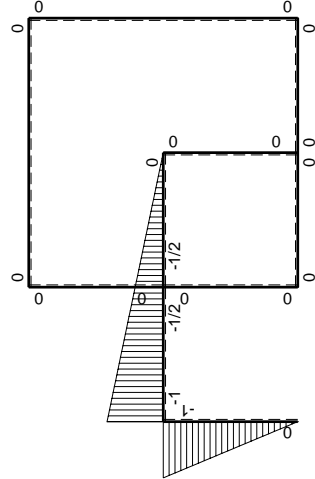


Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | $-Fb/EJ$ | $Fx-1/2Fx^2/b$           | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$         | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$             | 0        | $1/2Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $7/24Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $-7/24Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

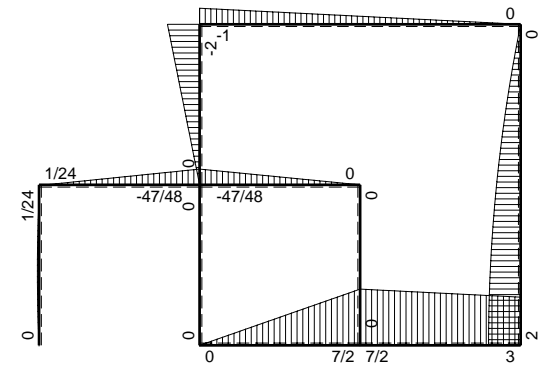
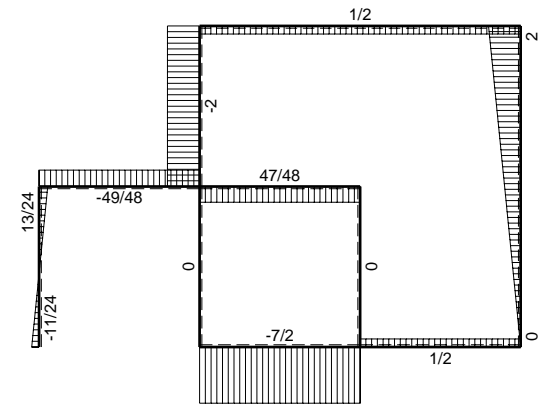
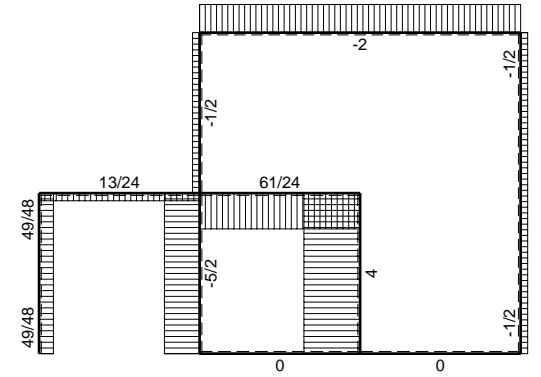
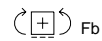
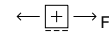
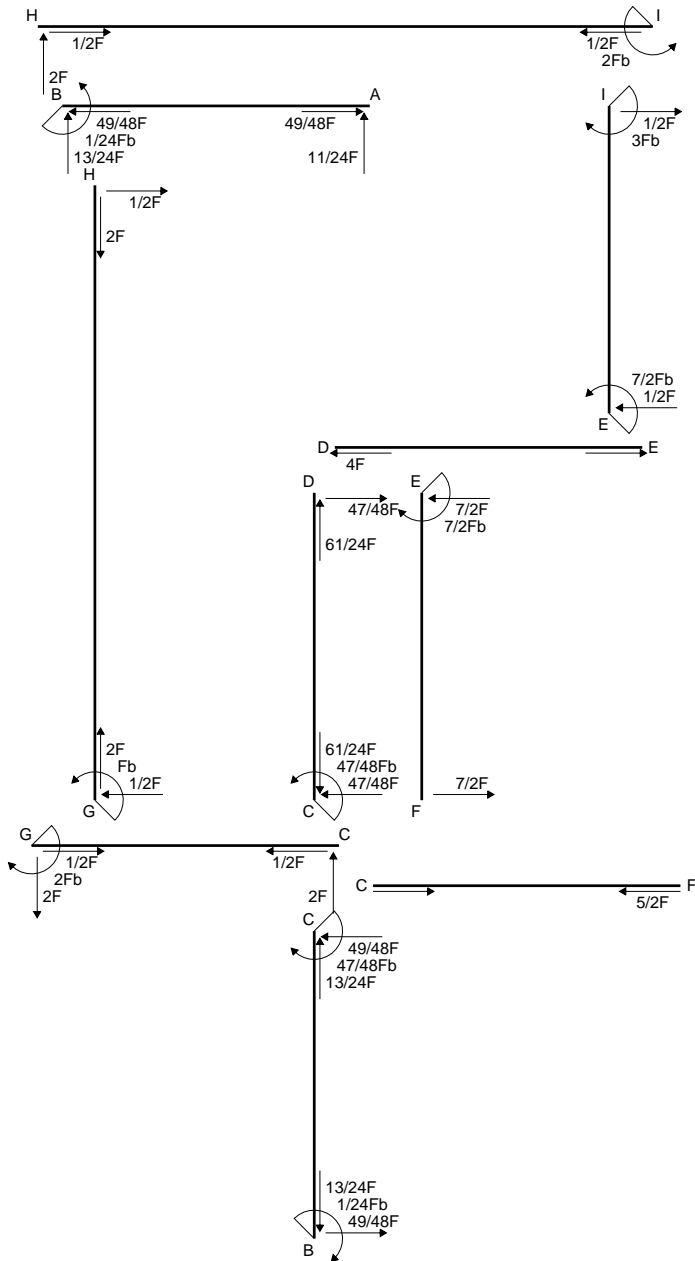
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

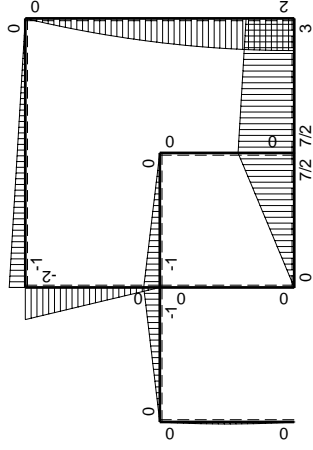
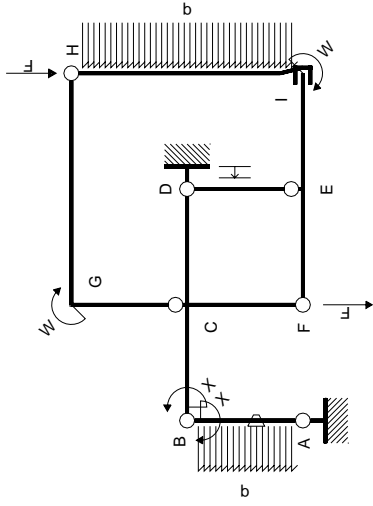
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

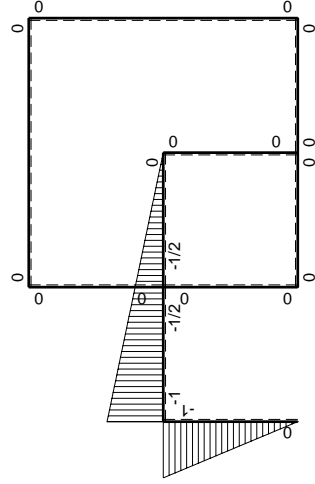
$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$         | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $Fx$             | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0             | 0                       |                             |                        |  |
| FC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| CF b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |  |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0             | 0                       |                             |                        |  |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0             | 0                       |                             |                        |  |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                        |  |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0             | 0                       |                             |                        |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |               |                         | $-Fb^2/EJ$                  |                        |  |
|       | totali                      |                  |          |                          |               |                         | $1/24Fb^2/EJ$               | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         | $-1/24Fb$                   |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

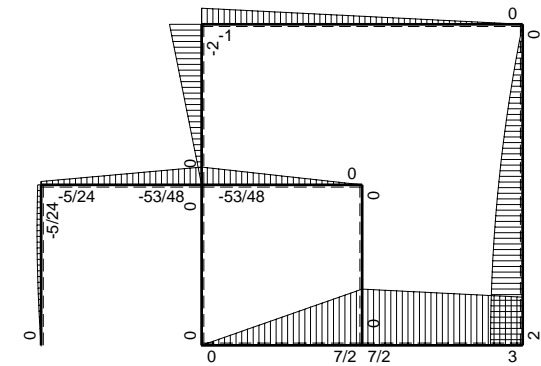
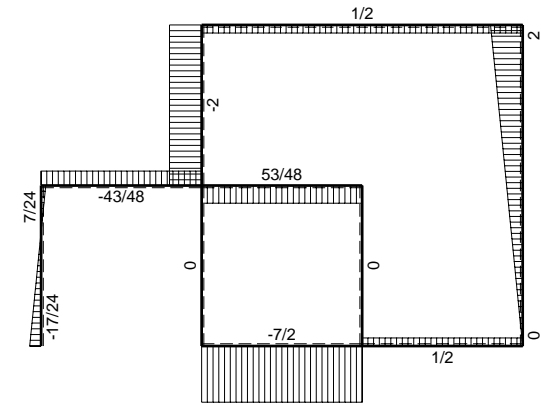
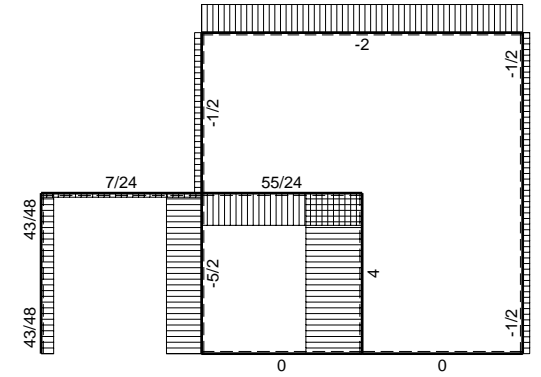
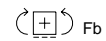
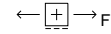
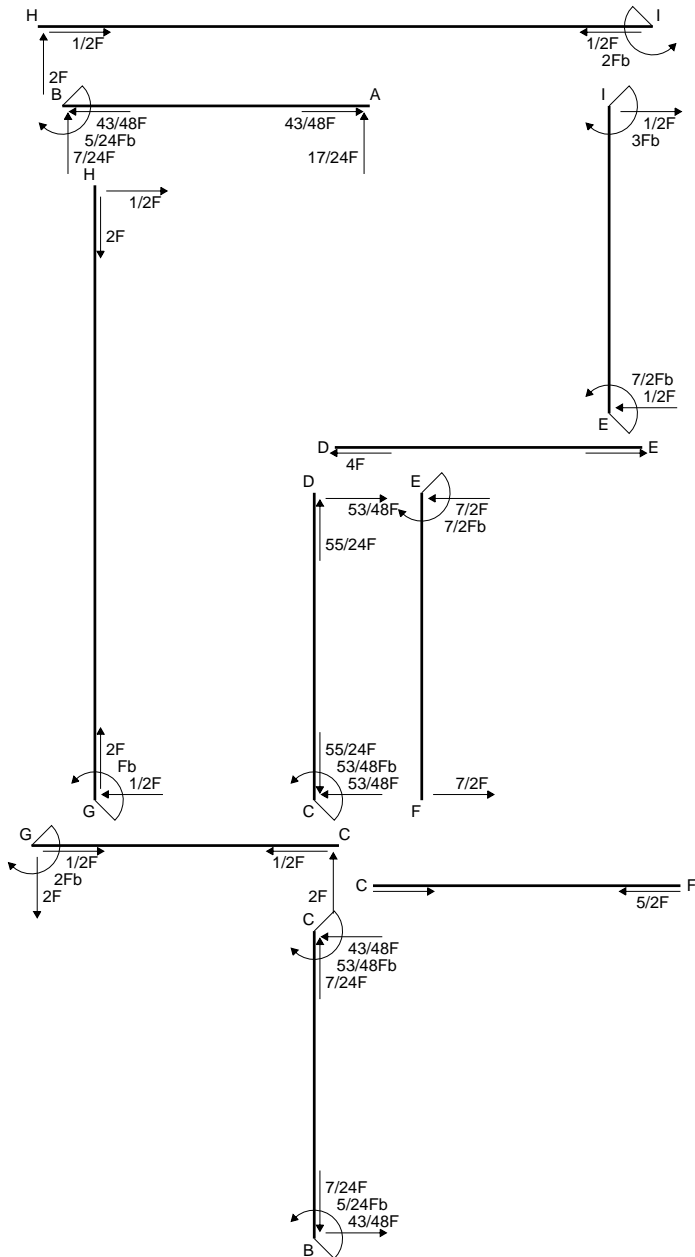
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

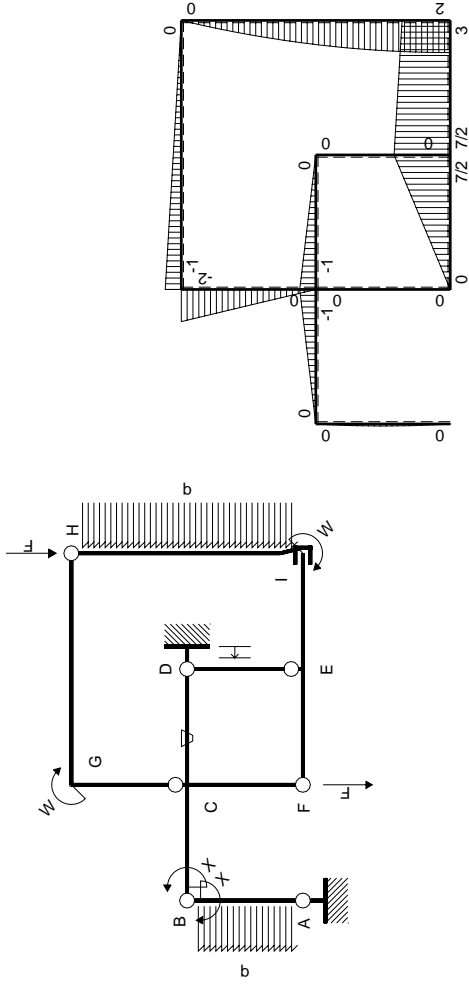
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

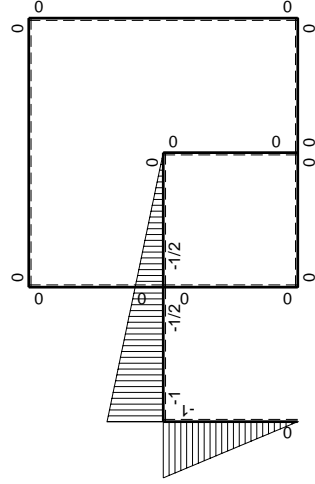






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$           | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$         | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$     | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $Fx$             | $Fb/EJ$  | $1/2Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                          |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                          |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                          |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0                   | 0                       |                             |                          |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                          |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                          |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                          |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         | $-Fb^2/EJ$                  |                          |
|       | totali                      |                  |          |                          |                     |                         | $-5/24Fb^2/EJ$              | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $5/24Fb$                    |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

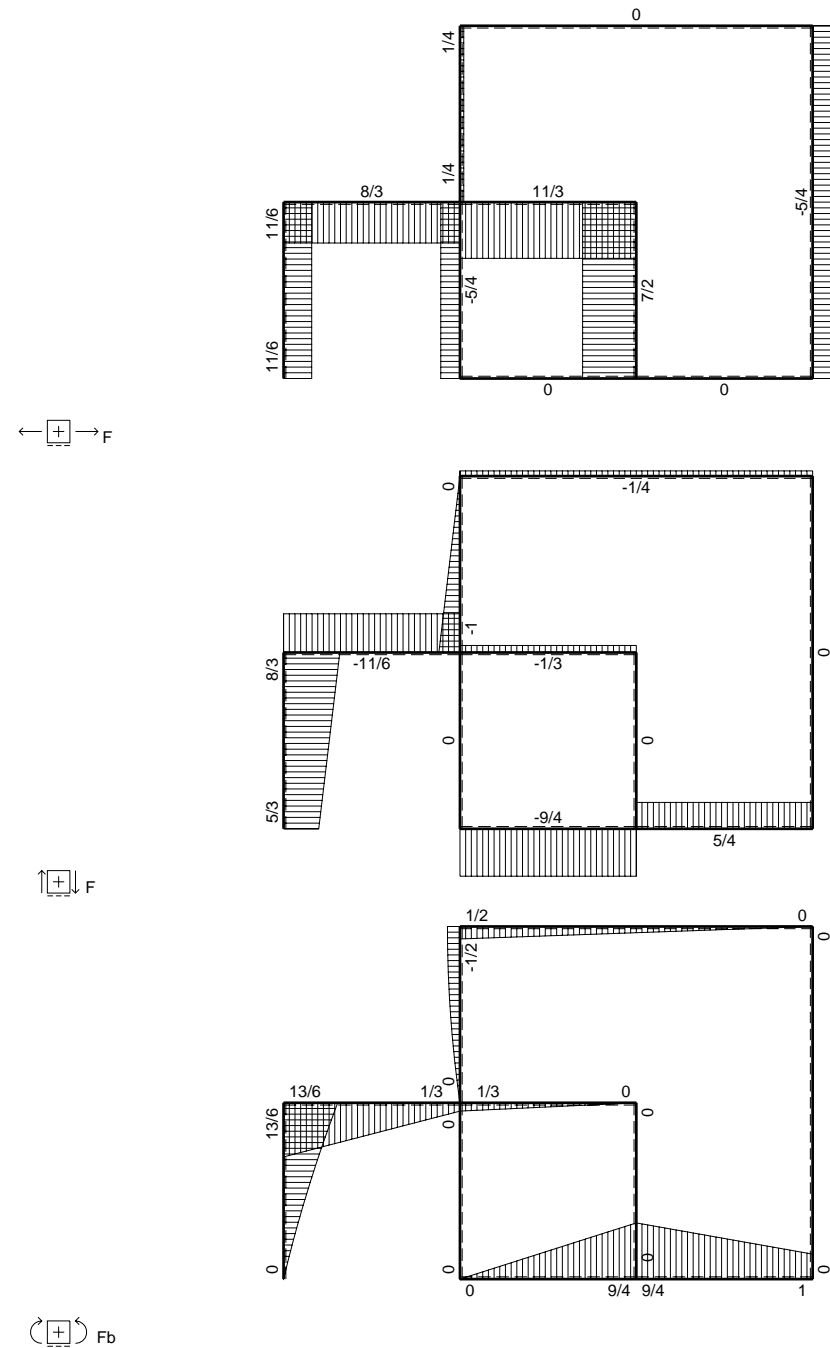
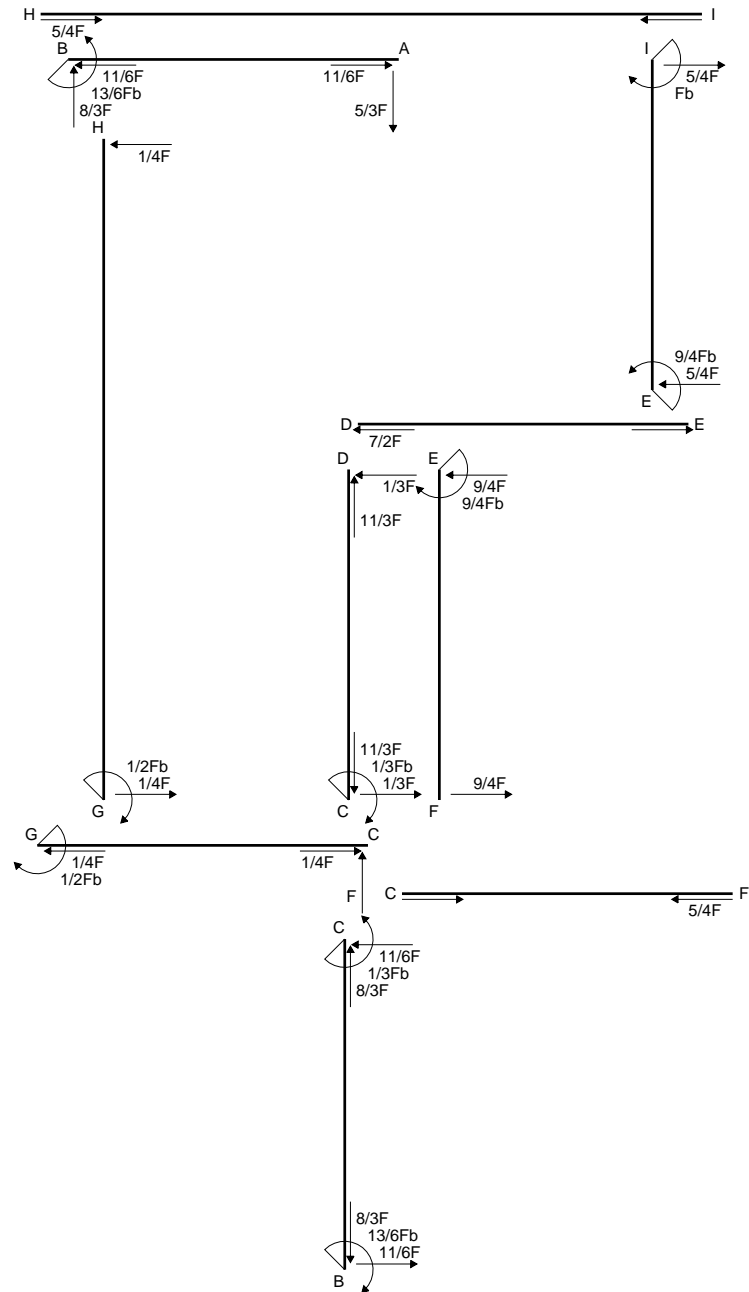
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

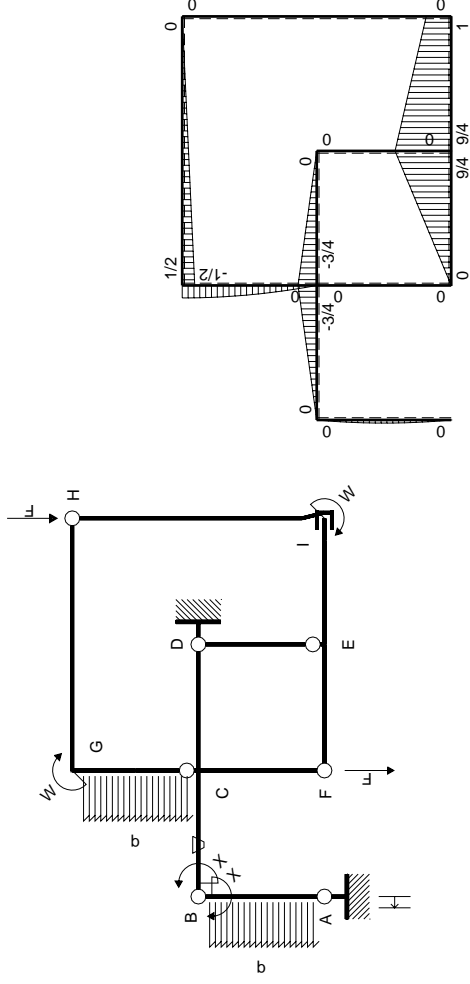
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

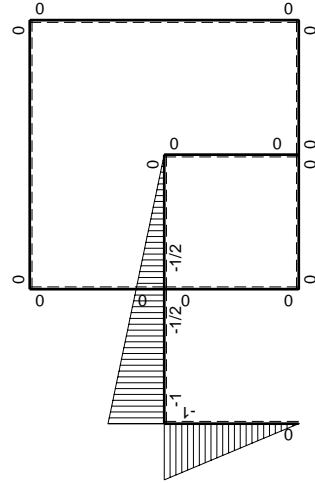
$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_X$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                          |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

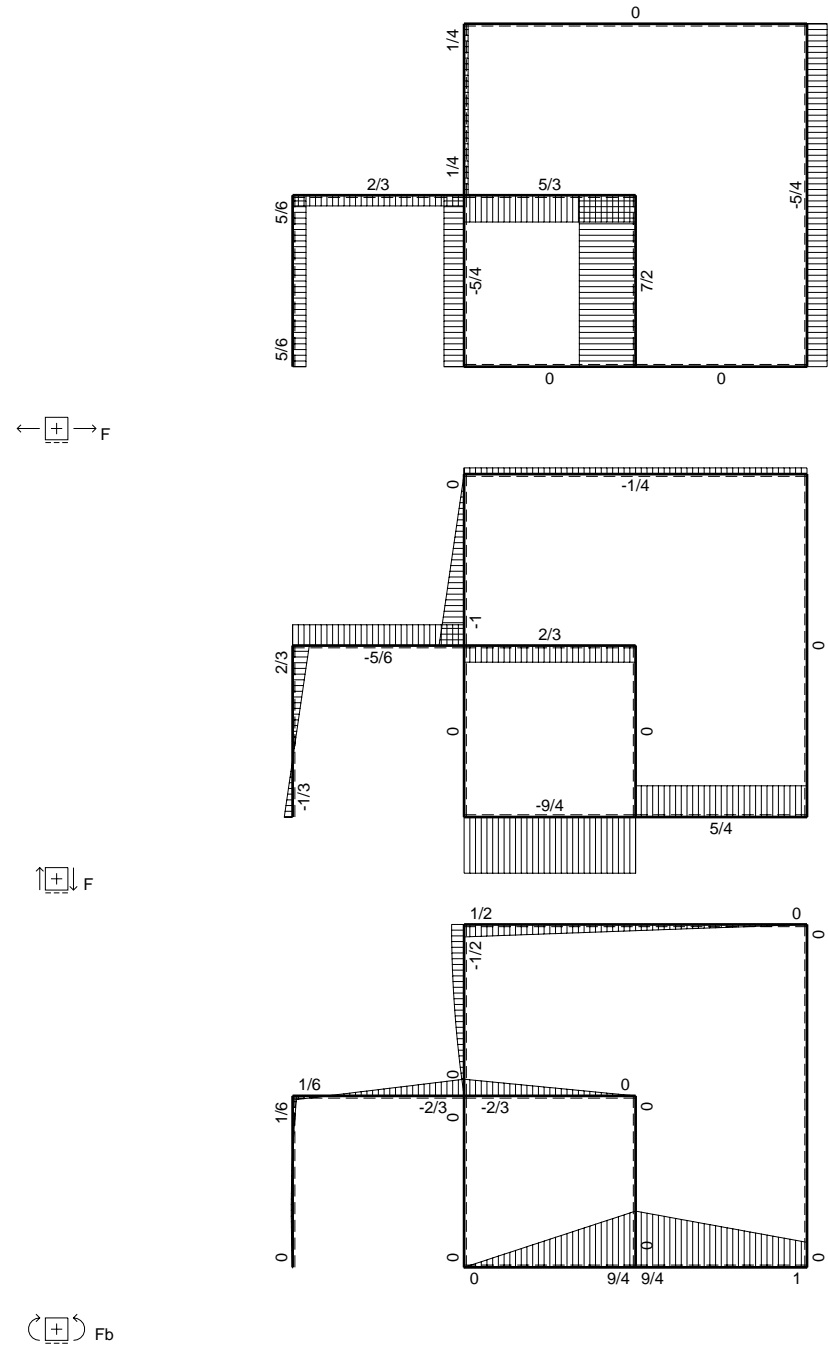
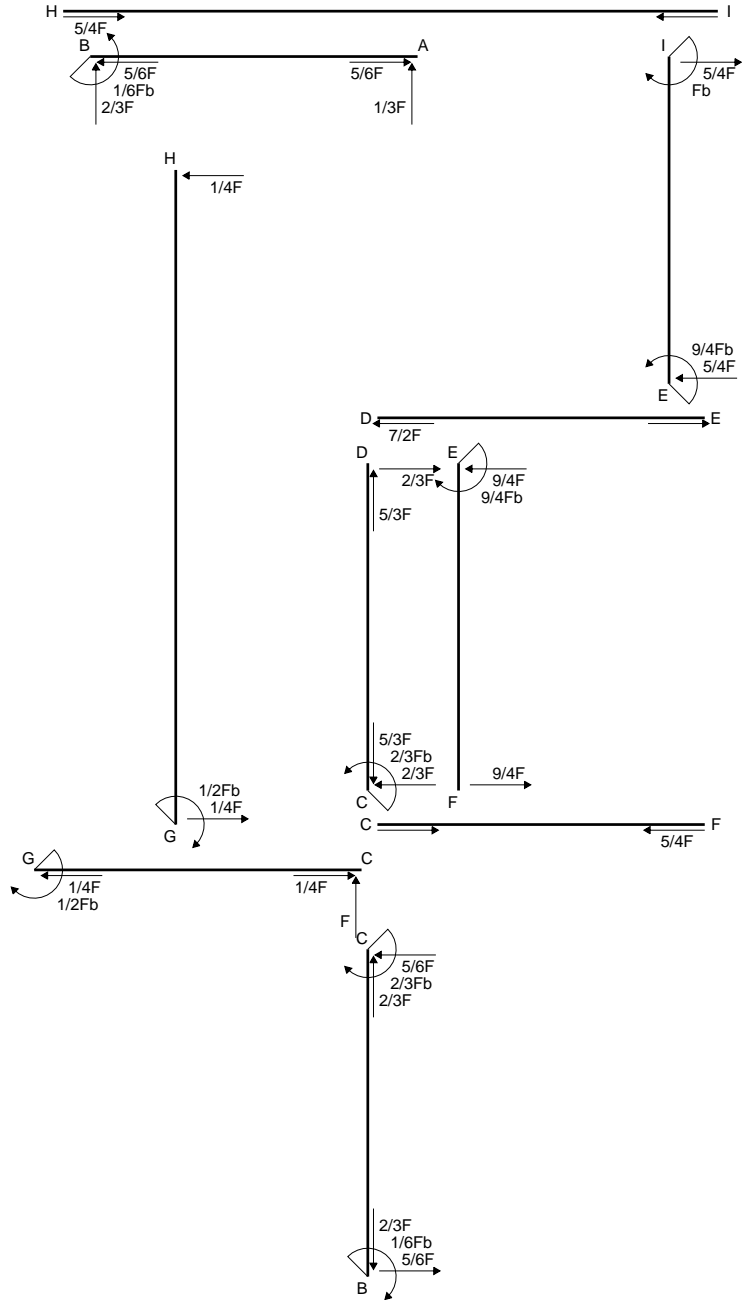
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

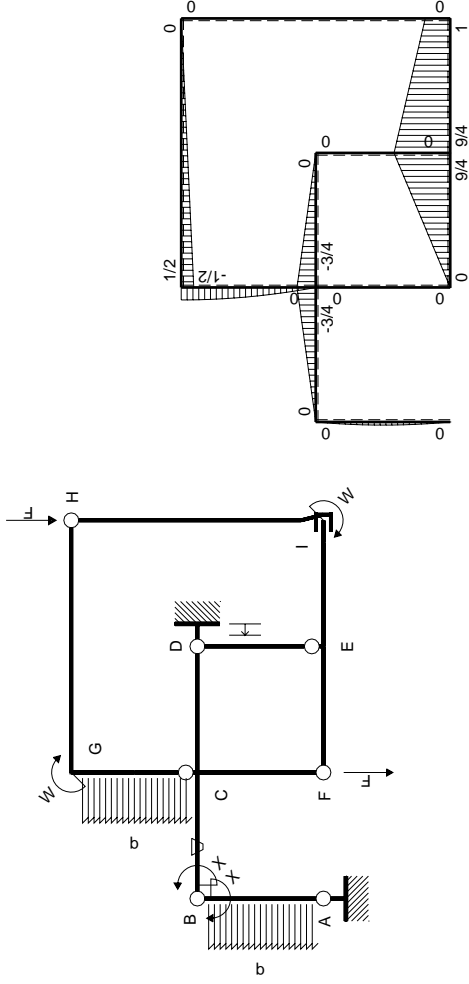
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

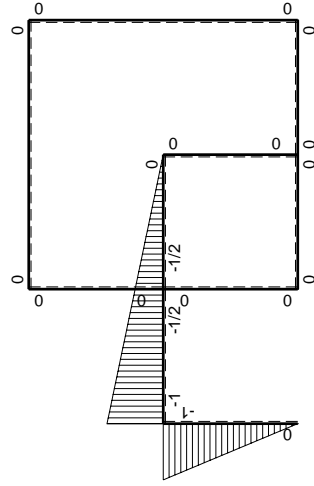






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

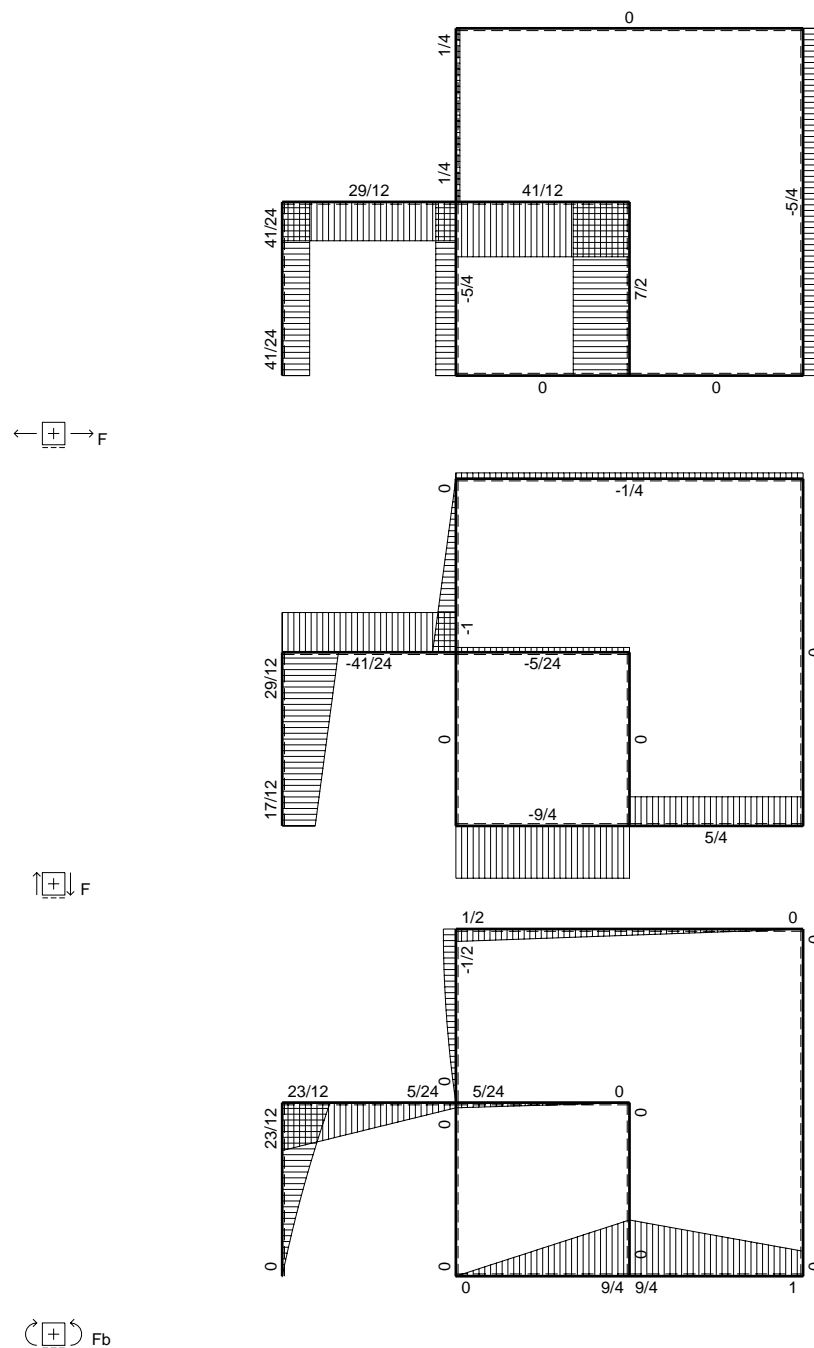
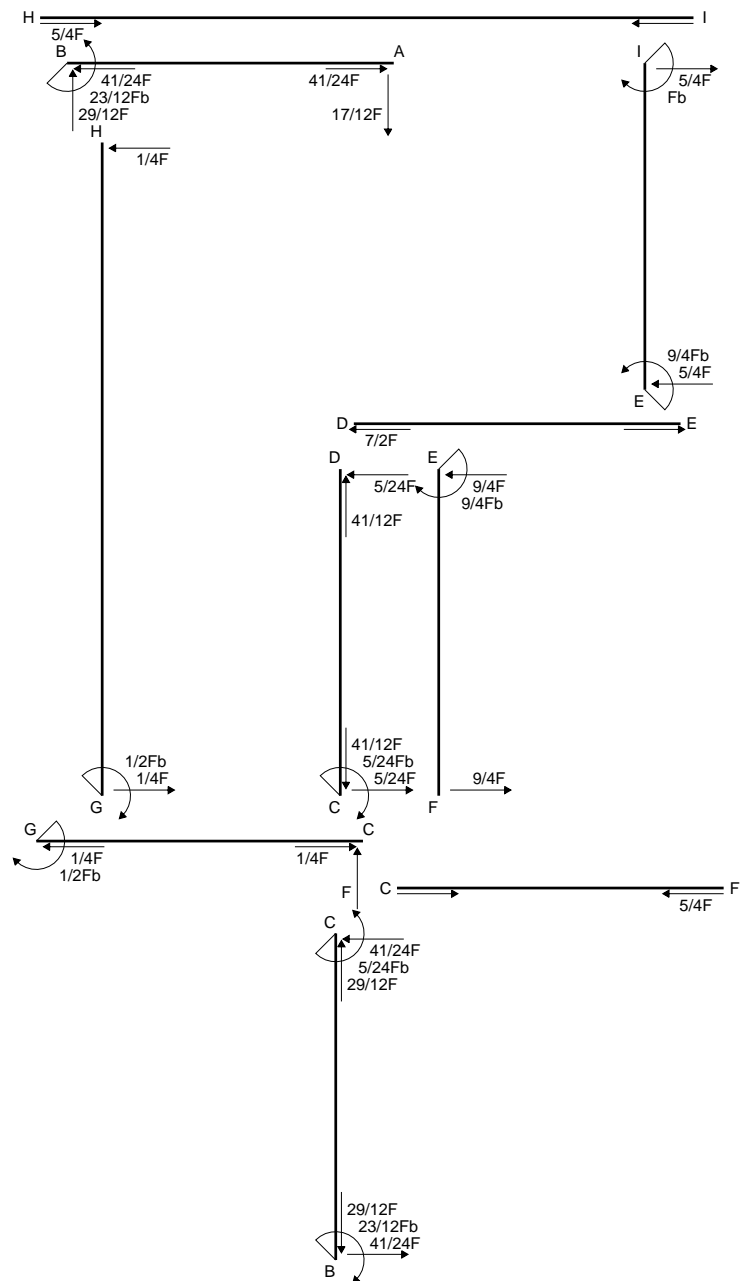
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

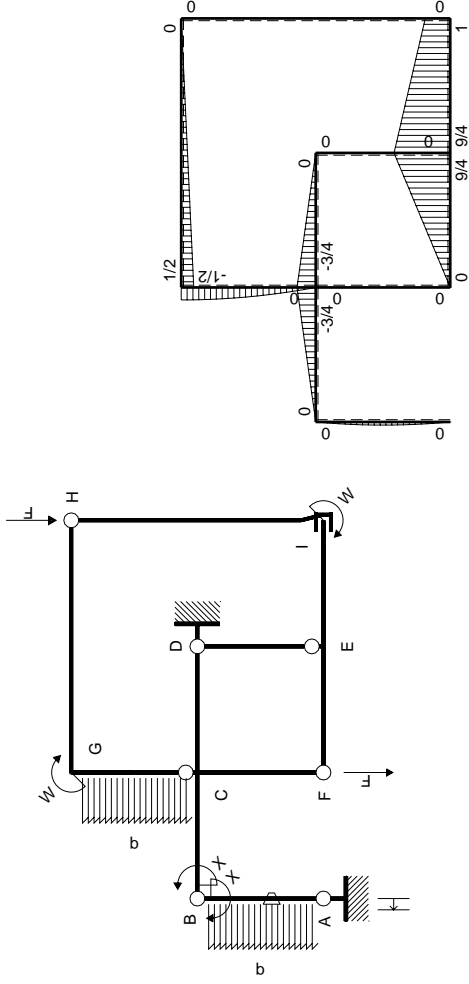
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| FC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0             | 0                       |                             |                          |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| HI 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |               |                         | $Fb^2/EJ$                   |                          |
|       | totali                      |                  |          |                          |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         | $-23/12Fb$                  |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

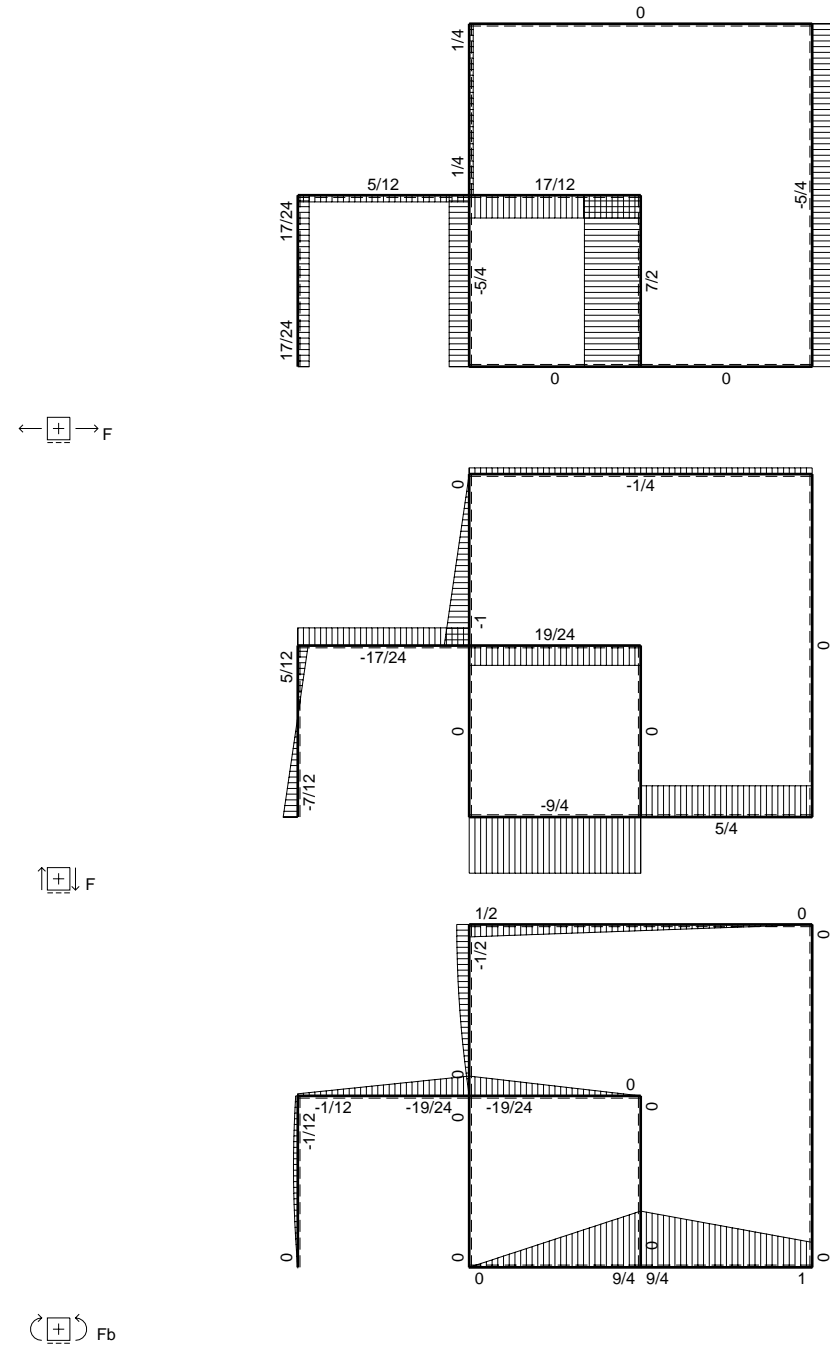
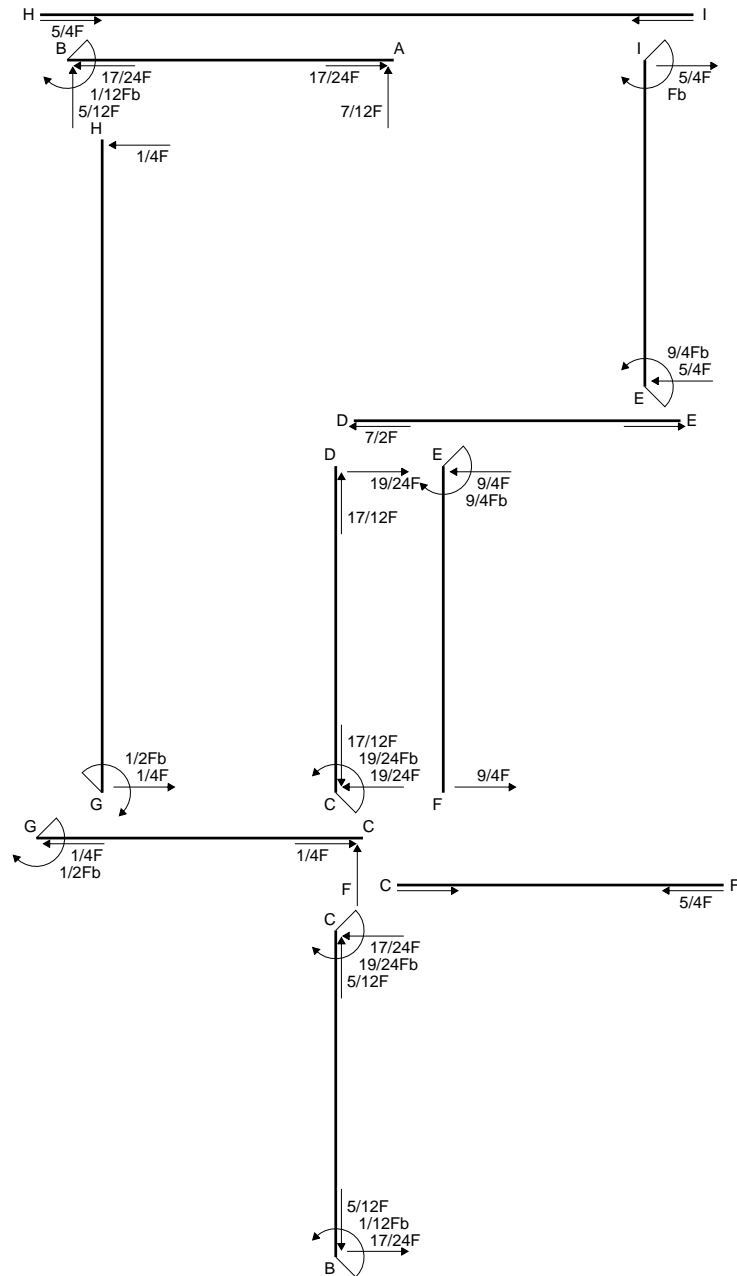
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

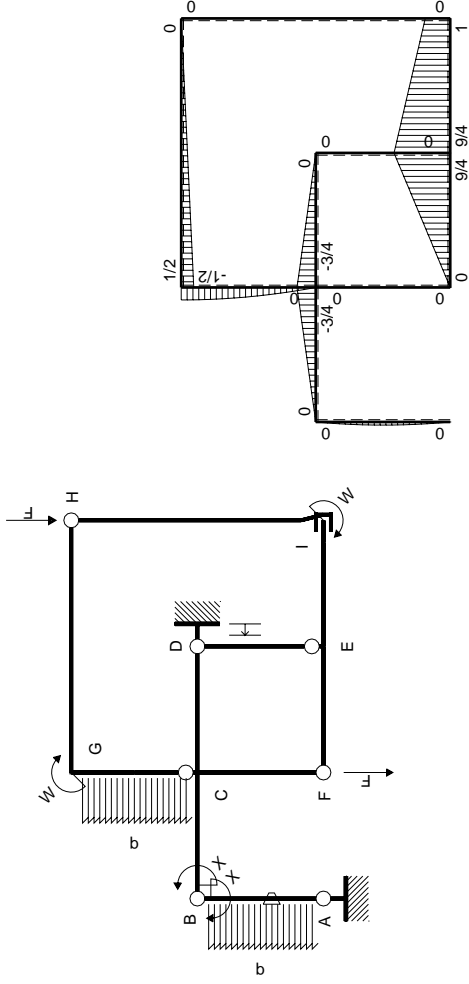
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|--------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$               |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$              |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                          |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0             | 0                       |                             |                          |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |               |                         |                             | $-Fb^2/EJ$               |         |
|       | totali                      |                  |          |                          |               |                         |                             | $-1/12Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         |                             | $1/12Fb$                 |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

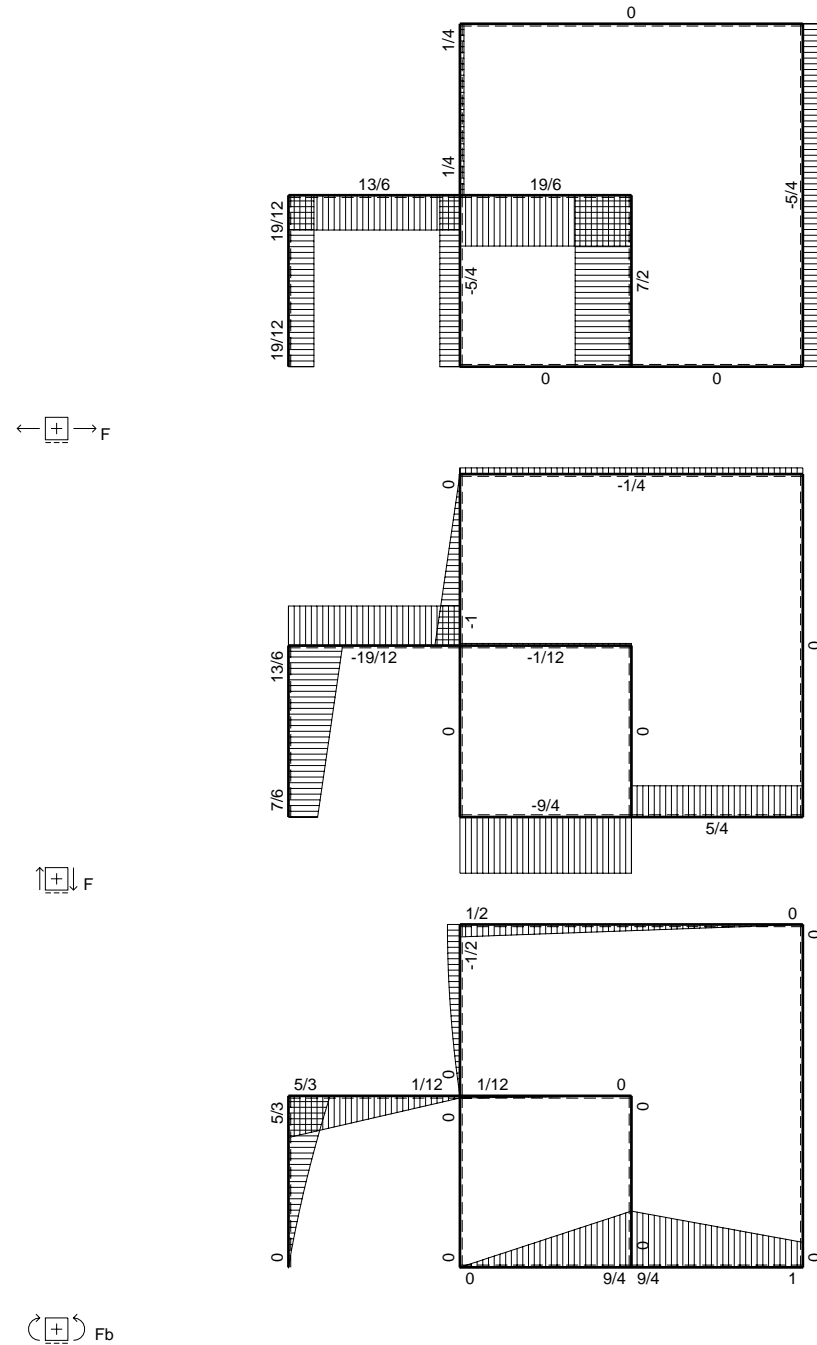
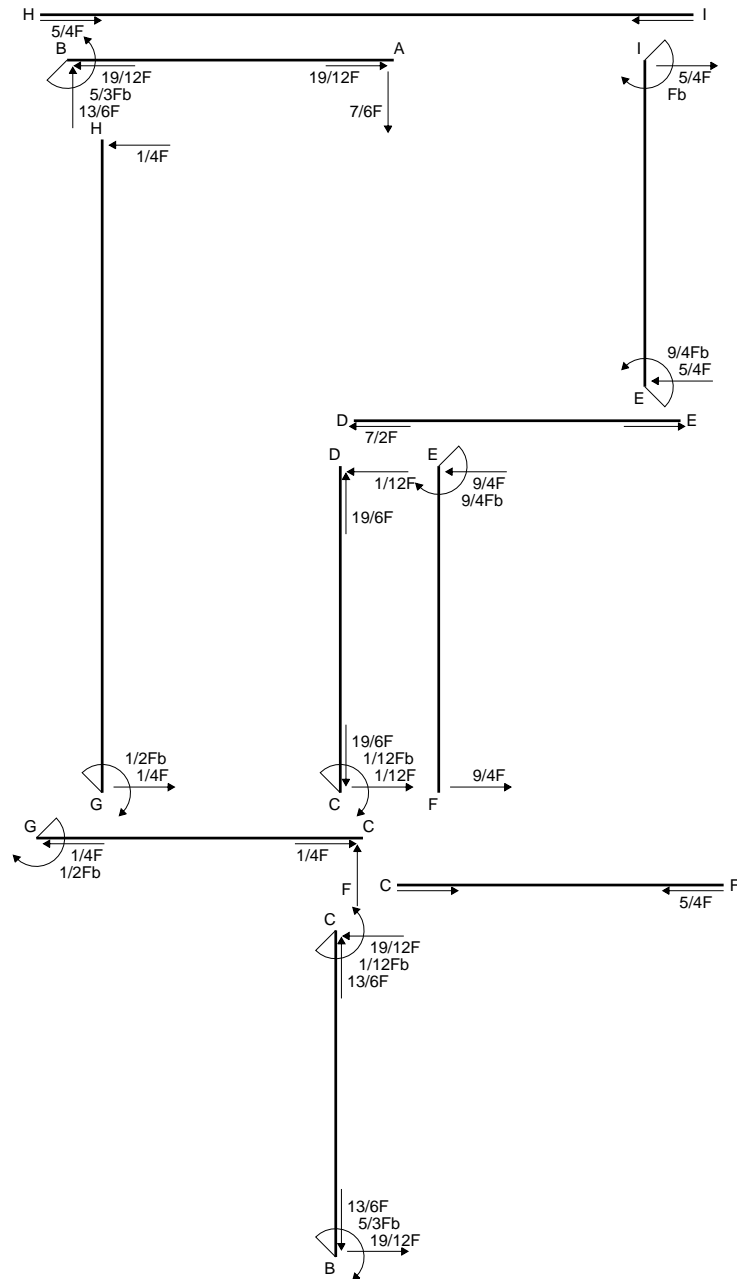
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

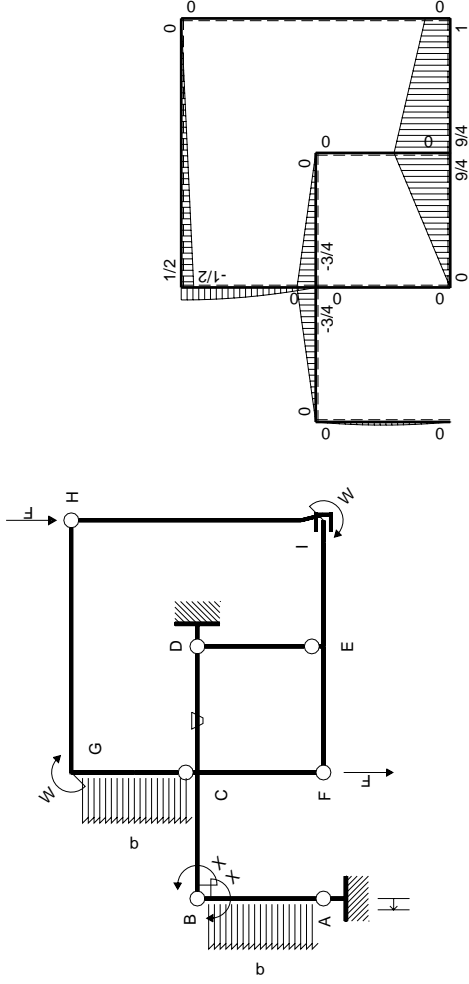
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$



$\boxed{+}$   $\curvearrowright$   $F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                          |                     |                         | $5/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $-5/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

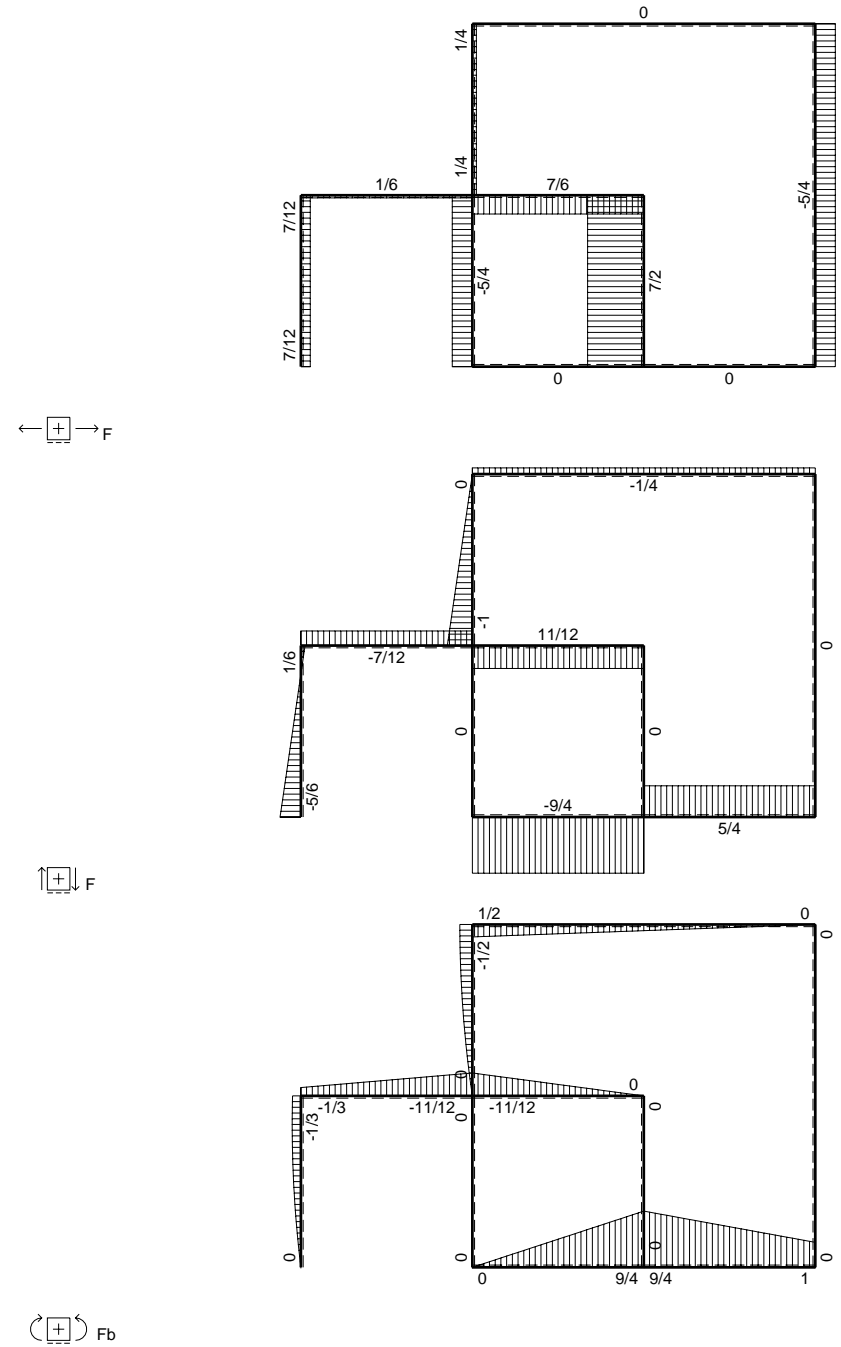
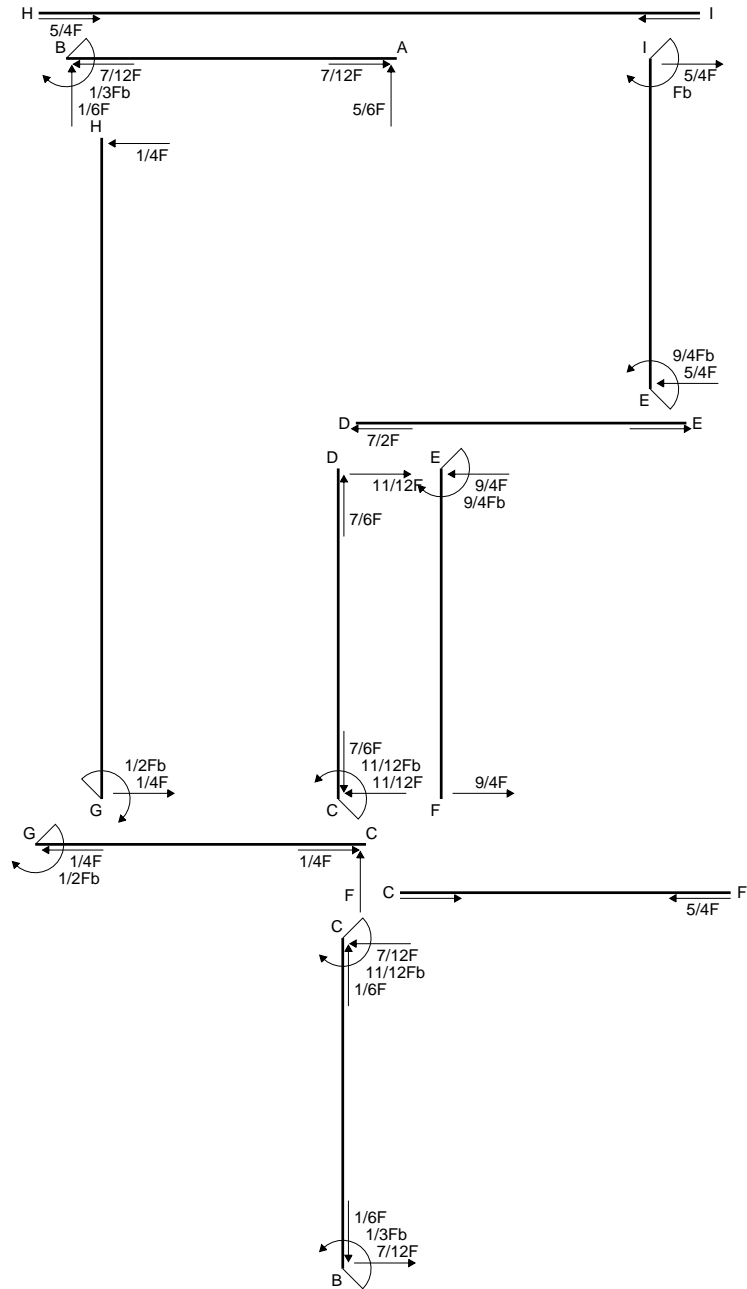
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

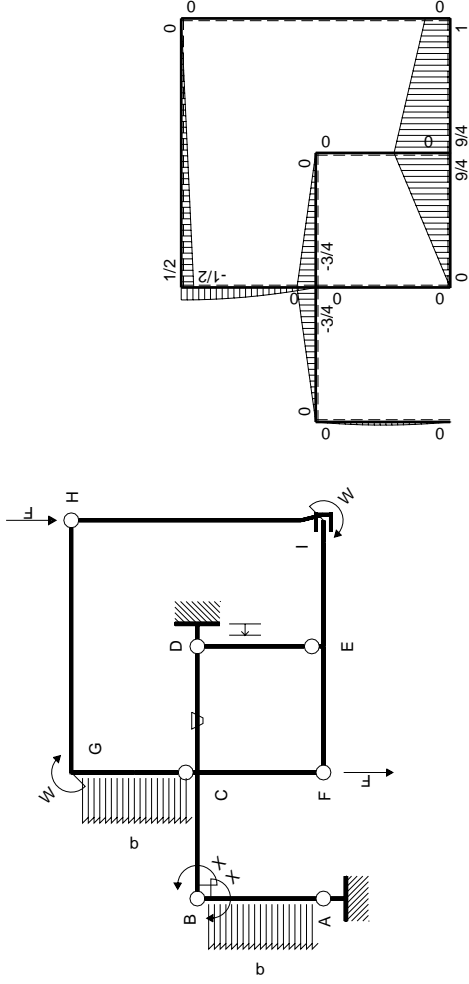
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

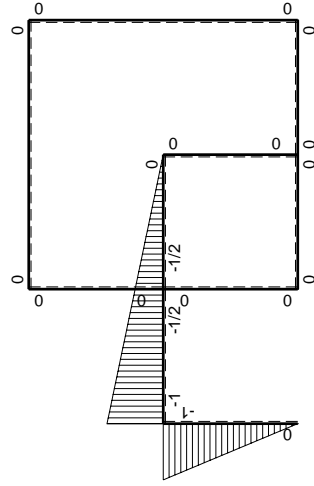






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $-1/3Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $1/3Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

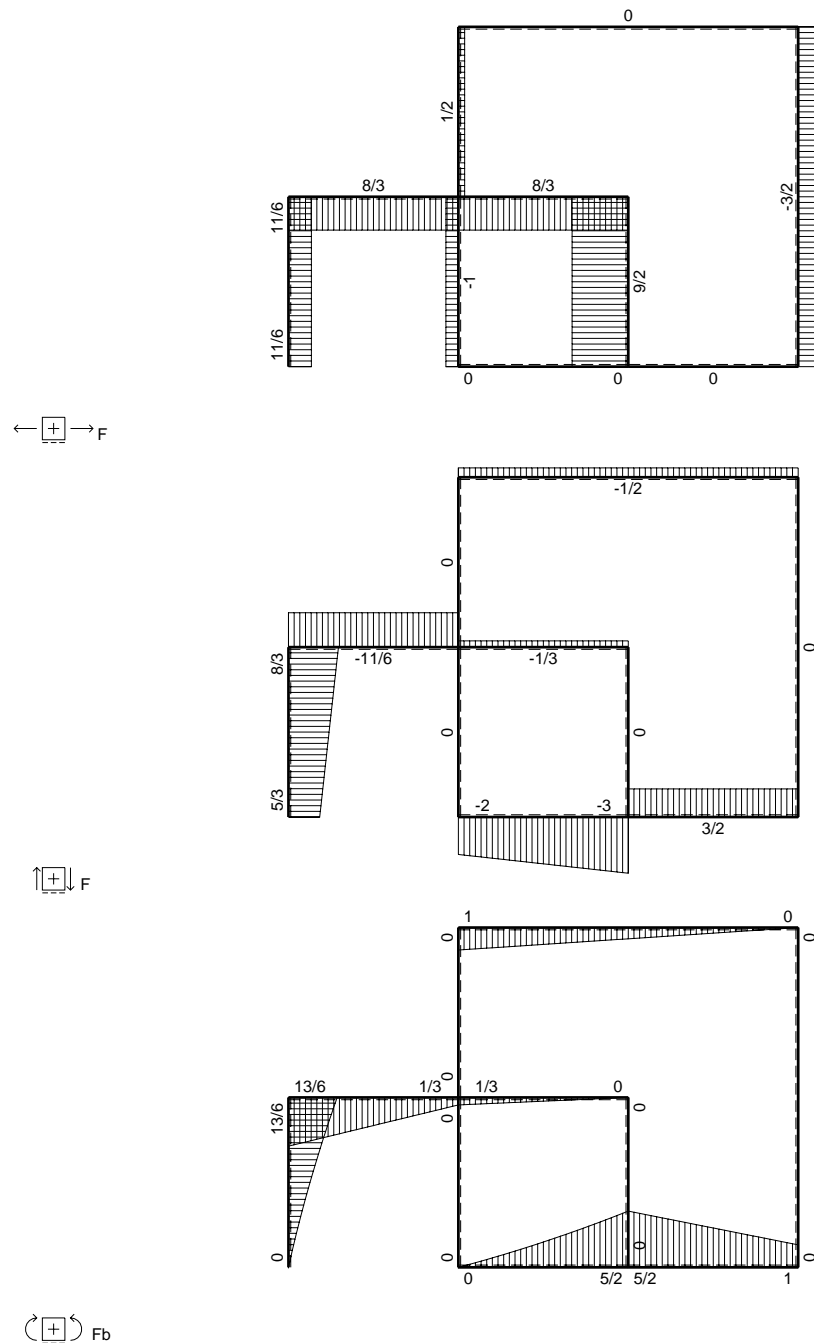
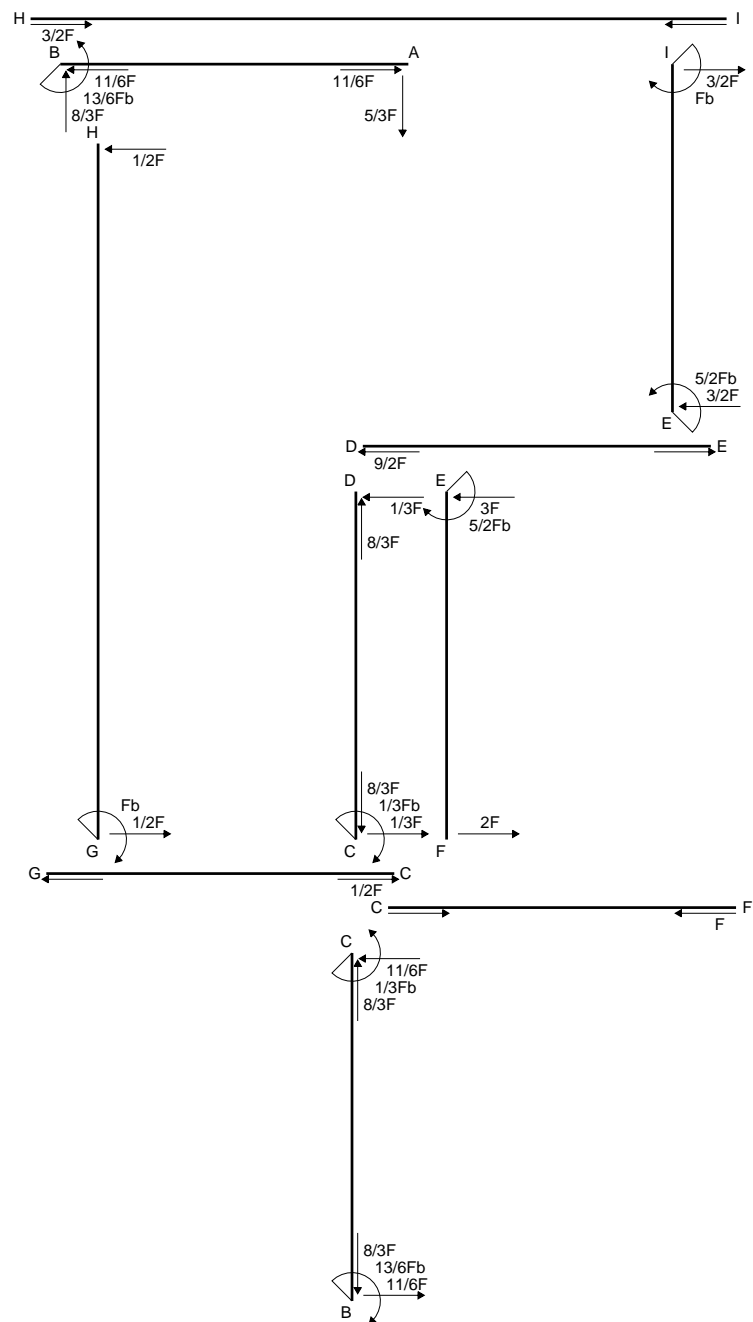
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

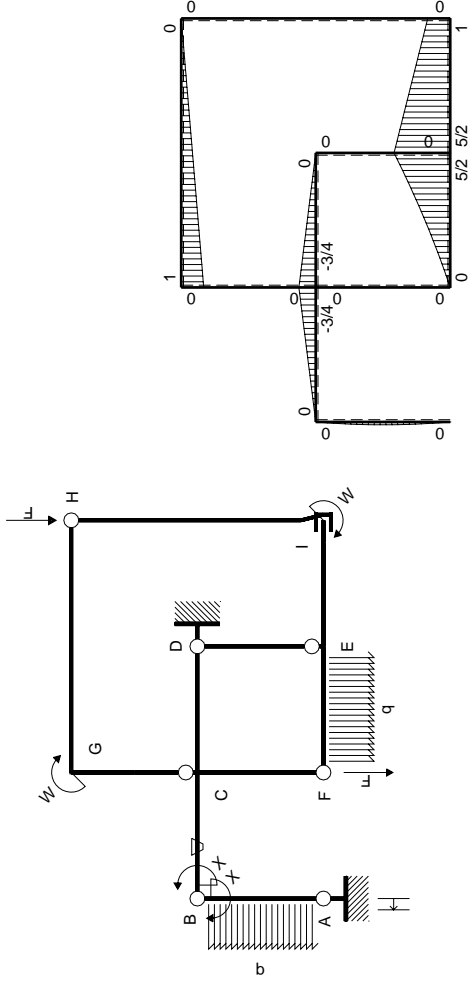
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

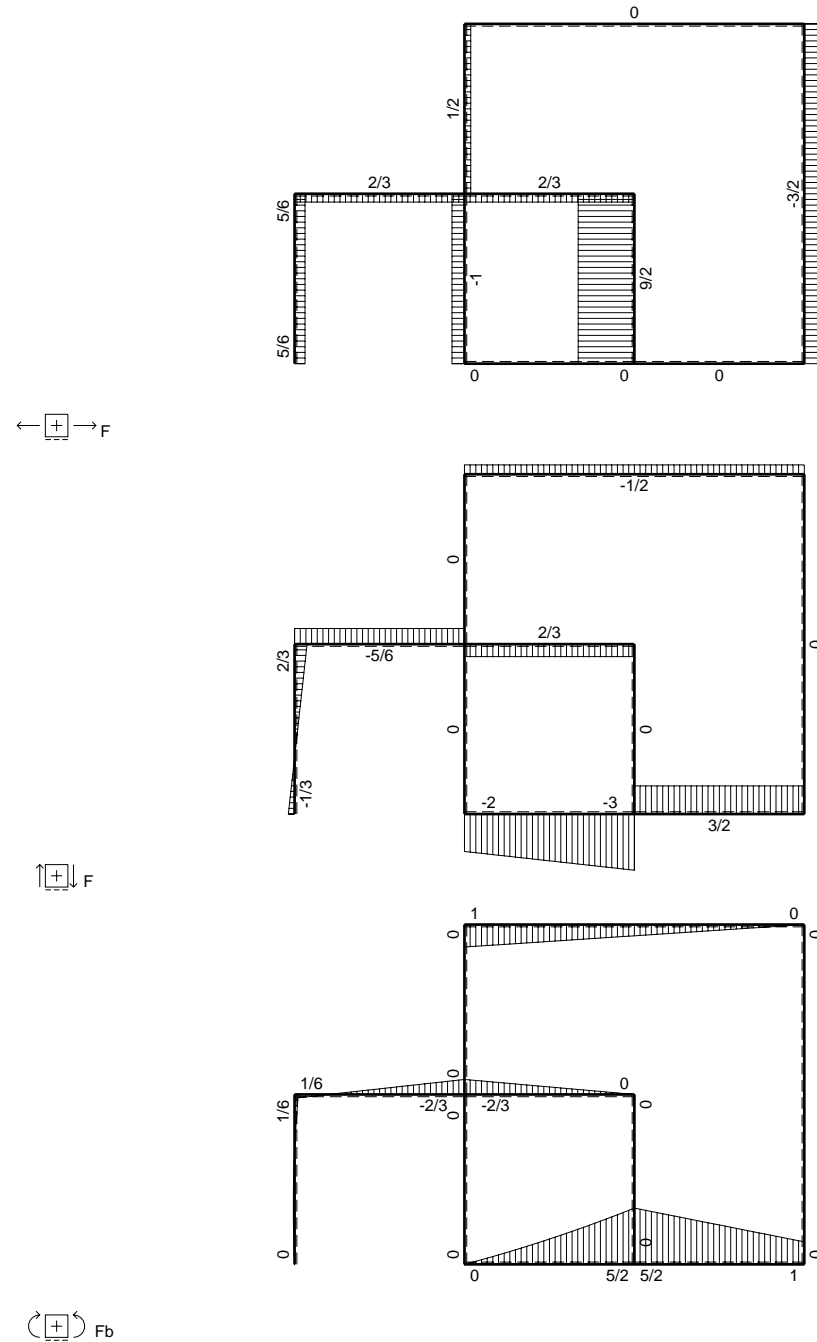
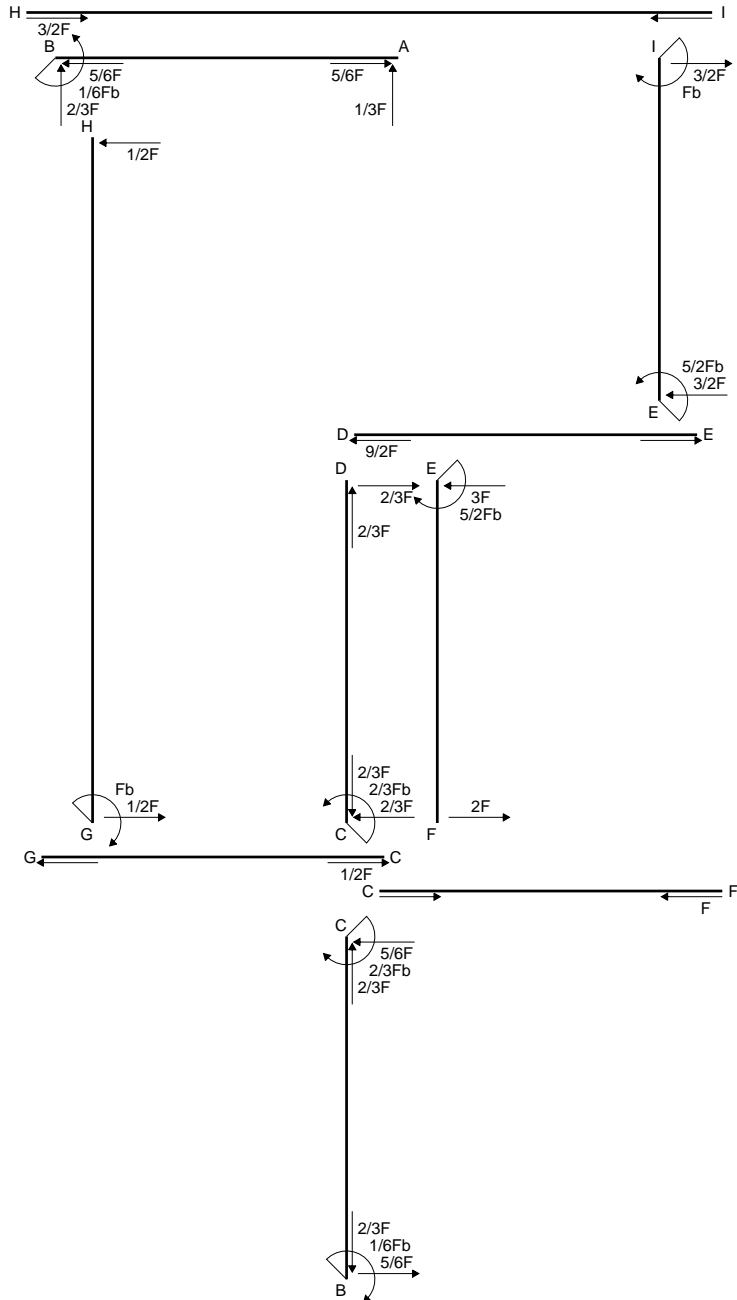
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

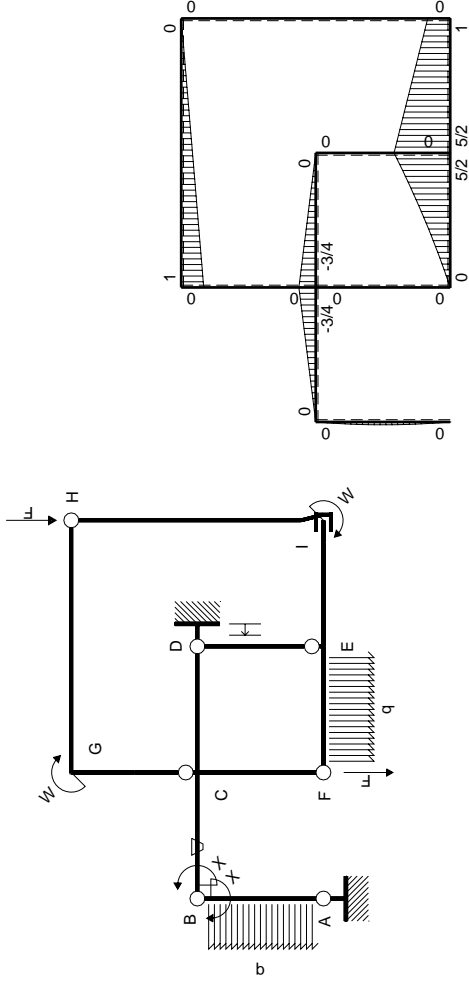
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





⤵ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                          |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

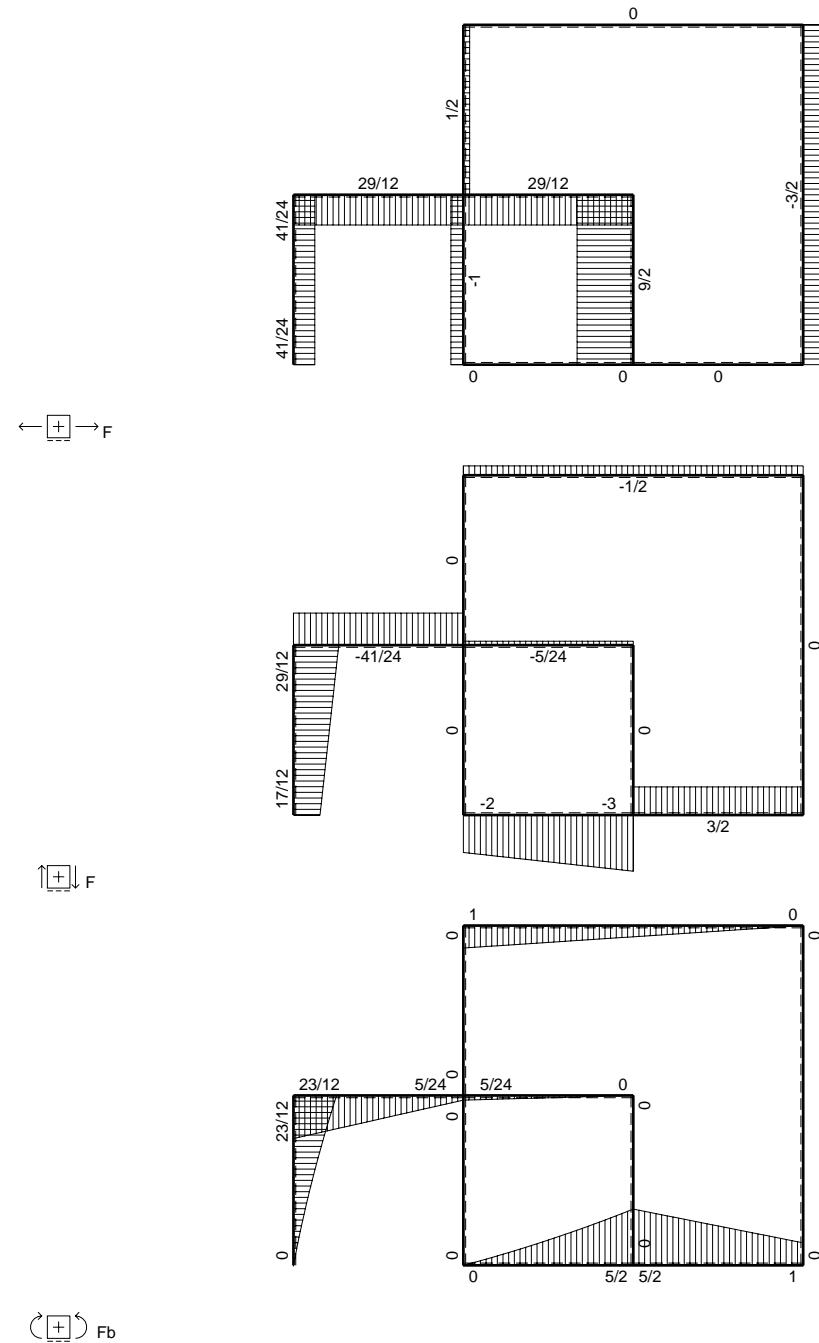
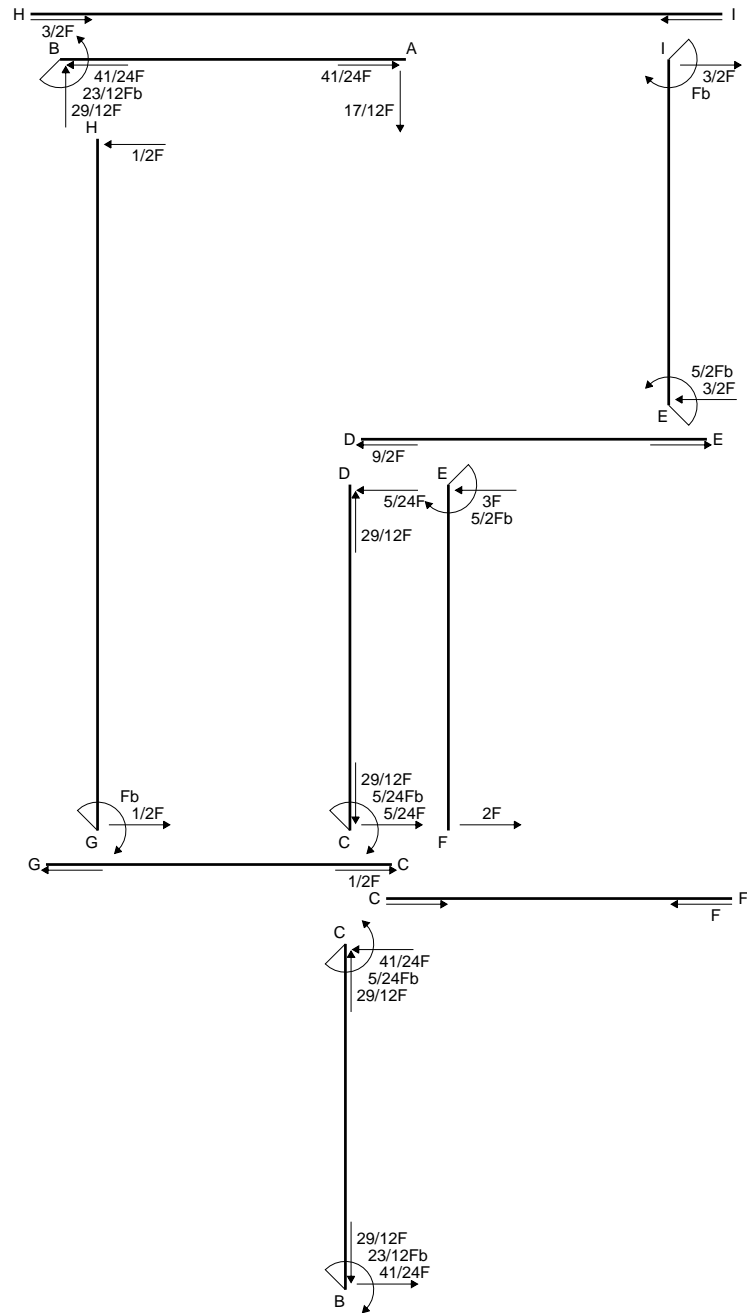
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

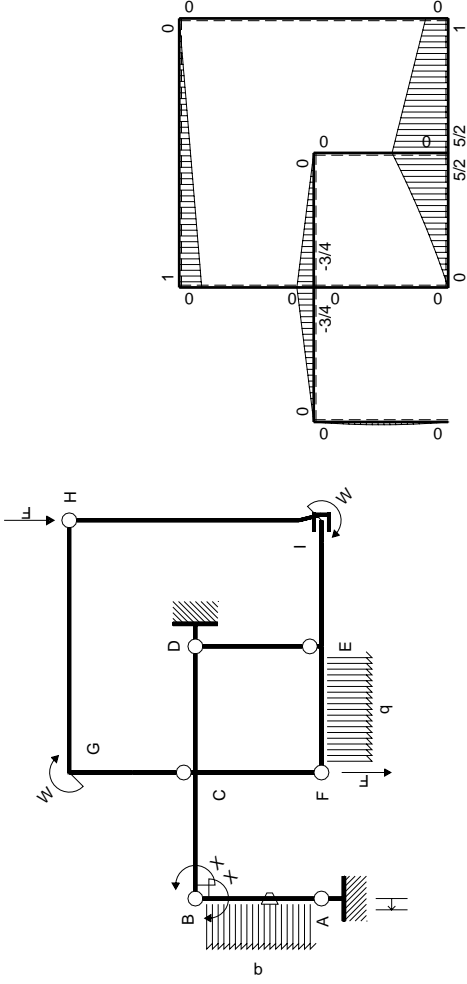
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

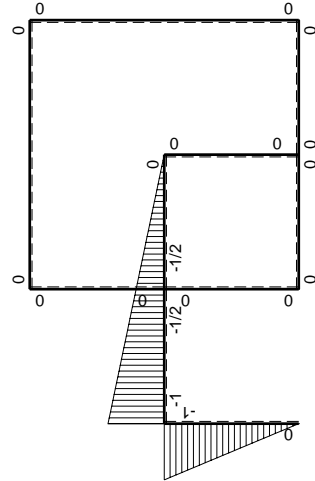
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |               |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

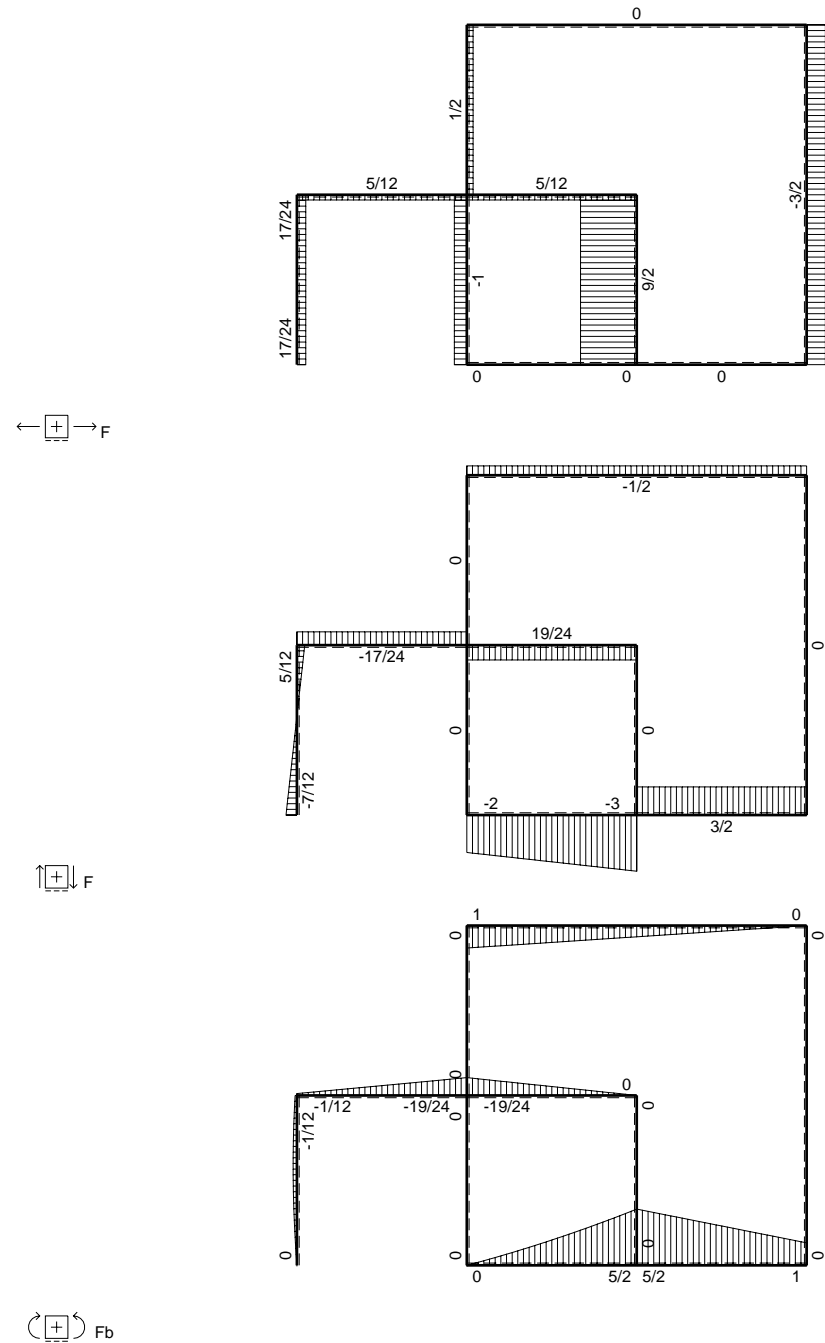
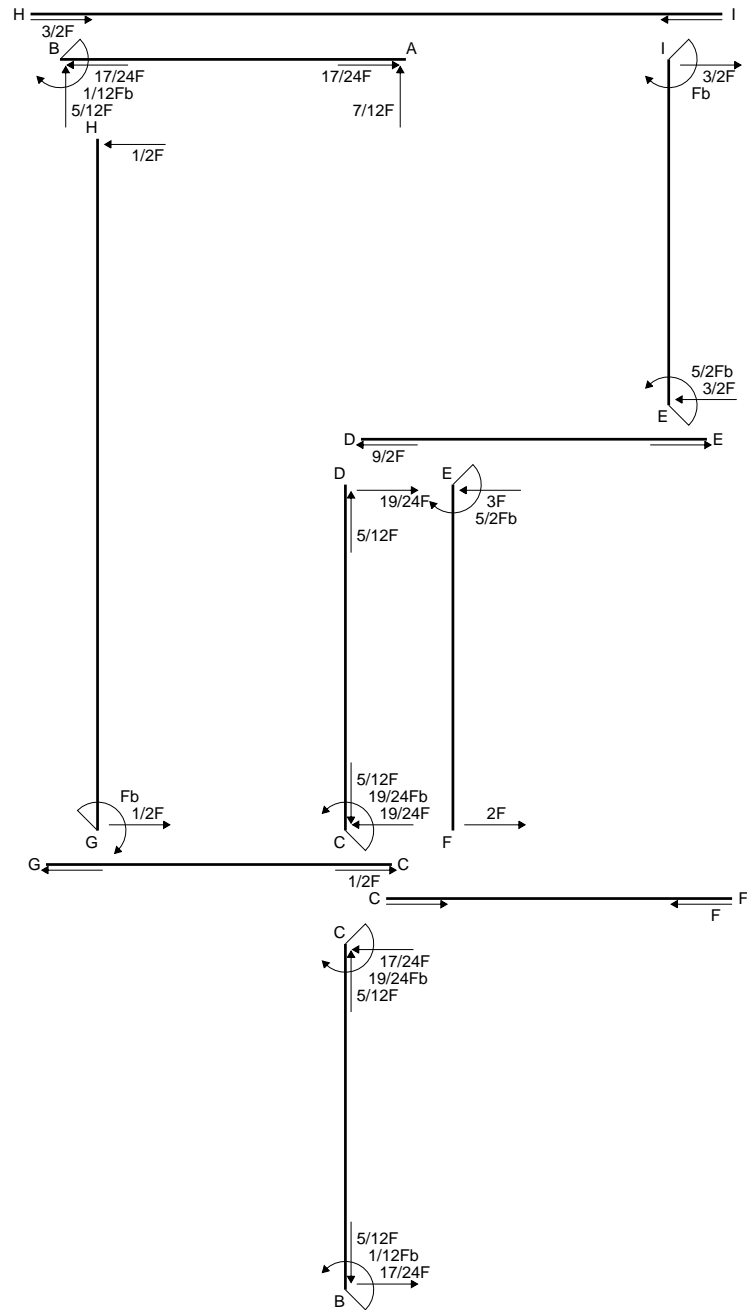
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

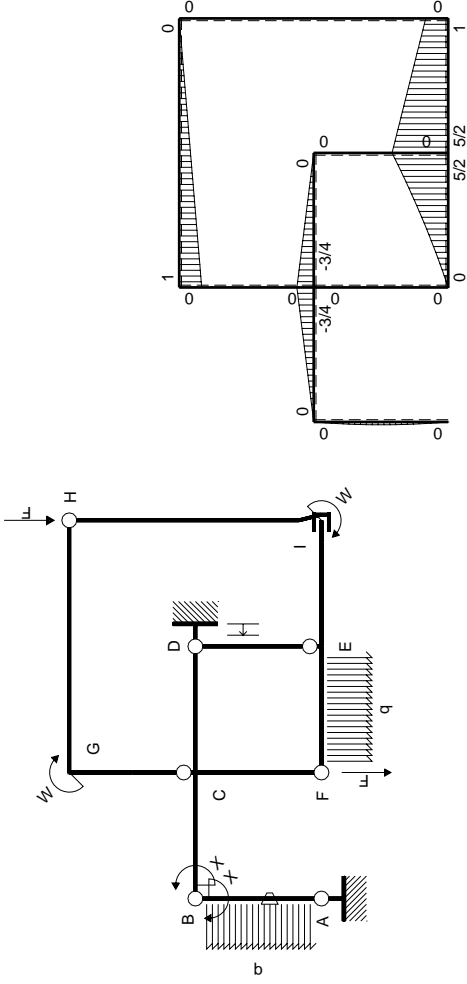
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$



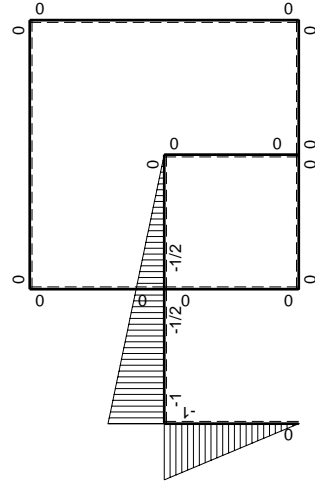


⊕ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                          |               |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |               |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

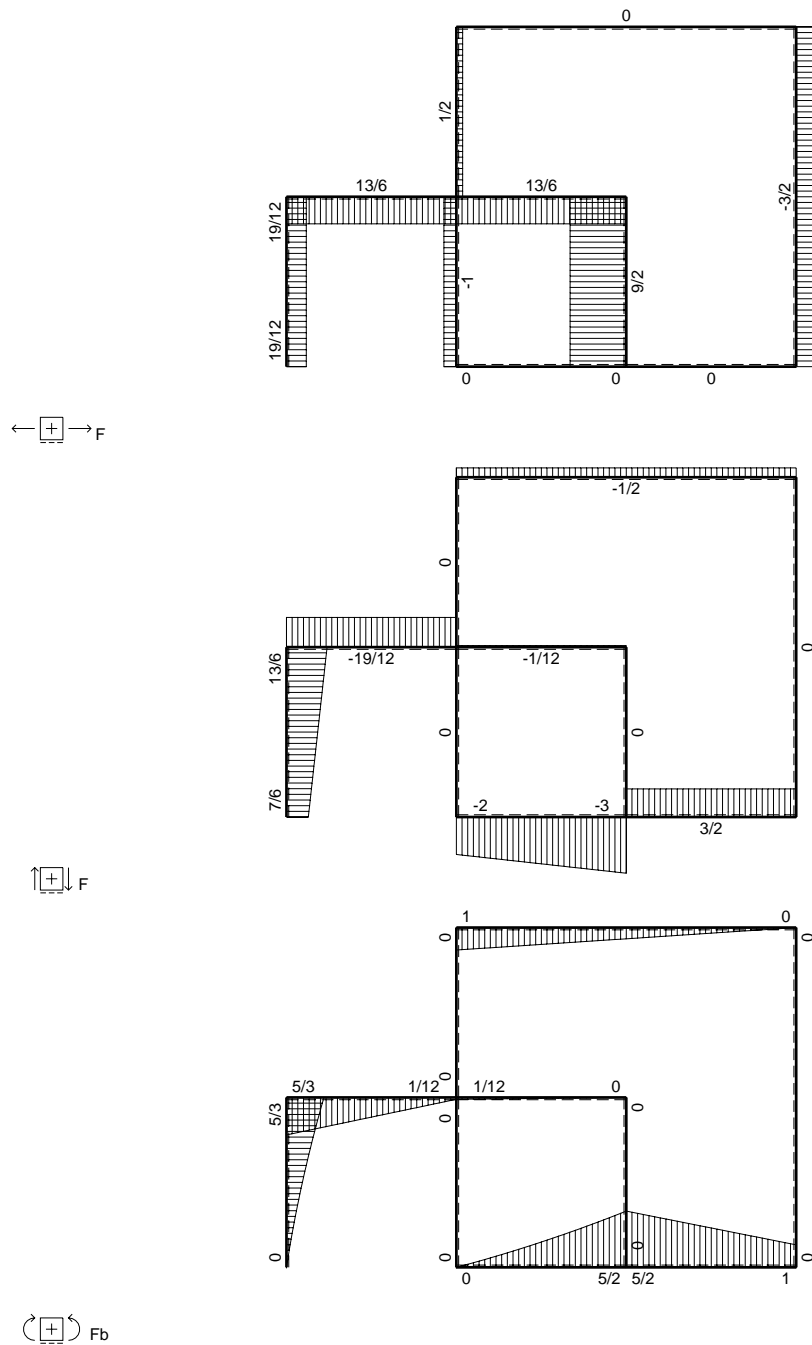
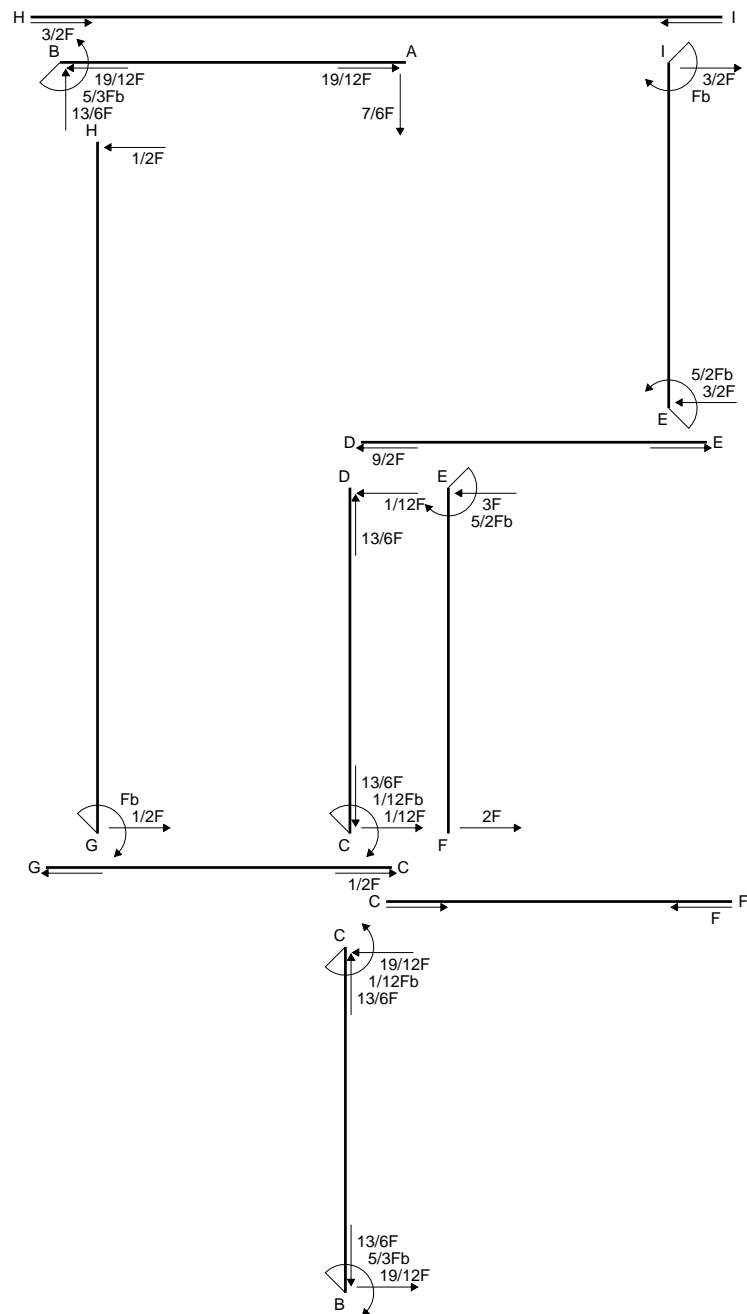
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

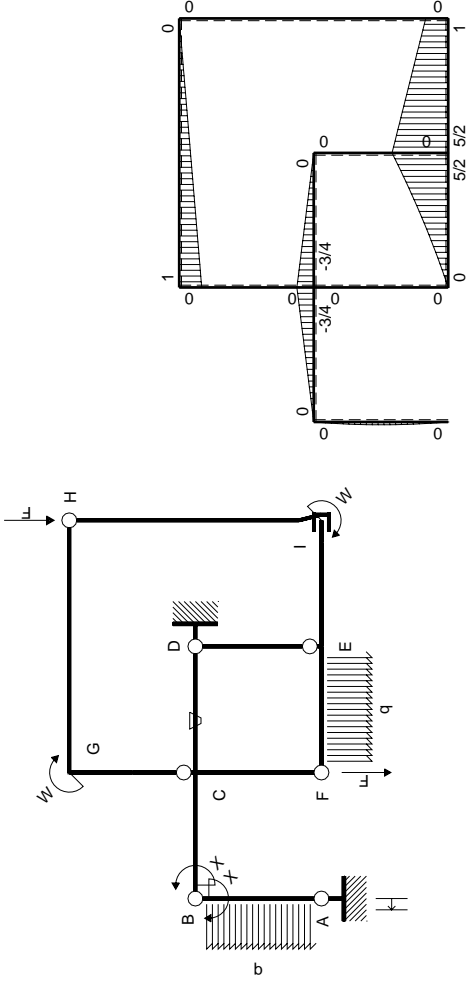
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |                     |                         | $5/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-5/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

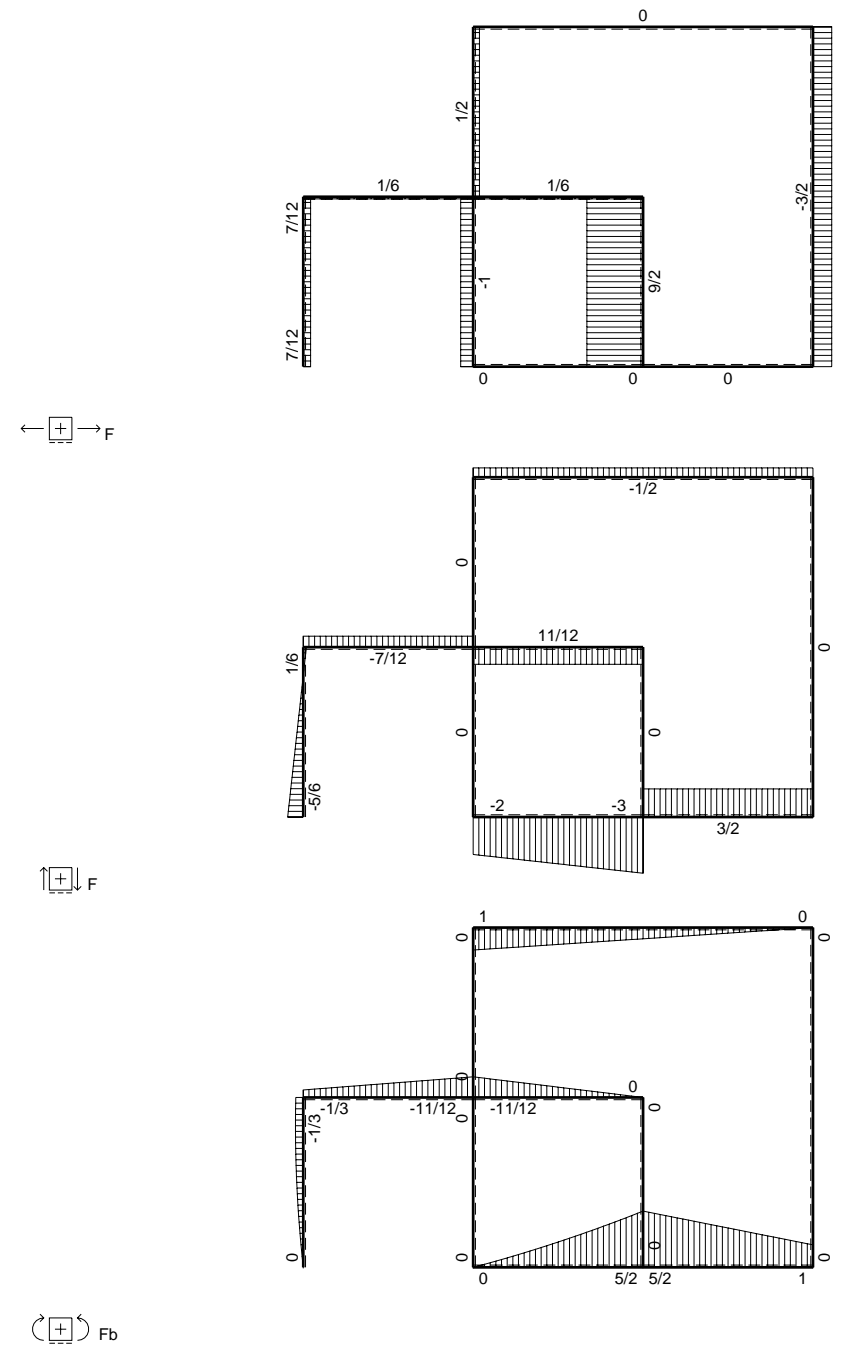
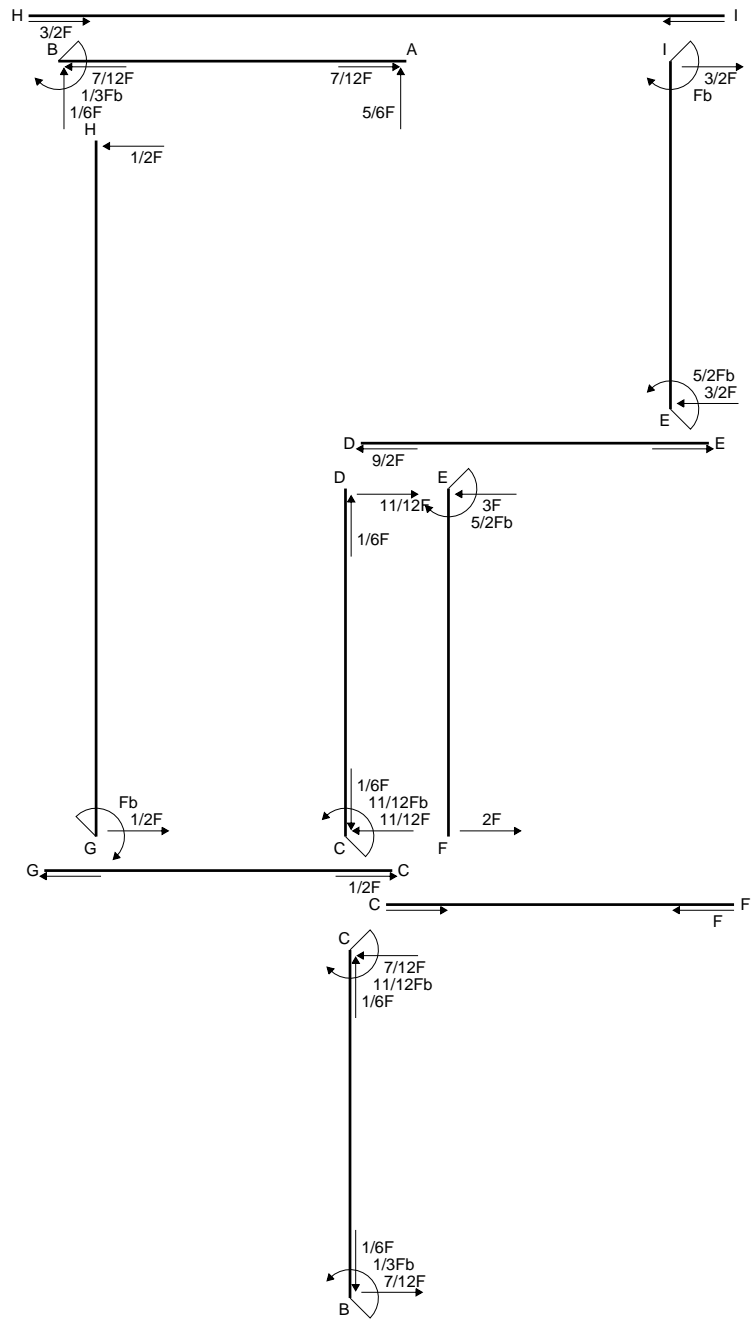
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

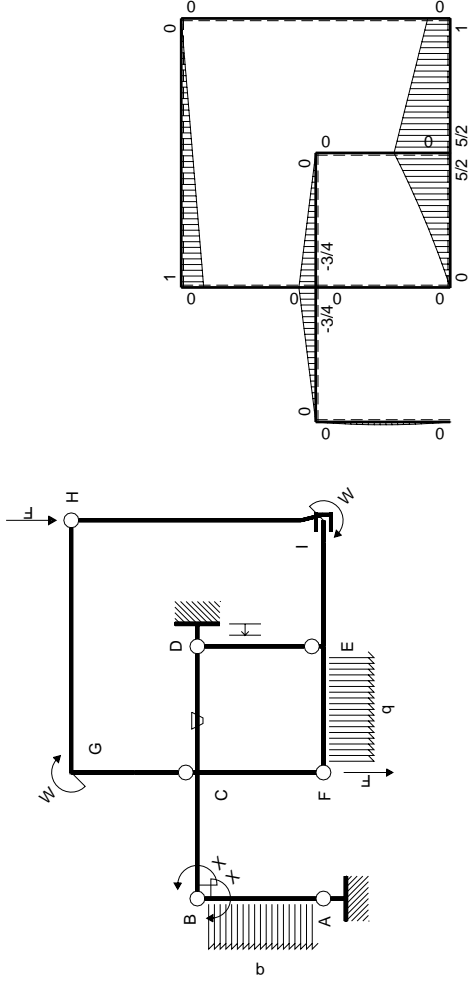
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                          |                     |                         | $-1/3Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $1/3Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

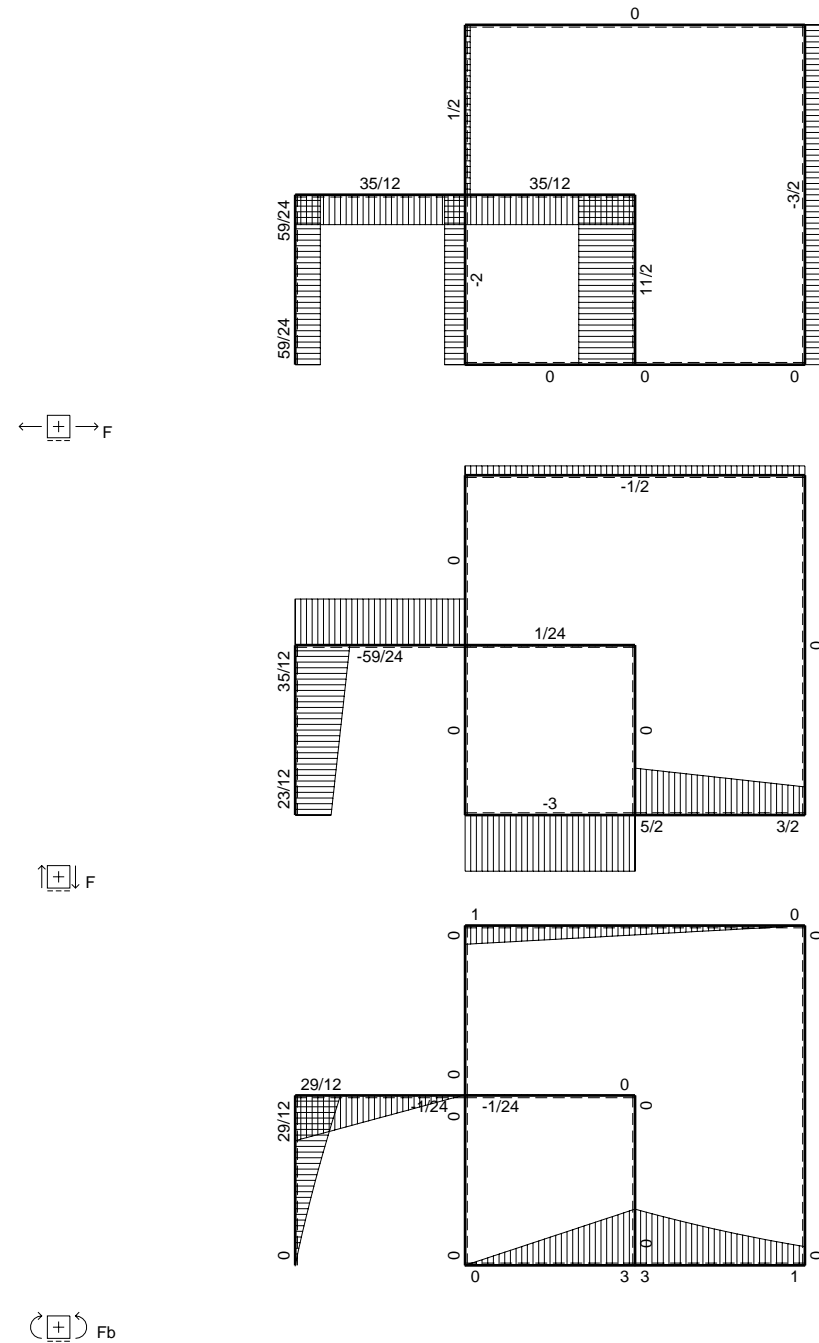
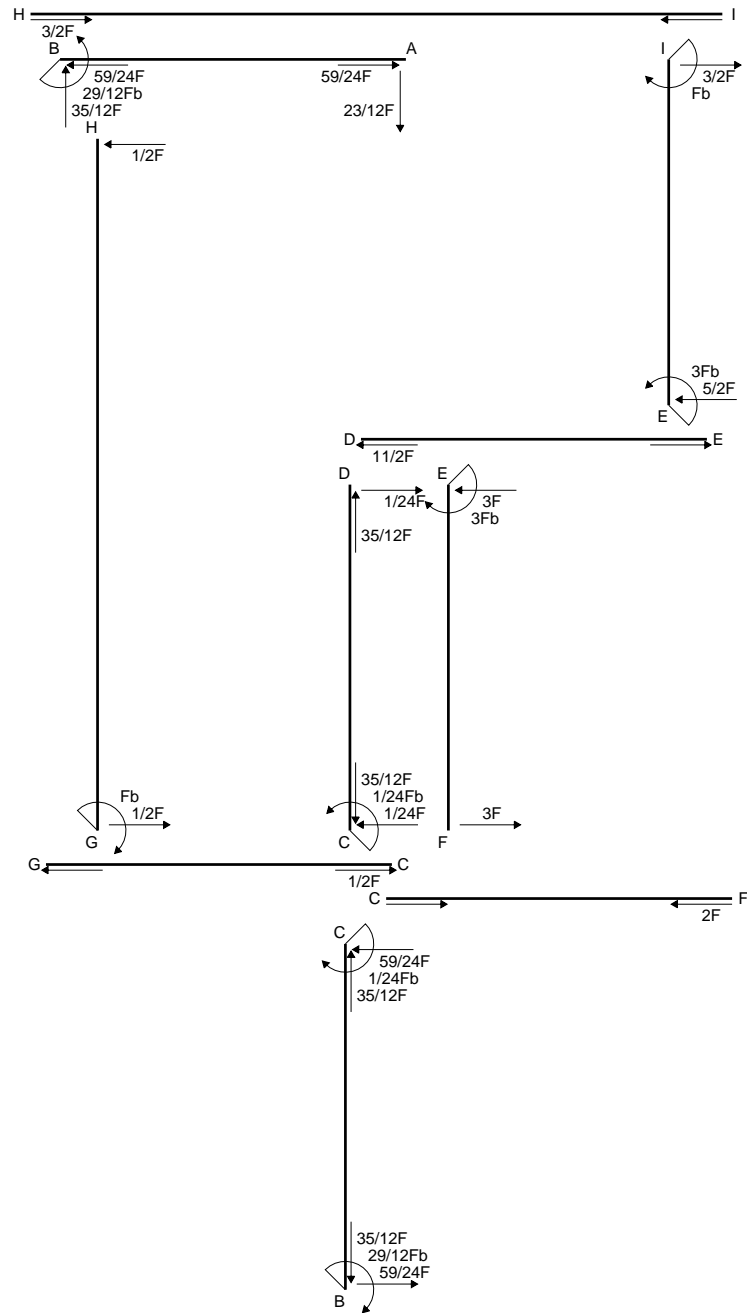
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

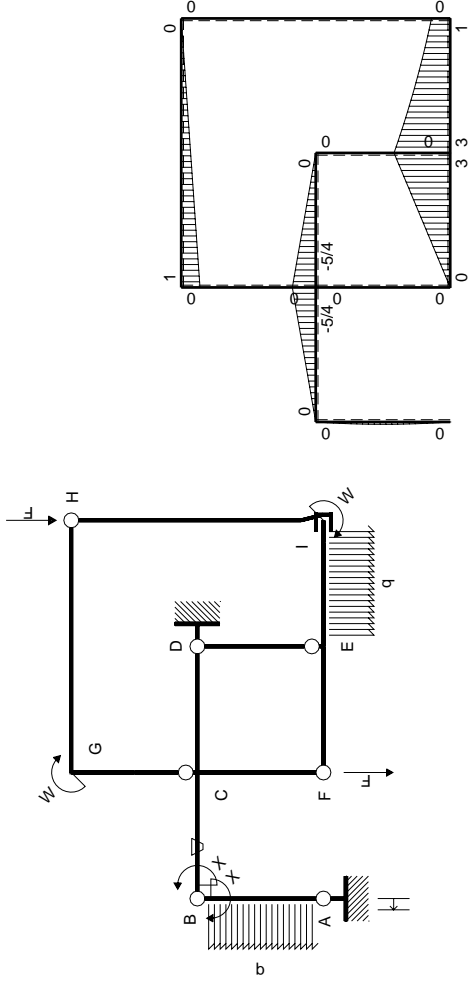
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                          |                     |                         | $29/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         | $-29/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

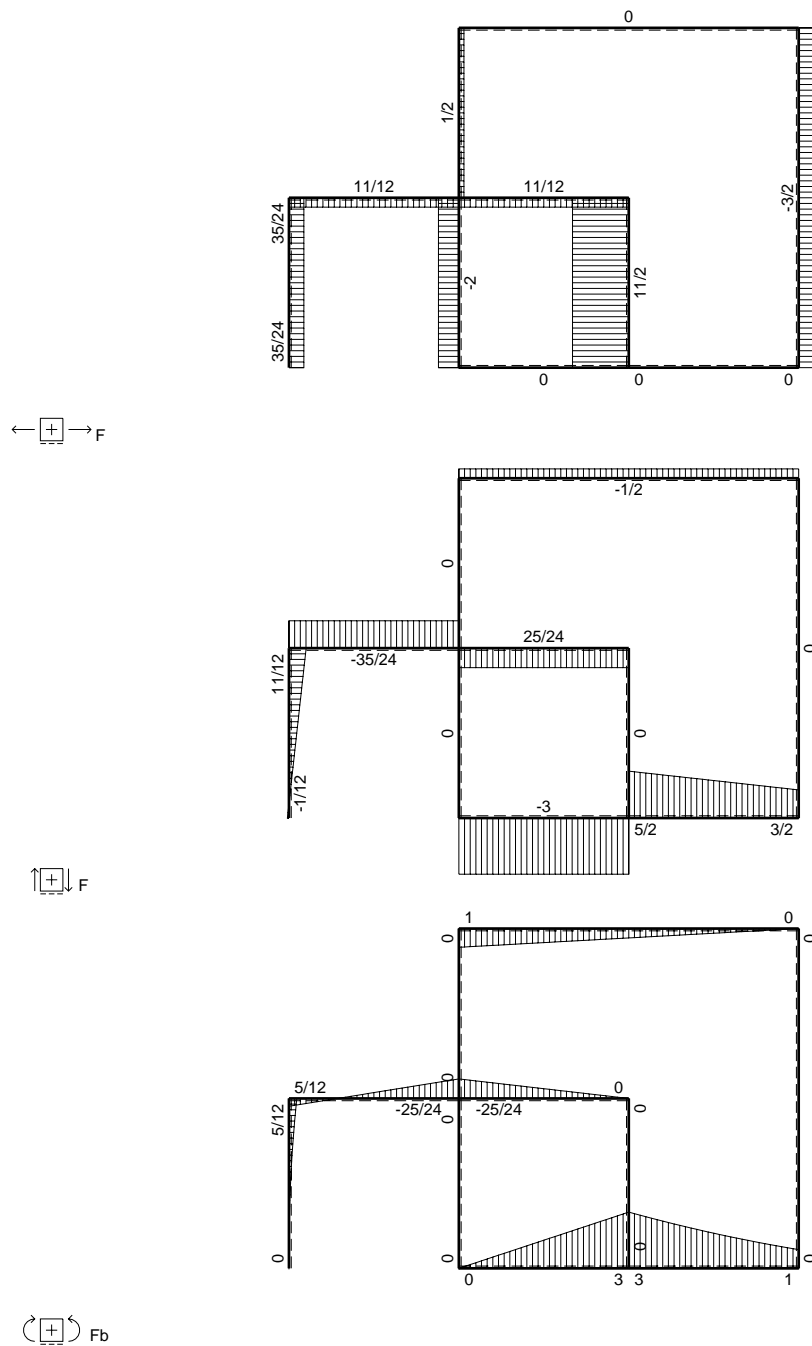
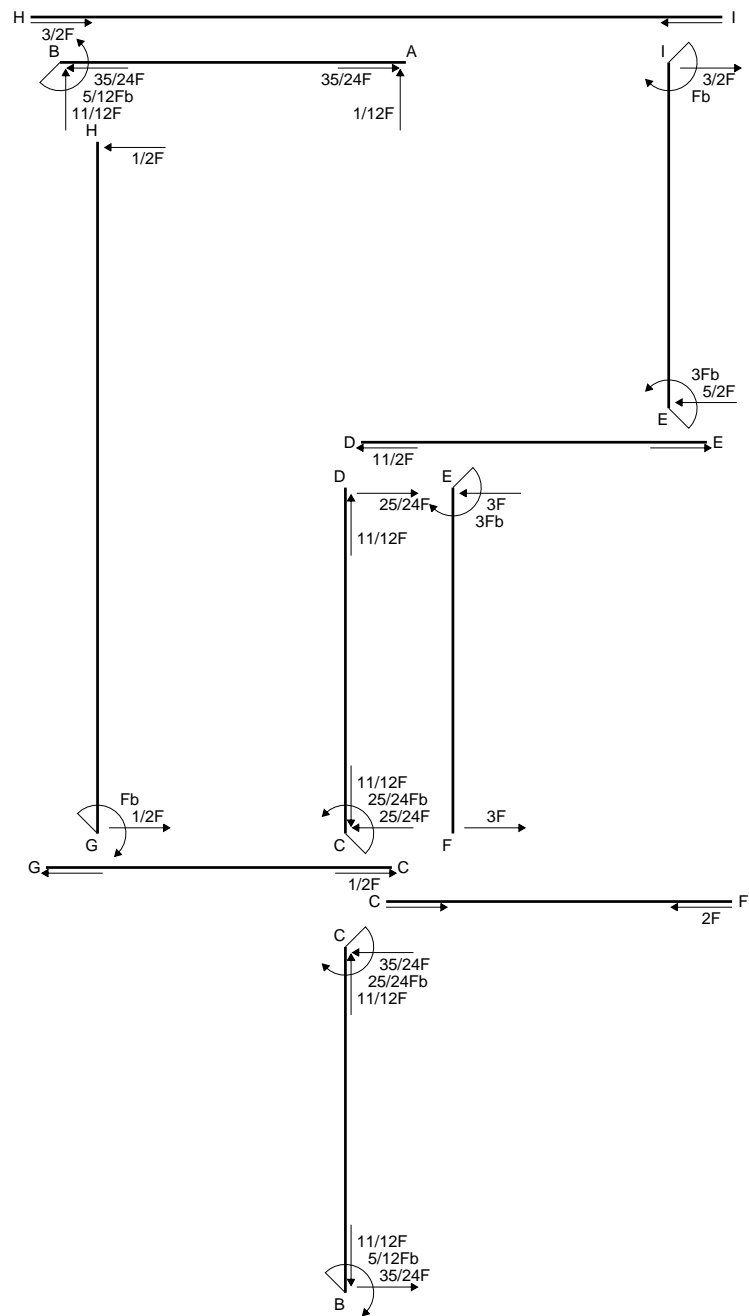
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

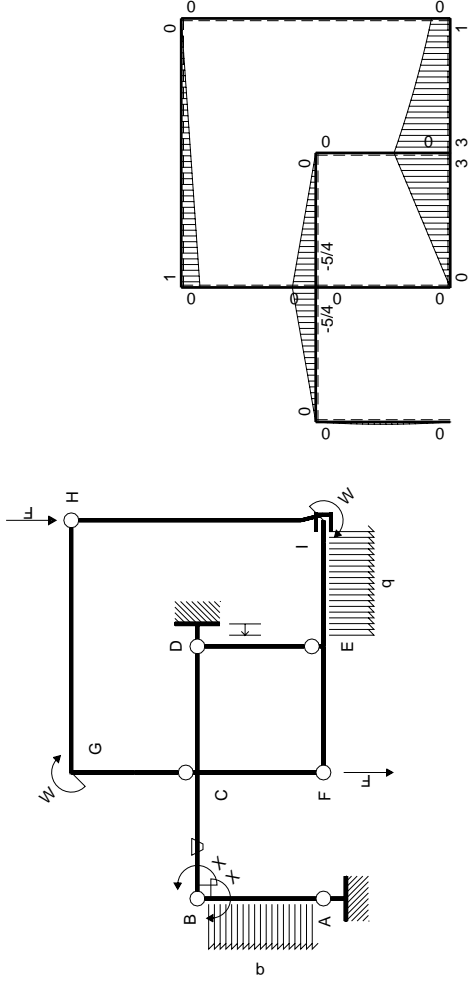
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                          |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                          |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                          |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                          |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                          |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                          |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                          |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |                     |                         | $-Fb^2/EJ$                  |                          |
|       | totali                      |                      |          |                          |                     |                         | $5/12Fb^2/EJ$               | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         | $-5/12Fb$                   |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

