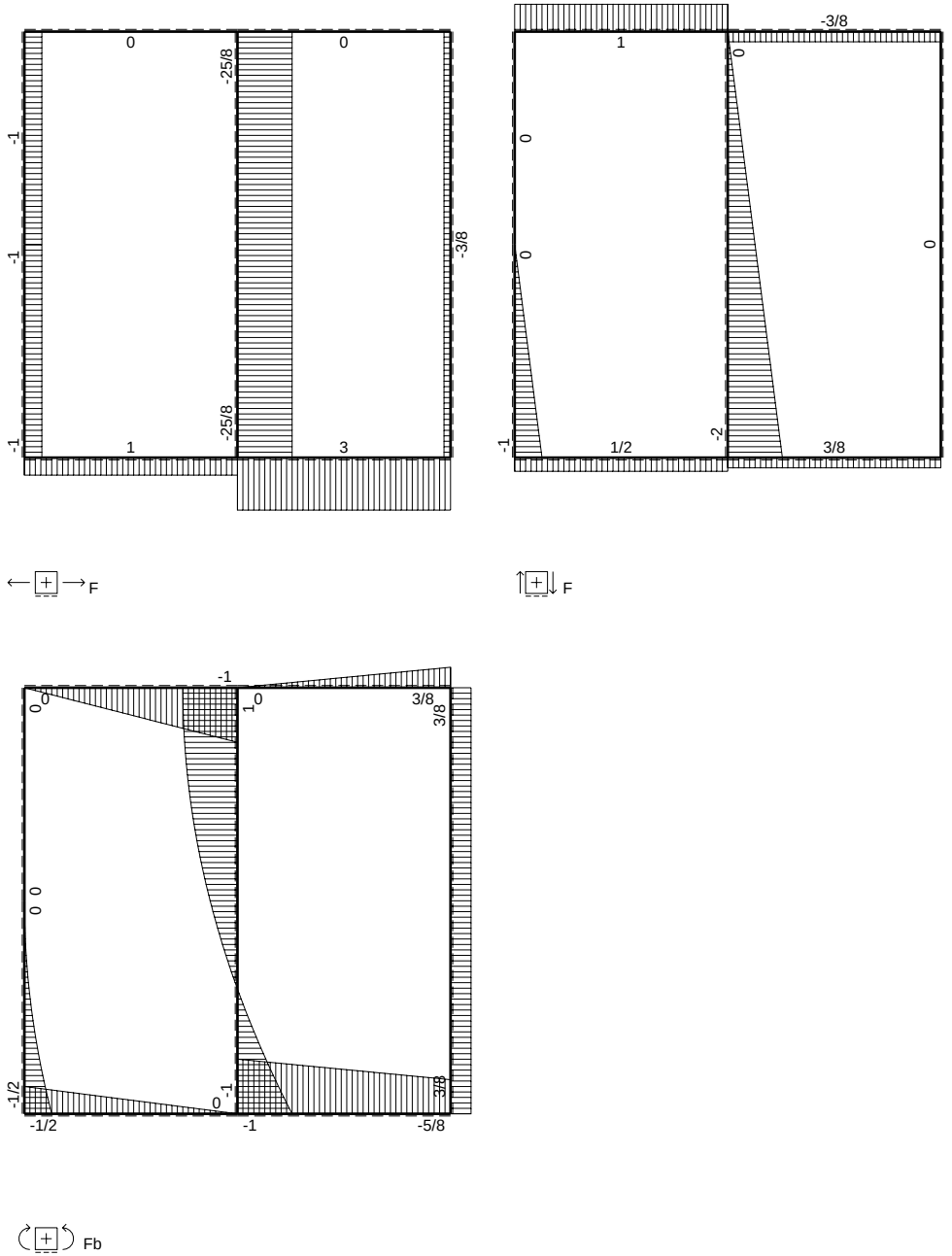
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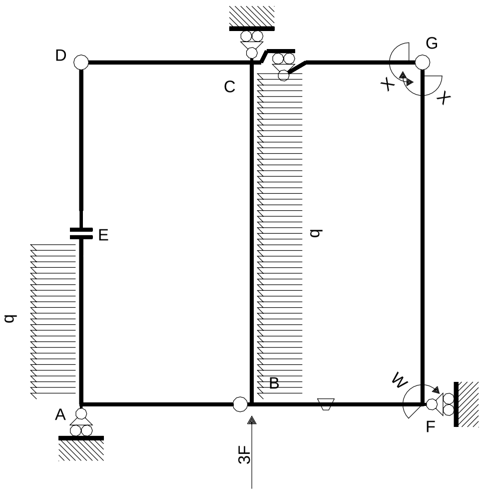
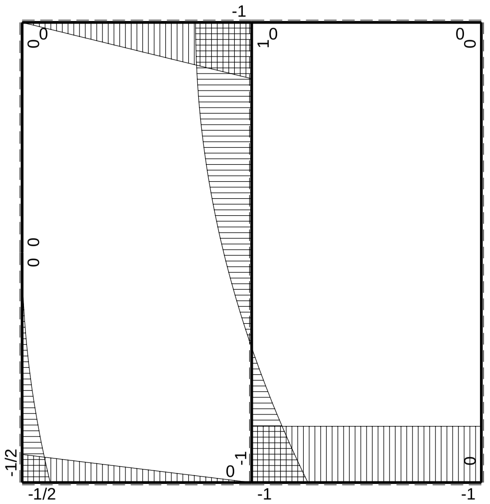
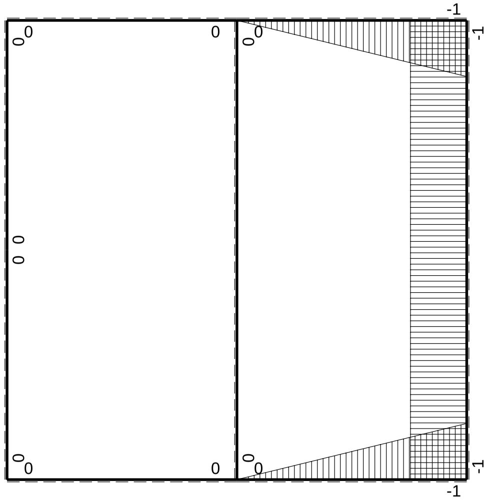


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

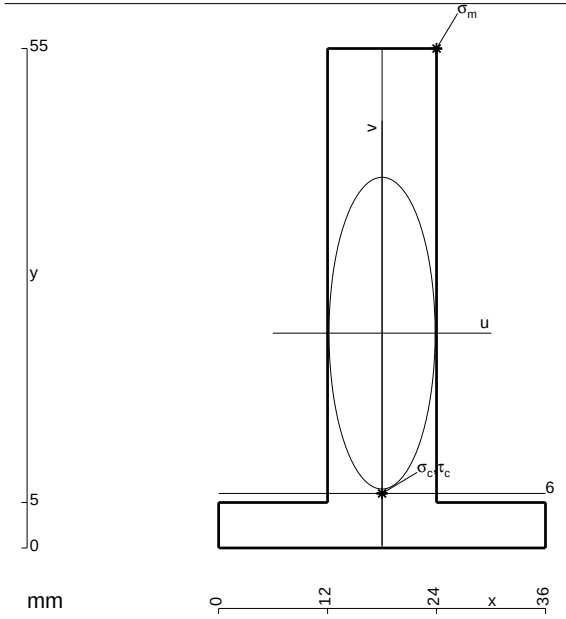
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 780. mm^2
J_u = 230087. mm^4
J_v = 26640. mm^4
y_g = 23.65 mm
N = -7156. N
T_y = -4580. N
M_x = -1603000. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 6. mm
v_m = 31.35 mm
$\sigma_m = N/A - Mv/J_u$ = 209.2 N/mm^2
x_c = 18. mm
y_c = 6. mm
v_c = -17.65 mm
$\sigma_c = N/A - Mv/J_u$ = -132.2 N/mm^2
τ_c = 6.678 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.7 N/mm^2
S^* = 4026. mm^3

ESΣ8.126

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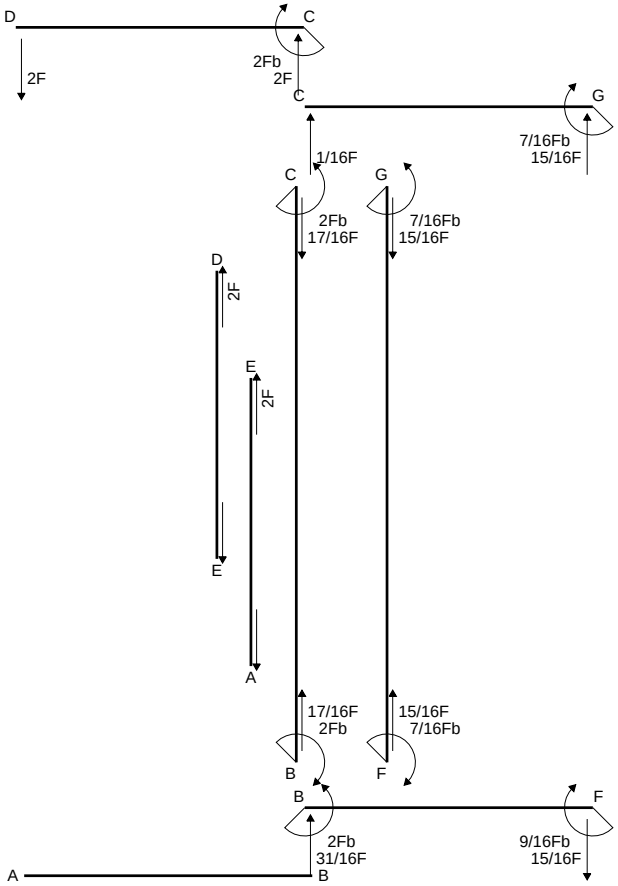
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2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
17/16F
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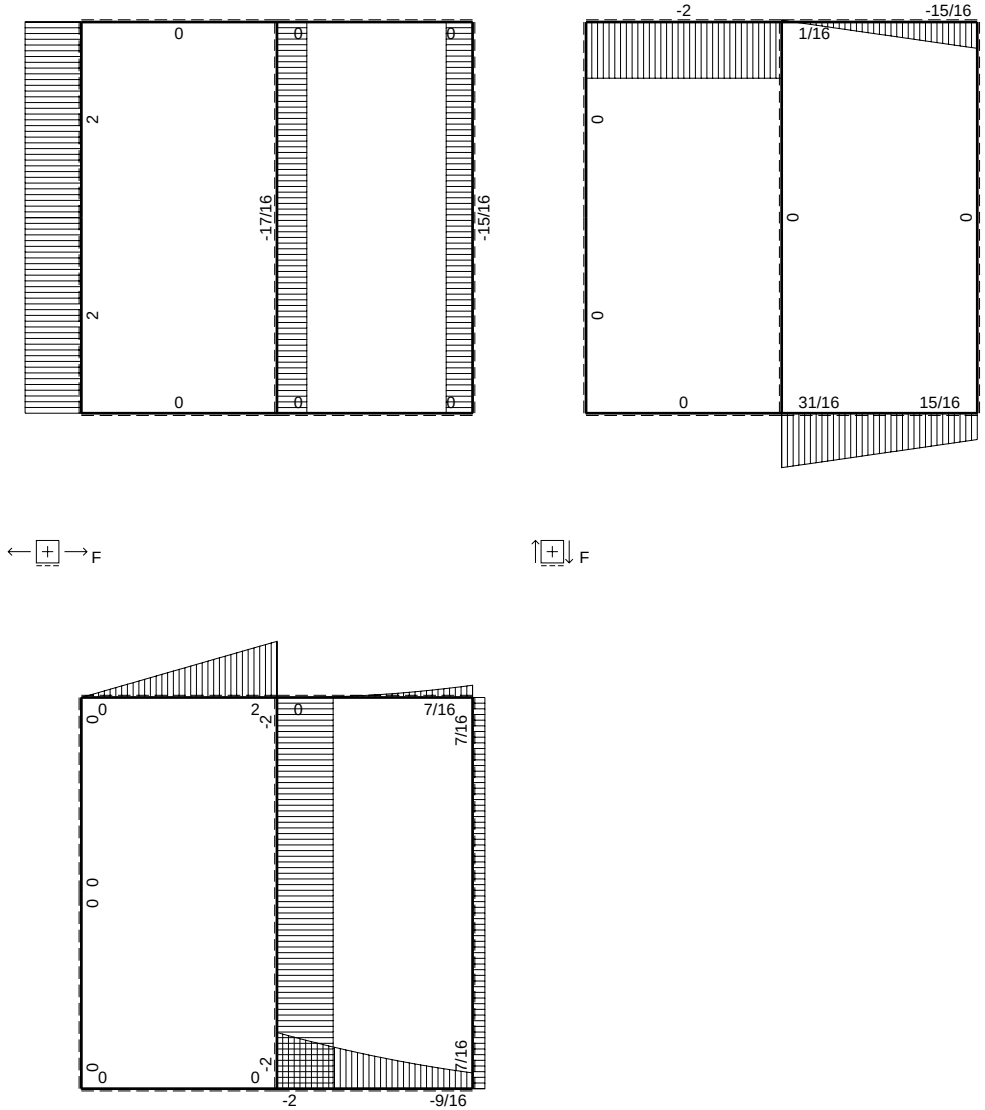
15/16F
D
2F
E
2F
E
A
17/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B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2
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0
31/16
15/16
F
F
0
2
-2
0
7/16
7/16
0
0
0
0
-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						7/6Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

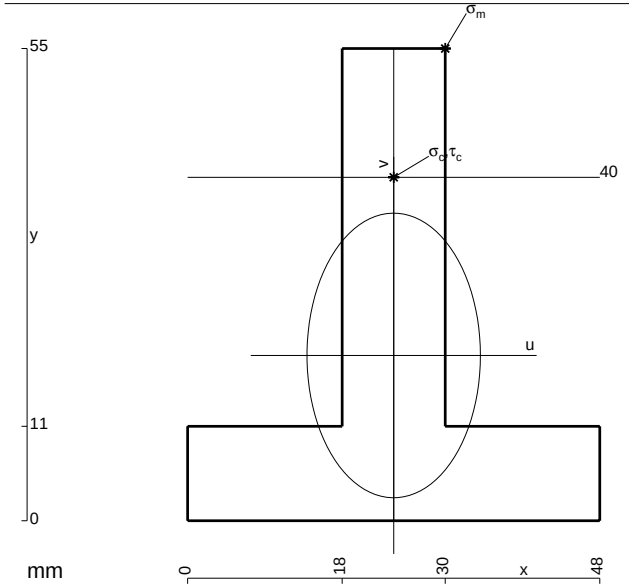
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 1056. mm^2
J_u = 290158. mm^4
J_v = 107712. mm^4
y_g = 19.25 mm
T_y = -2400. N
M_x = 1776000. Nmm
x_m = 30. mm
y_m = 55. mm
u_m = 6. mm
v_m = 35.75 mm
$\sigma_m = -Mv/J_u$ = -218.8 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 20.75 mm
$\sigma_c = -Mv/J_u$ = -127. N/mm^2
τ_c = 3.505 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.2 N/mm^2
S^* = 5085. mm^3

ESΣ8.127

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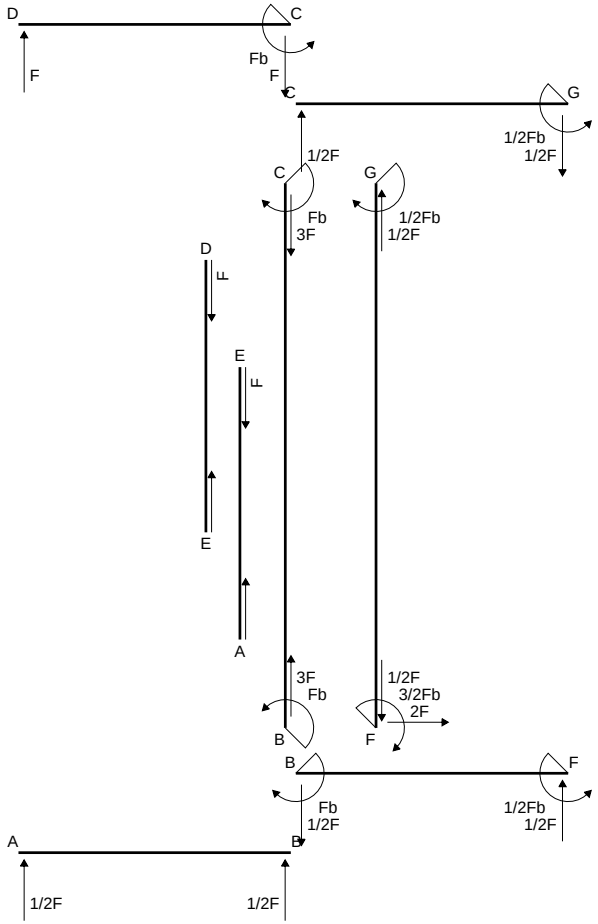
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C
G
1/2Fb
1/2F
1/2F
C
G
Fb
3F
1/2Fb
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E
F
E
A
3F
Fb
1/2F
3/2Fb
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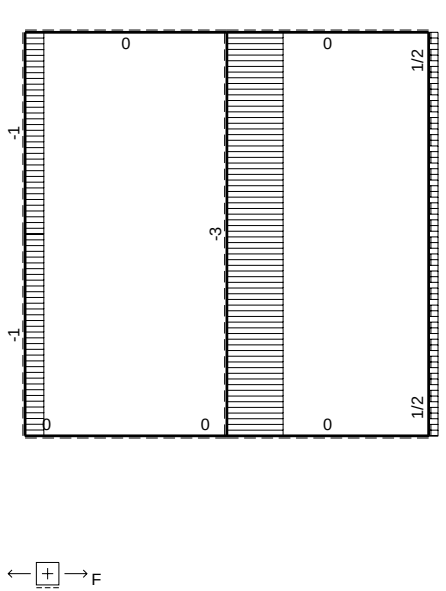
A
B
1/2F
1/2F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-3
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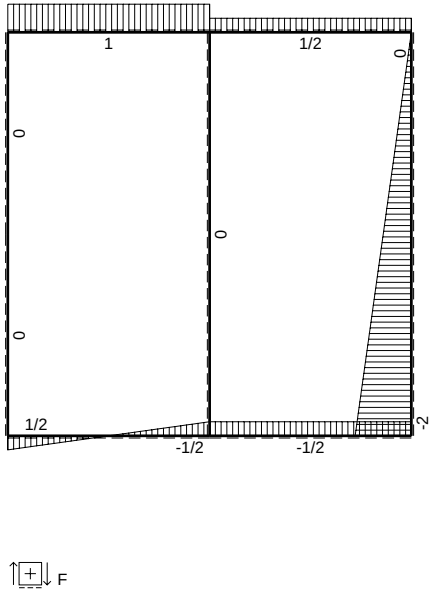
0
1/2
F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1/2
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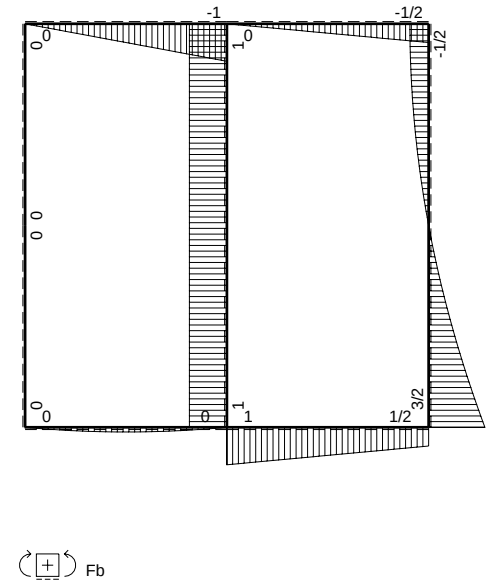
1/2
-1/2
-1/2
-2
F


-1
-1/2
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-1/2
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1
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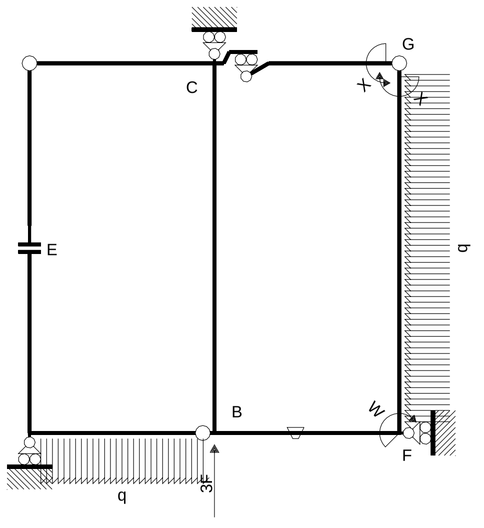
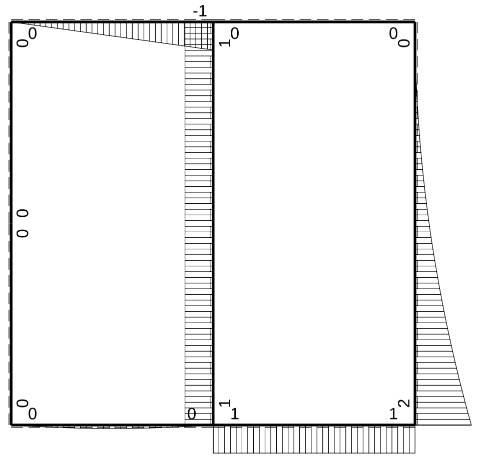
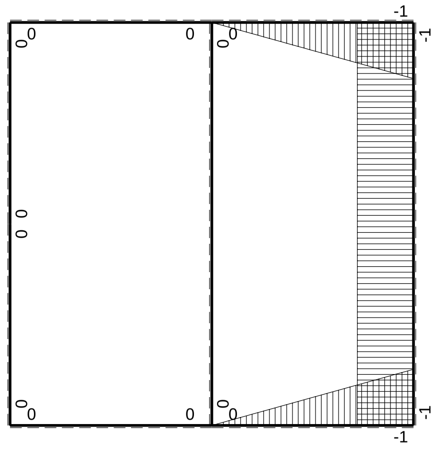
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

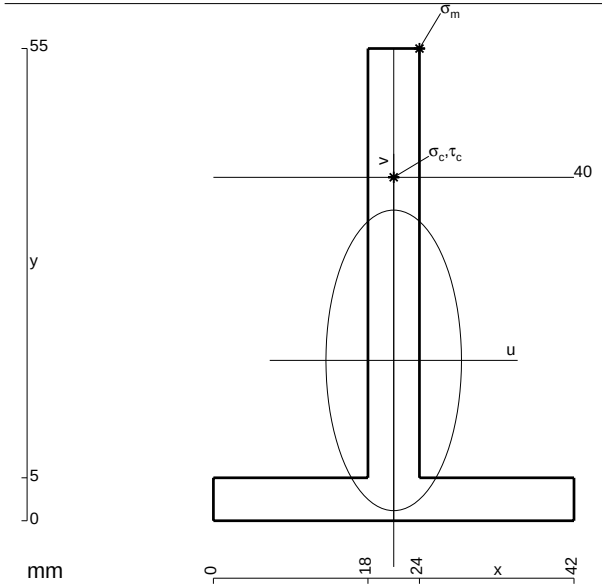
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 510. mm^2
J_u = 156357. mm^4
J_v = 31770. mm^4
y_g = 18.68 mm
T_y = 1250. N
M_x = -987500. Nmm
x_m = 24. mm
y_m = 55. mm
u_m = 3. mm
v_m = 36.32 mm
$\sigma_m = -Mv/J_u$ = 229.4 N/mm^2
x_c = 21. mm
y_c = 40. mm
v_c = 21.32 mm
$\sigma_c = -Mv/J_u$ = 134.7 N/mm^2
τ_c = 3.456 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 134.8 N/mm^2
S^* = 2594. mm^3

ESΣ8.128

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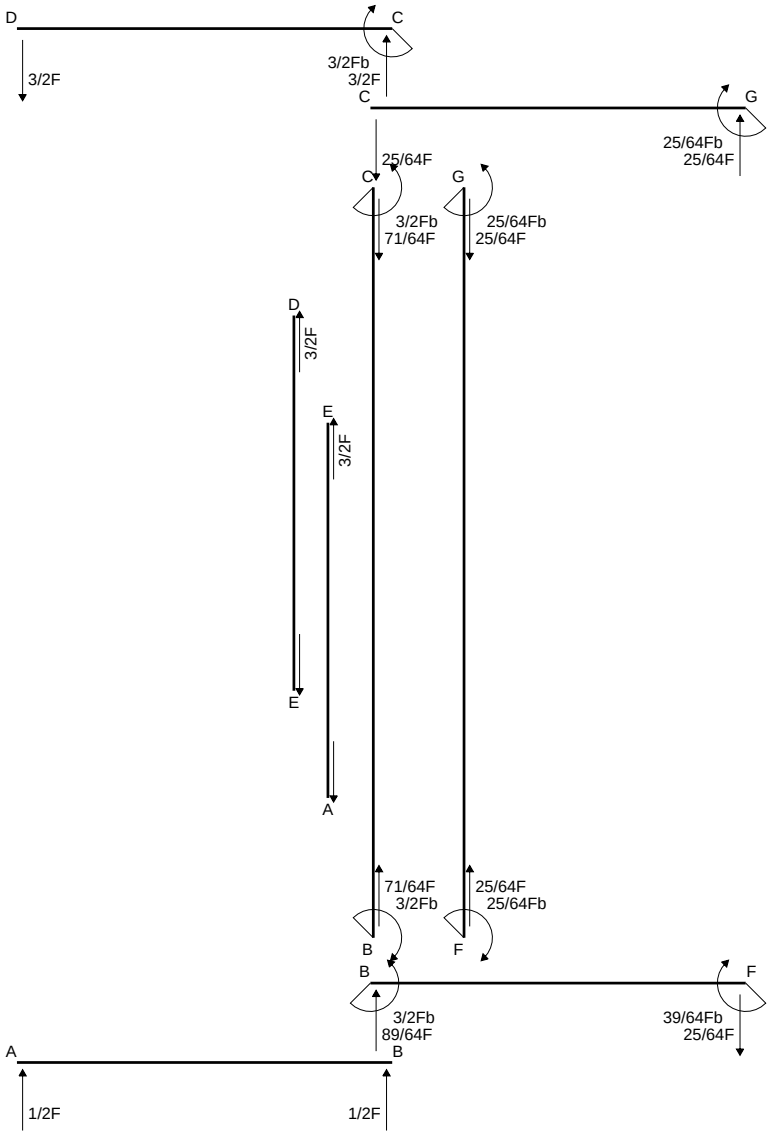
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3/2Fb
3/2F
C
G
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25/64Fb
25/64F
C
G
3/2Fb
71/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
A
71/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
3/2Fb
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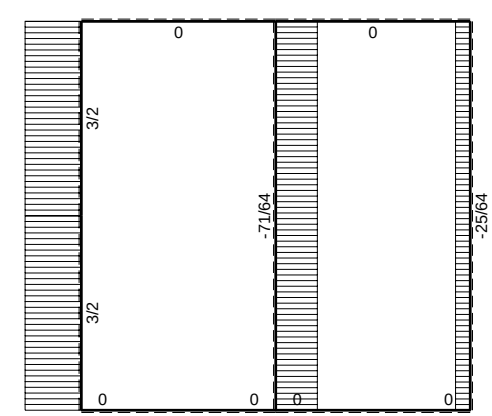
39/64Fb
25/64F
B
A
1/2F
1/2F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3/2
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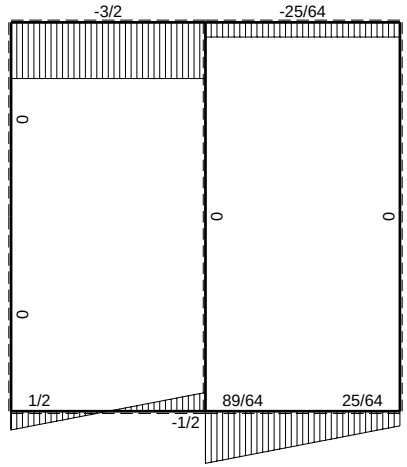
0

F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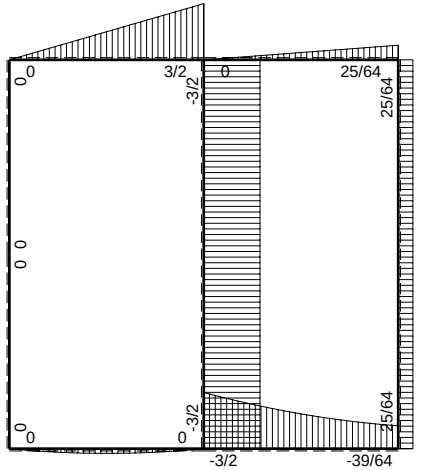
1/2
89/64
25/64
-1/2

F


0
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3/2
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0
25/64
25/64
0
0
0
0
-3/2
25/64
-3/2
-39/64

Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

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14.11.23

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0
DC b	0	$-3/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$25/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-25/64Fb	

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14.11.23

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

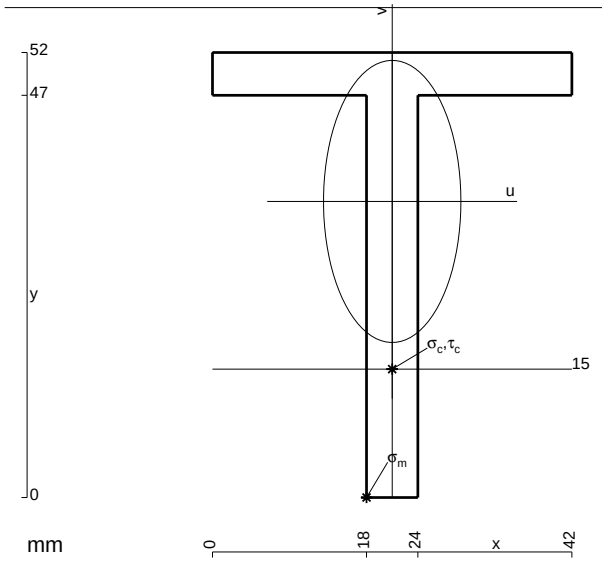
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 492. mm^2
J_u = 133716. mm^4
J_v = 31716. mm^4
y_g = 34.6 mm
T_y = -1200. N
M_x = 924000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -34.6 mm
$\sigma_m = -Mv/J_u$ = 239.1 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -19.6 mm
$\sigma_c = -Mv/J_u$ = 135.4 N/mm^2
τ_c = 3.648 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.6 N/mm^2
S^* = 2439. mm^3

ESΣ8.129

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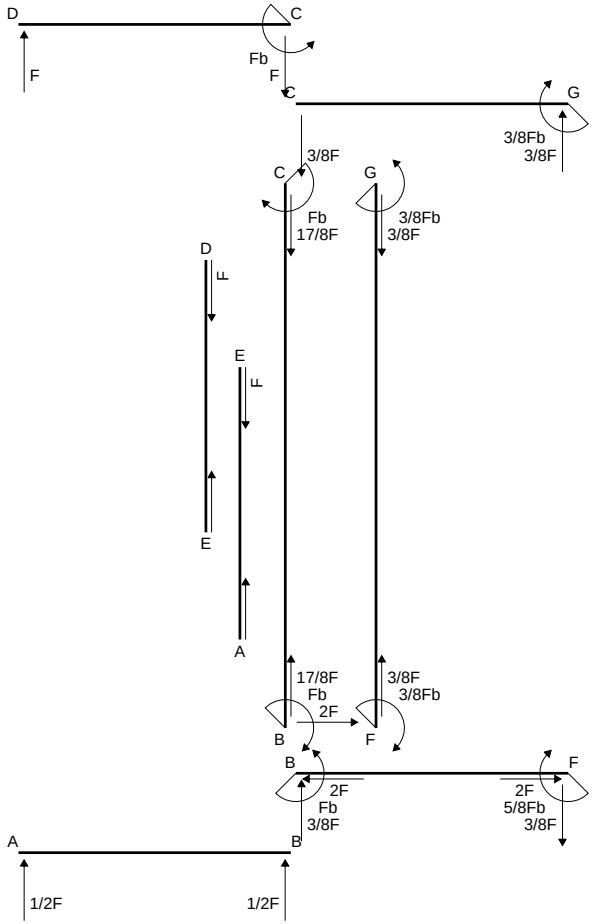
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Fb
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C
G
3/8Fb
3/8F
3/8F
C
G
Fb
17/8F
3/8Fb
3/8F
D
F
E
F
E
A
17/8F
Fb
2F
3/8F
3/8Fb
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Fb
3/8F
2F
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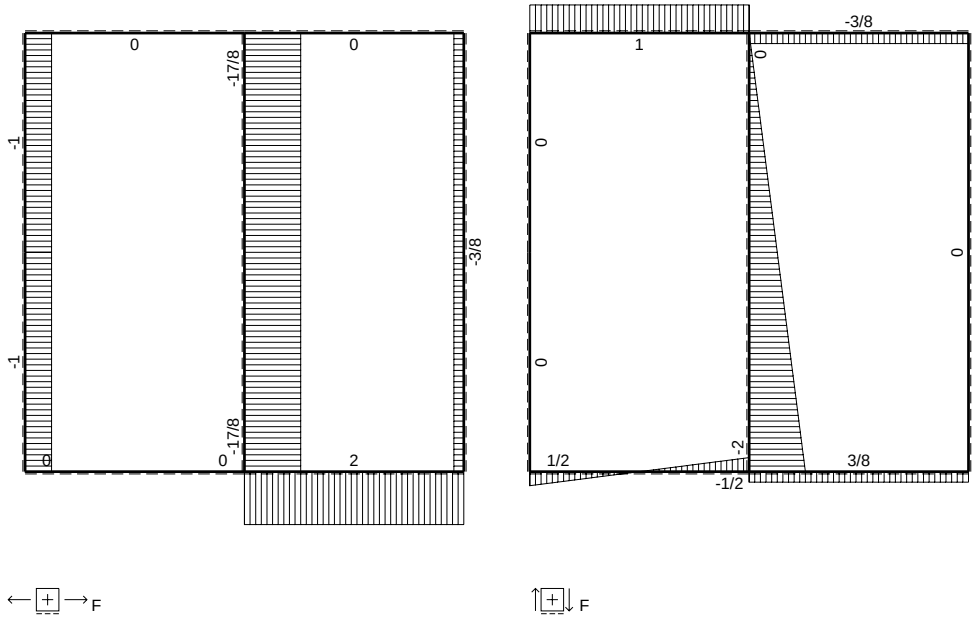
A
B
1/2F
1/2F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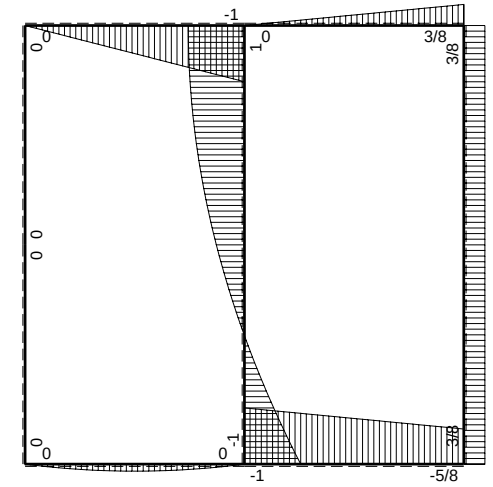
3/8
-1/2
F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3/8
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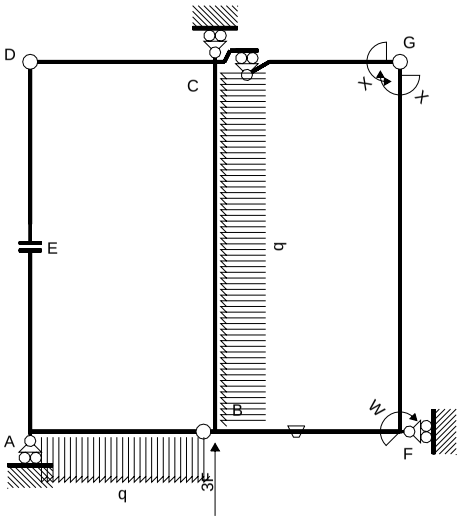
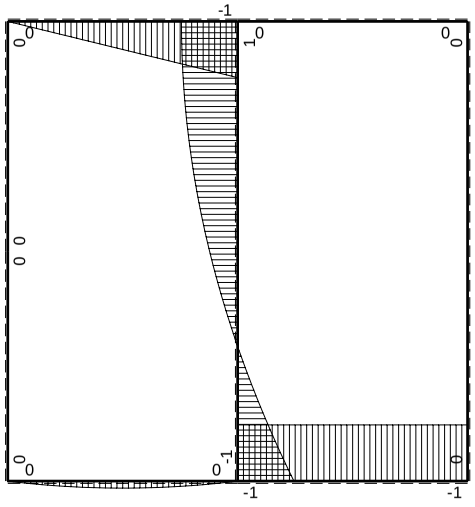
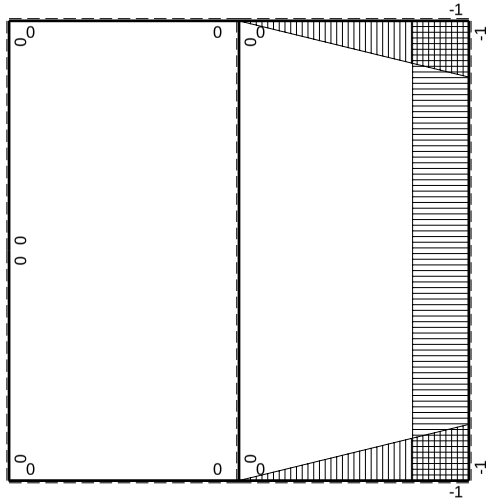
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.130

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

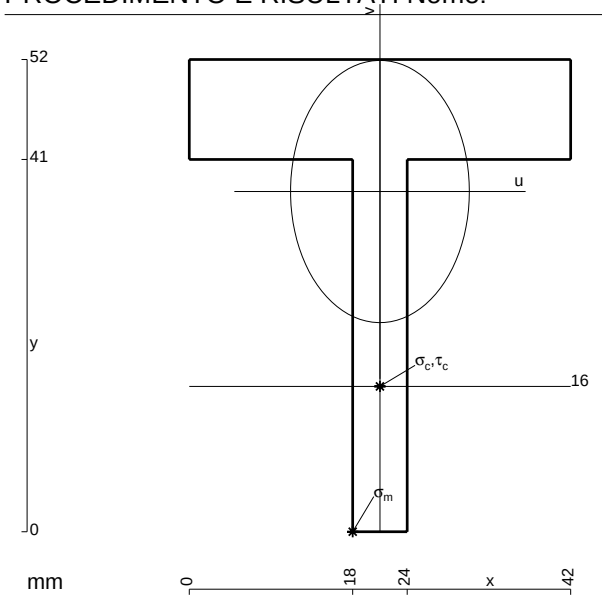
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 708. mm^2
J_u = 147634. mm^4
J_v = 68652. mm^4
y_g = 37.47 mm
N = -3953. N
T_y = -3720. N
M_x = -762600. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.47 mm
$\sigma_m = N/A - Mv/J_u$ = -199.1 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.47 mm
$\sigma_c = N/A - Mv/J_u$ = -116.5 N/mm^2
τ_c = 11.88 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.3 N/mm^2
S^* = 2829. mm^3

ESΣ8.130

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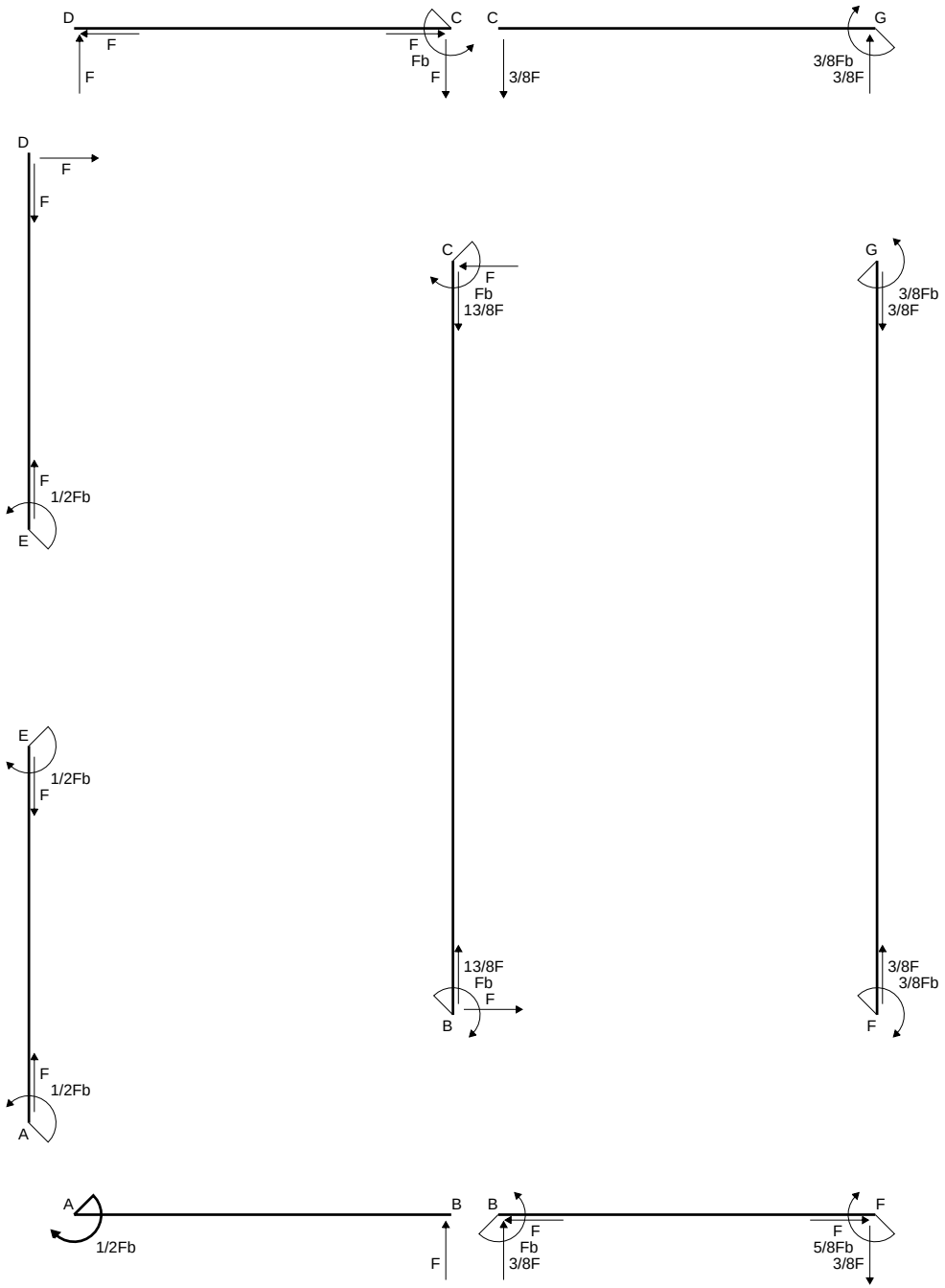
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C
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3/8F
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3/8F
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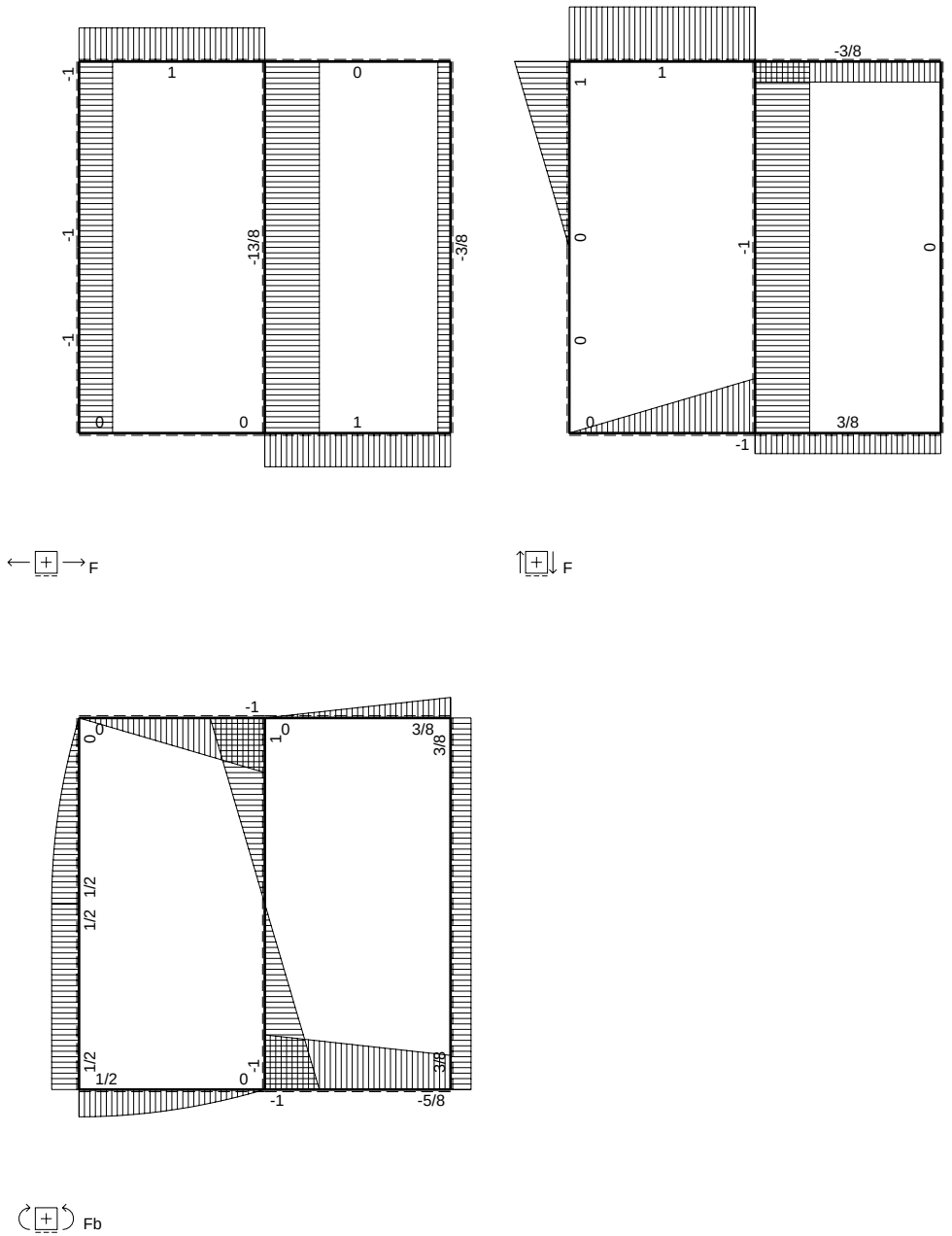
5/8Fb
3/8F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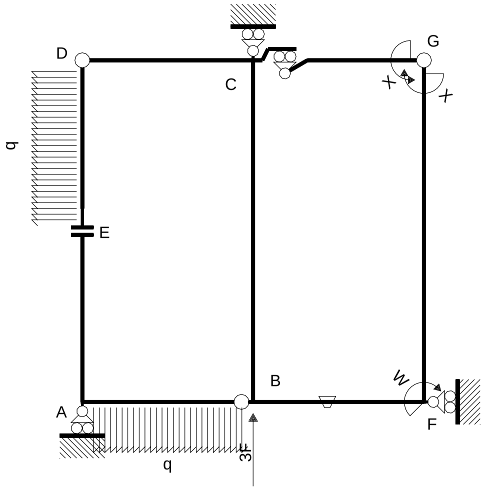
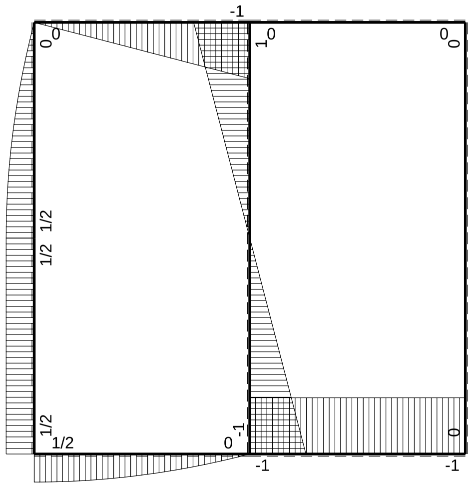
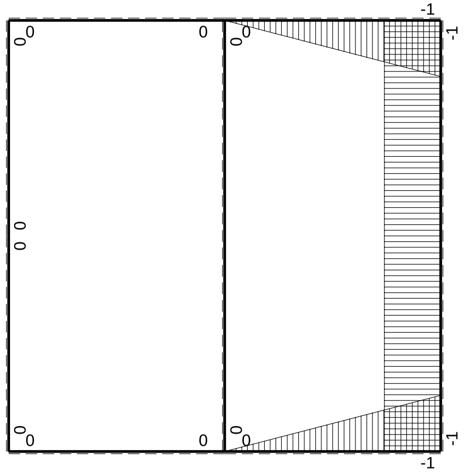
Fb



Schema di calcolo iperstatico



M$_o$ flessione da carichi assegnati



M$_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W$_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W$_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

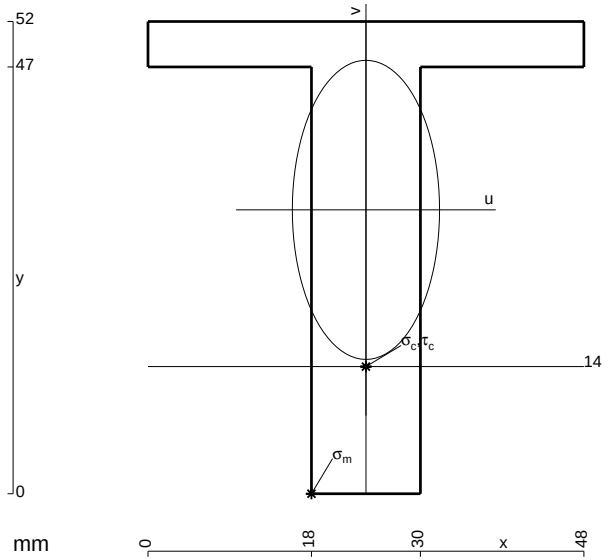
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 804. mm^2
J_u = 218133. mm^4
J_v = 52848. mm^4
y_g = 31.26 mm
N = -5119. N
T_y = -3150. N
M_x = -1417500. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -31.26 mm
$\sigma_m = N/A - Mv/J_u$ = -209.5 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -17.26 mm
$\sigma_c = N/A - Mv/J_u$ = -118.5 N/mm^2
τ_c = 4.905 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.8 N/mm^2
S^* = 4076. mm^3

ESΣ8.131

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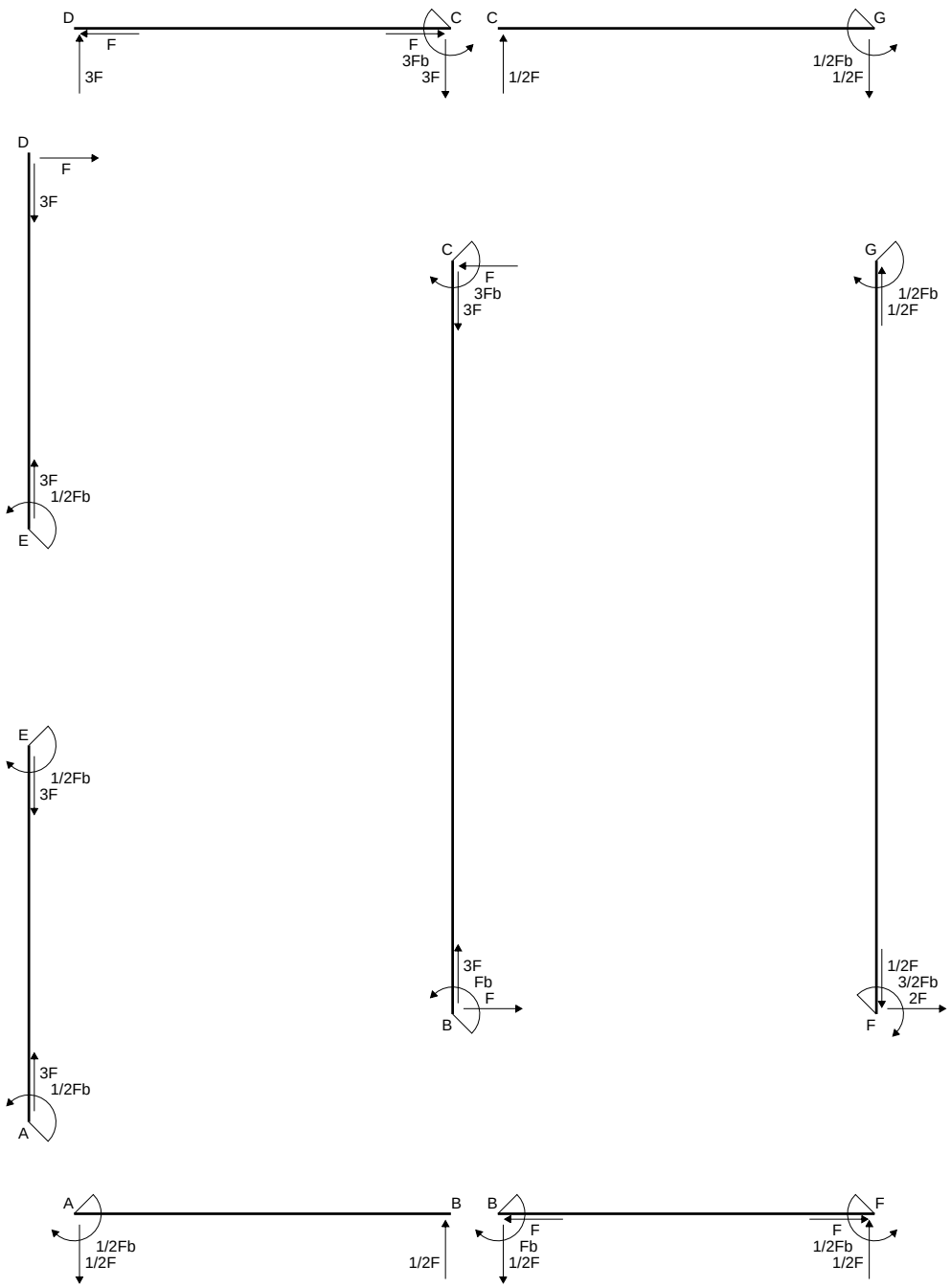
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3F
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A
A
B
B
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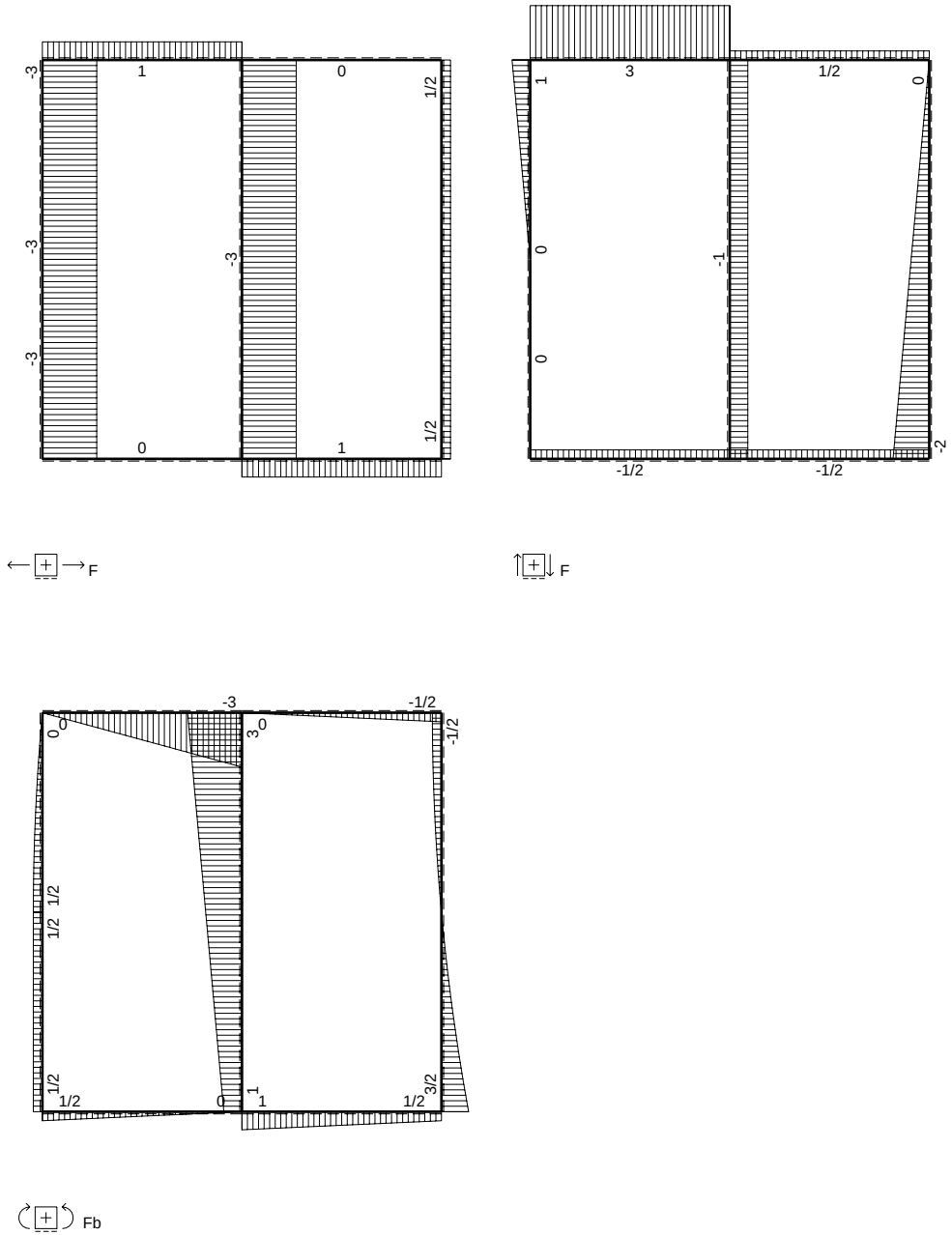
F
Fb
1/2F
F
1/2Fb
1/2F


-3
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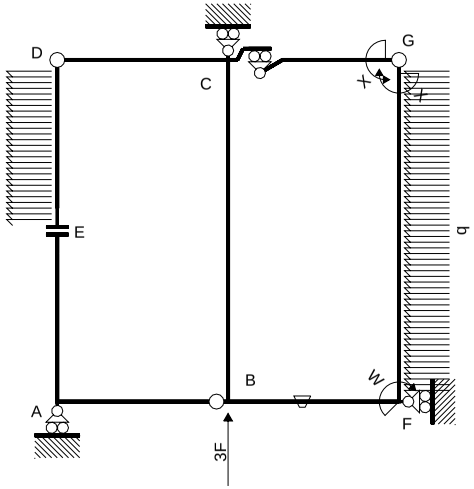
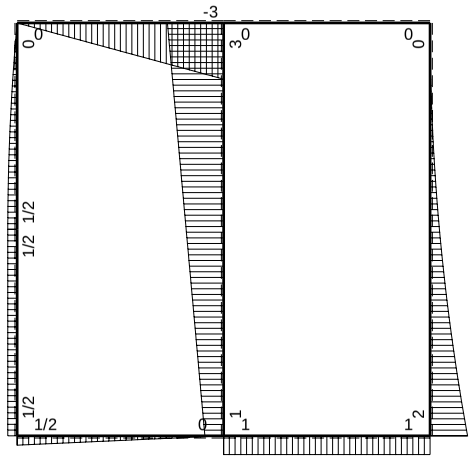
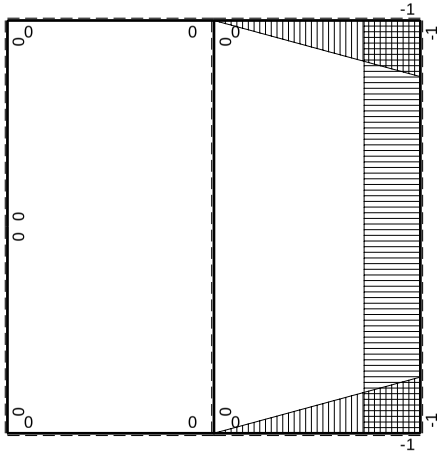
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
		totali					-4/3Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.132

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L_{FB}^{Xo} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

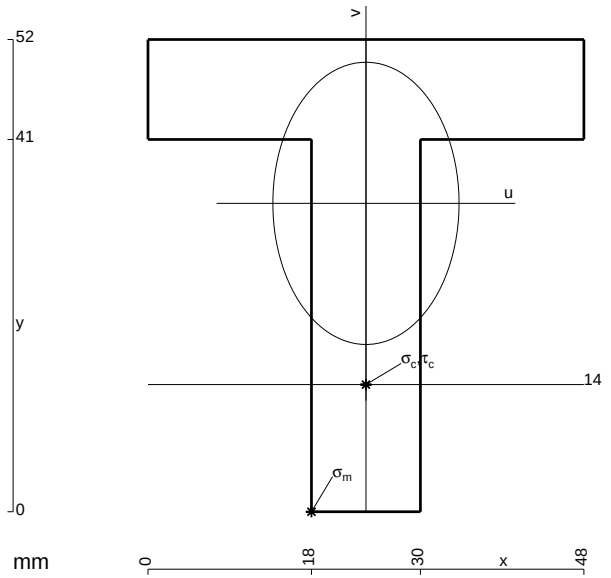
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L_{FG}^{Xo} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L_{GF}^{Xo} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1020. mm^2
J_u = 246410. mm^4
J_v = 107280. mm^4
y_g = 33.96 mm
N = 1090. N
T_y = 3270. N
M_x = -1602300. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.96 mm
$\sigma_m = N/A - Mv/J_u$ = -219.8 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.96 mm
$\sigma_c = N/A - Mv/J_u$ = -128.7 N/mm^2
τ_c = 5.009 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 129. N/mm^2
S^* = 4529. mm^3

ESΣ8.132

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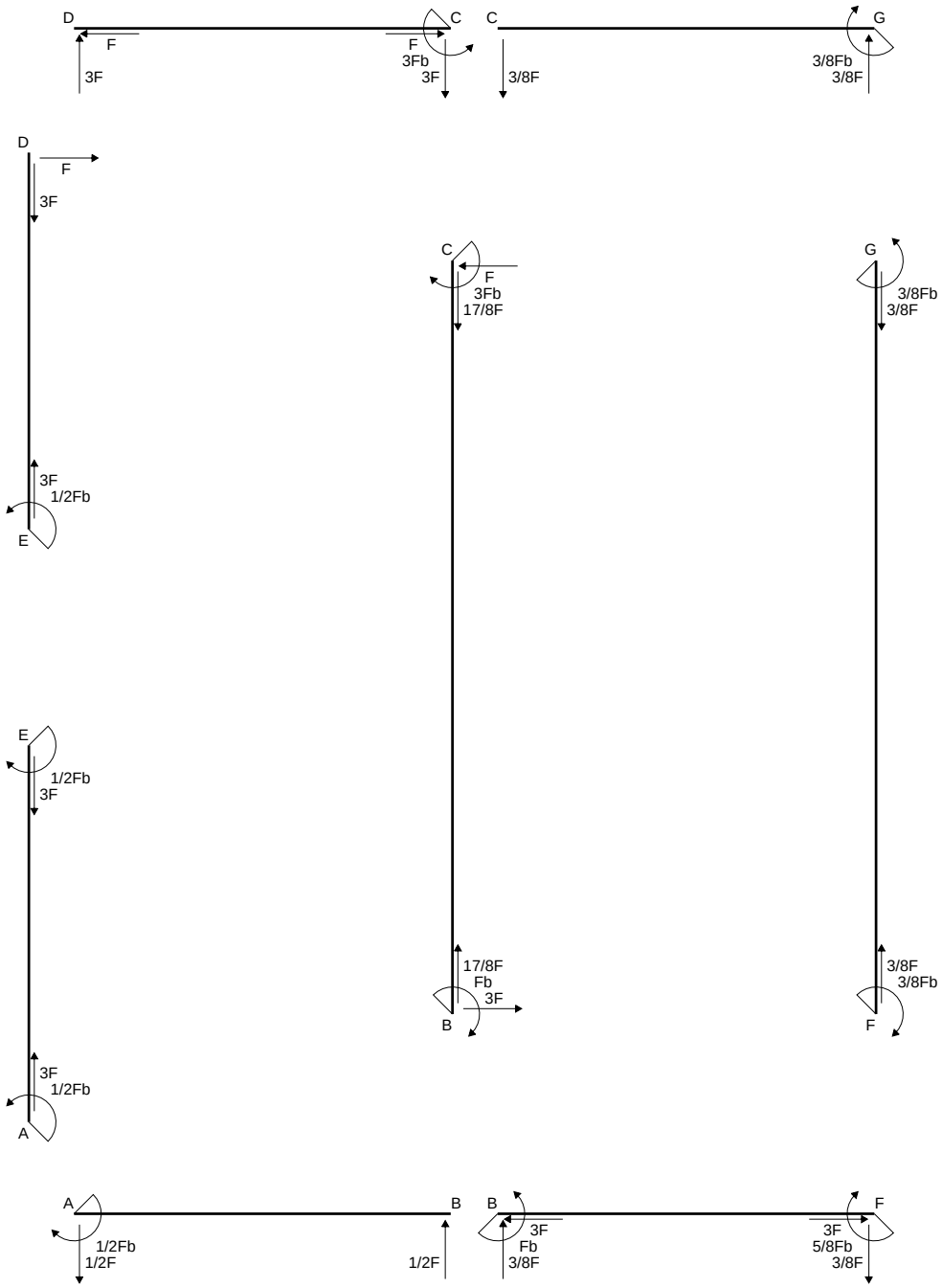
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3Fb
3F
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3/8F
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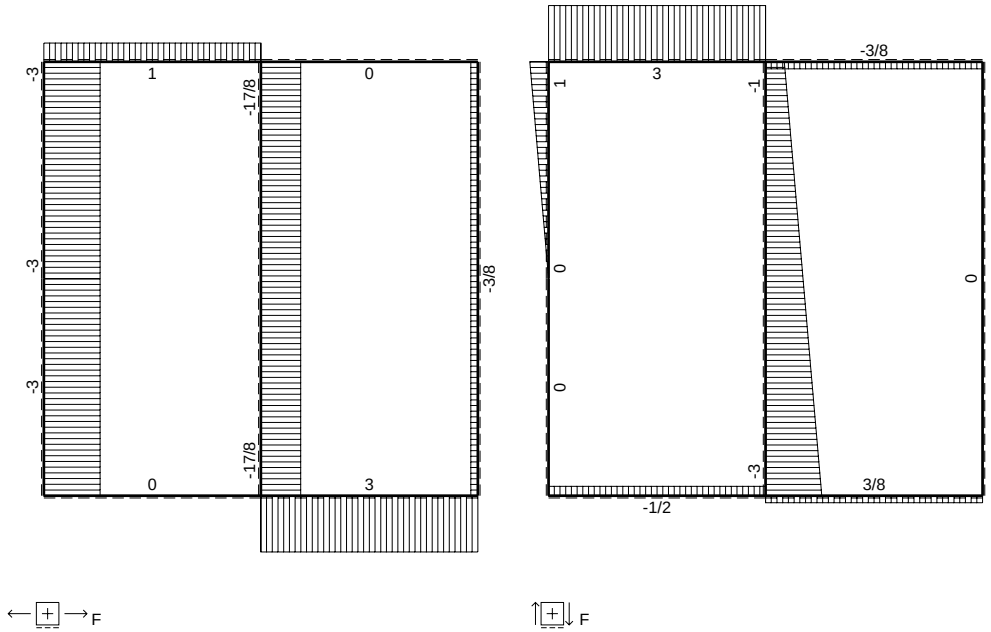
3F
5/8Fb
3/8F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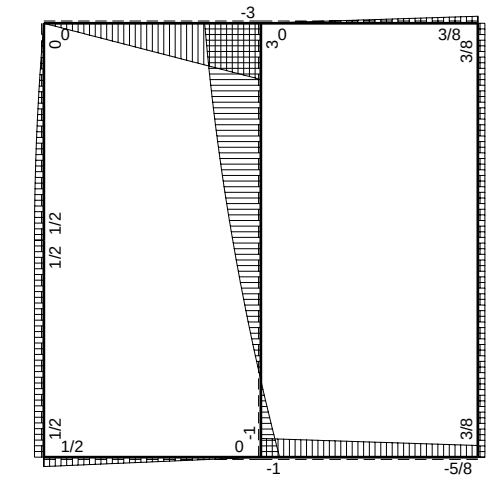
0
-3
3/8
-1/2
F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3
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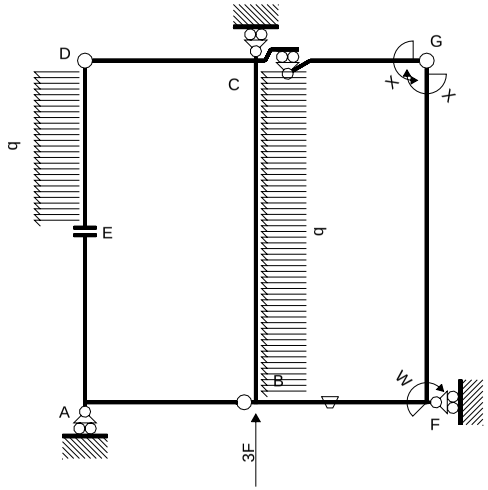
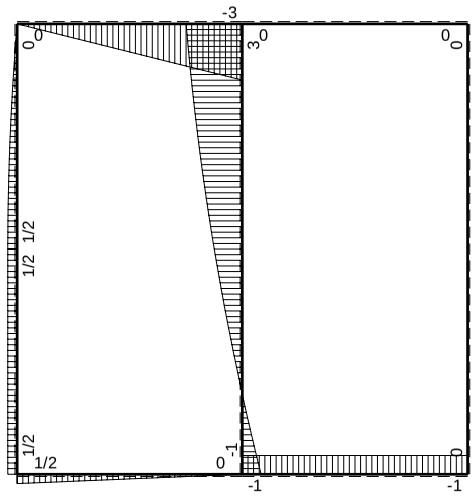
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8

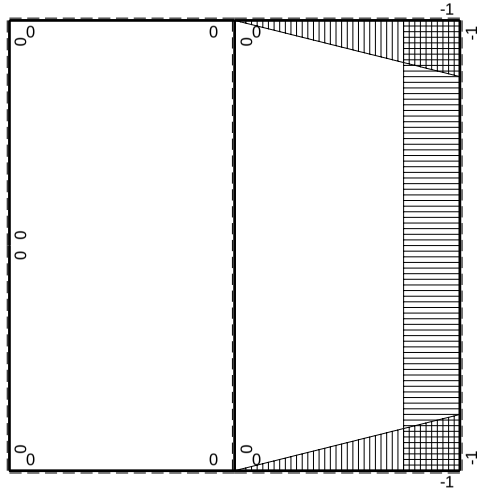

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$3Fb-Fx-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-3Fx+1/2qx^2$	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.133

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

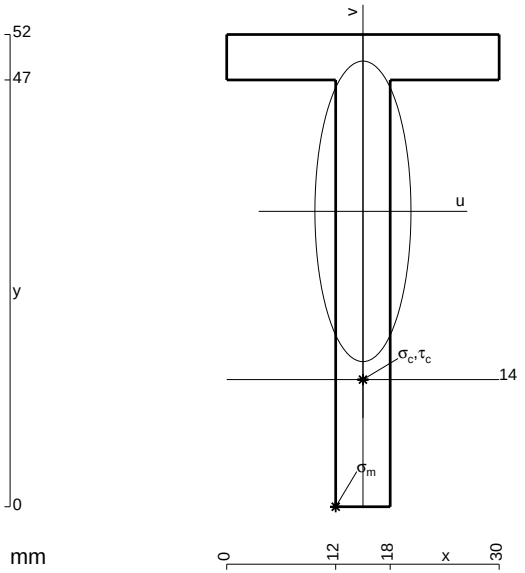
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 432. mm^2
J_u = 118416. mm^4
J_v = 12096. mm^4
y_g = 32.53 mm
N = 520. N
T_y = 1560. N
M_x = -826800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -32.53 mm
$\sigma_m = N/A - Mv/J_u$ = -225.9 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -18.53 mm
$\sigma_c = N/A - Mv/J_u$ = -128.2 N/mm^2
τ_c = 4.708 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.4 N/mm^2
S^* = 2144. mm^3

ESΣ8.133

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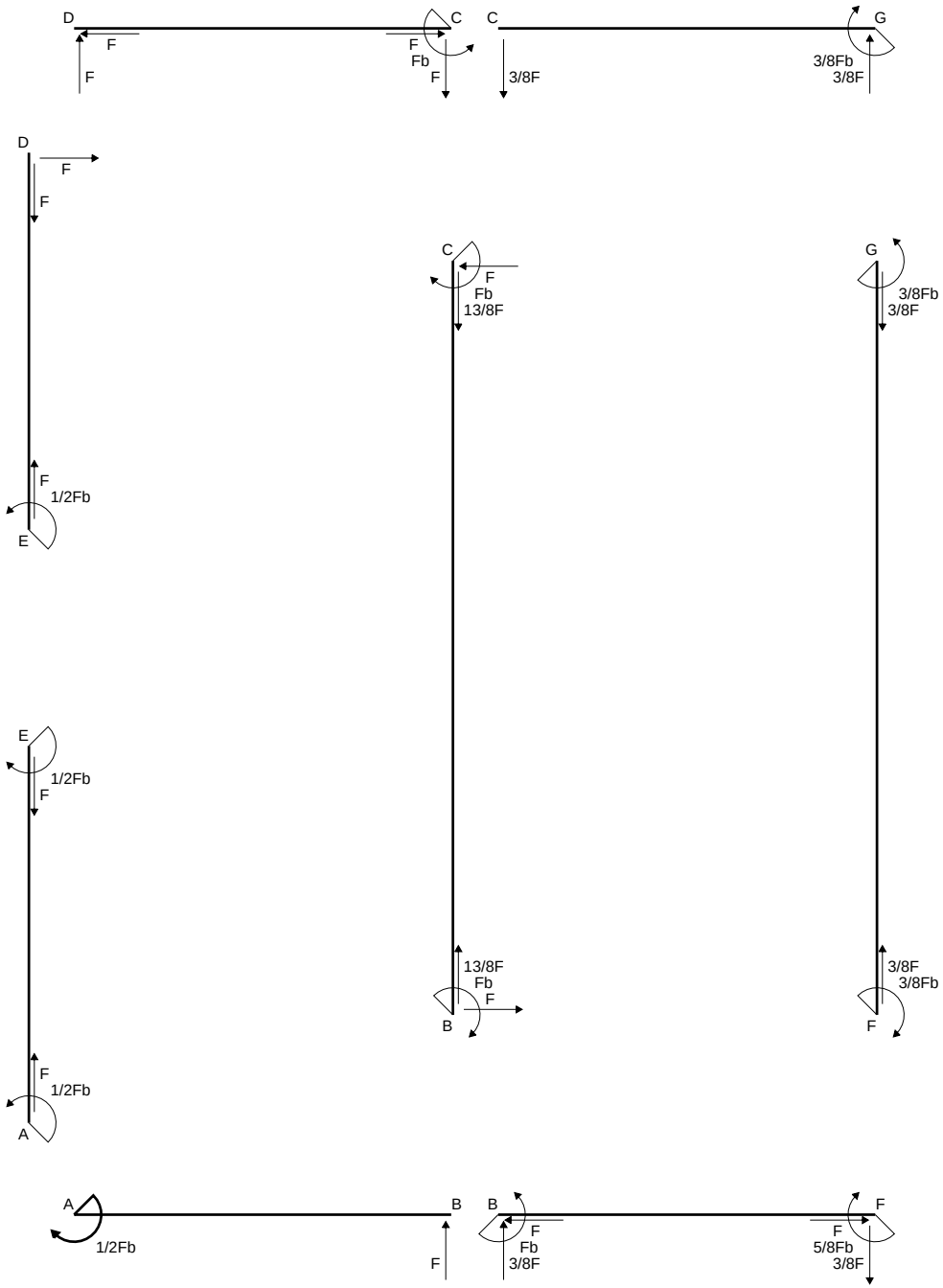
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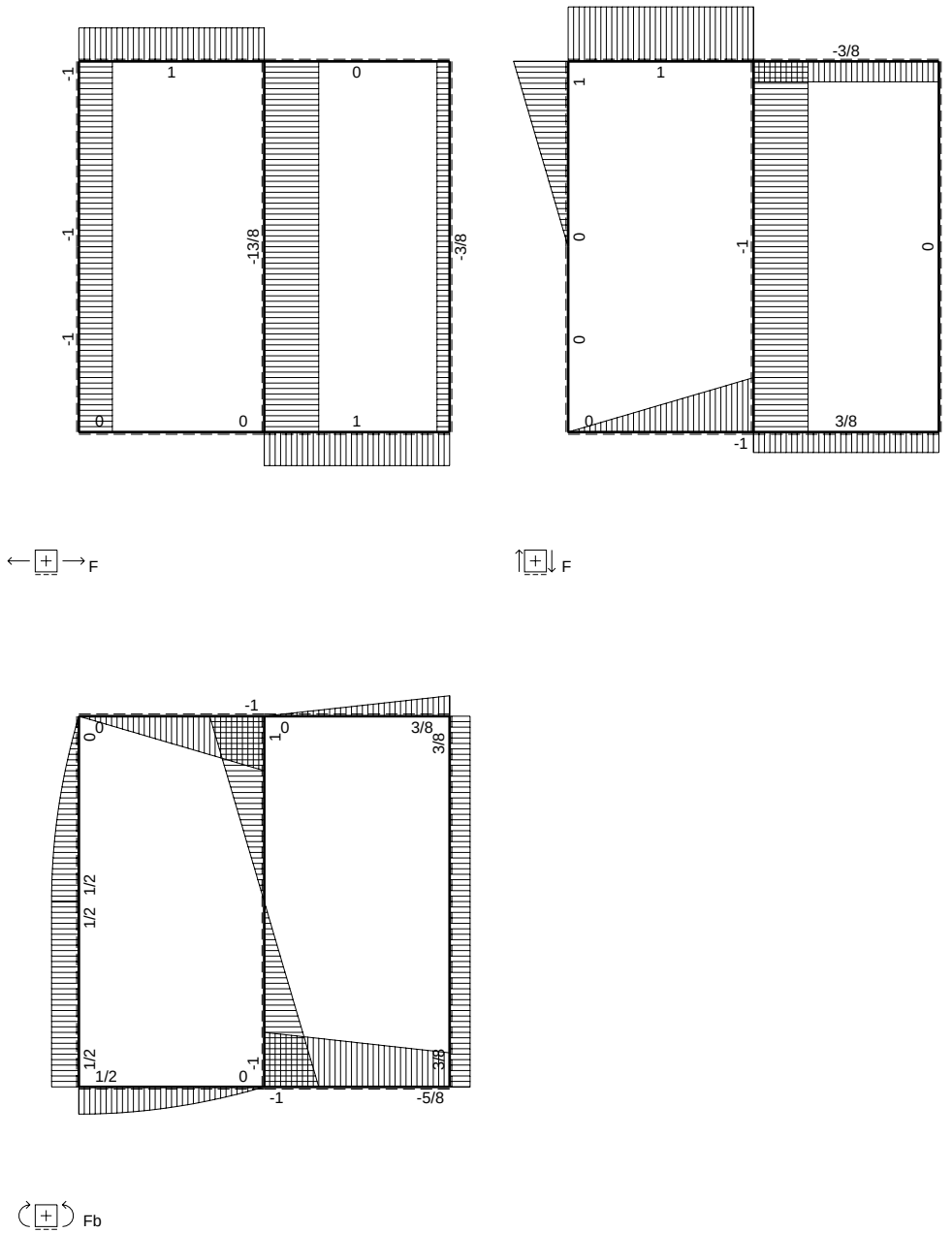
3/8F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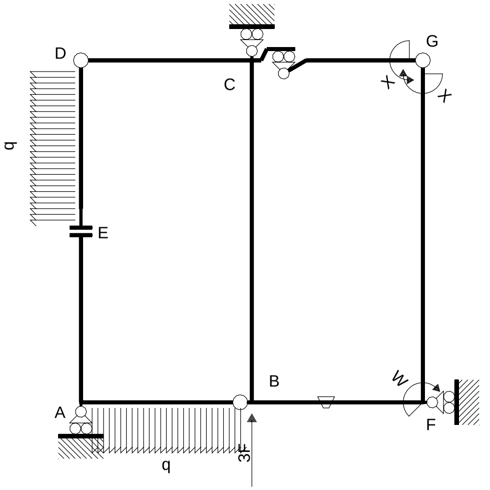
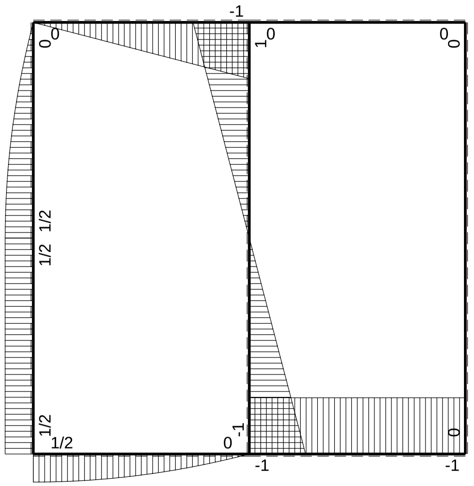
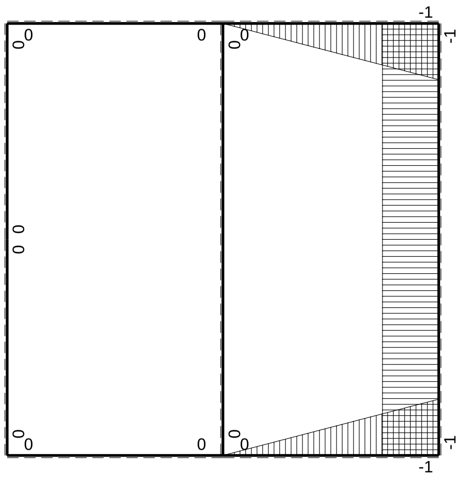
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

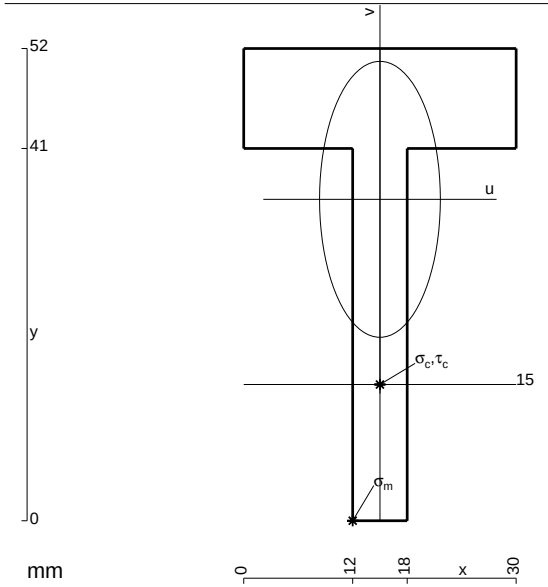
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 576. mm^2
J_u = 133062. mm^4
J_v = 25488. mm^4
y_g = 35.4 mm
N = -2519. N
T_y = -1550. N
M_x = -883500. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.4 mm
$\sigma_m = N/A - Mv/J_u$ = -239.4 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.4 mm
$\sigma_c = N/A - Mv/J_u$ = -139.8 N/mm^2
τ_c = 4.874 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.1 N/mm^2
S^* = 2511. mm^3

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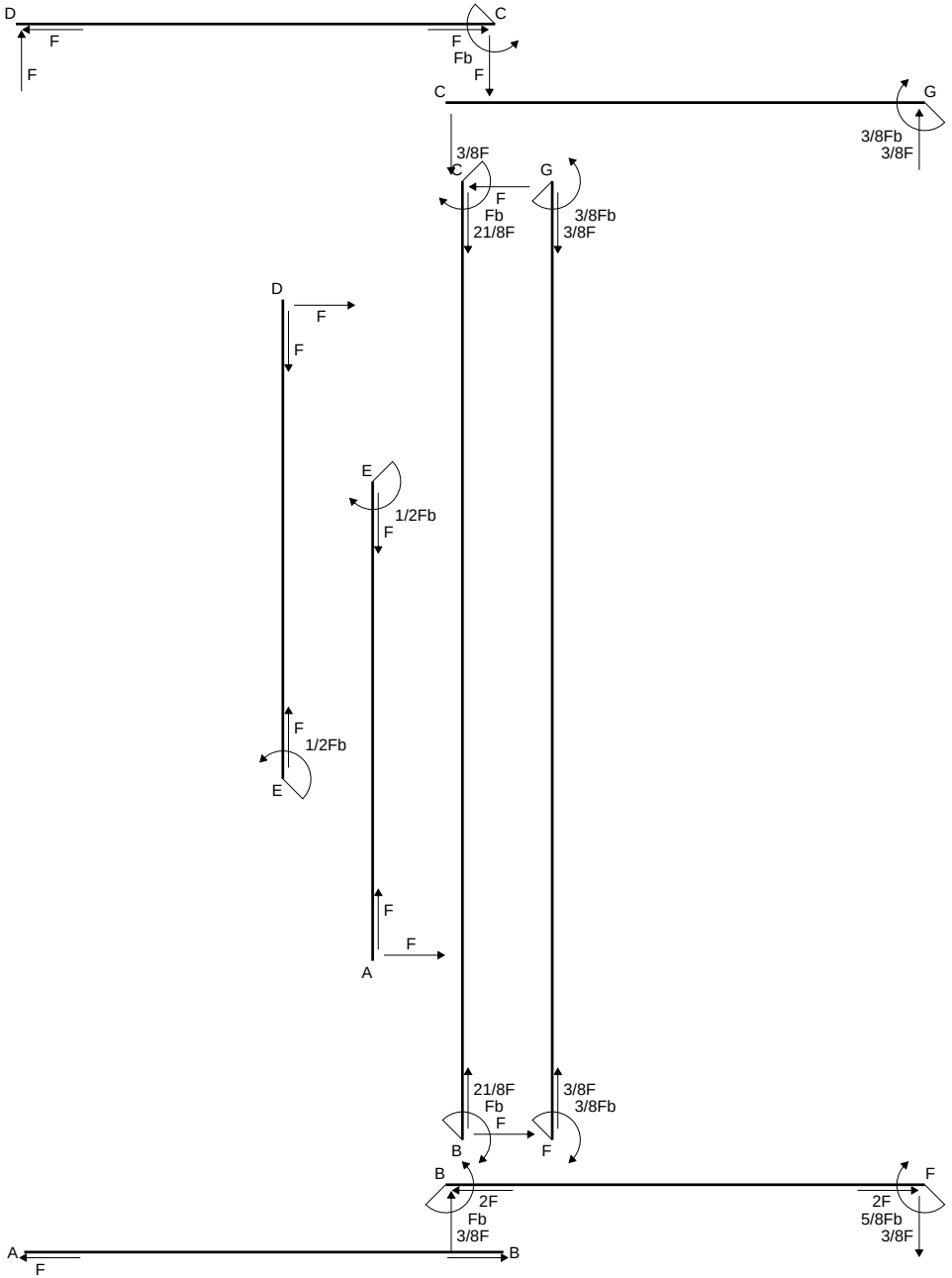
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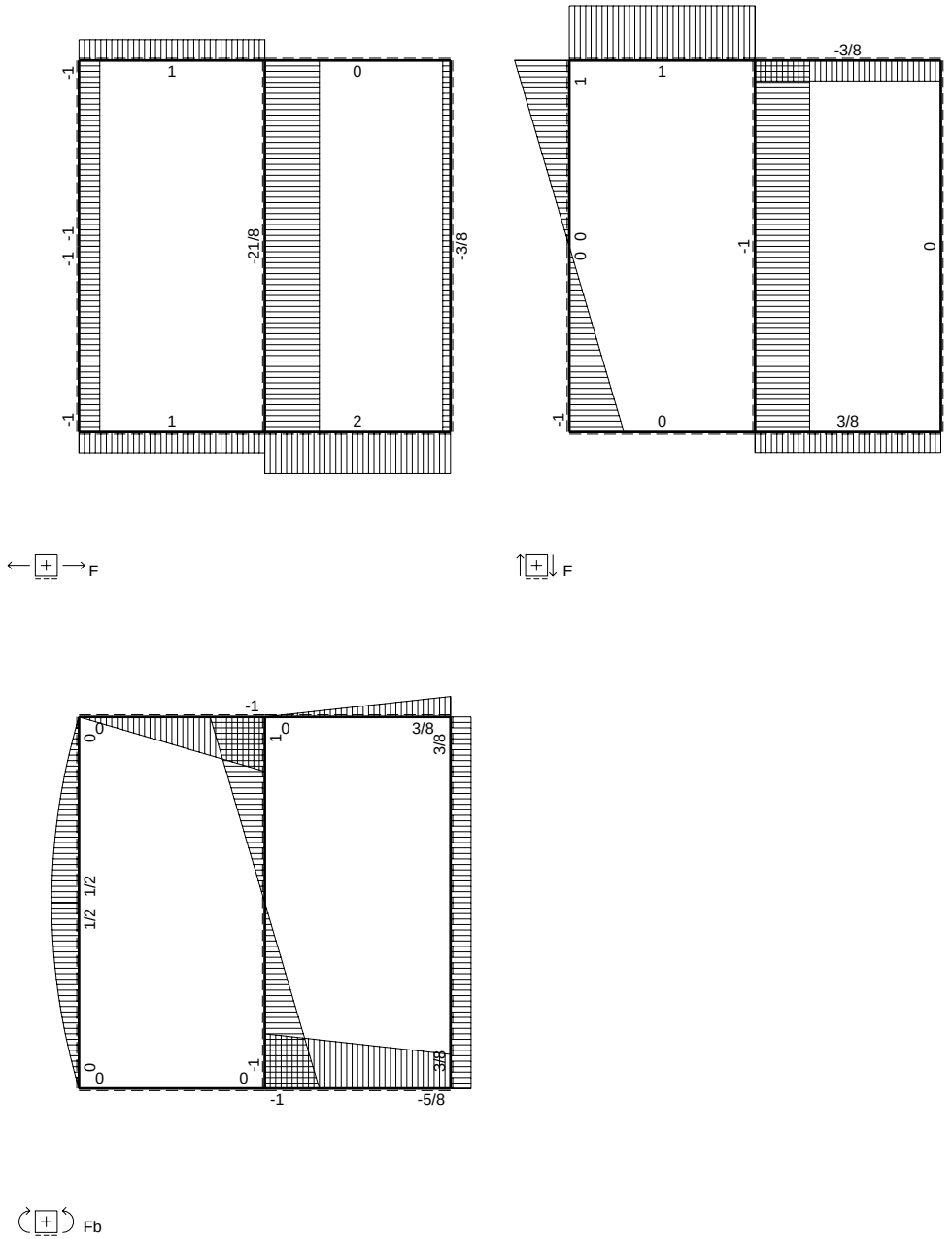
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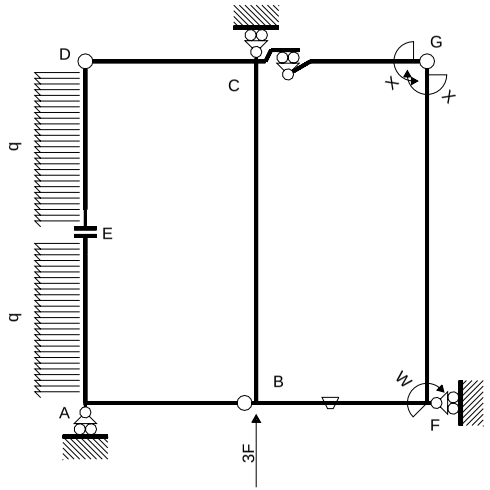
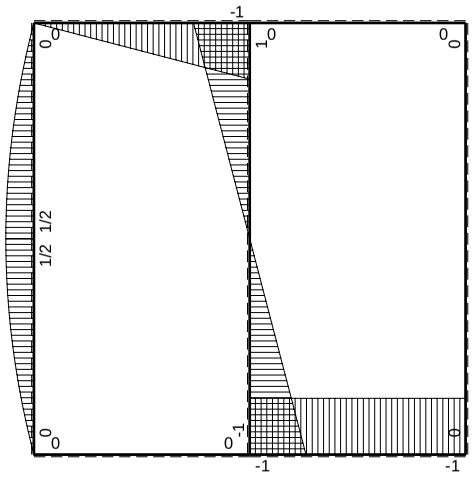
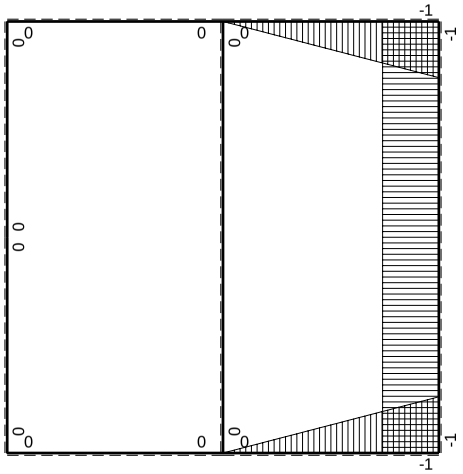
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb-1/2qx^2	0	0	0	0	0+0	0
AE b	0	-Fx+1/2qx^2	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
				totali			Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.135