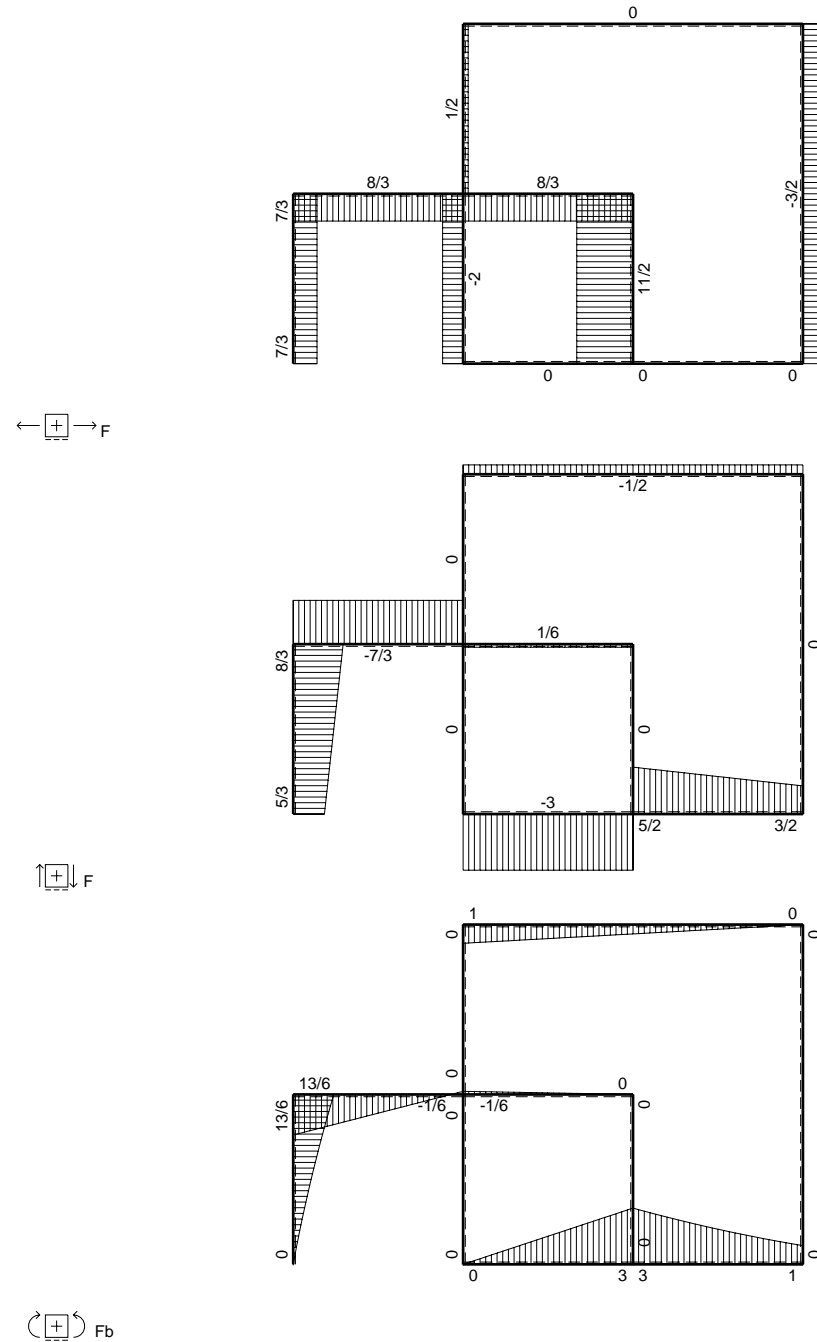
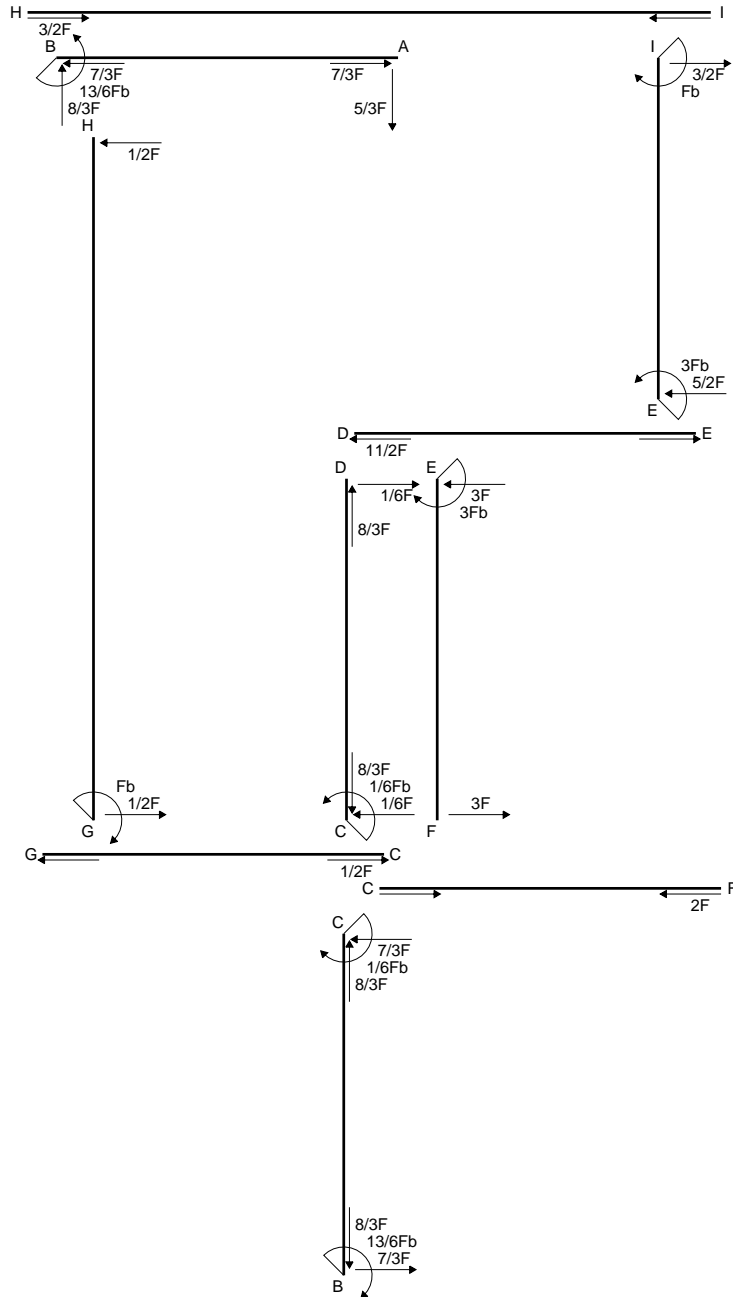
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

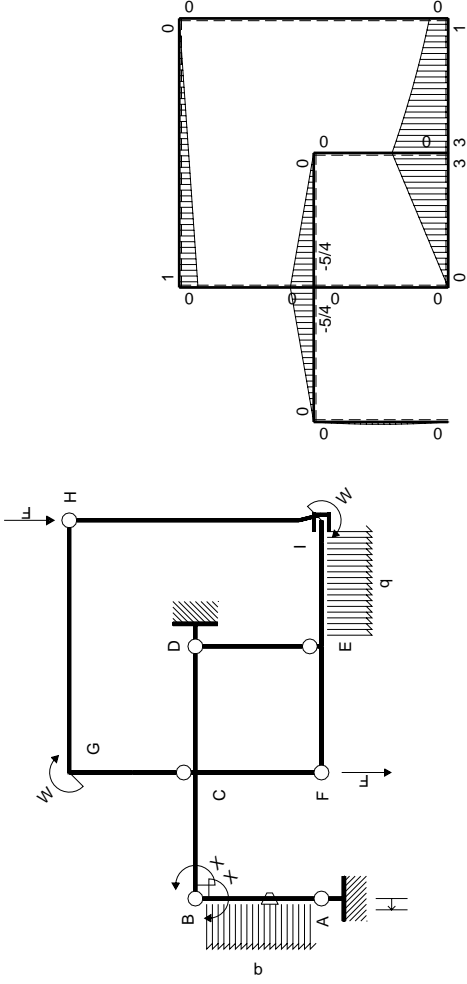
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

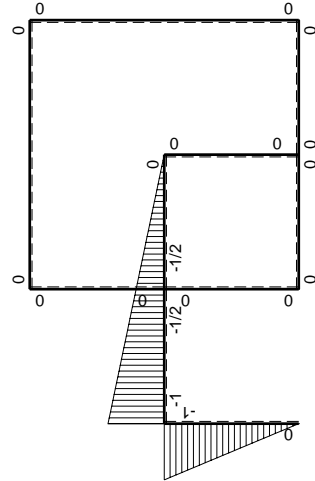


$\curvearrowright (+) Fb$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                          |               |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |               |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

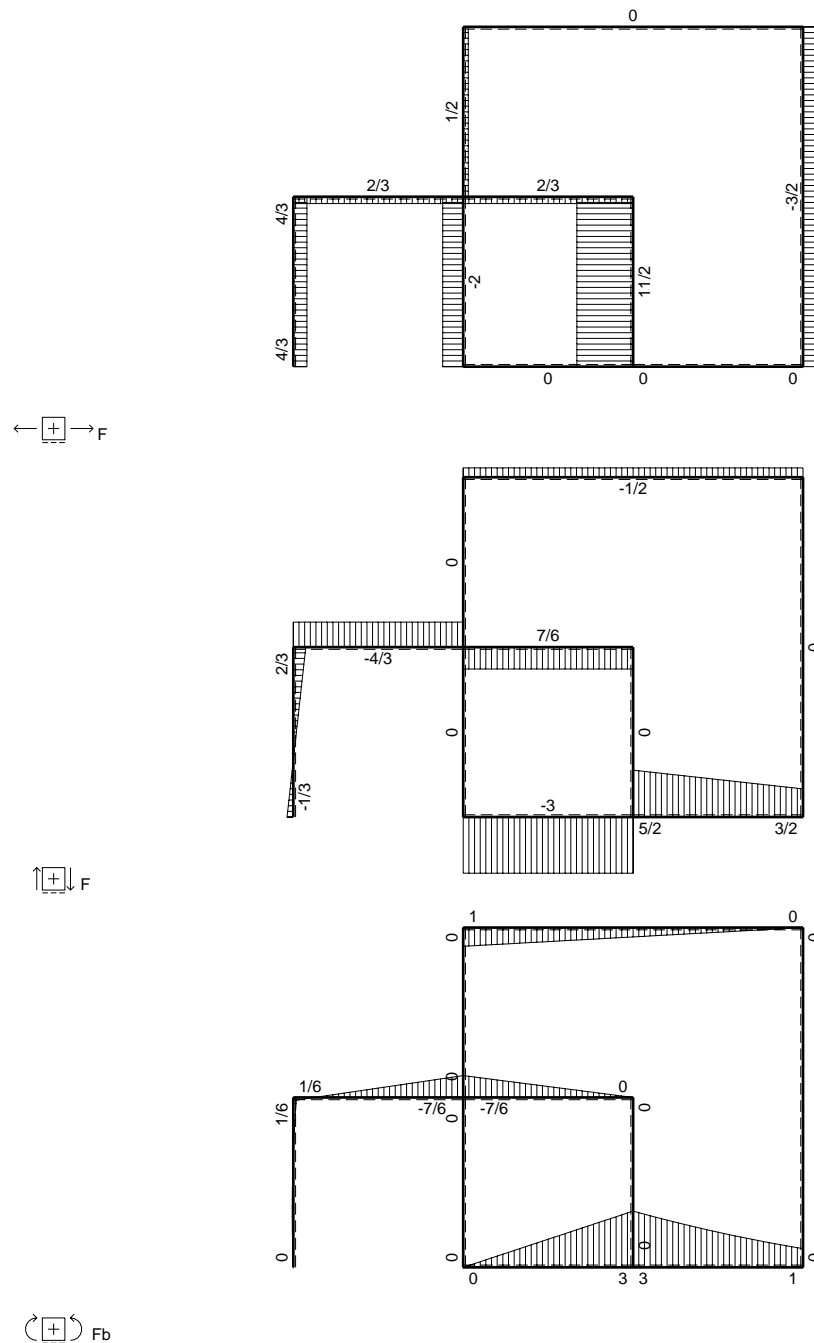
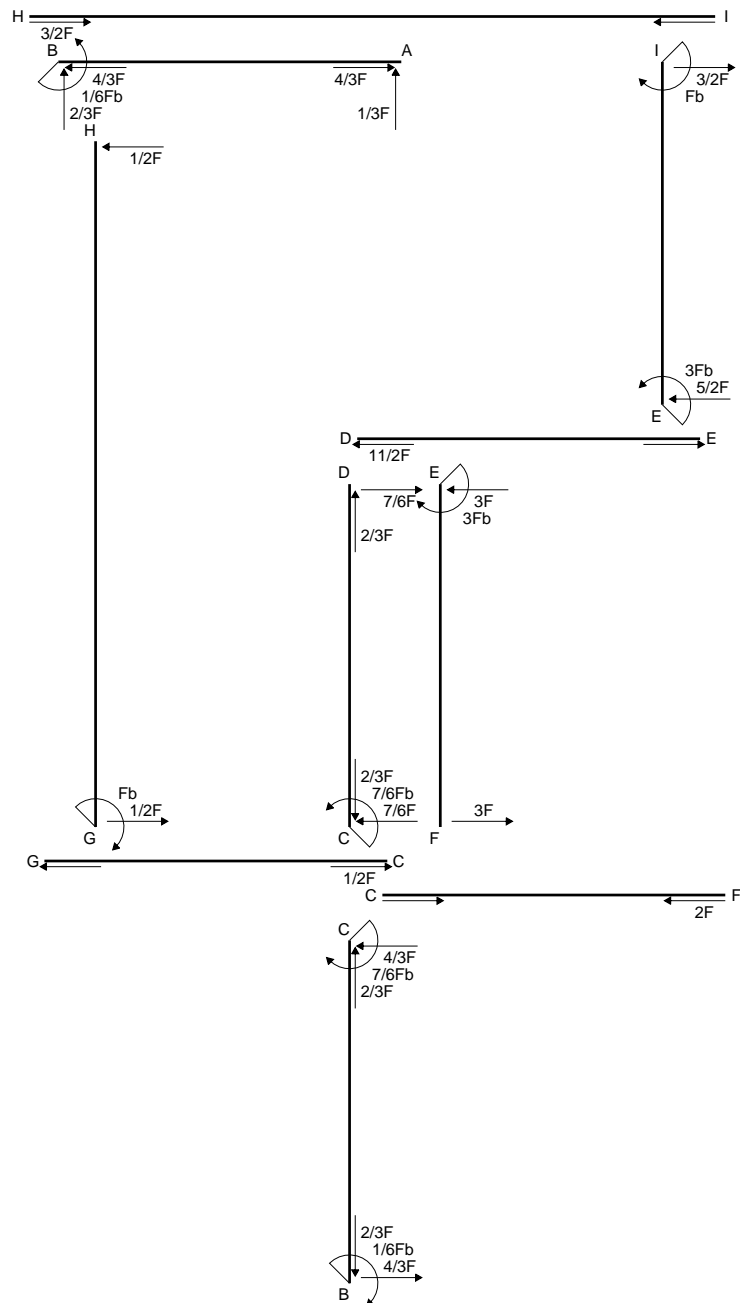
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

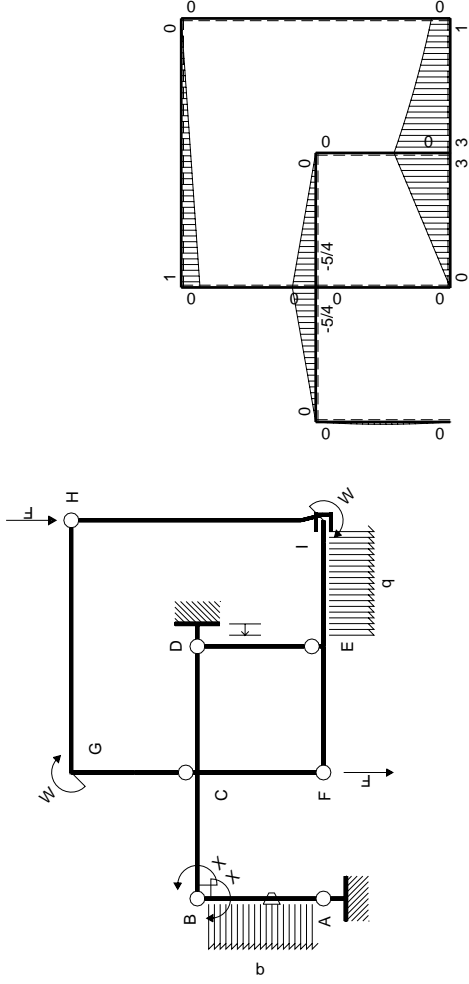
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                          |               |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |               |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

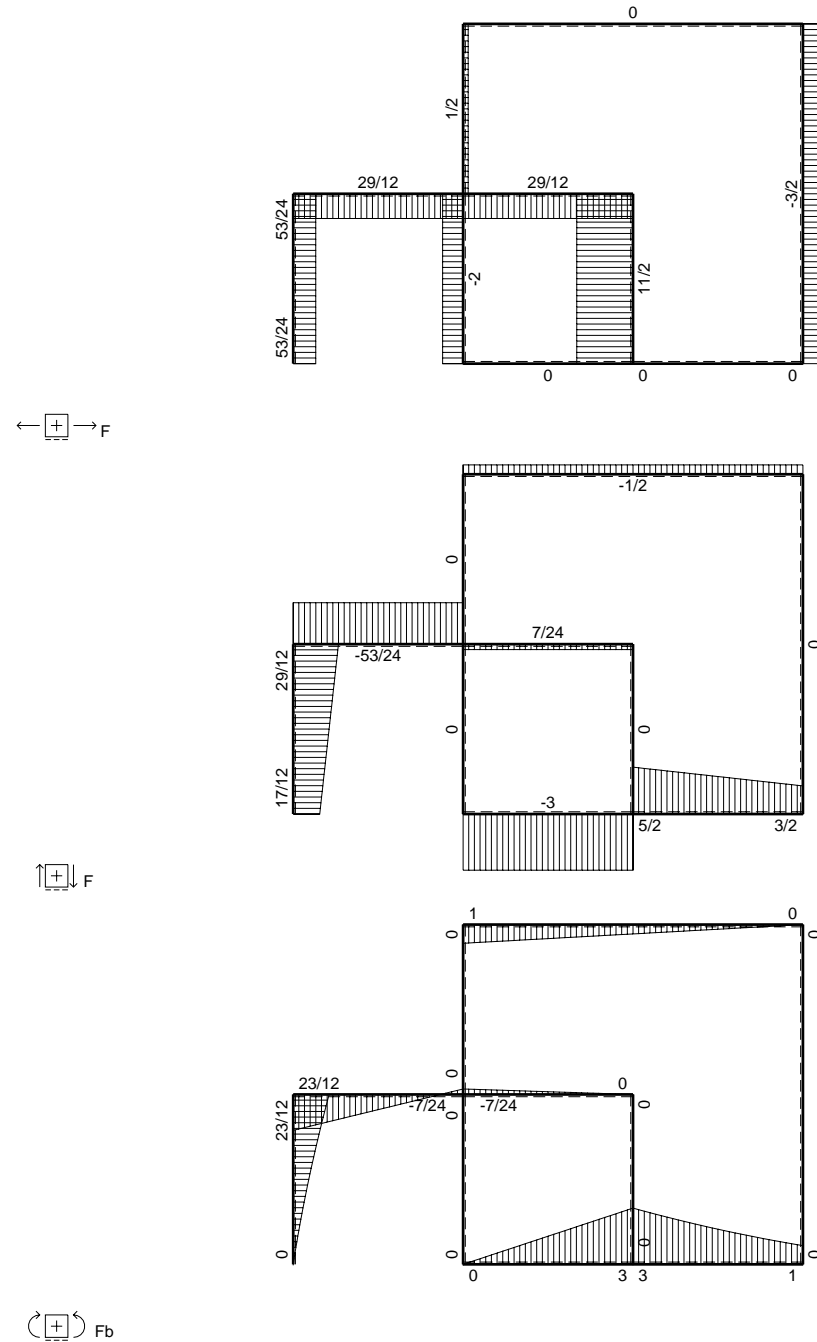
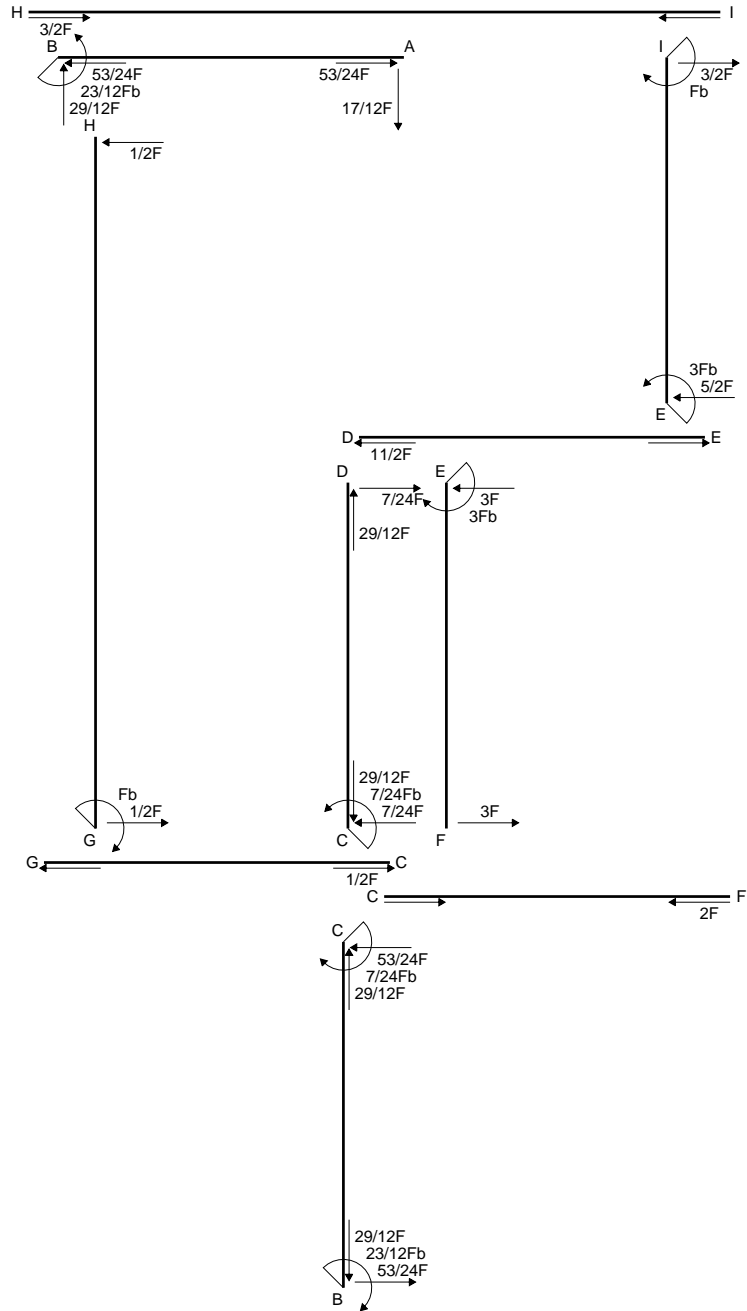
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

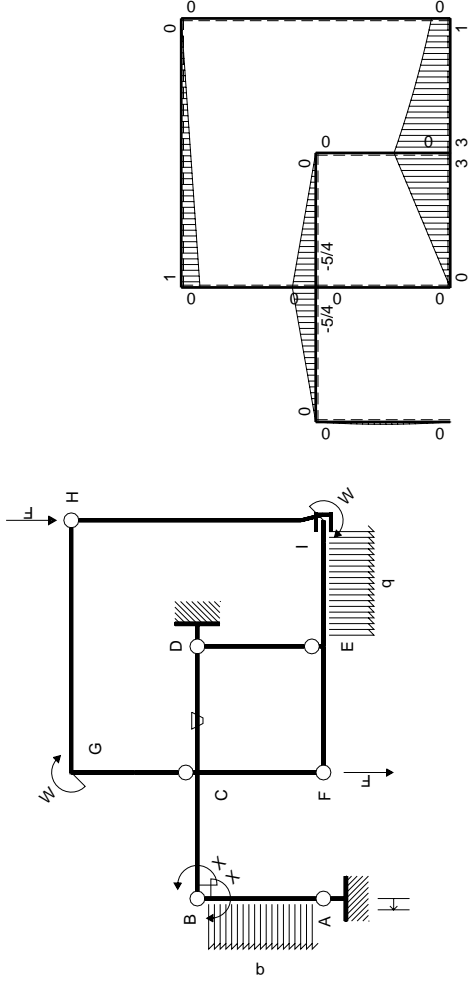
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                          |                     |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

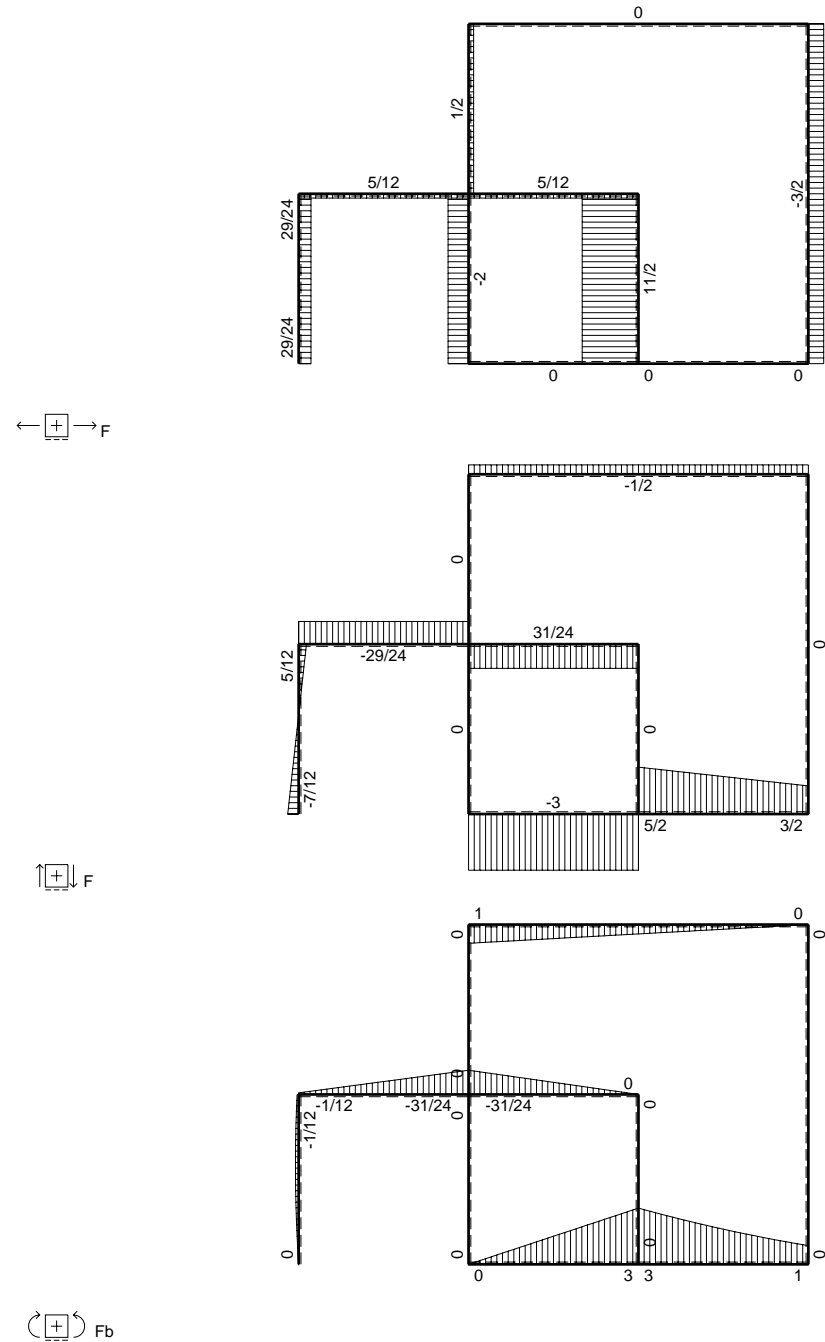
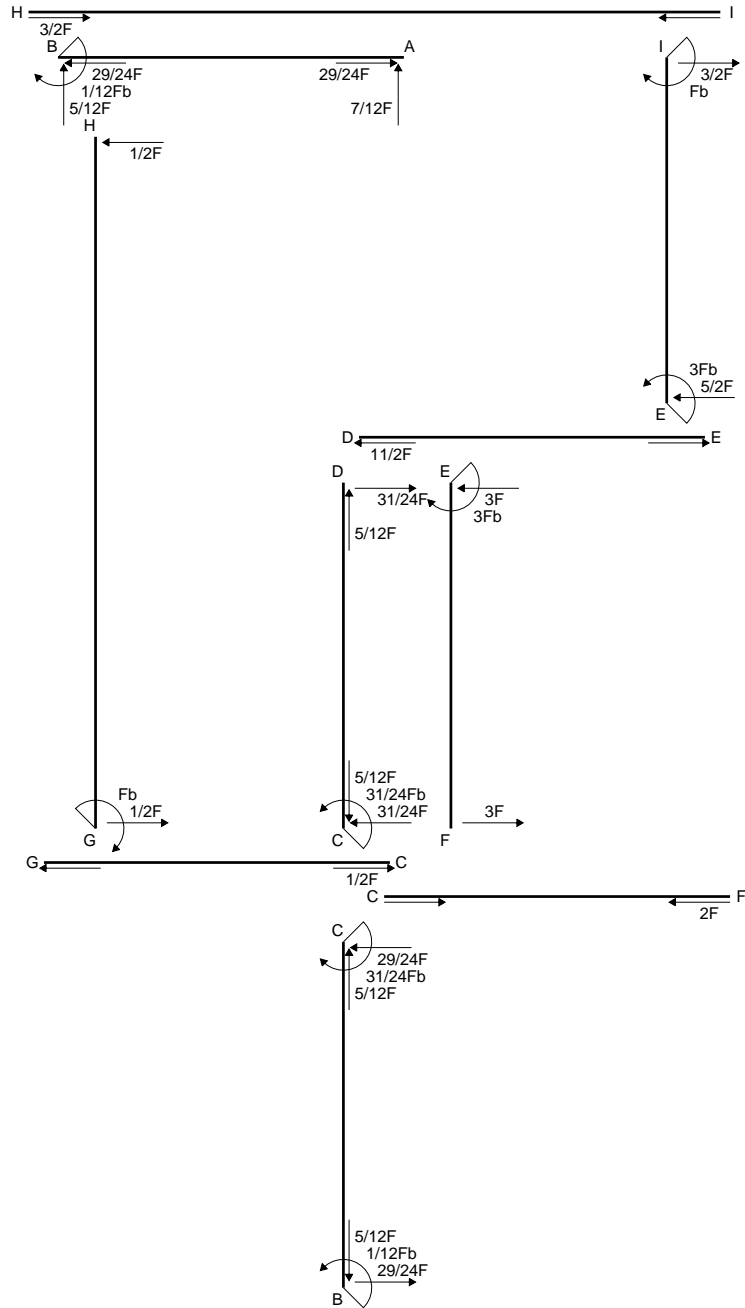
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

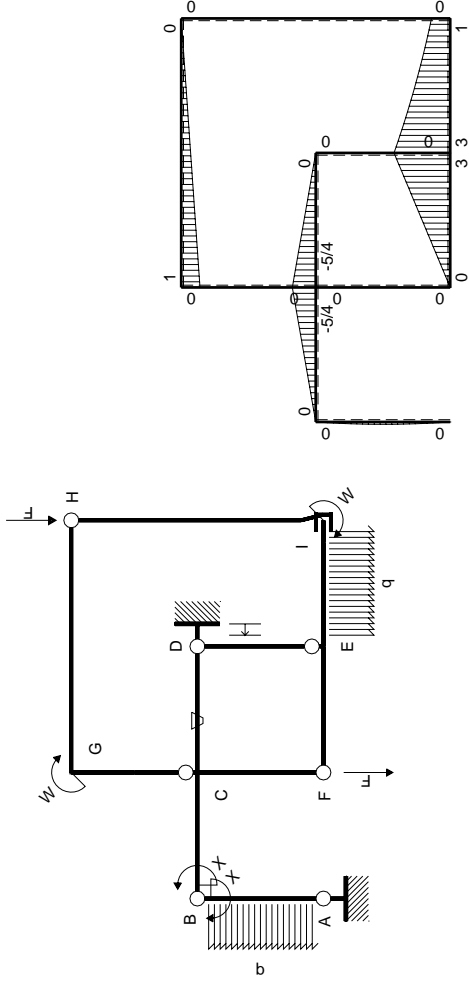
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                          |                     |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

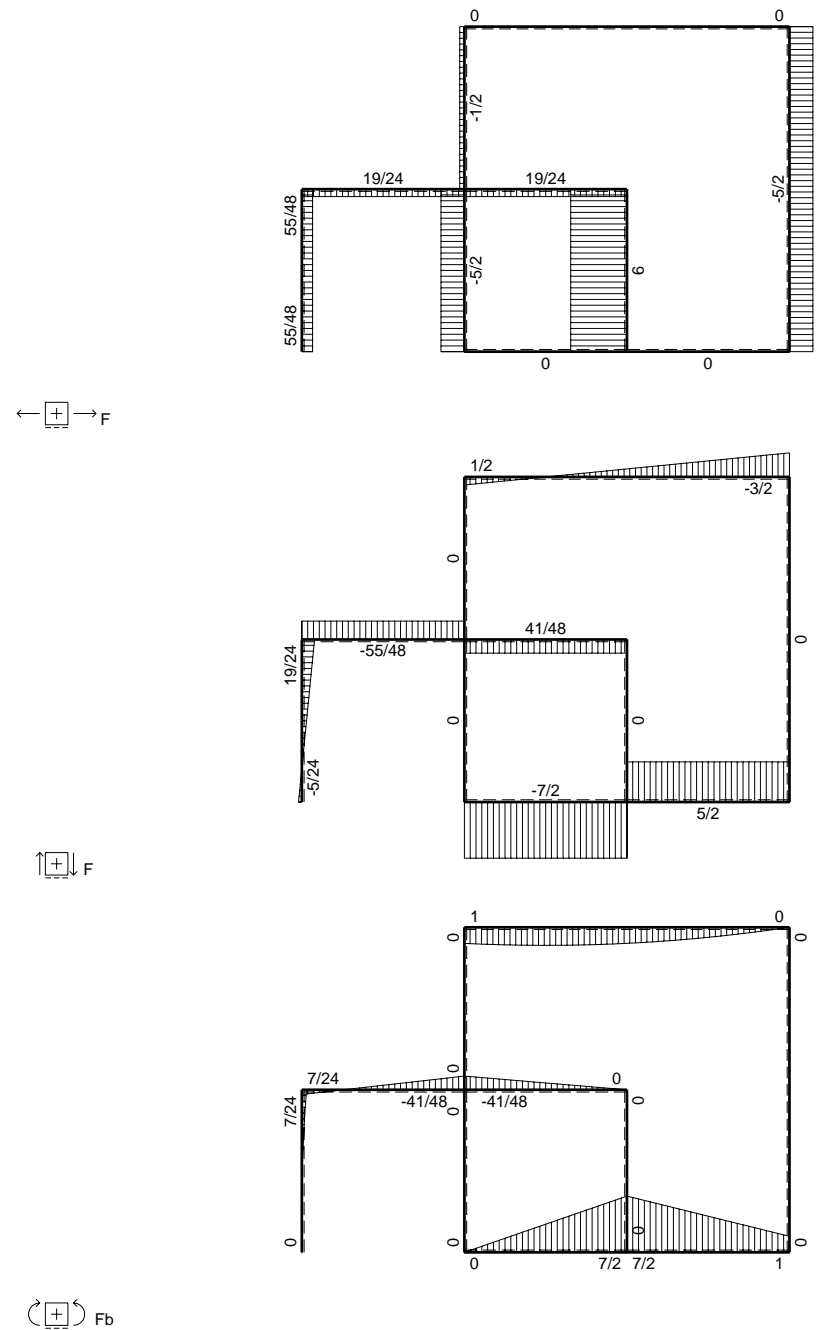
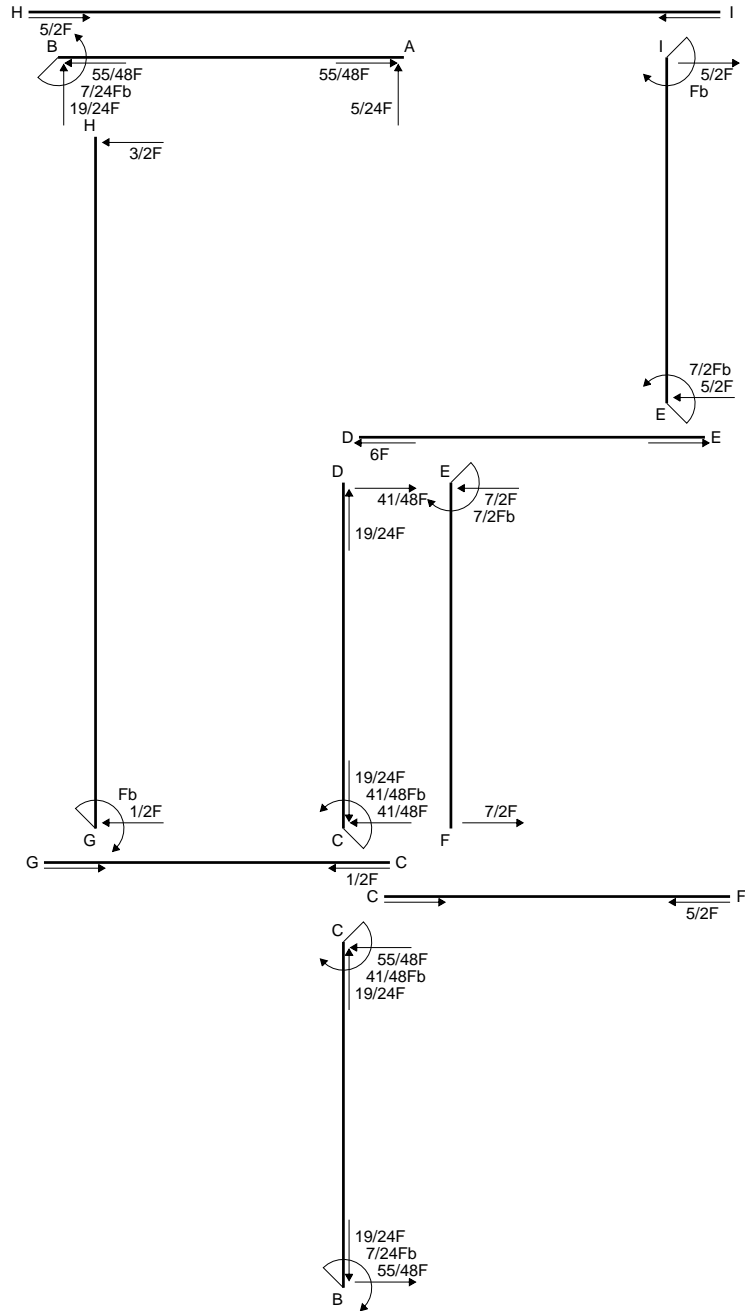
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

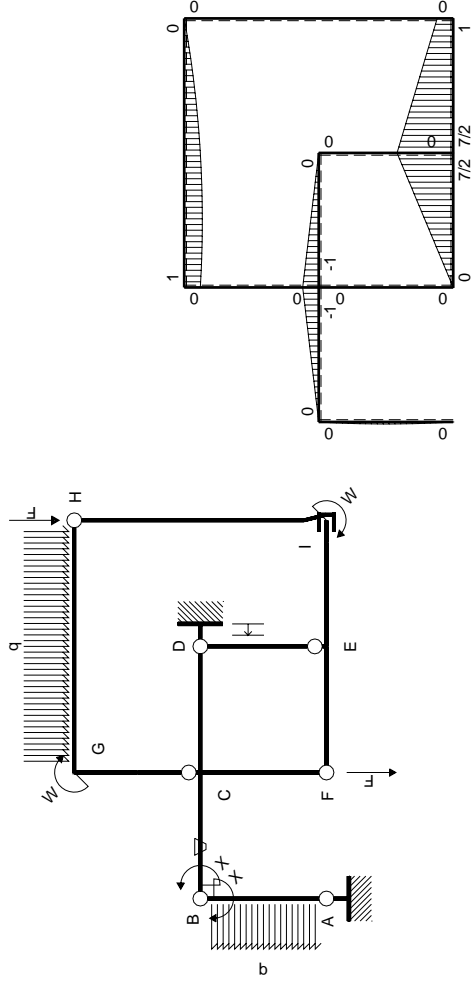
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

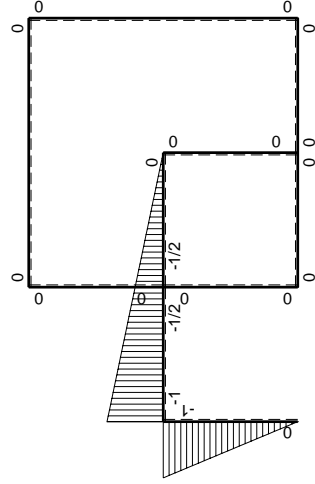
$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | $-Fb/EJ$ | $Fx-1/2Fx^2/b$           | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| ED b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| FC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| CF b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| CG b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| GC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |                     |                         | $-Fb^2/EJ$                  |                        |  |
|       | totali                      |                    |          |                          |                     |                         | $7/24Fb^2/EJ$               | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |                     |                         | $-7/24Fb$                   |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

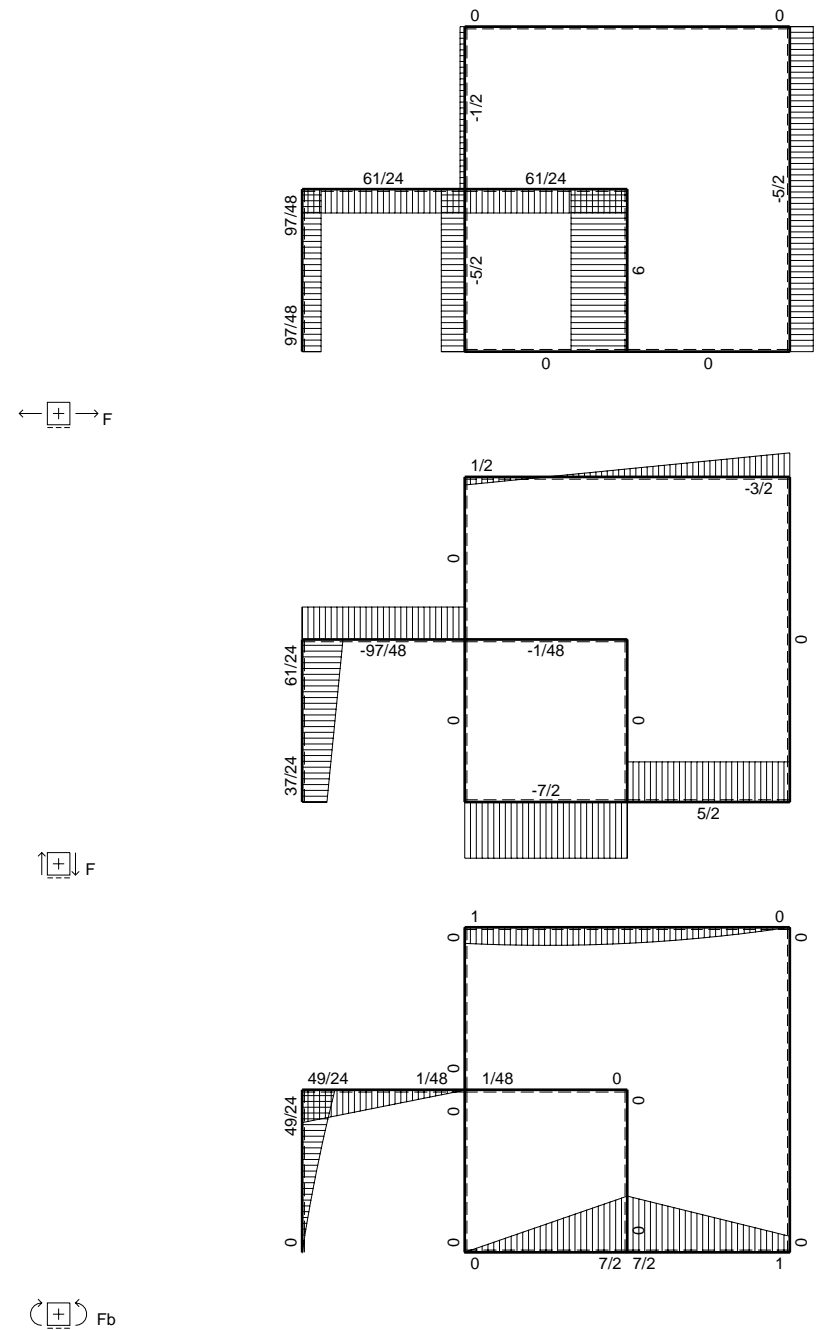
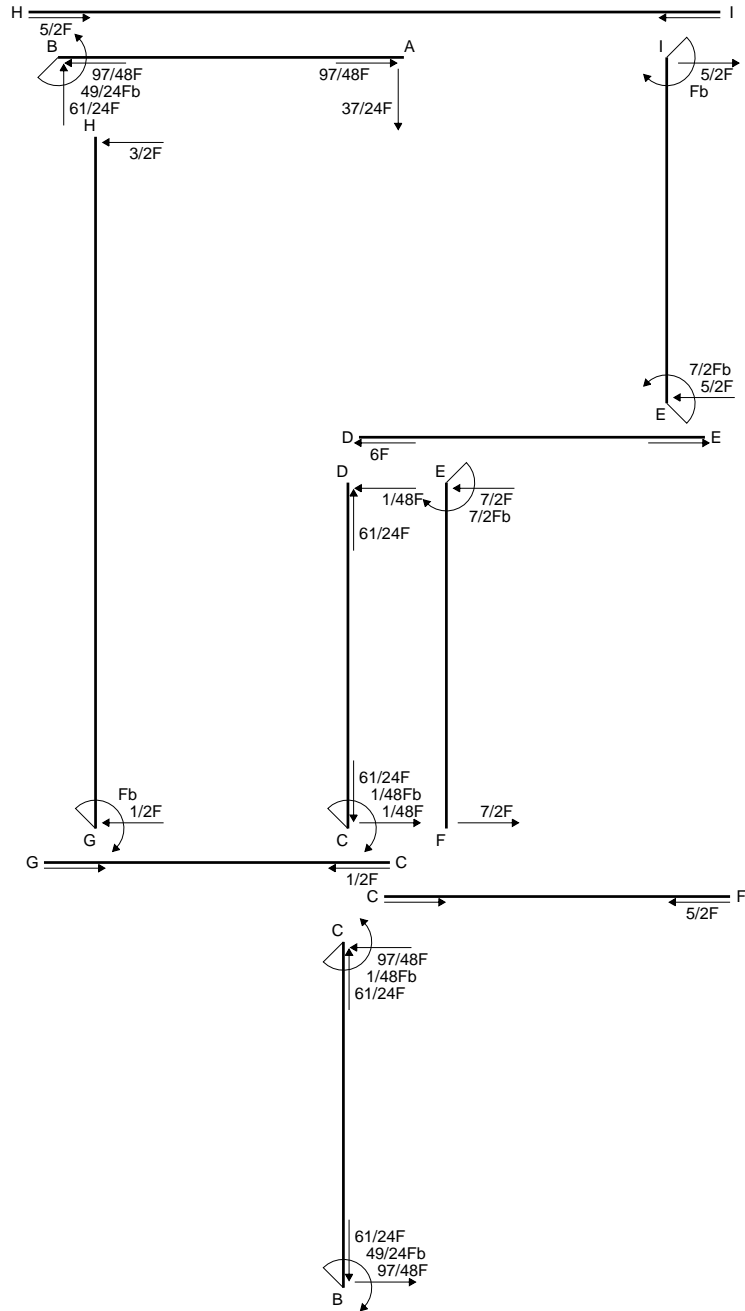
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

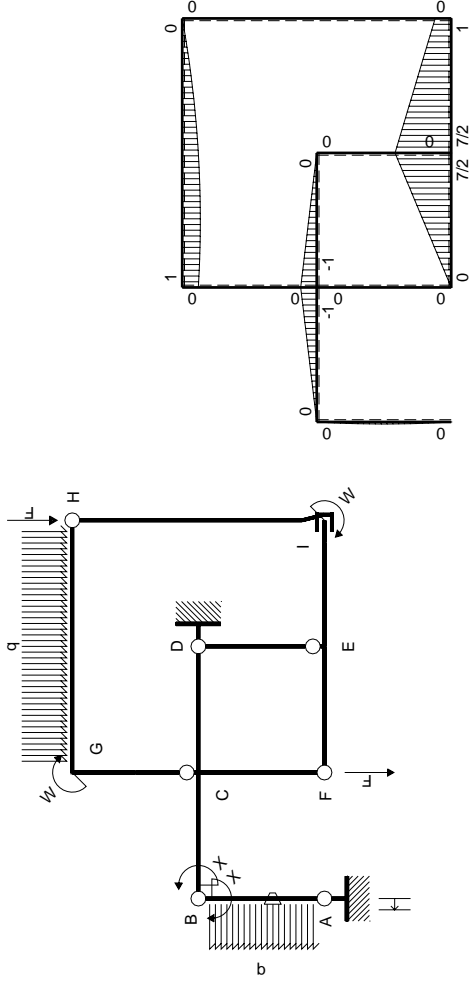
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$           |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$           |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |
| DE b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| ED b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0             | 0                       |                             |                       |
| FC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| CF b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |
| CG b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| GC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                       |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0             | 0                       |                             |                       |
| A     | cedimento nodo $-H_{1A}u_A$ |                    |          |                          |               |                         | $Fb^2/EJ$                   |                       |
|       | totali                      |                    |          |                          |               |                         | $49/24Fb^2/EJ$              | $Xb/EJ$               |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |               |                         | $-49/24Fb$                  |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

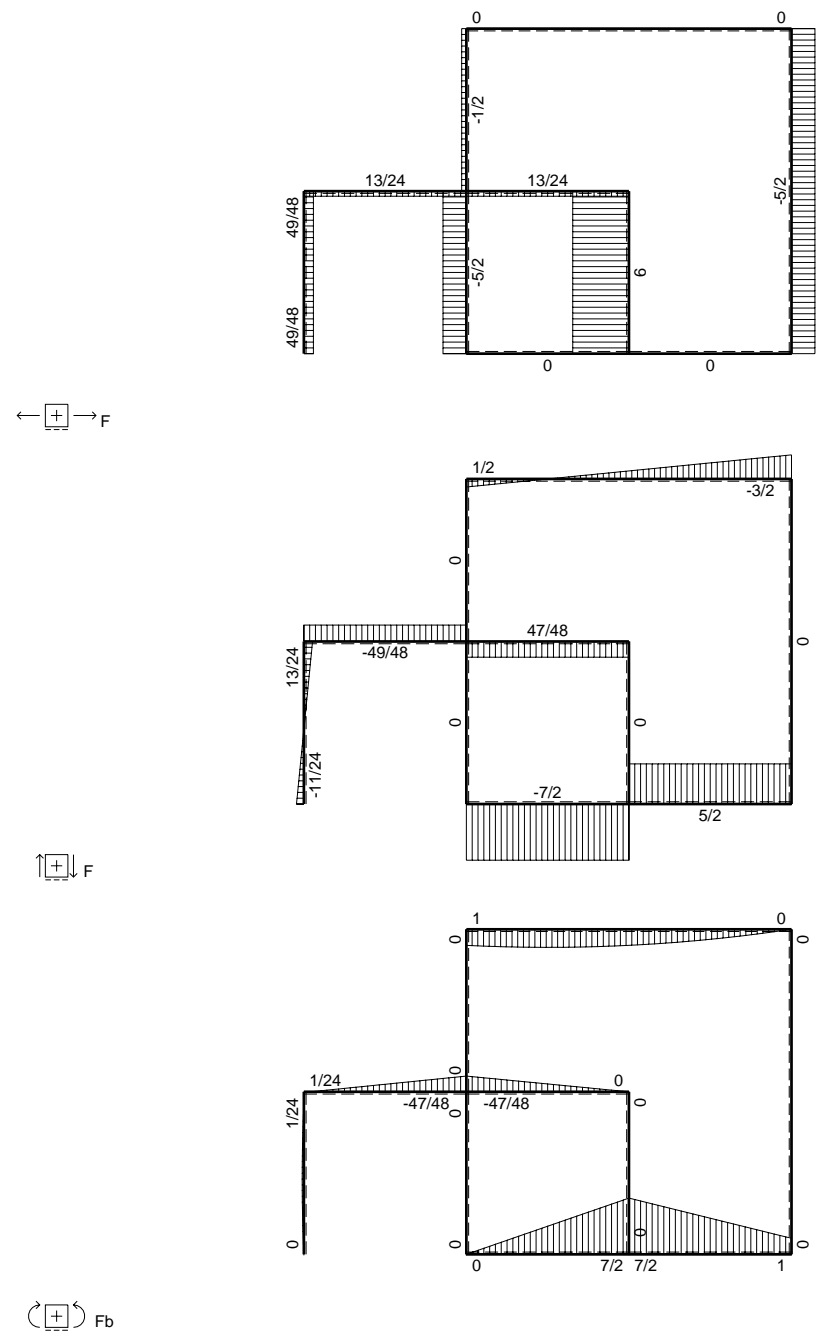
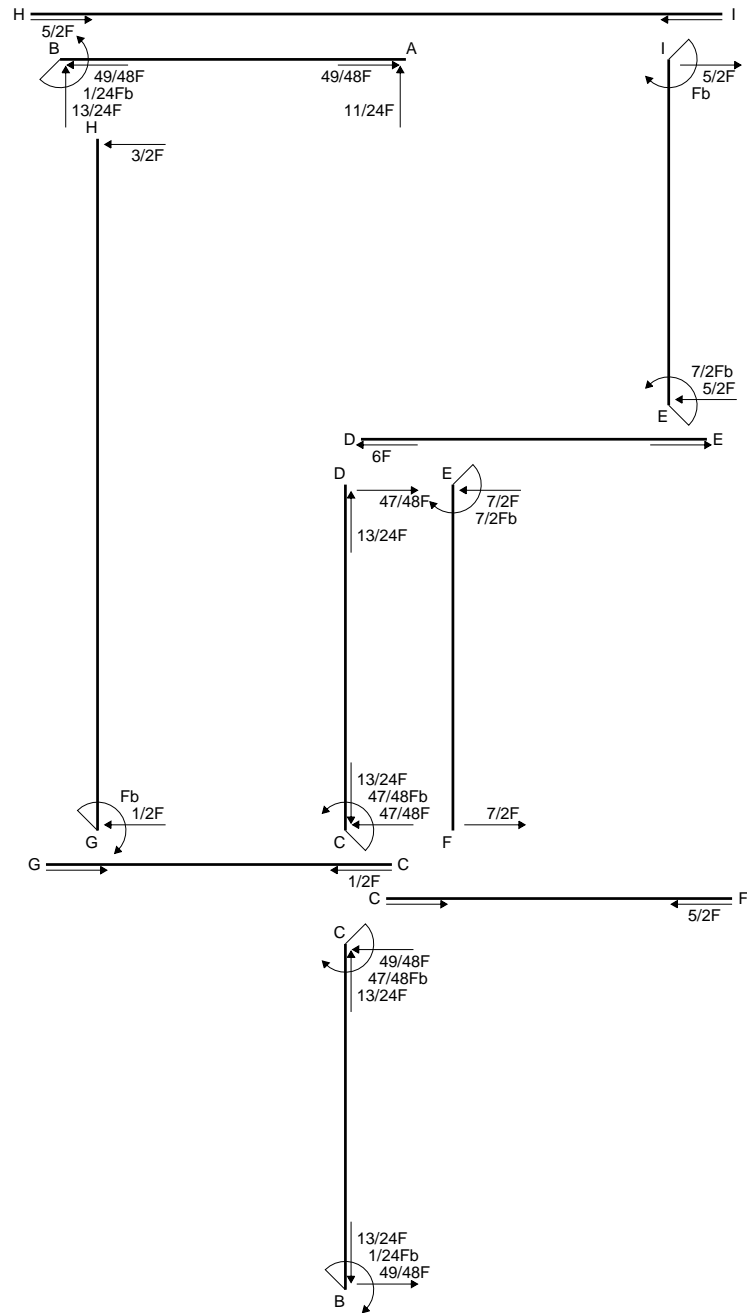
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

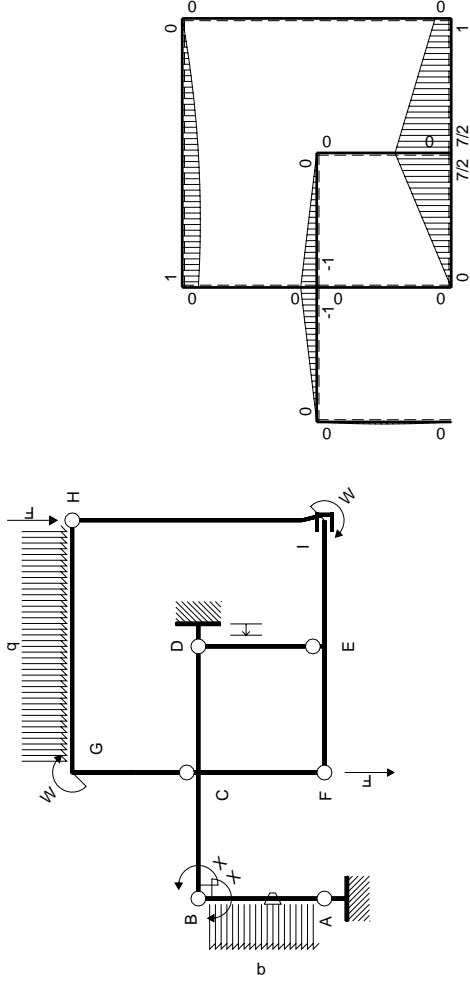
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |  |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$           |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$           |  |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |  |
| DE b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| ED b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0             | 0                       |                             |                       |  |
| FC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| CF b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| CG b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| GC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                       |  |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0             | 0                       |                             |                       |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |               |                         | $-Fb^2/EJ$                  |                       |  |
|       | totali                      |                    |          |                          |               |                         | $1/24Fb^2/EJ$               | $Xb/EJ$               |  |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |               |                         | $-1/24Fb$                   |                       |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

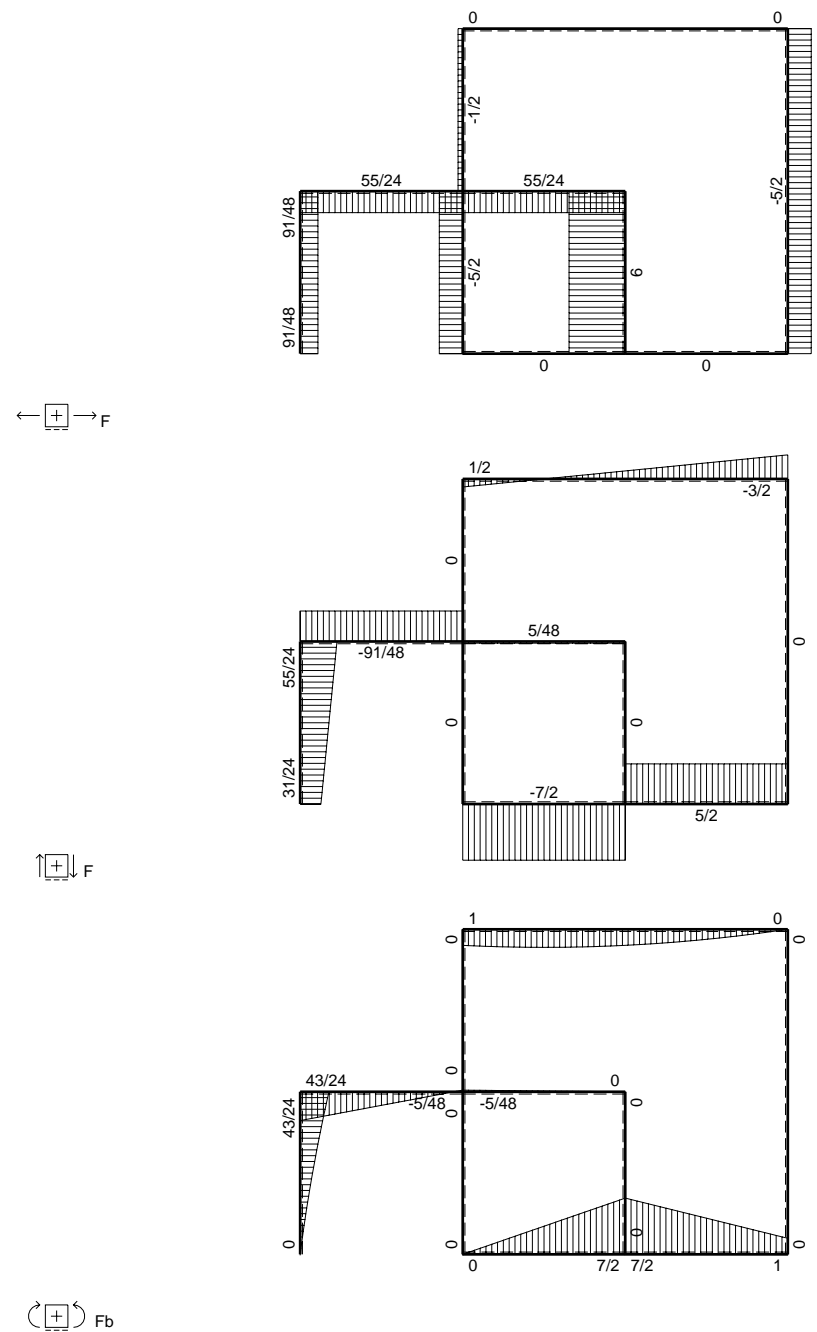
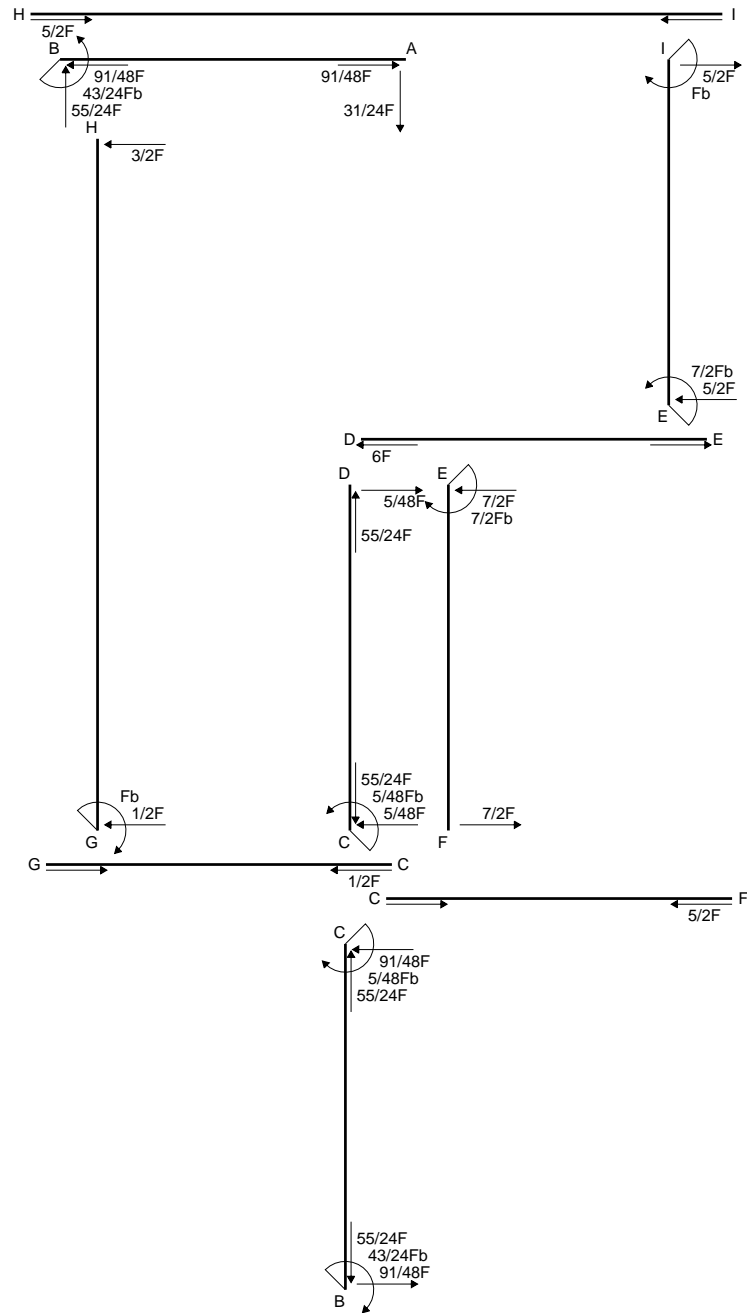
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

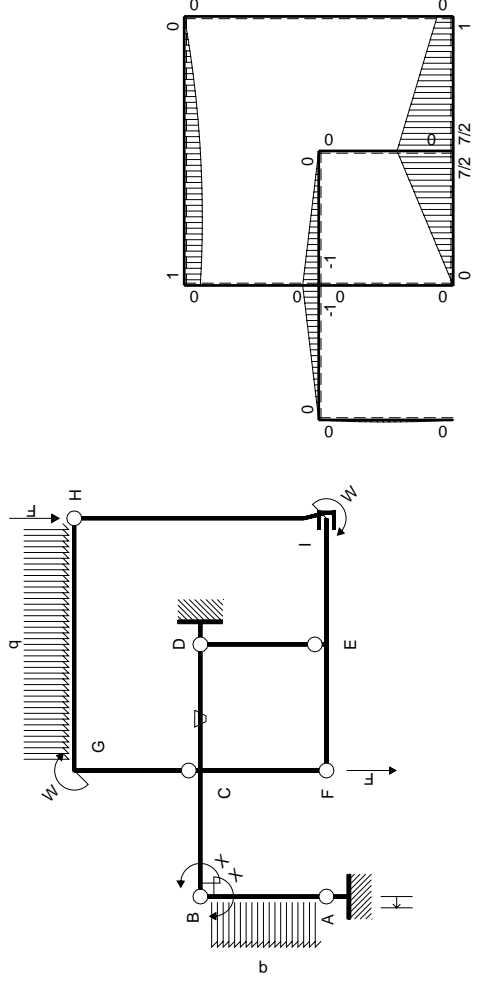
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

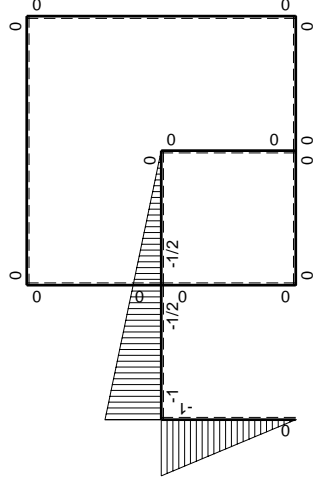






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$     | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$               | $Fb/EJ$  | $1/2Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                    |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                    |          |                          |                     |                         | $43/24Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |                     |                         | $-43/24Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

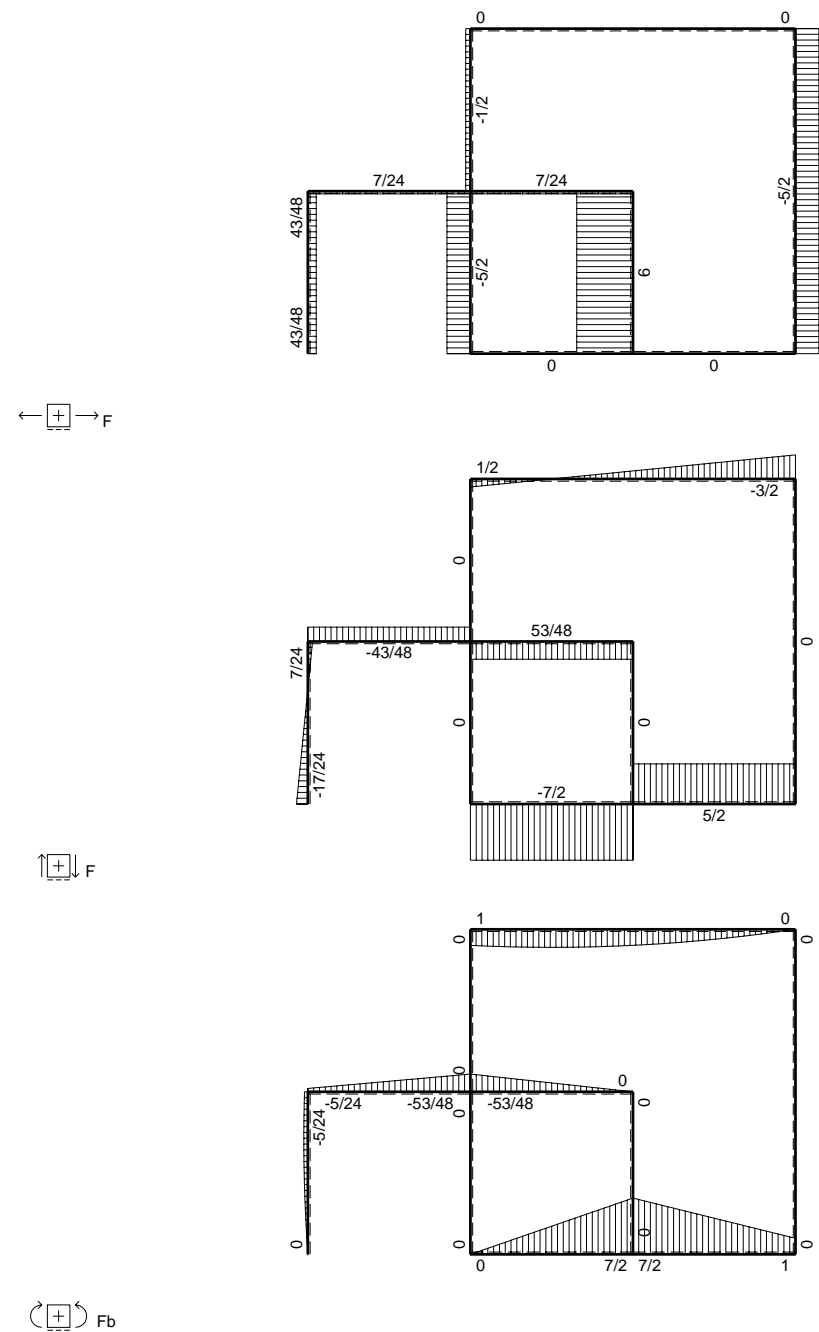
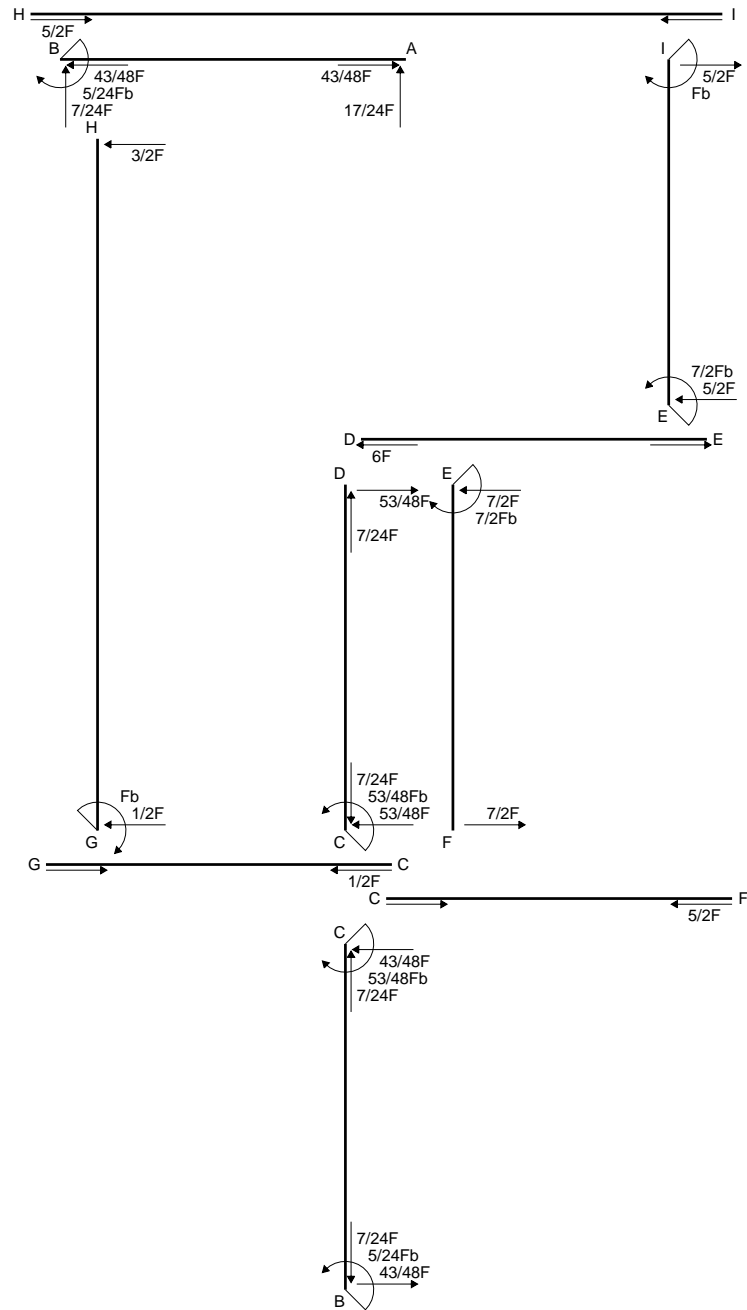
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

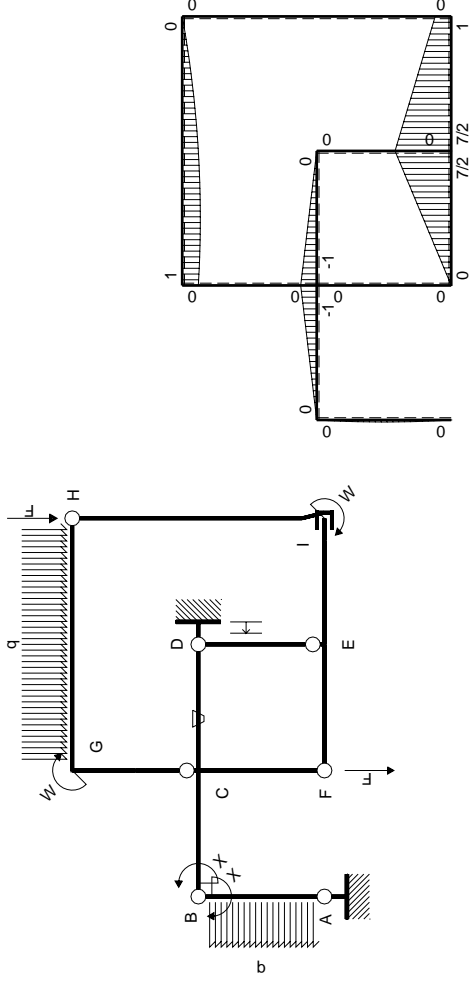
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$     | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$               | $Fb/EJ$  | $1/2Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                    |          |                          |                     |                         |                             | $-5/24Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |                     |                         |                             | $5/24Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

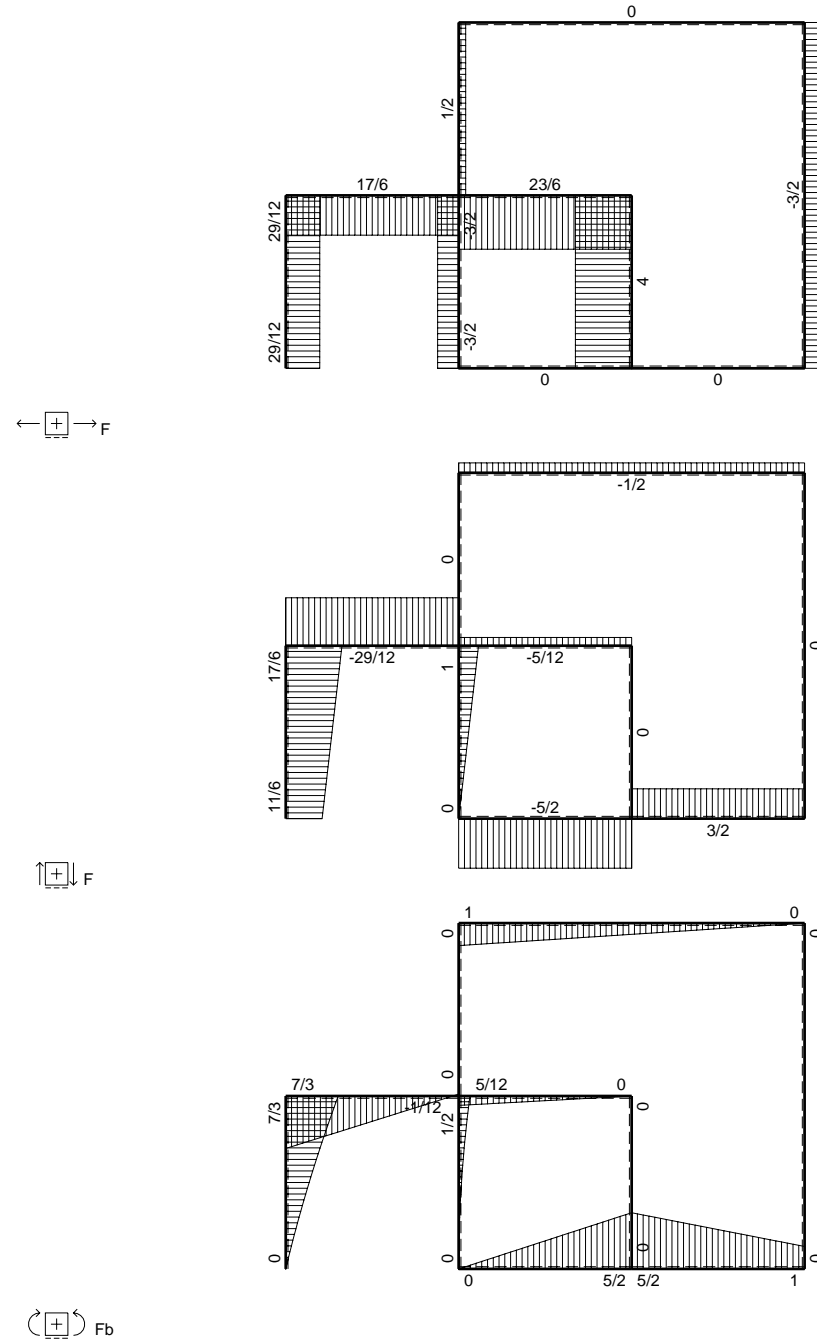
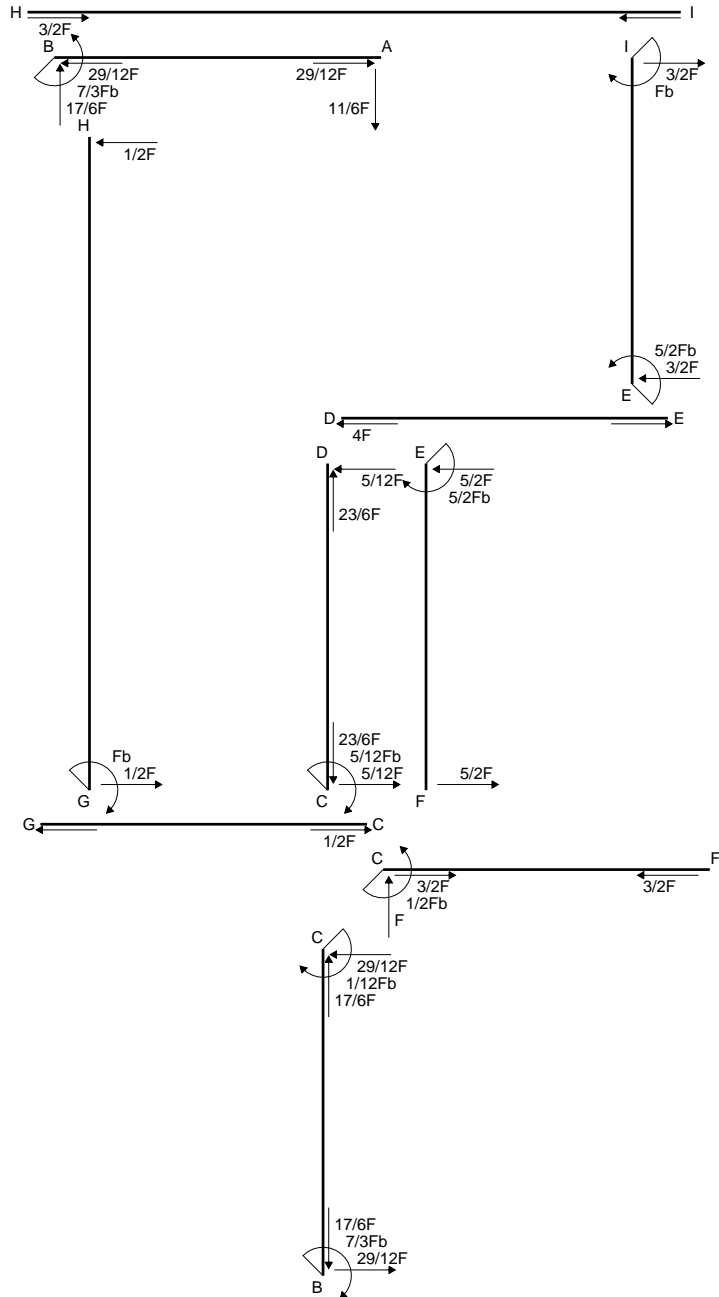
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

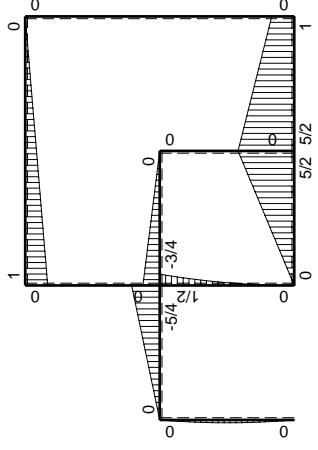
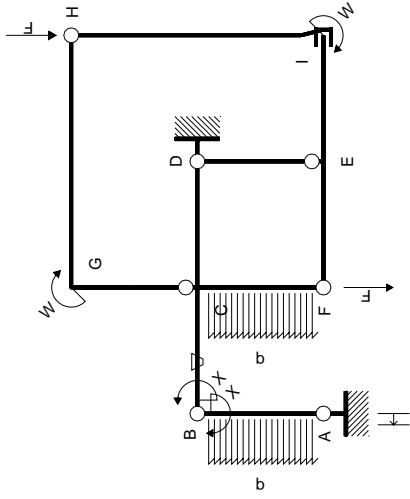
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$

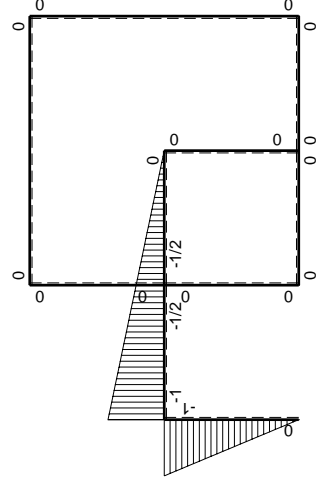






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                          |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                          |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                          |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                          |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                          |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                          |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                     |                         | $Fb^2/EJ$                   |                          |
|       | totali                      |                     |          |                          |                     |                         | $7/3Fb^2/EJ$                | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-7/3Fb$                    |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

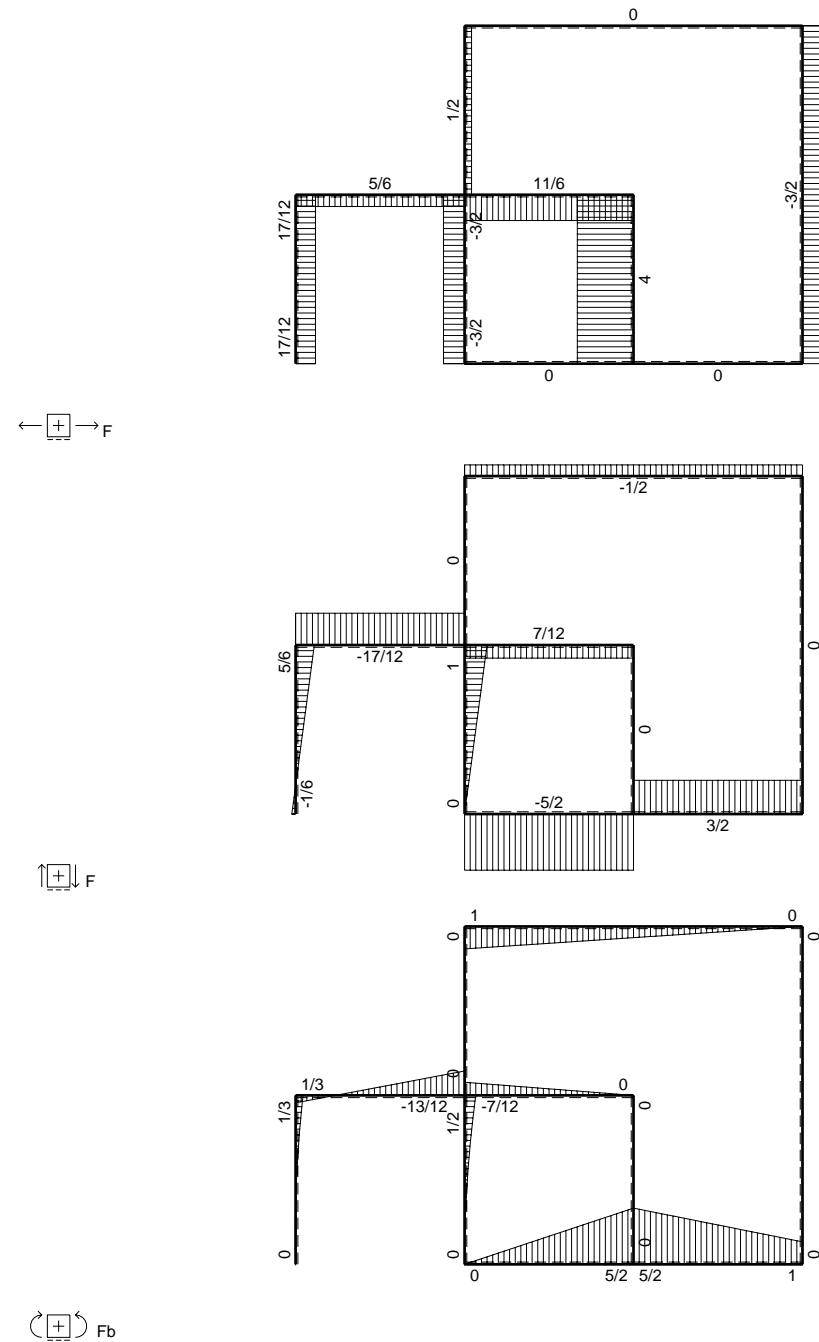
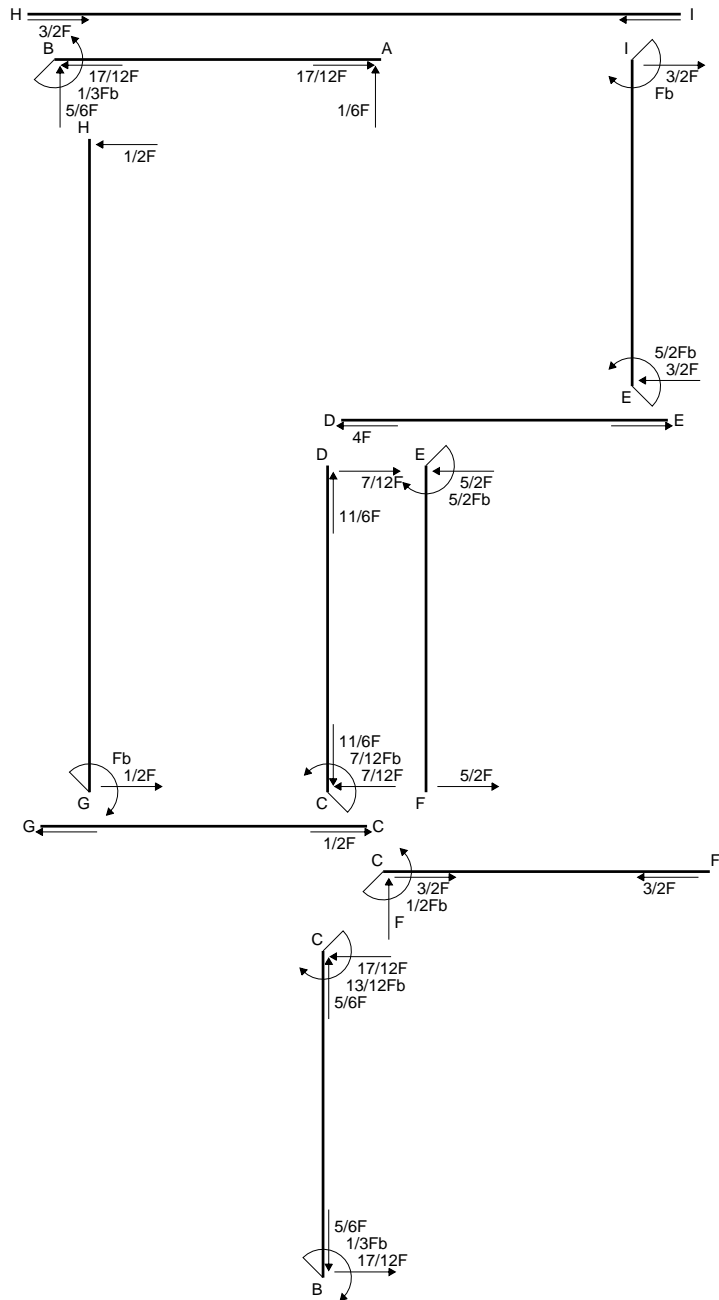
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

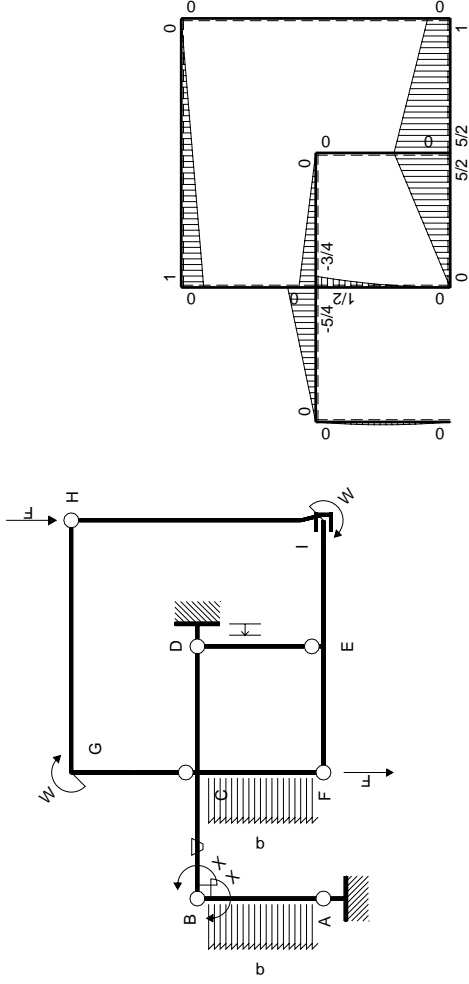
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_X$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                          |                     |                         | $1/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-1/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

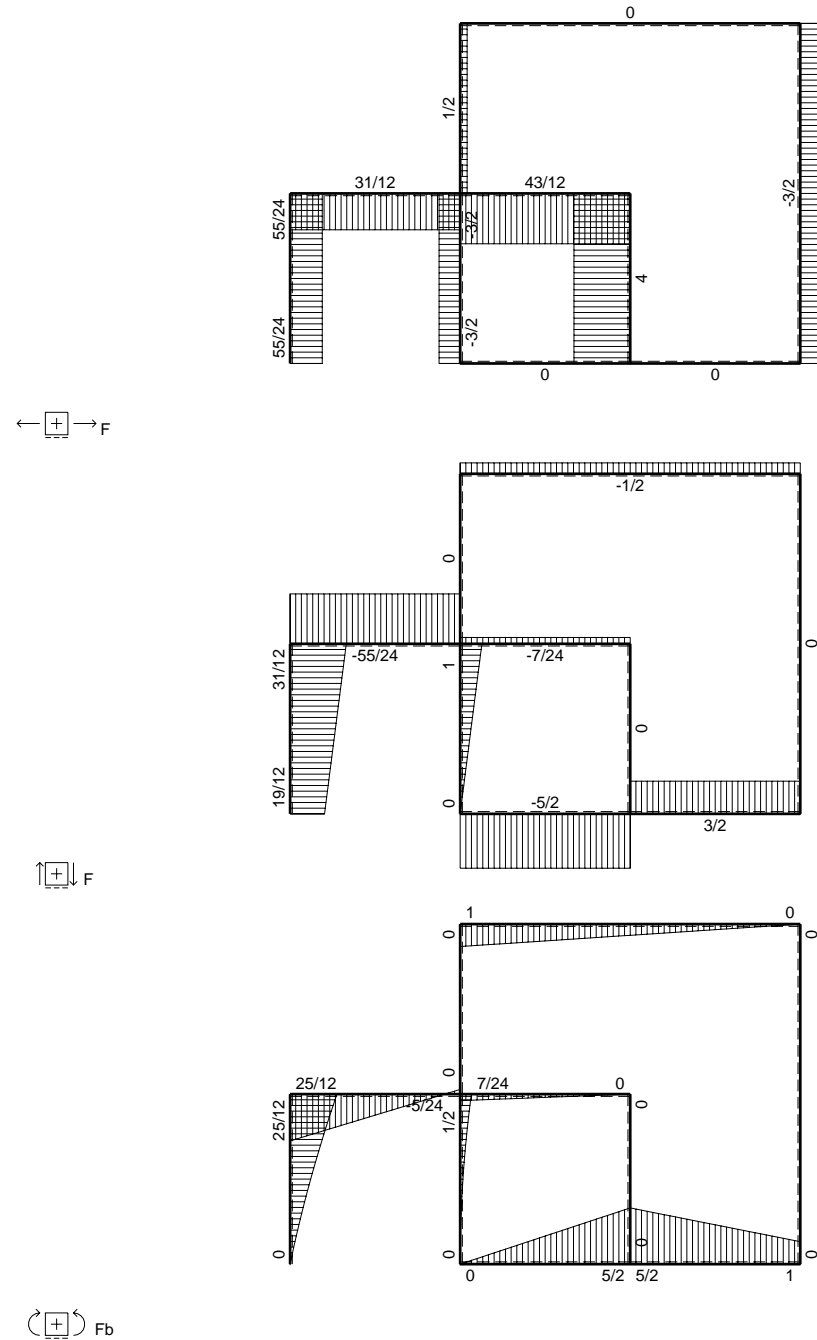
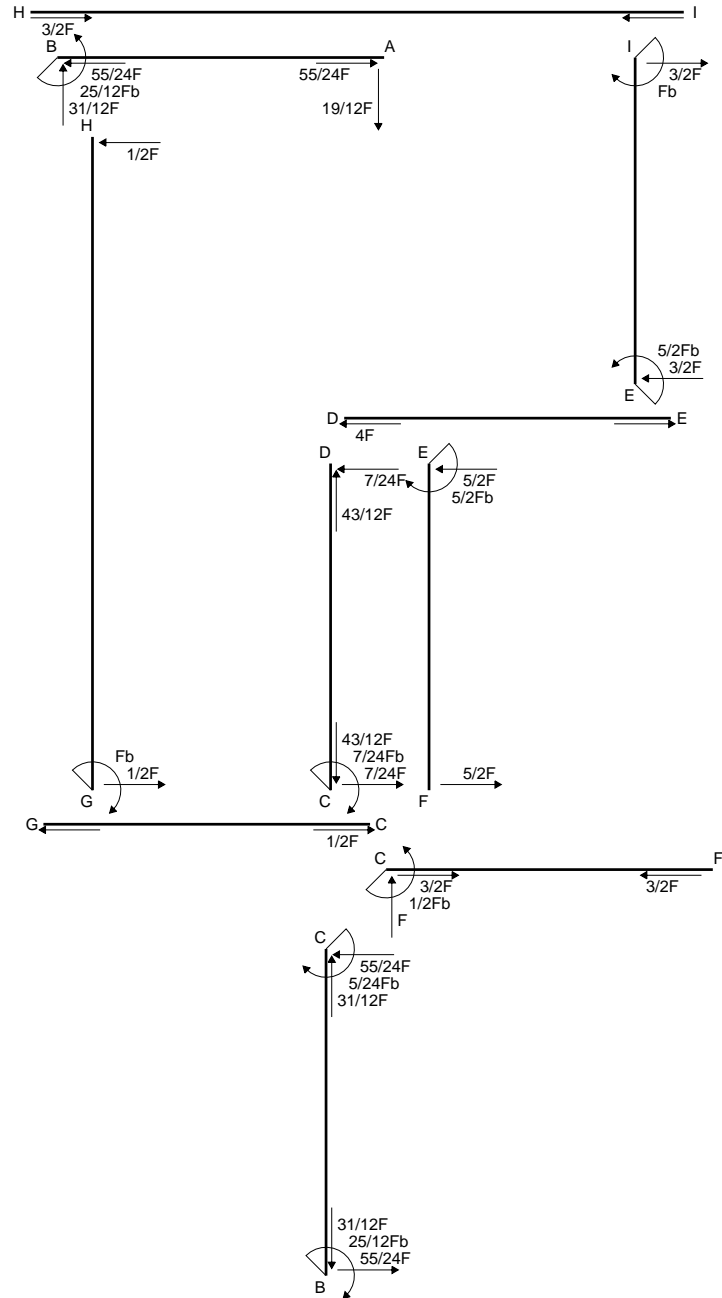
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

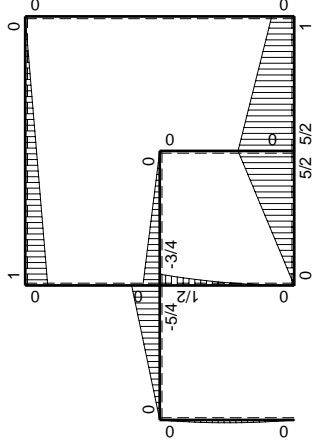
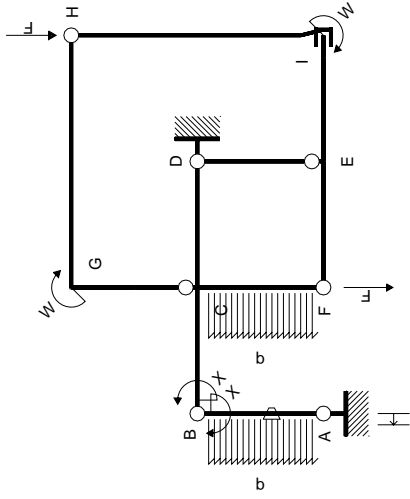
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

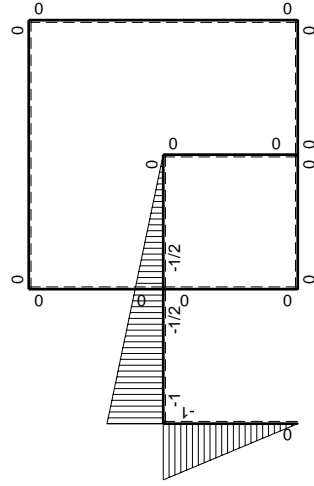






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$           |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$           |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |
| DE b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| ED b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                       |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                       |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                       |
| CG b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| GC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                       |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                       |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                       |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0             | 0                       |                             |                       |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |               |                         | $Fb^2/EJ$                   |                       |
|       | totali                      |                     |          |                          |               |                         | $25/12Fb^2/EJ$              | $Xb/EJ$               |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |               |                         | $-25/12Fb$                  |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

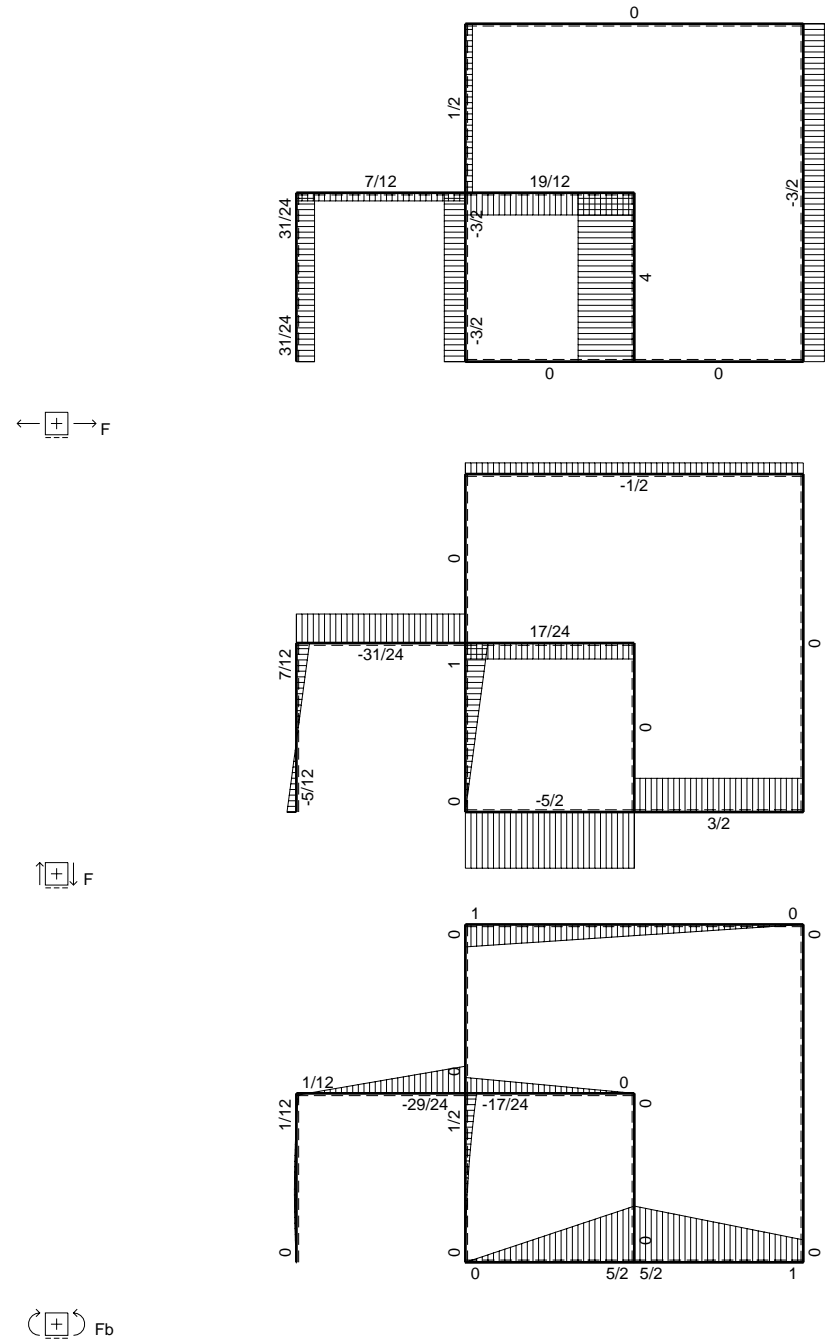
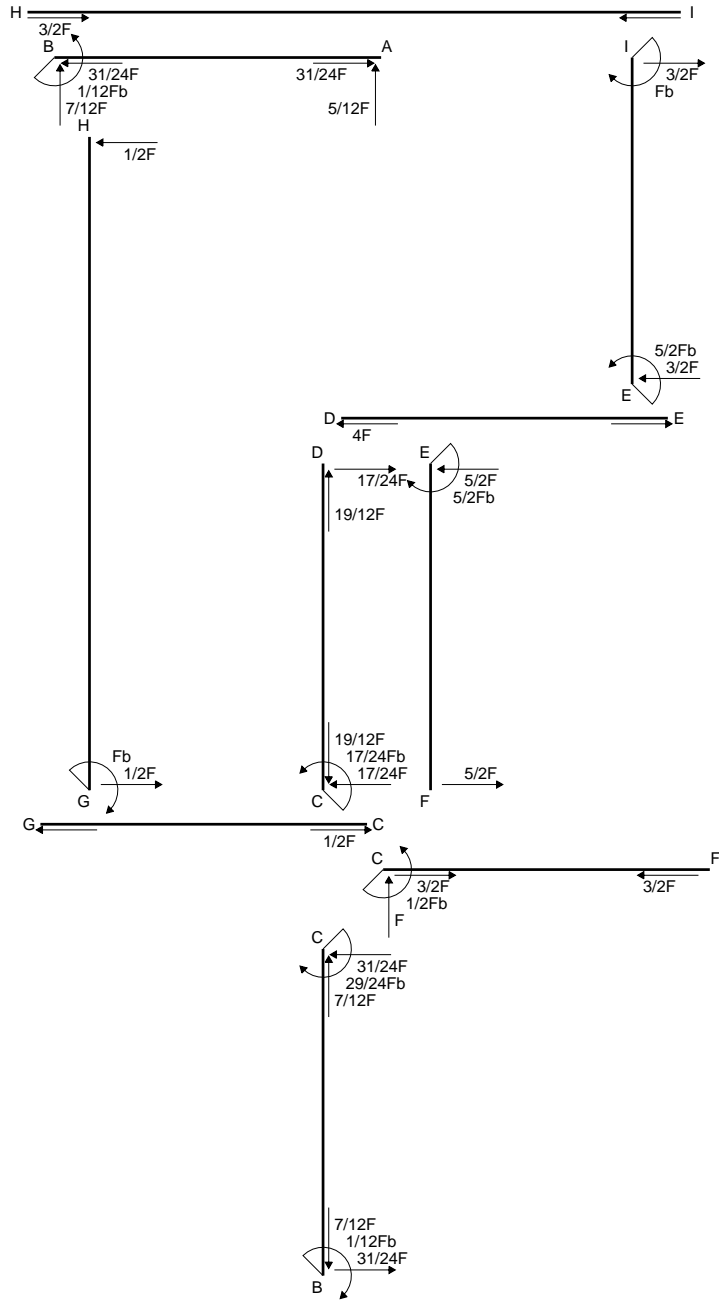
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

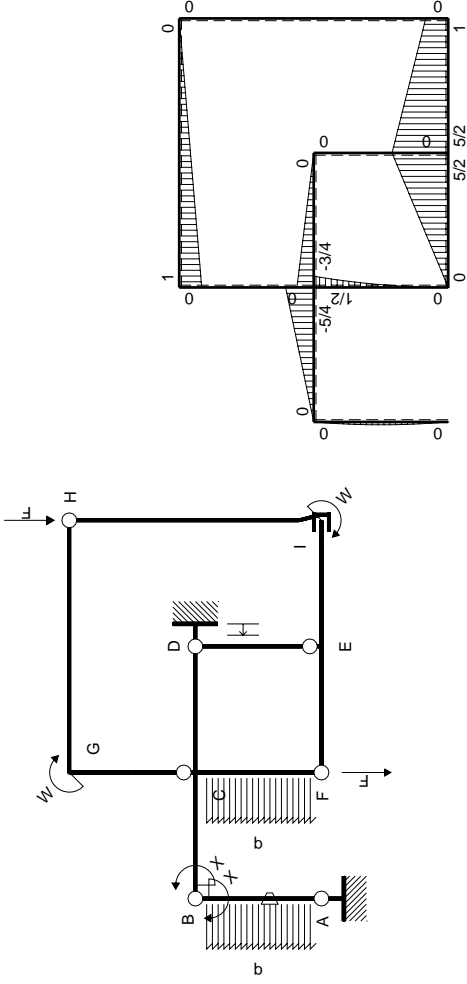
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |         |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$           |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$           |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |         |
| DE b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| ED b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                       |         |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                       |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                       |         |
| CG b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| GC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                       |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                       |         |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                       |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0             | 0                       |                             |                       |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |               |                         |                             | $-Fb^2/EJ$            |         |
|       | totali                      |                     |          |                          |               |                         |                             | $1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |               |                         |                             | $-1/12Fb$             |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

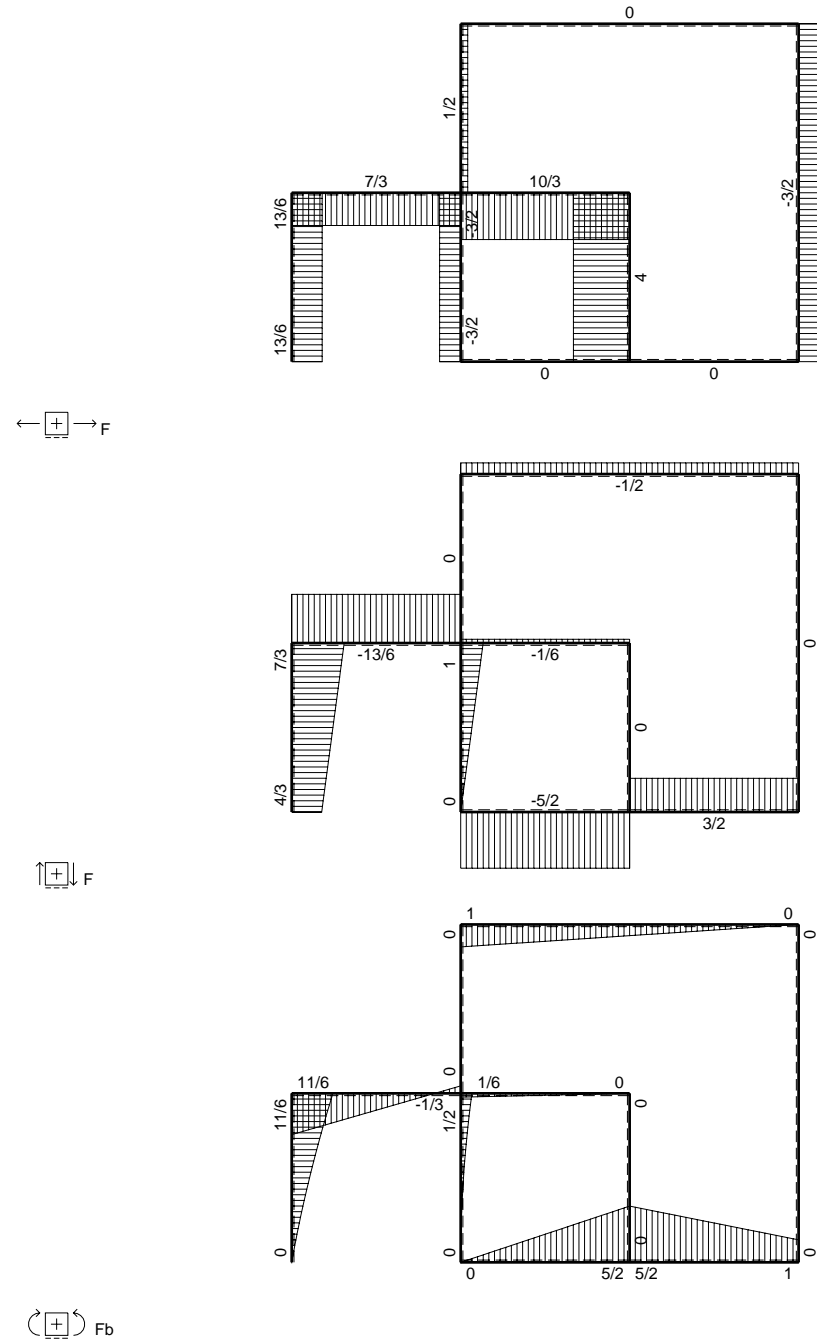
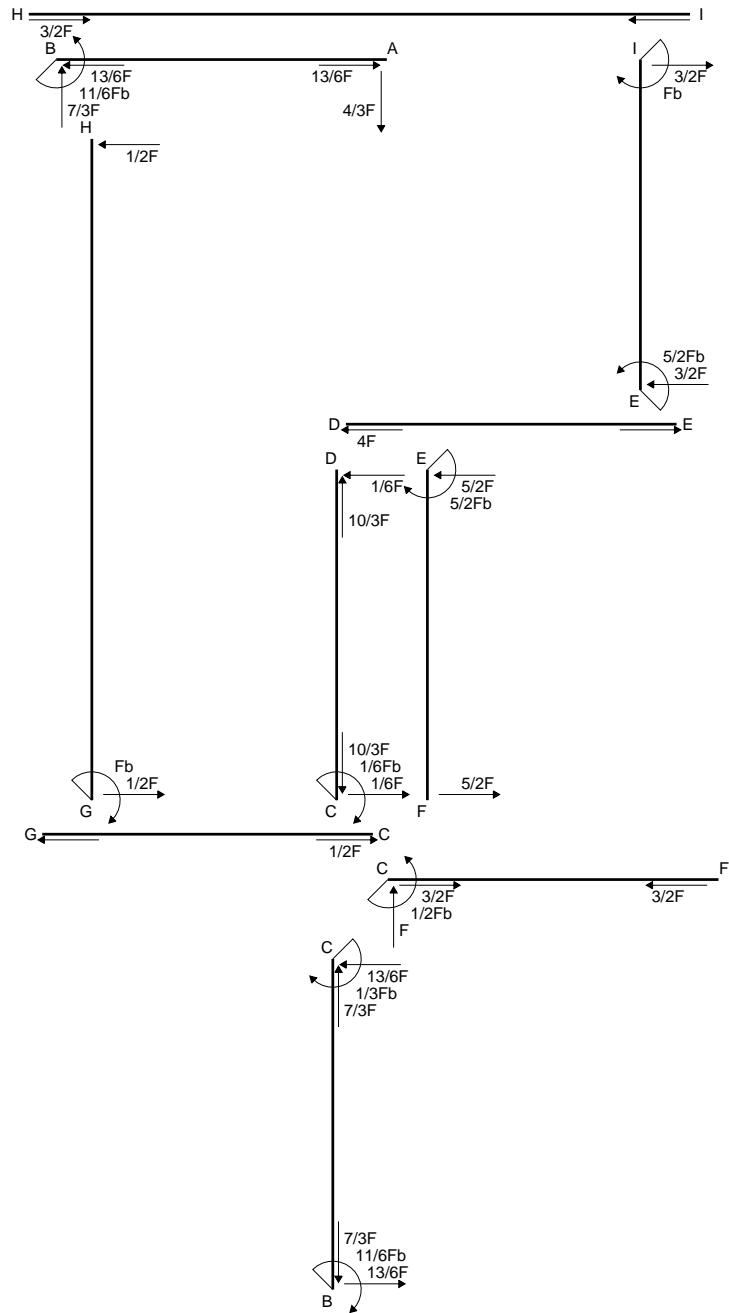
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

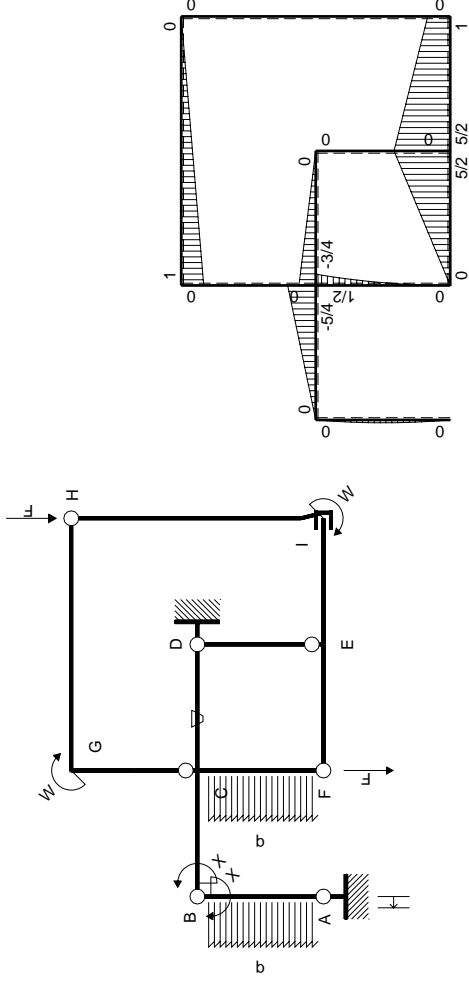
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |                     |                         | $11/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-11/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

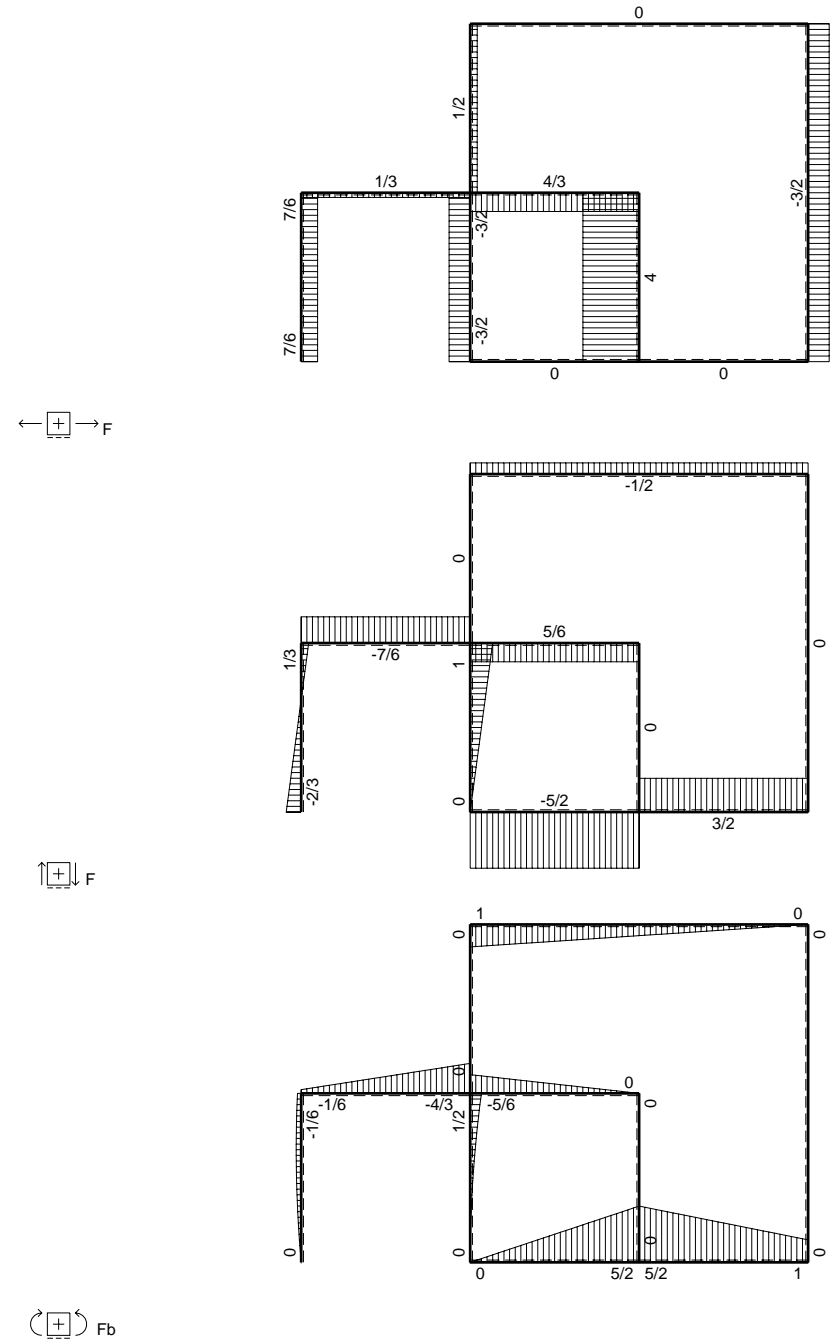
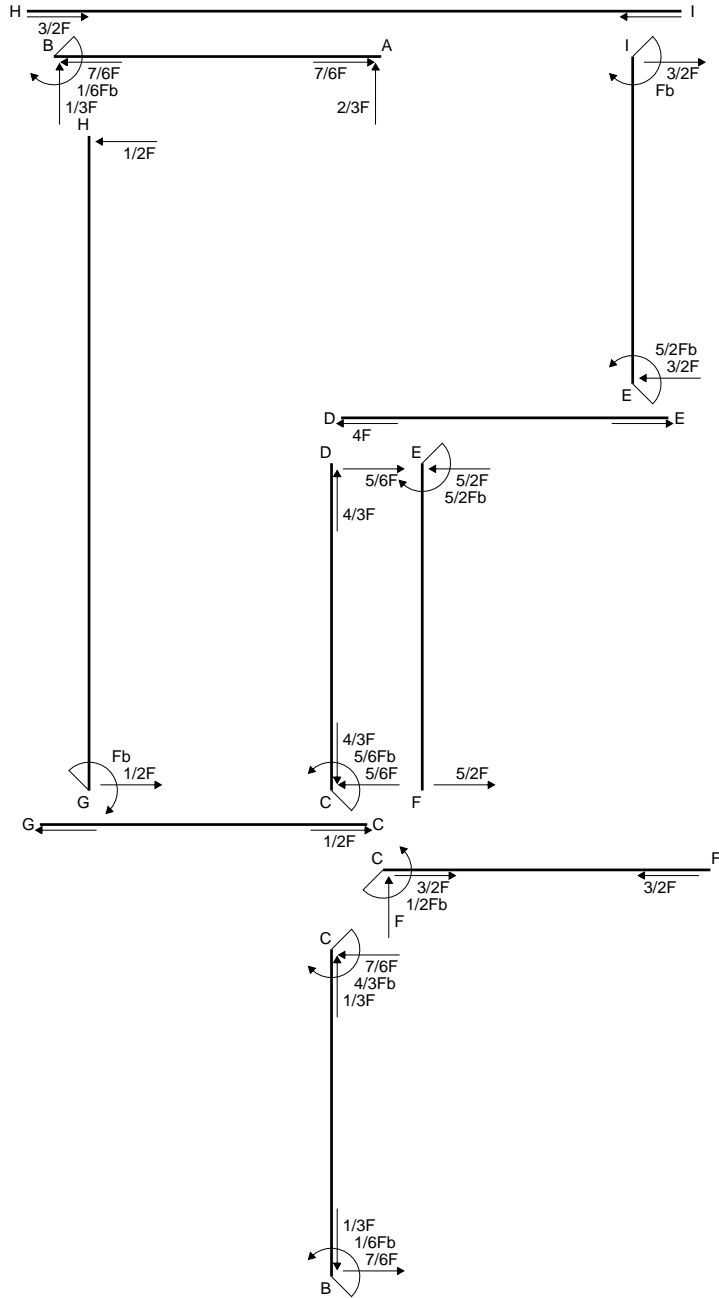
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

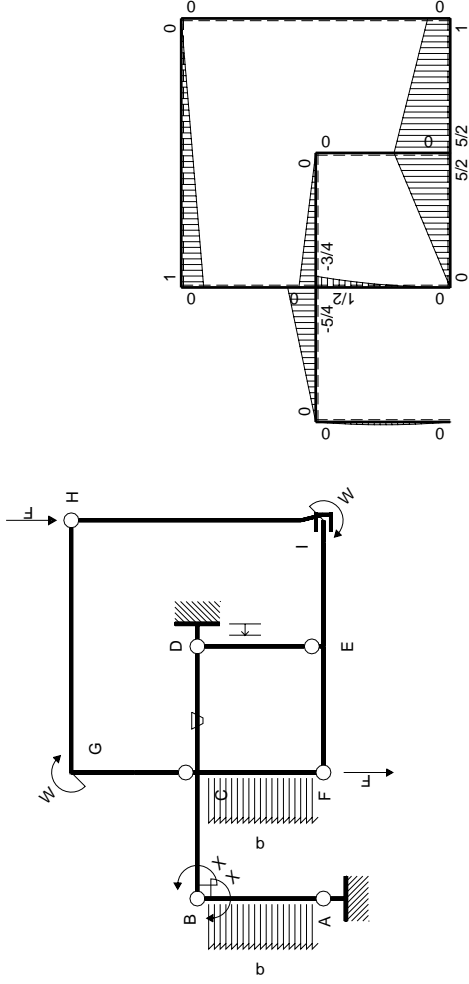
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-5/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-5/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                          |                     |                         |                             | $-1/6Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         |                             | $1/6Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

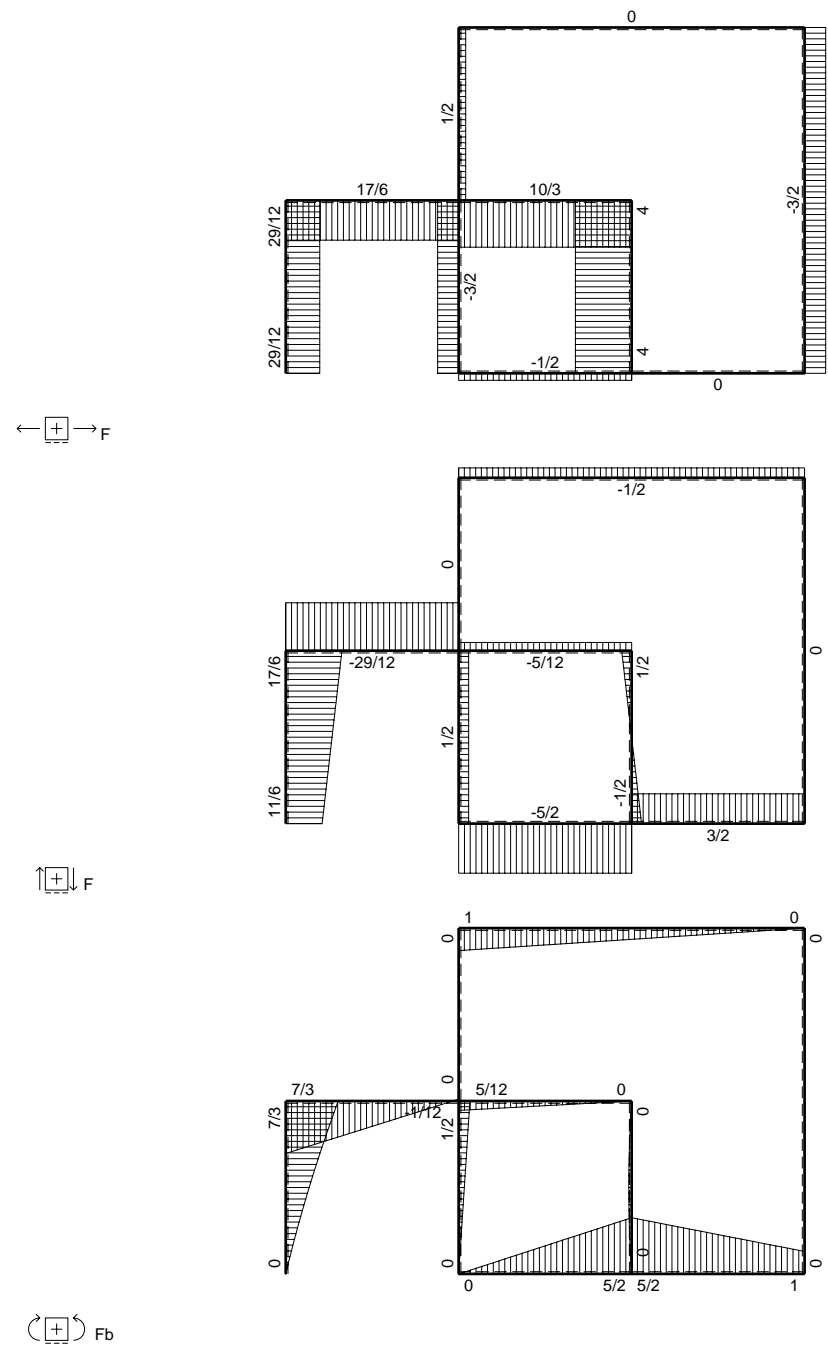
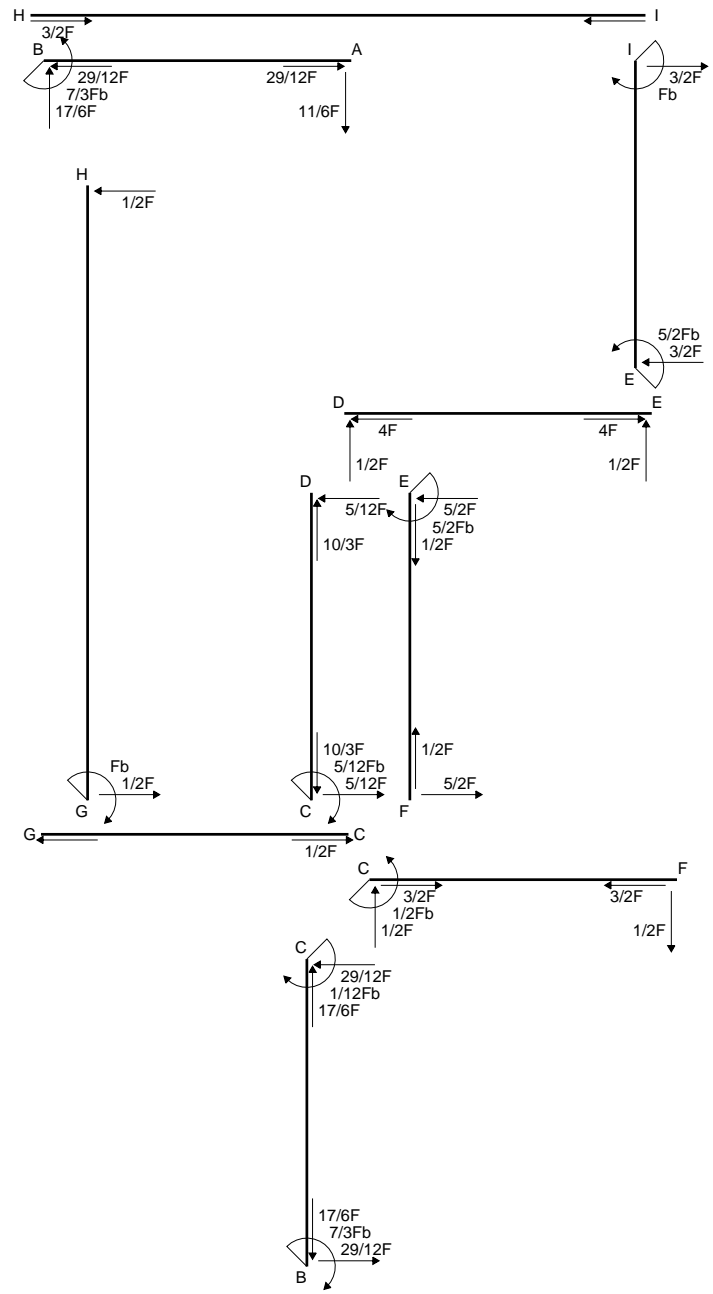
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

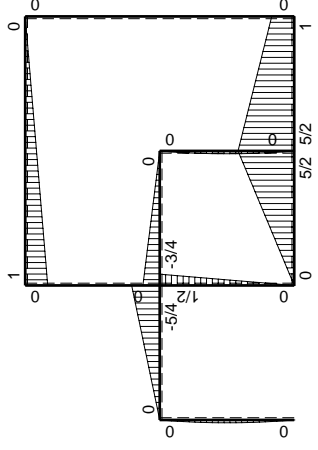
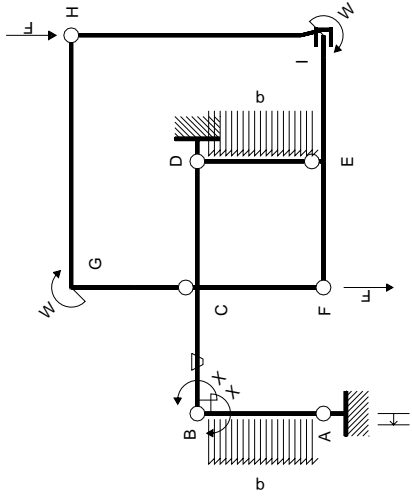
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

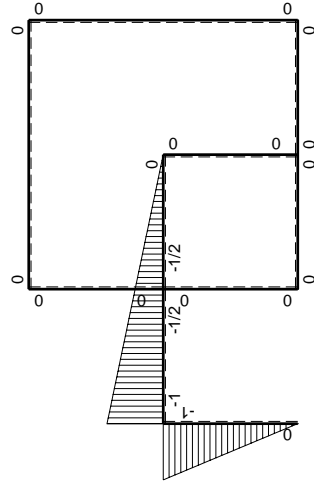






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-5/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                          |                     |                         | $7/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $-7/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

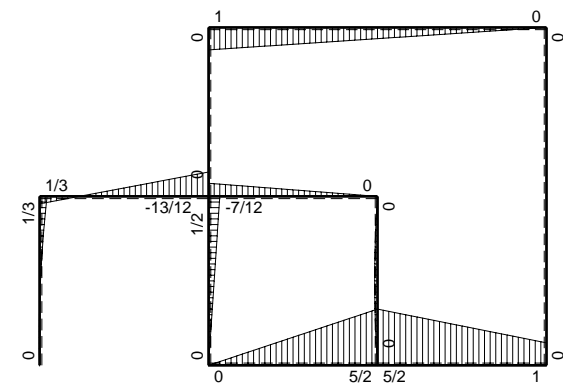
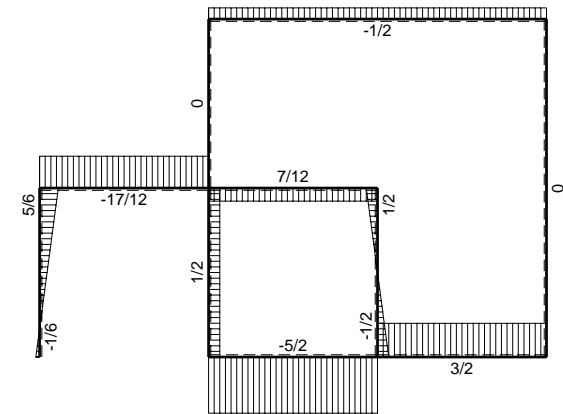
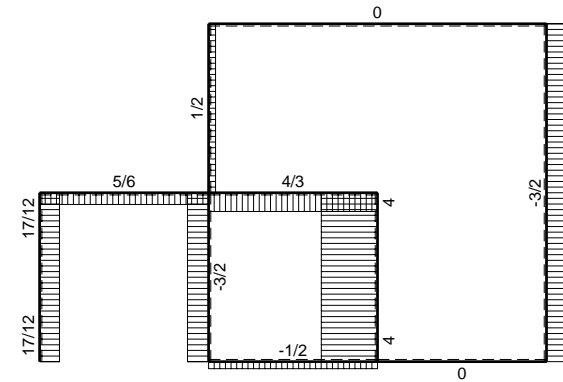
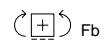
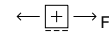
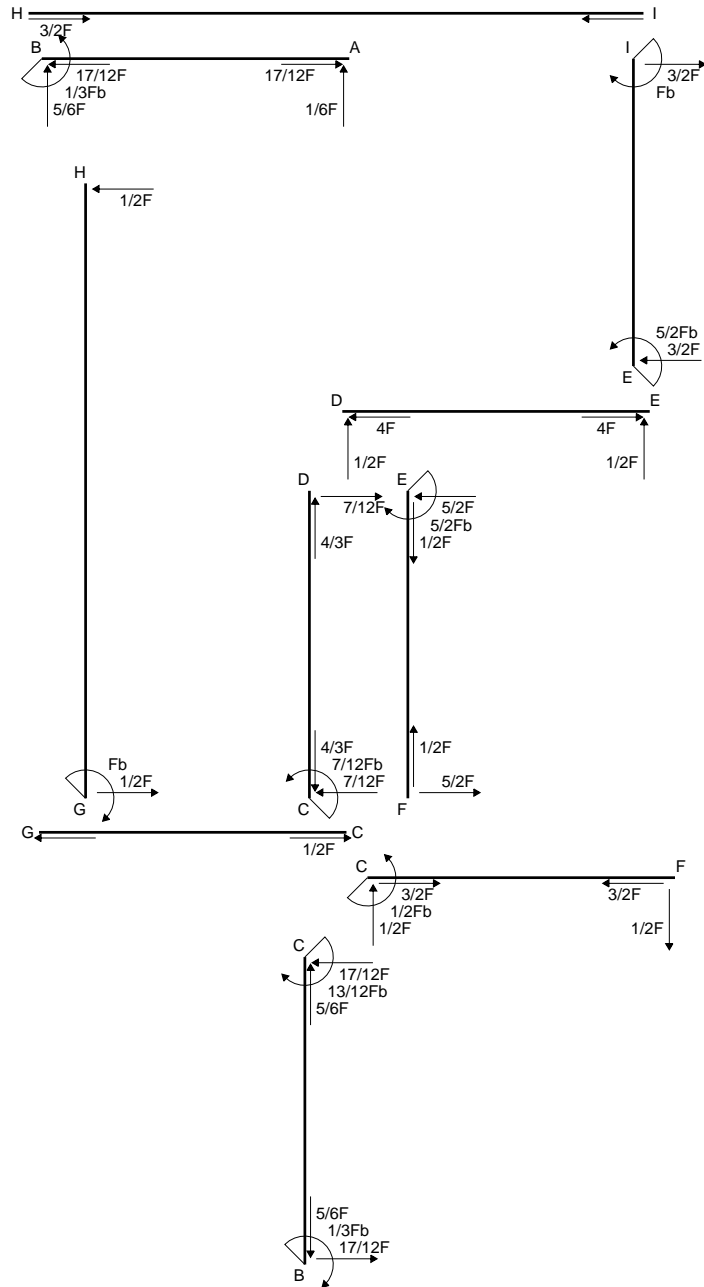
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

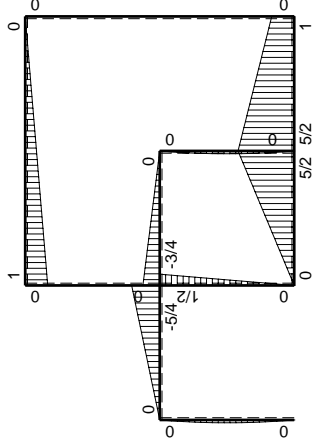
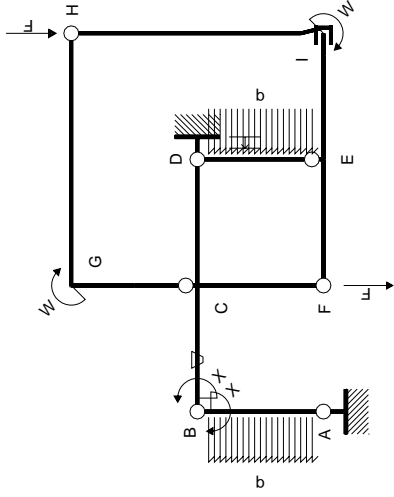
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

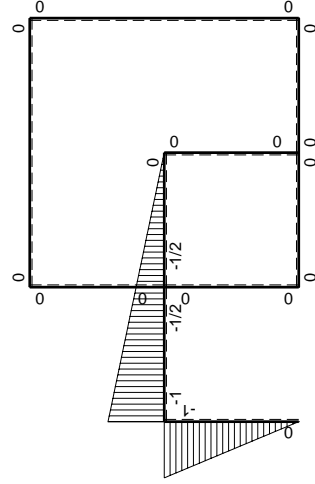
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-5/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-5/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $1/3Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $-1/3Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

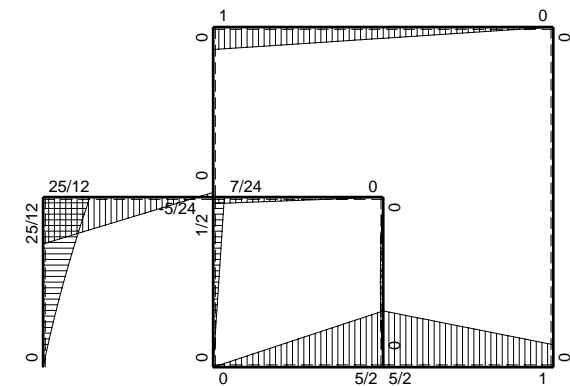
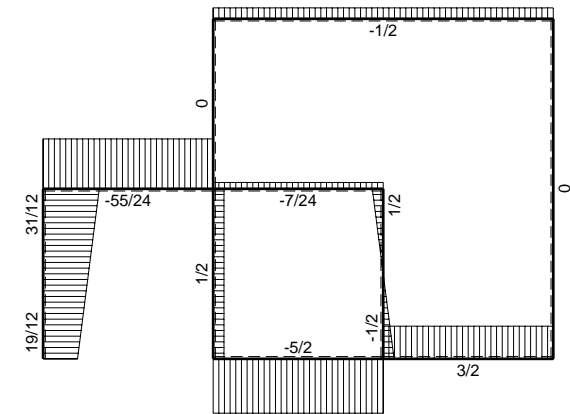
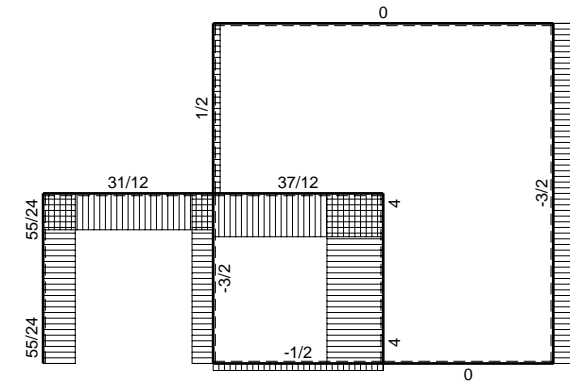
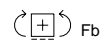
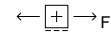
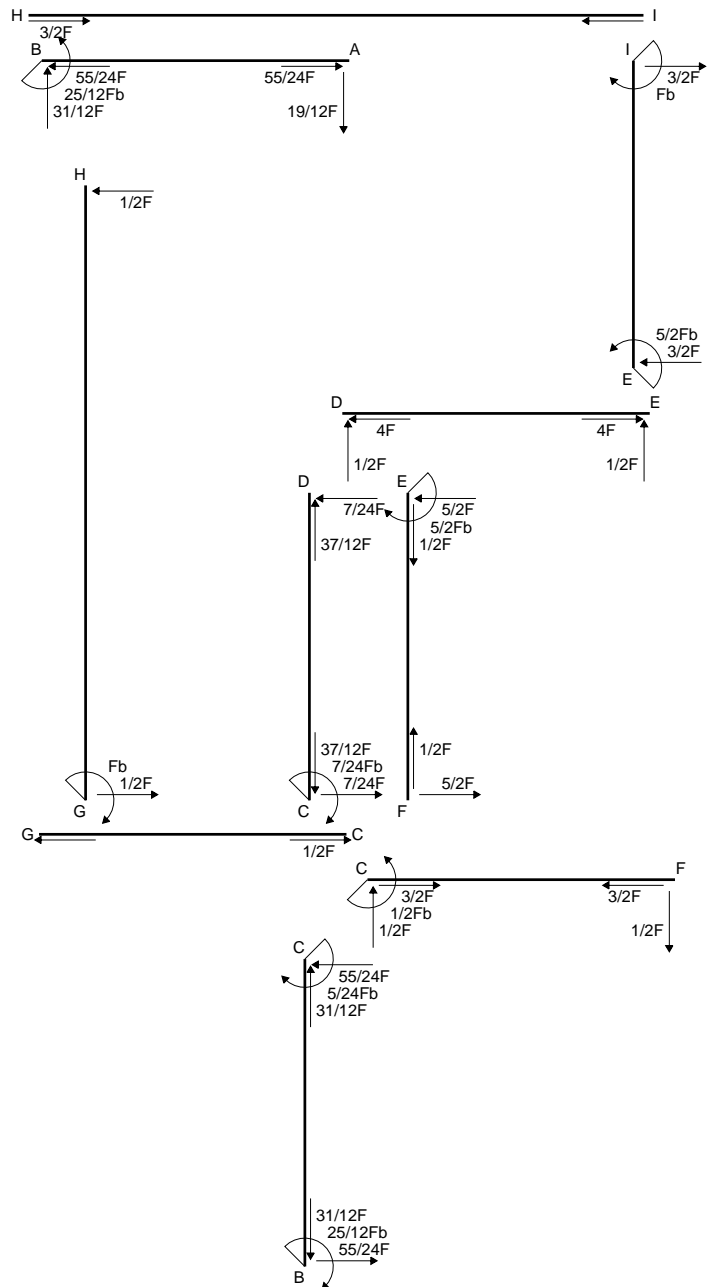
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

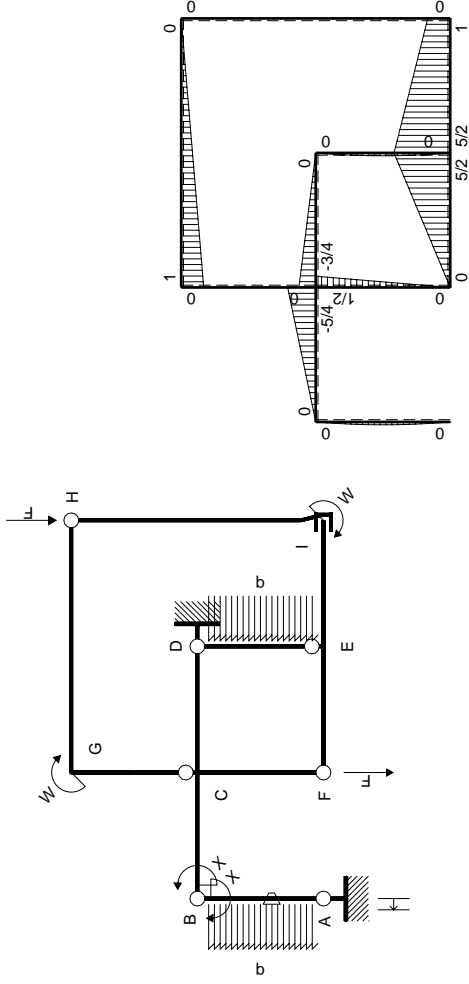
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                          |
| EF b  | 0                           | $5/2Fb-5/2Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-5/2Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |
| CG b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| GH 2b | 0                           | $Fb-1/2Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/2Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| HI 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| IE b  | 0                           | $Fb+3/2Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-5/2Fb+3/2Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |               |                         | $Fb^2/EJ$                   |                          |
|       | totali                      |                  |          |                          |               |                         | $25/12Fb^2/EJ$              | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         | $-25/12Fb$                  |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

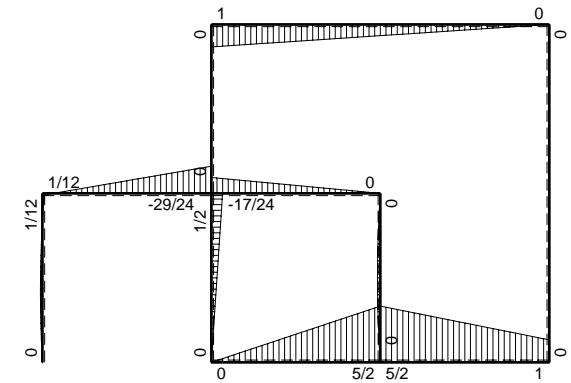
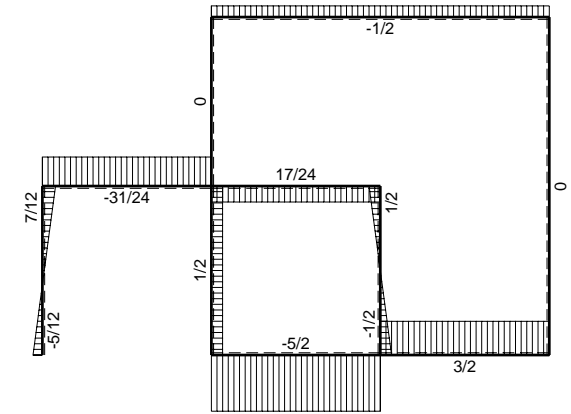
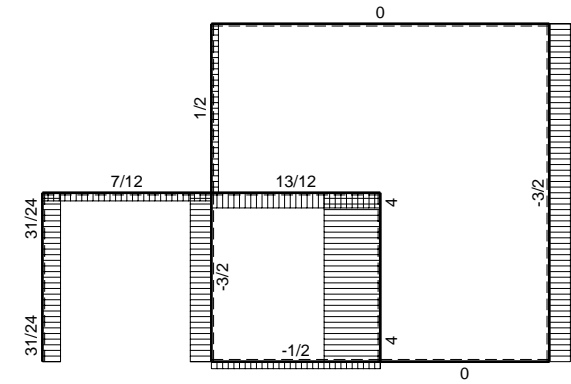
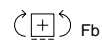
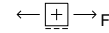
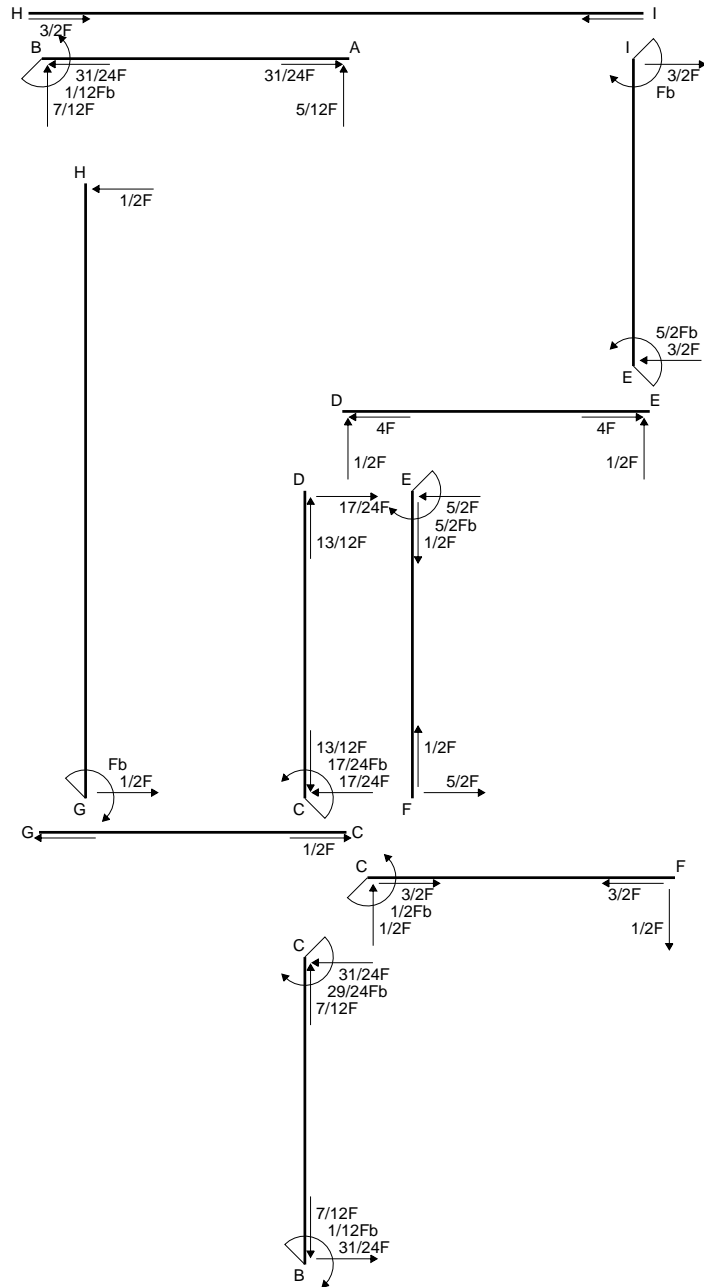
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

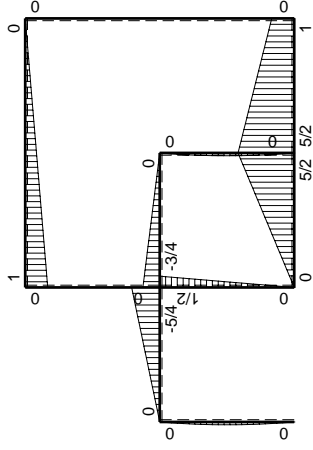
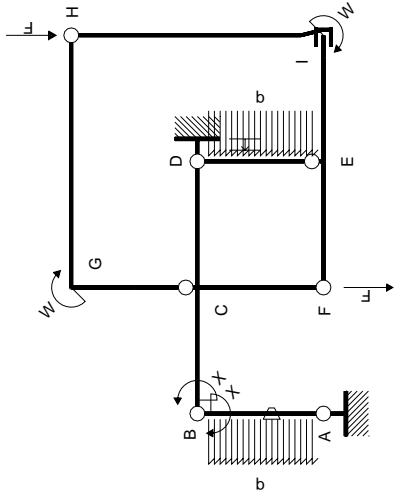
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

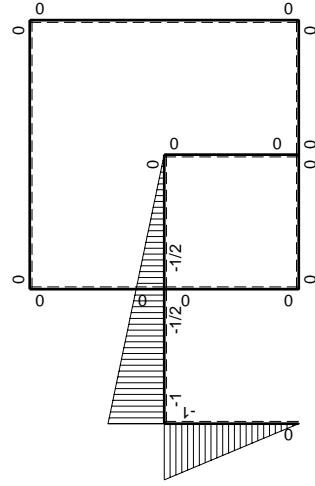
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                          |
| EF b  | 0                           | $5/2Fb-5/2Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-5/2Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |
| CG b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| GH 2b | 0                           | $Fb-1/2Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/2Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| HI 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| IE b  | 0                           | $Fb+3/2Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-5/2Fb+3/2Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |               |                         | $-Fb^2/EJ$                  |                          |
|       | totali                      |                  |          |                          |               |                         | $1/12Fb^2/EJ$               | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         | $-1/12Fb$                   |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

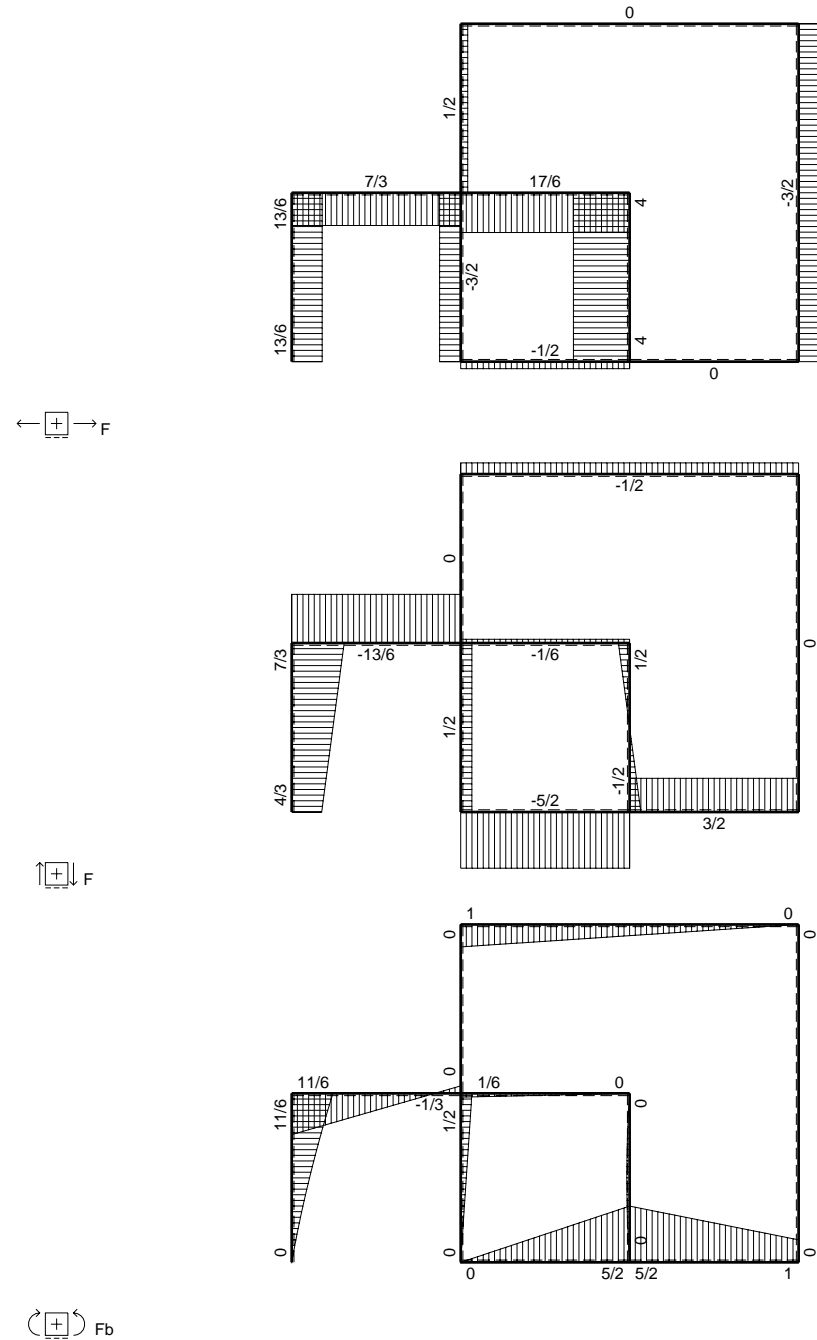
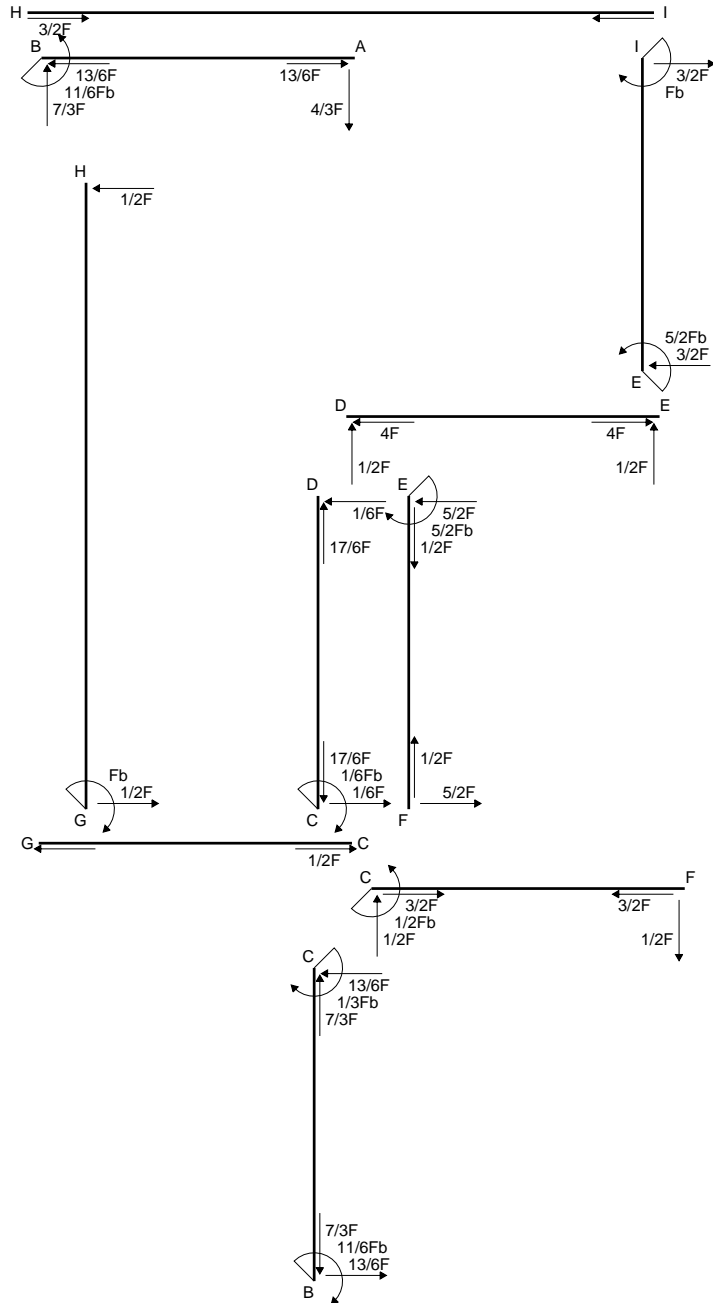
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

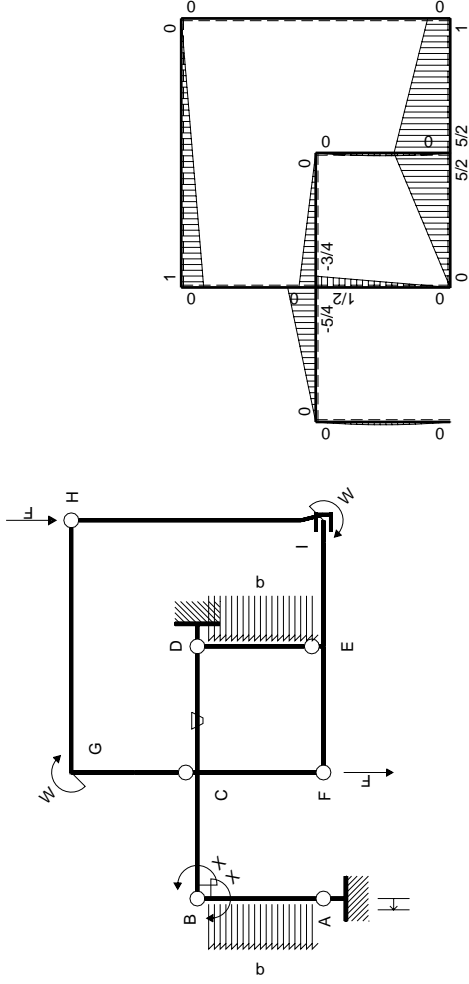
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

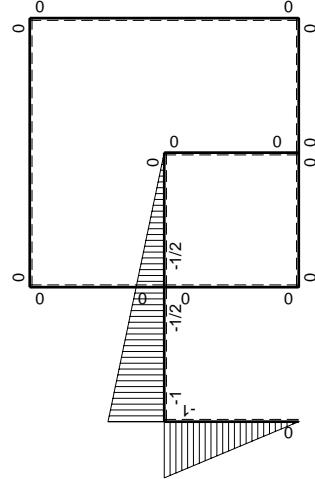






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-5/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                          |                     |                         | $11/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $-11/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

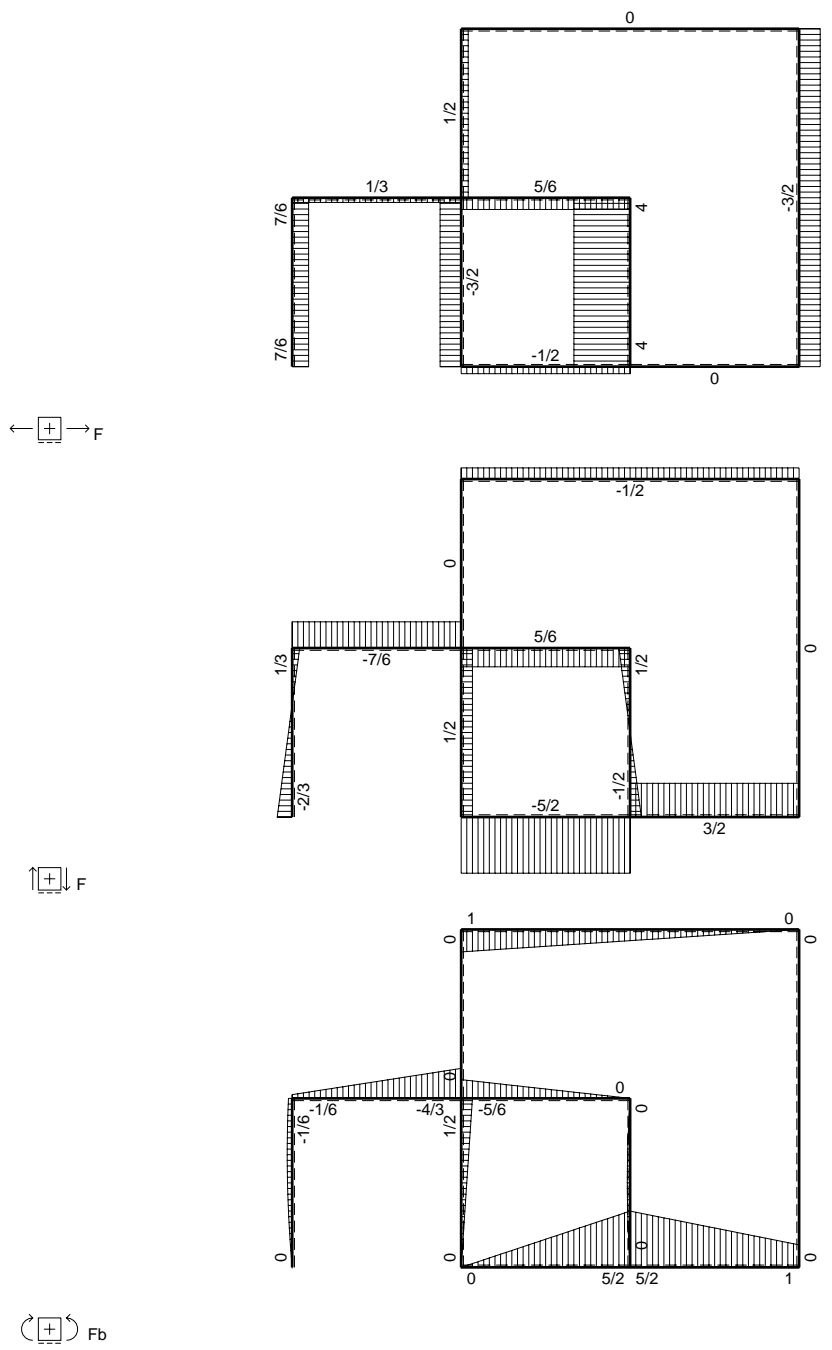
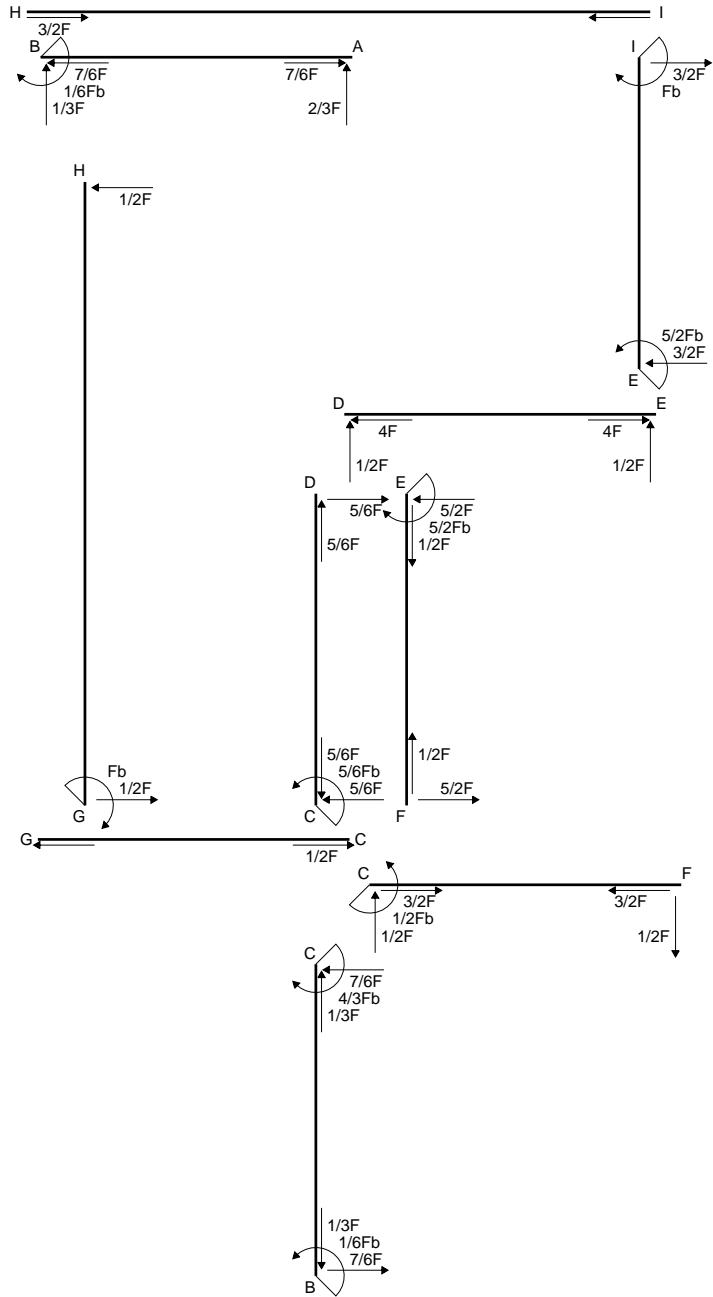
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

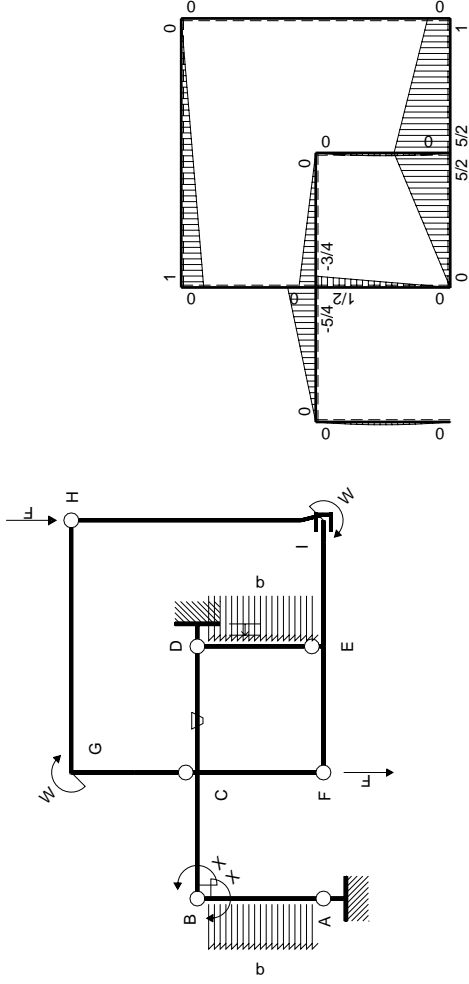
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-5/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-5/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $-1/6Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $1/6Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

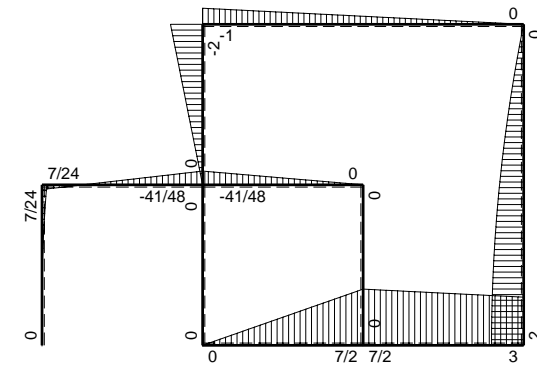
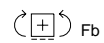
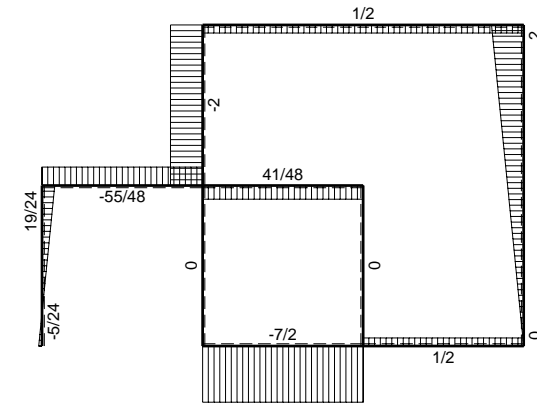
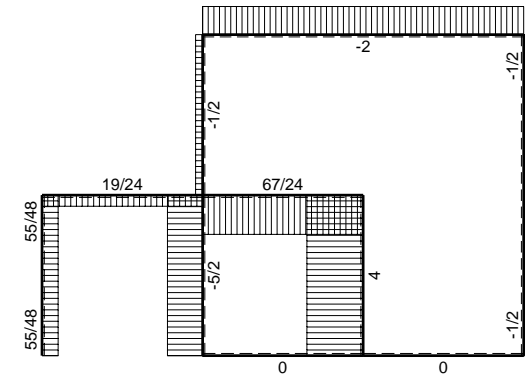
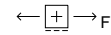
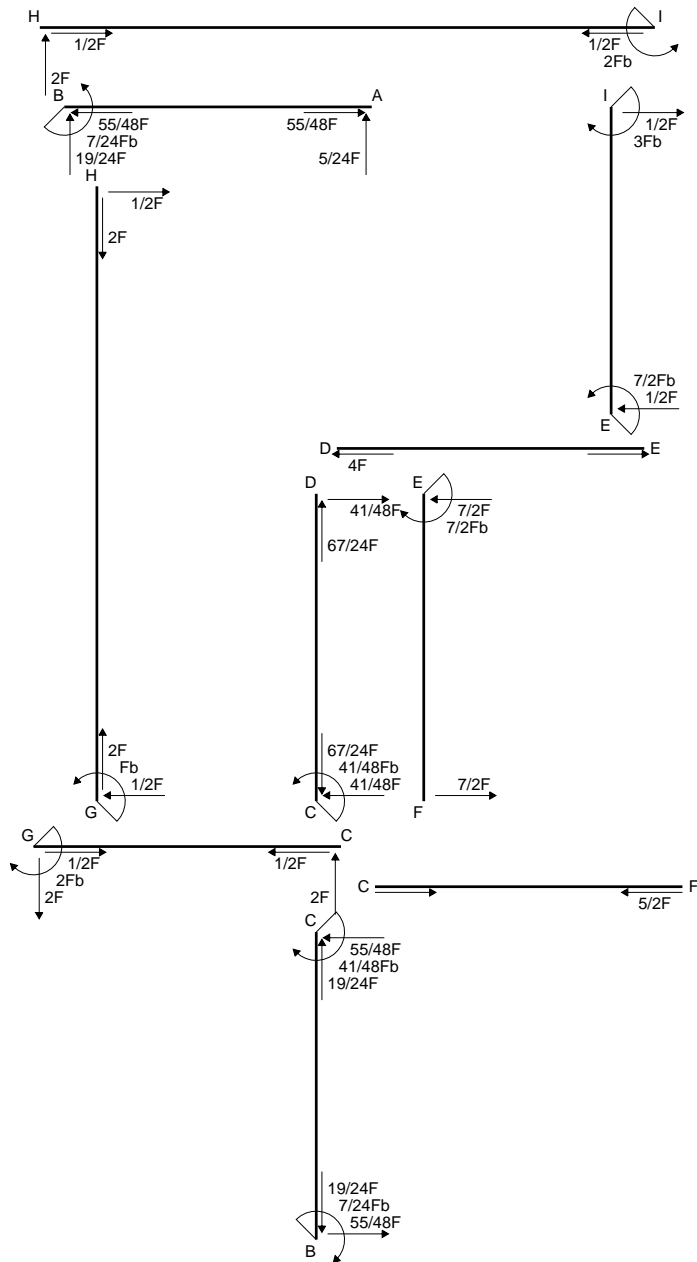
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

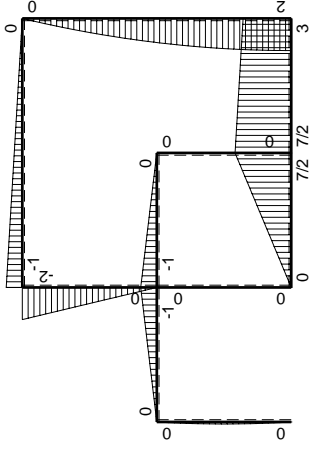
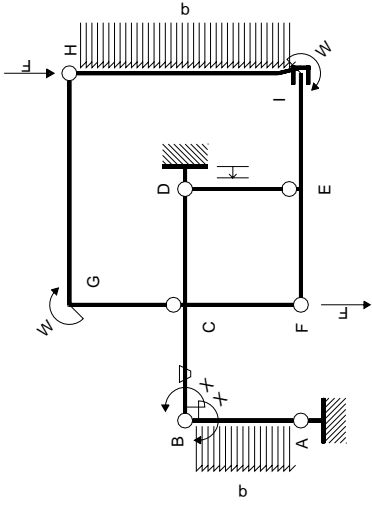
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

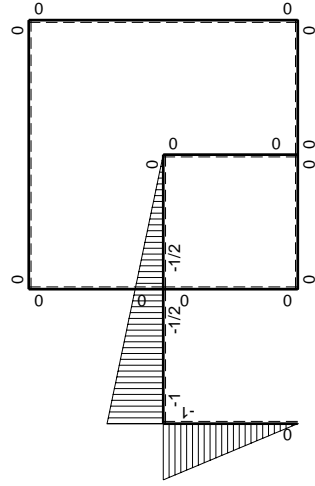






Schema di calcolo iperstatico

M<sub>0</sub> flessione da carichi assegnati



M<sub>x</sub> flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | $-Fb/EJ$ | $Fx-1/2Fx^2/b$           | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$         | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $Fx$             | 0        | $1/2Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |  |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         | $-Fb^2/EJ$                  |                        |  |
|       | totali                      |                  |          |                          |                     |                         | $7/24Fb^2/EJ$               | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $-7/24Fb$                   |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

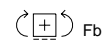
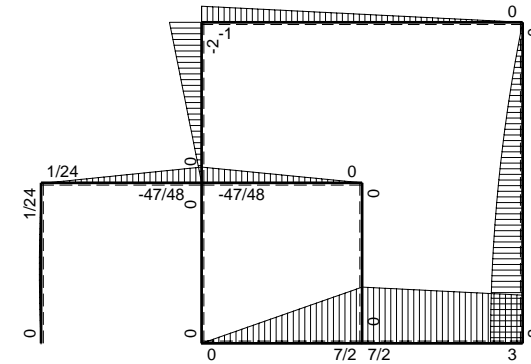
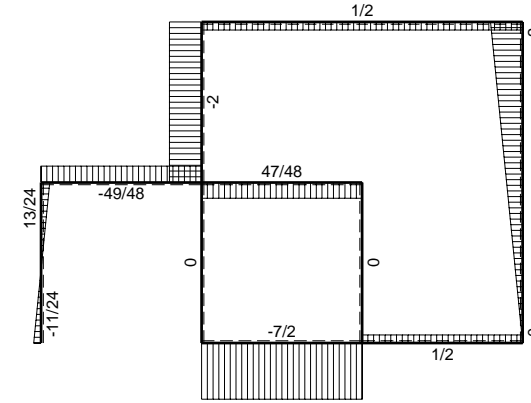
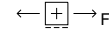
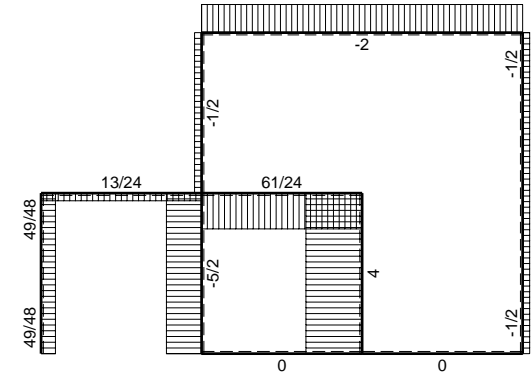
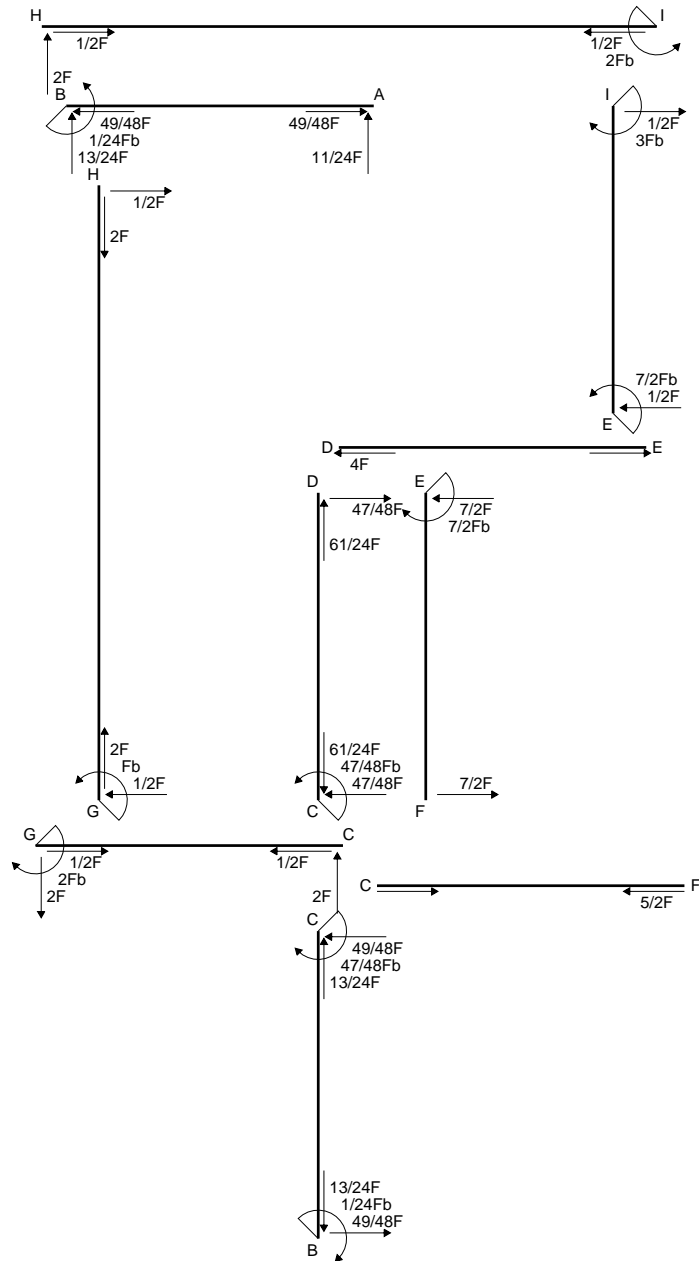
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

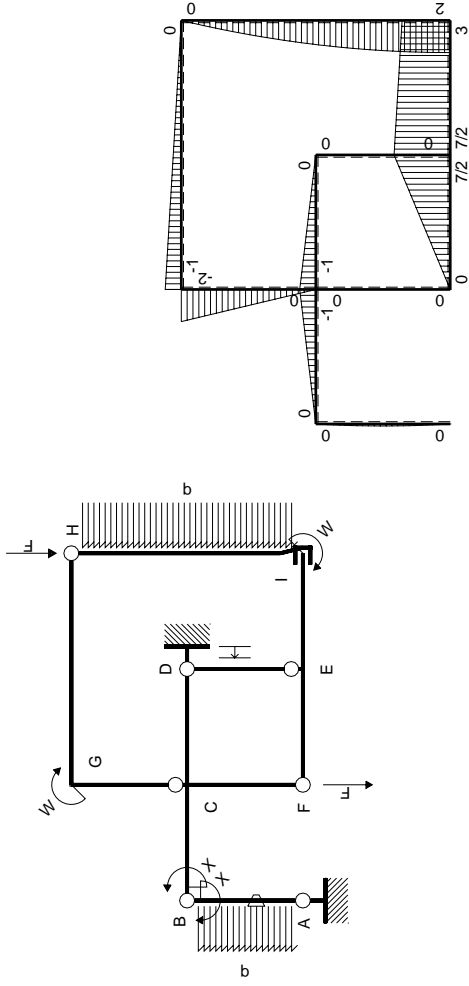
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

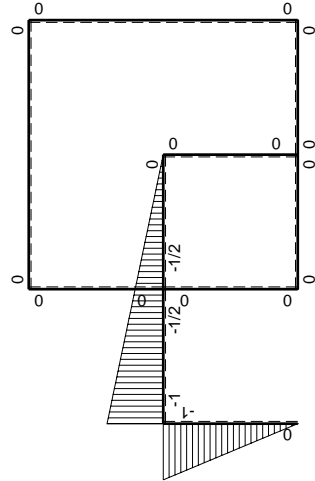
$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$         | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $Fx$             | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0             | 0                       |                             |                        |  |
| FC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| CF b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |  |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0             | 0                       |                             |                        |  |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0             | 0                       |                             |                        |  |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                        |  |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |  |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0             | 0                       |                             |                        |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |               |                         | $-Fb^2/EJ$                  |                        |  |
|       | totali                      |                  |          |                          |               |                         | $1/24Fb^2/EJ$               | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         | $-1/24Fb$                   |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

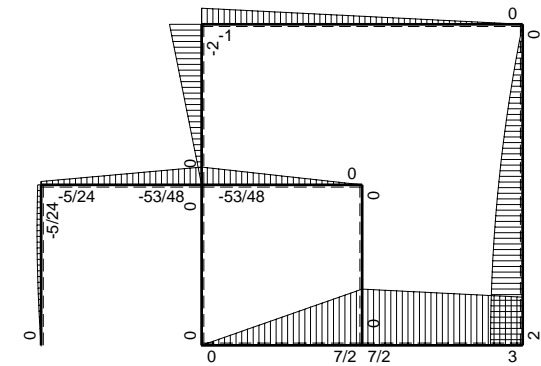
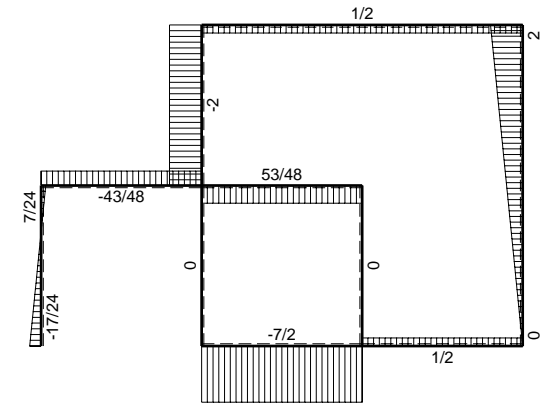
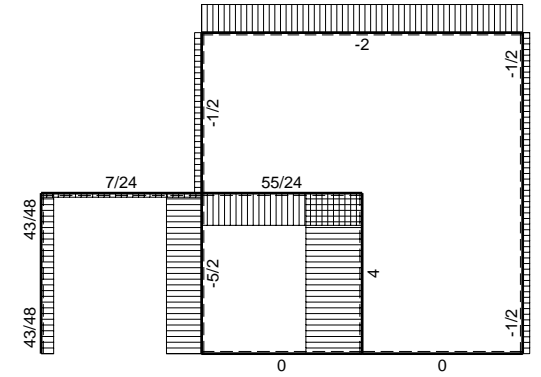
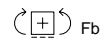
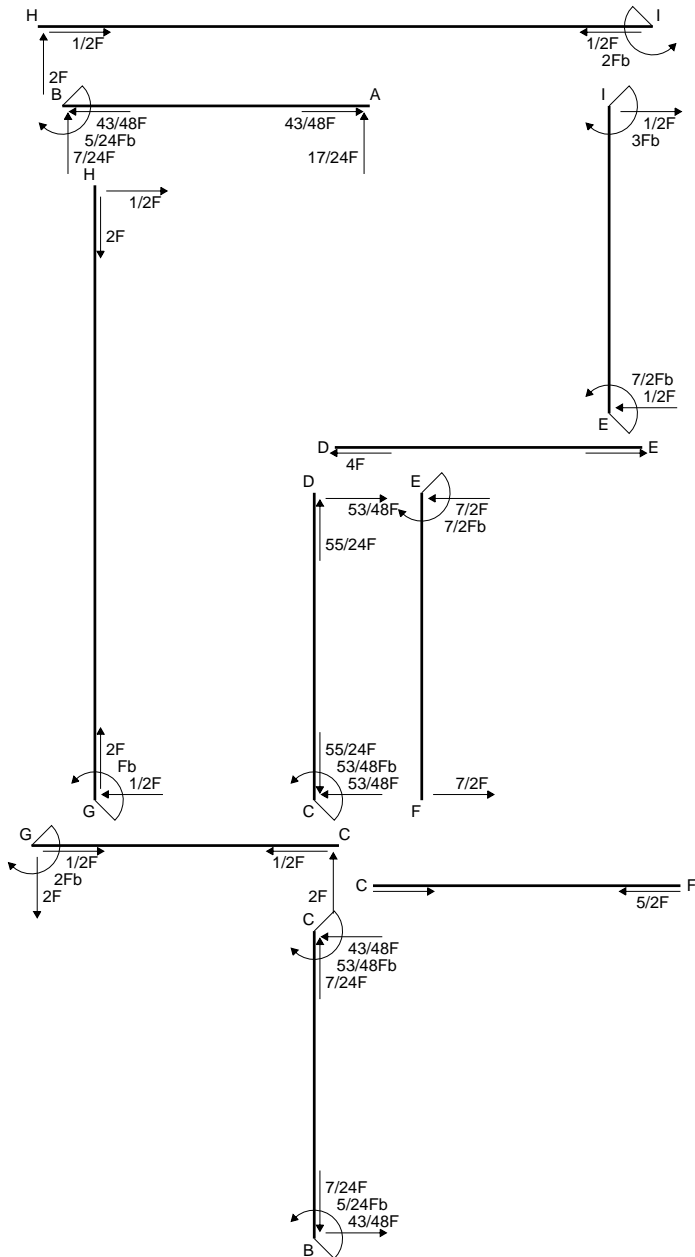
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

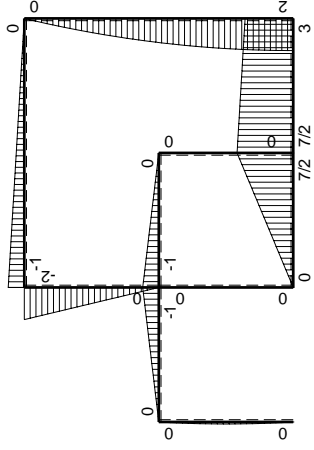
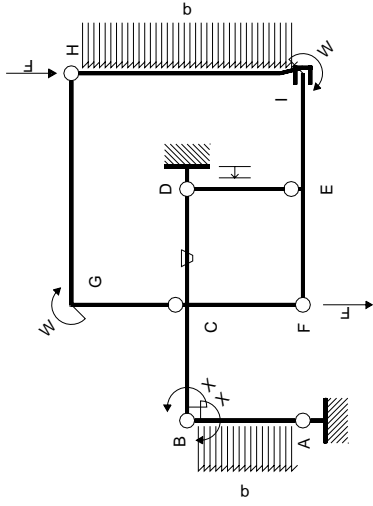
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

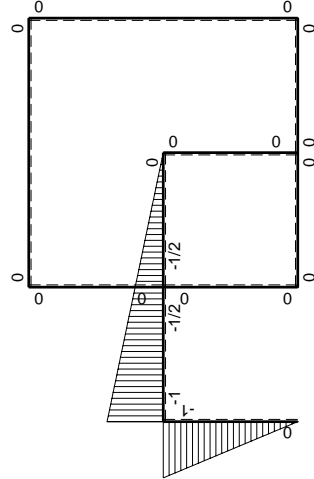






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|--------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$               |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                          |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$           | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$              |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$         | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$     | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$              |         |
| DC b  | $1/2x/b$                    | $Fx$             | $Fb/EJ$  | $1/2Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                          |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| CG b  | 0                           | $-2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                        |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                        | 0                   | 0                       |                             |                          |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$               |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $-5/24Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $5/24Fb$                 |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

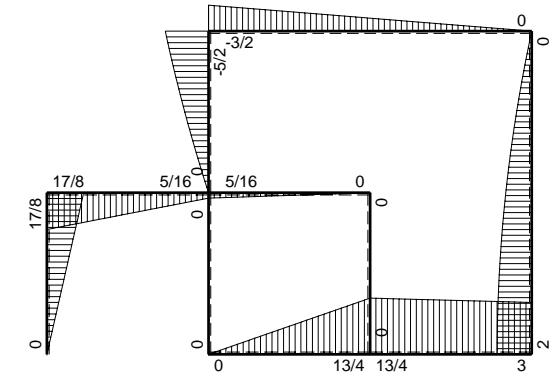
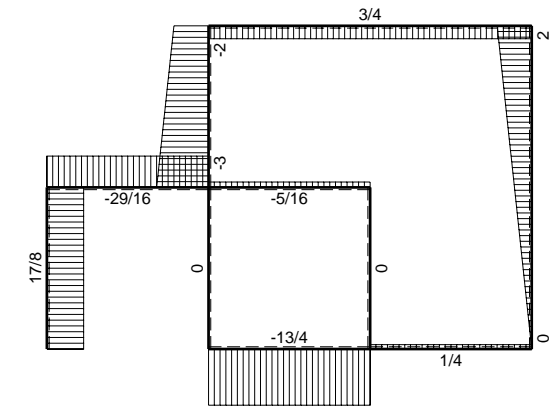
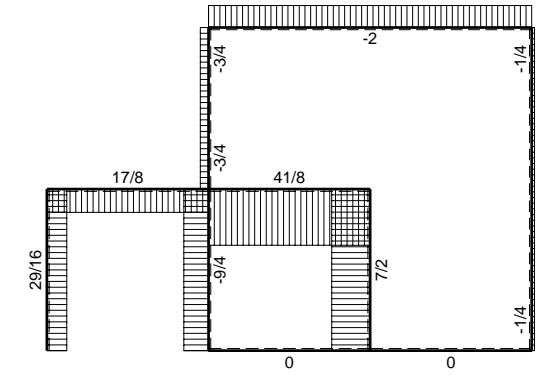
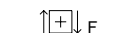
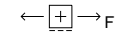
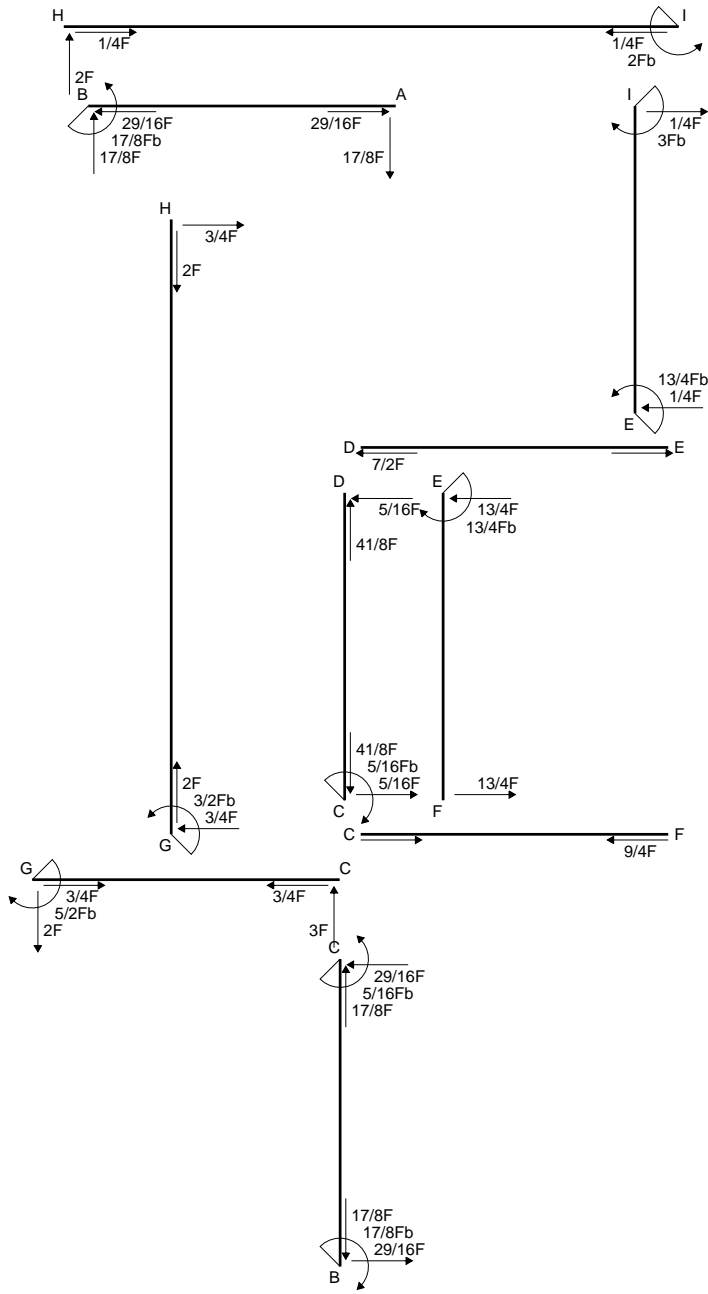
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

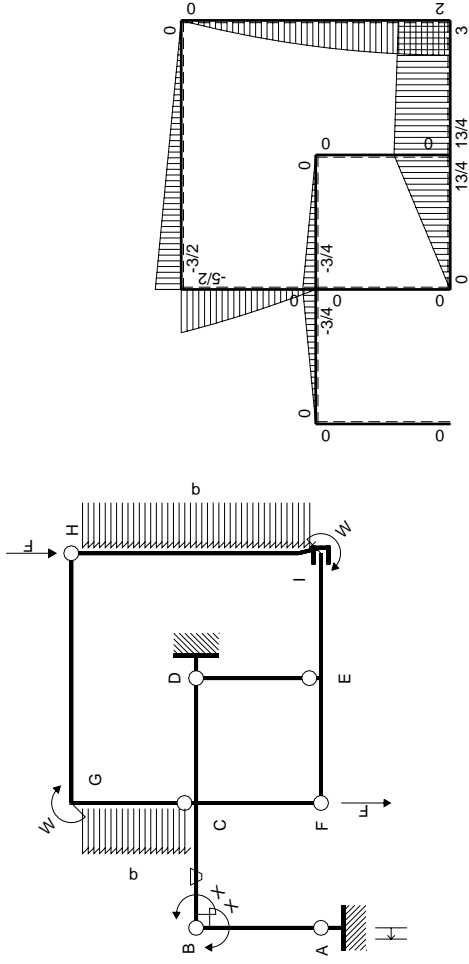
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

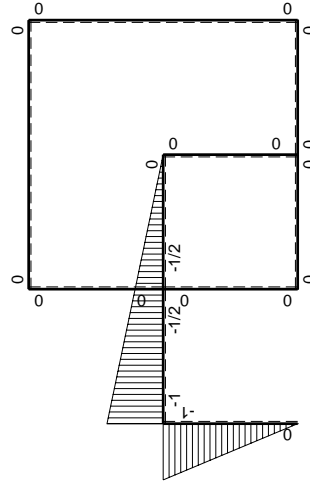
$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

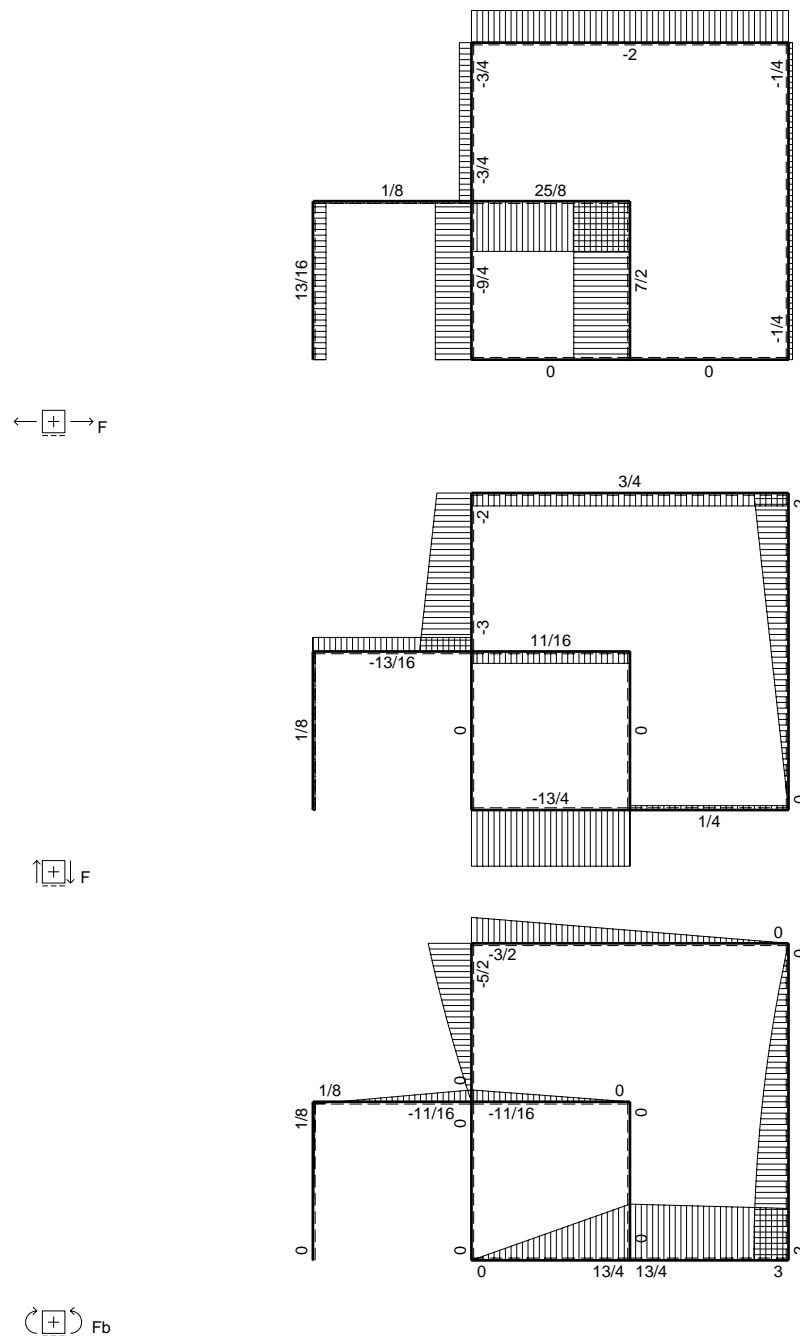
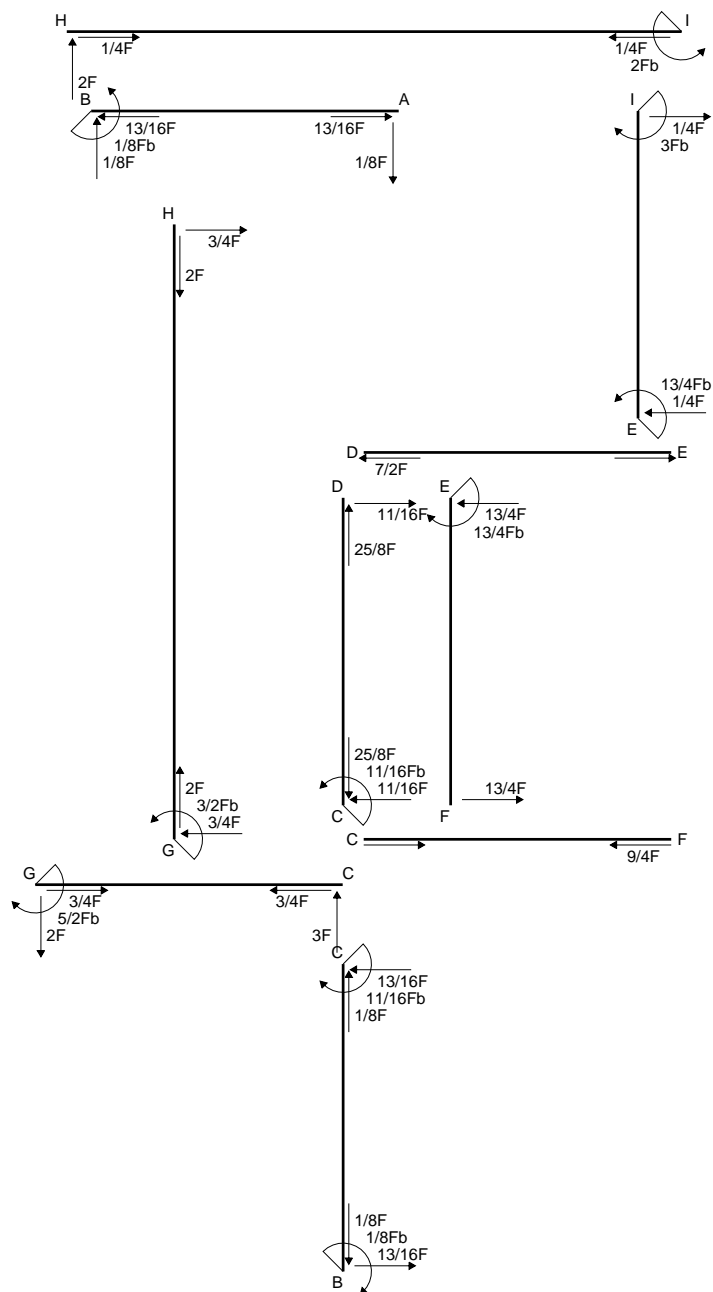
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

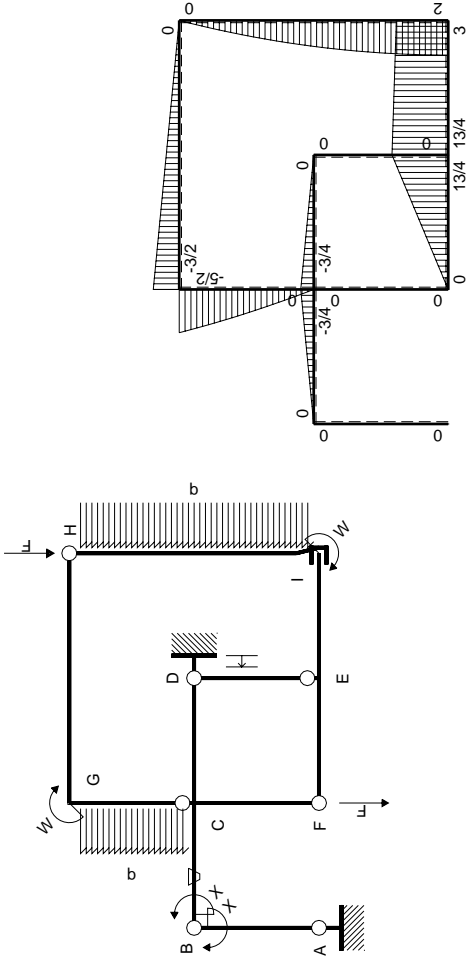
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

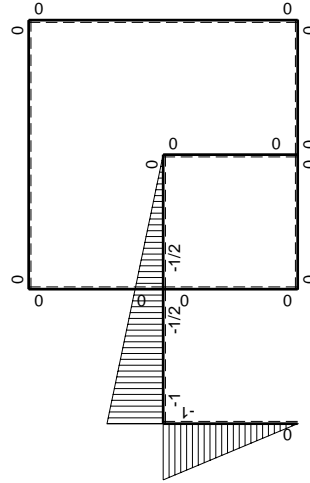






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |                     |                         | $1/8Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-1/8Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

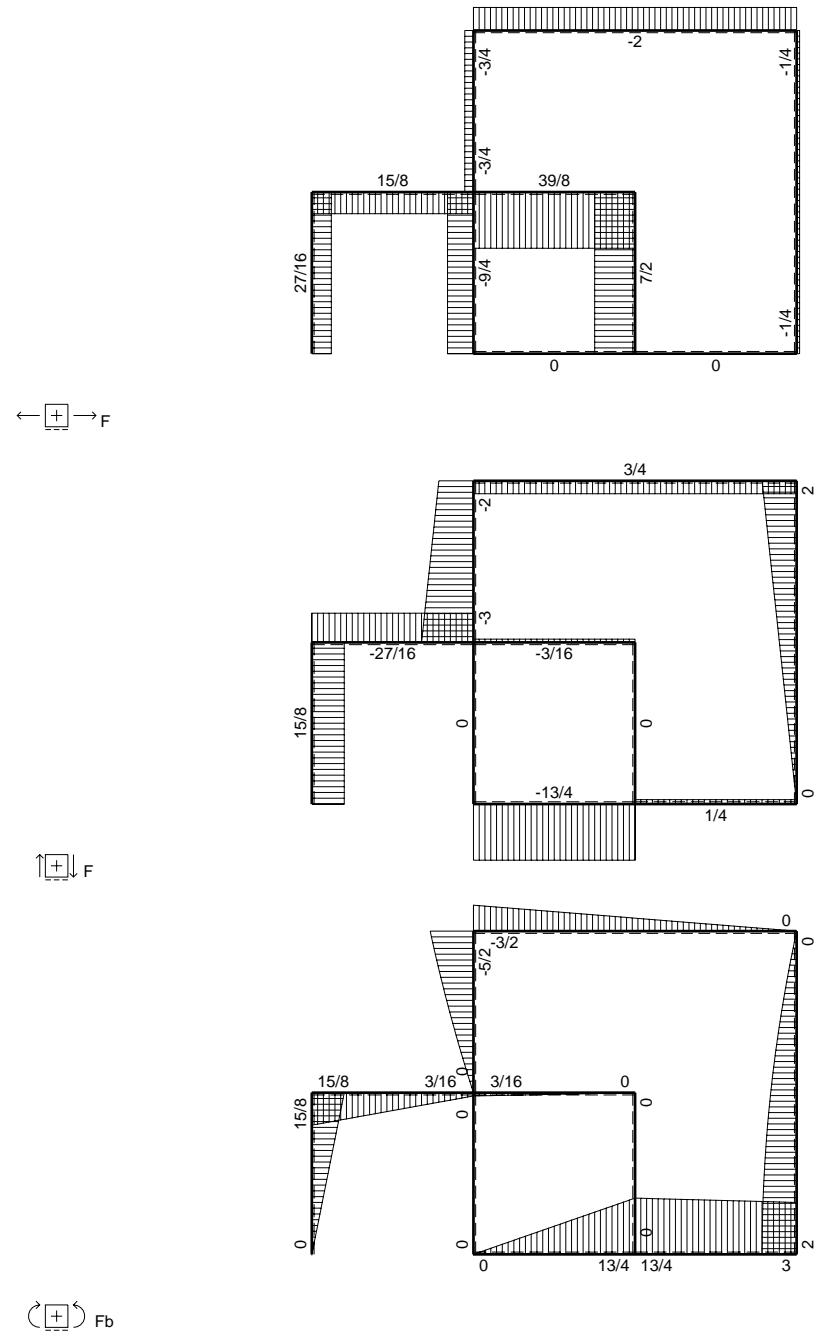
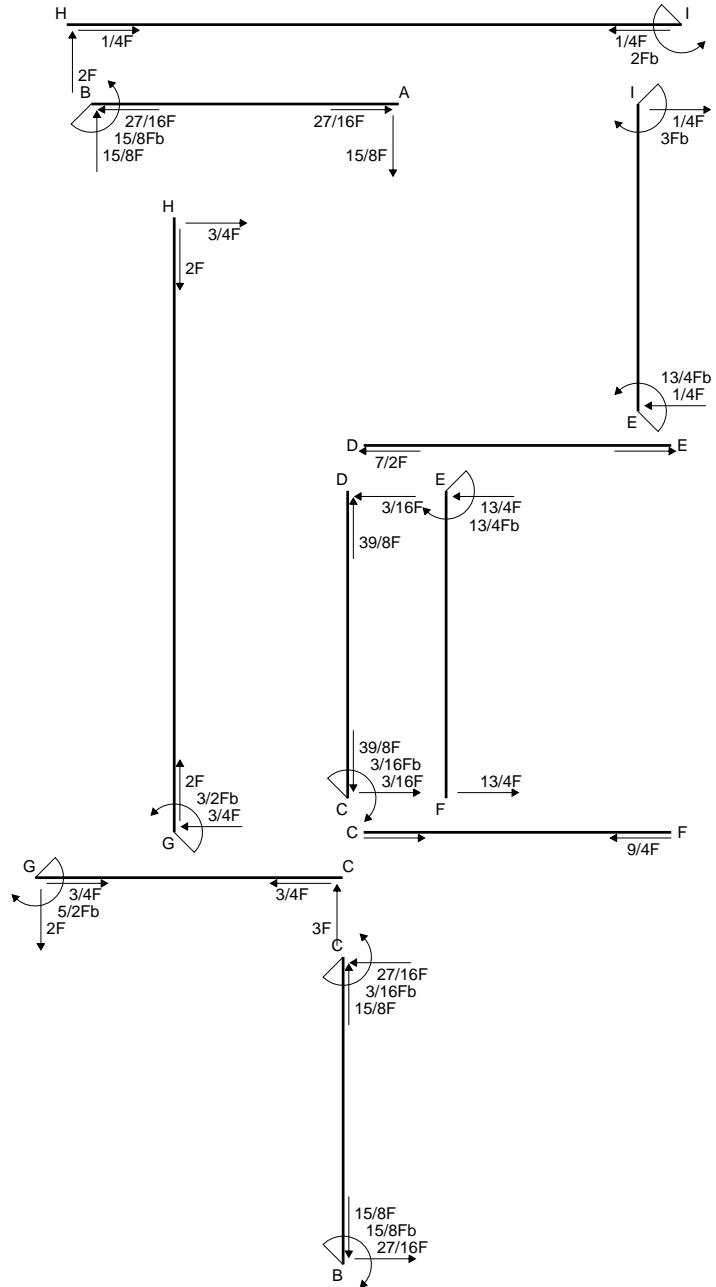
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

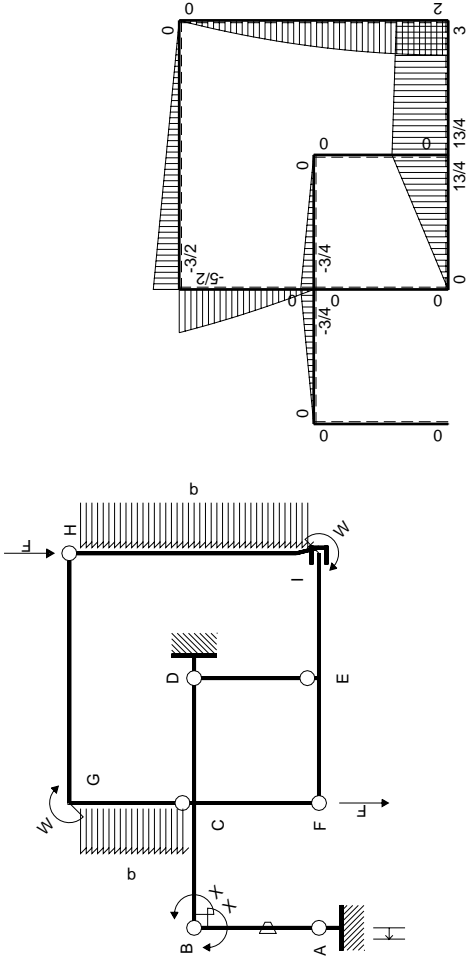
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

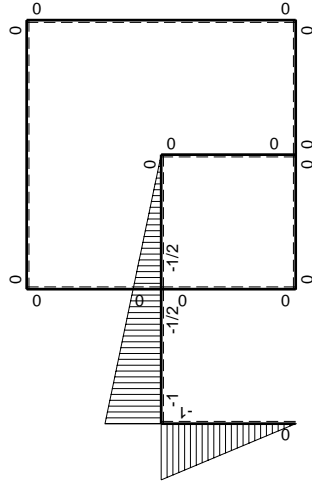
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |               |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-15/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

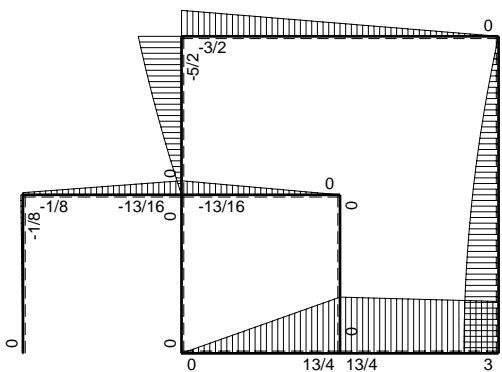
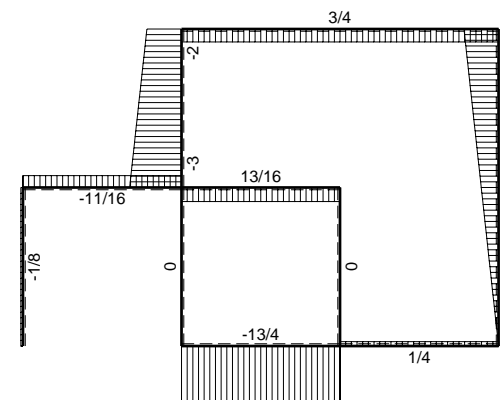
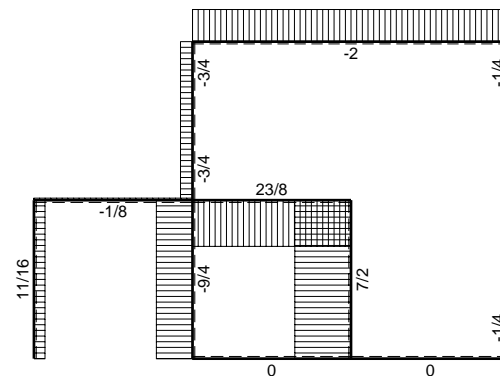
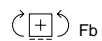
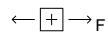
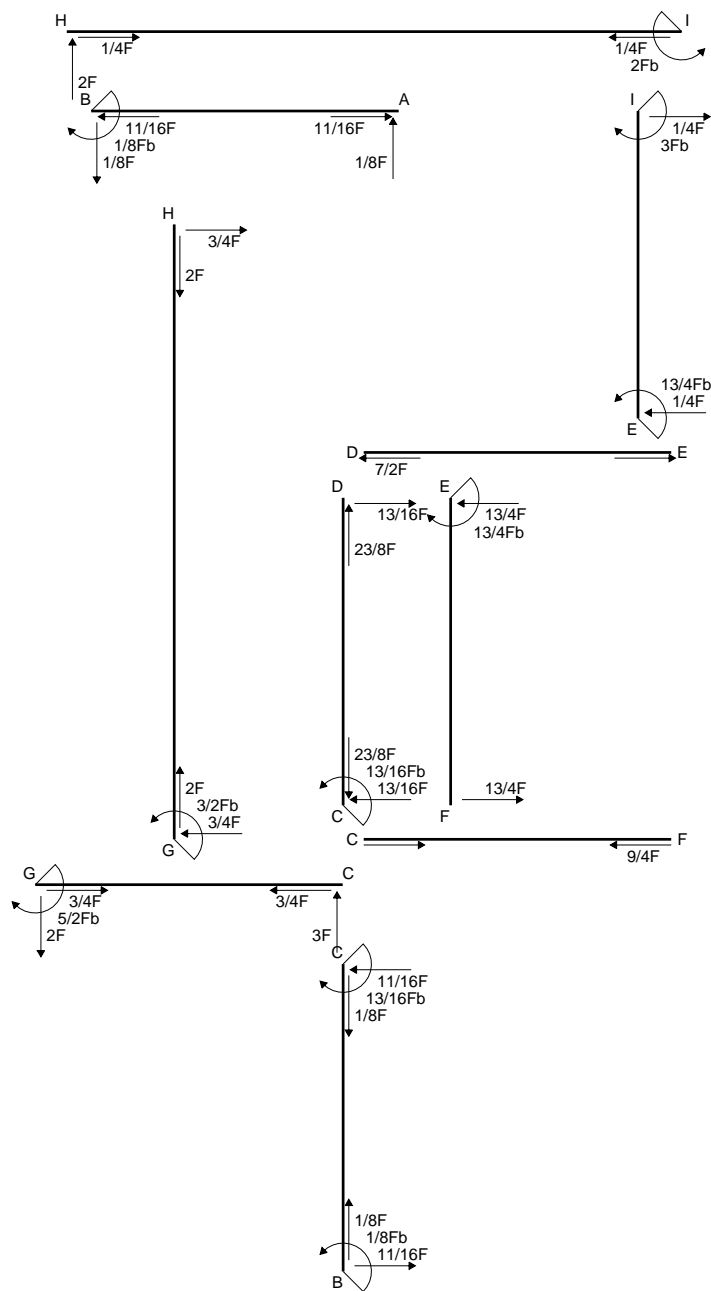
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

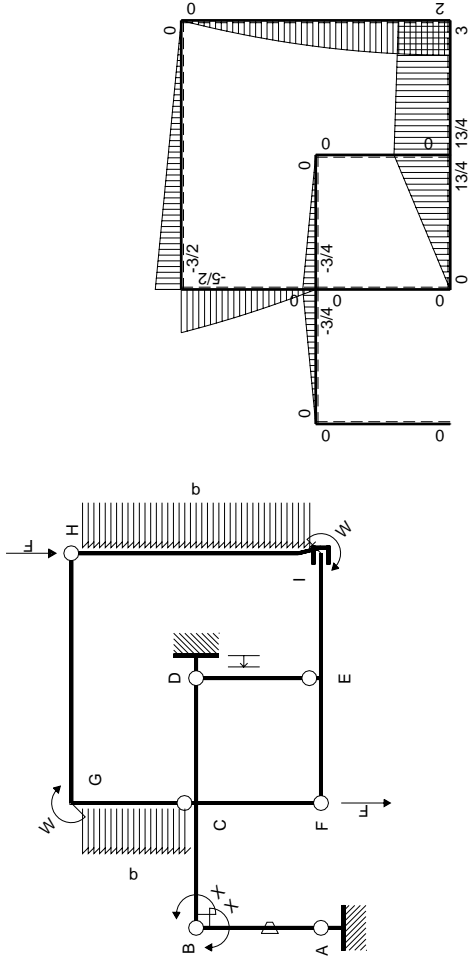
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

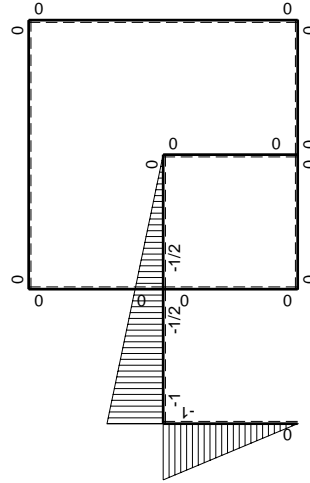






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |               |                         | $-1/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $1/8Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

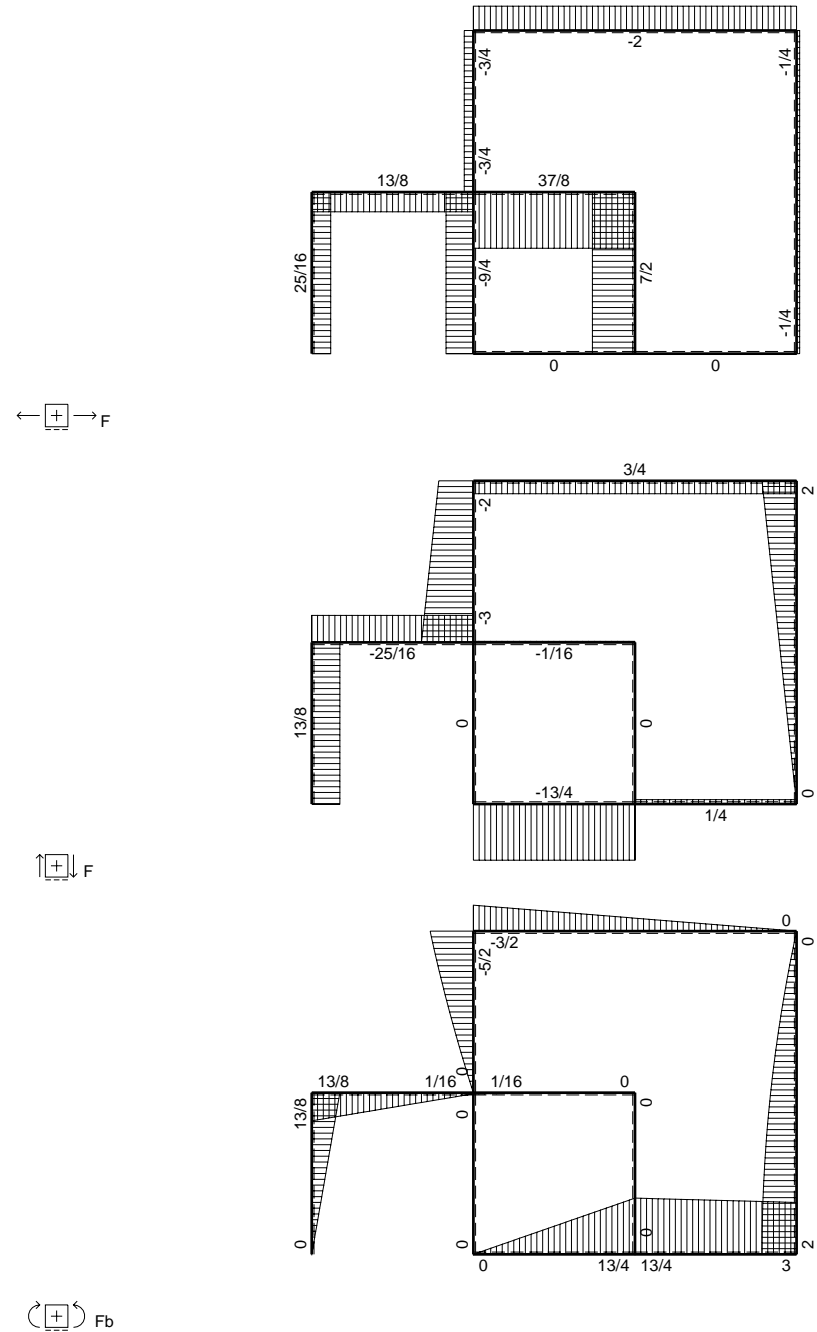
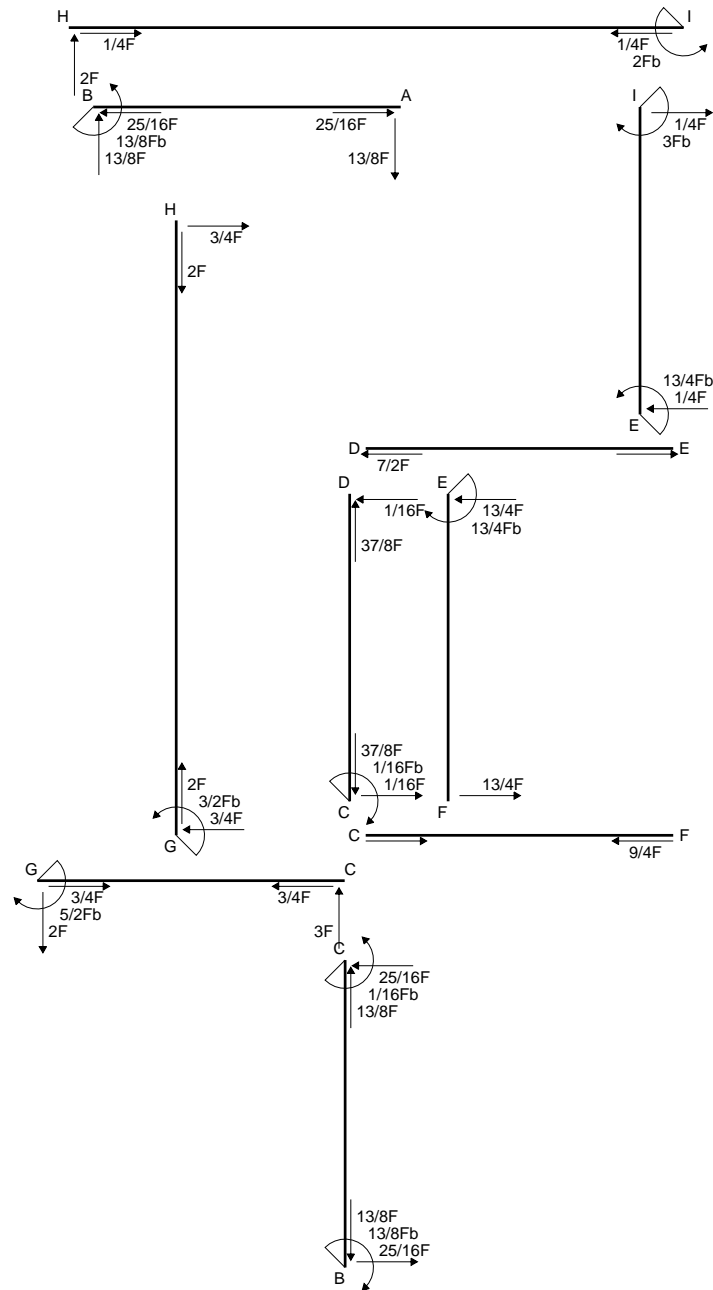
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

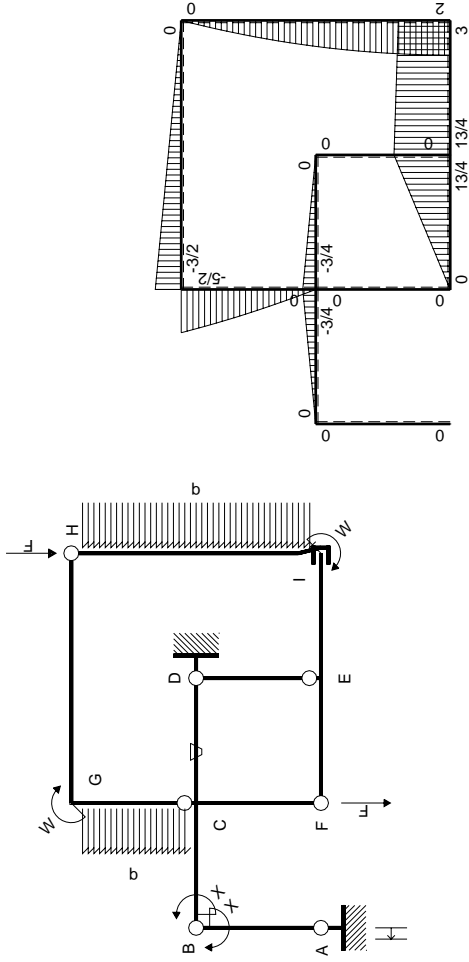
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

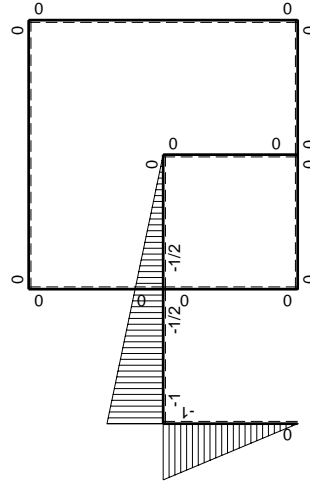
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $13/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-13/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

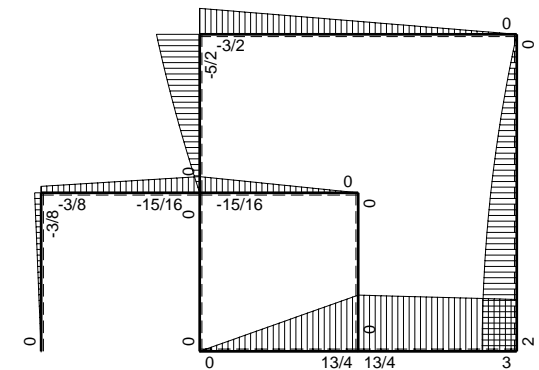
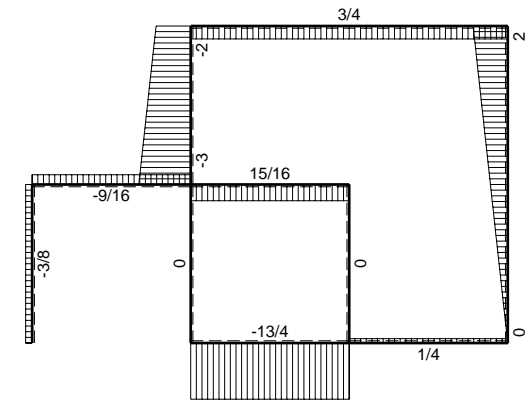
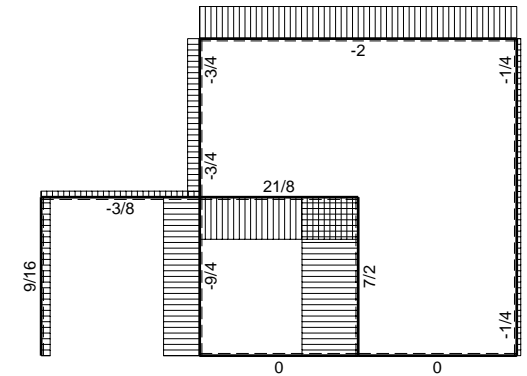
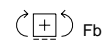
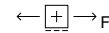
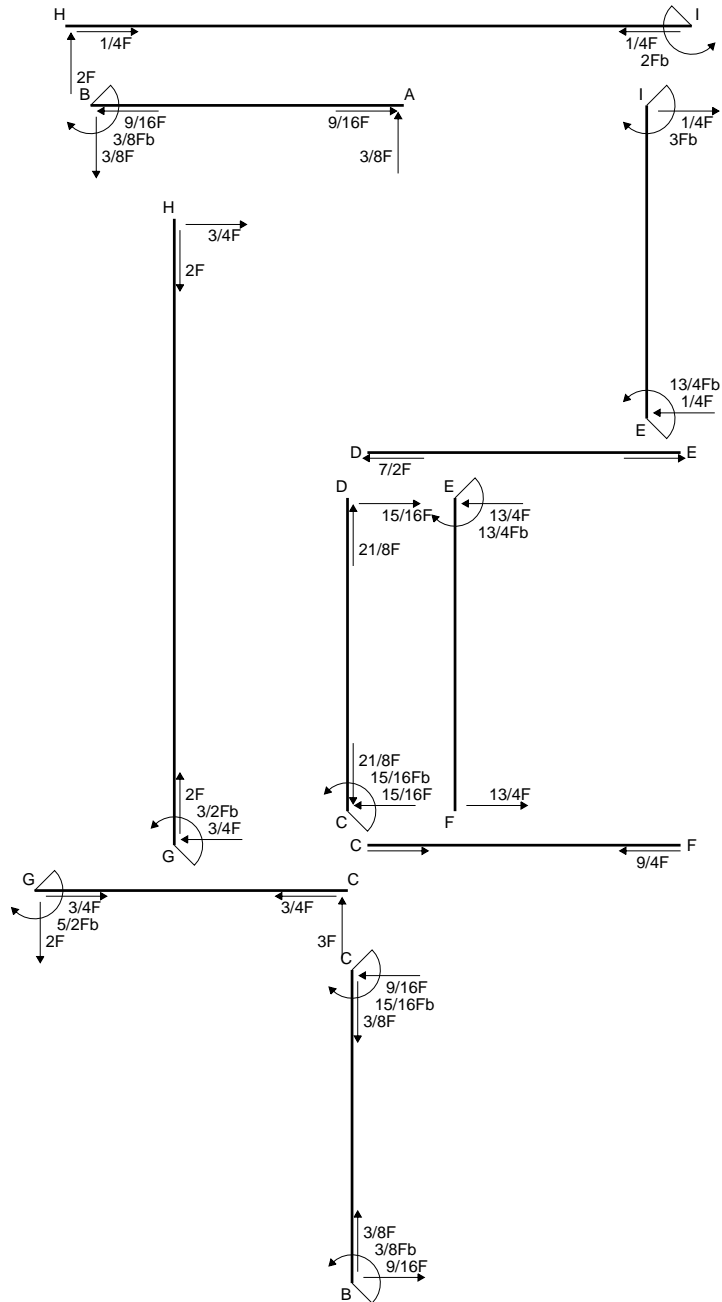
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

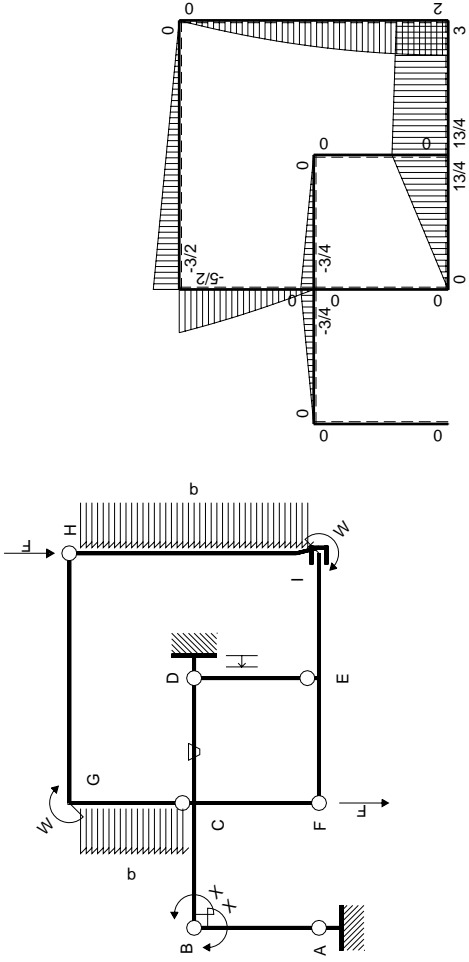
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

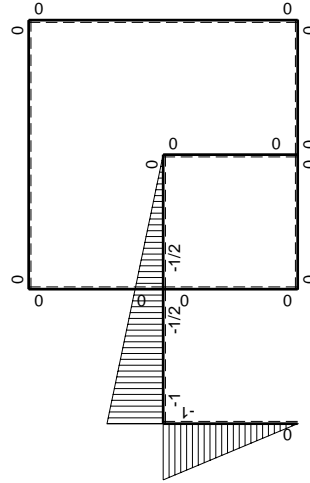






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{iD}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-3/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $3/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

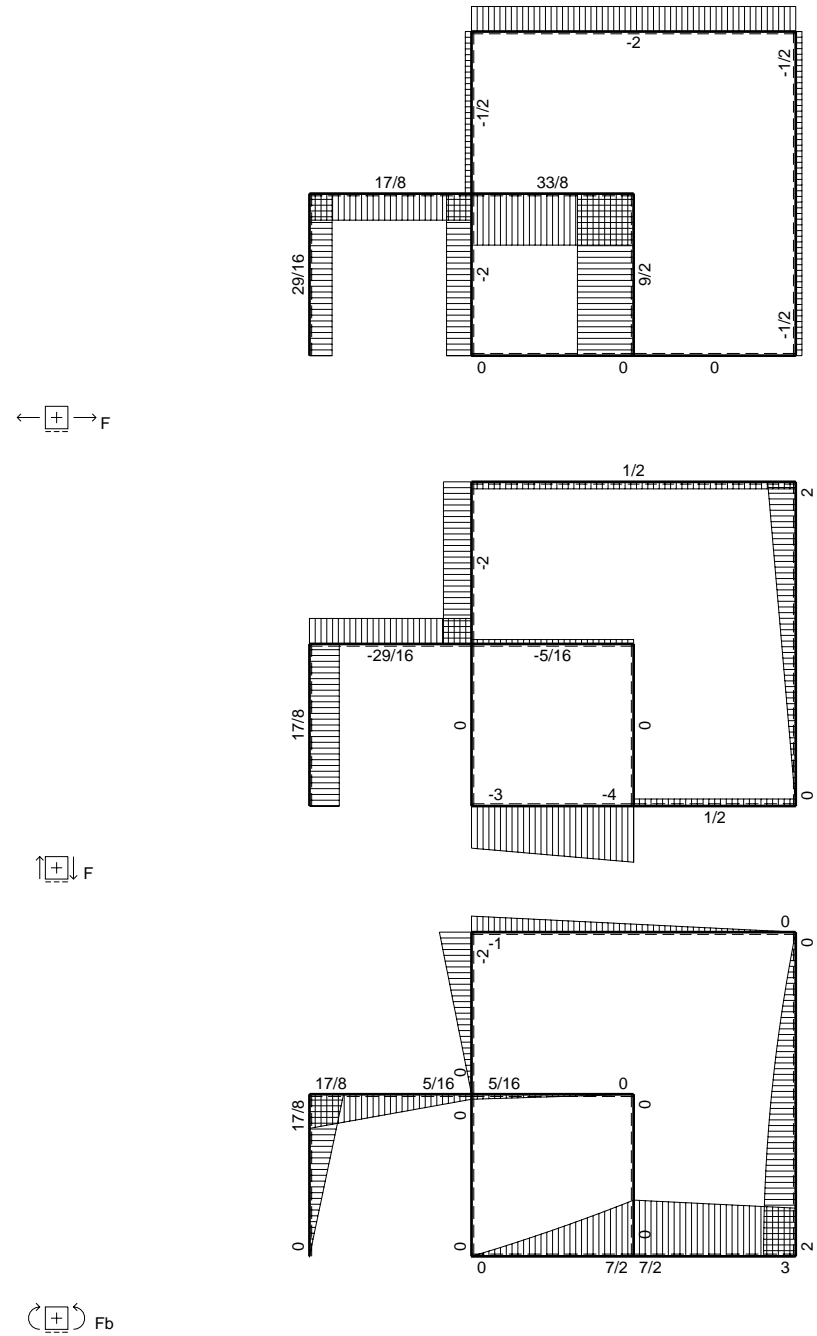
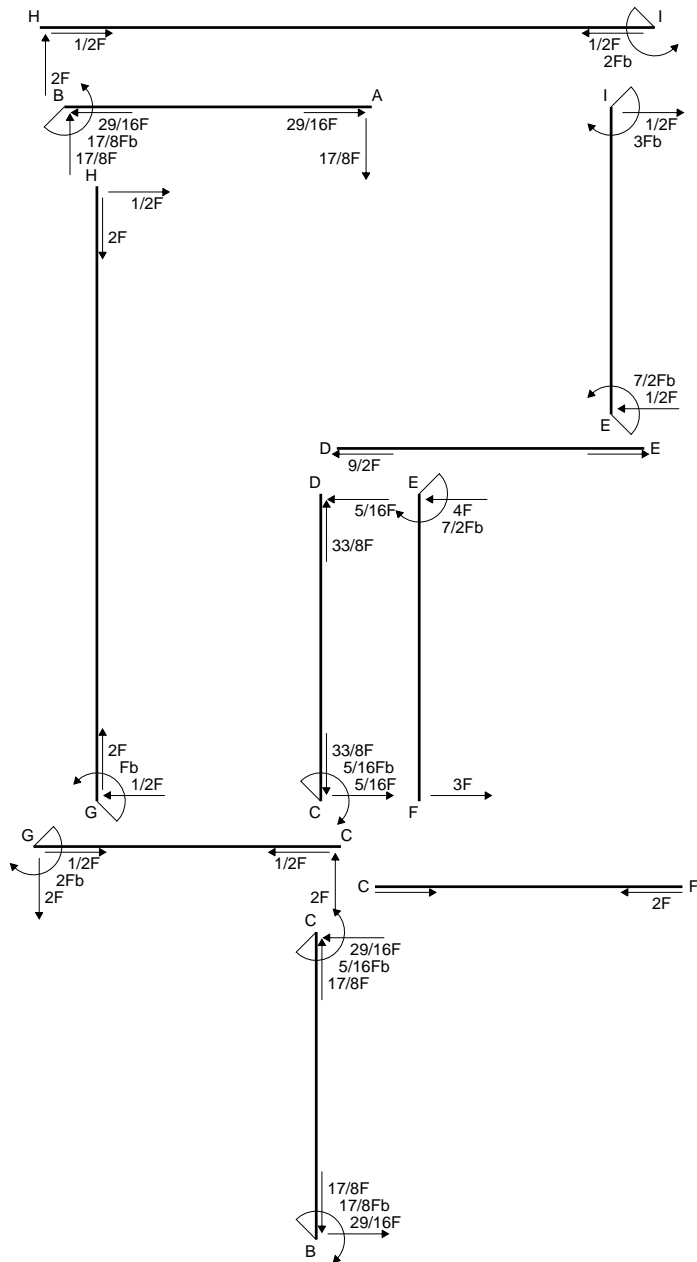
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

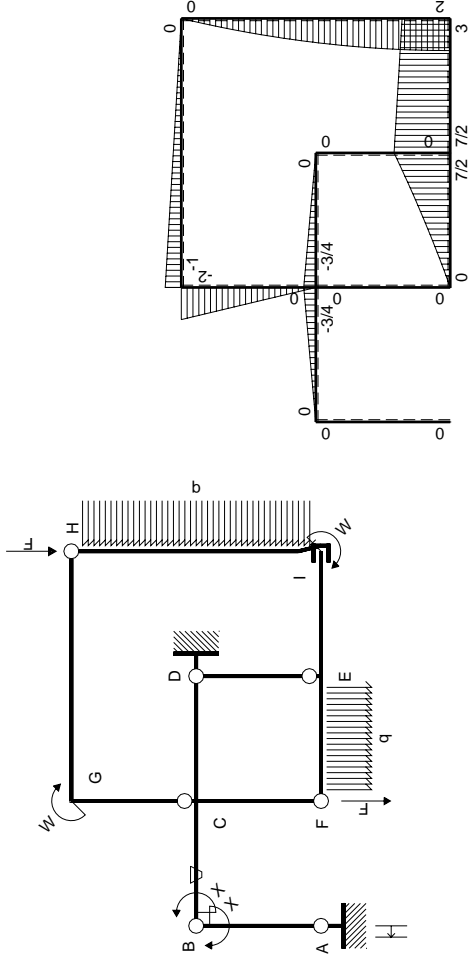
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

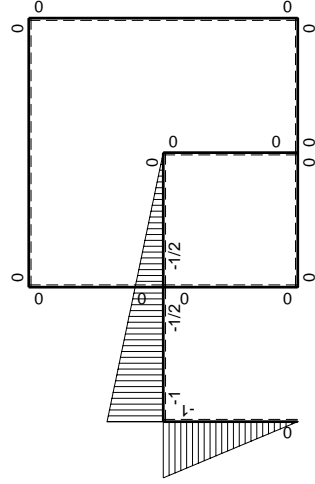
$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

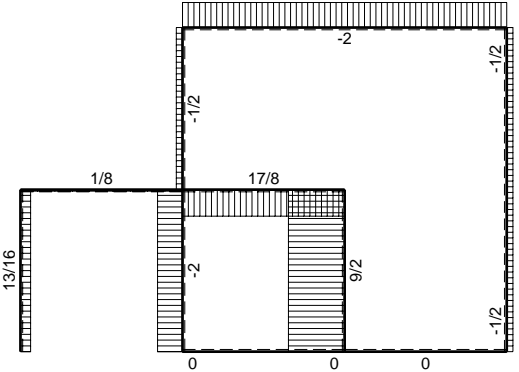
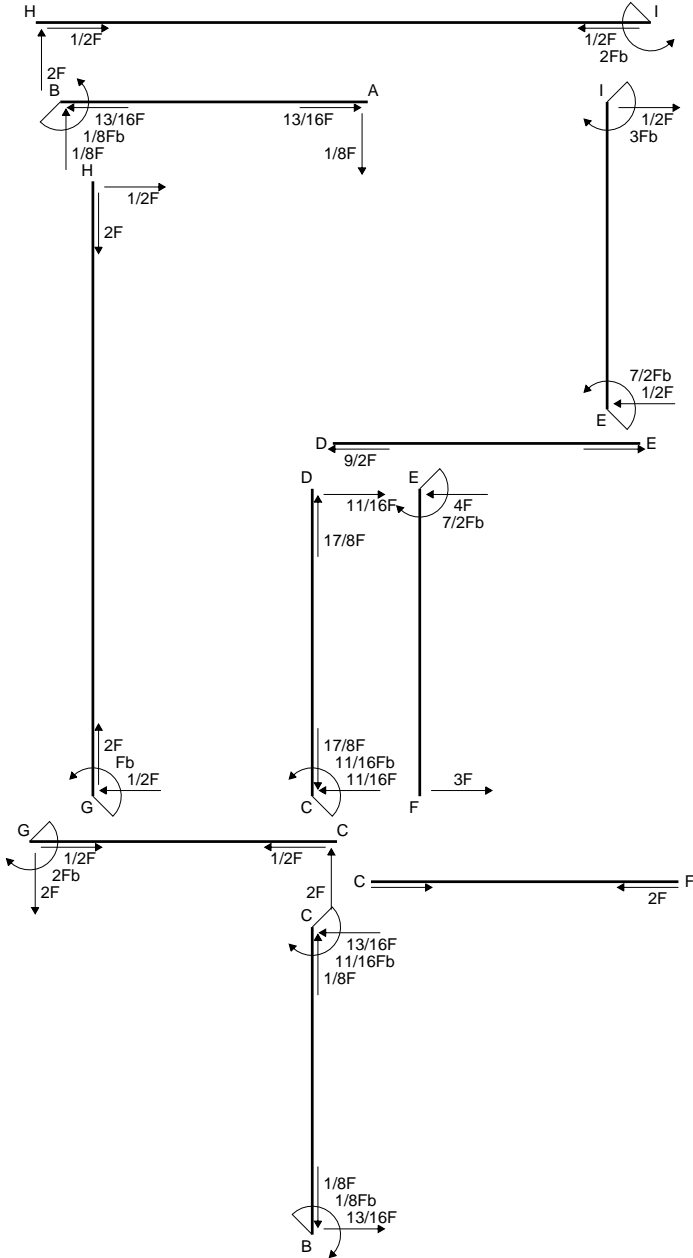
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

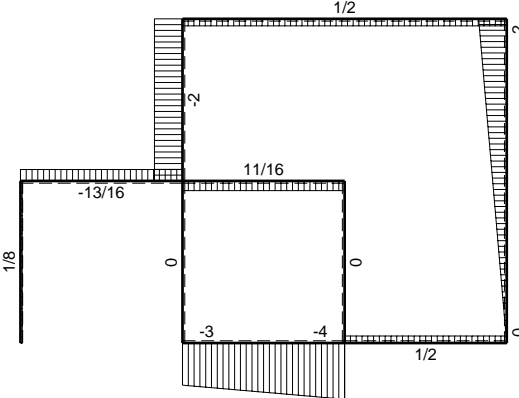
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

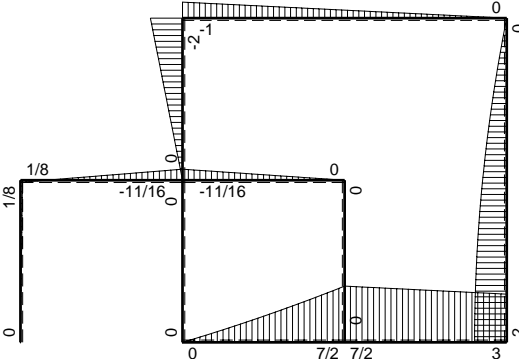




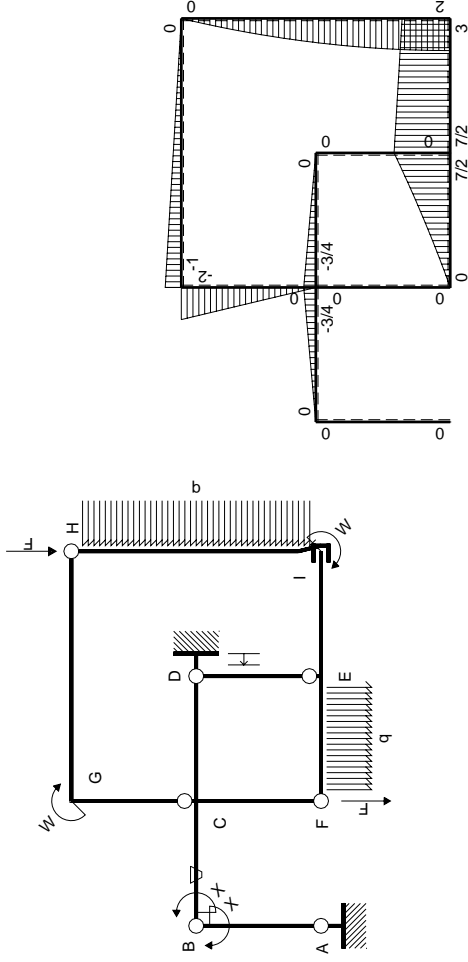
$\leftarrow \boxed{+} \rightarrow F$



$\uparrow \boxed{+} \downarrow F$

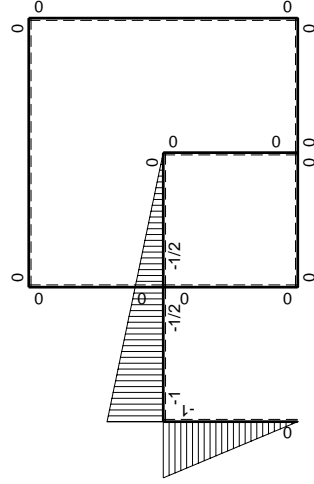


$\curvearrowright \boxed{+} \curvearrowleft F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|-----------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$            |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                       |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$           |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$           |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                       |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                       |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$            |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-1/8Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

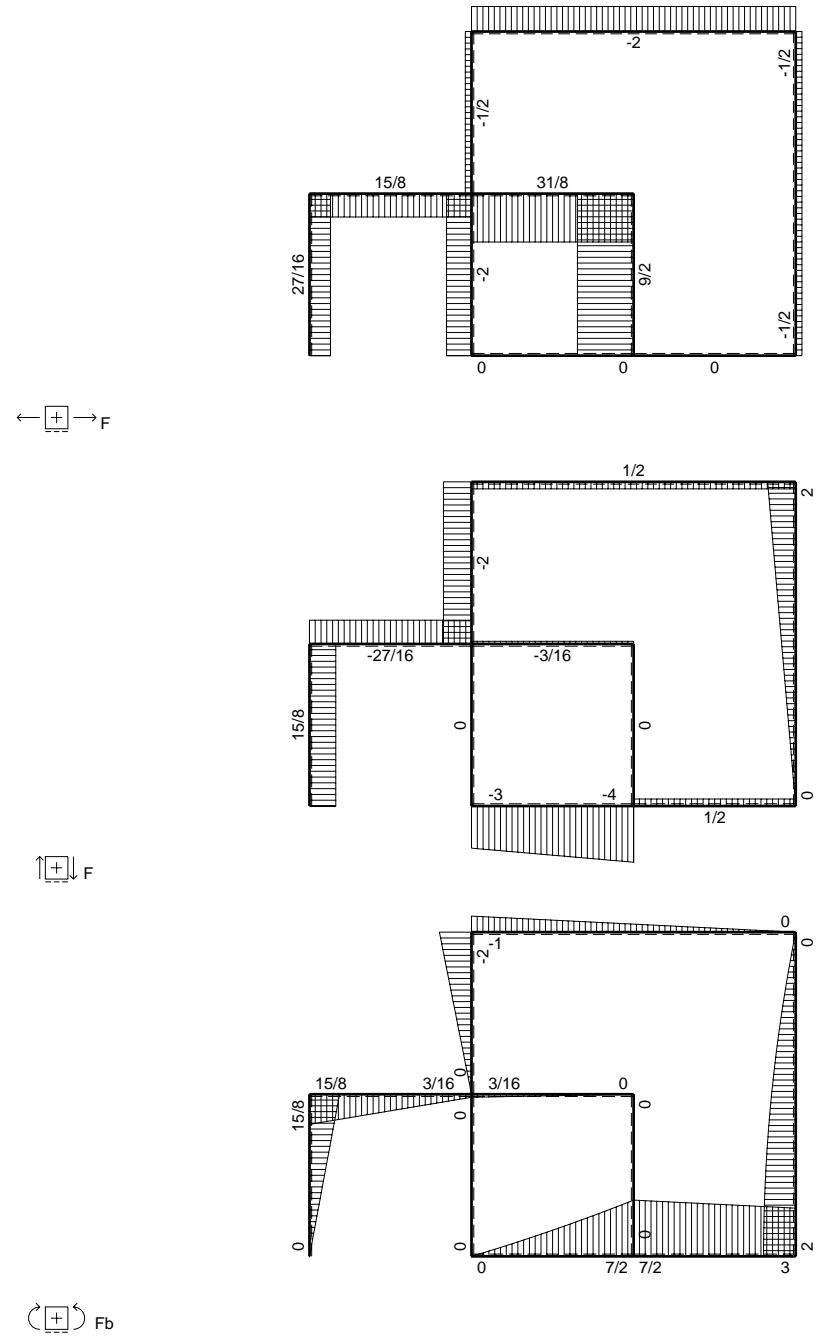
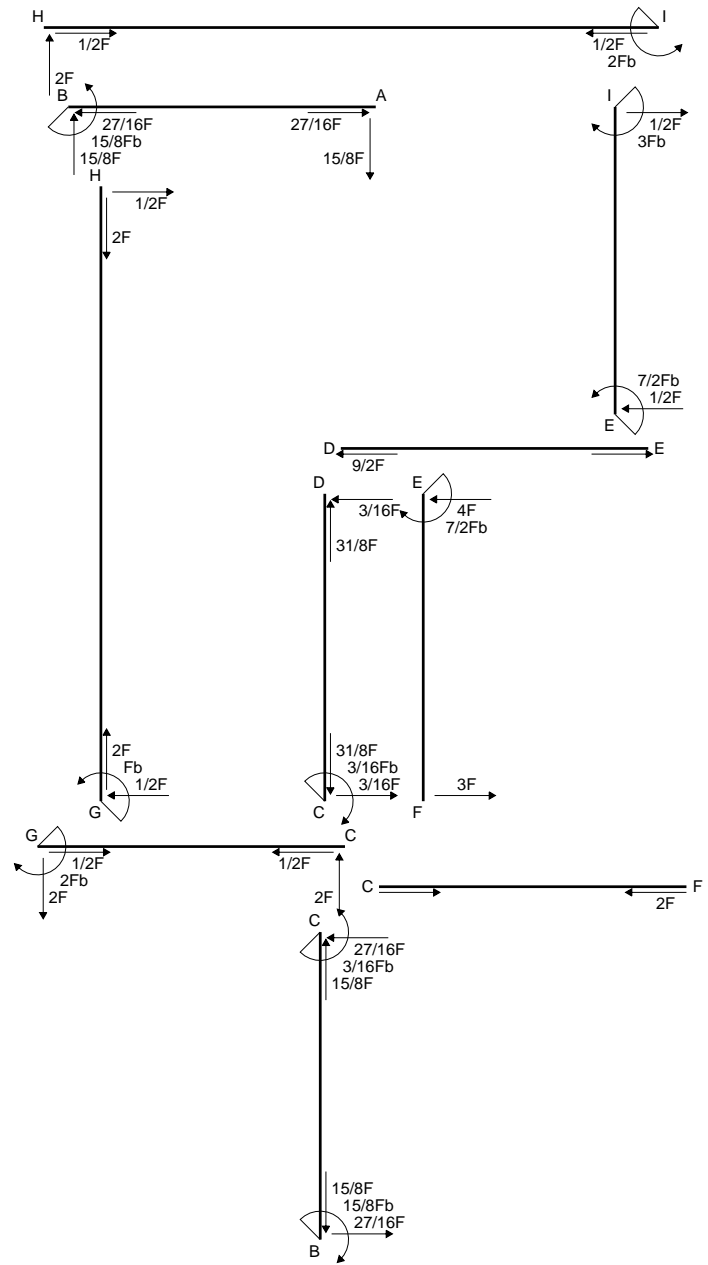
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

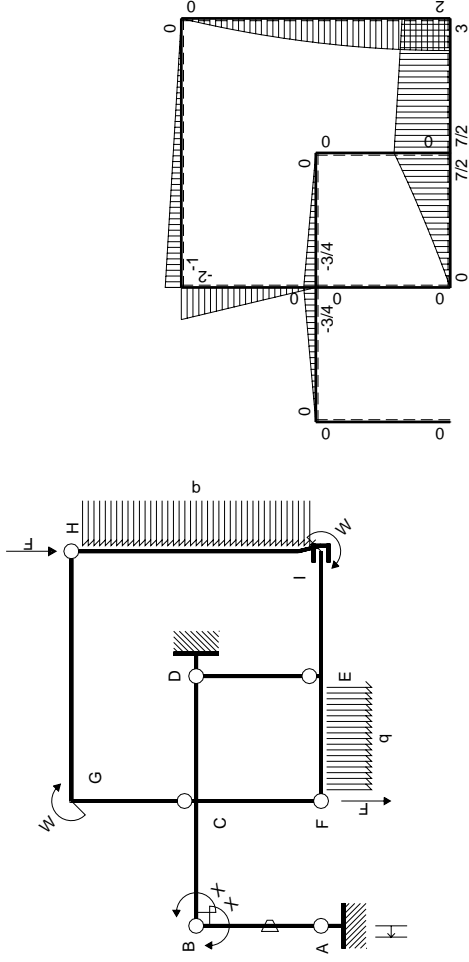
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

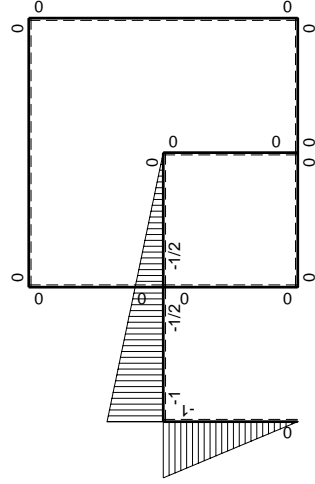
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |               |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-15/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

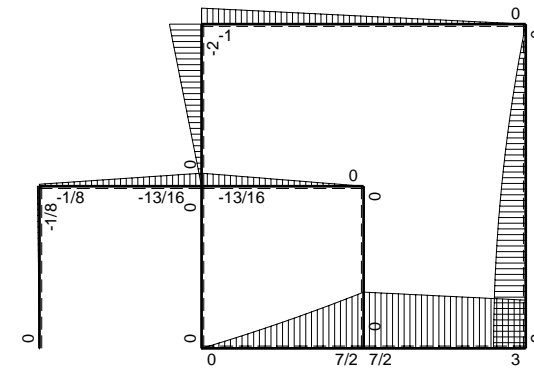
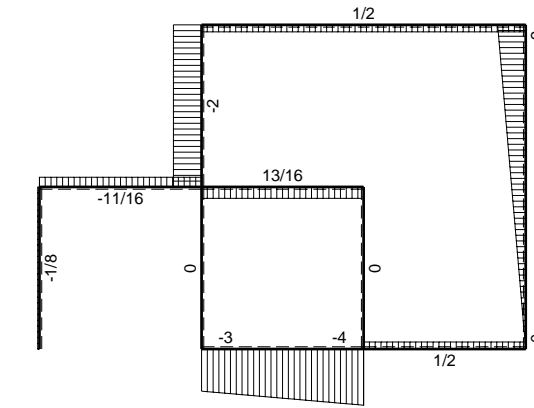
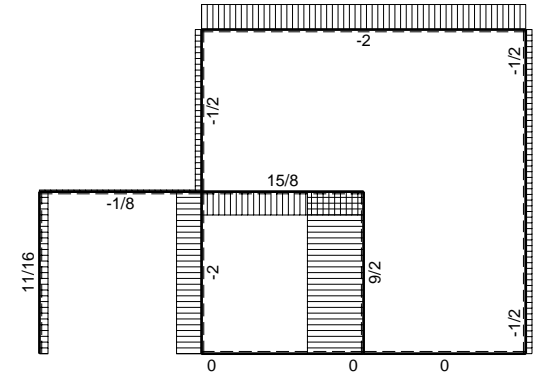
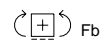
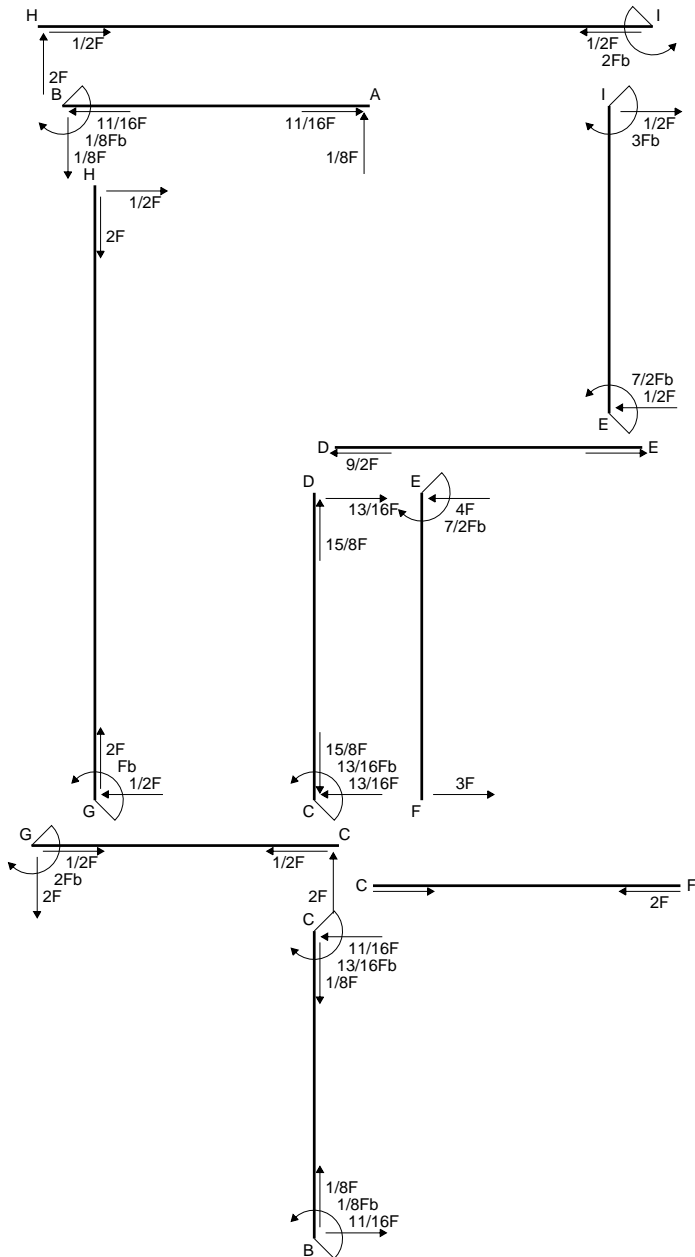
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

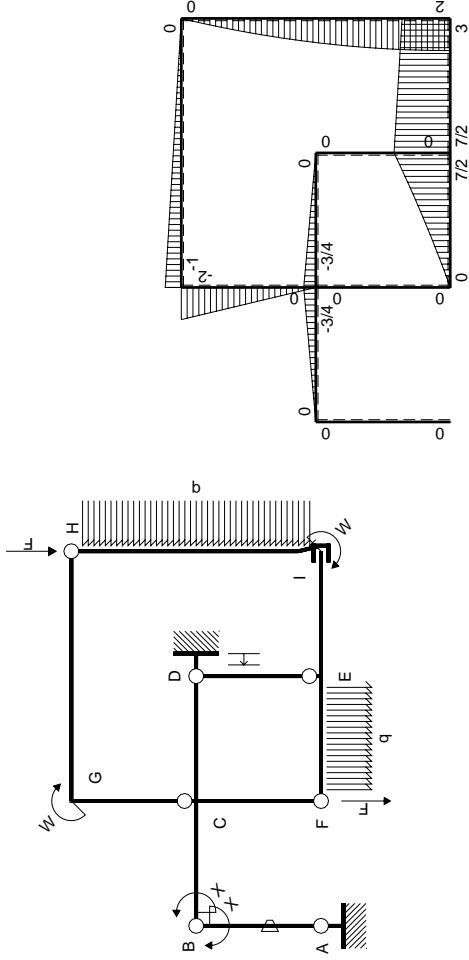
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

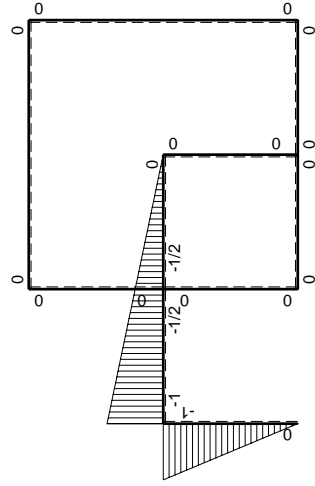






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |               |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