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

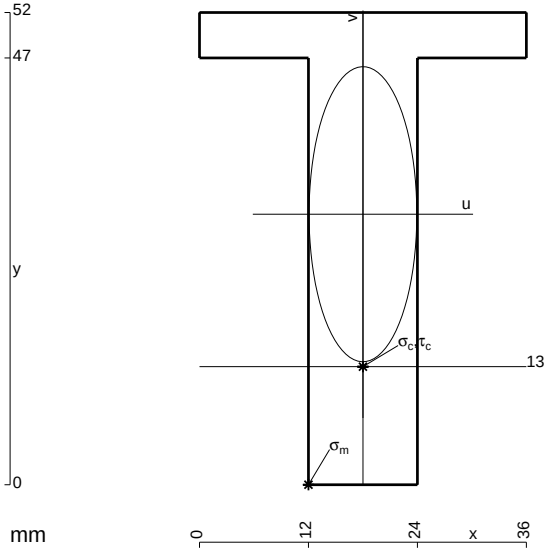
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 744. mm^2
J_u = 196439. mm^4
J_v = 26208. mm^4
y_g = 29.79 mm
N = -5460. N
T_y = -2080. N
M_x = -1268800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -29.79 mm
$\sigma_m = N/A - Mv/J_u$ = -199.8 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -16.79 mm
$\sigma_c = N/A - Mv/J_u$ = -115.8 N/mm^2
τ_c = 3.206 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.9 N/mm^2
S^* = 3633. mm^3

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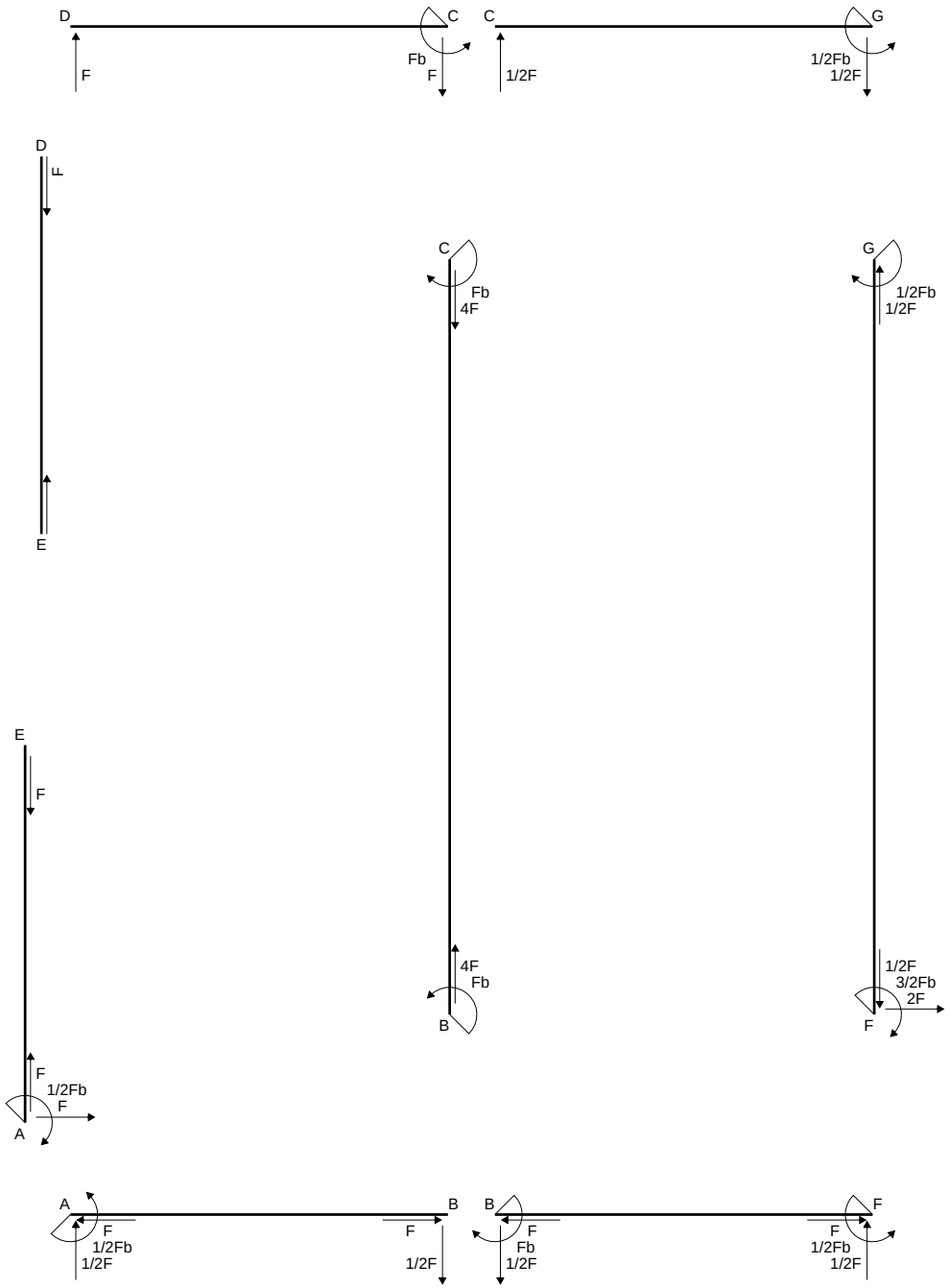
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Fb
1/2F
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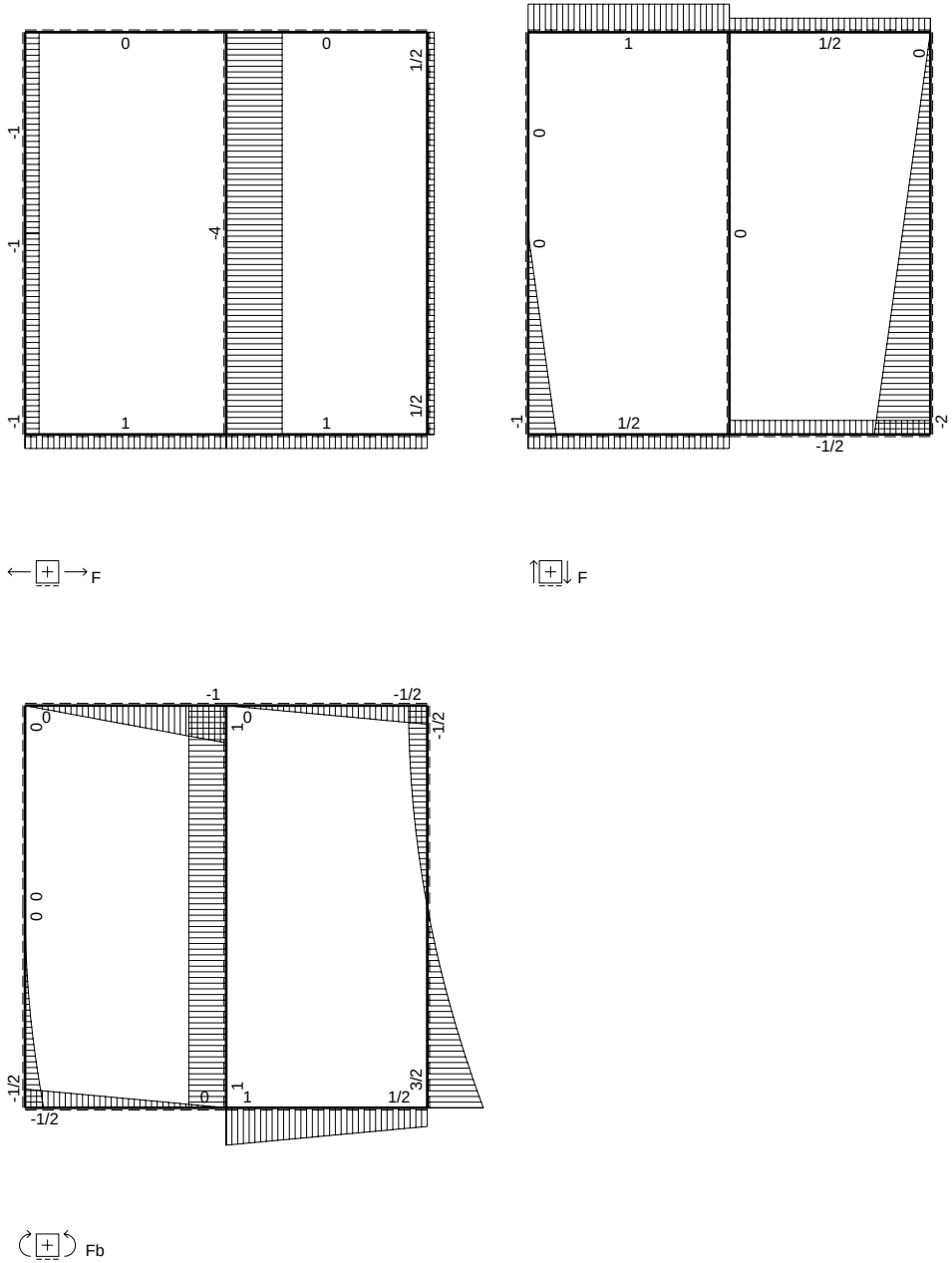
1/2Fb
1/2F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Fb

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

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14.11.23

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

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14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

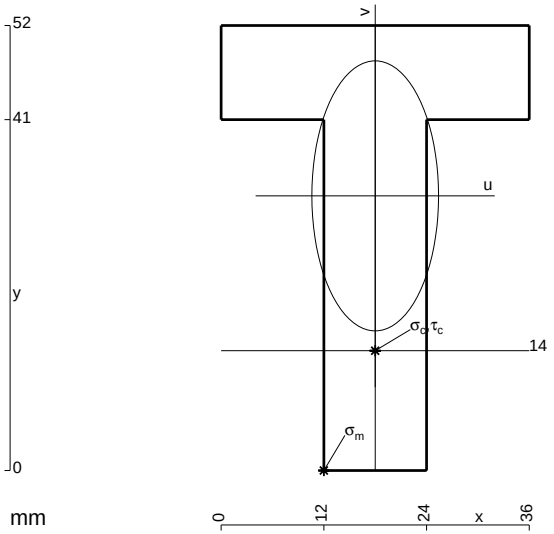
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 888. mm^2
J_u = 221232. mm^4
J_v = 48672. mm^4
y_g = 32.09 mm
T_y = 2220. N
M_x = -1443000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.09 mm
$\sigma_m = -Mv/J_u$ = -209.3 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.09 mm
$\sigma_c = -Mv/J_u$ = -118. N/mm^2
τ_c = 3.525 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 118.2 N/mm^2
S^* = 4216. mm^3

ESΣ8.136

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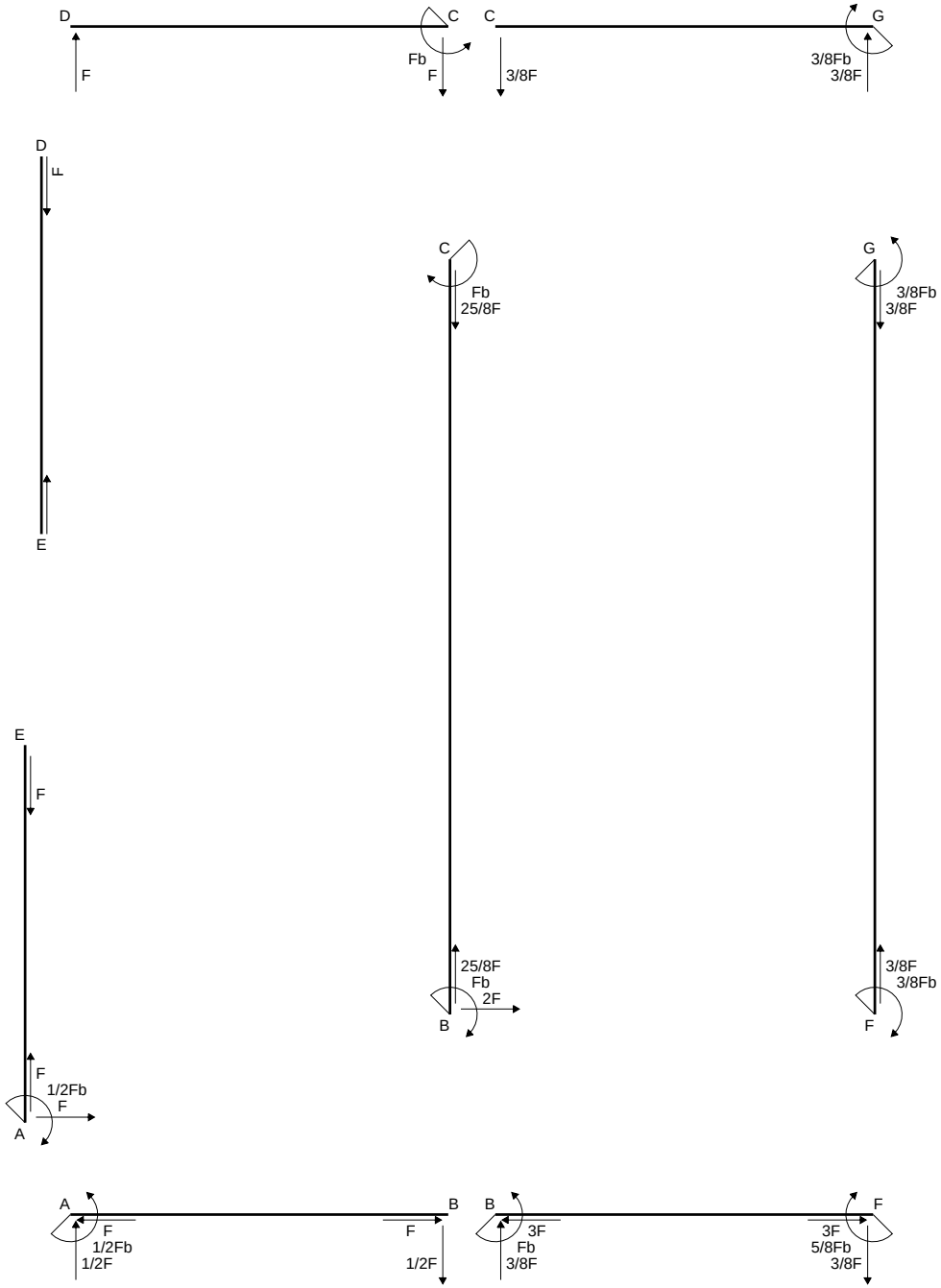
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5/8Fb
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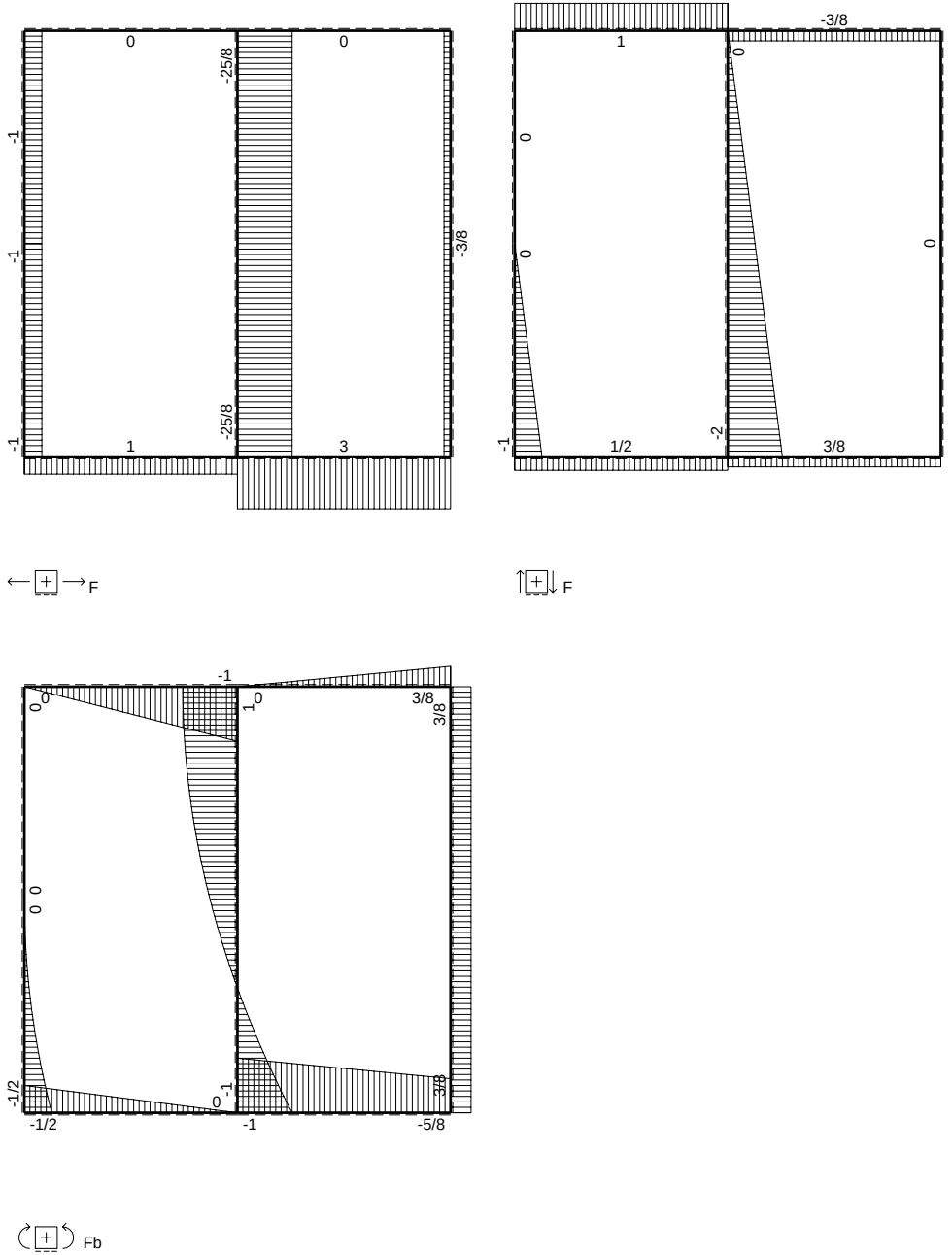
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3/8
0 0
-1/2
-1
0
-1
3/8
-1/2
-1
-5/8
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb²/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Schema di calcolo iperstatico

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

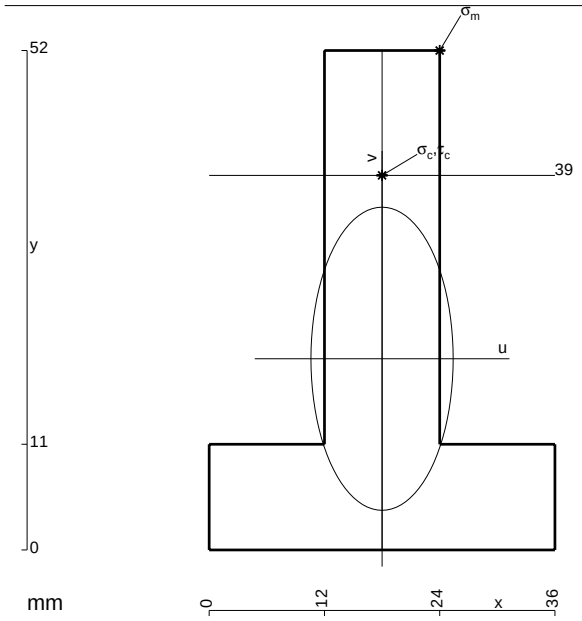
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 888. mm^2
J_u = 221232. mm^4
J_v = 48672. mm^4
y_g = 19.91 mm
N = -7094. N
T_y = -4540. N
M_x = -1566300. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 32.09 mm
$\sigma_m = N/A - Mv/J_u$ = 219.2 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 19.09 mm
$\sigma_c = N/A - Mv/J_u$ = 127.2 N/mm^2
τ_c = 6.828 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127.7 N/mm^2
S^* = 3993. mm^3

ESΣ8.137

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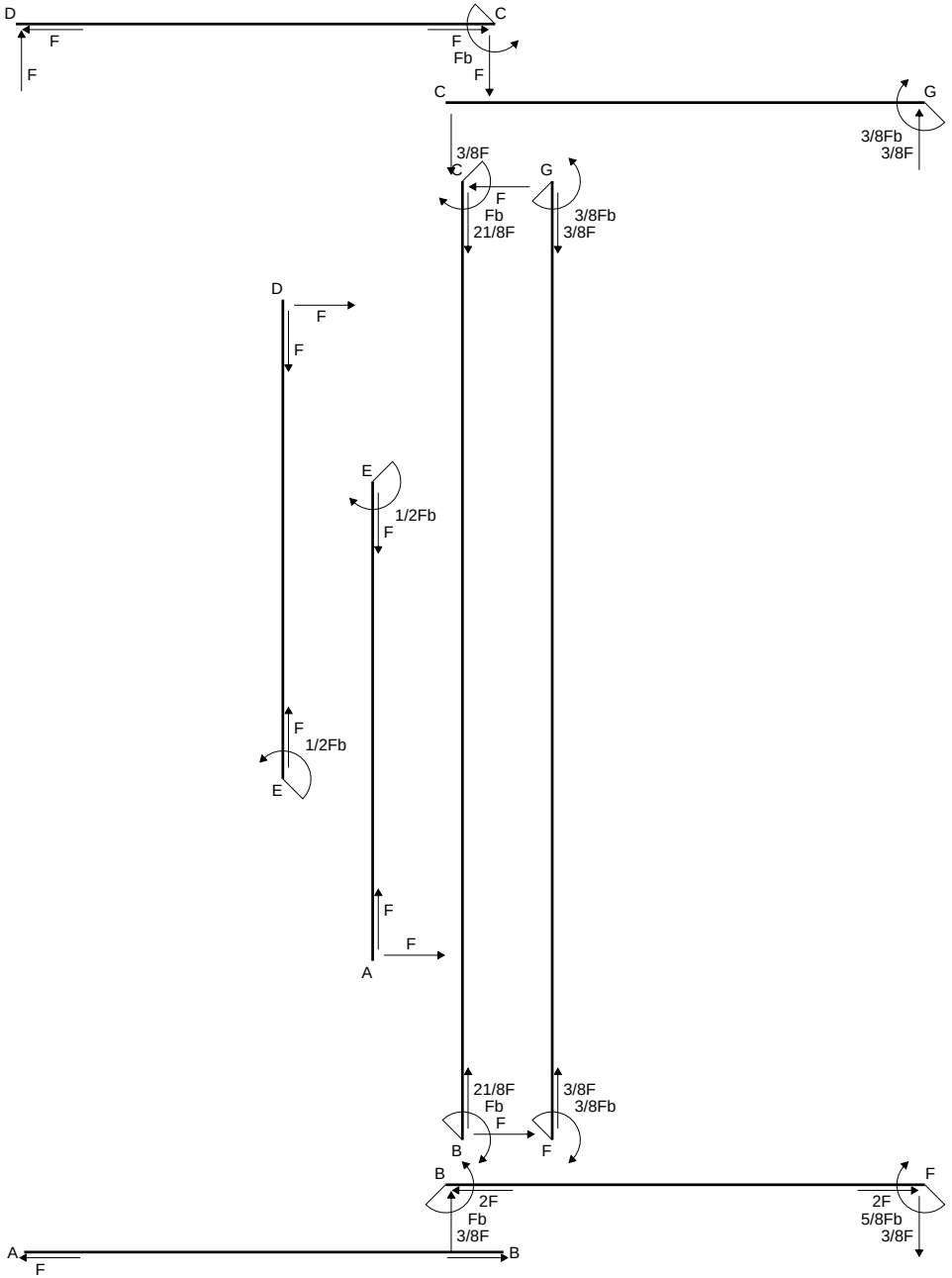
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Fb
F
3/8F
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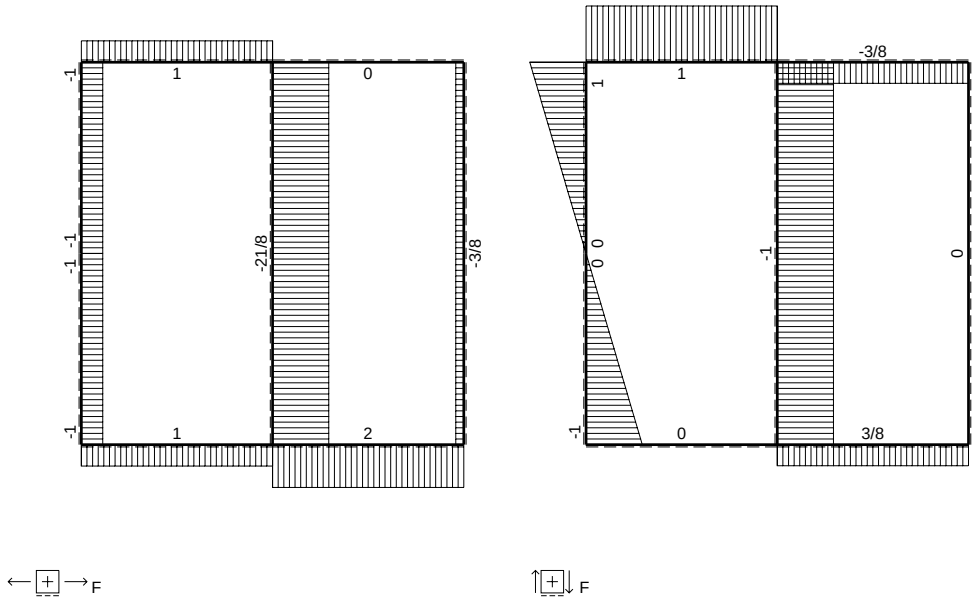
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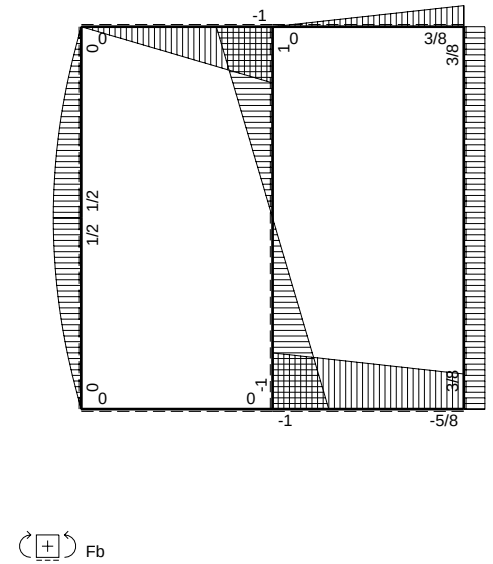
F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3/8
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-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

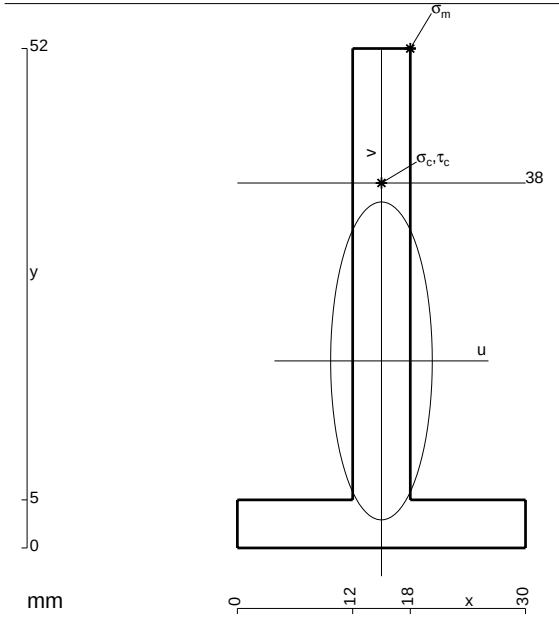
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 432. mm^2
J_u = 118416. mm^4
J_v = 12096. mm^4
y_g = 19.47 mm
N = -2914. N
T_y = -1110. N
M_x = 810300. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 32.53 mm
$\sigma_m = N/A - Mv/J_u$ = -229.3 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 18.53 mm
$\sigma_c = N/A - Mv/J_u$ = -133.5 N/mm^2
τ_c = 3.35 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.7 N/mm^2
S^* = 2144. mm^3

ESΣ8.138

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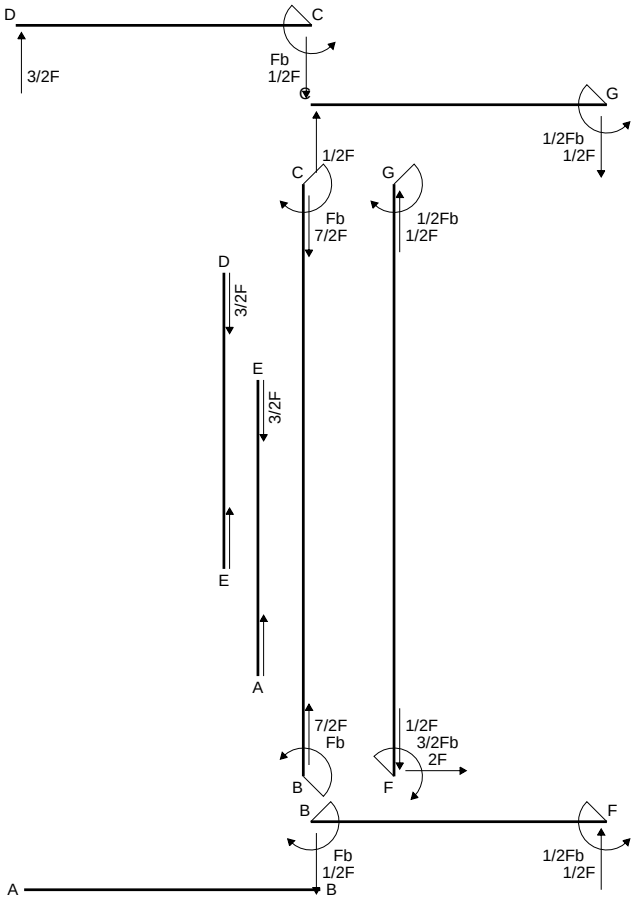
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1/2F
G
1/2Fb
1/2F
1/2F
C
G
Fb
7/2F
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1/2F
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3/2F
E
3/2F
E
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7/2F
Fb
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Fb
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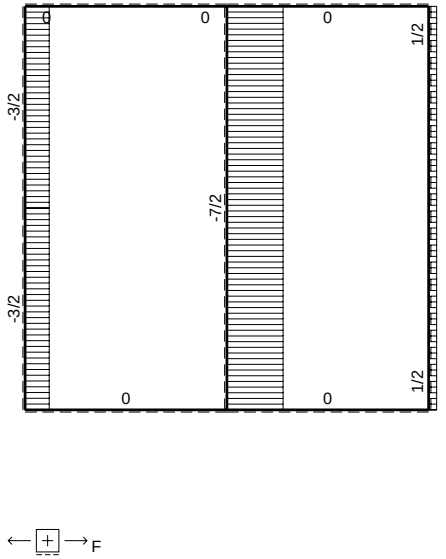
1/2Fb
1/2F
A
B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0
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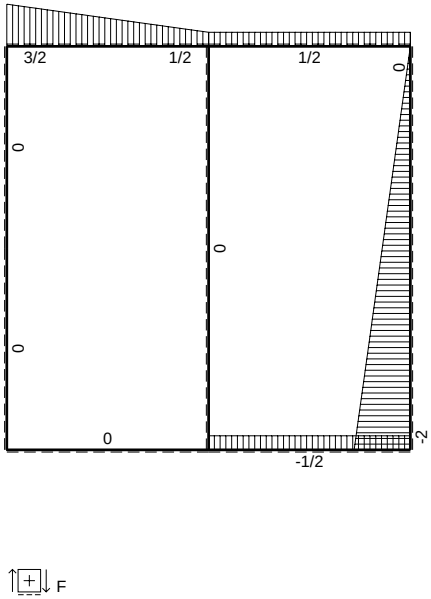
F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1/2
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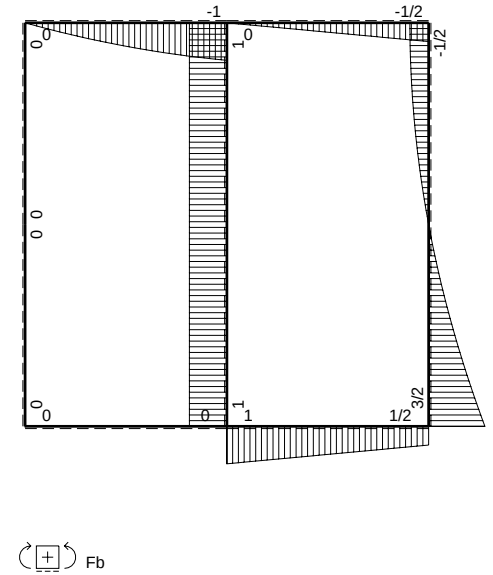
F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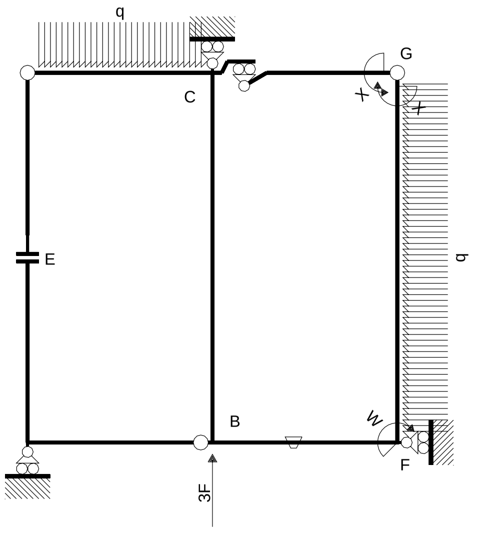
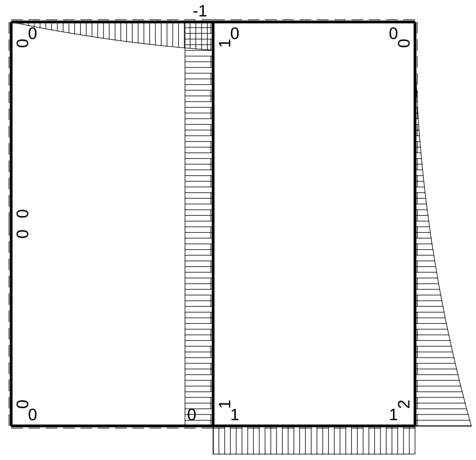
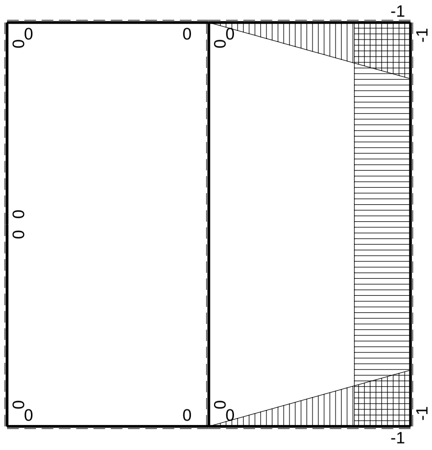
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$1/2Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

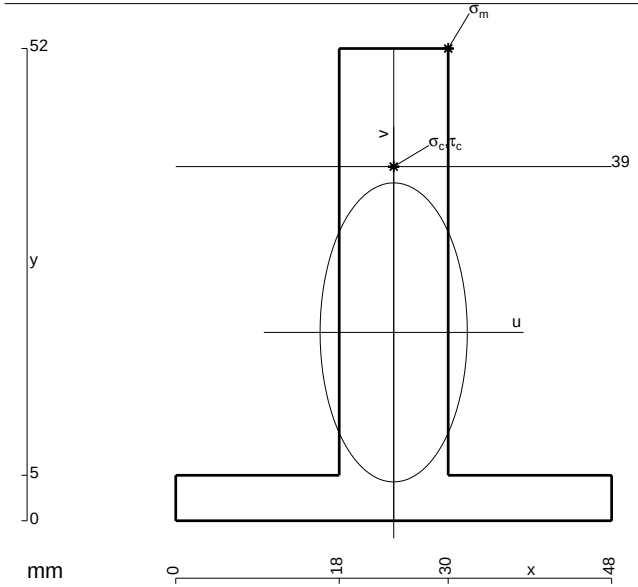
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 804. mm^2
J_u = 218133. mm^4
J_v = 52848. mm^4
y_g = 20.74 mm
T_y = 3470. N
M_x = -1669940. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 31.26 mm
$\sigma_m = -Mv/J_u$ = 239.3 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 18.26 mm
$\sigma_c = -Mv/J_u$ = 139.8 N/mm^2
τ_c = 5.121 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.1 N/mm^2
S^* = 3863. mm^3

ESΣ8.139

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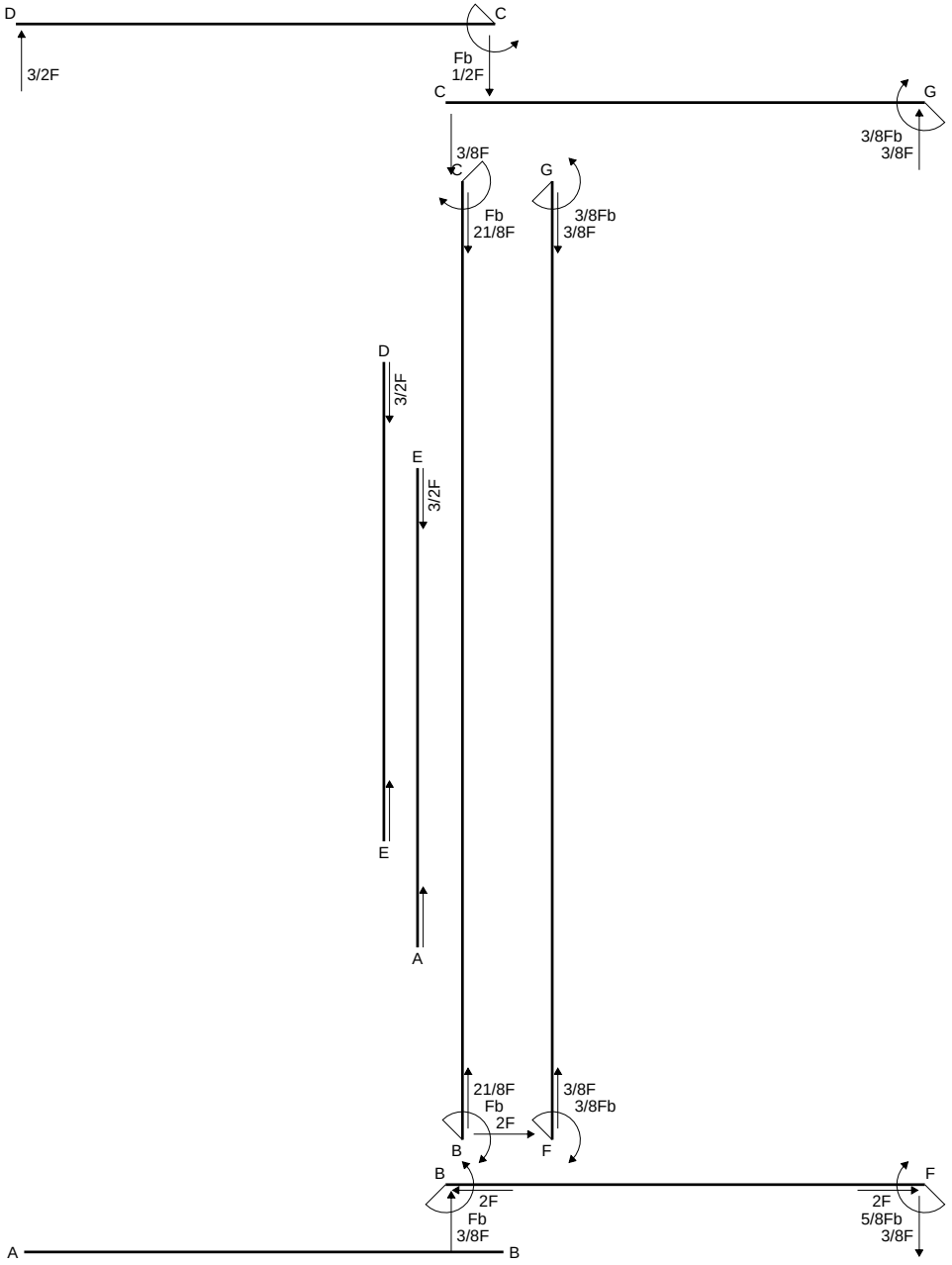
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Fb
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2F
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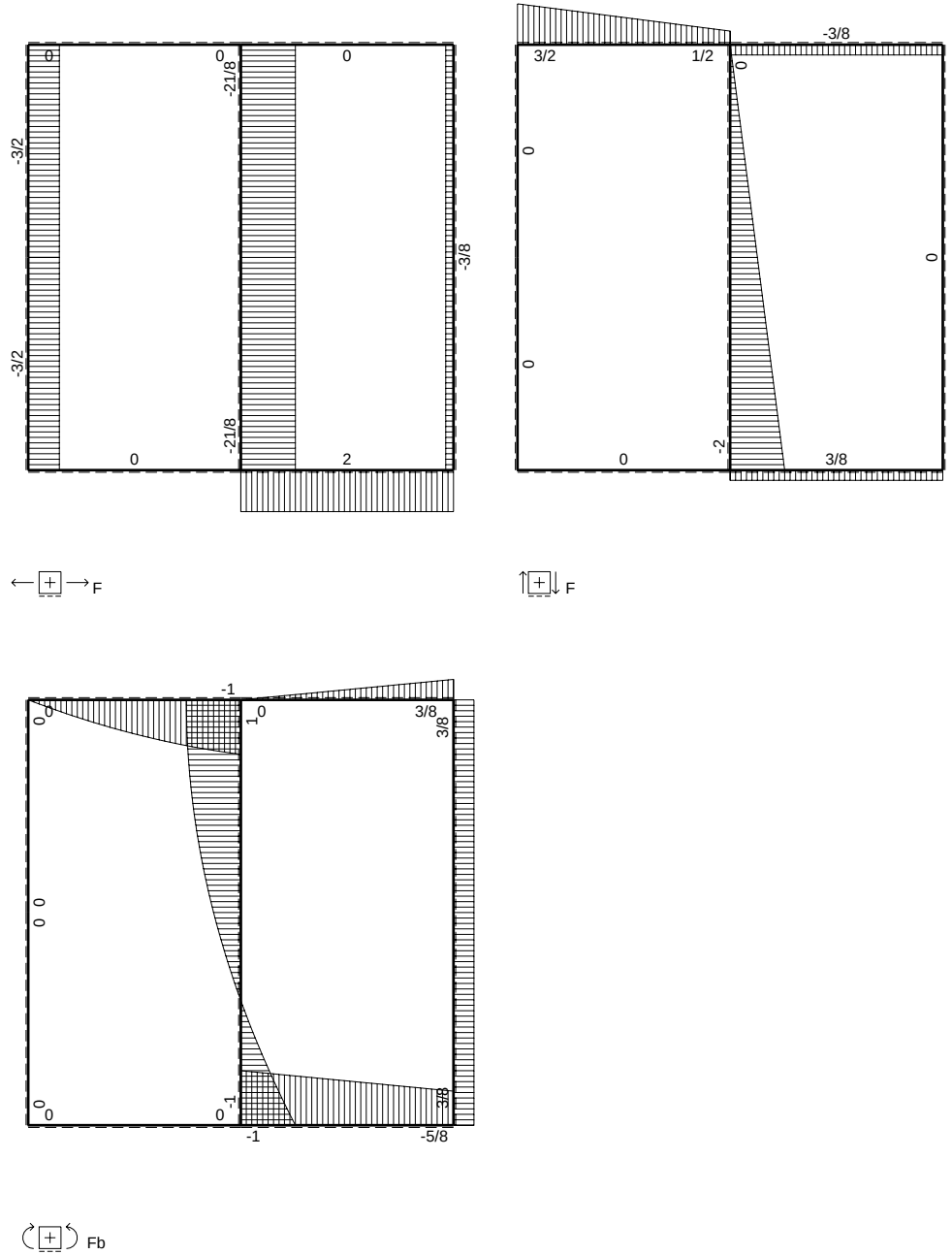
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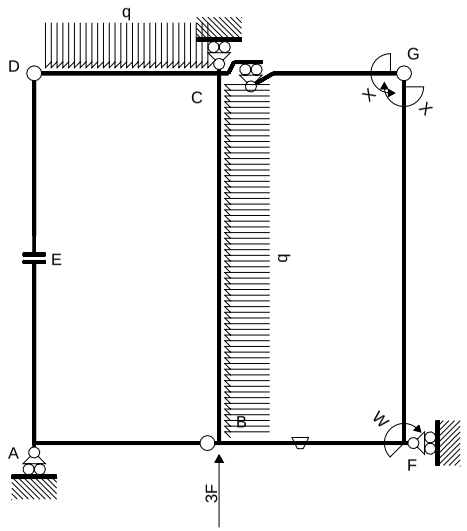
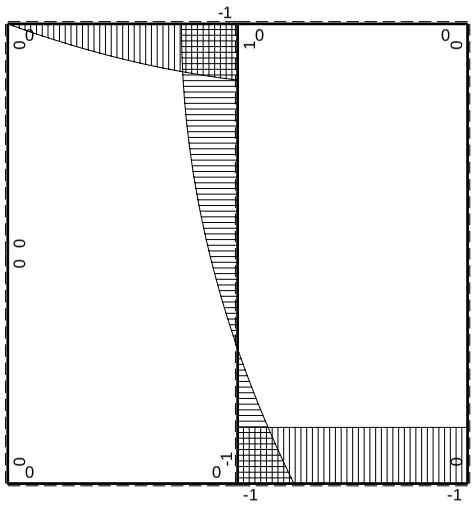
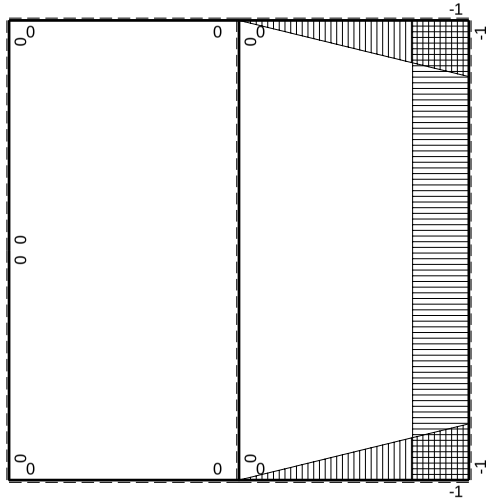
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.140

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

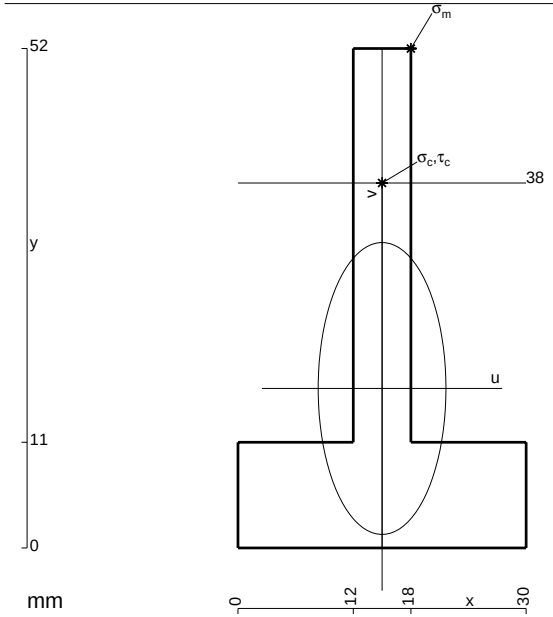
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 576. mm^2
J_u = 133062. mm^4
J_v = 25488. mm^4
y_g = 16.6 mm
N = -5014. N
T_y = -3820. N
M_x = -783100. Nmm
x_m = 18. mm
y_m = 52. mm
u_m = 3. mm
v_m = 35.4 mm
$\sigma_m = N/A - Mv/J_u$ = 199.6 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.4 mm
$\sigma_c = N/A - Mv/J_u$ = 117.2 N/mm^2
τ_c = 11.41 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.9 N/mm^2
S^* = 2385. mm^3

ESΣ8.140

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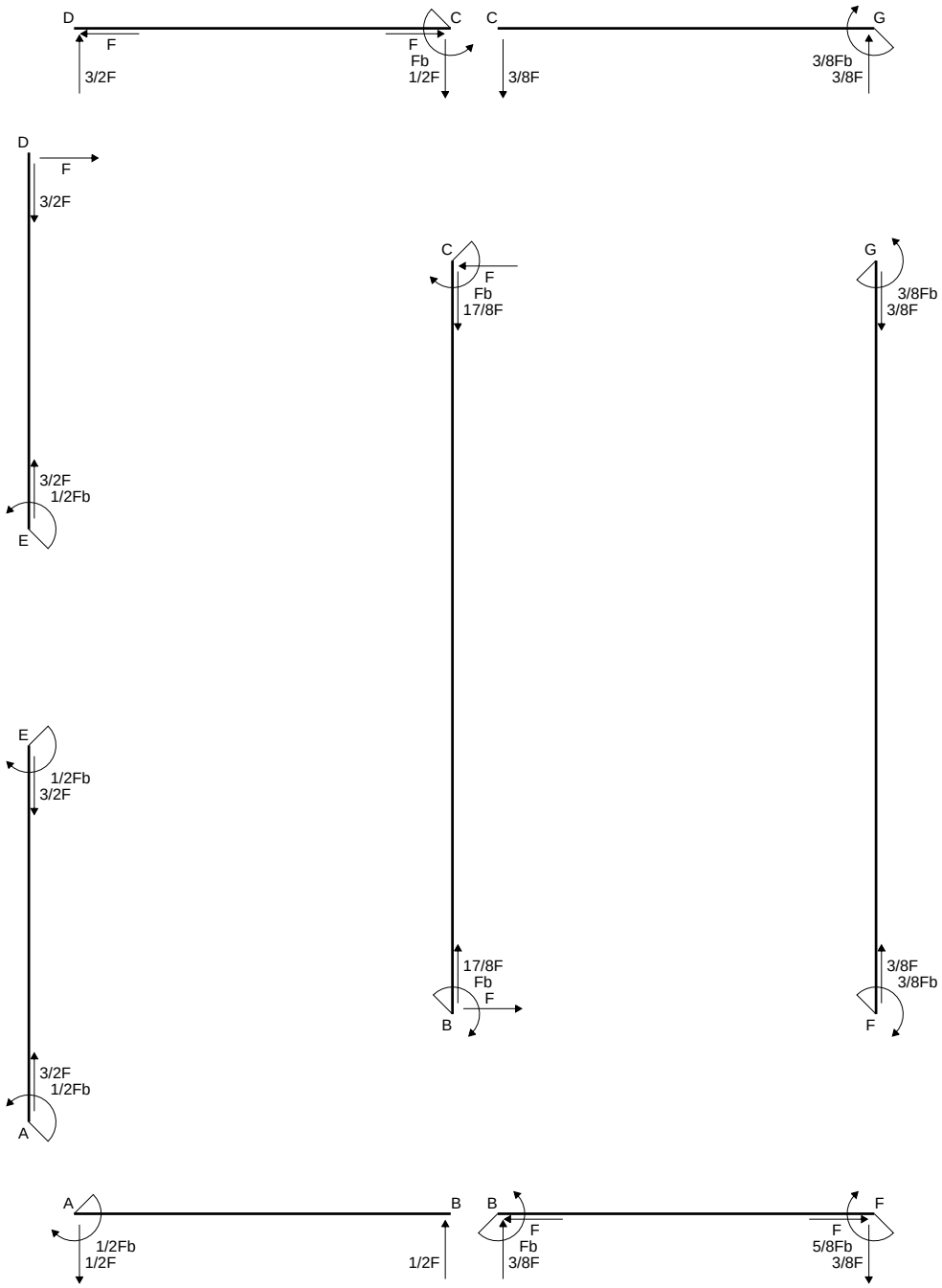
D
F
3/2F
C
F
Fb
1/2F
C
3/8F
G
3/8Fb
3/8F
D
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3/2F
1/2Fb
E
C
F
Fb
17/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
17/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
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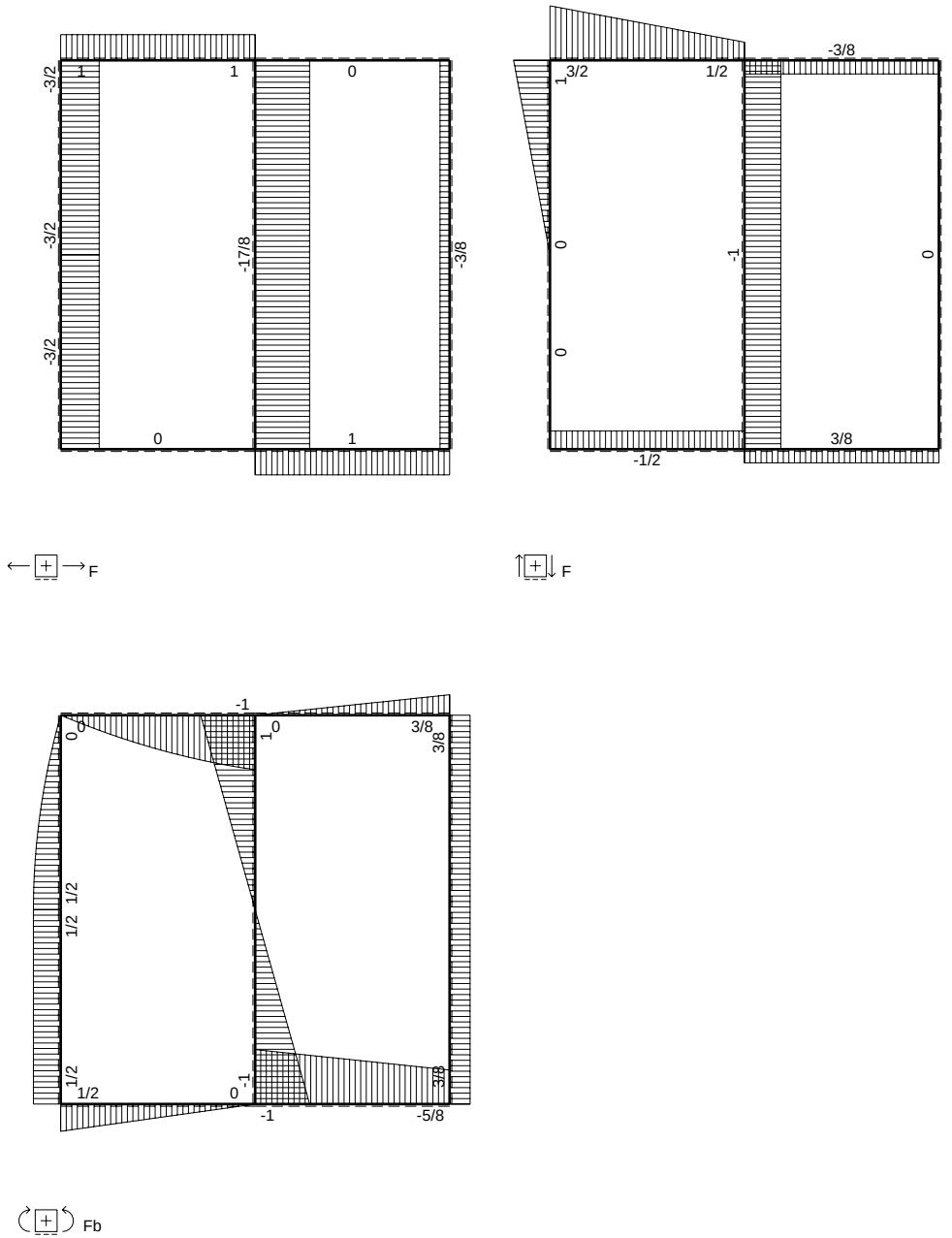
Fb
3/8F
F
F
5/8Fb
3/8F


-3/2
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-3/2
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-3/8
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F
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3/2
1/2
0
0
-1
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-1/2
3/8
F
-1
0
0
1
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3/8
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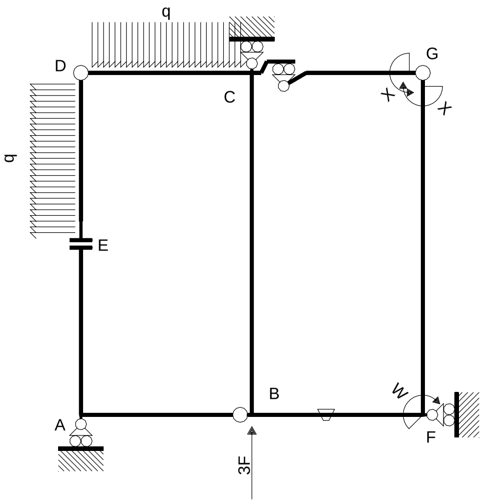
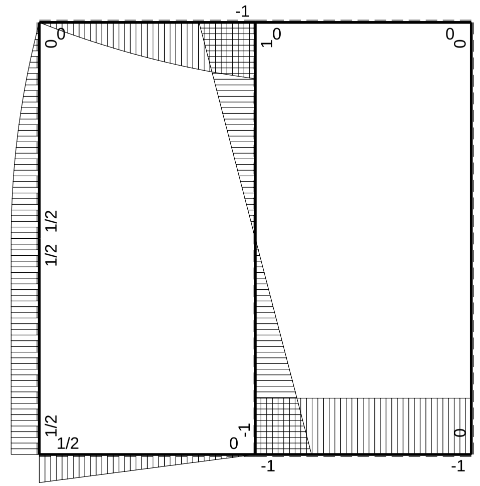
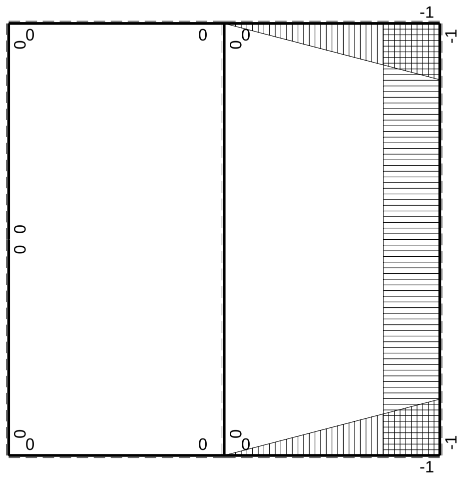
-1
-1
3/8
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

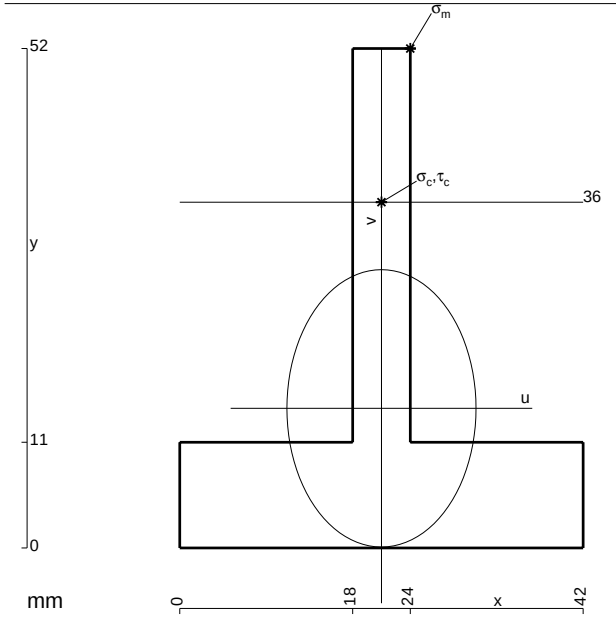
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 708. mm^2
J_u = 147634. mm^4
J_v = 68652. mm^4
y_g = 14.53 mm
N = -3804. N
T_y = -1790. N
M_x = 805500. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 37.47 mm
$\sigma_m = N/A - Mv/J_u$ = -209.8 N/mm^2
x_c = 21. mm
y_c = 36. mm
v_c = 21.47 mm
$\sigma_c = N/A - Mv/J_u$ = -122.5 N/mm^2
τ_c = 5.716 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.9 N/mm^2
S^* = 2829. mm^3

ESΣ8.141

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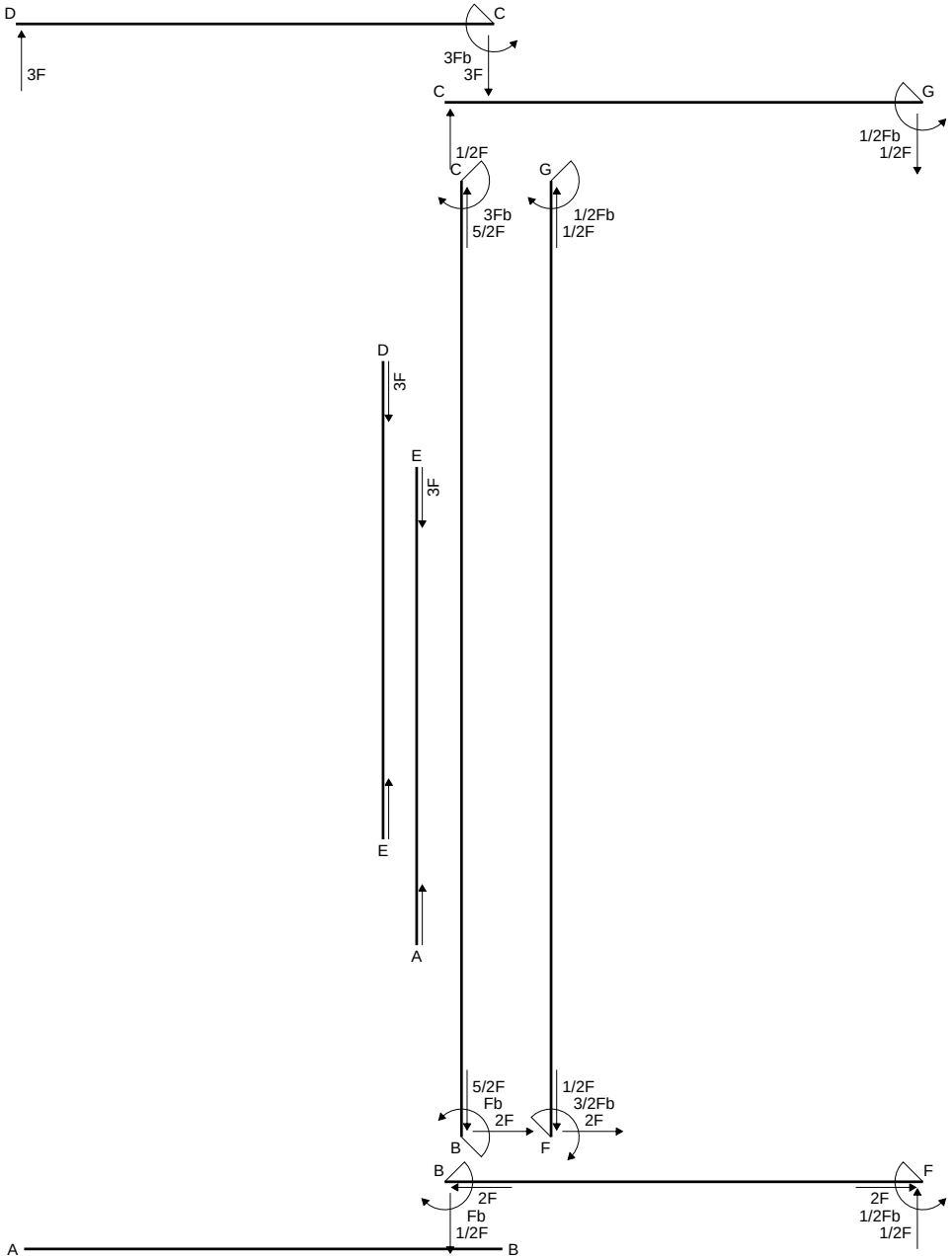
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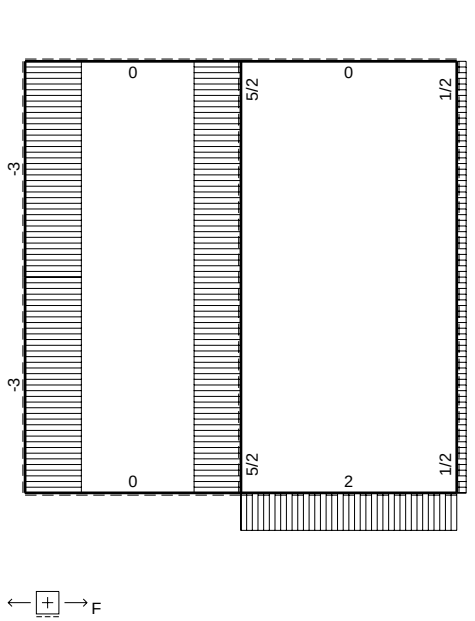
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B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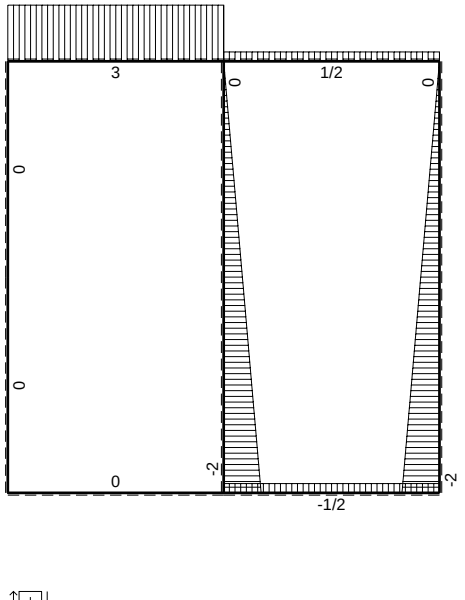
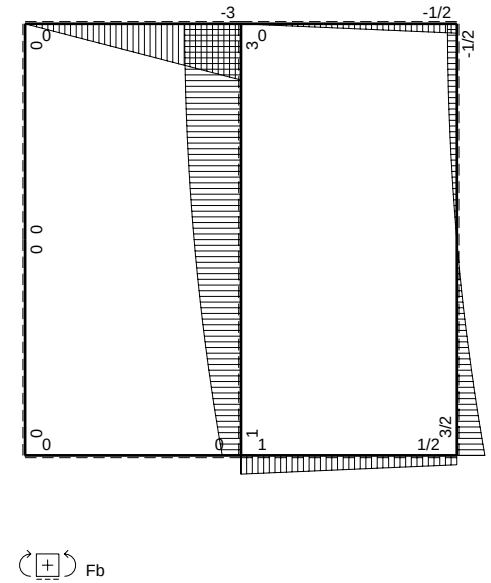
5/2
0
1/2
-3
-3
0
5/2
2
1/2
F


3
0
1/2
0
0
0
-2
0
-2
-1/2
F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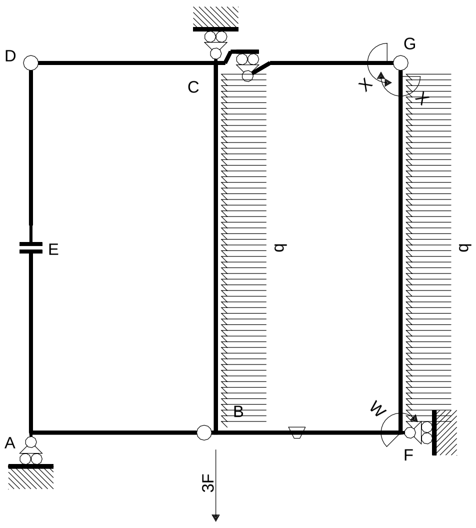
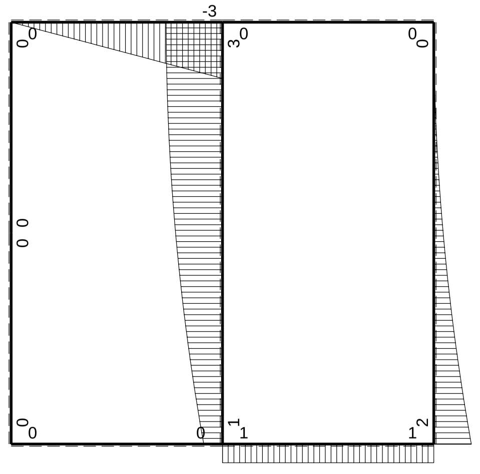
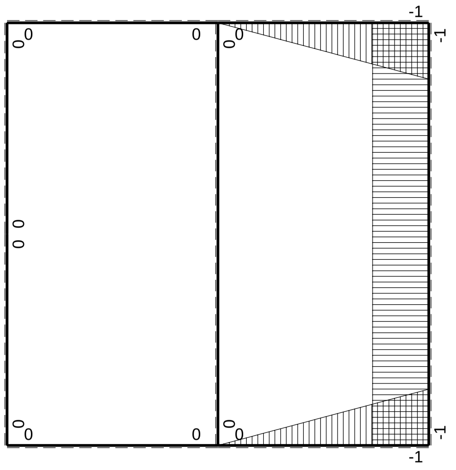
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

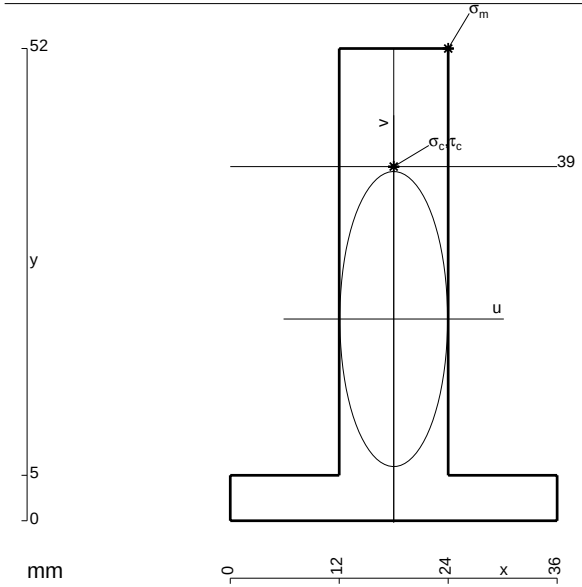
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 744. mm^2
J_u = 196439. mm^4
J_v = 26208. mm^4
y_g = 22.21 mm
T_y = 2940. N
M_x = -1440600. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 6. mm
v_m = 29.79 mm
$\sigma_m = -Mv/J_u$ = 218.5 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 16.79 mm
$\sigma_c = -Mv/J_u$ = 123.1 N/mm^2
τ_c = 4.531 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.4 N/mm^2
S^* = 3633. mm^3

ESΣ8.142

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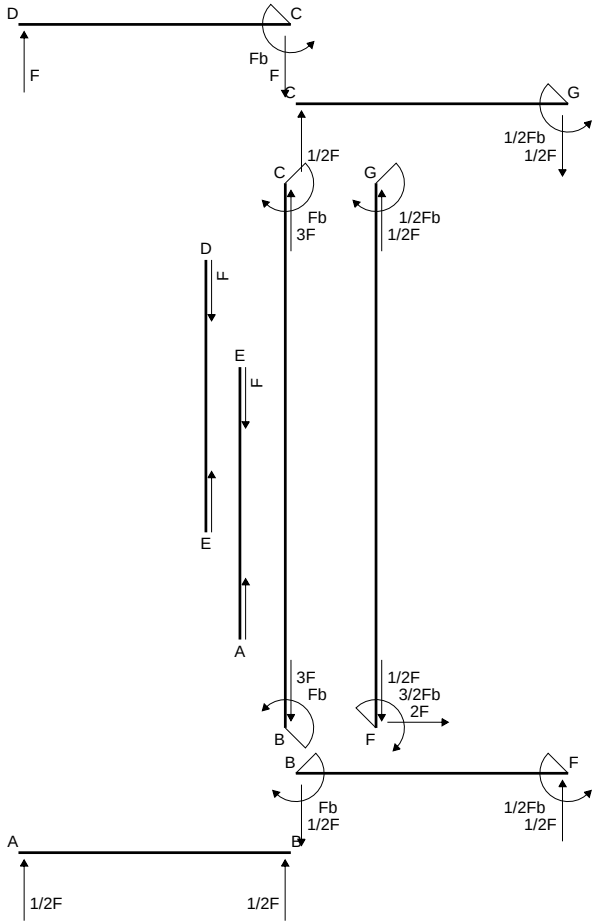
D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
3F
1/2Fb
1/2F
D
F
E
F
E
A
3F
Fb
1/2F
3/2Fb
2F
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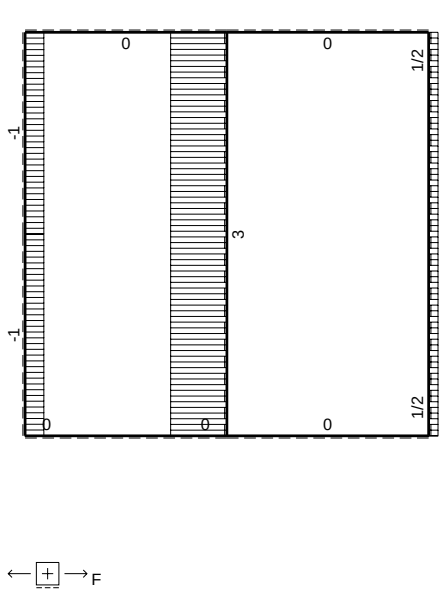
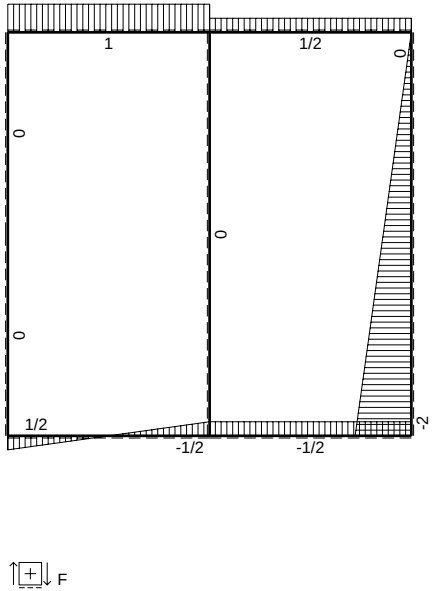
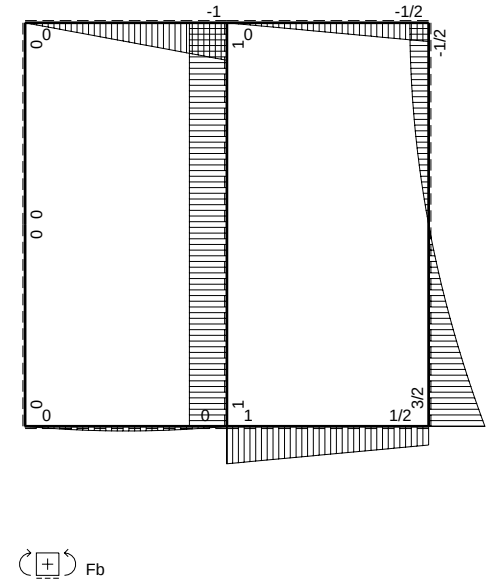
F
B
F
Fb
1/2F
1/2Fb
1/2F
A
B
1/2F
1/2F


0
0
1/2
-1
3
-1
0
0
0
1/2
F


1
1/2
0
0
0
0
1/2
-1/2
-1/2
-2
F


-1
-1/2
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0
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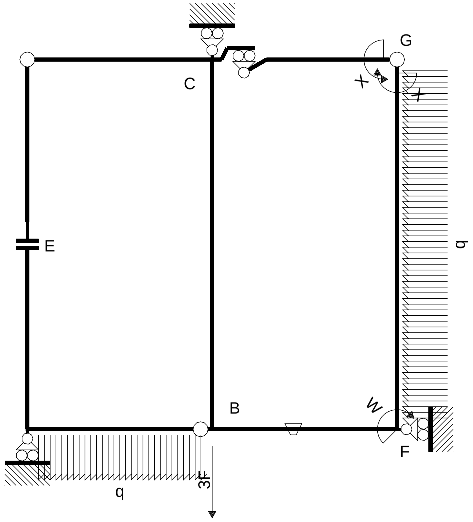
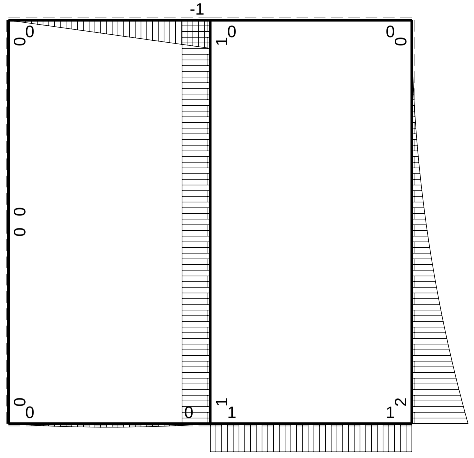
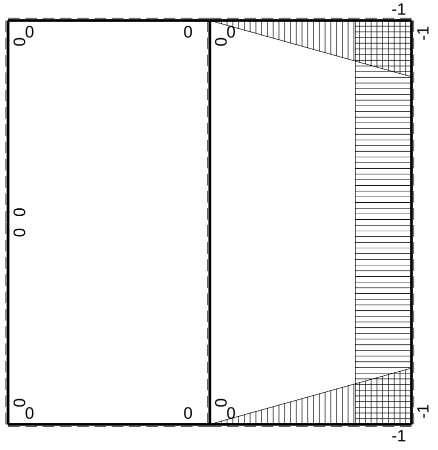
0
0
0
0
1
0
1
1/2
3/2
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

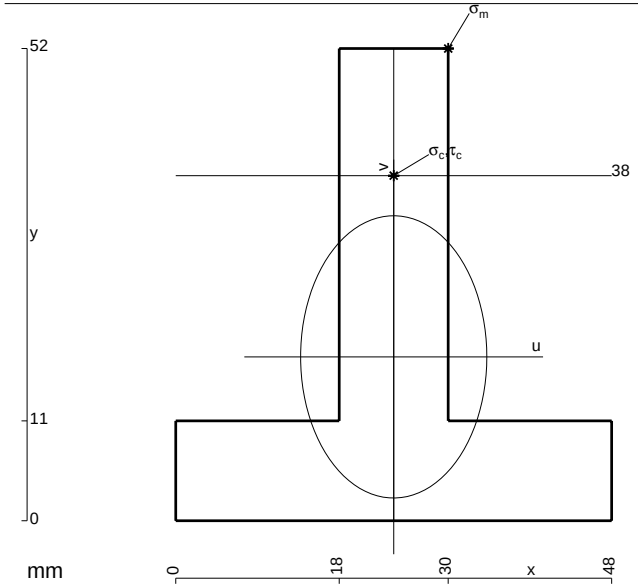
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1020. mm^2
J_u = 246410. mm^4
J_v = 107280. mm^4
y_g = 18.04 mm
T_y = 3140. N
M_x = -1664200. Nmm
x_m = 30. mm
y_m = 52. mm
u_m = 6. mm
v_m = 33.96 mm
$\sigma_m = -Mv/J_u$ = 229.4 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.96 mm
$\sigma_c = -Mv/J_u$ = 134.8 N/mm^2
τ_c = 4.81 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.1 N/mm^2
S^* = 4529. mm^3

ESΣ8.143

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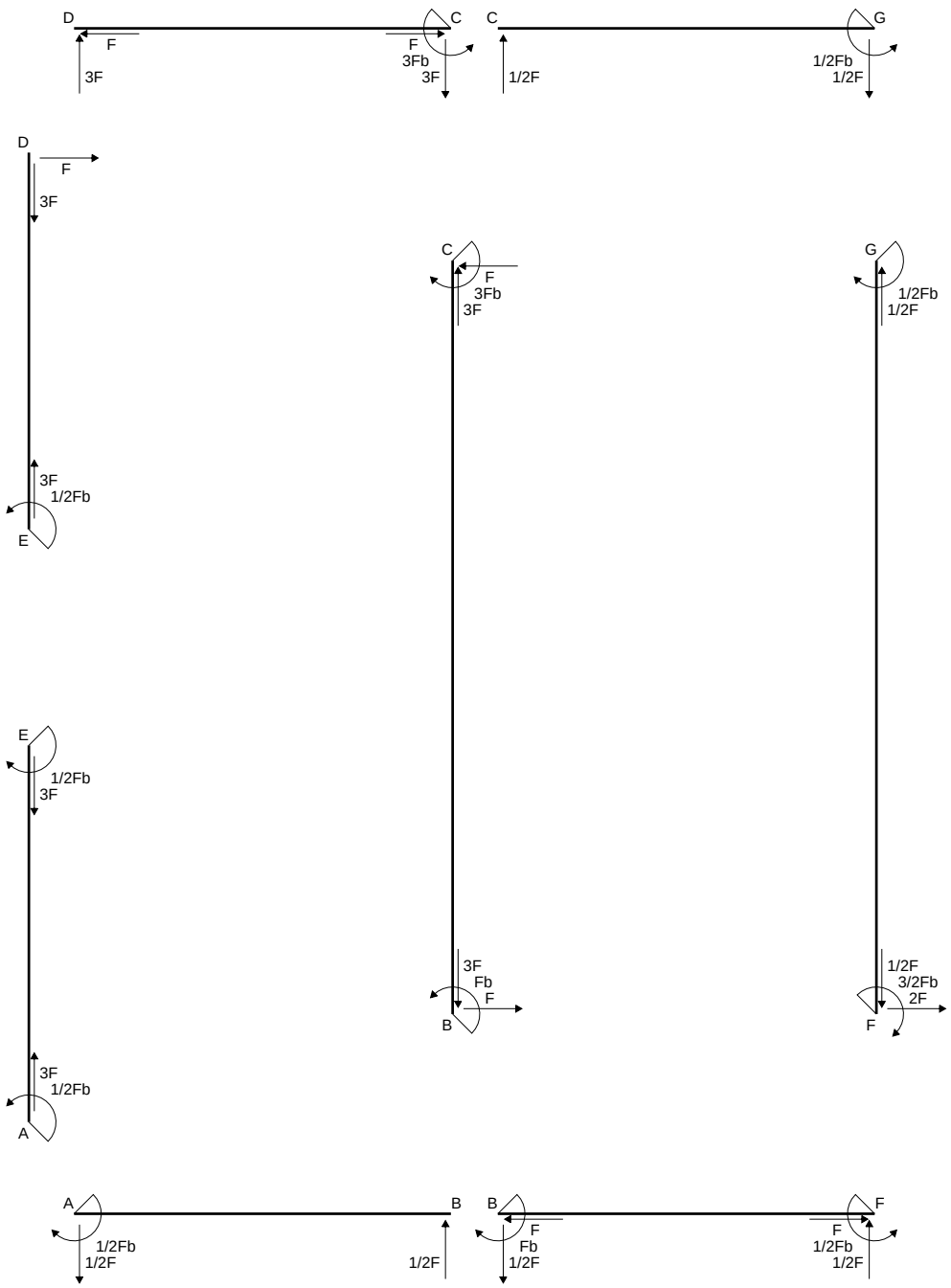
D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
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C
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3Fb
3F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
3F
Fb
F
B
1/2F
3/2Fb
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F
3F
1/2Fb
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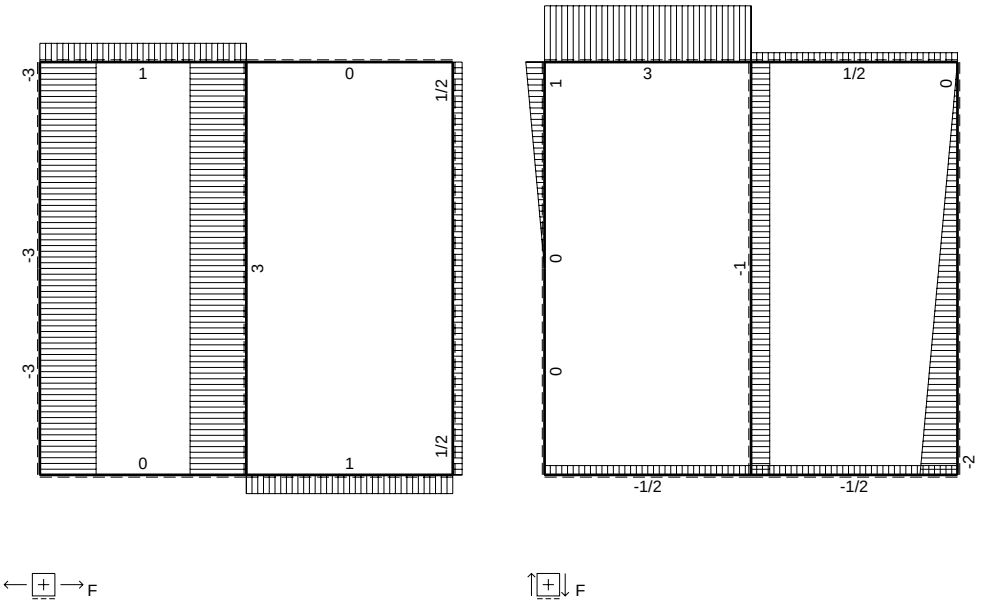
F
1/2Fb
1/2F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1/2
-3
3
-3
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1
1/2
F
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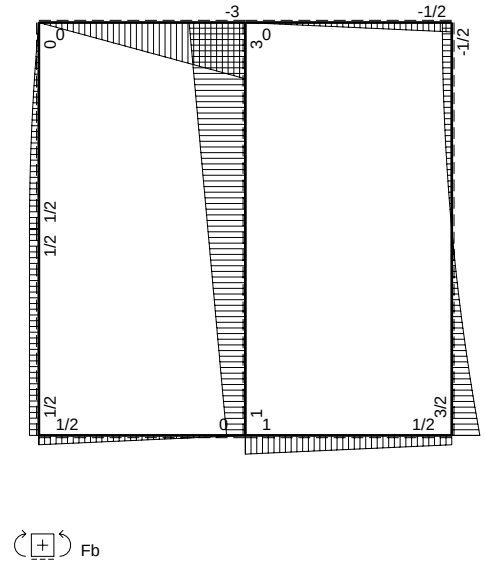
3
1/2
0
0
0
-1
-2
-1/2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x θ$	$M_x M_x$	$\int M_x(M_0/EJ+θ)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2XbEJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3XbEJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