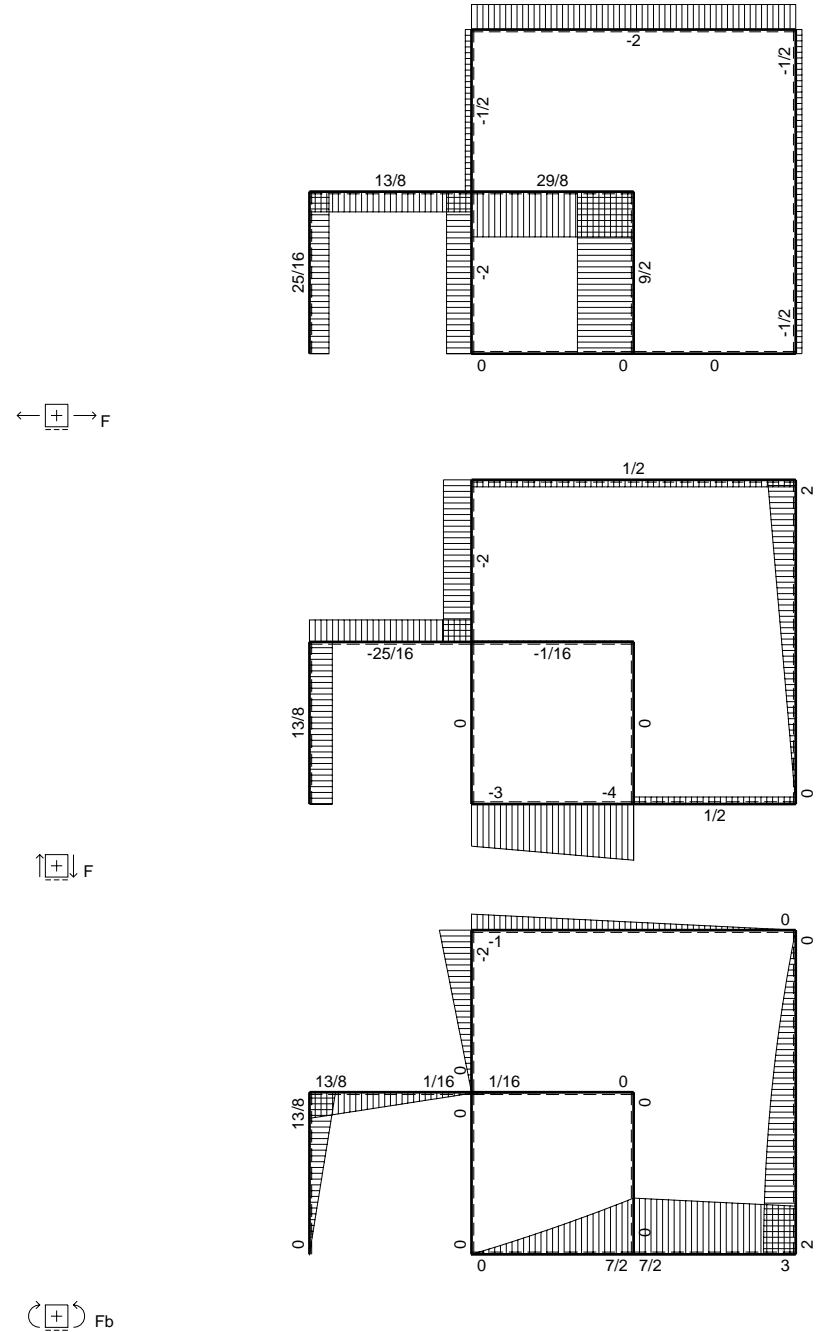
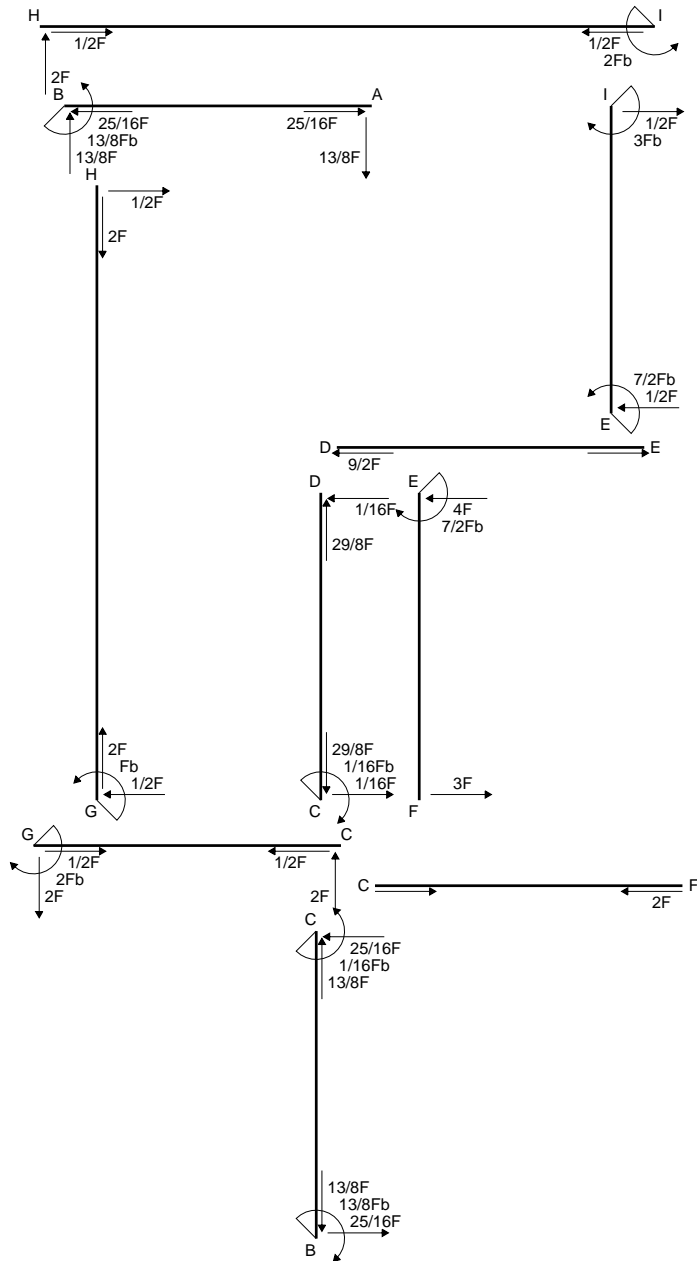
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

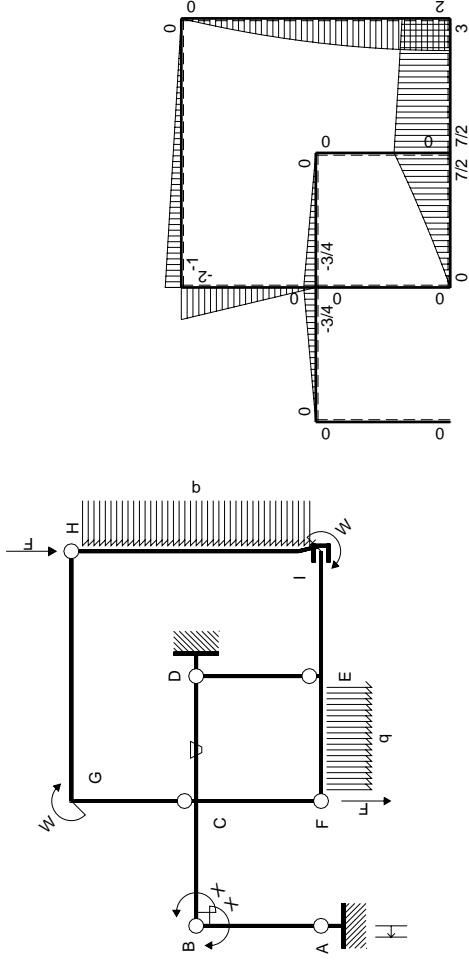
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

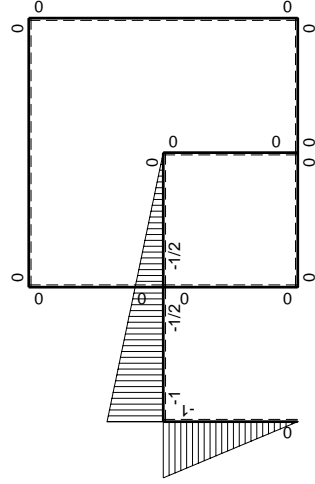
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $13/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-13/8Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

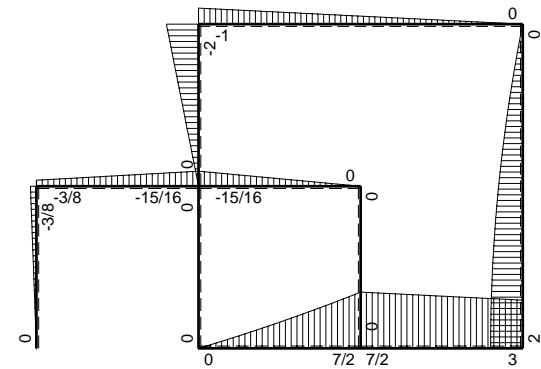
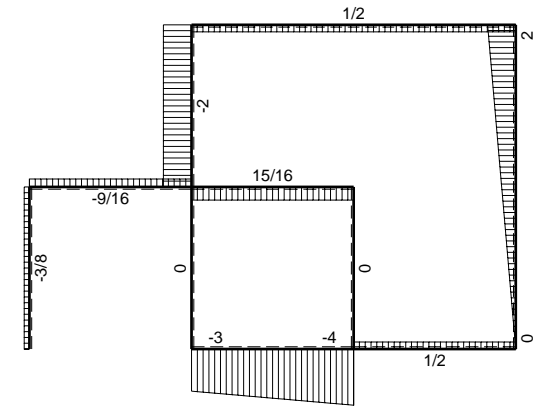
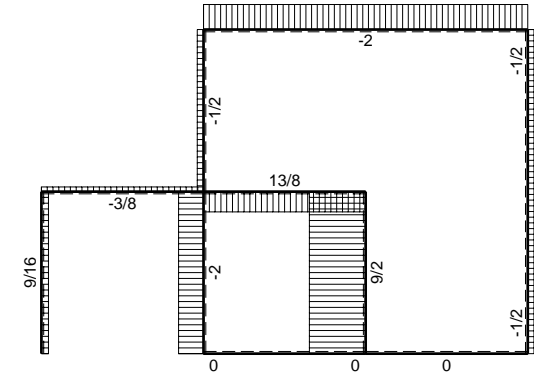
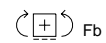
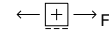
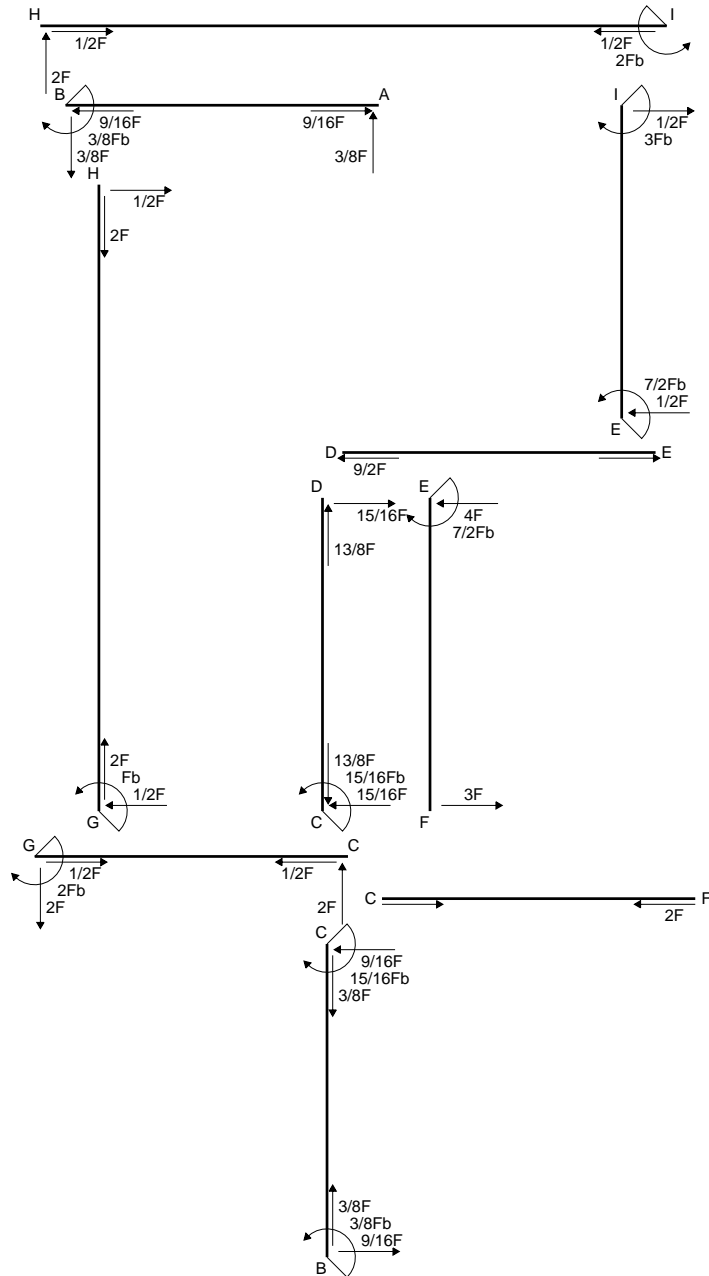
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

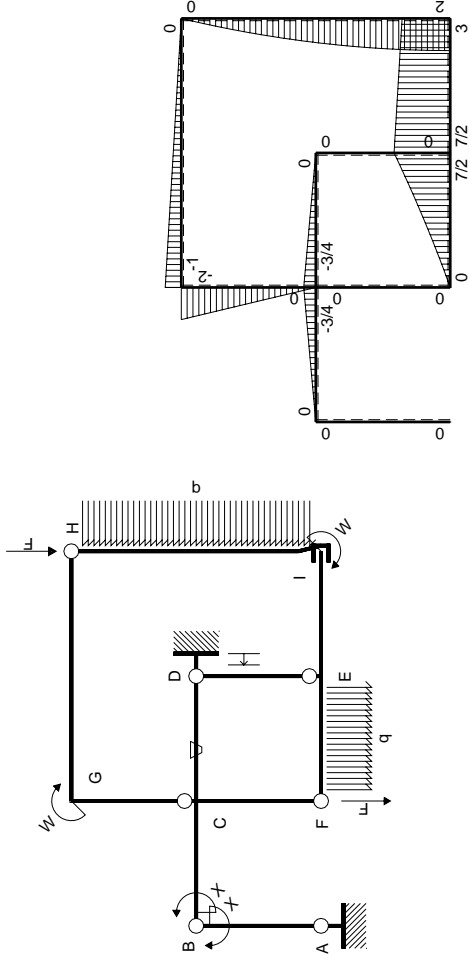
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

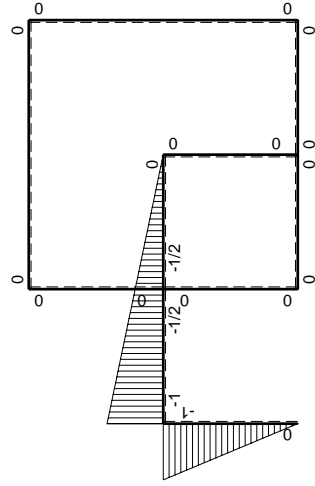






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-3/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $3/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

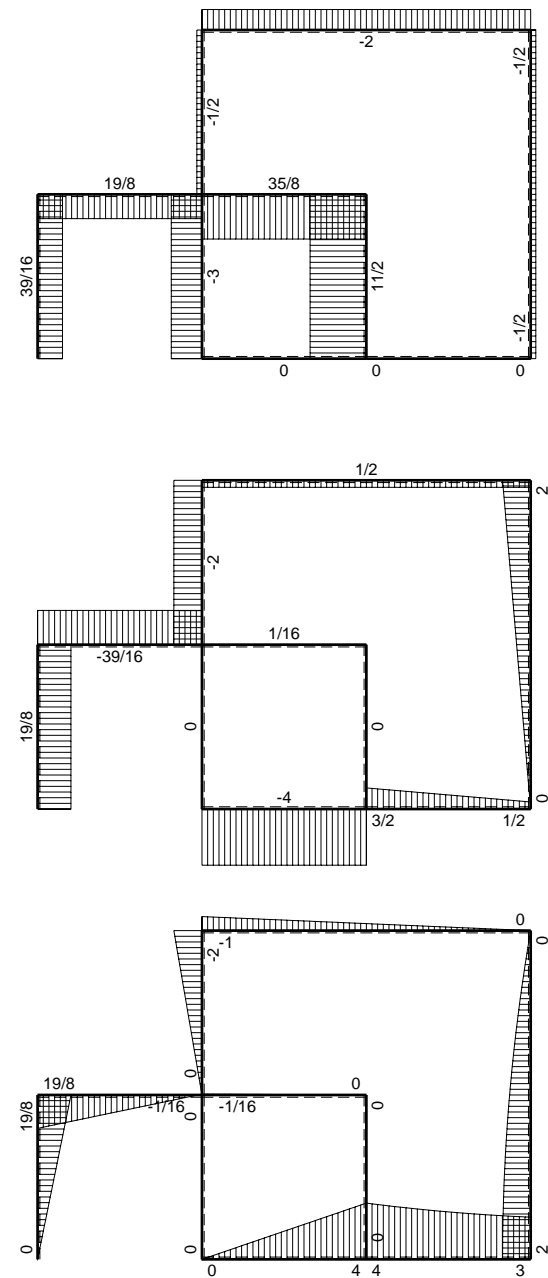
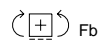
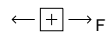
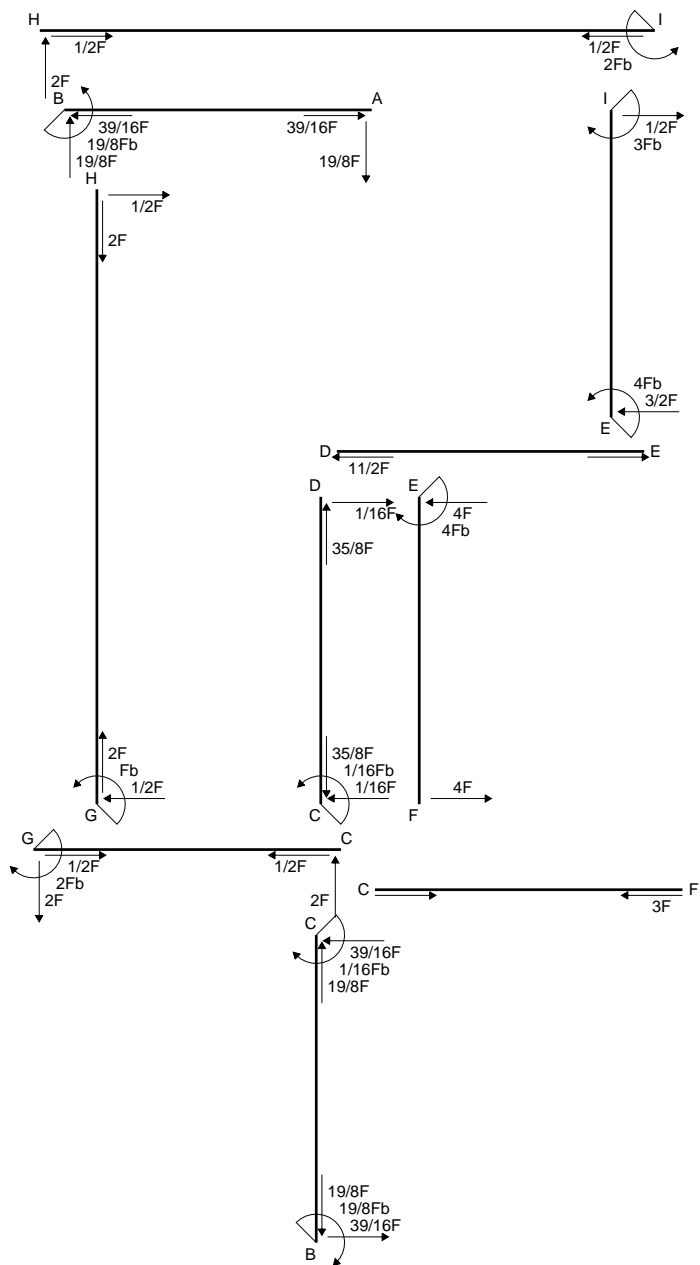
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

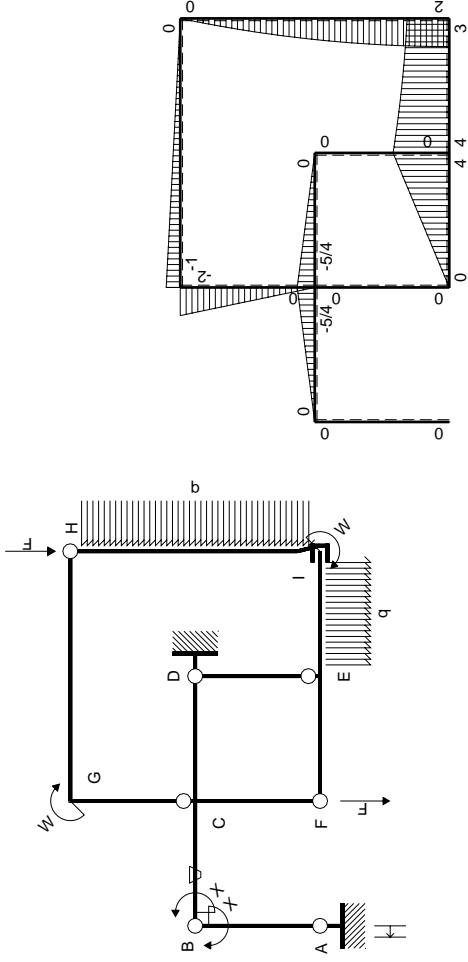
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

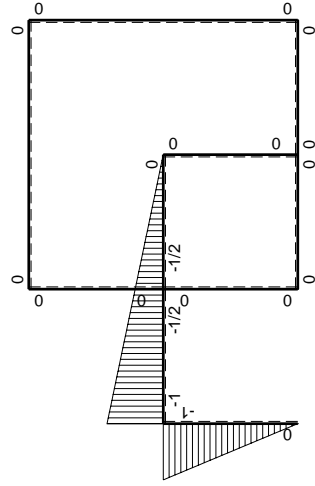
$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |                     |                         | $19/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-19/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

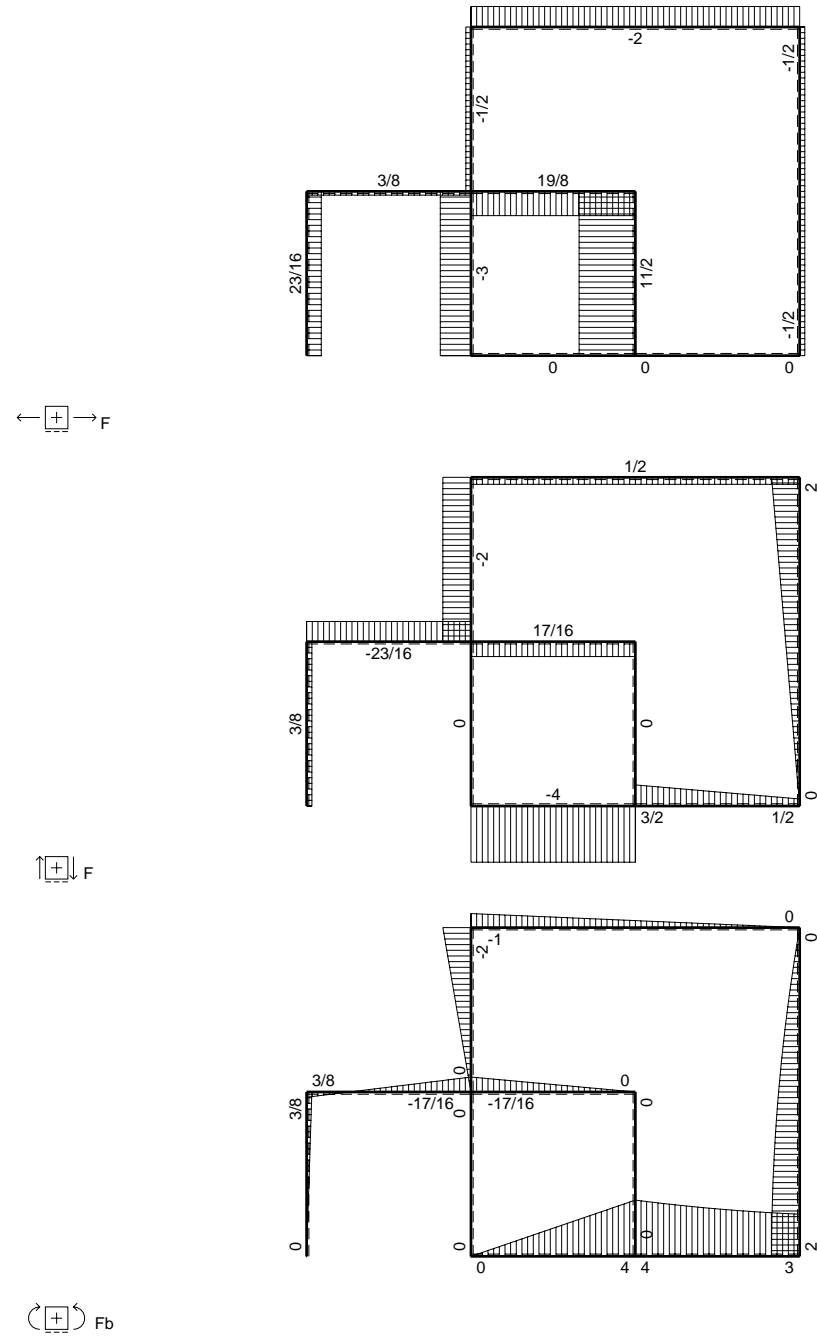
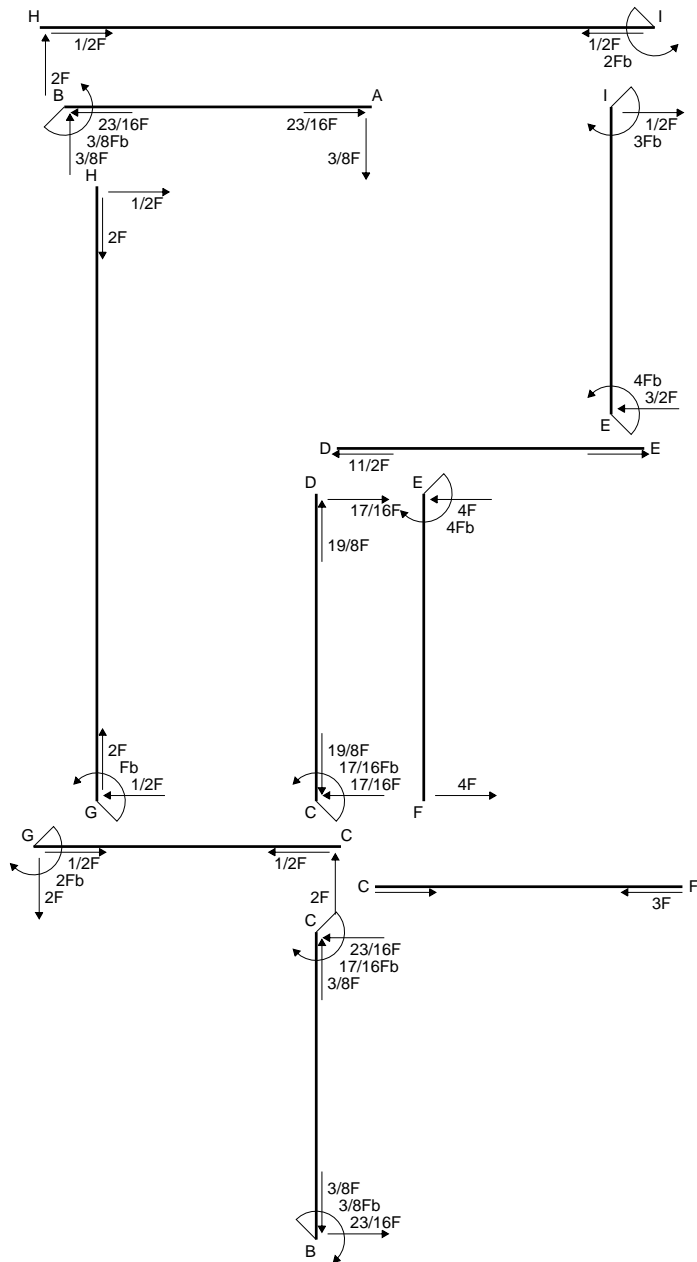
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

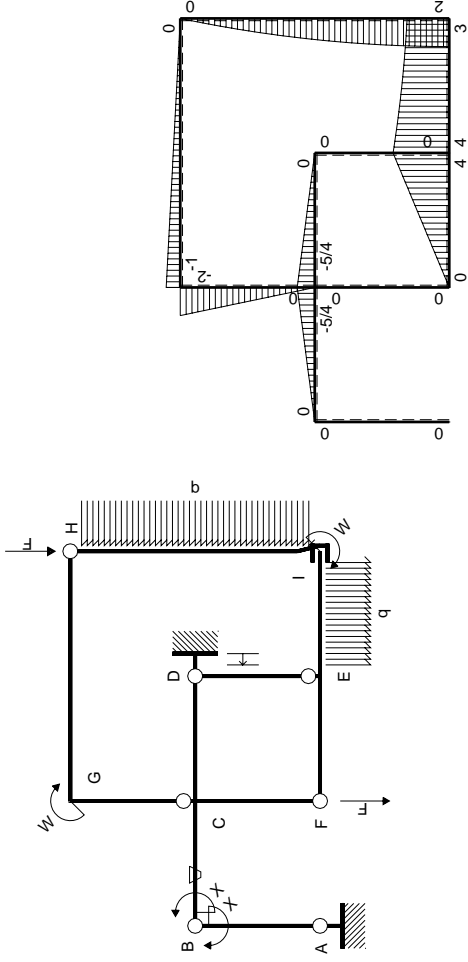
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

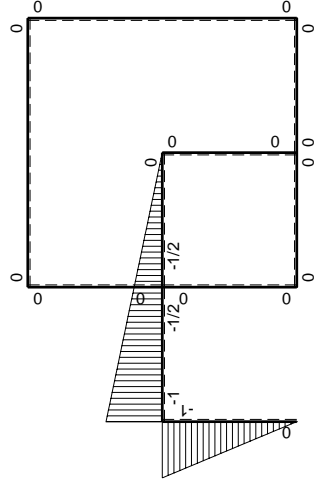






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |                     |                         |                             | $3/8Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         |                             | $-3/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

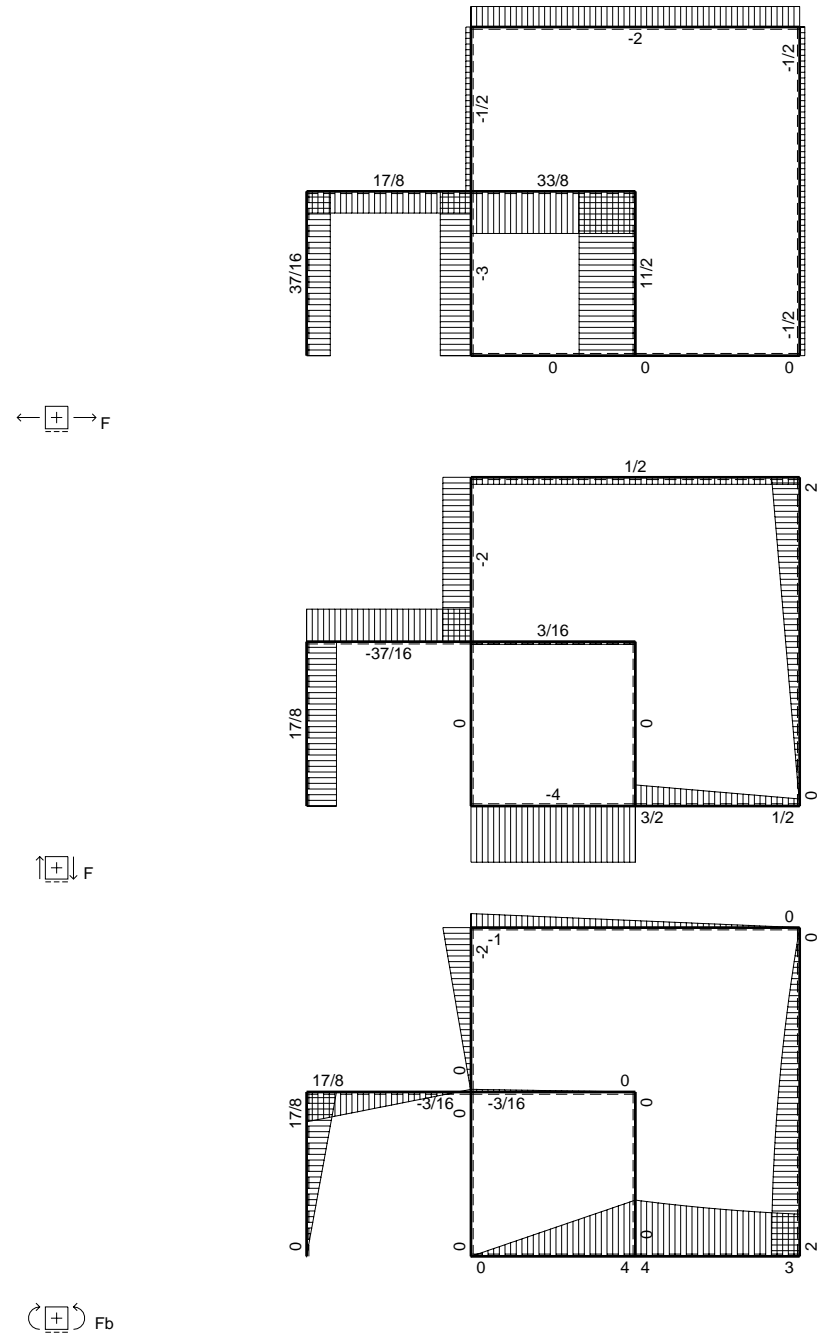
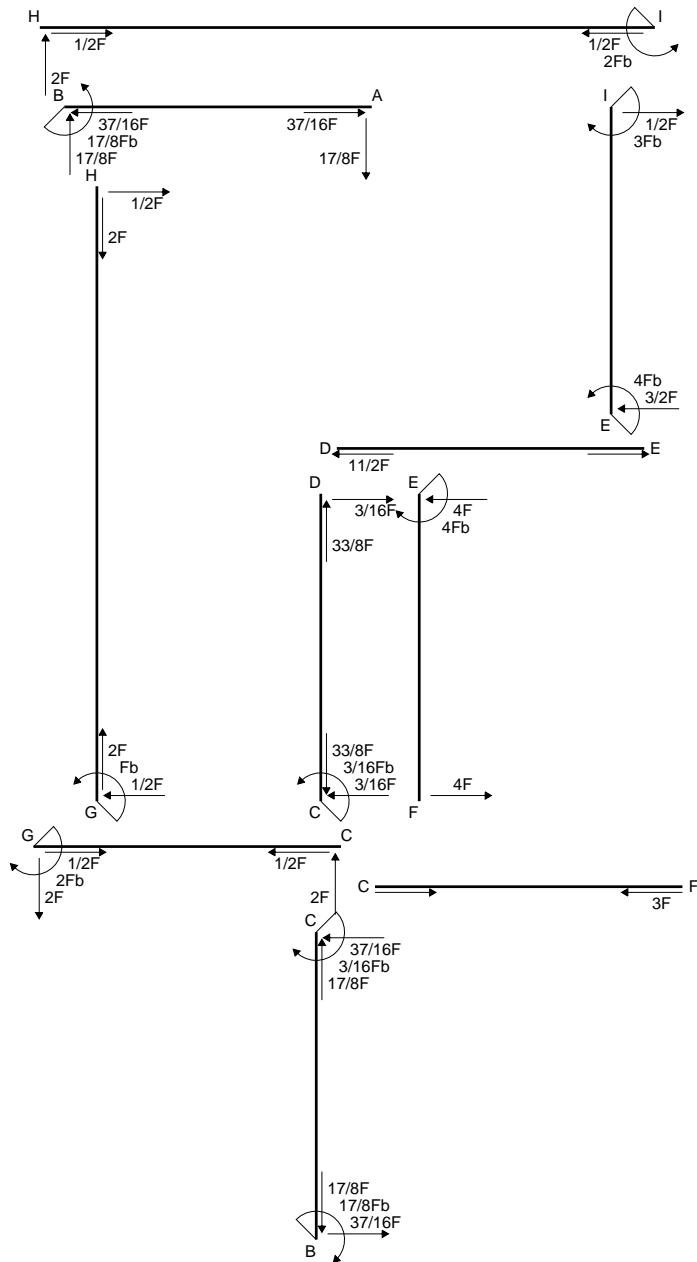
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

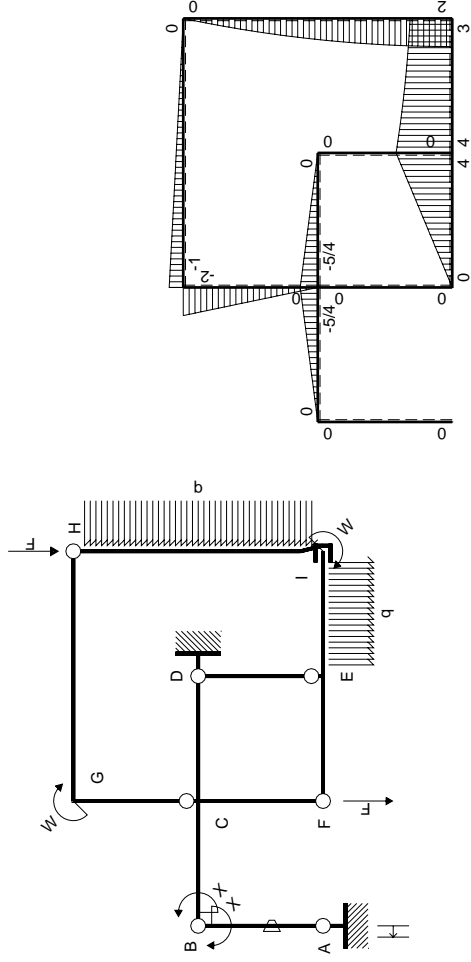
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

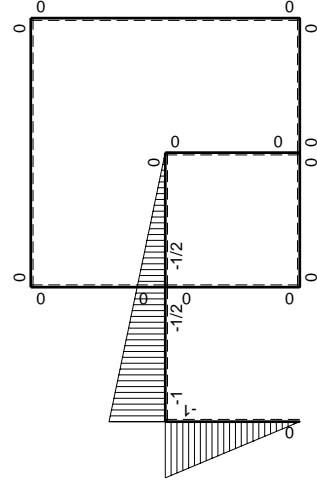


⊕ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |               |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |               |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

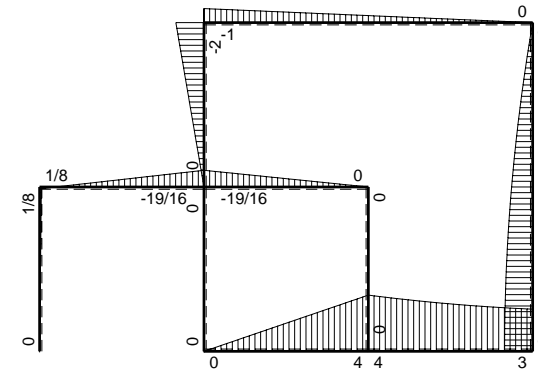
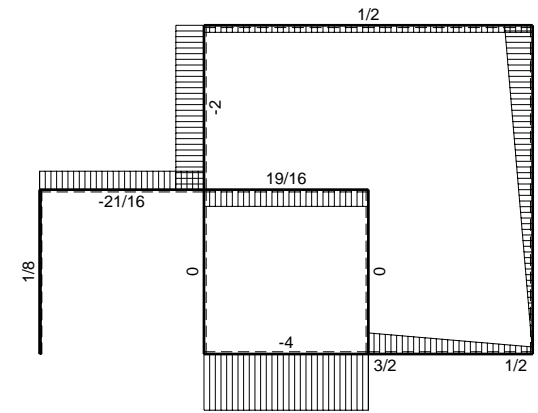
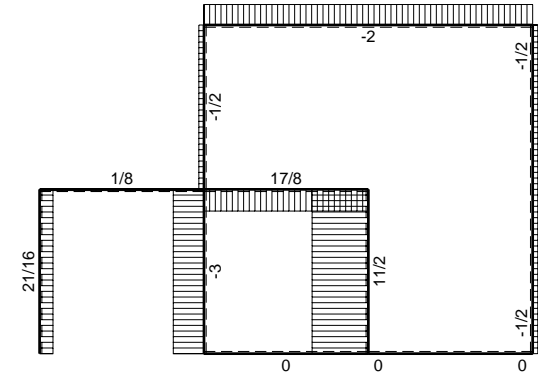
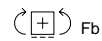
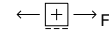
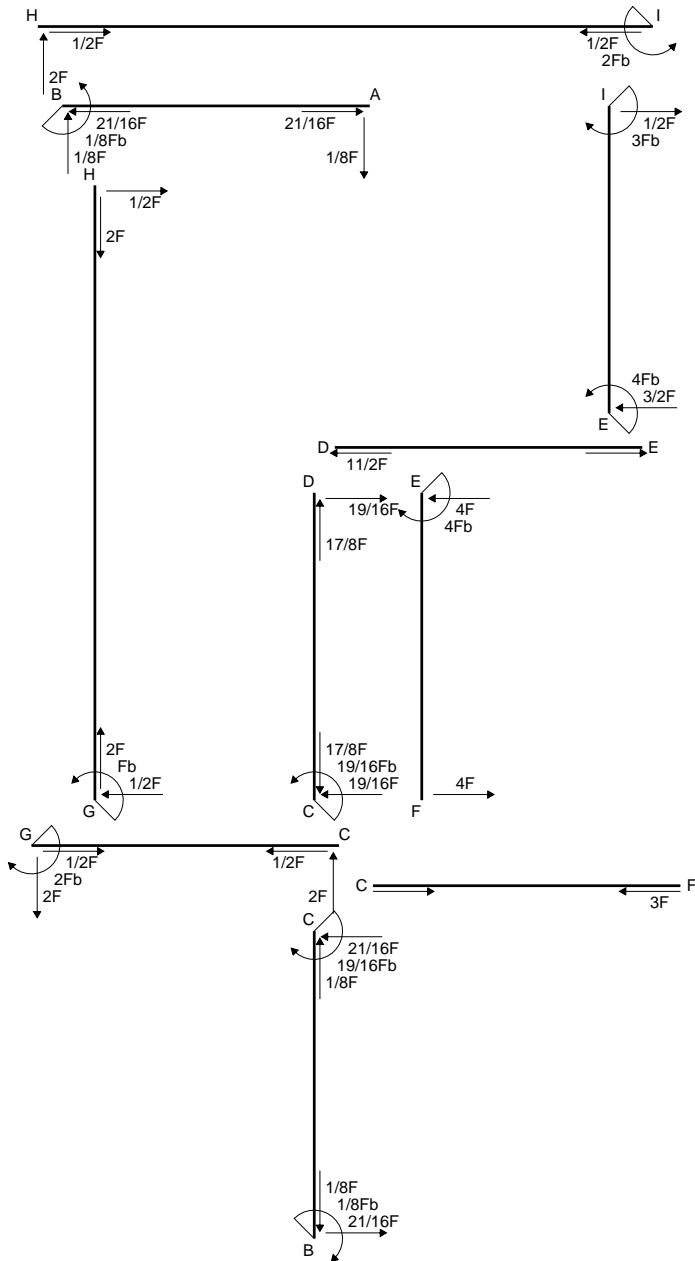
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

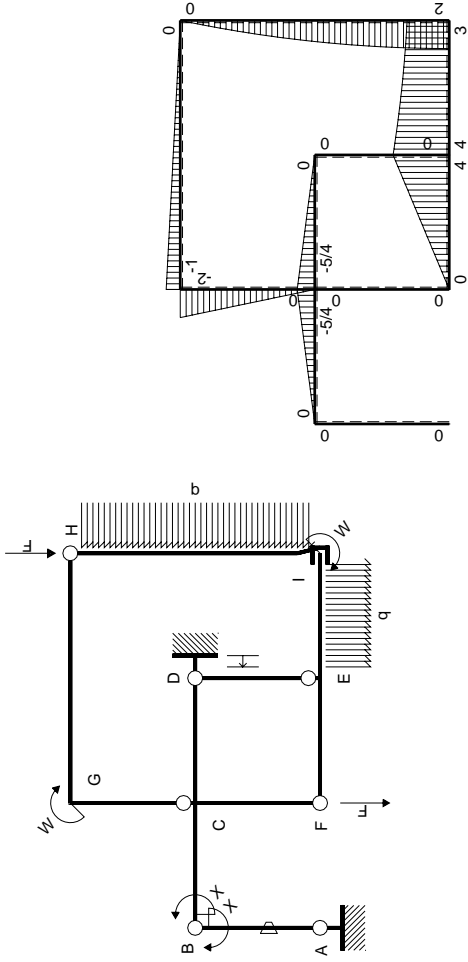
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

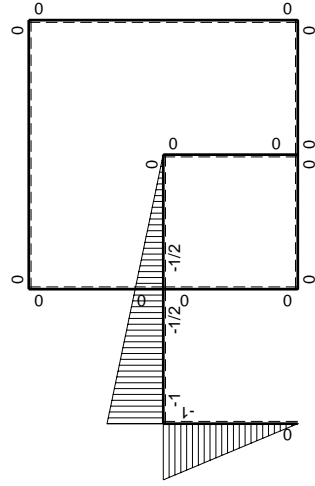






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |               |                         |                             | $1/8Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |               |                         |                             | $-1/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

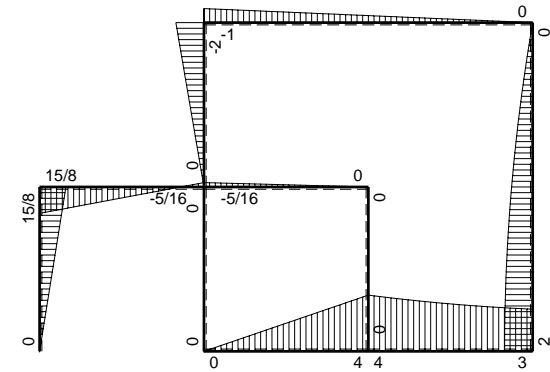
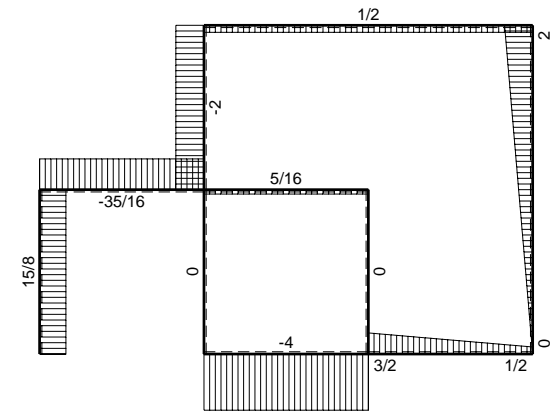
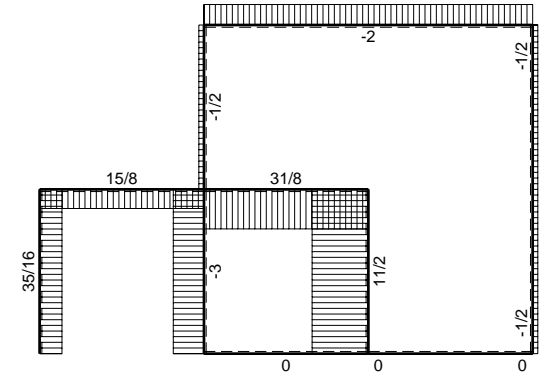
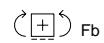
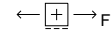
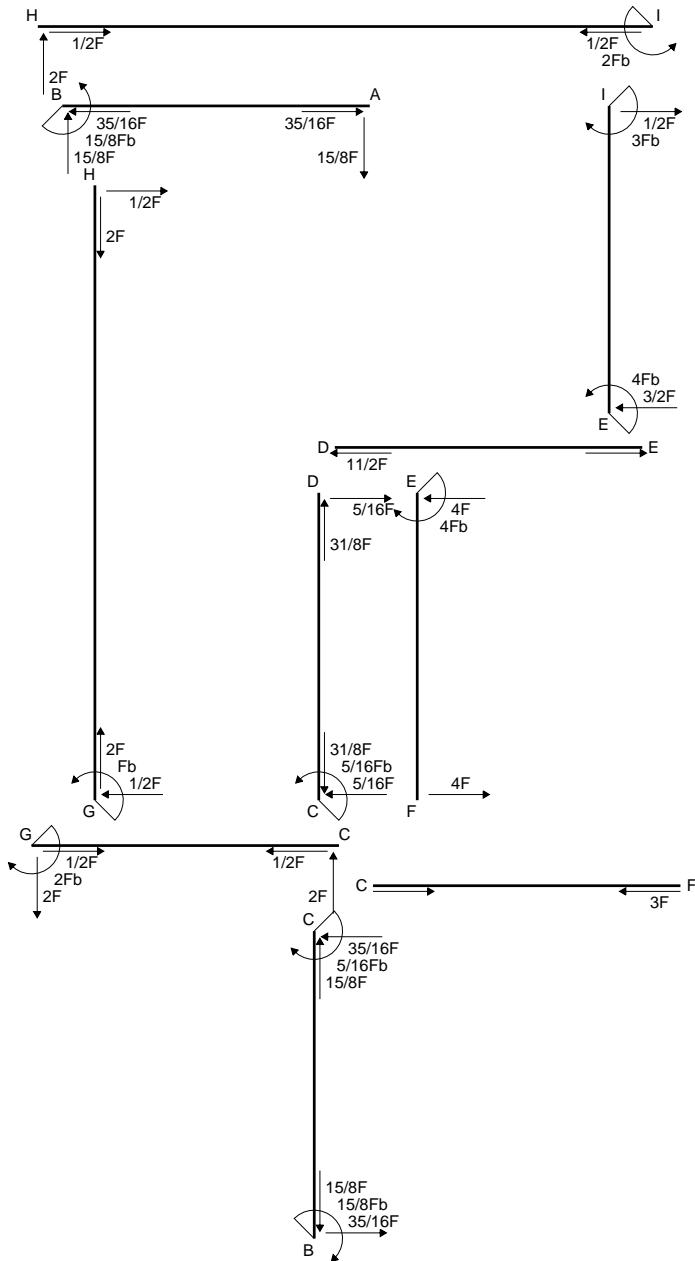
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

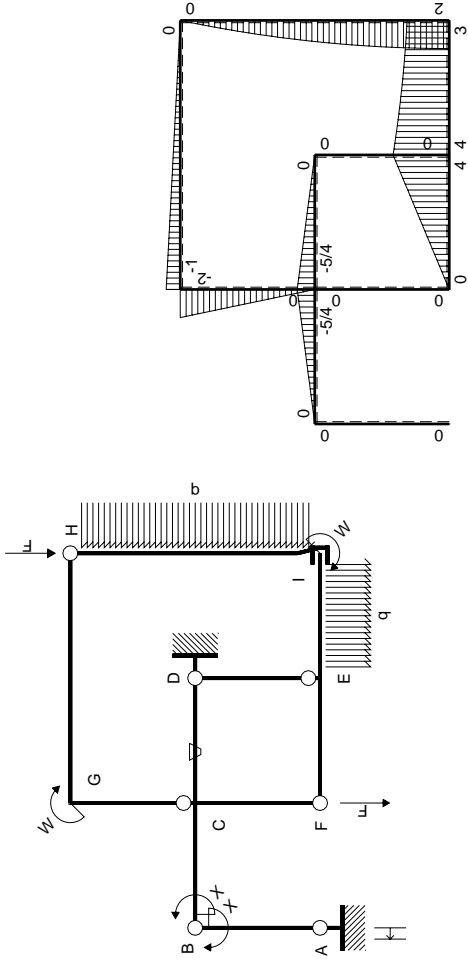
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

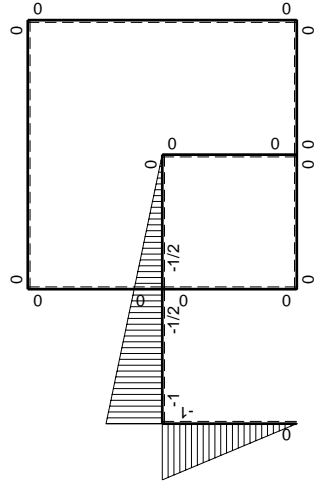
$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |                     |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-15/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

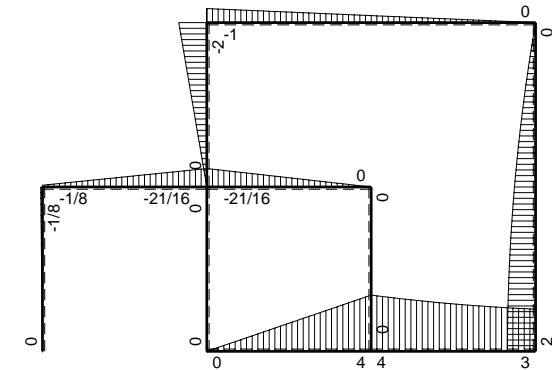
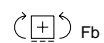
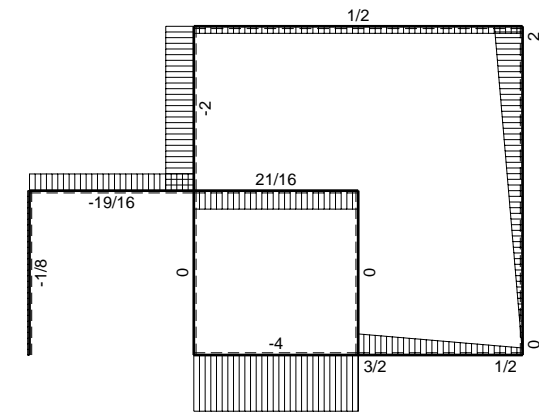
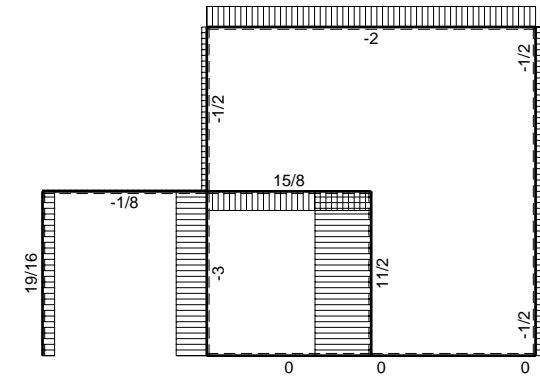
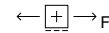
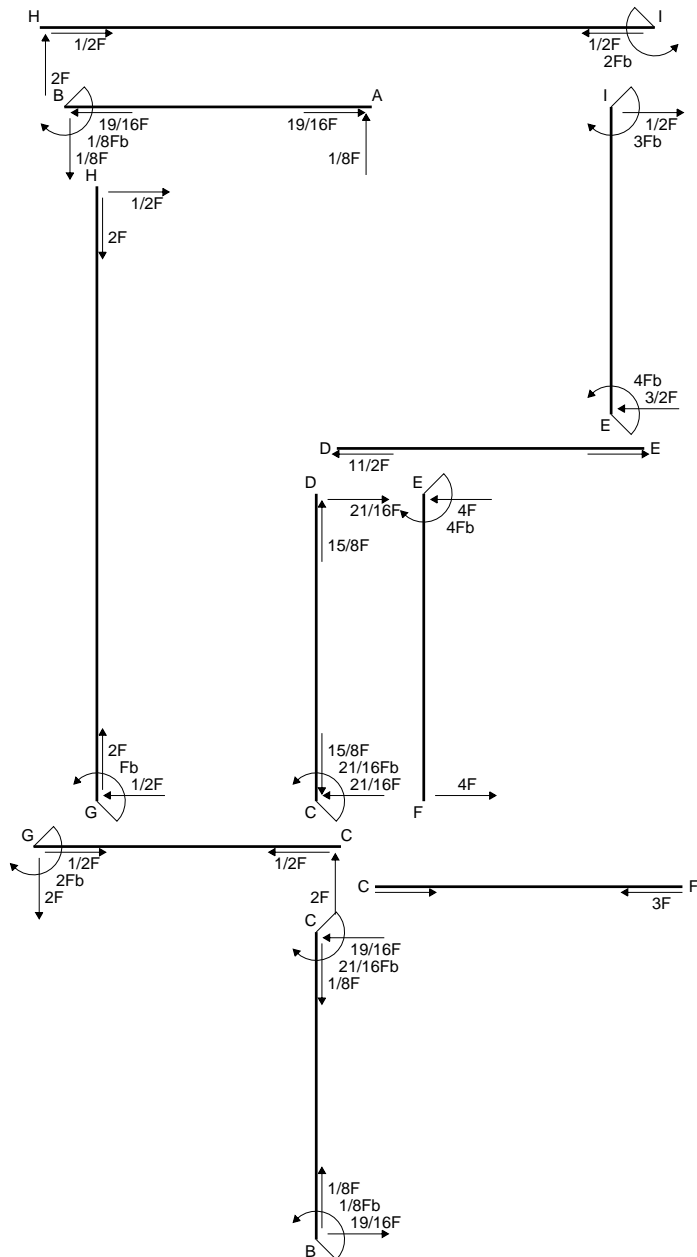
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

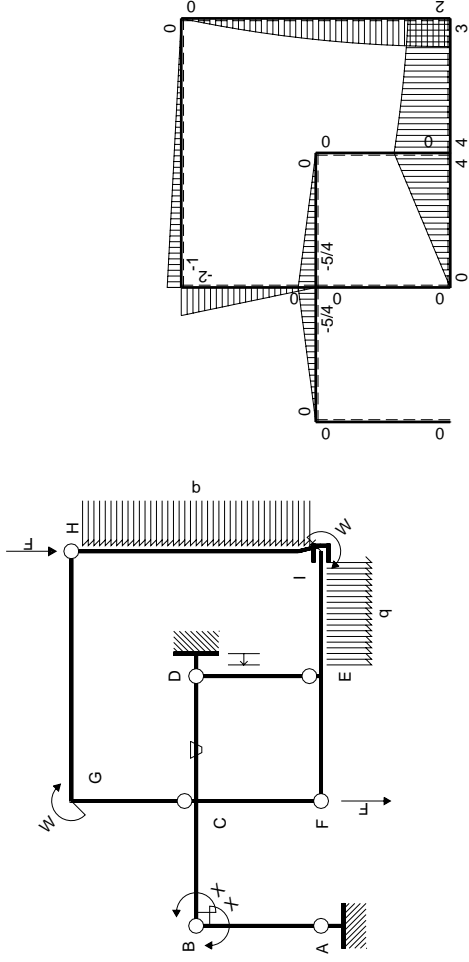
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$

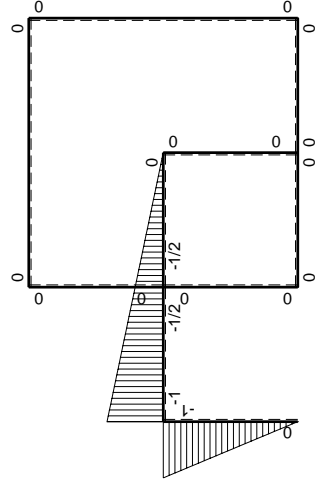






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |                     |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

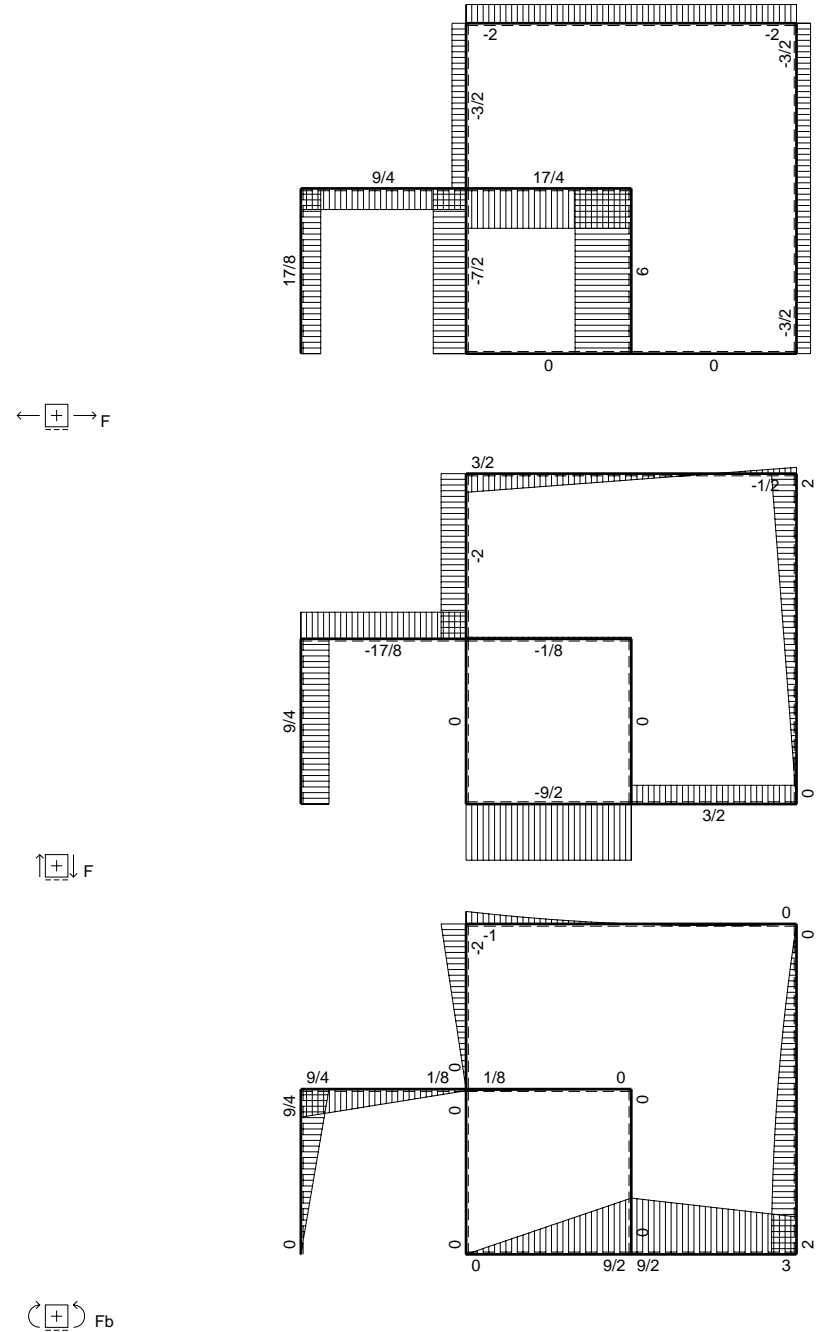
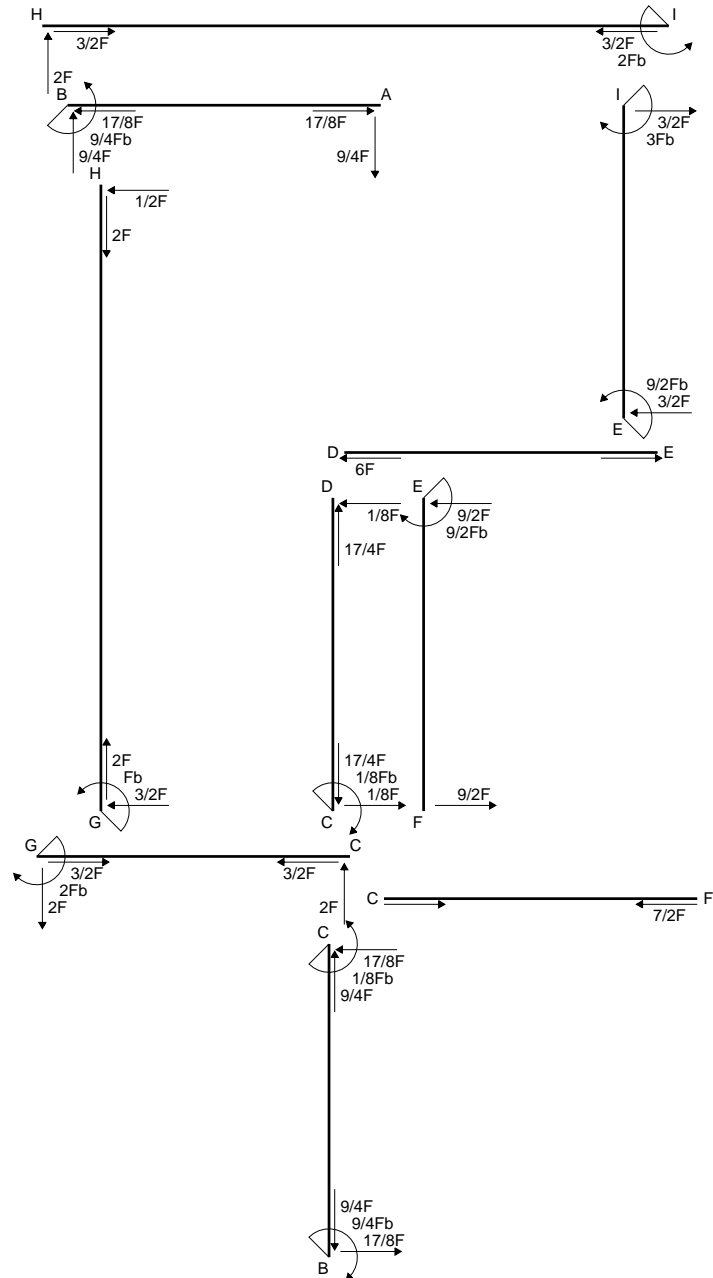
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

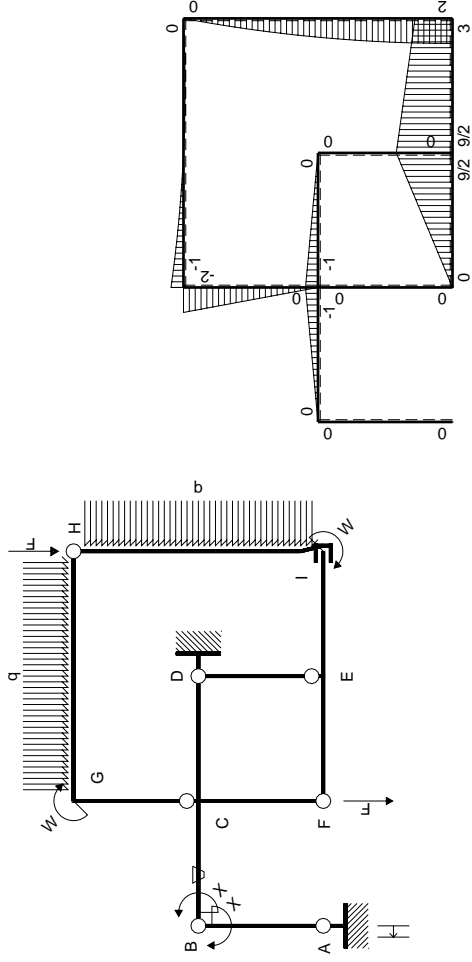
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

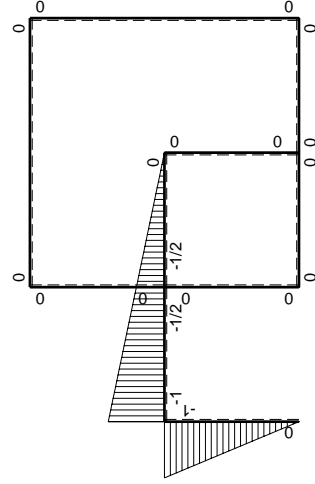
$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_X$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$            | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                      |                     |                         | $9/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                      |                     |                         | $-9/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

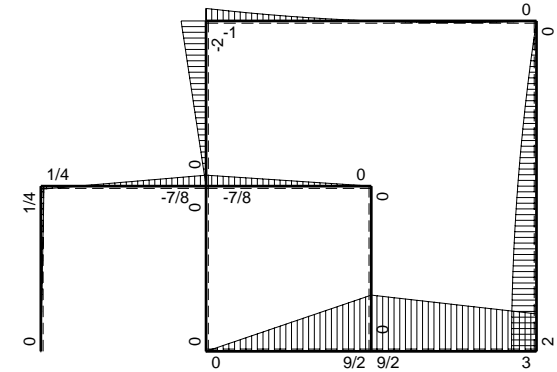
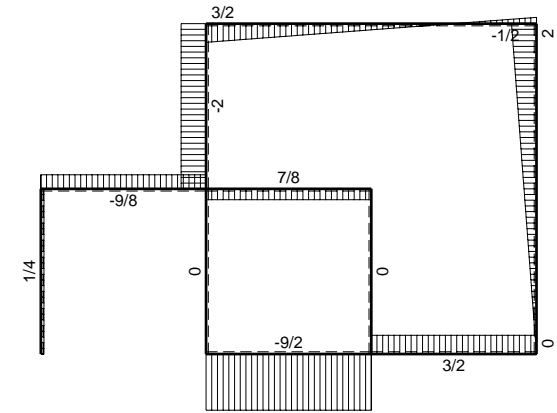
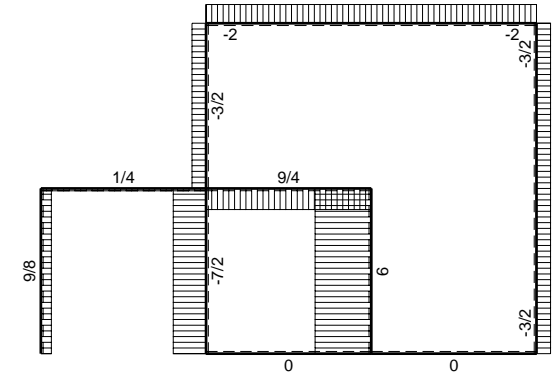
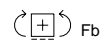
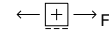
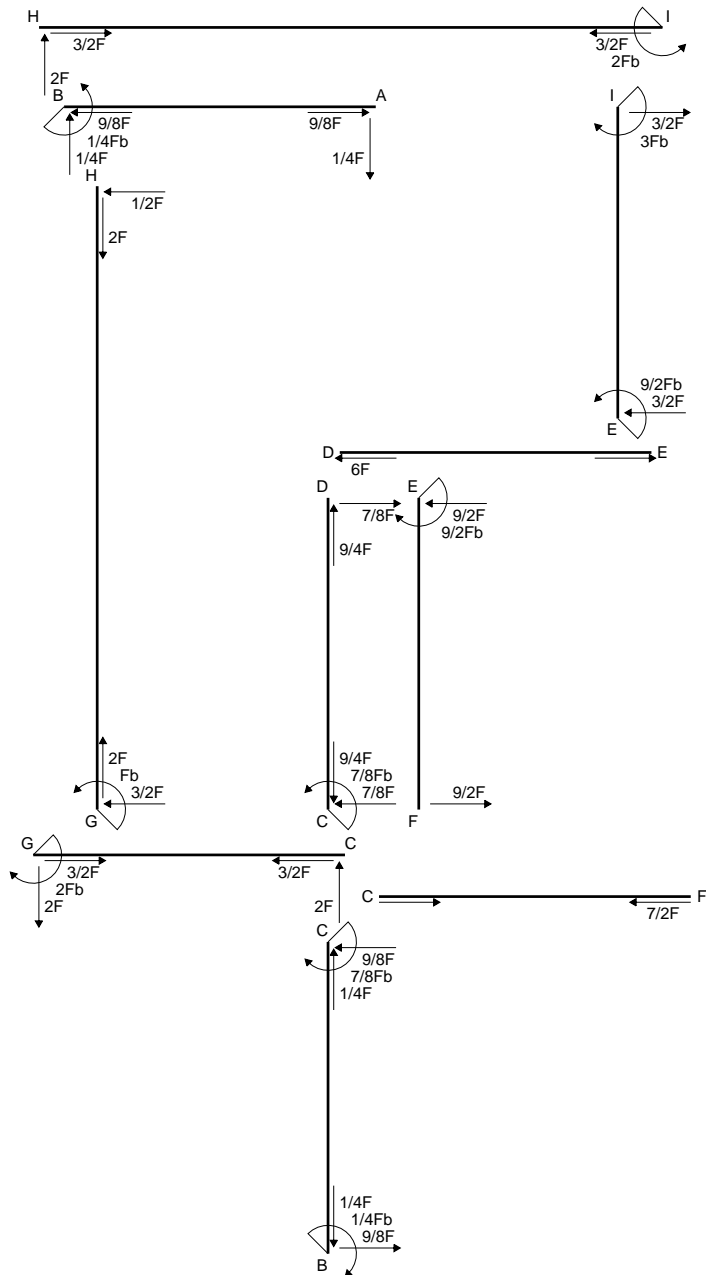
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

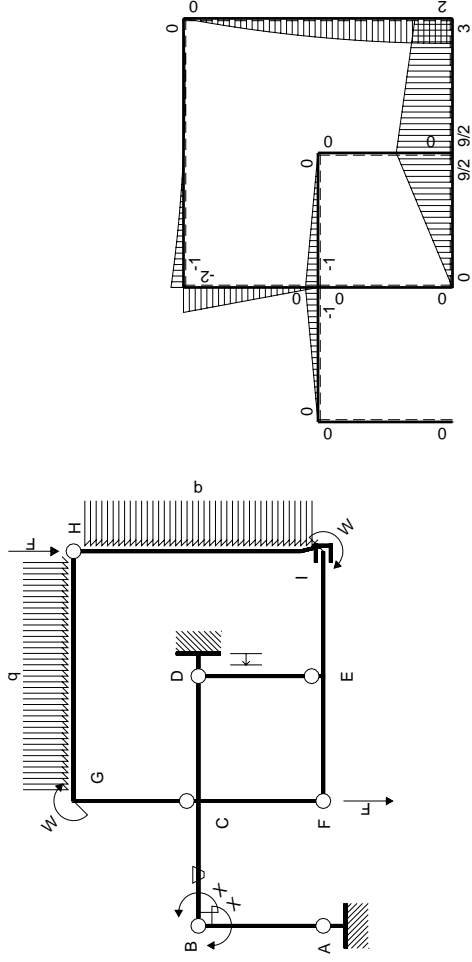
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

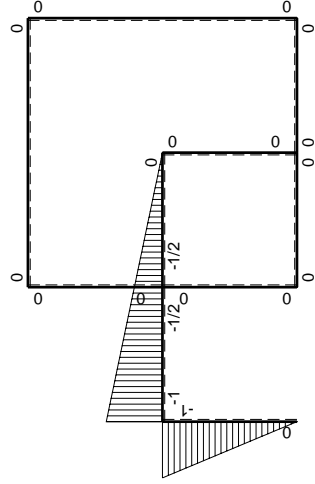






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$            | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                      |                     |                         |                             | $1/4Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                      |                     |                         |                             | $-1/4Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

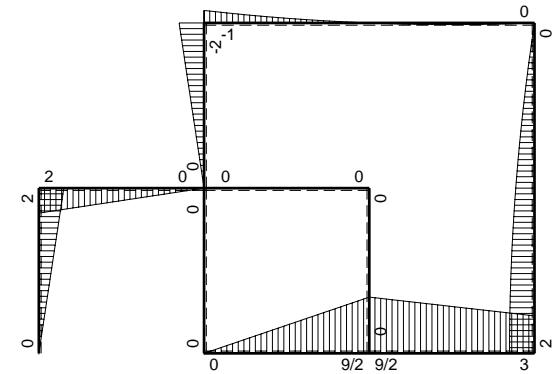
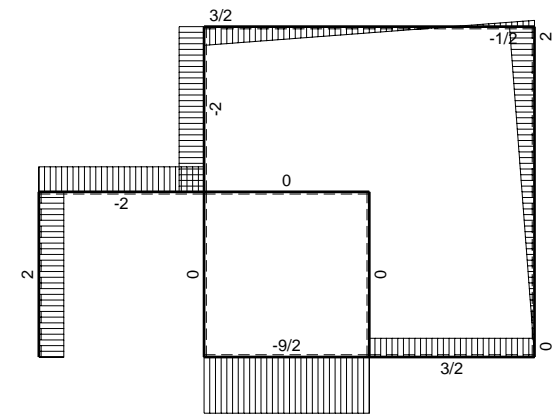
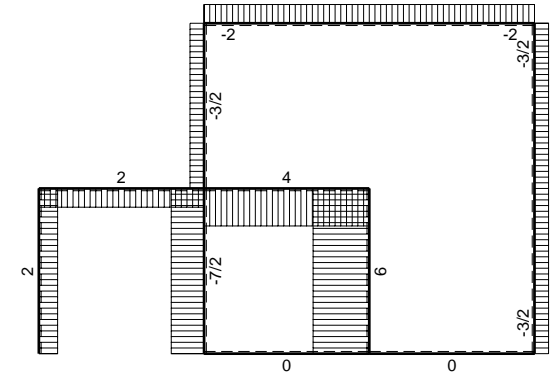
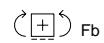
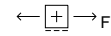
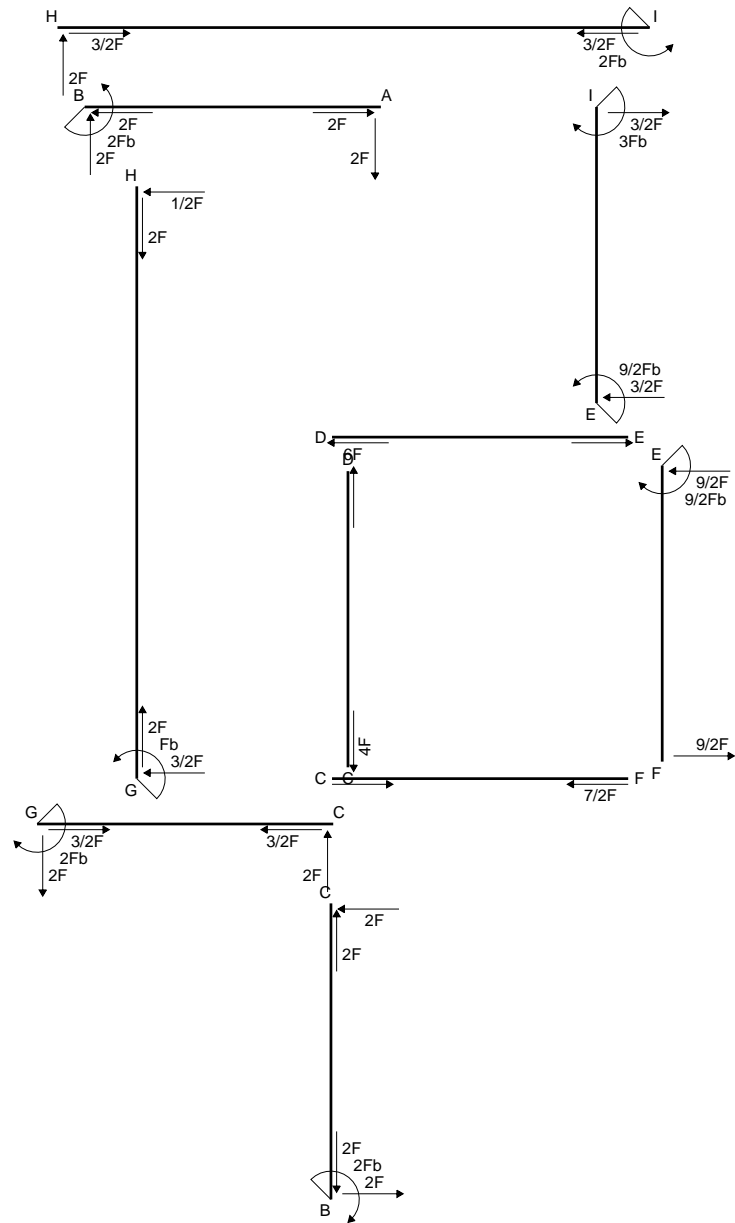
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

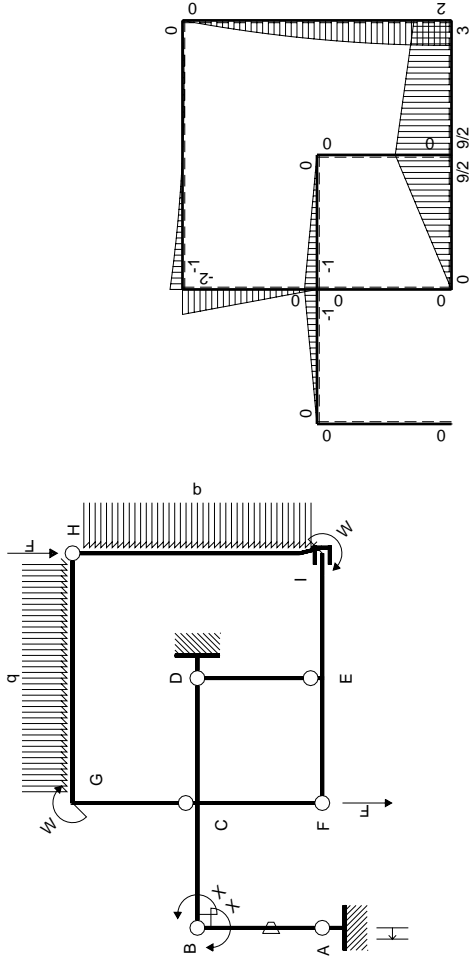
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

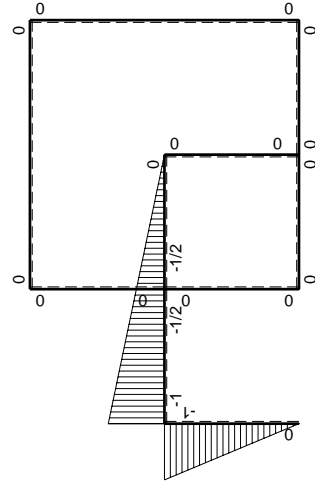
$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$            | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                      |               |                         | $2Fb^2/EJ$                  | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                      |               |                         | $-2Fb$                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

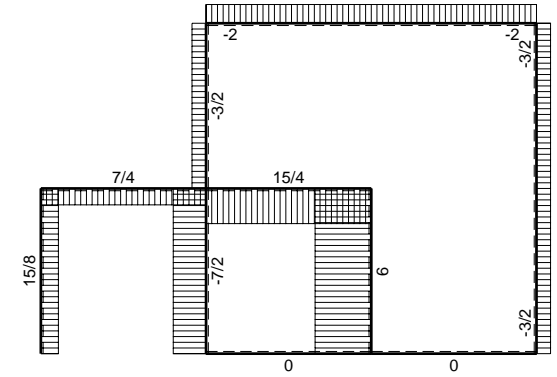
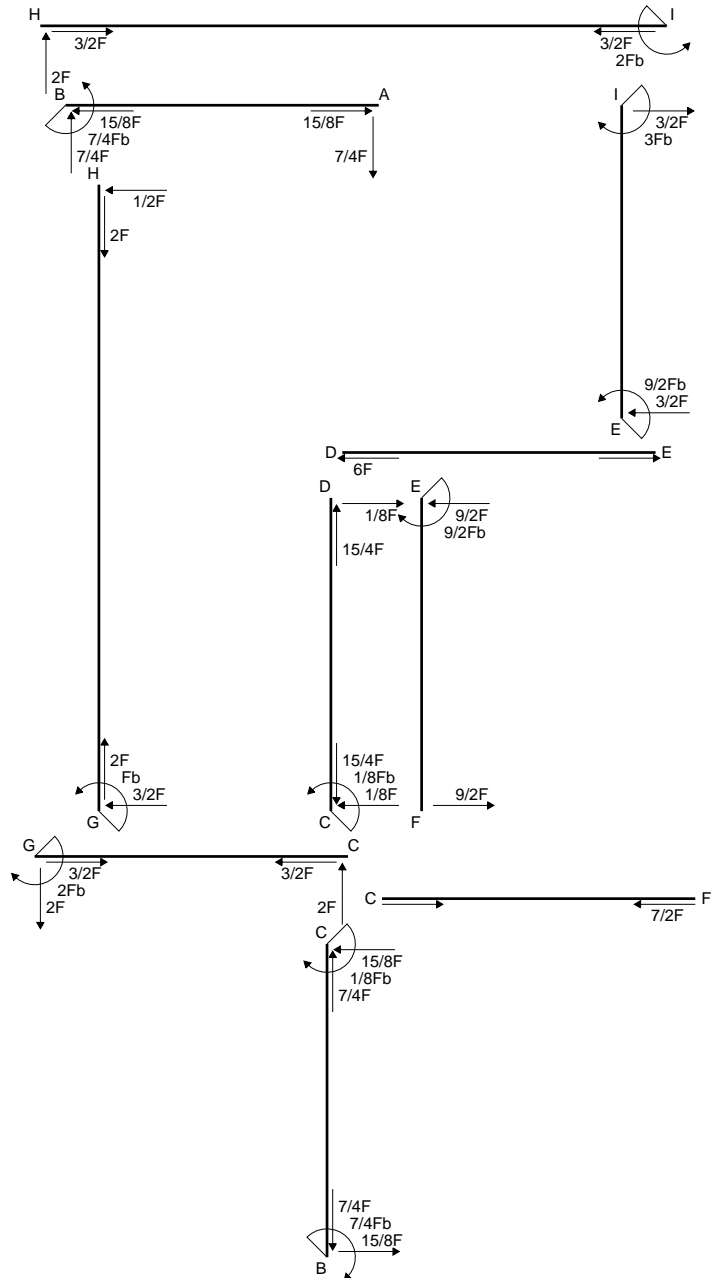
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

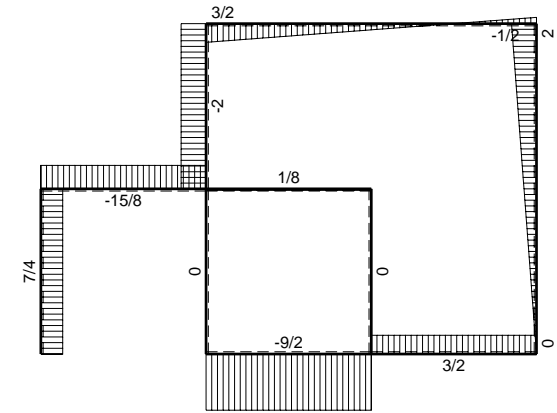
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

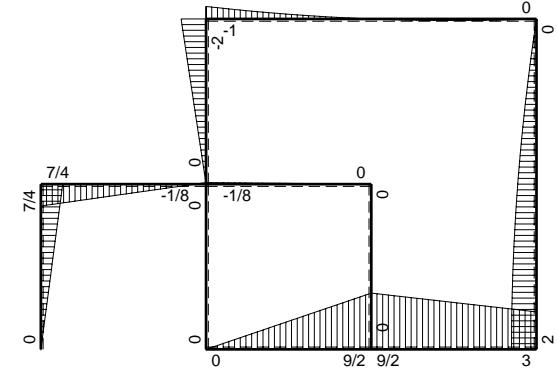




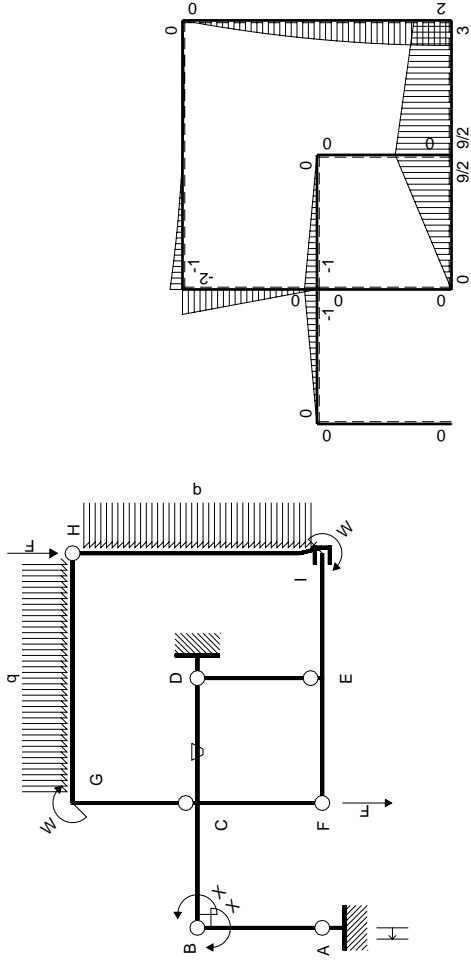
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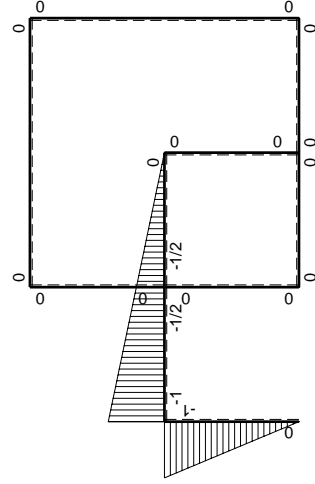


⊕ ⊖ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$            | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                      |                     |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                     |          |                      |                     |                         |                             | $7/4Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                      |                     |                         |                             | $-7/4Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

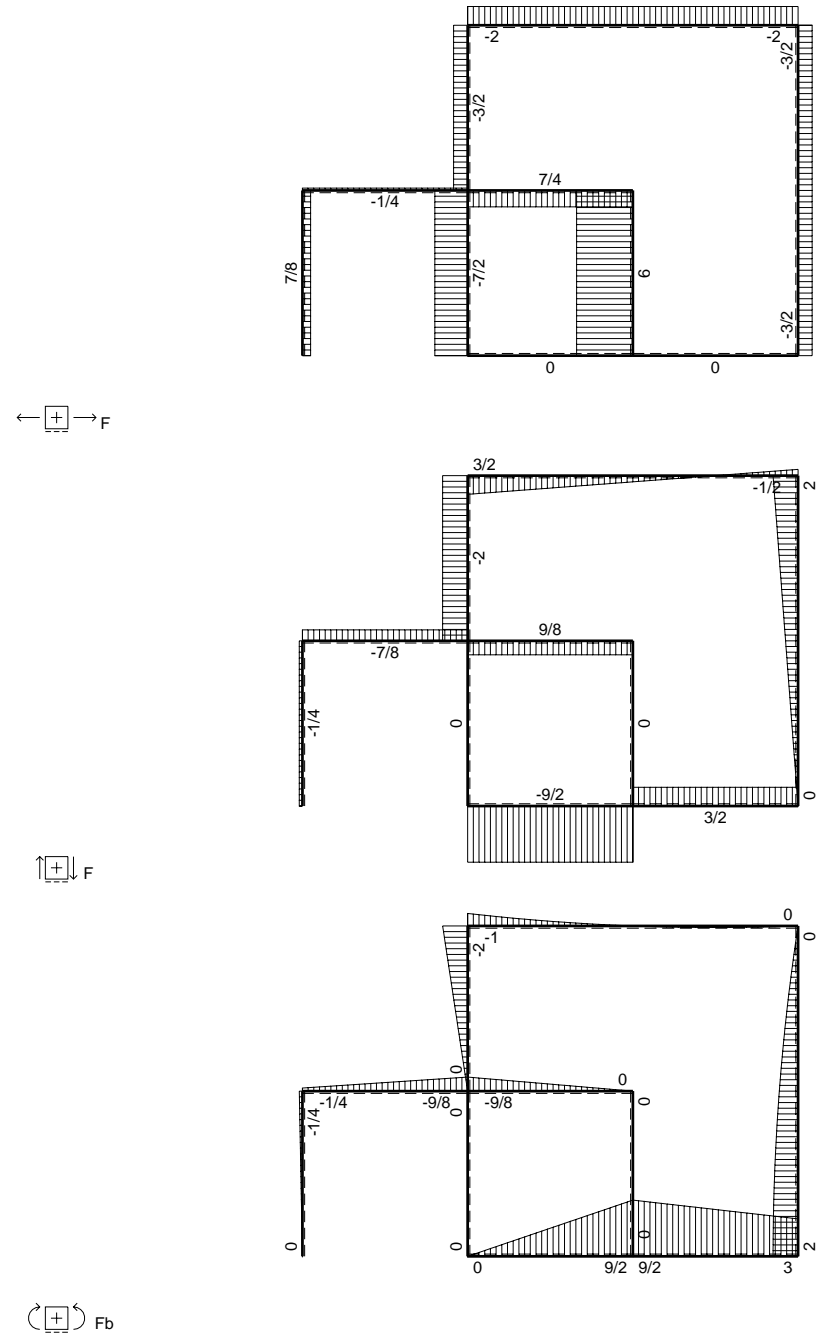
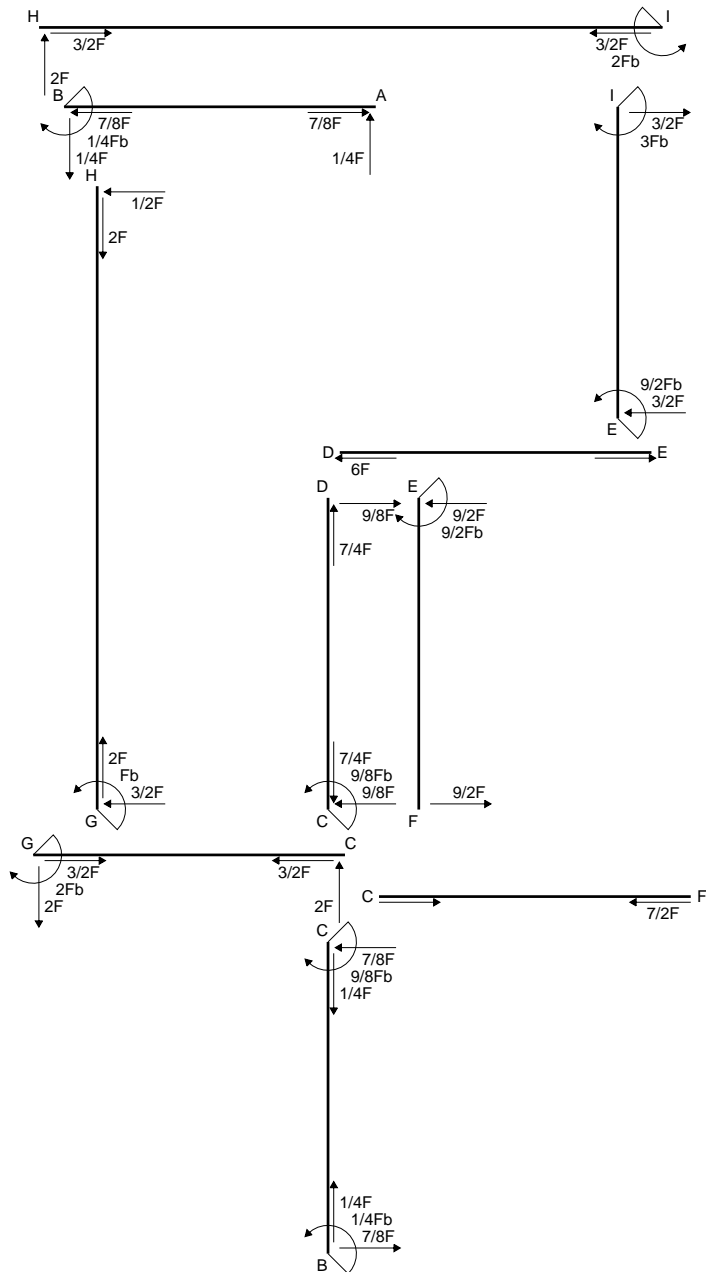
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

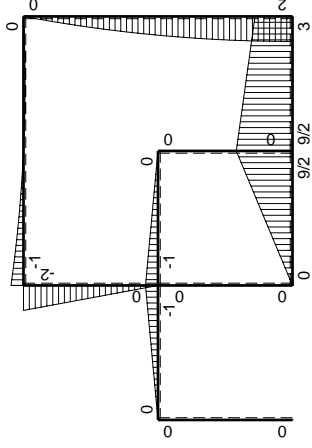
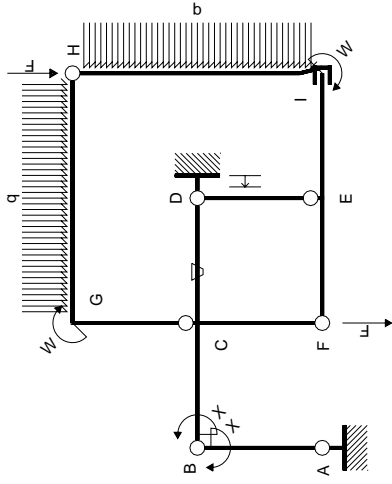
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$

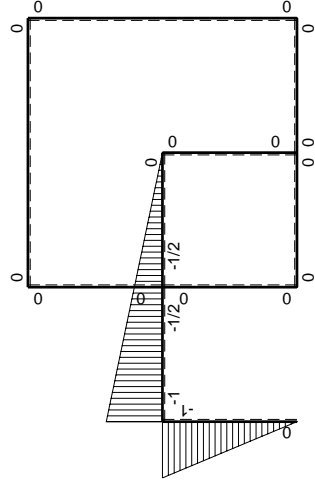


⊕ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$            | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/2Fb-9/2Fx$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/2Fx$            | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+3/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+3/2Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/2Fb+3/2Fx$      | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                      |                     |                         |                             | $-1/4Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                      |                     |                         |                             | $1/4Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

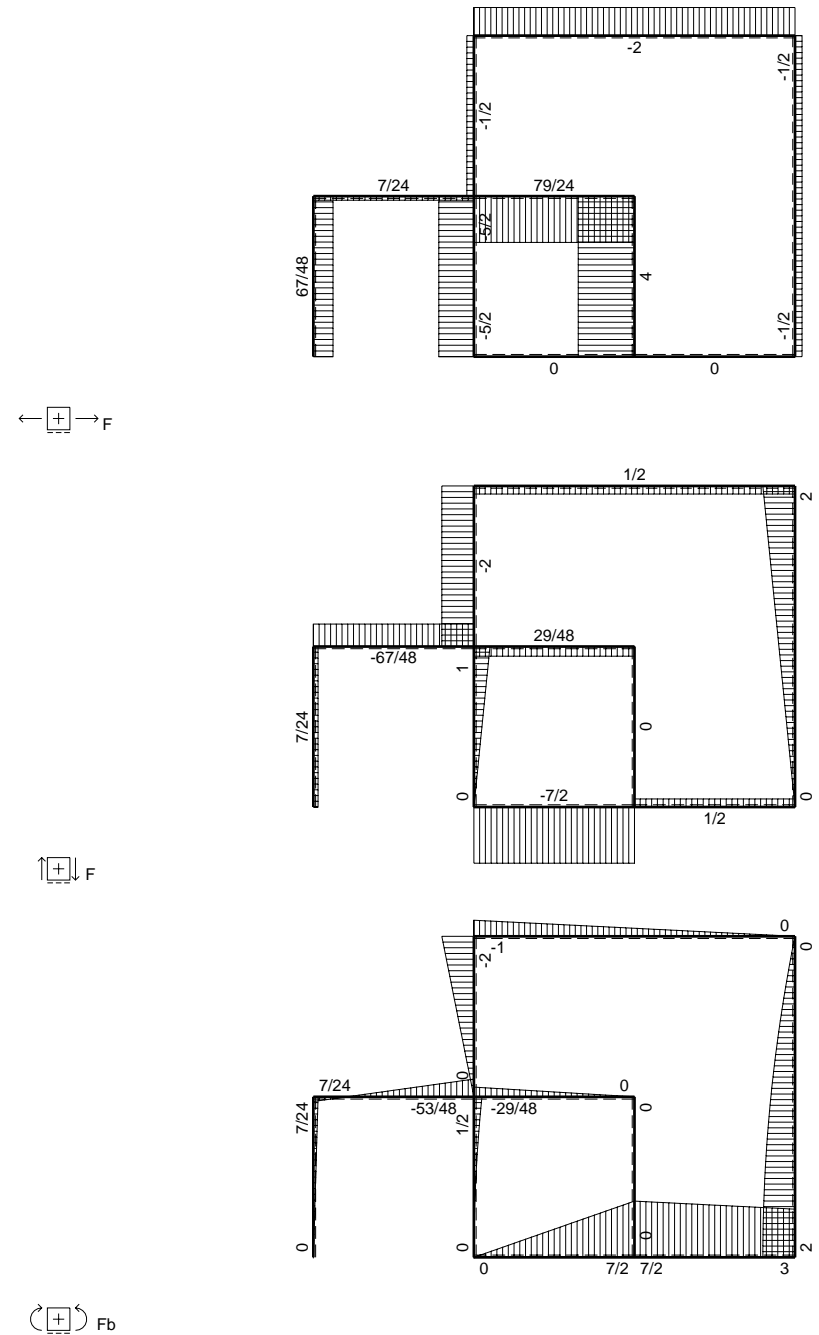
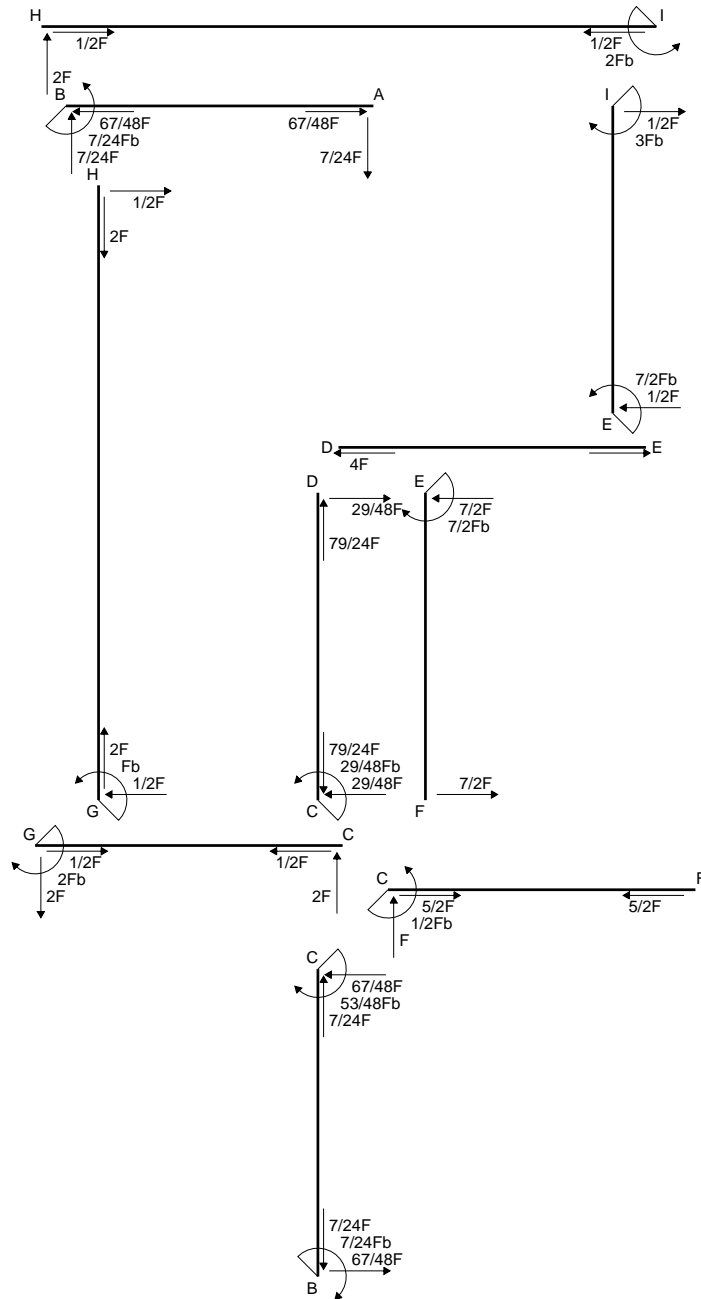
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

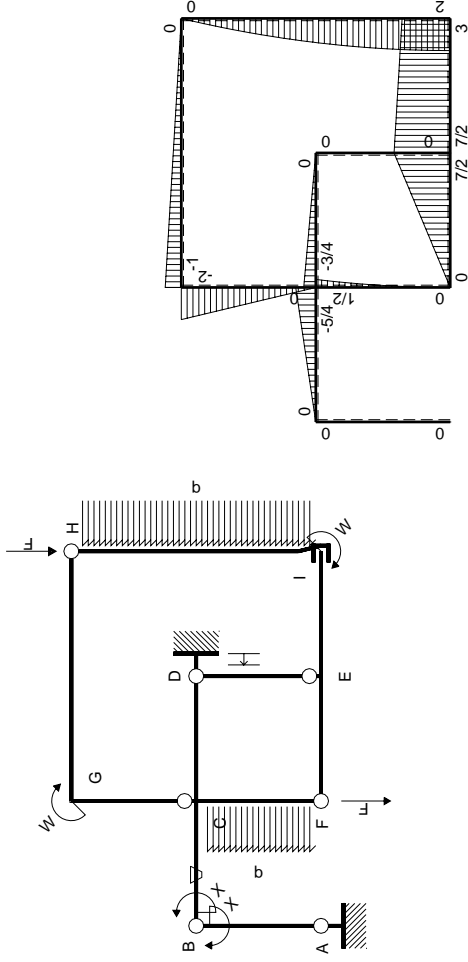
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$

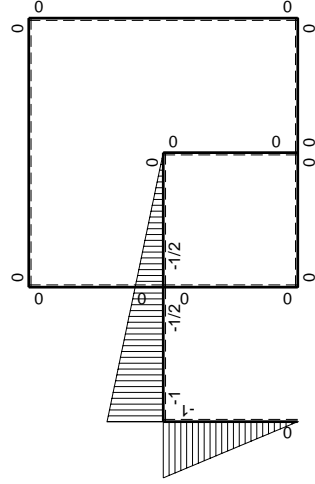






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |                     |                         | $7/24Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-7/24Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

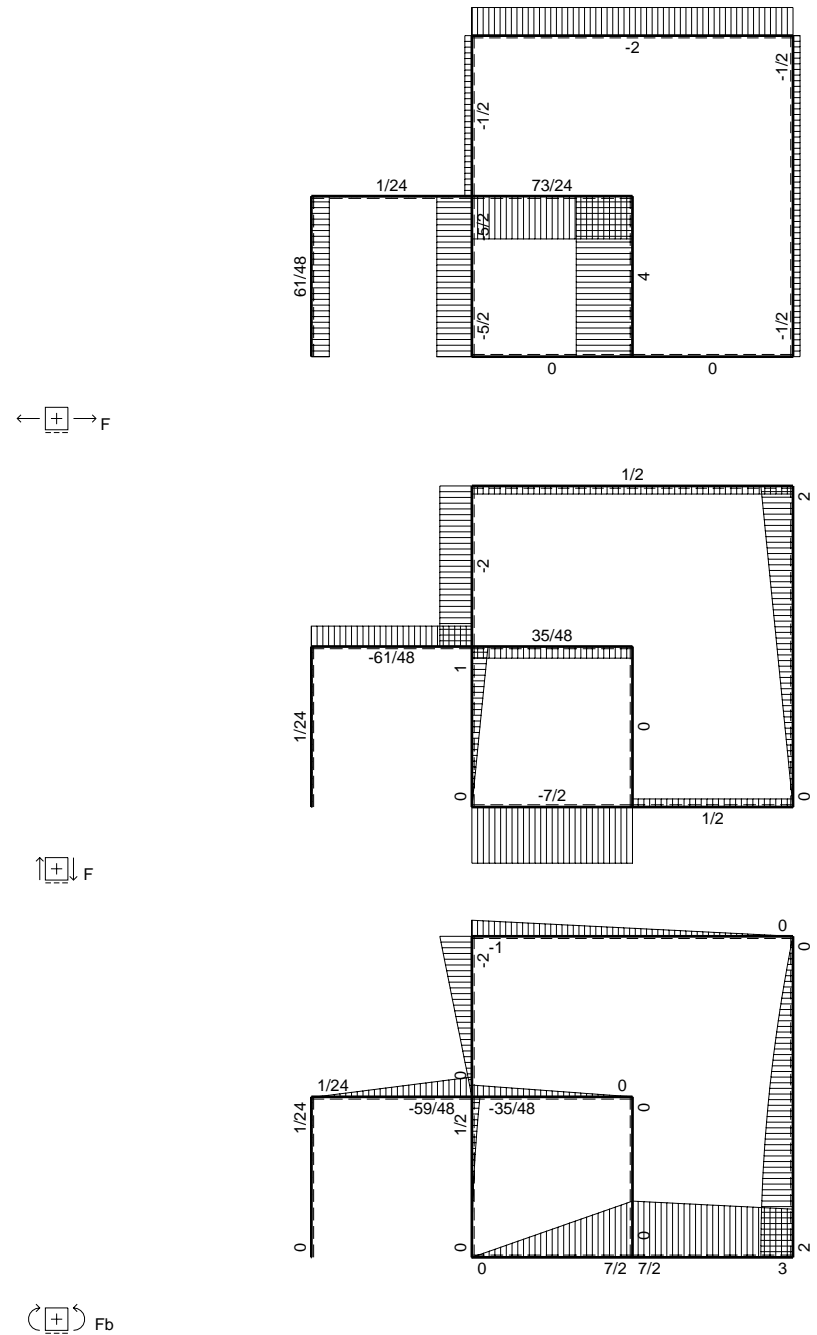
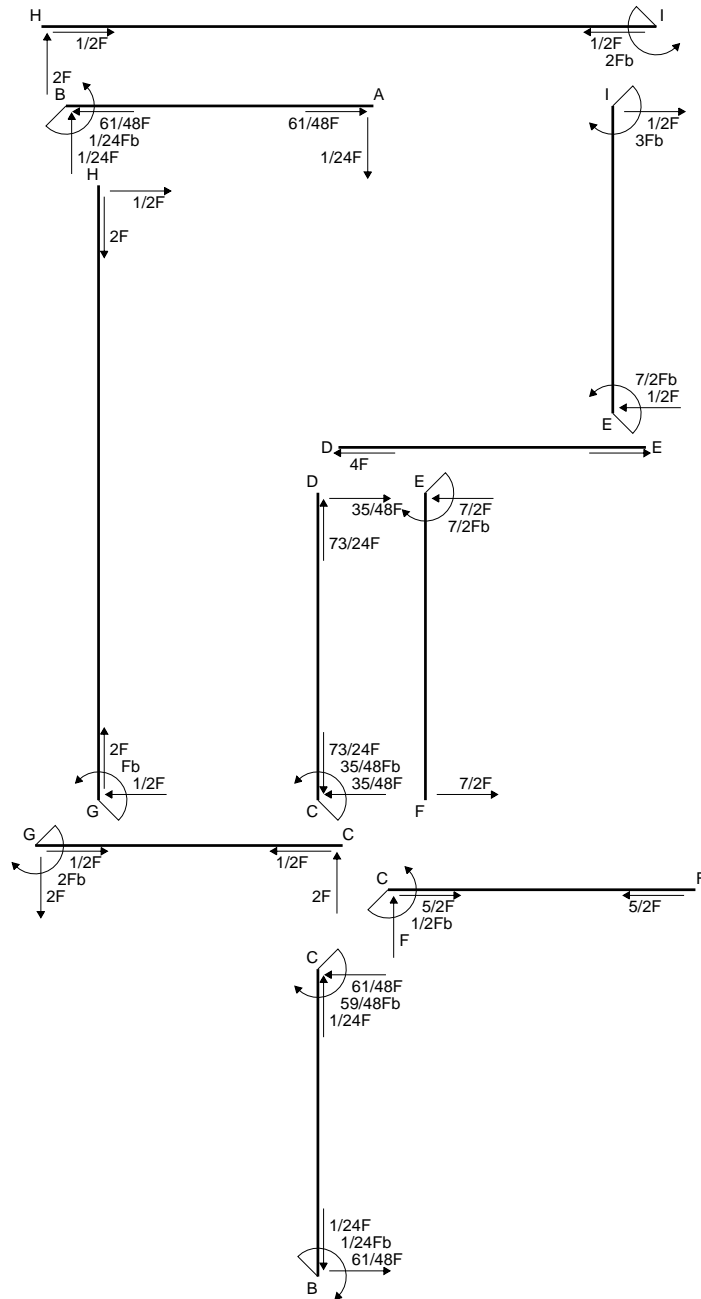
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |               |                         | $1/24Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-1/24Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

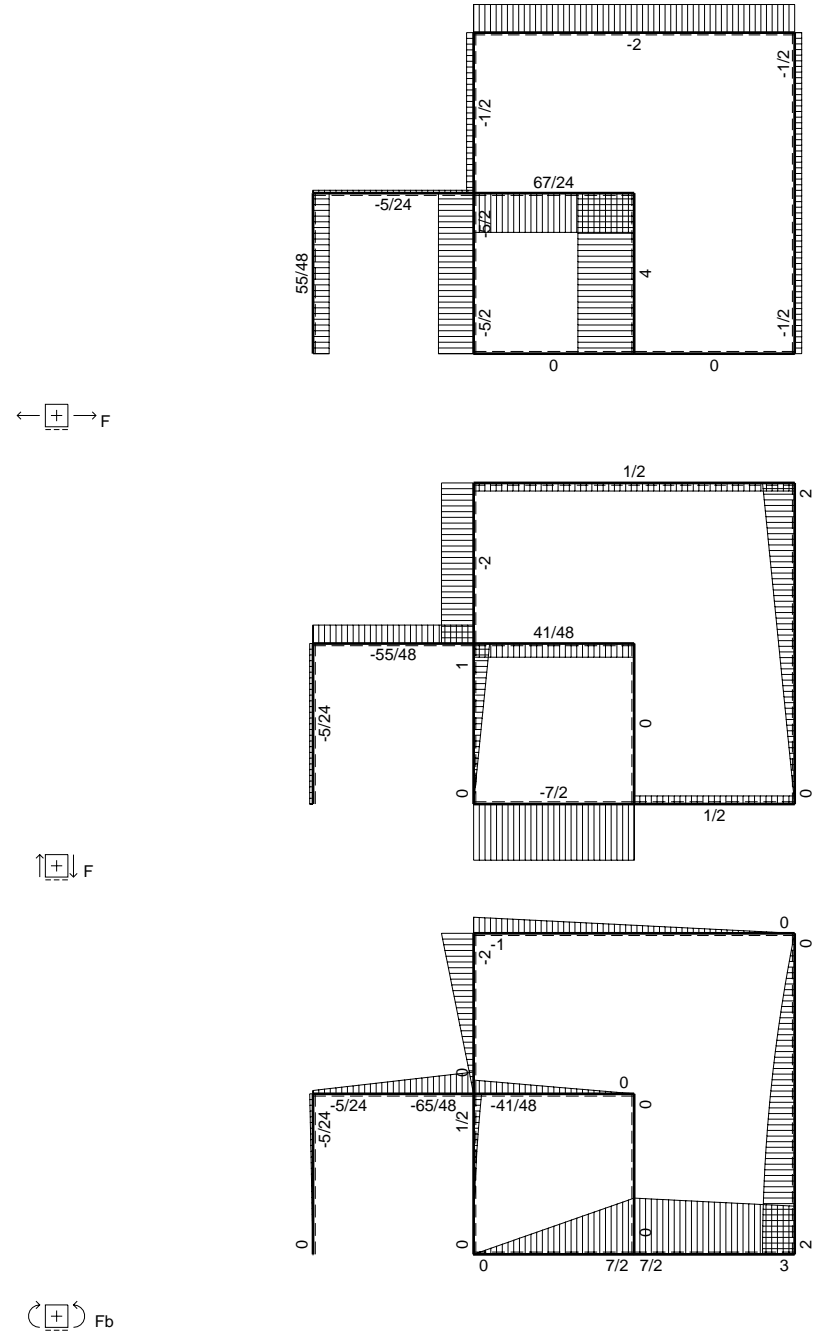
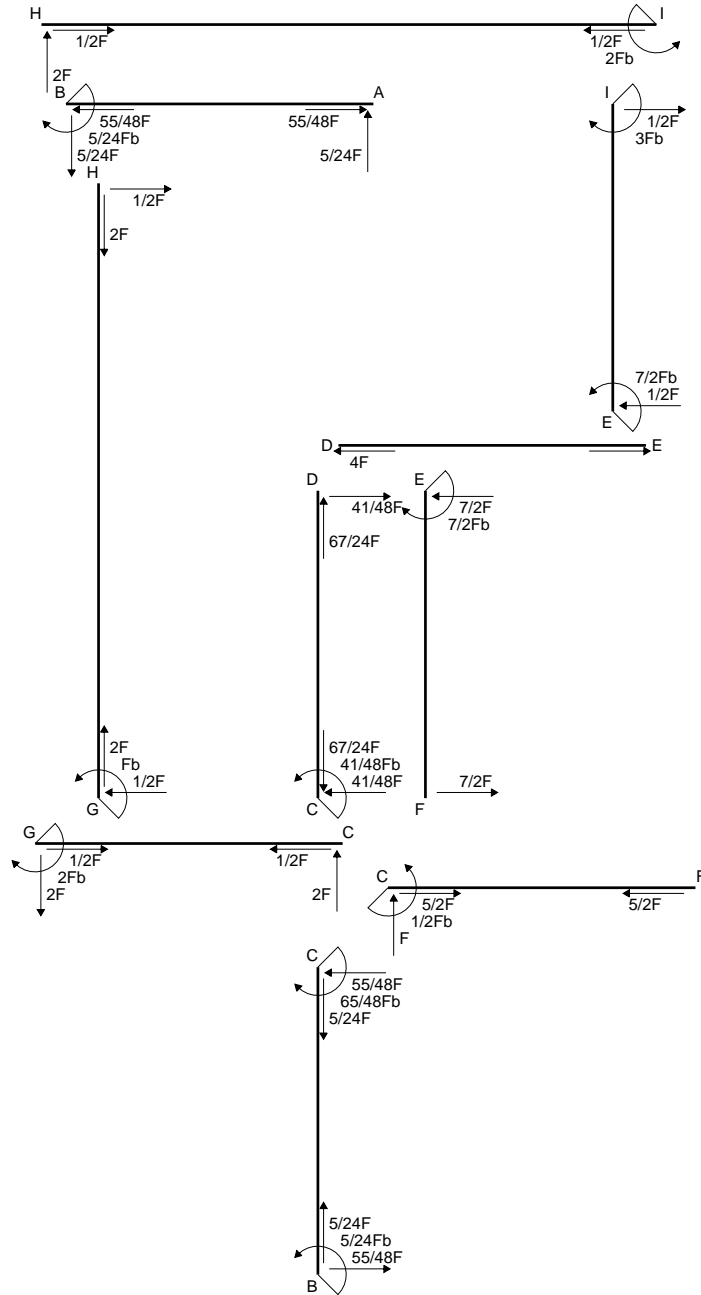
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

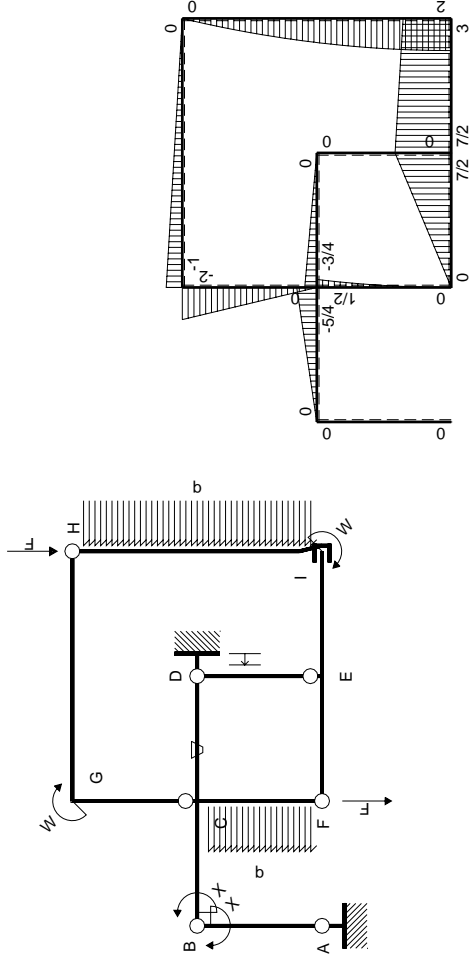
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

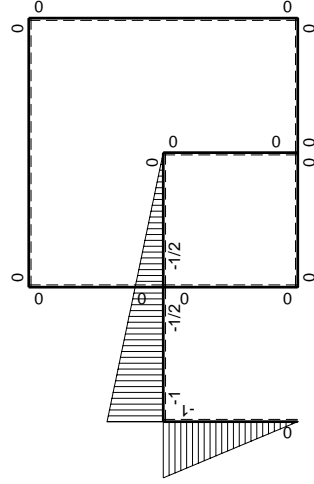






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$            | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$       | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-7/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-5/24Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $5/24Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

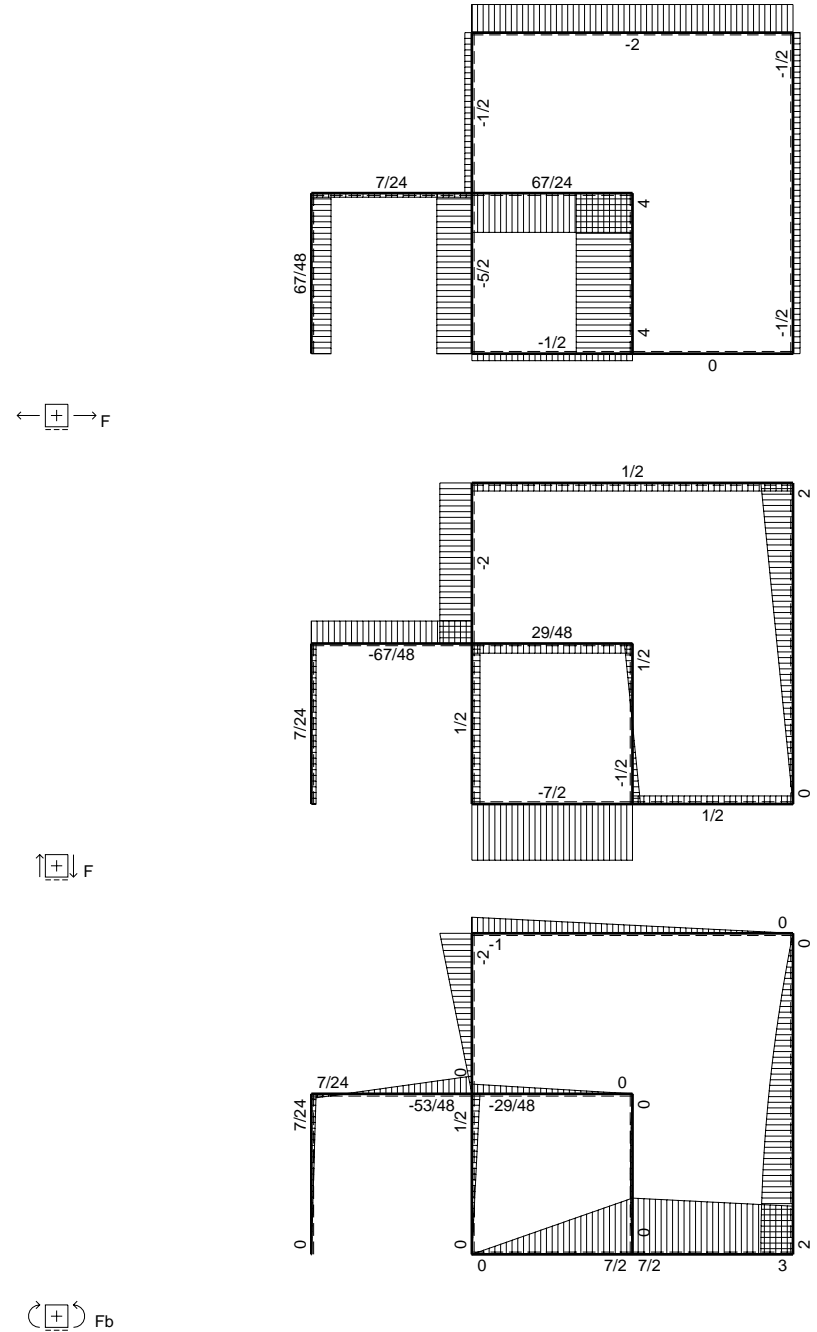
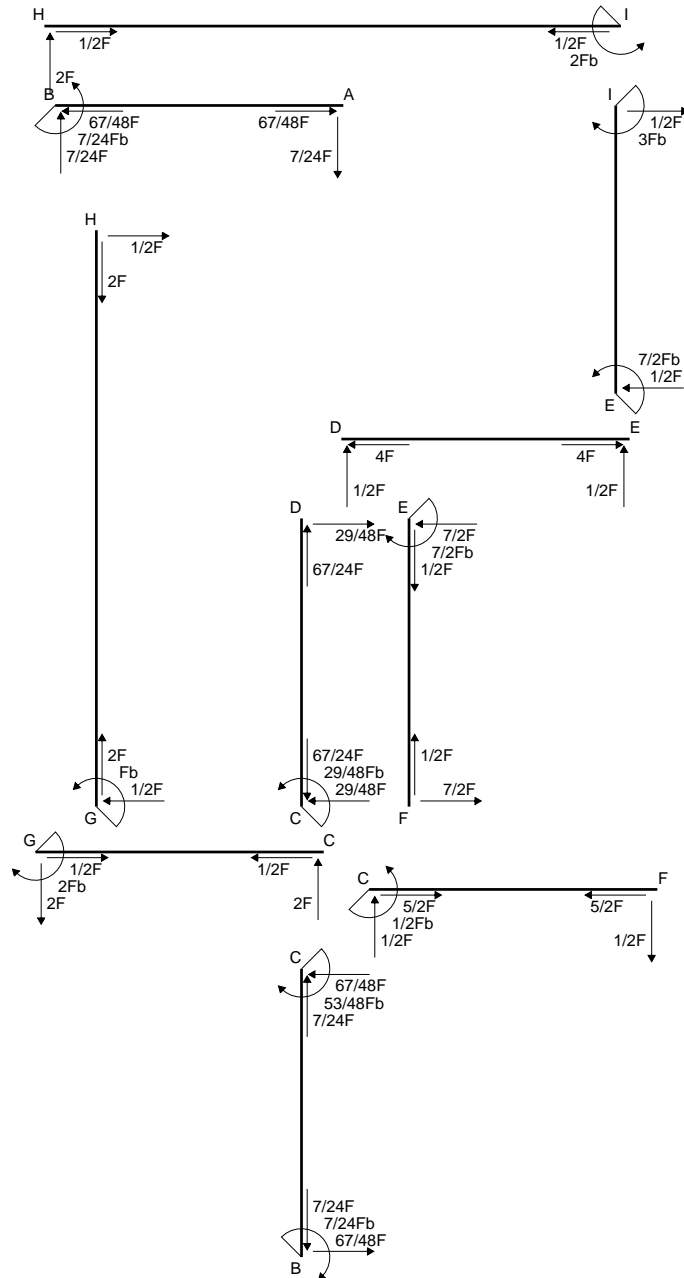
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

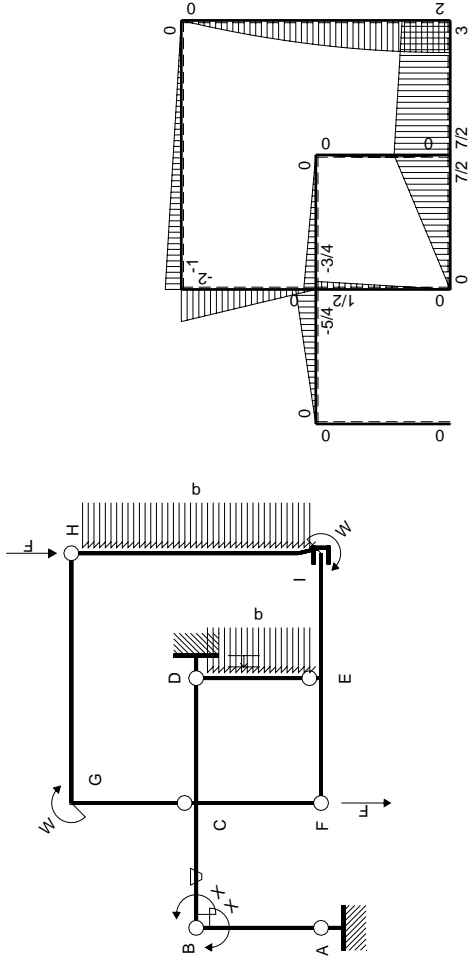
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

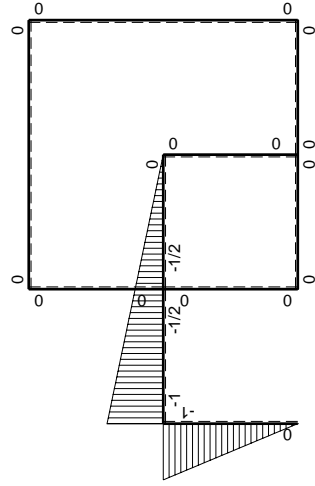
$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                         |                     |                         |                             | $7/24Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |                     |                         |                             | $-7/24Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( x^2/b^2 \right) 1/EJ dx = \left[ 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2x/b + x^2/b^2 \right) 1/EJ dx = \left[ x - x^2/b + 1/3 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - x/b + 1/4 x^2/b^2 \right) 1/EJ dx = \left[ x - 1/2 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b \left( 1/4 + 1/2 x/b + 1/4 x^2/b^2 \right) 1/EJ dx = \left[ 1/4 x + 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b \left( 1/4 - 1/2 x/b + 1/4 x^2/b^2 \right) 1/EJ dx = \left[ 1/4 x - 1/4 x^2/b + 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b \left( 1/4 x^2/b^2 \right) 1/EJ dx = \left[ 1/12 x^3/b^2 \right]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b \left( 5/4 x/b - 5/8 x^2/b^2 \right) Fb 1/EJ dx + \int_0^b \left( 1 - 1/2 x/b \right) \theta dx$$

$$= \left[ 5/8 x^2/b - 5/24 x^3/b^2 \right]_0^b Fb 1/EJ + \left[ x - 1/4 x^2/b \right]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b \left( 5/8 - 5/8 x^2/b^2 \right) Fb 1/EJ dx + \int_0^b \left( -1/2 - 1/2 x/b \right) \theta dx$$

$$= \left[ 5/8 x - 5/24 x^3/b^2 \right]_0^b Fb 1/EJ + \left[ -1/2 x - 1/4 x^2/b \right]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

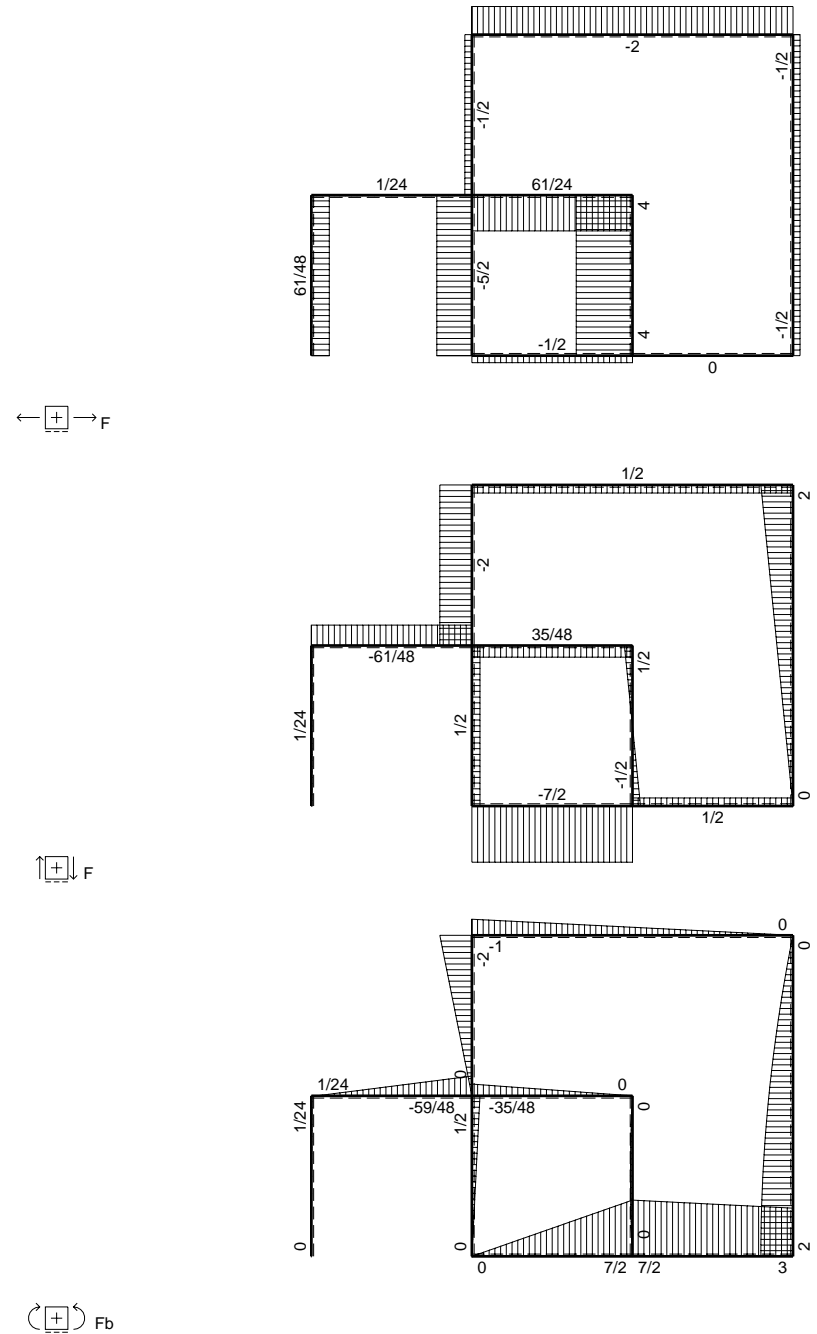
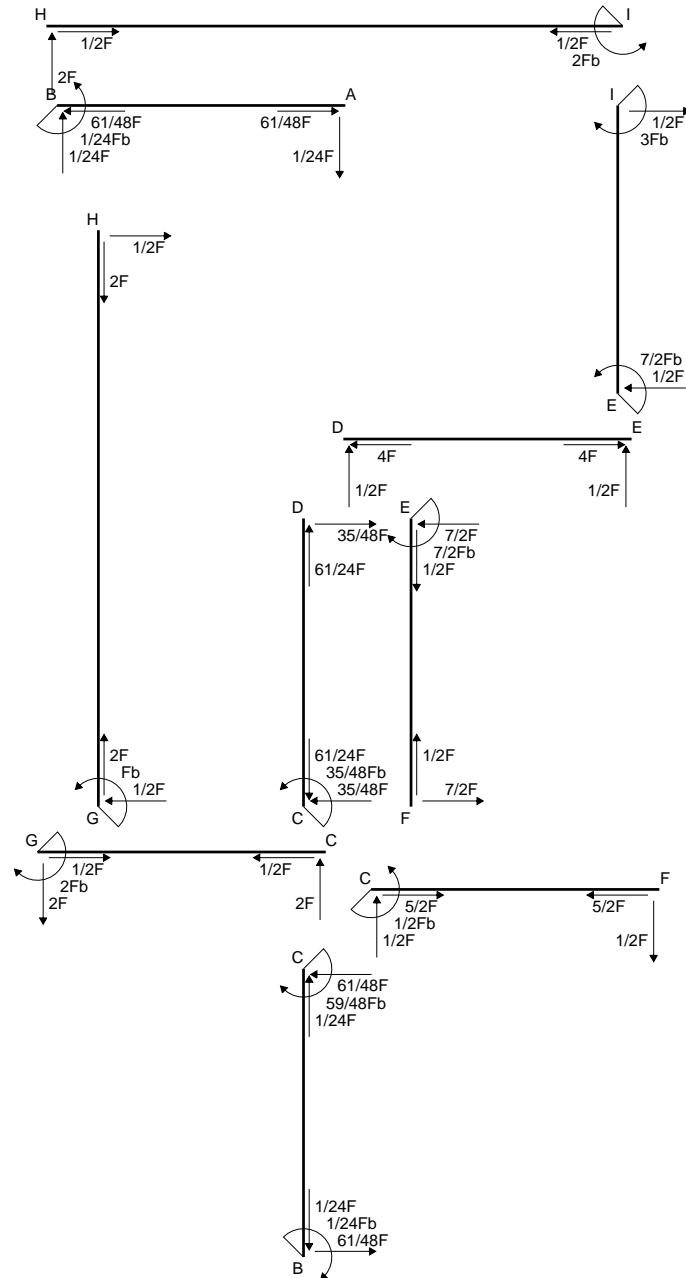
$$L_{CD}^{xo} = \int_0^b \left( 3/8 - 3/4 x/b + 3/8 x^2/b^2 \right) Fb 1/EJ dx = \left[ 3/8 x - 3/8 x^2/b + 1/8 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b \left( 3/8 x^2/b^2 \right) Fb 1/EJ dx = \left[ 1/8 x^3/b^2 \right]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$







Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$           | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                  |          |                         |               |                         | $1/24Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |               |                         | $-1/24Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

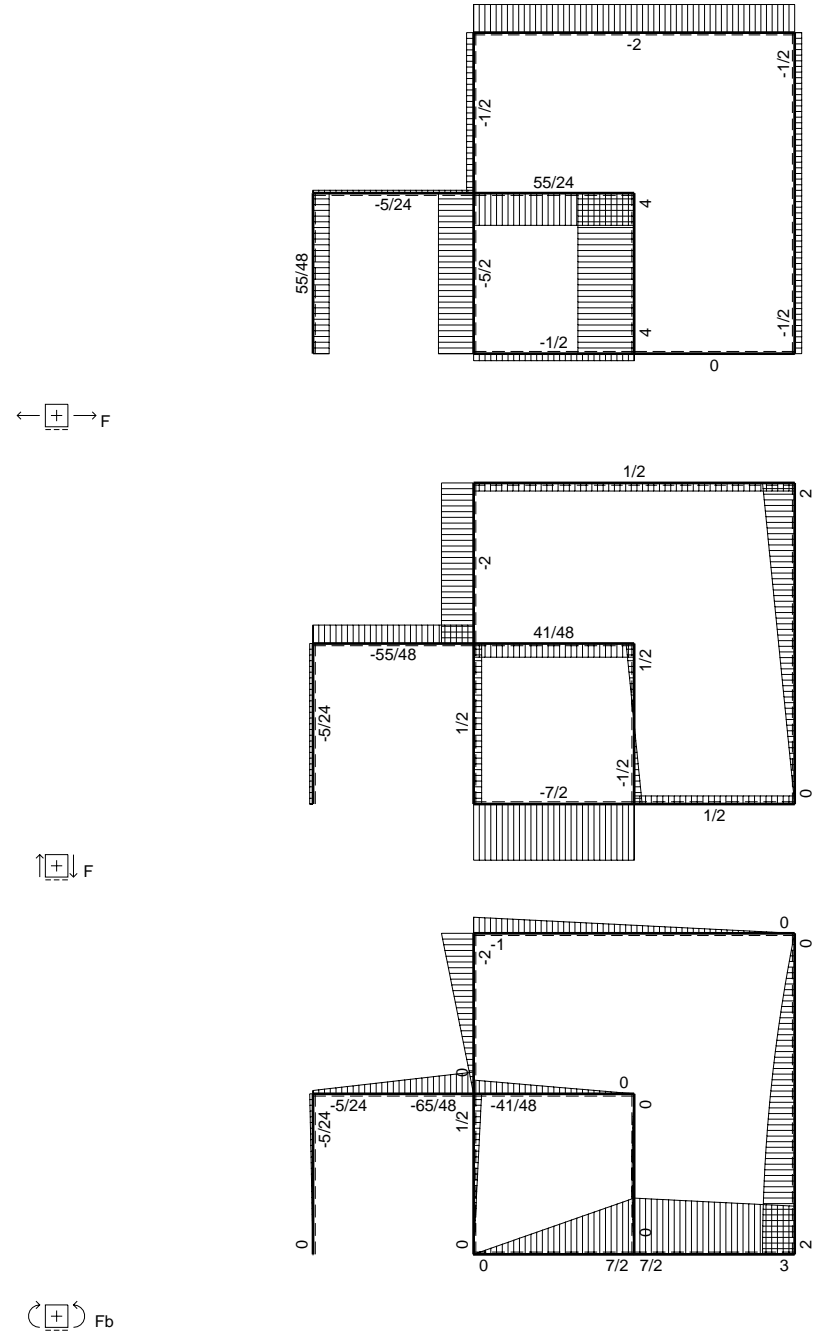
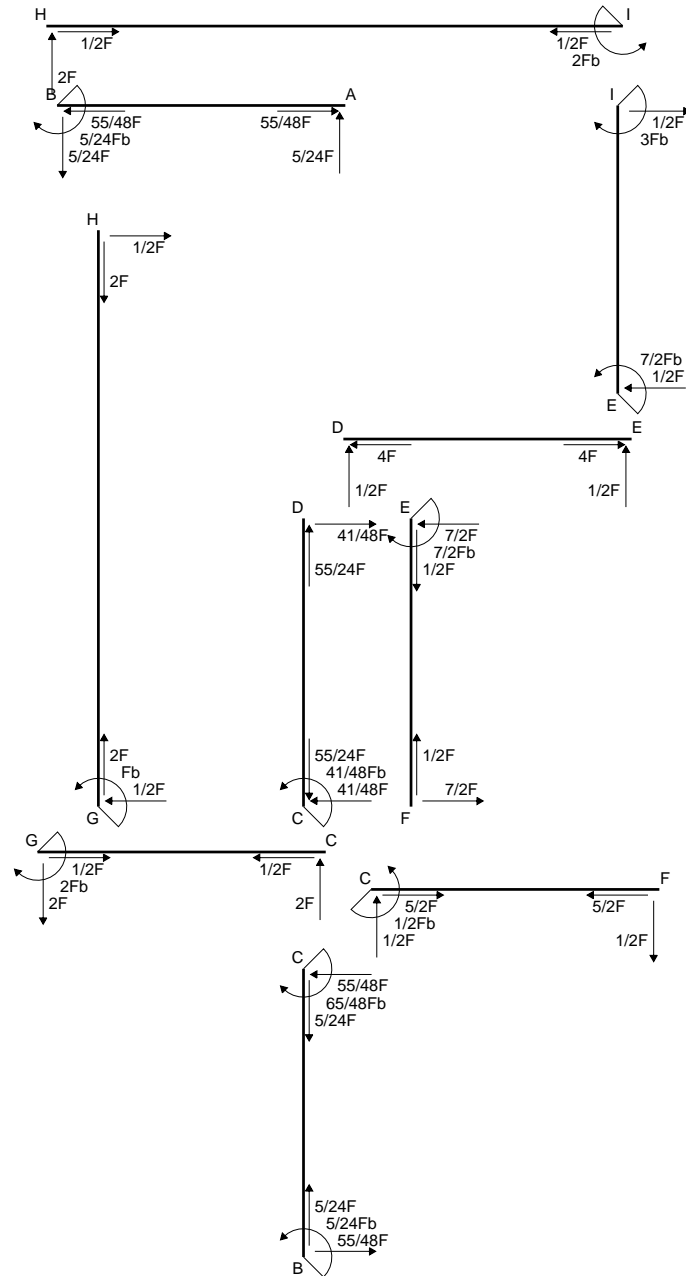
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

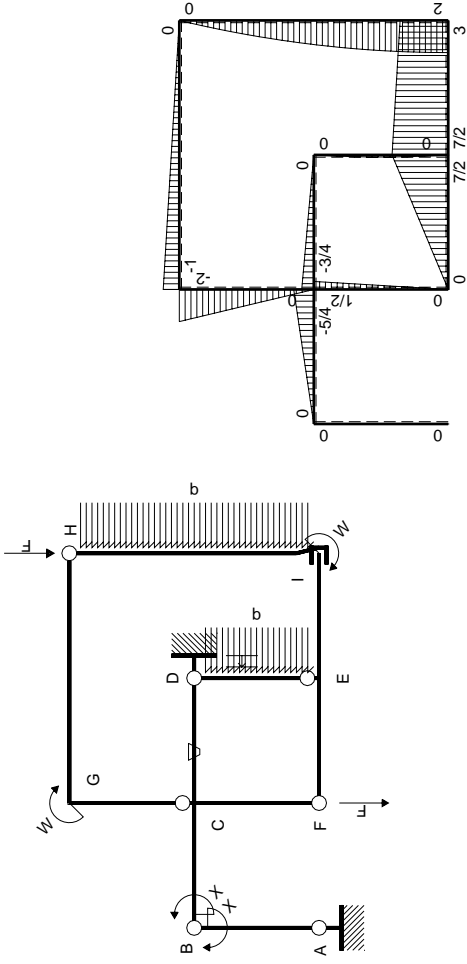
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

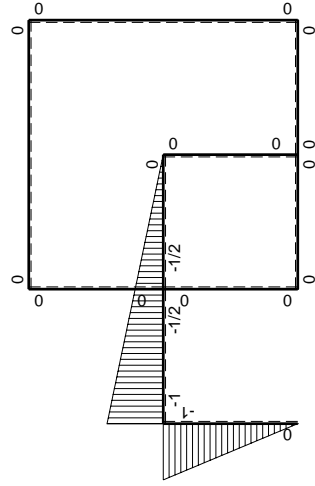
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$         | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$    | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$        | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                  |          |                         |                     |                         | $-5/24Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |                     |                         | $5/24Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

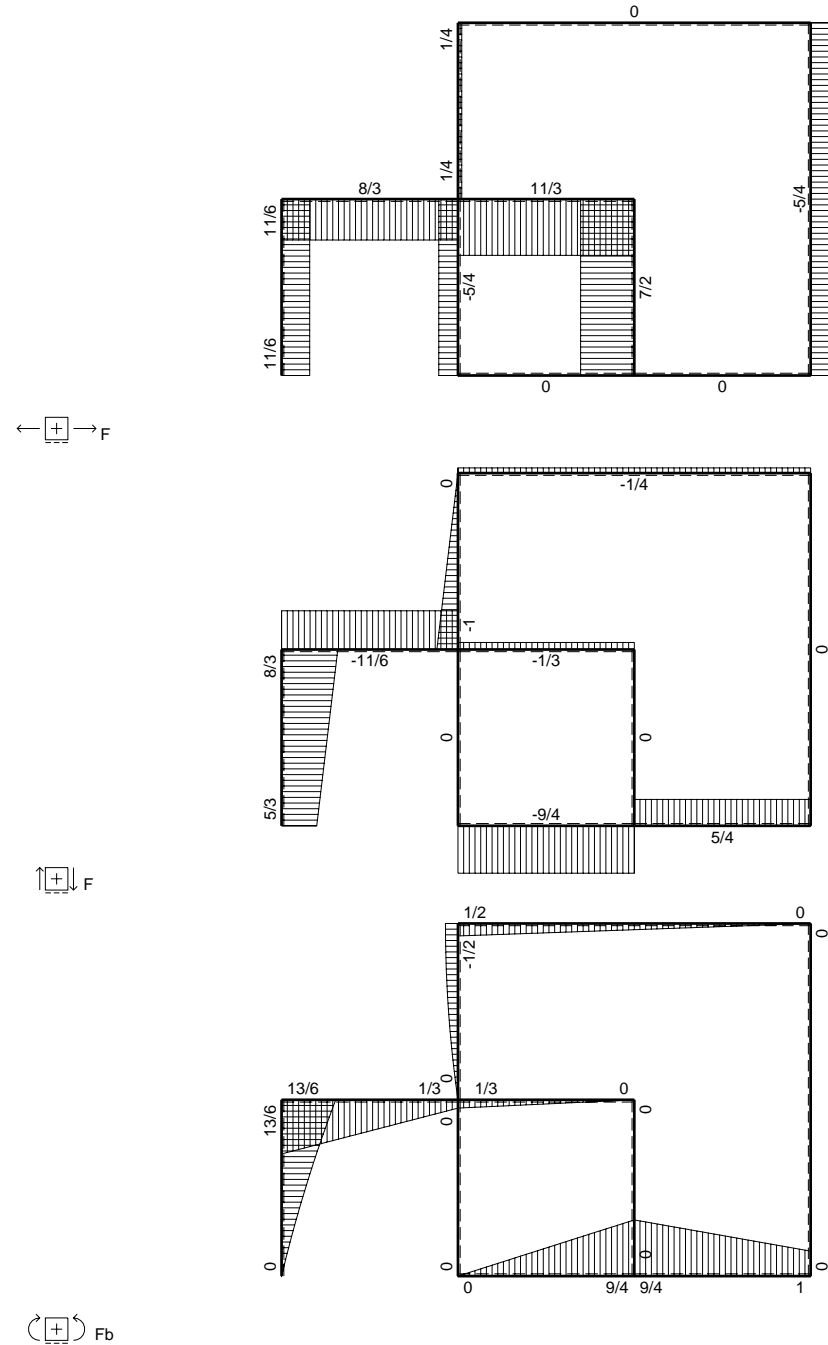
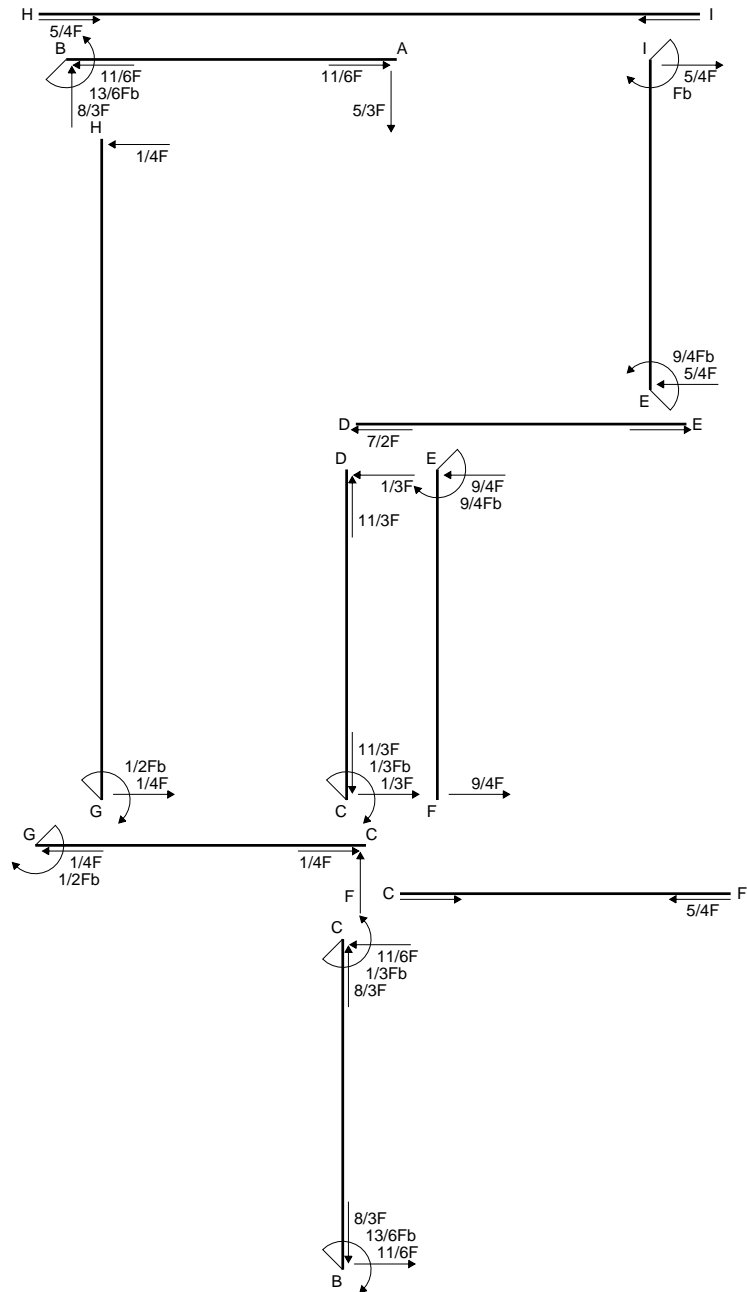
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

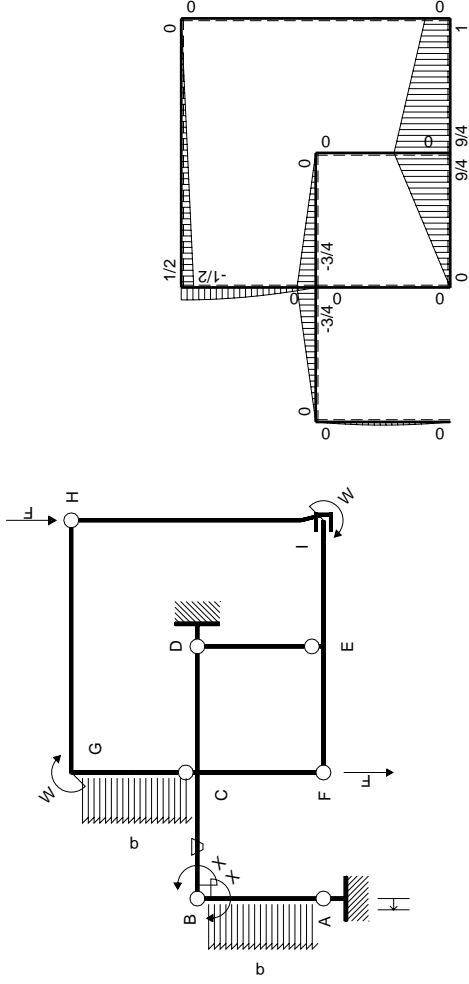
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                          |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

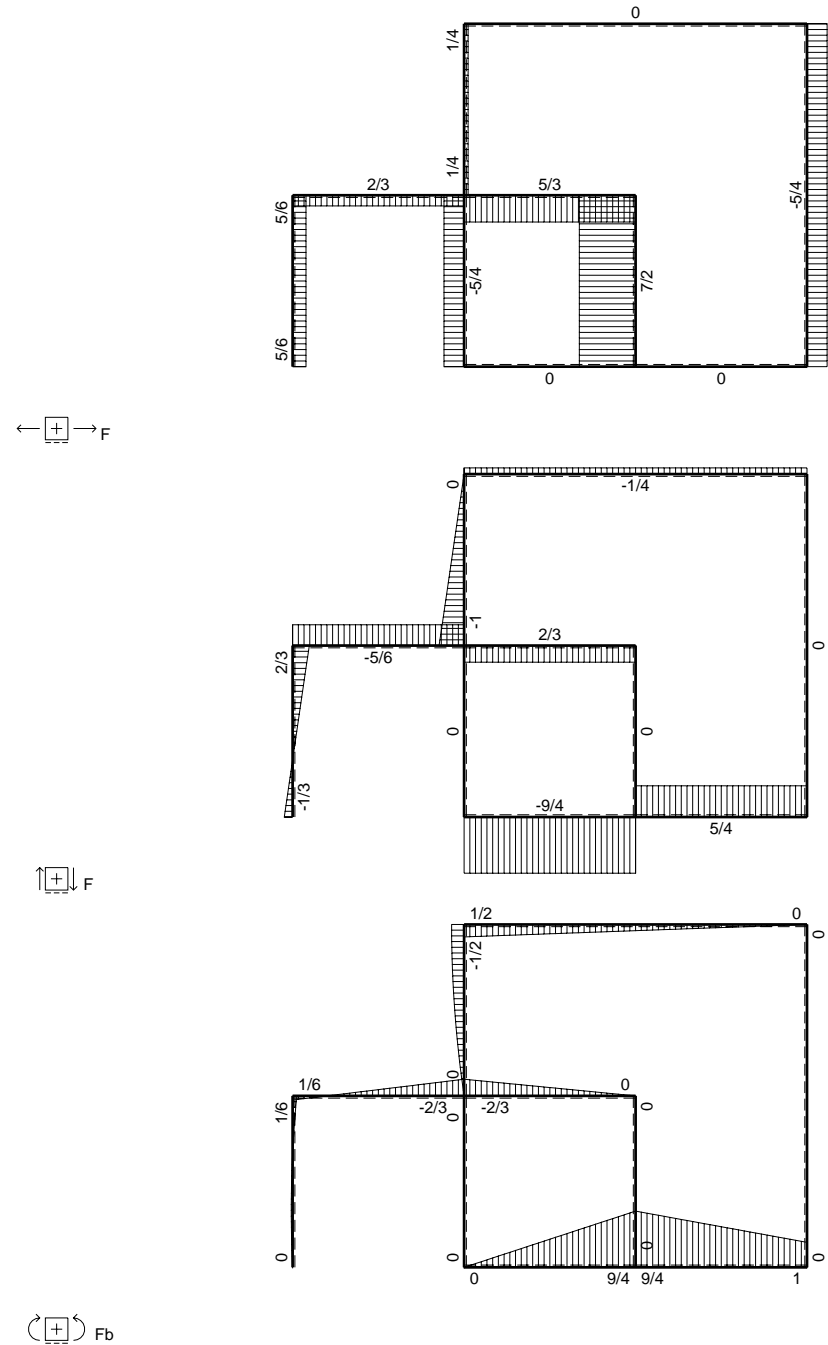
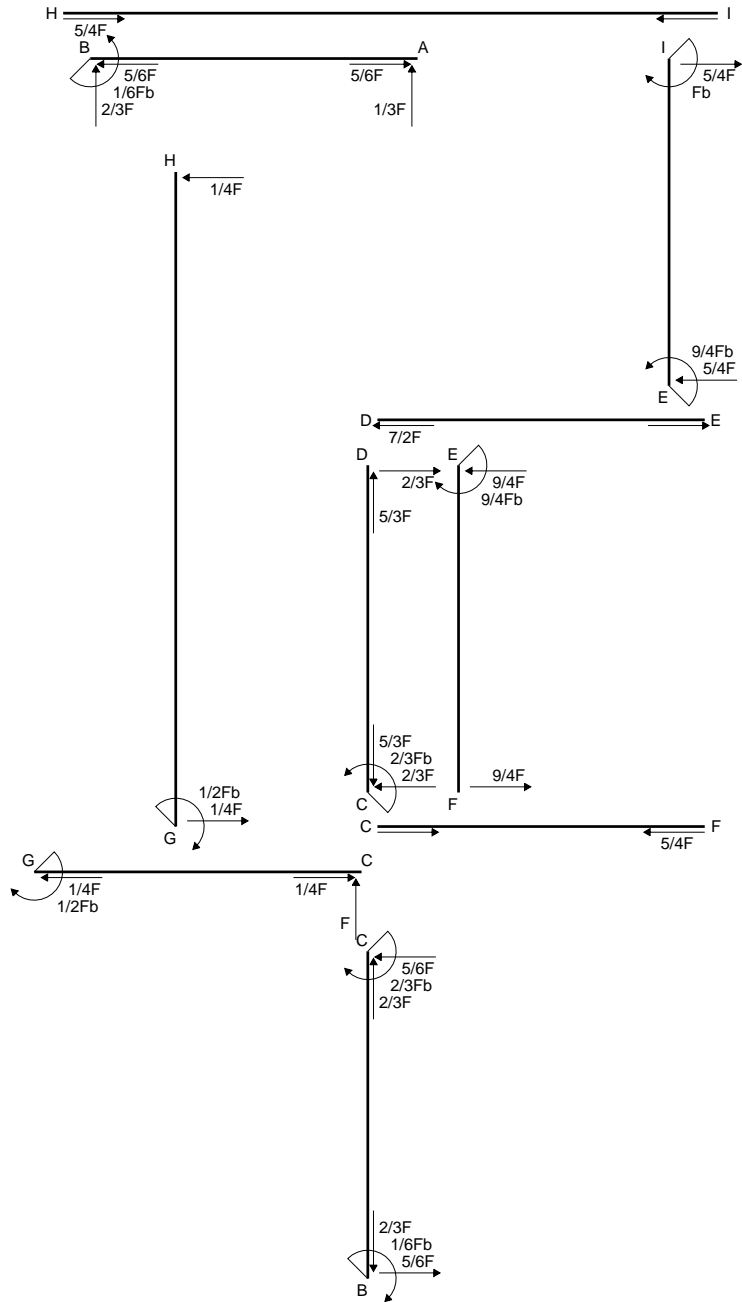
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

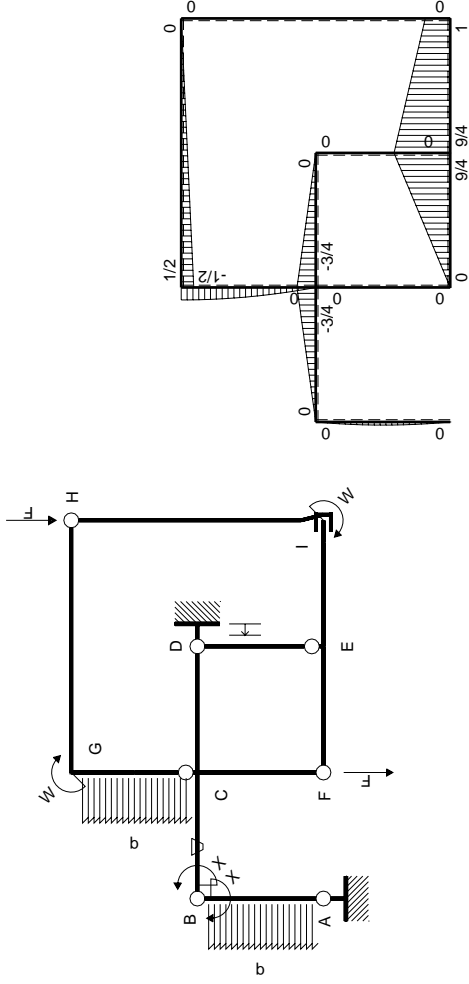
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$







Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| FC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0             | 0                       |                             |                          |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                          |
| HI 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                          |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0             | 0                       |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |               |                         | $Fb^2/EJ$                   |                          |
|       | totali                      |                  |          |                          |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         | $-23/12Fb$                  |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

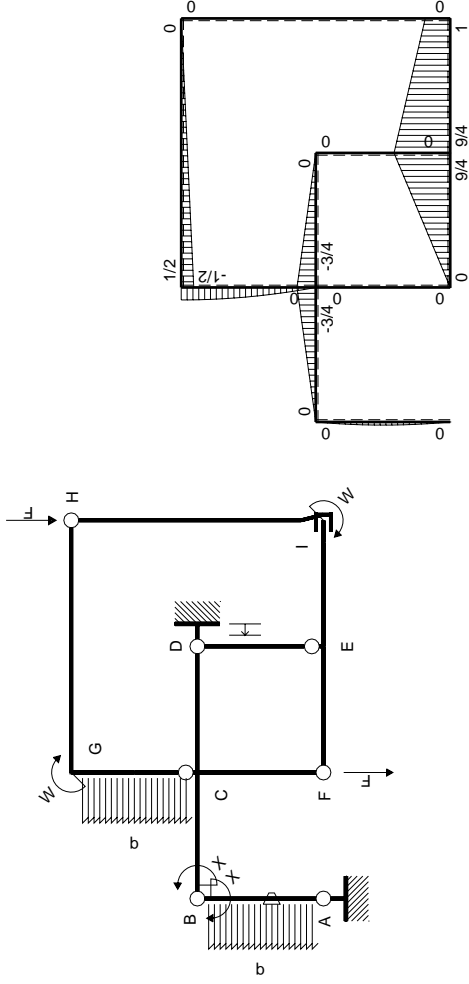
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |               |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |               |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

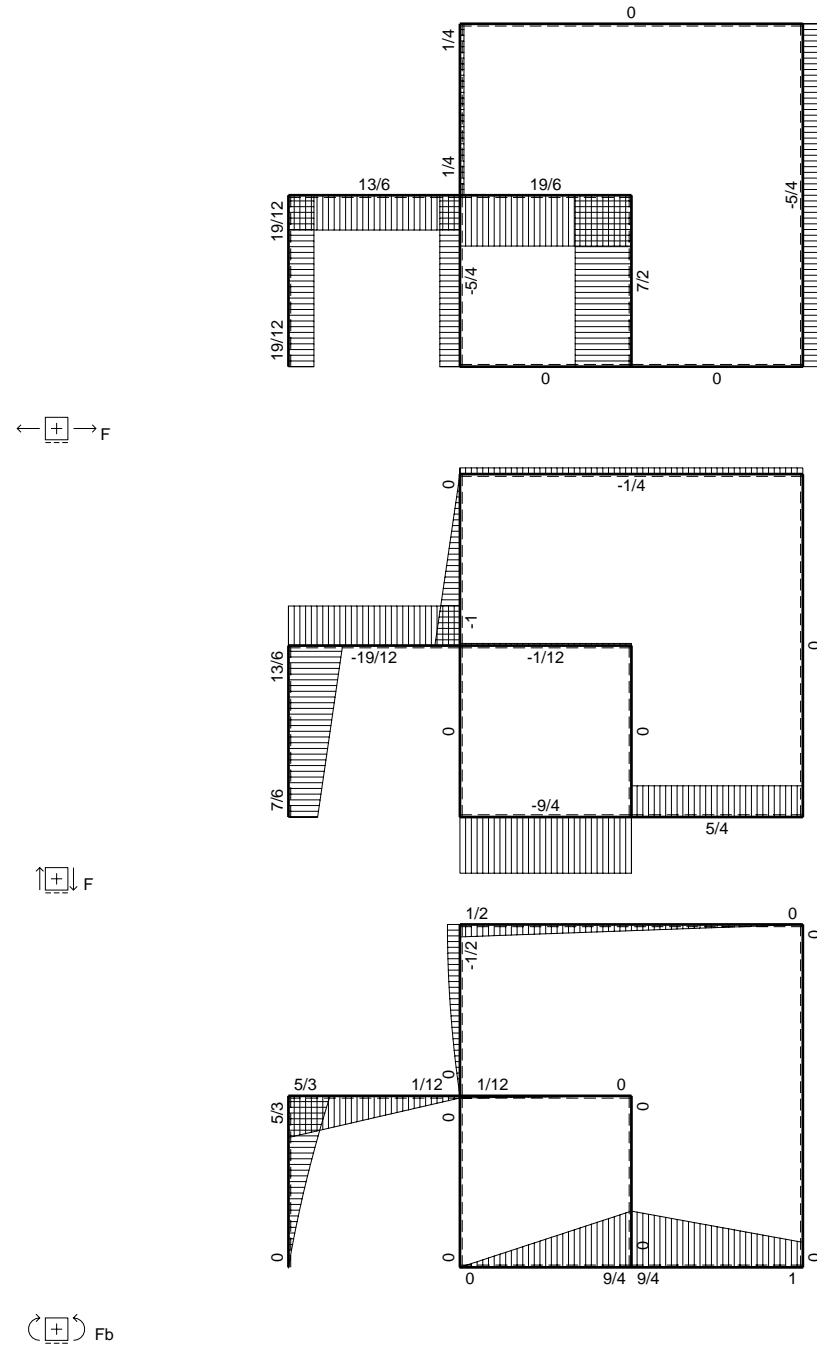
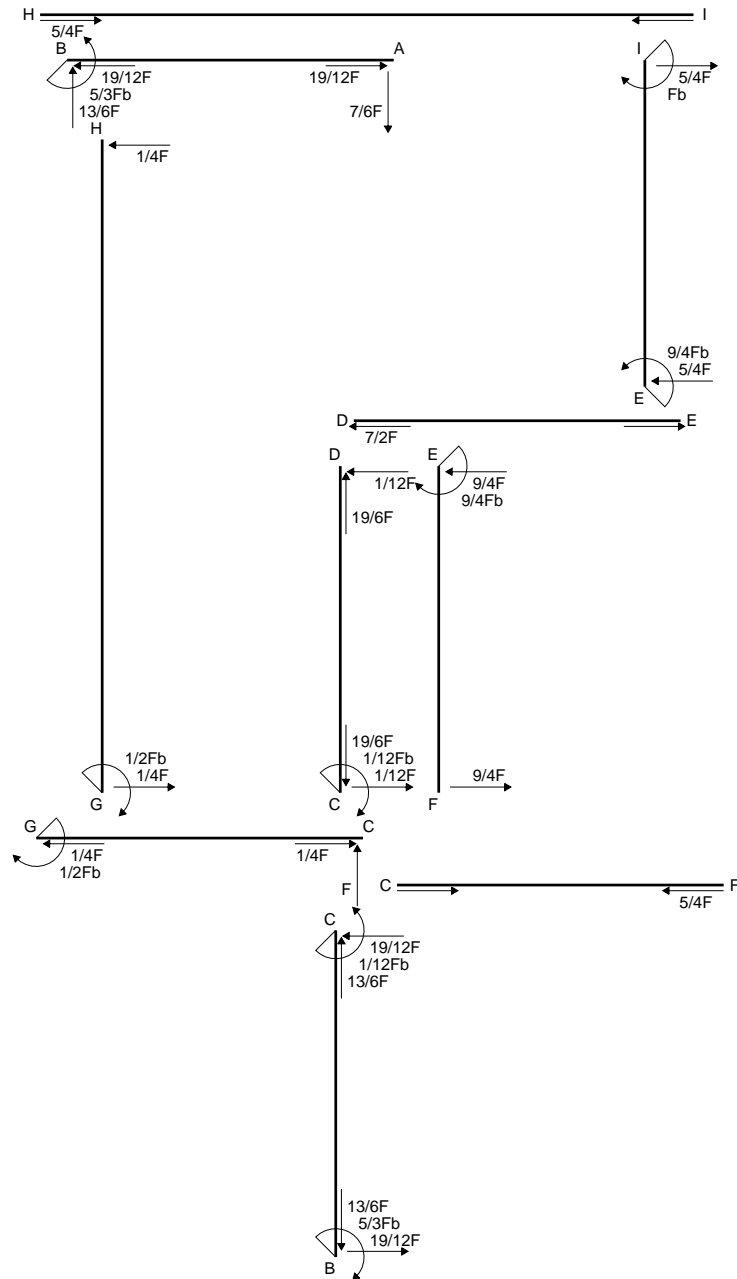
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

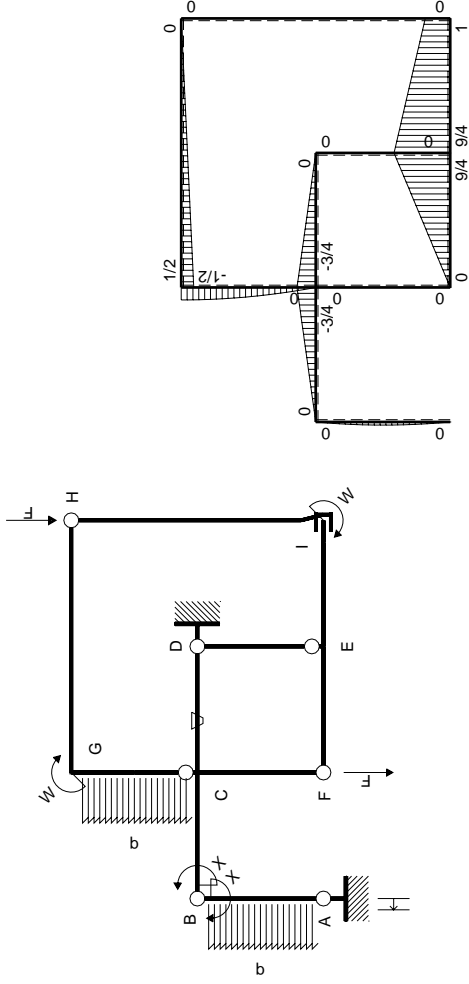
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

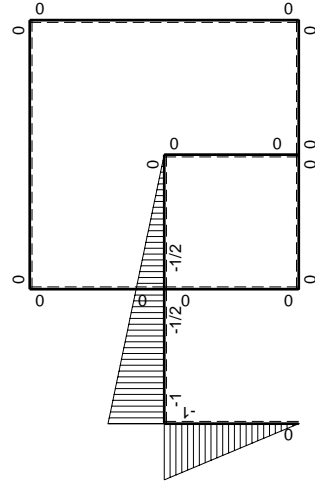






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                          |                     |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $5/3Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $-5/3Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

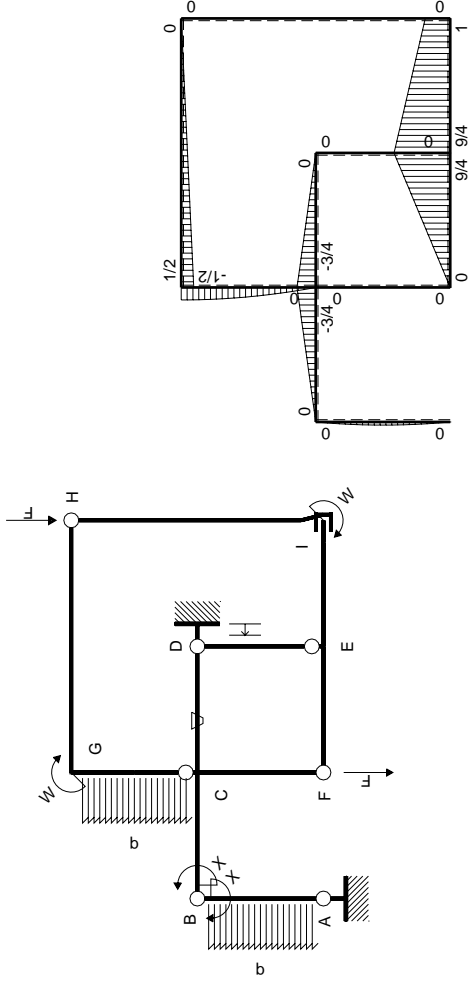
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$ | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$  | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$         | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$    | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$   | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$          | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                  |          |                          |                     |                         |                             | $-1/3Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                  |          |                          |                     |                         |                             | $1/3Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

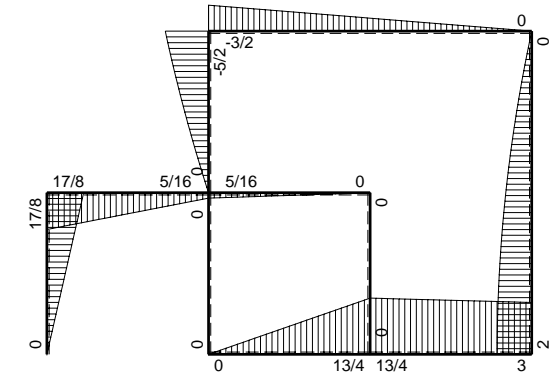
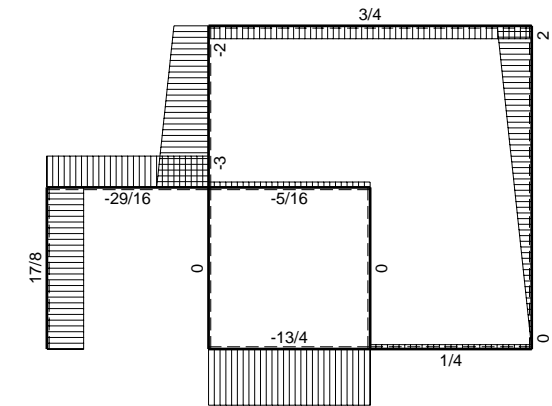
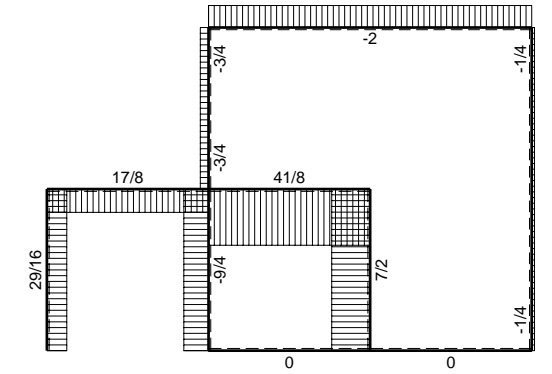
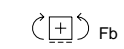
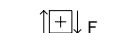
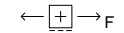
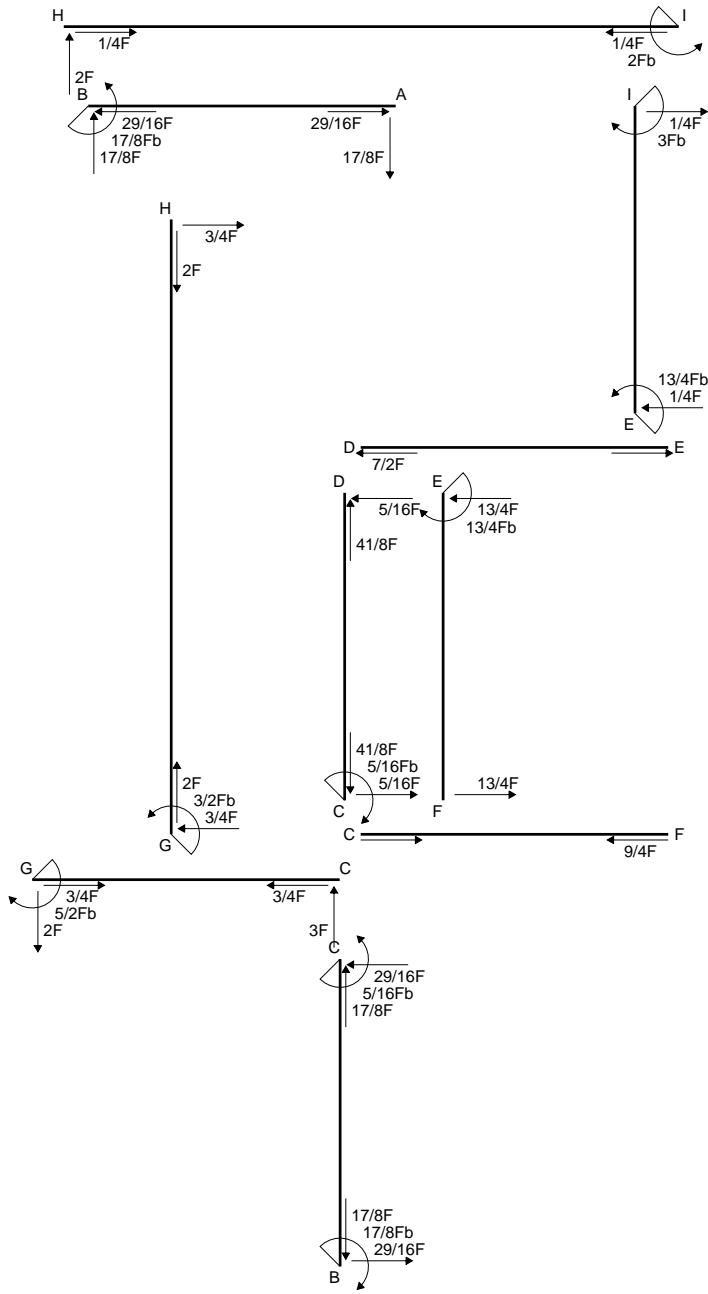
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

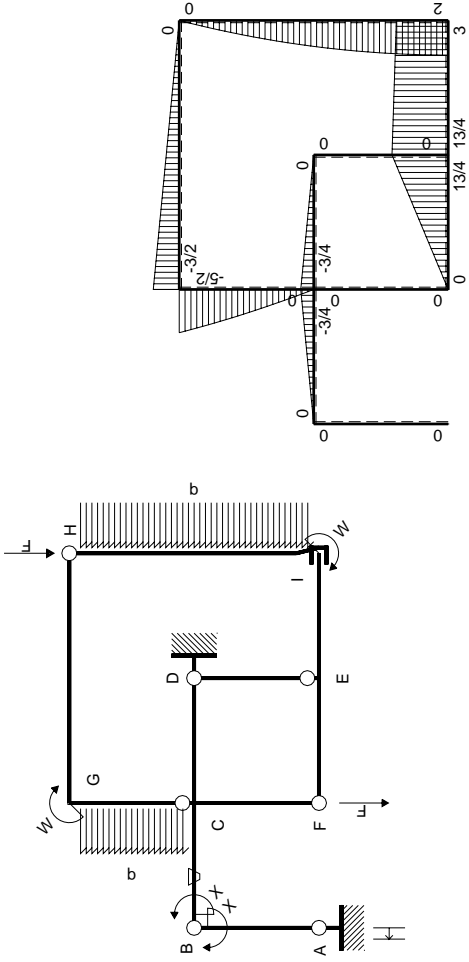
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

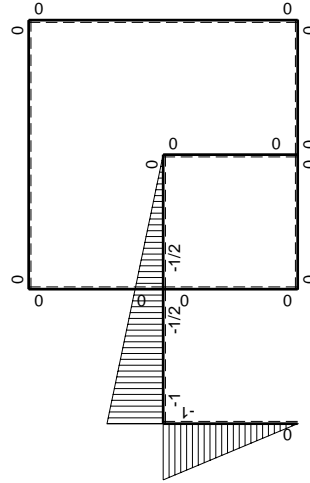






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

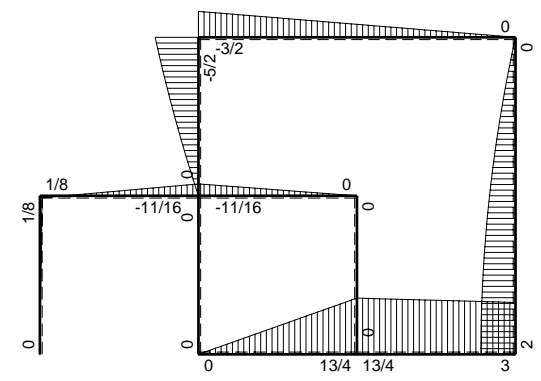
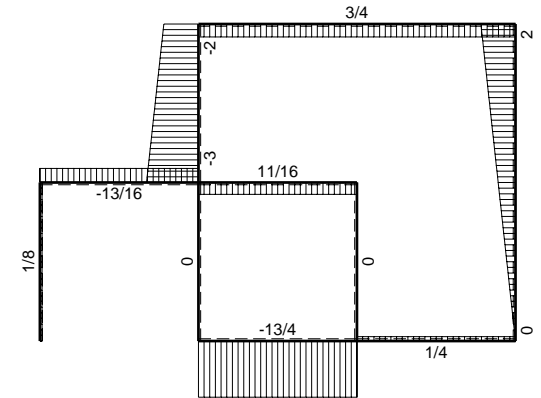
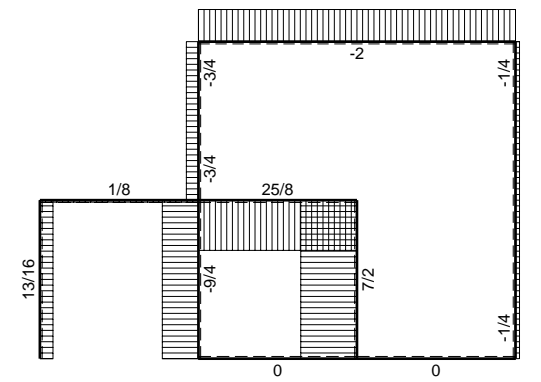
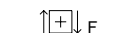
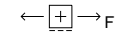
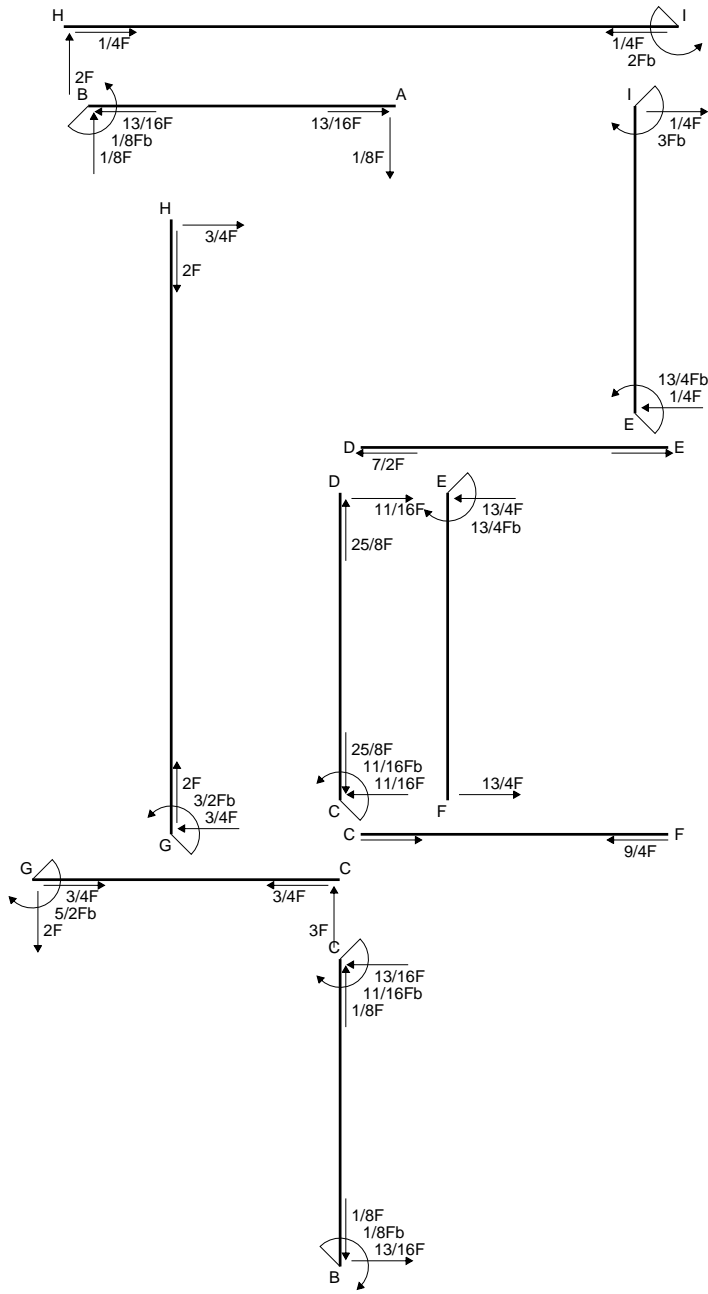
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

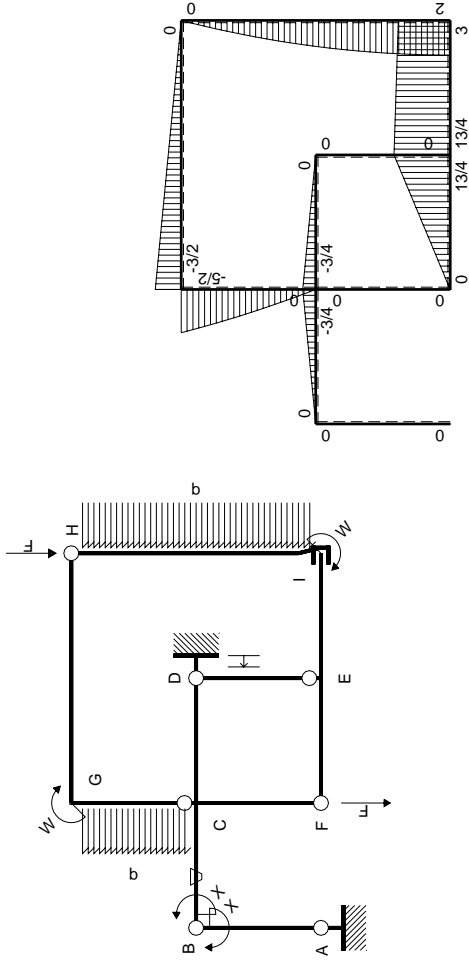
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

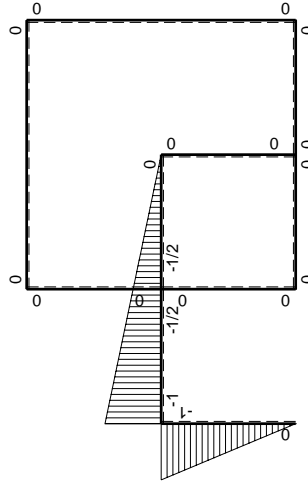
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |                     |                         | $1/8Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-1/8Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

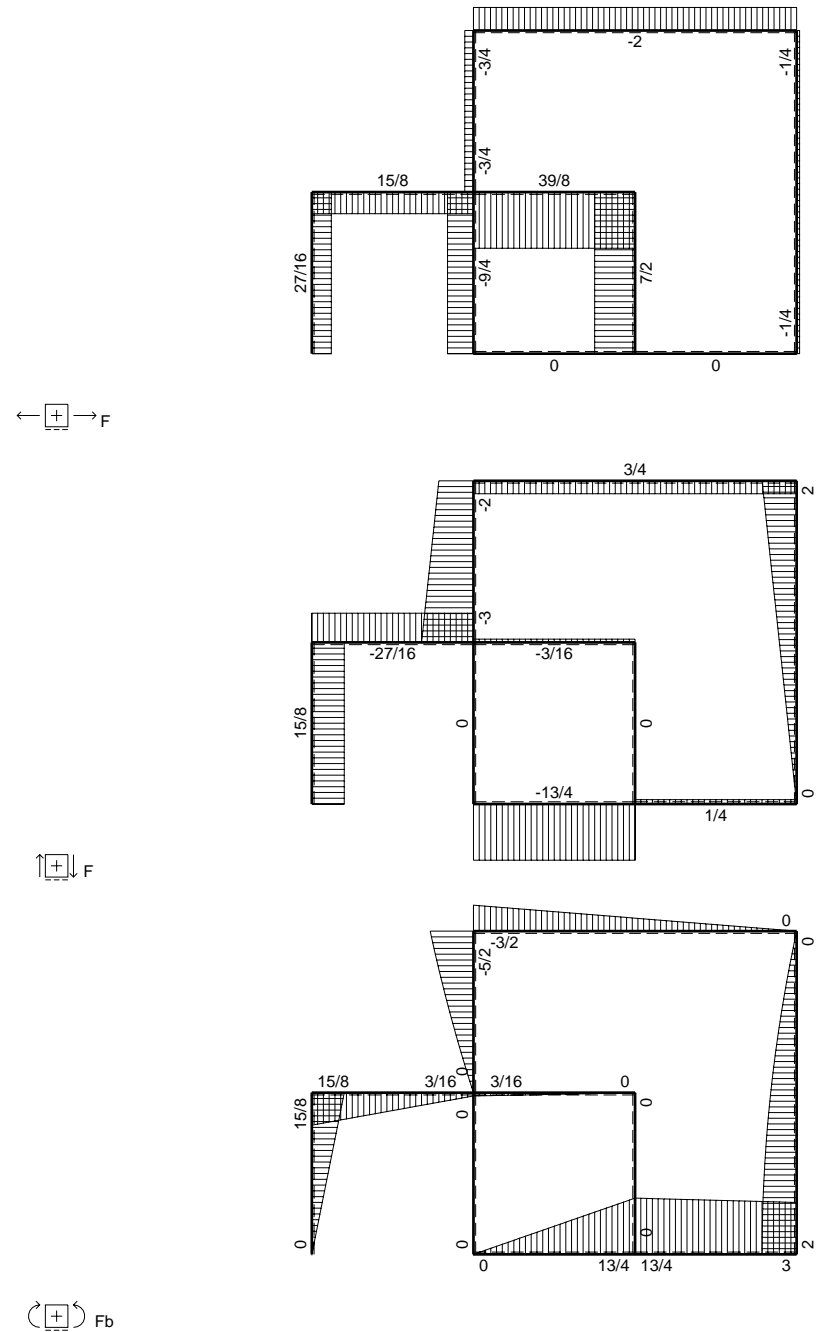
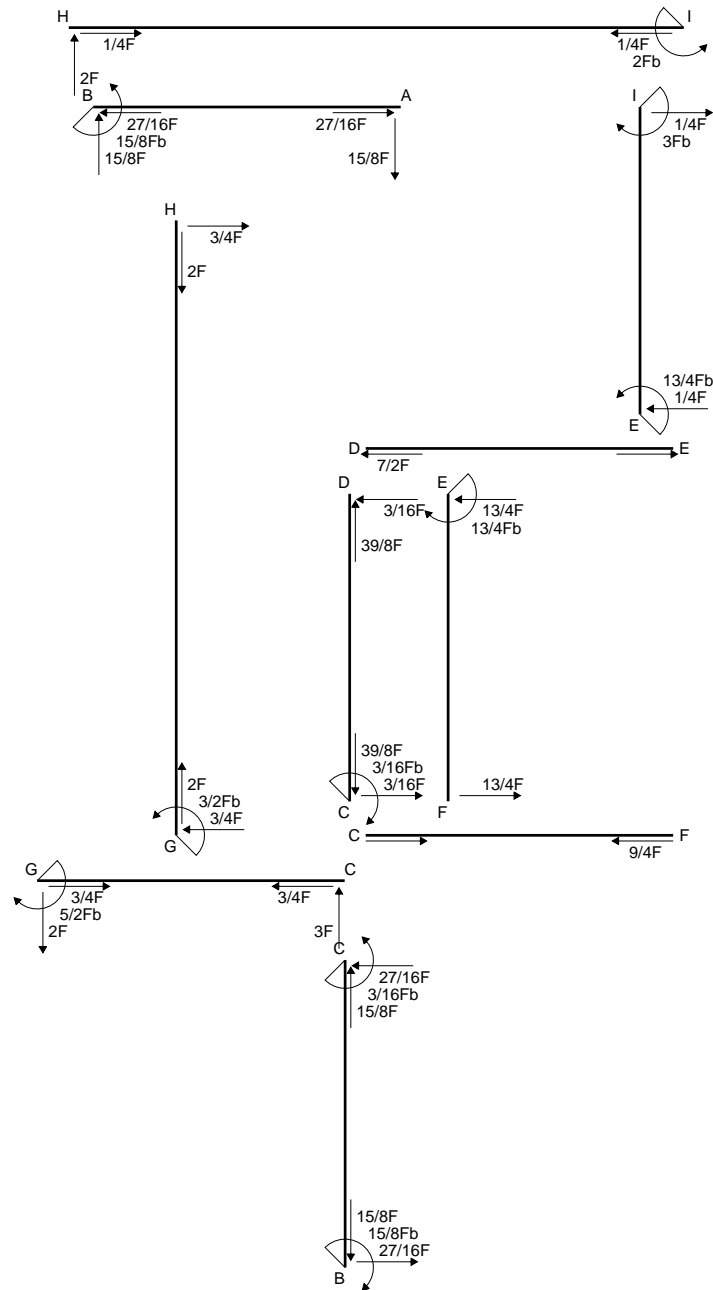
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

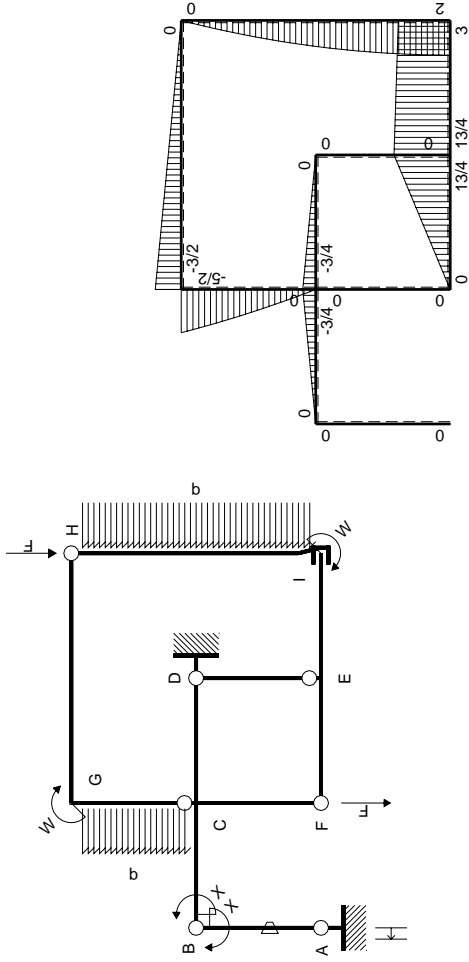
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





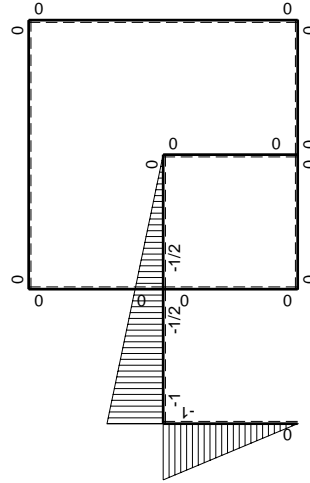
Legend for internal force diagrams:

- $\leftarrow \boxed{+} \rightarrow F$
- $\uparrow \boxed{+} \downarrow F_b$
- $\curvearrowright \boxed{+} \curvearrowleft F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |               |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-15/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

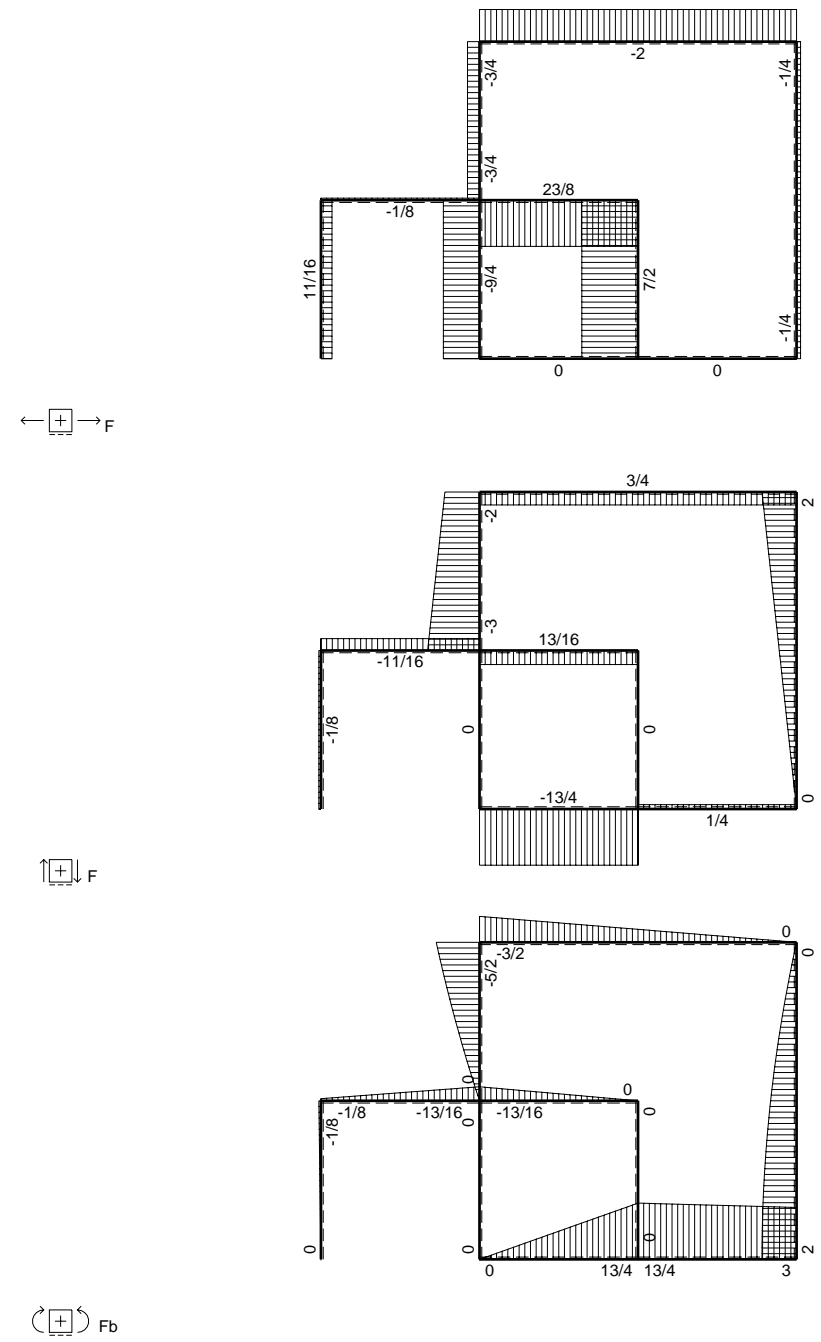
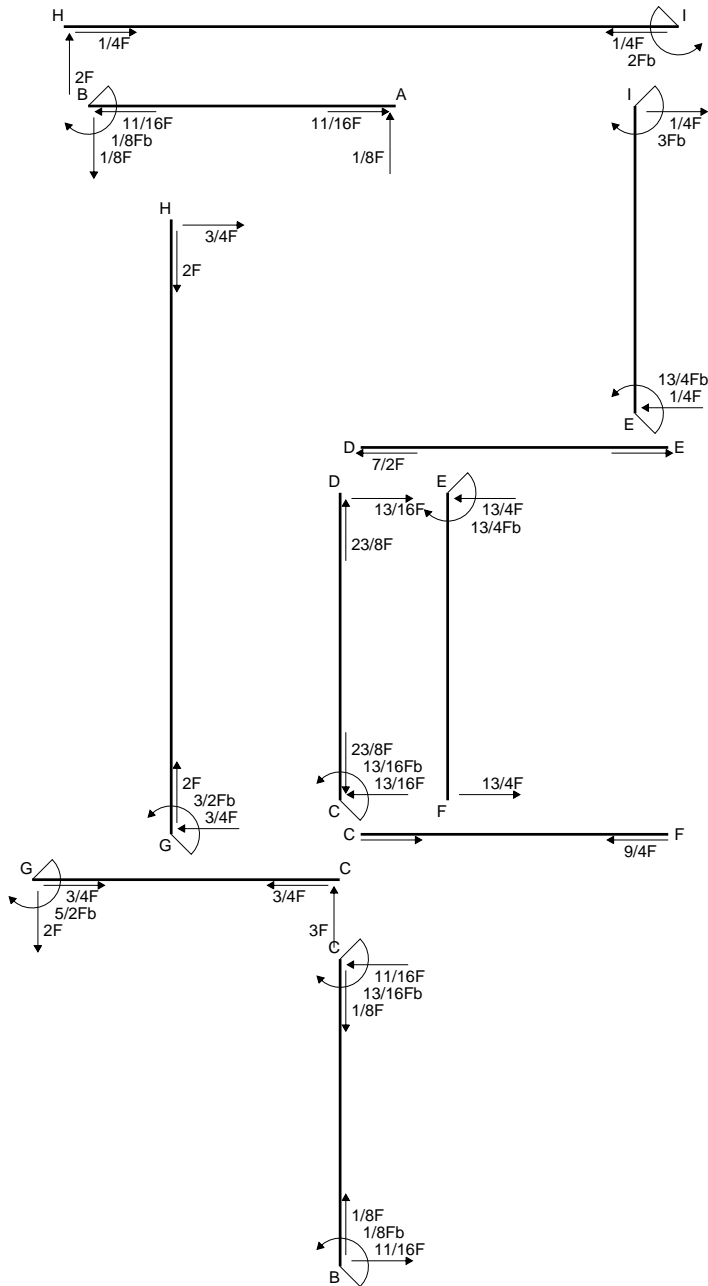
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

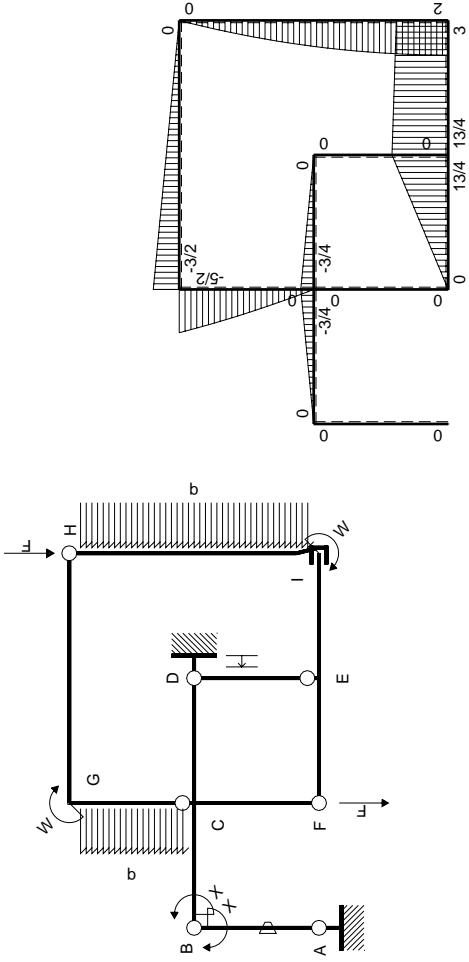
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

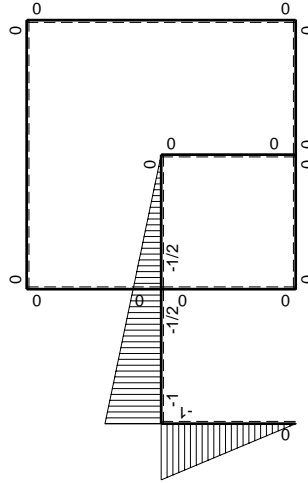
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|-----------------------|---------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$            |         |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$           |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$           |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                       |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                       |         |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0             | 0                       |                             |                       |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                       |         |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                       |         |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                       |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                       |         |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                     |         |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0             | 0                       |                             |                       |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         |                             | $-Fb^2/EJ$            |         |
|       | totali                      |                     |          |                         |               |                         |                             | $-1/8Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         |                             | $1/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

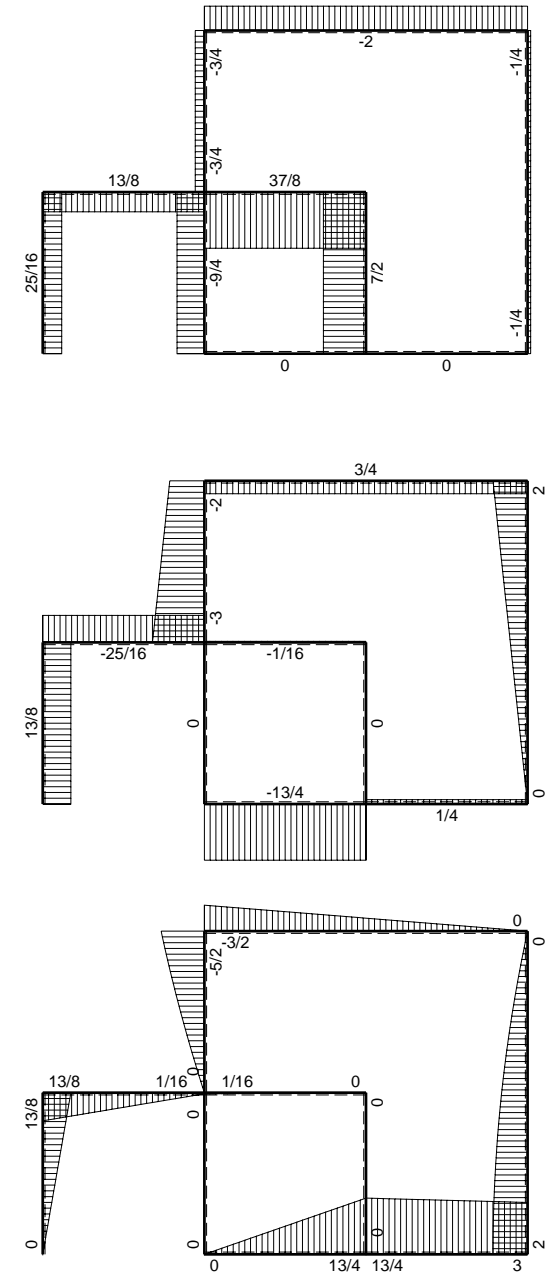
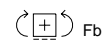
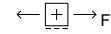
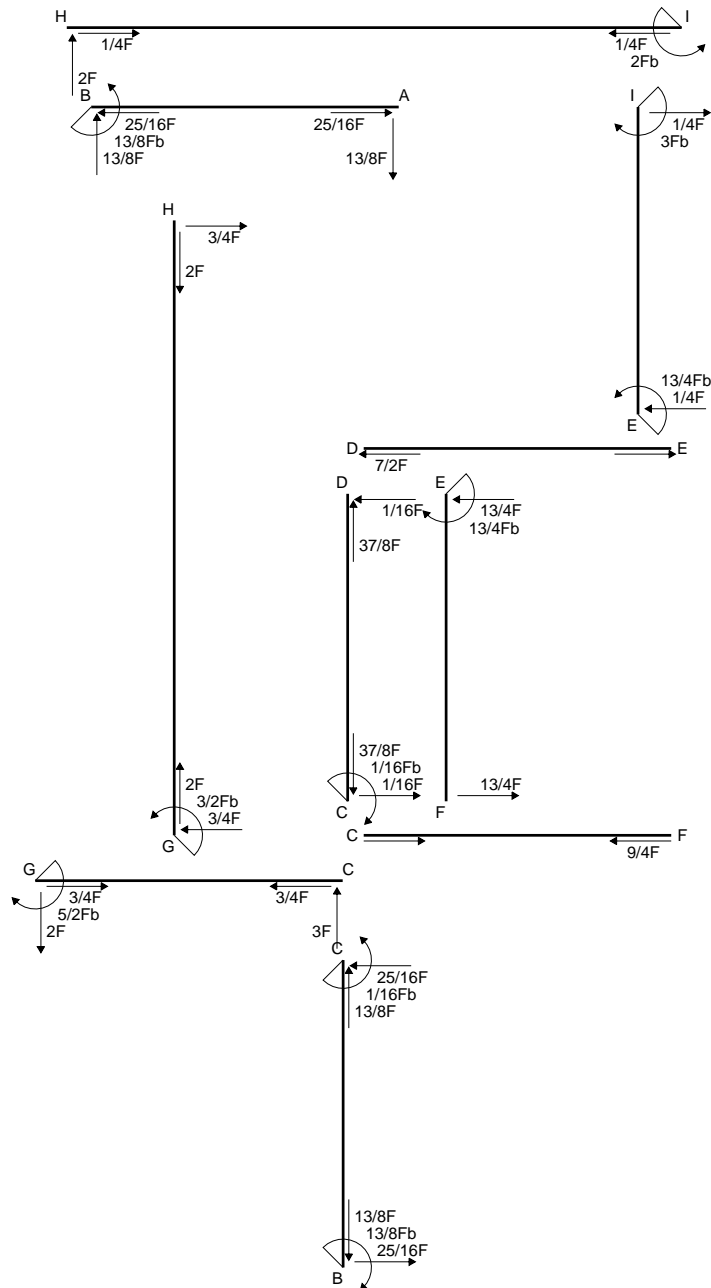
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

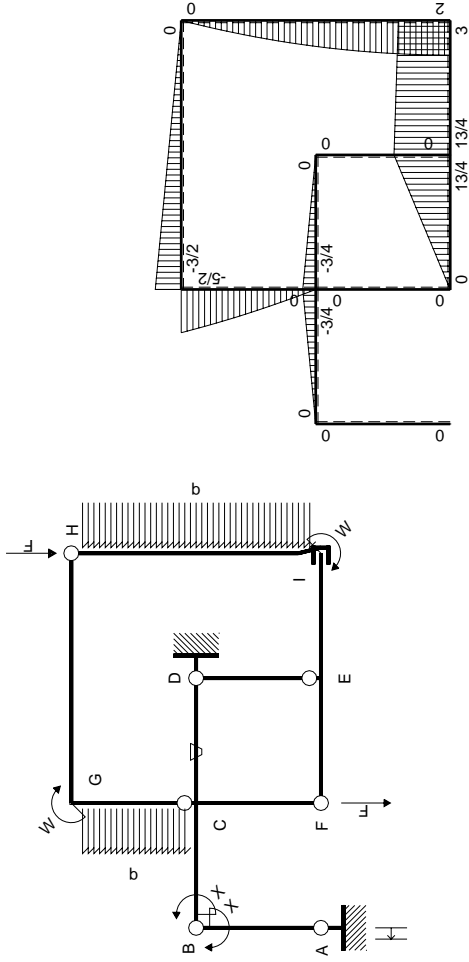
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

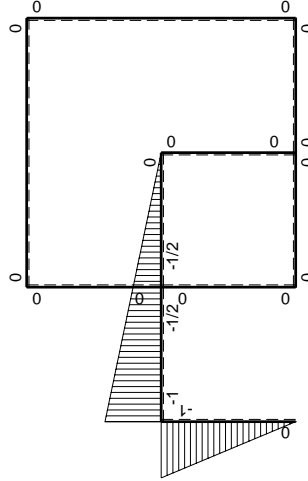






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $13/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-13/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

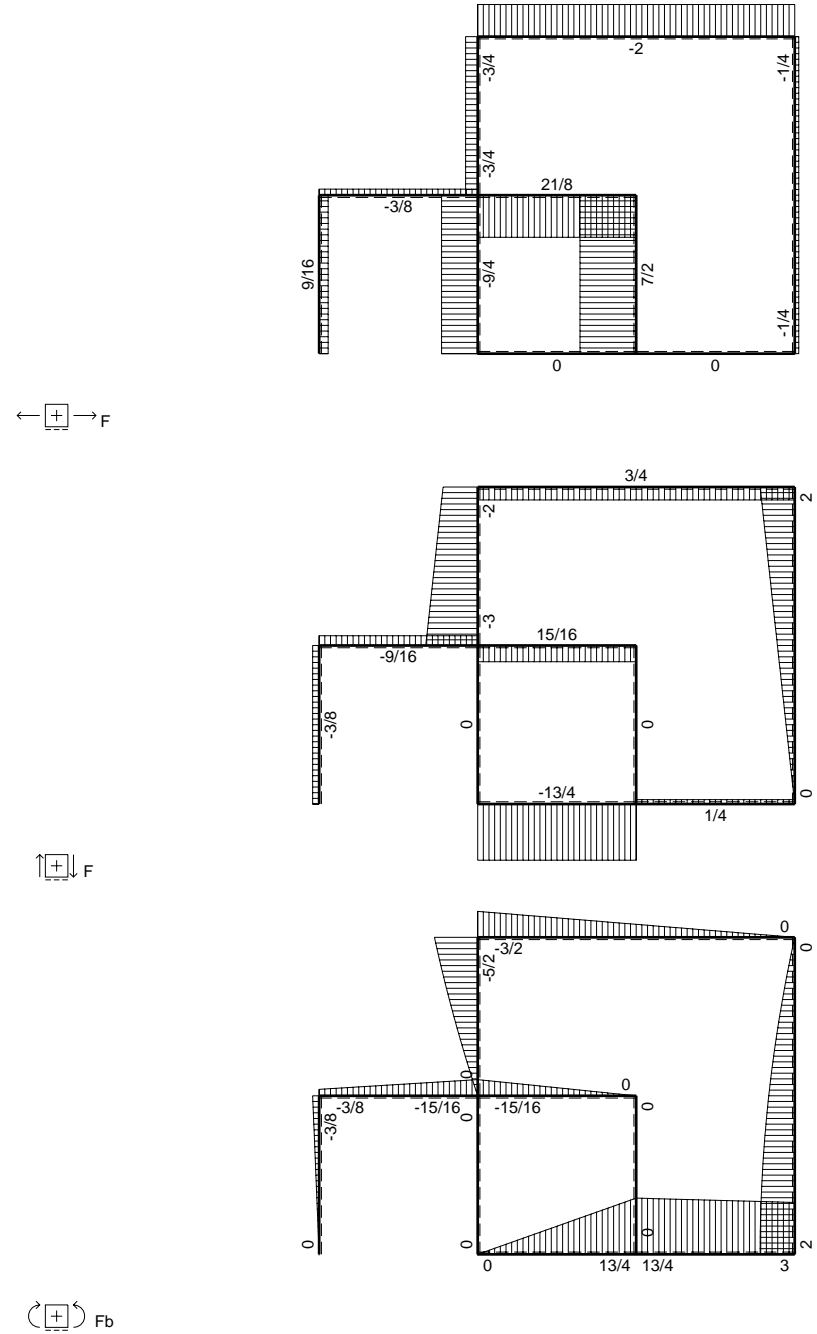
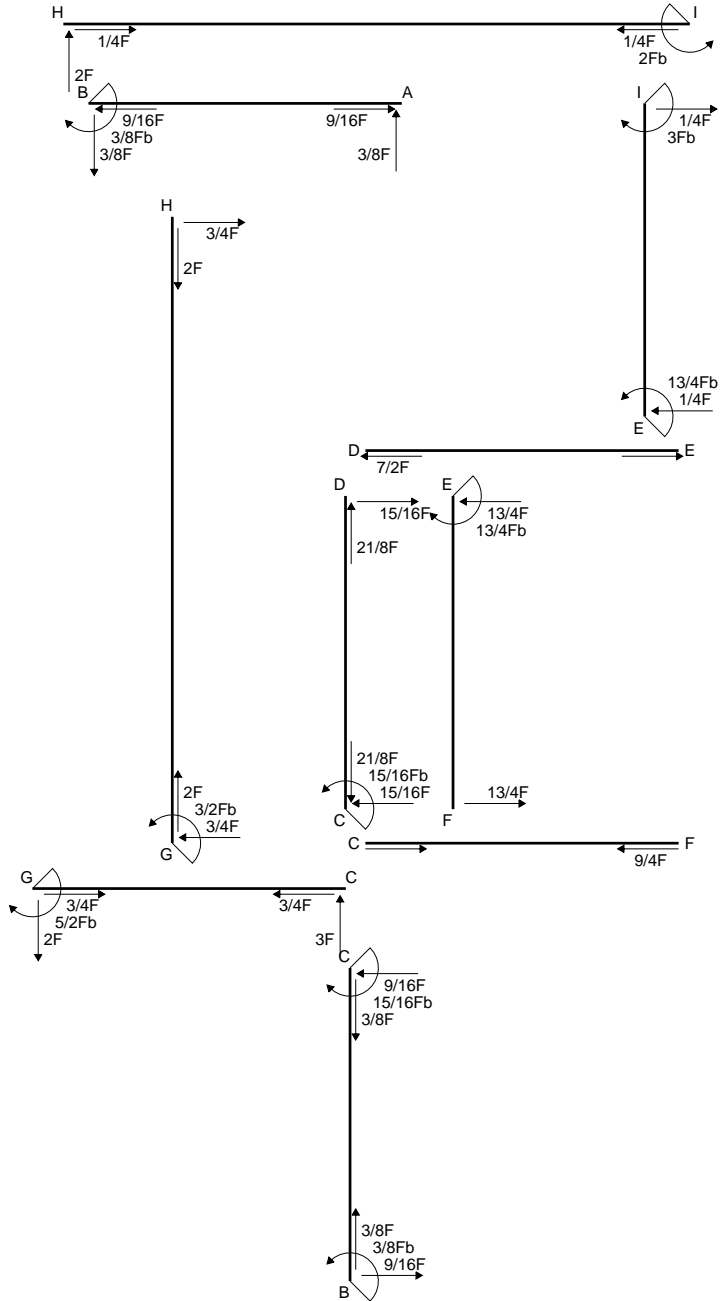
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

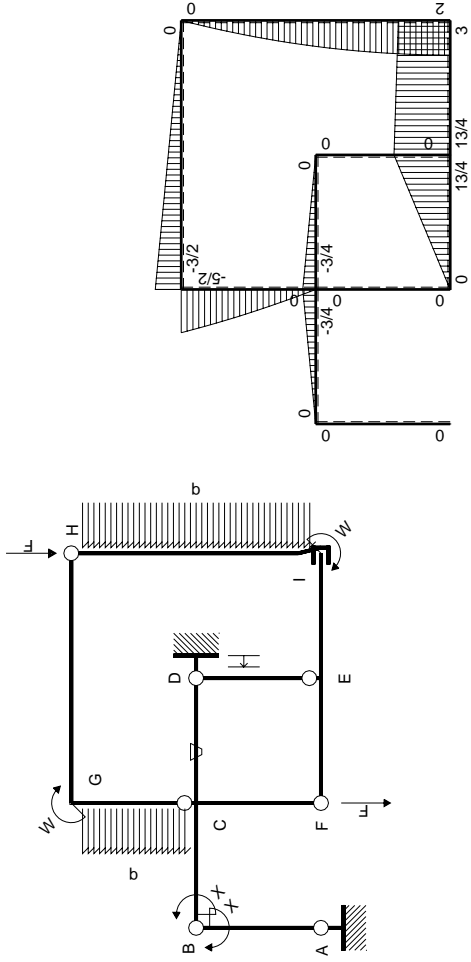
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

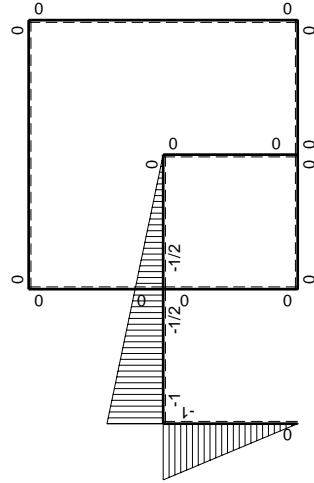
$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-3Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $5/2Fb-2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-3/2Fb+3/4Fx$      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $3/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/4Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+1/4Fx$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |                     |                         | $-3/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $3/8Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

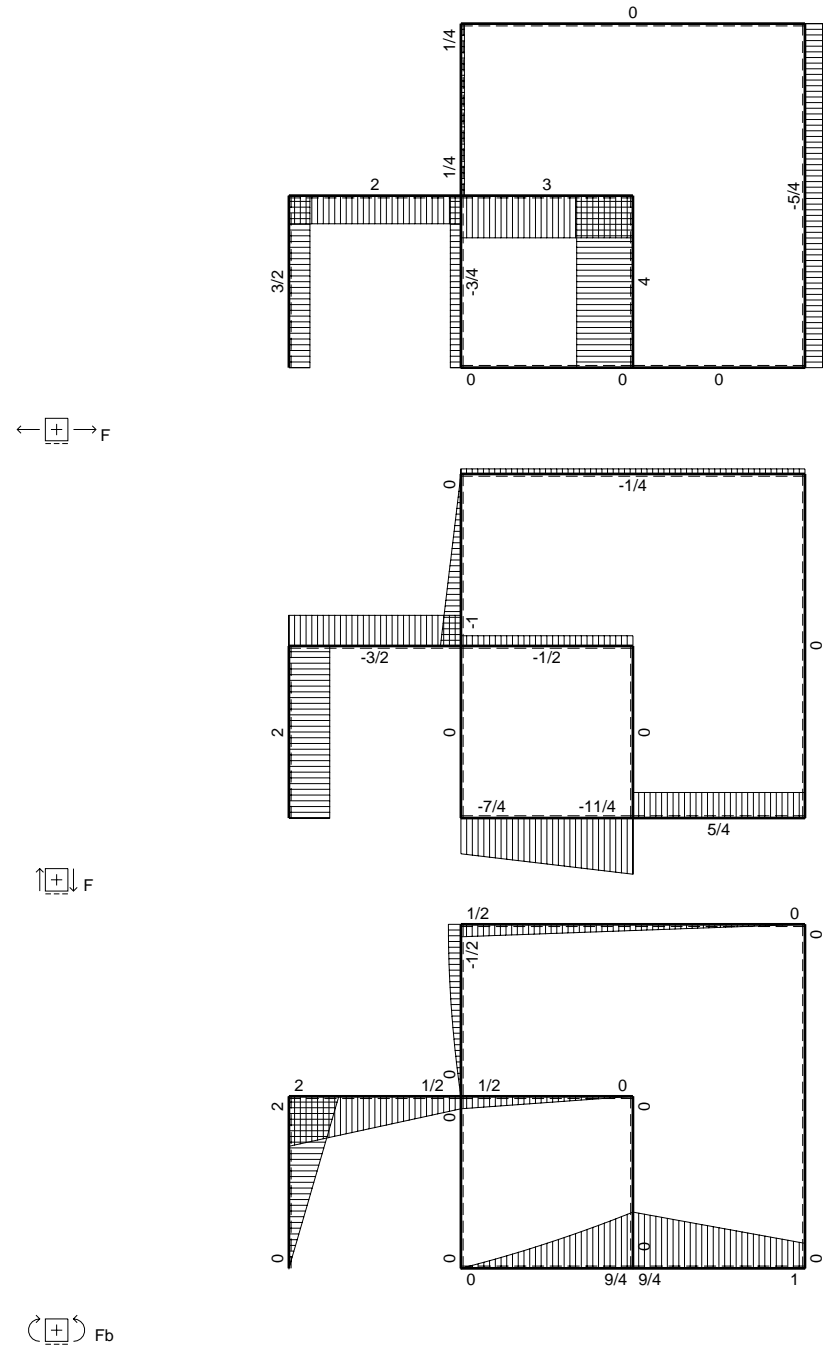
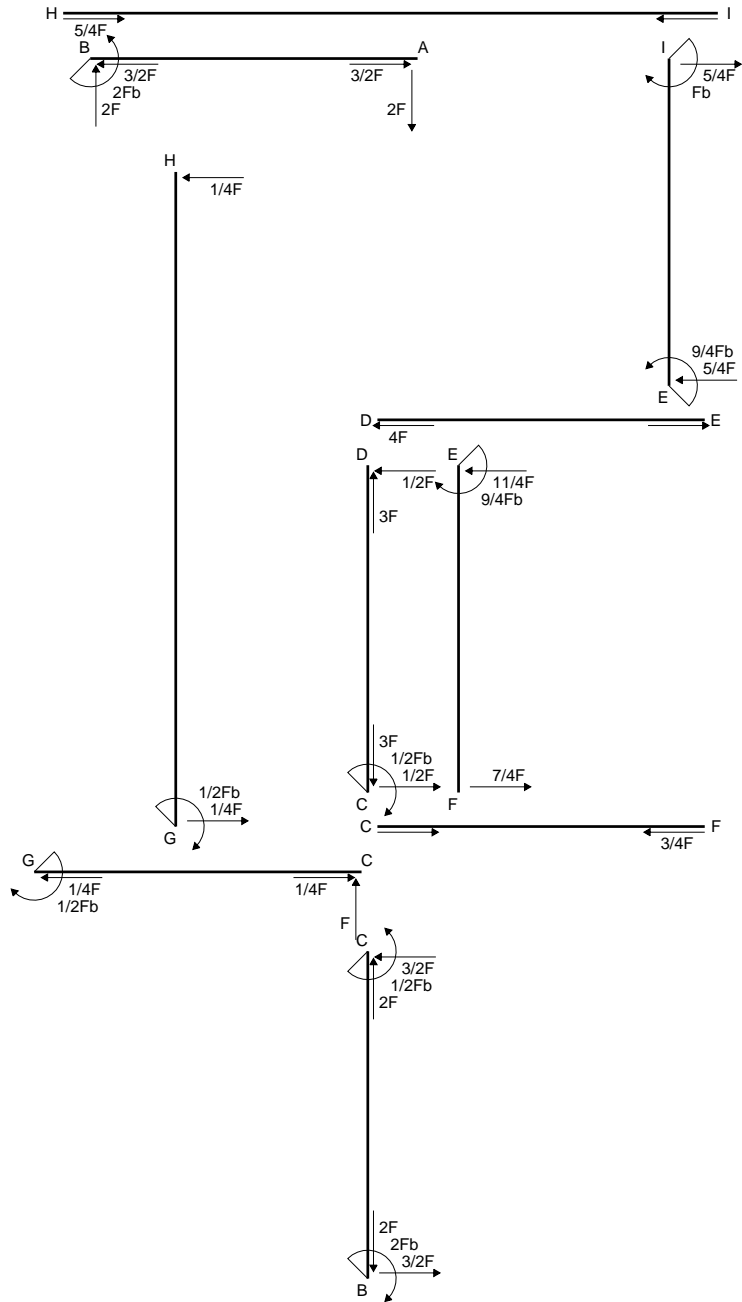
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

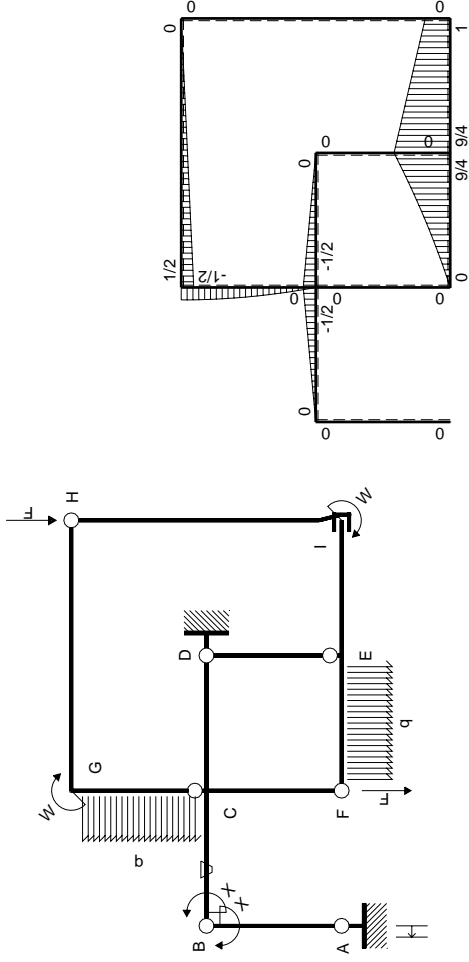
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$



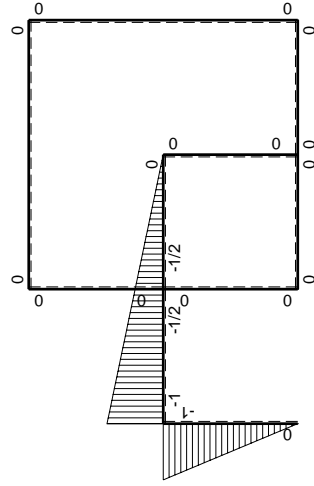


⊕ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | $-Fb/EJ$ | $1/2Fx-1/4Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/6+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | $Fb/EJ$  | $1/4Fb-1/4Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                        |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                        |          |                         |                     |                         | $2Fb^2/EJ$                  | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |                     |                         | $-2Fb$                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

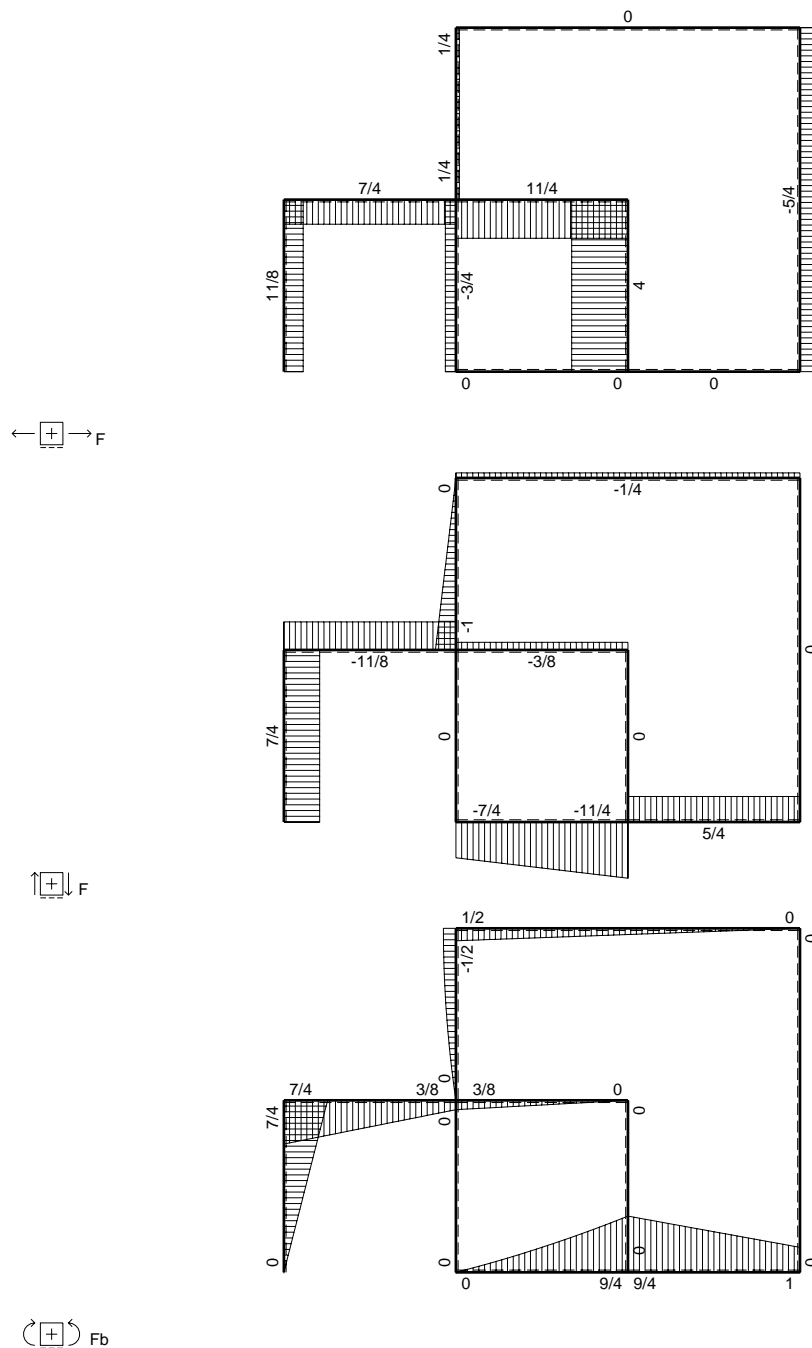
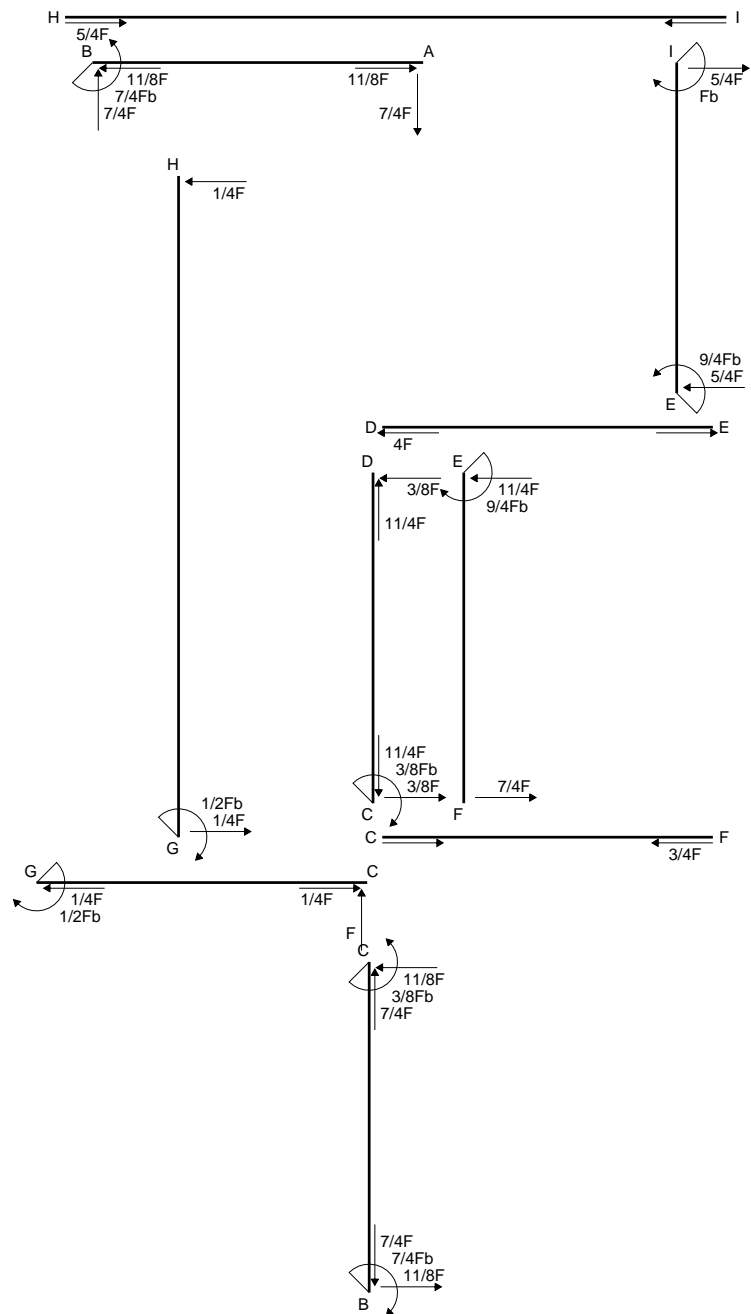
$$= (1/4 b - 1/12 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

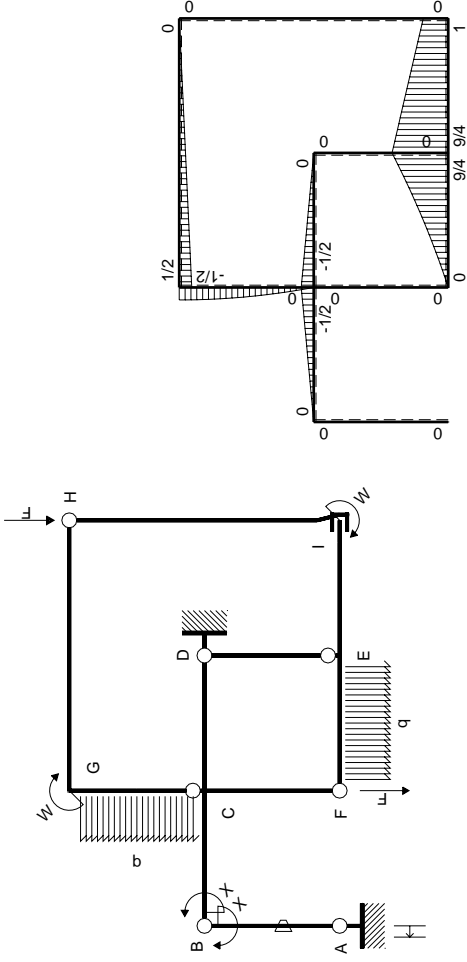
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                        |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                        |          |                         |               |                         | $7/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |               |                         | $-7/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) \frac{1}{EJ} = \frac{1}{3} \frac{b}{EJ}$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2 \frac{x}{b} + \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ x - \frac{x^2}{b} + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) \frac{1}{EJ} = \frac{1}{3} \frac{b}{EJ}$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) \frac{1}{EJ} = \frac{7}{12} \frac{b}{EJ}$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) \frac{1}{EJ} = \frac{7}{12} \frac{b}{EJ}$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) \frac{1}{EJ} = \frac{1}{12} \frac{b}{EJ}$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) \frac{1}{EJ} = \frac{1}{12} \frac{b}{EJ}$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{x}{b} \right) \theta dx = \left[ \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b \right) \theta = \frac{1}{2} \frac{Fb^2}{EJ}$$

$$L_{BA}^{xo} = \int_0^b \left( -1 + \frac{x}{b} \right) \theta dx = \left[ -x + \frac{1}{2} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( -b + \frac{1}{2} b \right) \theta = \frac{1}{2} \frac{Fb^2}{EJ}$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x}{b} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{4} \frac{x^2}{b} - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{12} b \right) Fb \frac{1}{EJ} = \frac{1}{6} \frac{Fb^2}{EJ}$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{4} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{12} b \right) Fb \frac{1}{EJ} = \frac{1}{6} \frac{Fb^2}{EJ}$$

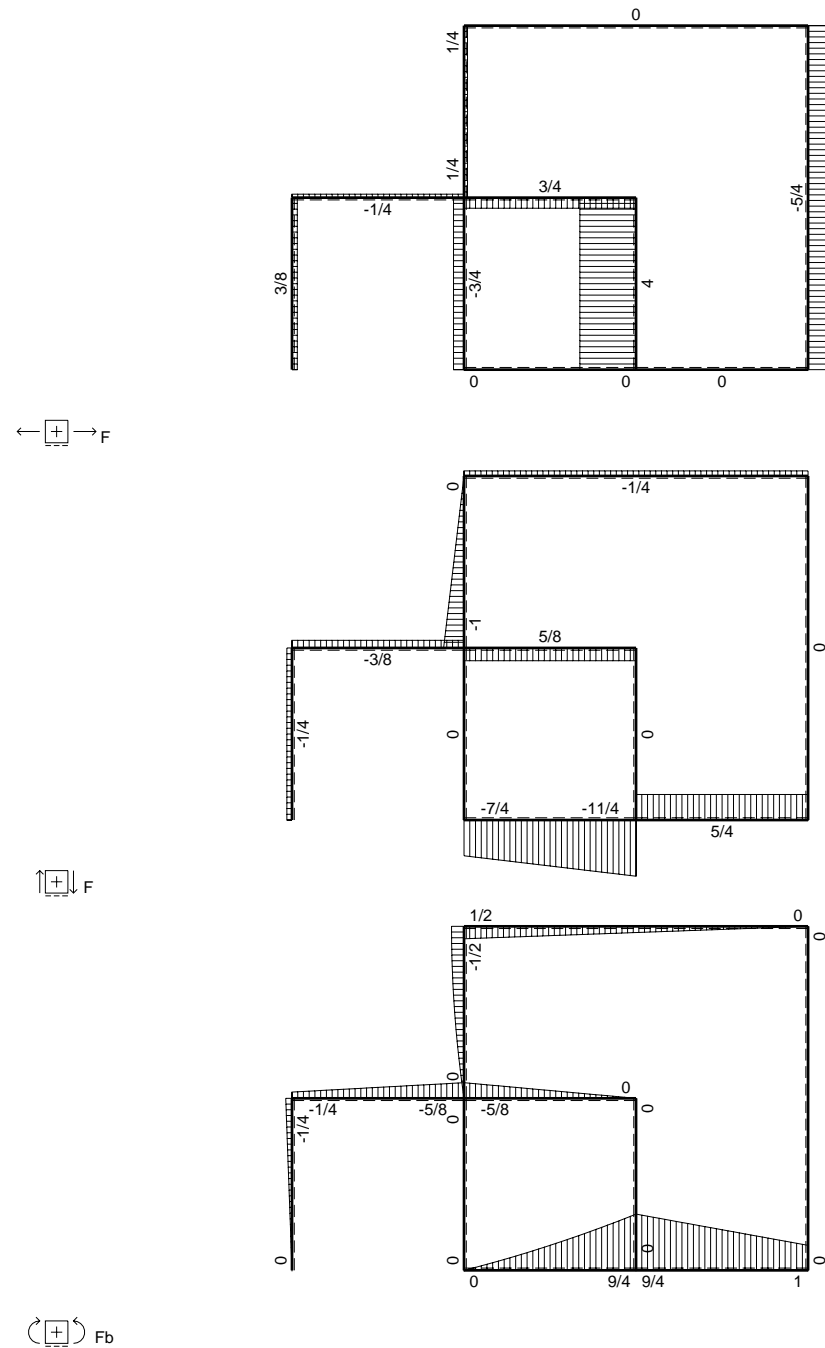
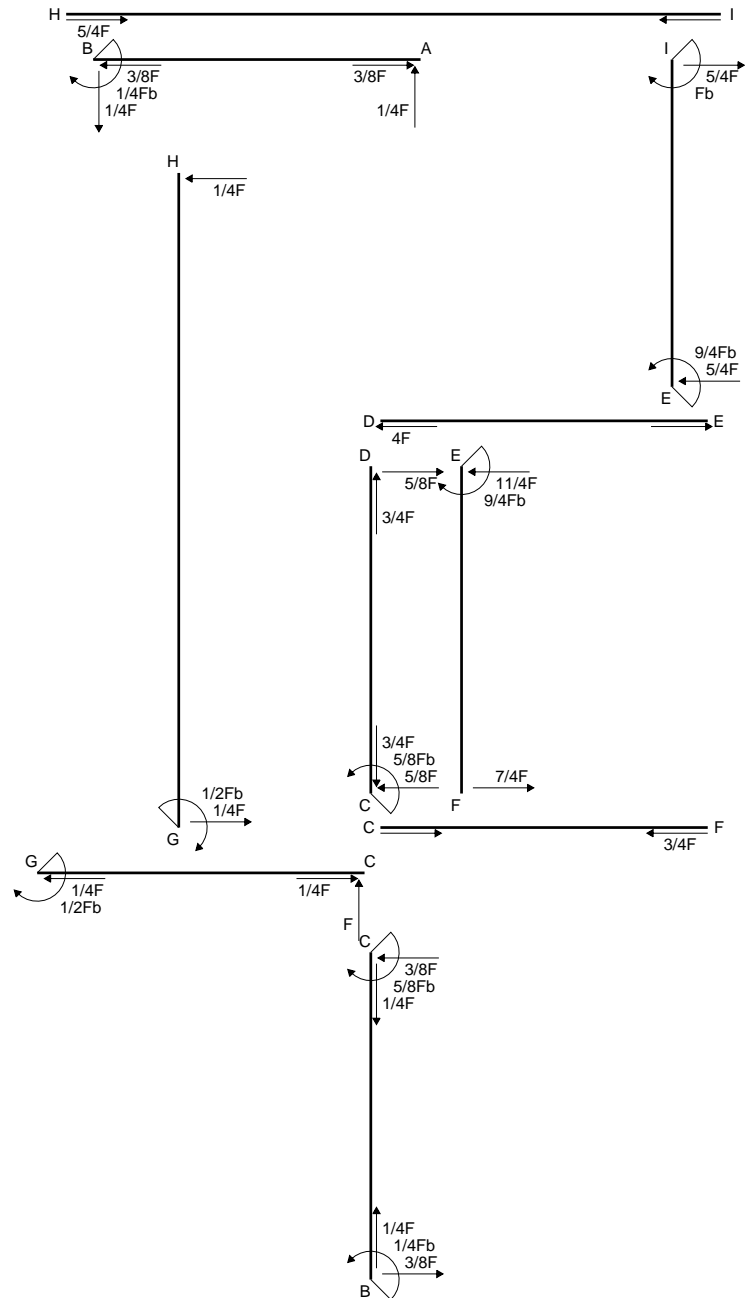
$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ}$$

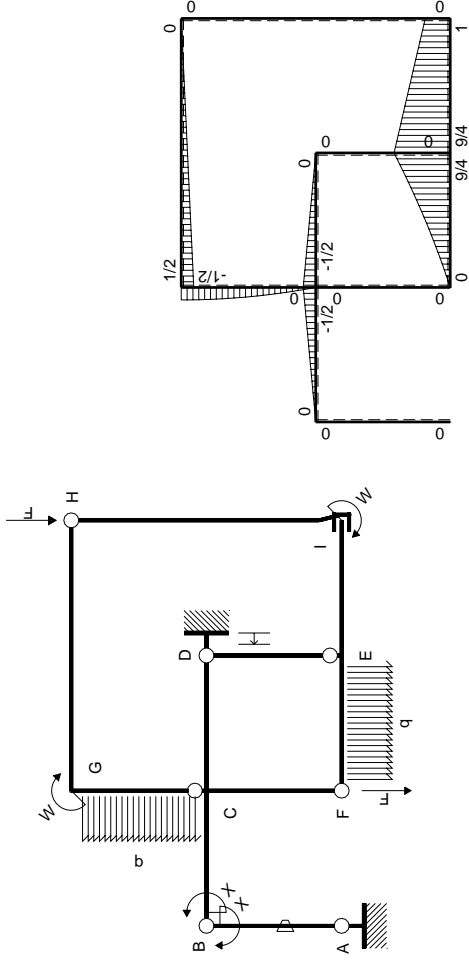
$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) Fb \frac{1}{EJ} = \frac{1}{12} \frac{Fb^2}{EJ}$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) Fb \frac{1}{EJ} = \frac{1}{12} \frac{Fb^2}{EJ}$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                        |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                        |          |                         |               |                         | $-1/4Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |               |                         | $1/4Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

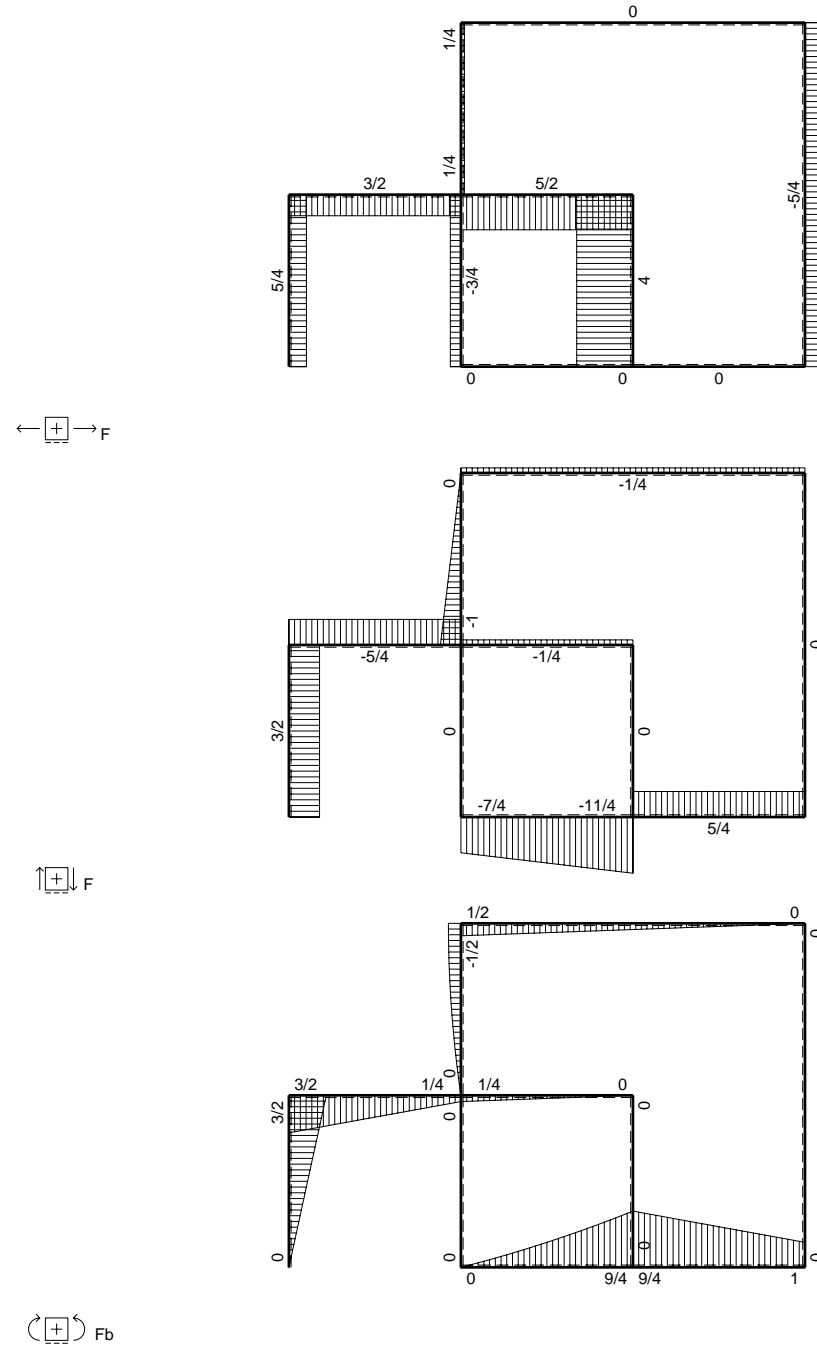
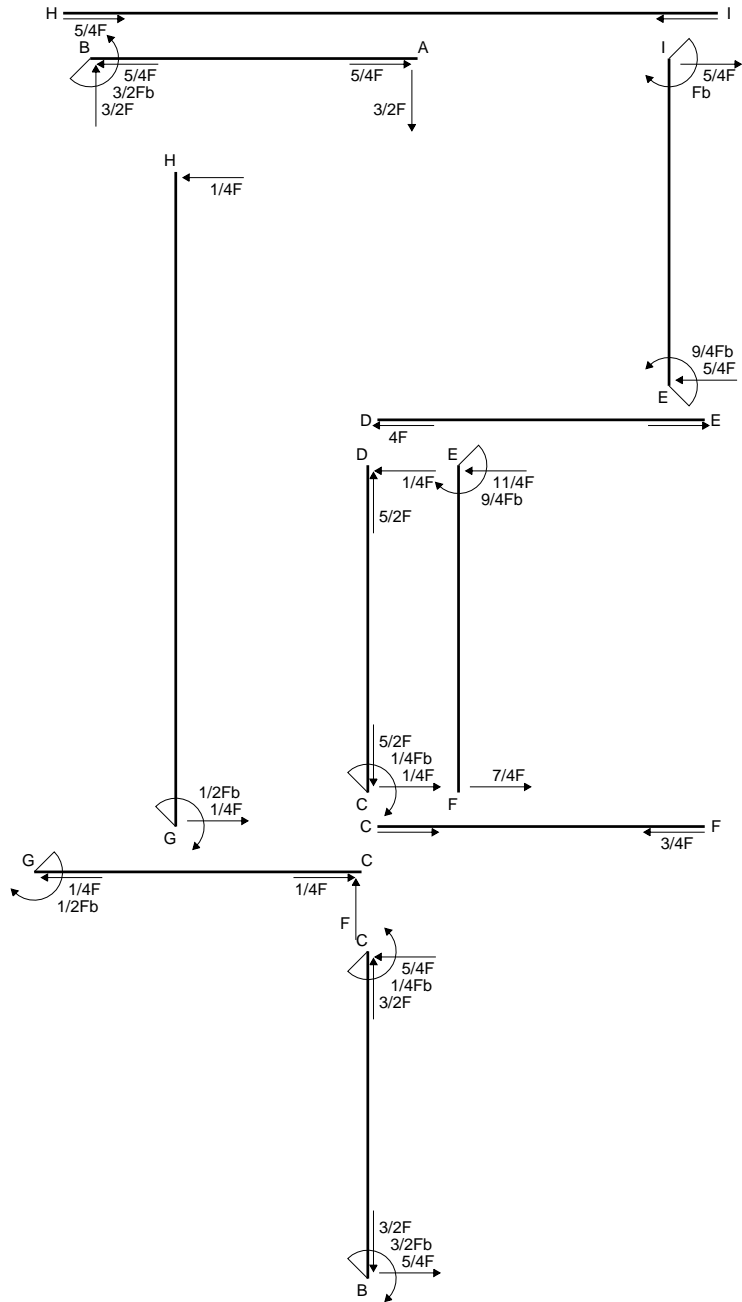
$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

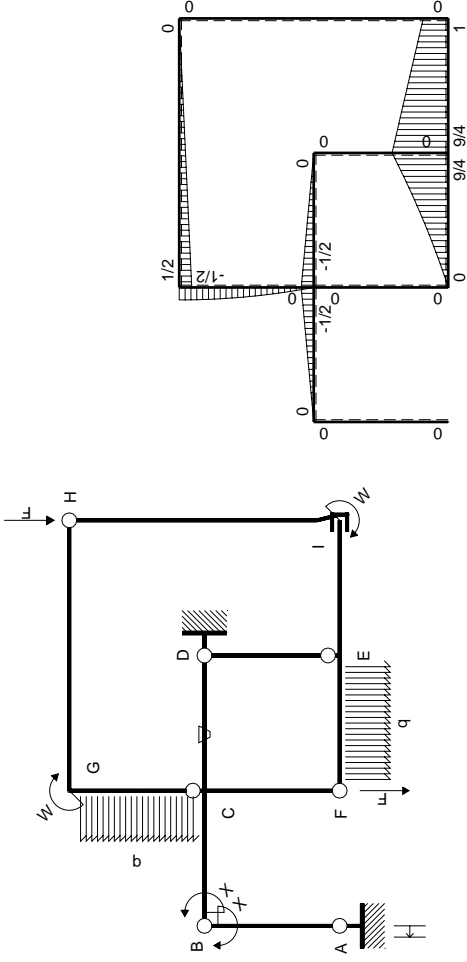
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                        |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                        |          |                         |                     |                         | $3/2Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |                     |                         | $-3/2Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

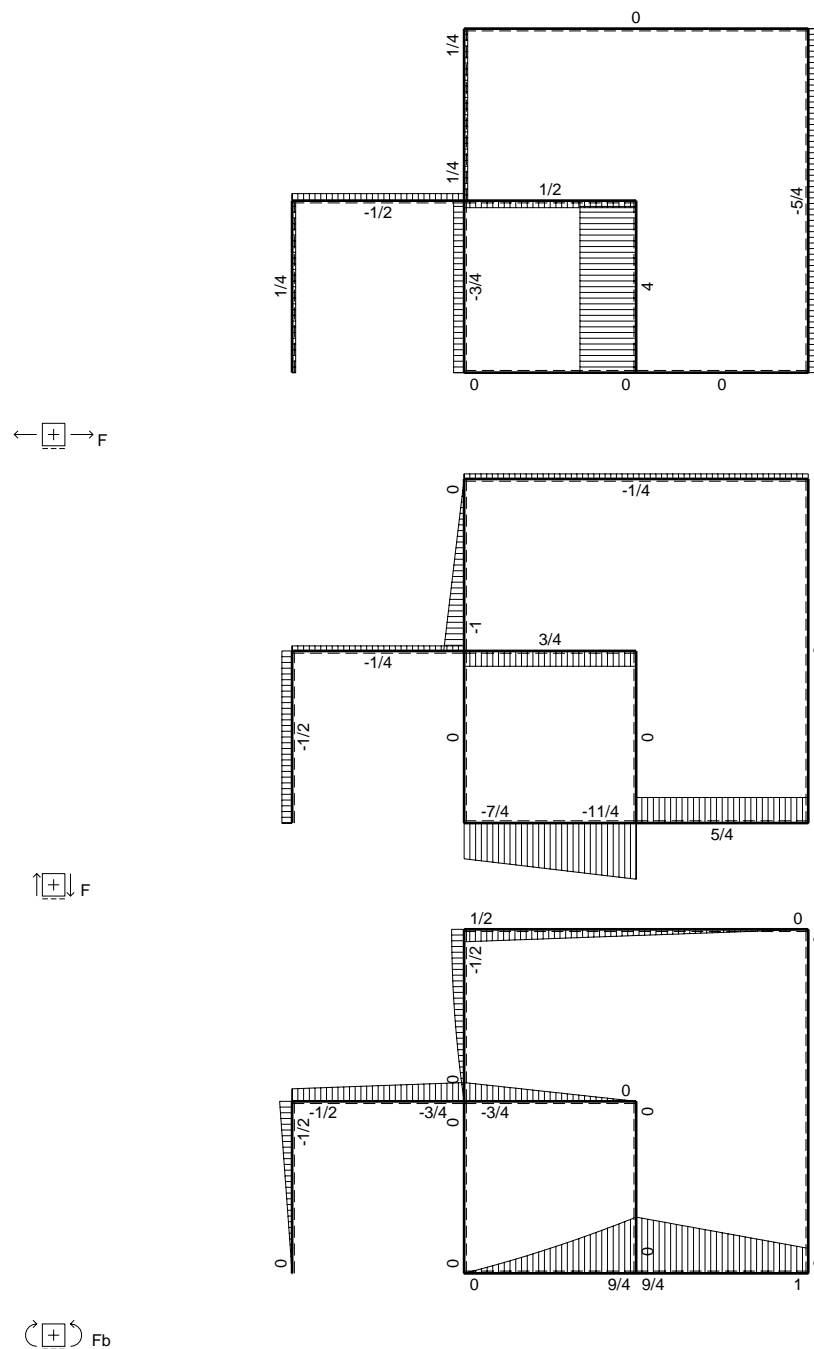
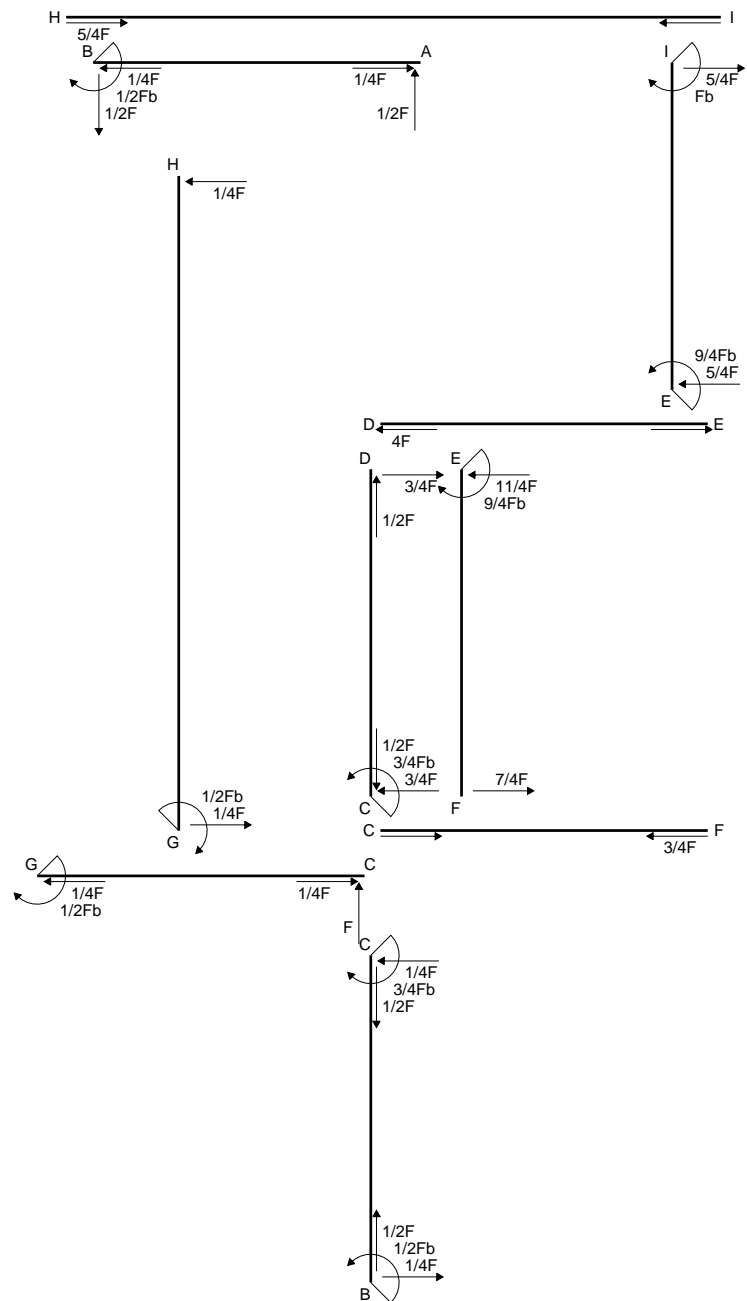
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

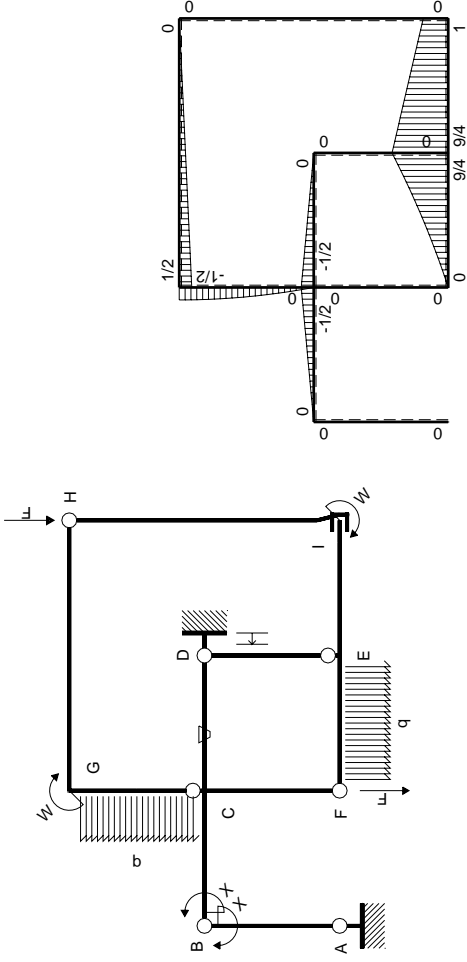
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                        |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                        |          |                         |                     |                         | $-1/2Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |                     |                         | $1/2Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

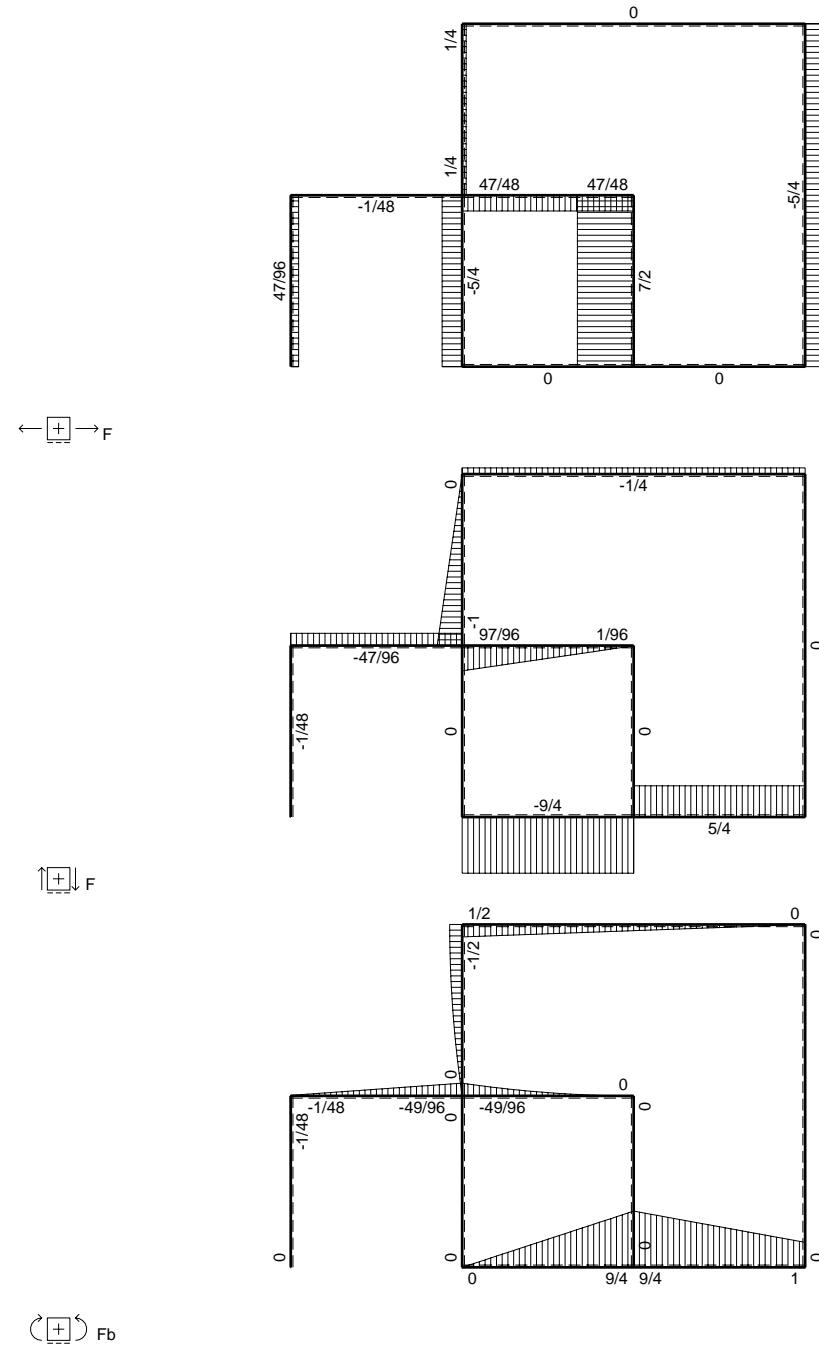
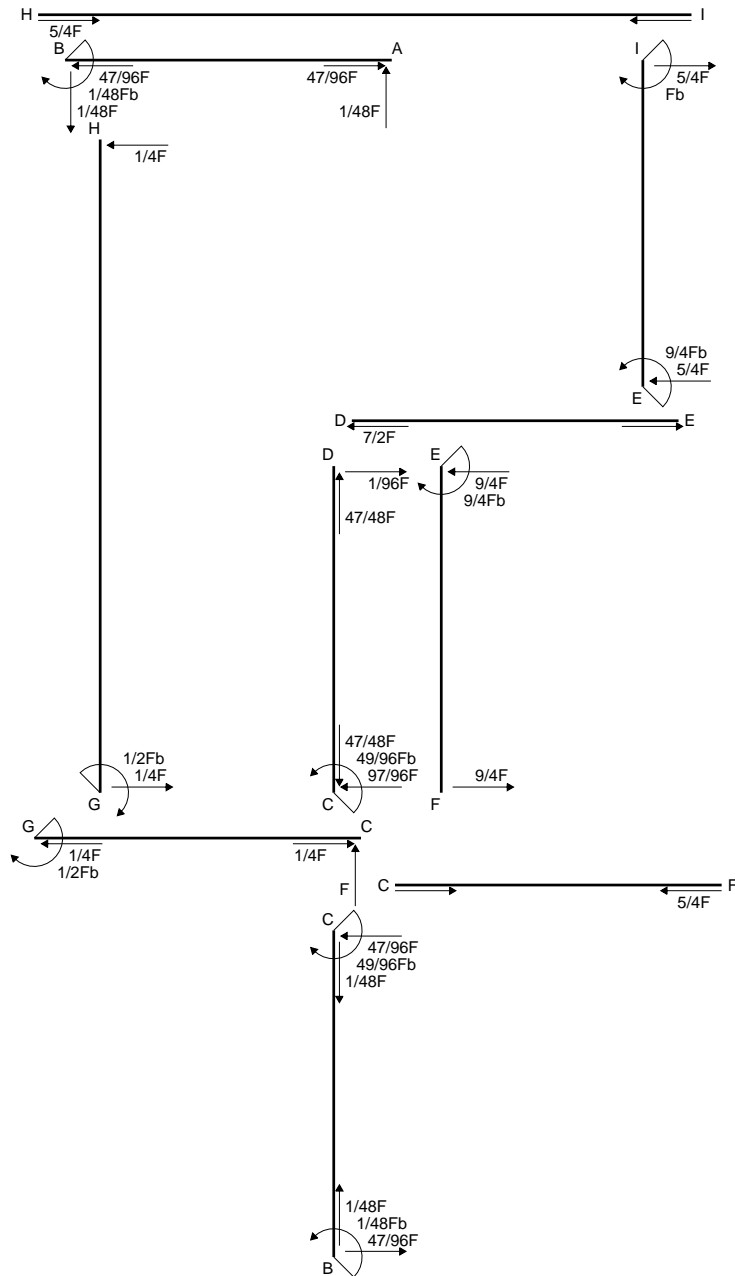
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

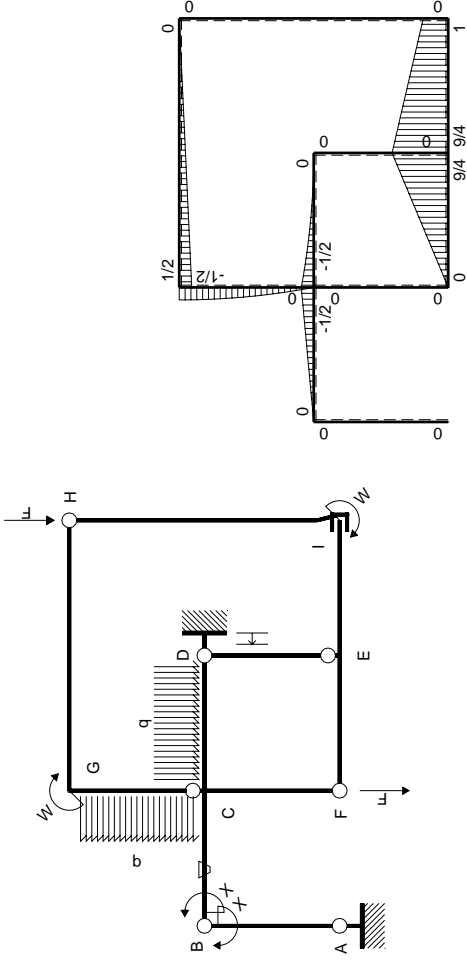
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-----------------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                                 | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                                 | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$            | $-Fb/EJ$ | $1/2Fx-1/4Fx^2/b$                 | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/6+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$       | $Fb/EJ$  | $1/4Fb-1/4Fx^2/b$                 | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+Fx-1/2qx^2$ | 0        | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2qx^2$           | 0        | $1/4qx^3/b$                       | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                                   |                     |                         |                             | $-1/48Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                                   |                     |                         |                             | $1/48Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 3/4 x/b + 3/4 x^2/b^2 - 1/4 x^3/b^3) Fb 1/EJ dx$$

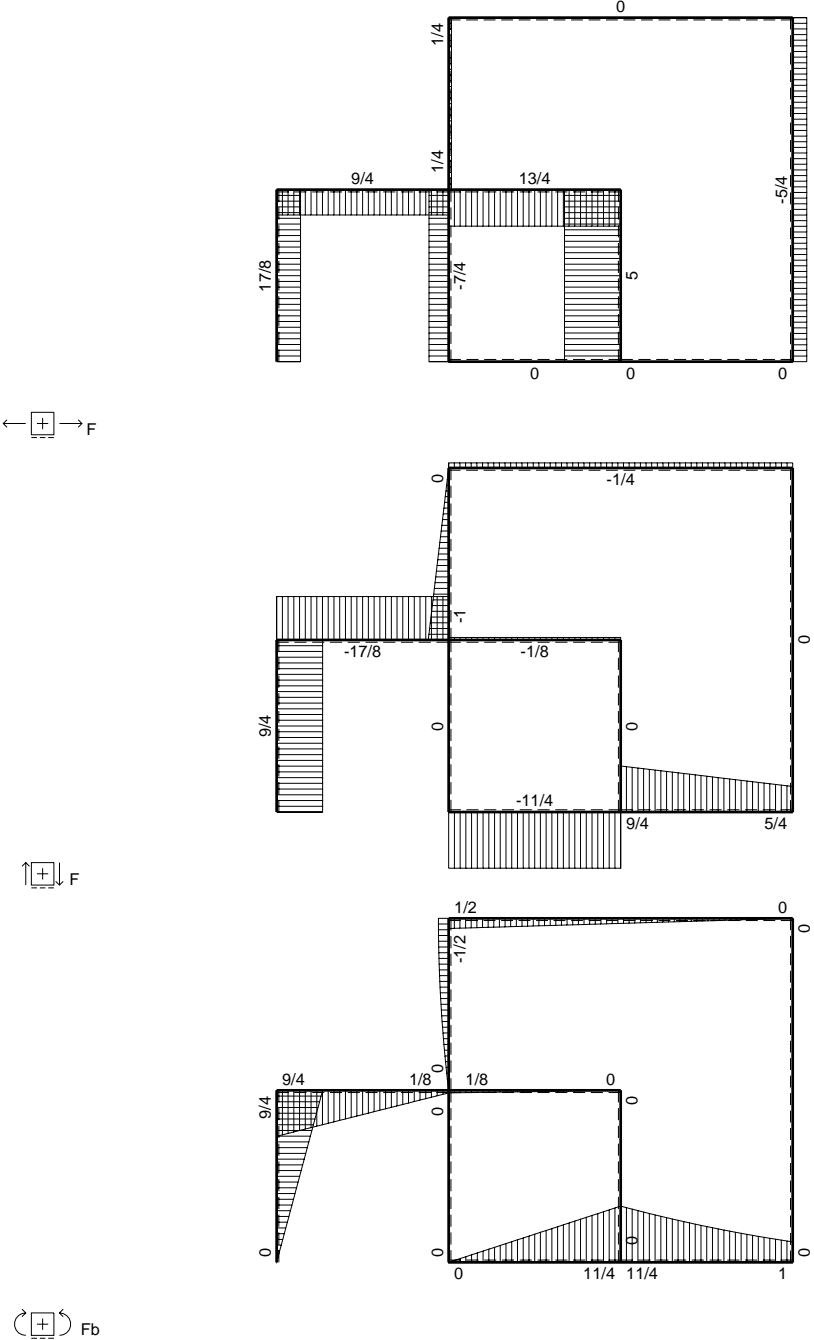
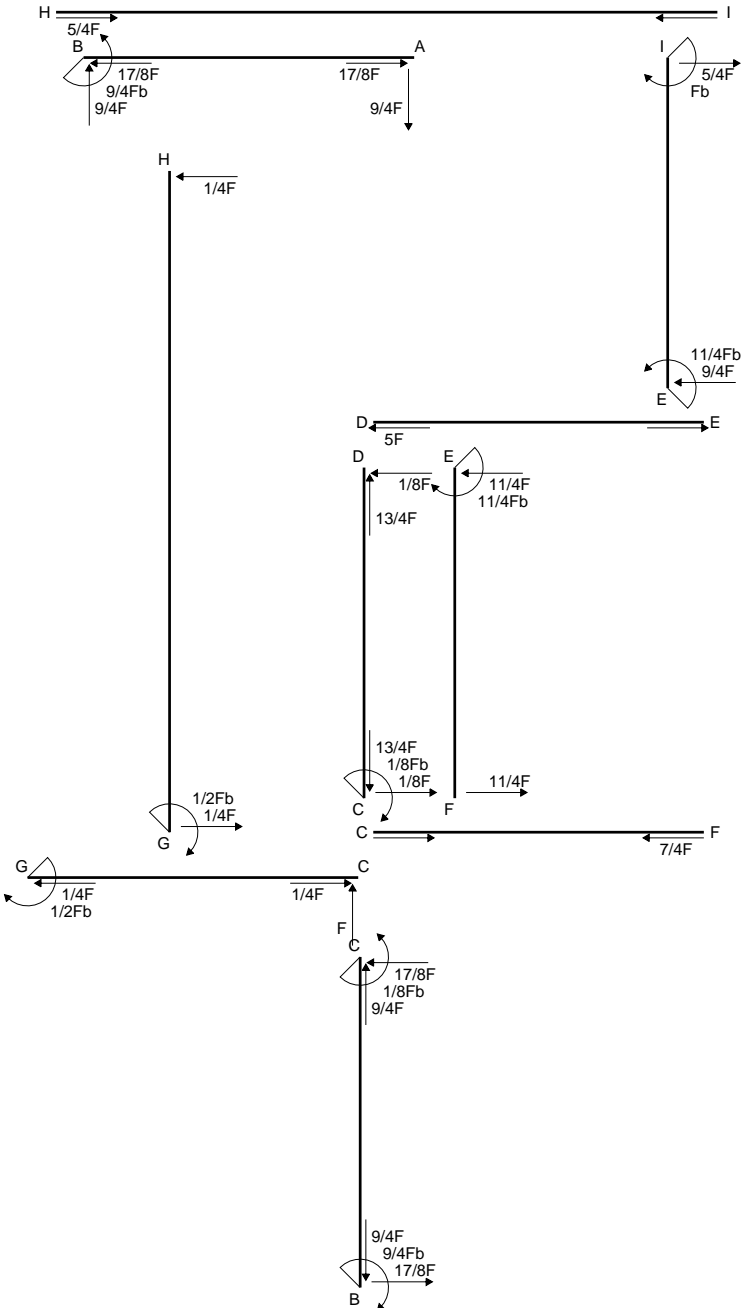
$$= [1/4 x - 3/8 x^2/b + 1/4 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb 1/EJ$$

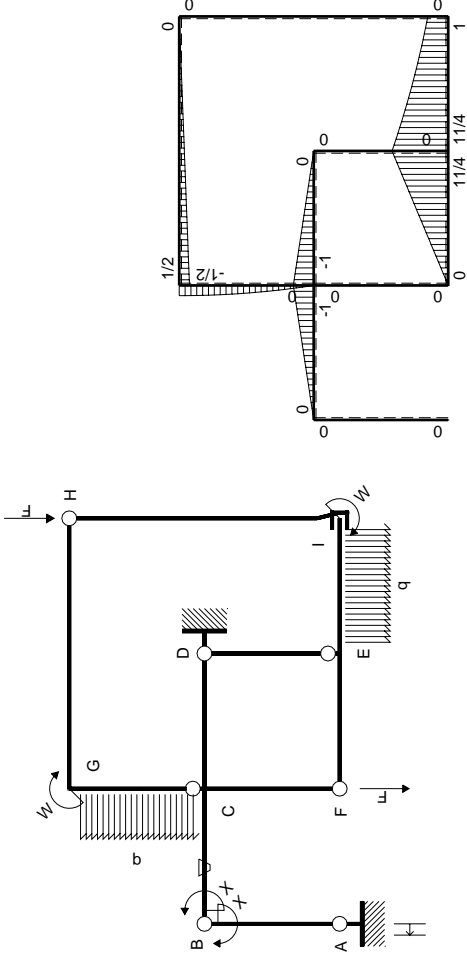
$$= (1/4 b - 3/8 b + 1/4 b - 1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^3/b^3) Fb 1/EJ dx = [1/16 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$

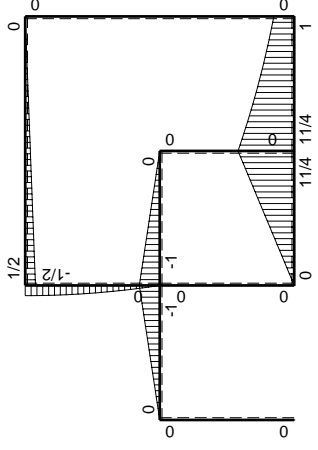
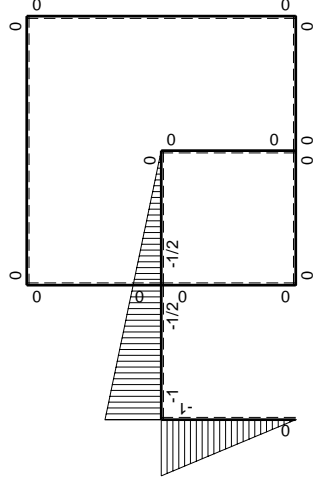






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | -Fx                     | -Fb/EJ   | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | Fb-Fx                   | Fb/EJ    | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | -Fb+Fx                  | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | Fx                      | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                         |          |                      |                     |                         | $9/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         | $-9/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

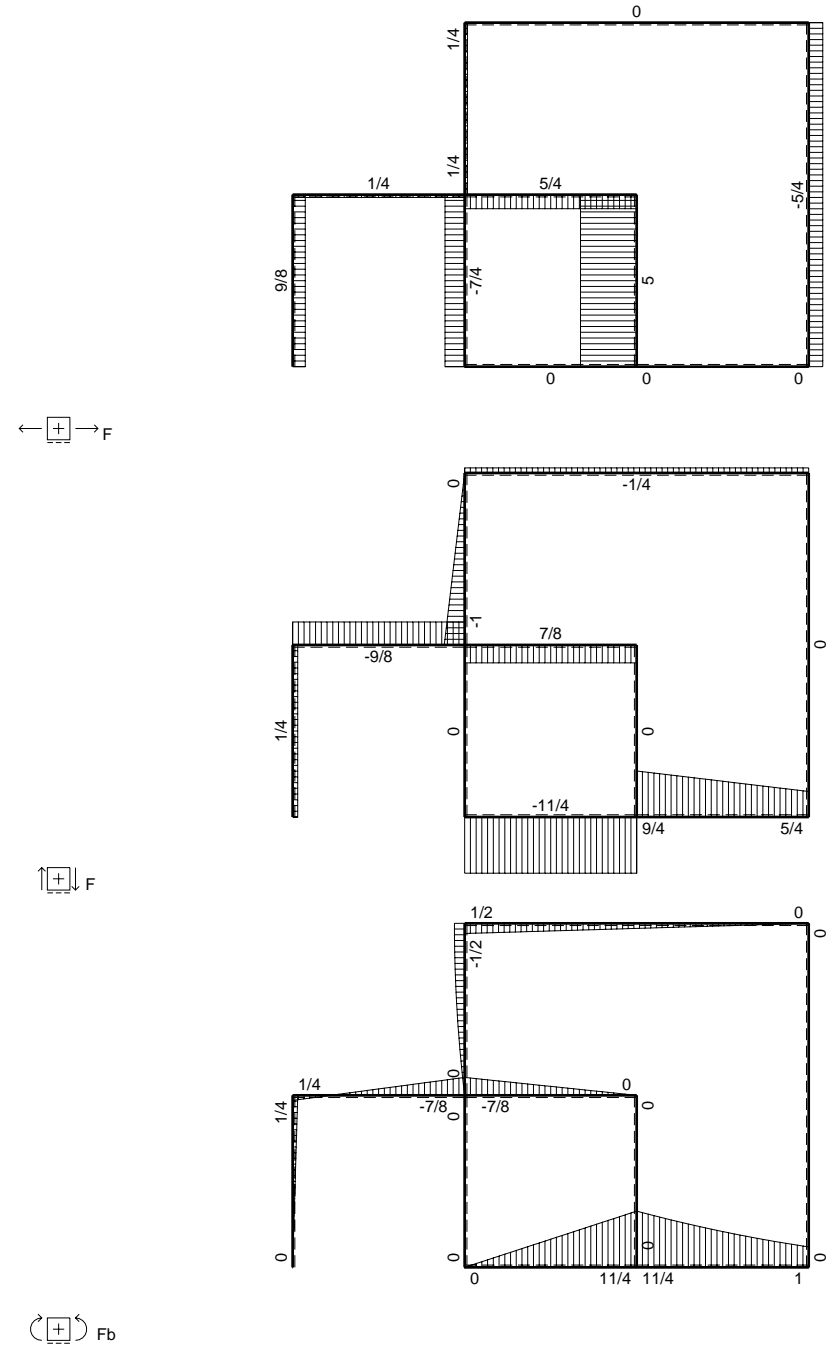
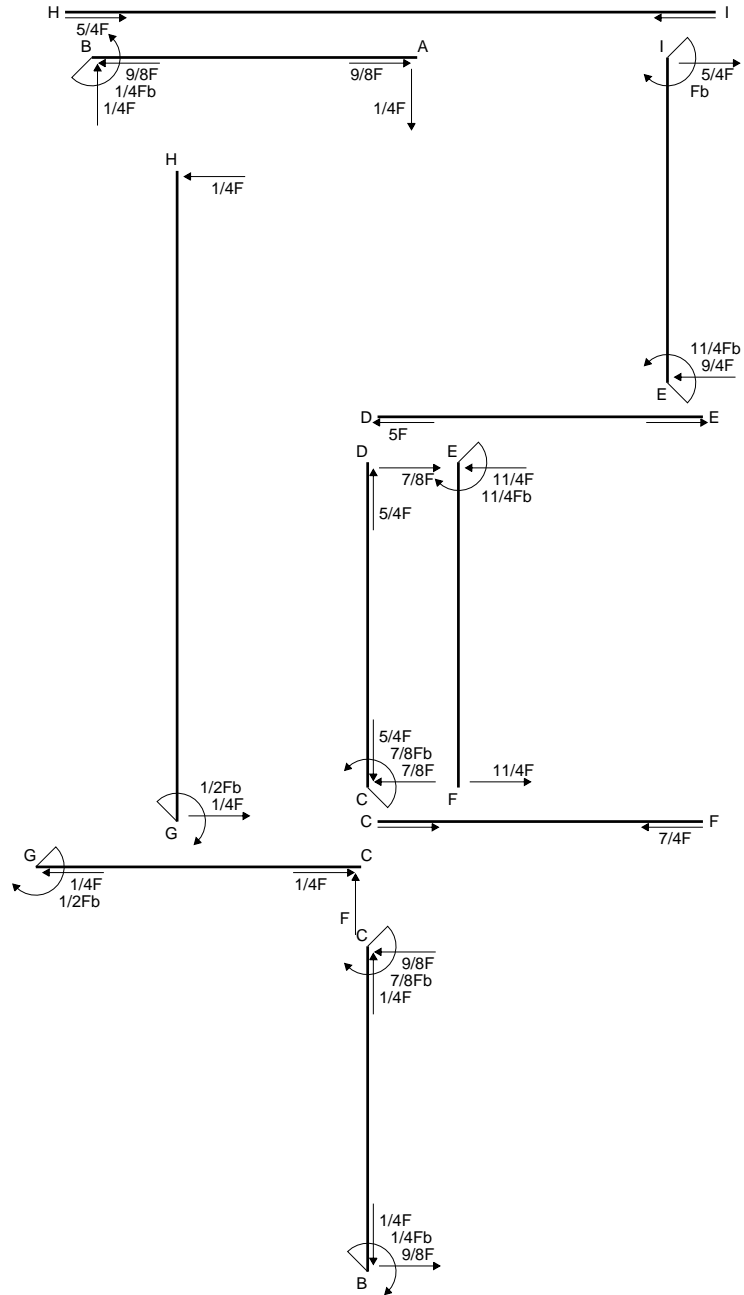
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

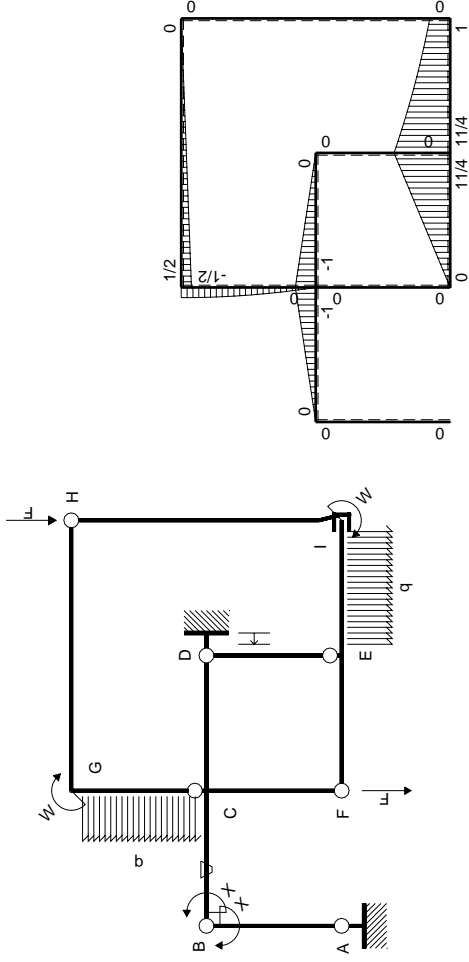
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | -Fx                     | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | Fx                      | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                         |          |                      |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                         |          |                      |                     |                         | $1/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         | $-1/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

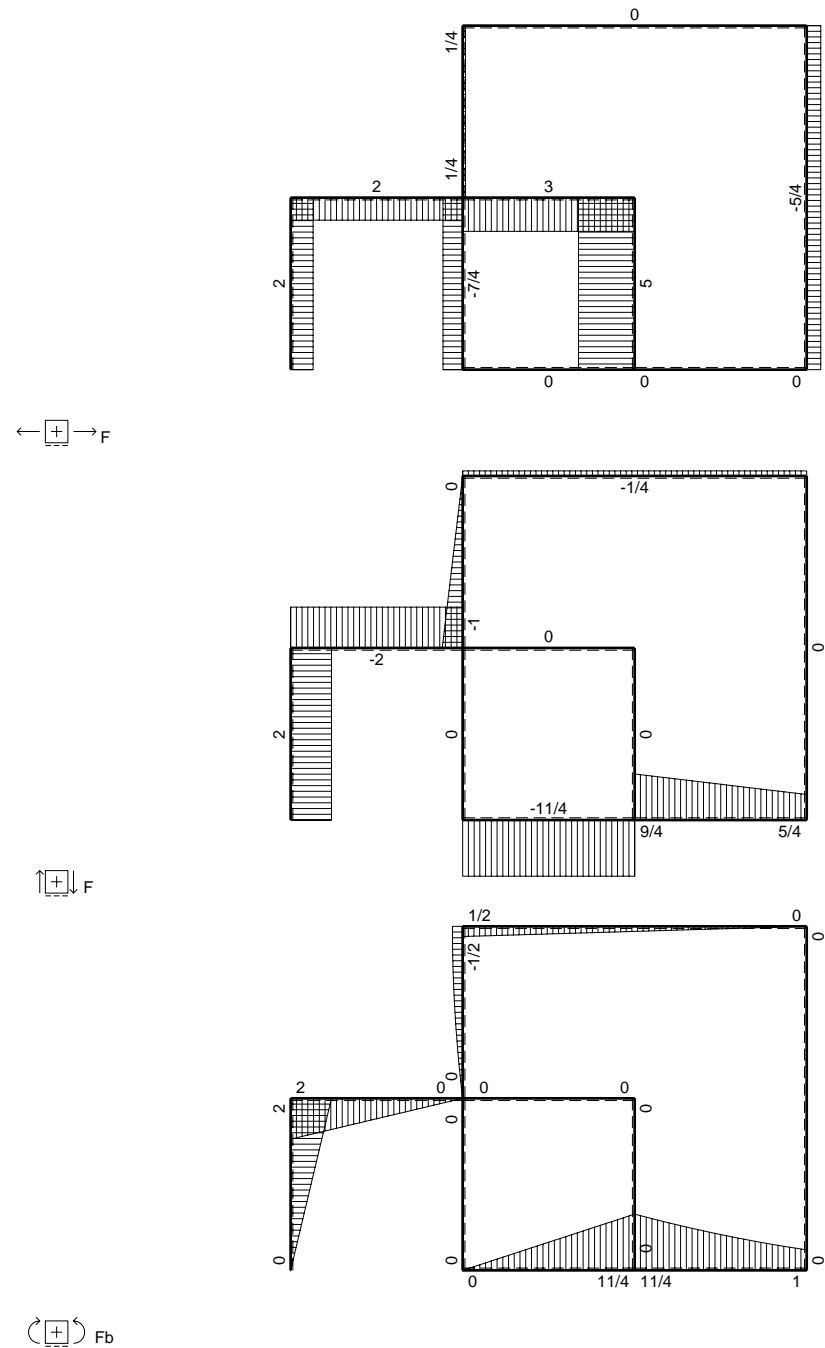
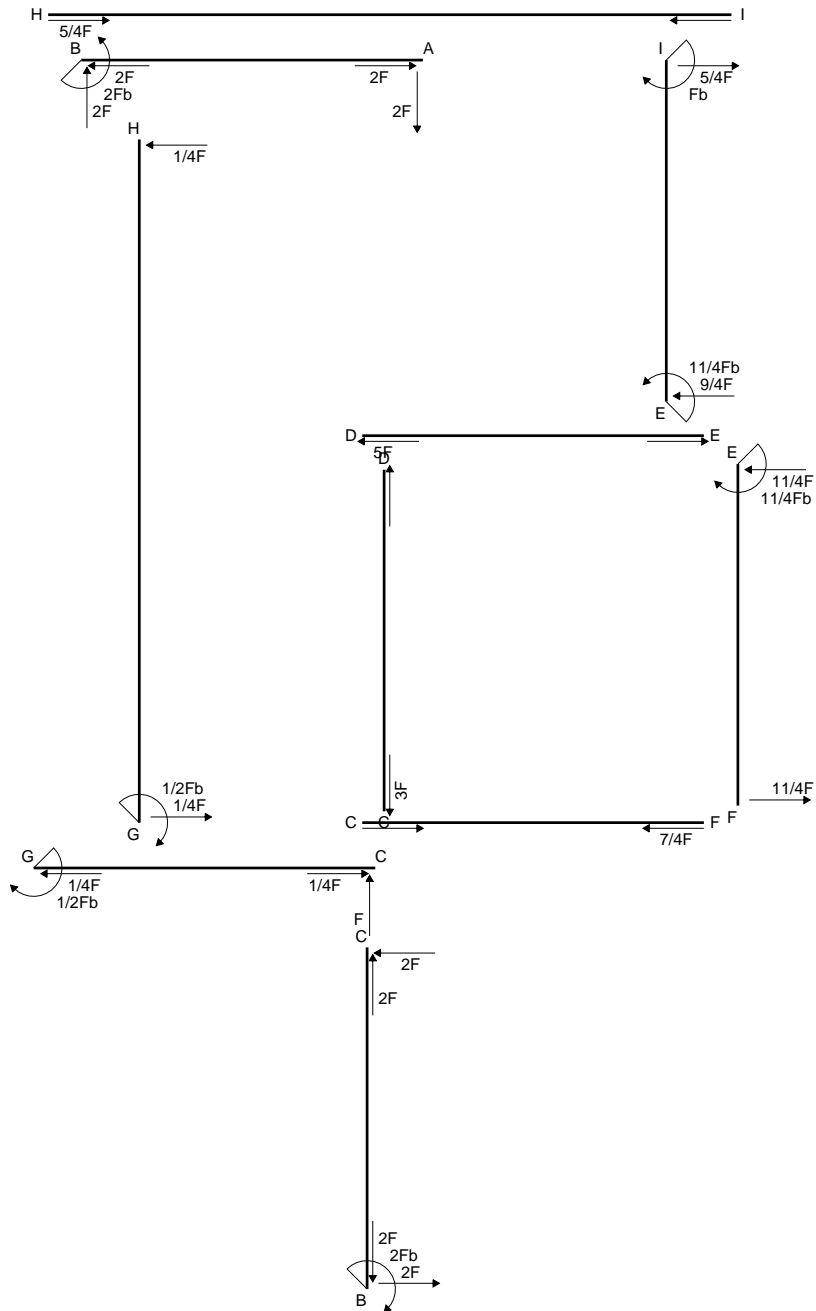
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

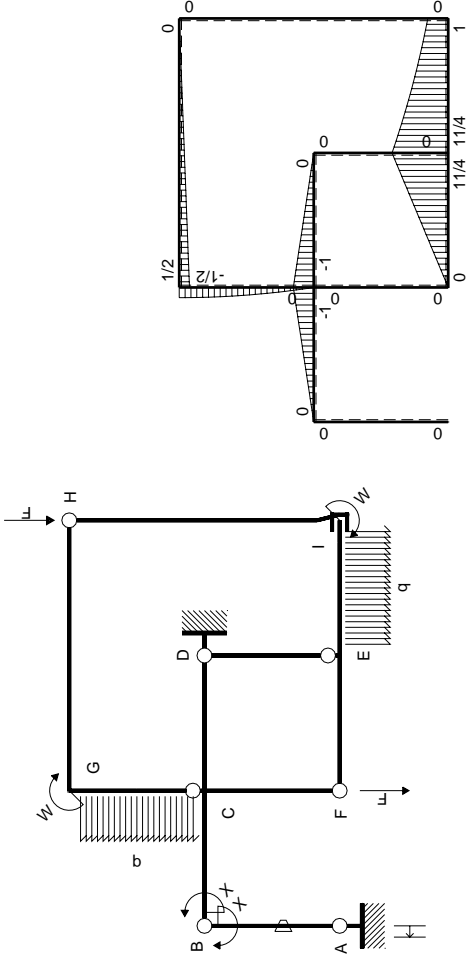
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | 0        | $Fx-1/2Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | 0        | $1/2Fb-1/2Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                    | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                         |          |                      |               |                         | $2Fb^2/EJ$                  | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |               |                         | $-2Fb$                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

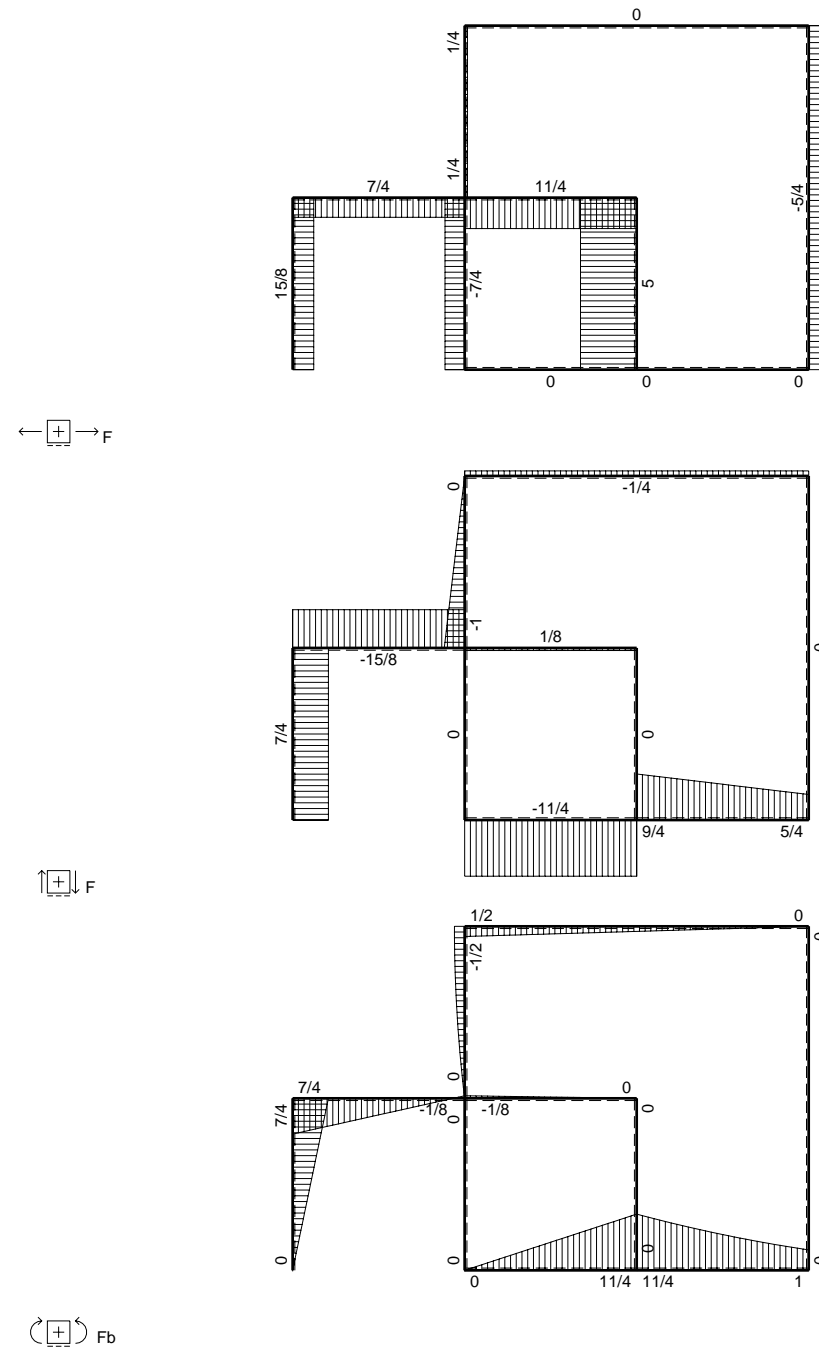
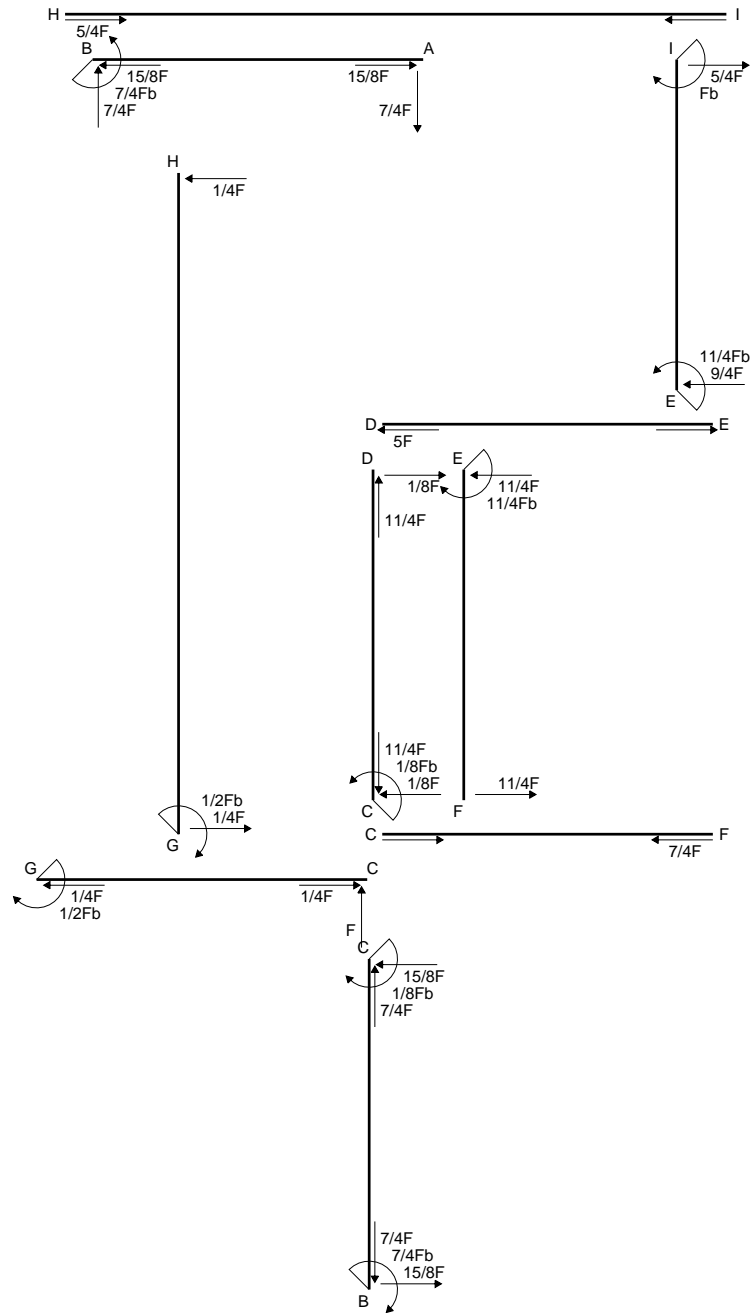
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

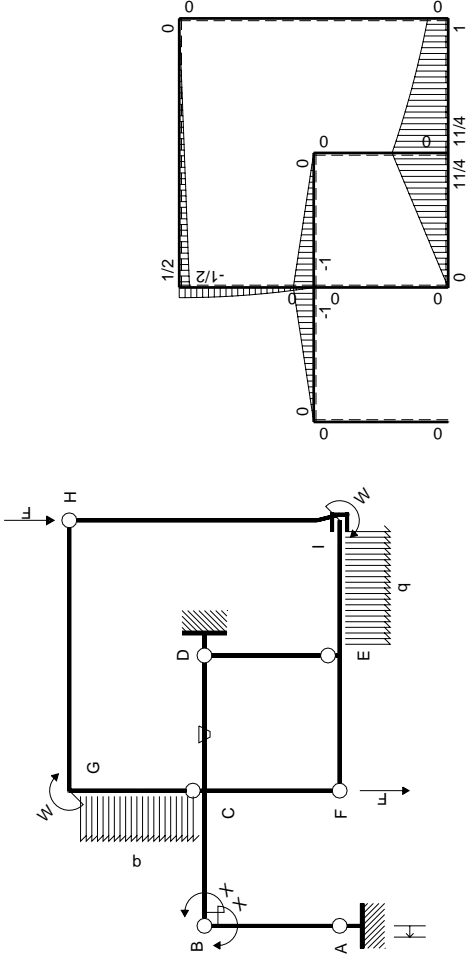
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                    | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                         |          |                      |                     |                         | $7/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         | $-7/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

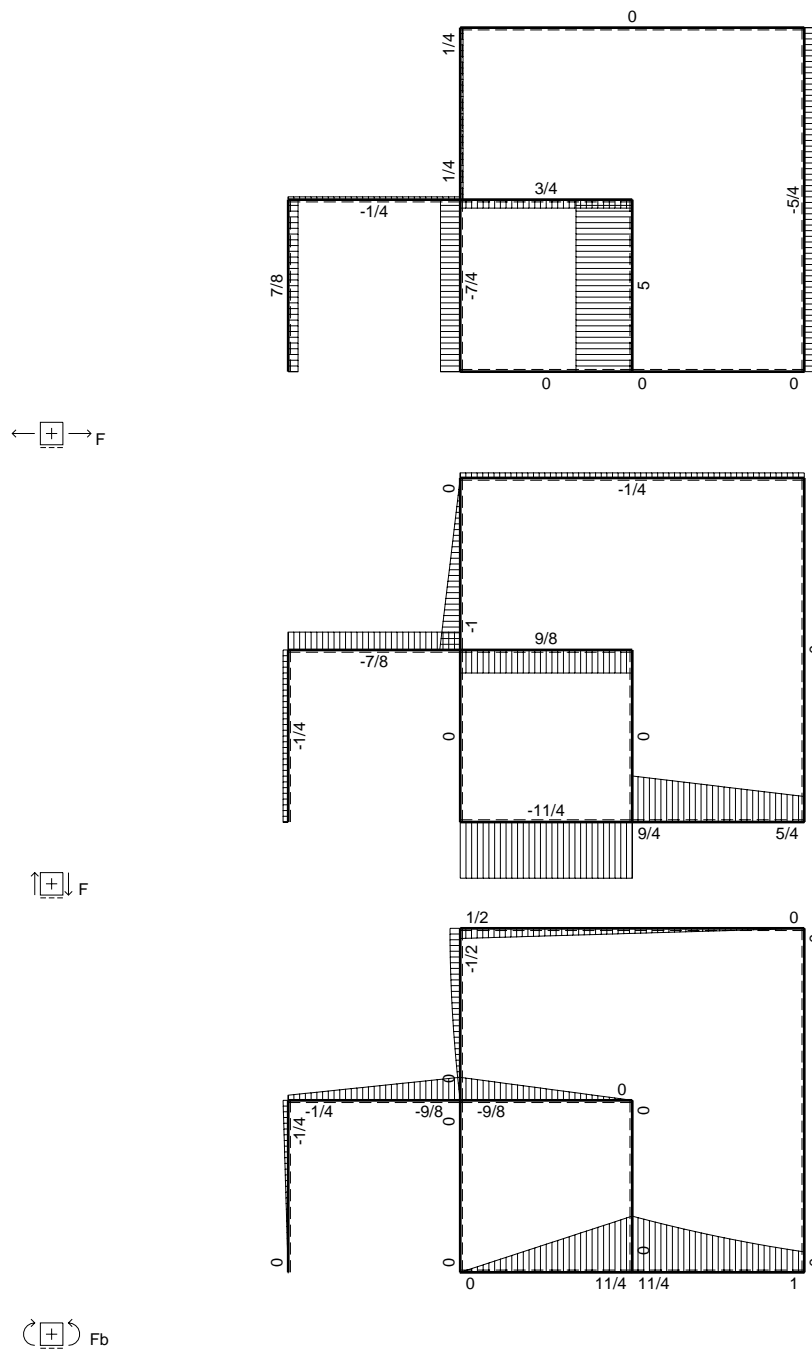
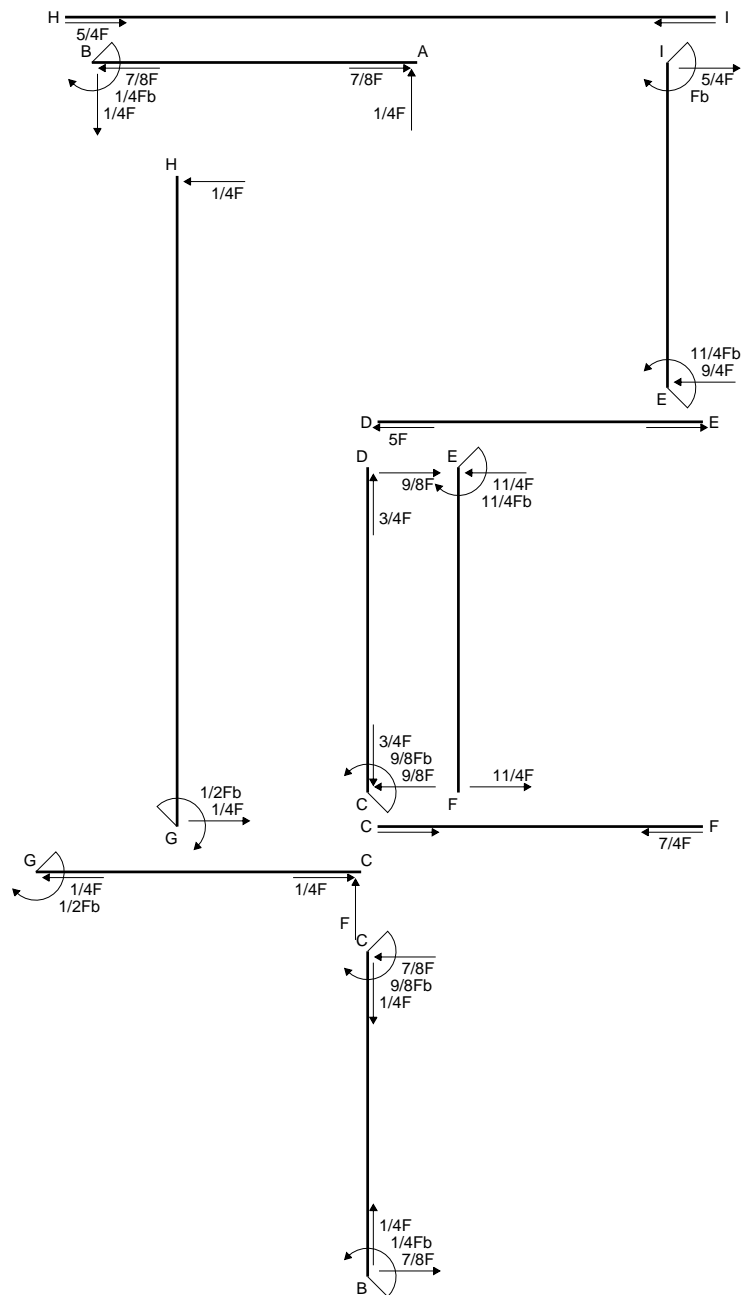
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

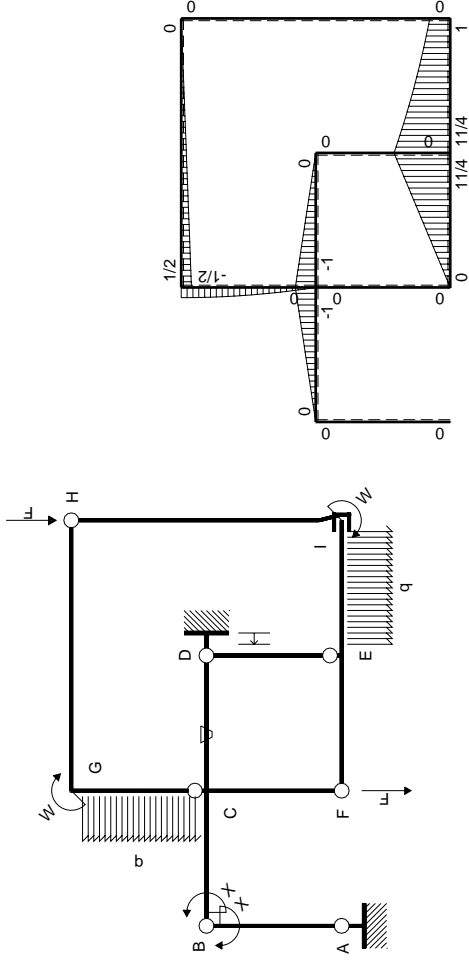
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                    | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                         |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                         |          |                      |                     |                         |                             | $-1/4Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         |                             | $1/4Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

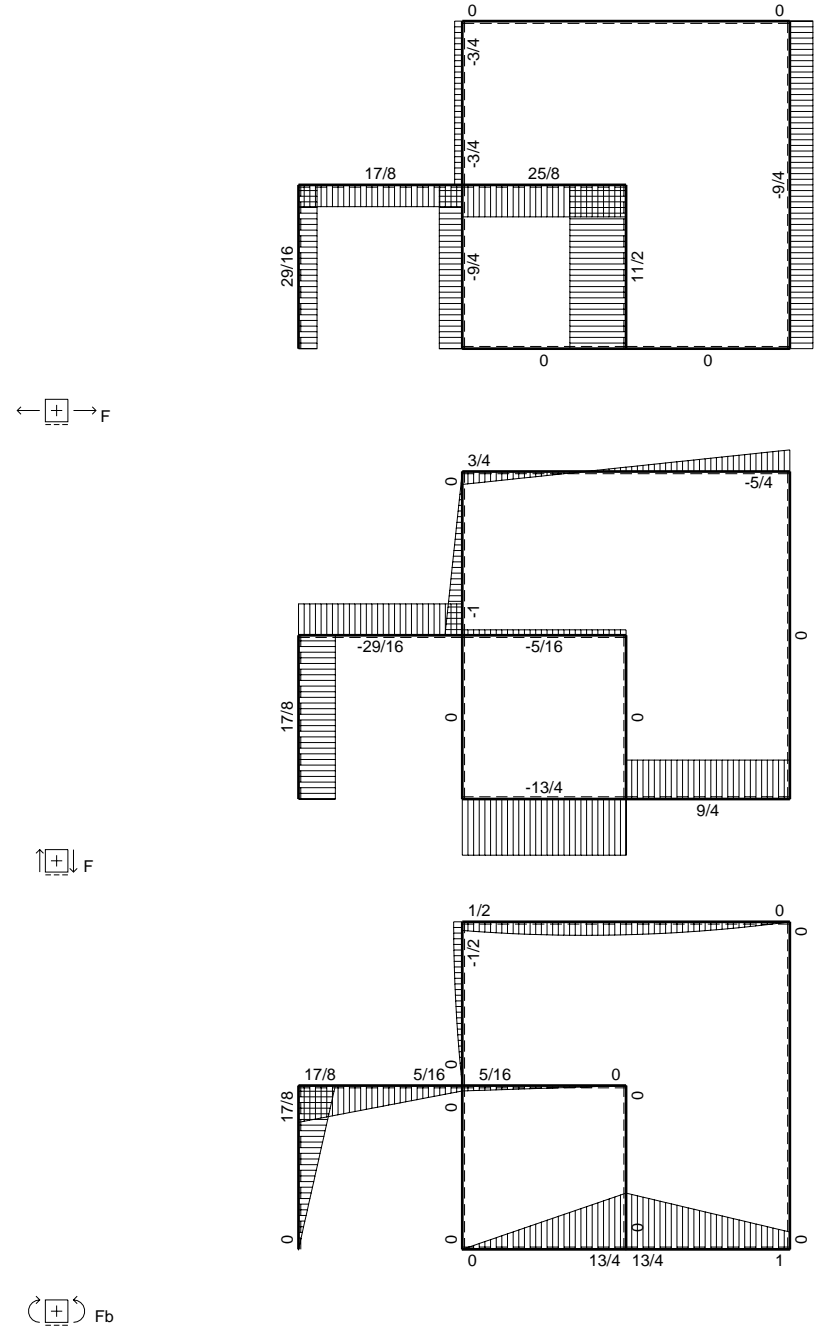
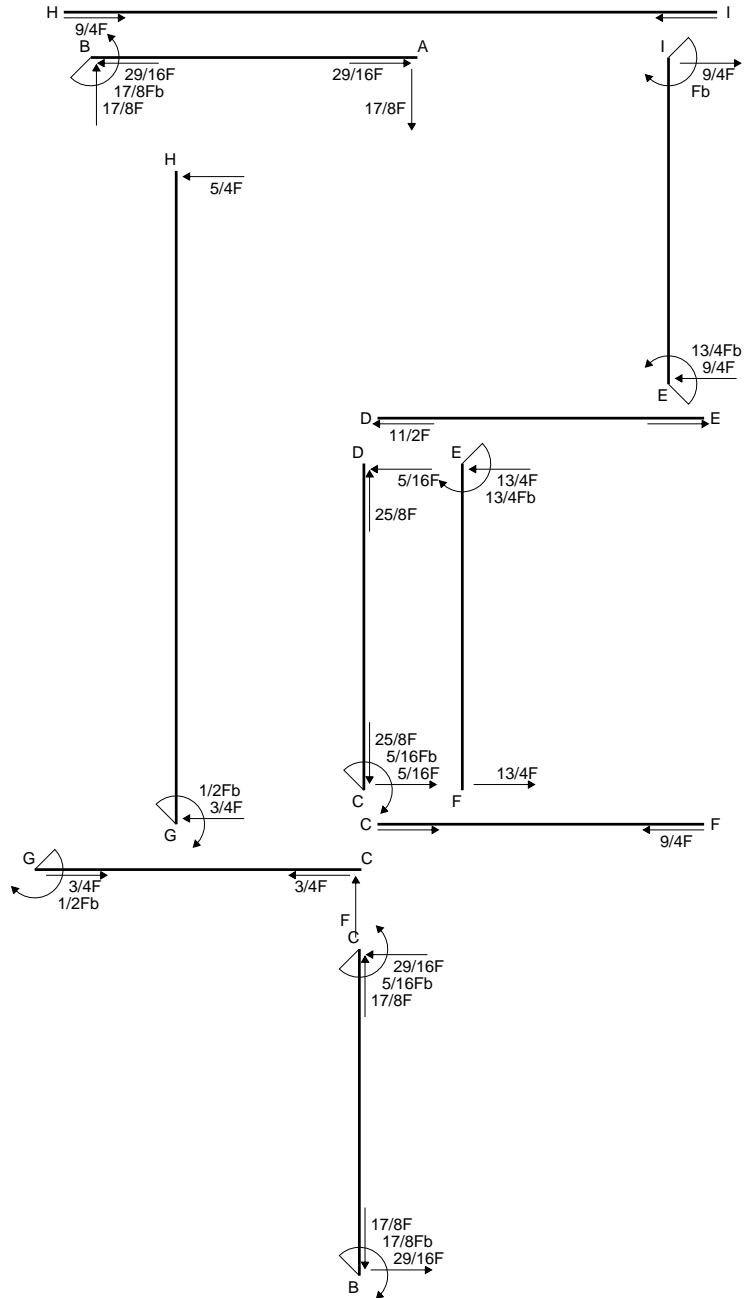
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

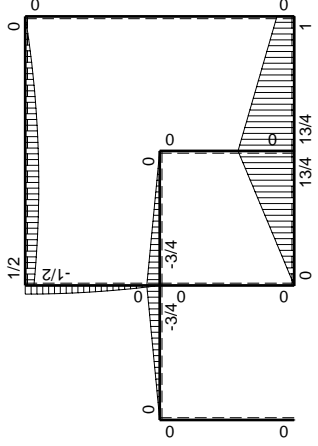
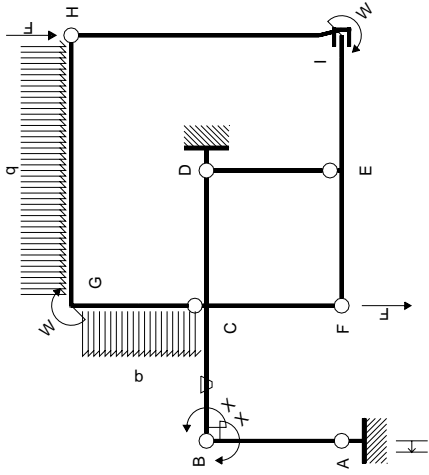
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

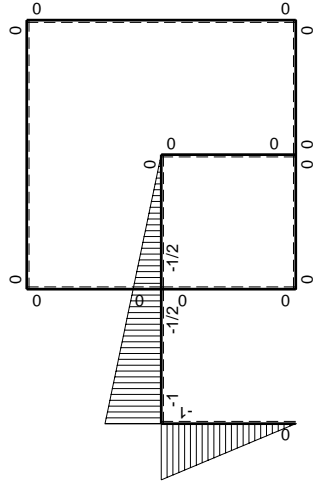
$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                     | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                     | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$              | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$         | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$        | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$               | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                       |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                       |          |                         |                     |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                       |          |                         |                     |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

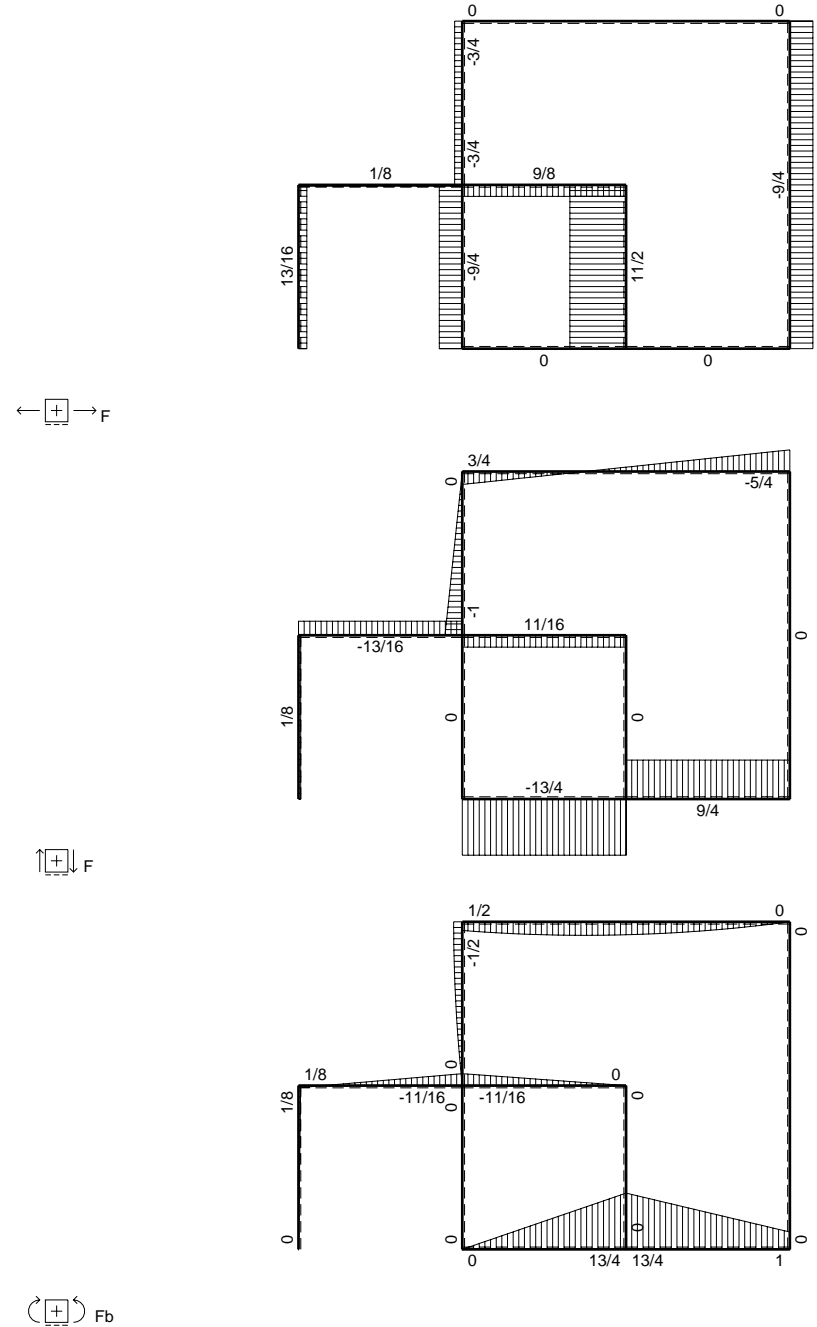
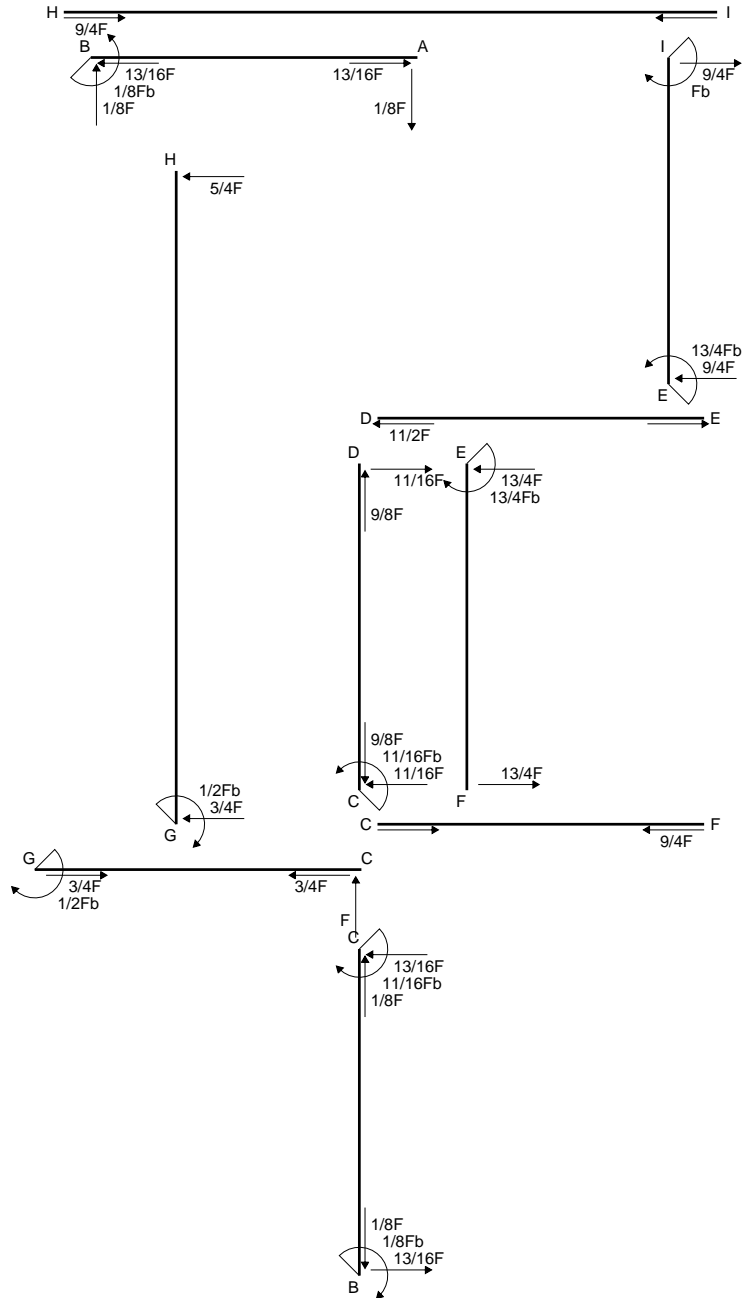
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

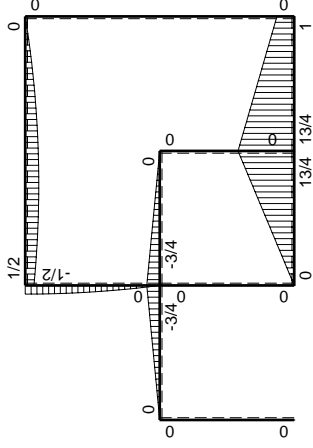
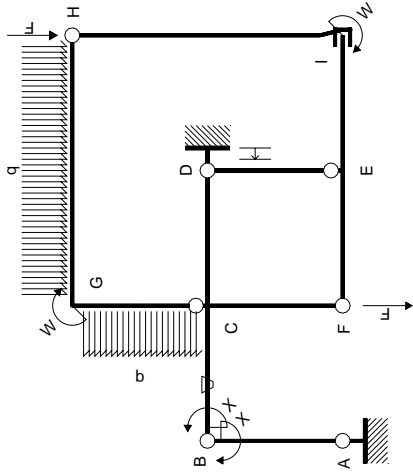
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

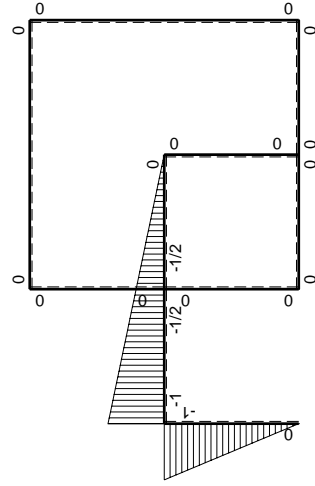






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|-----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | 0                     | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | 0                     | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$              | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$         | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$        | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $3/4Fx$               | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| FC b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| CF b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| HI 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                       |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |  |
|       | totali                      |                       |          |                         |                     |                         | $1/8Fb^2/EJ$                | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                       |          |                         |                     |                         | $-1/8Fb$                    |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

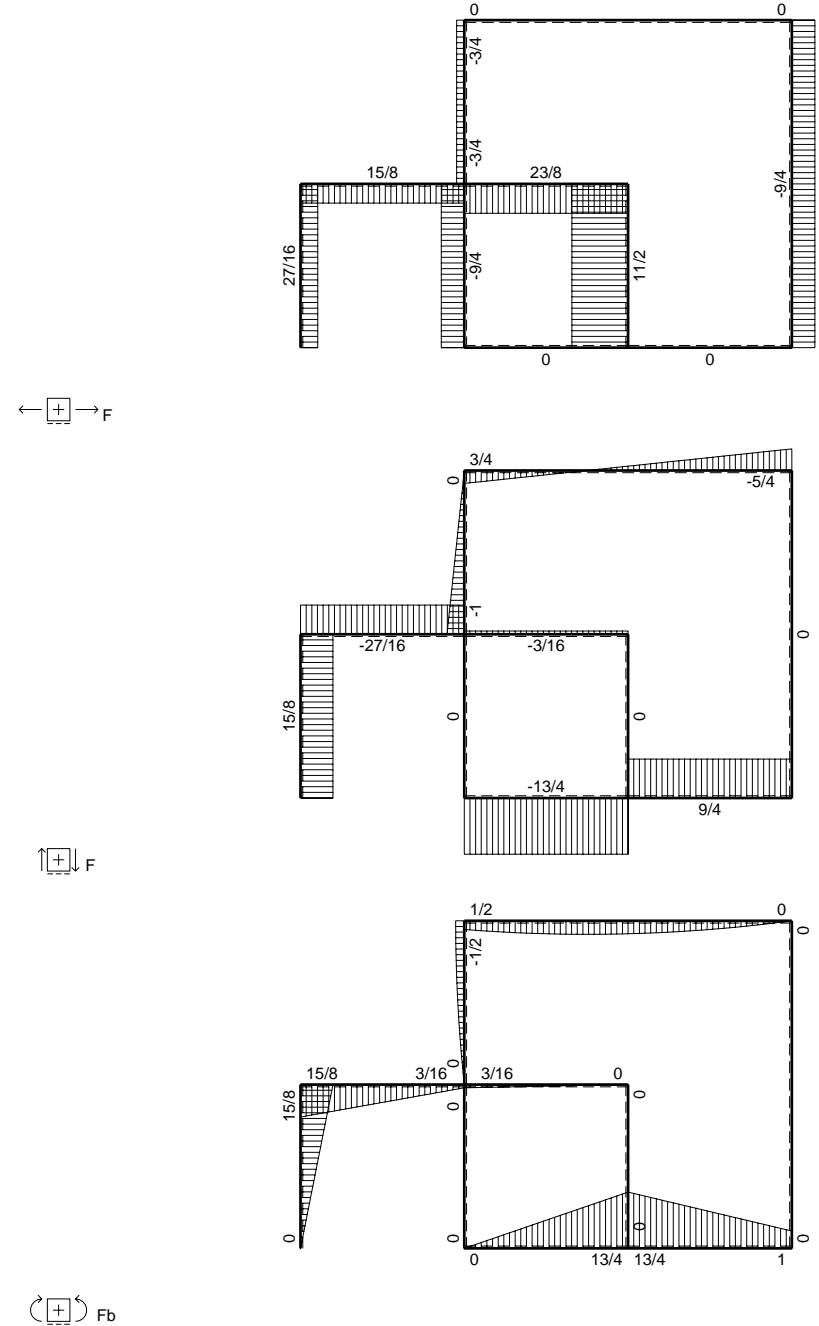
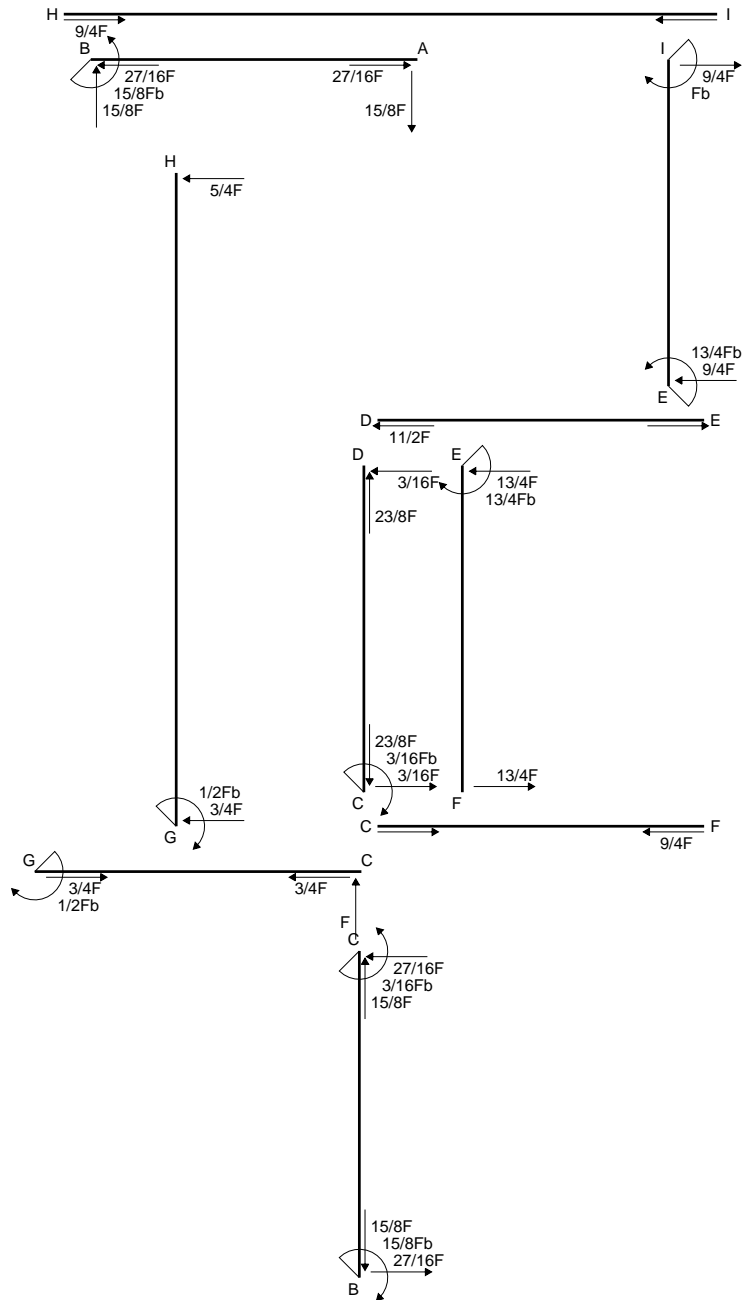
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

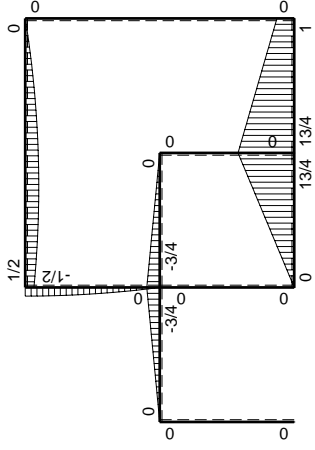
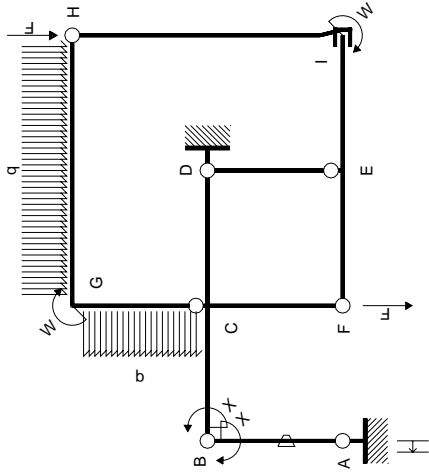
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

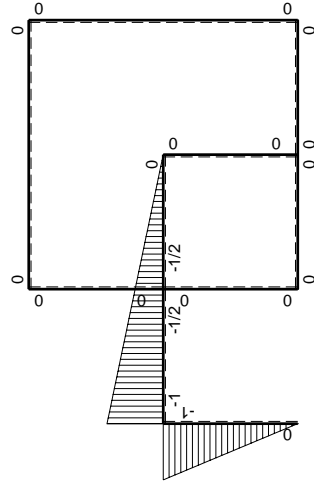
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                     | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                     | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$              | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$         | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$        | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$               | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                     | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                       |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                       |          |                         |               |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                       |          |                         |               |                         | $-15/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

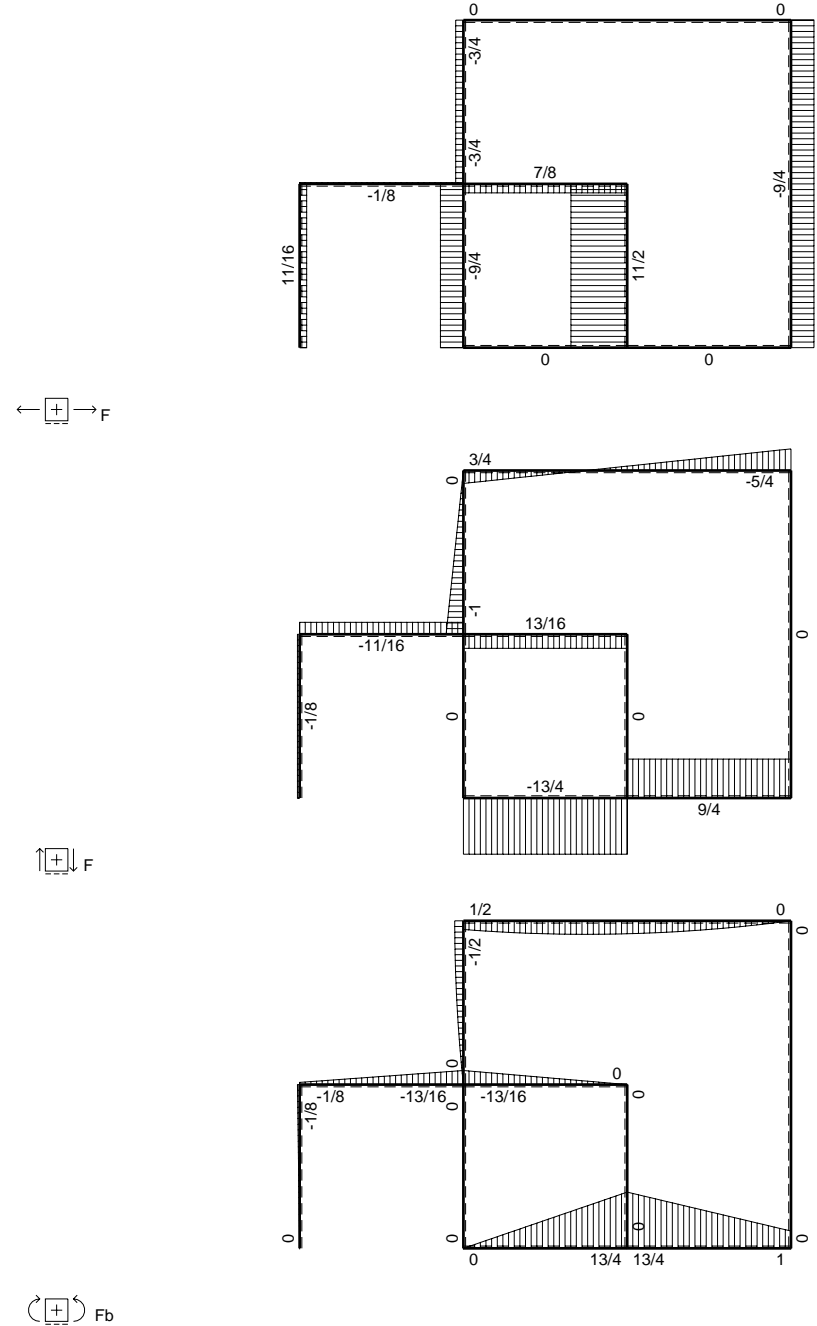
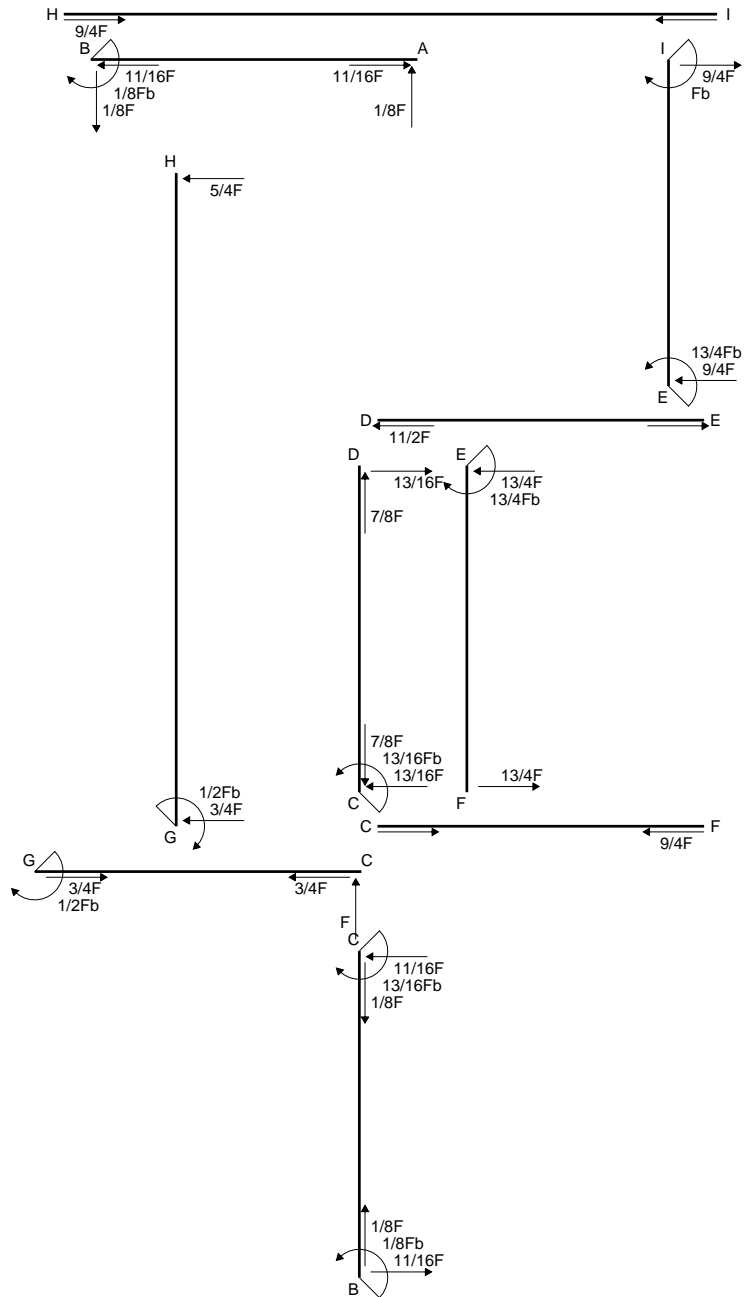
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

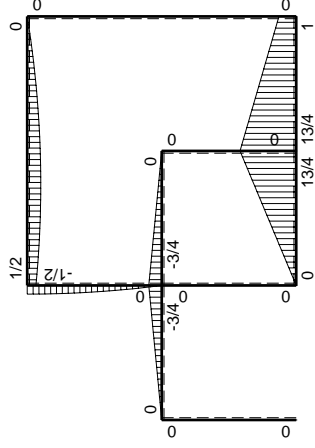
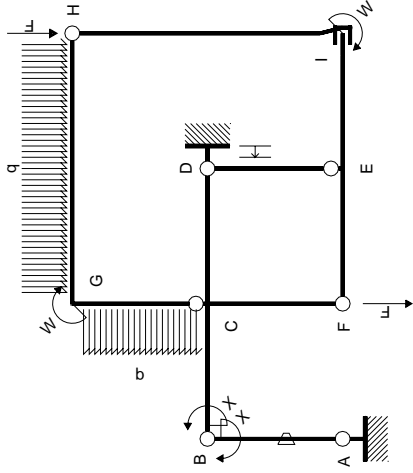
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

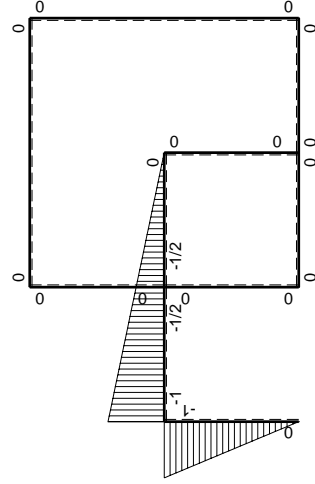






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                     | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                     | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$              | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$         | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$        | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$               | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                     | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                     | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                       |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                       |          |                         |               |                         | $-1/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                       |          |                         |               |                         | $1/8Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

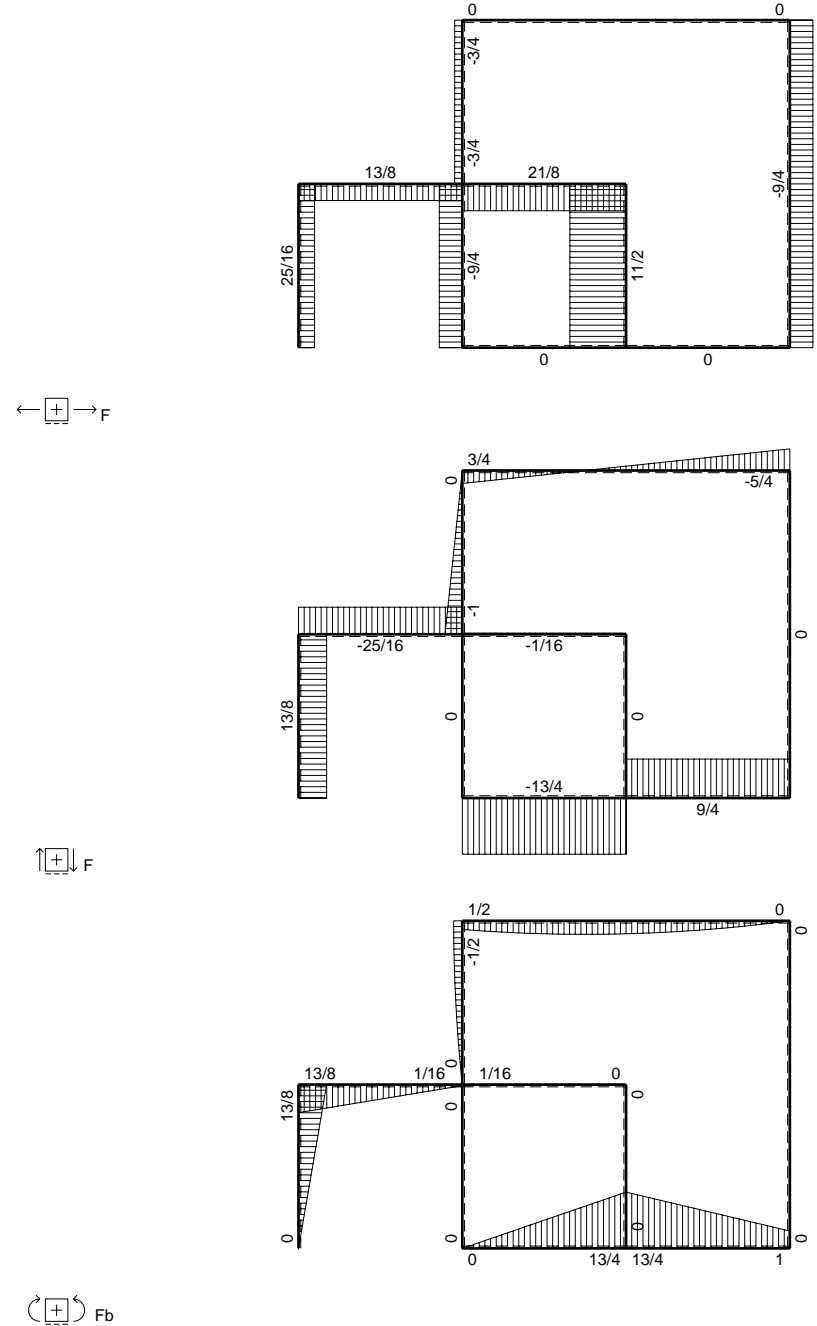
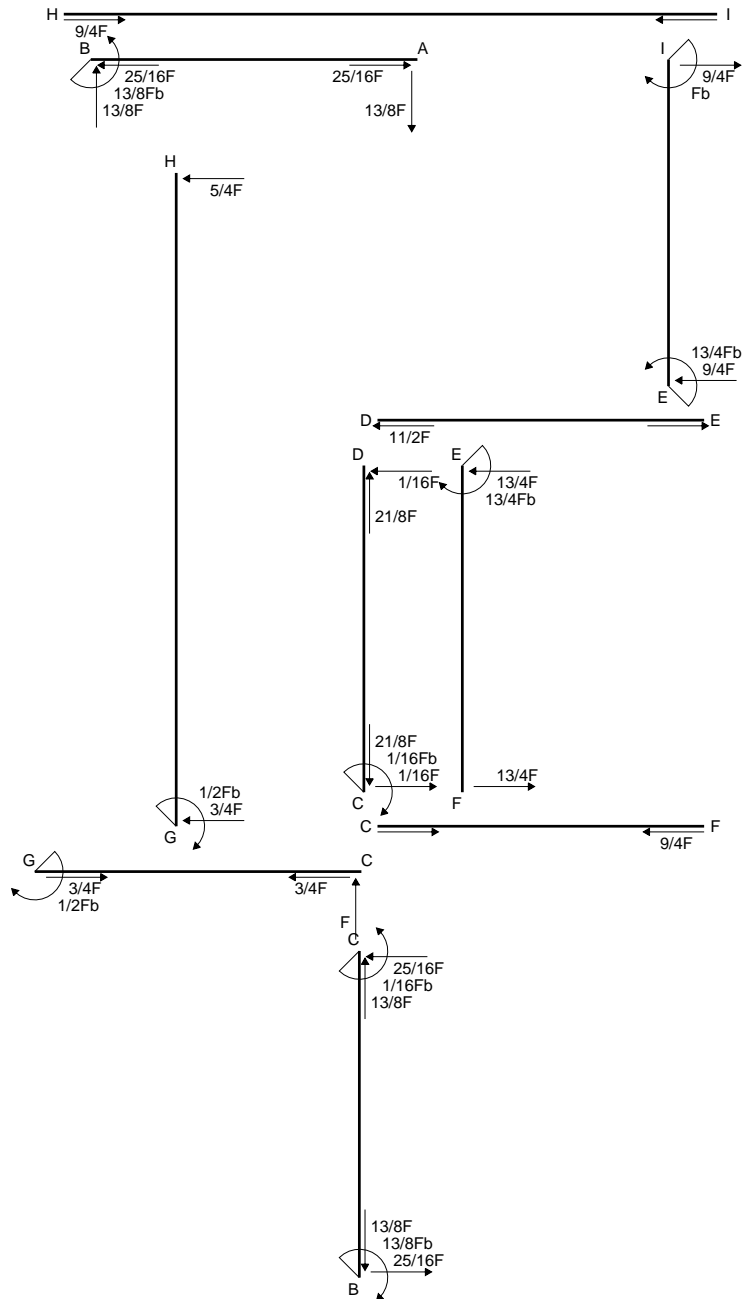
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

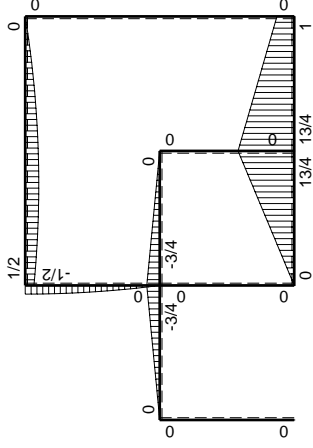
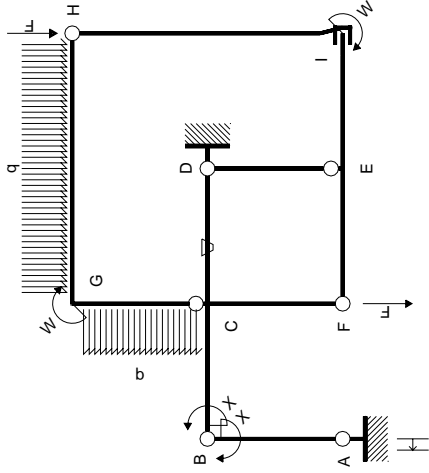
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

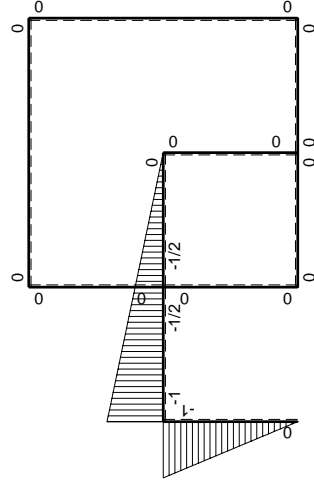
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|-----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|-----------------------|
| AB b  | $-x/b$                      | 0                     | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$            |
| BA b  | $1-x/b$                     | 0                     | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                       |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$              | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$           |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$         | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$        | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$           |
| DC b  | $1/2x/b$                    | $3/4Fx$               | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                       |
| DE b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| ED b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                       |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                       |
| FC b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| CF b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                       |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                       |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                       |
| HI 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| IH 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                       |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                       | 0                   | 0                       |                             |                       |
| A     | cedimento nodo $-H_{1A}u_A$ |                       |          |                         |                     |                         | $Fb^2/EJ$                   |                       |
|       | totali                      |                       |          |                         |                     |                         | $13/8Fb^2/EJ$               | $Xb/EJ$               |
|       | iperstatica $X=W_{BC}$      |                       |          |                         |                     |                         | $-13/8Fb$                   |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

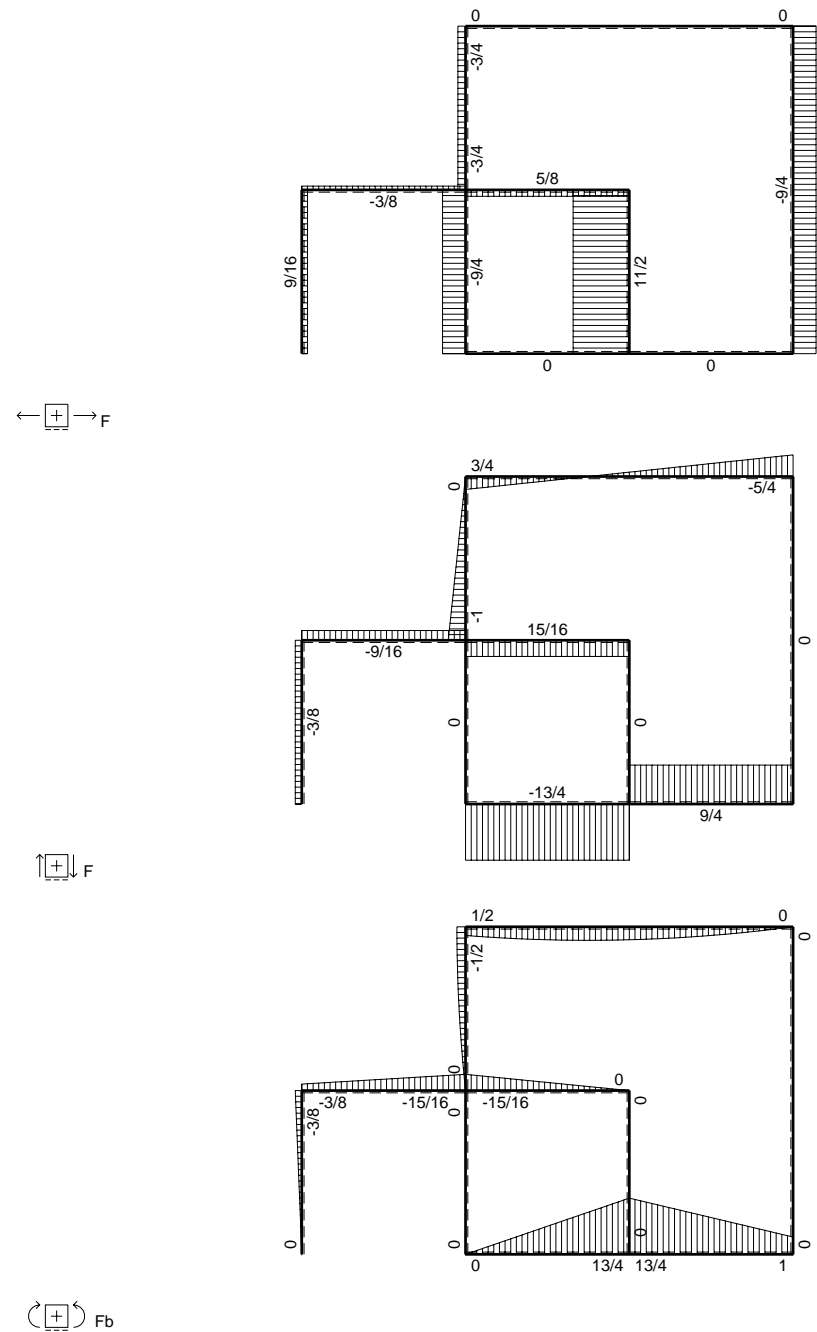
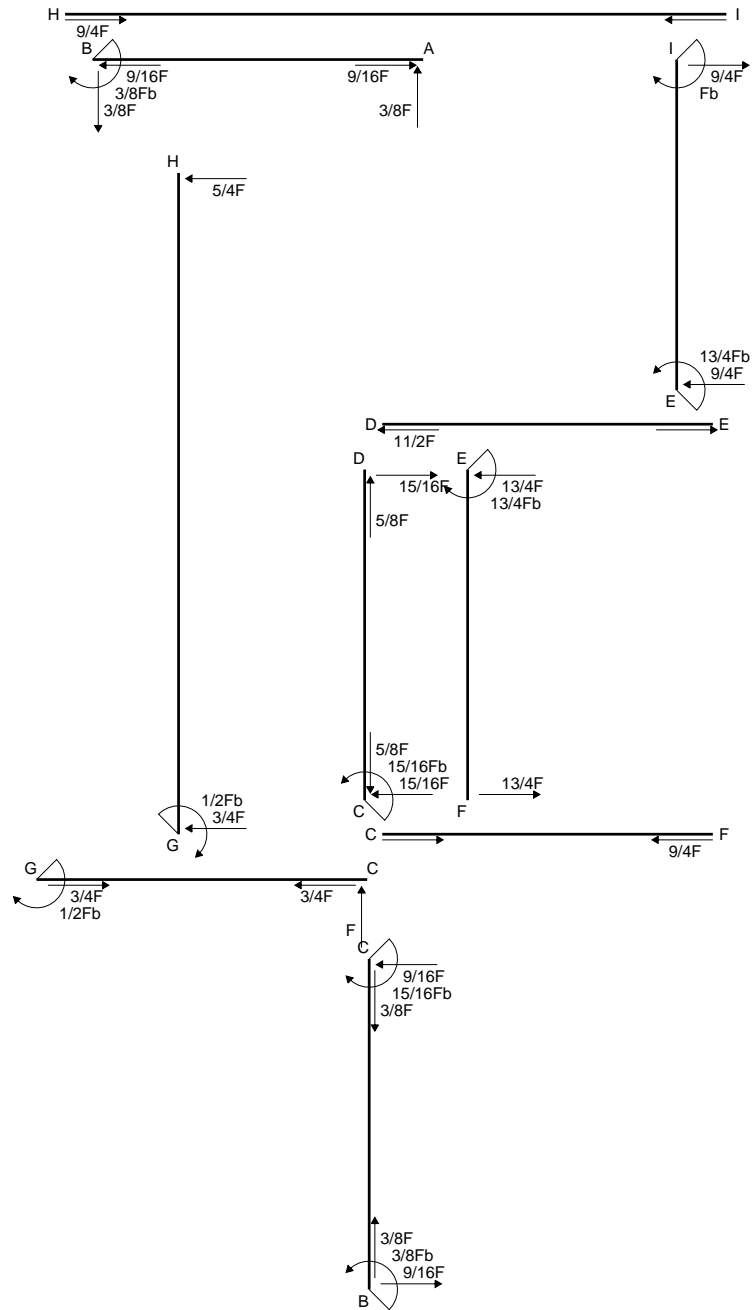
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

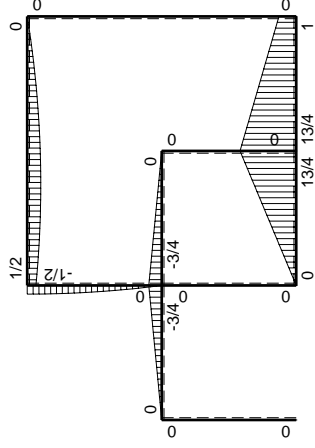
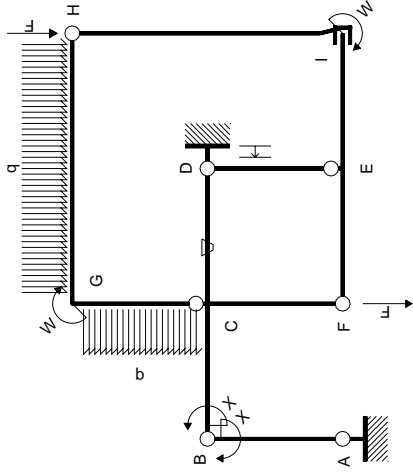
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

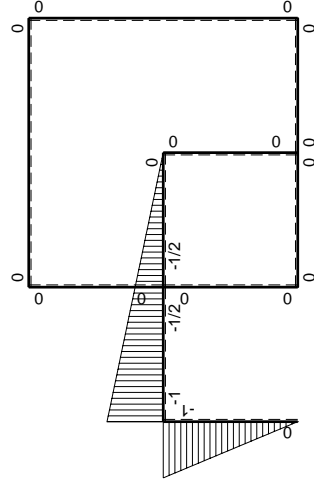






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$              | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|-----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|-----------------------|
| AB b  | $-x/b$                      | 0                     | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$            |
| BA b  | $1-x/b$                     | 0                     | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                       |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$              | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$           |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$         | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$        | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$           |
| DC b  | $1/2x/b$                    | $3/4Fx$               | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                       |
| DE b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| ED b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                       |
| EF b  | 0                           | $13/4Fb-13/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| FE b  | 0                           | $-13/4Fx$             | 0        | 0                       | 0                   | 0                       |                             |                       |
| FC b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| CF b  | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                       |
| CG b  | 0                           | $-Fx+1/2qx^2$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| GC b  | 0                           | $1/2Fb-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                       |
| GH 2b | 0                           | $1/2Fb+3/4Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| HG 2b | 0                           | $-5/4Fx+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                       |
| HI 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| IH 2b | 0                           | 0                     | 0        | 0                       | 0                   | 0                       |                             |                       |
| IE b  | 0                           | $Fb+9/4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                     |
| EI b  | 0                           | $-13/4Fb+9/4Fx$       | 0        | 0                       | 0                   | 0                       |                             |                       |
| D     | cedimento nodo $-H_{1D}u_D$ |                       |          |                         |                     |                         | $-Fb^2/EJ$                  |                       |
|       | totali                      |                       |          |                         |                     |                         | $-3/8Fb^2/EJ$               | $Xb/EJ$               |
|       | iperstatica $X=W_{BC}$      |                       |          |                         |                     |                         | $3/8Fb$                     |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

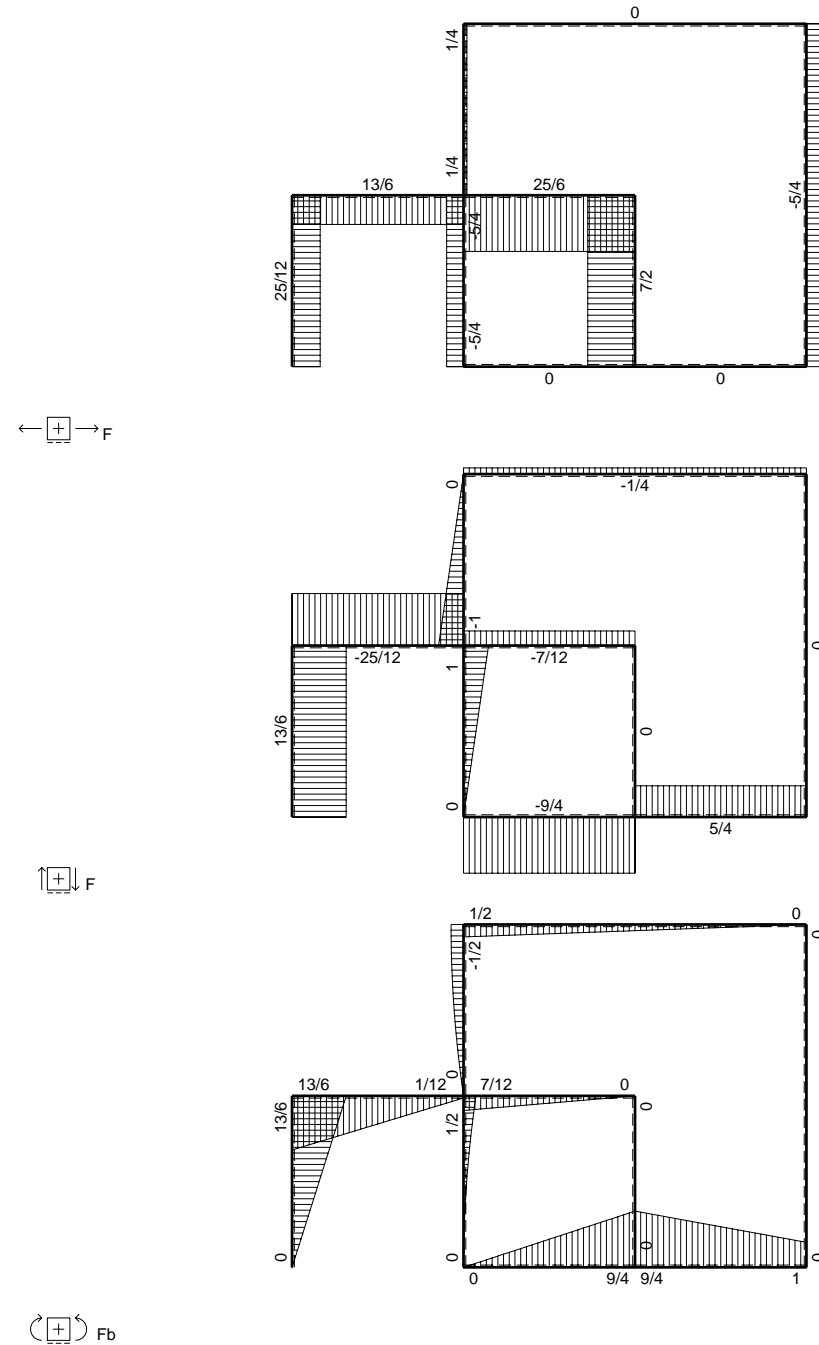
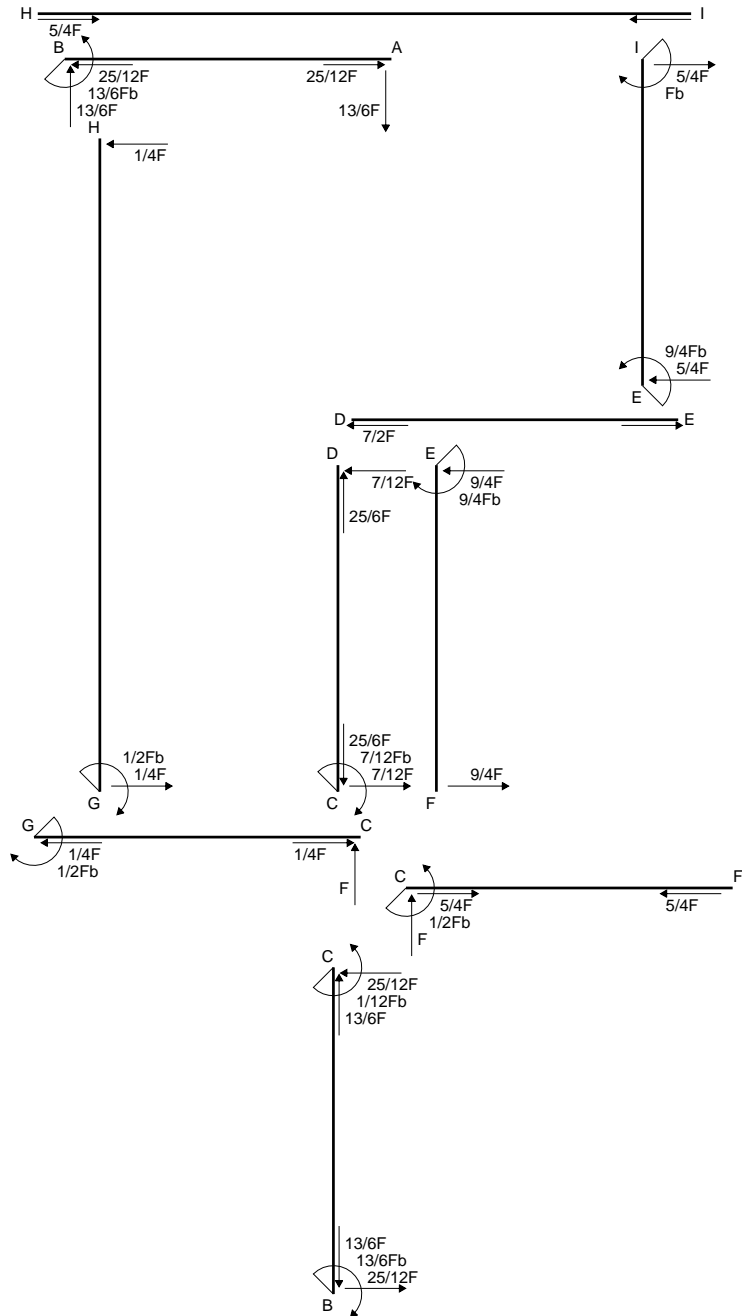
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

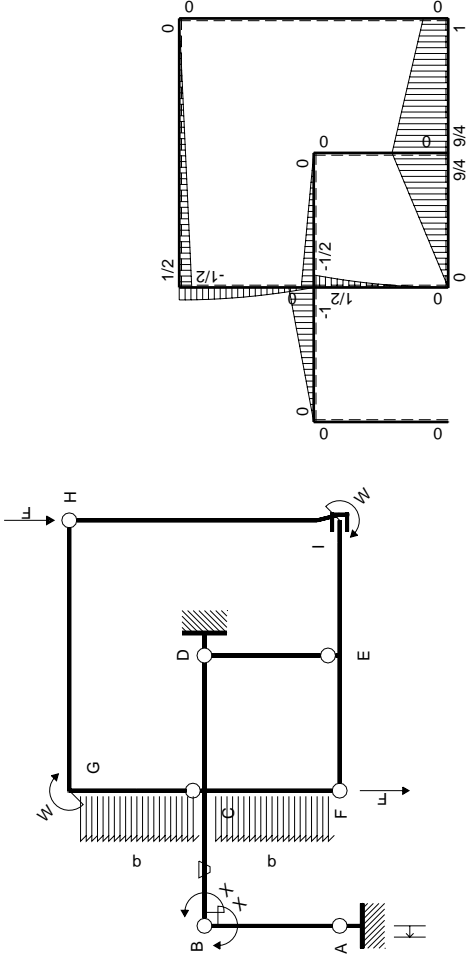
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

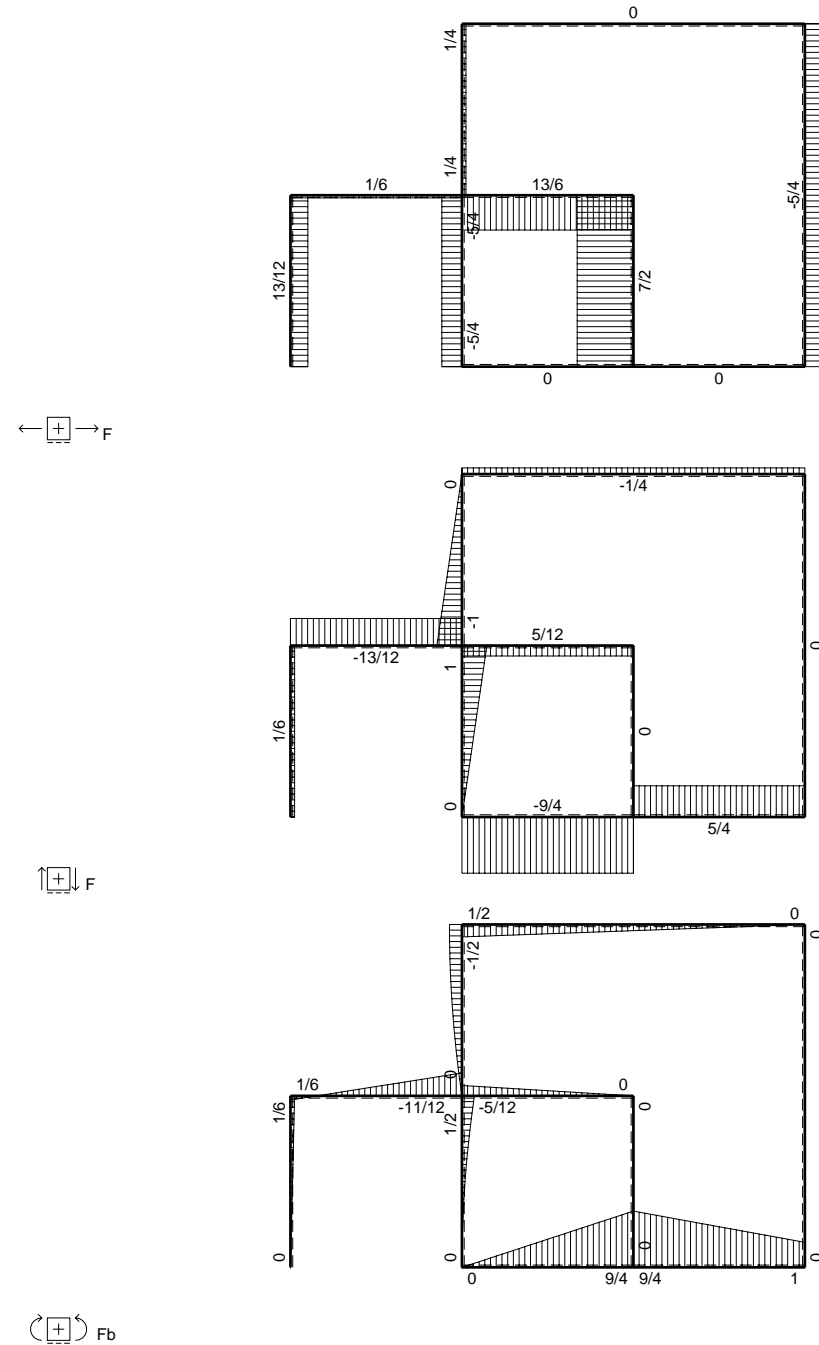
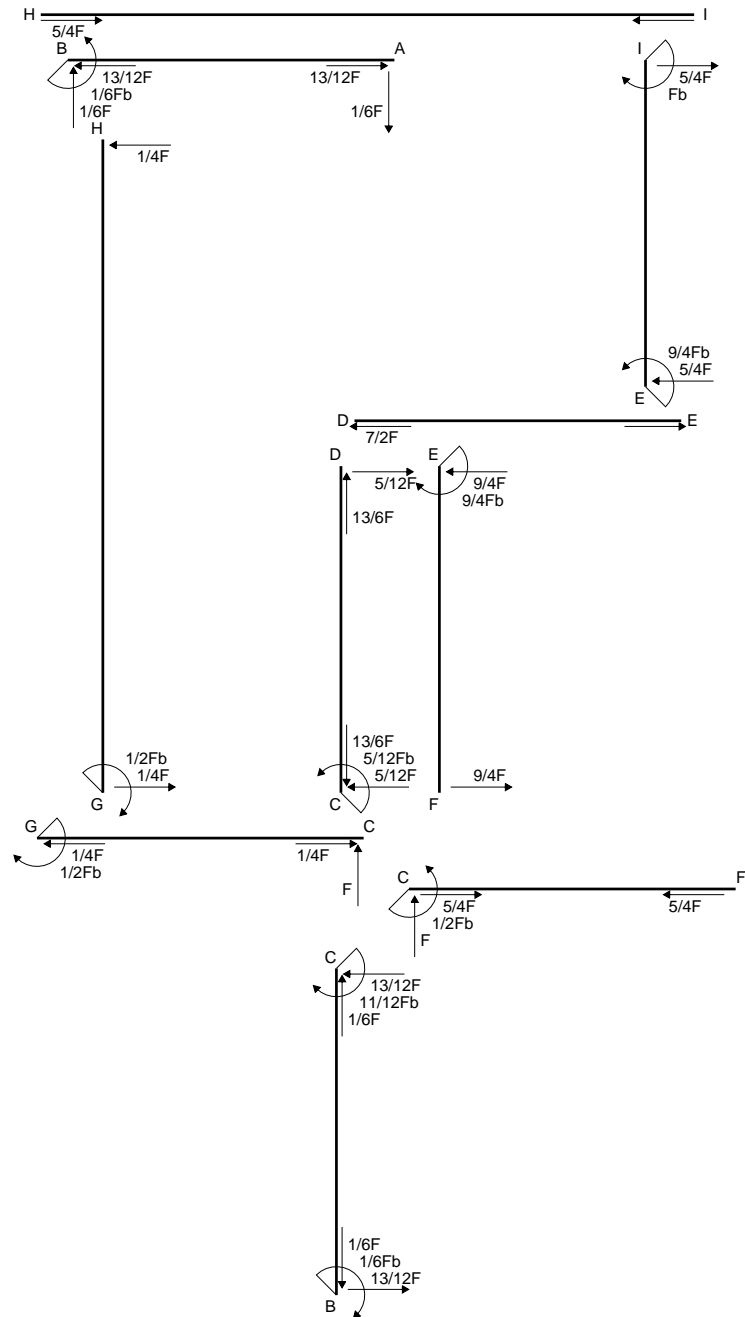
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

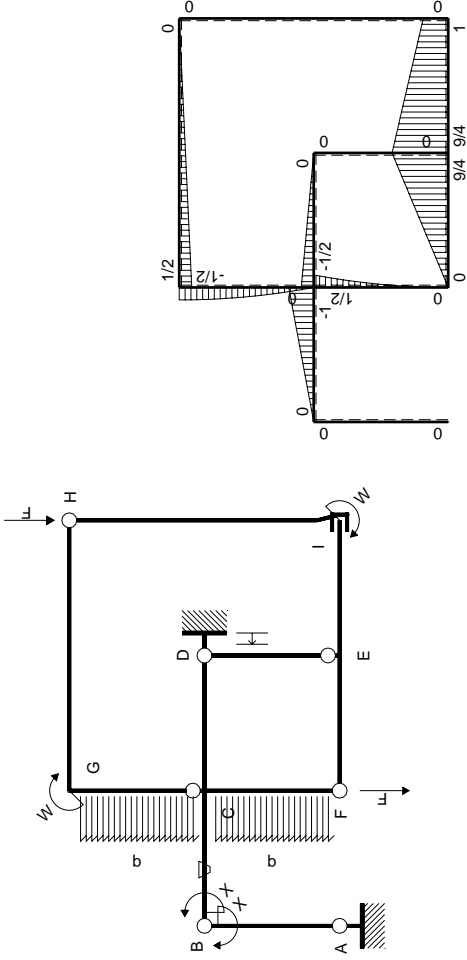
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





⊕ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

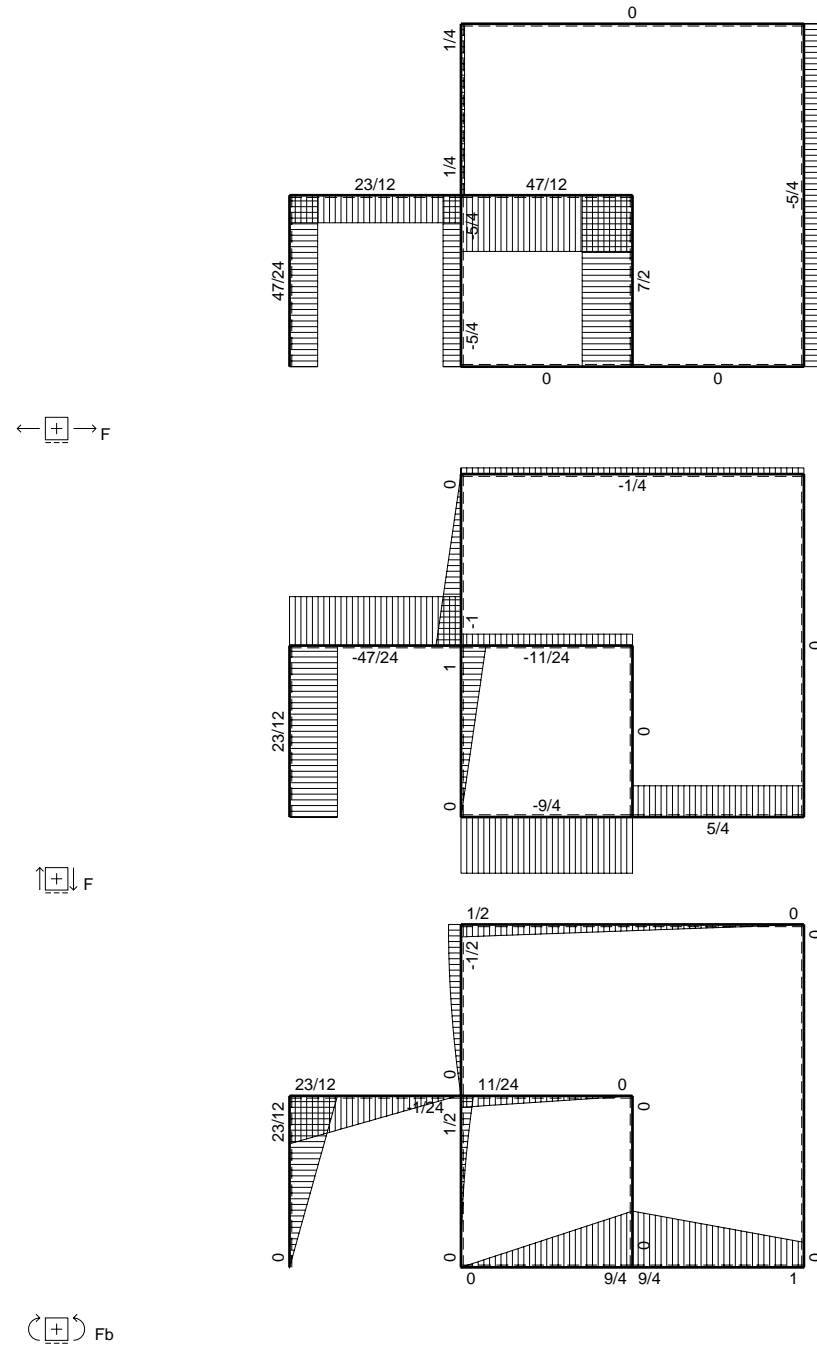
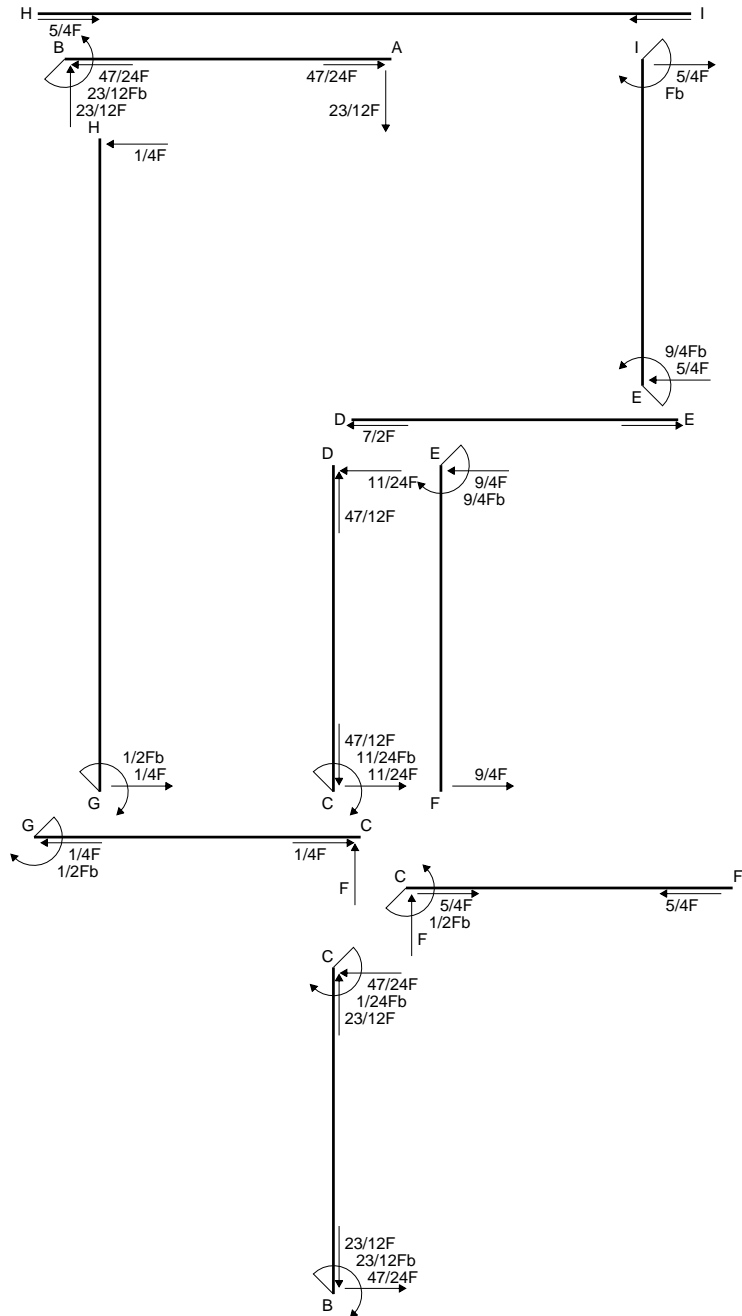
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

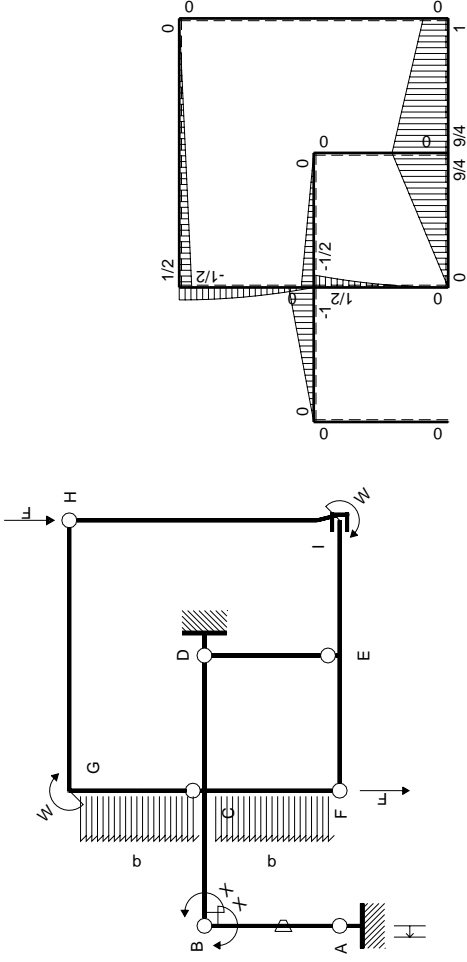
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$ | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|--------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | -x/b                        | 0                   | -Fb/EJ   | 0                       | Fx/EJ        | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | 1-x/b                       | 0                   | Fb/EJ    | 0                       | Fb/EJ-Fx/EJ  | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | -1+1/2x/b                   | -Fx                 | 0        | $Fx-1/2Fx^2/b$          | 0            | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | 1/2+1/2x/b                  | Fb-Fx               | 0        | $1/2Fb-1/2Fx^2/b$       | 0            | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | -1/2+1/2x/b                 | -1/2Fb+1/2Fx        | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0            | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | 1/2x/b                      | 1/2Fx               | 0        | $1/4Fx^2/b$             | 0            | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0            | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | -9/4Fx              | 0        | 0                       | 0            | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0            | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                       | 0            | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | -1/4Fx              | 0        | 0                       | 0            | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0            | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                       | 0            | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                       | 0            | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |              |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                     |          |                         |              |                         |                             | $23/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |              |                         |                             | $-23/12Fb$             |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

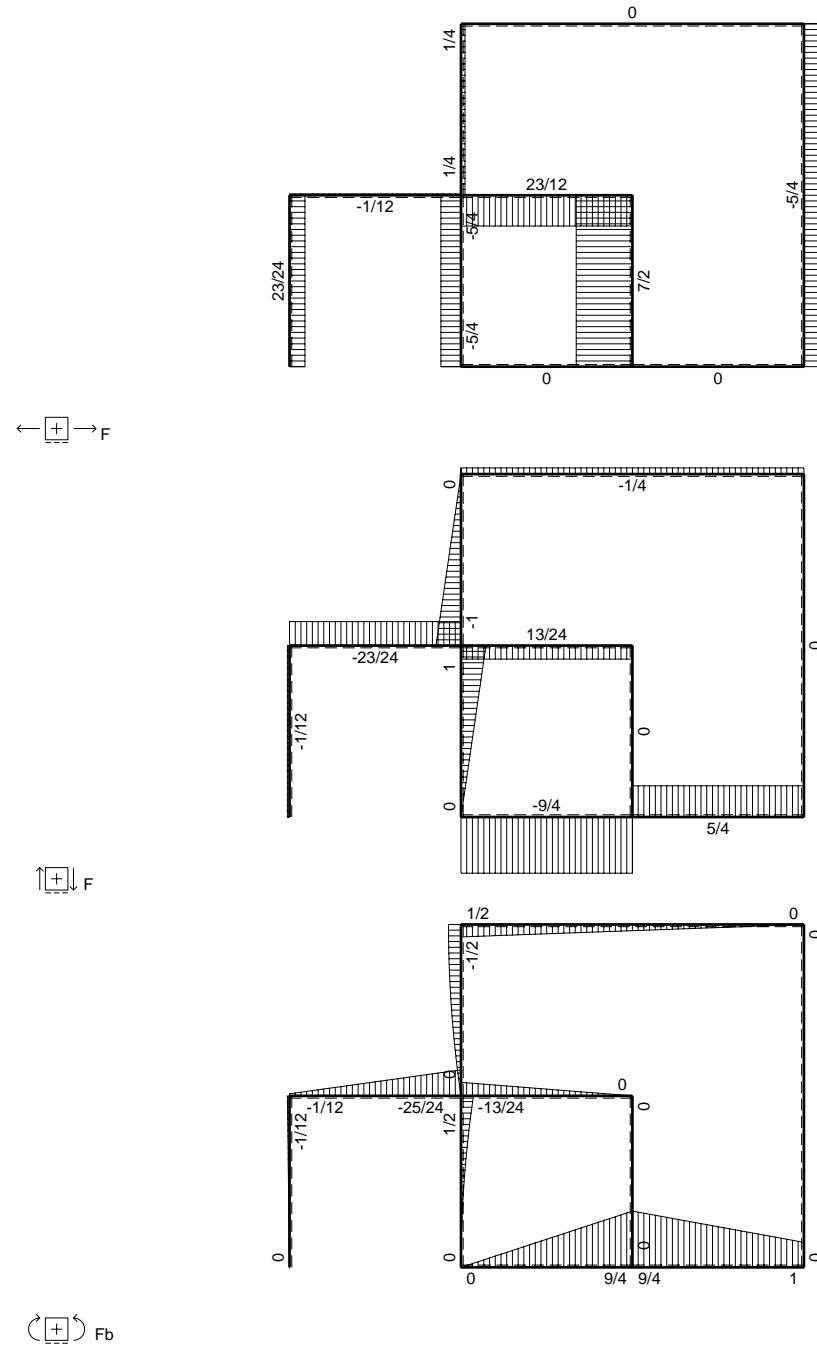
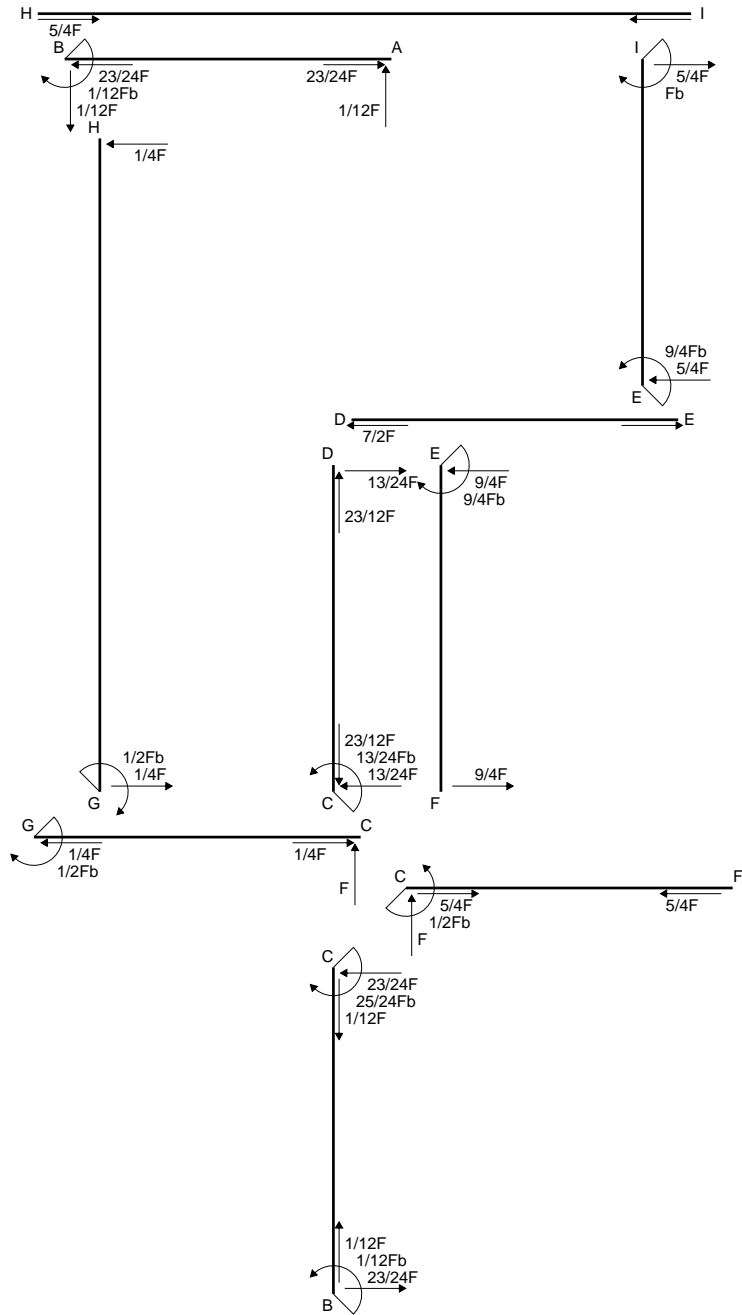
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

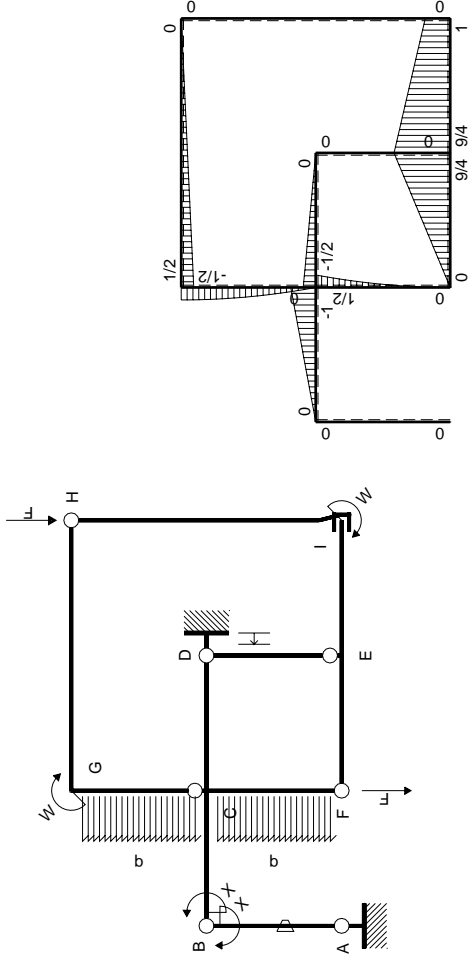
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |               |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

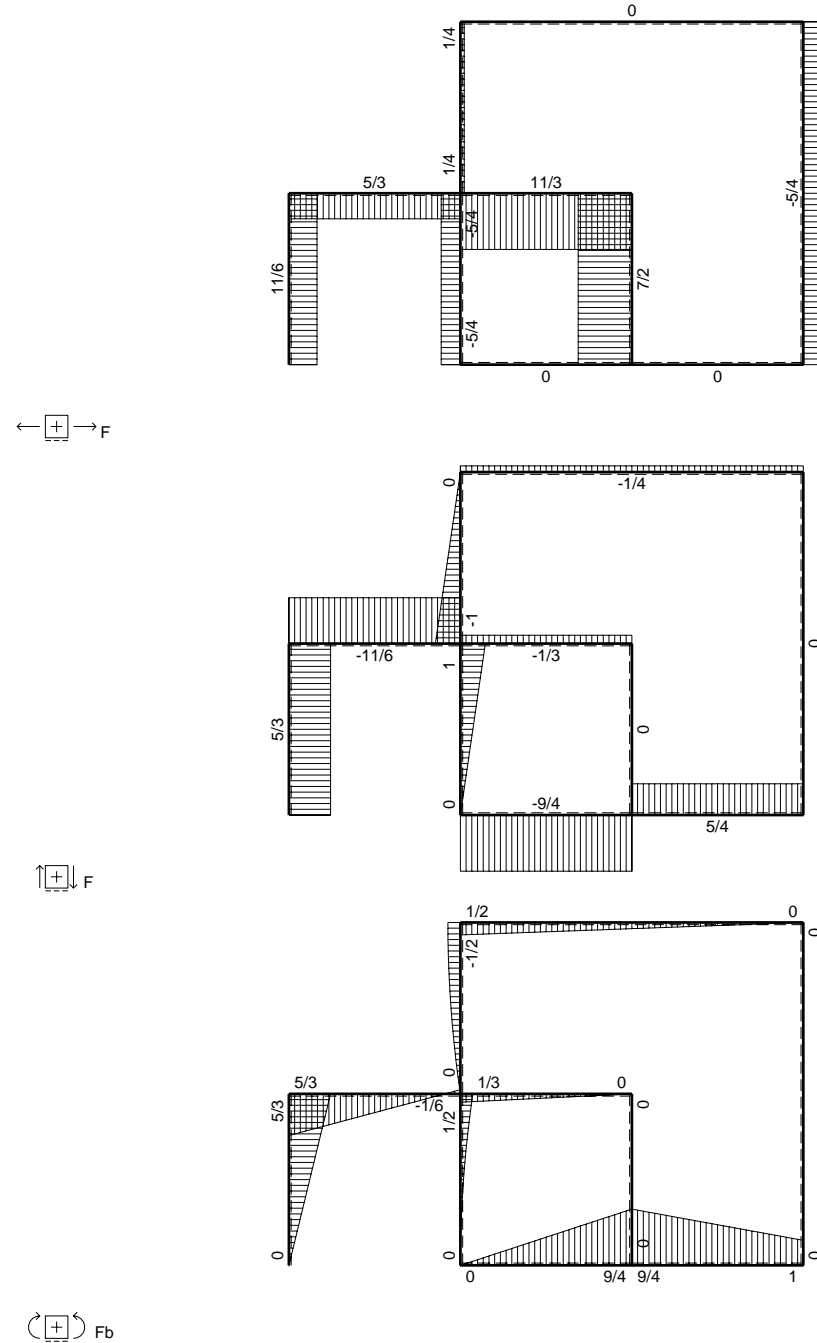
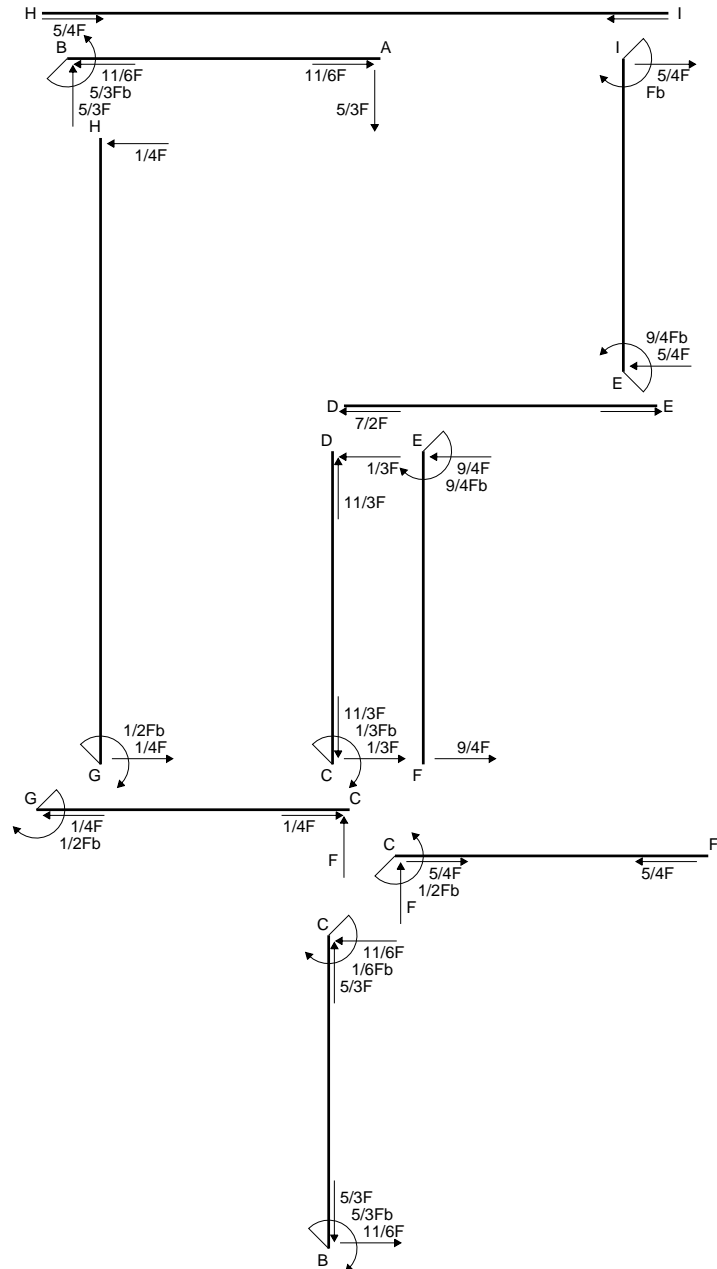
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