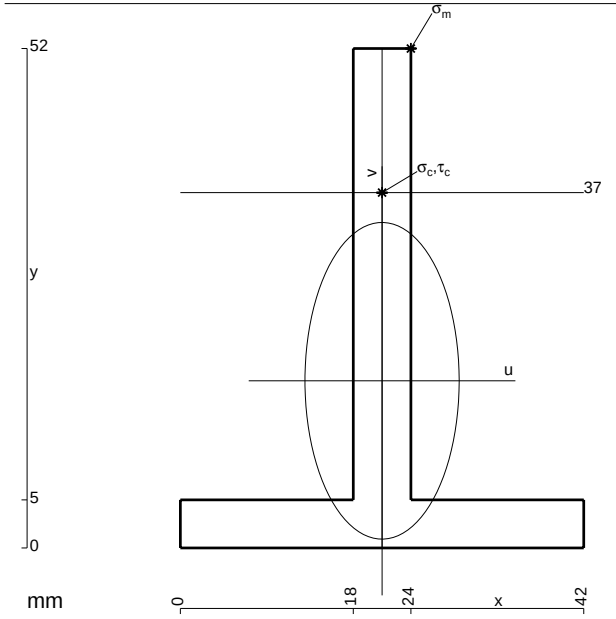
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 492. mm^2
J_u = 133716. mm^4
J_v = 31716. mm^4
y_g = 17.4 mm
N = 530. N
T_y = 1590. N
M_x = -906300. Nmm
x_m = 24. mm
y_m = 52. mm
u_m = 3. mm
v_m = 34.6 mm
$\sigma_m = N/A - Mv/J_u$ = 235.6 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 19.6 mm
$\sigma_c = N/A - Mv/J_u$ = 133.9 N/mm^2
τ_c = 4.833 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 134.2 N/mm^2
S^* = 2439. mm^3

ESΣ8.144

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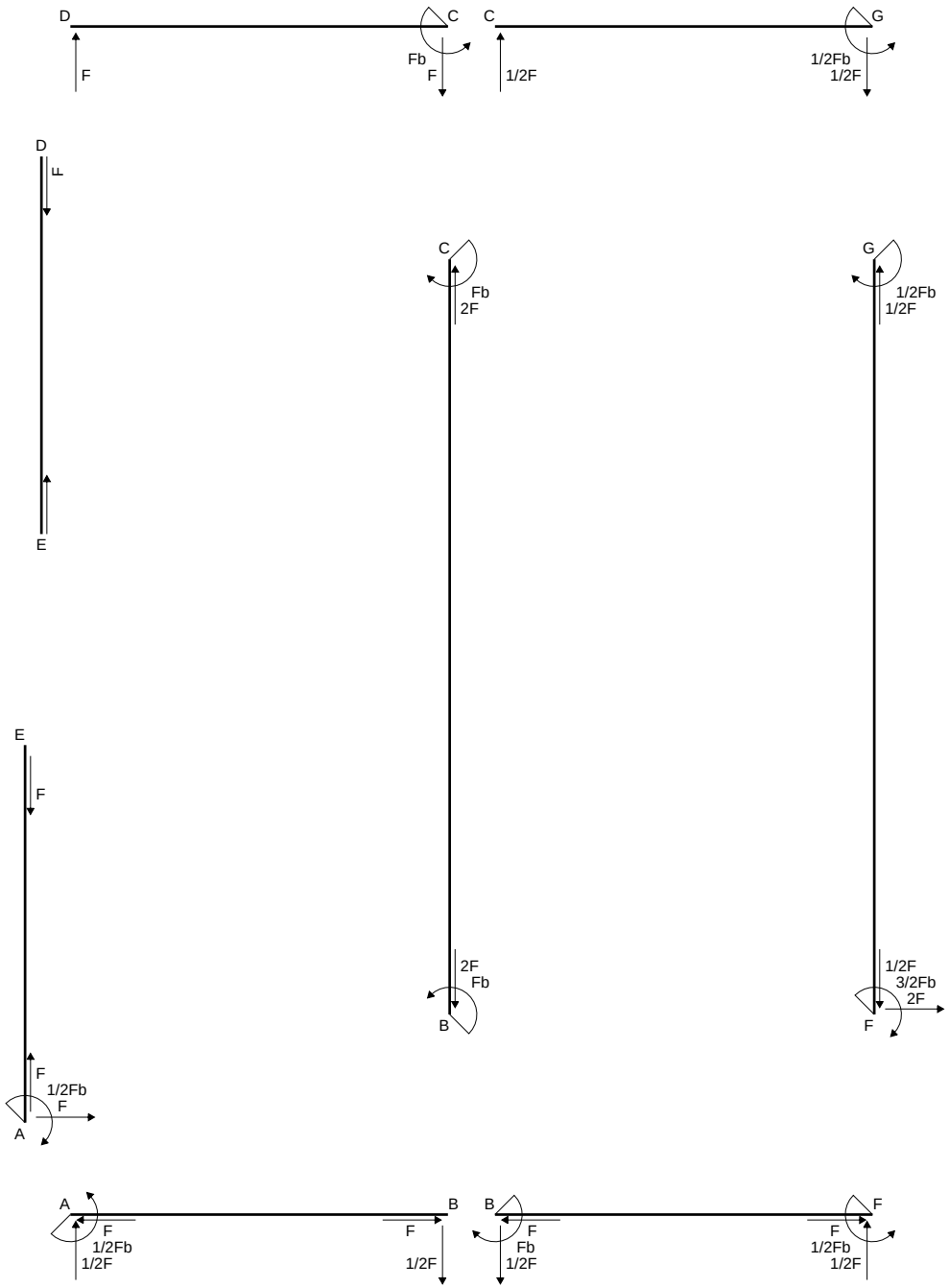
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1/2Fb
1/2F
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Fb
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1/2F
2F
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3/2Fb
2F
F
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F
F
1/2Fb
F
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1/2F
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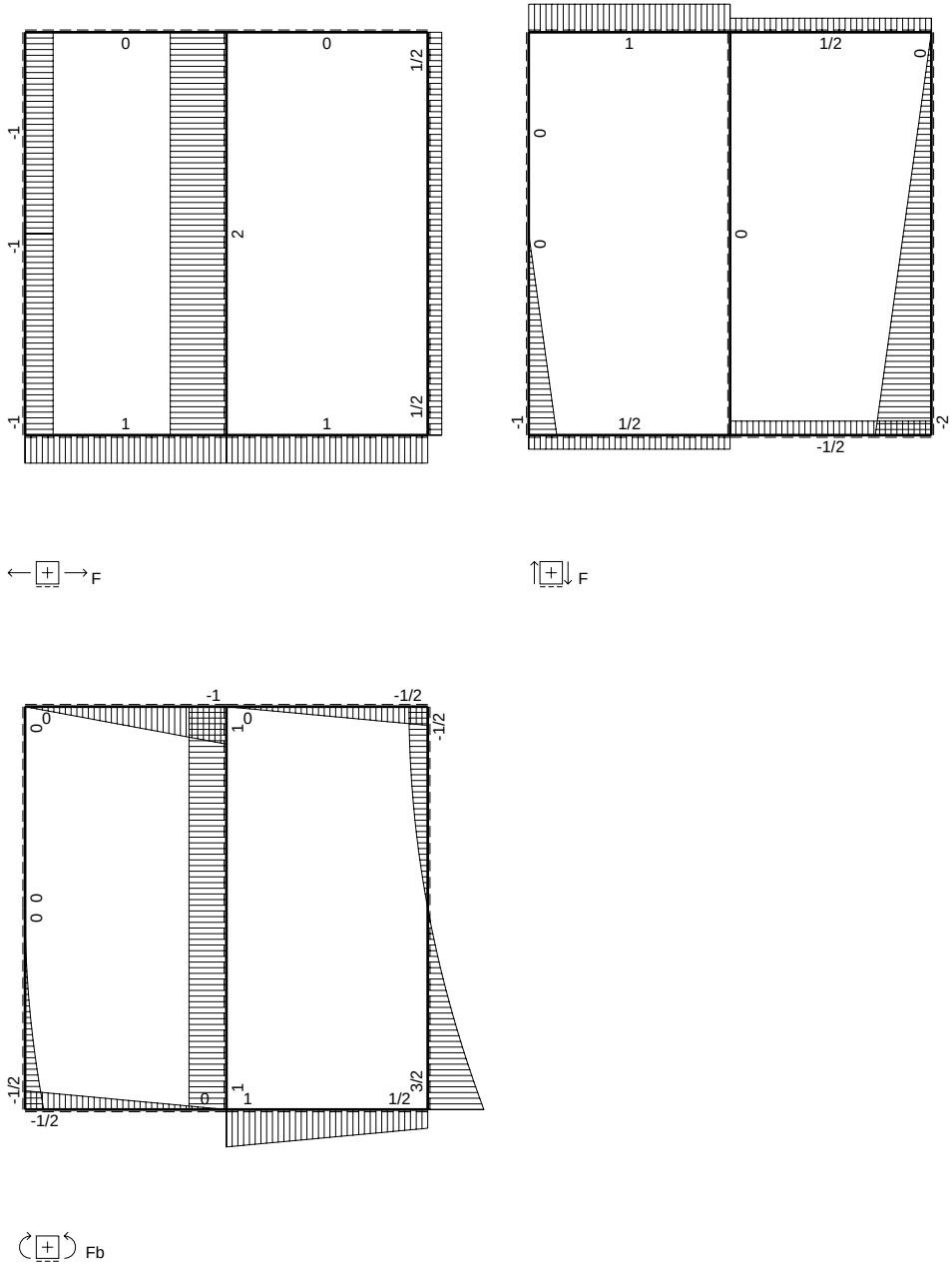
1/2Fb
1/2F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-2
-1/2
F
-1
-1/2
0
0
1
0
-1/2
0 0
-1/2
0
1
1
1/2
3/2
-1/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

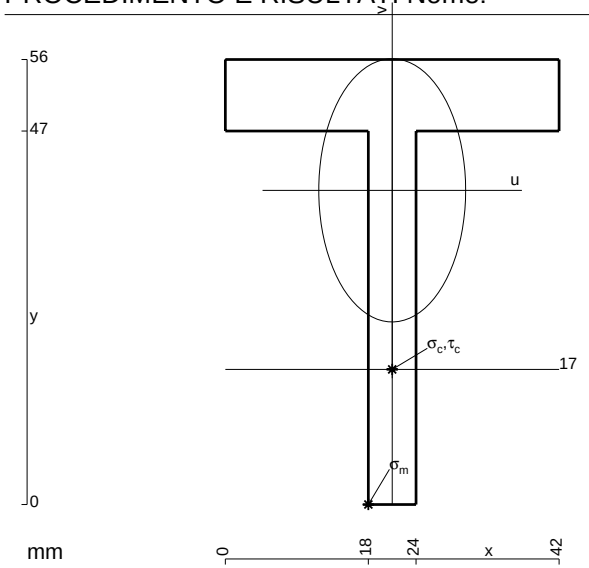
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 660. mm^2
J_u = 181086. mm^4
J_v = 56412. mm^4
y_g = 39.54 mm
T_y = 1380. N
M_x = -910800. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.54 mm
$\sigma_m = -Mv/J_u$ = -198.9 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.54 mm
$\sigma_c = -Mv/J_u$ = -113.3 N/mm^2
τ_c = 4.021 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 113.6 N/mm^2
S^* = 3166. mm^3

ESΣ8.145

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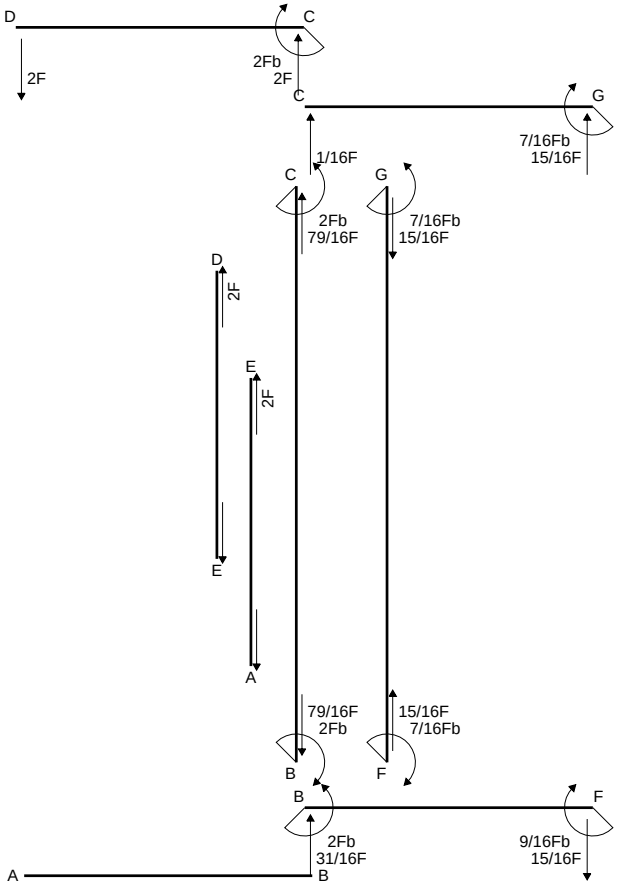
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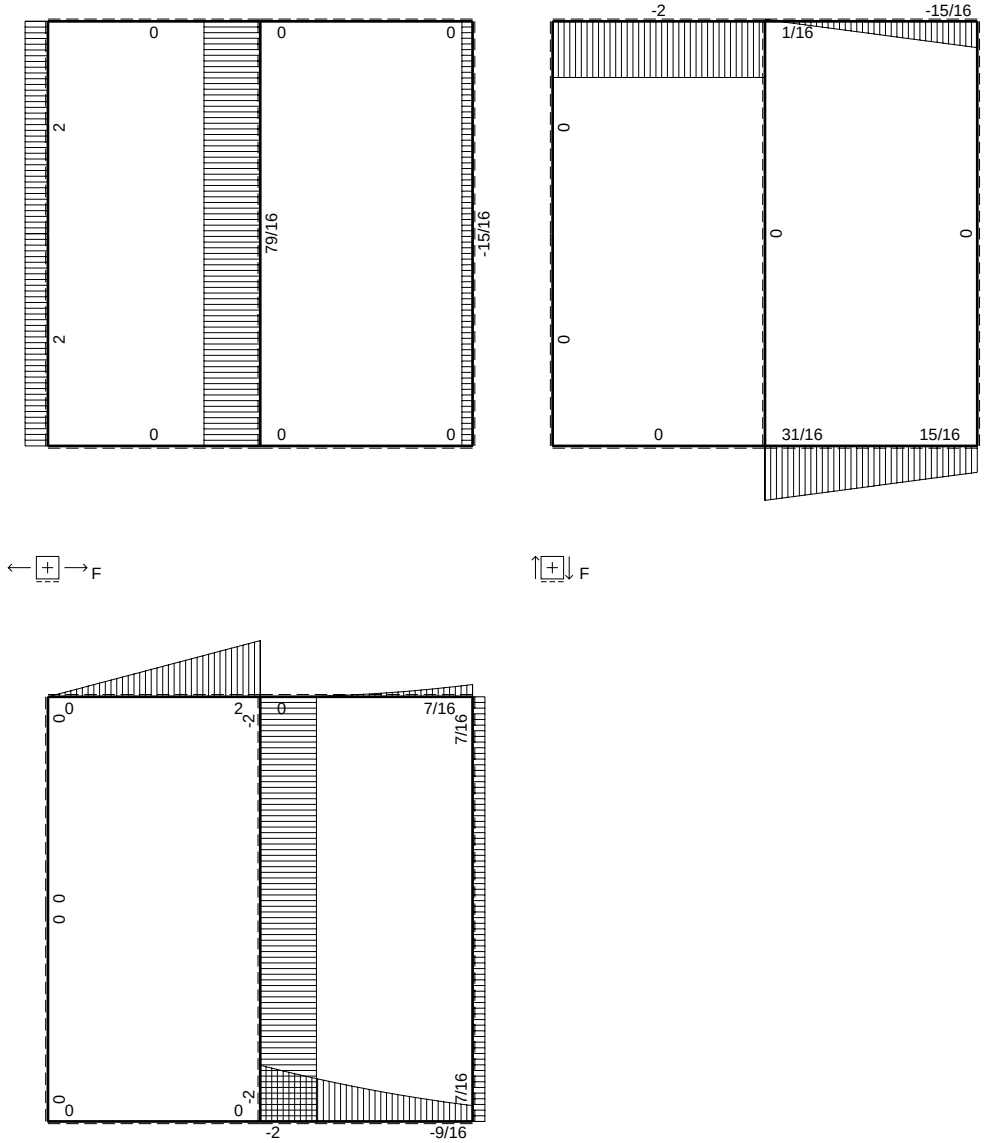
C
2Fb
2F
2F
C
G
7/16Fb
15/16F
1/16F
C
G
2Fb
79/16F
7/16Fb
15/16F
D
2F
E
2F
E
A
79/16F
2Fb
15/16F
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B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79/16
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0
0
31/16
15/16
F
F
0
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

14.11.23

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

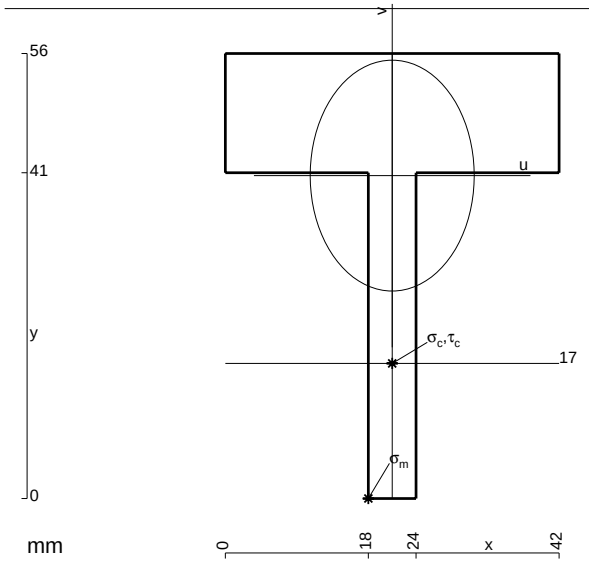
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.146



A = 876. mm^2
J_u = 184977. mm^4
J_v = 93348. mm^4
y_g = 40.64 mm
T_y = -1360. N
M_x = 952000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -40.64 mm
$\sigma_m = -Mv/J_u$ = 209.1 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -23.64 mm
$\sigma_c = -Mv/J_u$ = 121.7 N/mm^2
τ_c = 4.017 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 121.8 N/mm^2
S^* = 3278. mm^3

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ESΣ8.146

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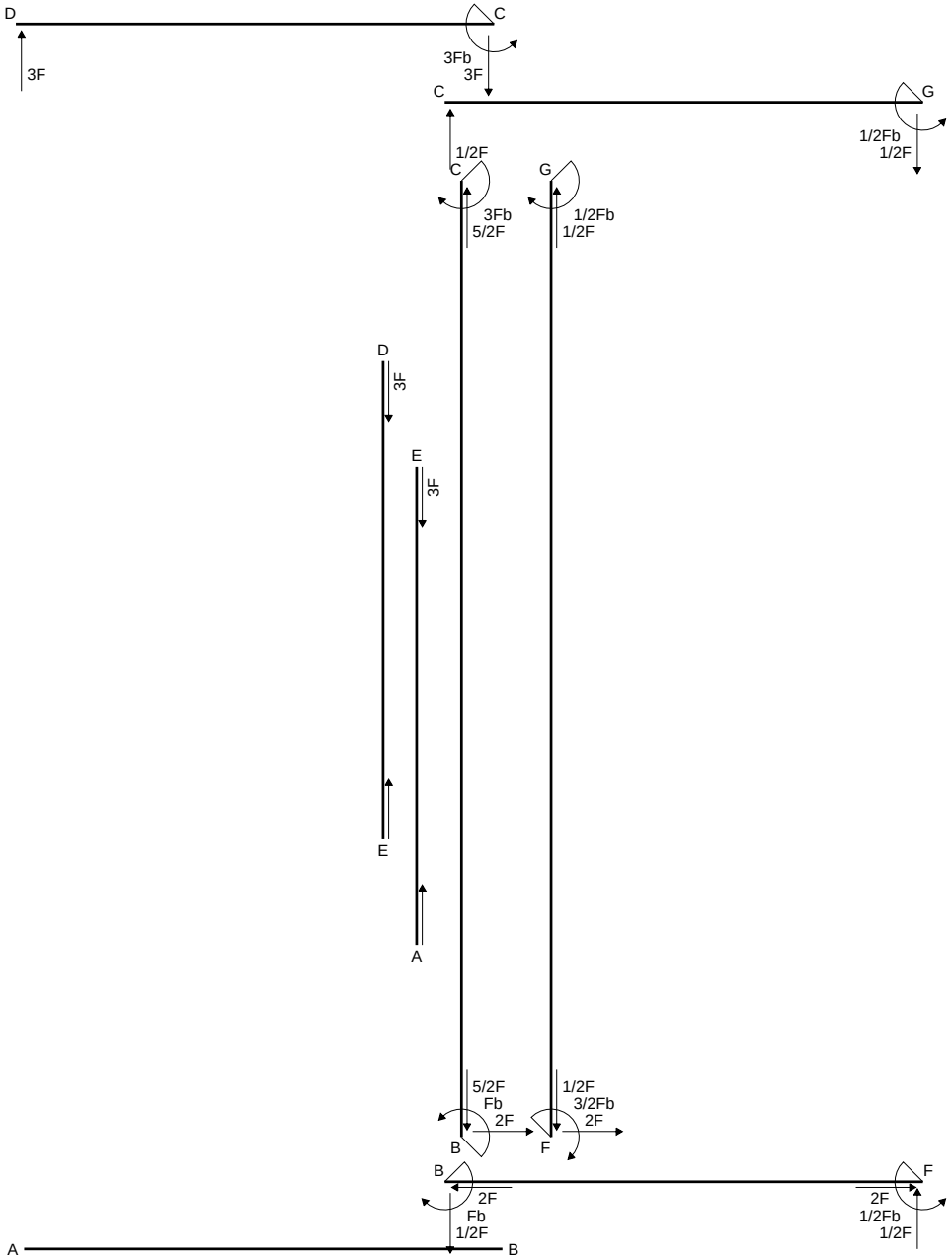
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3Fb
3F
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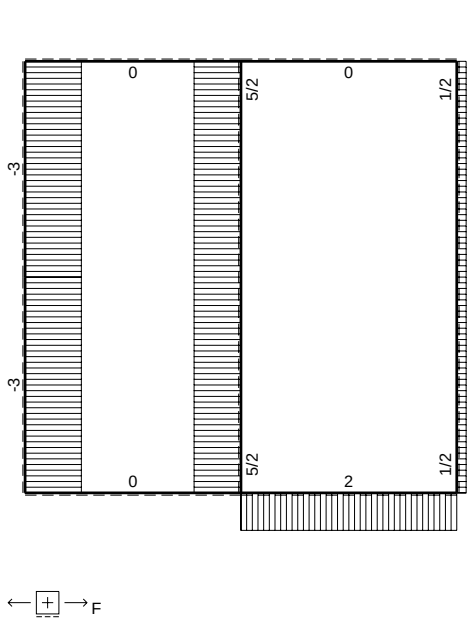
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B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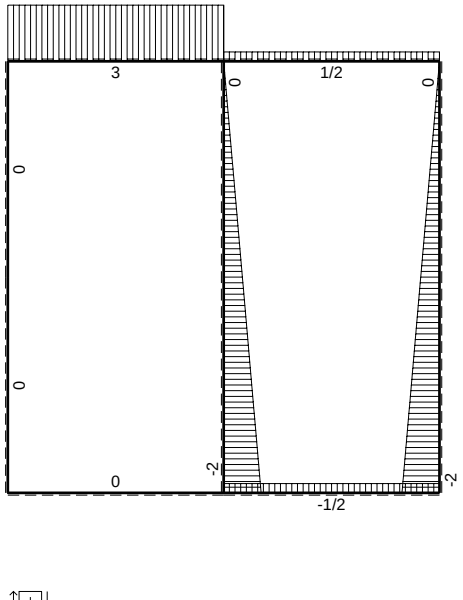
0
1/2
-3
-3
0
5/2
2
1/2
F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0
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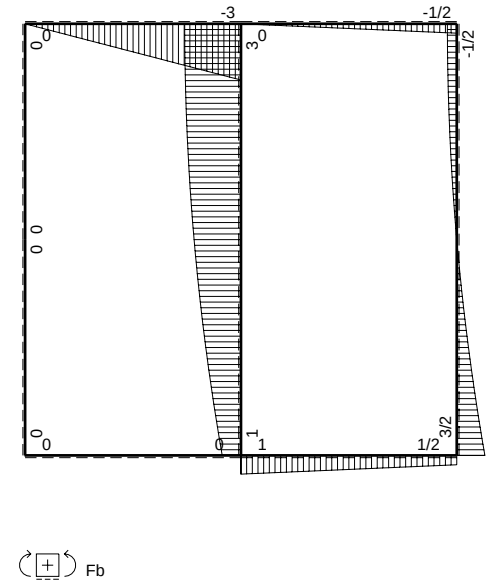
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

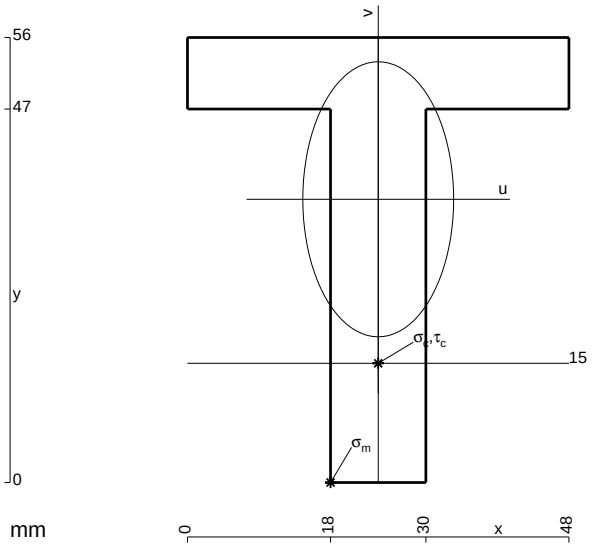
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 996. mm^2
J_u = 298526. mm^4
J_v = 89712. mm^4
y_g = 35.64 mm
T_y = 2460. N
M_x = -1820400. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.64 mm
$\sigma_m = -Mv/J_u$ = -217.4 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.64 mm
$\sigma_c = -Mv/J_u$ = -125.9 N/mm^2
τ_c = 3.479 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126. N/mm^2
S^* = 5066. mm^3

ESΣ8.147

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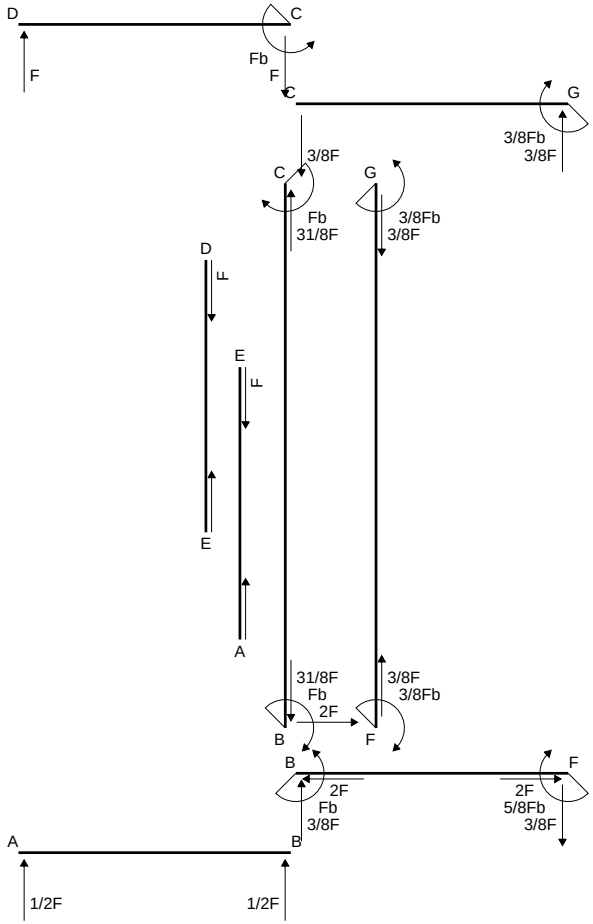
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G
Fb
31/8F
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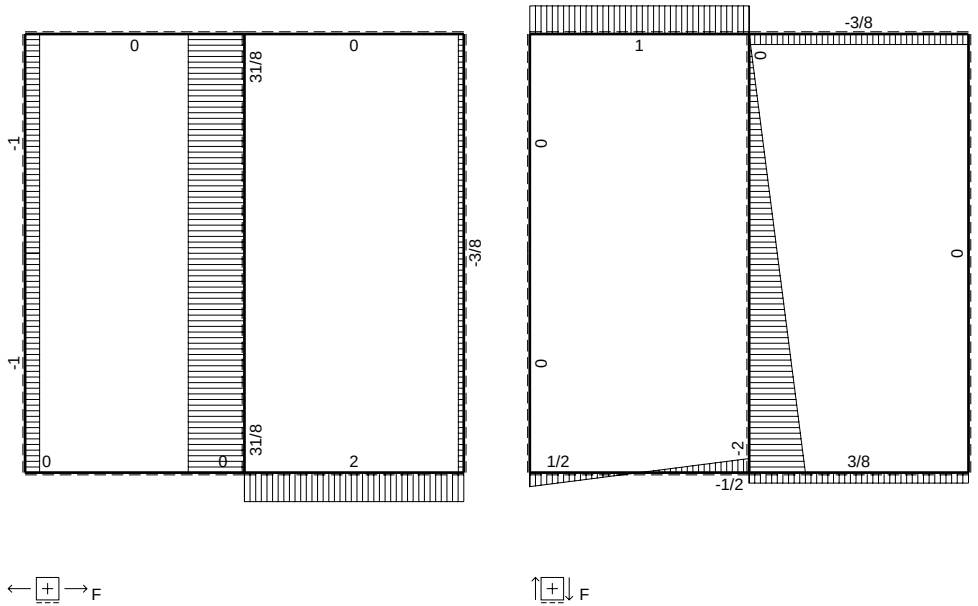
B
1/2F
1/2F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-3/8
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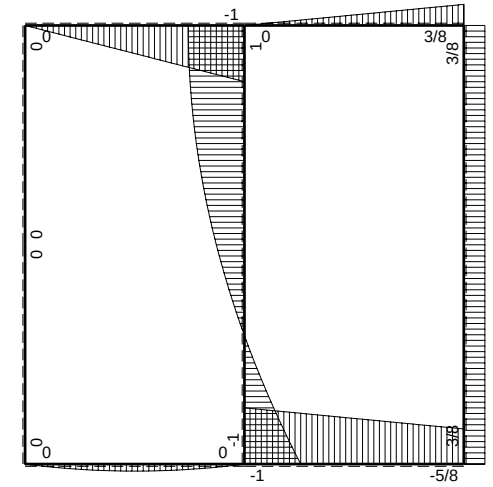
3/8
-1/2
F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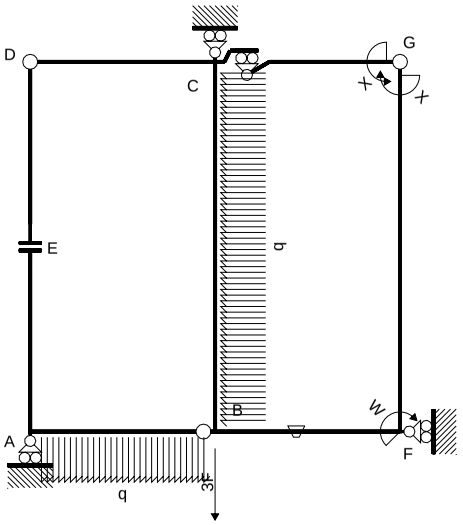
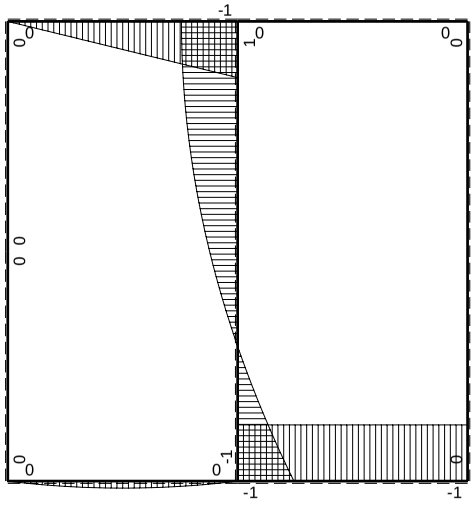
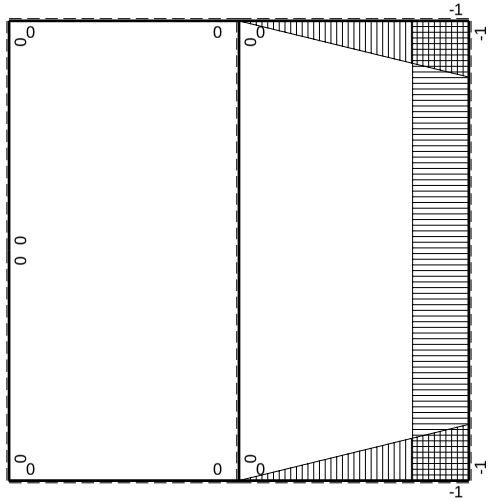
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
						totali	Fb^2/EJ	8/3Xb/EJ
						iperstatica $X=W_{GC}$	-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.148

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

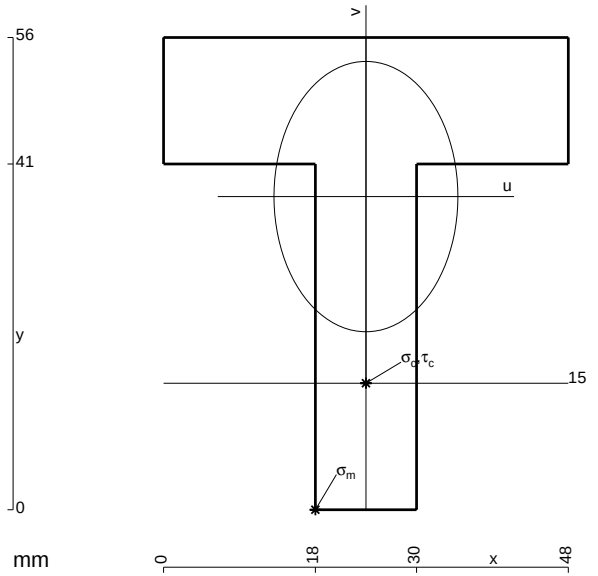
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1212. mm^2
J_u = 311566. mm^4
J_v = 144144. mm^4
y_g = 37.13 mm
N = 9765. N
T_y = -5040. N
M_x = -1990800. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -37.13 mm
$\sigma_m = N/A - Mv/J_u$ = -229.2 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -22.13 mm
$\sigma_c = N/A - Mv/J_u$ = -133.4 N/mm^2
τ_c = 7.19 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 133.9 N/mm^2
S^* = 5334. mm^3

ESΣ8.148

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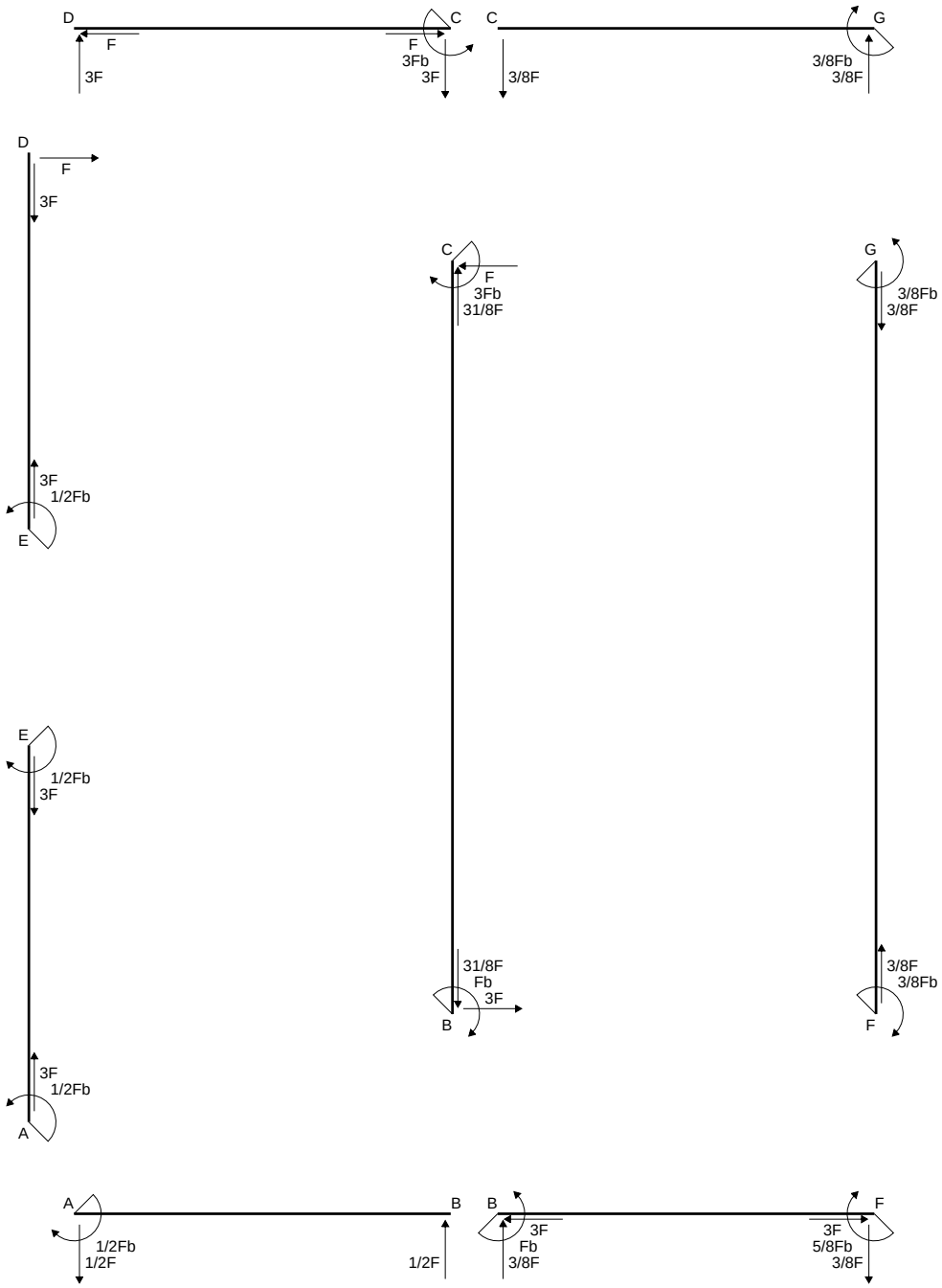
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3/8F
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3F
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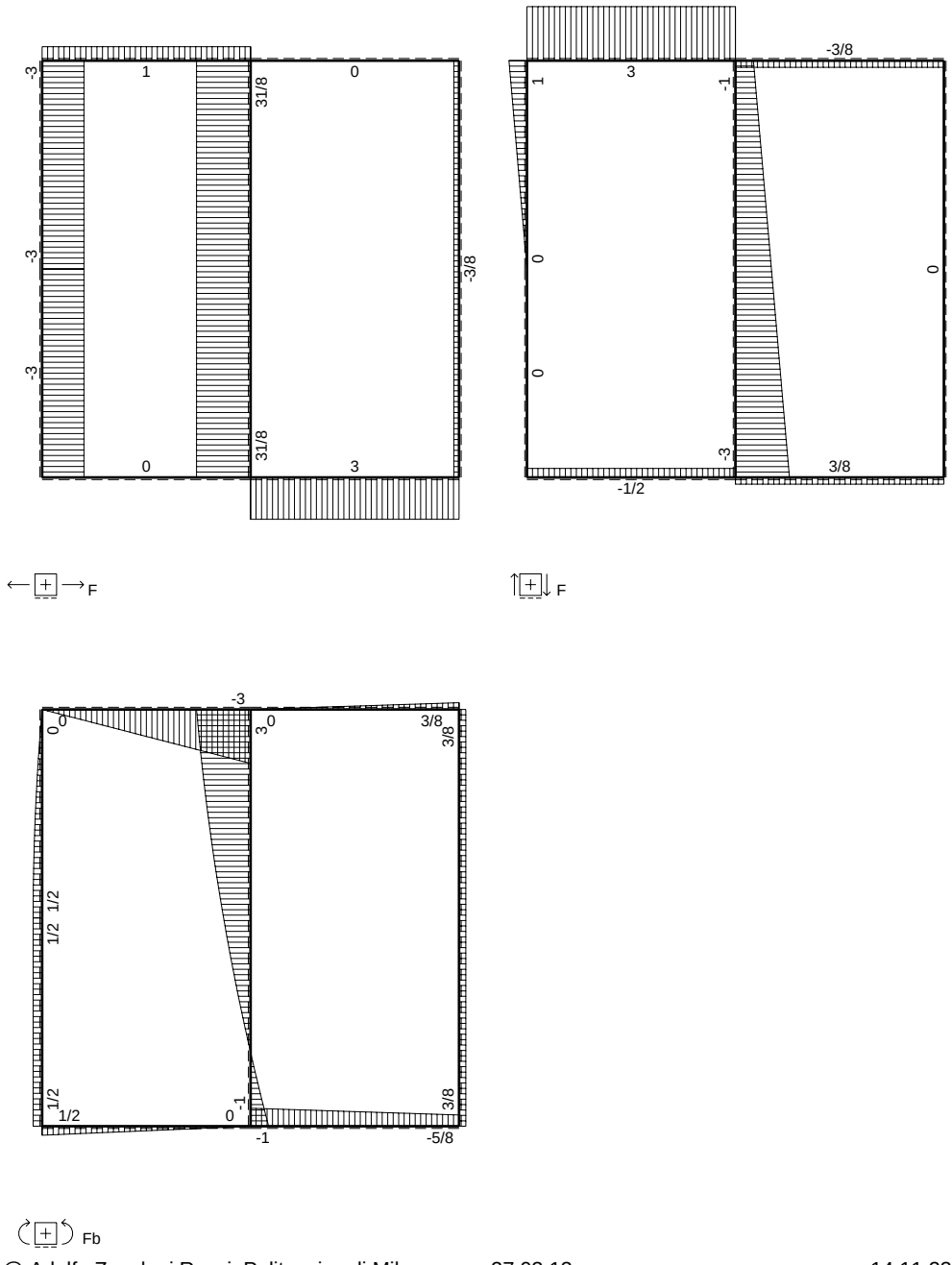
3/8F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-1/2
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F
-3
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0
3/8
3/8
1/2
1/2
1/2
1/2
0
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

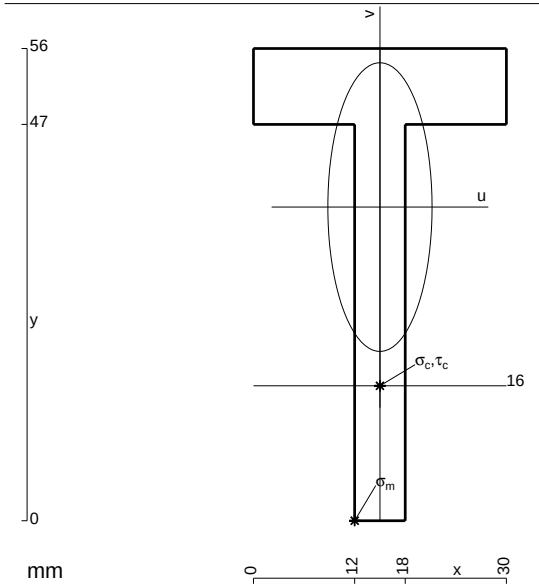
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 552. mm^2
J_u = 161875. mm^4
J_v = 21096. mm^4
y_g = 37.2 mm
N = 420. N
T_y = 1260. N
M_x = -1045800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.2 mm
$\sigma_m = N/A - Mv/J_u$ = -239.5 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.2 mm
$\sigma_c = N/A - Mv/J_u$ = -136.2 N/mm^2
τ_c = 3.636 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.3 N/mm^2
S^* = 2803. mm^3

ESΣ8.149

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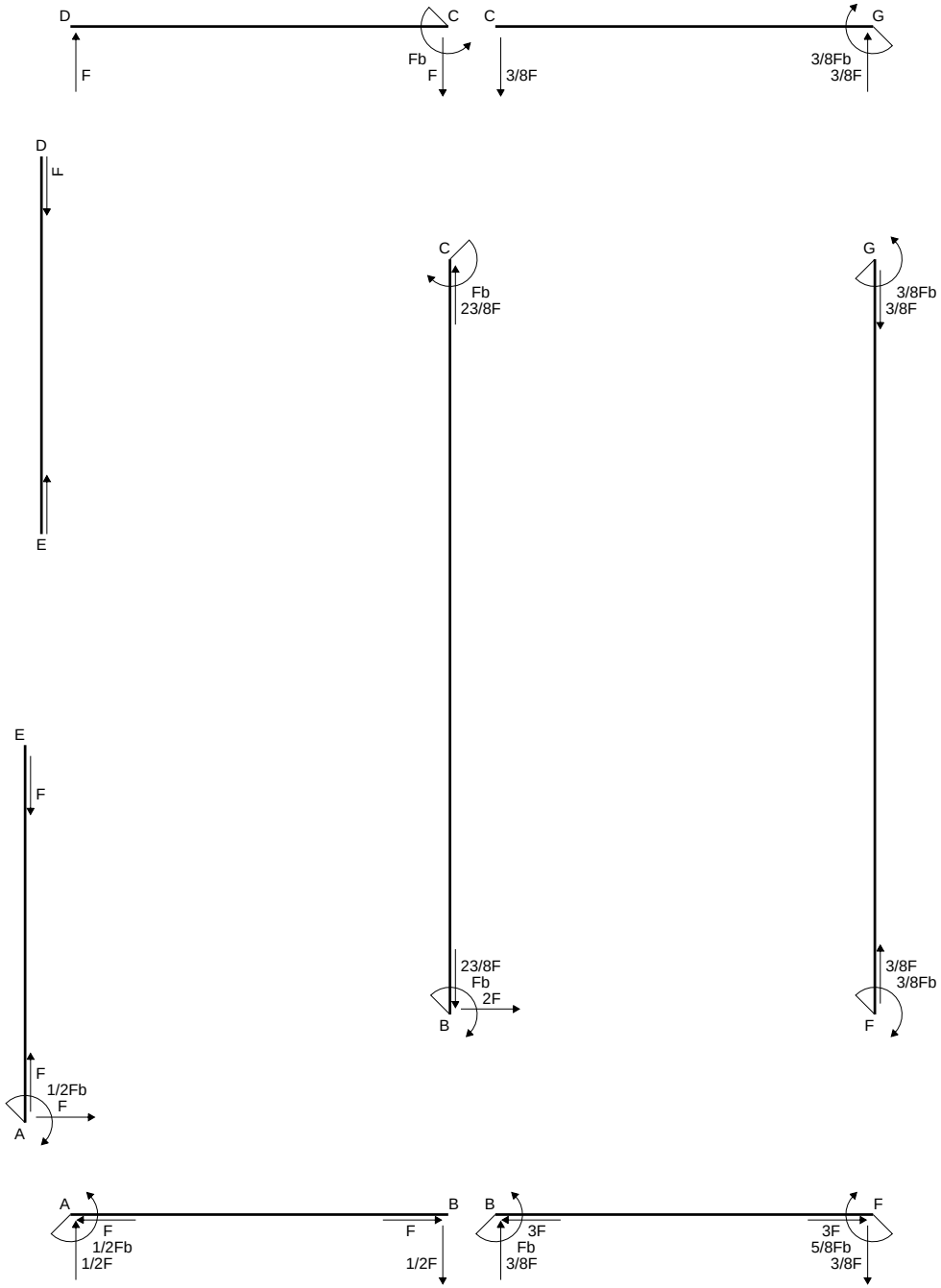
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3/8F
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Fb
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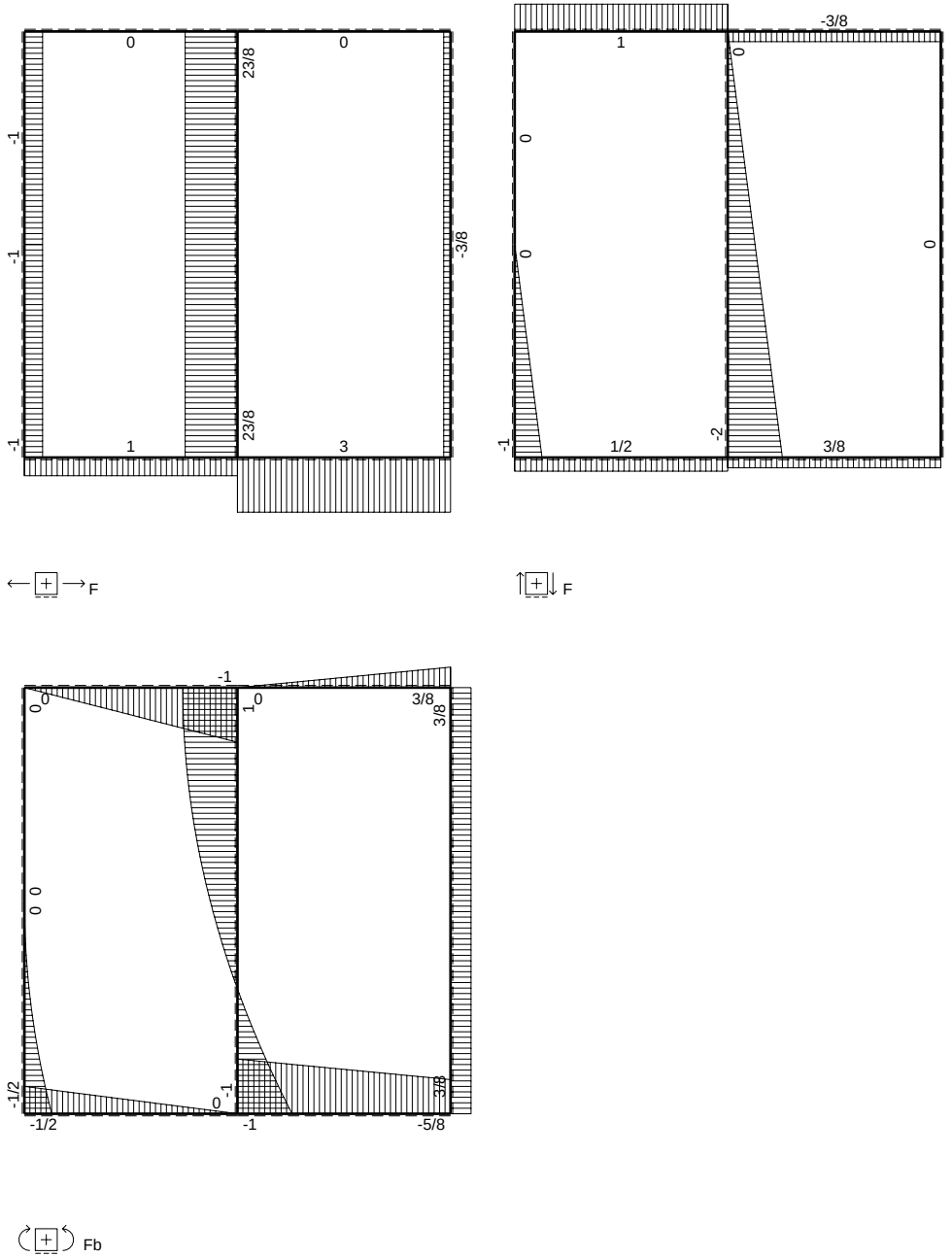
Fb
3/8F
3F
5/8Fb
3/8F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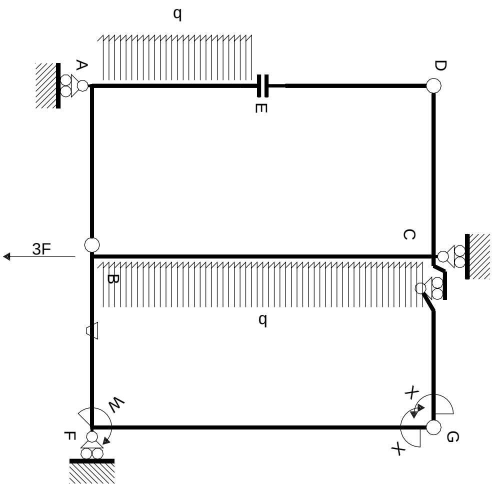
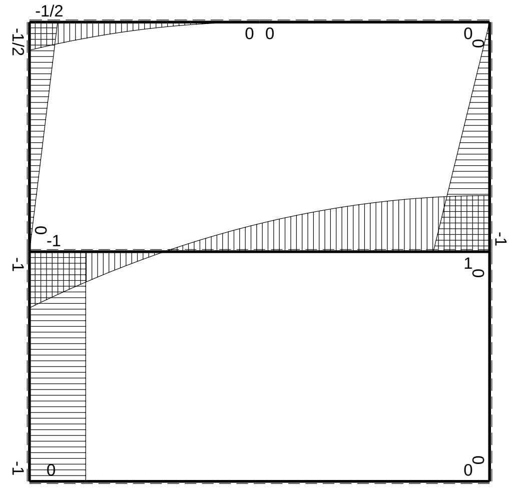
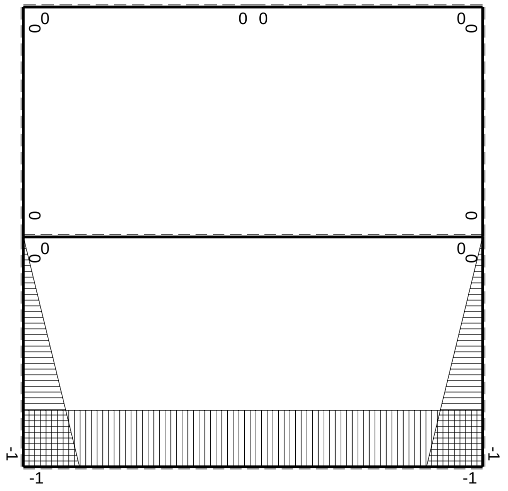
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

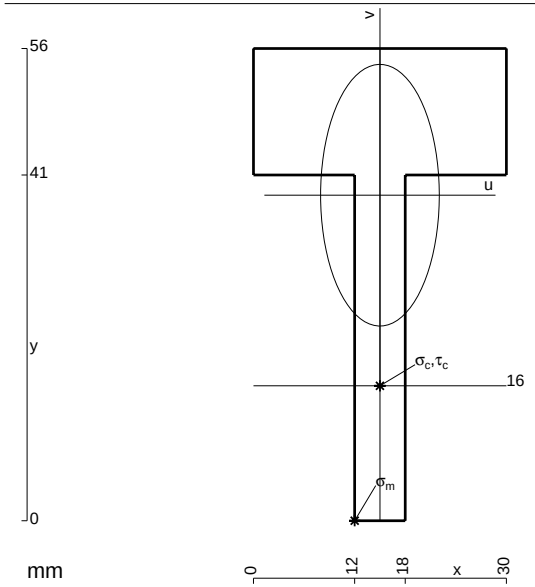
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 696. mm^2
J_u = 167595. mm^4
J_v = 34488. mm^4
y_g = 38.6 mm
N = 5894. N
T_y = -4100. N
M_x = -902000. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -38.6 mm
$\sigma_m = N/A - Mv/J_u$ = -199.3 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -22.6 mm
$\sigma_c = N/A - Mv/J_u$ = -113.2 N/mm^2
τ_c = 11.98 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.1 N/mm^2
S^* = 2938. mm^3

ESΣ8.150

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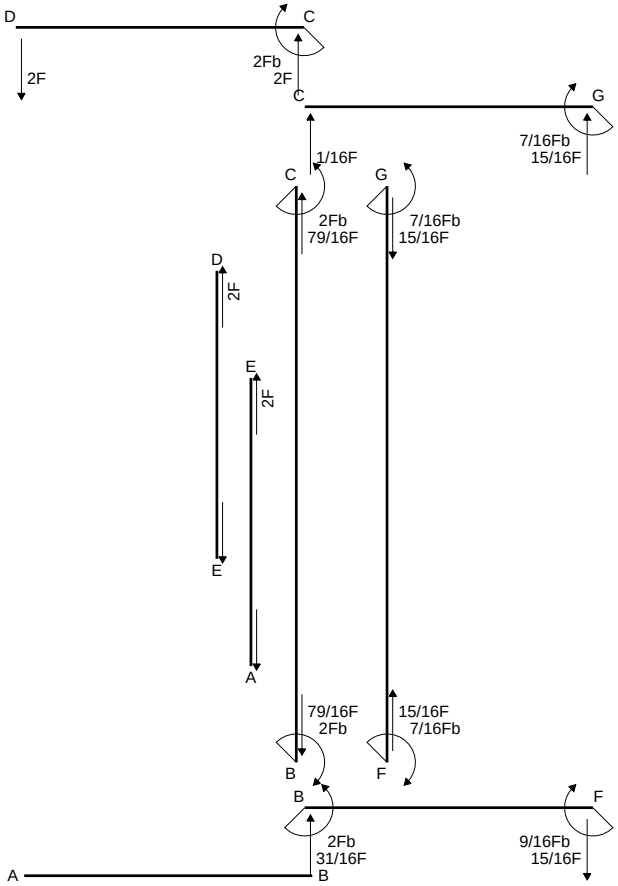
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7/16Fb
B
F
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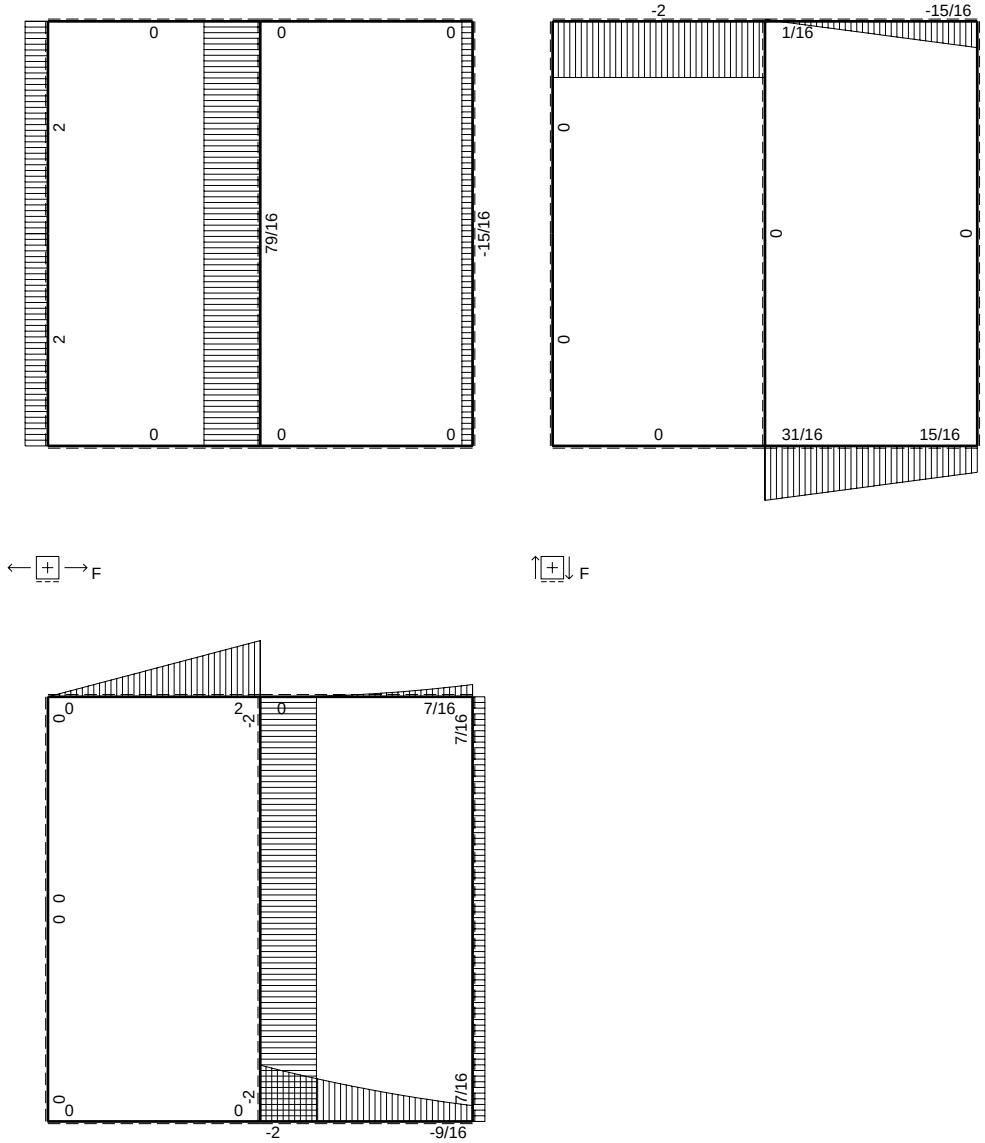
2Fb
31/16F
9/16Fb
15/16F
A
B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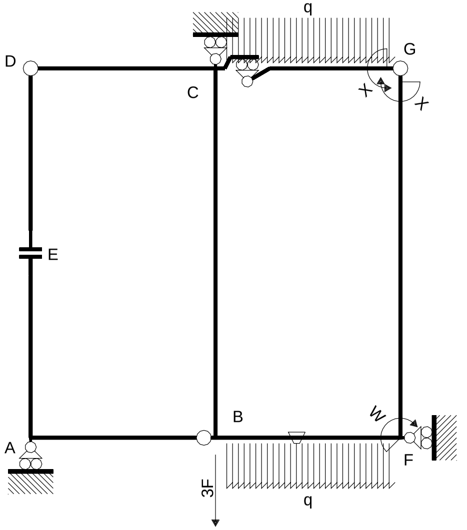
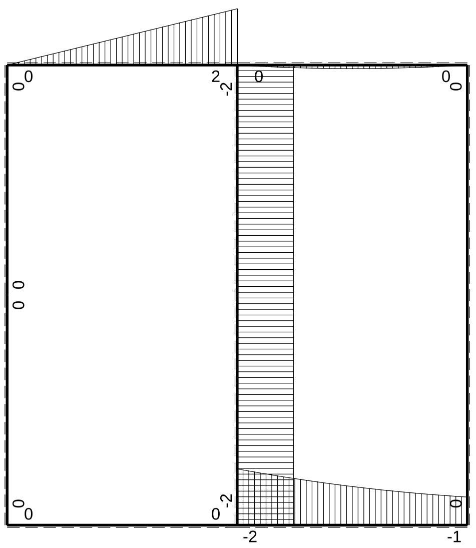
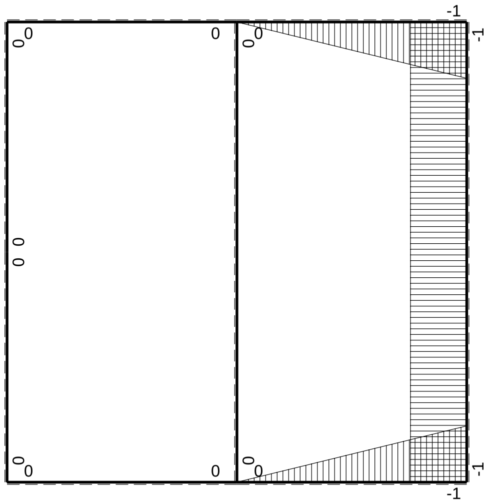
Fb



Schema di calcolo iperstatico



M$_o$ flessione da carichi assegnati



M$_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
totali							$7/6Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

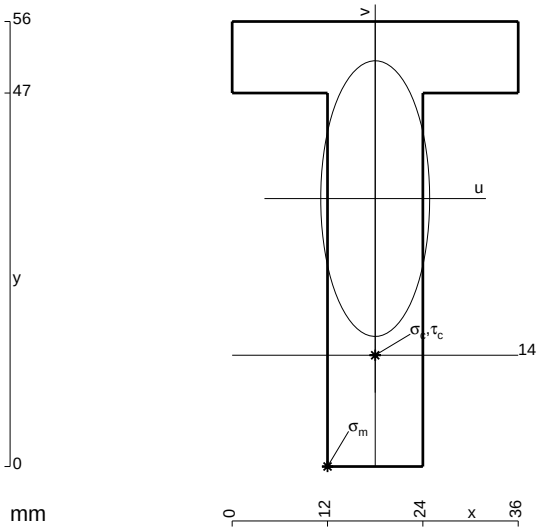
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 888. mm^2
J_u = 267344. mm^4
J_v = 41760. mm^4
y_g = 33.72 mm
T_y = -3460. N
M_x = 1660800. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.72 mm
$\sigma_m = -Mv/J_u$ = 209.5 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.72 mm
$\sigma_c = -Mv/J_u$ = 122.5 N/mm^2
τ_c = 4.841 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 122.8 N/mm^2
S^* = 4488. mm^3

ESΣ8.151

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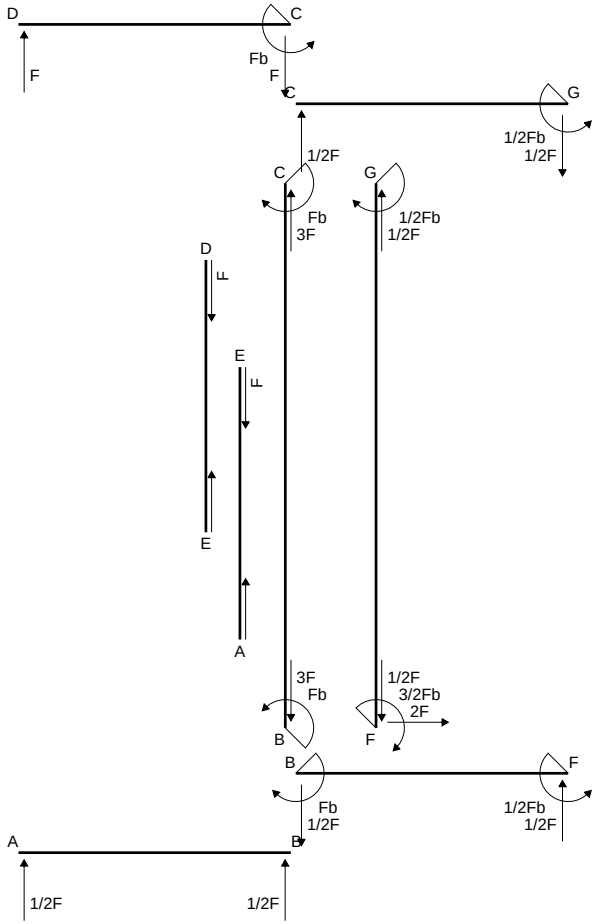
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C
G
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1/2F
1/2F
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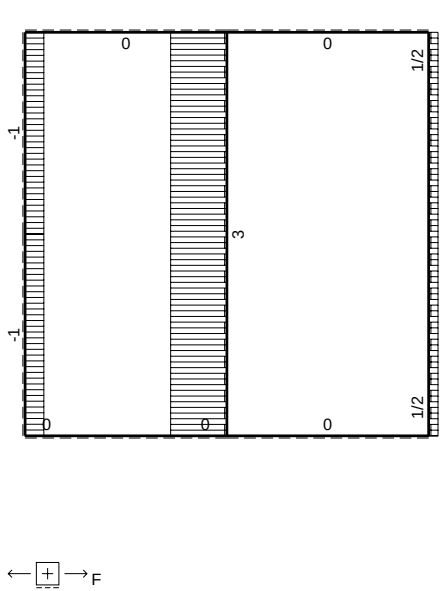
B
1/2F
1/2F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1/2
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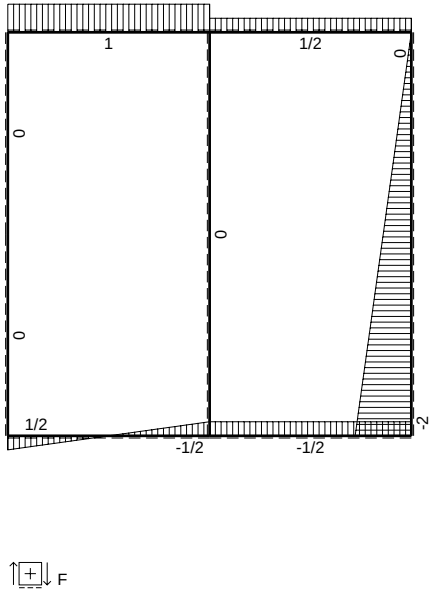
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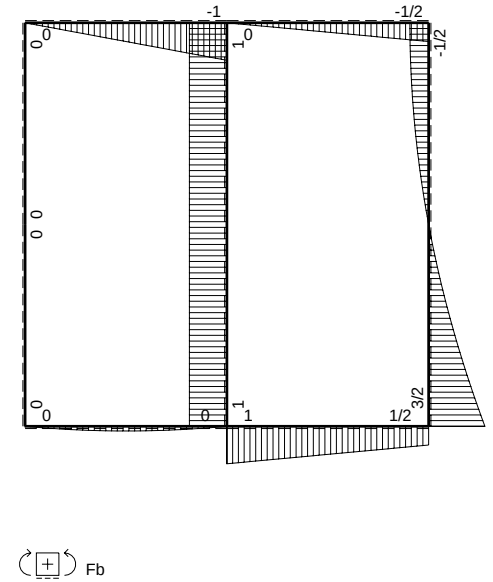
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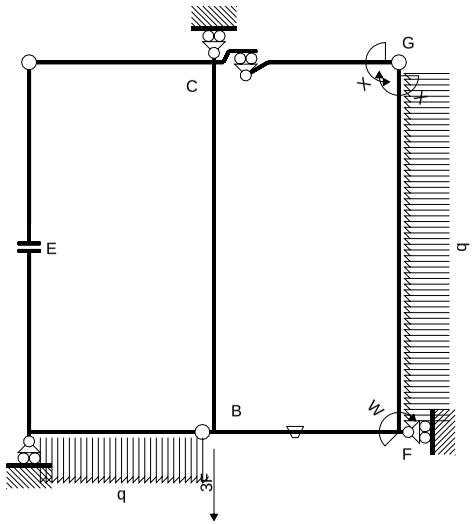
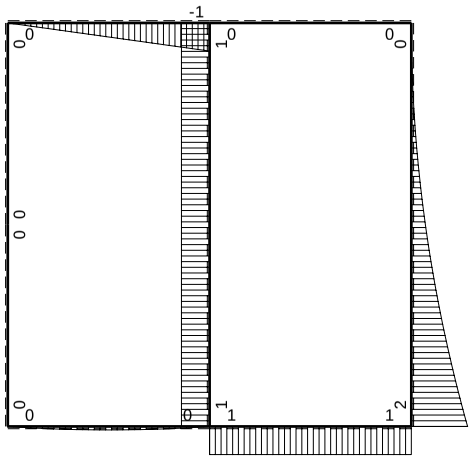
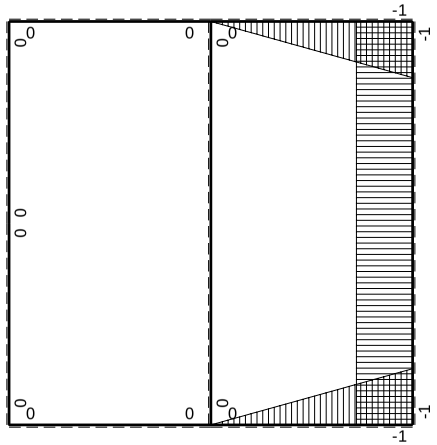
1/2
3/2
Fb





M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fx-1/2qx^2	0	0	0	0	0+0	0
BA b	0	-1/2Fx+1/2qx^2	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
		totali					-4/3Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					1/2Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.152

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

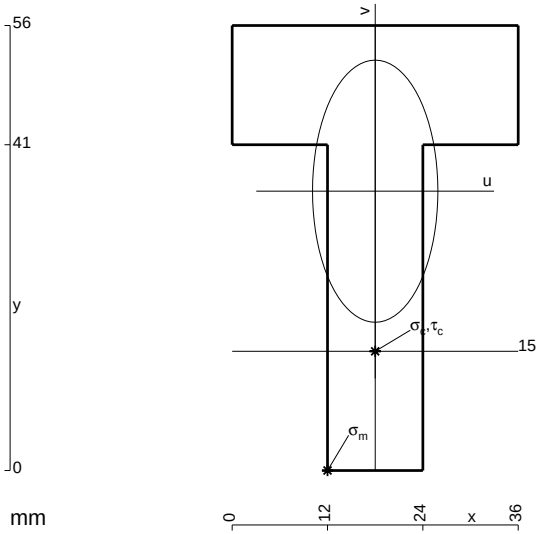
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1032. mm^2
J_u = 280881. mm^4
J_v = 64224. mm^4
y_g = 35.15 mm
T_y = 3380. N
M_x = -1757600. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -35.15 mm
$\sigma_m = -Mv/J_u$ = -220. N/mm^2
x_c = 18. mm
y_c = 15. mm
v_c = -20.15 mm
$\sigma_c = -Mv/J_u$ = -126.1 N/mm^2
τ_c = 4.991 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.4 N/mm^2
S^* = 4977. mm^3

ESΣ8.152

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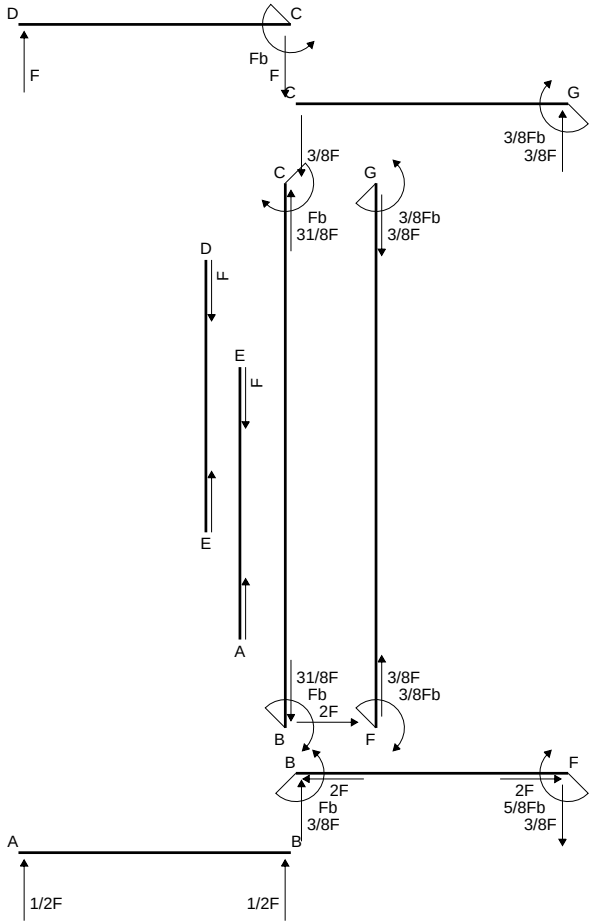
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D
F
E
F
E
A
31/8F
Fb
2F
3/8F
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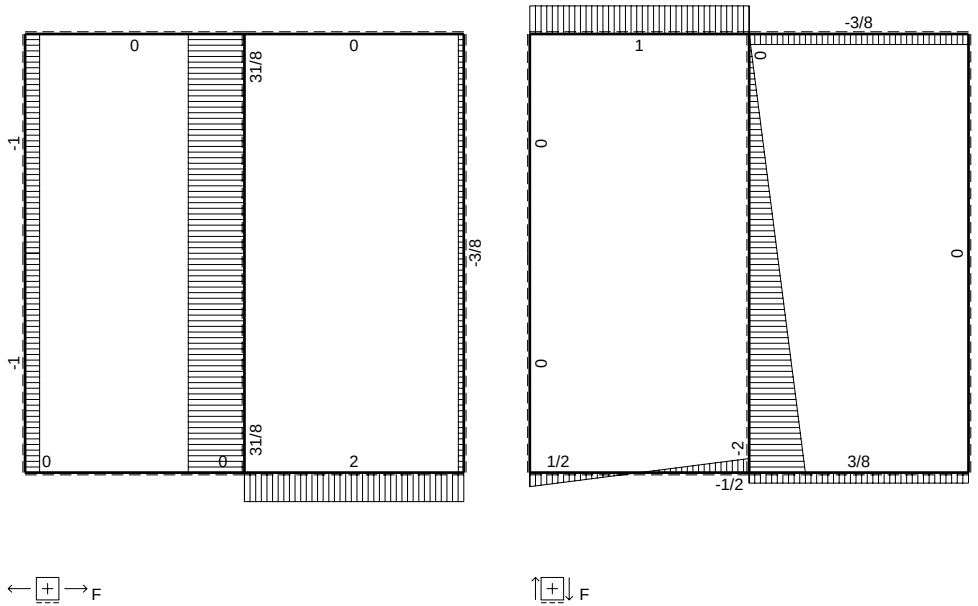
1/2F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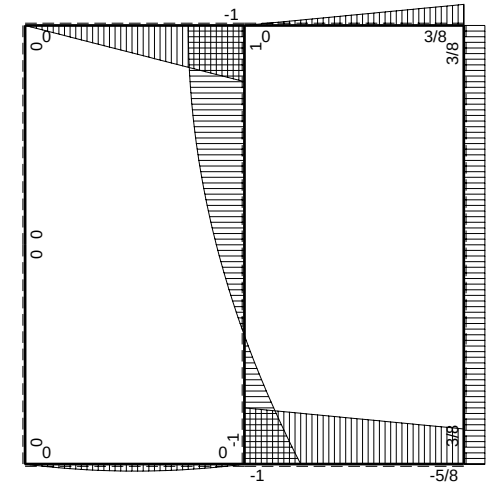
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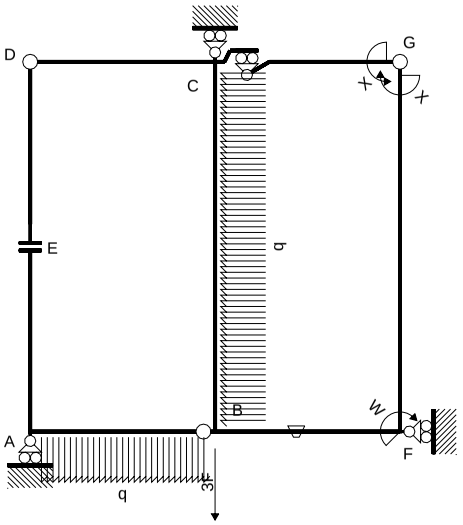
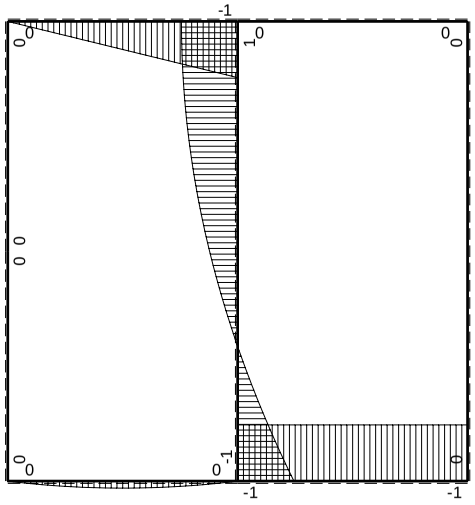
-5/8

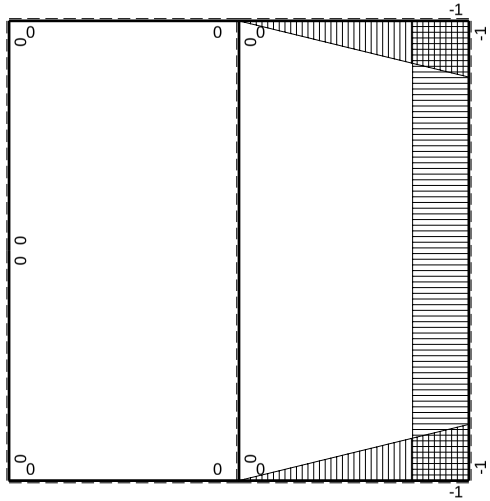
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-1/2qx²	0	0	0	0	0+0	0
BC 2b	0	Fb-2Fx+1/2qx²	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.153

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

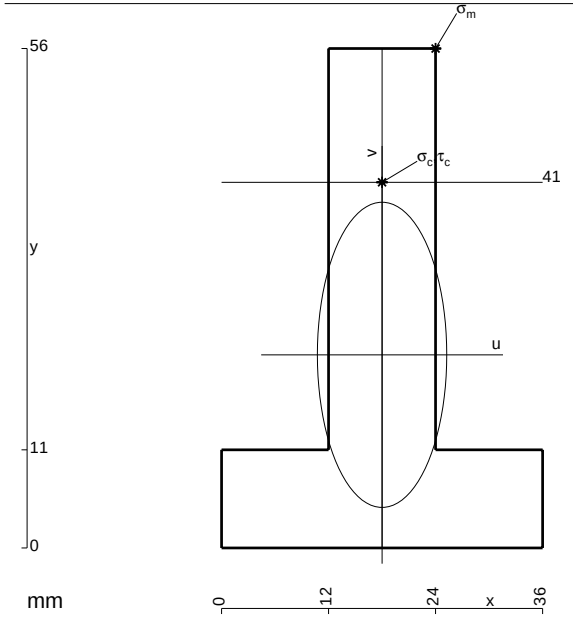
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 936. mm^2
J_u = 274232. mm^4
J_v = 49248. mm^4
y_g = 21.65 mm
N = 11780. N
T_y = -6080. N
M_x = -1732800. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 34.35 mm
$\sigma_m = N/A - Mv/J_u$ = 229.6 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 19.35 mm
$\sigma_c = N/A - Mv/J_u$ = 134.8 N/mm^2
τ_c = 8.928 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.7 N/mm^2
S^* = 4832. mm^3

ESΣ8.153

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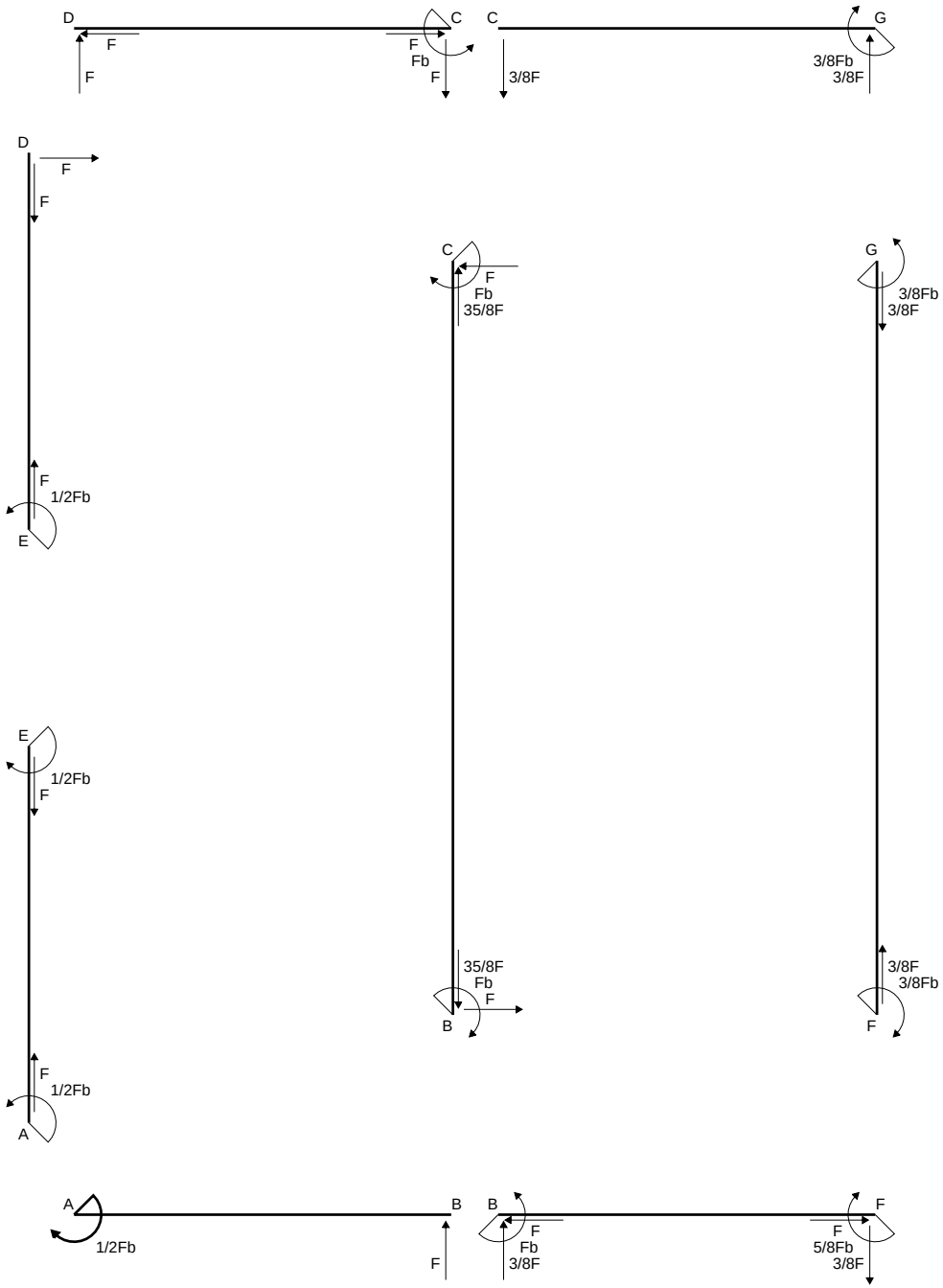
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35/8F
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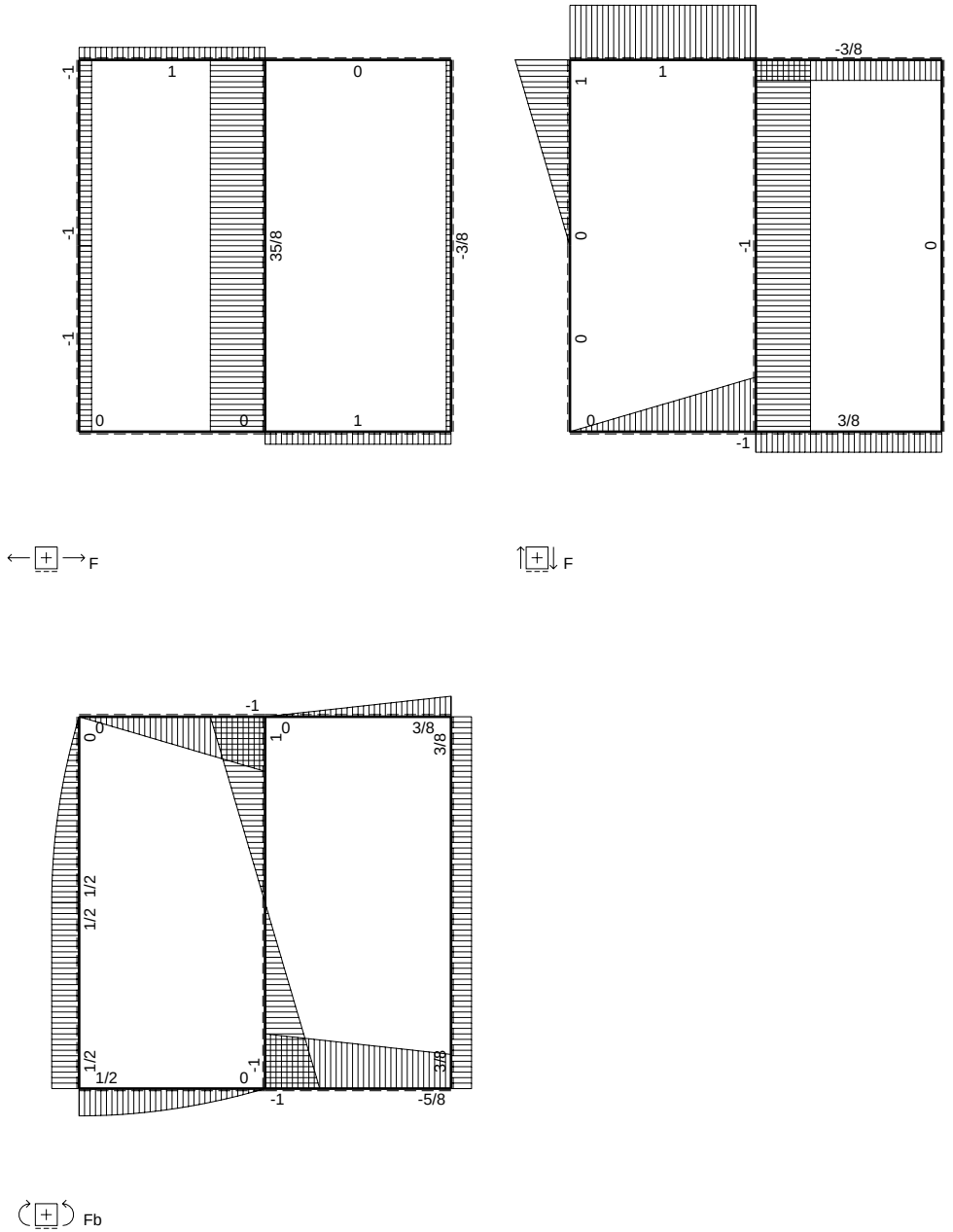
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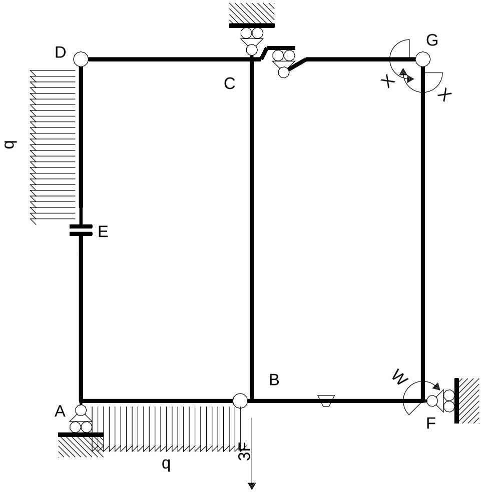
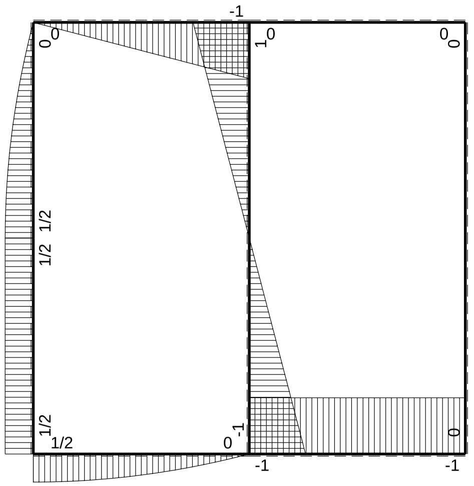
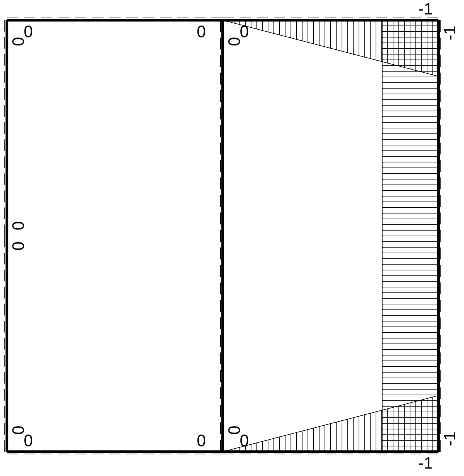
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
totali							Fb^2/EJ	8/3Xb/EJ
iperstatica X=W_{GC}							-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