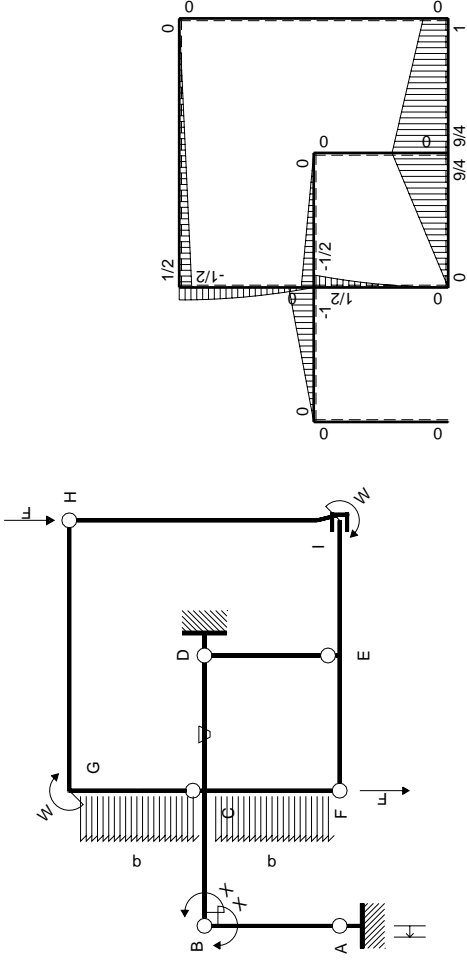
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $5/3Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-5/3Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

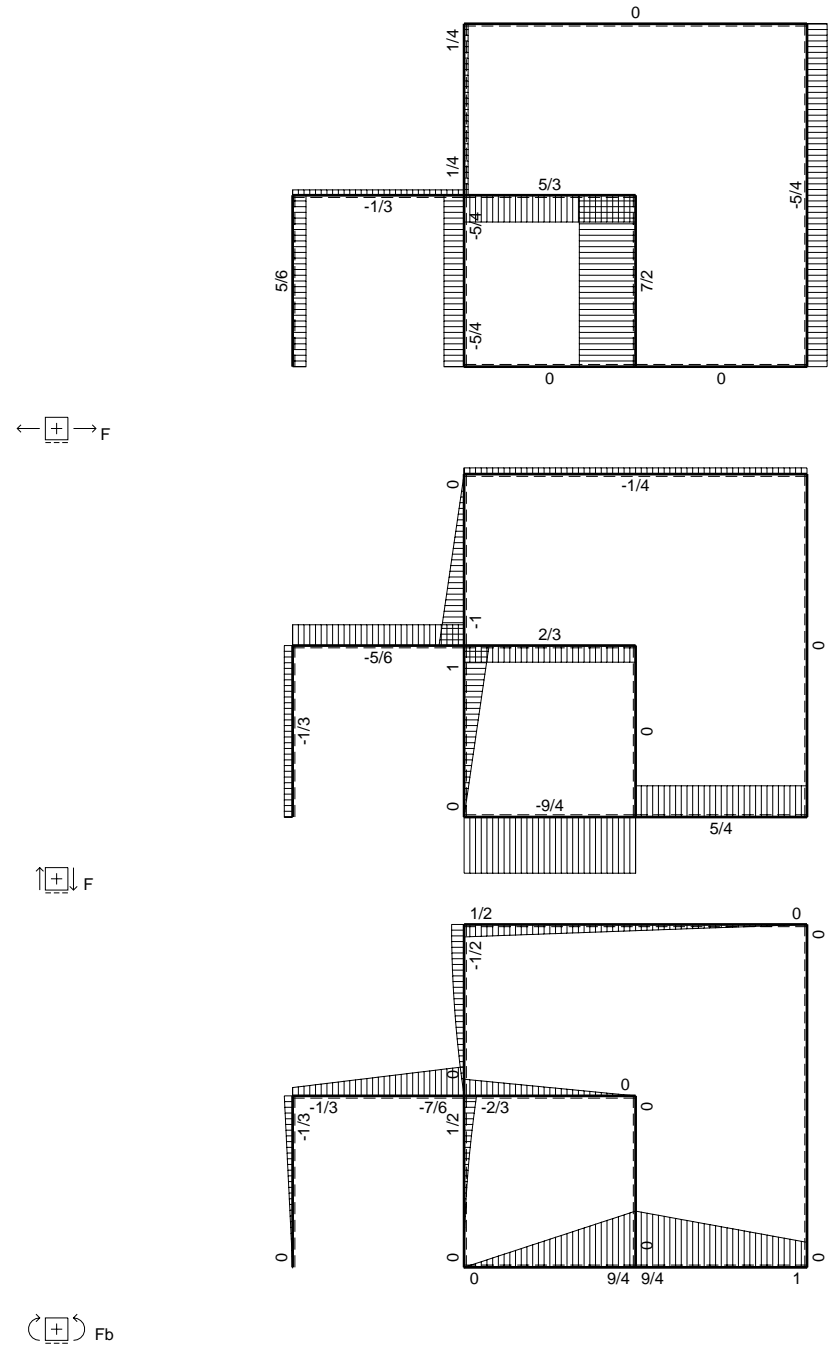
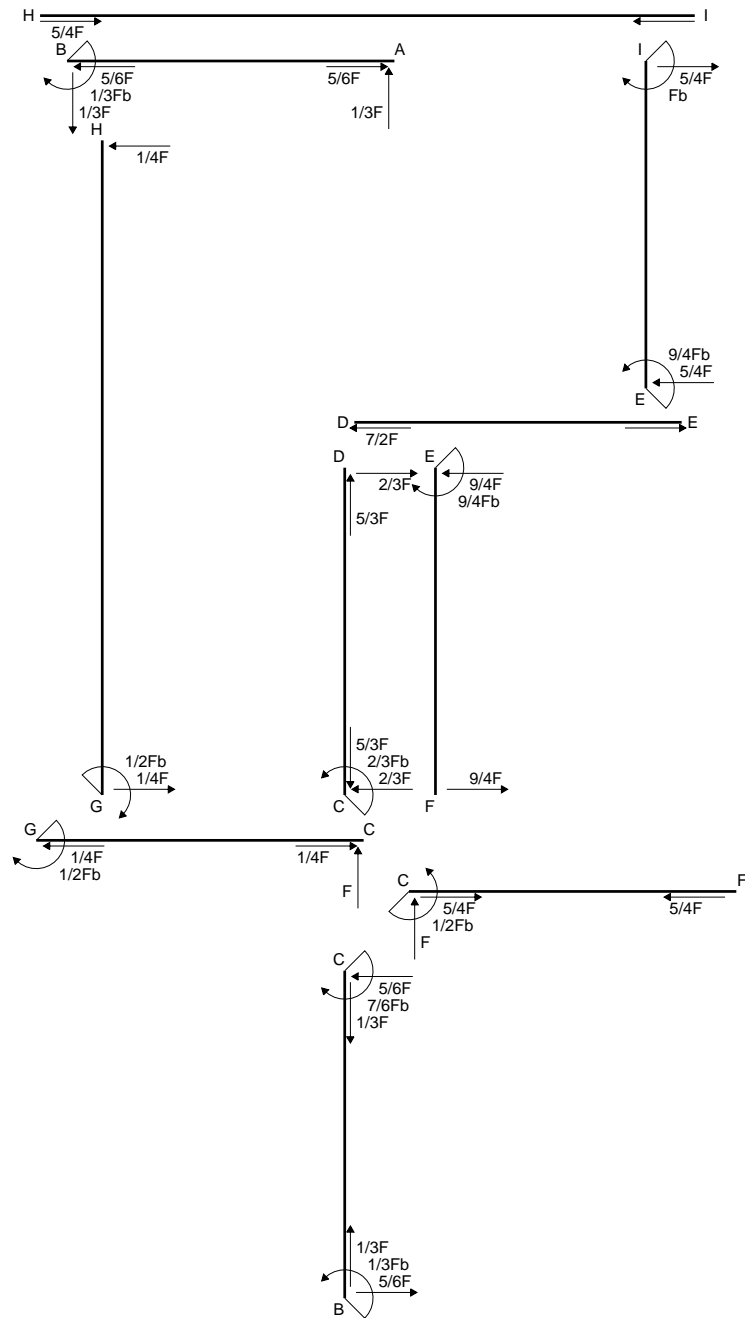
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

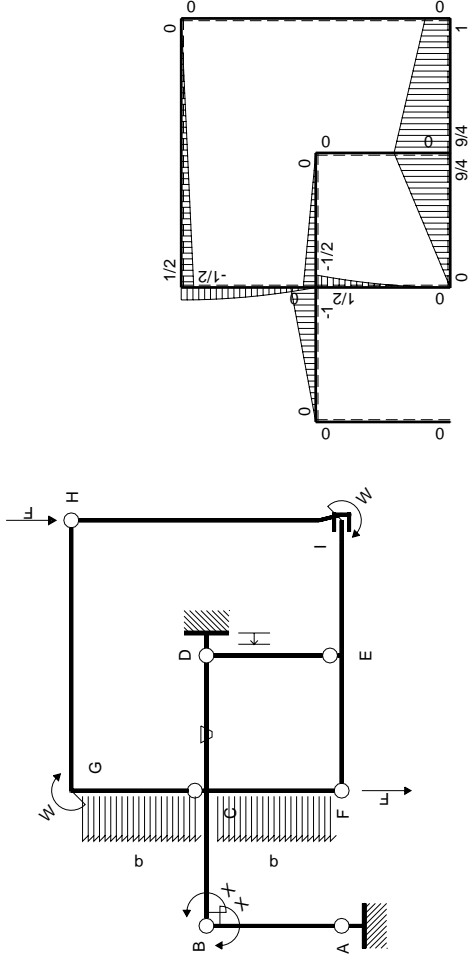
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-1/3Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $1/3Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

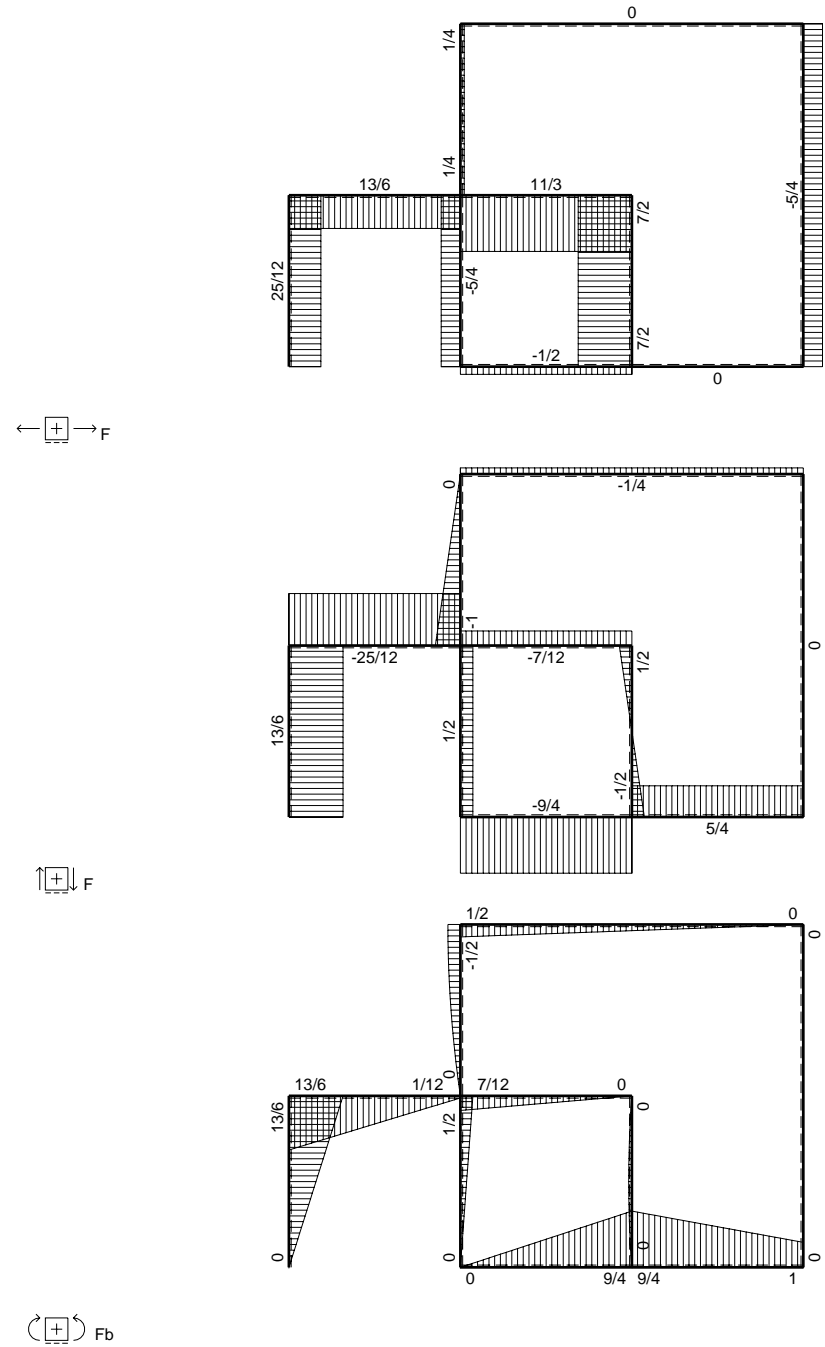
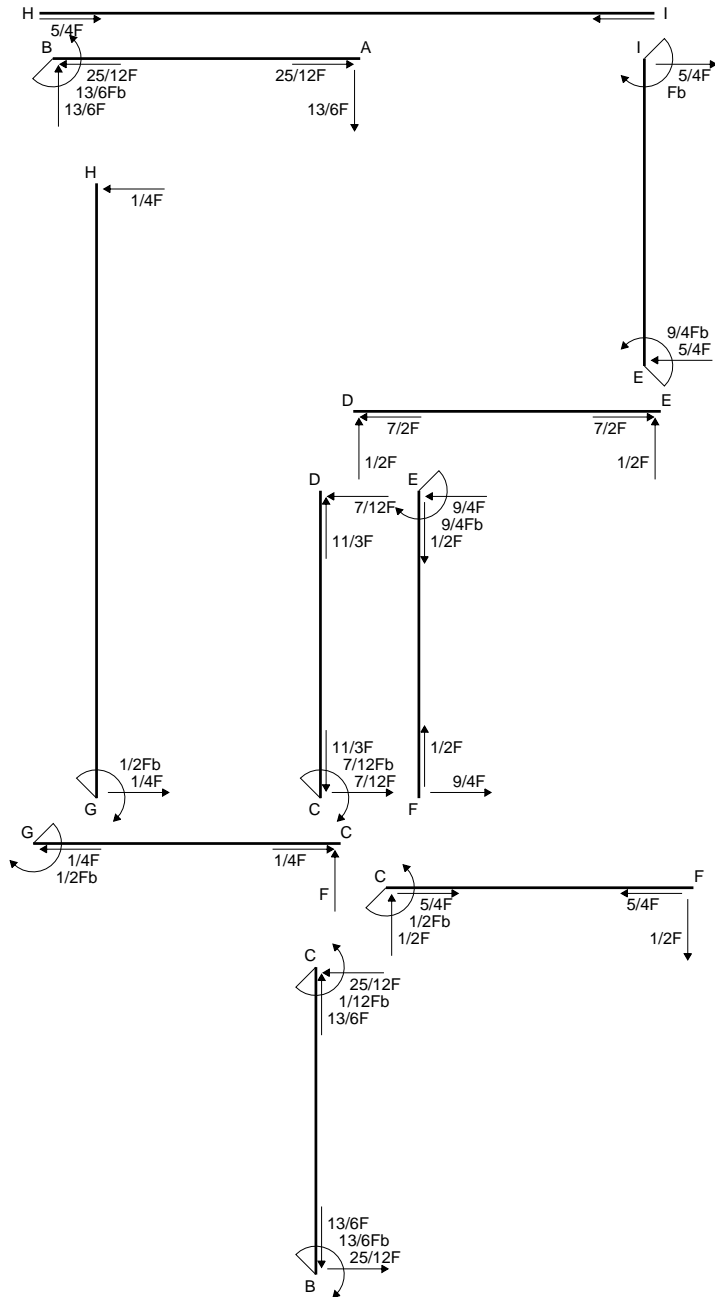
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

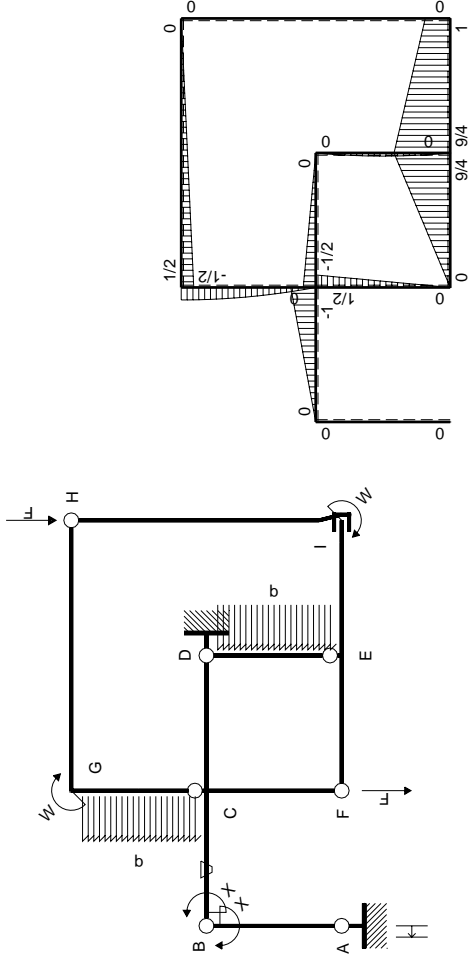
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

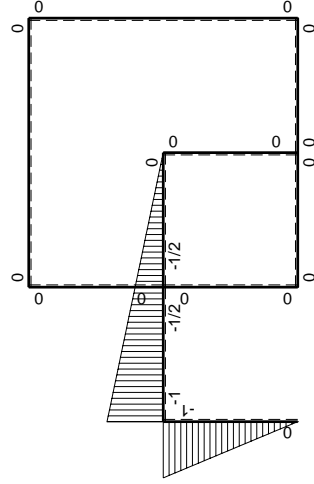
$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$   | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$          | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                         |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

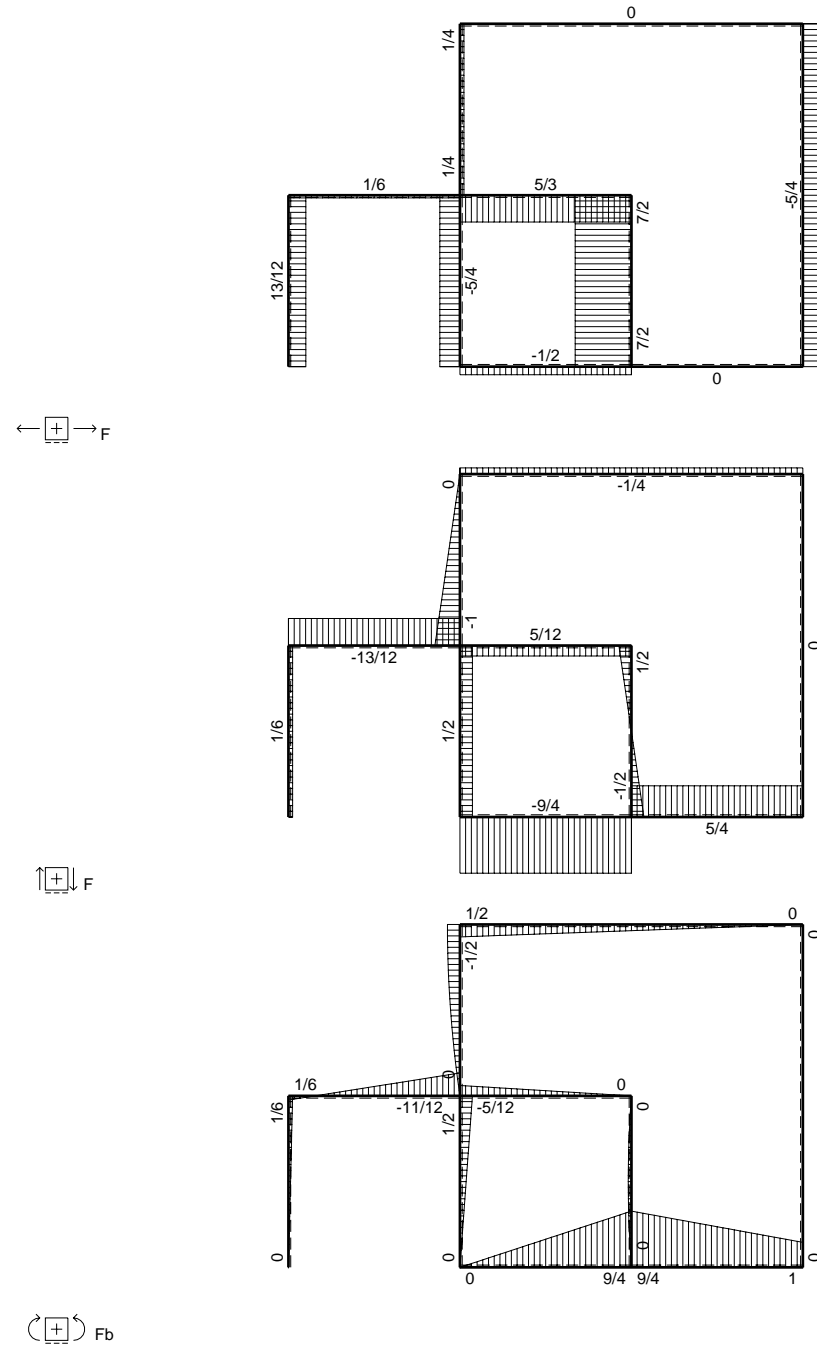
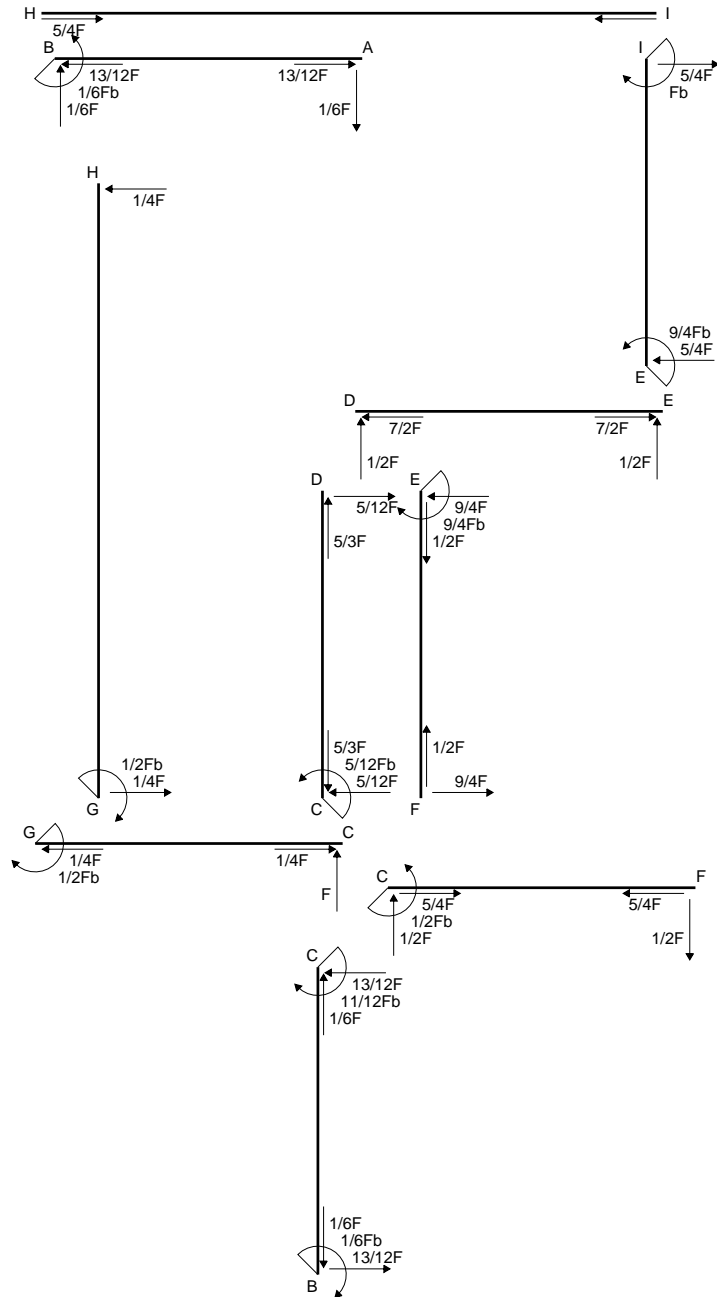
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

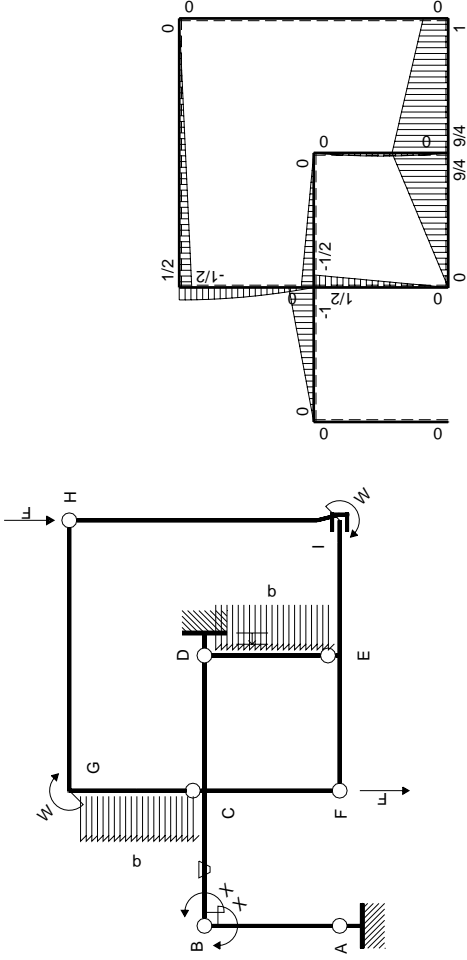
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





$\boxed{+}$   $\curvearrowright$   $F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$   | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$          | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                  |          |                         |                     |                         | $1/6Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |                     |                         | $-1/6Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

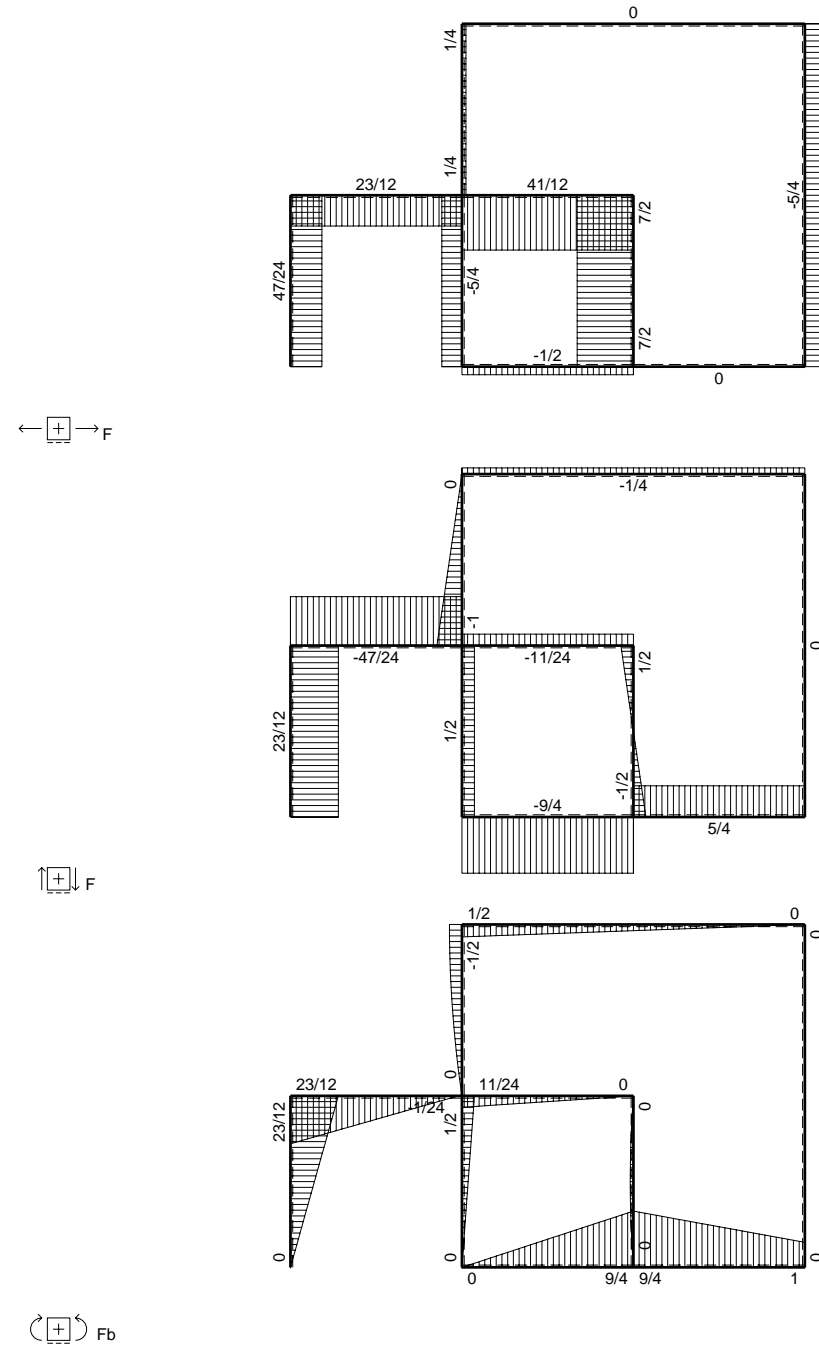
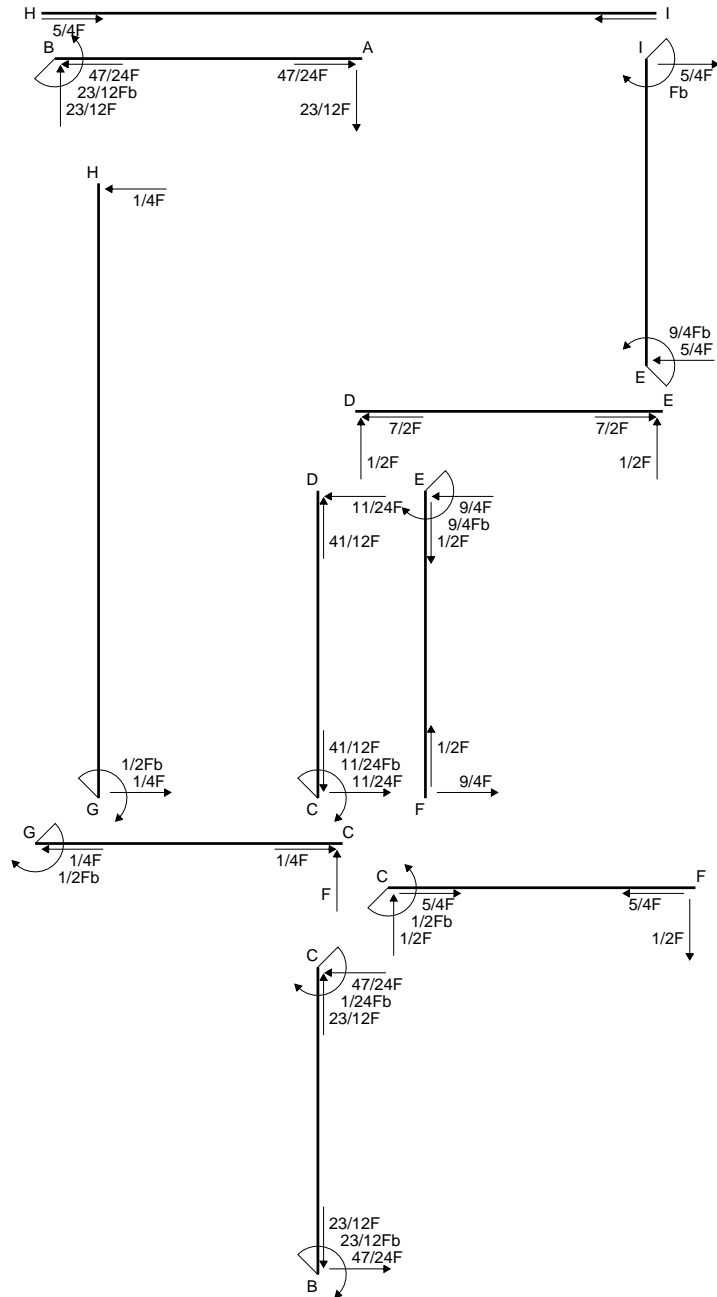
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

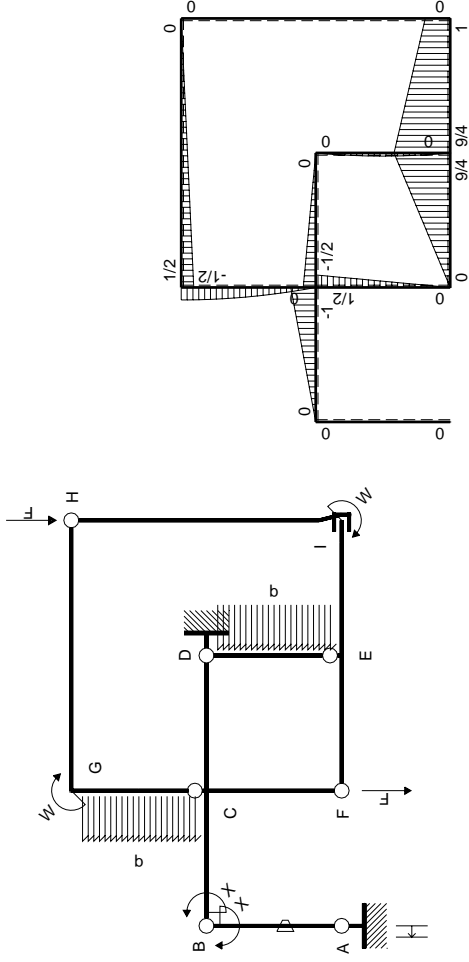
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$   | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$          | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                         |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |               |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

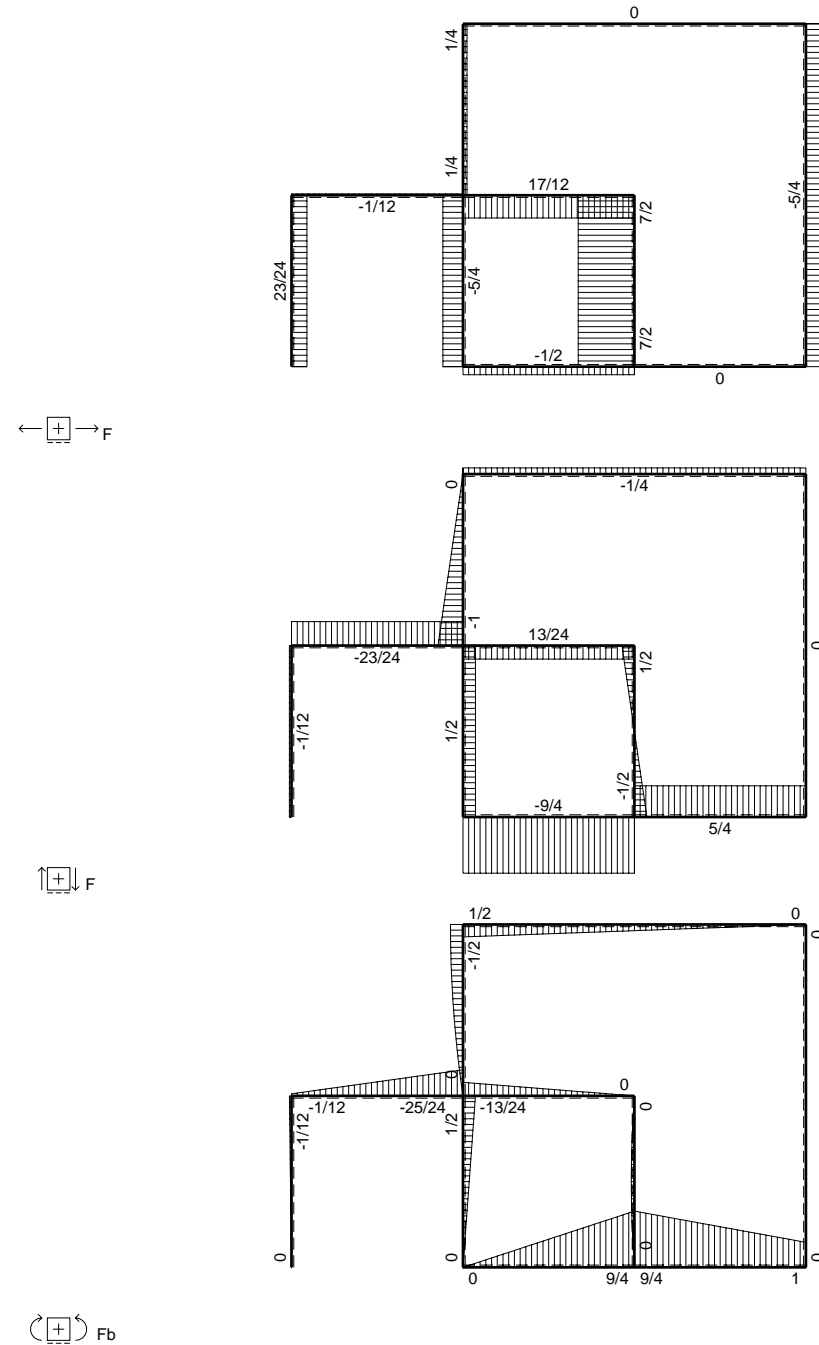
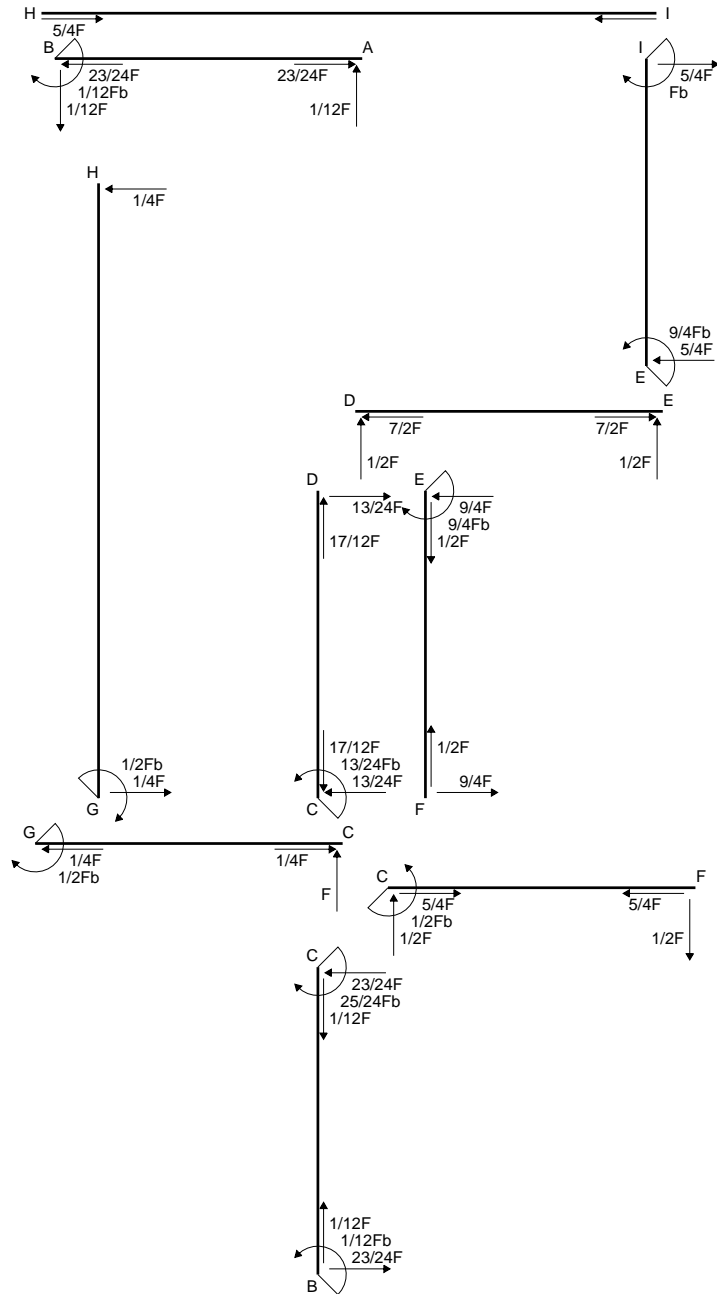
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

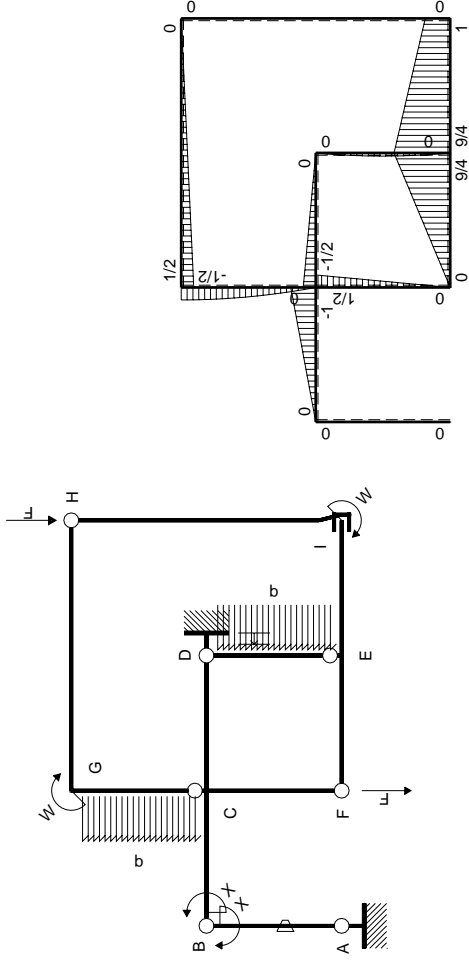
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$   | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$          | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                  |          |                         |               |                         | $-1/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |               |                         | $1/12Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

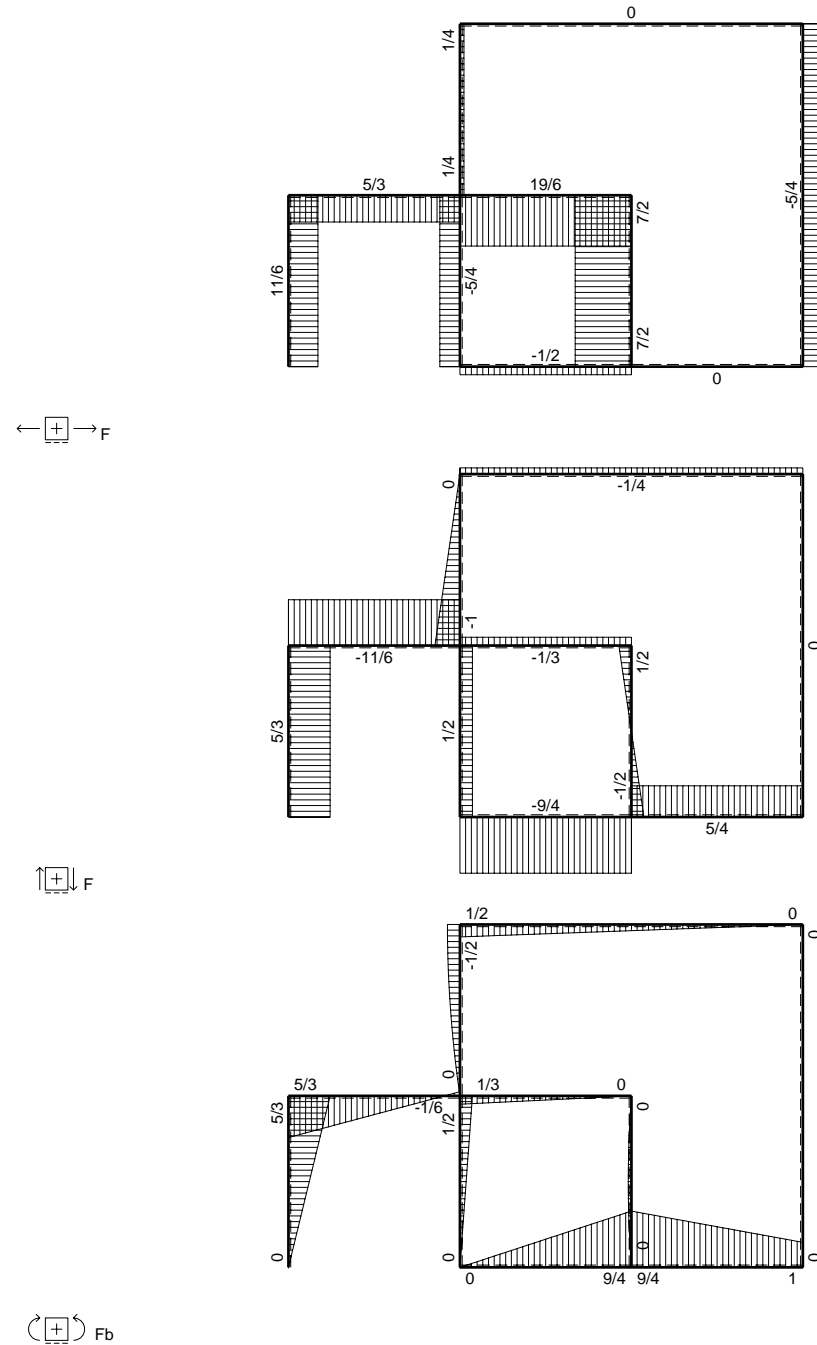
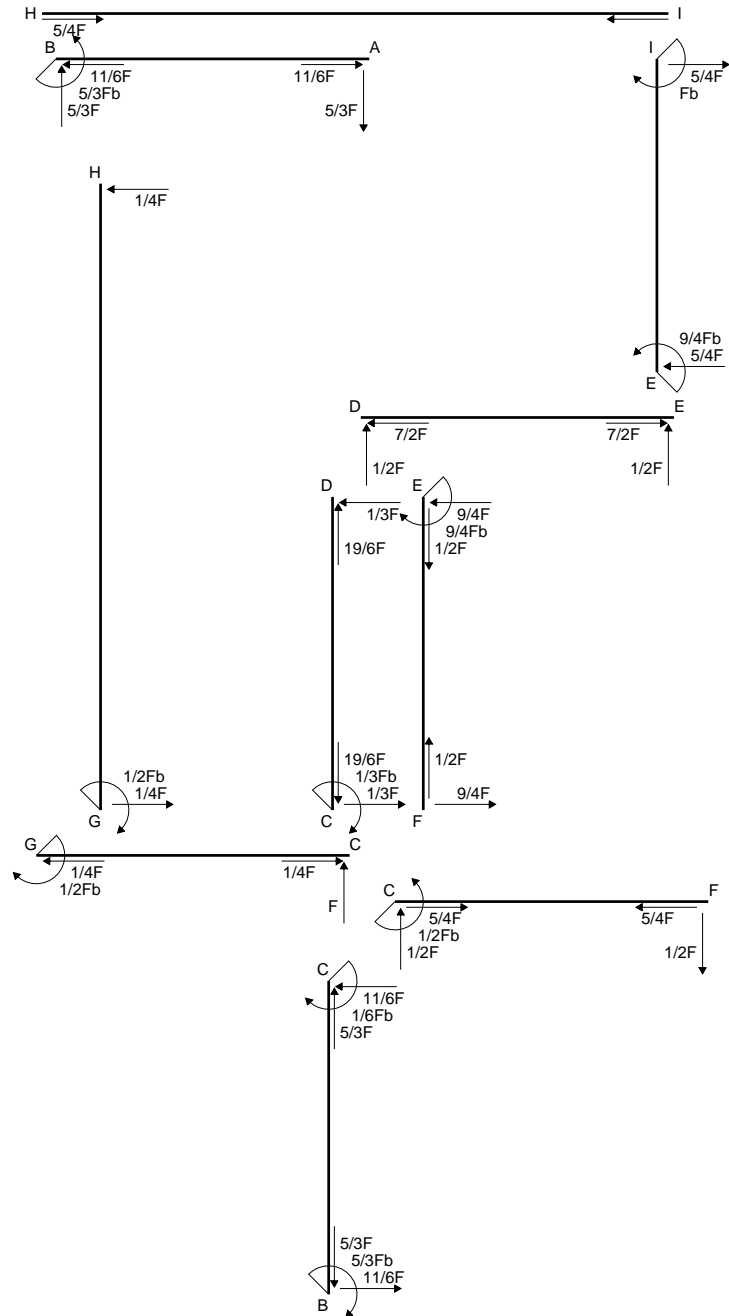
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

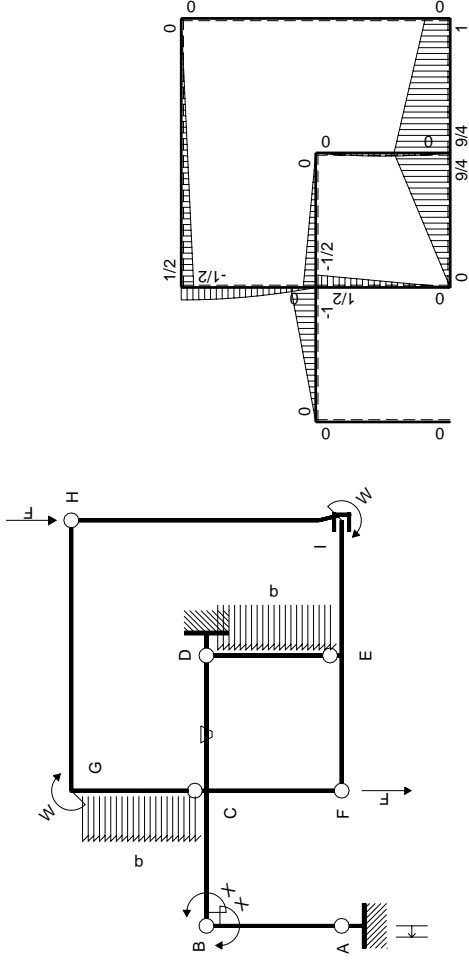
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int x M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$            | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$          | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$   | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$          | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                  |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                  |          |                         |                     |                         | $5/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |                     |                         | $-5/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

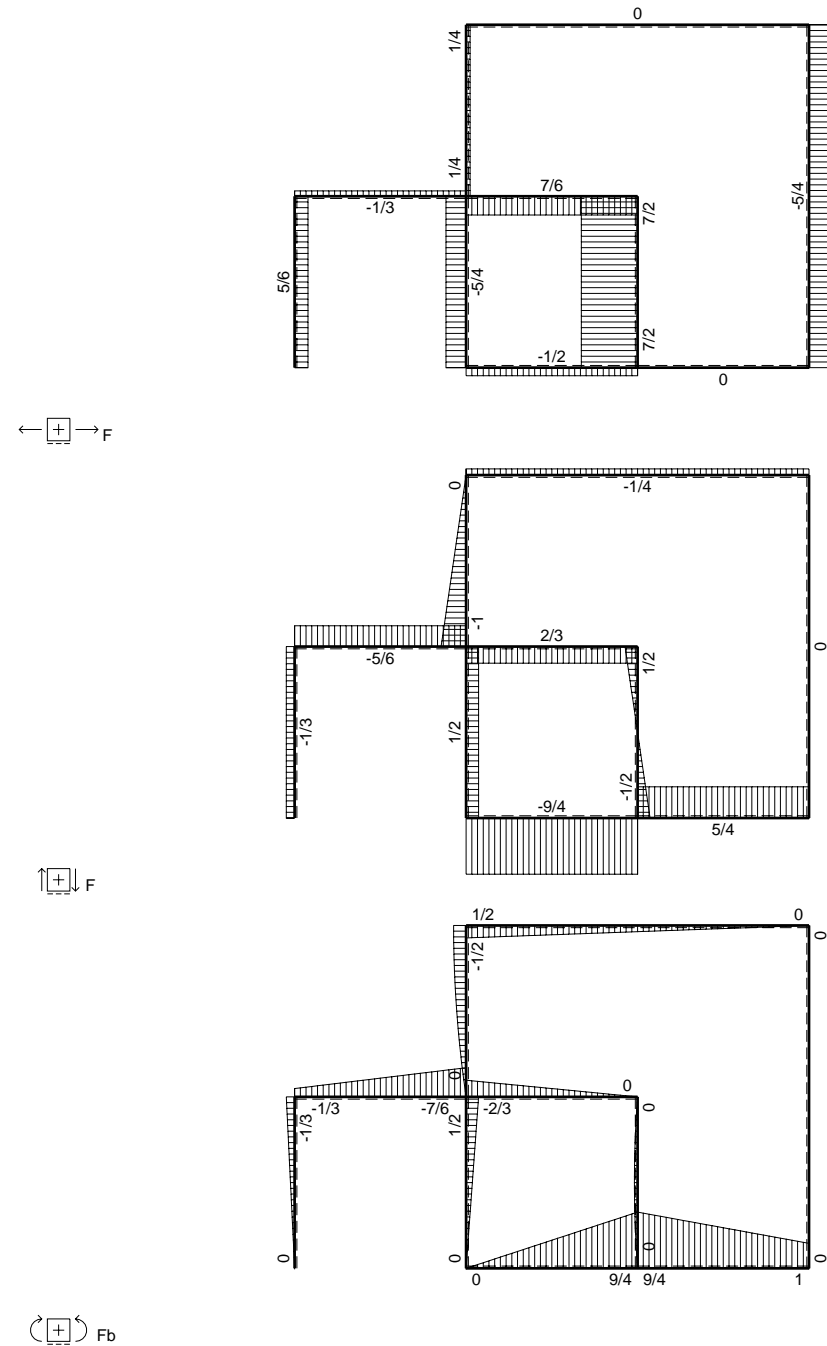
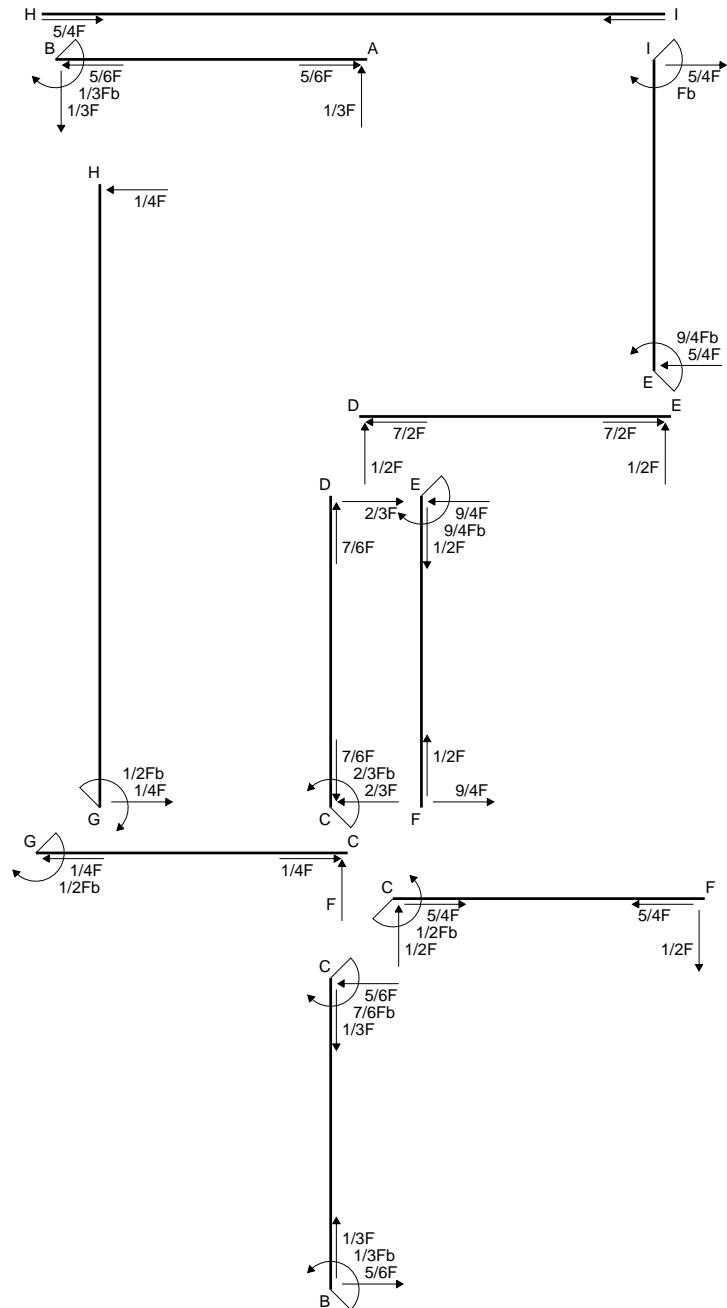
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

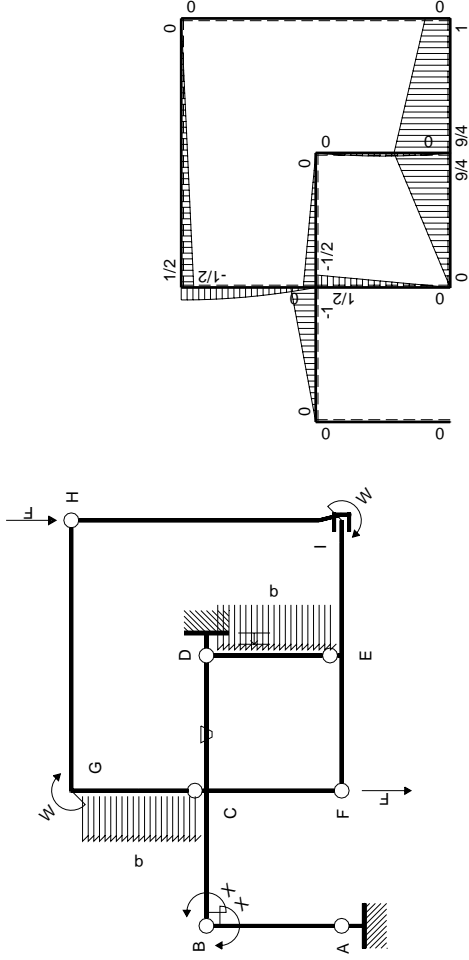
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$         | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int x M_x M_x/EJ dx$ |       |
|-------|-----------------------------|------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|-------|
| AB b  | $-x/b$                      | 0                | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | 1/3Xb/EJ               |       |
| BA b  | $1-x/b$                     | 0                | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |       |
| BC b  | $-1+1/2x/b$                 | -Fx              | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | 7/12Xb/EJ              |       |
| CB b  | $1/2+1/2x/b$                | Fb-Fx            | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |       |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$   | -Fb/EJ   | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | 1/12Xb/EJ              |       |
| DC b  | $1/2x/b$                    | $1/2Fx$          | Fb/EJ    | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |       |
| DE b  | 0                           | $1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| EF b  | 0                           | $9/4Fb-9/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| FE b  | 0                           | $-9/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| FC b  | 0                           | $1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| CF b  | 0                           | $-1/2Fb+1/2Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| CG b  | 0                           | $-Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| GC b  | 0                           | $1/2Fb-1/2qx^2$  | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| GH 2b | 0                           | $1/2Fb-1/4Fx$    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| HG 2b | 0                           | $-1/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| HI 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| IH 2b | 0                           | 0                | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| IE b  | 0                           | $Fb+5/4Fx$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |       |
| EI b  | 0                           | $-9/4Fb+5/4Fx$   | 0        | 0                       | 0                   | 0                       |                             |                        |       |
| D     | cedimento nodo $-H_{1D}u_D$ |                  |          |                         |                     |                         |                             | $-Fb^2/EJ$             |       |
|       | totali                      |                  |          |                         |                     |                         |                             | $-1/3Fb^2/EJ$          | Xb/EJ |
|       | iperstatica $X=W_{BC}$      |                  |          |                         |                     |                         |                             | 1/3Fb                  |       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

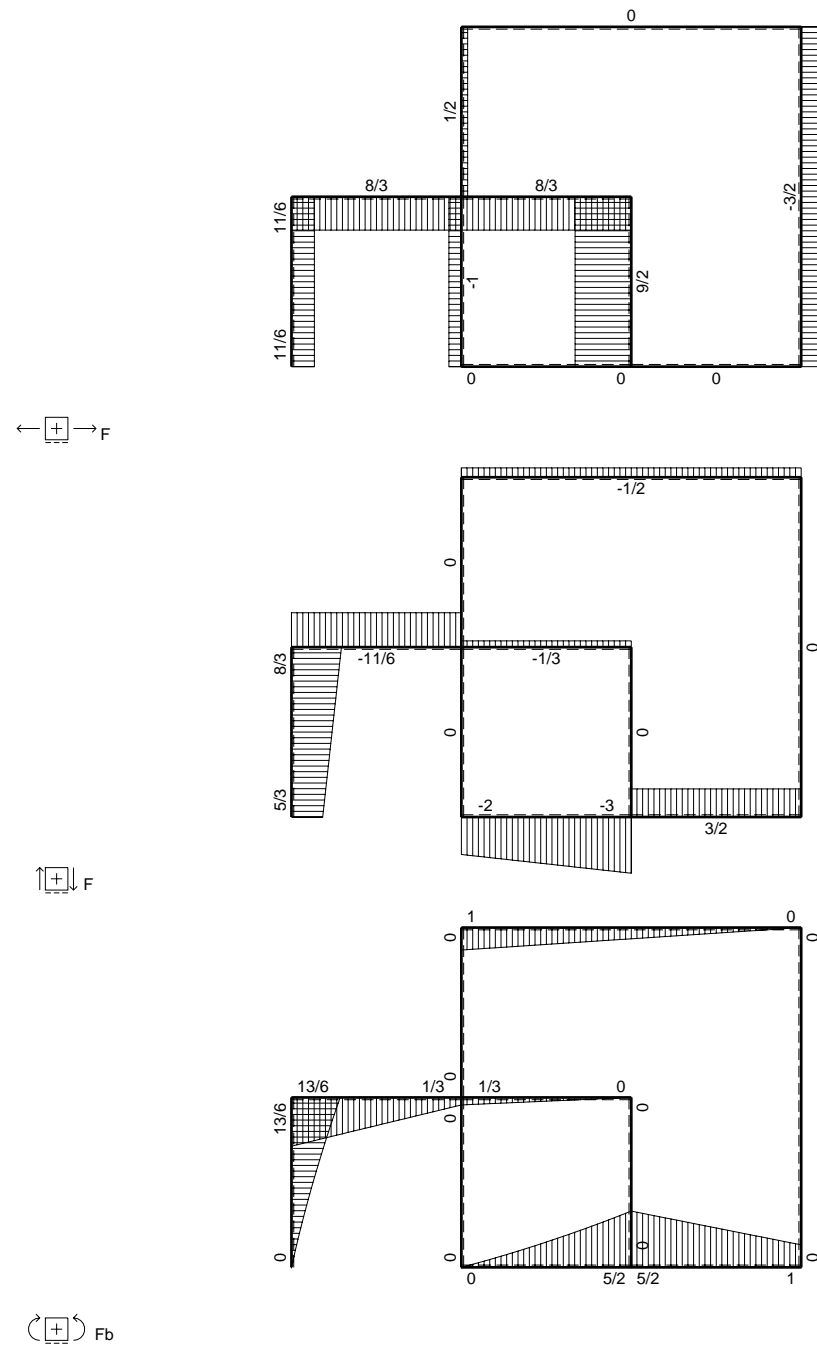
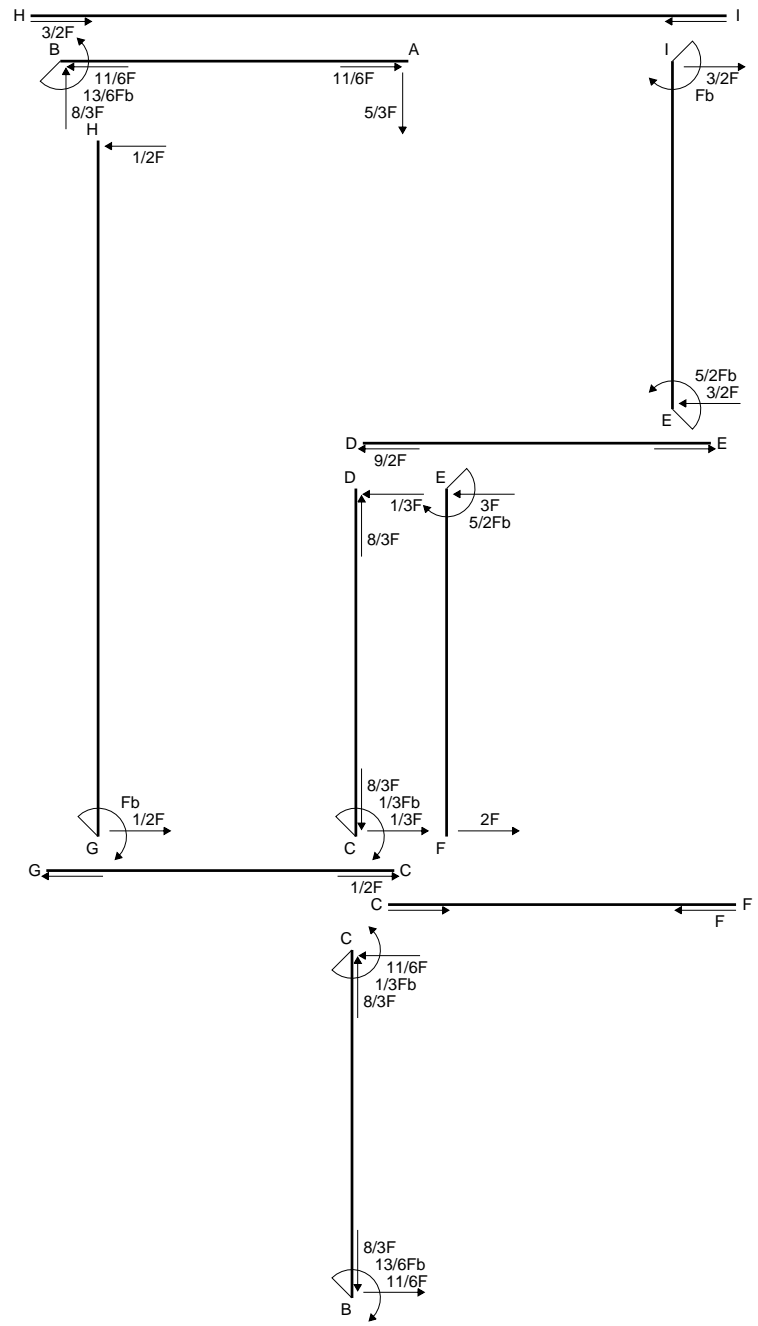
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

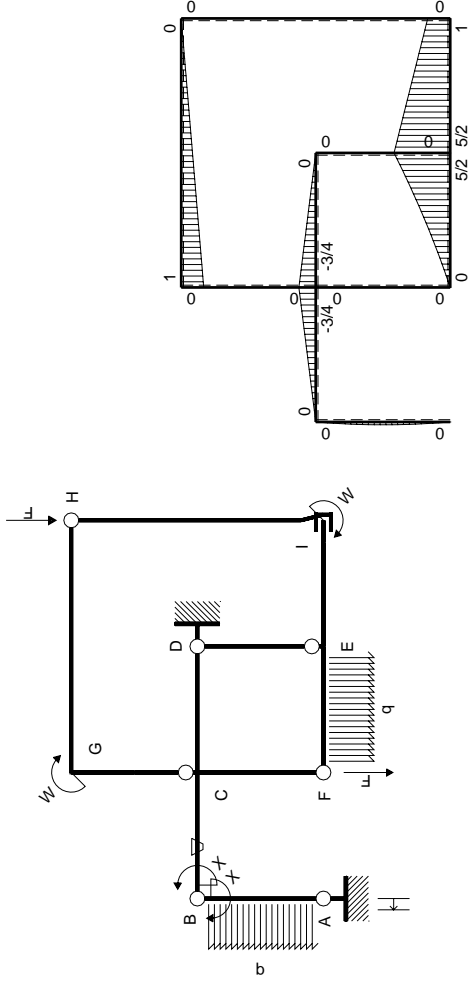
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

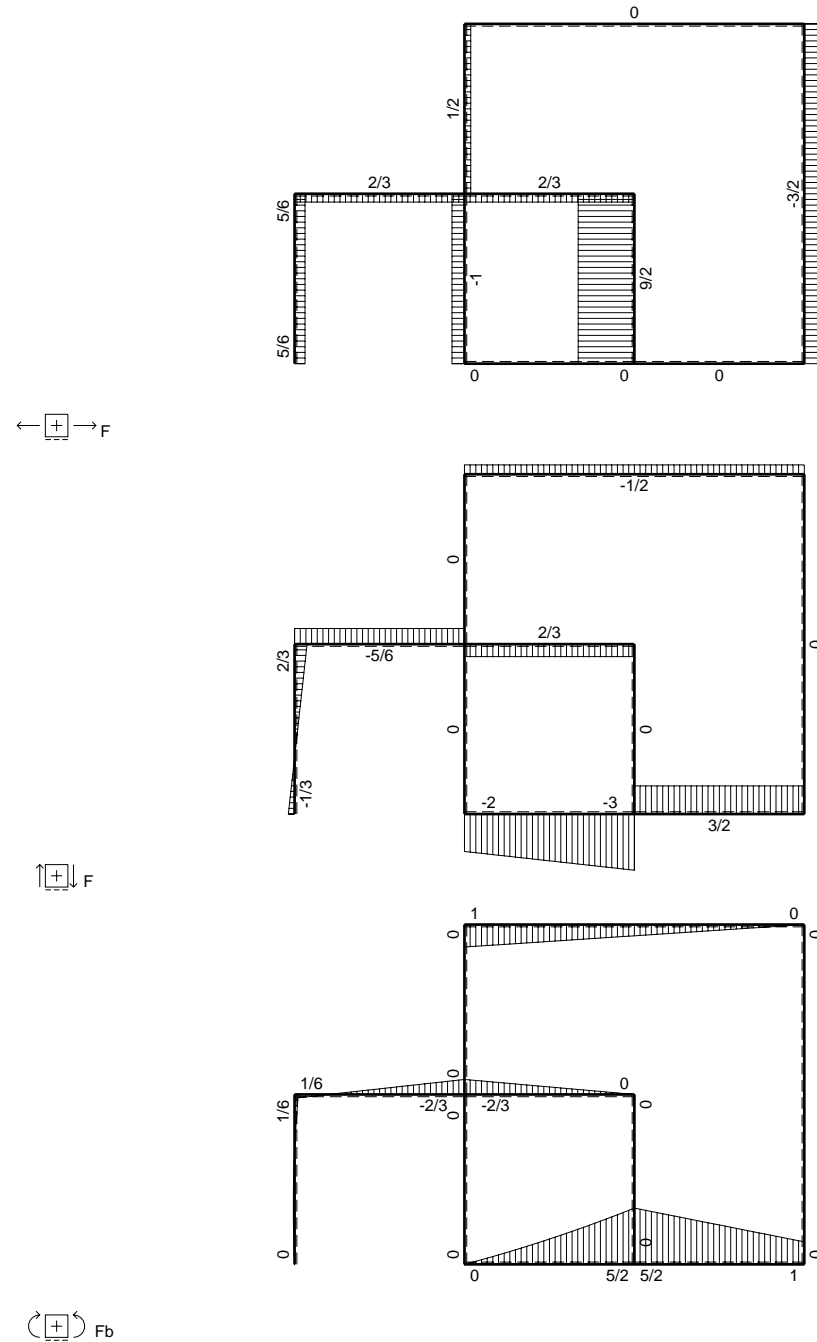
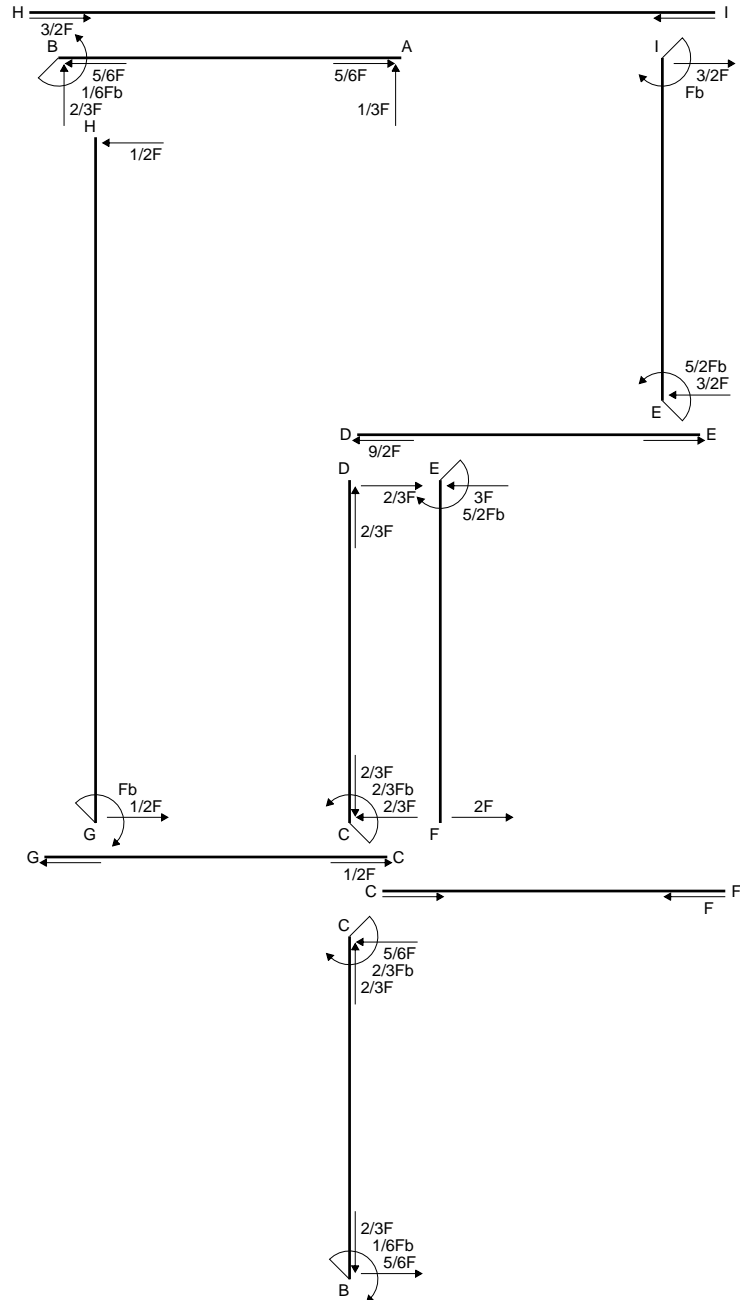
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

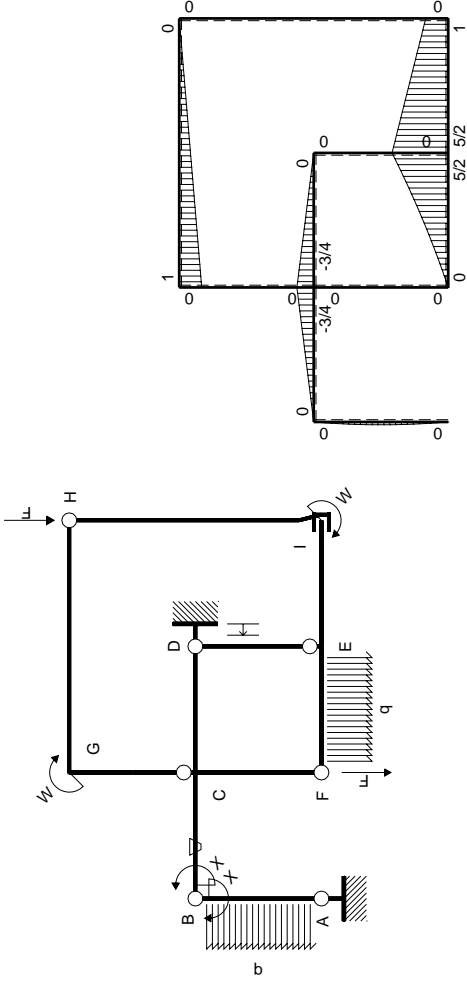
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

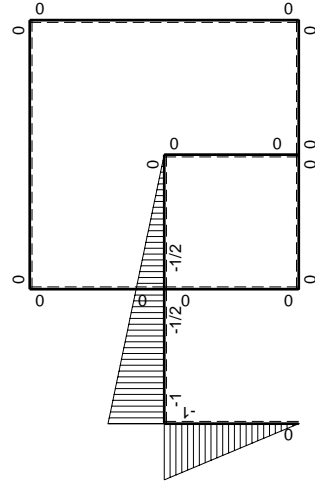






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                          |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

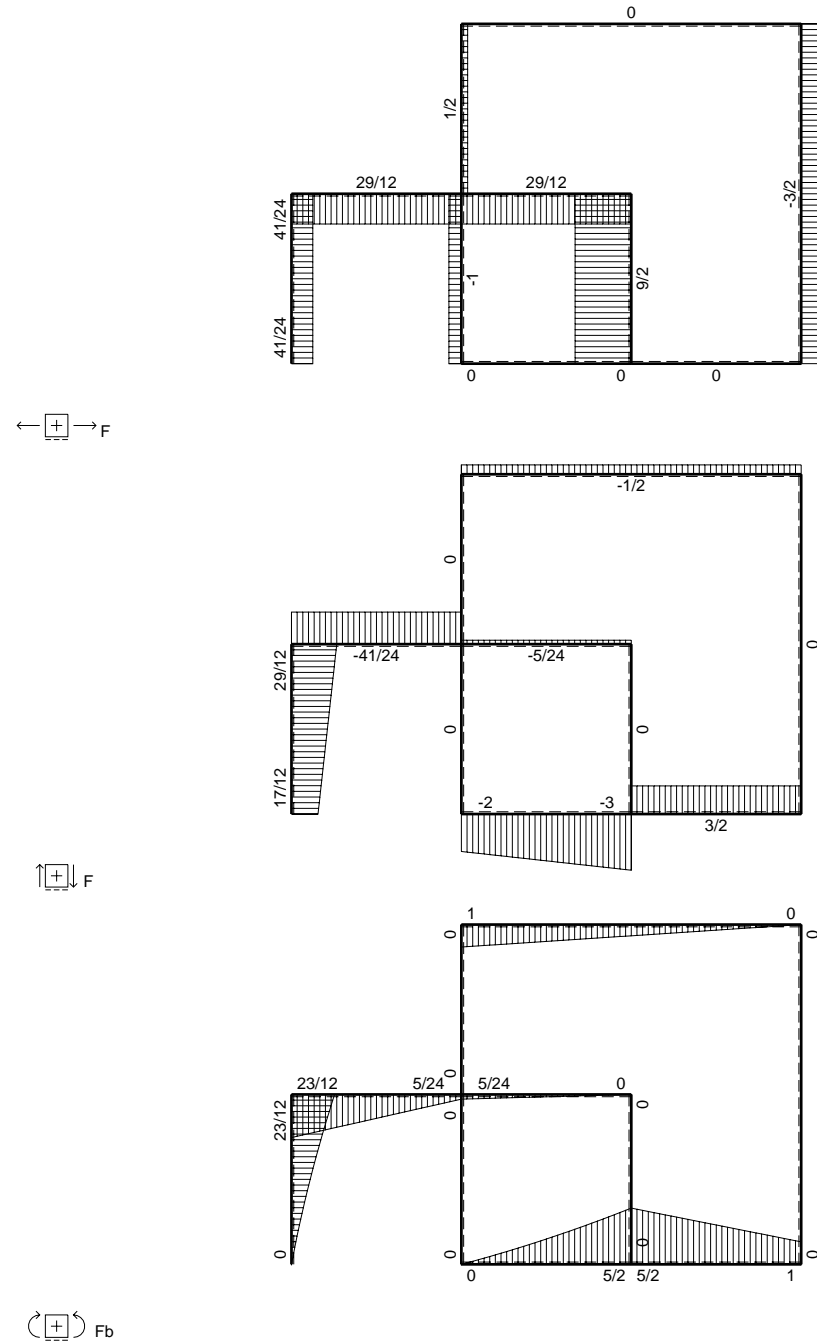
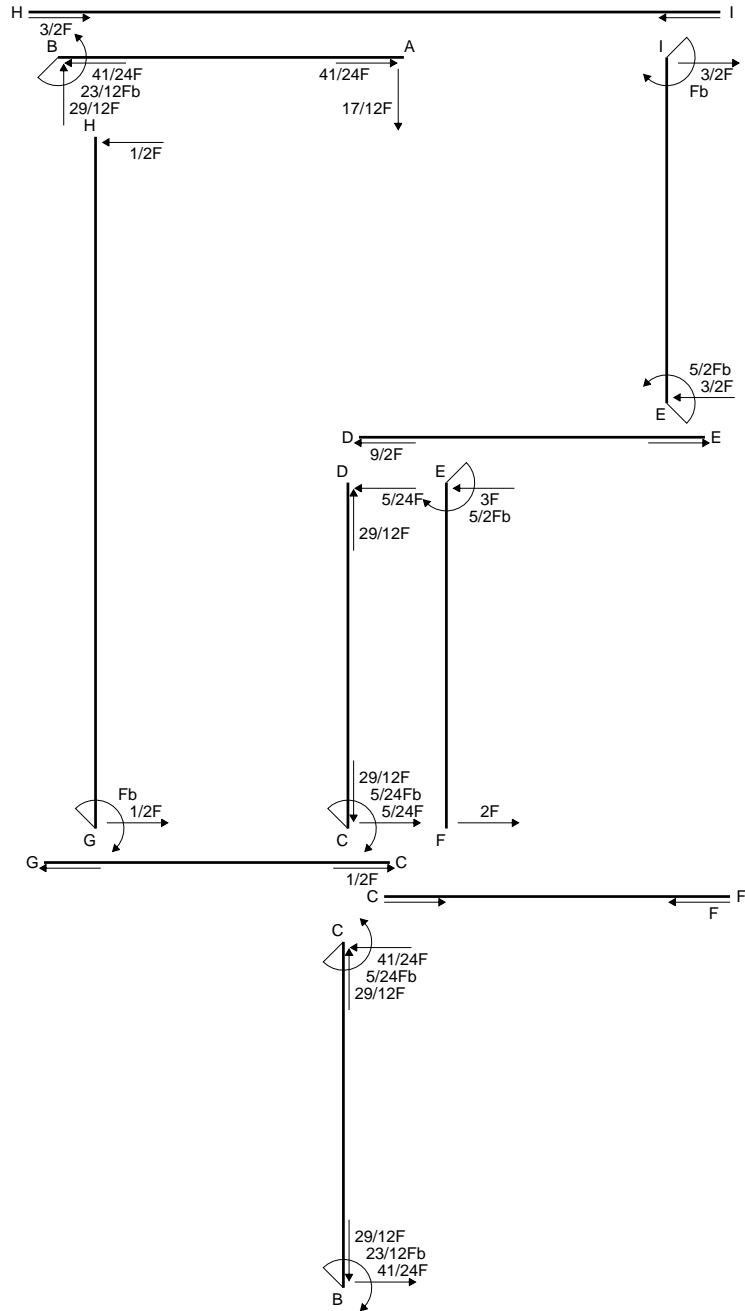
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

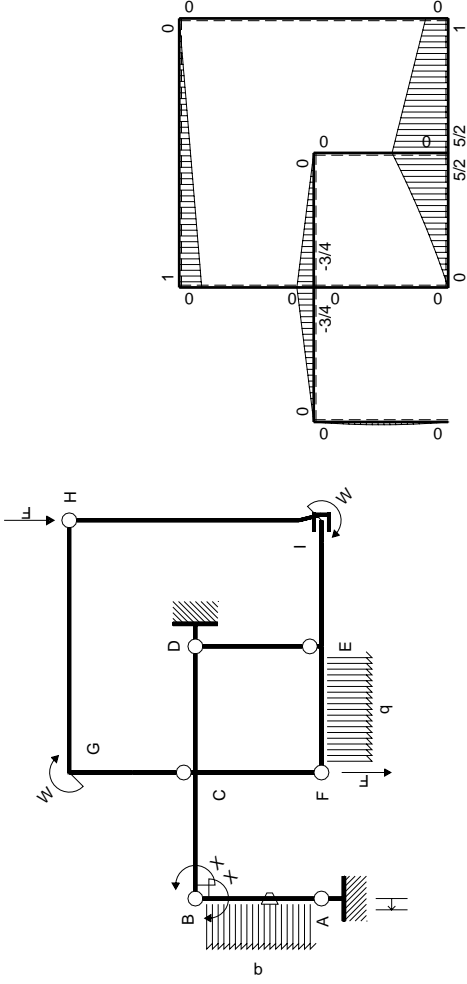
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |               |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

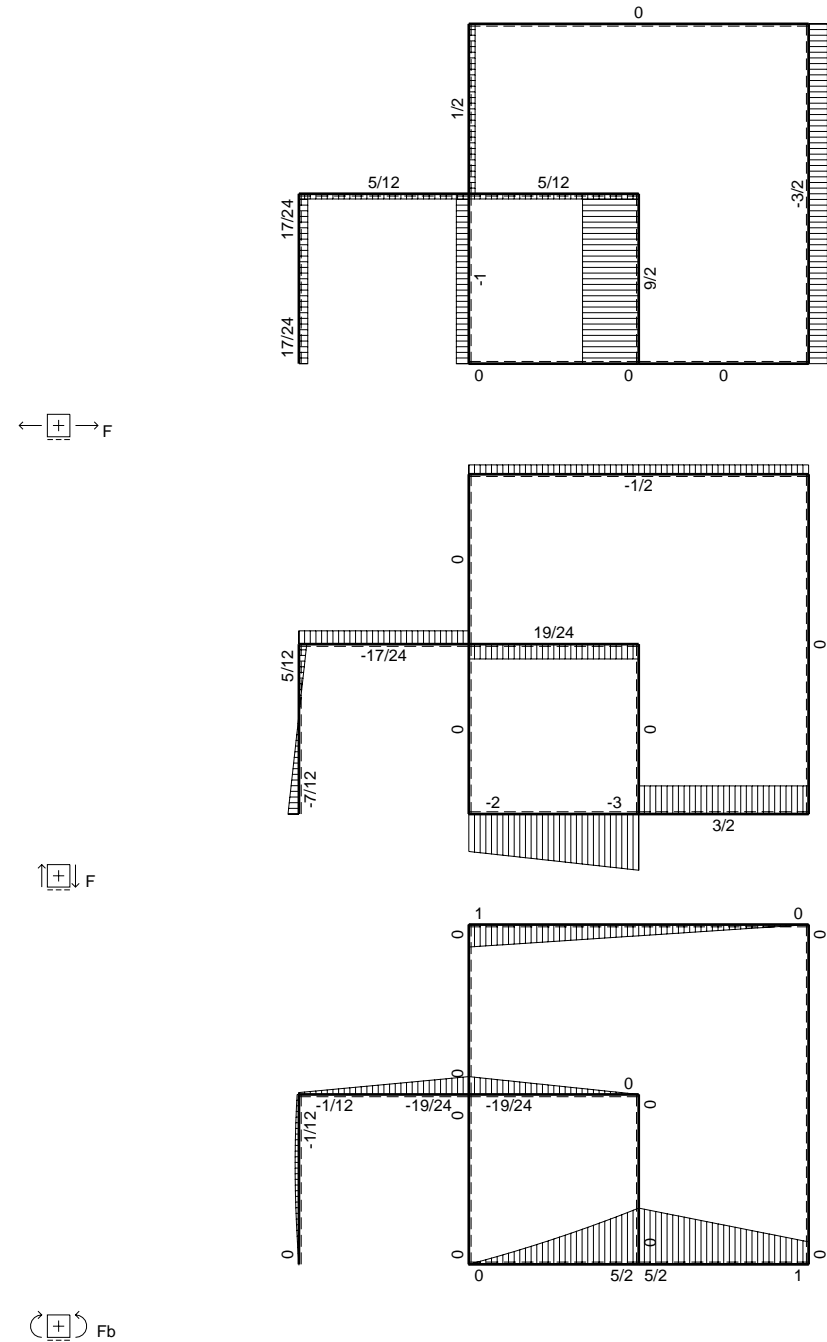
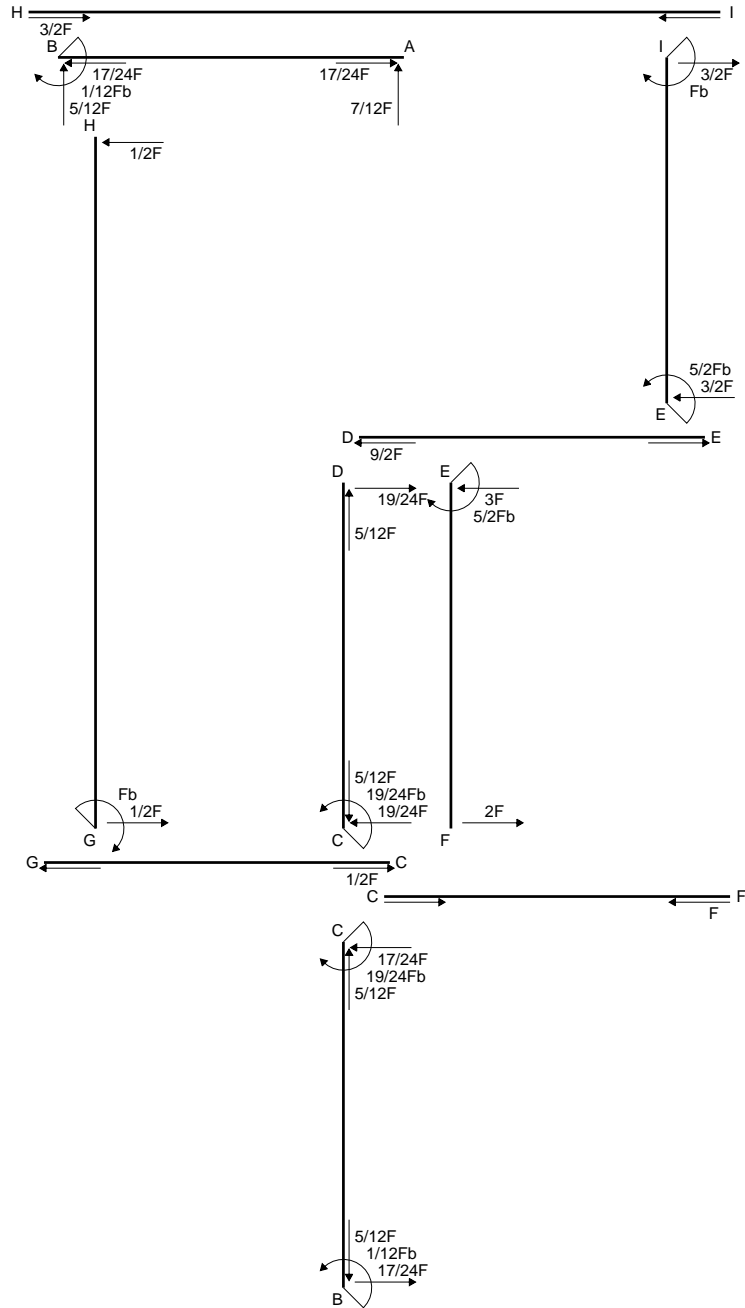
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

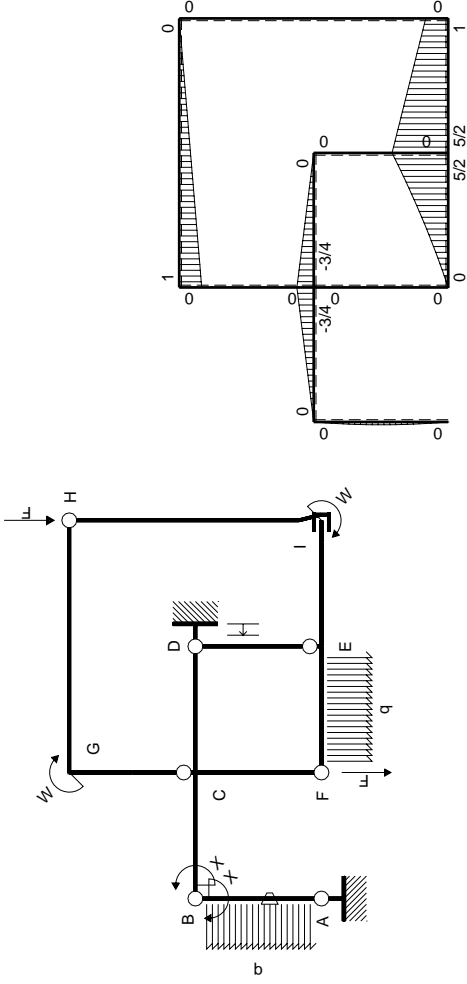
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

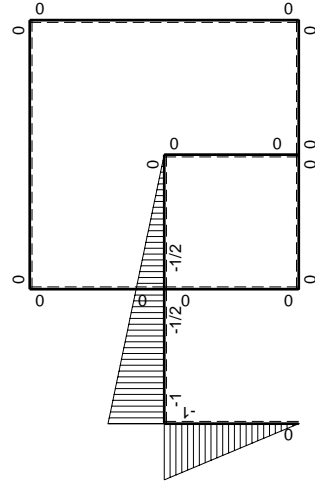






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                          |               |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |               |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

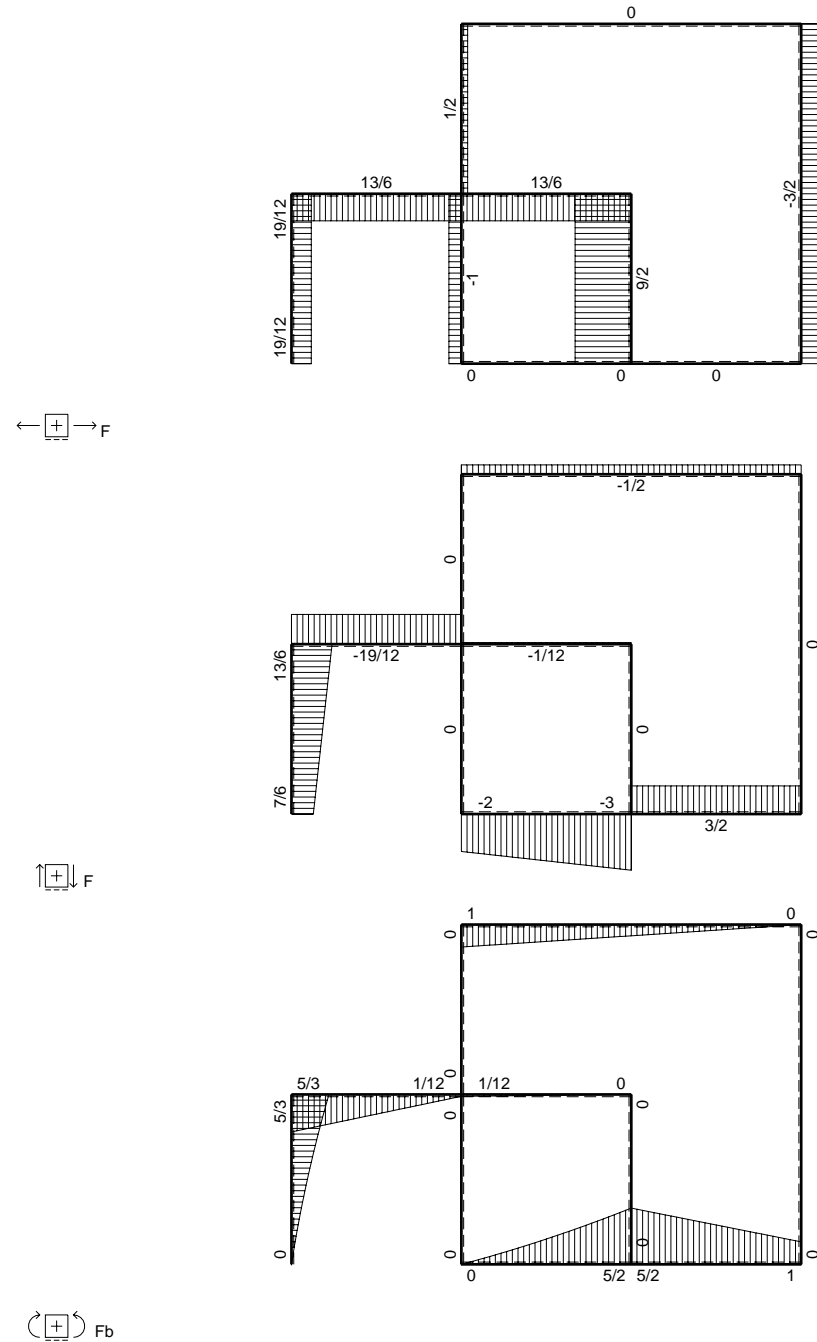
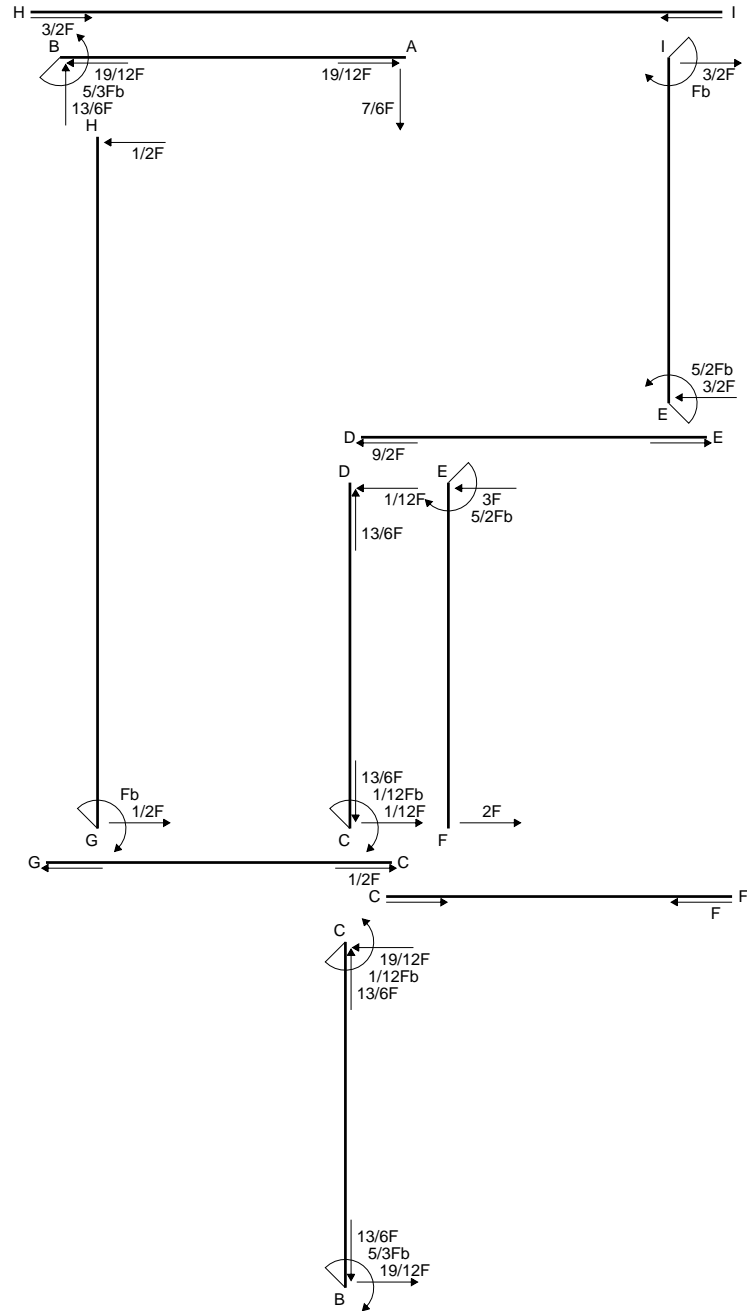
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

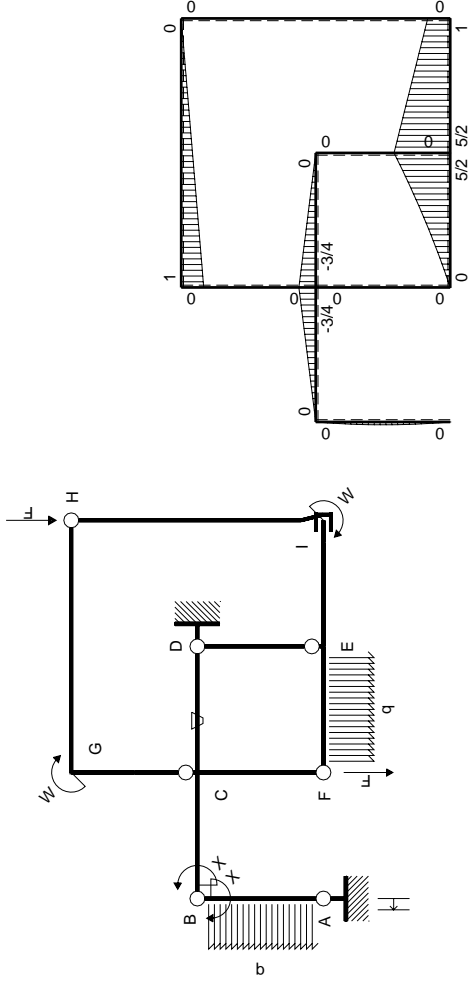
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                          |                     |                         | $5/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $-5/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

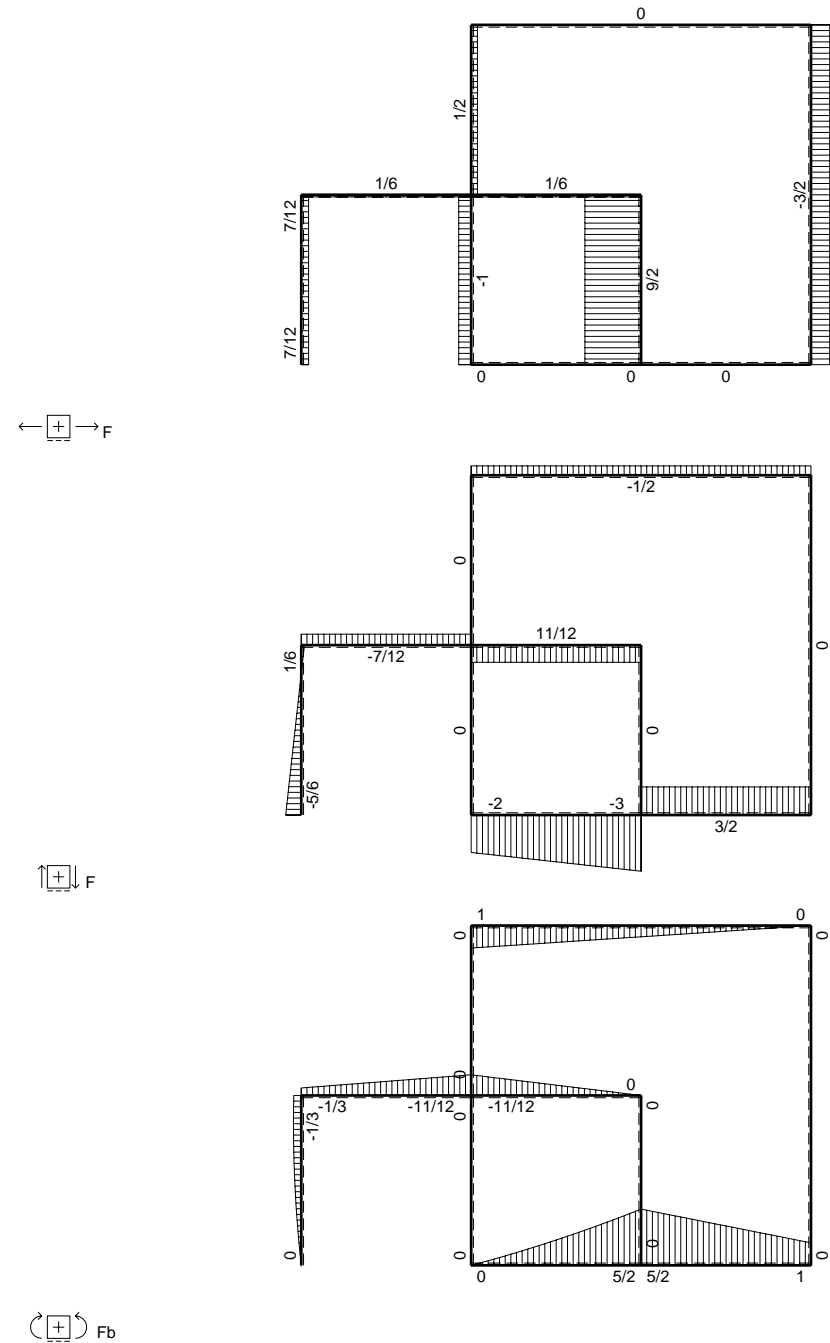
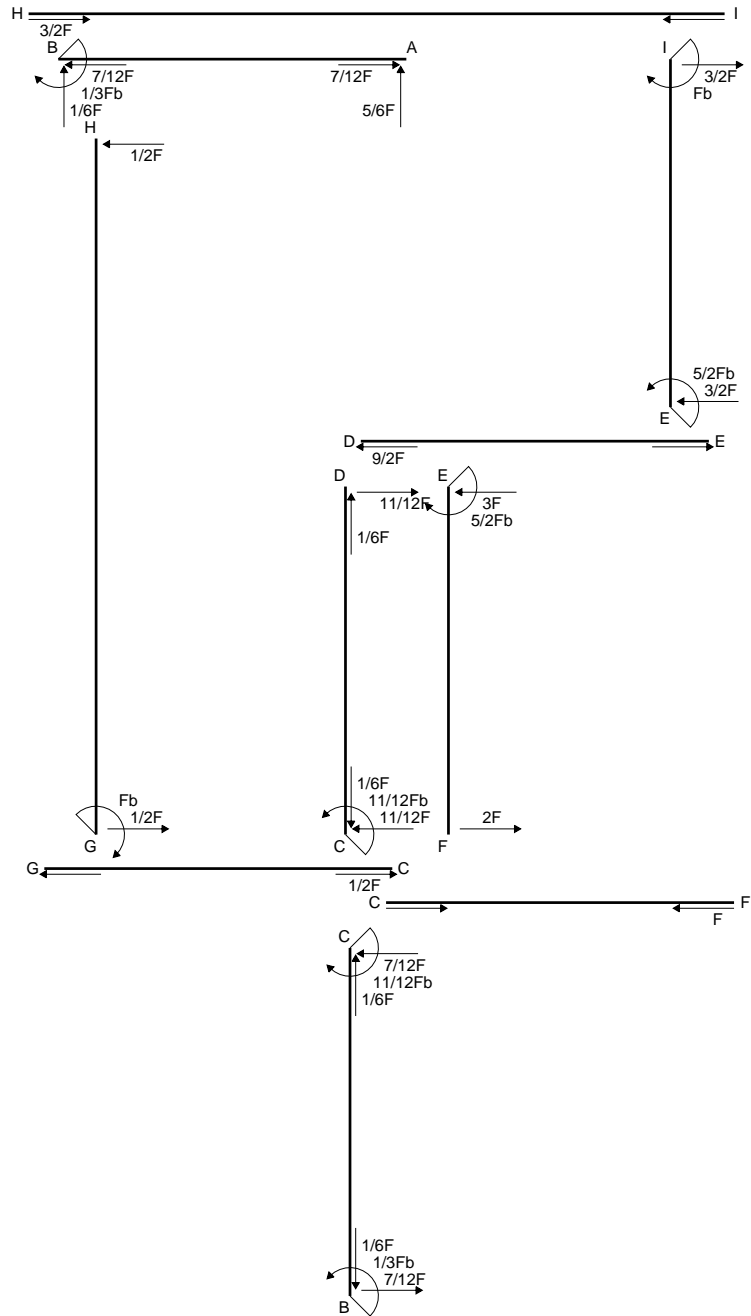
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

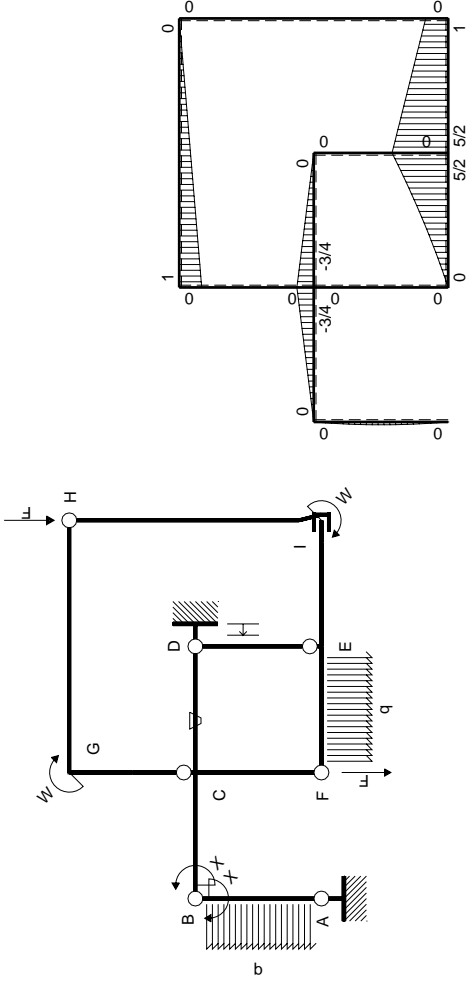
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$    | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$     | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                          |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                          |                     |                         | $-1/3Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                          |                     |                         | $1/3Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

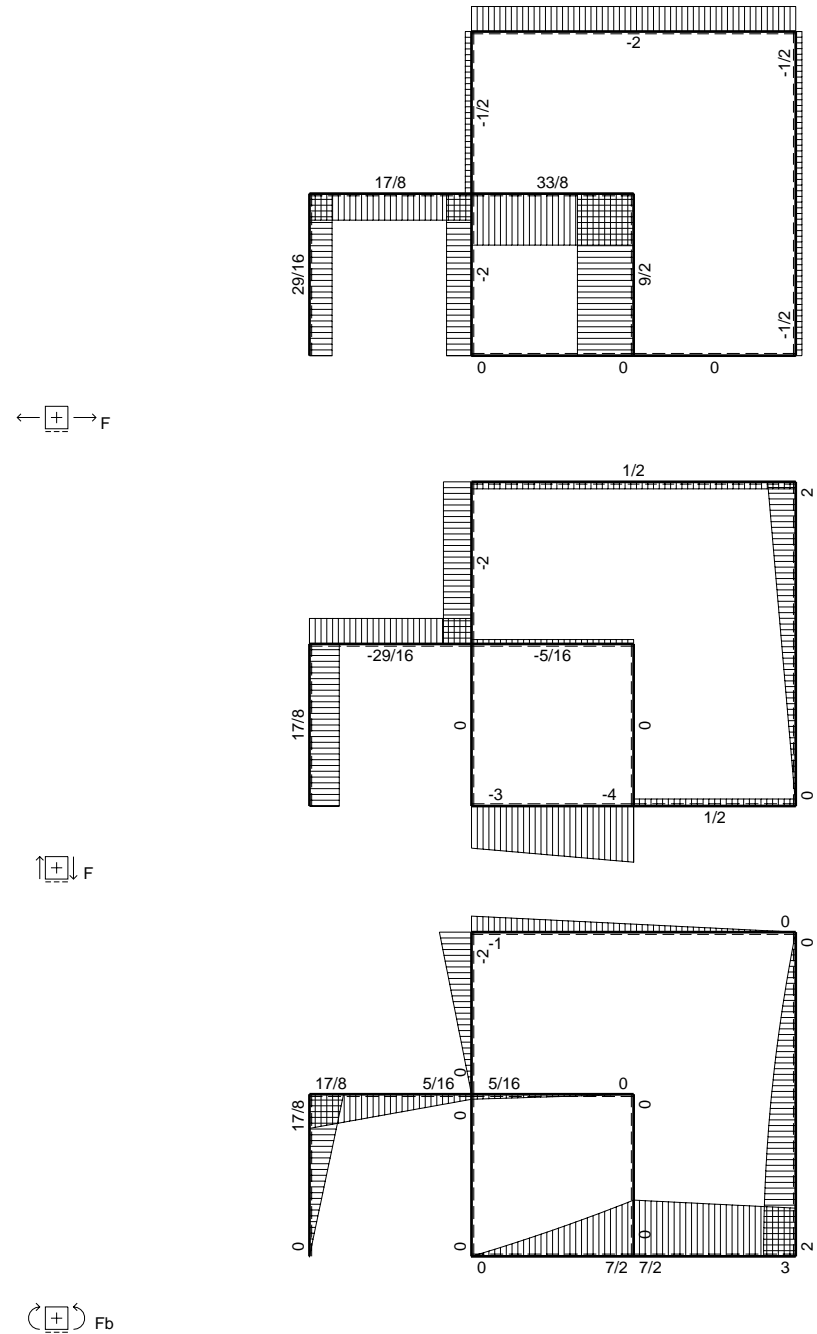
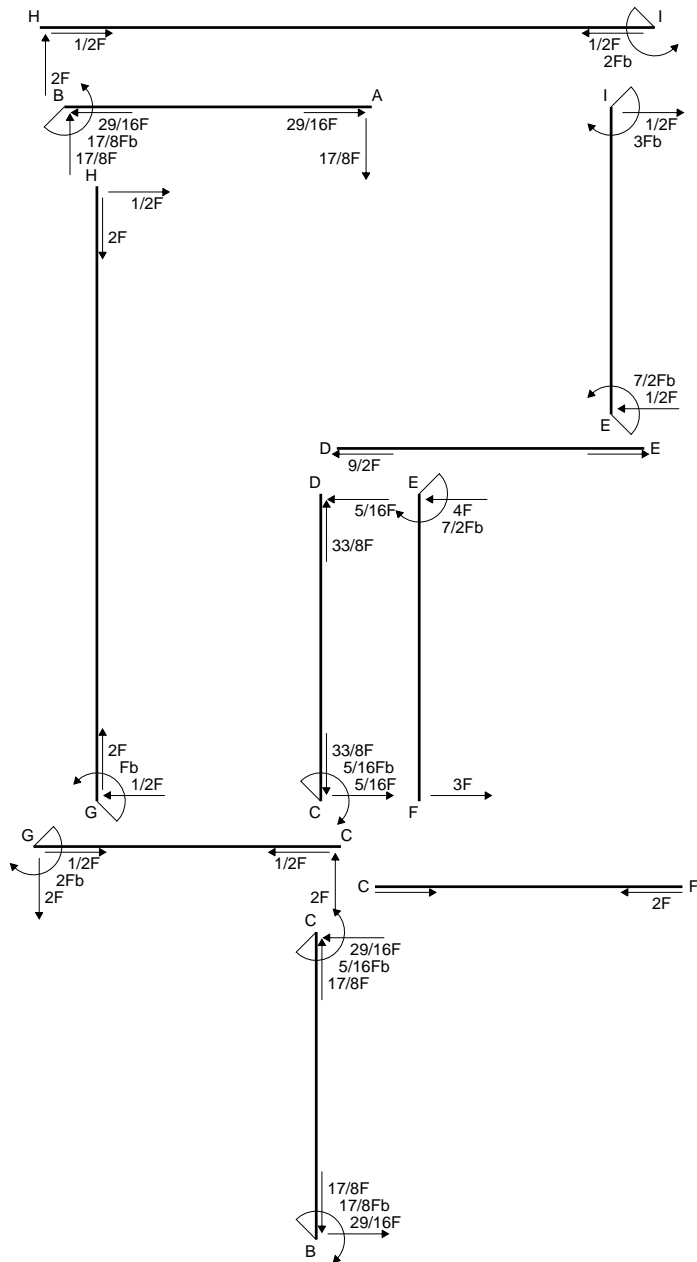
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

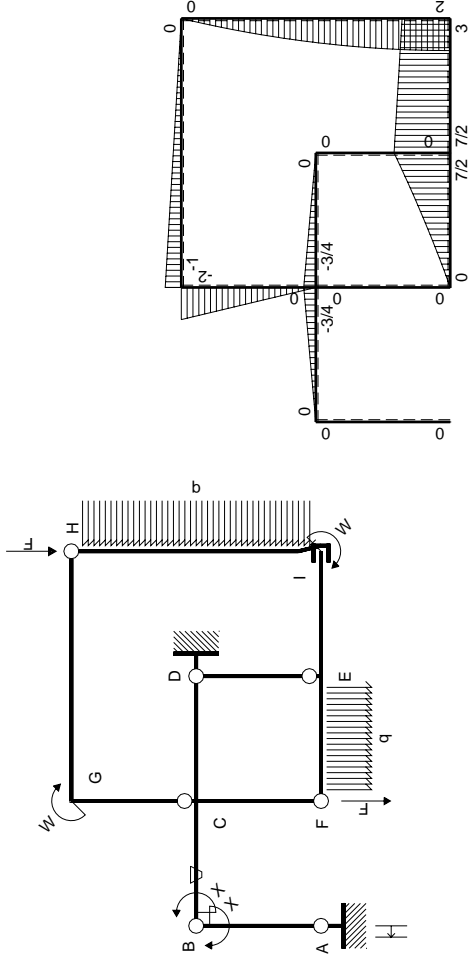
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

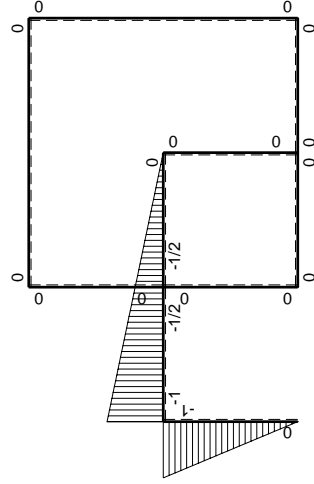
$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

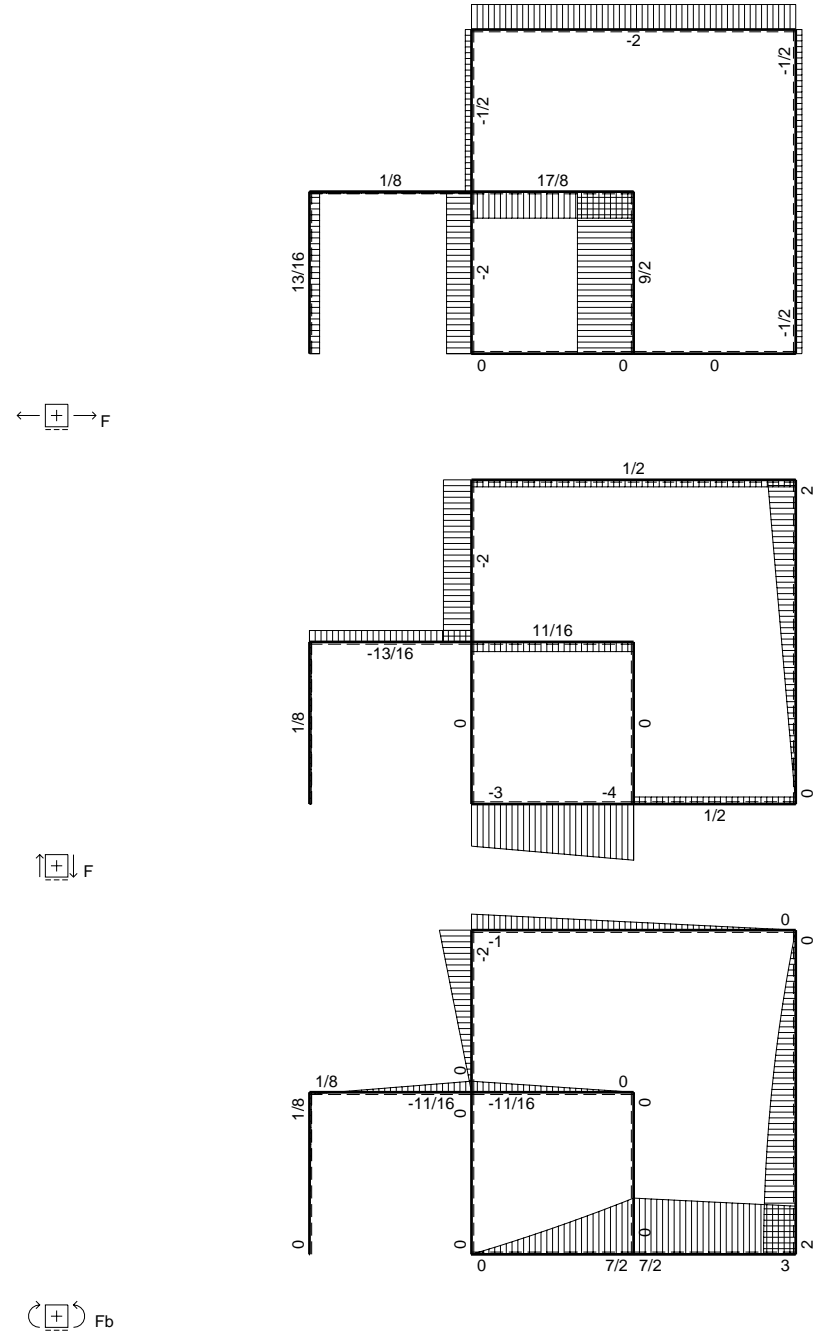
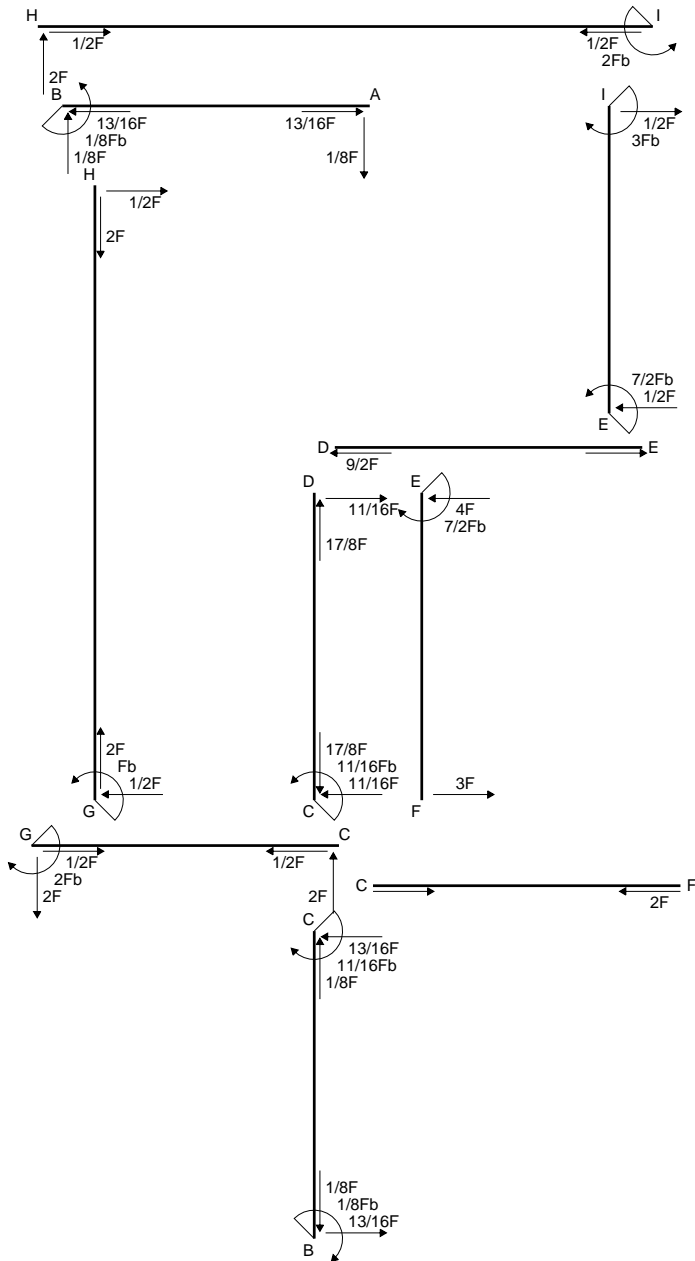
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

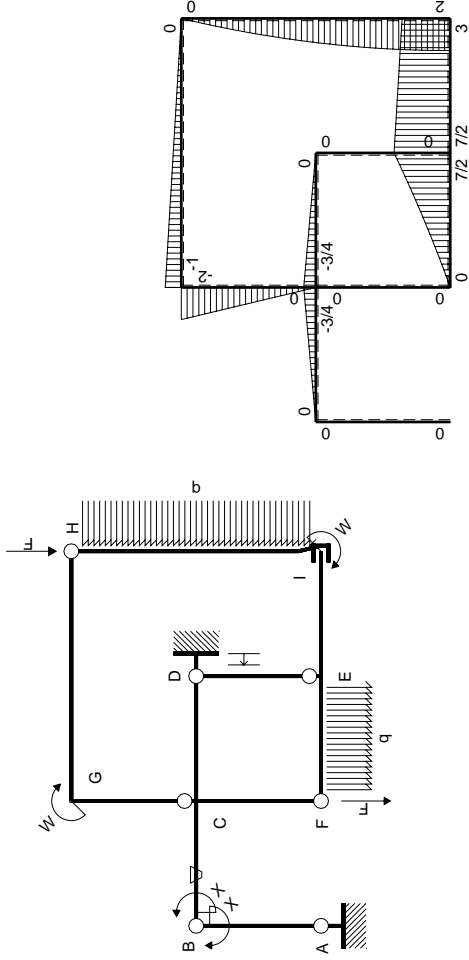
$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$



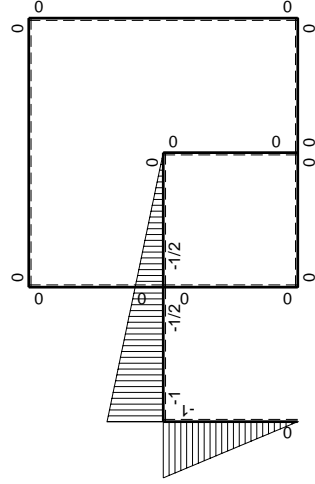


⊕ ⊖ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $1/8Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-1/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

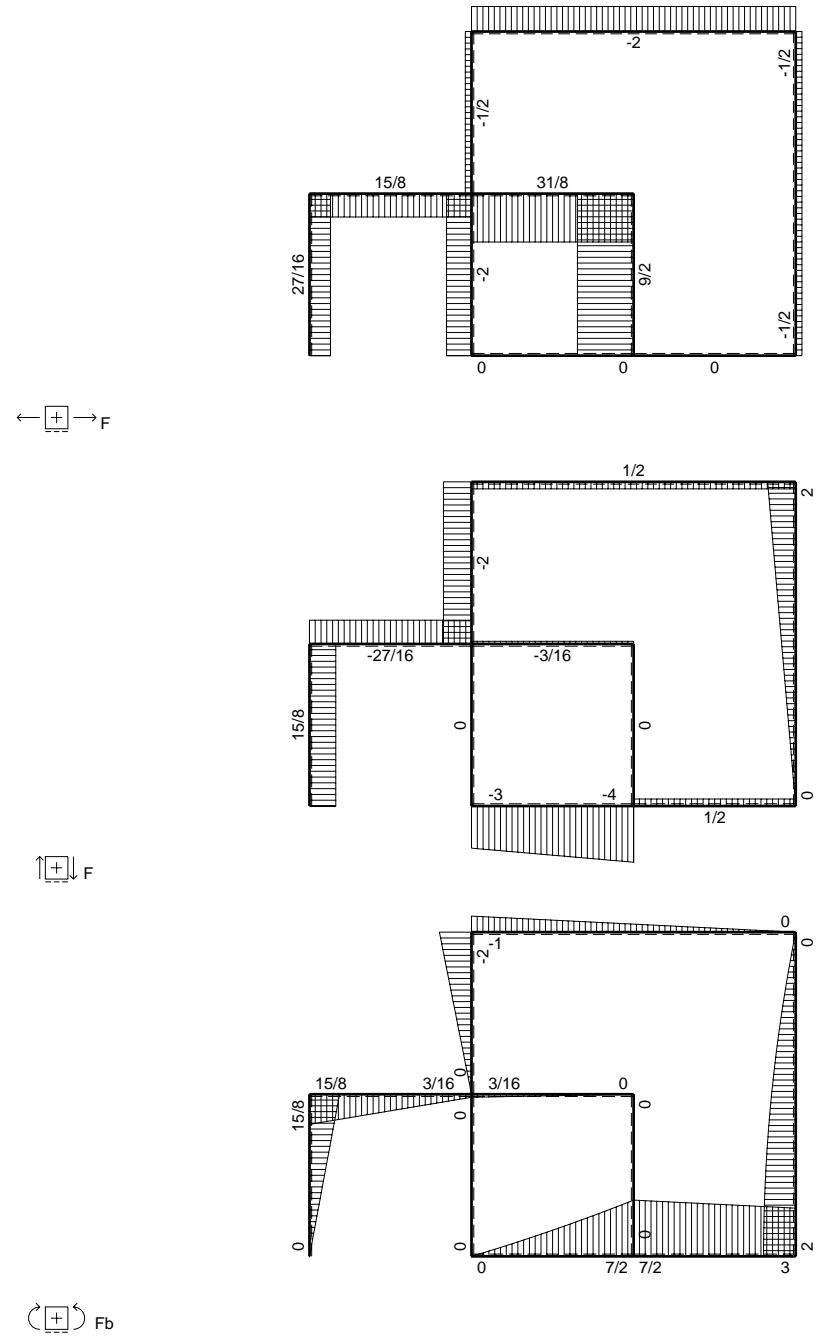
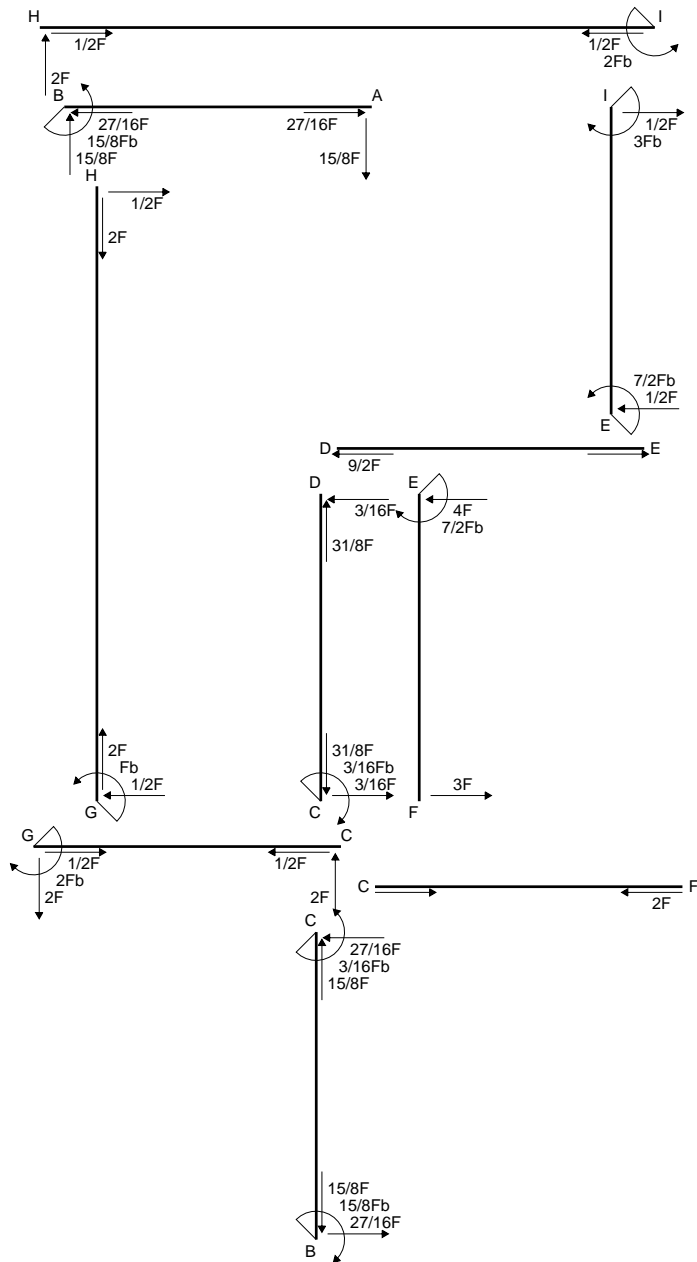
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