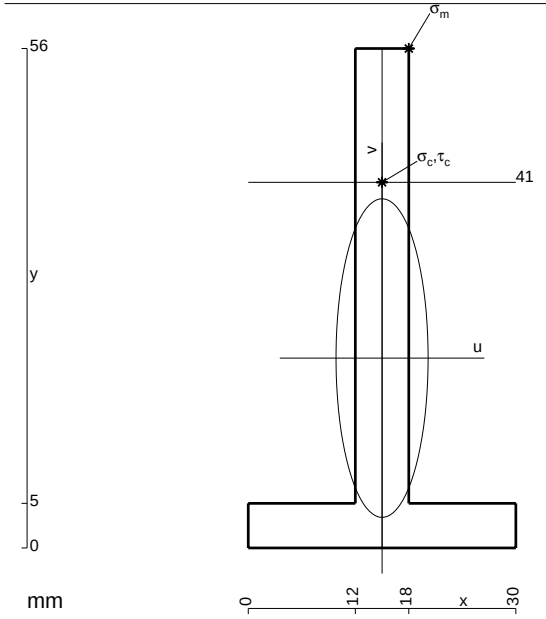
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 456. mm^2
J_u = 145554. mm^4
J_v = 12168. mm^4
y_g = 21.29 mm
N = 6738. N
T_y = -1540. N
M_x = -939400. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 34.71 mm
$\sigma_m = N/A - Mv/J_u$ = 238.8 N/mm^2
x_c = 15. mm
y_c = 41. mm
v_c = 19.71 mm
$\sigma_c = N/A - Mv/J_u$ = 142. N/mm^2
τ_c = 4.318 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 142.2 N/mm^2
S^* = 2449. mm^3

ESΣ8.154

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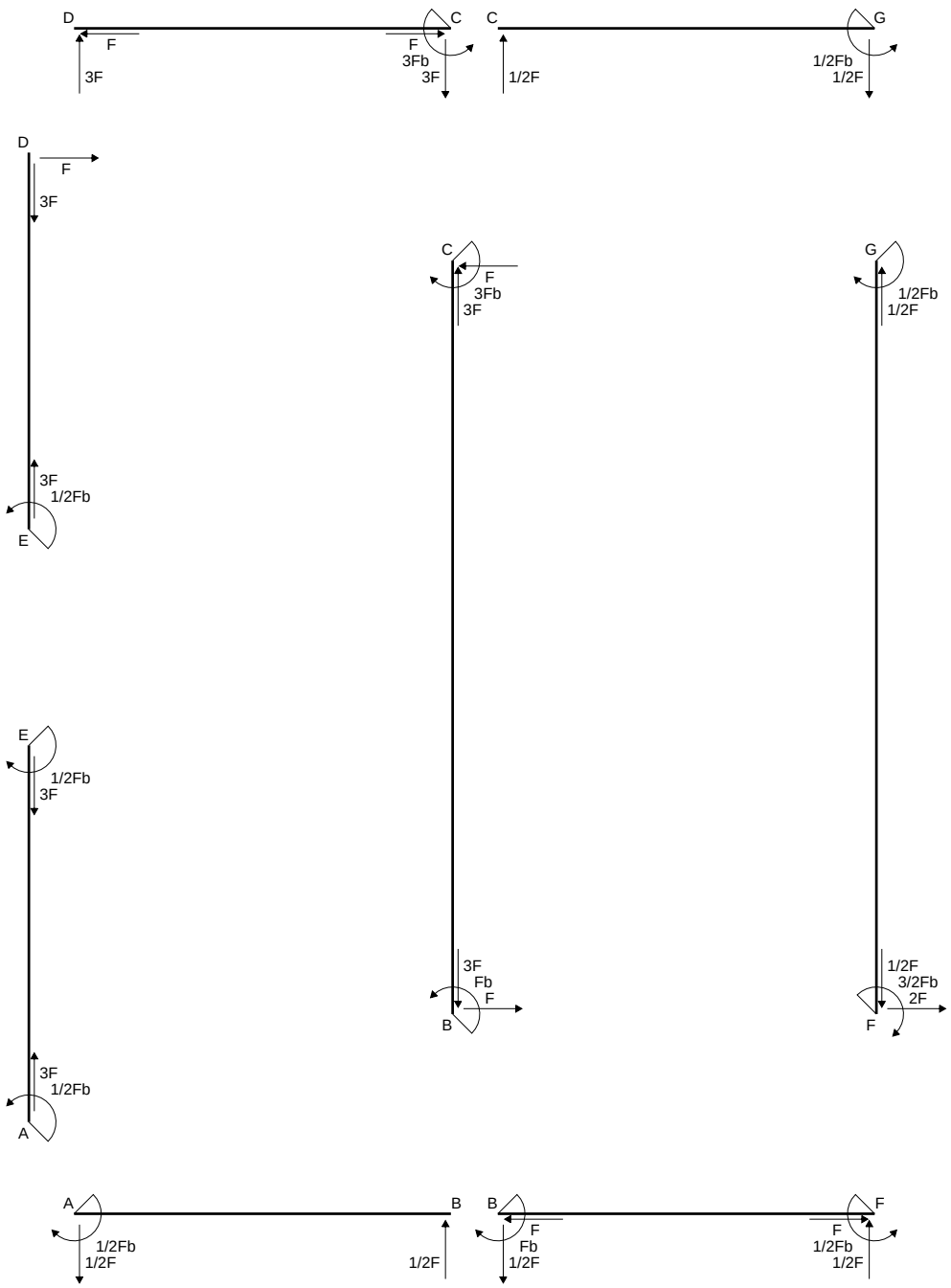
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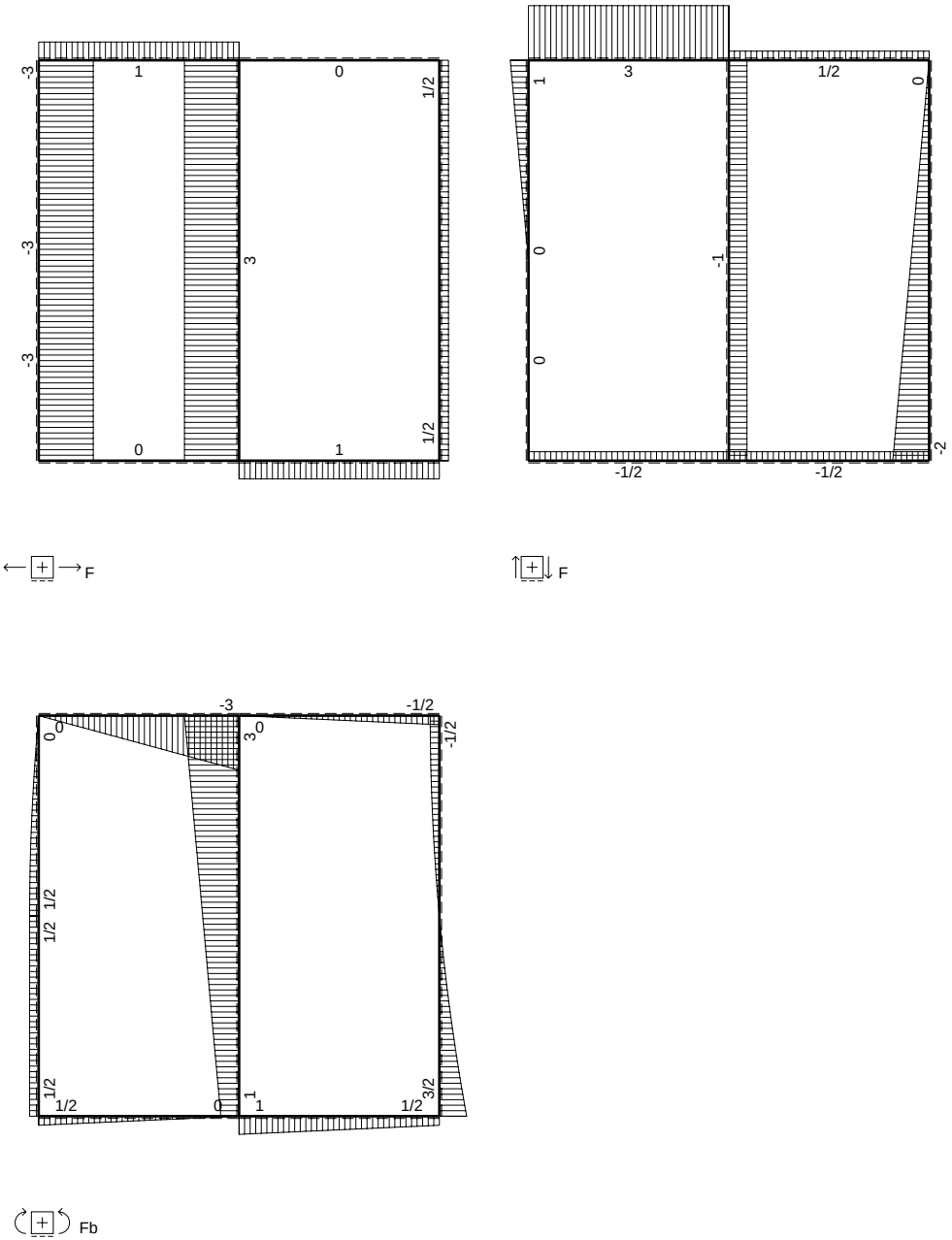
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1/2
1/2
1/2
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1
1/2
3/2
Fb

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x\theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

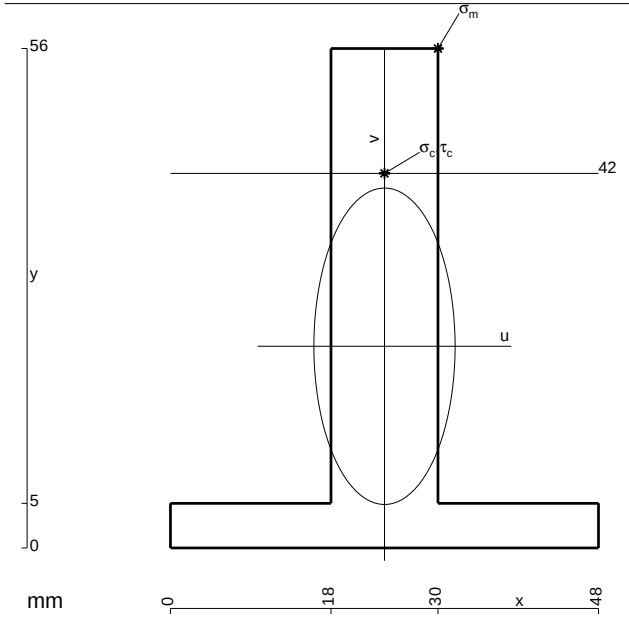
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 852. mm^2
J_u = 268308. mm^4
J_v = 53424. mm^4
y_g = 22.61 mm
N = 800. N
T_y = 2400. N
M_x = -1584000. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 33.39 mm
$\sigma_m = N/A - Mv/J_u$ = 198. N/mm^2
x_c = 24. mm
y_c = 42. mm
v_c = 19.39 mm
$\sigma_c = N/A - Mv/J_u$ = 115.4 N/mm^2
τ_c = 3.304 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 115.5 N/mm^2
S^* = 4433. mm^3

ESΣ8.155

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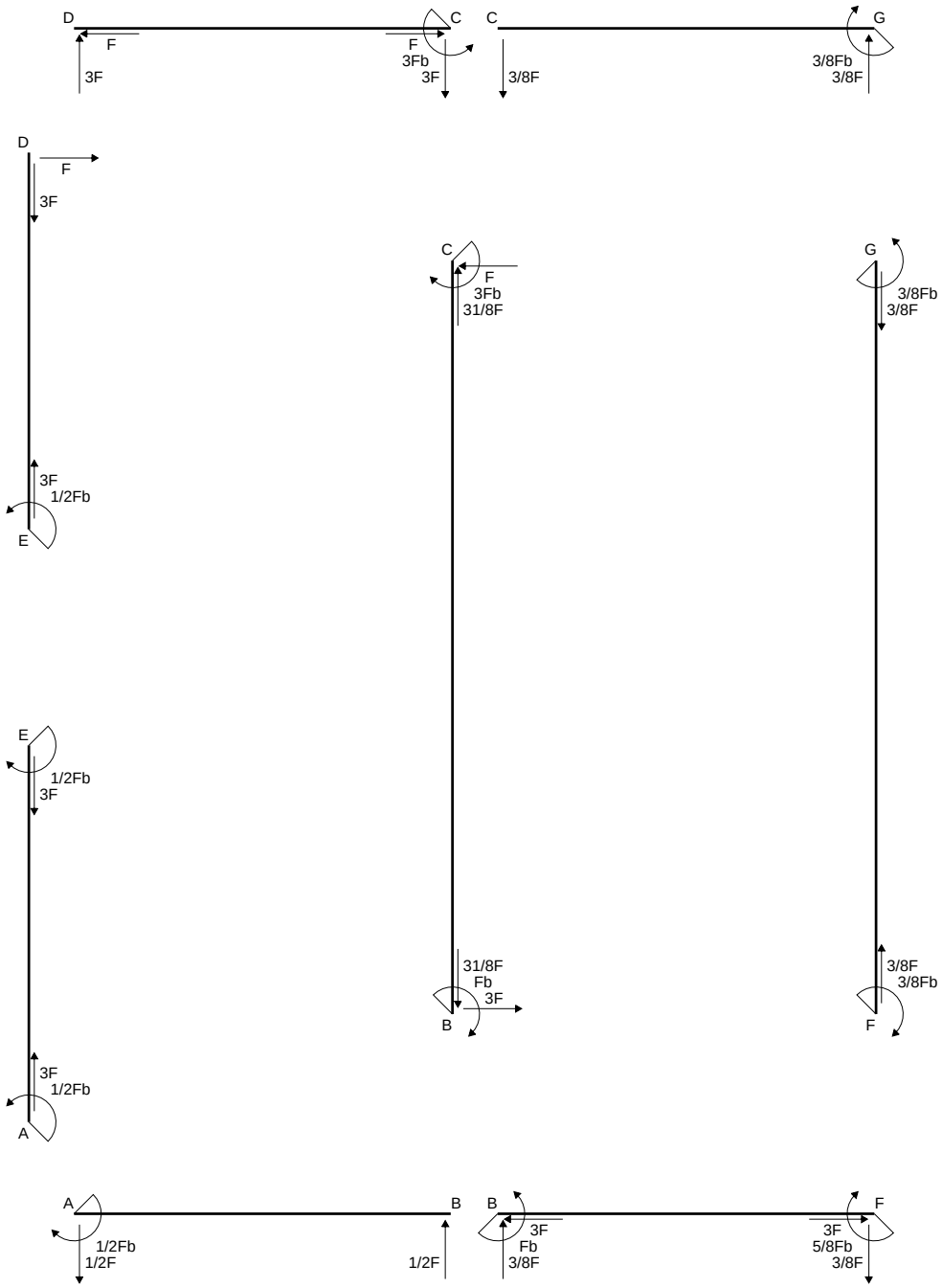
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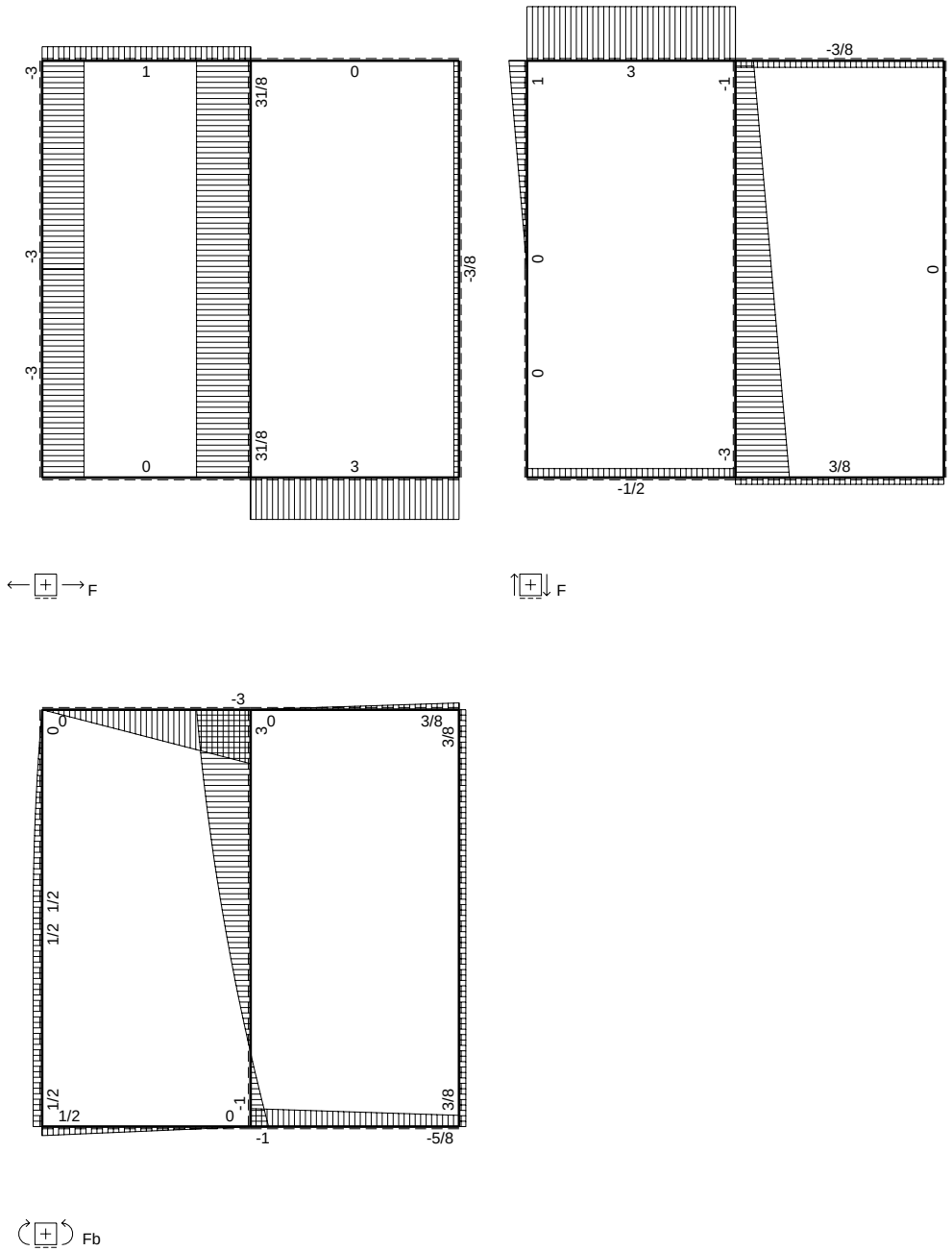
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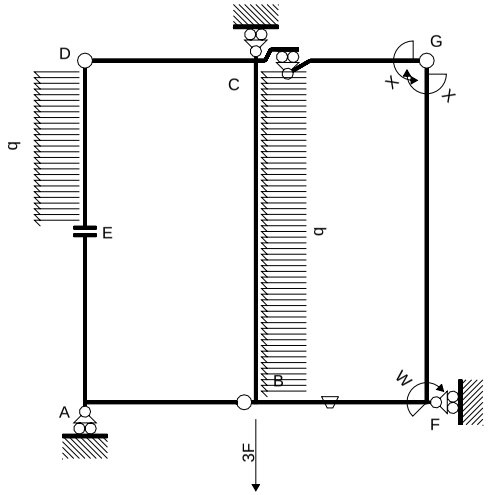
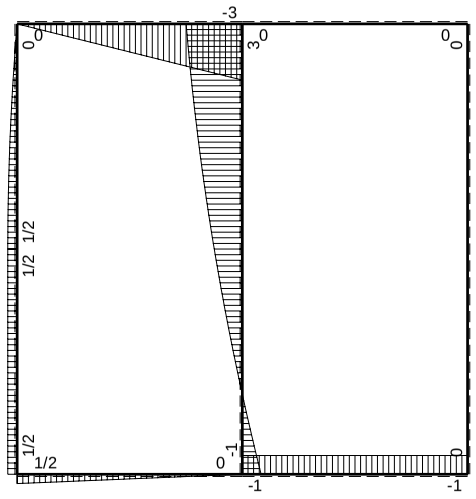
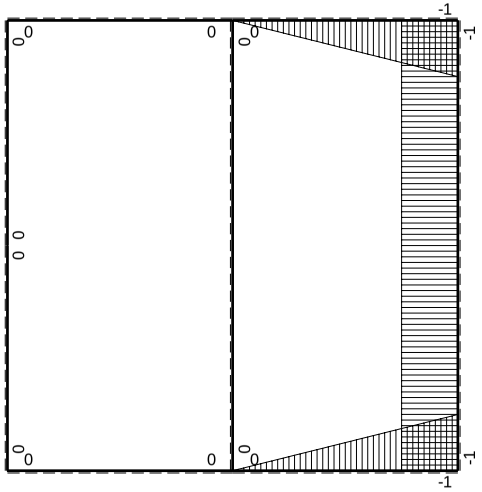
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
		totali					Fb^2/EJ	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.156

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

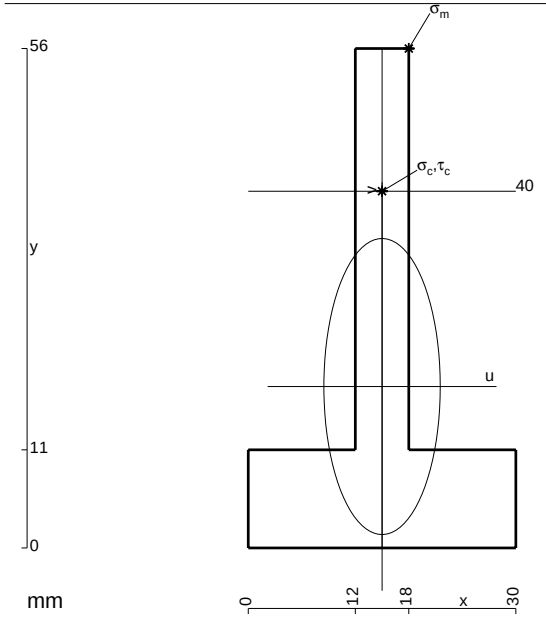
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 600. mm^2
J_u = 165314. mm^4
J_v = 25560. mm^4
y_g = 18.1 mm
N = 430. N
T_y = 1290. N
M_x = -903000. Nmm
x_m = 18. mm
y_m = 56. mm
u_m = 3. mm
v_m = 37.9 mm
$\sigma_m = N/A - Mv/J_u$ = 207.7 N/mm^2
x_c = 15. mm
y_c = 40. mm
v_c = 21.9 mm
$\sigma_c = N/A - Mv/J_u$ = 120.3 N/mm^2
τ_c = 3.733 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 120.5 N/mm^2
S^* = 2870. mm^3

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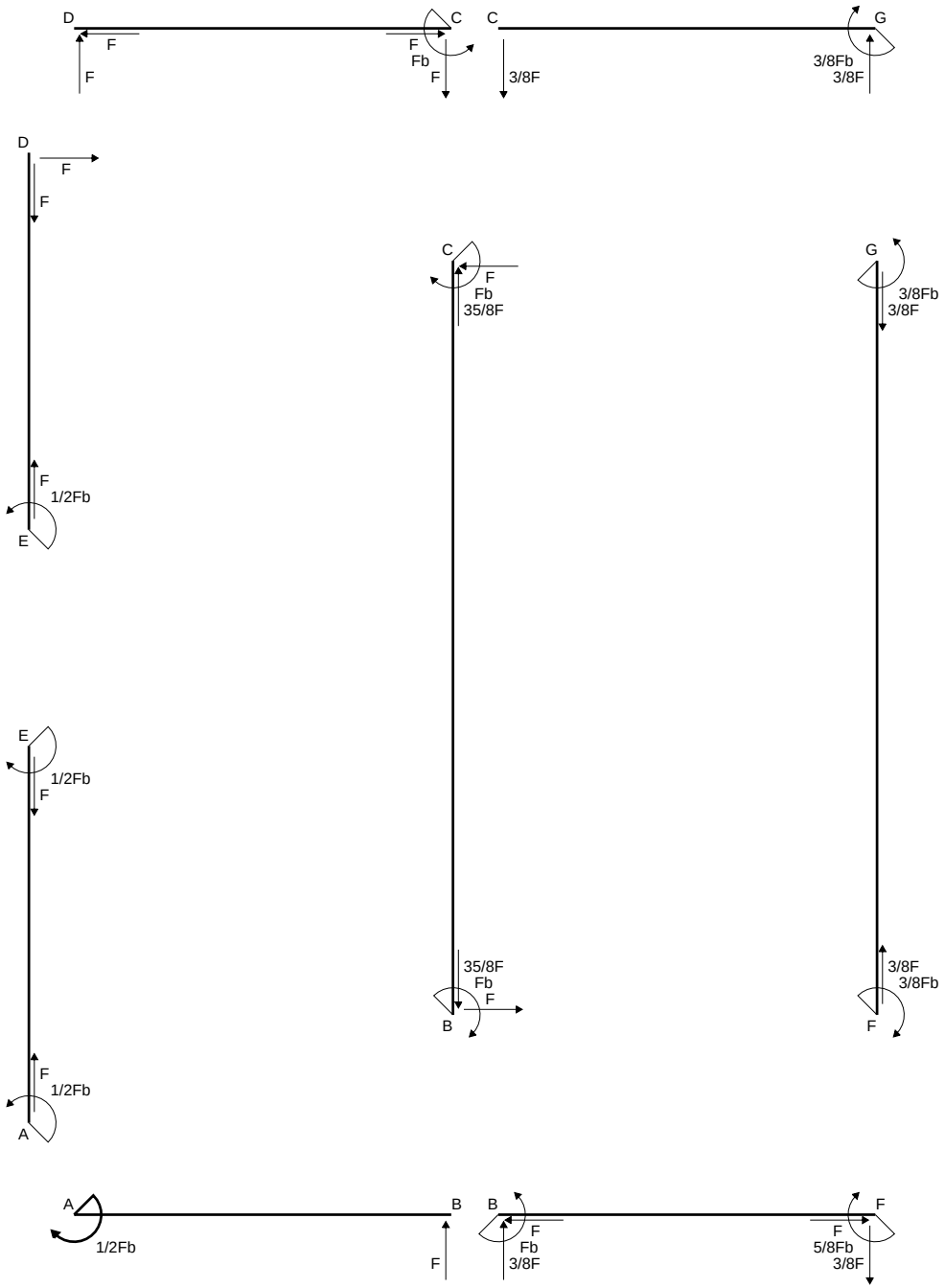
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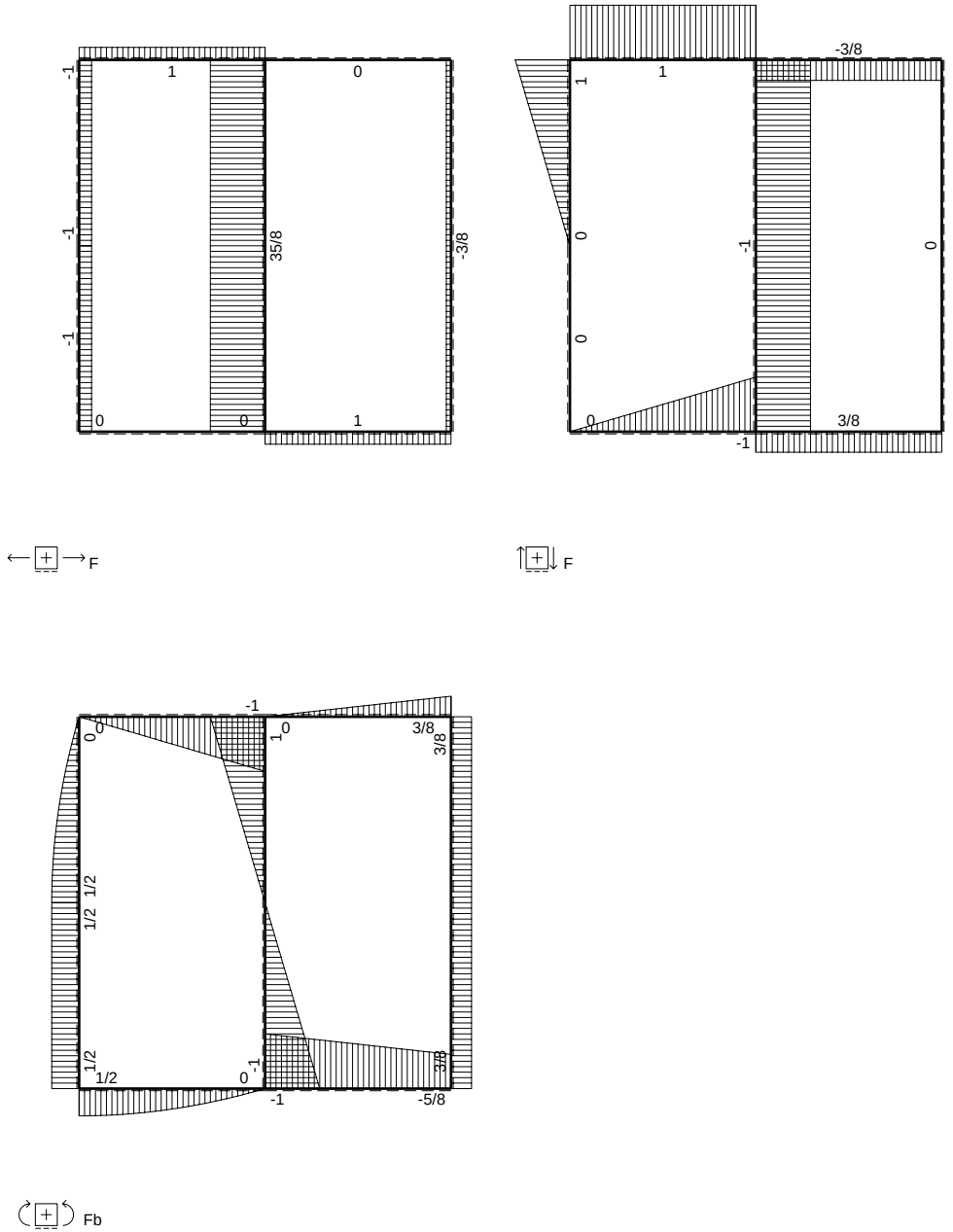
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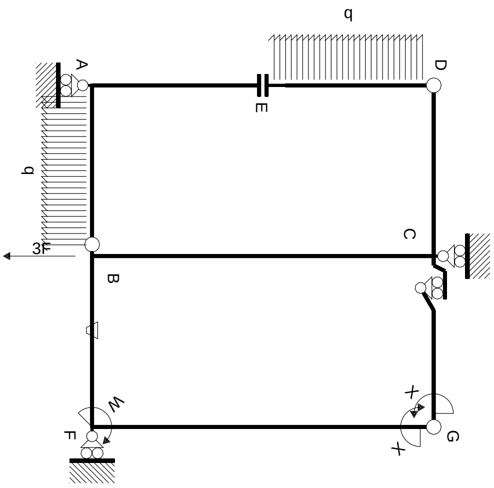
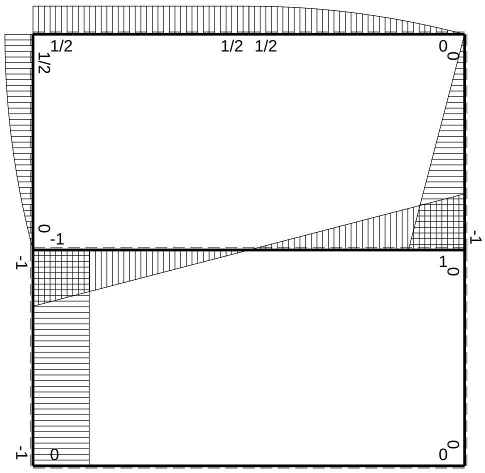
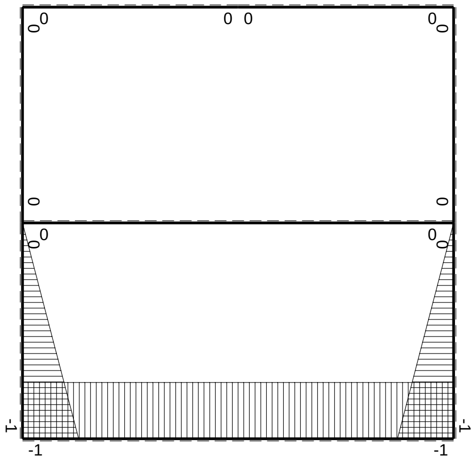


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

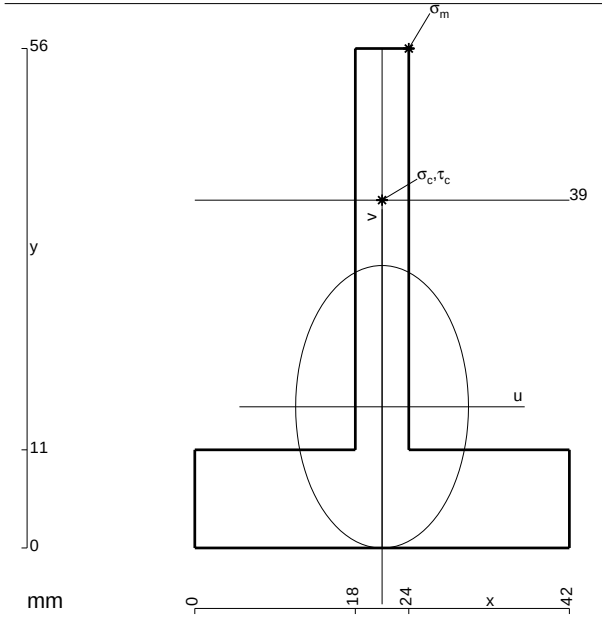
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 732. mm^2
J_u = 183822. mm^4
J_v = 68724. mm^4
y_g = 15.83 mm
N = 5731. N
T_y = -1310. N
M_x = -969400. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 40.17 mm
$\sigma_m = N/A - Mv/J_u$ = 219.7 N/mm^2
x_c = 21. mm
y_c = 39. mm
v_c = 23.17 mm
$\sigma_c = N/A - Mv/J_u$ = 130. N/mm^2
τ_c = 3.837 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.2 N/mm^2
S^* = 3231. mm^3

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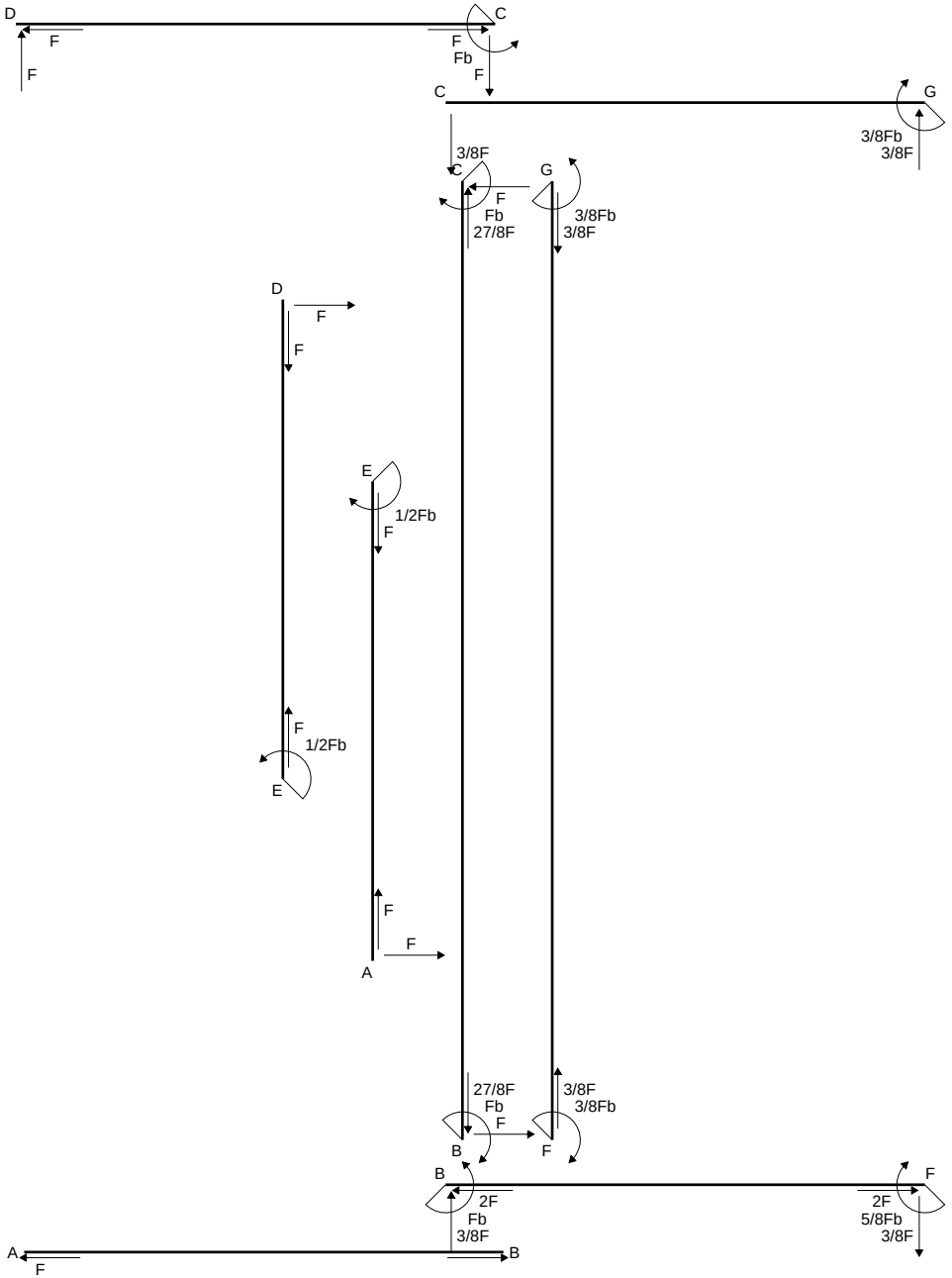
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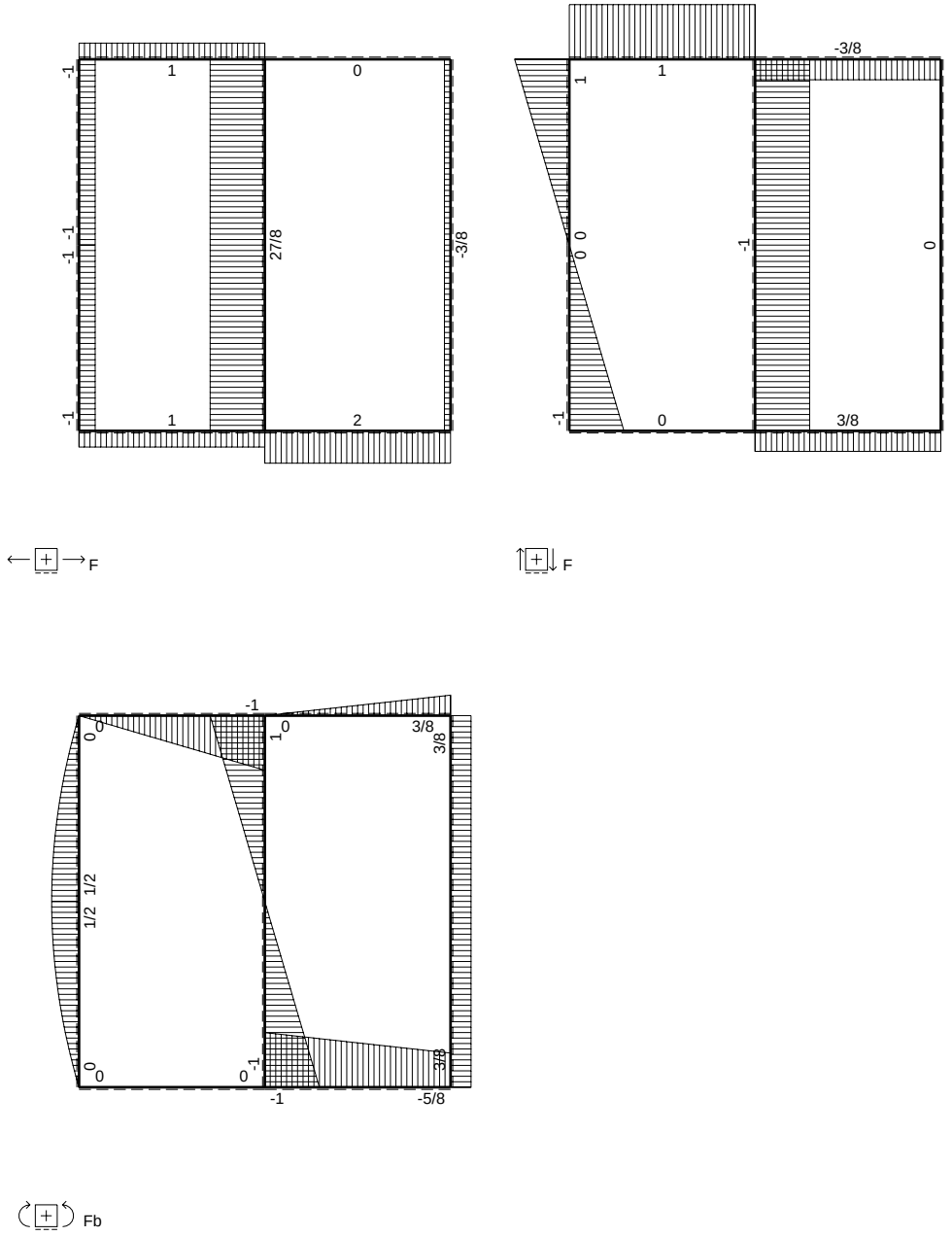
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Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

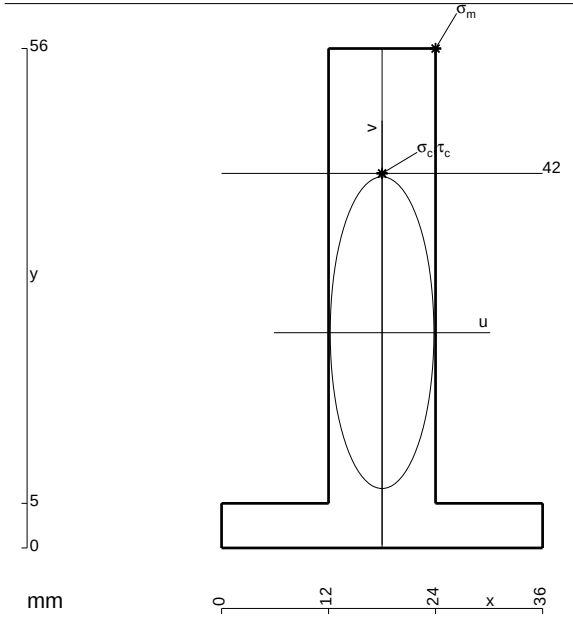
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 792. mm^2
J_u = 242073. mm^4
J_v = 26784. mm^4
y_g = 24.14 mm
N = 7155. N
T_y = -2120. N
M_x = -1674800. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 6. mm
v_m = 31.86 mm
$\sigma_m = N/A - Mv/J_u$ = 229.5 N/mm^2
x_c = 18. mm
y_c = 42. mm
v_c = 17.86 mm
$\sigma_c = N/A - Mv/J_u$ = 132.6 N/mm^2
τ_c = 3.048 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.7 N/mm^2
S^* = 4177. mm^3

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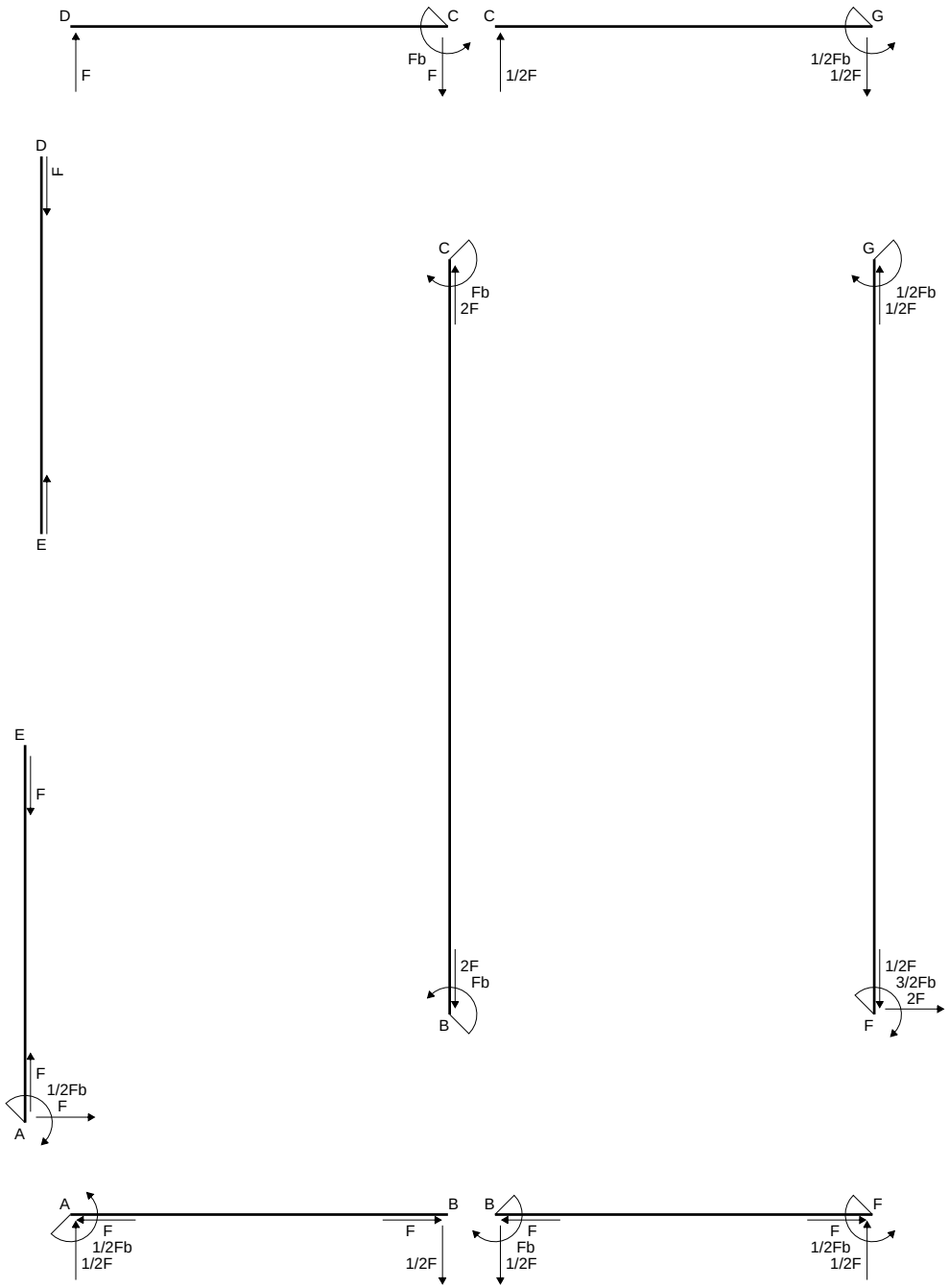
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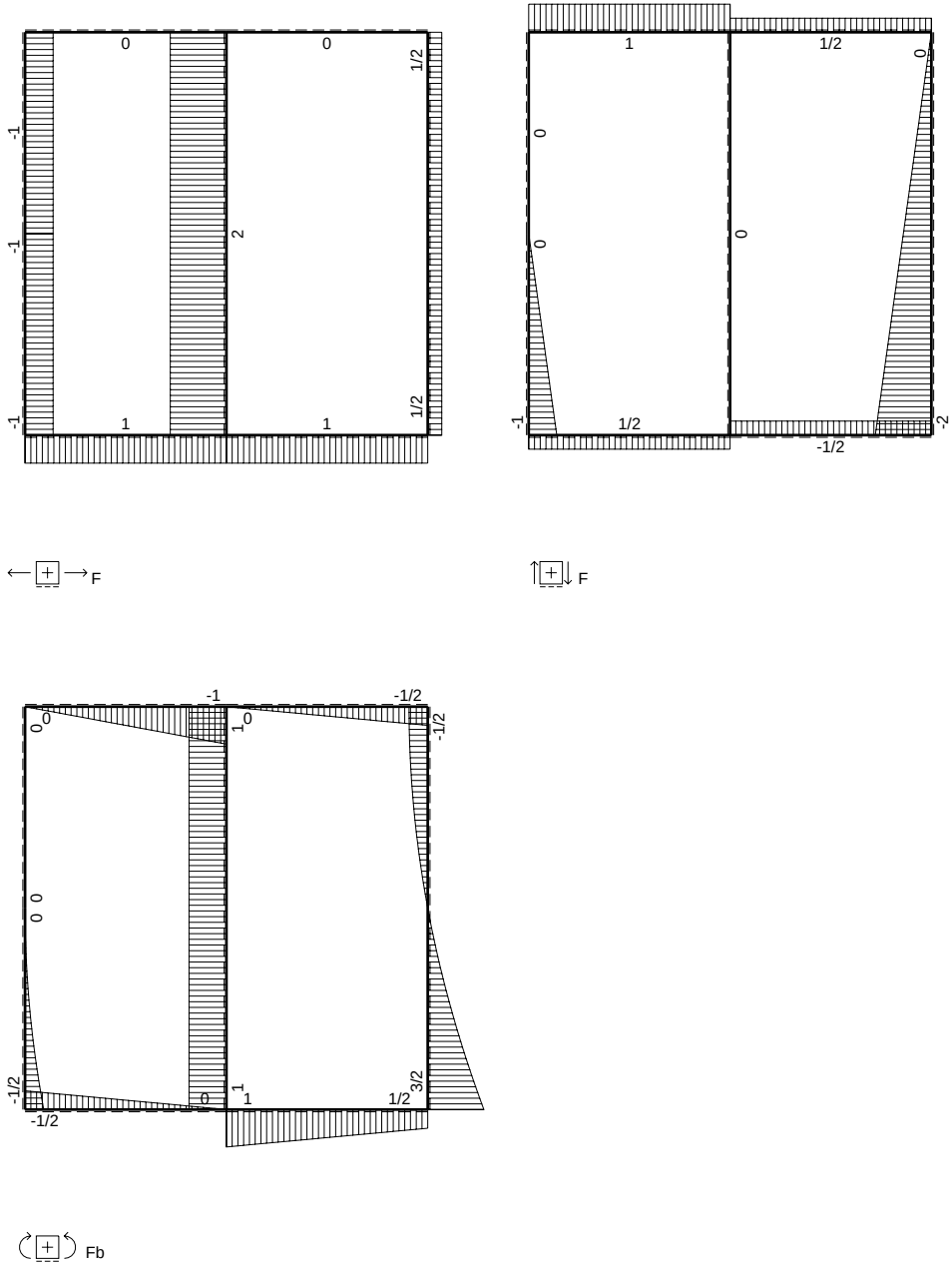
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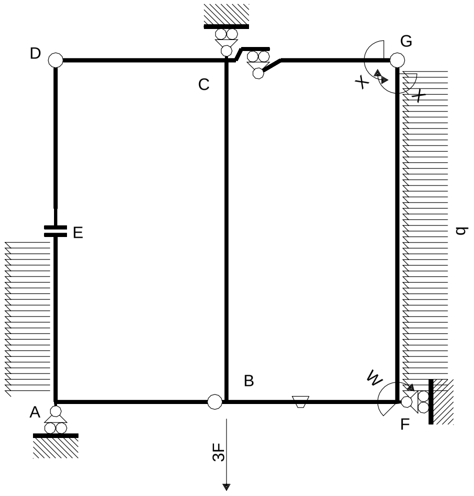
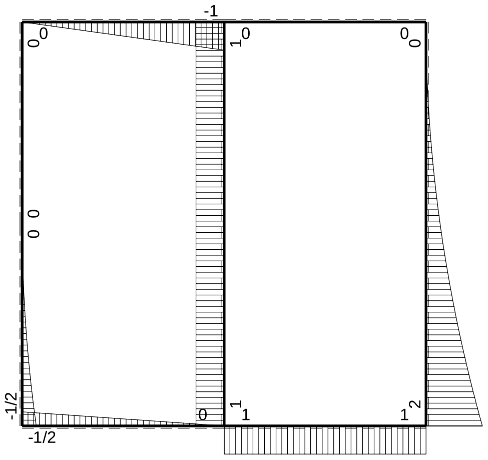
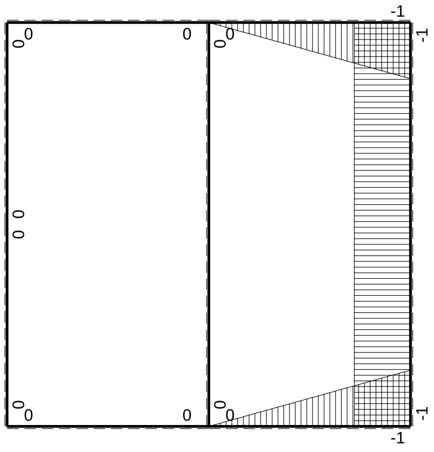




M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

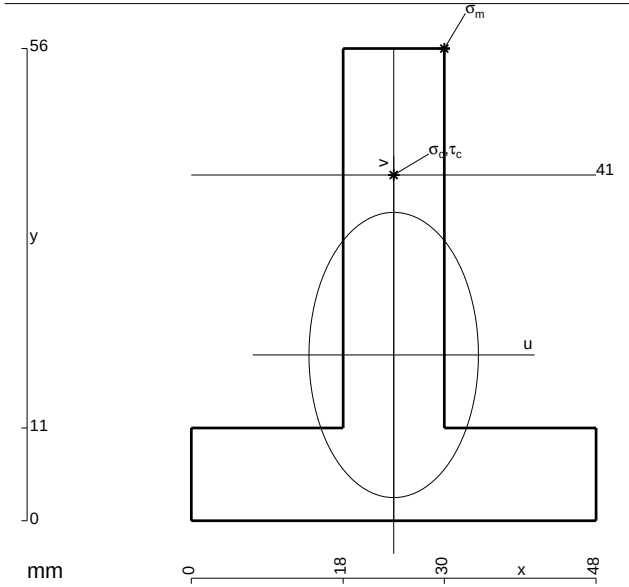
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.159



A = 1068. mm^2
J_u = 305751. mm^4
J_v = 107856. mm^4
y_g = 19.66 mm
T_y = 2430. N
M_x = -2016900. Nmm
x_m = 30. mm
y_m = 56. mm
u_m = 6. mm
v_m = 36.34 mm
$\sigma_m = -Mv/J_u$ = 239.7 N/mm^2
x_c = 24. mm
y_c = 41. mm
v_c = 21.34 mm
$\sigma_c = -Mv/J_u$ = 140.8 N/mm^2
τ_c = 3.438 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.9 N/mm^2
S^* = 5192. mm^3

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ESΣ8.159

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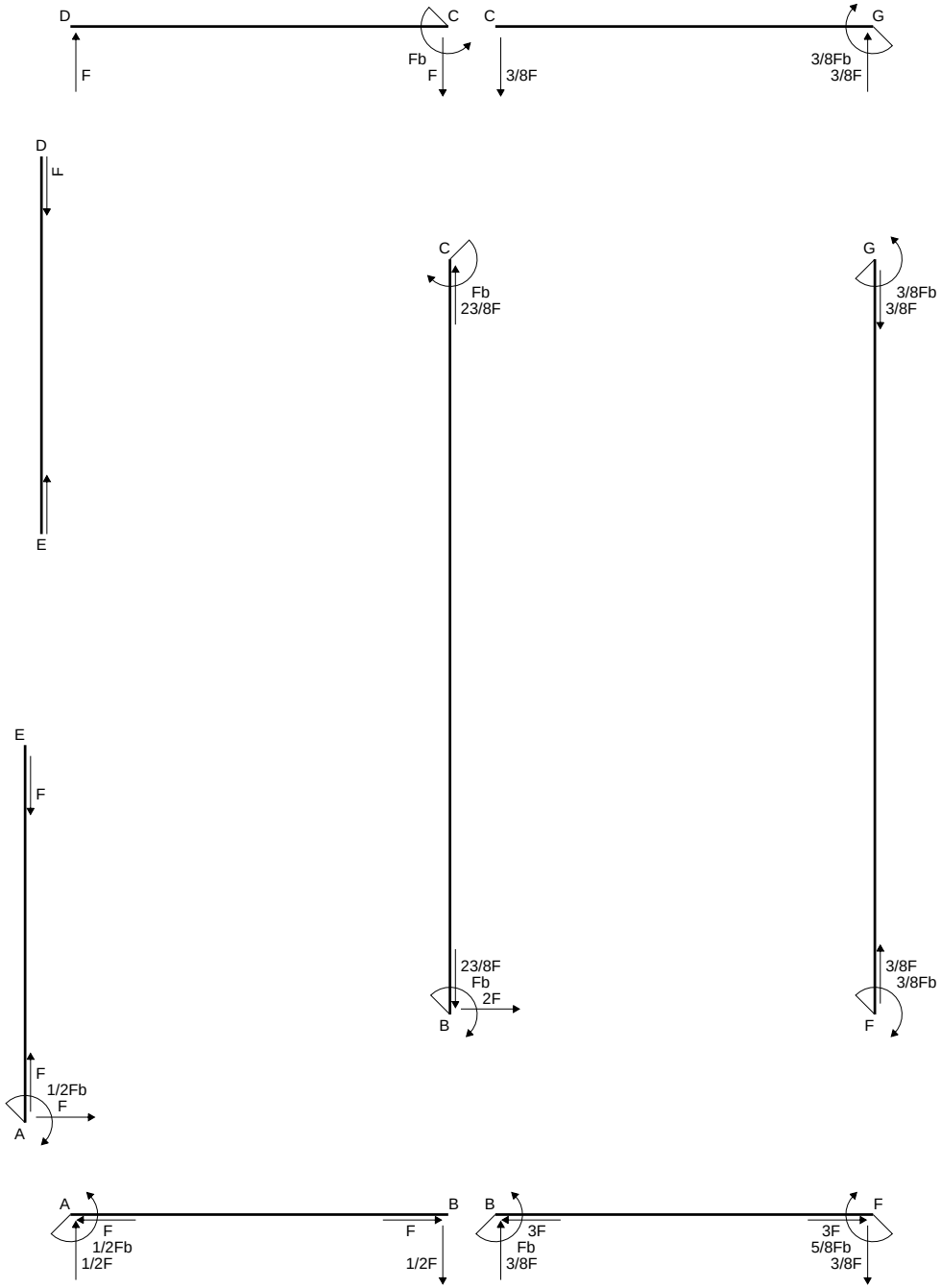
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3/8Fb
3/8F
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Fb
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3/8F
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1/2Fb
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A
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Fb
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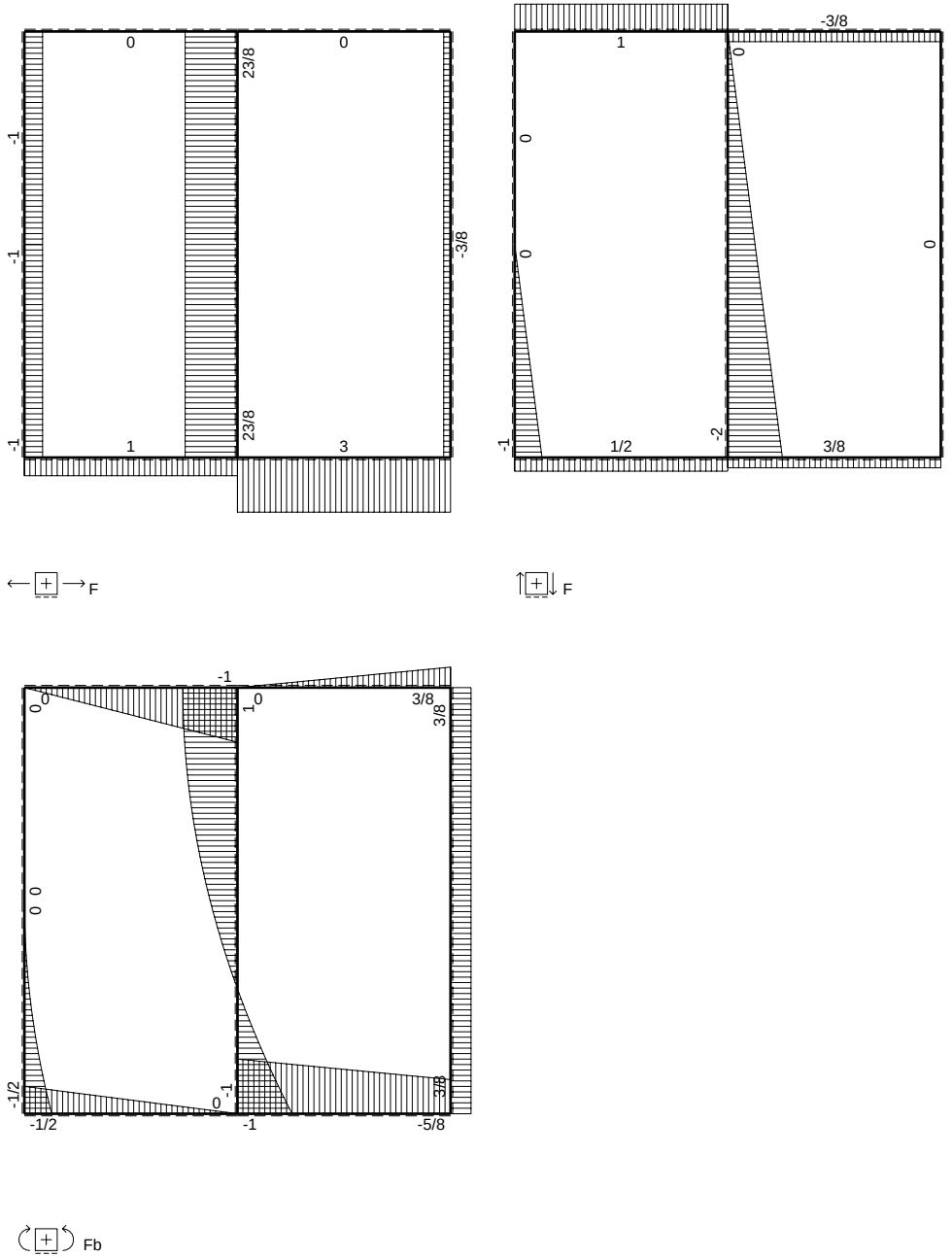
3F
5/8Fb
3/8F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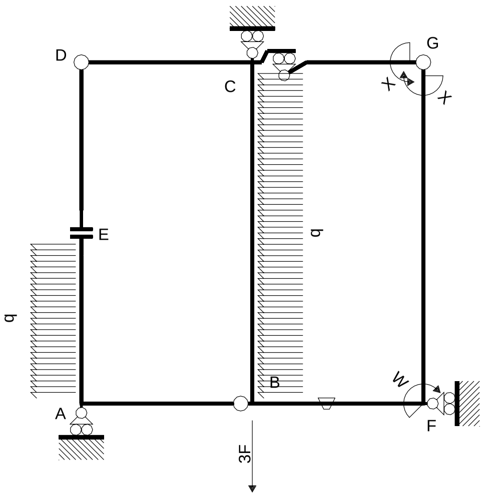
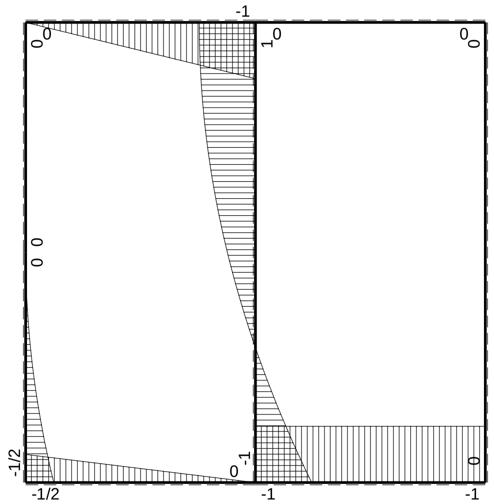
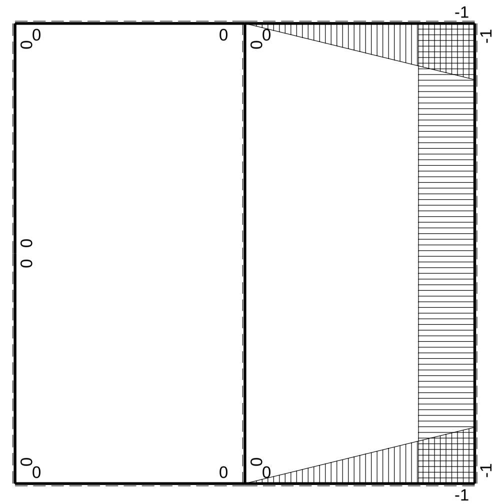
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	$-Fb$	$-Fb/EJ$	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	Fb	Fb/EJ	$Fb-Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$-3/8Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

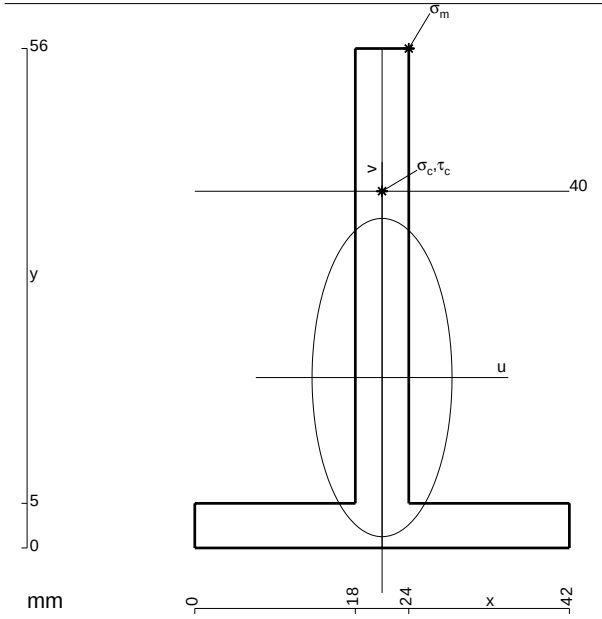
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 516. mm^2
J_u = 164398. mm^4
J_v = 31788. mm^4
y_g = 19.1 mm
N = 5491. N
T_y = -3820. N
M_x = -840400. Nmm
x_m = 24. mm
y_m = 56. mm
u_m = 3. mm
v_m = 36.9 mm
$\sigma_m = N/A - Mv/J_u$ = 199.3 N/mm^2
x_c = 21. mm
y_c = 40. mm
v_c = 20.9 mm
$\sigma_c = N/A - Mv/J_u$ = 117.5 N/mm^2
τ_c = 10.74 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.9 N/mm^2
S^* = 2774. mm^3

ESΣ8.160

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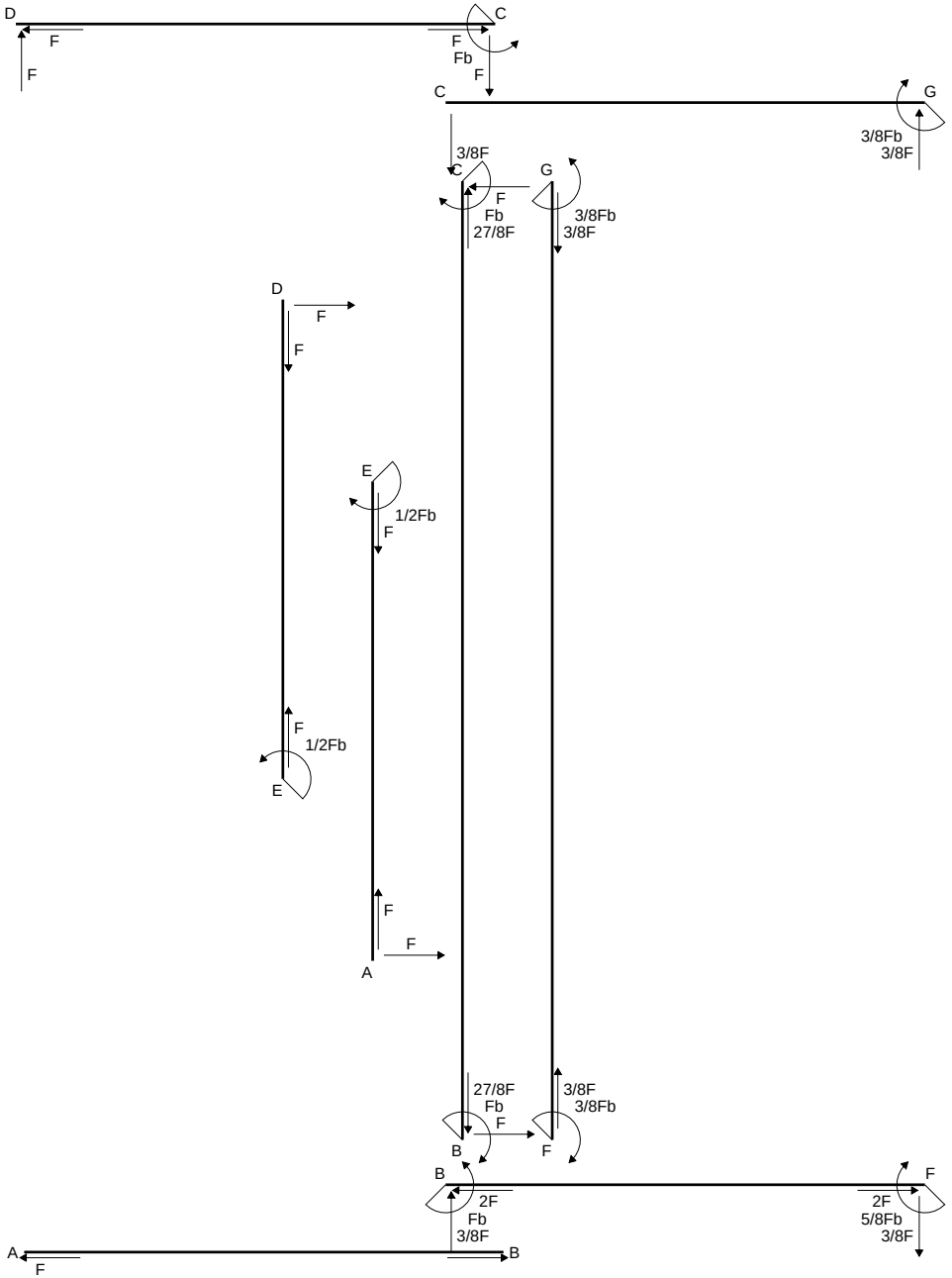
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Fb
F
3/8F
3/8Fb
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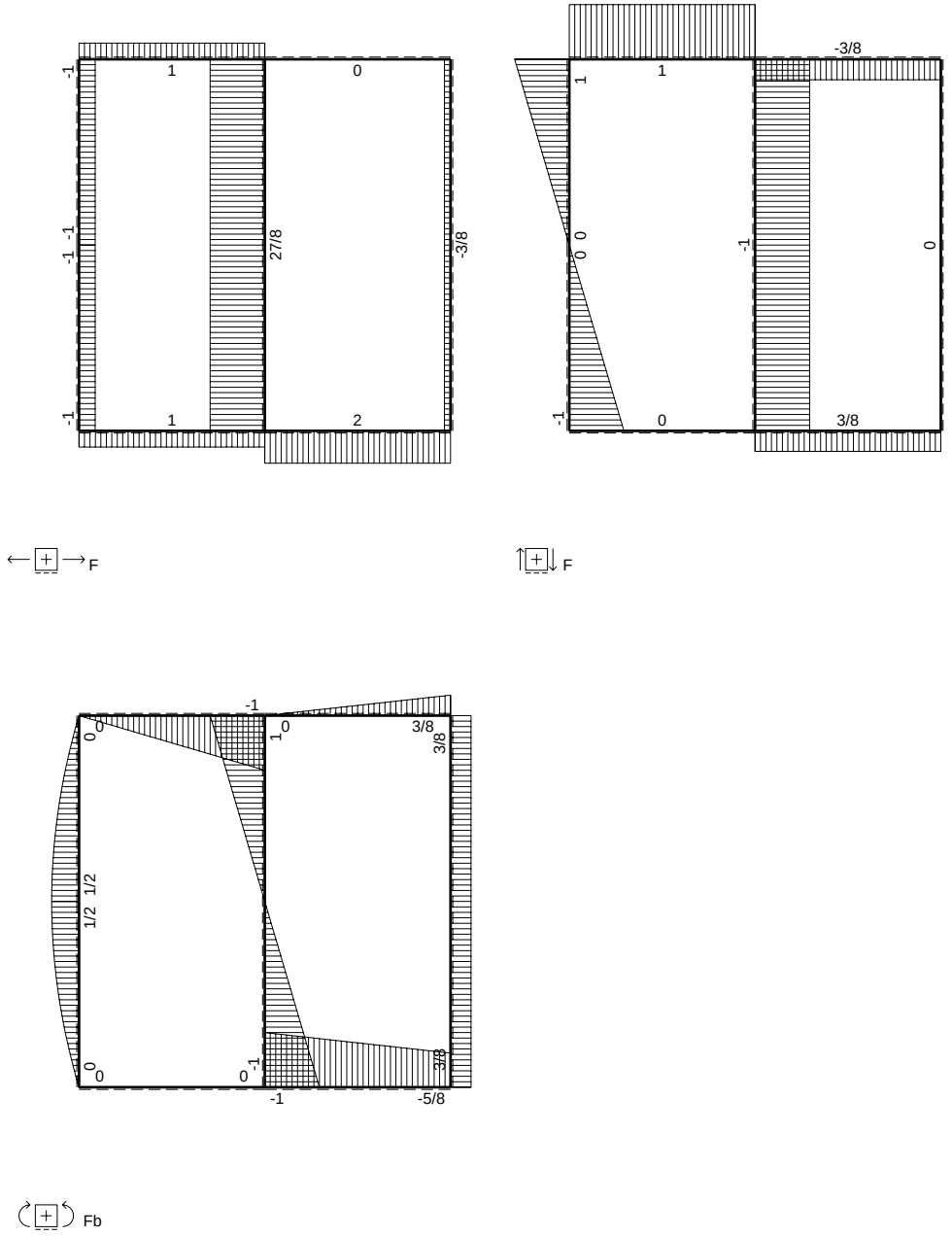
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F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1/2
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-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

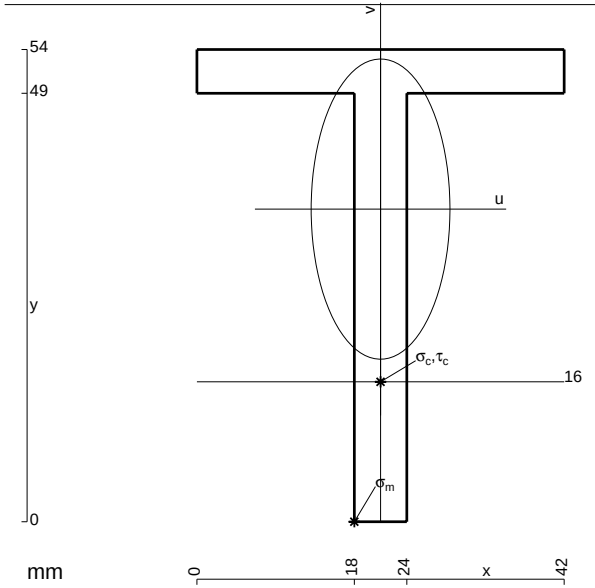
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 504. mm^2
J_u = 148565. mm^4
J_v = 31752. mm^4
y_g = 35.75 mm
N = 5906. N
T_y = -1750. N
M_x = 822500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -35.75 mm
$\sigma_m = N/A - Mv/J_u$ = 209.6 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -19.75 mm
$\sigma_c = N/A - Mv/J_u$ = 121.1 N/mm^2
τ_c = 5.23 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.4 N/mm^2
S^* = 2664. mm^3

ESΣ8.161

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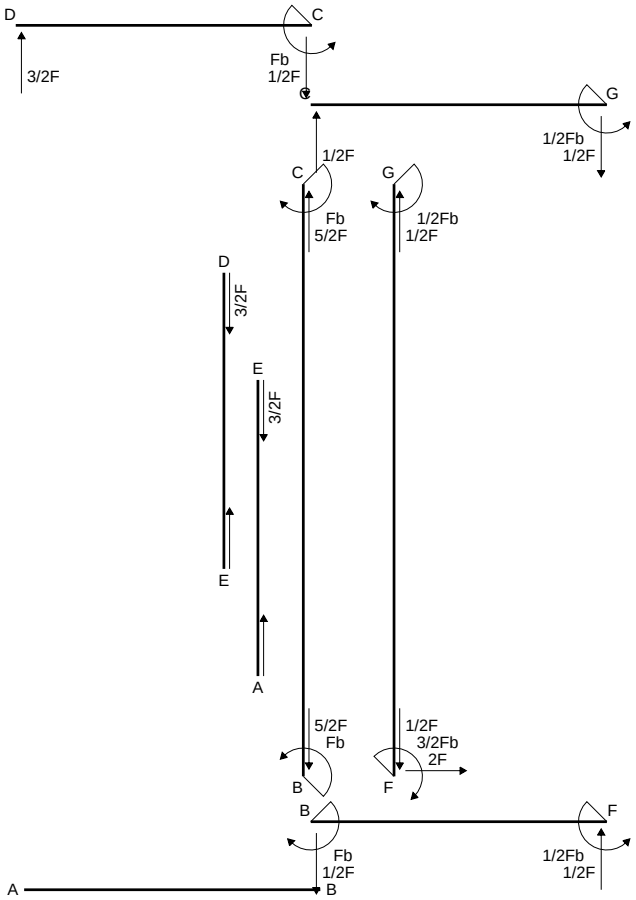
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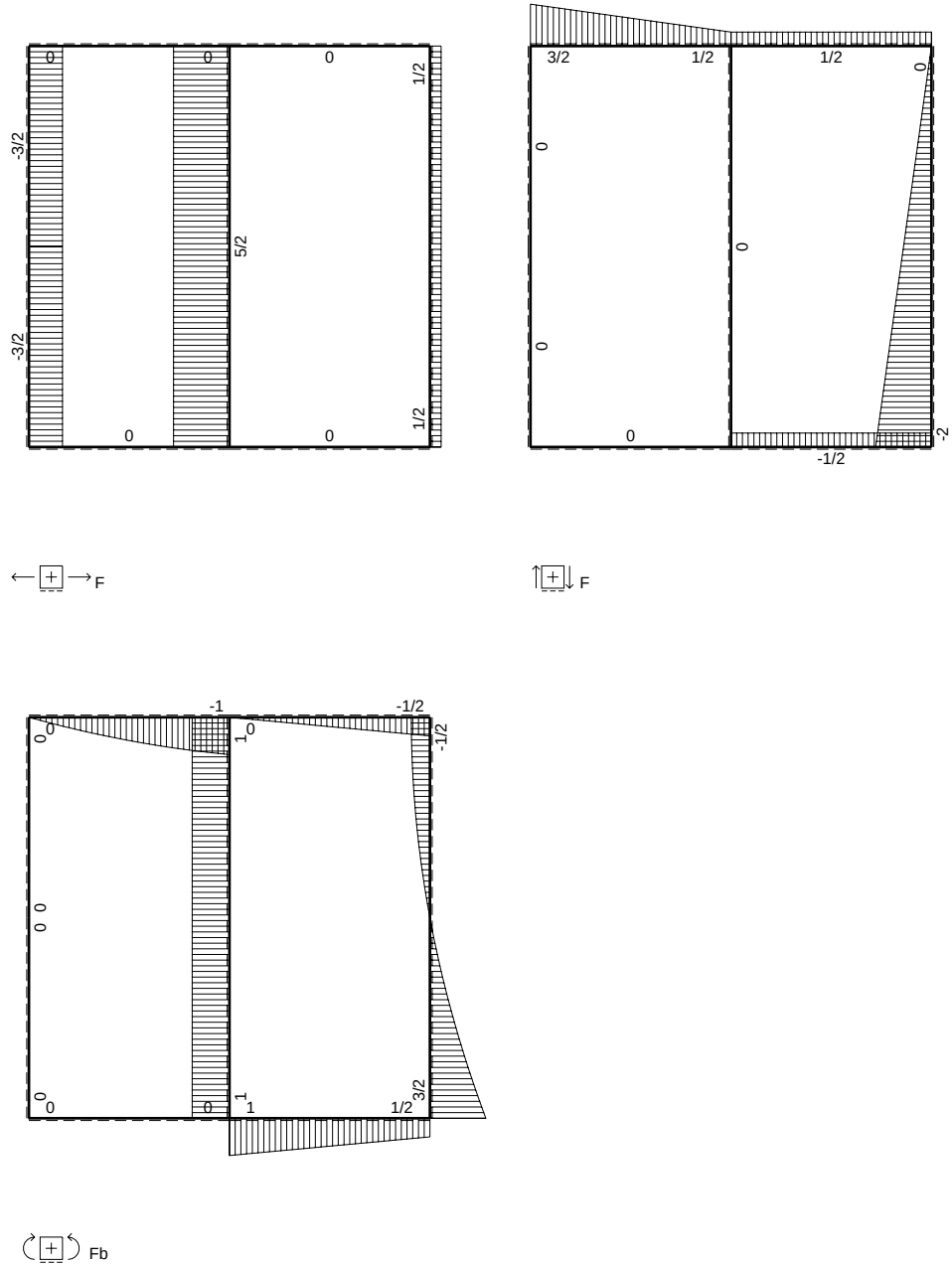
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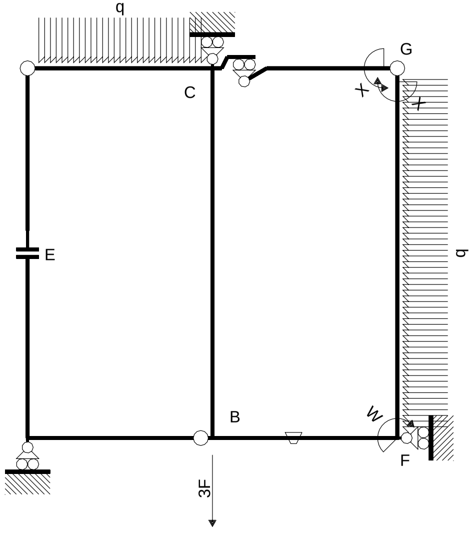
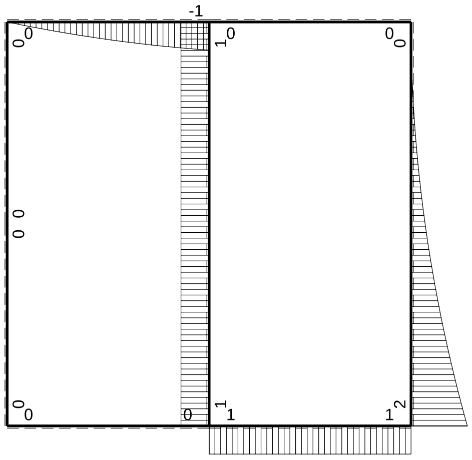
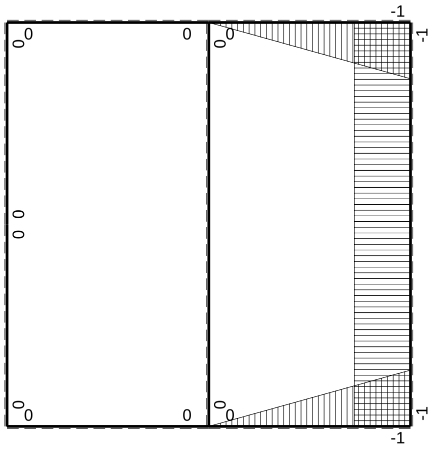




M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

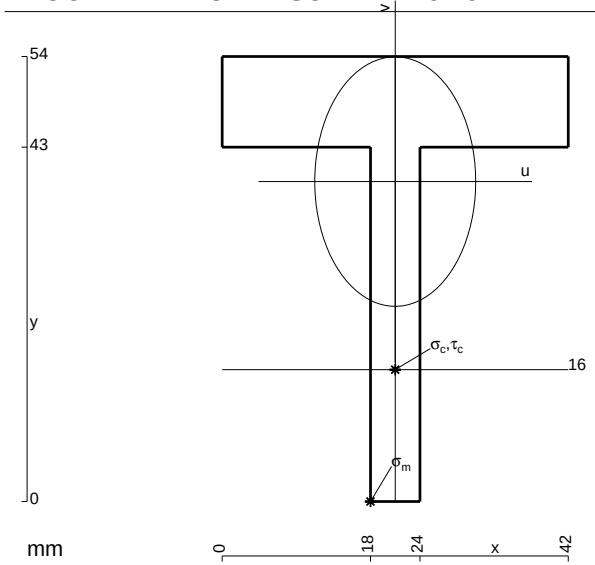
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 720. mm^2
J_u = 165098. mm^4
J_v = 68688. mm^4
y_g = 38.83 mm
T_y = 2930. N
M_x = -933938. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -38.83 mm
$\sigma_m = -Mv/J_u$ = -219.6 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -22.83 mm
$\sigma_c = -Mv/J_u$ = -129.1 N/mm^2
τ_c = 8.753 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130. N/mm^2
S^* = 2959. mm^3

ESΣ8.162

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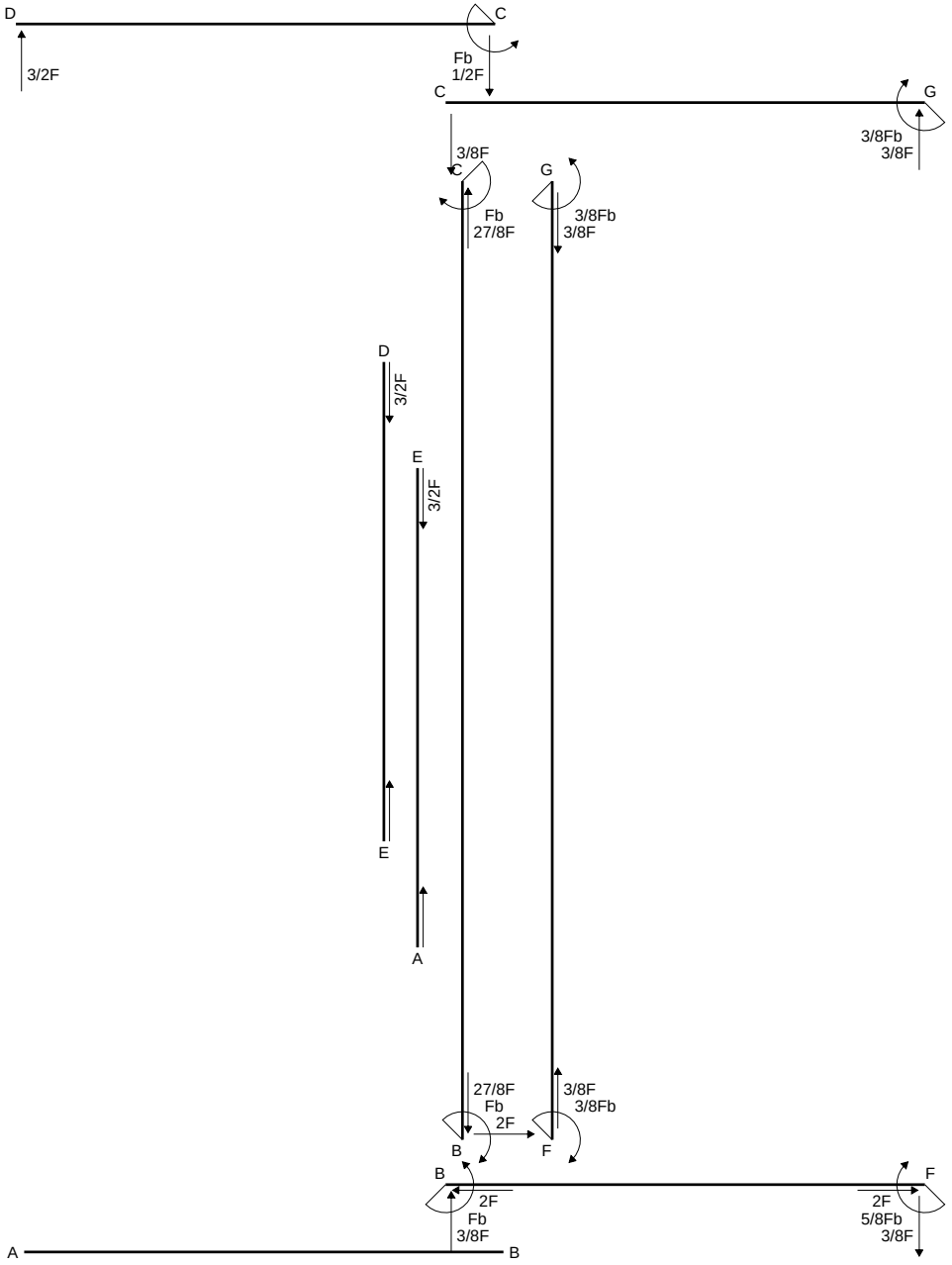
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C
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Fb
1/2F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
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3/8Fb
3/8F
D
3/2F
E
3/2F
E
A
27/8F
Fb
2F
3/8F
3/8Fb
B
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F
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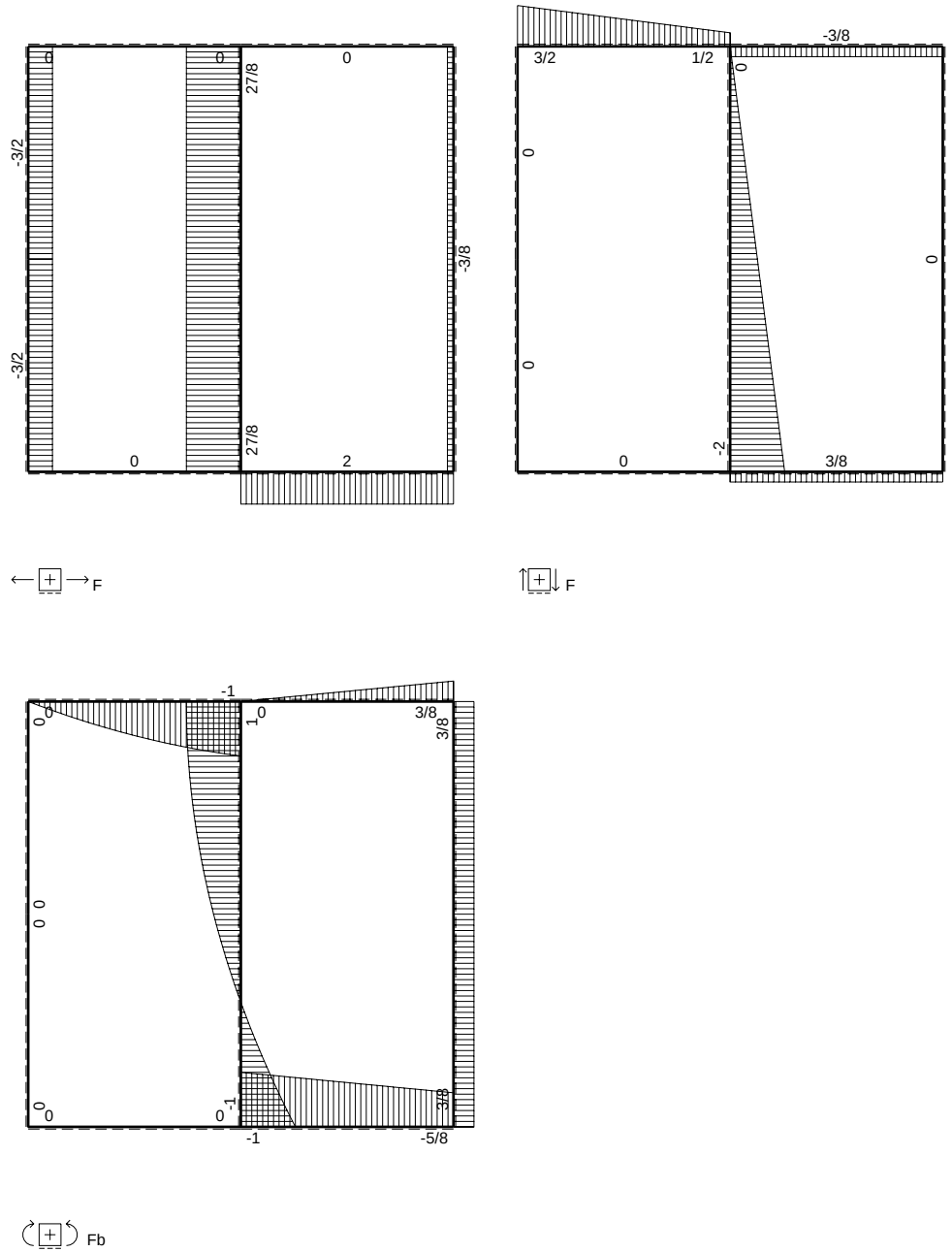
Fb
3/8F
2F
5/8Fb
3/8F
A
B


0
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27/8
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27/8
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2
F
-3/8
3/2
1/2
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3/8
F
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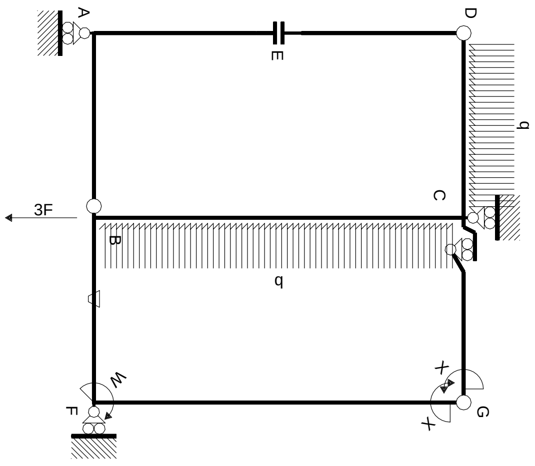
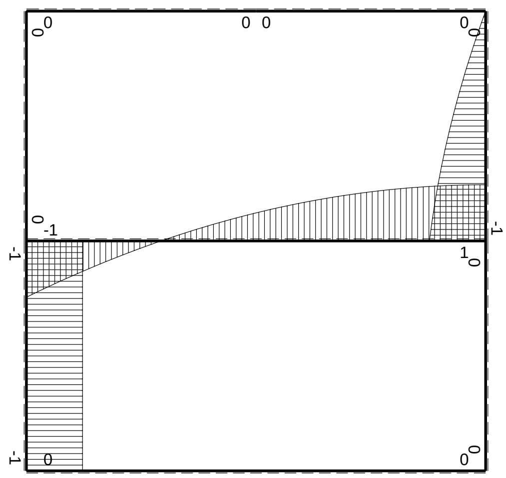
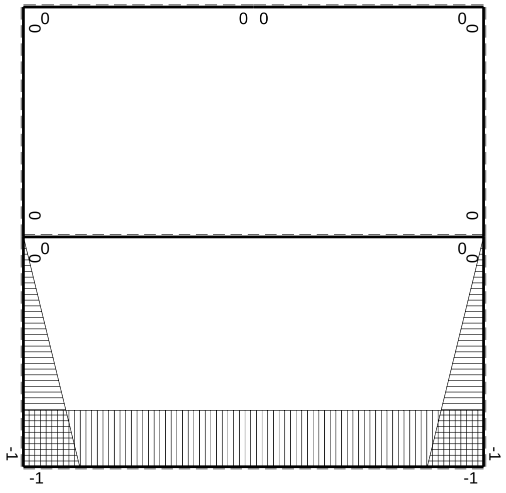
-1
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

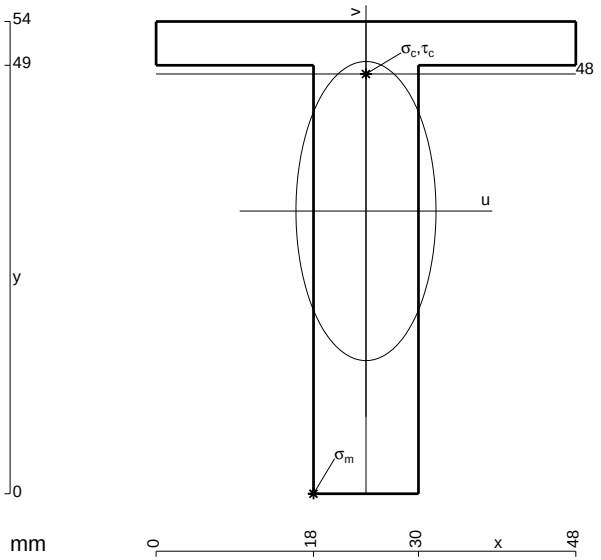
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.163



A = 828. mm^2
J_u = 242396. mm^4
J_v = 53136. mm^4
y_g = 32.33 mm
N = 11205. N
T_y = -6640. N
M_x = -1826000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -32.33 mm
$\sigma_m = N/A - Mv/J_u$ = -230. N/mm^2
x_c = 24. mm
y_c = 48. mm
v_c = 15.67 mm
$\sigma_c = N/A - Mv/J_u$ = 131.6 N/mm^2
τ_c = 10.95 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133. N/mm^2
S^* = 4796. mm^3

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ESΣ8.163

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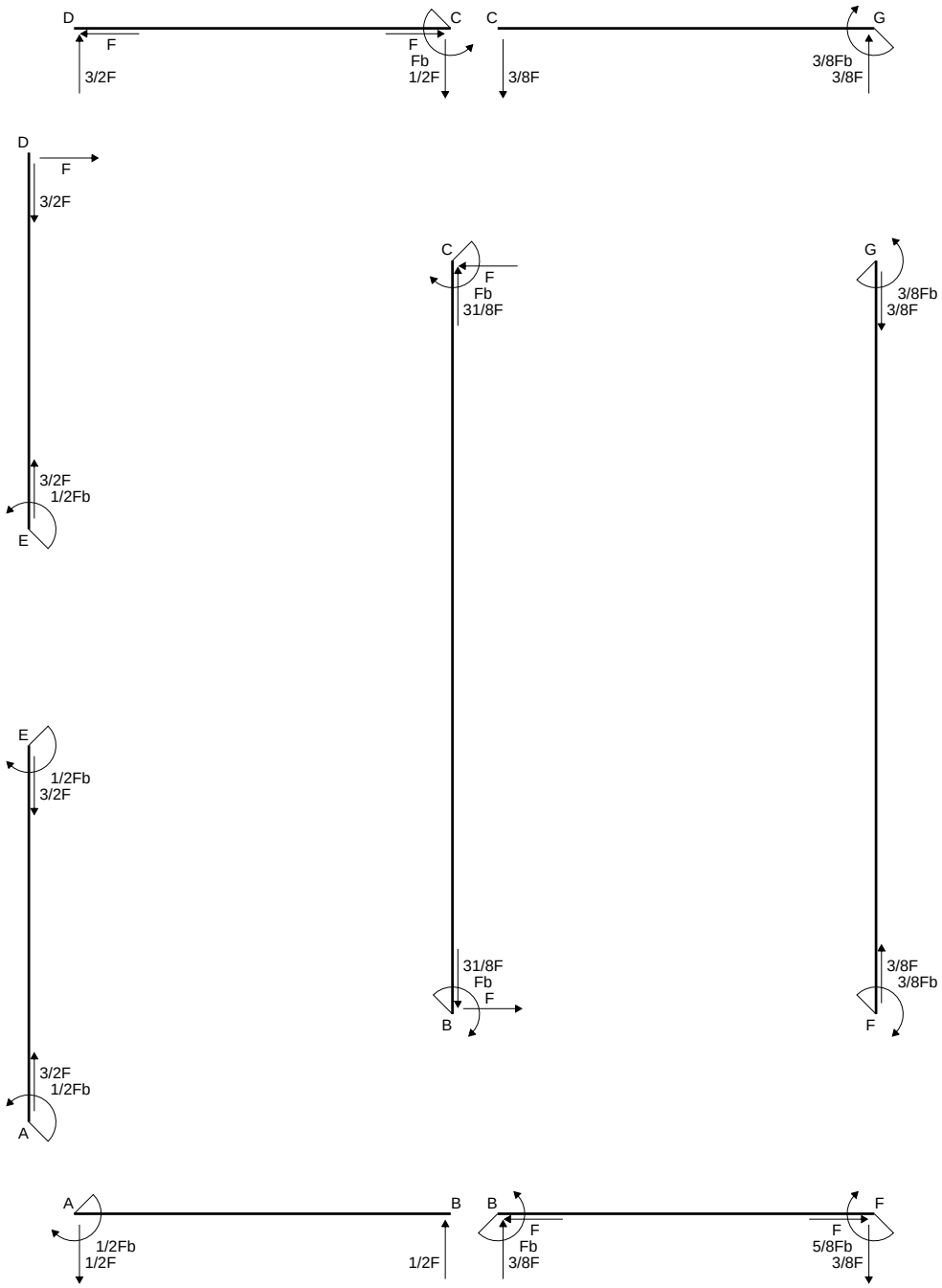
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F
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C
F
Fb
1/2F
C
3/8F
G
3/8Fb
3/8F
D
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3/2F
1/2Fb
E
C
F
Fb
31/8F
G
3/8Fb
3/8F
E
1/2Fb
3/2F
31/8F
Fb
F
B
3/8F
3/8Fb
F
3/2F
1/2Fb
A
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1/2Fb
1/2F
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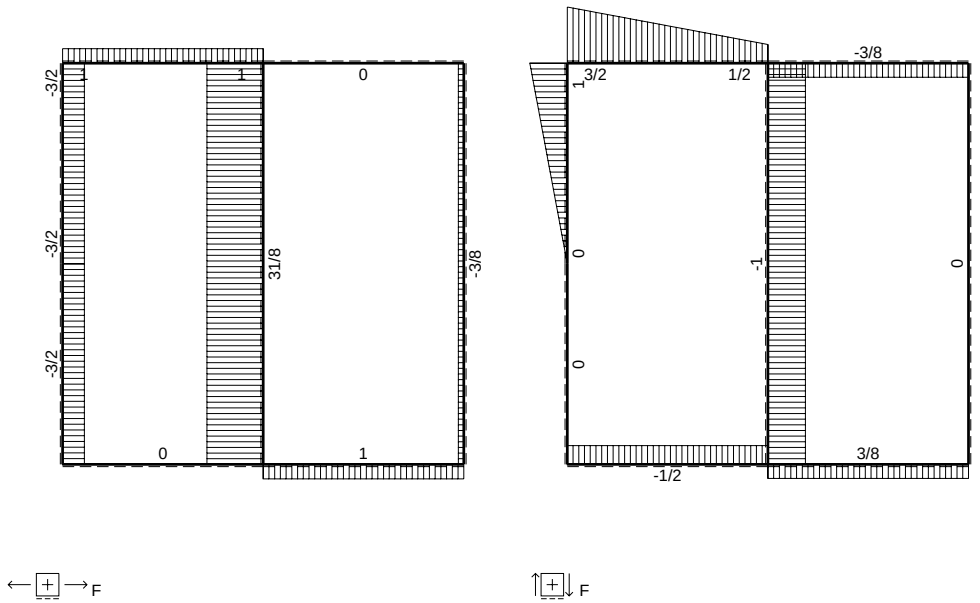
F
5/8Fb
3/8F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-3/2
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1
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31/8
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-3/8
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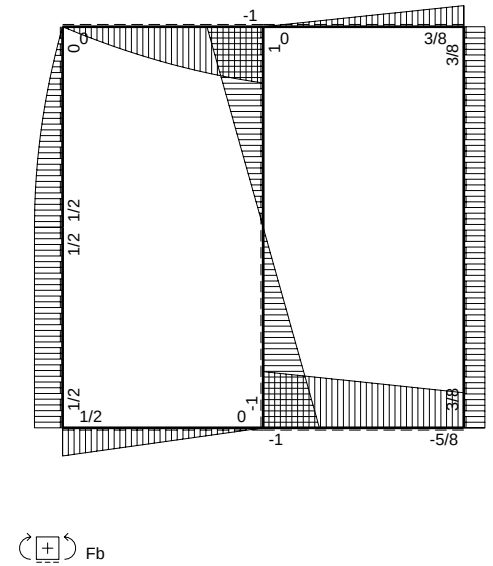
1/2
0
0
-1
0
3/8
-1/2
F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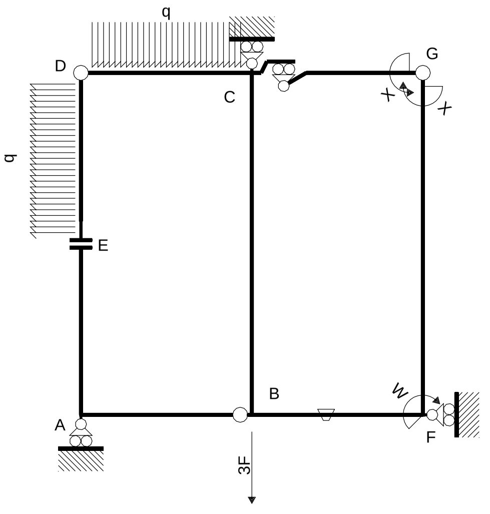
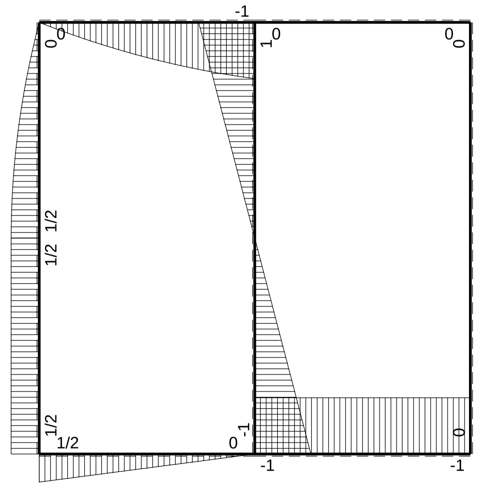
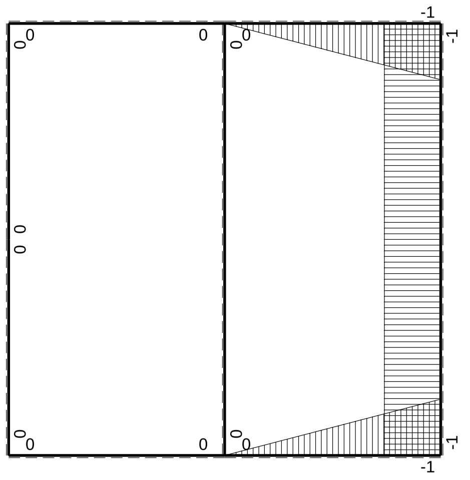
-1
-1
3/8
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

↑	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

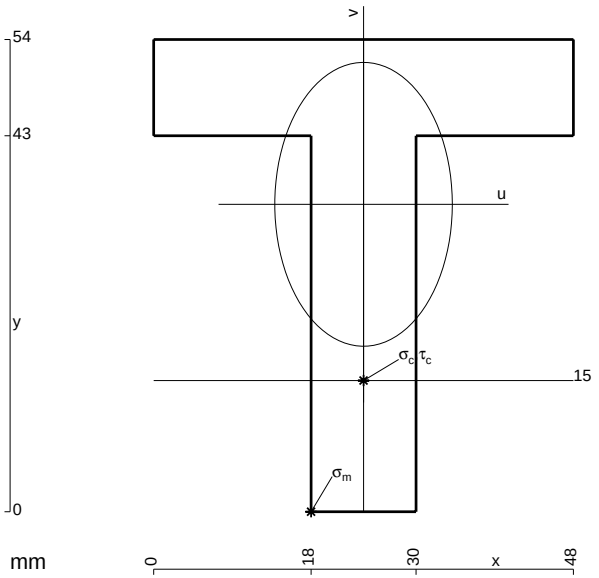
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1044. mm^2
J_u = 275075. mm^4
J_v = 107568. mm^4
y_g = 35.16 mm
N = 11548. N
T_y = -2980. N
M_x = 1788000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -35.16 mm
$\sigma_m = N/A - Mv/J_u$ = 239.6 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -20.16 mm
$\sigma_c = N/A - Mv/J_u$ = 142.1 N/mm^2
τ_c = 4.494 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 142.3 N/mm^2
S^* = 4978. mm^3

ESΣ8.164

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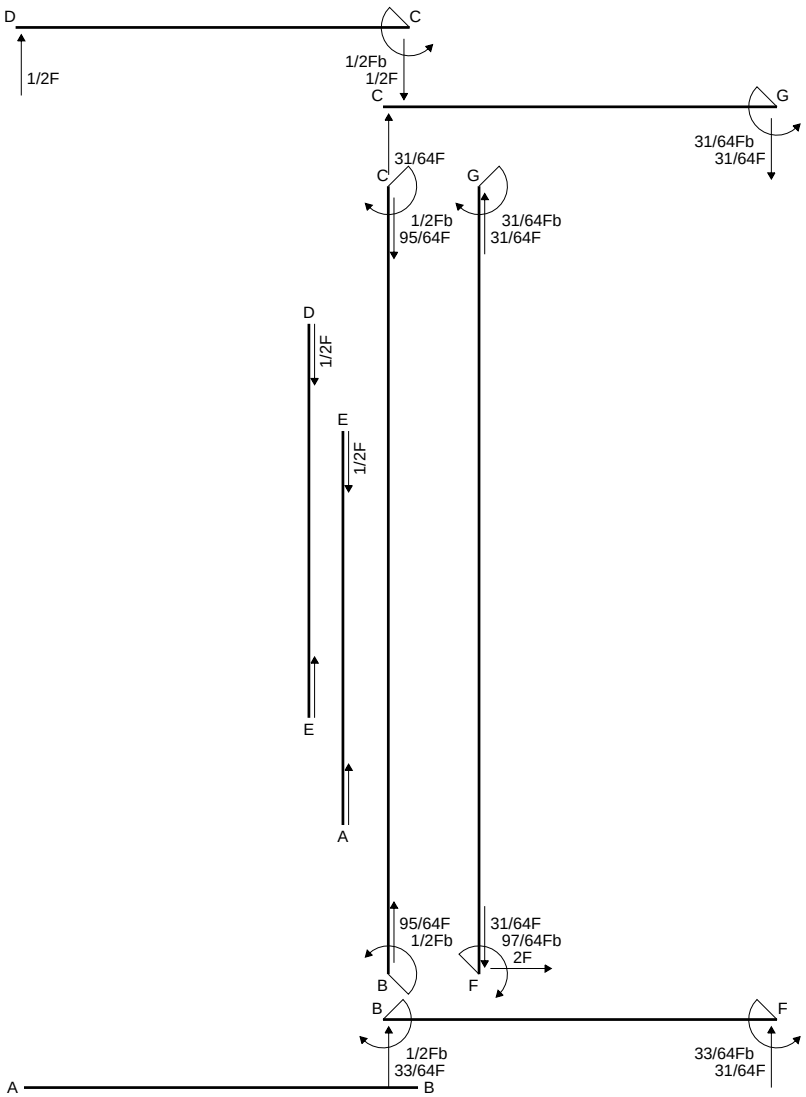
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C
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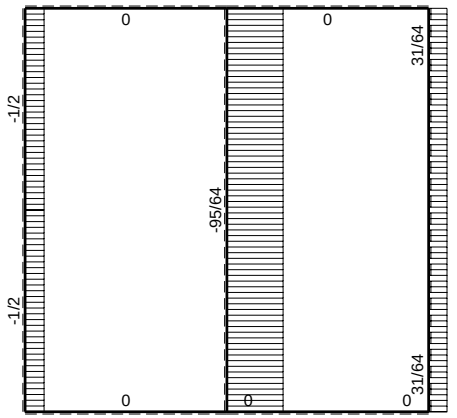
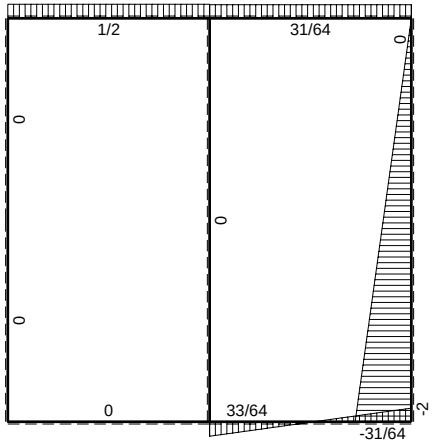
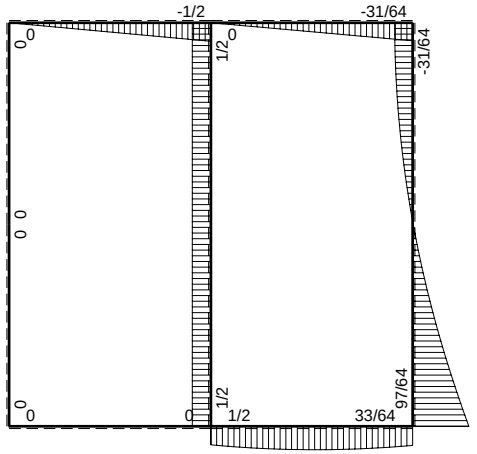
1/2Fb
1/2F
C
G
31/64Fb
31/64F
C
31/64F
G
1/2Fb
95/64F
31/64Fb
31/64F
D
1/2F
E
1/2F
E
A
95/64F
1/2Fb
31/64F
97/64Fb
2F
B
F
B
F
1/2Fb
33/64F
33/64Fb
31/64F
A
B


0
0
31/64
-1/2
-95/64
-1/2
0
0
0
31/64

F


1/2
31/64
0
0
0
0
0
33/64
-2
-31/64

F


-1/2
-31/64
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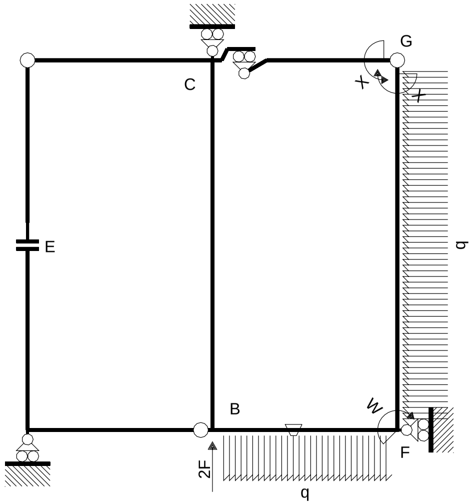
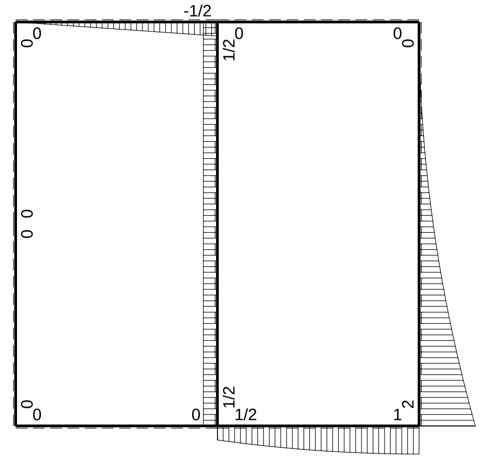
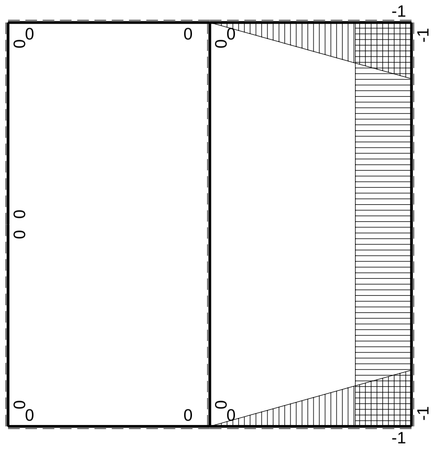
0
1/2
0
-31/64
0
0
0
0
1/2
0
1/2
33/64
97/64

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$1/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	$-1/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$1/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$-1/2Fb$	0	0	0	0		
	totali						$-31/24Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$31/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

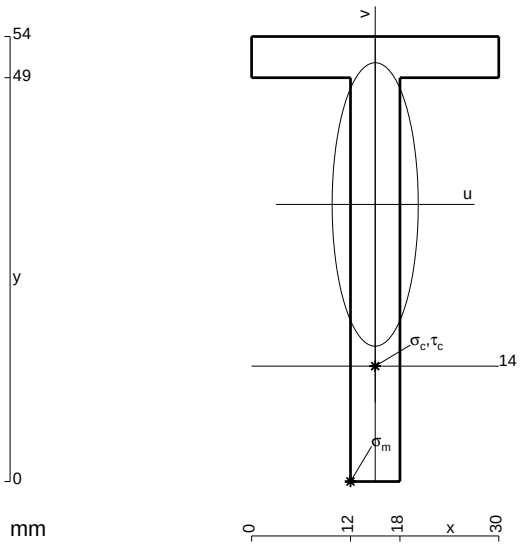
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 444. mm^2
J_u = 131544. mm^4
J_v = 12132. mm^4
y_g = 33.62 mm
T_y = 1220. N
M_x = -780800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -33.62 mm
$\sigma_m = -Mv/J_u$ = -199.6 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -19.62 mm
$\sigma_c = -Mv/J_u$ = -116.5 N/mm^2
τ_c = 3.457 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 116.6 N/mm^2
S^* = 2236. mm^3

ESΣ8.165

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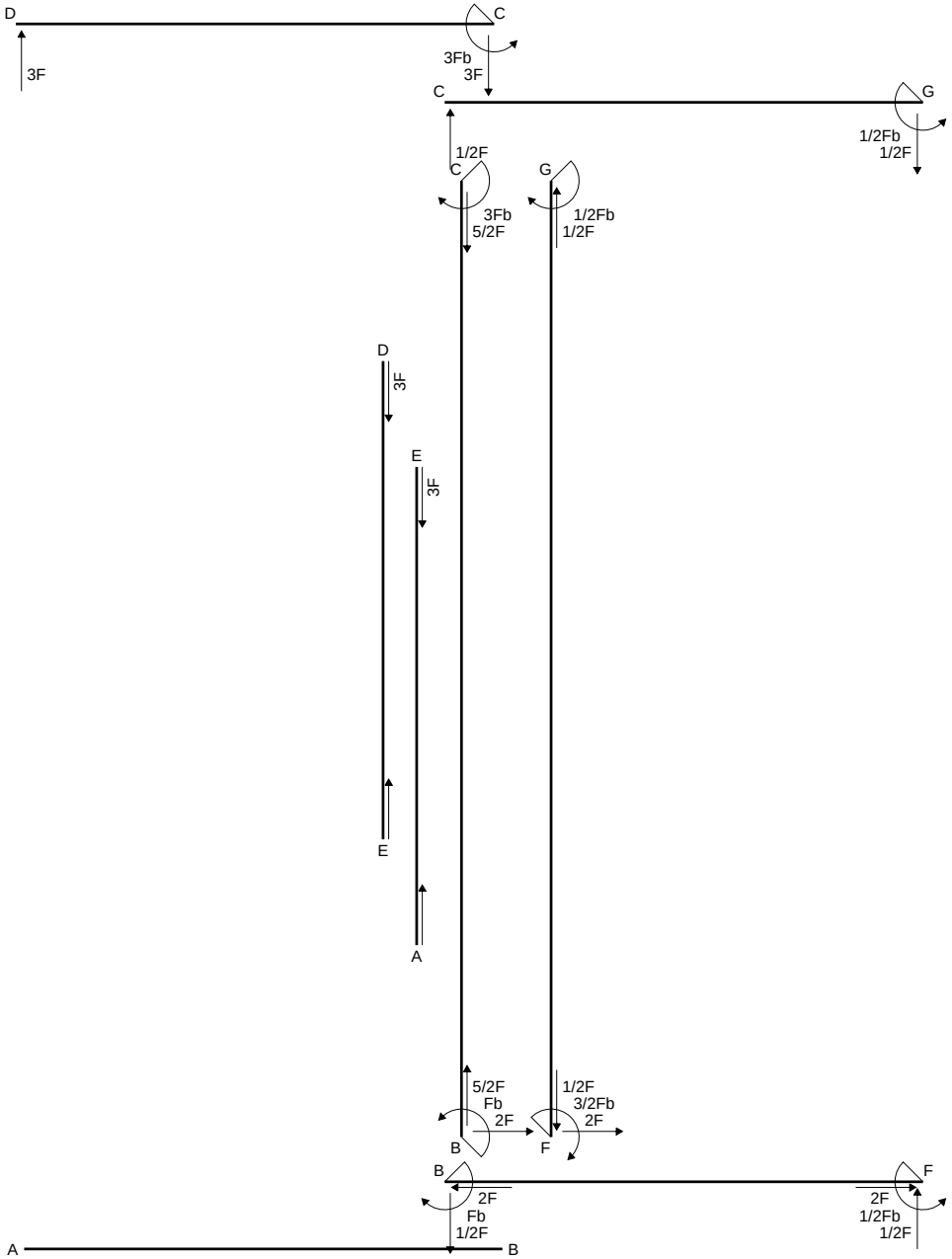
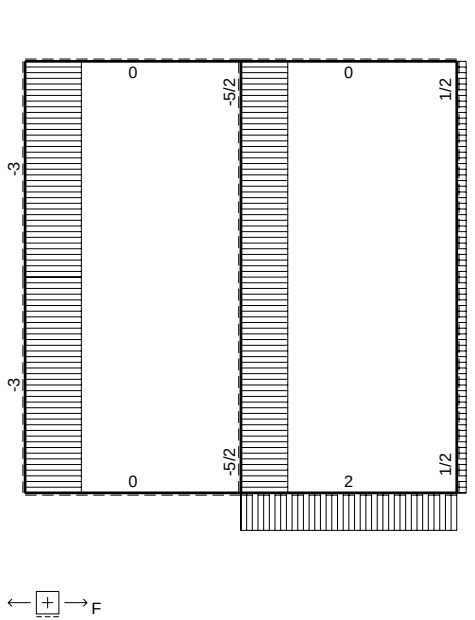
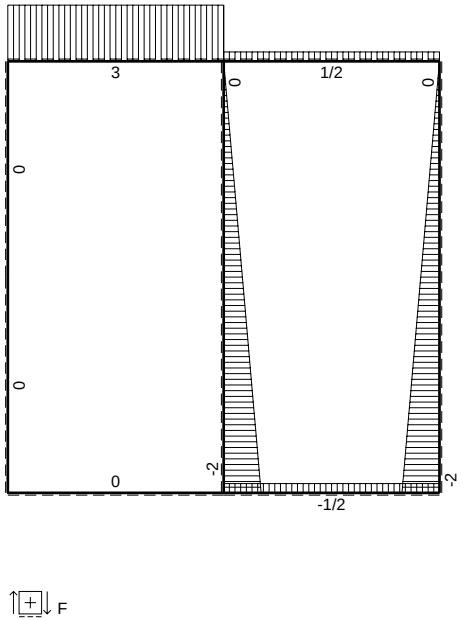
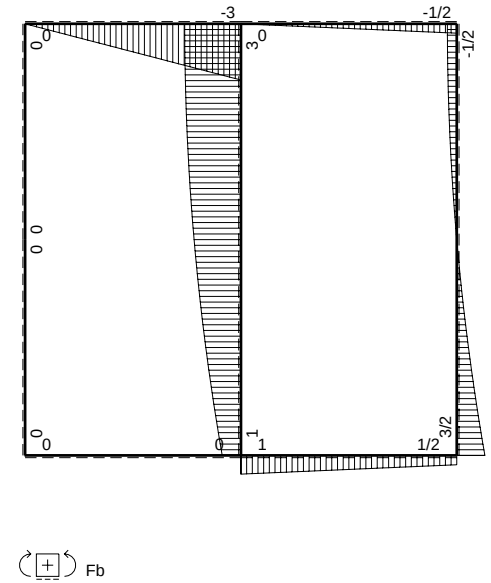

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B


0
-5/2
0
1/2
-3
-3
0
-5/2
2
1/2
F


3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
0
-1/2
0
0
0
0
0
1
1
1/2
3/2
Fb

M_x flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	$3Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$-Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						1/2Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

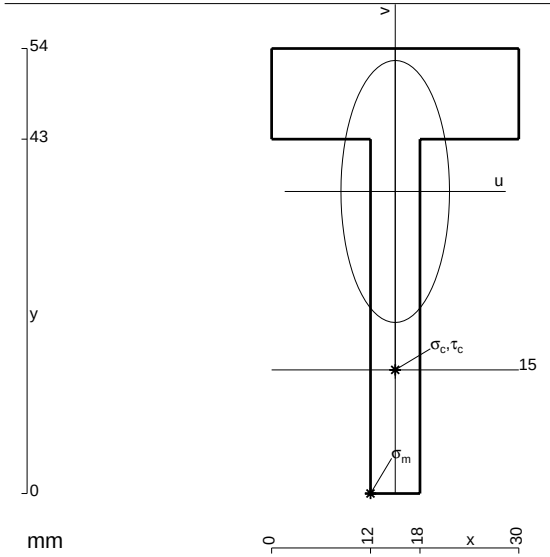
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.166



mm

A = 588. mm^2
J_u = 148637. mm^4
J_v = 25524. mm^4
y_g = 36.65 mm
T_y = 1230. N
M_x = -836400. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -36.65 mm
$\sigma_m = -Mv/J_u$ = -206.3 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -21.65 mm
$\sigma_c = -Mv/J_u$ = -121.8 N/mm^2
τ_c = 3.619 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122. N/mm^2
S^* = 2624. mm^3

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ESΣ8.166

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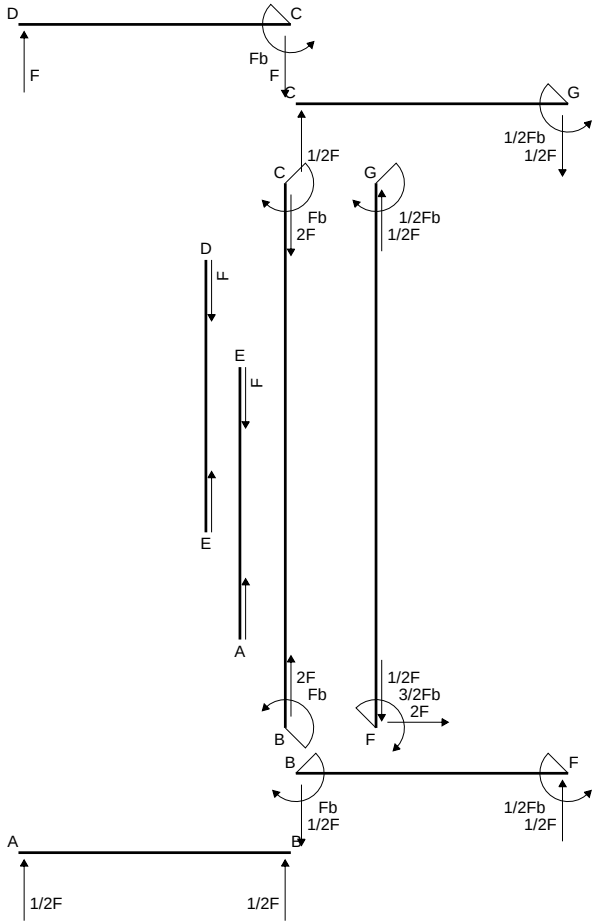
D
C
F
Fb
F
C
G
1/2Fb
1/2F
1/2F
C
G
Fb
2F
1/2Fb
1/2F
D
F
E
F
E
A
2F
Fb
1/2F
3/2Fb
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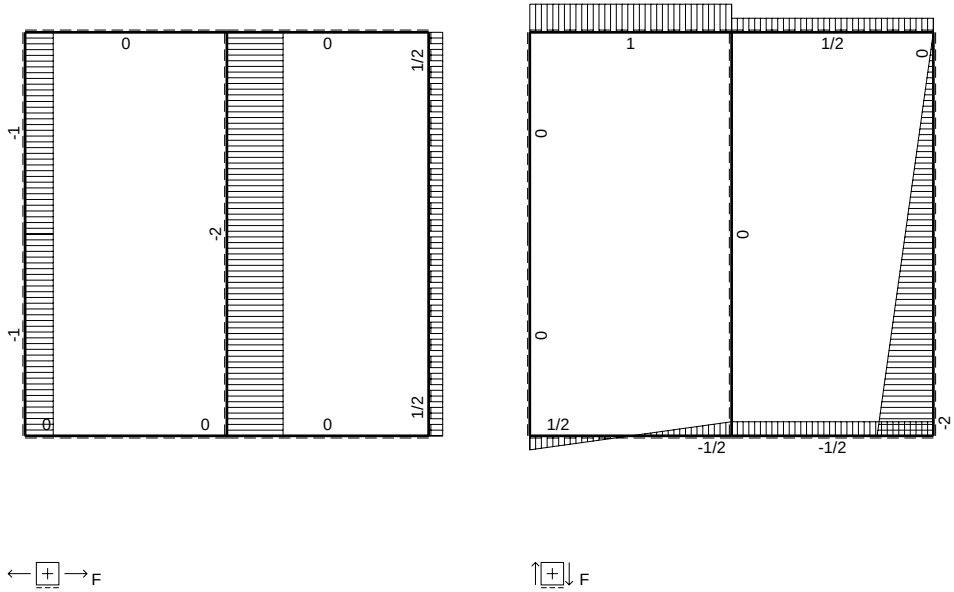
A
B
1/2F
1/2F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0
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-2
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1/2
0
0
0
0
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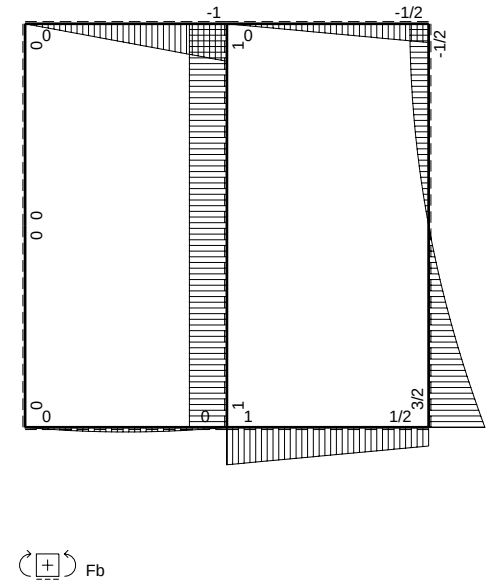
-2
-1/2
-1/2
F
F


-1
-1/2
0
1
0
-1/2
0
0
0
0
1
0
1
1/2
3/2
Fb

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ8.167

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14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

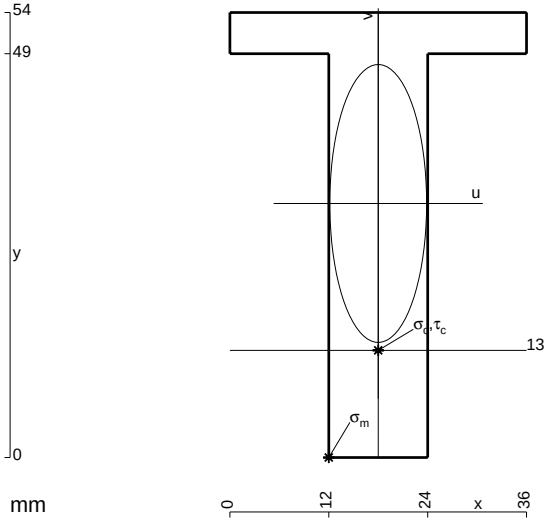
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 768. mm^2
J_u = 218489. mm^4
J_v = 26496. mm^4
y_g = 30.83 mm
T_y = 2130. N
M_x = -1554900. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -30.83 mm
$\sigma_m = -Mv/J_u$ = -219.4 N/mm^2
x_c = 18. mm
y_c = 13. mm
v_c = -17.83 mm
$\sigma_c = -Mv/J_u$ = -126.9 N/mm^2
τ_c = 3.083 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 127. N/mm^2
S^* = 3795. mm^3

ESΣ8.167

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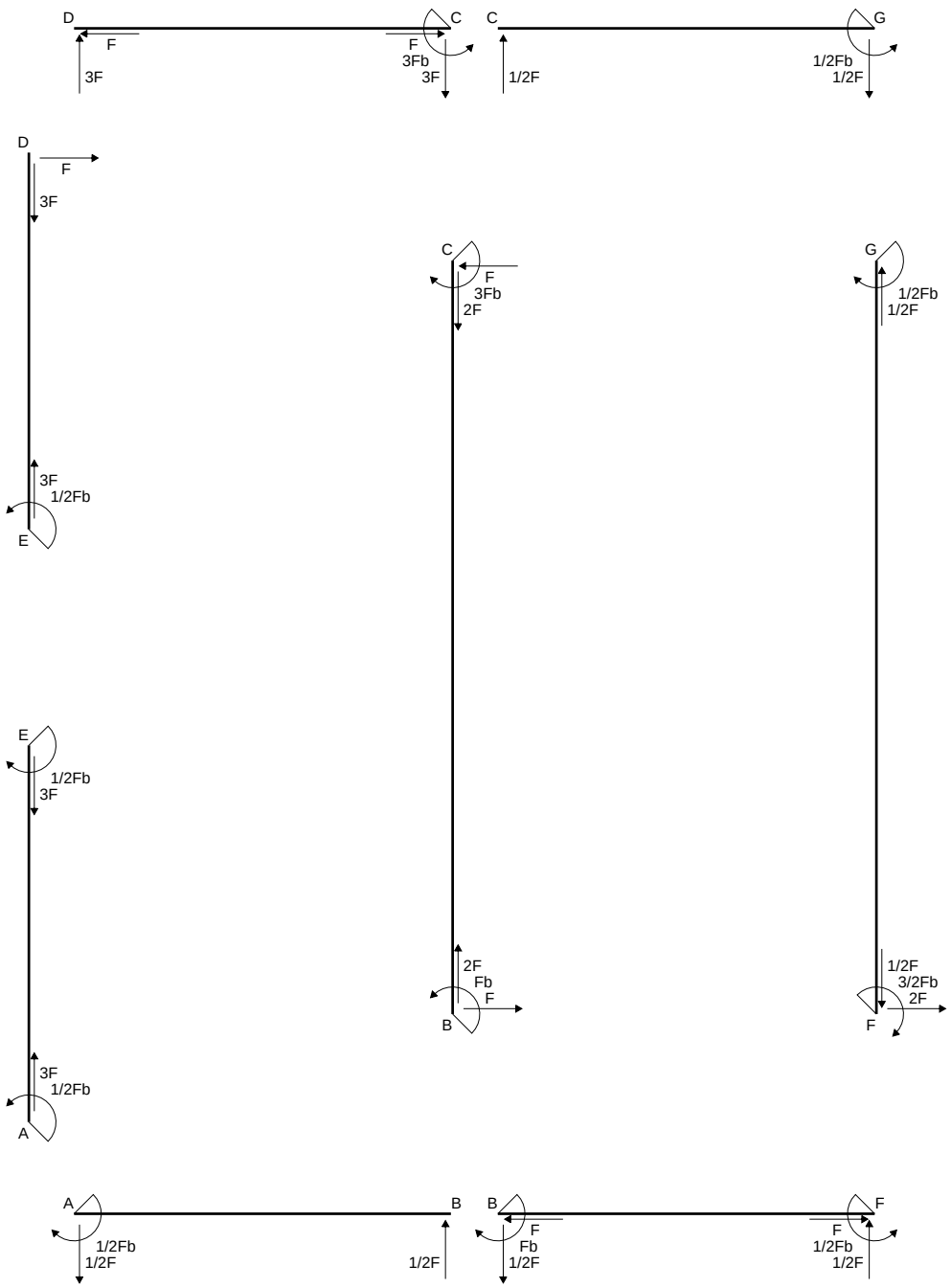
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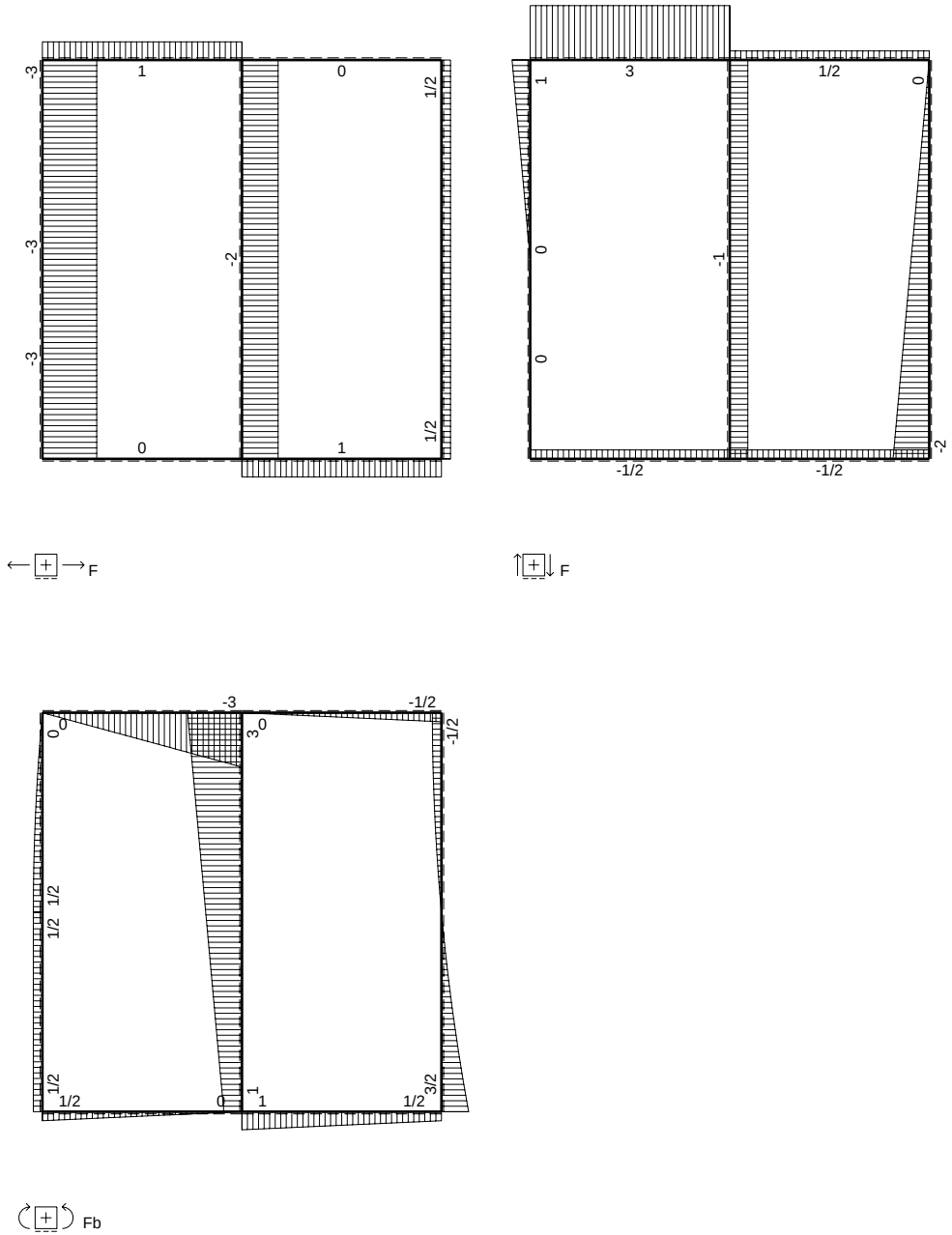
1/2F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F
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-1/2
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0
3
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-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^{2}b^2	(-1/2+1/2)Fb^2/EJ	1/3XbEJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3XbEJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2XbEJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							-4/3Fb^2/EJ	8/3XbEJ
	iperstatica $X=W_{GC}$						1/2Fb	

PROCEDIMENTO E RISULTATI Nome: ESΣ8.168

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14.11.23

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

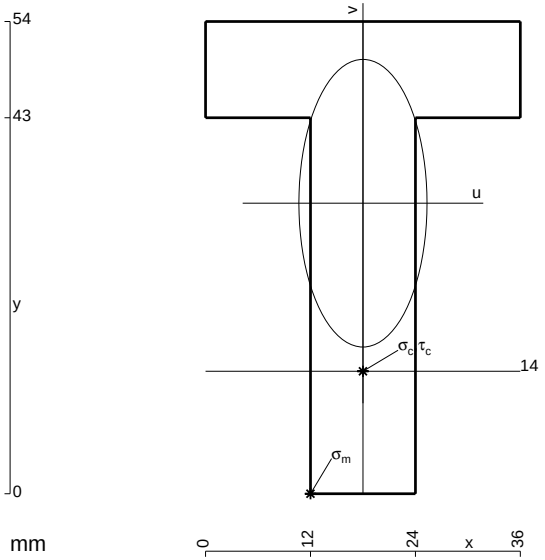
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 912. mm^2
J_u = 246834. mm^4
J_v = 48960. mm^4
y_g = 33.22 mm
N = 740. N
T_y = 2220. N
M_x = -1709400. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -33.22 mm
$\sigma_m = N/A - Mv/J_u$ = -229.3 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -19.22 mm
$\sigma_c = N/A - Mv/J_u$ = -132.3 N/mm^2
τ_c = 3.302 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 132.4 N/mm^2
S^* = 4406. mm^3

ESΣ8.168

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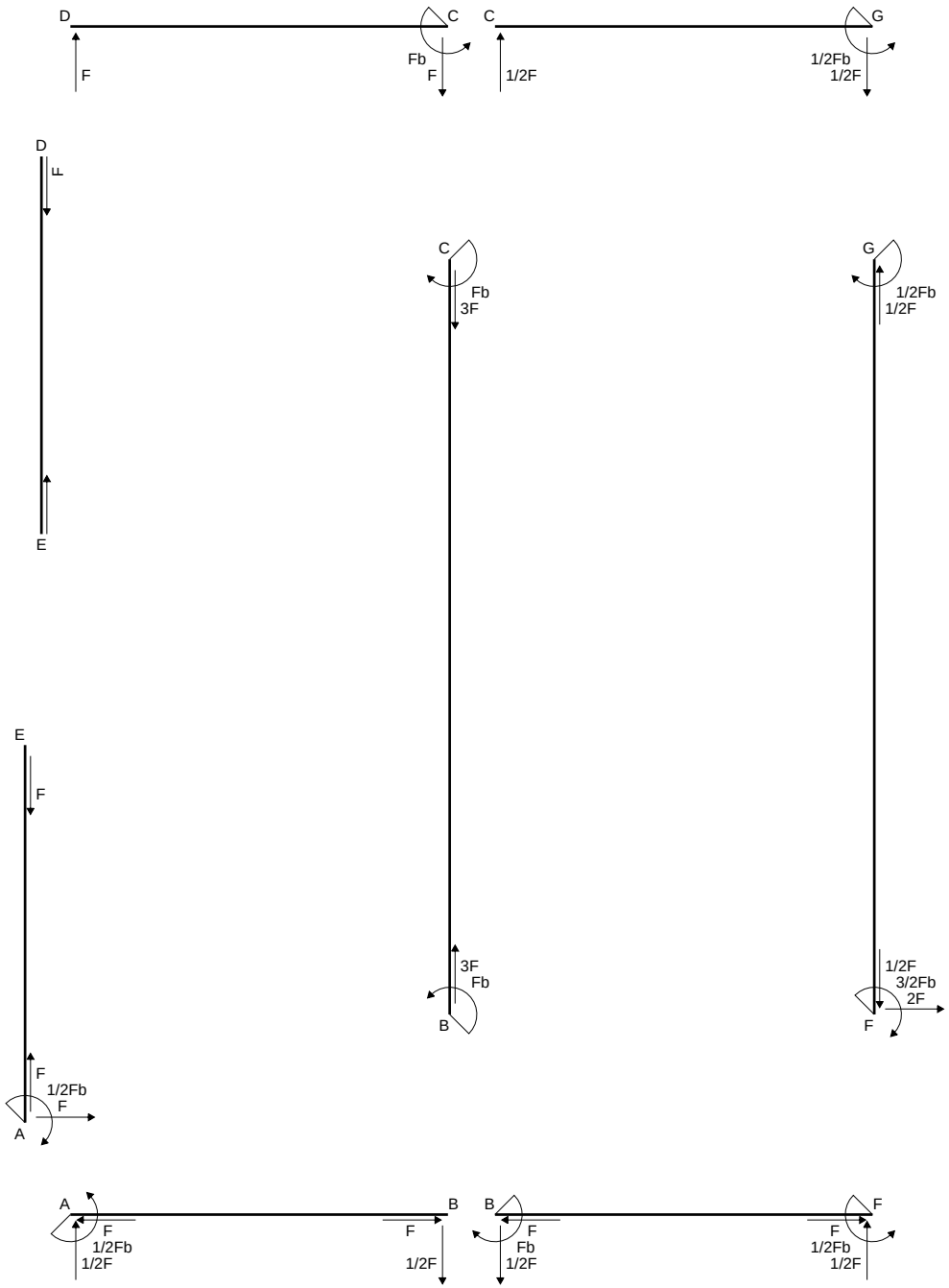
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E
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Fb
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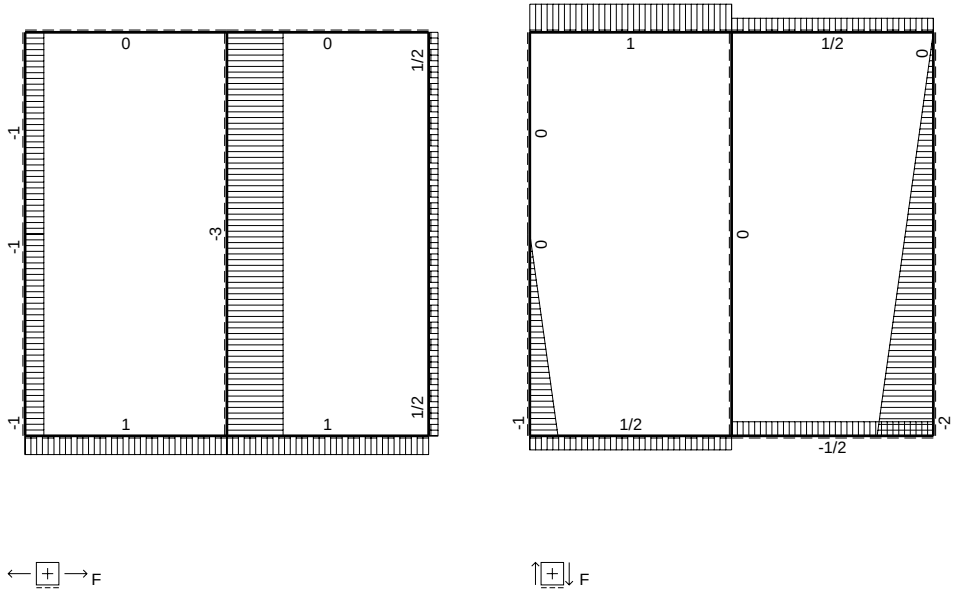
1/2Fb
1/2F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-1/2
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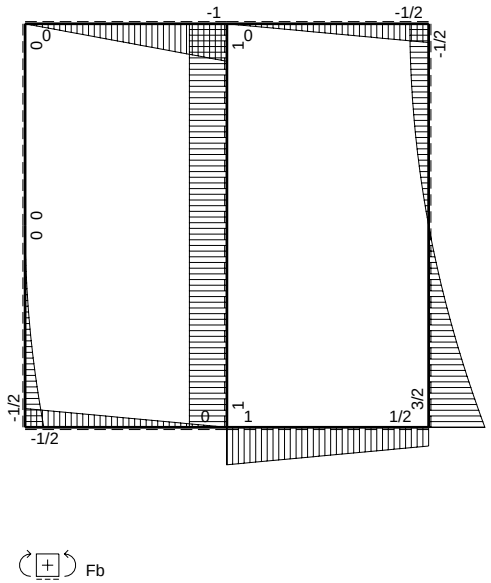
F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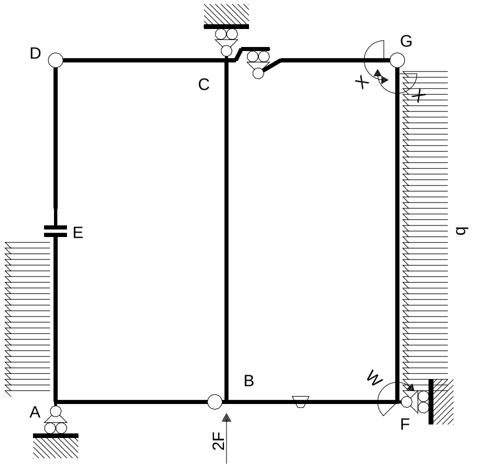
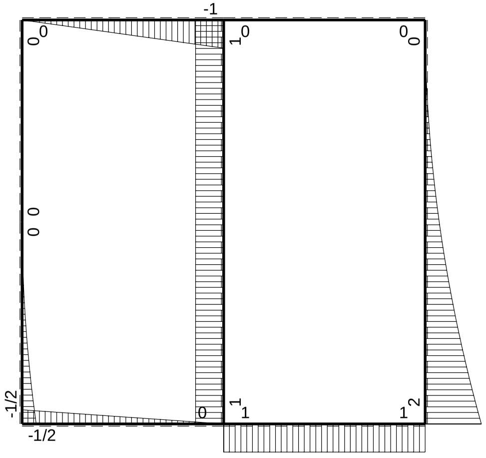
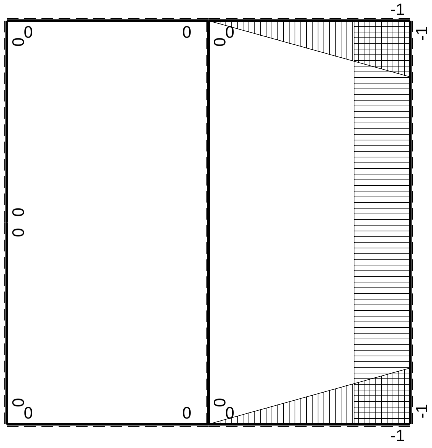
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$1/2Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

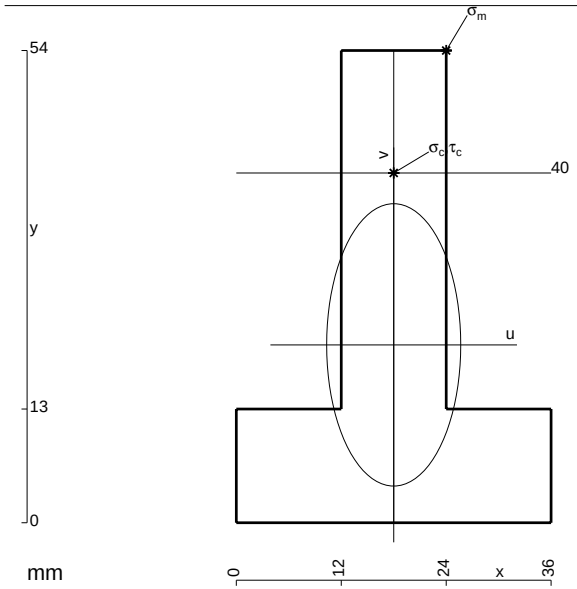
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 960. mm^2
J_u = 250363. mm^4
J_v = 56448. mm^4
y_g = 20.34 mm
T_y = 2200. N
M_x = -1782000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.66 mm
$\sigma_m = -Mv/J_u$ = 239.6 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 19.66 mm
$\sigma_c = -Mv/J_u$ = 140. N/mm^2
τ_c = 3.28 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 140.1 N/mm^2
S^* = 4479. mm^3

ESΣ8.169

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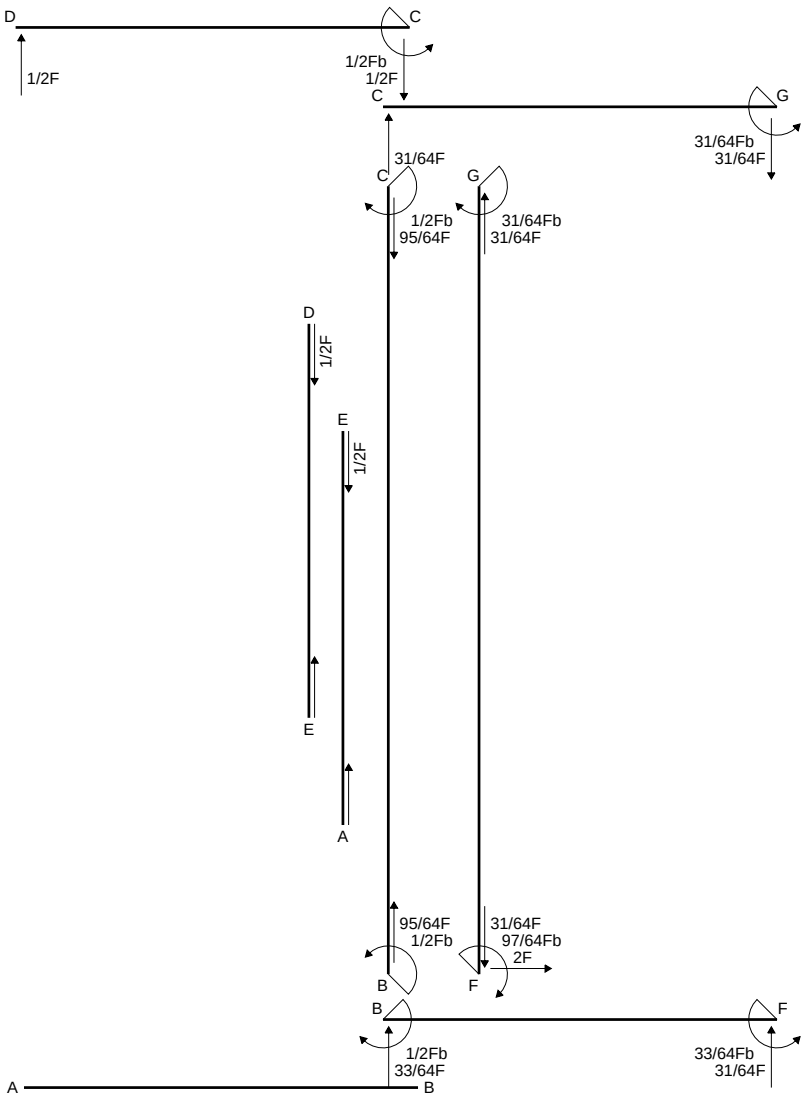
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1/2Fb
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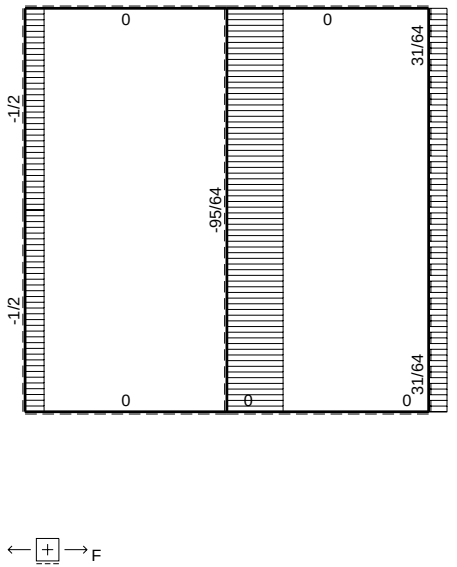
B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31/64

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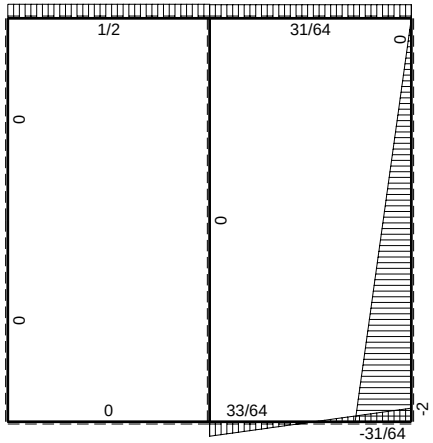
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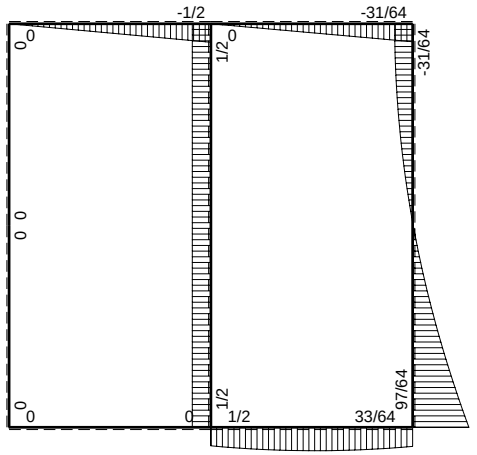
-31/64

F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33/64
97/64

Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$1/2Fb+Fx-1/2qx^2$	-Fb/EJ	$-1/2Fx-Fx^2/b+1/2qx^3/b$	FX/EJ	x^2/b^2	$(-11/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$-Fb+1/2qx^2$	Fb/EJ	$-Fb+Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-FX/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	1/2Fb	0	0	0	0	0+0	0
BC 2b	0	-1/2Fb	0	0	0	0		
	totali						-31/24Fb²/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						31/64Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ8.170

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [-1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

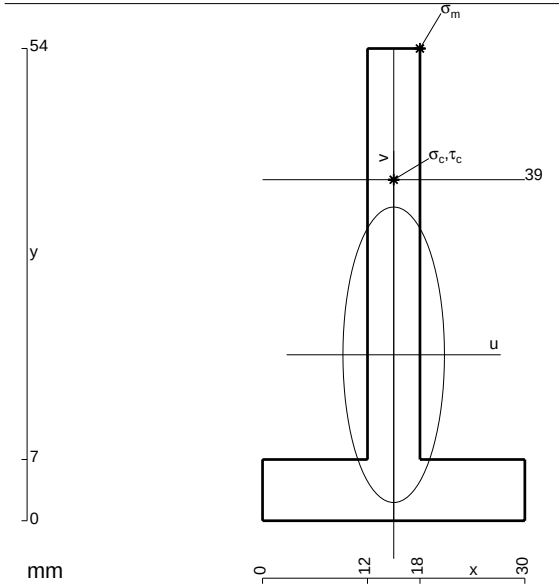
$= (-\ b +1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 492. mm^2
J_u = 140516. mm^4
J_v = 16596. mm^4
y_g = 18.98 mm
T_y = 1865. N
M_x = -801950. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 35.02 mm
$\sigma_m = -Mv/J_u$ = 199.9 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 20.02 mm
$\sigma_c = -Mv/J_u$ = 114.3 N/mm^2
τ_c = 5.48 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 114.7 N/mm^2
S^* = 2477. mm^3

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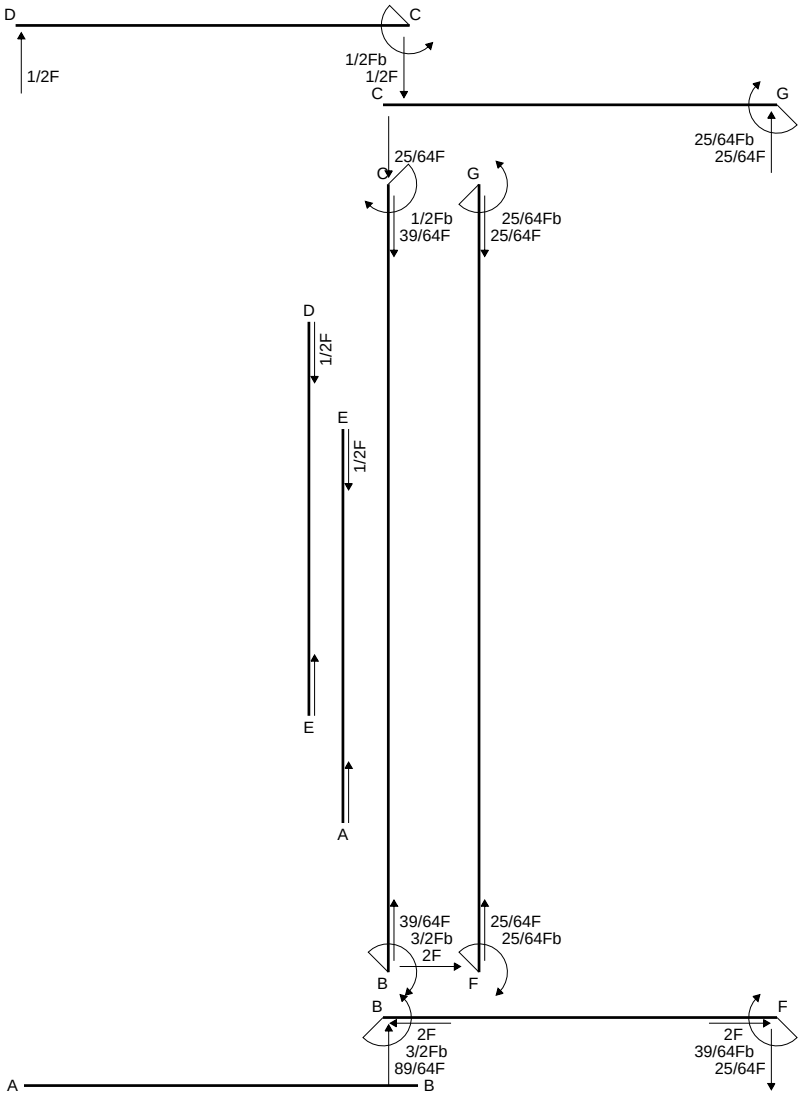
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1/2Fb
1/2F
C
G
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25/64F
C
25/64F
G
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25/64Fb
25/64F
D
1/2F
E
1/2F
E
A
39/64F
3/2Fb
2F
25/64F
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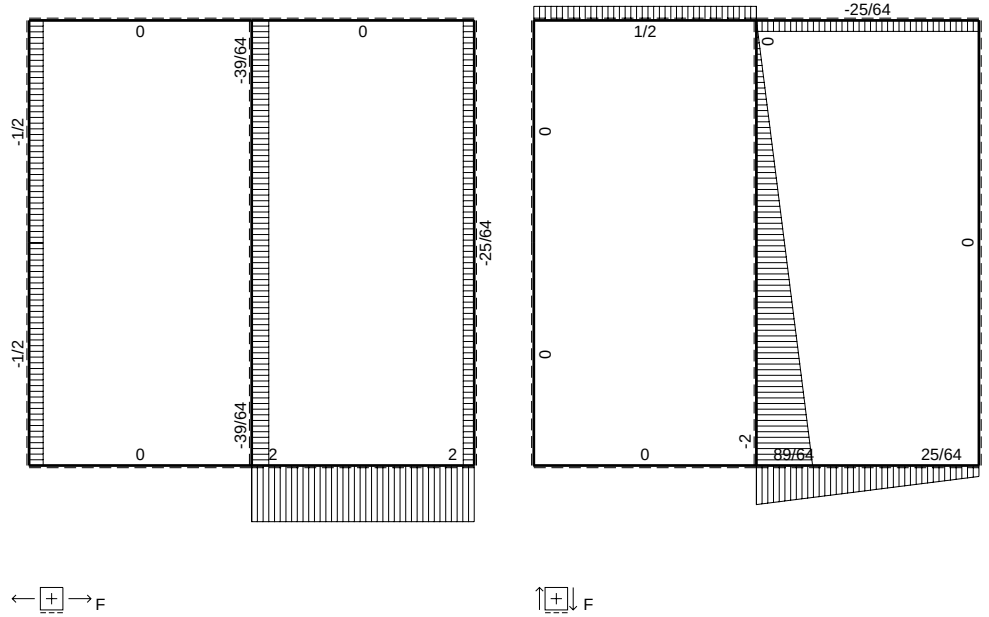
A
B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1/2
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0
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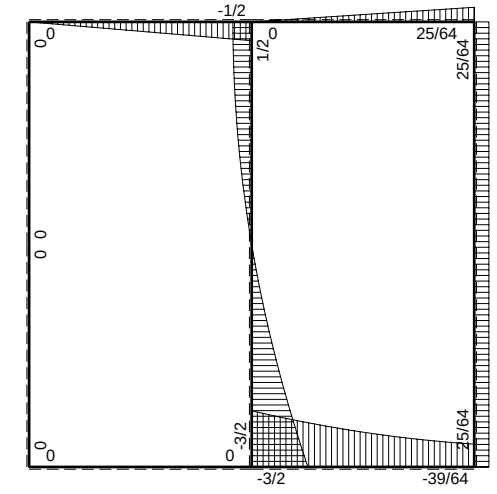
89/64
25/64
F
F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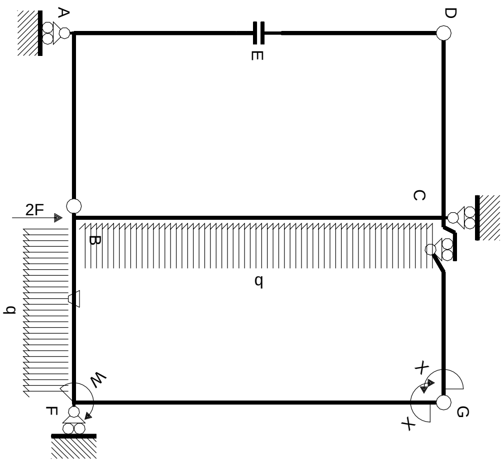
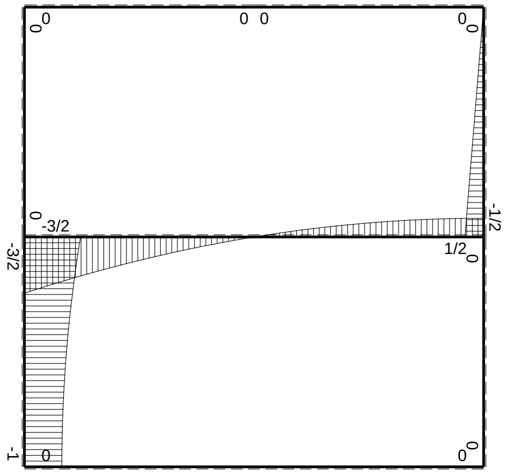
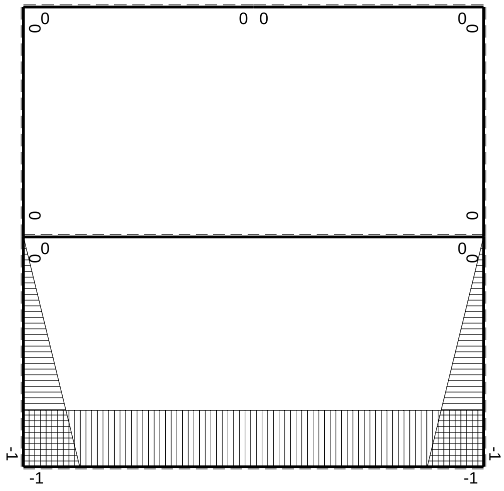
-3/2
-39/64
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$25/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-25/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b)\ \theta\ dx$

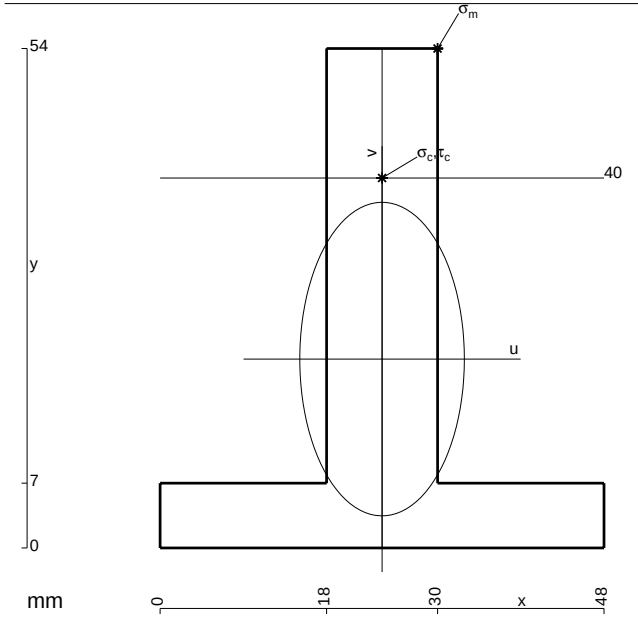
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 900. mm^2
J_u = 258693. mm^4
J_v = 71280. mm^4
y_g = 20.42 mm
N = -1408. N
T_y = -4620. N
M_x = -1628550. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 33.58 mm
$\sigma_m = N/A - Mv/J_u$ = 209.8 N/mm^2
x_c = 24. mm
y_c = 40. mm
v_c = 19.58 mm
$\sigma_c = N/A - Mv/J_u$ = 121.7 N/mm^2
τ_c = 6.646 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 122.2 N/mm^2
S^* = 4465. mm^3

ESΣ8.171

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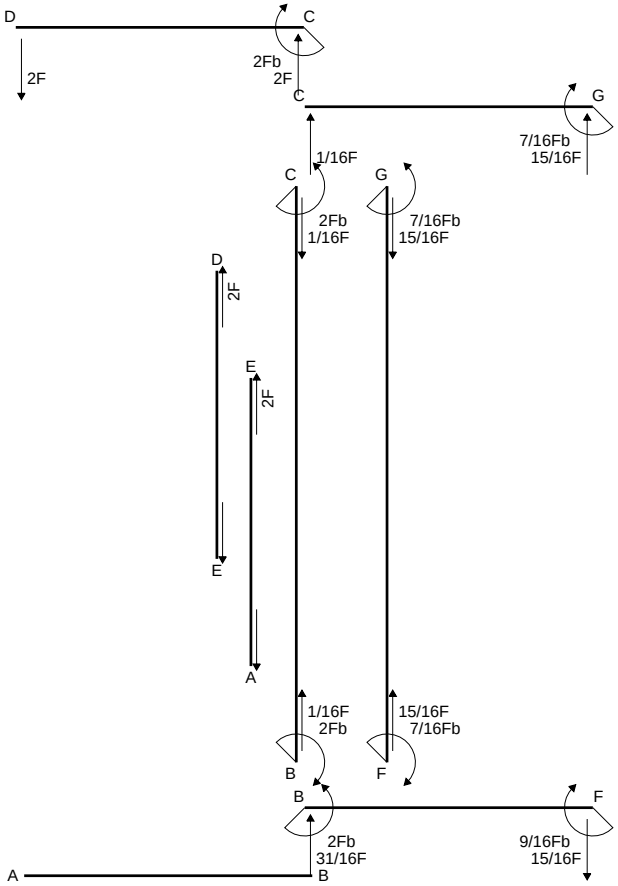
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2F
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G
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C
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F
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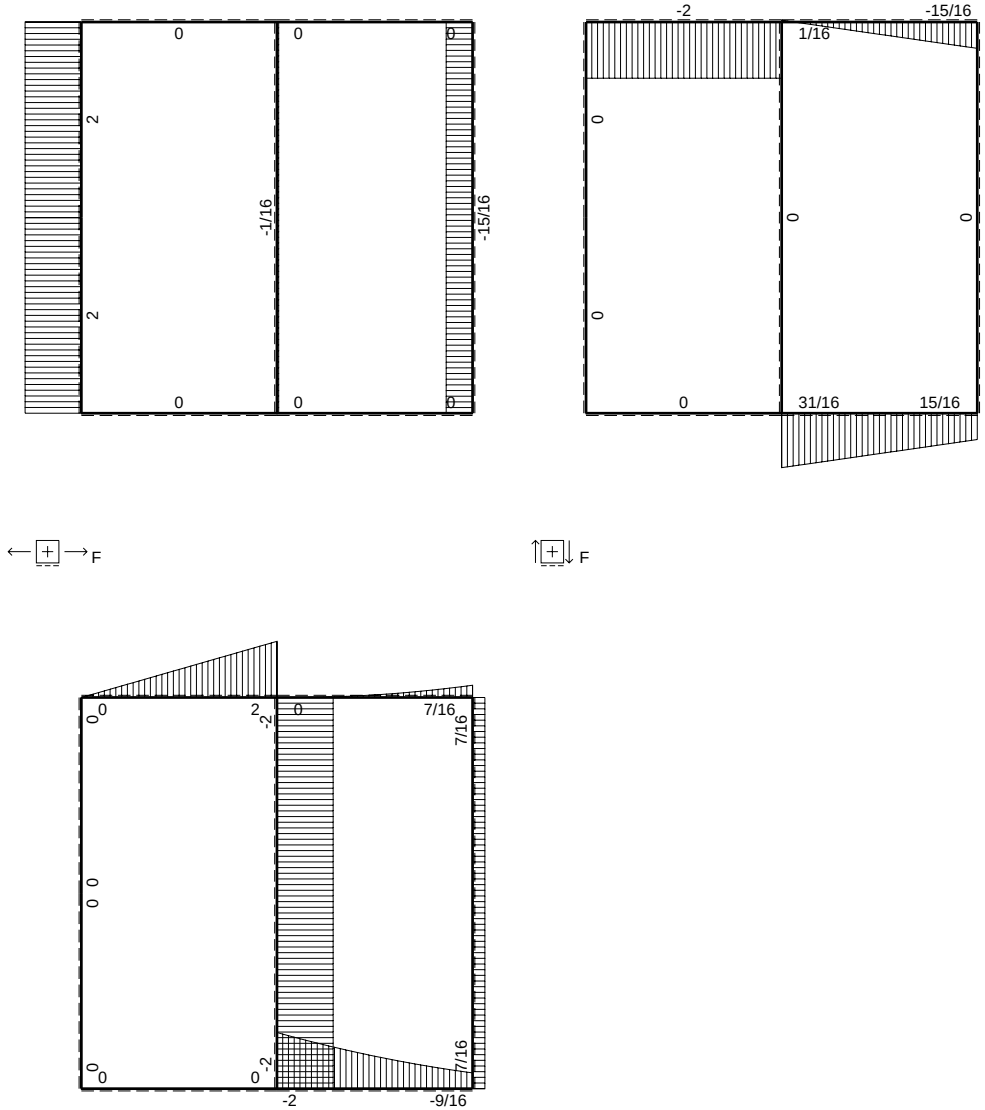
9/16Fb
15/16F
A
B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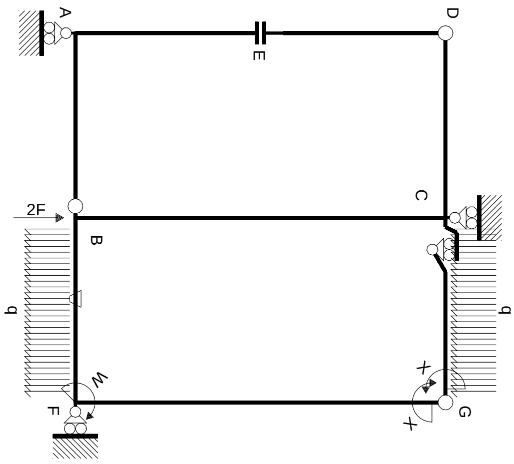
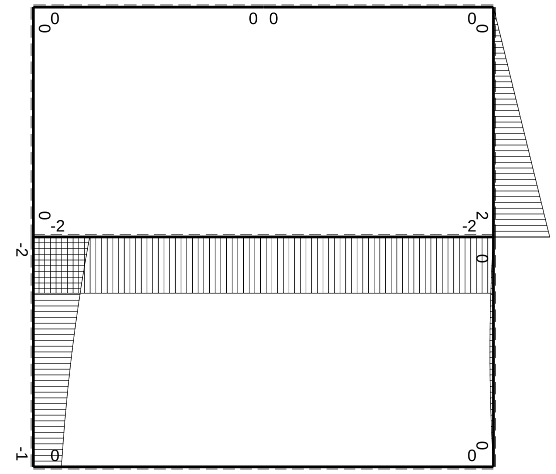
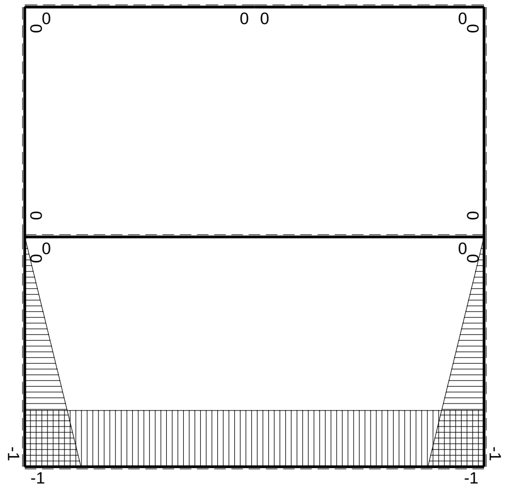
-9/16
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	2Fb-2Fx	0	0	0	0	0+0	0
DC b	0	-2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-2Fb+3/2Fx-1/2qx^2$	-Fb/EJ	$2Fx-3/2Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(5/8+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2Fx+1/2qx^2$	Fb/EJ	$Fb-1/2Fx-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-2Fb	0	0	0	0	0+0	0
BC 2b	0	2Fb	0	0	0	0		
	totali						$7/6Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-7/16Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

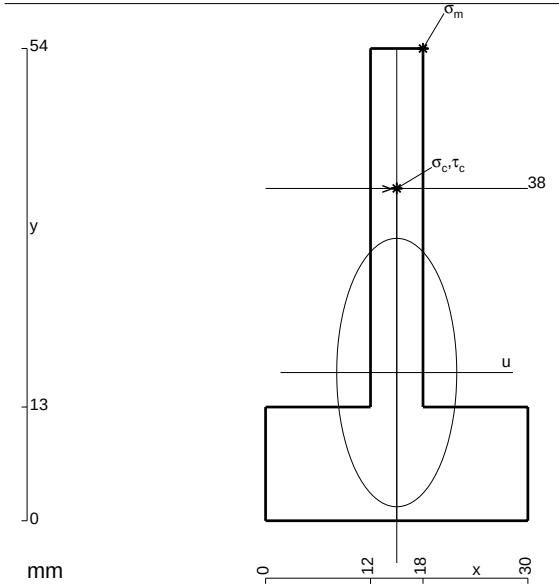
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 636. mm^2
J_u = 149922. mm^4
J_v = 29988. mm^4
y_g = 16.94 mm
T_y = -1740. N
M_x = 887400. Nmm
x_m = 18. mm
y_m = 54. mm
u_m = 3. mm
v_m = 37.06 mm
$\sigma_m = -Mv/J_u$ = -219.3 N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.06 mm
$\sigma_c = -Mv/J_u$ = -124.6 N/mm^2
τ_c = 5.396 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 125. N/mm^2
S^* = 2789. mm^3

ESΣ8.172

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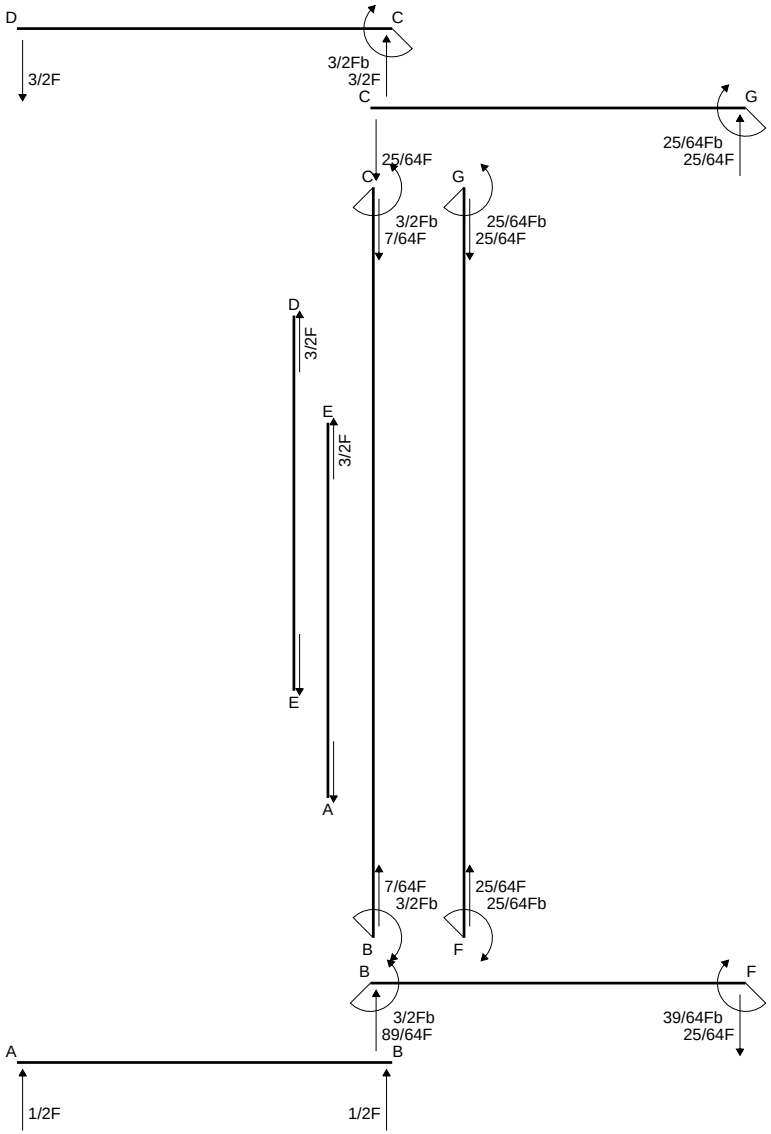
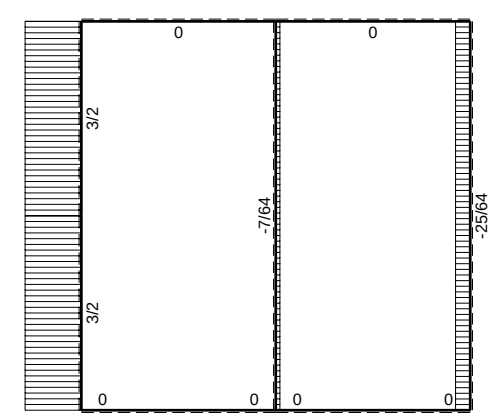
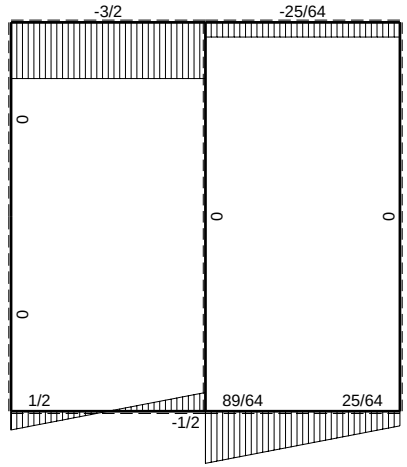

D
C
3/2F
3/2Fb
3/2F
C
G
25/64Fb
25/64F
25/64F
C
G
3/2Fb
7/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
A
7/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
3/2Fb
89/64F
39/64Fb
25/64F
B
A
1/2F
1/2F


0
0
3/2
-7/64
-25/64
3/2
0
0
0
0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-25/64
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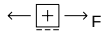
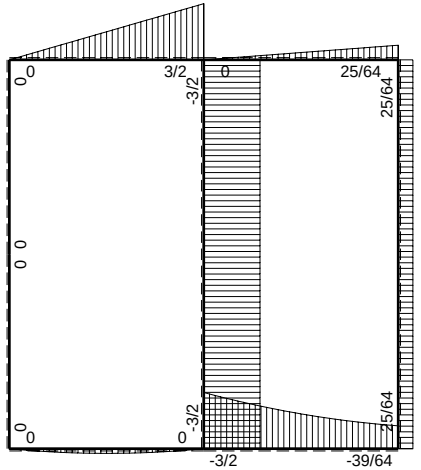
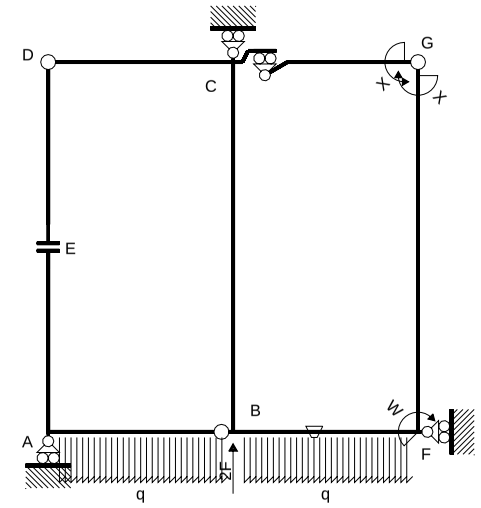
0
0
0
1/2
89/64
25/64
-1/2


F

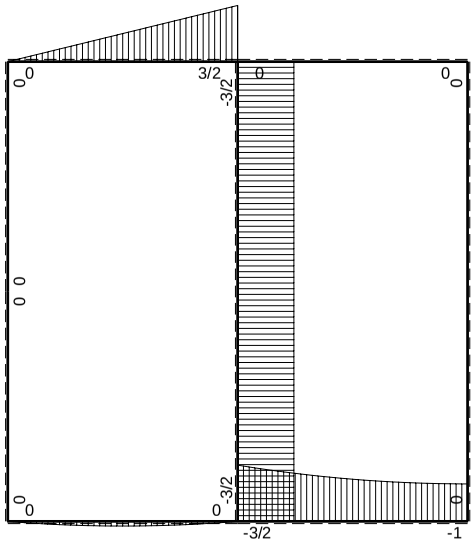

F


0
0
3/2
0
25/64
-3/2
25/64
0
0
0
0
-3/2
25/64
-3/2
-39/64

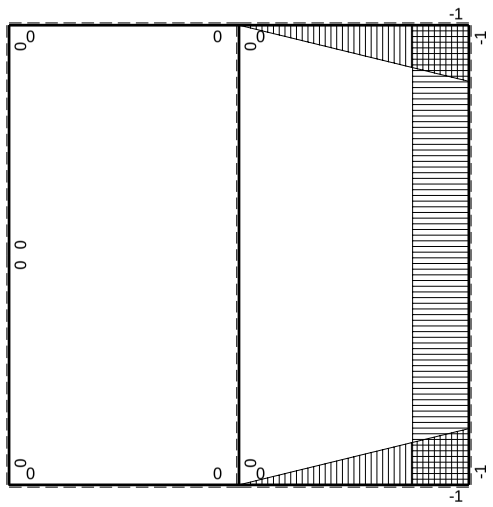

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
		totali					$25/24Fb^2/EJ$	8/3Xb/EJ
		iperstatica $X=W_{GC}$					-25/64Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.173

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

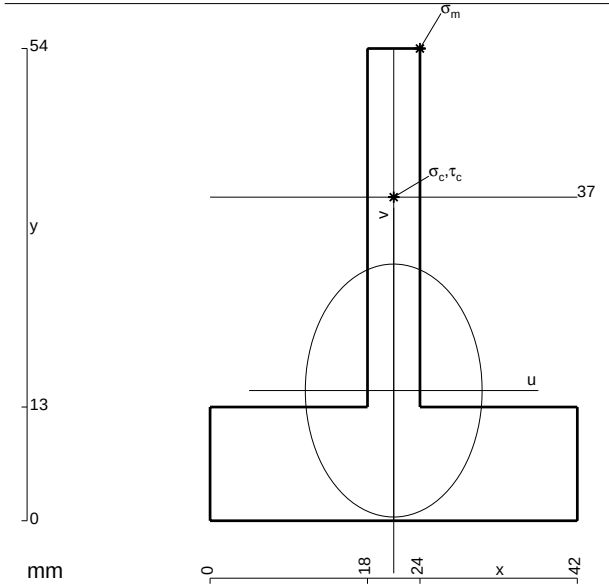
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 792. mm^2
J_u = 165782. mm^4
J_v = 81000. mm^4
y_g = 14.89 mm
T_y = -1770. N
M_x = 973500. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 39.11 mm
$\sigma_m = -Mv/J_u$ = -229.7 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.11 mm
$\sigma_c = -Mv/J_u$ = -129.9 N/mm^2
τ_c = 5.556 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 130.2 N/mm^2
S^* = 3123. mm^3

ESΣ8.173

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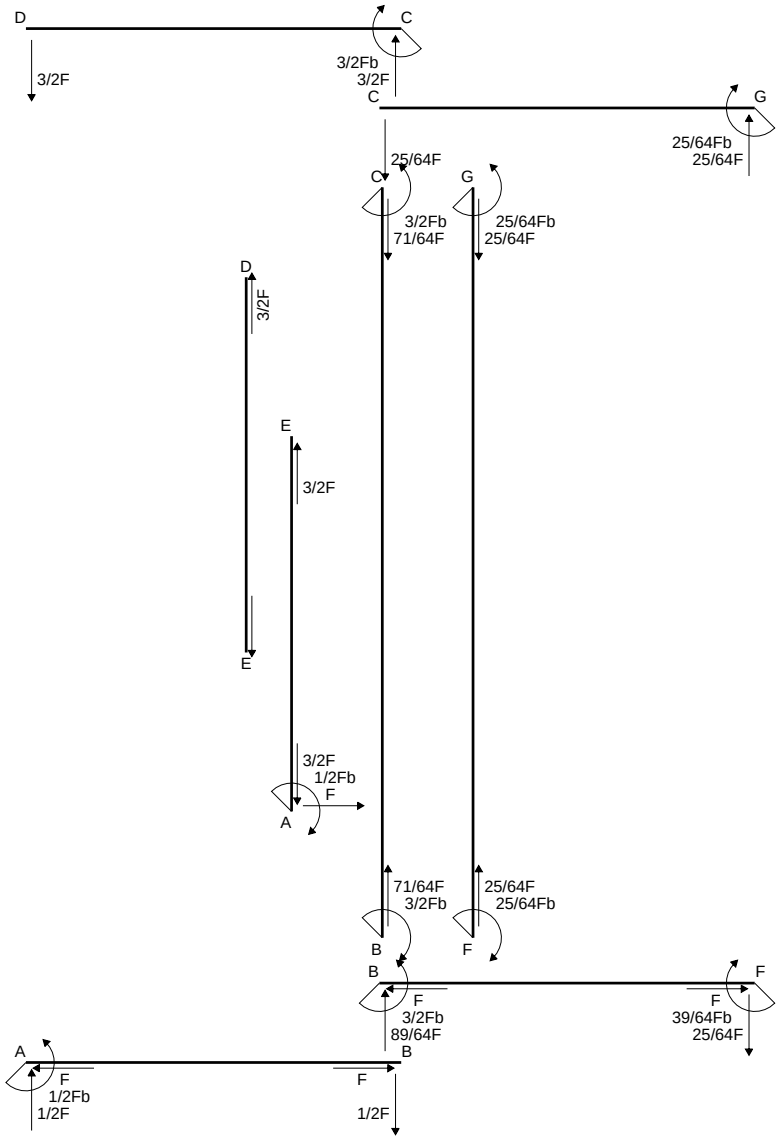
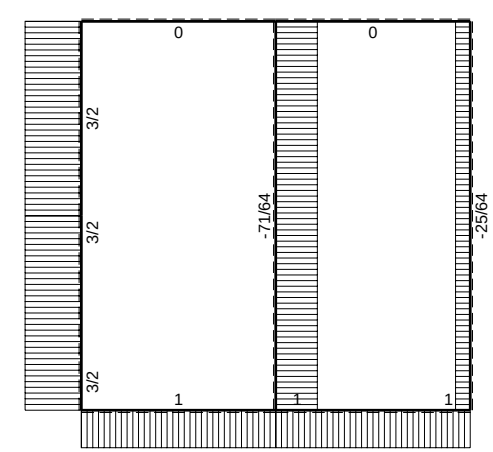

D
C
3/2F
3/2Fb
3/2F
C
G
25/64F
25/64Fb
25/64F
C
G
3/2Fb
71/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
71/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
F
3/2Fb
89/64F
F
39/64Fb
25/64F
B
A
F
1/2Fb
1/2F
F
1/2F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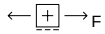
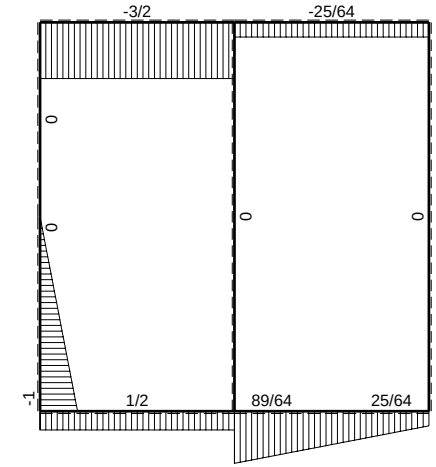
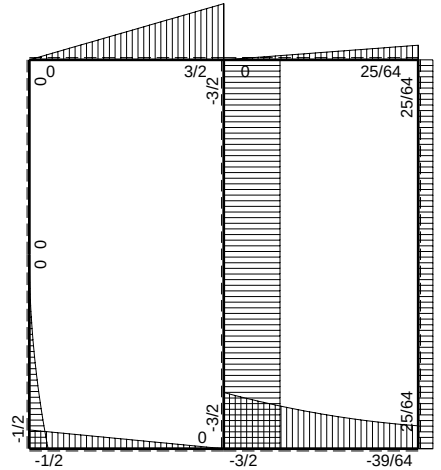
0
3/2
3/2
3/2
-71/64
-25/64
1
1
1

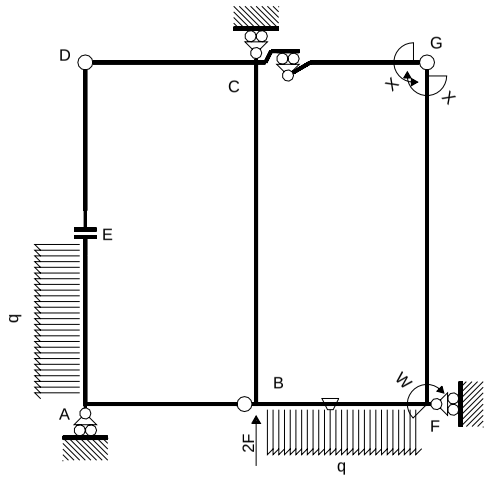

F


-3/2
-25/64
0
0
0
0
0
-1
1/2
89/64
25/64

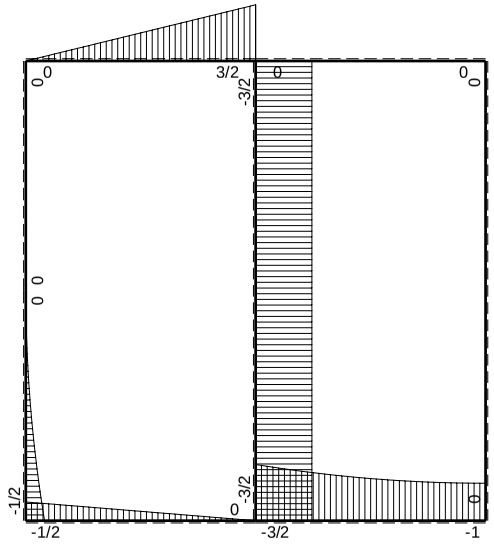

F


0
0
3/2
-3/2
0
25/64
25/64
0
0
0
-1/2
-3/2
0
25/64
-1/2
-3/2
-39/64

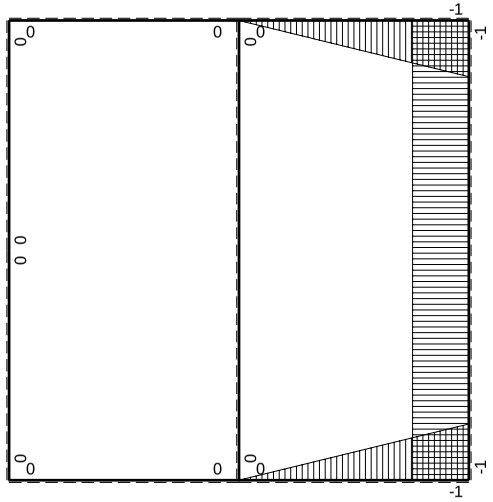

Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$25/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-25/64Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.174

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2\ )\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2\ )\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1\ )\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b\ )\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

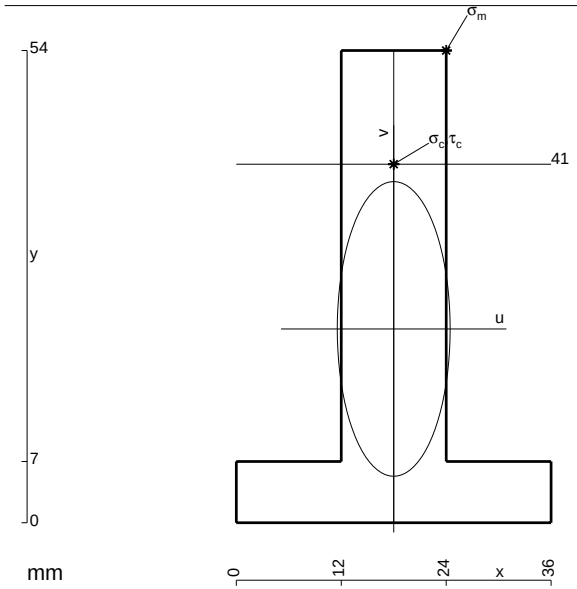
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 816. mm^2
J_u = 231827. mm^4
J_v = 33984. mm^4
y_g = 22.16 mm
T_y = -2910. N
M_x = 1746000. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 6. mm
v_m = 31.84 mm
$\sigma_m = -Mv/J_u$ = -239.8 N/mm^2
x_c = 18. mm
y_c = 41. mm
v_c = 18.84 mm
$\sigma_c = -Mv/J_u$ = -141.9 N/mm^2
τ_c = 4.135 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 142.1 N/mm^2
S^* = 3953. mm^3

ESΣ8.174

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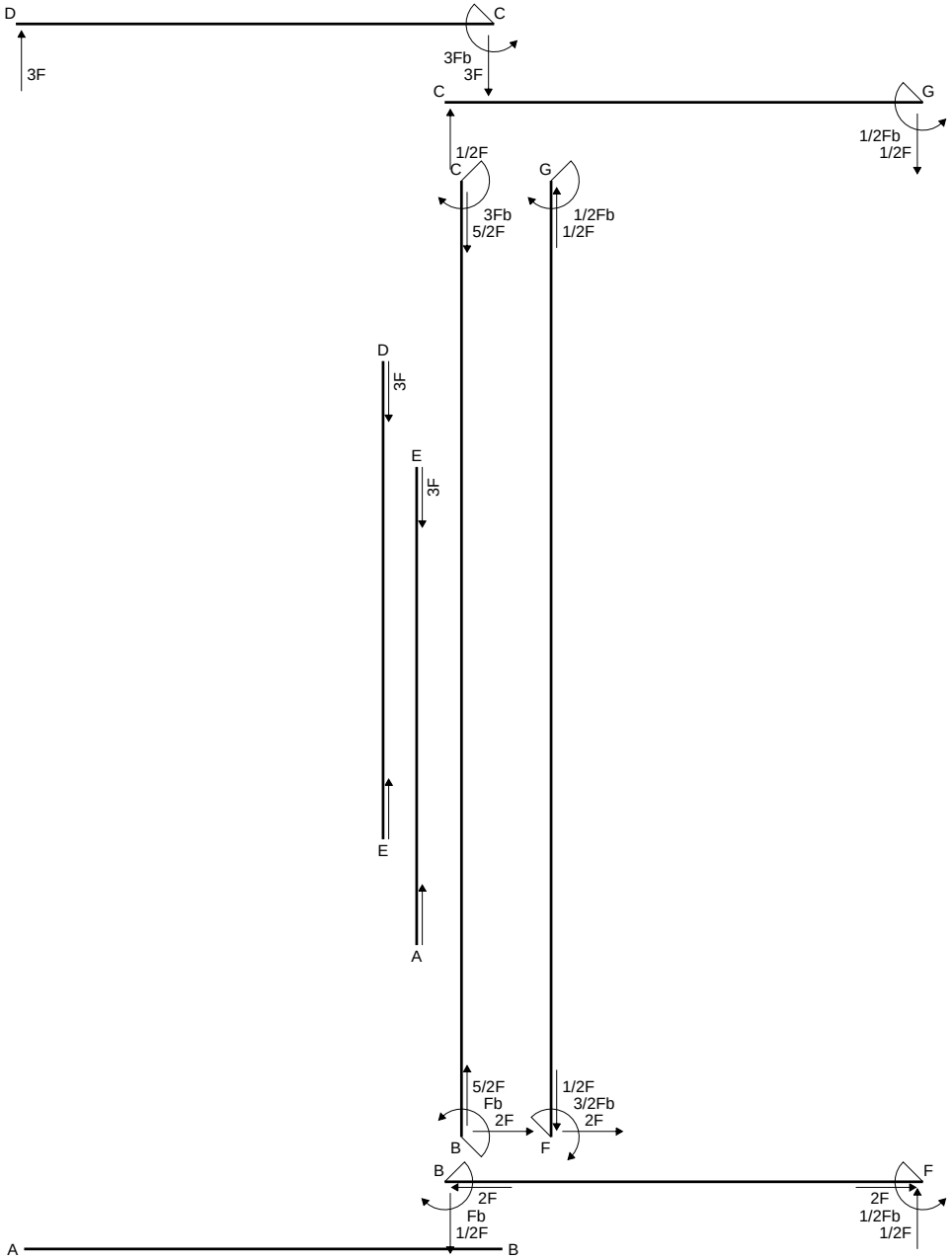
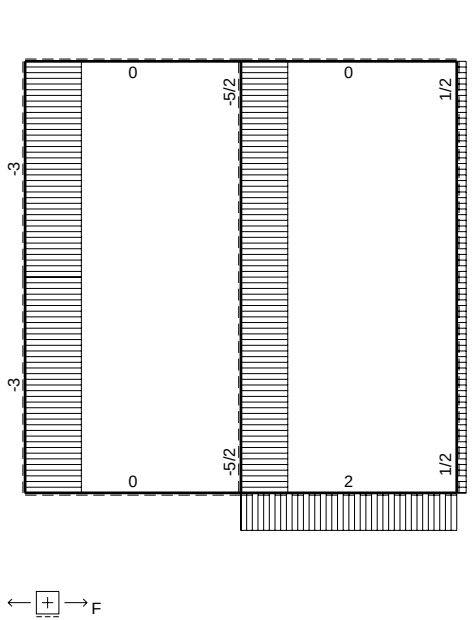

D
C
3F
3Fb
3F
C
G
1/2Fb
1/2F
1/2F
C
G
3Fb
5/2F
1/2Fb
1/2F
D
3F
E
3F
E
A
5/2F
Fb
2F
1/2F
3/2Fb
2F
B
F
B
F
2F
Fb
1/2F
2F
1/2Fb
1/2F
A
B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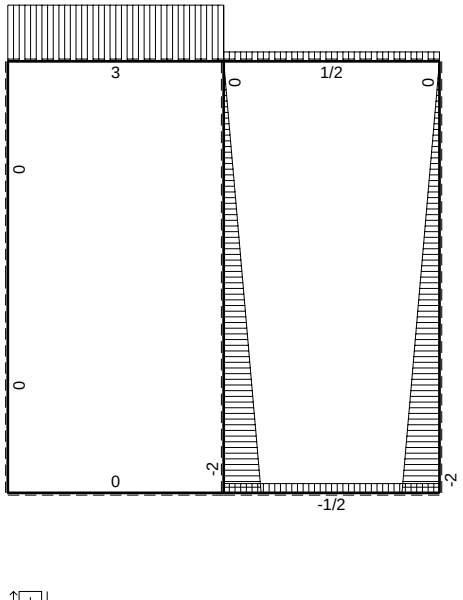
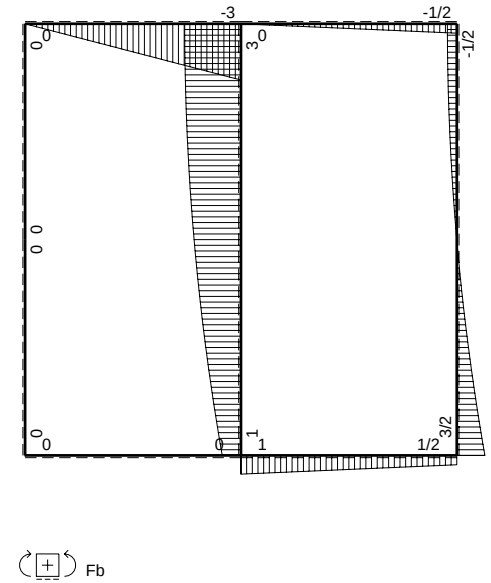
-5/2
0
1/2
-3
-3
0
-5/2
2
1/2
F


3
0
1/2
0
0
0
-2
0
-2
-1/2
F


-3
-1/2
0
0
3
-1/2
0
0
0
0
1
1
1/2
3/2
Fb

M_x flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	(-1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	2Fb-2Fx+1/2qx^2	0	-2Fb+2Fx-1/2Fx^2/b	0	1	(-4/3+0)Fb^2/EJ	2Xb/EJ
GF 2b	1	-1/2qx^2	0	-1/2Fx^2/b	0	1		
CB 2b	0	3Fb-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	-Fb-2Fx+1/2qx^2	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

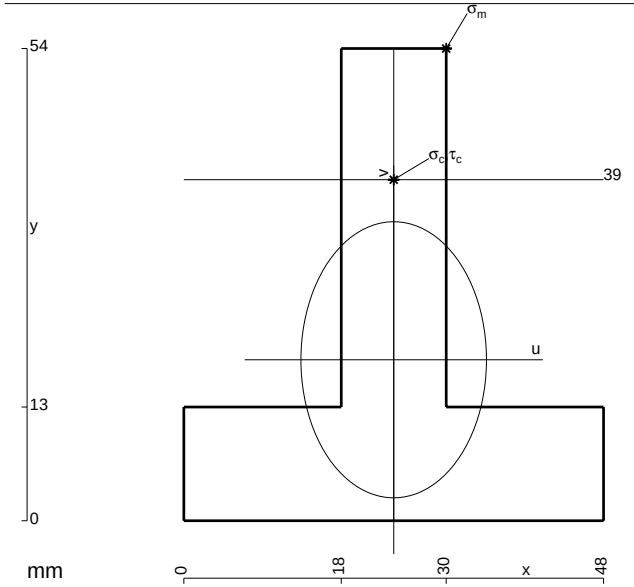
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 1116. mm^2
J_u = 278255. mm^4
J_v = 125712. mm^4
y_g = 18.4 mm
T_y = 2430. N
M_x = -1555200. Nmm
x_m = 30. mm
y_m = 54. mm
u_m = 6. mm
v_m = 35.6 mm
$\sigma_m = -Mv/J_u$ = 199. N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 20.6 mm
$\sigma_c = -Mv/J_u$ = 115.1 N/mm^2
τ_c = 3.681 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.3 N/mm^2
S^* = 5057. mm^3

ESΣ8.175

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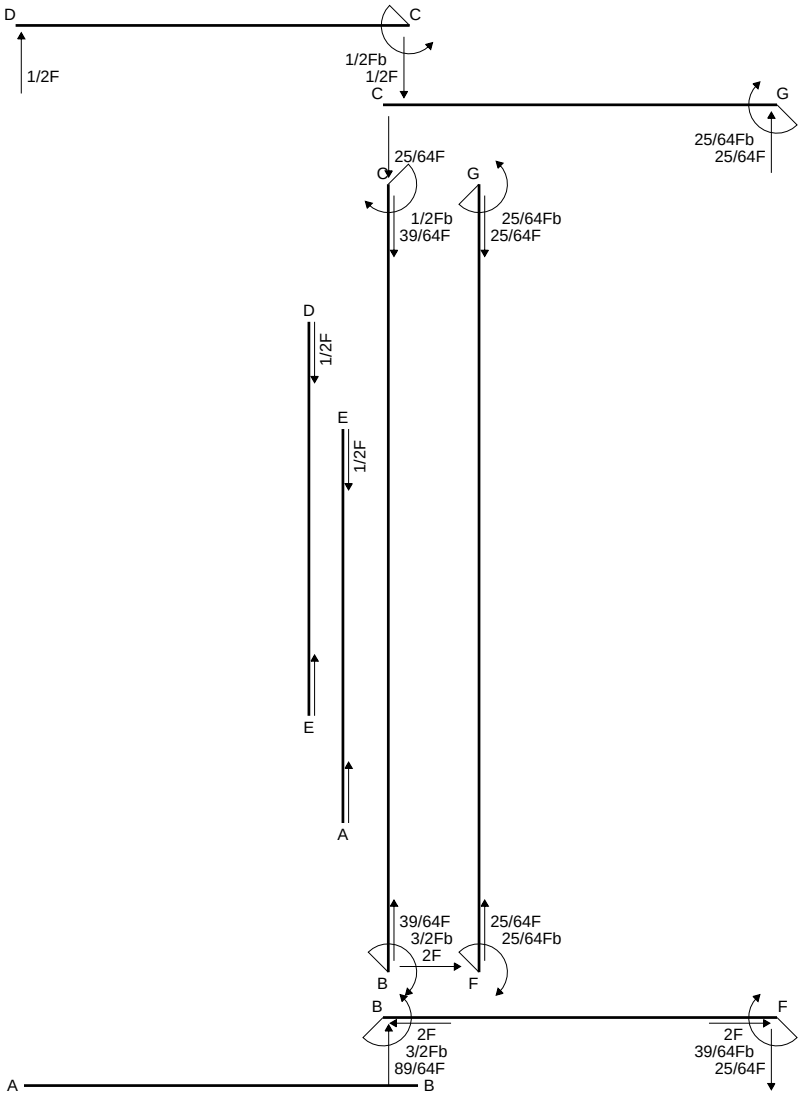
D
C
1/2F
1/2Fb
1/2F
C
G
25/64Fb
25/64F
C
25/64F
G
1/2Fb
39/64F
25/64Fb
25/64F
D
1/2F
E
1/2F
E
A
39/64F
3/2Fb
2F
25/64F
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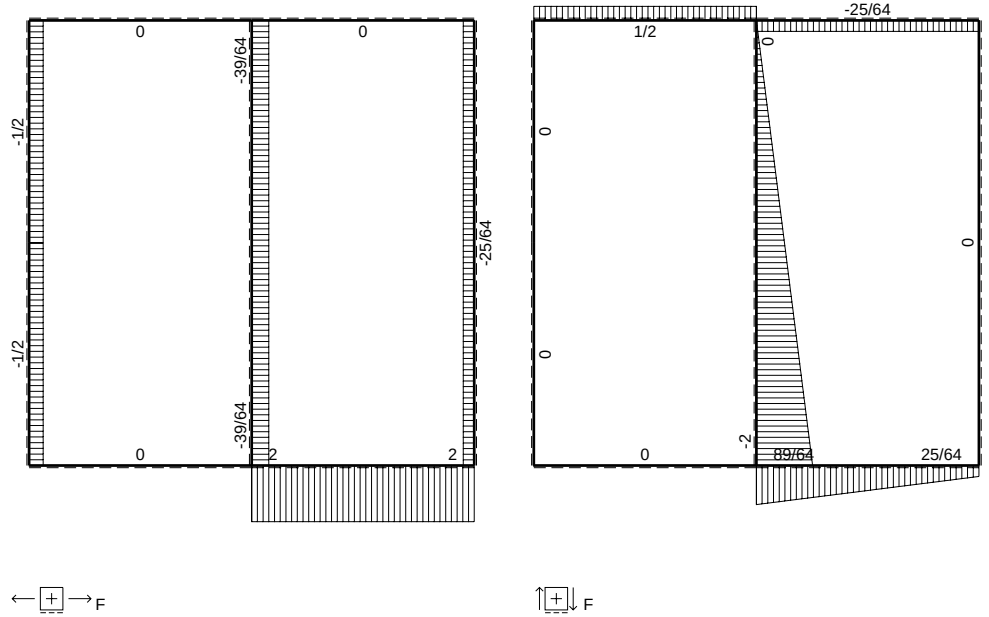
25/64F
A
B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-25/64
-1/2
-39/64
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-25/64
1/2
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0
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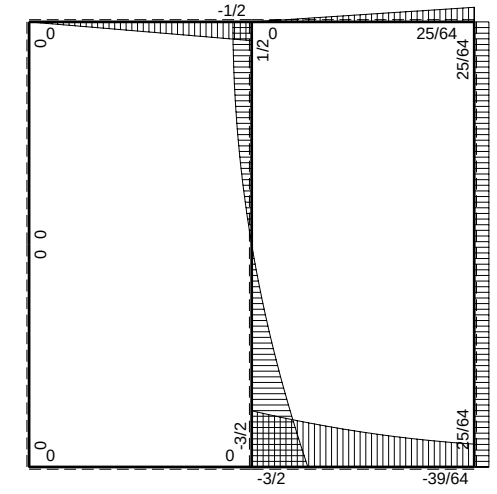
-2
89/64
25/64
F


-1/2
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1/2
25/64
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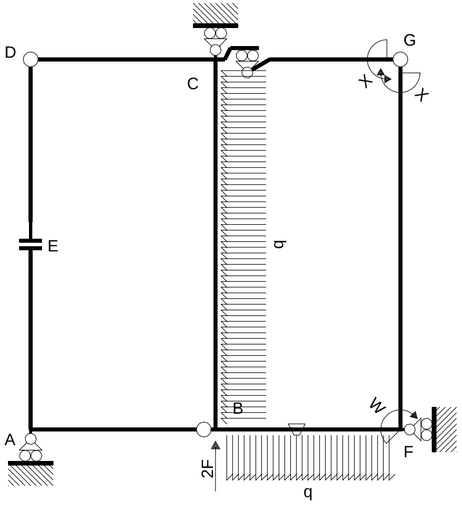
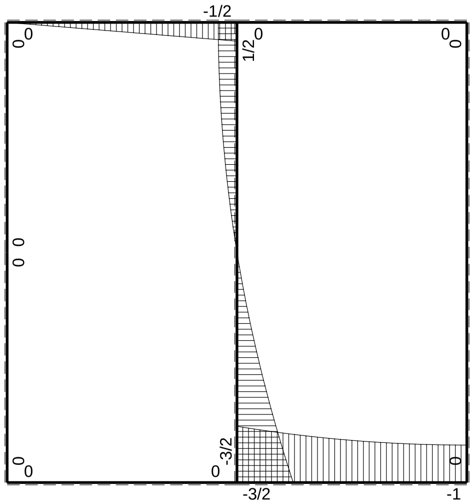
0
0
0
-3/2
25/64
-3/2
-39/64

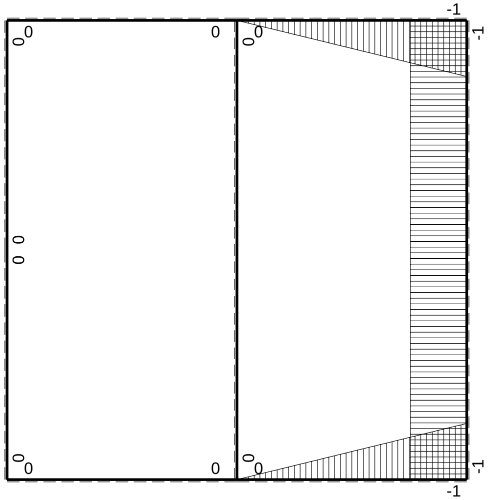
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$-3/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$25/24Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$-25/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

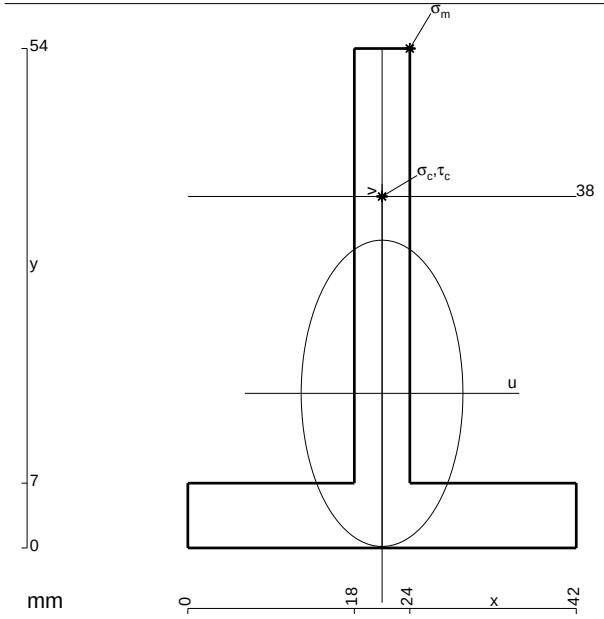
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 576. mm^2
J_u = 158042. mm^4
J_v = 44064. mm^4
y_g = 16.72 mm
N = -530.2 N
T_y = -1740. N
M_x = -887400. Nmm
x_m = 24. mm
y_m = 54. mm
u_m = 3. mm
v_m = 37.28 mm
$\sigma_m = N/A - Mv/J_u$ = 208.4 N/mm^2
x_c = 21. mm
y_c = 38. mm
v_c = 21.28 mm
$\sigma_c = N/A - Mv/J_u$ = 118.6 N/mm^2
τ_c = 5.158 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 118.9 N/mm^2
S^* = 2811. mm^3

ESΣ8.176

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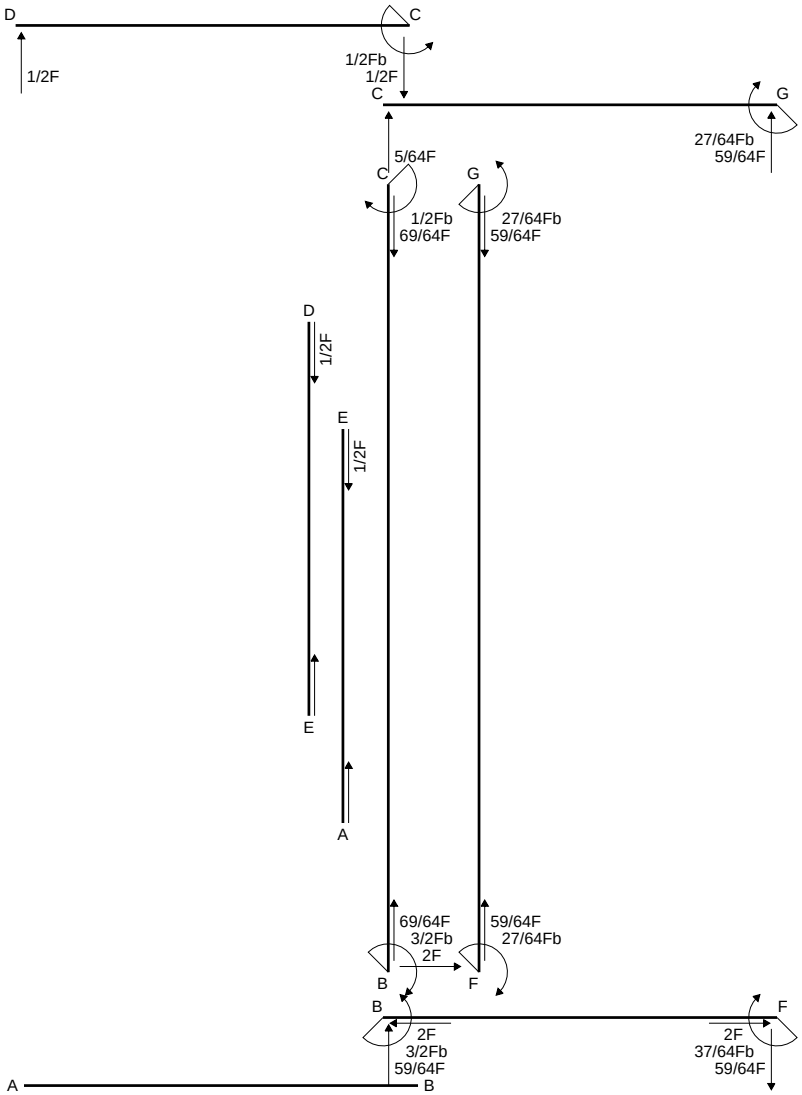
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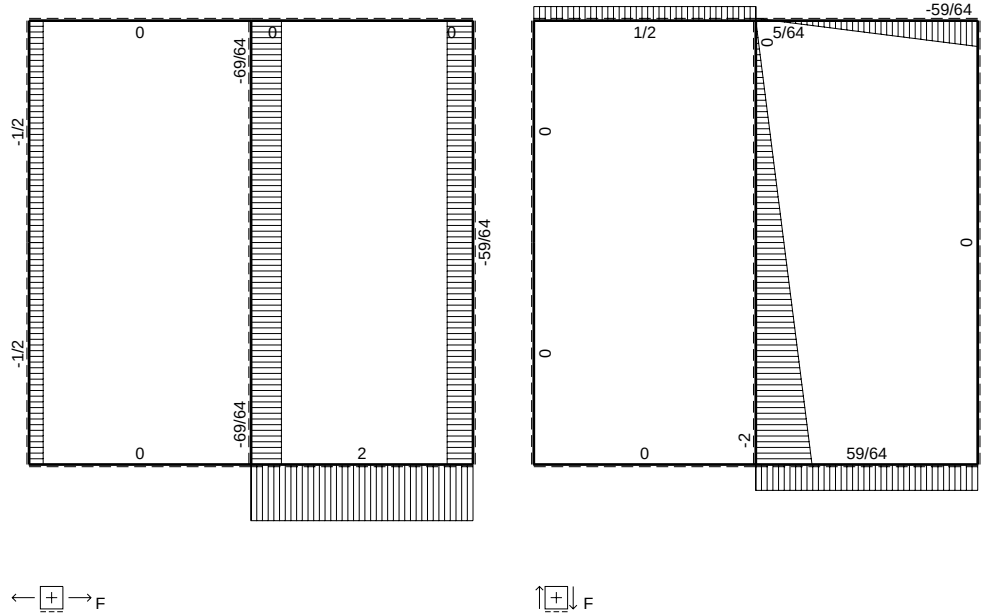
1/2Fb
1/2F
C
G
27/64Fb
59/64F
5/64F
C
G
1/2Fb
69/64F
27/64Fb
59/64F
D
1/2F
E
1/2F
E
A
69/64F
3/2Fb
2F
59/64F
27/64Fb
B
F
B
F
2F
3/2Fb
59/64F
2F
37/64Fb
59/64F
A
B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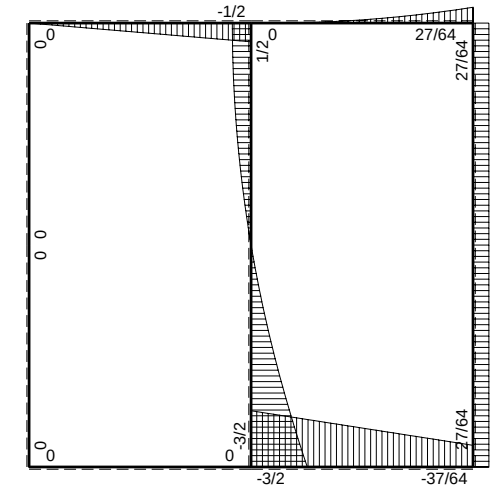
59/64
F
F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27/64
1/2
0
27/64
0
0
0
0
-3/2
0
27/64
-3/2
-37/64


Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

M_0 flessione da carichi assegnati

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
DC b	0	1/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-3/2Fb+1/2Fx	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	(7/12+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	(1/24+0)Fb^2/EJ	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0		
totali							$9/8Fb^2$/EJ	8/3Xb/EJ
iperstatica $X=W_{GC}$							-27/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

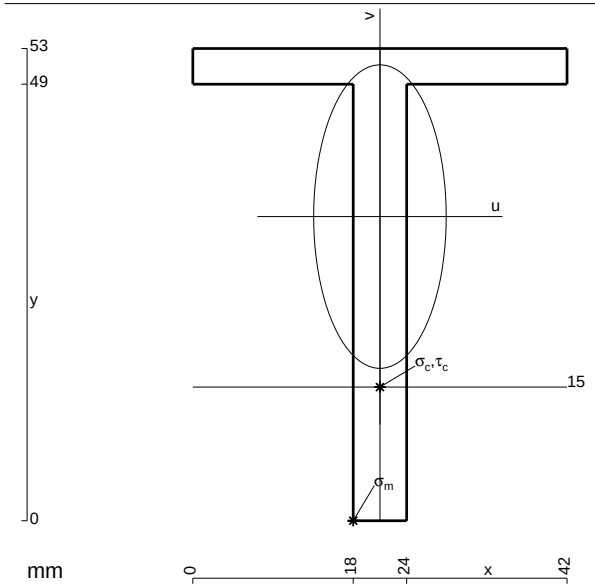
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.177



A = 462. mm^2
J_u = 134125. mm^4
J_v = 25578. mm^4
y_g = 34.14 mm
N = -862.5 N
T_y = -1600. N
M_x = -852000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -34.14 mm
$\sigma_m = N/A - Mv/J_u$ = -218.7 N/mm^2
x_c = 21. mm
y_c = 15. mm
v_c = -19.14 mm
$\sigma_c = N/A - Mv/J_u$ = -123.4 N/mm^2
τ_c = 4.766 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 123.7 N/mm^2
S^* = 2397. mm^3

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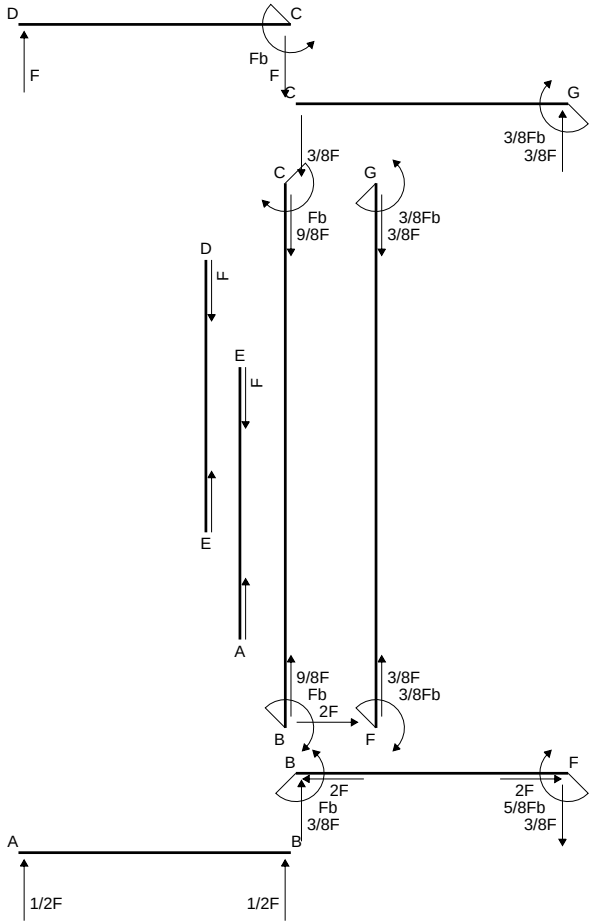
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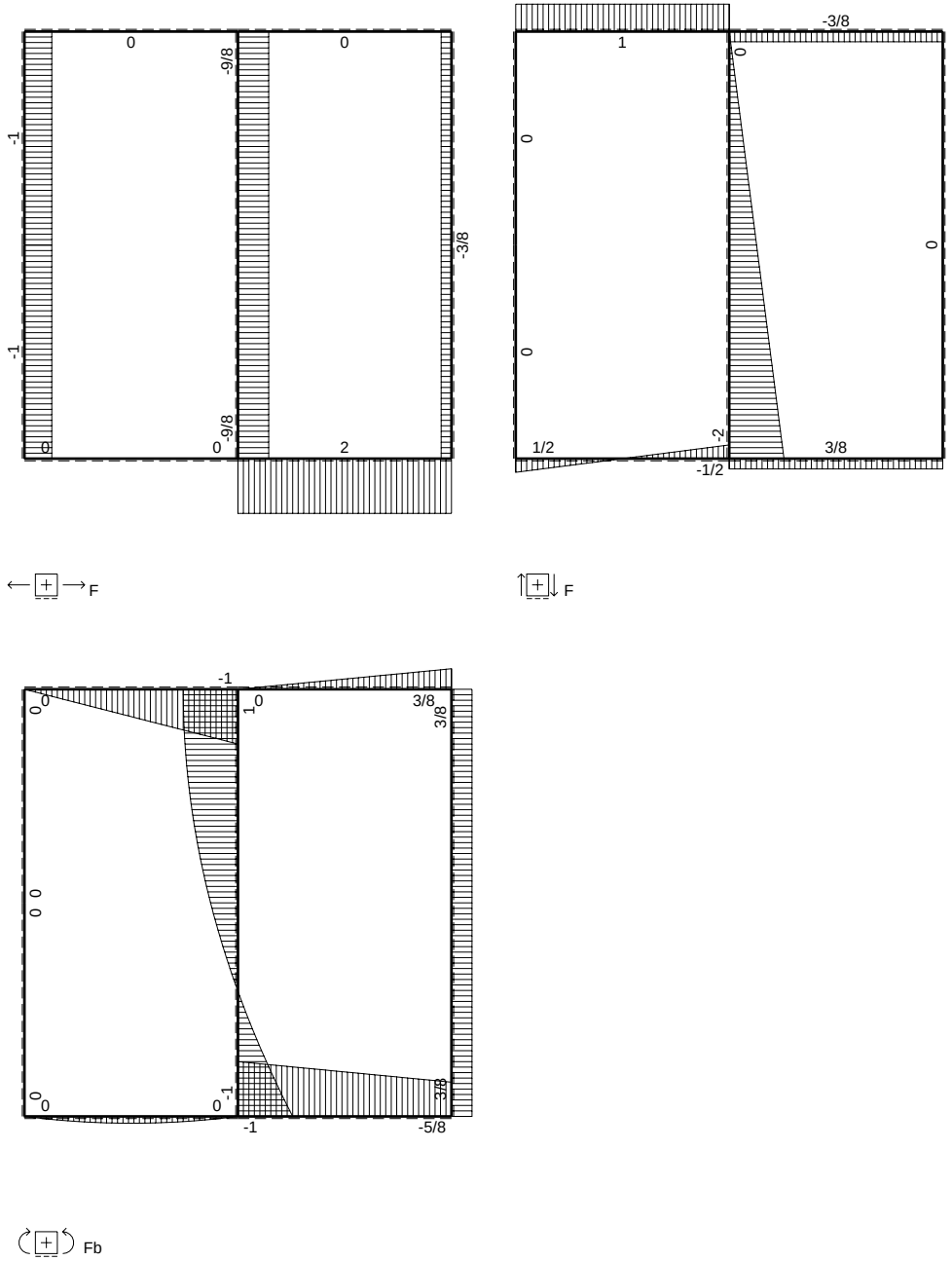
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-1
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Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

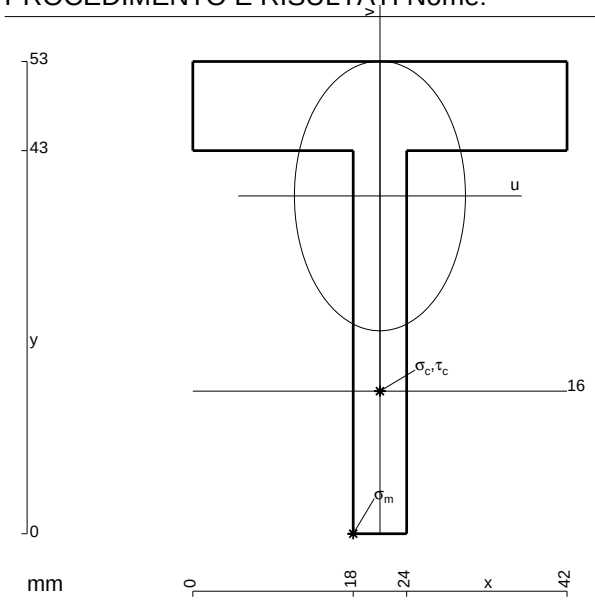
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 678. mm^2
J_u = 155489. mm^4
J_v = 62514. mm^4
y_g = 37.92 mm
N = -1395. N
T_y = -2480. N
M_x = -930000. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.92 mm
$\sigma_m = N/A - Mv/J_u$ = -228.8 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.92 mm
$\sigma_c = N/A - Mv/J_u$ = -133.1 N/mm^2
τ_c = 7.634 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 133.8 N/mm^2
S^* = 2872. mm^3

ESΣ8.178

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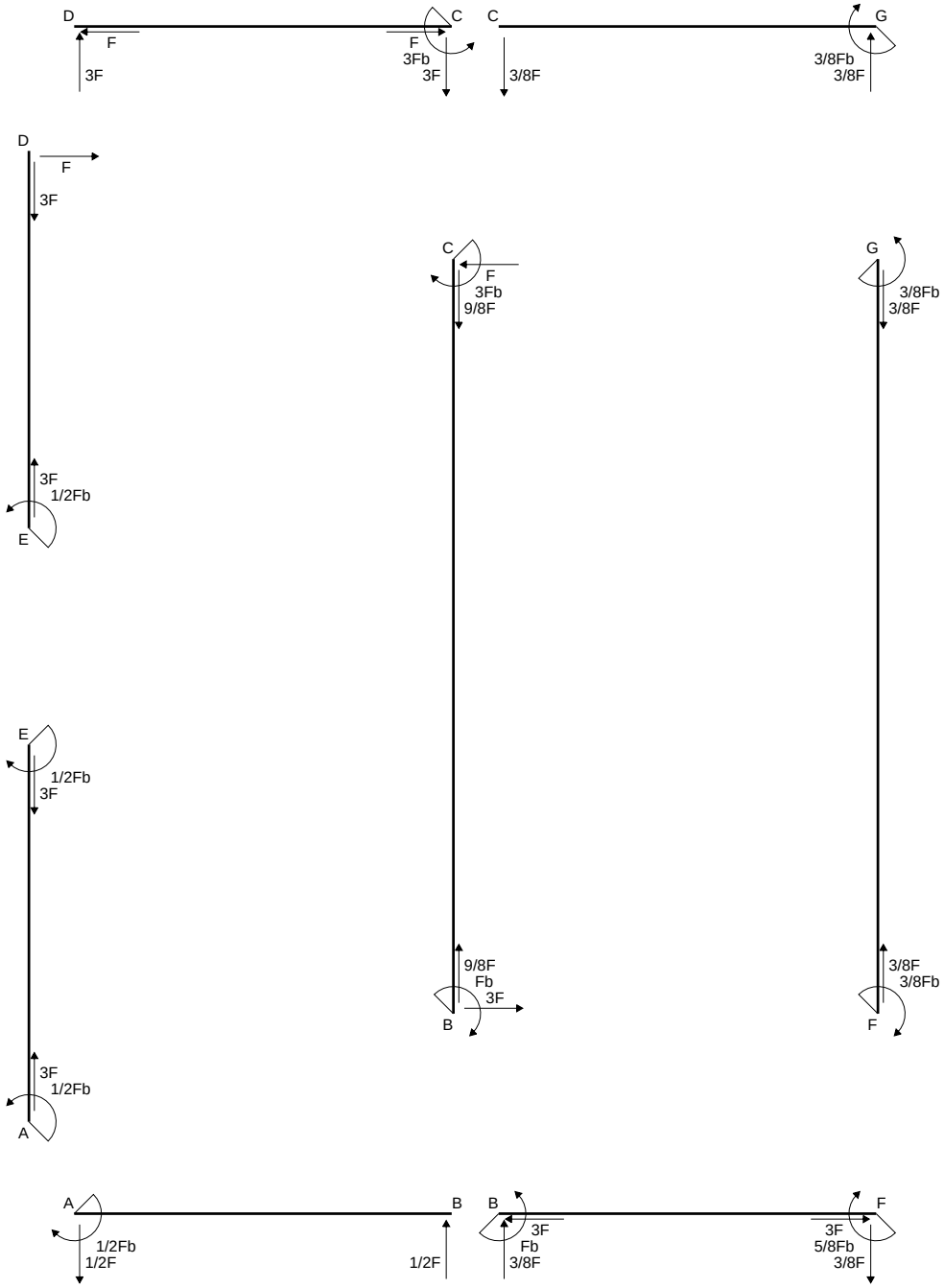
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5/8Fb
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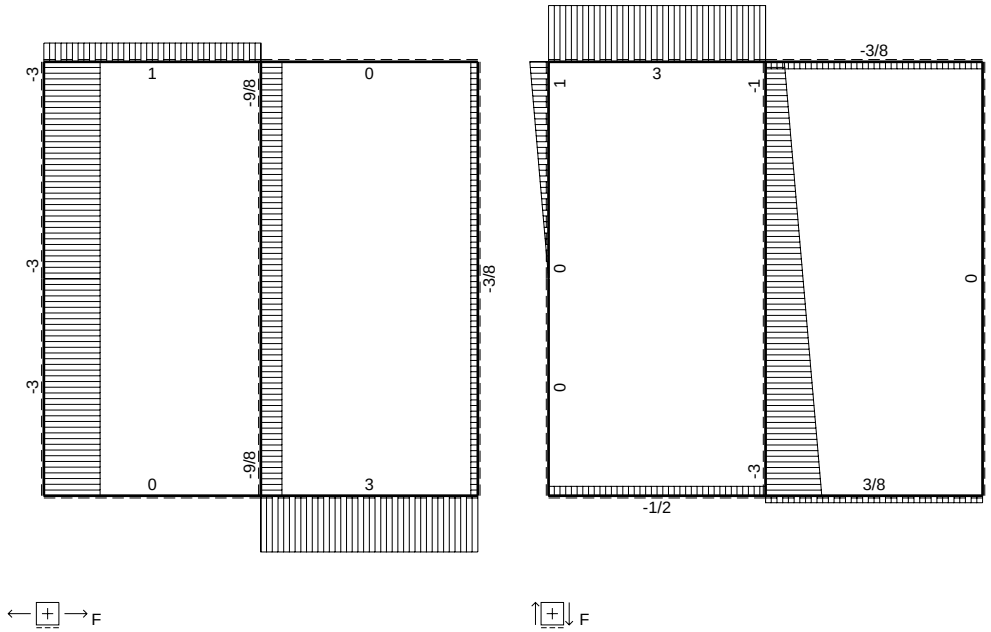
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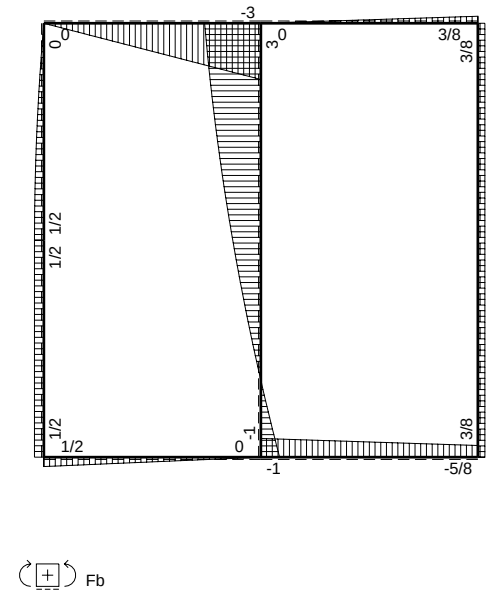
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-1
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Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

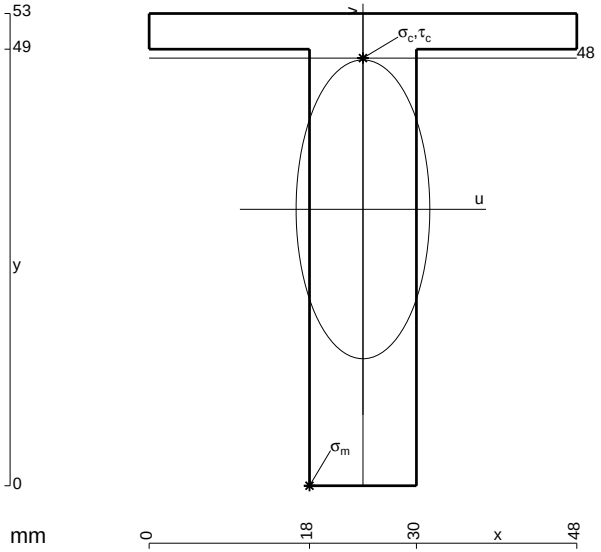
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 780. mm^2
J_u = 219548. mm^4
J_v = 43920. mm^4
y_g = 31.02 mm
N = 710. N
T_y = 2130. N
M_x = -1682700. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -31.02 mm
$\sigma_m = N/A - Mv/J_u$ = -236.9 N/mm^2
x_c = 24. mm
y_c = 48. mm
v_c = 16.98 mm
$\sigma_c = N/A - Mv/J_u$ = 131. N/mm^2
τ_c = 3.271 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.2 N/mm^2
S^* = 4045. mm^3

ESΣ8.179

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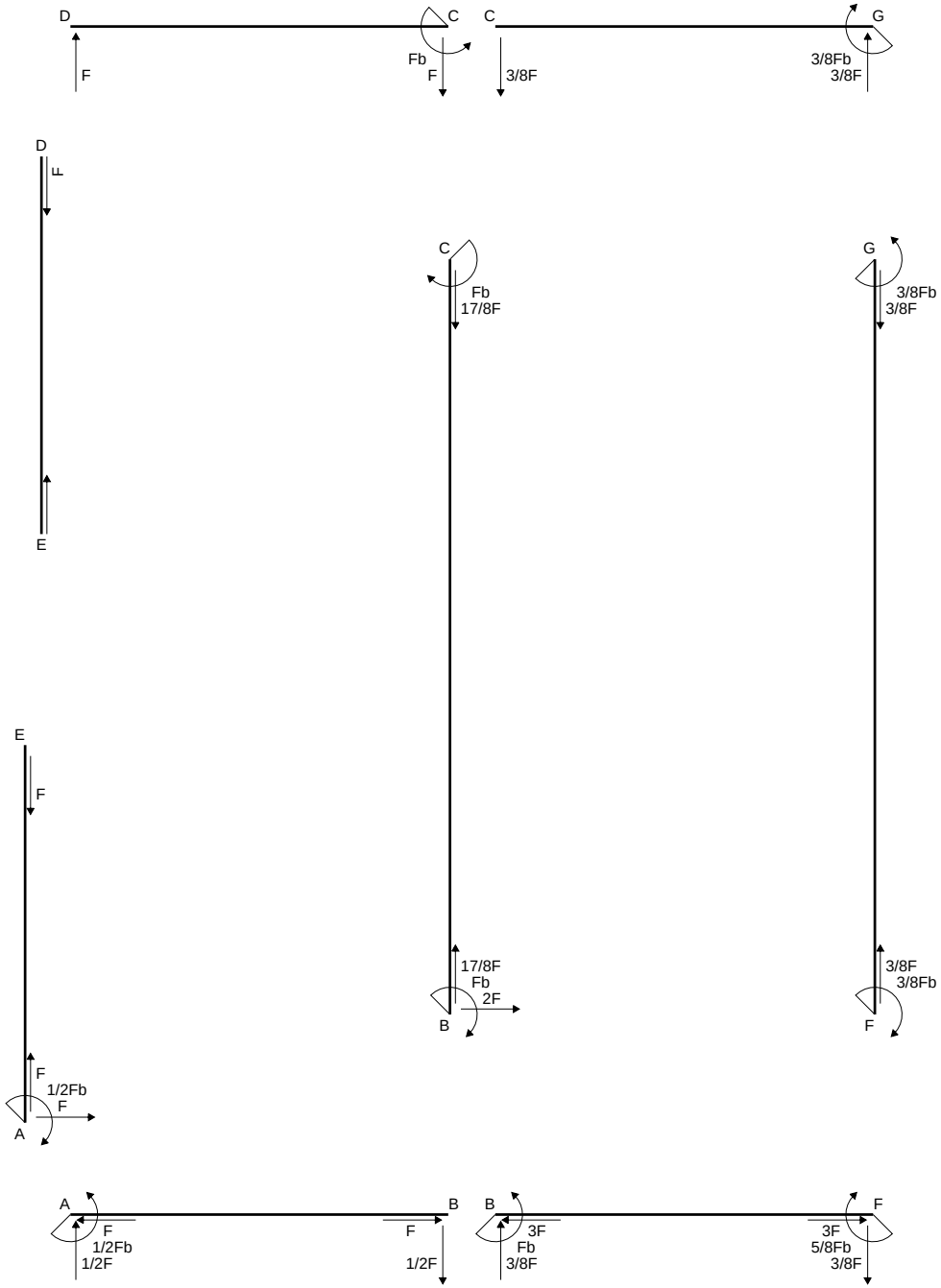
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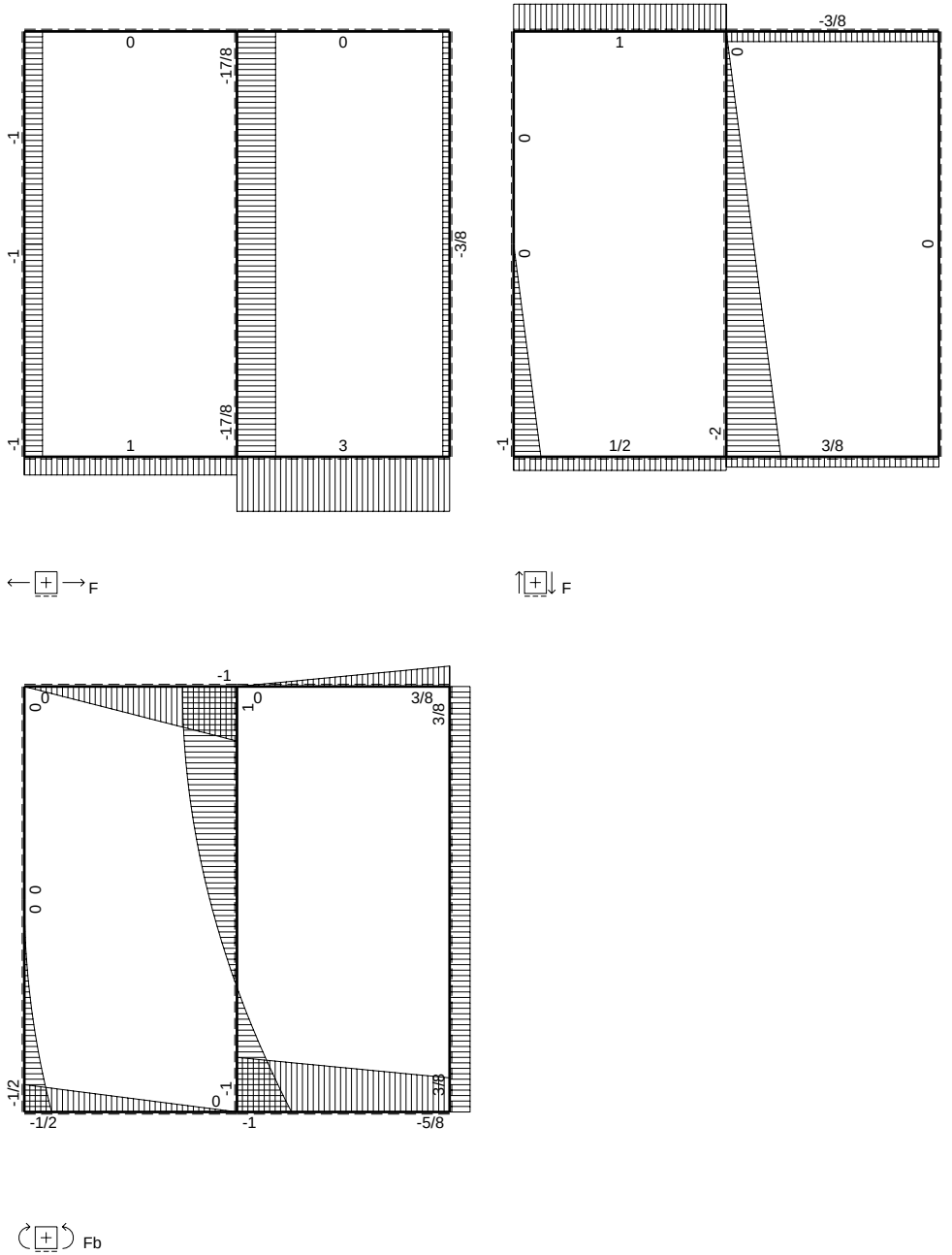
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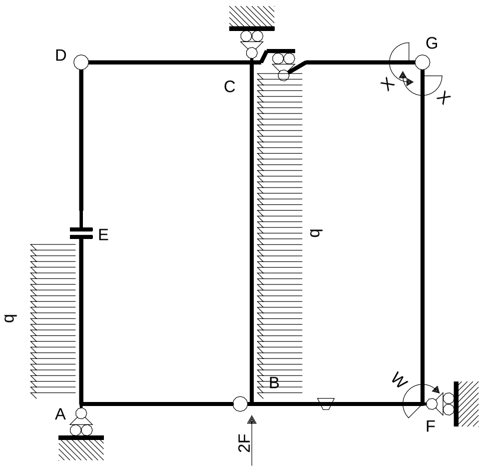
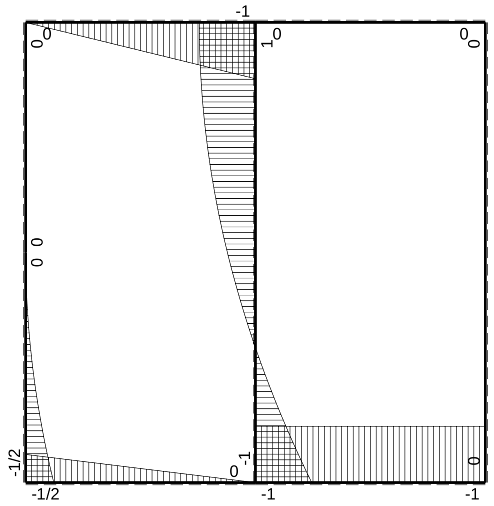
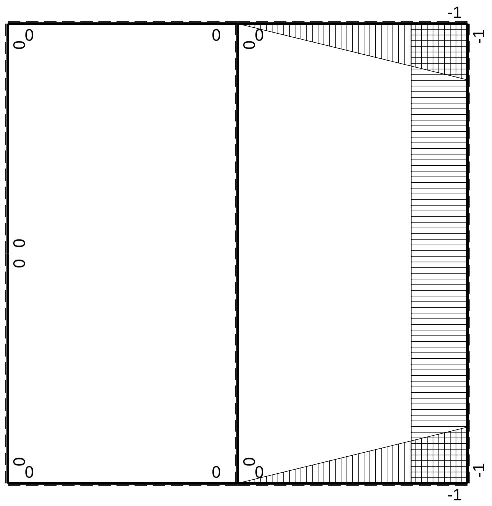
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$-Fb+Fx$	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	$-Fb$	$-Fb/EJ$	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	Fb	Fb/EJ	$Fb-Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	$8/3Xb/EJ$
	iperstatica X=W_{GC}						$-3/8Fb$	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

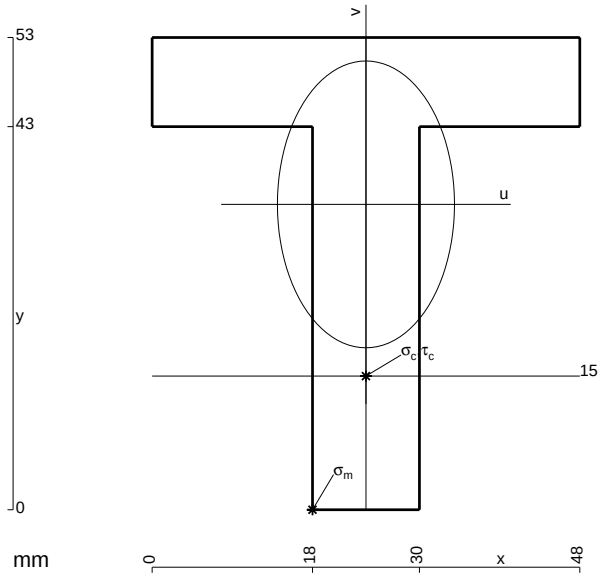
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 996. mm^2
J_u = 258139. mm^4
J_v = 98352. mm^4
y_g = 34.27 mm
N = -7331. N
T_y = -6900. N
M_x = -1449000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -34.27 mm
$\sigma_m = N/A - Mv/J_u$ = -199.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -19.27 mm
$\sigma_c = N/A - Mv/J_u$ = -115.5 N/mm^2
τ_c = 10.73 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117. N/mm^2
S^* = 4819. mm^3

ESΣ8.180

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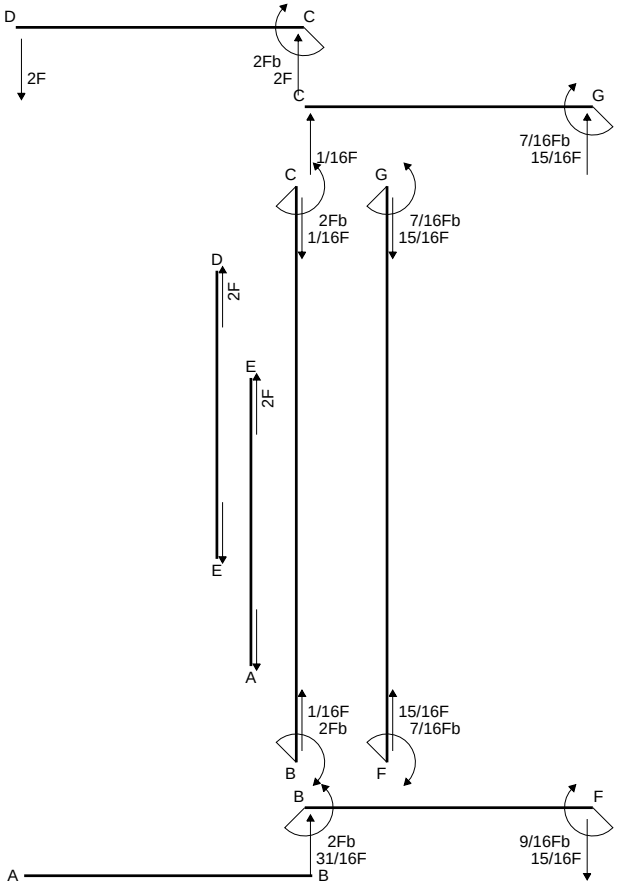
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2F
2F
C
G
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15/16F
1/16F
C
G
2Fb
1/16F
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15/16F
D
2F
E
2F
E
A
1/16F
2Fb
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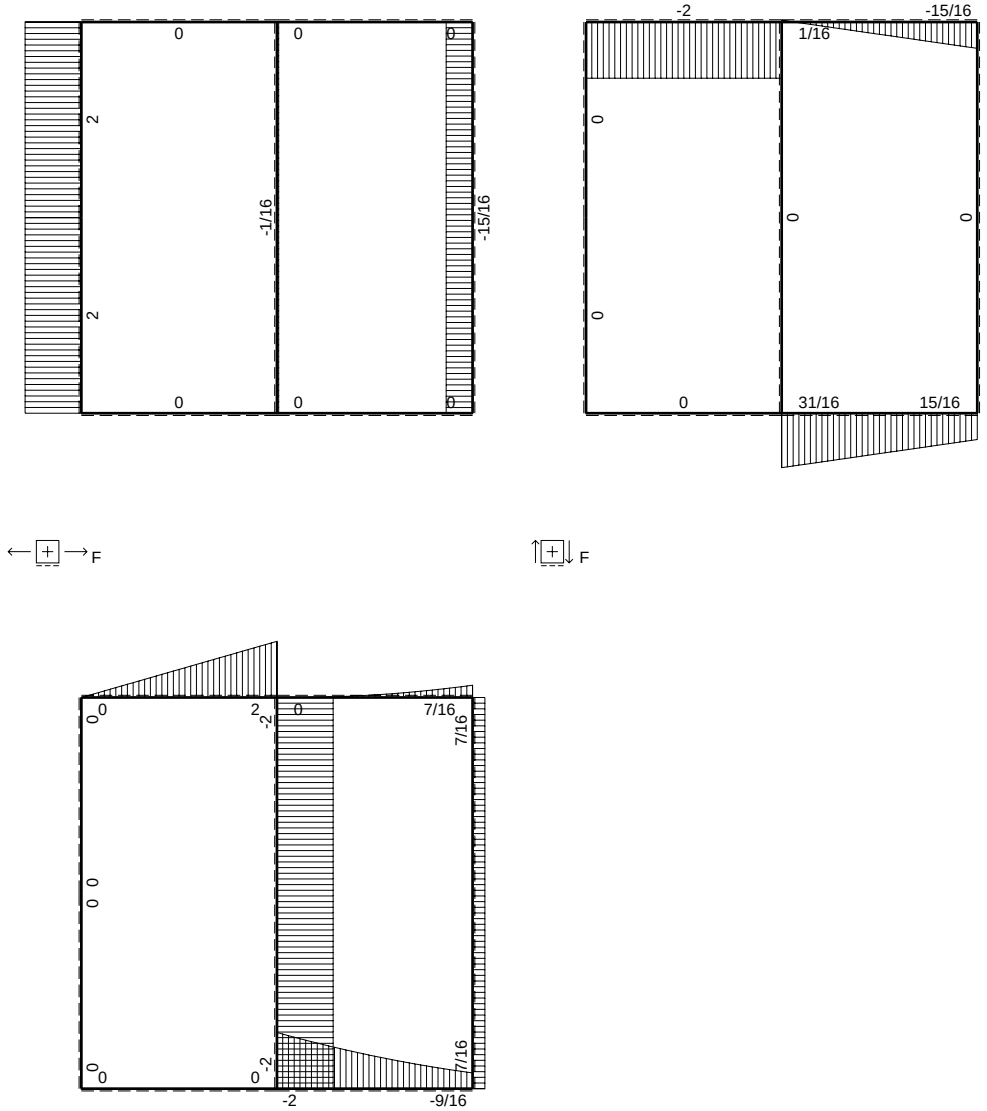
7/16Fb
B
F
B
F
2Fb
31/16F
9/16Fb
15/16F
A
B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0
31/16
15/16
F
F
0
2
-2
0
7/16
7/16
0
0
0
0
0
-2
7/16
-2
-9/16
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	←	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b		0	0	0	0	0	0	0+0	0
BA b		0	0	0	0	0	0		
CD b		0	2Fb-2Fx	0	0	0	0	0+0	0
DC b		0	-2Fx	0	0	0	0		
DE b		0	0	0	0	0	0	0+0	0
ED b		0	0	0	0	0	0		
EA b		0	0	0	0	0	0	0+0	0
AE b		0	0	0	0	0	0		
BF b		-x/b	-2Fb+3/2Fx-1/2qx^2	-Fb/EJ	2Fx-3/2Fx^2/b+1/2qx^3/b	Fx/EJ	x^2/b^2	(5/8+1/2)Fb^2/EJ	1/3XbJ/EJ
FB b		1-x/b	Fb+1/2Fx+1/2qx^2	Fb/EJ	Fb-1/2Fx-1/2qx^3/b	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b		-1+x/b	-1/2Fx+1/2qx^2	0	1/2Fx-Fx^2/b+1/2qx^3/b	0	1-2x/b+x^2/b^2	(1/24+0)Fb^2/EJ	1/3XbJ/EJ
CG b		x/b	1/2Fx-1/2qx^2	0	1/2Fx^2/b-1/2qx^3/b	0	x^2/b^2		
FG 2b		-1	0	0	0	0	1	0+0	2XbJ/EJ
GF 2b		1	0	0	0	0	1		
CB 2b		0	-2Fb	0	0	0	0	0+0	0
BC 2b		0	2Fb	0	0	0	0		
		totali						7/6Fb^2/EJ	8/3XbJ/EJ
		iperstatica $X=W_{GC}$						-7/16Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (2\ x/b -3/2\ x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [\ x^2/b -1/2\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

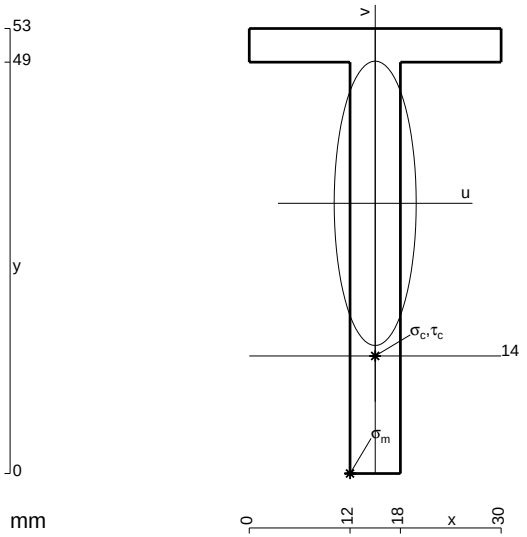
$= (\ b -1/4\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 9/8\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 414. mm^2
J_u = 118828. mm^4
J_v = 9882. mm^4
y_g = 32.18 mm
T_y = -1680. N
M_x = 772800. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -32.18 mm
$\sigma_m = -Mv/J_u$ = 209.3 N/mm^2
x_c = 15. mm
y_c = 14. mm
v_c = -18.18 mm
$\sigma_c = -Mv/J_u$ = 118.2 N/mm^2
τ_c = 4.984 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 118.6 N/mm^2
S^* = 2115. mm^3

ESΣ8.181

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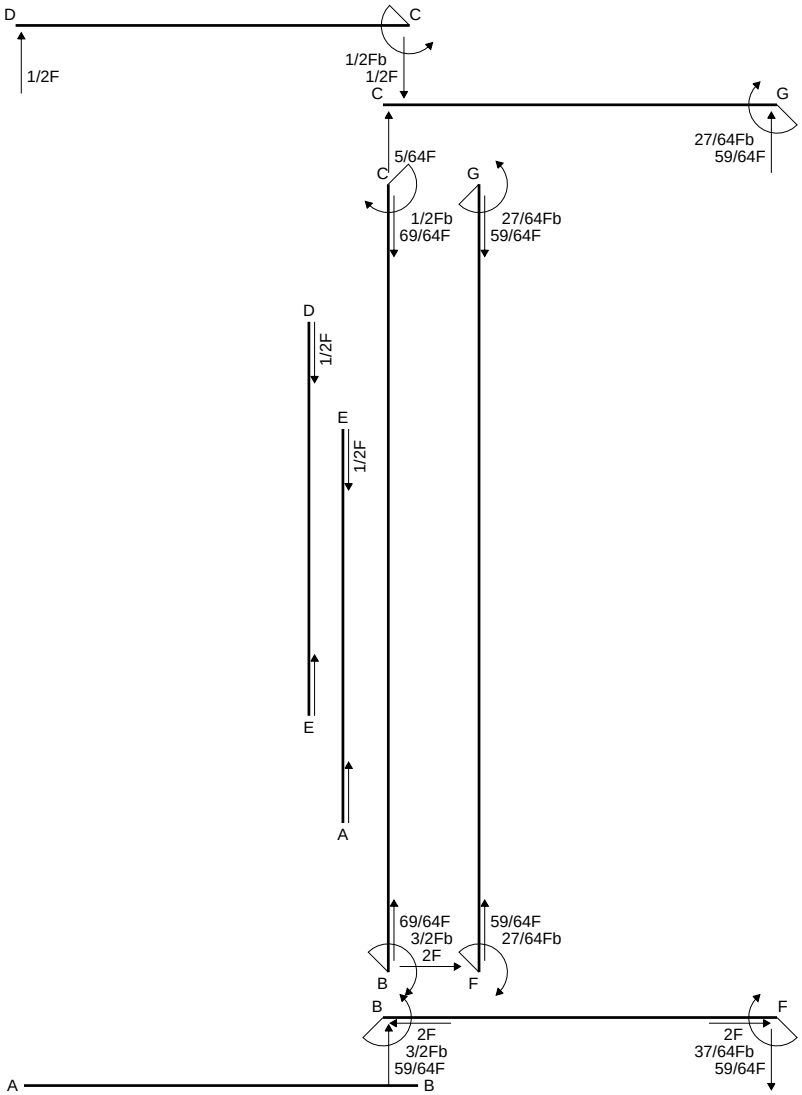
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1/2Fb
1/2F
C
G
27/64Fb
59/64F
5/64F
C
G
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27/64Fb
59/64F
D
1/2F
E
1/2F
E
A
69/64F
3/2Fb
2F
59/64F
27/64Fb
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B
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2F
3/2Fb
59/64F
2F
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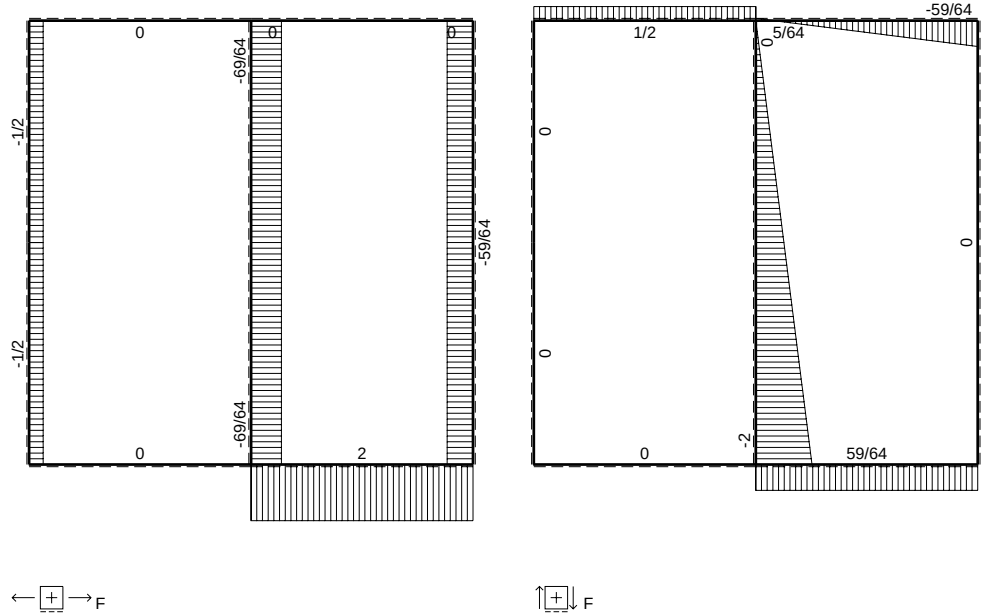
59/64F
A
B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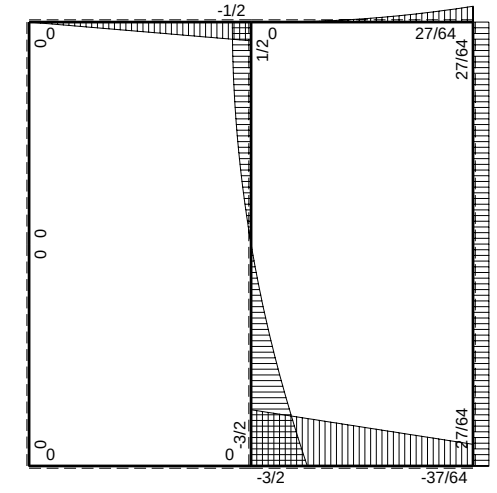
59/64
F
F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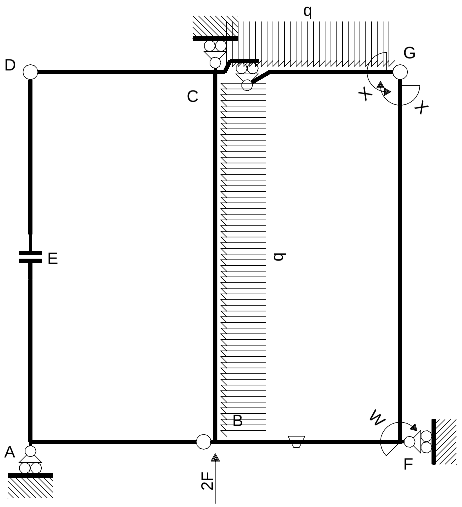
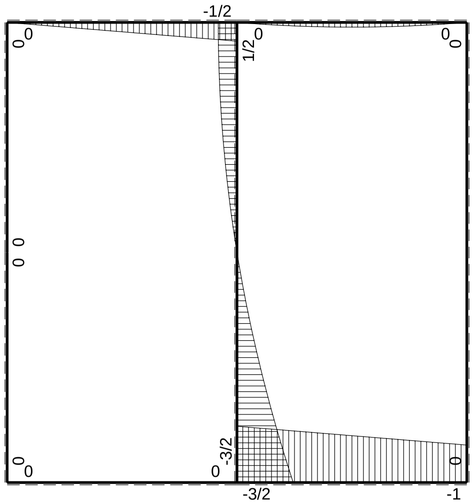
0
27/64
-3/2
-37/64

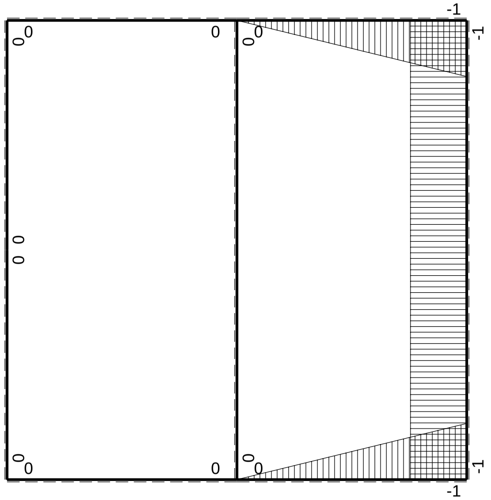

Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
DC b	0	$1/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	$-3/2Fb+1/2Fx$	$-Fb/EJ$	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$Fb+1/2Fx$	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	$1/3Xb/EJ$
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						$9/8Fb^2/EJ$	$8/3Xb/EJ$
	iperstatica $X=W_{GC}$						$-27/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

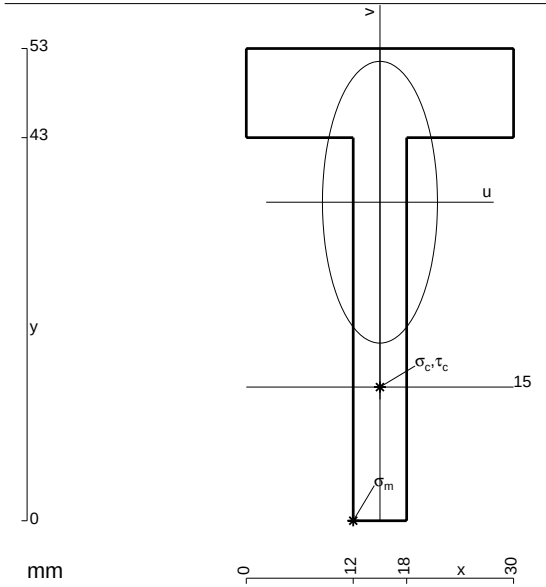
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 558. mm^2
J_u = 139662. mm^4
J_v = 23274. mm^4
y_g = 35.75 mm
N = -1218. N
T_y = -2260. N
M_x = -847500. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -35.75 mm
$\sigma_m = N/A - Mv/J_u$ = -219.1 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -20.75 mm
$\sigma_c = N/A - Mv/J_u$ = -128.1 N/mm^2
τ_c = 6.856 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 128.6 N/mm^2
S^* = 2542. mm^3

ESΣ8.182

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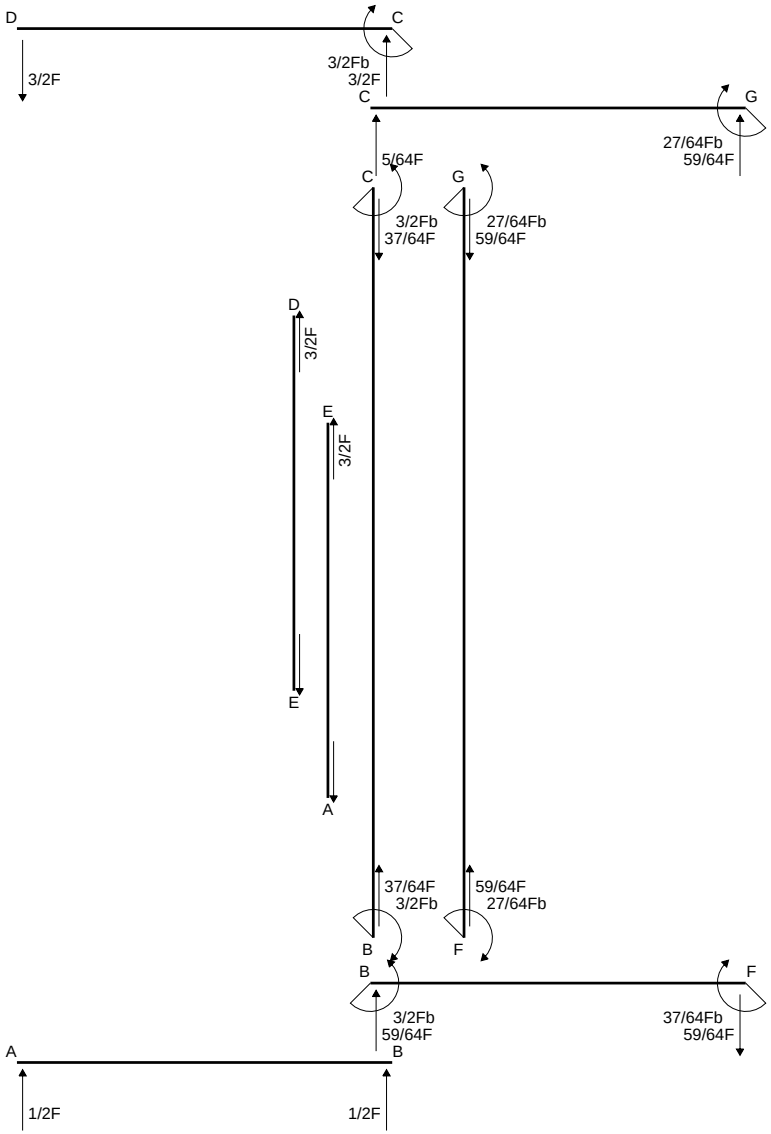
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G
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59/64F
5/64F
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G
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27/64Fb
59/64F
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E
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E
A
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3/2Fb
59/64F
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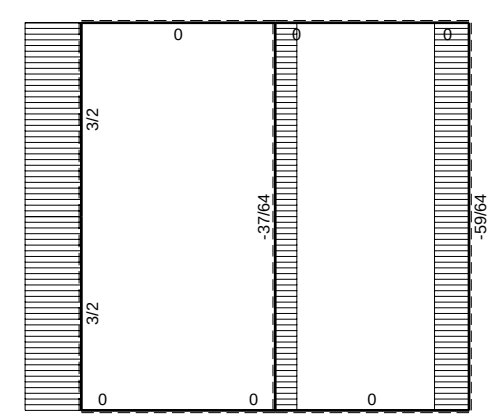
1/2F
1/2F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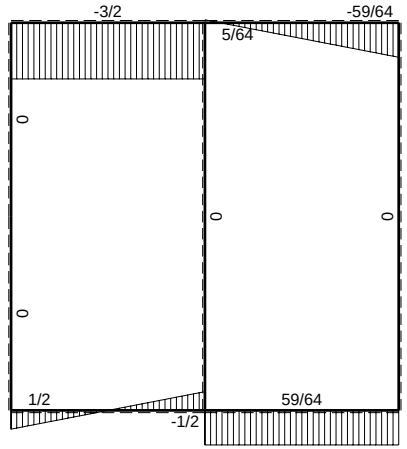
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5/64
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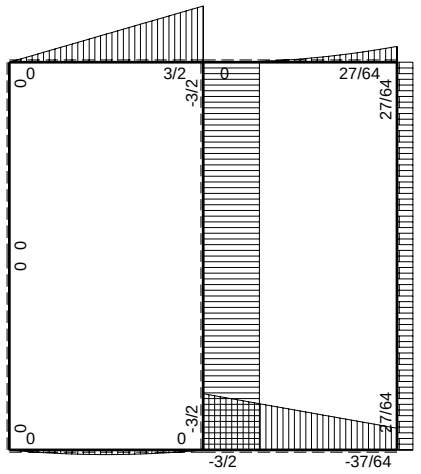
1/2
59/64
-1/2

F

F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27/64
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+1/2Fx$	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

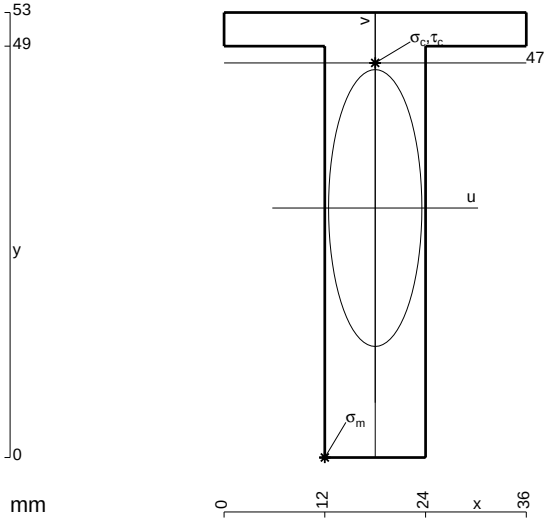
$L^{Xo}_{GC} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L^{Xo}_{CG} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.183



A = 732. mm^2
J_u = 199072. mm^4
J_v = 22608. mm^4
y_g = 29.71 mm
T_y = -2850. N
M_x = 1539000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -29.71 mm
$\sigma_m = -Mv/J_u$ = 229.7 N/mm^2
x_c = 18. mm
y_c = 47. mm
v_c = 17.29 mm
$\sigma_c = -Mv/J_u$ = -133.6 N/mm^2
τ_c = 4.181 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 133.8 N/mm^2
S^* = 3504. mm^3

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ESΣ8.183

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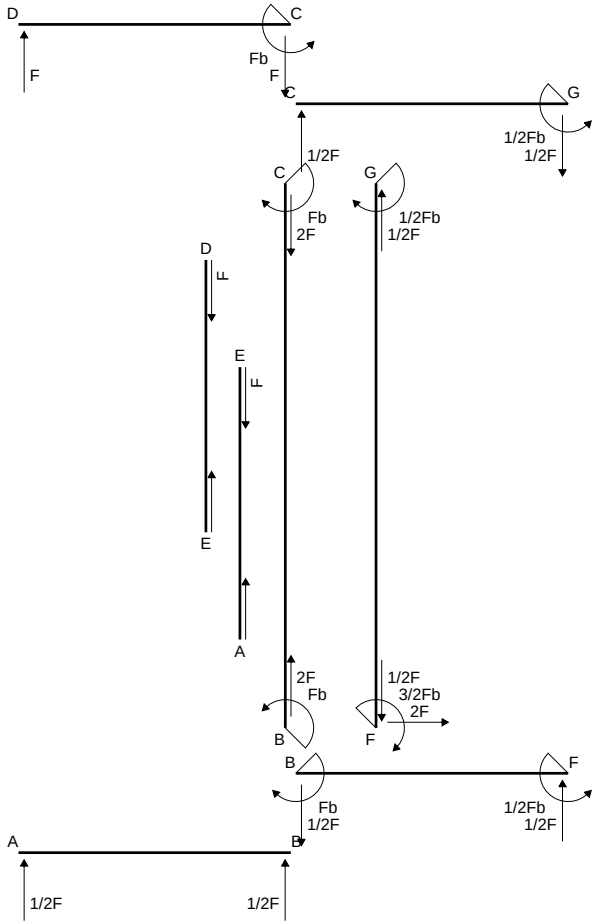
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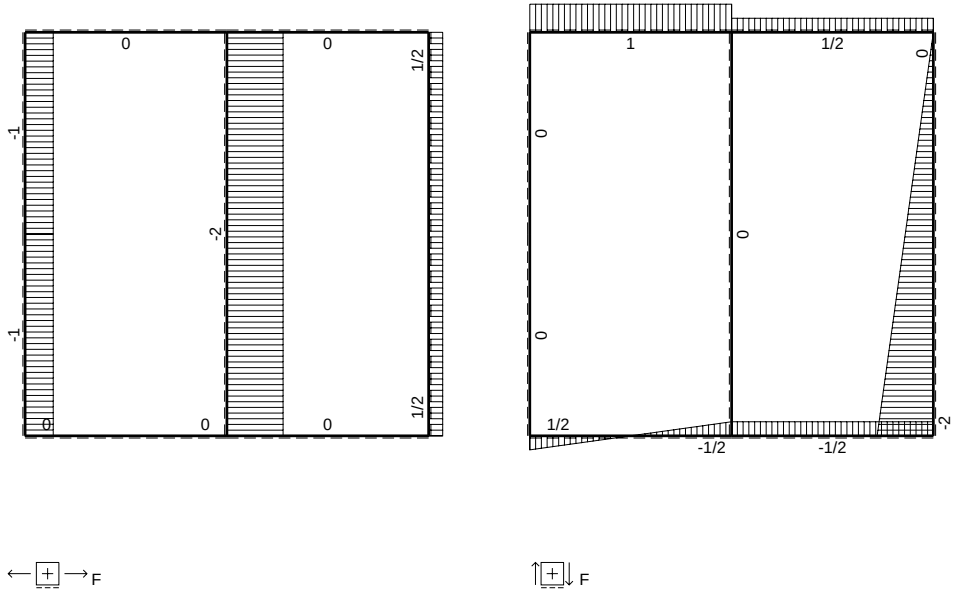
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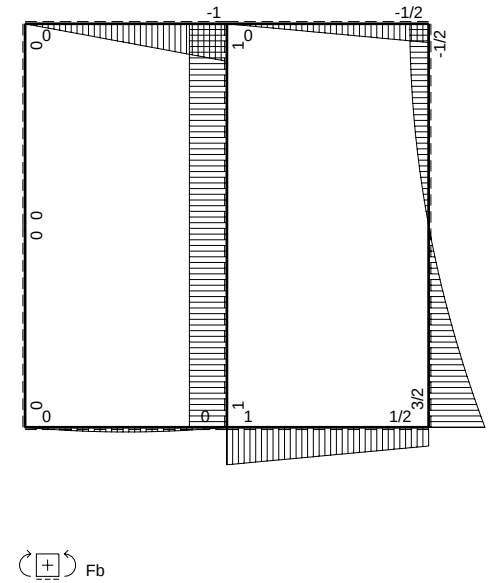
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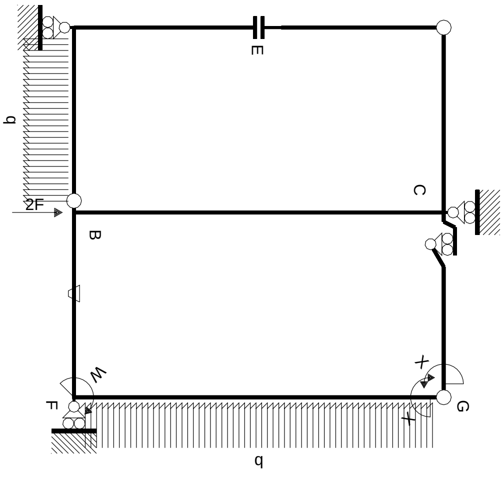
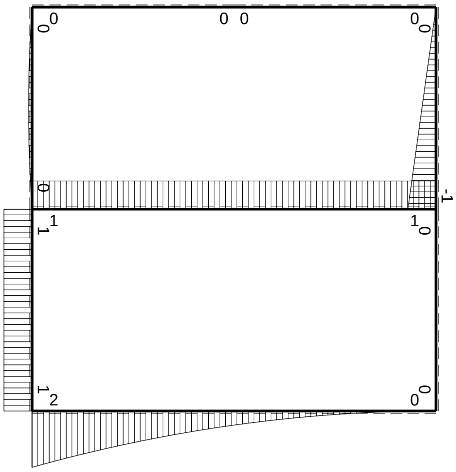
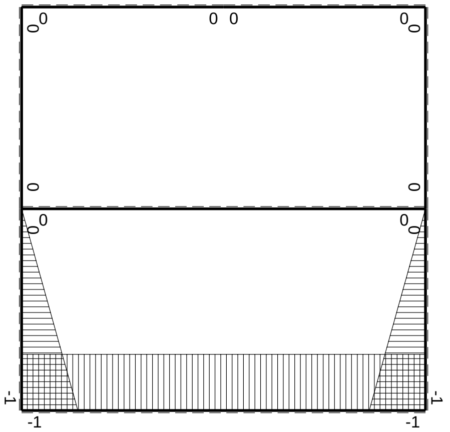




M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

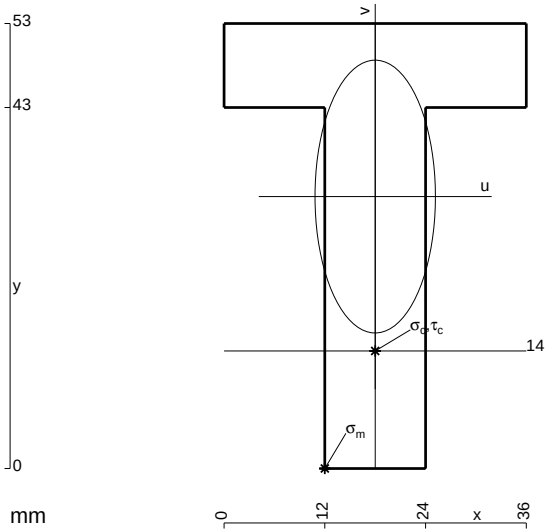
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 876. mm^2
J_u = 231422. mm^4
J_v = 45072. mm^4
y_g = 32.39 mm
T_y = 2950. N
M_x = -1711000. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -32.39 mm
$\sigma_m = -Mv/J_u$ = -239.5 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -18.39 mm
$\sigma_c = -Mv/J_u$ = -136. N/mm^2
τ_c = 4.531 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.2 N/mm^2
S^* = 4266. mm^3

ESΣ8.184

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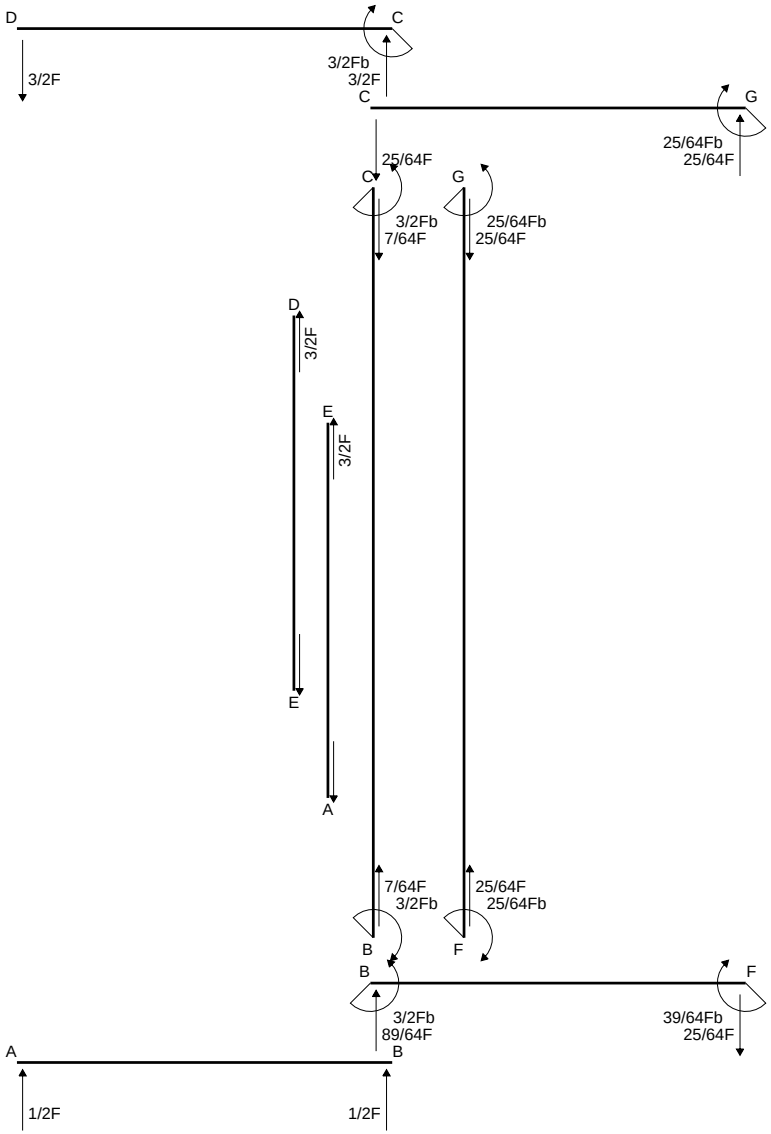
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G
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25/64Fb
25/64F
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A
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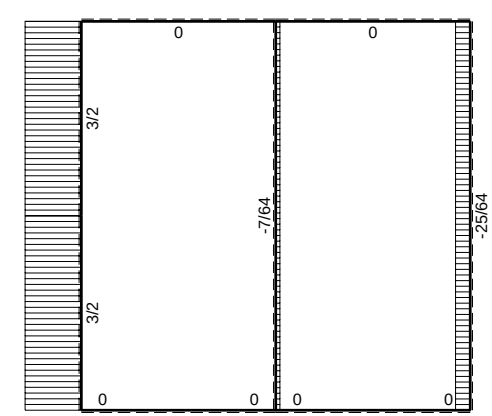
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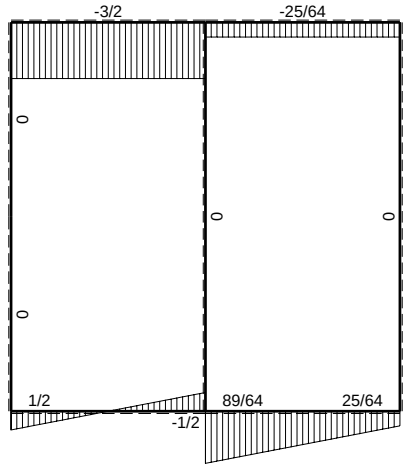
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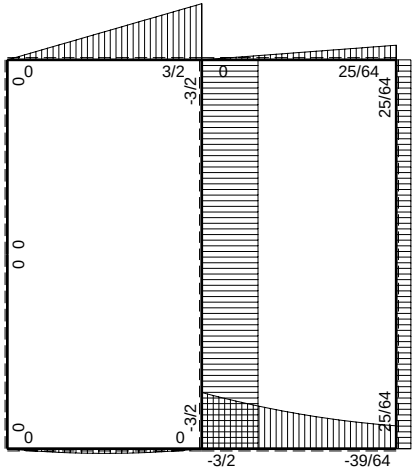
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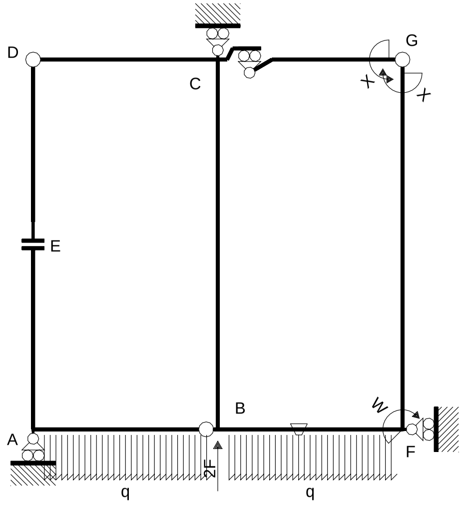
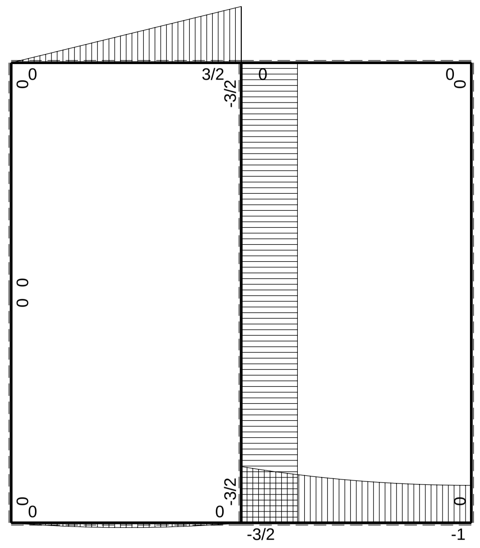
-3/2
-39/64

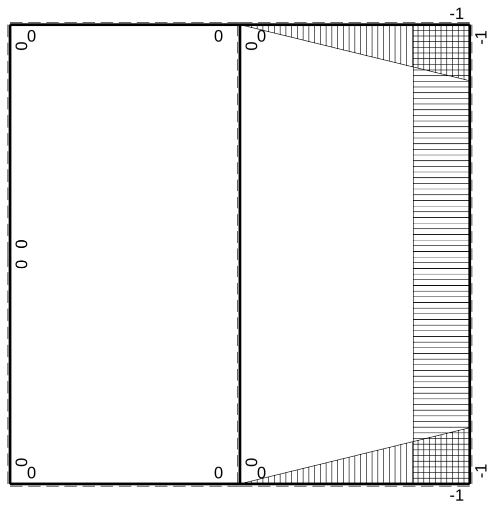
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
totali							$25/24Fb^2/EJ$	8/3Xb/EJ
iperstatica $X=W_{GC}$							-25/64Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

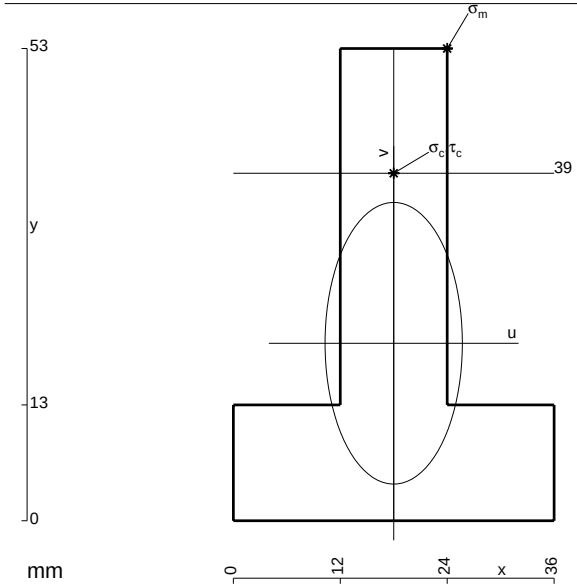
$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$



A = 948. mm^2
J_u = 236998. mm^4
J_v = 56304. mm^4
y_g = 19.92 mm
T_y = -2265. N
M_x = 1426950. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 33.08 mm
$\sigma_m = -Mv/J_u$ = -199.2 N/mm^2
x_c = 18. mm
y_c = 39. mm
v_c = 19.08 mm
$\sigma_c = -Mv/J_u$ = -114.9 N/mm^2
τ_c = 3.49 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 115.1 N/mm^2
S^* = 4382. mm^3

ESΣ8.185

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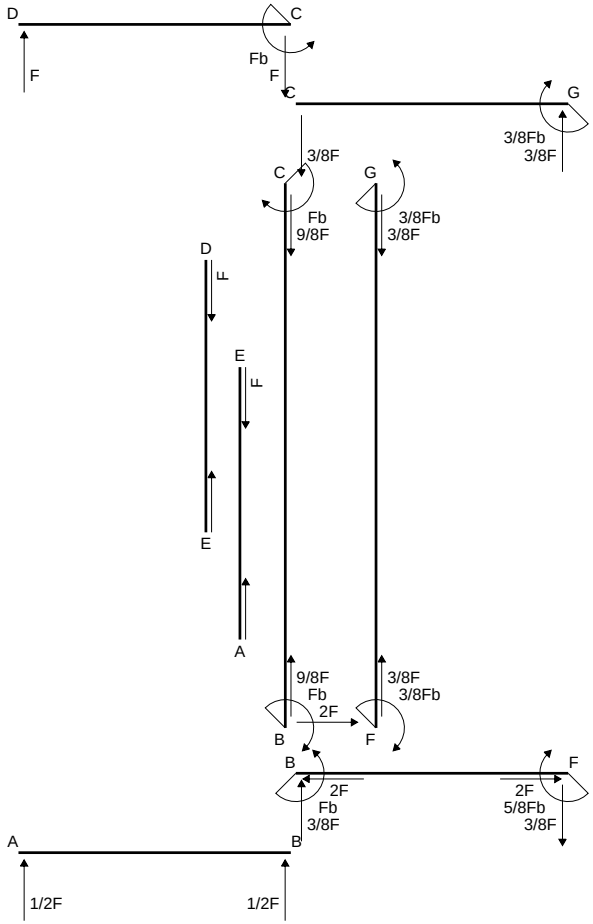
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Fb
F
C
G
3/8Fb
3/8F
3/8F
C
G
Fb
9/8F
3/8Fb
3/8F
D
F
E
F
E
A
9/8F
Fb
2F
3/8F
3/8Fb
B
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3/8F
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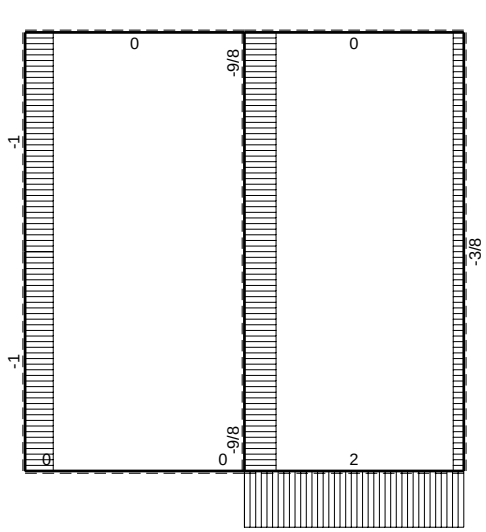
B
1/2F
1/2F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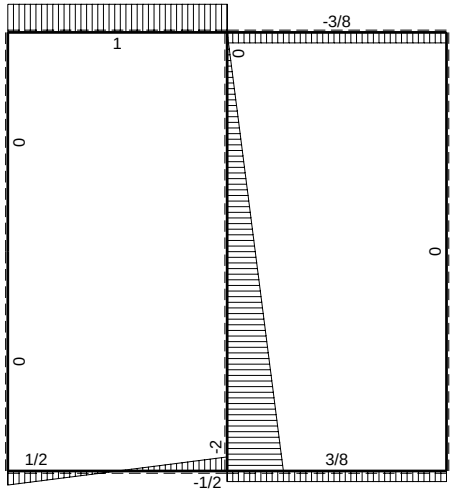
2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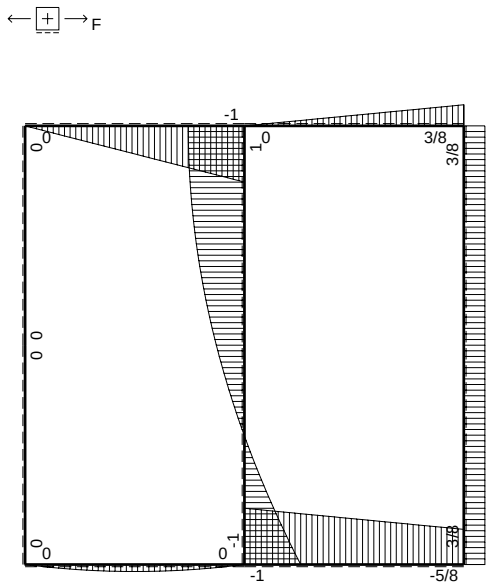
3/8
-1/2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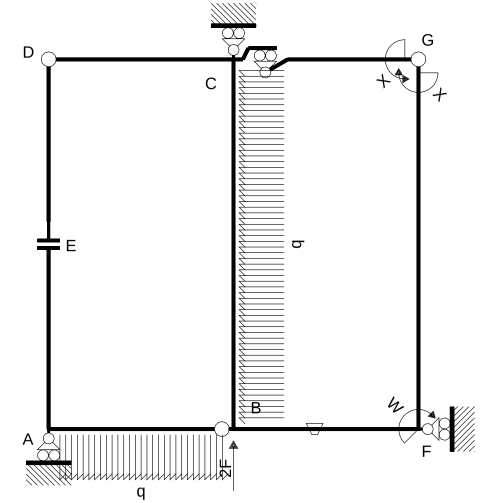
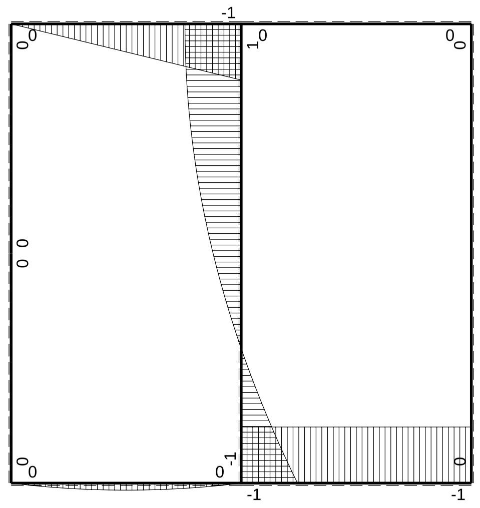
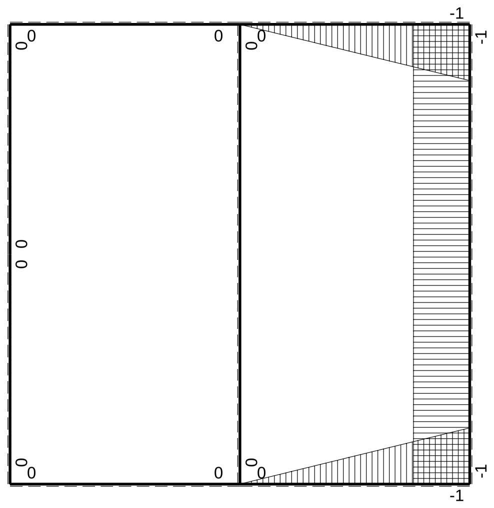


Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W_{GC}						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

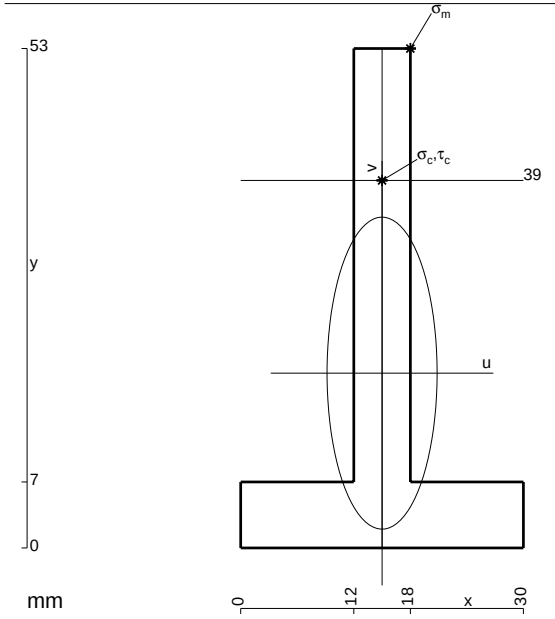
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 486. mm^2
J_u = 133275. mm^4
J_v = 16578. mm^4
y_g = 18.55 mm
N = -1373. N
T_y = -2440. N
M_x = -817400. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 34.45 mm
$\sigma_m = N/A - Mv/J_u$ = 208.5 N/mm^2
x_c = 15. mm
y_c = 39. mm
v_c = 20.45 mm
$\sigma_c = N/A - Mv/J_u$ = 122.6 N/mm^2
τ_c = 7.036 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 123.2 N/mm^2
S^* = 2306. mm^3

ESΣ8.186

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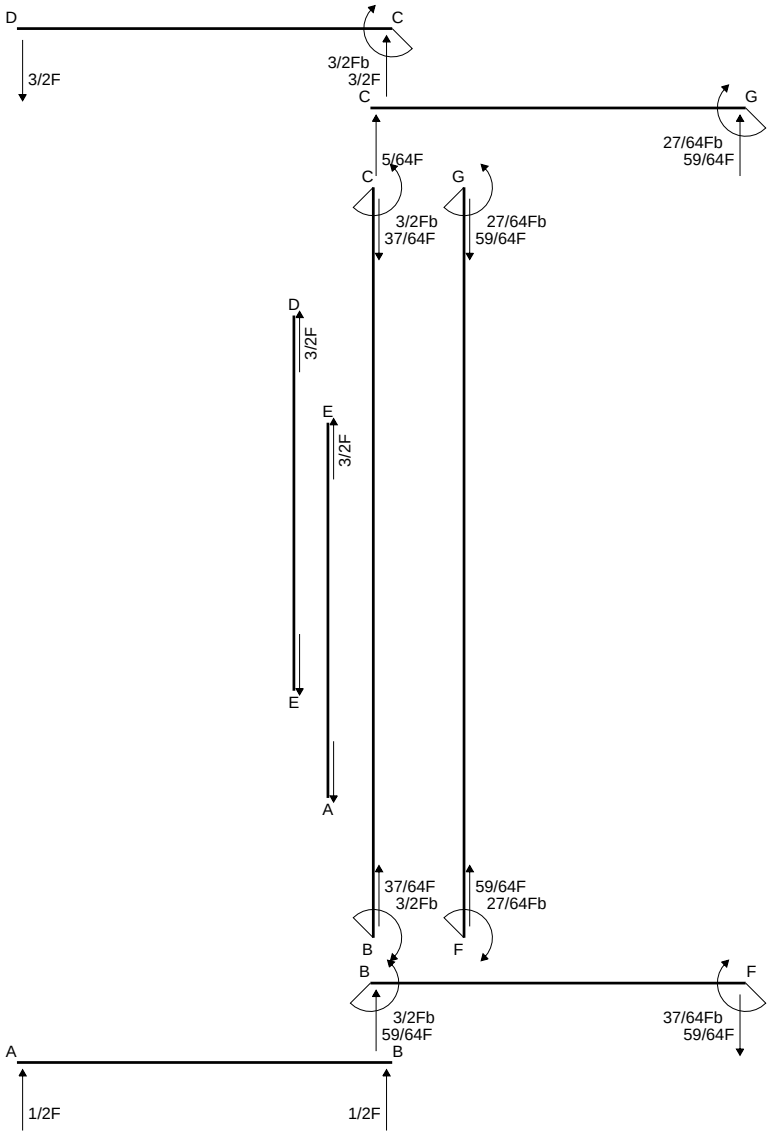
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3/2Fb
3/2F
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G
27/64Fb
59/64F
5/64F
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G
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27/64Fb
59/64F
D
3/2F
E
3/2F
E
A
37/64F
3/2Fb
59/64F
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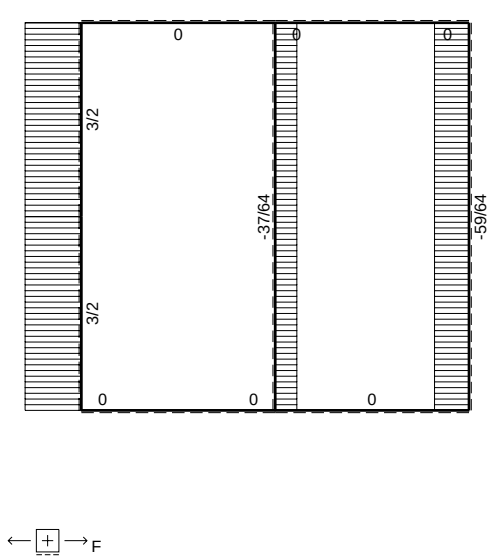
B
1/2F
1/2F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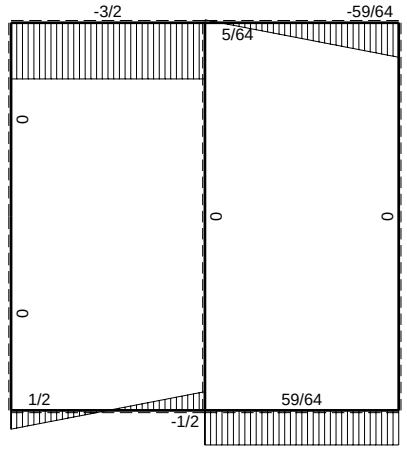
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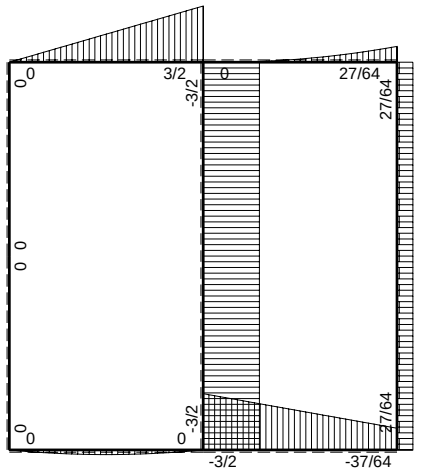
1/2
59/64
-1/2

F


0
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3/2
-3/2
0
27/64
27/64
0
0
0
0
-3/2
27/64
-3/2
-37/64

Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_0(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fx-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-1/2Fx+1/2qx^2$	0	0	0	0		
CD b	0	3/2Fb-3/2Fx	0	0	0	0	0+0	0
DC b	0	-3/2Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+1/2Fx$	-Fb/EJ	$3/2Fx-1/2Fx^2/b$	Fx/EJ	x^2/b^2	$(7/12+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb+1/2Fx	Fb/EJ	$Fb-1/2Fx-1/2Fx^2/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	$-1/2Fx+1/2qx^2$	0	$1/2Fx-Fx^2/b+1/2qx^3/b$	0	$1-2x/b+x^2/b^2$	$(1/24+0)Fb^2/EJ$	1/3Xb/EJ
CG b	x/b	$1/2Fx-1/2qx^2$	0	$1/2Fx^2/b-1/2qx^3/b$	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$9/8Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-27/64Fb	

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (3/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (x/b)\ \theta\ dx$

$= [3/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (3/4\ b -1/6\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 -1/2\ x/b -1/2\ x^2/b^2)\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b)\ \theta\ dx$

$= [\ x -1/4\ x^2/b -1/6\ x^3/b^2\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

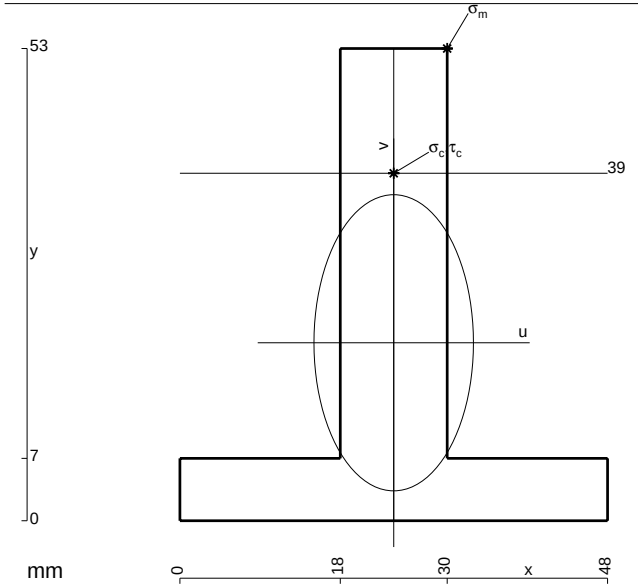
$= (\ b -1/4\ b -1/6\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b)\ \theta\ = 13/12\ Fb^2/EJ$

$L_{GC}^{Xo} = \int_o^b (1/2\ x/b - x^2/b^2 +1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/4\ b -1/3\ b +1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$

$L_{CG}^{Xo} = \int_o^b (1/2\ x^2/b^2 -1/2\ x^3/b^3)\ Fb\ 1/EJ\ dx = [1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ$

$= (1/6\ b -1/8\ b)\ Fb\ 1/EJ\ = 1/24\ Fb^2/EJ$



A = 888. mm^2
J_u = 245383. mm^4
J_v = 71136. mm^4
y_g = 19.97 mm
T_y = -2295. N
M_x = 1629450. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 33.03 mm
$\sigma_m = -Mv/J_u$ = -219.3 N/mm^2
x_c = 24. mm
y_c = 39. mm
v_c = 19.03 mm
$\sigma_c = -Mv/J_u$ = -126.3 N/mm^2
τ_c = 3.408 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126.5 N/mm^2
S^* = 4373. mm^3

ESΣ8.187

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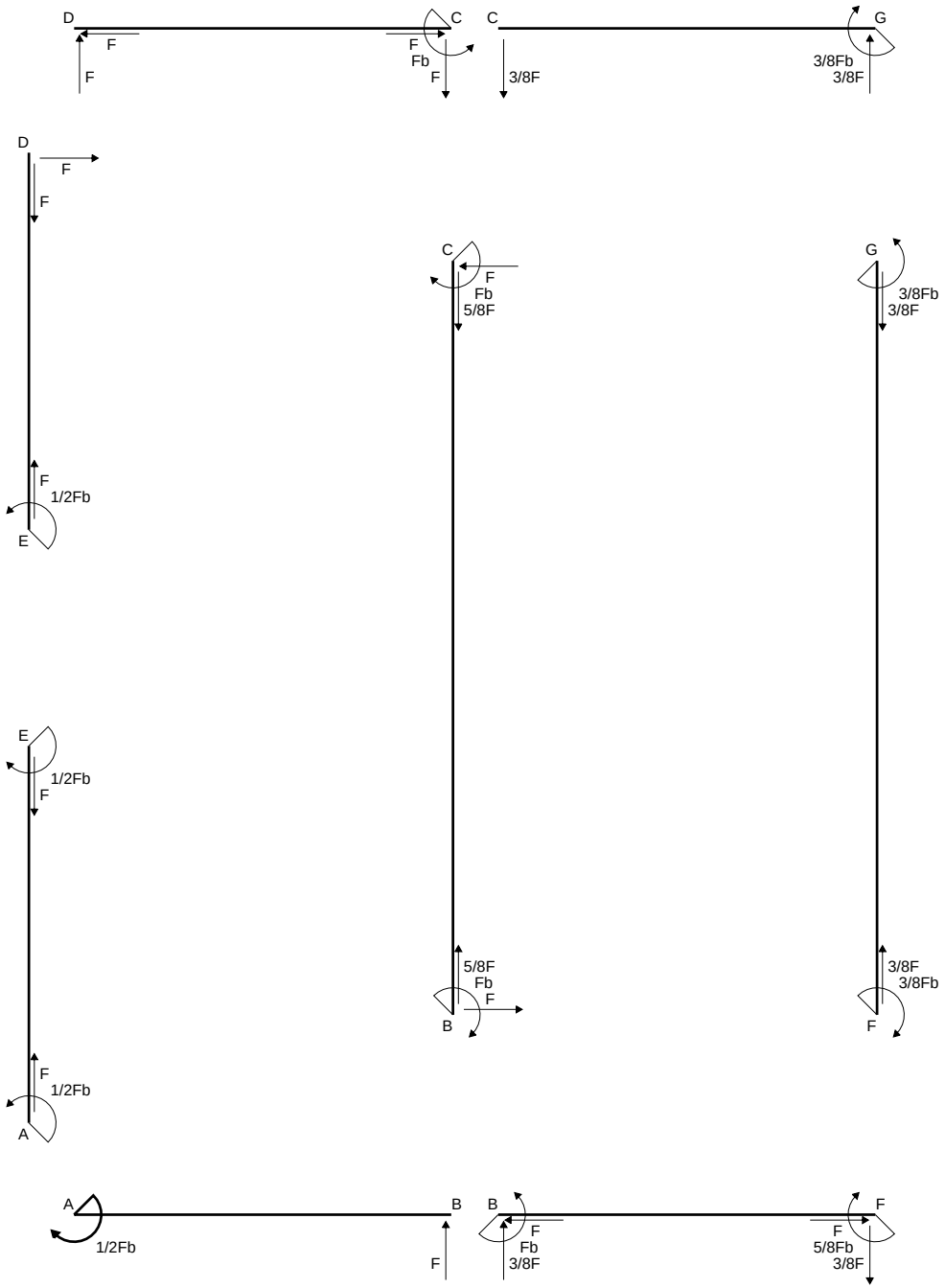
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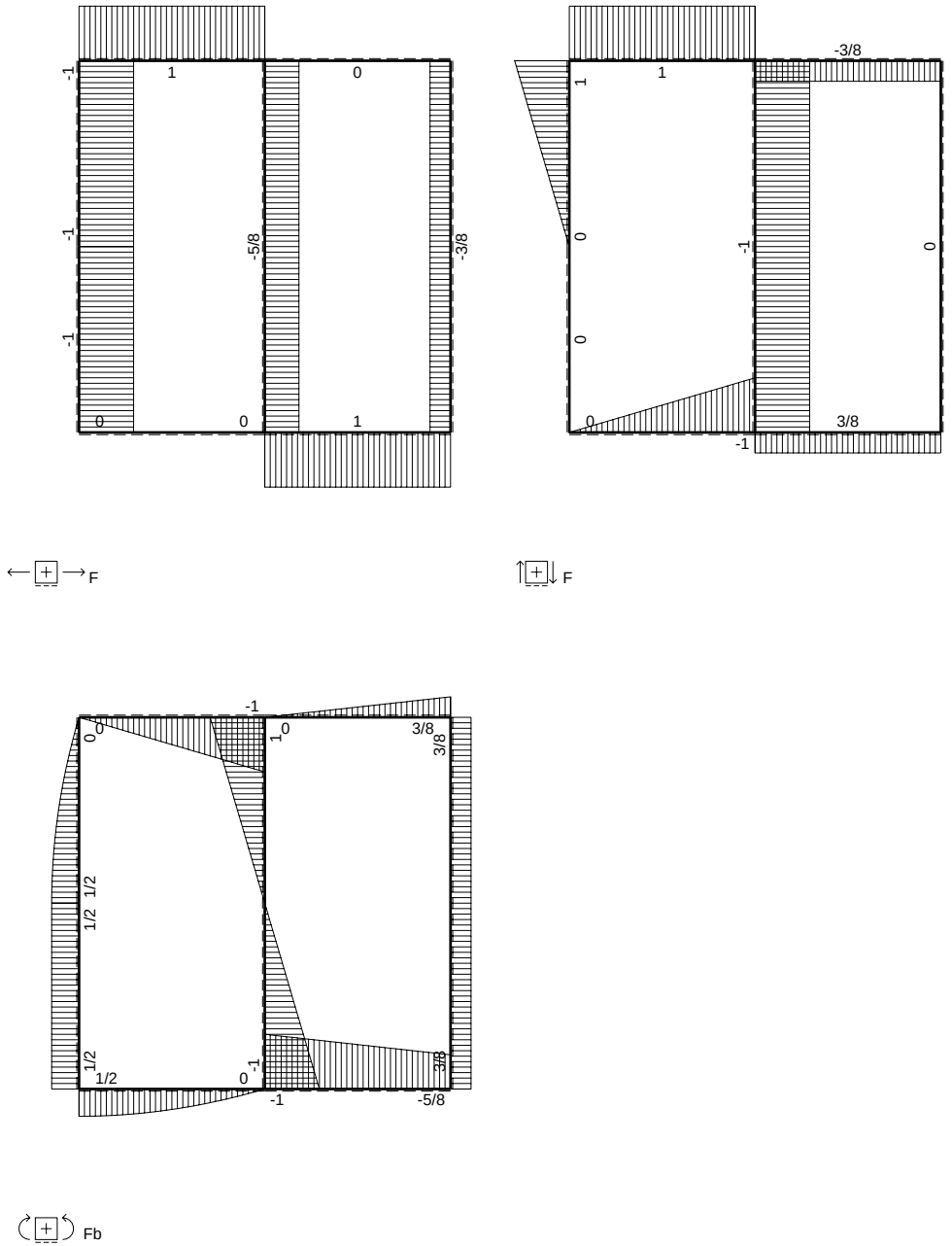
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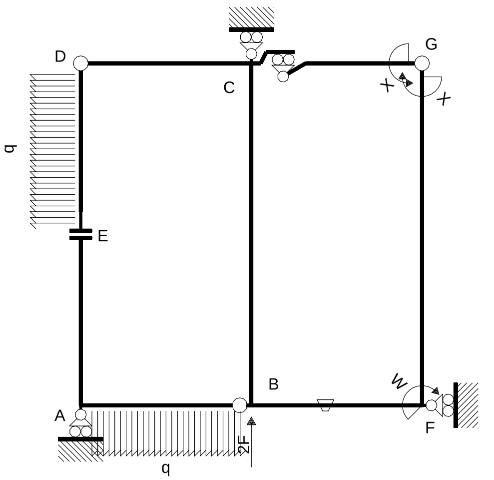
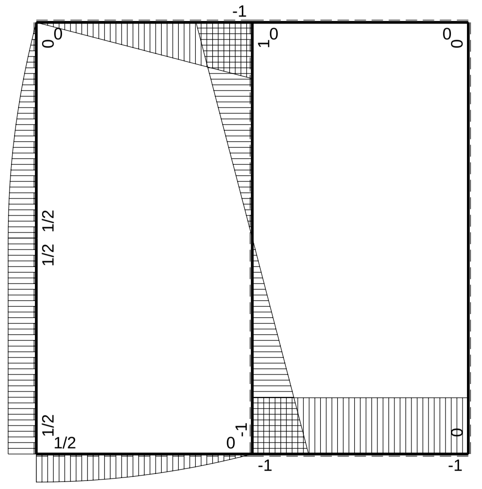
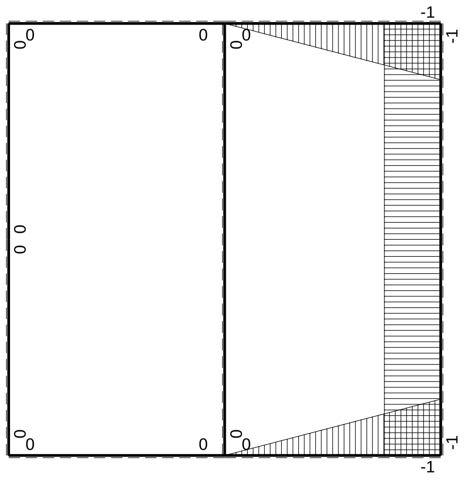
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

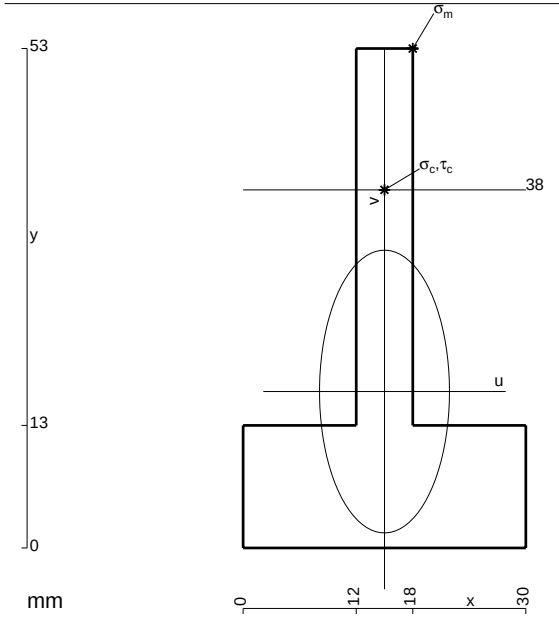
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 630. mm^2
J_u = 141827. mm^4
J_v = 29970. mm^4
y_g = 16.6 mm
N = 1180. N
T_y = 1180. N
M_x = -885000. Nmm
x_m = 18. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.4 mm
$\sigma_m = N/A - Mv/J_u$ = 229. N/mm^2
x_c = 15. mm
y_c = 38. mm
v_c = 21.4 mm
$\sigma_c = N/A - Mv/J_u$ = 135.4 N/mm^2
τ_c = 3.607 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 135.6 N/mm^2
S^* = 2601. mm^3

ESΣ8.188

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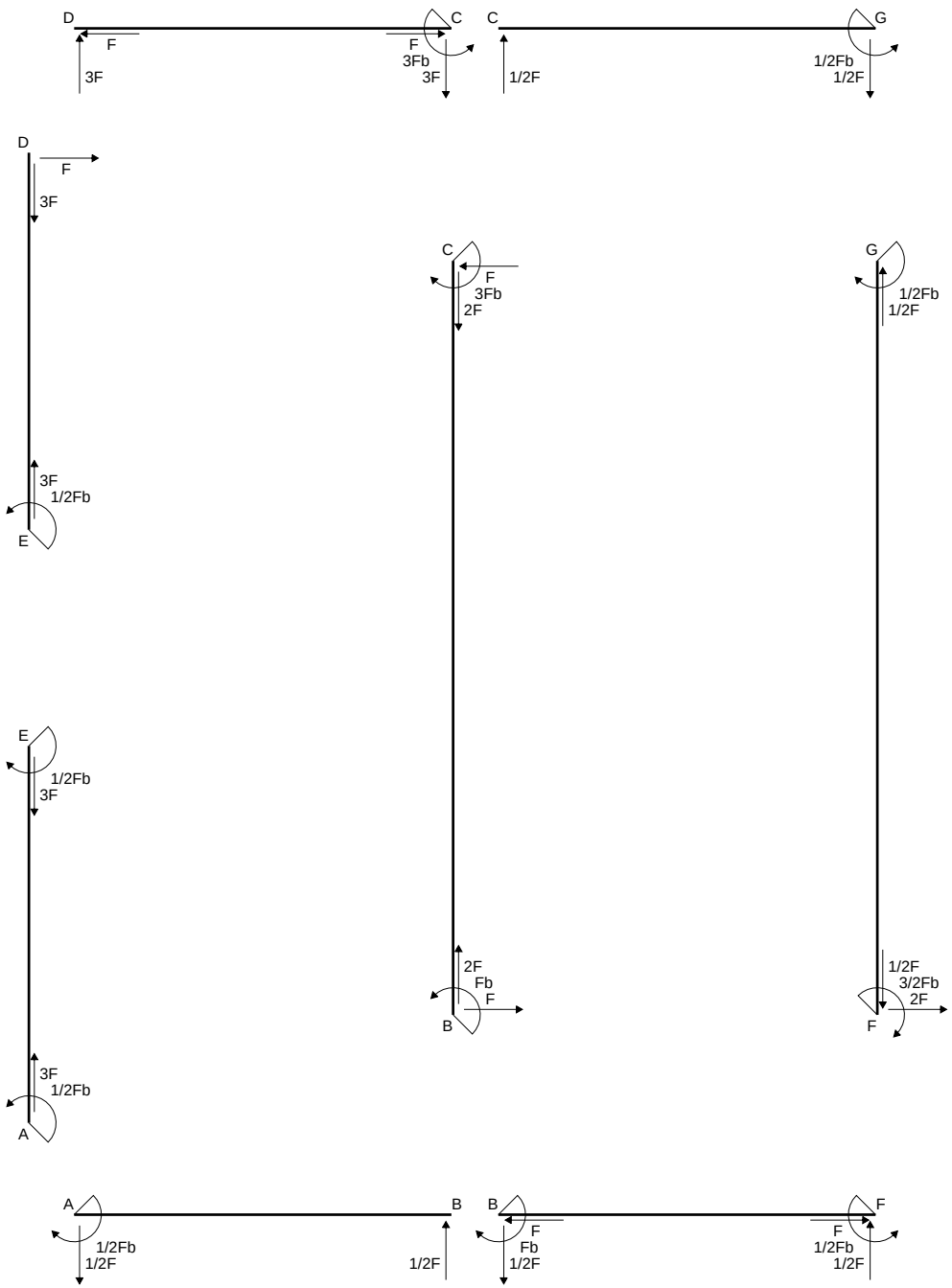
D
C
C
G
F
3F
F
3Fb
3F
1/2F
1/2Fb
1/2F
D
F
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3Fb
2F
G
1/2Fb
1/2F
3F
1/2Fb
E
E
1/2Fb
3F
2F
Fb
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B
1/2F
3/2Fb
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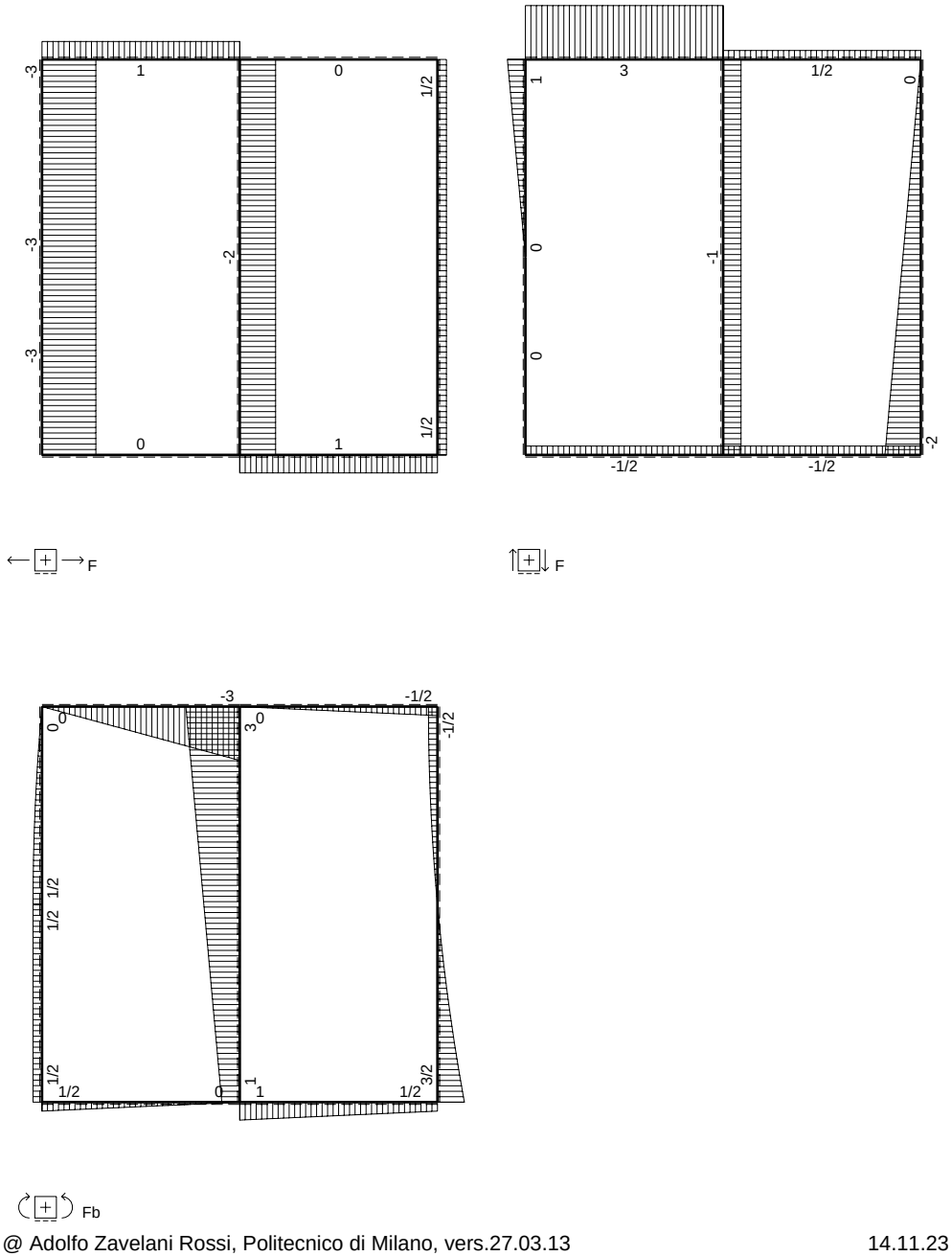
F
Fb
1/2F
F
1/2Fb
1/2F


-3
1
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1/2
-3
-2
-3
0
1
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F
1
3
1/2
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0
0
-1
-2
-1/2
-1/2
F
-3
-1/2
0
0
3
0
-1/2
1/2
1/2
1/2
1/2
0
1
1
1/2
3/2
Fb

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
	0	3Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
	0	-1/2Fb	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	3Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	-Fb-Fx	0	0	0	0		
totali							$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						1/2Fb	

Sviluppi di calcolo iperstatica

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

PROCEDIMENTO E RISULTATI Nome: ESΣ8.189

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$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

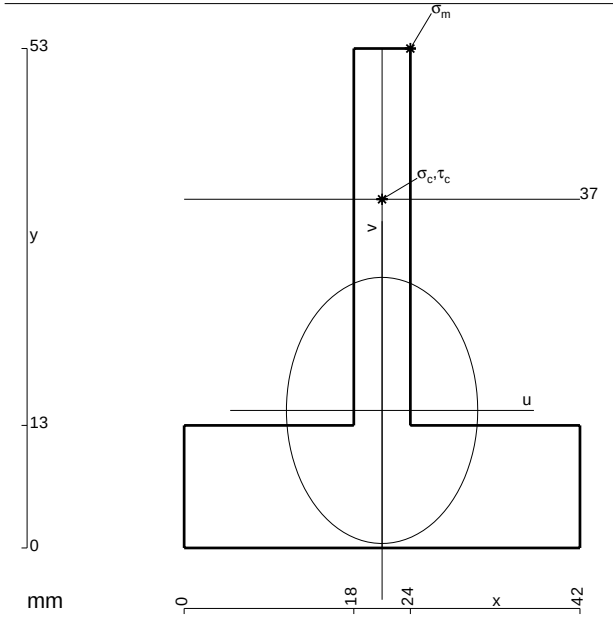
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 786. mm^2
J_u = 156767. mm^4
J_v = 80982. mm^4
y_g = 14.59 mm
N = 410. N
T_y = 1230. N
M_x = -971700. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 38.41 mm
$\sigma_m = N/A - Mv/J_u$ = 238.6 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 22.41 mm
$\sigma_c = N/A - Mv/J_u$ = 139.4 N/mm^2
τ_c = 3.817 N/mm^2
$\sigma_o = \sqrt{\sigma^2 + 3\tau^2}$ = 139.6 N/mm^2
S^* = 2919. mm^3

ESΣ8.189

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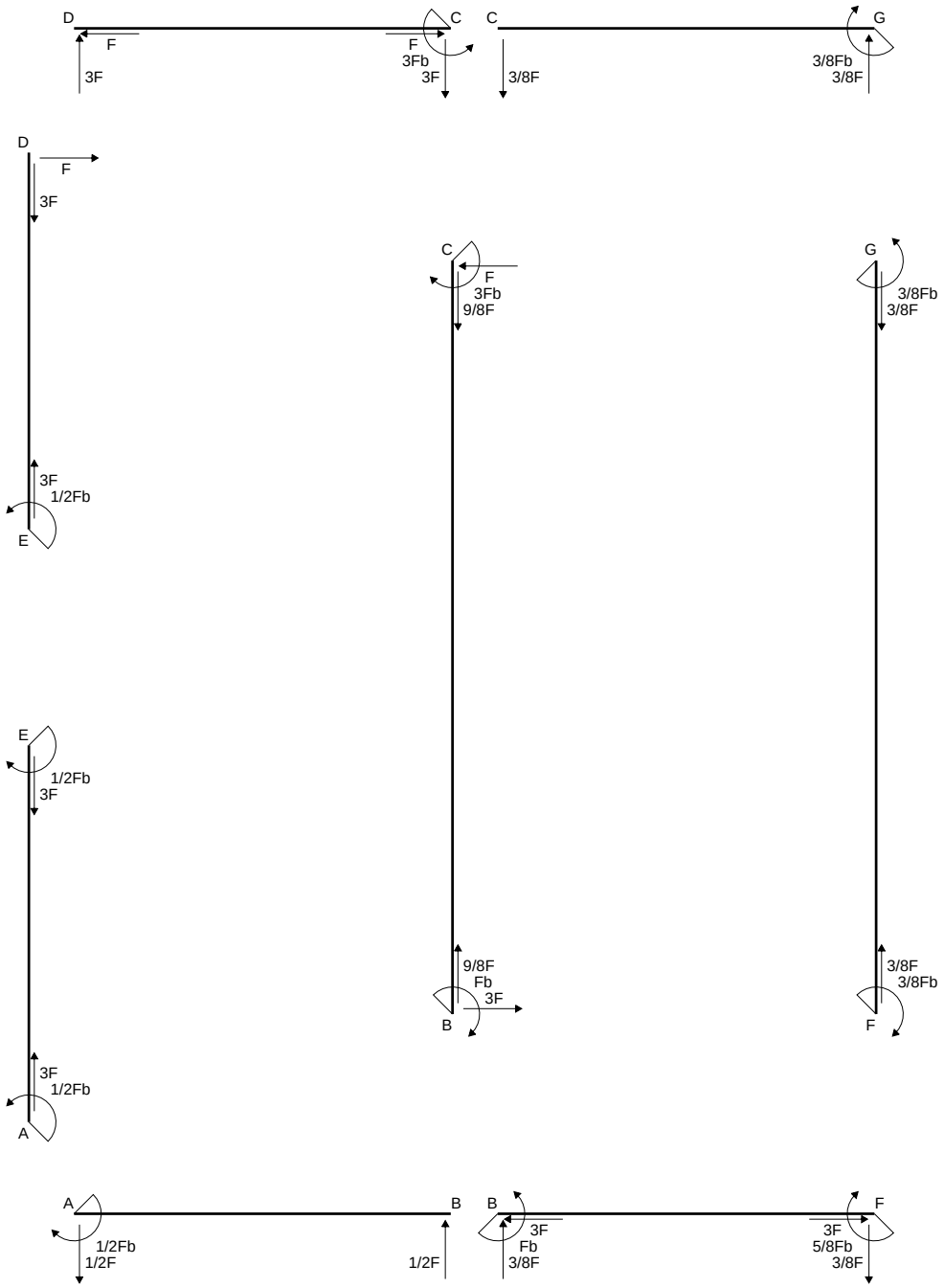
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3F
C
3/8F
G
3/8Fb
3/8F
D
F
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E
C
F
3Fb
9/8F
G
3/8Fb
3/8F
E
1/2Fb
3F
9/8F
Fb
3F
B
3/8F
3/8Fb
F
3F
1/2Fb
A
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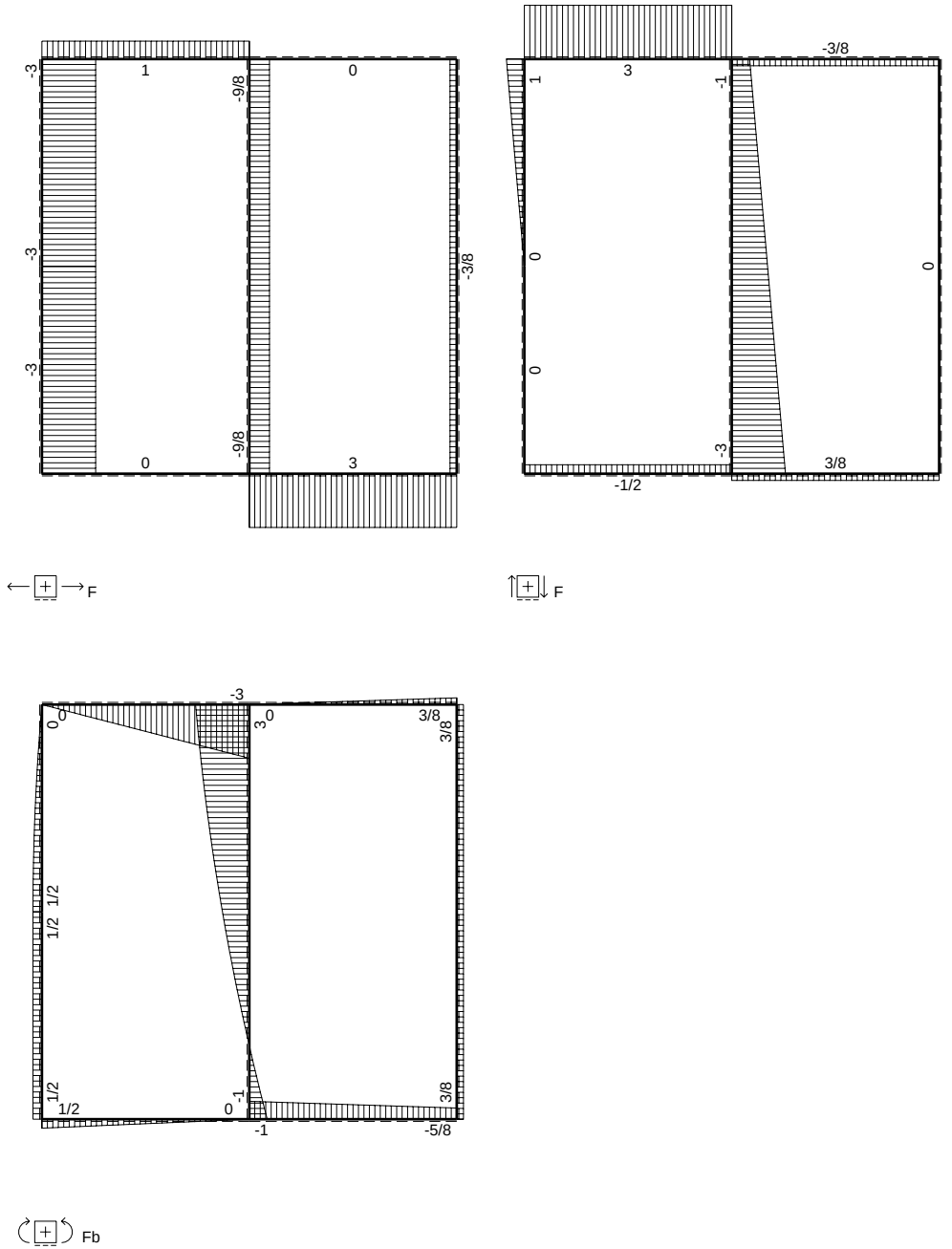
Fb
3/8F
F
3F
5/8Fb
3/8F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-9/8
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F
-3
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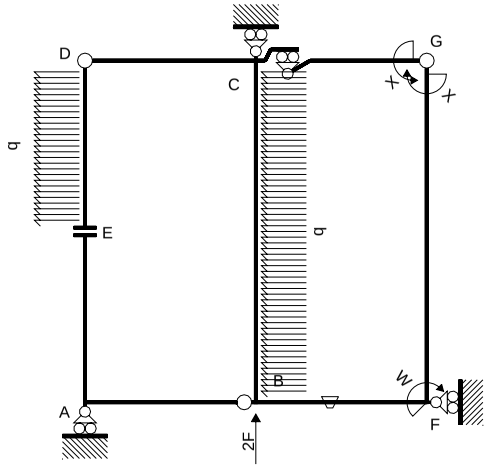
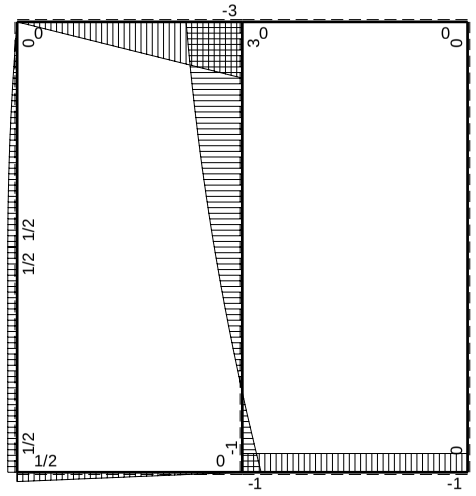
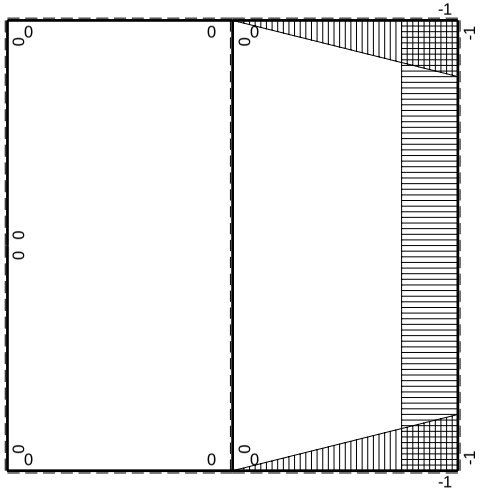
3/8
-1
-5/8
Fb



Schema di calcolo iperstatico



M_0 flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	1/2Fb-1/2Fx	0	0	0	0	0+0	0
BA b	0	-1/2Fx	0	0	0	0		
CD b	0	-3Fb+3Fx	0	0	0	0	0+0	0
DC b	0	3Fx	0	0	0	0		
DE b	0	Fx-1/2qx^2	0	0	0	0	0+0	0
ED b	0	-1/2Fb+1/2qx^2	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	1-2x/b+x^2/b^2		
GC b	-1+x/b	0	0	0	0	1-2x/b+x^2/b^2	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	3Fb-Fx-1/2qx^2	0	0	0	0	0+0	0
BC 2b	0	Fb-3Fx+1/2qx^2	0	0	0	0		
				totali			Fb^2/EJ	8/3Xb/EJ
				iperstatica $X=W_{GC}$			-3/8Fb	

Sviluppi di calcolo iperstatica

PROCEDIMENTO E RISULTATI Nome: ESΣ8.190

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$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

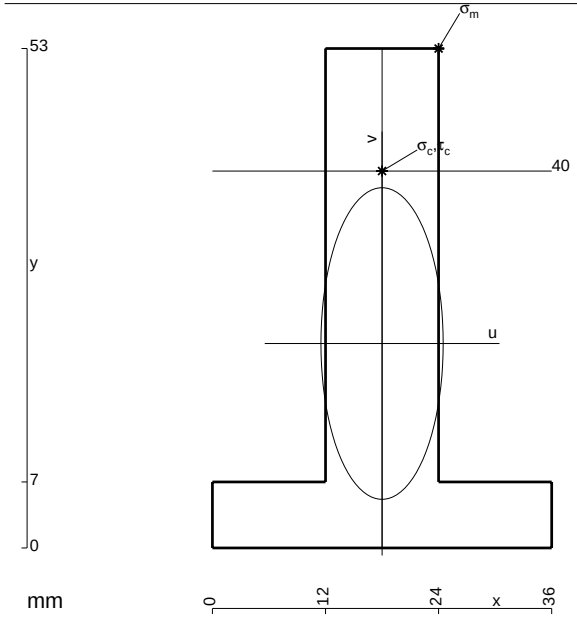
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 804. mm^2
J_u = 219865. mm^4
J_v = 33840. mm^4
y_g = 21.69 mm
N = 1100. N
T_y = 3300. N
M_x = -1386000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 6. mm
v_m = 31.31 mm
$\sigma_m = N/A - Mv/J_u$ = 198.7 N/mm^2
x_c = 18. mm
y_c = 40. mm
v_c = 18.31 mm
$\sigma_c = N/A - Mv/J_u$ = 116.8 N/mm^2
τ_c = 4.84 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 117.1 N/mm^2
S^* = 3870. mm^3

ESΣ8.190

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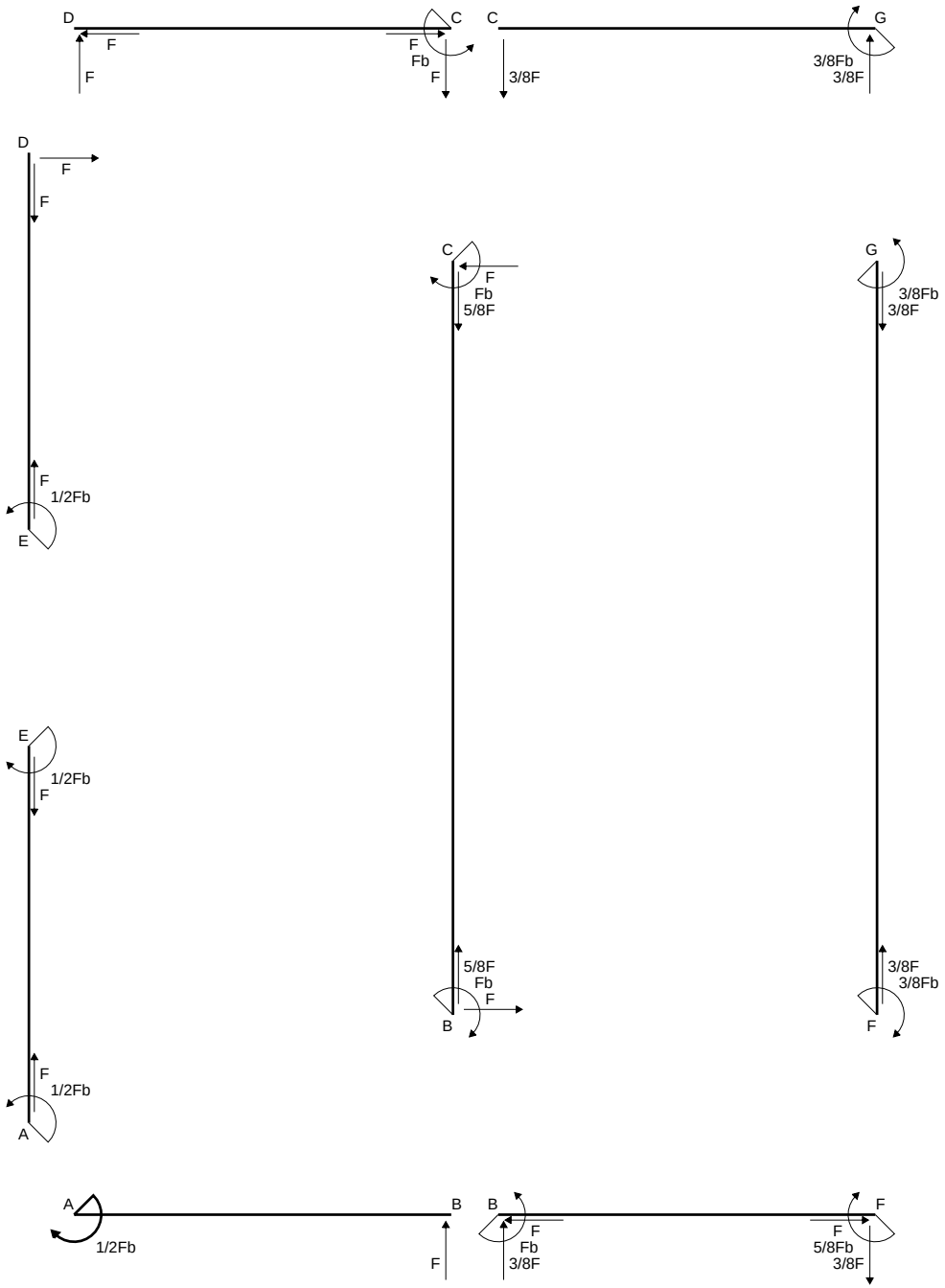
D
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F
Fb
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3/8F
3/8Fb
3/8F
D
F
F
C
F
Fb
5/8F
G
3/8Fb
3/8F
F
1/2Fb
E
E
1/2Fb
F
5/8F
Fb
F
B
3/8F
3/8Fb
F
F
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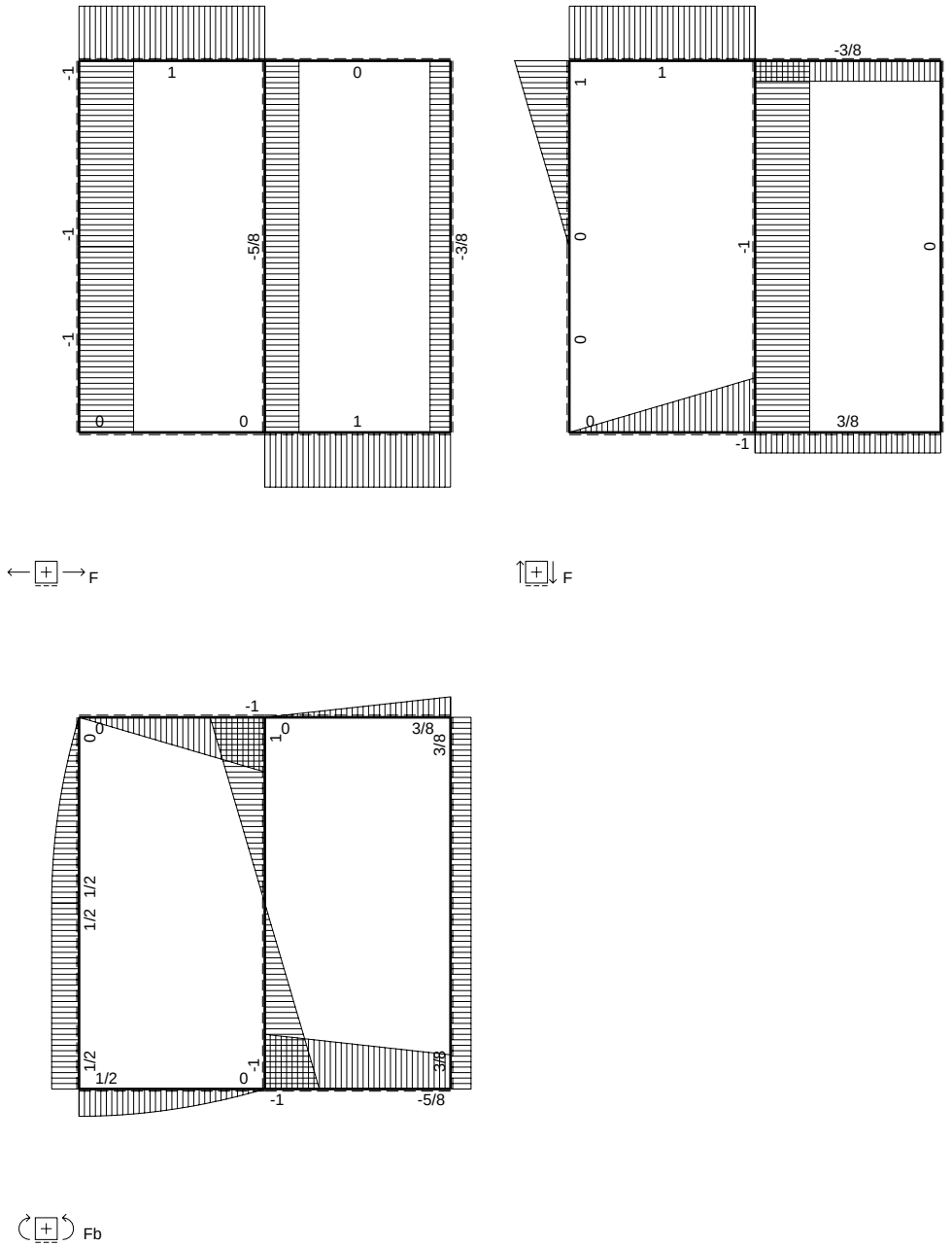
Fb
3/8F
F
5/8Fb
3/8F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-1
F
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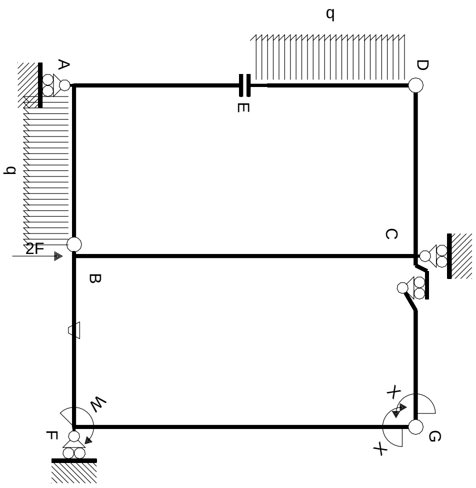
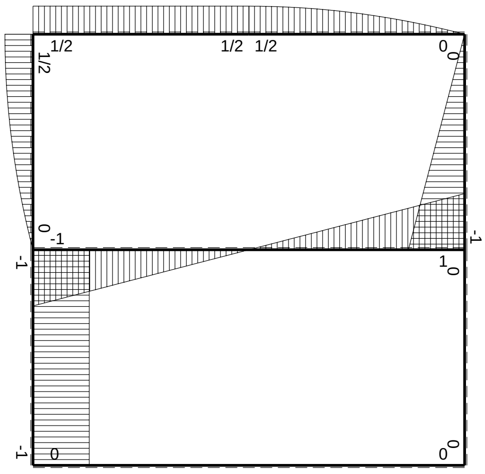
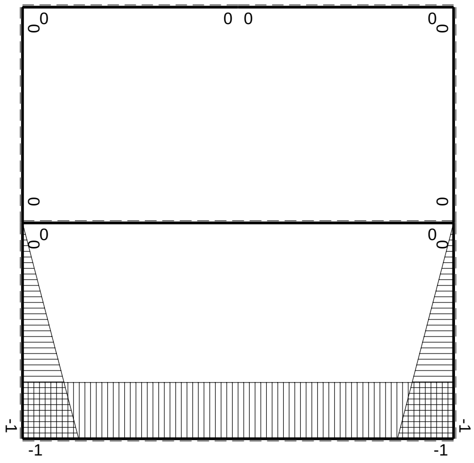
Fb



Schema di calcolo iperstatico



M$_o$ flessione da carichi assegnati



M$_x$ flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W$_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
BA b	0	$-Fx+1/2qx^2$	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	1/2Fb	0	0	0	0	0+0	0
AE b	0	-1/2Fb	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica X=W$_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

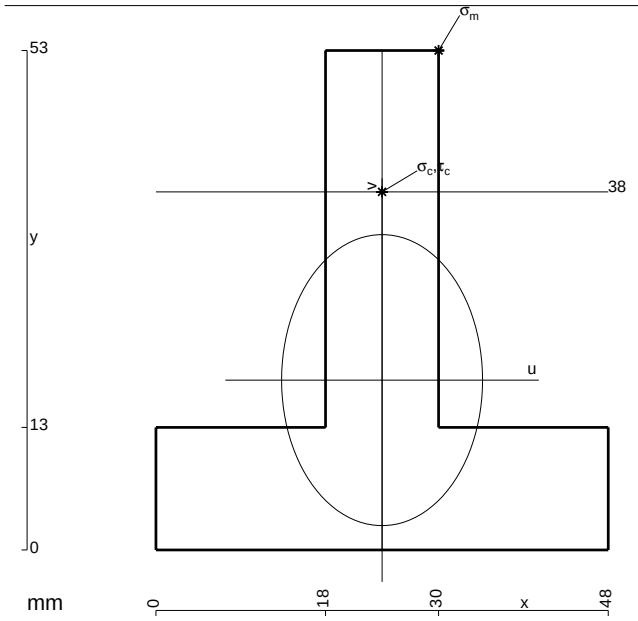
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 1104. mm^2
J_u = 263311. mm^4
J_v = 125568. mm^4
y_g = 18.02 mm
N = 3380. N
T_y = 3380. N
M_x = -1554800. Nmm
x_m = 30. mm
y_m = 53. mm
u_m = 6. mm
v_m = 34.98 mm
$\sigma_m = N/A - Mv/J_u$ = 209.6 N/mm^2
x_c = 24. mm
y_c = 38. mm
v_c = 19.98 mm
$\sigma_c = N/A - Mv/J_u$ = 121. N/mm^2
τ_c = 5.291 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 121.4 N/mm^2
S^* = 4946. mm^3

ESΣ8.191

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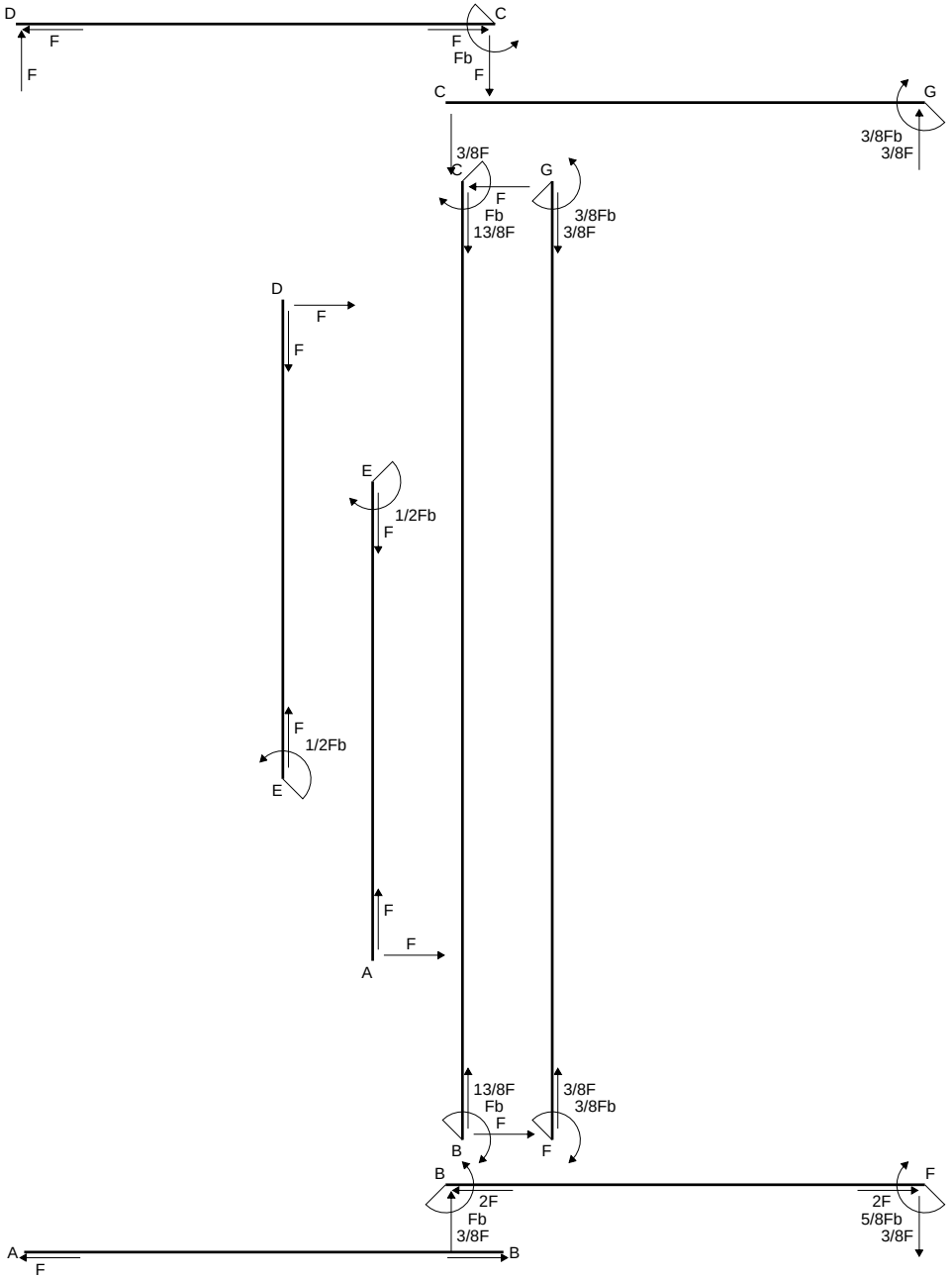
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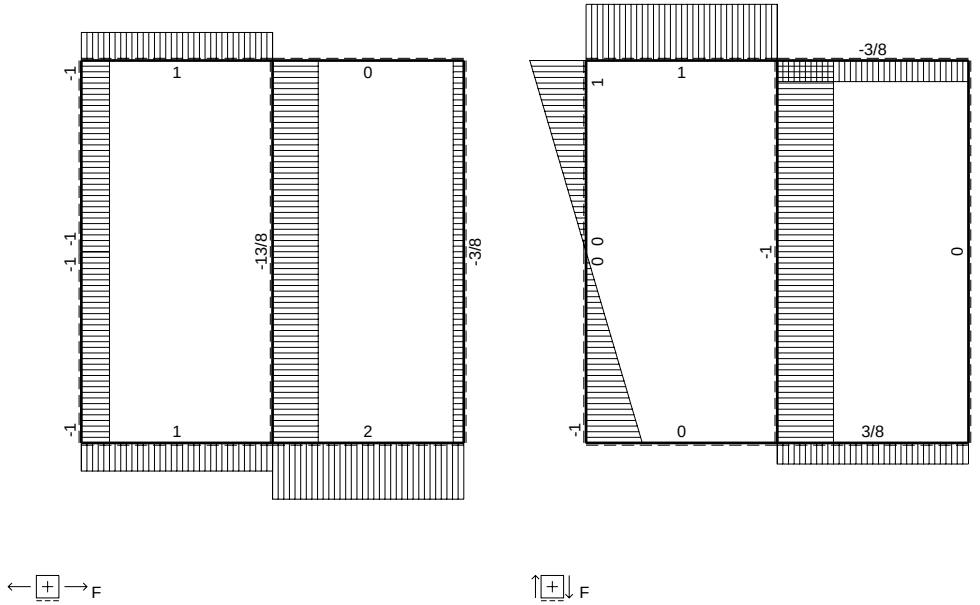
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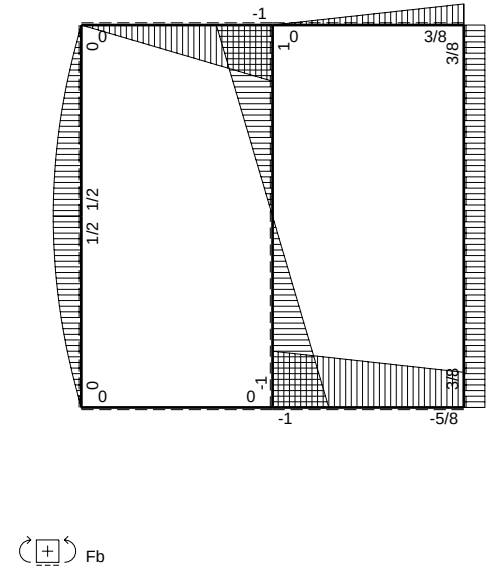
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3/8
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-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	$M_x M_0$	$M_x \theta$	$M_x M_x$	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

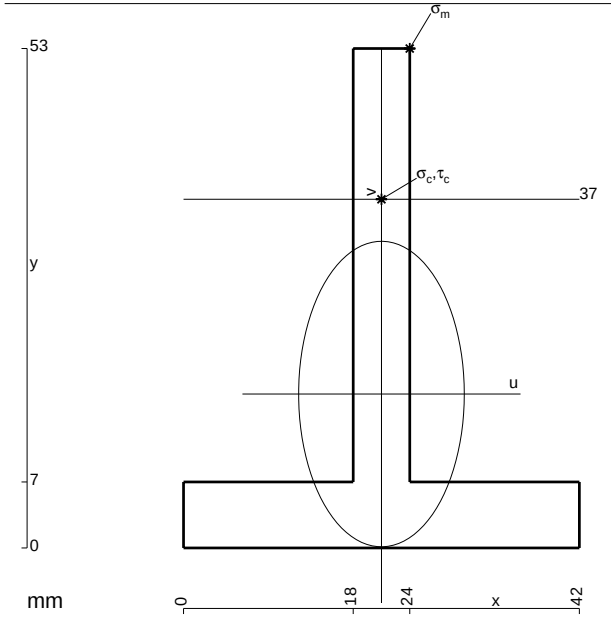
$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$



A = 570. mm^2
J_u = 149839. mm^4
J_v = 44046. mm^4
y_g = 16.33 mm
N = -2844. N
T_y = -1750. N
M_x = 875000. Nmm
x_m = 24. mm
y_m = 53. mm
u_m = 3. mm
v_m = 36.67 mm
$\sigma_m = N/A - Mv/J_u$ = -219.1 N/mm^2
x_c = 21. mm
y_c = 37. mm
v_c = 20.67 mm
$\sigma_c = N/A - Mv/J_u$ = -125.7 N/mm^2
τ_c = 5.357 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 126. N/mm^2
S^* = 2752. mm^3

ESΣ8.192

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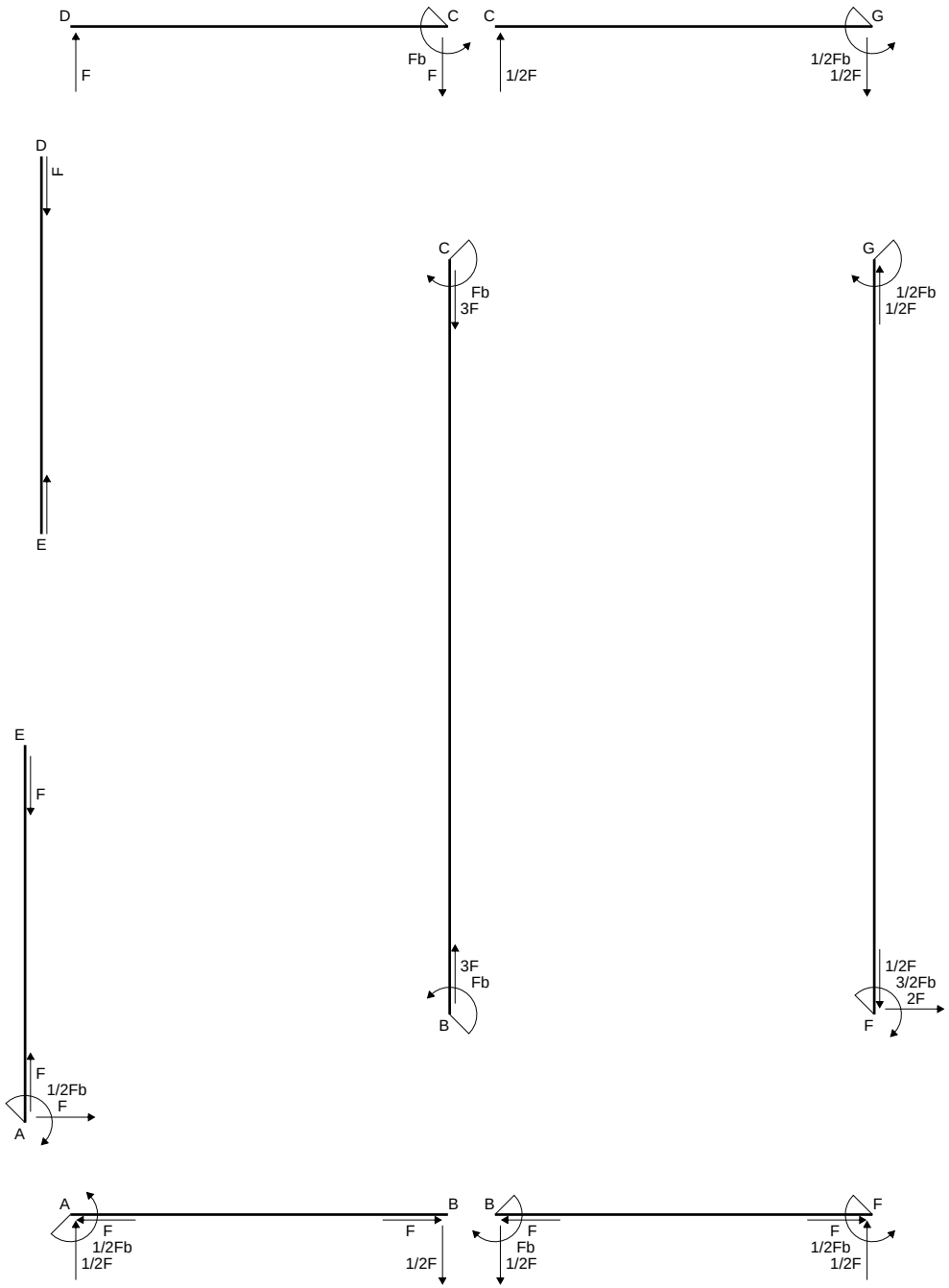
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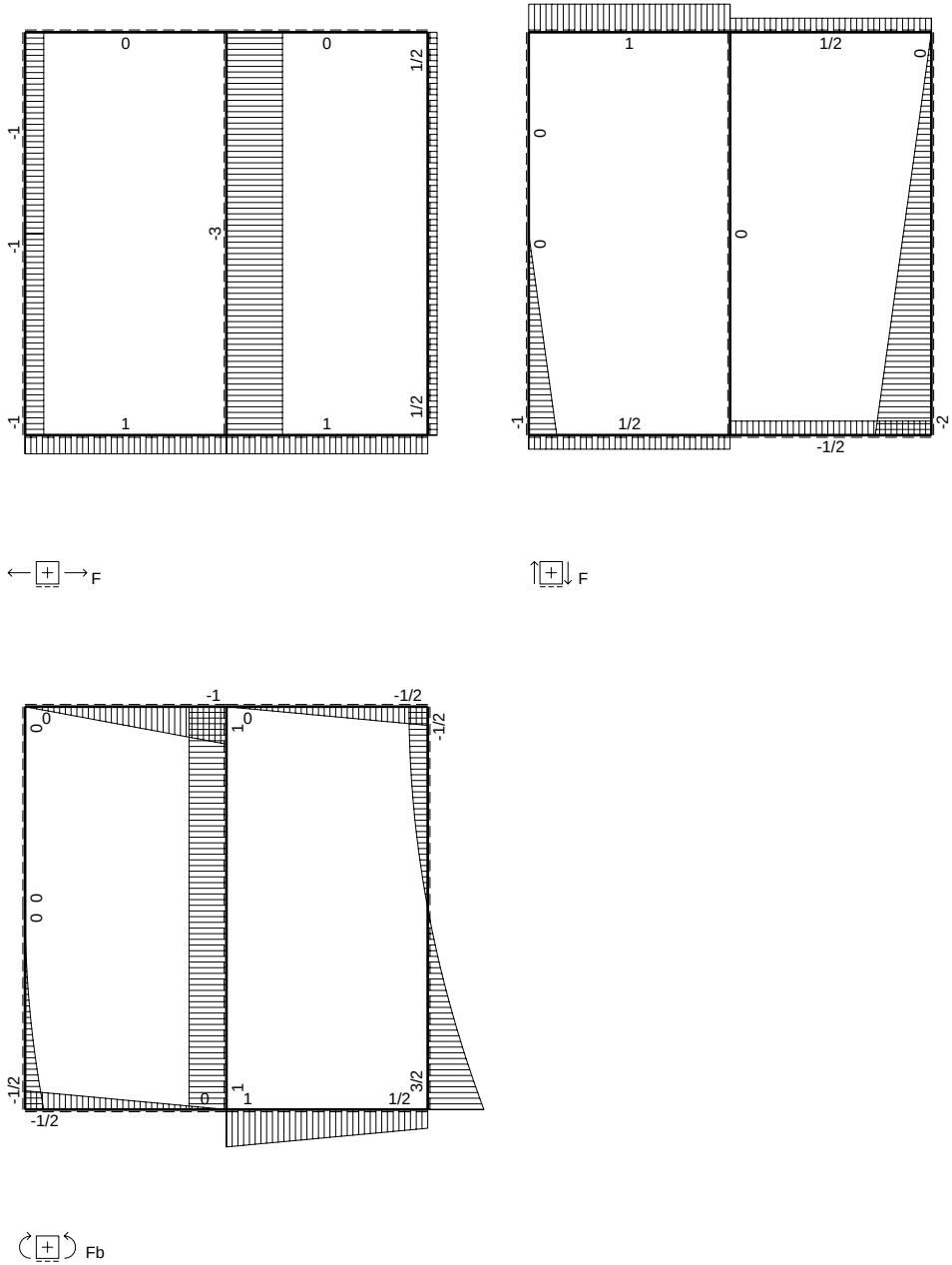
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F
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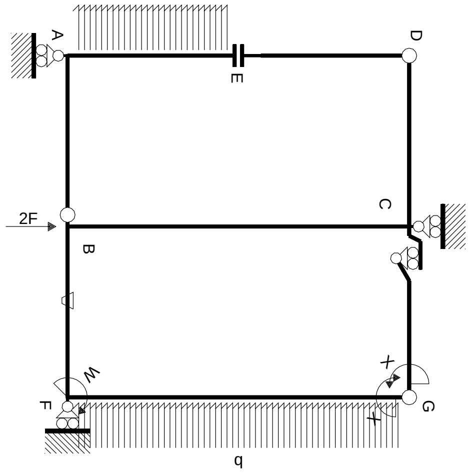
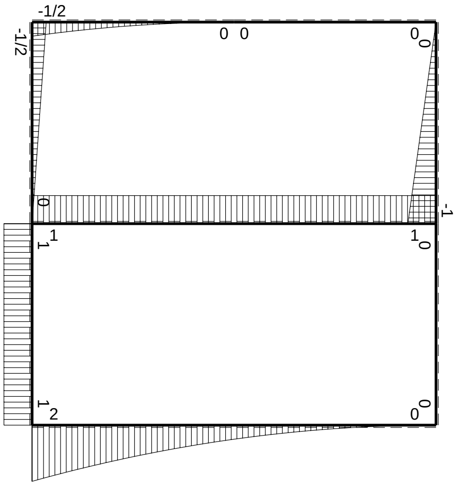
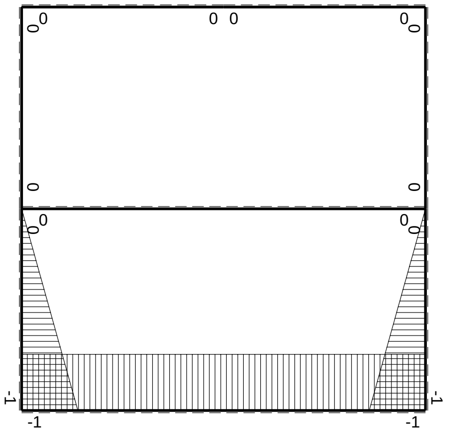
Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica X=W_{GC}

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	Fb	-Fb/EJ	-Fx	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	-Fb	Fb/EJ	-Fb+Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	2Xb/EJ
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	-Fb	0	0	0	0		
	totali						$-4/3Fb^2/EJ$	8/3Xb/EJ
	iperstatica X=W_{GC}						1/2Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

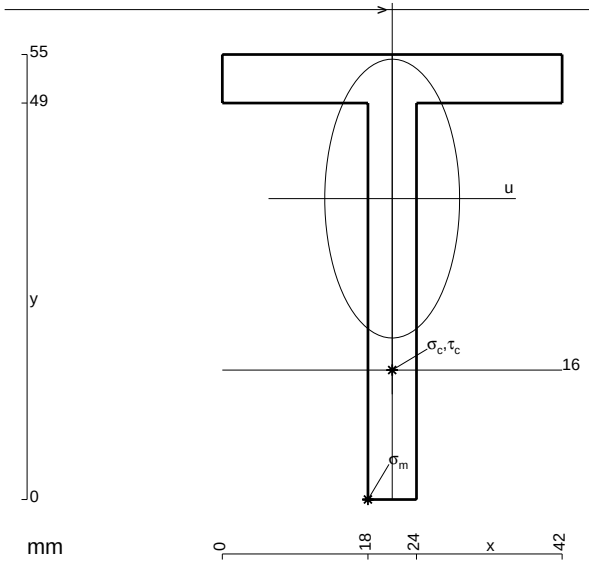
$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.193



A = 546. mm^2
J_u = 162198. mm^4
J_v = 37926. mm^4
y_g = 37.19 mm
T_y = 1750. N
M_x = -997500. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -37.19 mm
$\sigma_m = -Mv/J_u$ = -228.7 N/mm^2
x_c = 21. mm
y_c = 16. mm
v_c = -21.19 mm
$\sigma_c = -Mv/J_u$ = -130.3 N/mm^2
τ_c = 5.039 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 130.6 N/mm^2
S^* = 2802. mm^3

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ESΣ8.193

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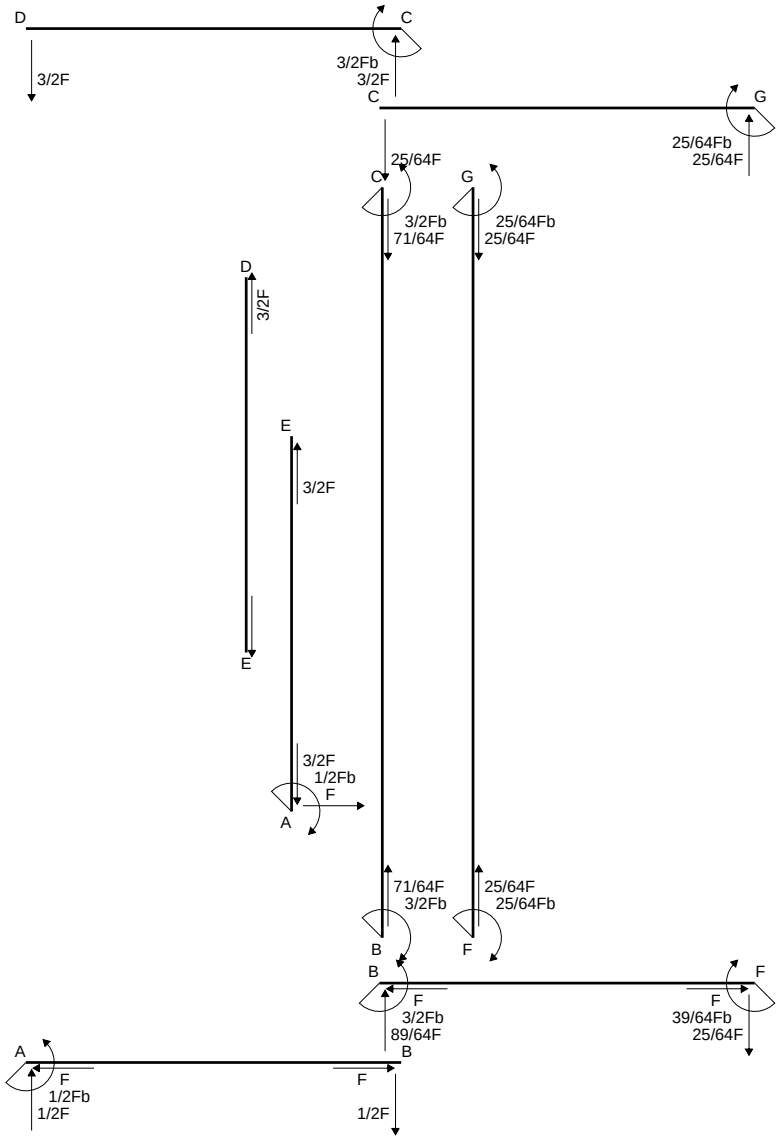
D
C
3/2F
3/2Fb
3/2F
C
G
25/64Fb
25/64F
25/64F
C
G
3/2Fb
71/64F
25/64Fb
25/64F
D
3/2F
E
3/2F
E
3/2F
1/2Fb
F
A
71/64F
3/2Fb
25/64F
25/64Fb
B
F
B
F
F
3/2Fb
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F
39/64Fb
25/64F
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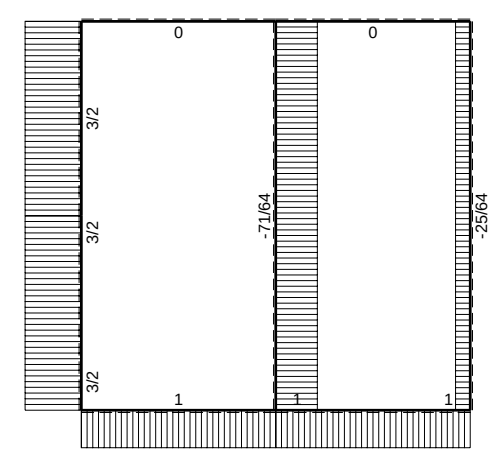
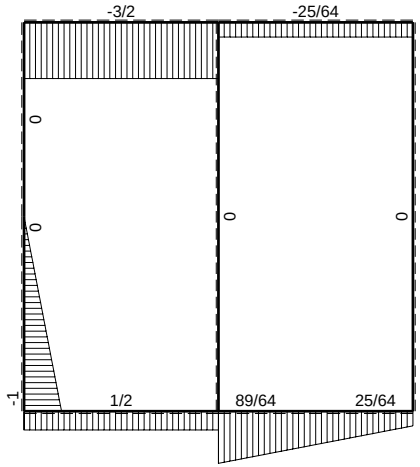
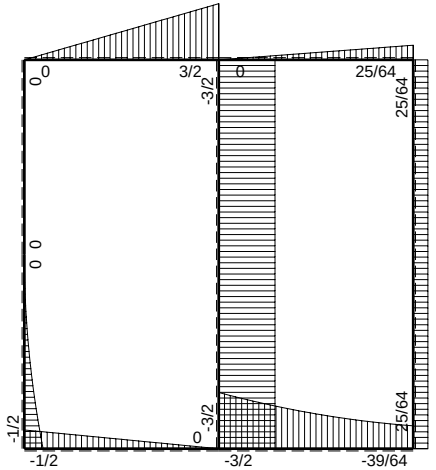
A
F
1/2Fb
1/2F
F
1/2F


0
0
3/2
3/2
3/2
-71/64
-25/64
1
1
1

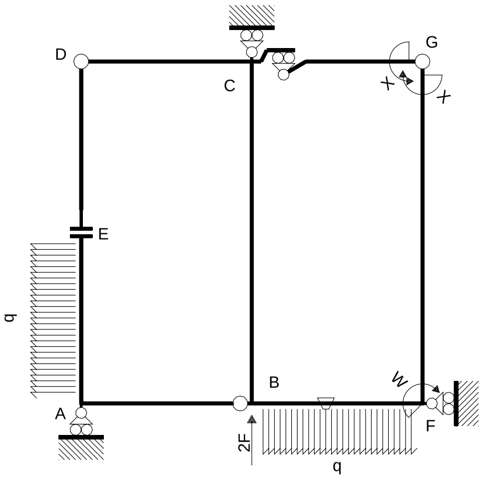
F


-3/2
-25/64
0
0
0
0
0
-1
1/2
89/64
25/64

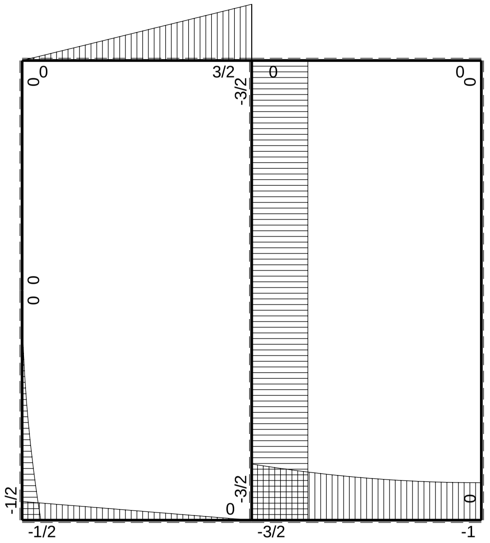
F


0
0
3/2
-3/2
0
25/64
25/64
0
0
-1/2
-3/2
0
-3/2
25/64
-1/2
-3/2
-39/64

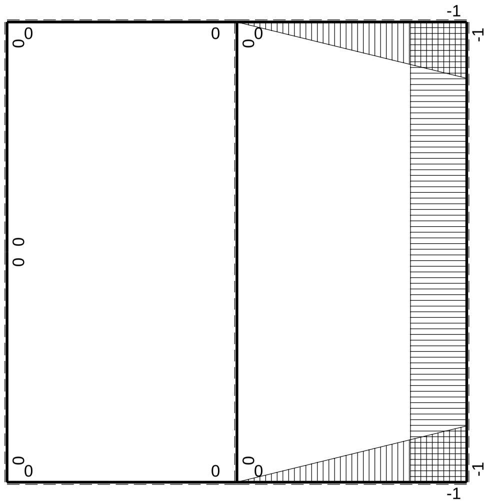
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	$-1/2Fb+1/2Fx$	0	0	0	0	0+0	0
BA b	0	$1/2Fx$	0	0	0	0		
CD b	0	$3/2Fb-3/2Fx$	0	0	0	0	0+0	0
DC b	0	$-3/2Fx$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	$-x/b$	$-3/2Fb+Fx-1/2qx^2$	$-Fb/EJ$	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	$2Xb/EJ$
GF 2b	1	0	0	0	0	1		
CB 2b	0	$-3/2Fb$	0	0	0	0	0+0	0
BC 2b	0	$3/2Fb$	0	0	0	0		
totali							$25/24Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$-25/64Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

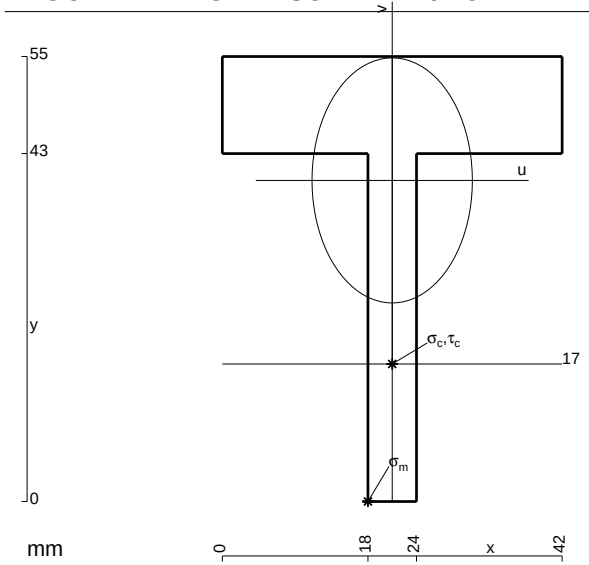
$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.194



A = 762. mm^2
J_u = 174852. mm^4
J_v = 74862. mm^4
y_g = 39.69 mm
T_y = -1725. N
M_x = 1052250. Nmm
x_m = 18. mm
u_m = -3. mm
v_m = -39.69 mm
$\sigma_m = -Mv/J_u$ = 238.8 N/mm^2
x_c = 21. mm
y_c = 17. mm
v_c = -22.69 mm
$\sigma_c = -Mv/J_u$ = 136.5 N/mm^2
τ_c = 5.231 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.8 N/mm^2
S^* = 3181. mm^3

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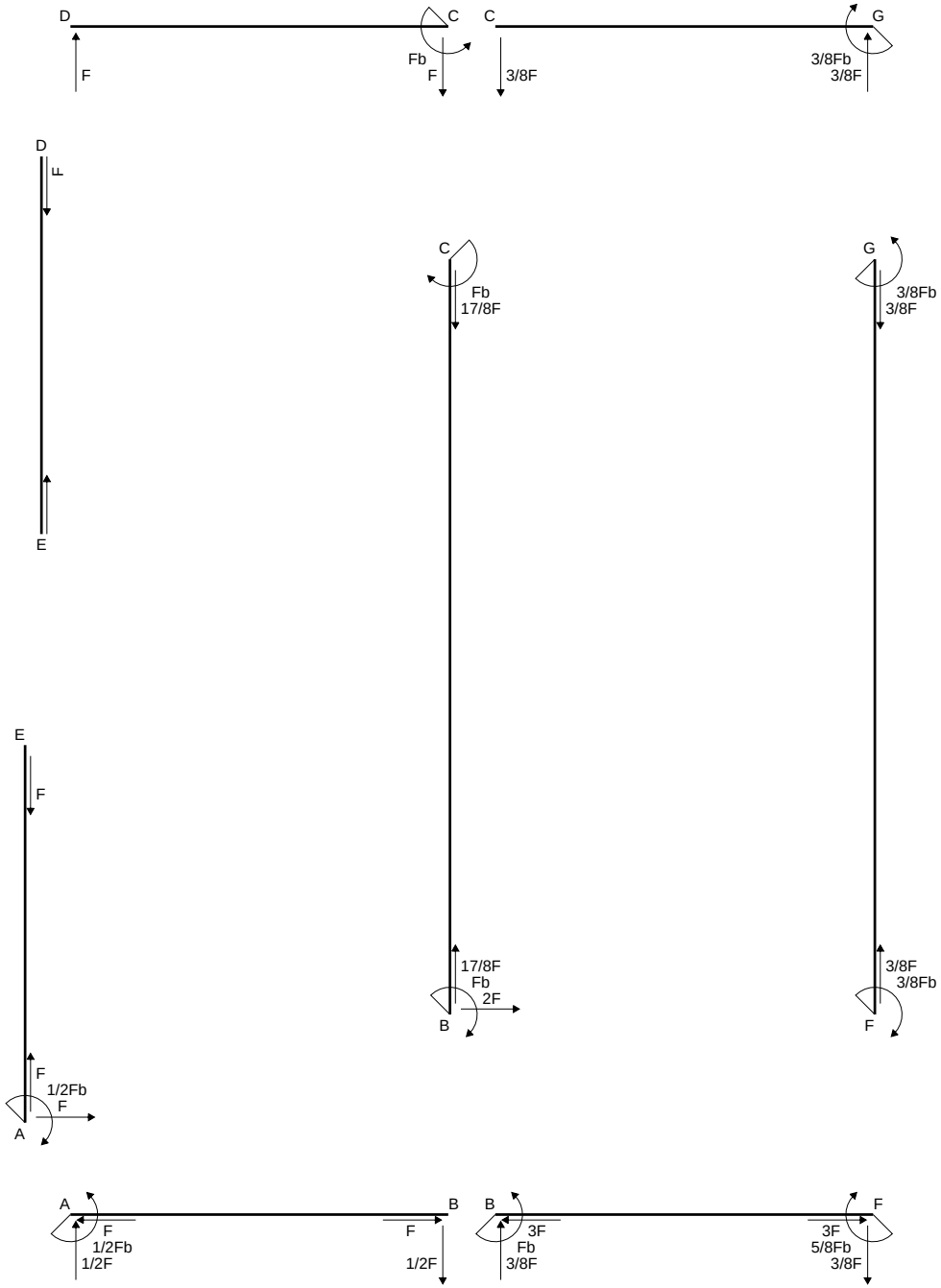
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C
C
G
F
Fb
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3/8Fb
3/8F
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Fb
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G
3/8Fb
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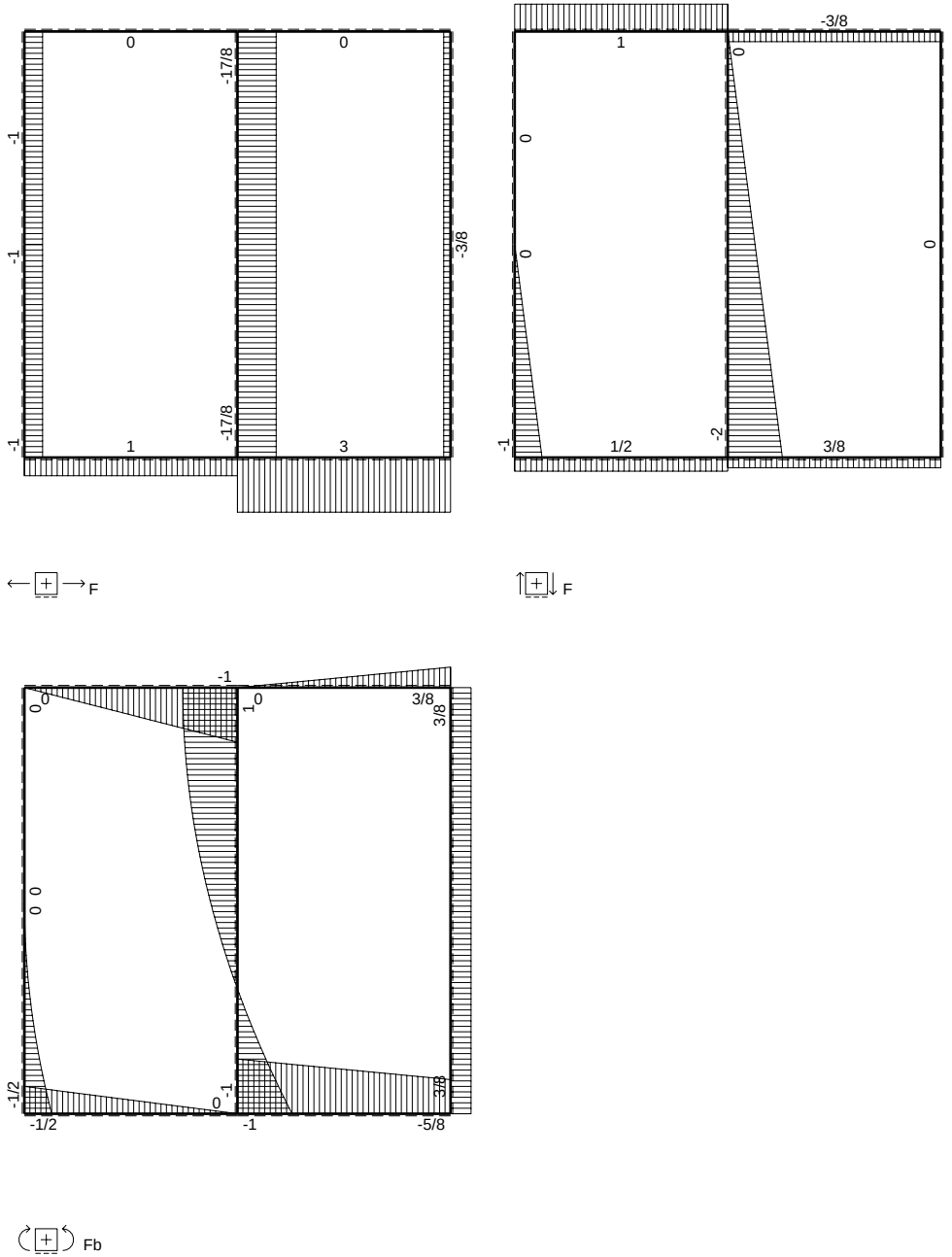
Fb
3/8F
3F
5/8Fb
3/8F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1/2
-2
3/8
F
-1
0
0
1
0
3/8
3/8
0 0
-1/2
0
-1
3/8
-1/2
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	-1/2Fb+1/2Fx	0	0	0	0	0+0	0
BA b	0	1/2Fx	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	$-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$1/2Fb-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	FxEJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

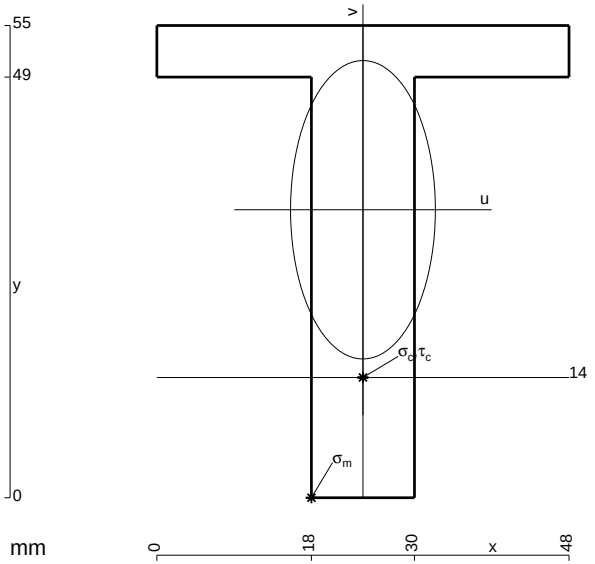
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.195



A = 876. mm^2
J_u = 264708. mm^4
J_v = 62352. mm^4
y_g = 33.54 mm
N = -4930. N
T_y = -4640. N
M_x = -1531200. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -33.54 mm
$\sigma_m = N/A - Mv/J_u$ = -199.6 N/mm^2
x_c = 24. mm
y_c = 14. mm
v_c = -19.54 mm
$\sigma_c = N/A - Mv/J_u$ = -118.7 N/mm^2
τ_c = 6.513 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 119.2 N/mm^2
S^* = 4459. mm^3

ESΣ8.195

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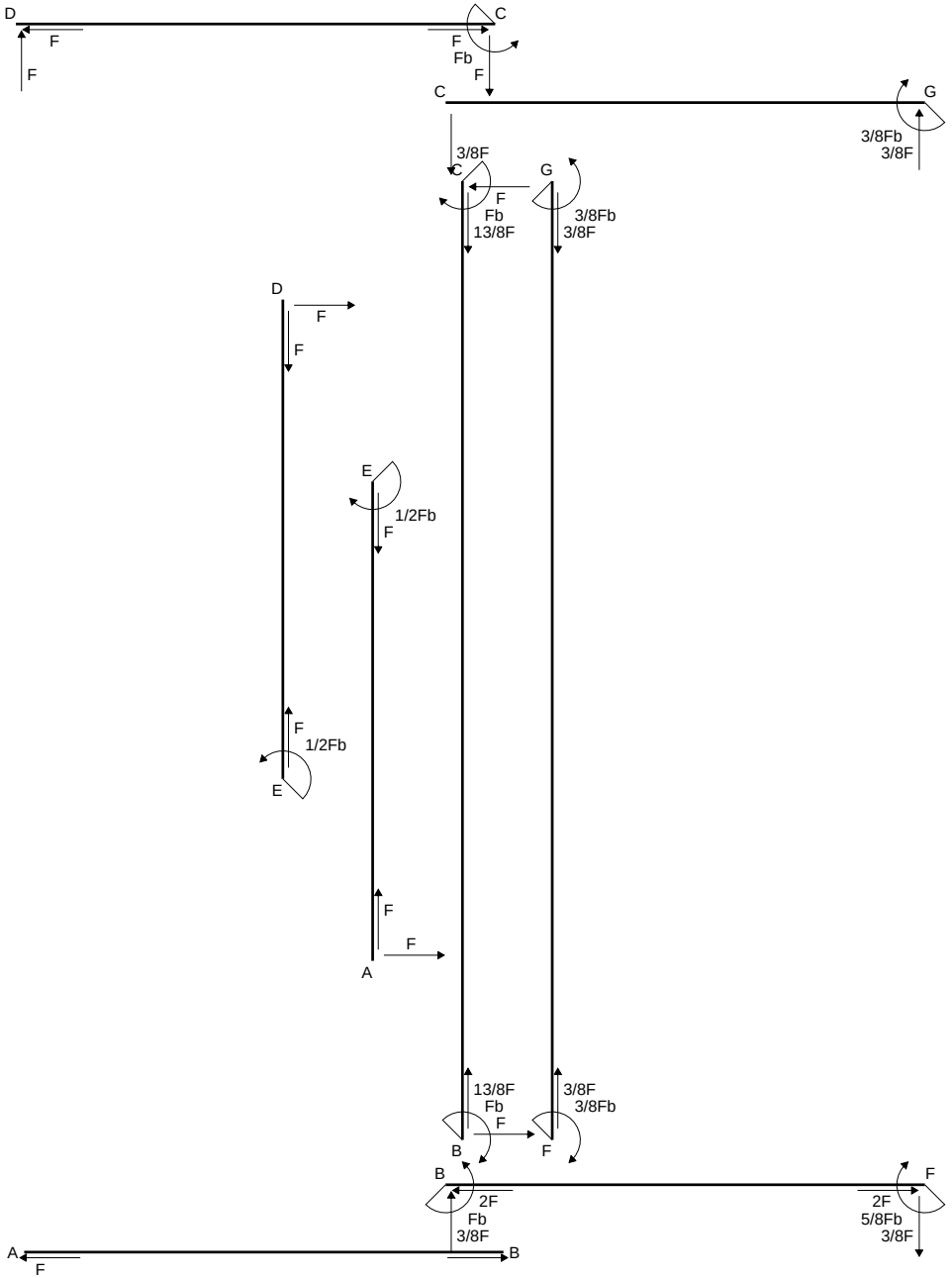
D
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3/8Fb
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G
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Fb
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3/8Fb
3/8F
D
F
F
E
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F
F
1/2Fb
E
F
F
A
13/8F
Fb
F
3/8F
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F
B
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F
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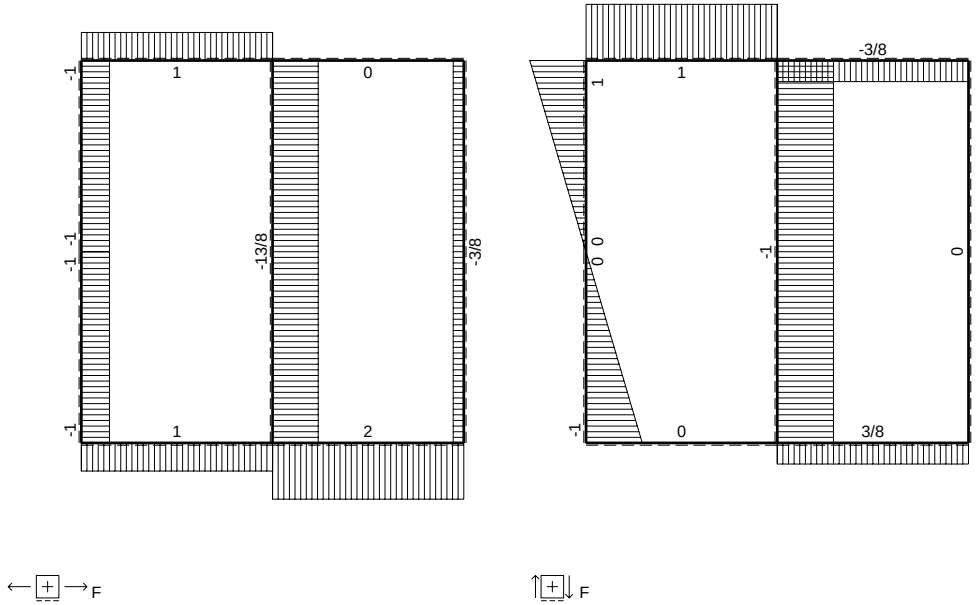
Fb
3/8F
2F
5/8Fb
3/8F
A
B
F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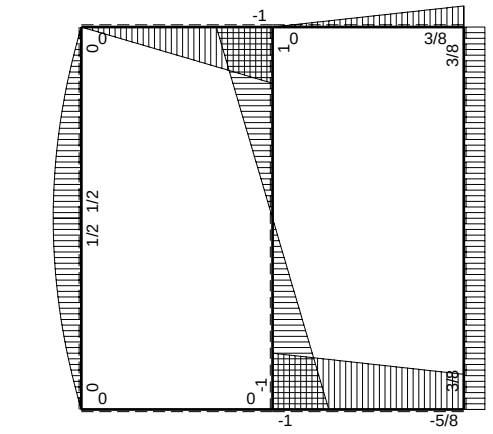
-1
0
3/8
F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1
0
3/8
3/8
1/2
1/2
0
0
-1
3/8
-1
-5/8
Fb

Schema di calcolo iperstatico

M_0 flessione da carichi assegnati

M_x flessione da iperstatica X=1

Sviluppi di calcolo iperstatica

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_0(x)$	θ	M_xM_0	$M_x\theta$	M_xM_x	$\int M_x(M_0/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	-Fb+Fx	0	0	0	0	0+0	0
DC b	0	Fx	0	0	0	0		
DE b	0	$Fx-1/2qx^2$	0	0	0	0	0+0	0
ED b	0	$-1/2Fb+1/2qx^2$	0	0	0	0		
EA b	0	$1/2Fb-1/2qx^2$	0	0	0	0	0+0	0
AE b	0	$-Fx+1/2qx^2$	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	(1/2+1/2)Fb^2/EJ	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	Fb-Fx	0	0	0	0	0+0	0
BC 2b	0	Fb-Fx	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

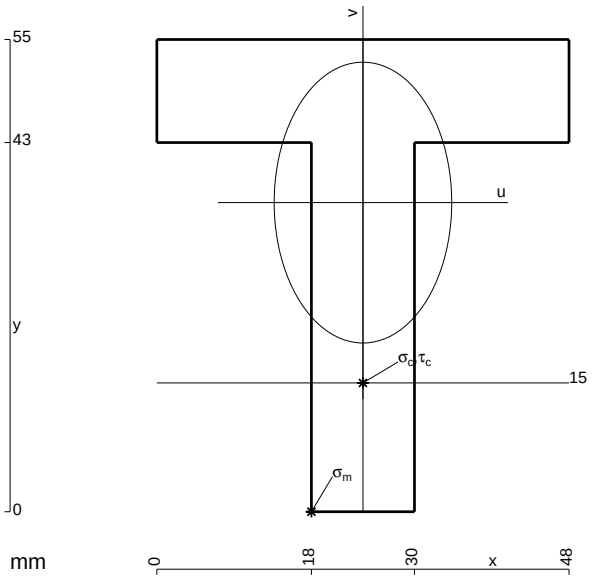
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b)\ Fb\ 1/EJ\ + (1/2\ b)\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.196



A = 1092. mm^2
J_u = 292252. mm^4
J_v = 116784. mm^4
y_g = 36.01 mm
N = -3884. N
T_y = -2390. N
M_x = -1673000. Nmm
x_m = 18. mm
u_m = -6. mm
v_m = -36.01 mm
$\sigma_m = N/A - Mv/J_u$ = -209.7 N/mm^2
x_c = 24. mm
y_c = 15. mm
v_c = -21.01 mm
$\sigma_c = N/A - Mv/J_u$ = -123.8 N/mm^2
τ_c = 3.497 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 124. N/mm^2
S^* = 5131. mm^3

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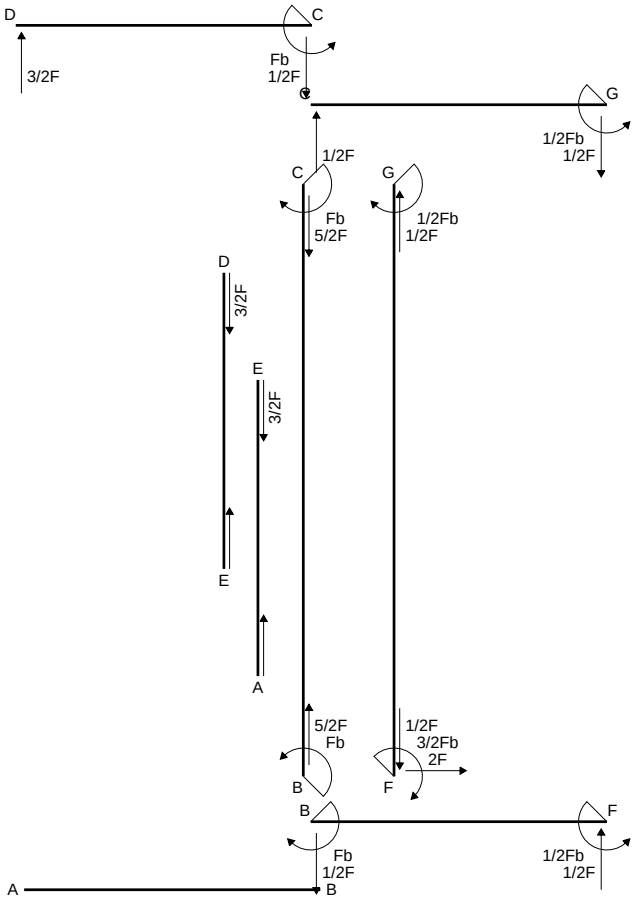
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C
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1/2F
G
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C
G
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5/2F
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A
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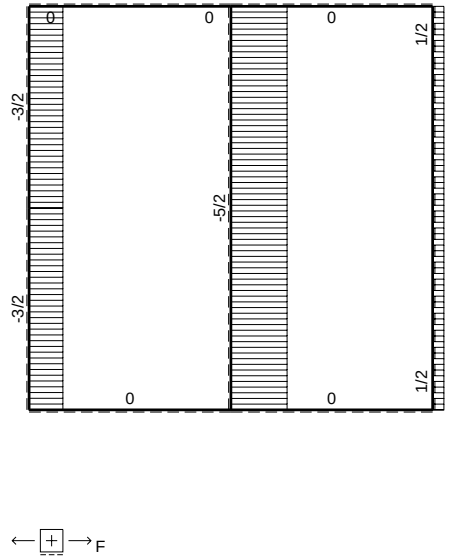
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1/2
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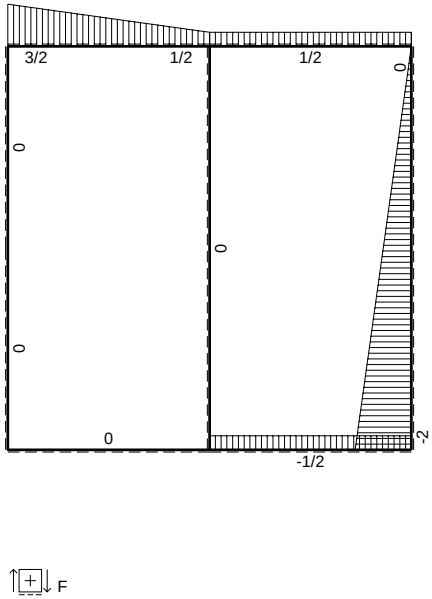
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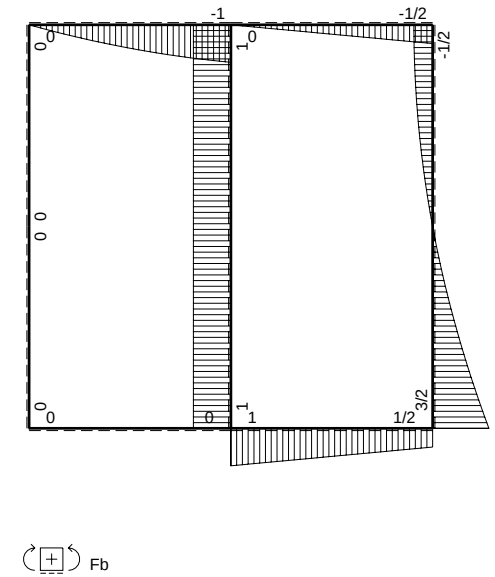
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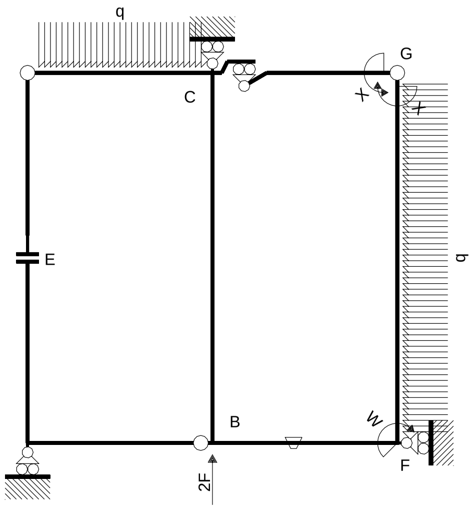
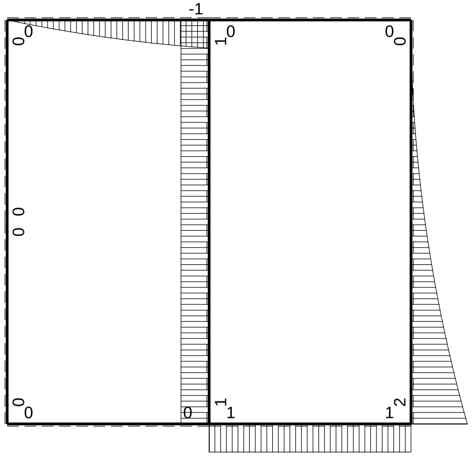
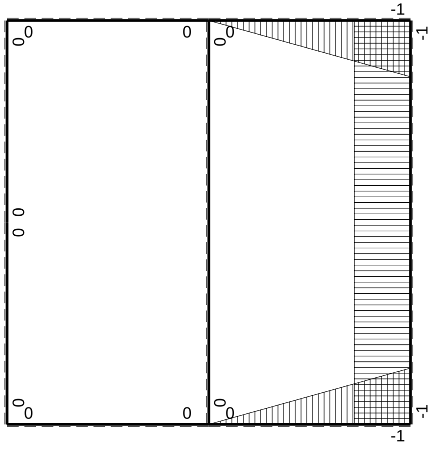
3/2

Fb





M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

→	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	$-x/b$	Fb	$-Fb/EJ$	$-Fx$	Fx/EJ	x^2/b^2	$(-1/2+1/2)Fb^2/EJ$	$1/3Xb/EJ$
FB b	$1-x/b$	$-Fb$	Fb/EJ	$-Fb+Fx$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	$-1+x/b$	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	$1/3Xb/EJ$
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	$2Fb-2Fx+1/2qx^2$	0	$-2Fb+2Fx-1/2Fx^2/b$	0	1	$(-4/3+0)Fb^2/EJ$	$2Xb/EJ$
GF 2b	1	$-1/2qx^2$	0	$-1/2Fx^2/b$	0	1		
CB 2b	0	Fb	0	0	0	0	0+0	0
BC 2b	0	$-Fb$	0	0	0	0		
totali							$-4/3Fb^2/EJ$	$8/3Xb/EJ$
iperstatica $X=W_{GC}$							$1/2Fb$	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{Xo}_{BF} = \int_o^b (-\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [-1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (-1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FB} = \int_o^b (-1 + x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [-\ x +1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

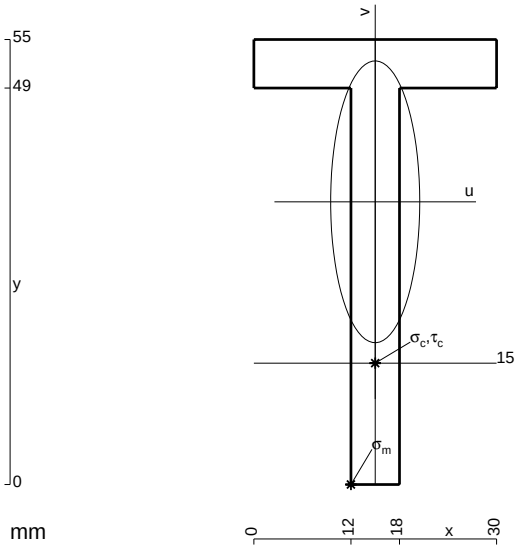
$= (-\ b +1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ = 0$

$L^{Xo}_{FG} = \int_o^{2b} (-2 +2\ x/b -1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-2\ x + x^2/b -1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4\ b +4\ b -4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$

$L^{Xo}_{GF} = \int_o^{2b} (-1/2\ x^2/b^2\ )\ Fb\ 1/EJ\ dx = [-1/6\ x^3/b^2\ ]_o^{2b}\ Fb\ 1/EJ$

$= (-4/3\ b\ )\ Fb\ 1/EJ\ = -4/3\ Fb^2/EJ$



A = 474. mm^2
J_u = 143796. mm^4
J_v = 14382. mm^4
y_g = 34.94 mm
T_y = 1950. N
M_x = -901875. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -34.94 mm
$\sigma_m = -Mv/J_u$ = -219.2 N/mm^2
x_c = 15. mm
y_c = 15. mm
v_c = -19.94 mm
$\sigma_c = -Mv/J_u$ = -125.1 N/mm^2
τ_c = 5.582 N/mm^2
$\sigma_0 = \sqrt{\sigma^2+3\tau^2}$ = 125.5 N/mm^2
S^* = 2470. mm^3

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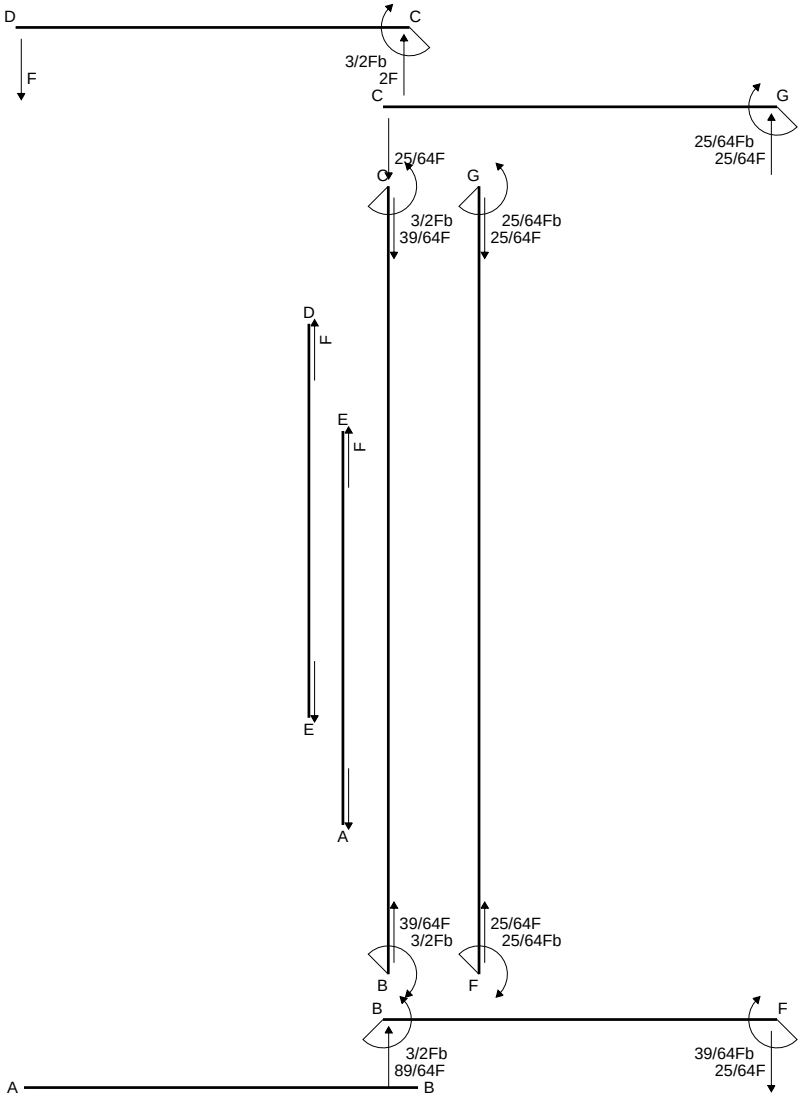
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2F
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C
G
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25/64F
C
25/64F
G
3/2Fb
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25/64Fb
25/64F
D
F
E
F
E
A
39/64F
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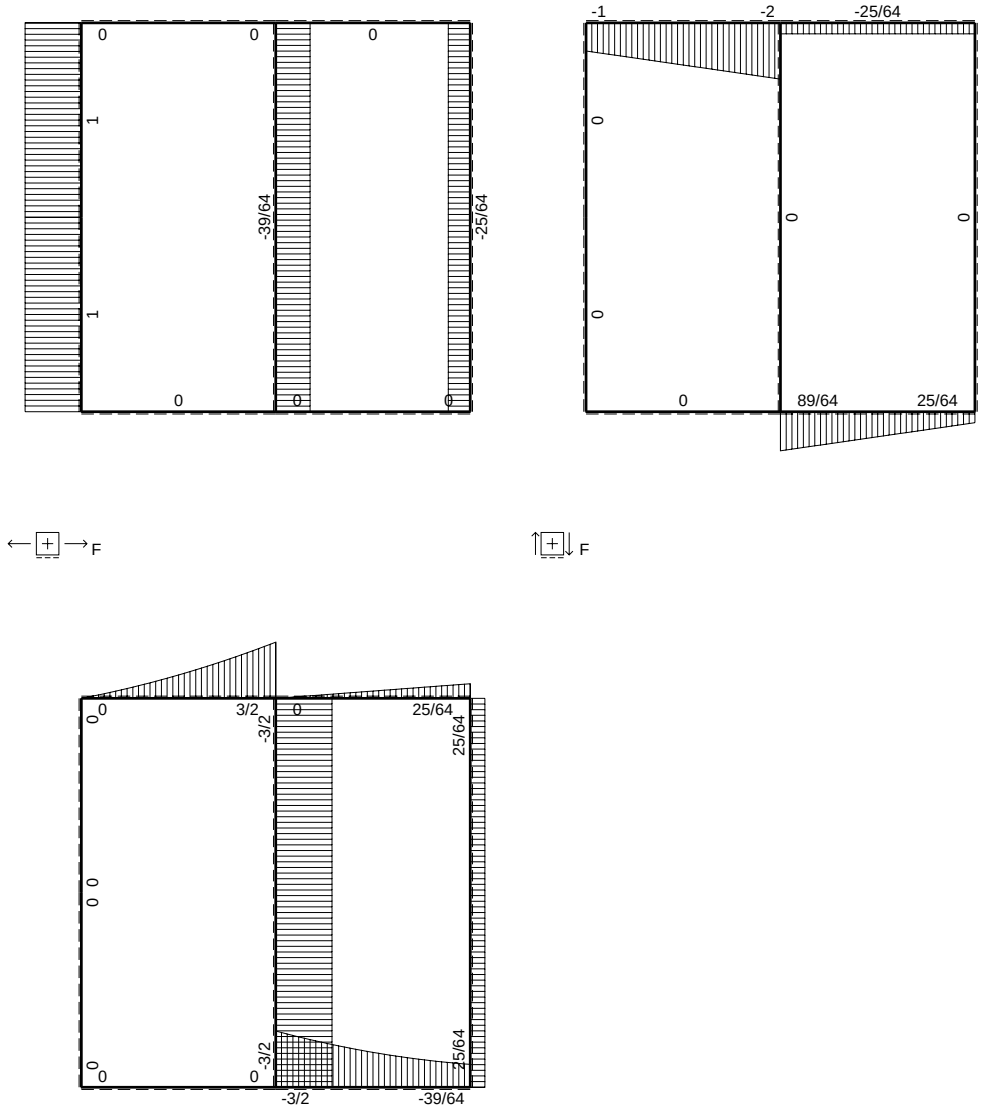
89/64F
39/64Fb
25/64F
A
B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-25/64
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F
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-2
-25/64
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F
0
0
3/2
-3/2
0
25/64
25/64
0
0
0
0
-3/2
-3/2
-39/64
Fb

Schema di calcolo iperstatico

M_x flessione da iperstatica X=1

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M_0 flessione da carichi assegnati

Quadro contributi PLV per iperstatica $X=W_{GC}$

	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$3/2Fb-2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$-Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	$-3/2Fb+Fx-1/2qx^2$	-Fb/EJ	$3/2Fx-Fx^2/b+1/2qx^3/b$	Fx/EJ	x^2/b^2	$(13/24+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	$Fb+1/2qx^2$	Fb/EJ	$Fb-Fx+1/2Fx^2/b-1/2qx^3/b$	$Fb/EJ-Fx/EJ$	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	-3/2Fb	0	0	0	0	0+0	0
BC 2b	0	3/2Fb	0	0	0	0		
	totali						$25/24Fb^2/EJ$	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-25/64Fb	

Sviluppi di calcolo iperstatica

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14.11.23

$L_{BF}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FB}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{GC}^{XX} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L_{CG}^{XX} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L_{FG}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{GF}^{XX} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L_{BF}^{Xo} = \int_o^b (3/2\ x/b - x^2/b^2 +1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx$

$= [3/4\ x^2/b -1/3\ x^3/b^2 +1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

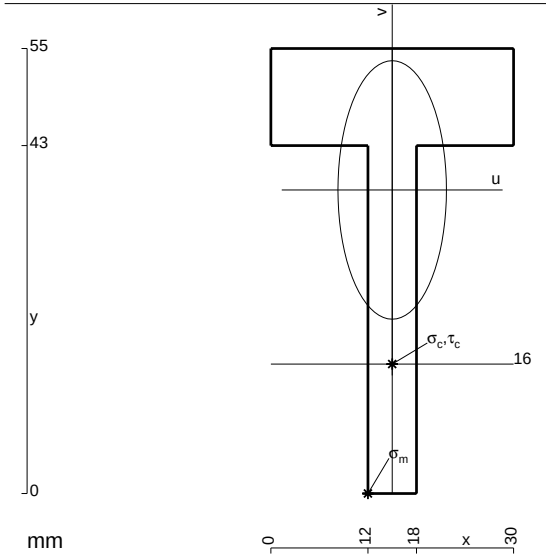
$= (3/4\ b -1/3\ b +1/8\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

$L_{FB}^{Xo} = \int_o^b (1 - x/b +1/2\ x^2/b^2 -1/2\ x^3/b^3\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx$

$= [\ x -1/2\ x^2/b +1/6\ x^3/b^2 -1/8\ x^4/b^3\ ]_o^b\ Fb\ 1/EJ\ + [- x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b +1/6\ b -1/8\ b\ )\ Fb\ 1/EJ\ + (- b +1/2\ b\ )\ \theta\ = 25/24\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.198



A = 618. mm^2
J_u = 157731. mm^4
J_v = 27774. mm^4
y_g = 37.52 mm
T_y = -1620. N
M_x = 959850. Nmm
x_m = 12. mm
u_m = -3. mm
v_m = -37.52 mm
$\sigma_m = -Mv/J_u$ = 228.3 N/mm^2
x_c = 15. mm
y_c = 16. mm
v_c = -21.52 mm
$\sigma_c = -Mv/J_u$ = 131. N/mm^2
τ_c = 4.851 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 131.2 N/mm^2
S^* = 2834. mm^3

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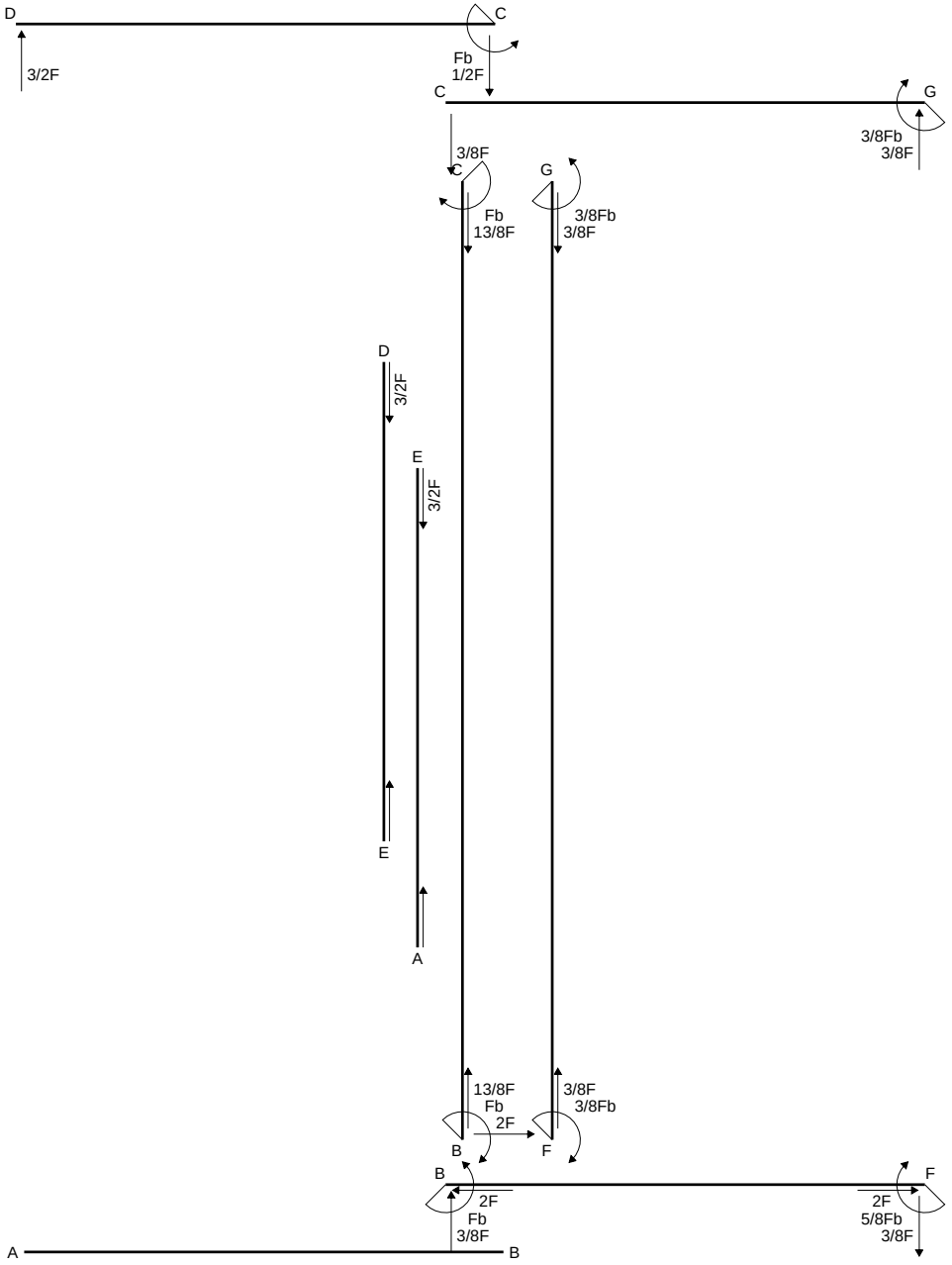
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G
Fb
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2F
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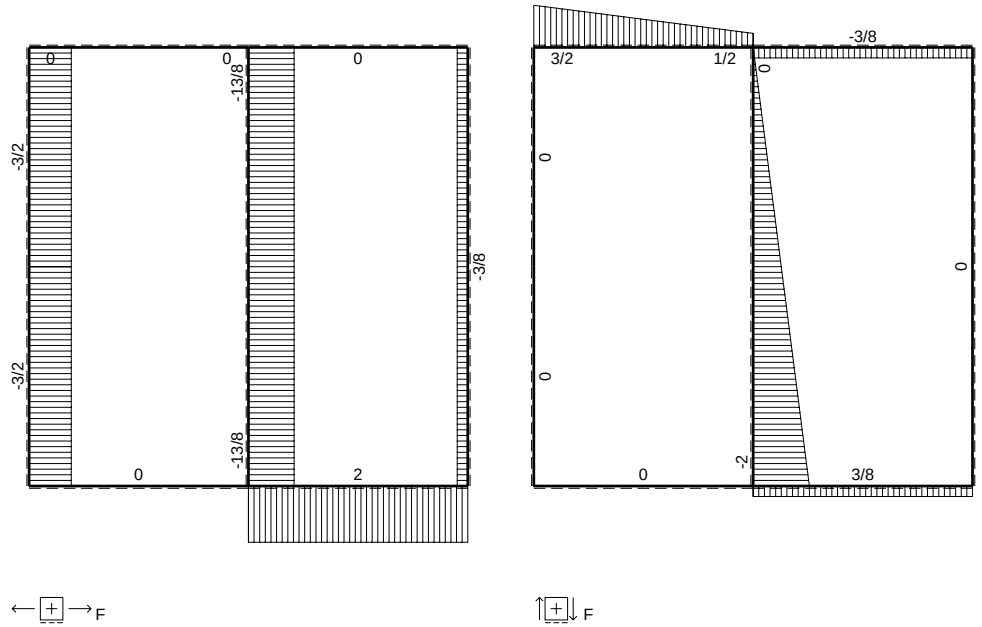
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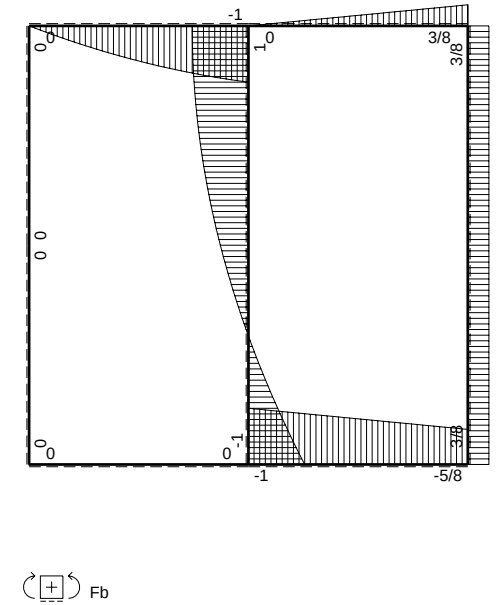
3/8
F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-1
3/8
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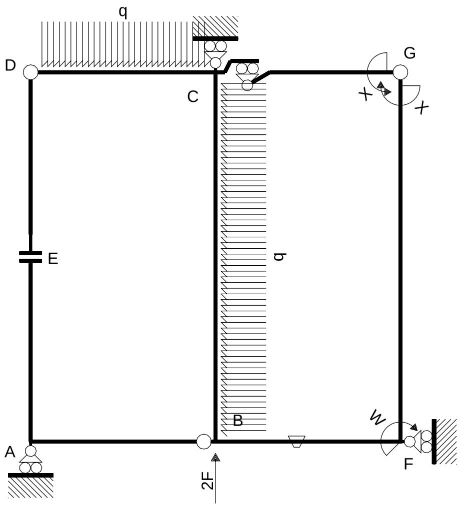
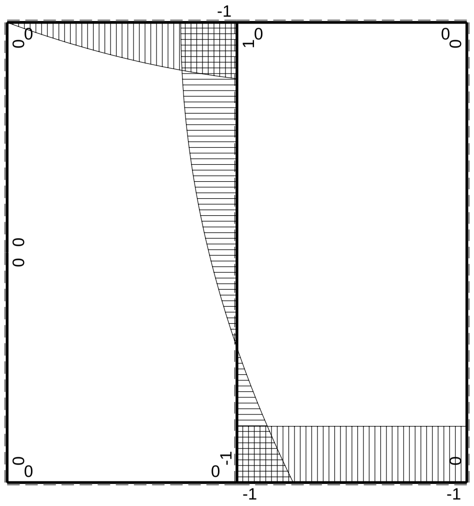
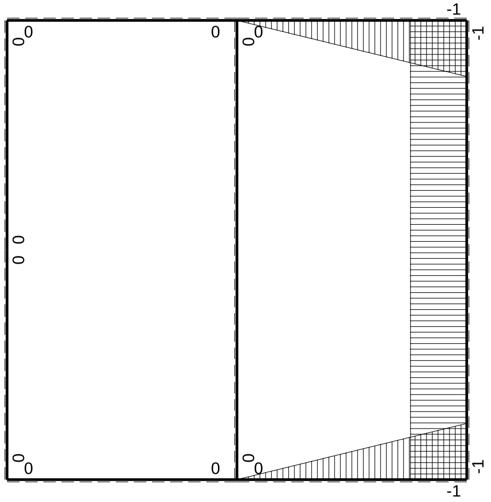
-5/8
Fb



Schema di calcolo iperstatico



M_o flessione da carichi assegnati



M_x flessione da iperstatica X=1

Quadro contributi PLV per iperstatica $X=W_{GC}$

↑	$M_x(x)$	$M_o(x)$	θ	M_xM_o	$M_x\theta$	M_xM_x	$\int M_x(M_o/EJ+\theta)dx$	$\int XM_xM_x/EJdx$
AB b	0	0	0	0	0	0	0+0	0
BA b	0	0	0	0	0	0		
CD b	0	$-Fb+1/2Fx+1/2qx^2$	0	0	0	0	0+0	0
DC b	0	$3/2Fx-1/2qx^2$	0	0	0	0		
DE b	0	0	0	0	0	0	0+0	0
ED b	0	0	0	0	0	0		
EA b	0	0	0	0	0	0	0+0	0
AE b	0	0	0	0	0	0		
BF b	-x/b	-Fb	-Fb/EJ	Fx	Fx/EJ	x^2/b^2	$(1/2+1/2)Fb^2/EJ$	1/3Xb/EJ
FB b	1-x/b	Fb	Fb/EJ	Fb-Fx	Fb/EJ-Fx/EJ	$1-2x/b+x^2/b^2$		
GC b	-1+x/b	0	0	0	0	$1-2x/b+x^2/b^2$	0+0	1/3Xb/EJ
CG b	x/b	0	0	0	0	x^2/b^2		
FG 2b	-1	0	0	0	0	1	0+0	2Xb/EJ
GF 2b	1	0	0	0	0	1		
CB 2b	0	$Fb-1/2qx^2$	0	0	0	0	0+0	0
BC 2b	0	$Fb-2Fx+1/2qx^2$	0	0	0	0		
	totali						Fb^2/EJ	8/3Xb/EJ
	iperstatica $X=W_{GC}$						-3/8Fb	

Sviluppi di calcolo iperstatica

$L^{XX}_{BF} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FB} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{GC} = \int_o^b (1 -2\ x/b + x^2/b^2)\ 1/EJ\ dx = [\ x - x^2/b +1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (\ b - b +1/3\ b\ )\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{CG} = \int_o^b (x^2/b^2)\ 1/EJ\ dx = [1/3\ x^3/b^2\ ]_o^b\ 1/EJ$

$= (1/3\ b)\ 1/EJ\ = 1/3\ b/EJ$

$L^{XX}_{FG} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

$L^{XX}_{GF} = \int_o^{2b} (1)\ 1/EJ\ dx = [\ x\ ]_o^{2b}\ 1/EJ$

$= (2\ b)\ 1/EJ\ = 2\ b/EJ$

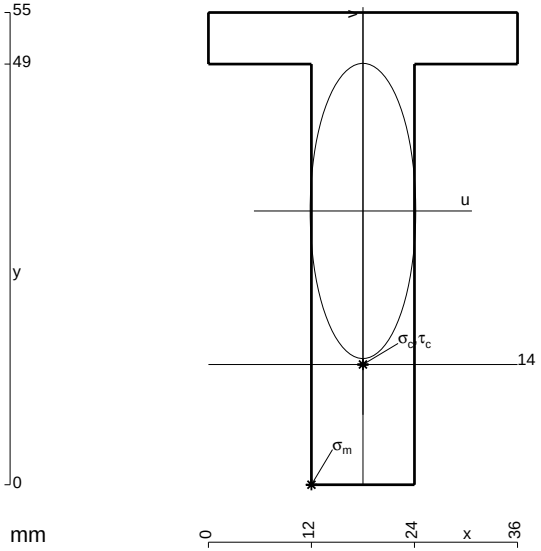
$L^{Xo}_{BF} = \int_o^b (\ x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (\ x/b\ )\ \theta\ dx = [1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [1/2\ x^2/b\ ]_o^b\ \theta$

$= (1/2\ b\ )\ Fb\ 1/EJ\ + (1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

$L^{Xo}_{FB} = \int_o^b (1 - x/b\ )\ Fb\ 1/EJ\ dx + \int_o^b (-1 + x/b\ )\ \theta\ dx = [\ x -1/2\ x^2/b\ ]_o^b\ Fb\ 1/EJ\ + [-\ x +1/2\ x^2/b\ ]_o^b\ \theta$

$= (\ b -1/2\ b\ )\ Fb\ 1/EJ\ + (-\ b +1/2\ b\ )\ \theta\ =\ Fb^2/EJ$

PROCEDIMENTO E RISULTATI Nome: ESΣ8.199



A = 804. mm^2
J_u = 237762. mm^4
J_v = 30384. mm^4
y_g = 31.89 mm
N = -3429. N
T_y = -4220. N
M_x = -1751300. Nmm
x_m = 12. mm
u_m = -6. mm
v_m = -31.89 mm
$\sigma_m = N/A - Mv/J_u$ = -239.1 N/mm^2
x_c = 18. mm
y_c = 14. mm
v_c = -17.89 mm
$\sigma_c = N/A - Mv/J_u$ = -136. N/mm^2
τ_c = 6.184 N/mm^2
$\sigma_o = \sqrt{\sigma^2+3\tau^2}$ = 136.4 N/mm^2
S^* = 4181. mm^3

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