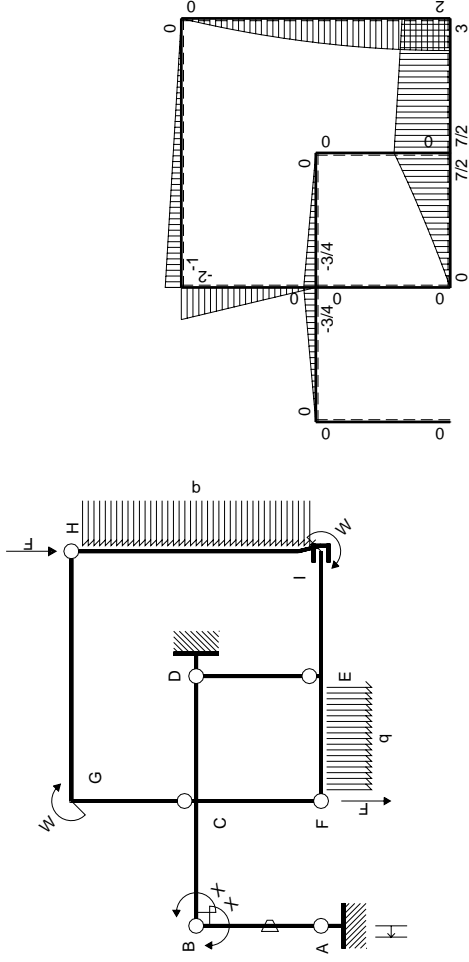
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

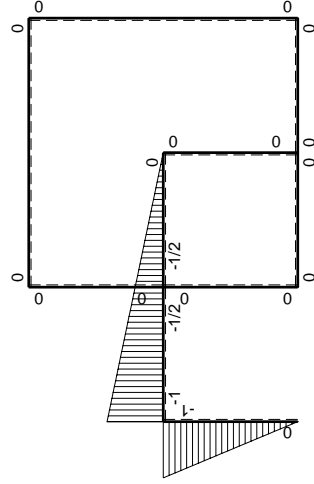
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                          |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                          |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                          |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0             | 0                       |                             |                          |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       |                             |                          |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                          |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                          |
|       | totali                      |                     |          |                         |               |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-15/8Fb$                   |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

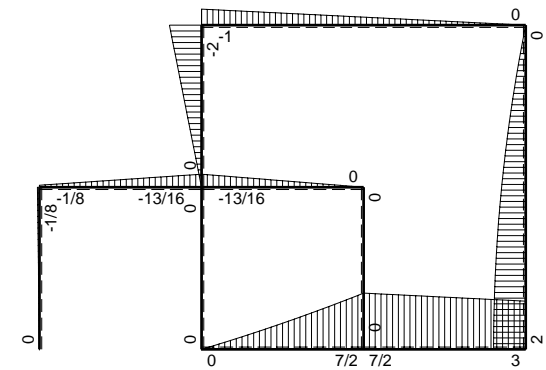
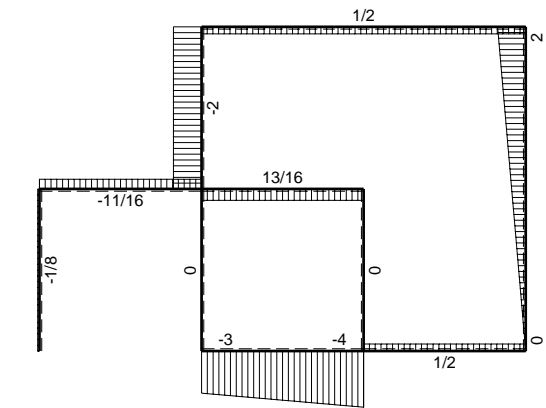
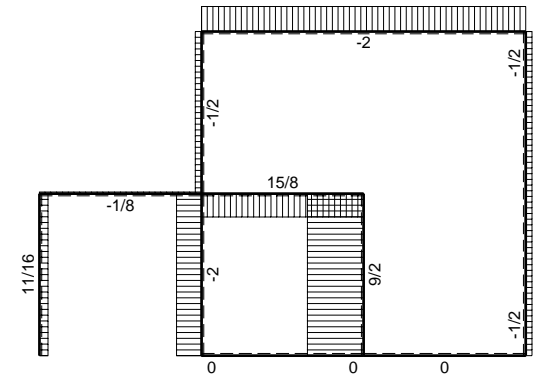
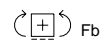
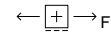
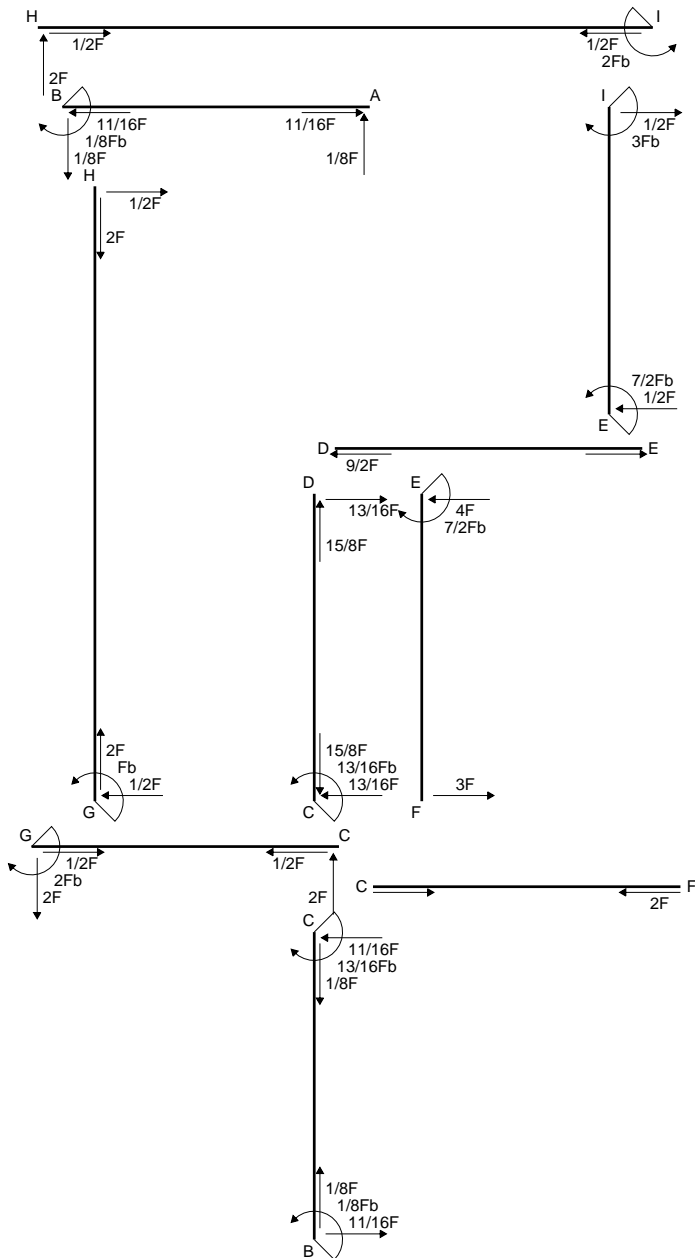
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

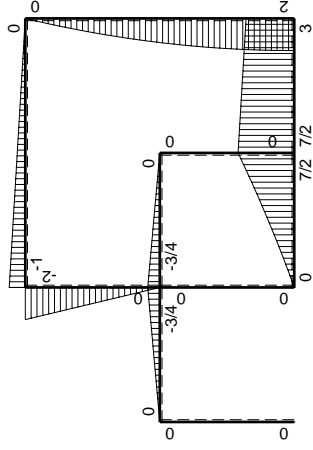
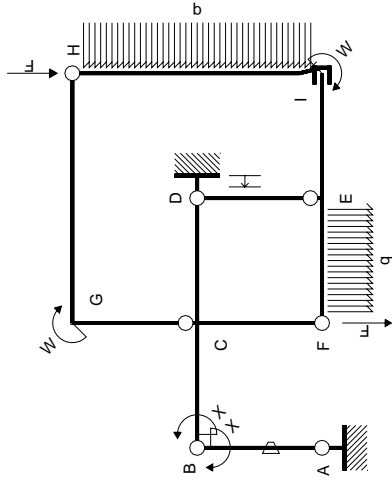
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

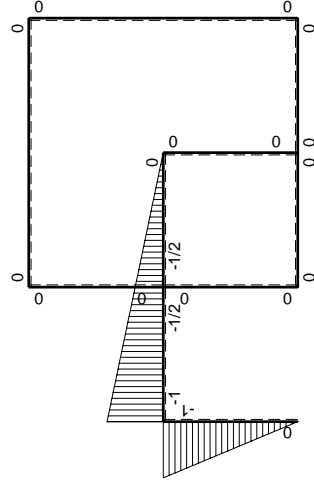






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |               |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

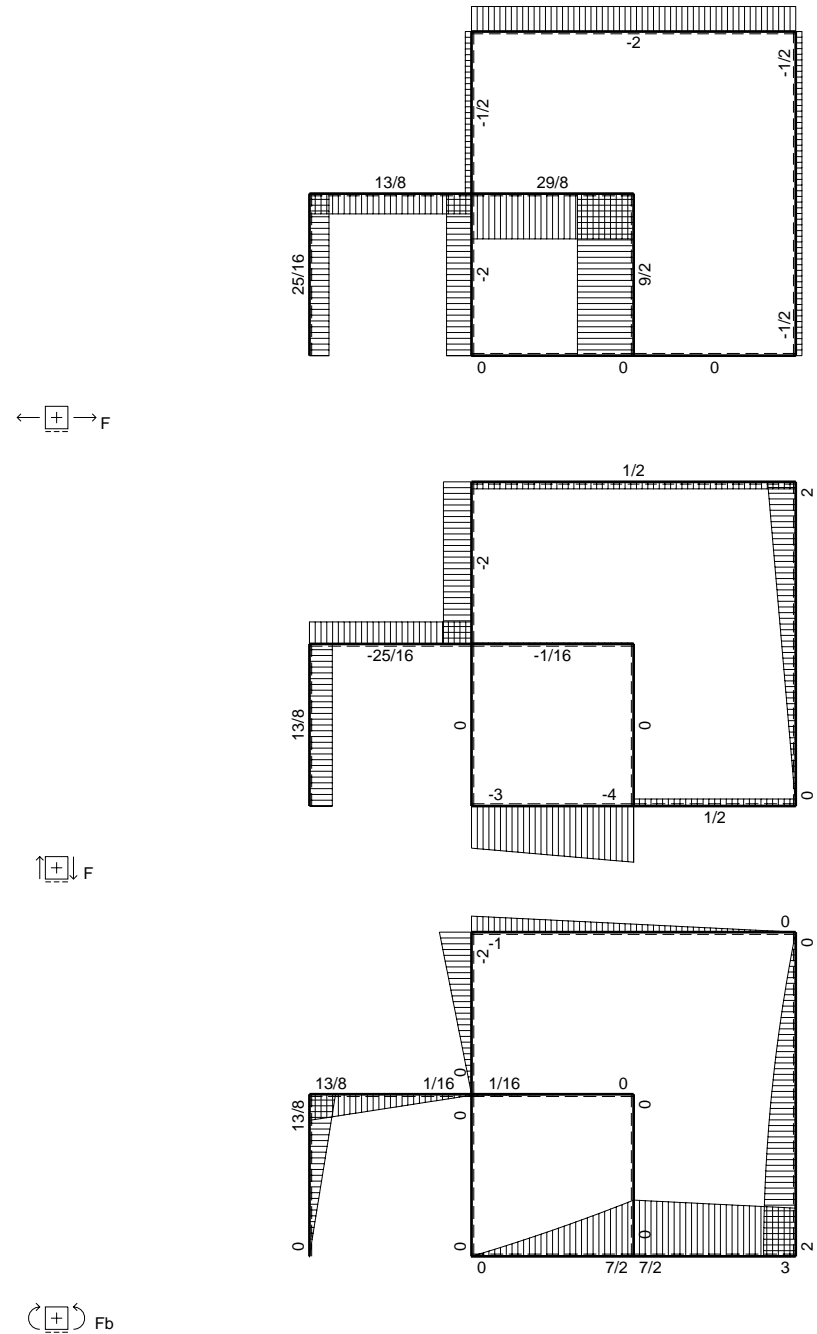
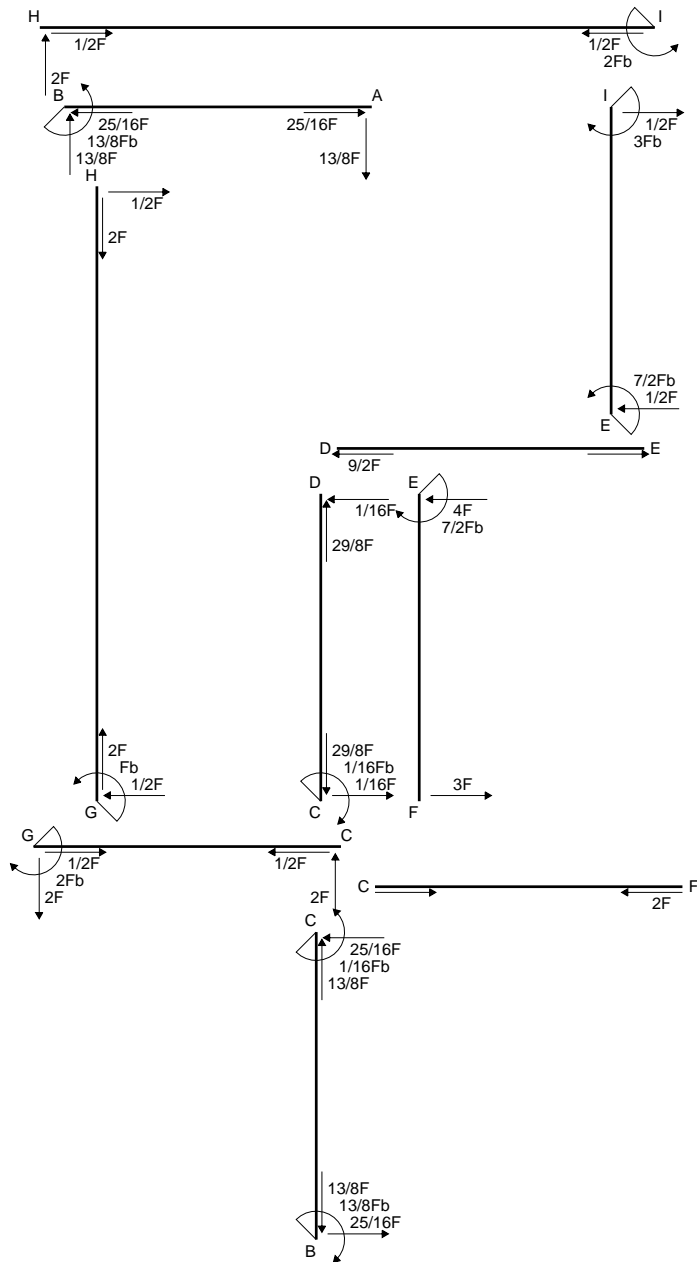
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

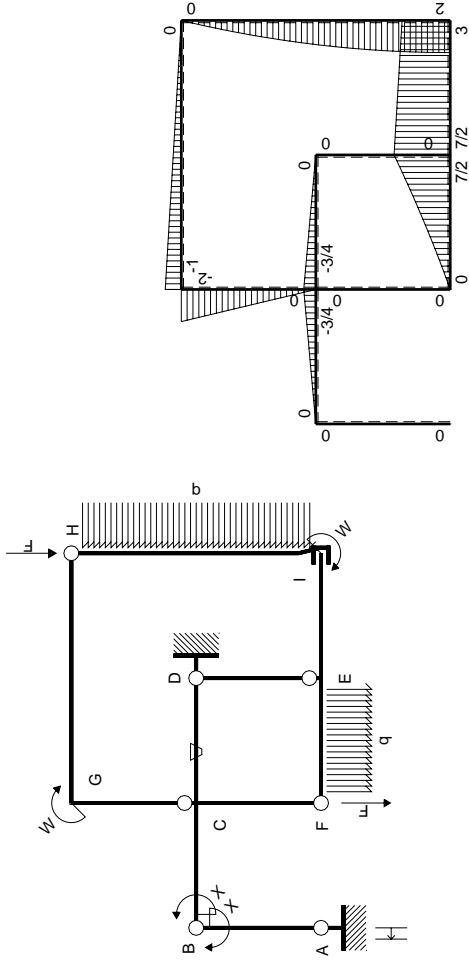
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

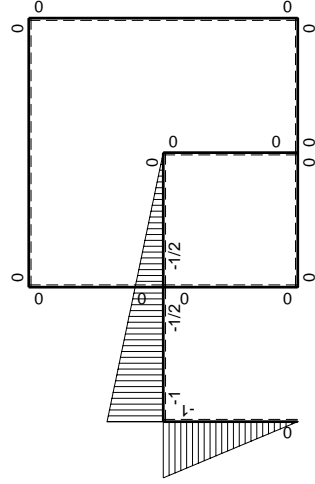
$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $13/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-13/8Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

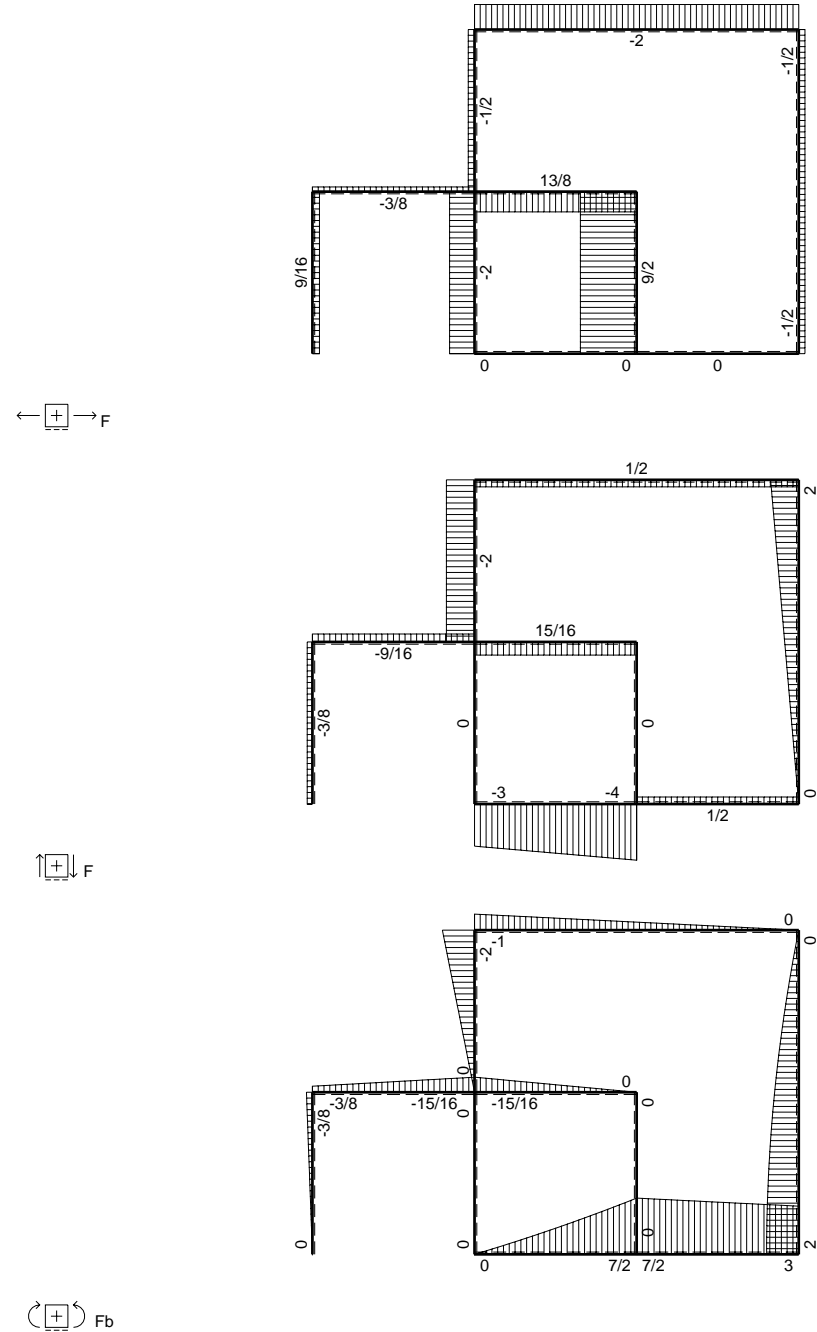
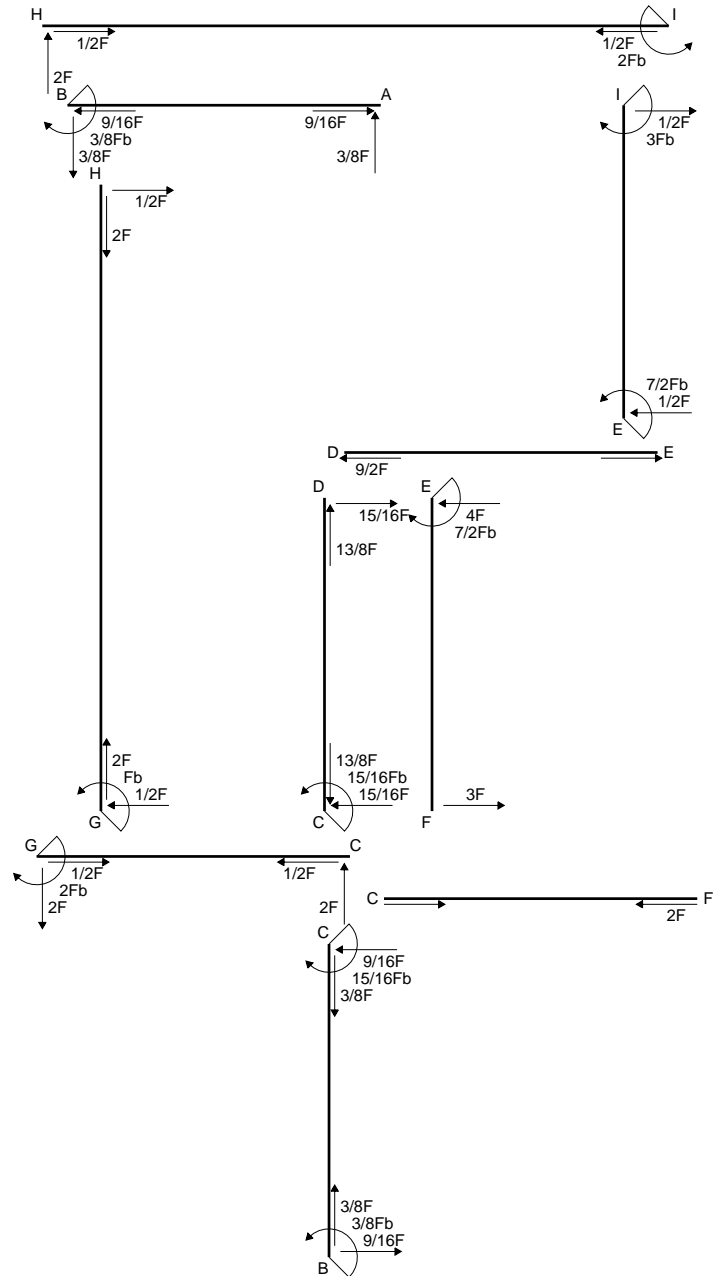
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

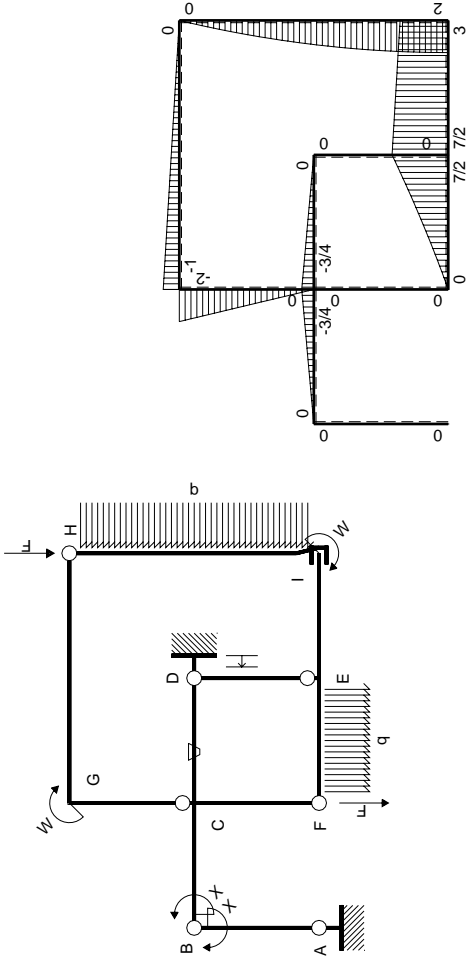
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$

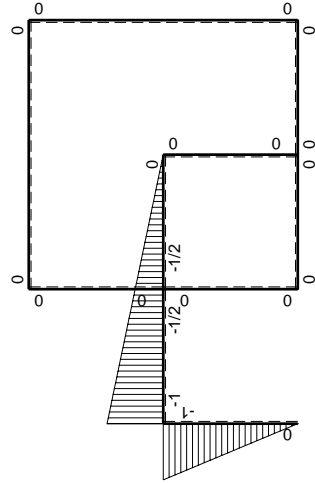






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$              | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$           | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx$         | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-3/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $3/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

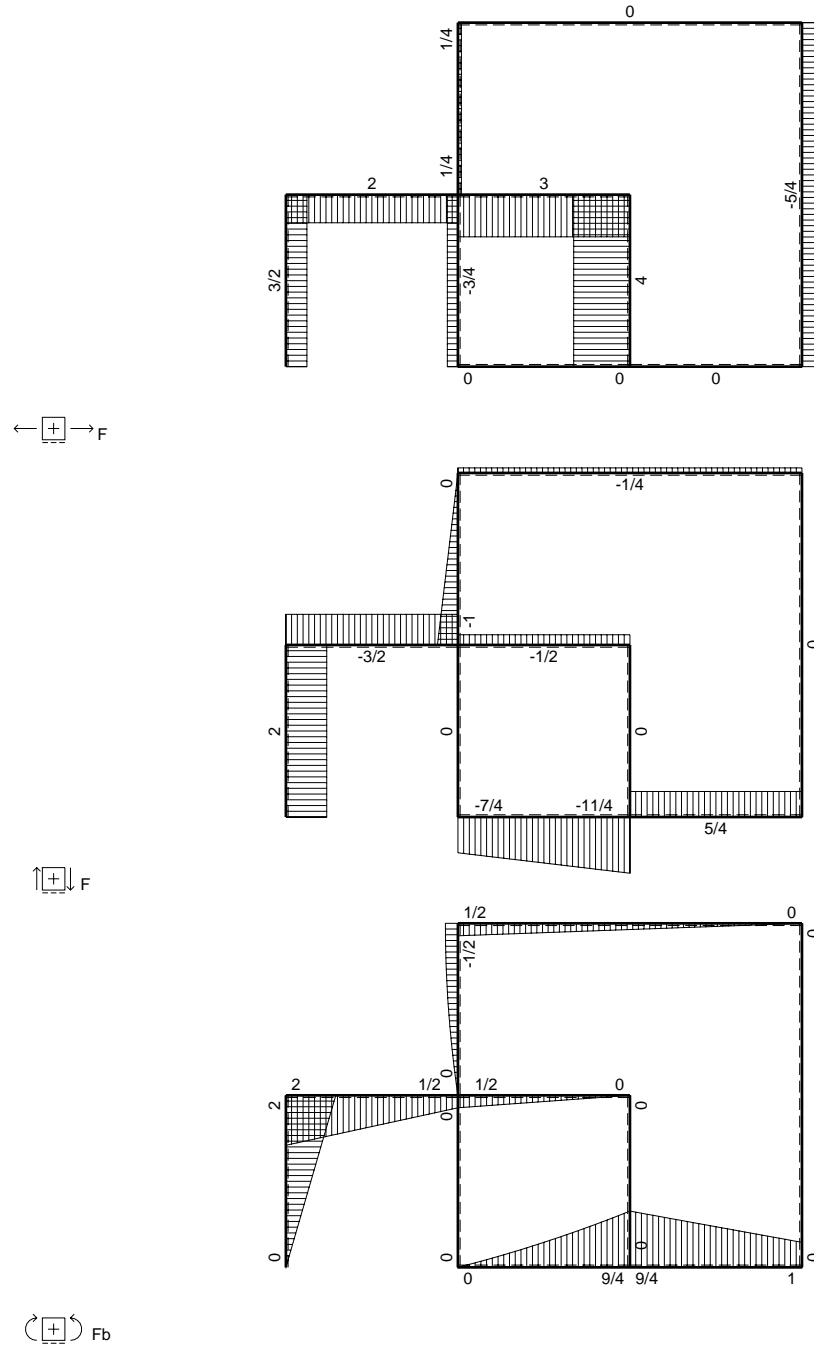
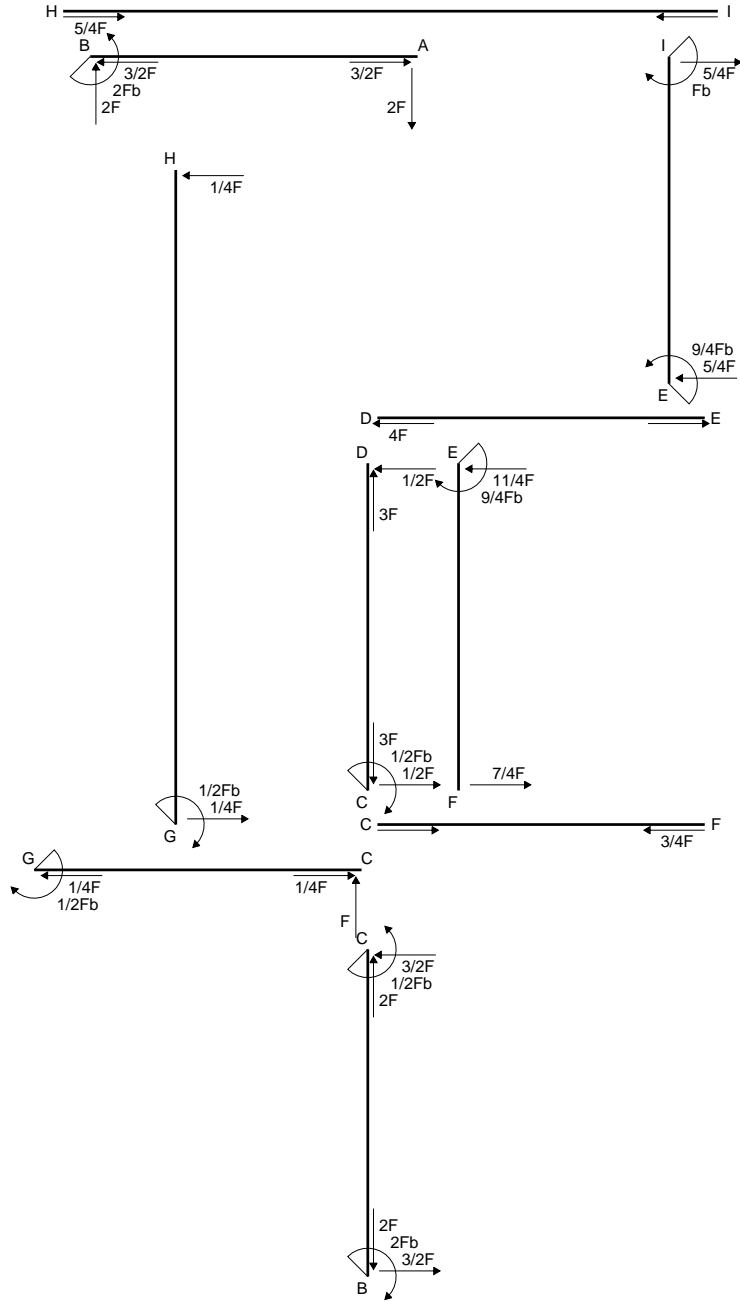
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

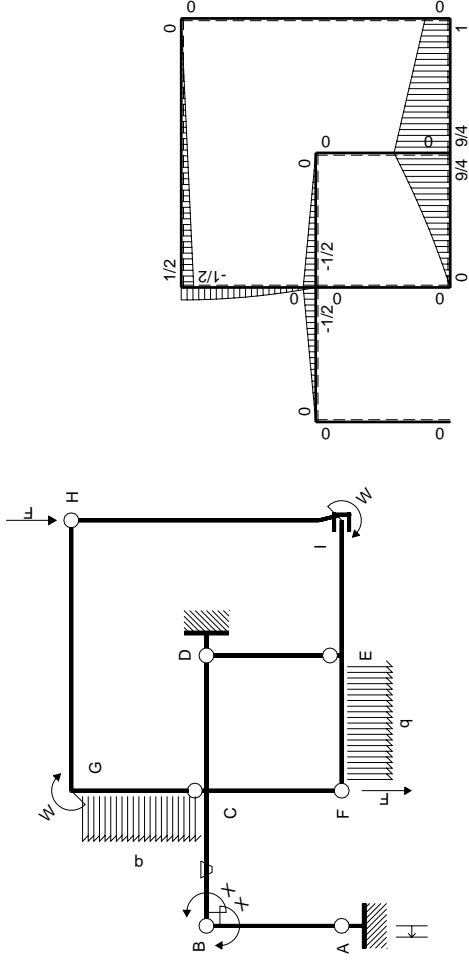
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                                       | $M_o(x)$                        | $\theta$ | $M_x M_o$                         | $M_x \theta$      | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|--|---------------------------------|----------|-----------------------------------|-------------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | -x/b   | 0                               | 0        | 0                                 | 0                 | $x^2/b^2$               | 0+0                         | 1/3Xb/EJ               |  |
| BA b  | 1-x/b  | 0                               | 0        | 0                                 | 0                 | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | -1+1/2x/b                                      | -1/2Fx                          | -Fb/EJ   | 1/2Fx-1/4Fx <sup>2</sup> /b       | Fb/EJ-1/2Fx/EJ    | $1-x/b+1/4x^2/b^2$      | $(1/6+3/4)Fb^2/EJ$          | 7/12Xb/EJ              |  |
| CB b  | 1/2+1/2x/b                                     | 1/2Fb-1/2Fx                     | Fb/EJ    | 1/4Fb-1/4Fx <sup>2</sup> /b       | 1/2Fb/EJ+1/2Fx/EJ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | -1/2+1/2x/b                                    | -1/2Fb+1/2Fx                    | 0        | 1/4Fb-1/2Fx+1/4Fx <sup>2</sup> /b | 0                 | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | 1/12Xb/EJ              |  |
| DC b  | 1/2x/b   | 1/2Fx                           | 0        | 1/4Fx <sup>2</sup> /b             | 0                 | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0  | 0                               | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| ED b  | 0  | 0                               | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| EF b  | 0  | 9/4Fb-11/4Fx+1/2qx <sup>2</sup> | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| FE b  | 0  | -7/4Fx-1/2qx <sup>2</sup>       | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| FC b  | 0  | 0                               | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| CF b  | 0  | 0                               | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| CG b  | 0  | -Fx+1/2qx <sup>2</sup>          | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| GC b  | 0  | 1/2Fb-1/2qx <sup>2</sup>        | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| GH 2b | 0  | 1/2Fb-1/4Fx                     | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0  | -1/4Fx                          | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| HI 2b | 0  | 0                               | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0  | 0                               | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| IE b  | 0  | Fb+5/4Fx                        | 0        | 0                                 | 0                 | 0                       | 0+0                         | 0                      |  |
| EI b  | 0  | -9/4Fb+5/4Fx                    | 0        | 0                                 | 0                 | 0                       |                             |                        |  |
| A     | cedimento nodo -H <sub>1A</sub> u <sub>A</sub> |                                 |          |                                   |                   |                         | Fb <sup>2</sup> /EJ         |                        |  |
|       | totali   |                                 |          |                                   |                   |                         | 2Fb <sup>2</sup> /EJ        | Xb/EJ                  |  |
|       | iperstatica X=W <sub>BC</sub>                  |                                 |          |                                   |                   |                         | -2Fb                        |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

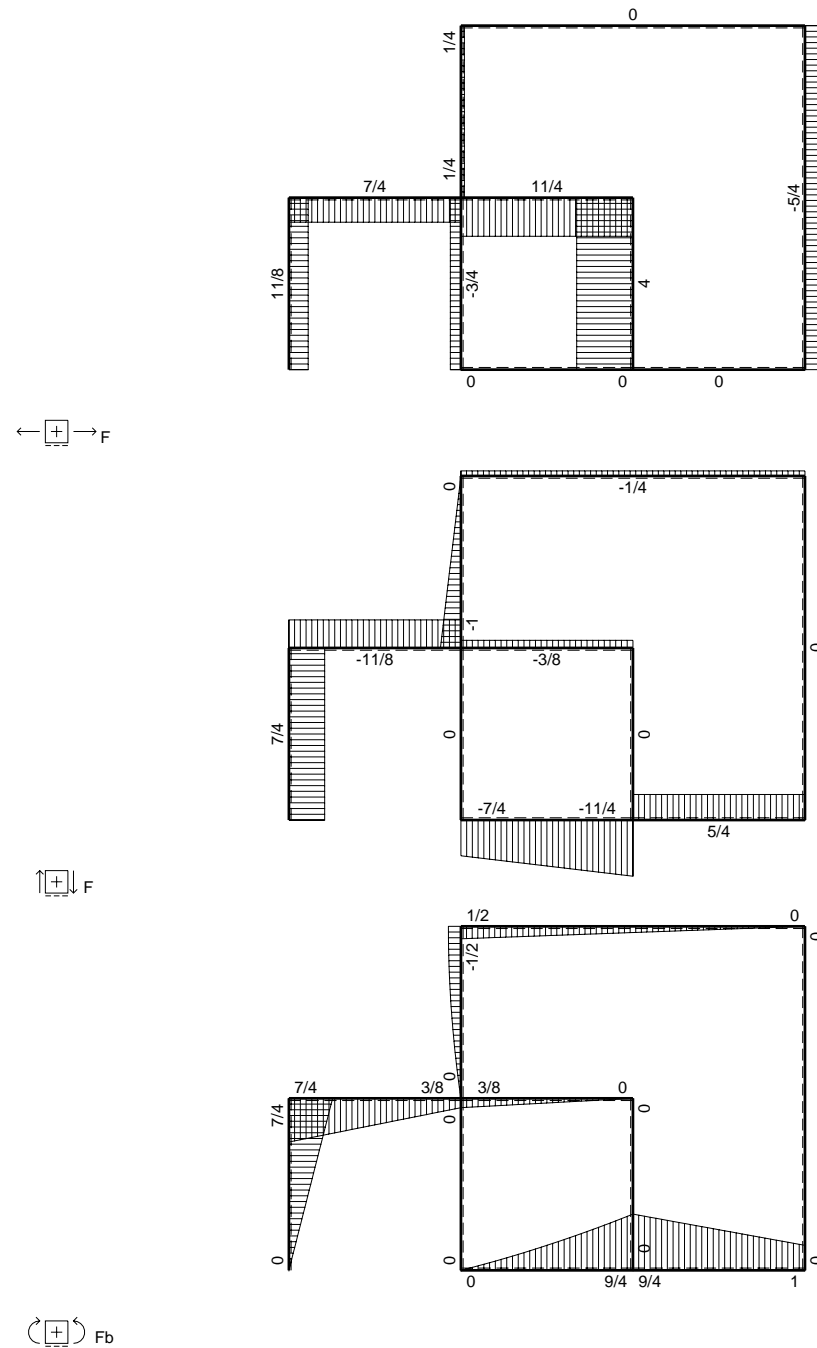
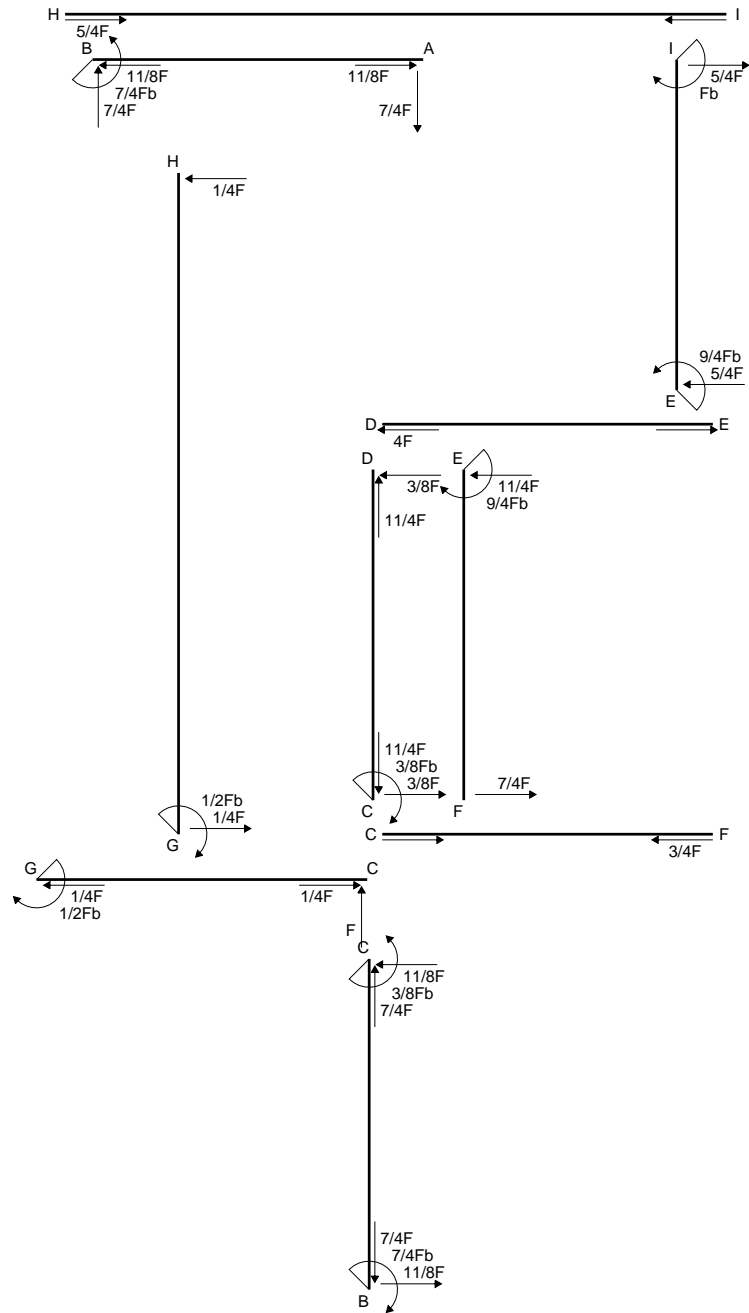
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

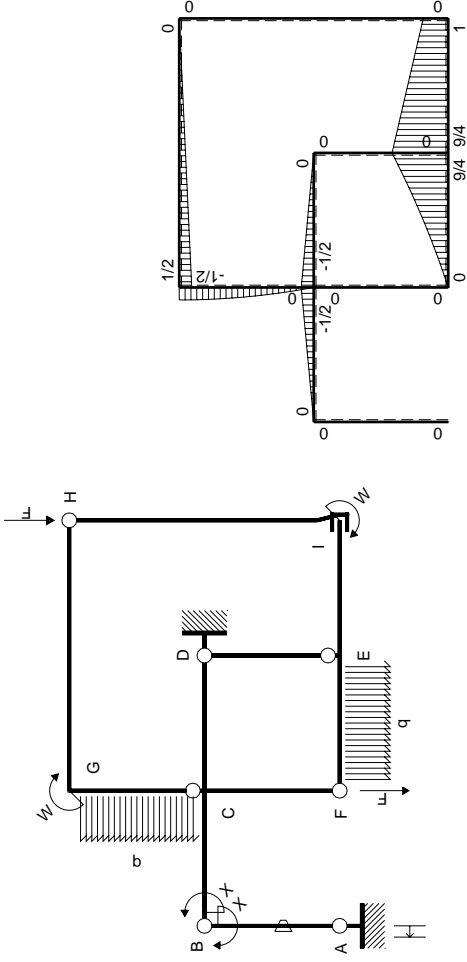
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                        |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                        |          |                         |               |                         | $7/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |               |                         | $-7/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

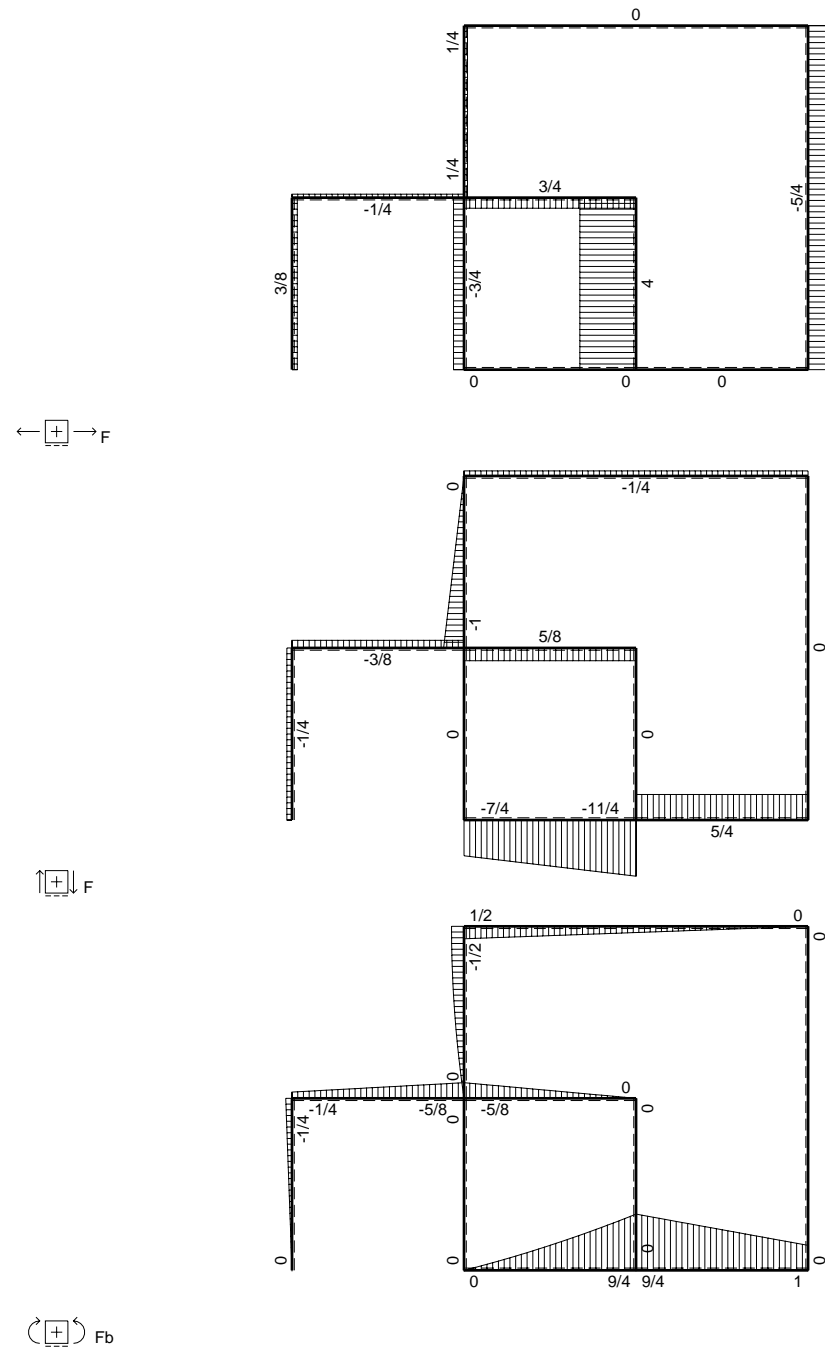
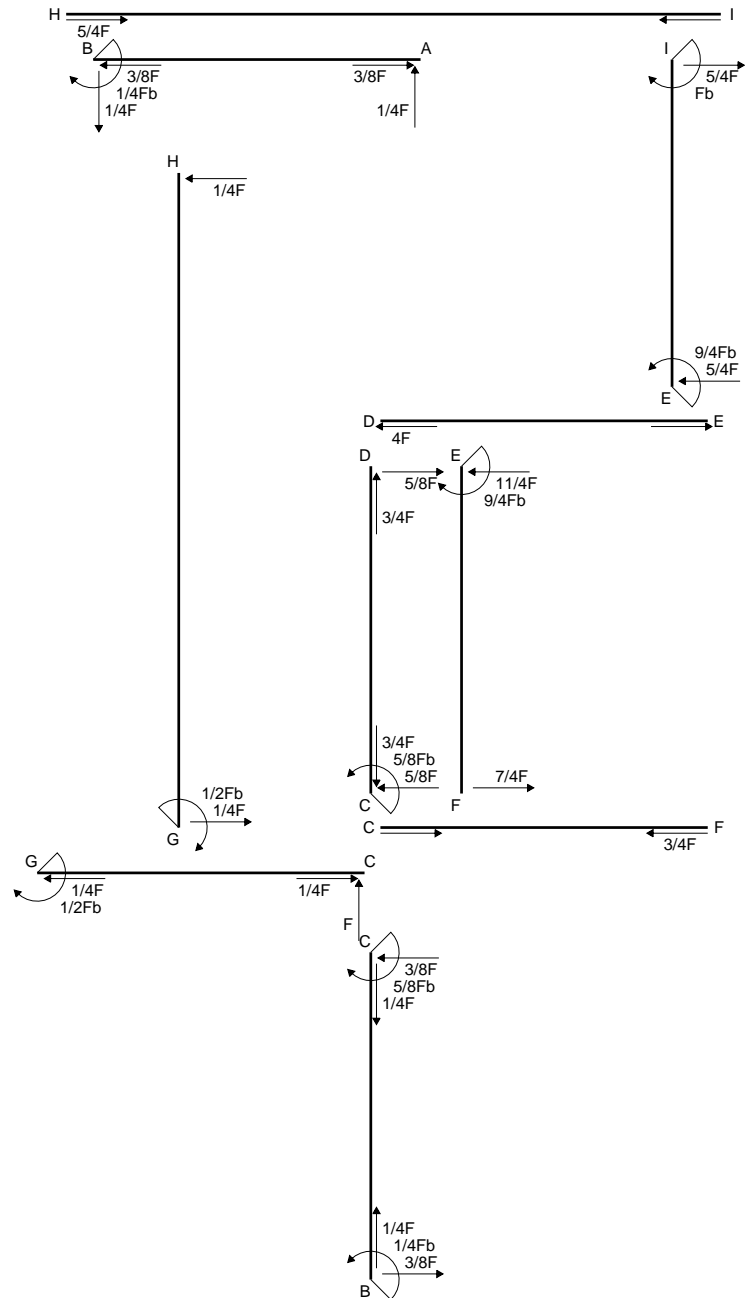
$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

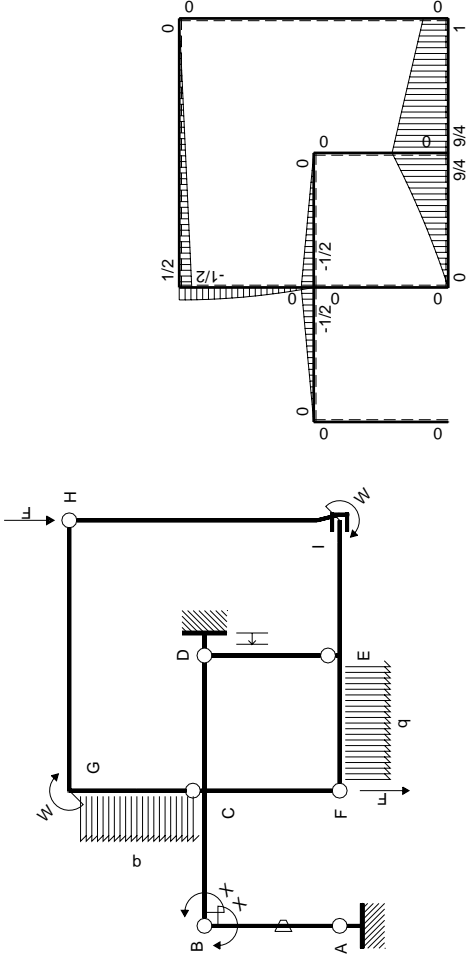
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                        |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                        |          |                         |               |                         | $-1/4Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |               |                         | $1/4Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

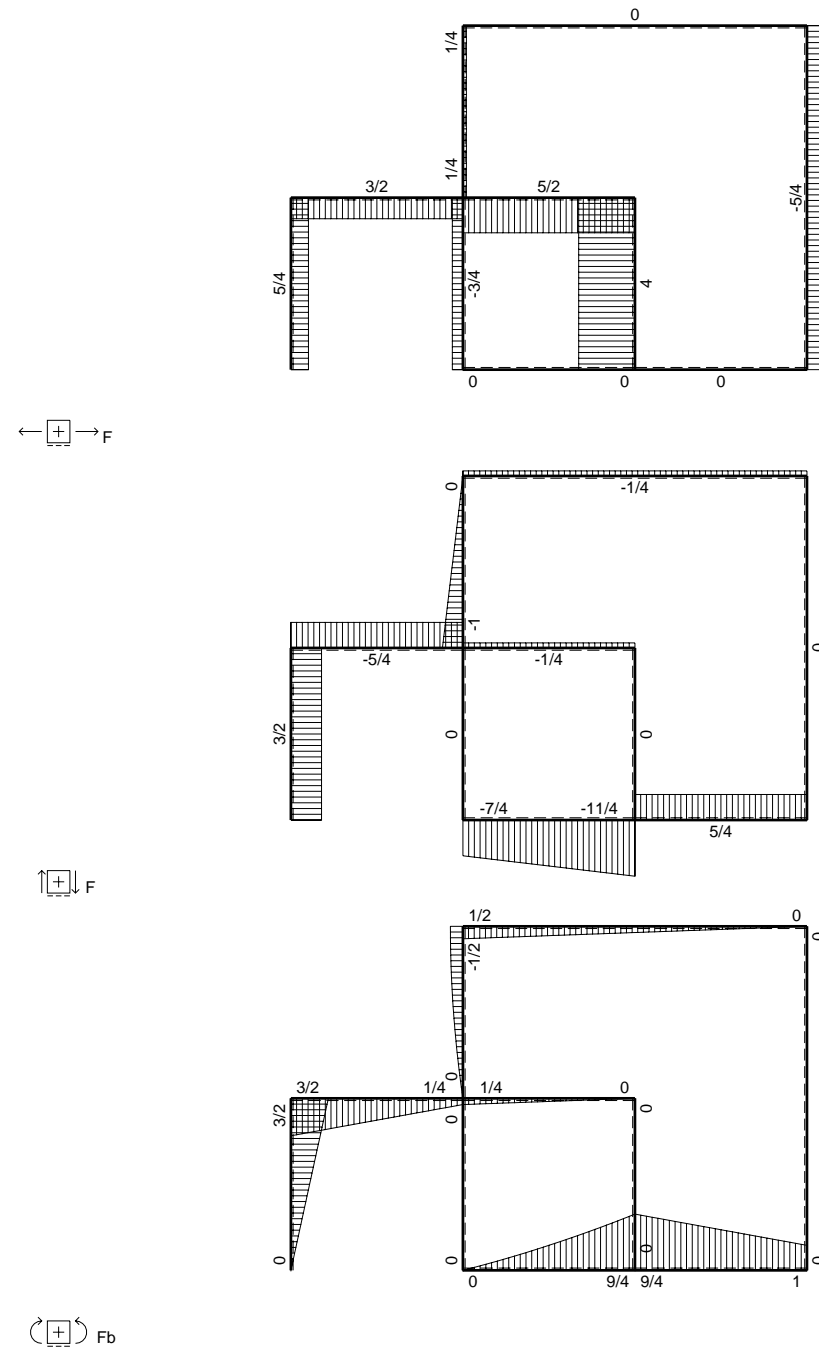
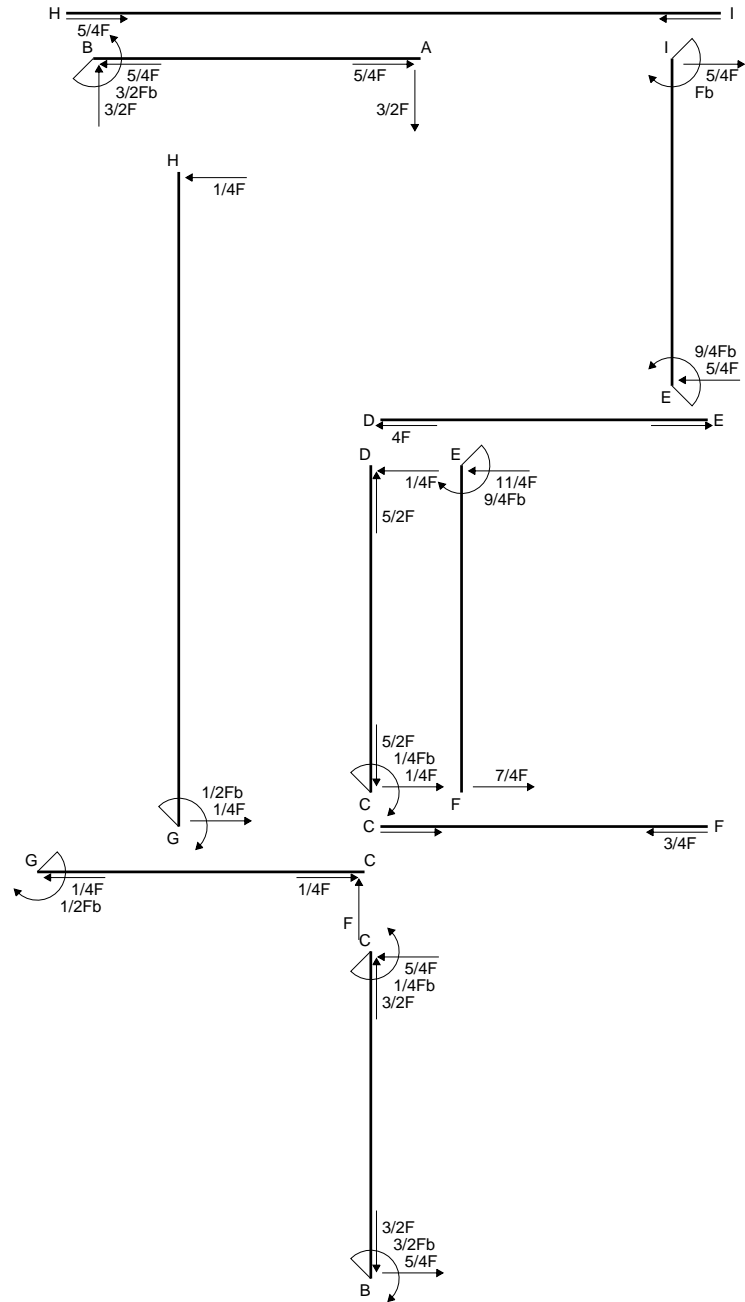
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

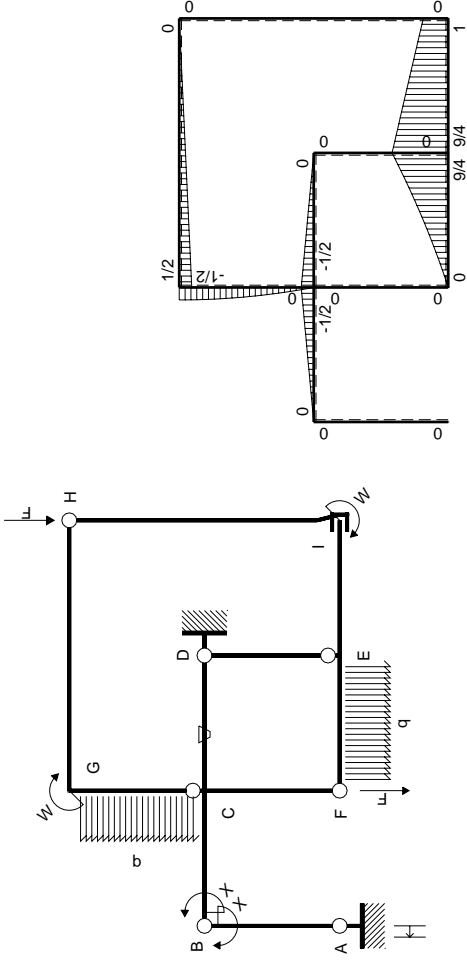
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                        |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                        |          |                         |                     |                         | $3/2Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |                     |                         | $-3/2Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

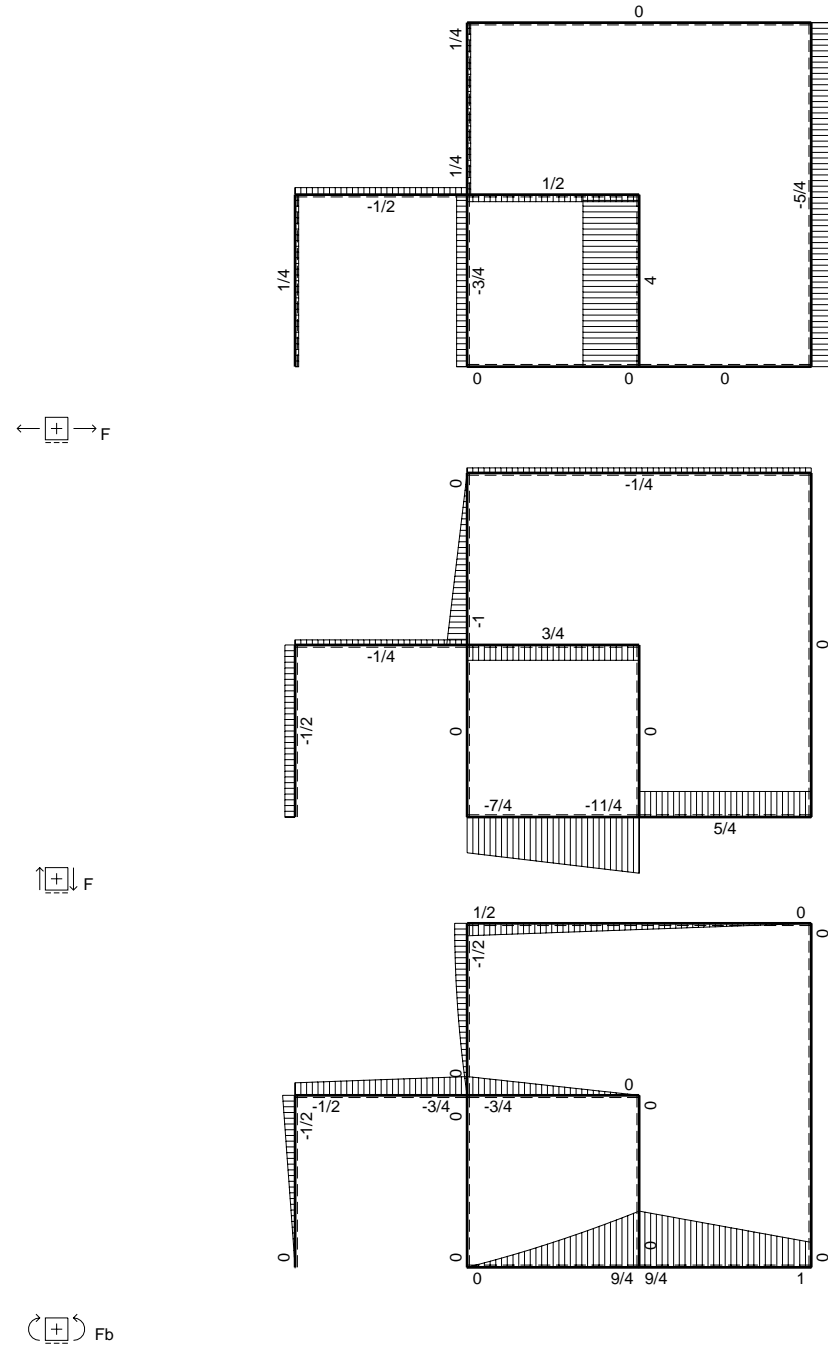
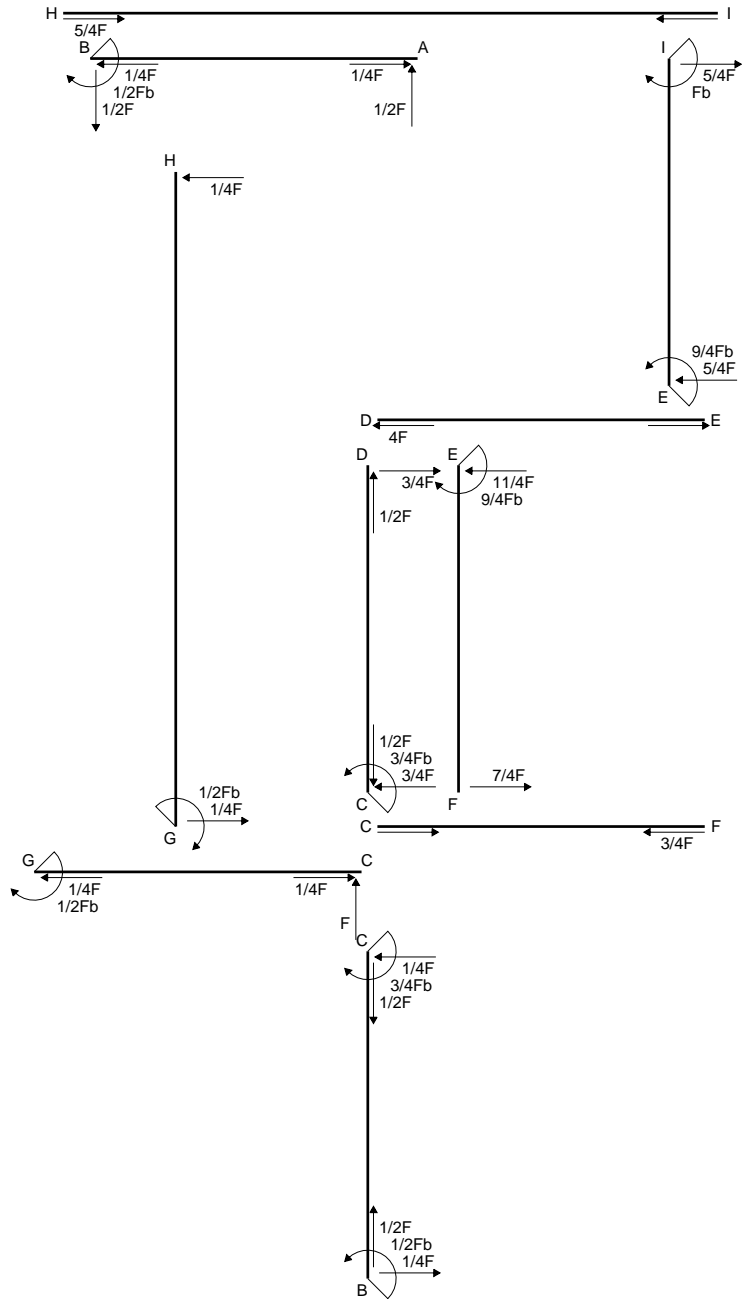
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

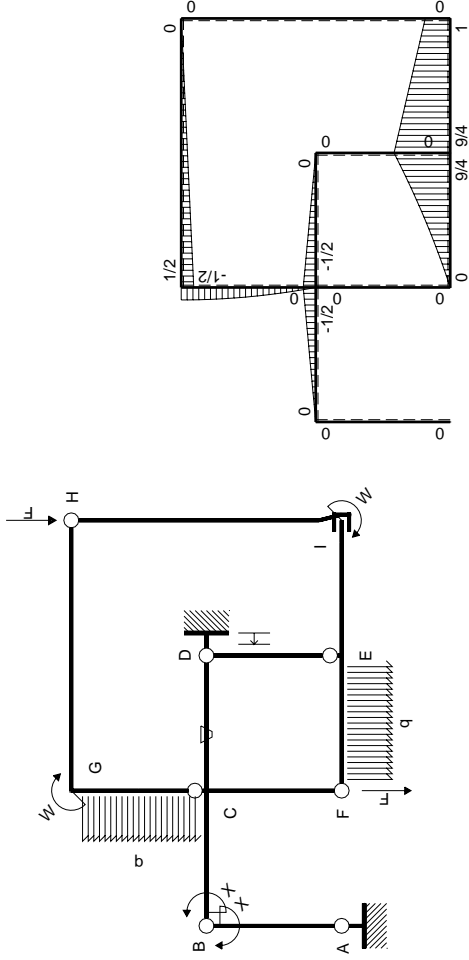
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$               | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|------------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                      | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                      | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$               | 0        | $1/2Fx-1/4Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/6+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$          | 0        | $1/4Fb-1/4Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$         | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$                | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $9/4Fb-11/4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/4Fx-1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$        | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                      | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-9/4Fb+5/4Fx$         | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                        |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                        |          |                         |                     |                         | $-1/2Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                        |          |                         |                     |                         | $1/2Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/12 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

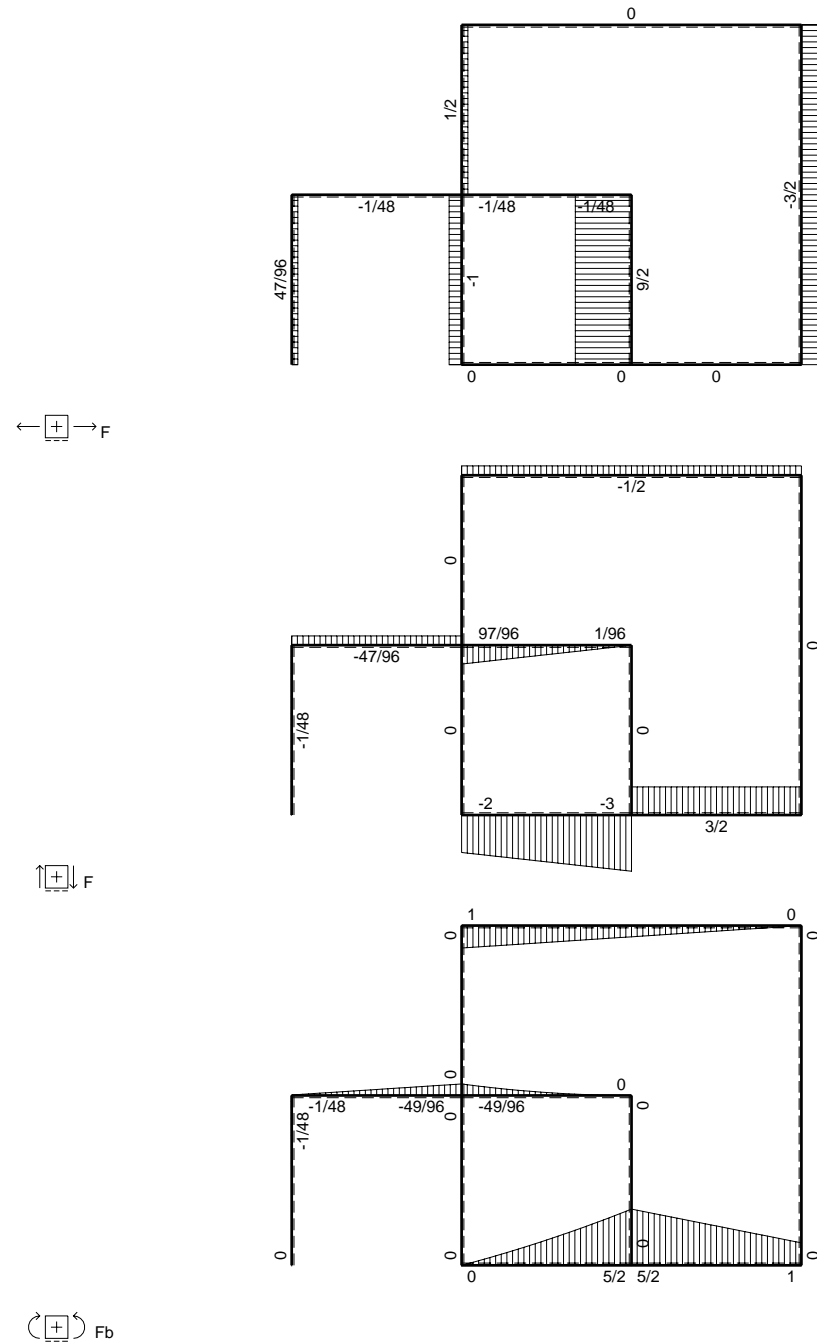
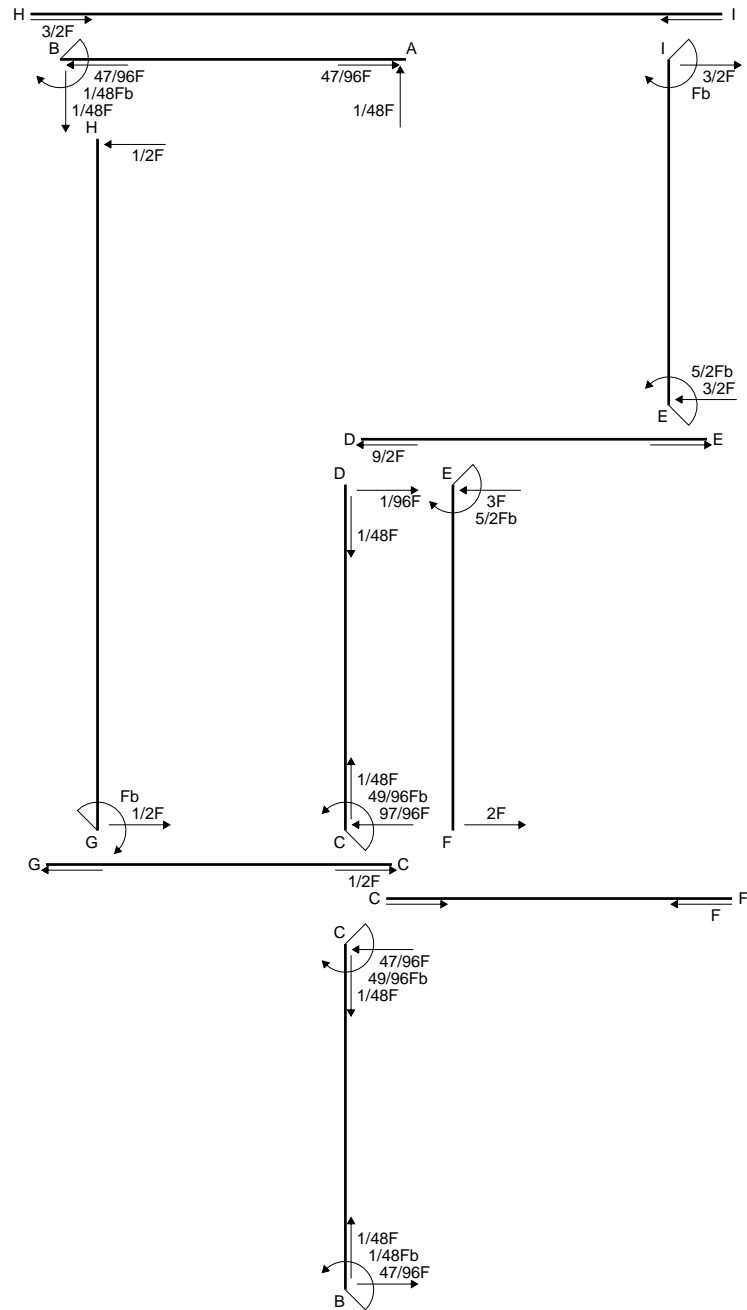
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

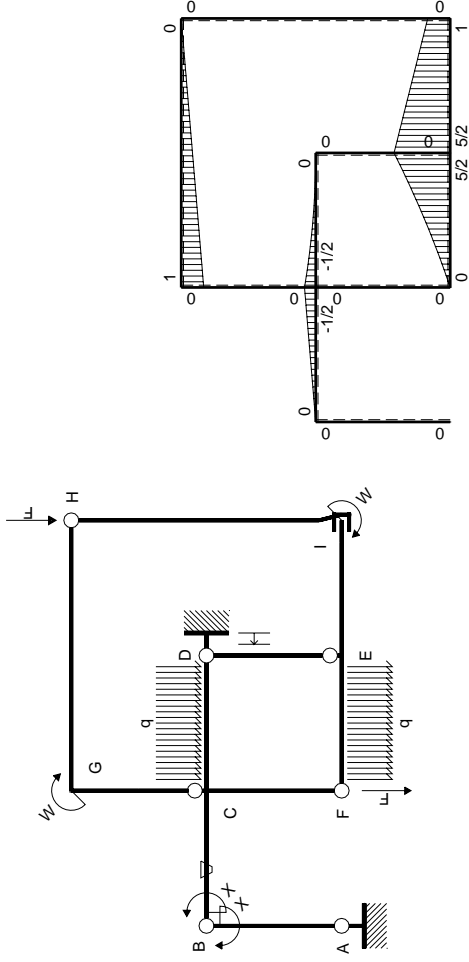
$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$





⊕ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-----------------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                                 | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                                 | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$            | $-Fb/EJ$ | $1/2Fx-1/4Fx^2/b$                 | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/6+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$       | $Fb/EJ$  | $1/4Fb-1/4Fx^2/b$                 | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+Fx-1/2qx^2$ | 0        | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2qx^2$           | 0        | $1/4qx^3/b$                       | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                                 | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                                 | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                                 | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                                   |                     |                         | $-1/48Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                                   |                     |                         | $1/48Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

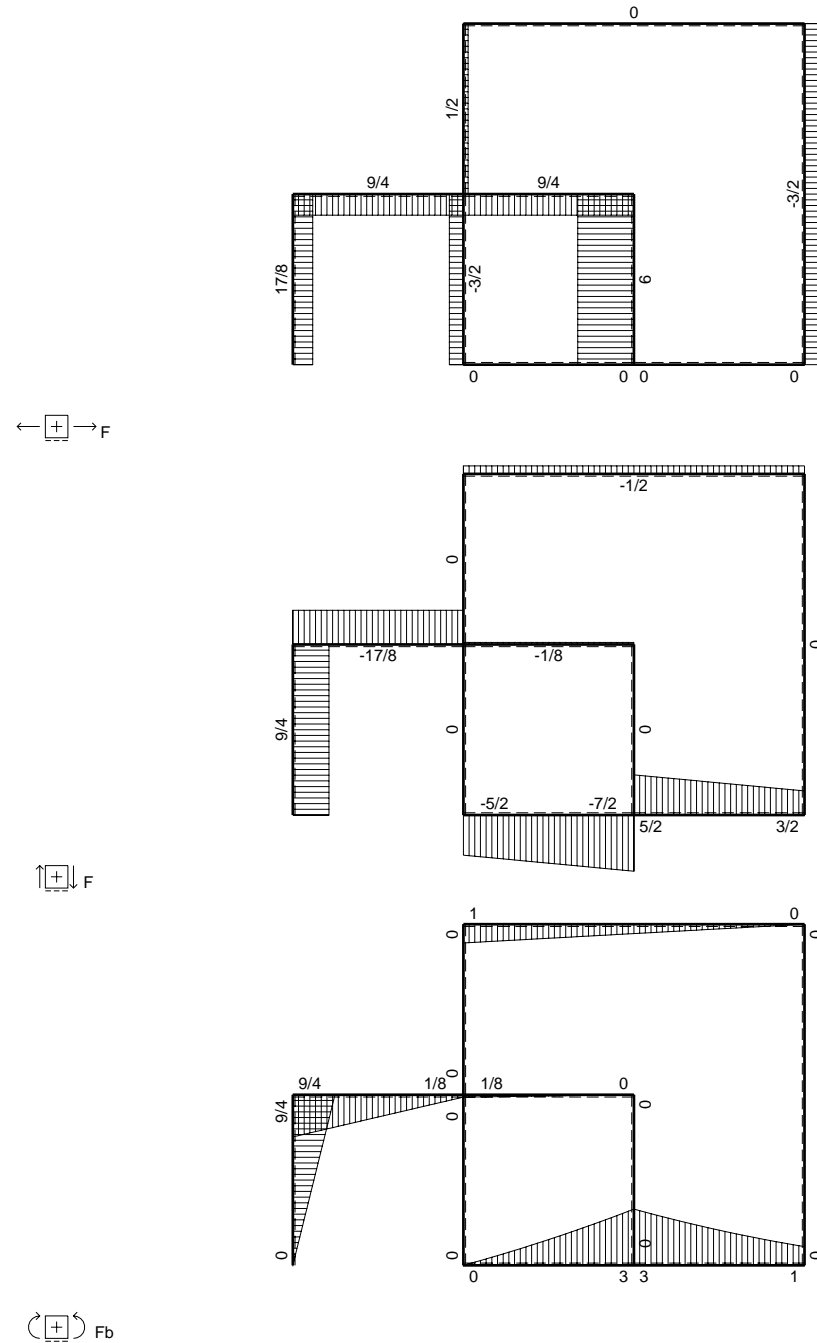
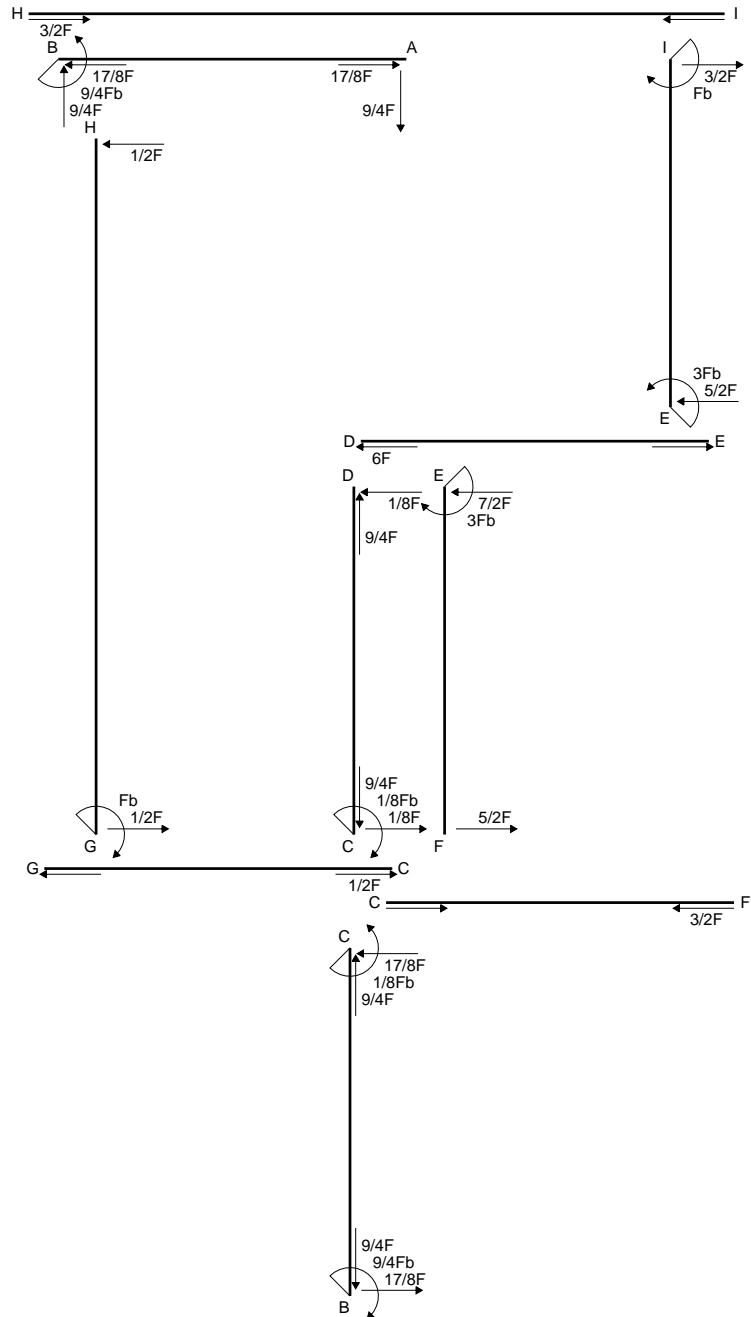
$$L_{CD}^{xo} = \int_0^b (1/4 - 3/4 x/b + 3/4 x^2/b^2 - 1/4 x^3/b^3) Fb 1/EJ dx$$

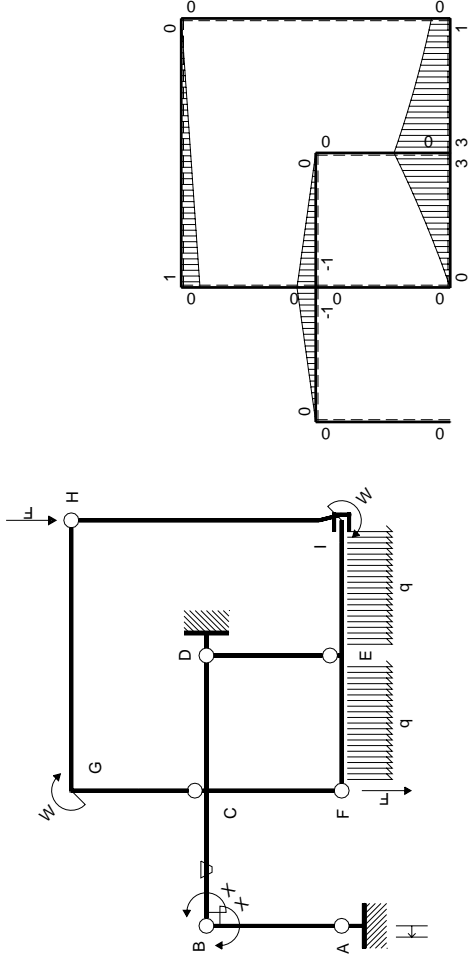
$$= [1/4 x - 3/8 x^2/b + 1/4 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 3/8 b + 1/4 b - 1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^3/b^3) Fb 1/EJ dx = [1/16 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |                     |                         | $9/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-9/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

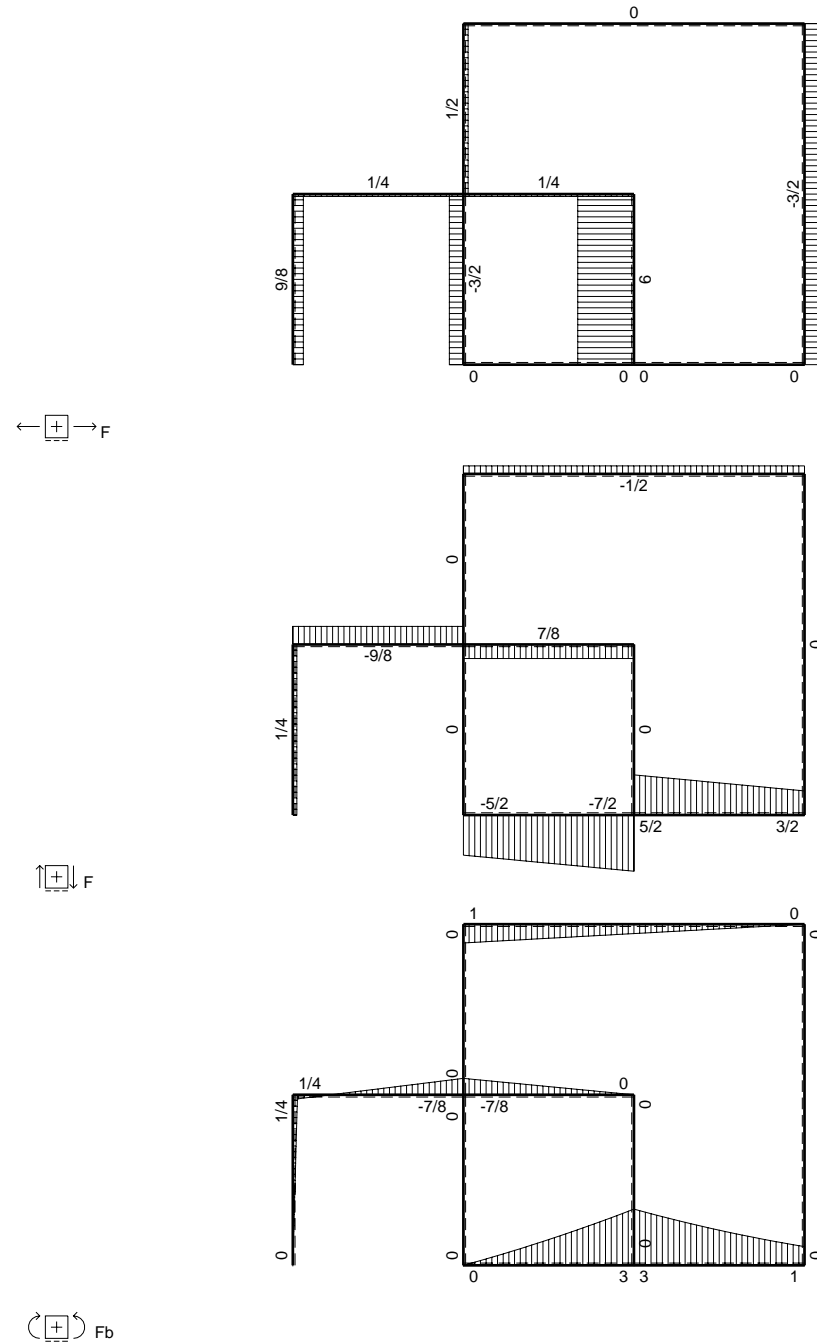
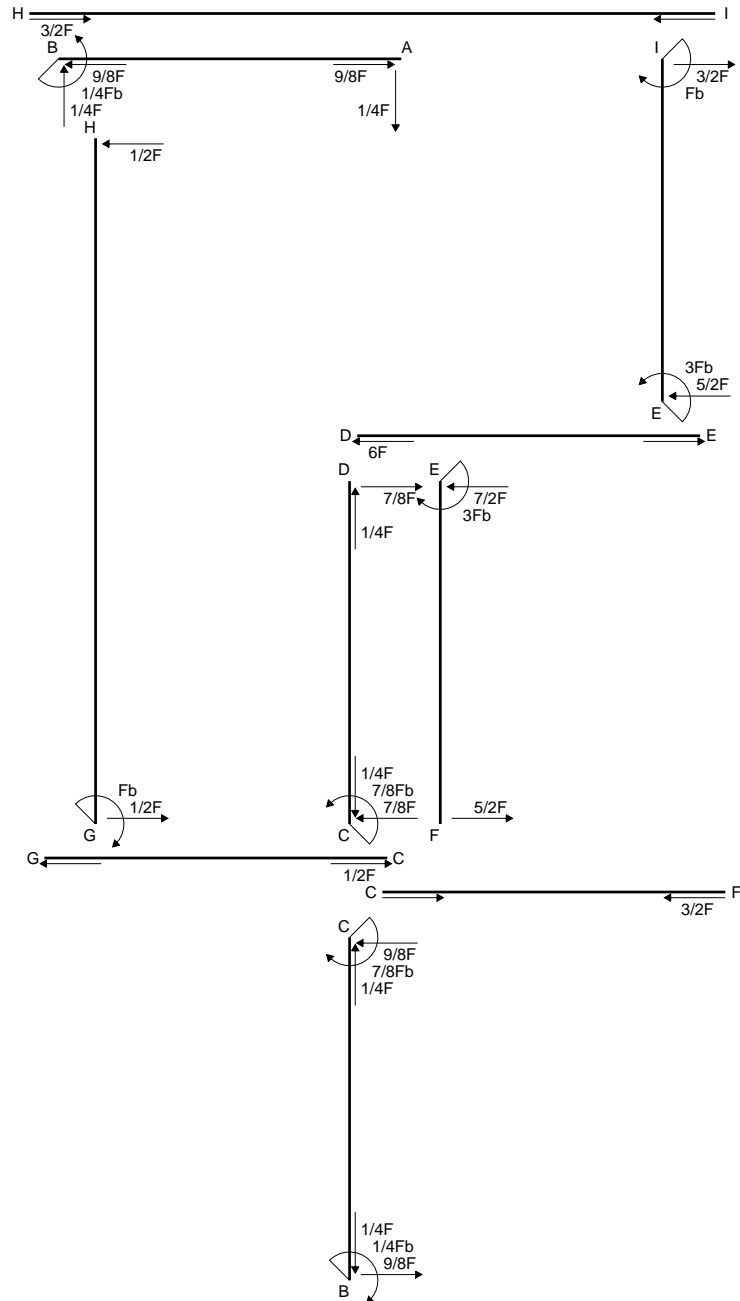
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

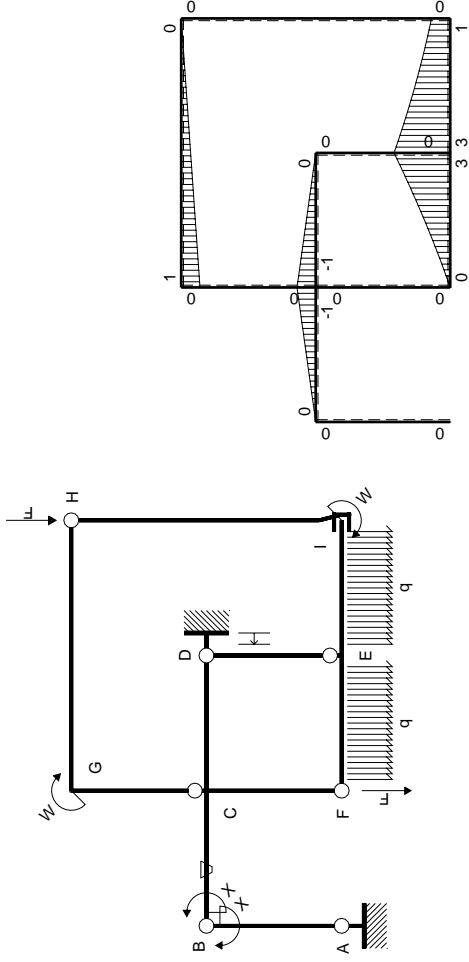
$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





⊕ ⊖ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $1/4Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $-1/4Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

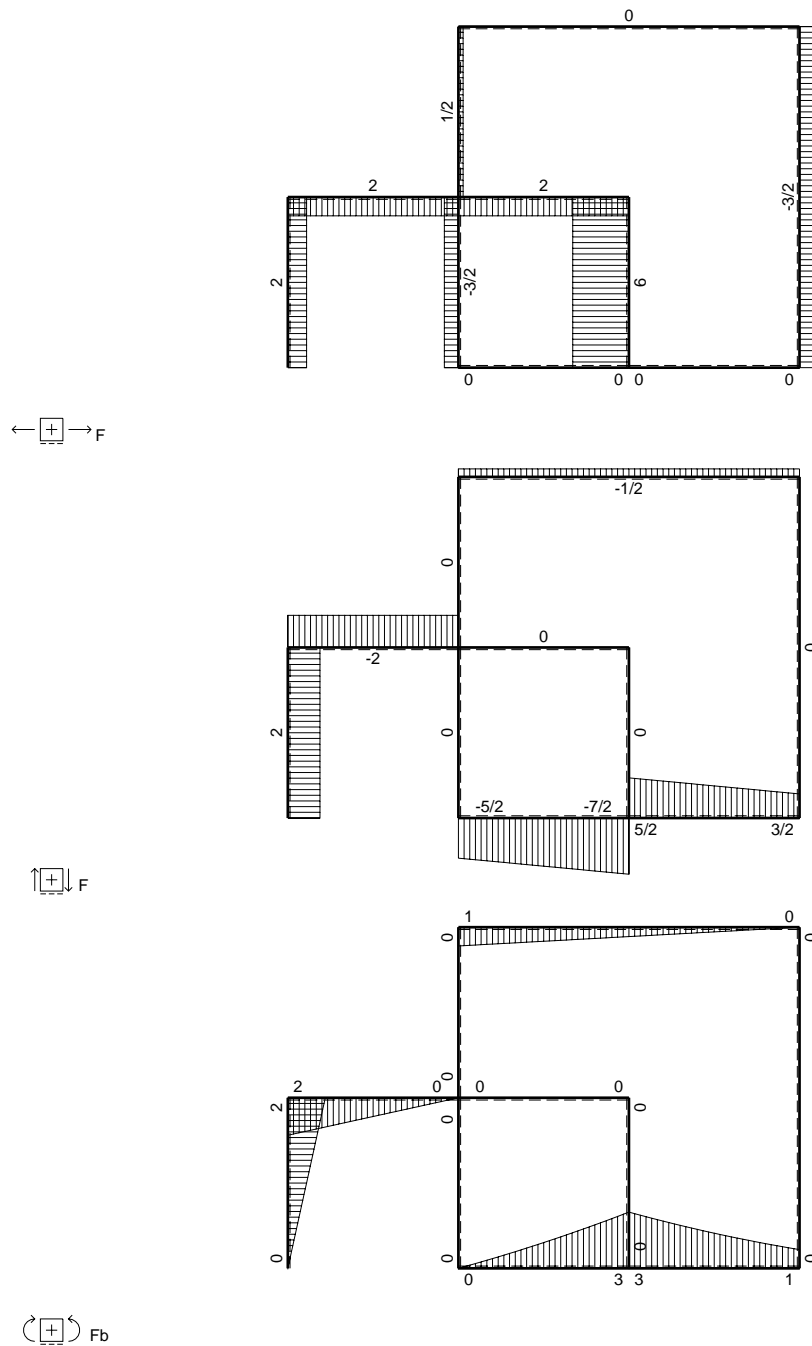
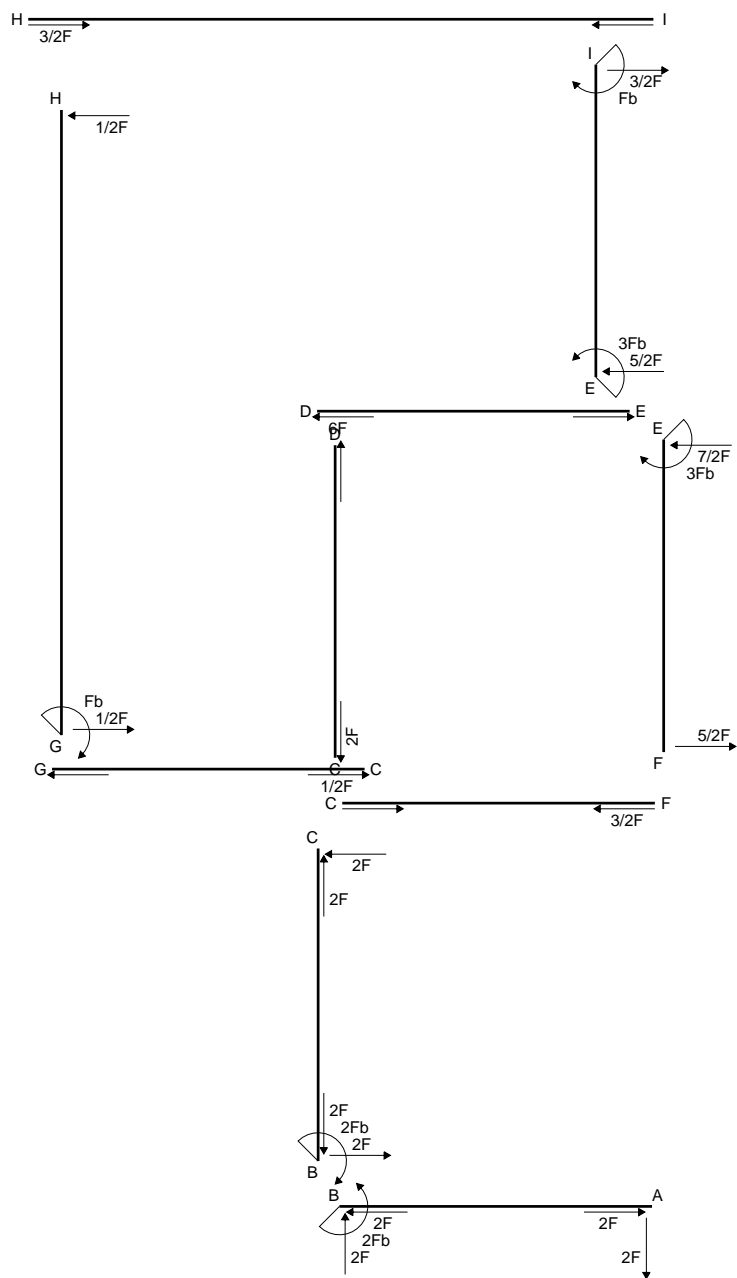
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

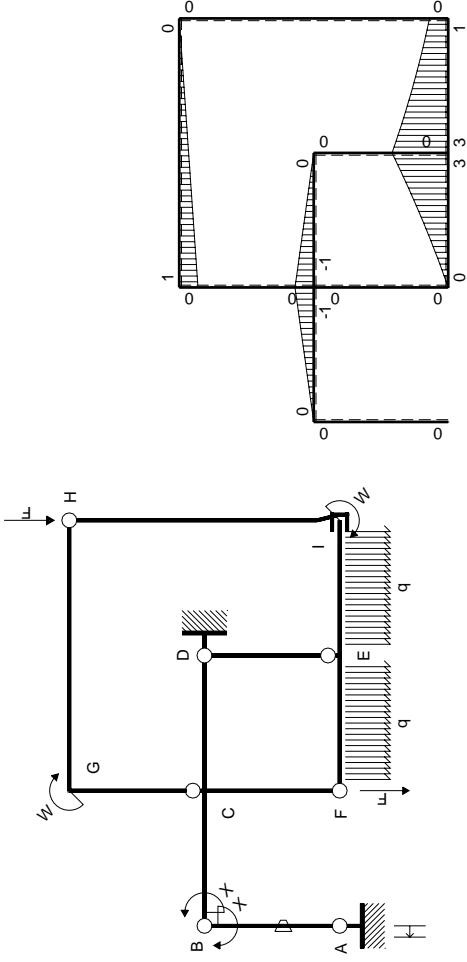
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | 0        | $Fx-1/2Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | 0        | $1/2Fb-1/2Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |               |                         | $2Fb^2/EJ$                  | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |               |                         | $-2Fb$                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

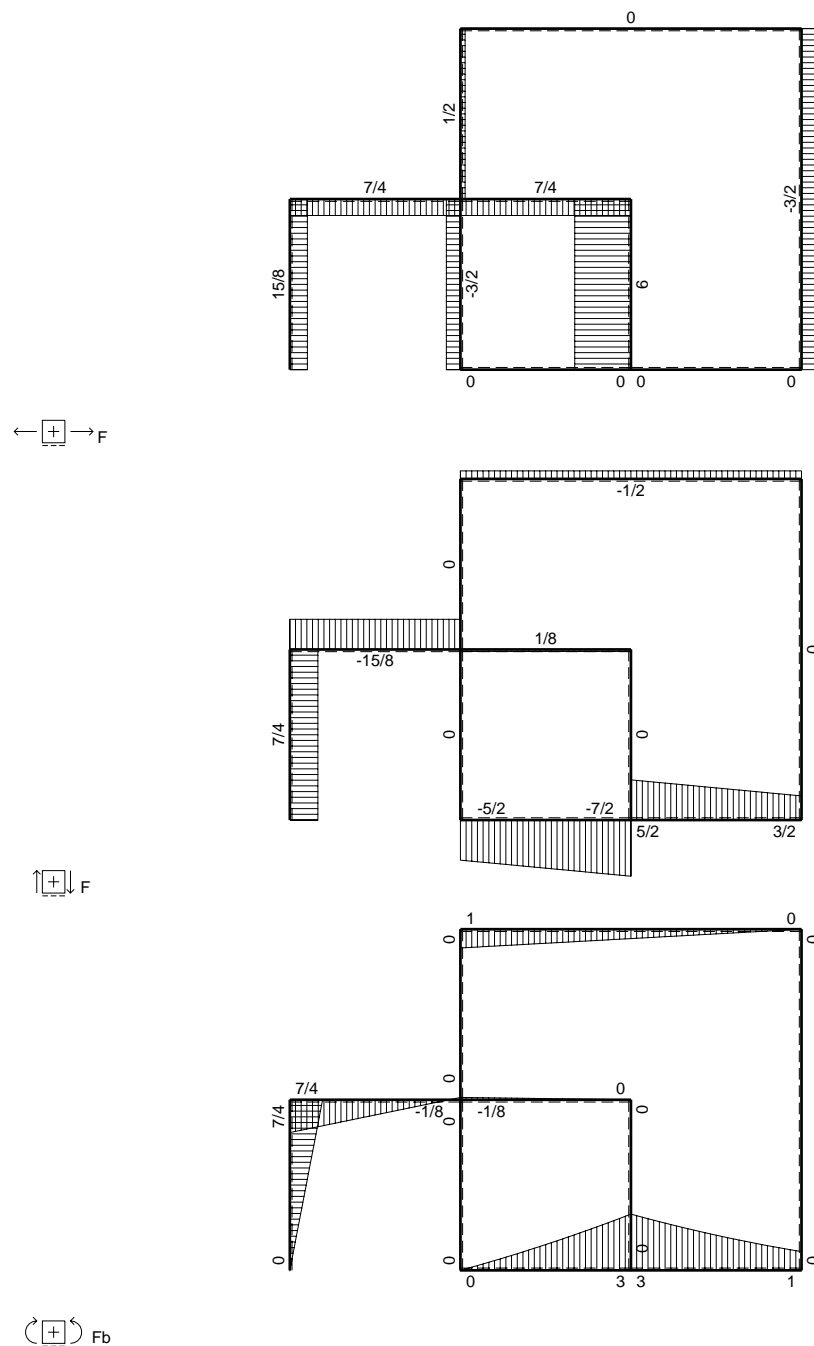
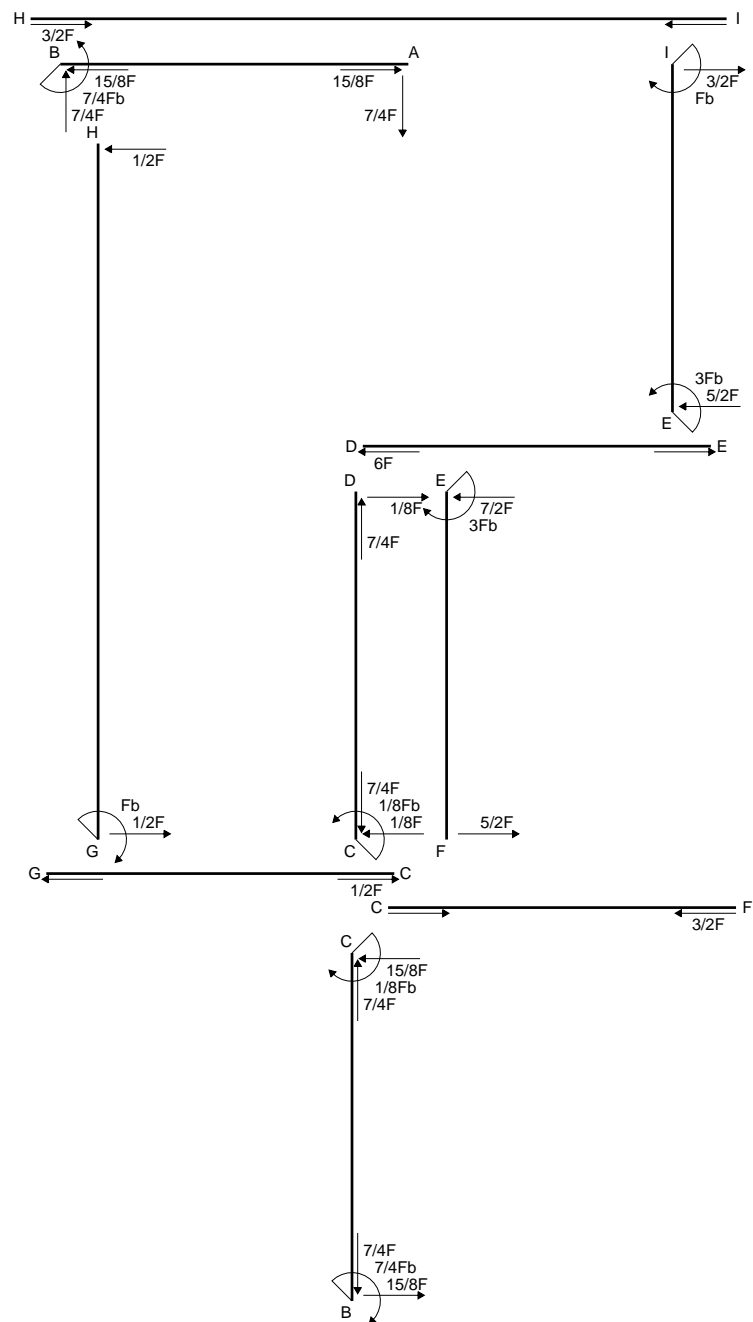
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

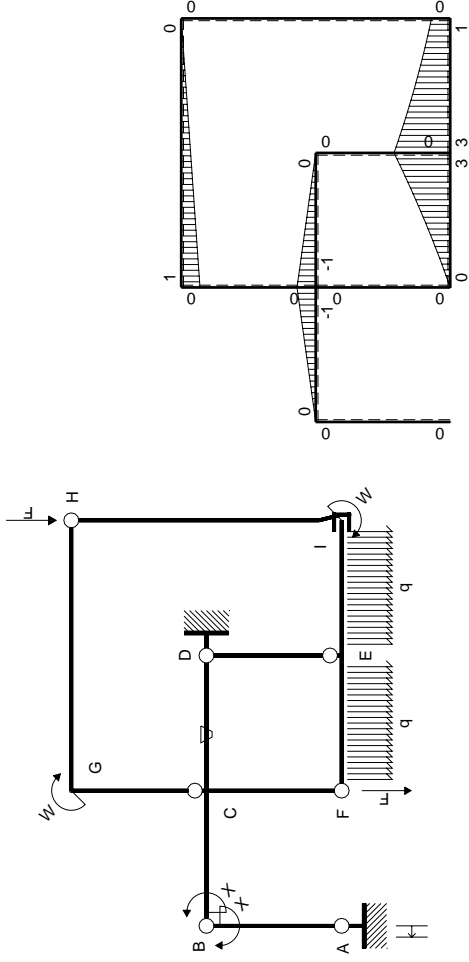
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |                     |                         | $7/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-7/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

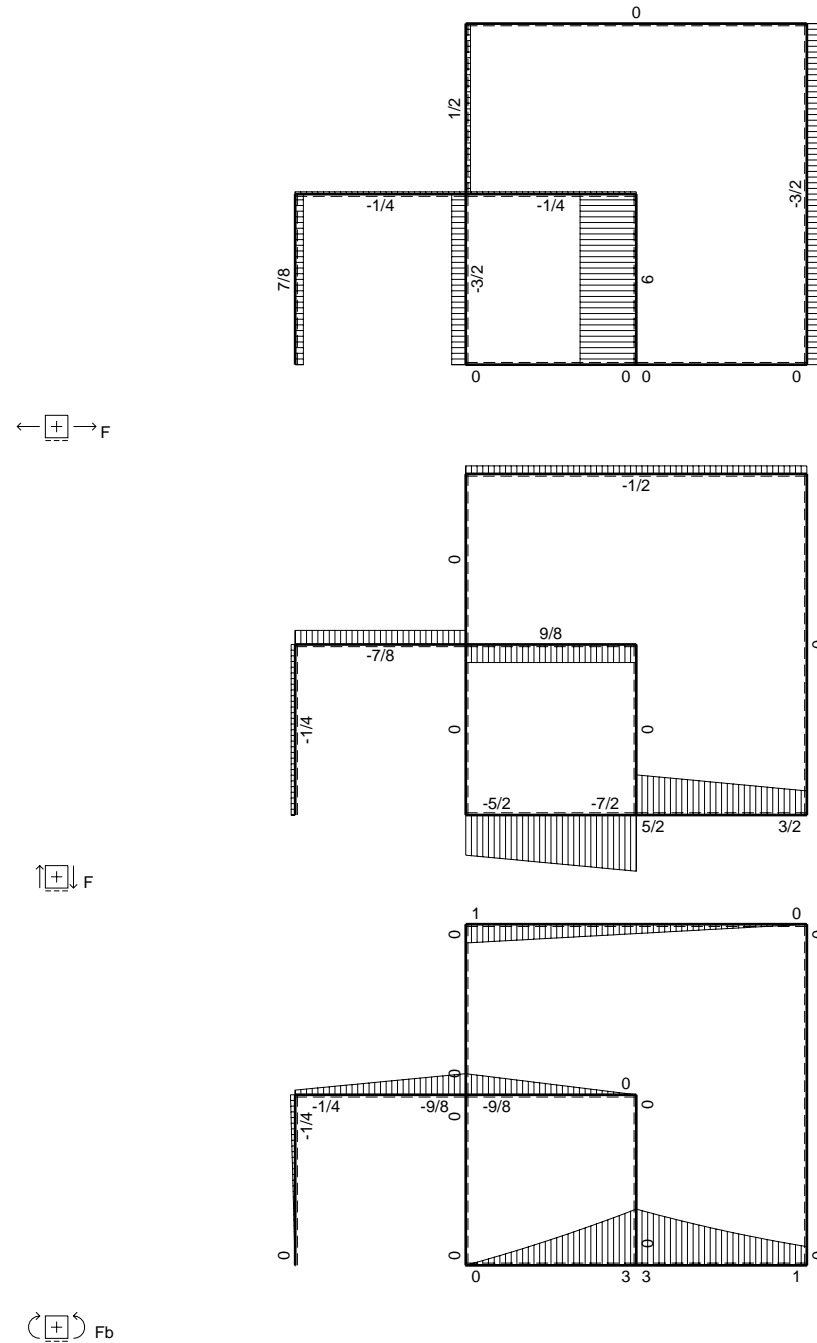
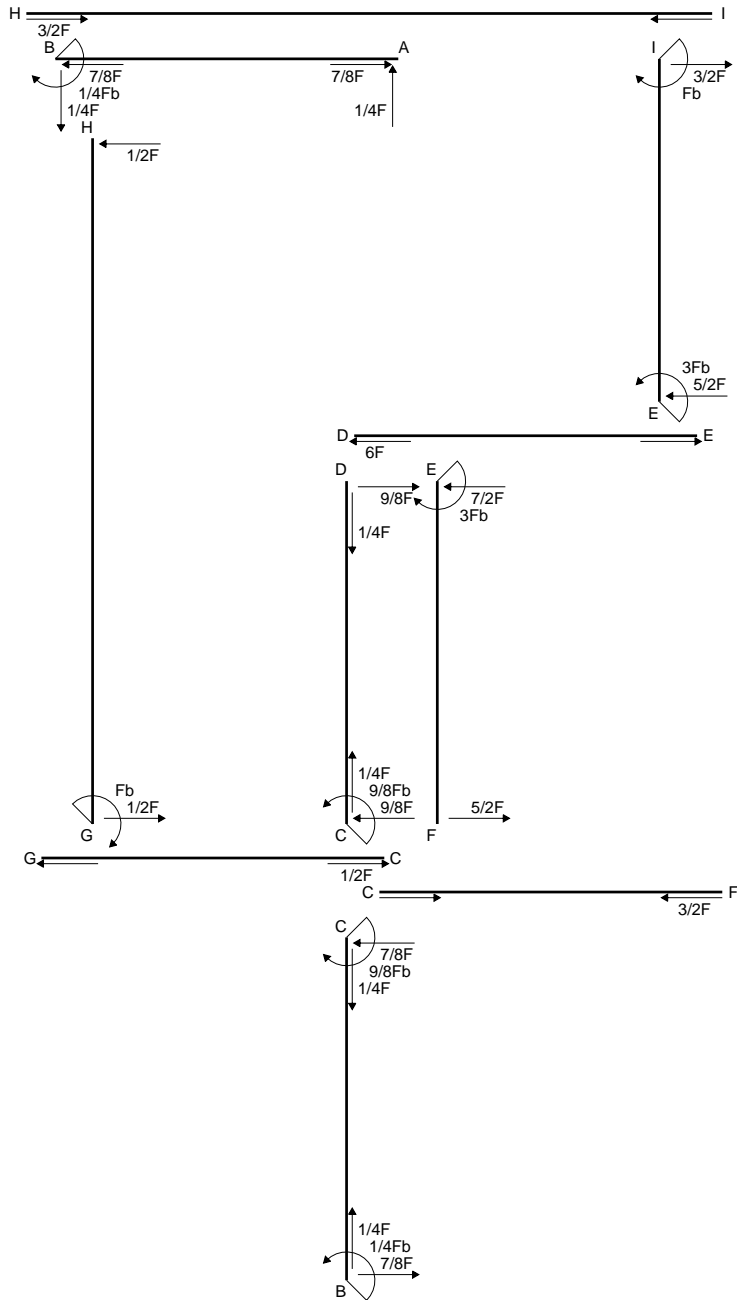
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

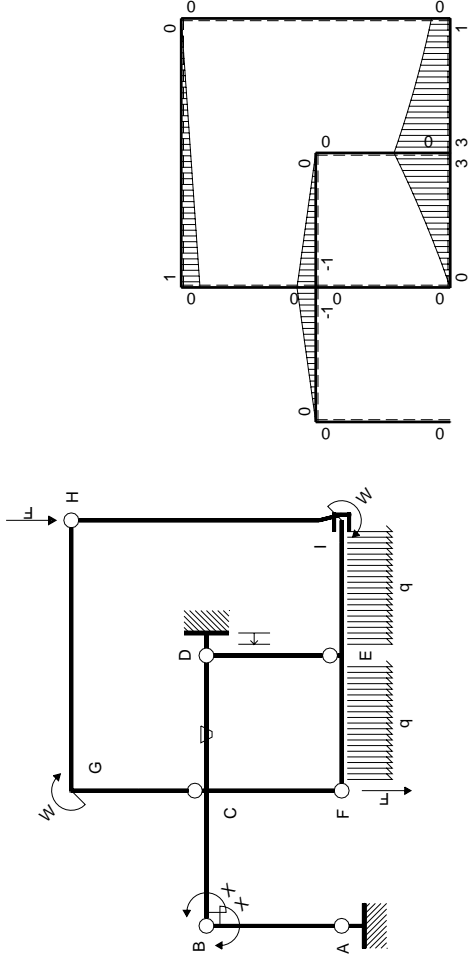
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                      |          |                      |                     |                         | $-1/4Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $1/4Fb$                     |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

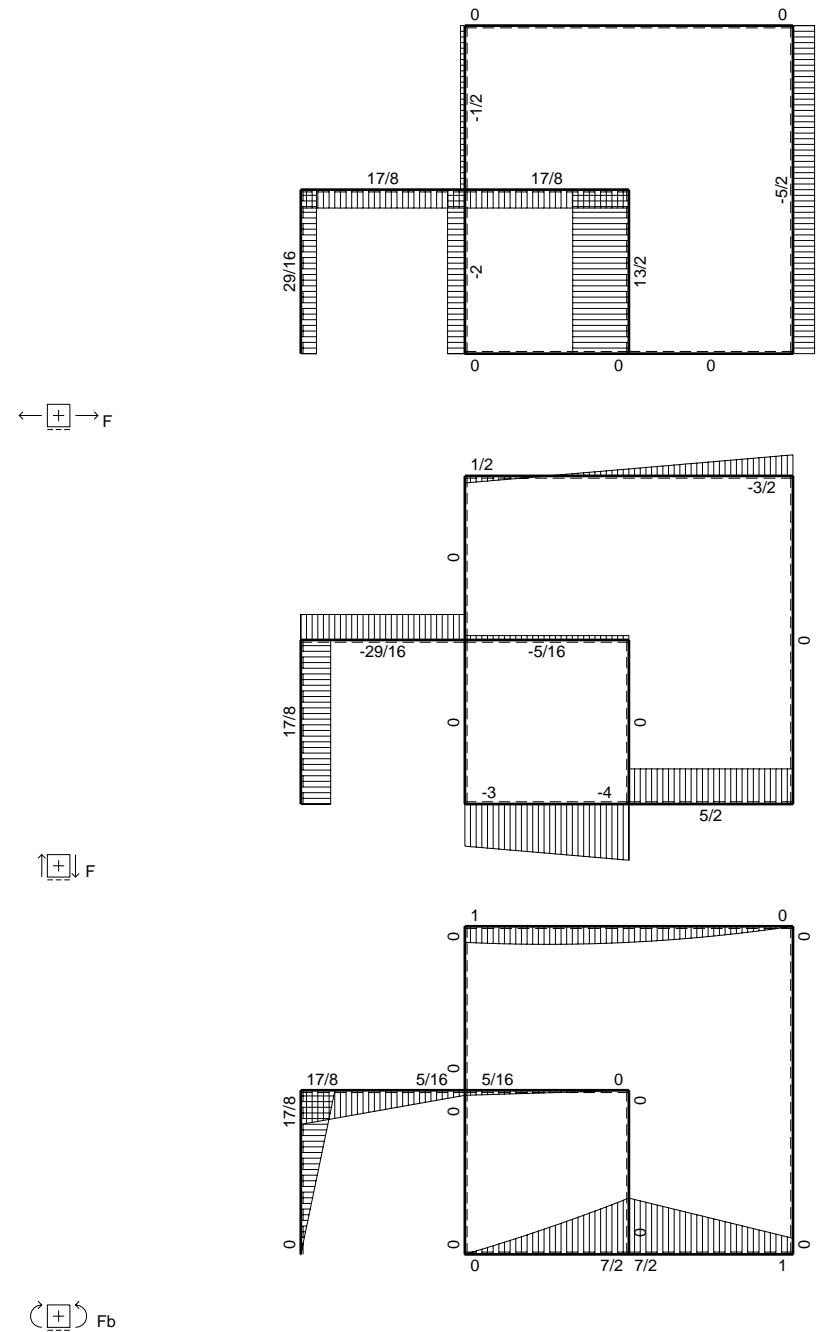
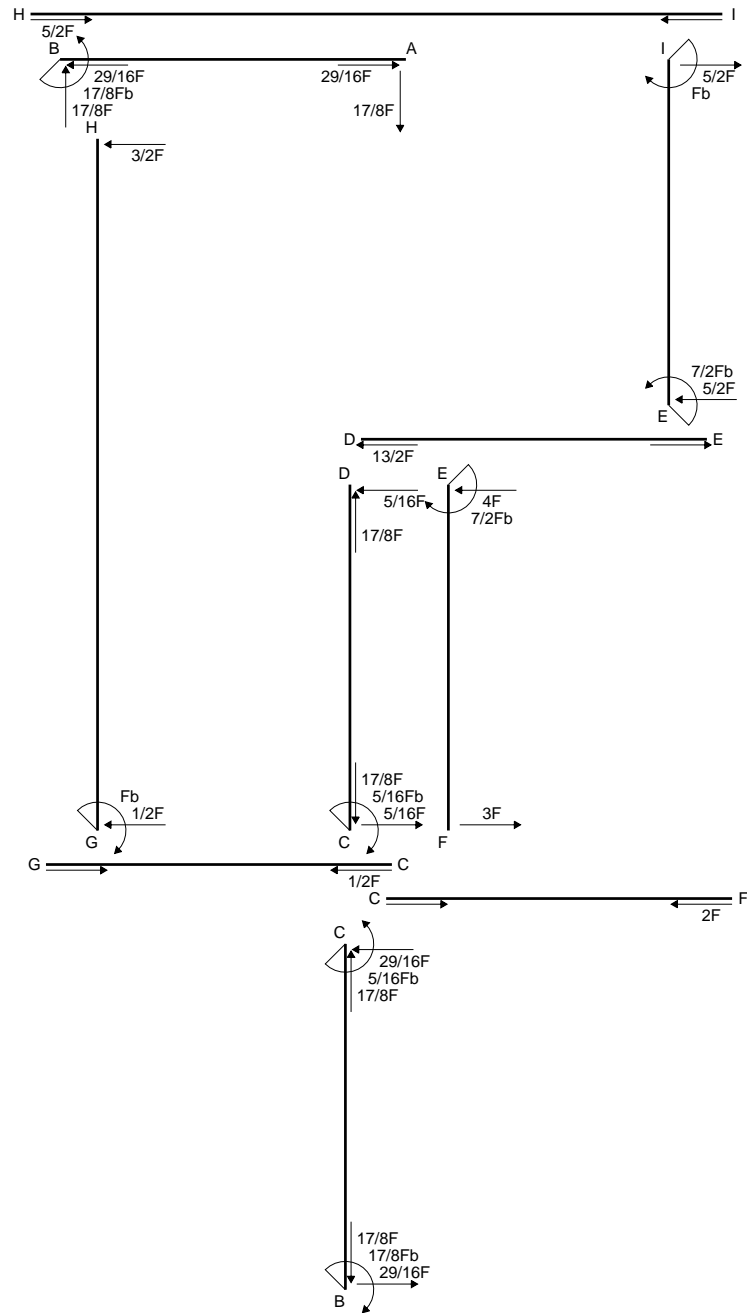
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

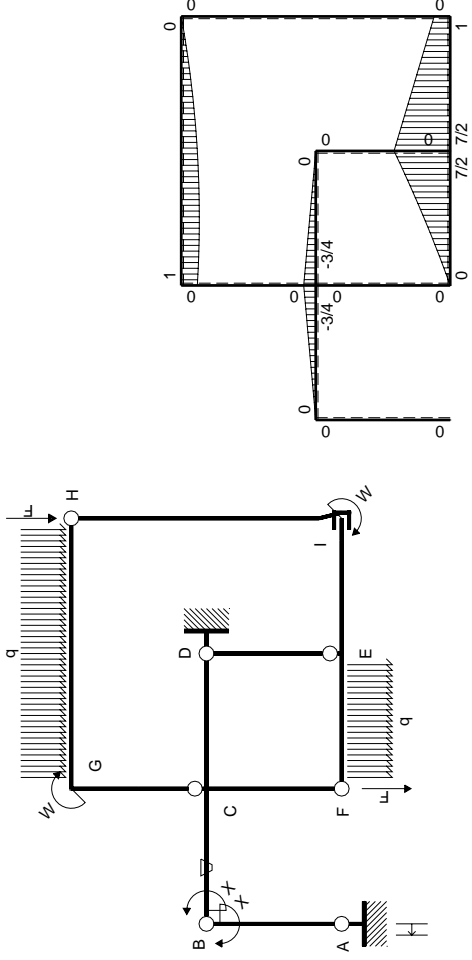
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

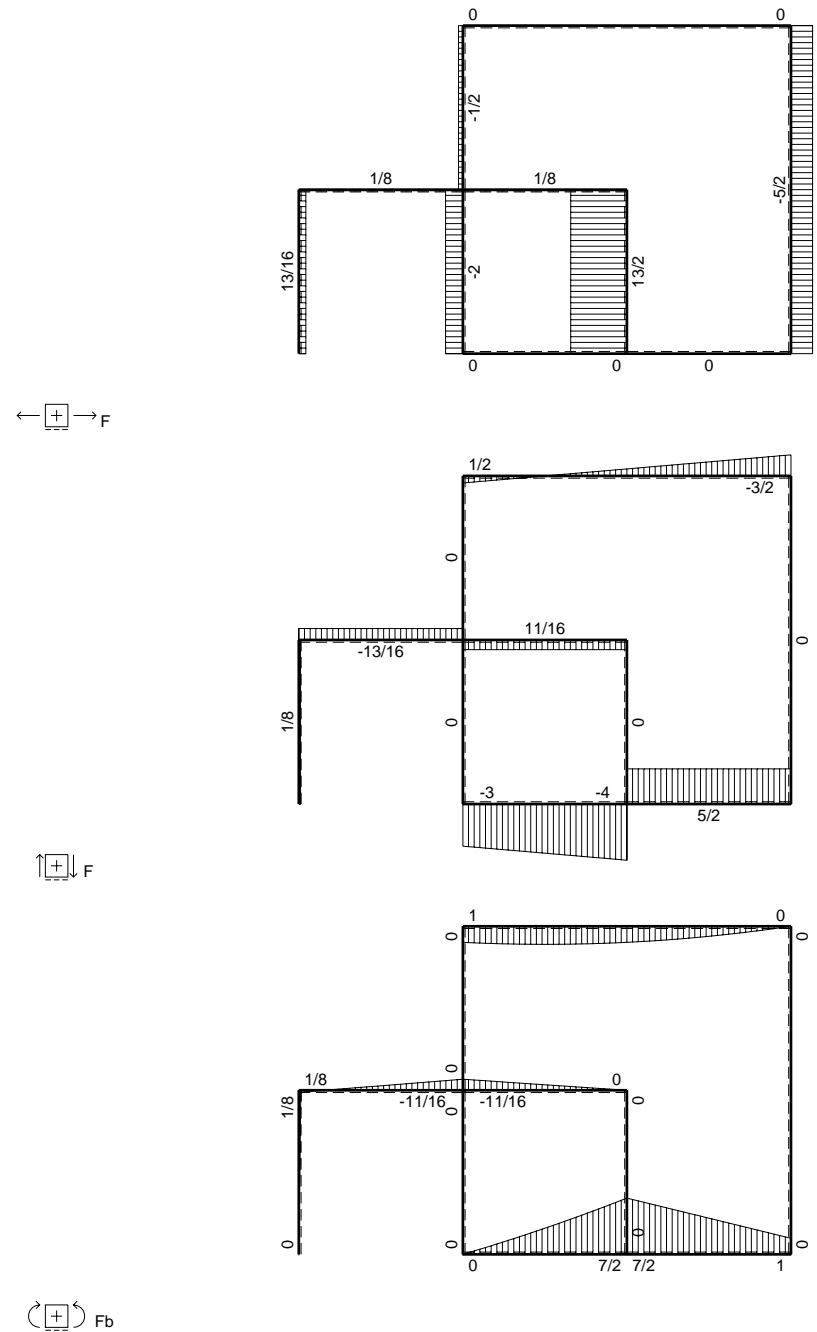
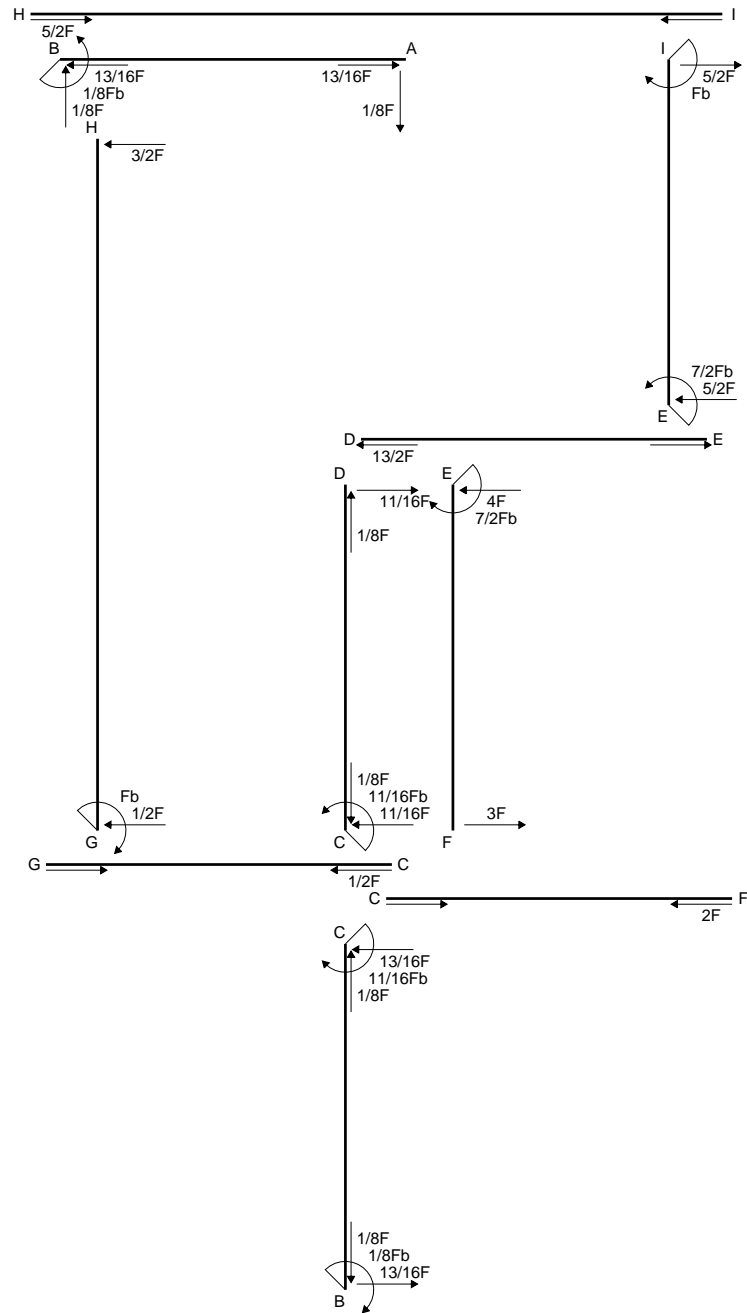
$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

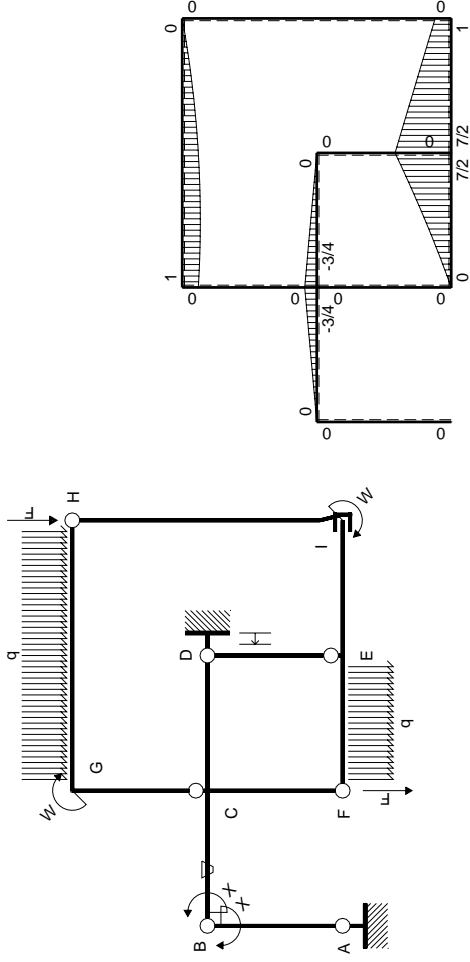
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | $-Fb/EJ$ | $3/4Fx-3/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/4+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | $Fb/EJ$  | $3/8Fb-3/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $1/8Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-1/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (b - 1/4 b) \theta = Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 1/8 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = Fb^2/EJ$$

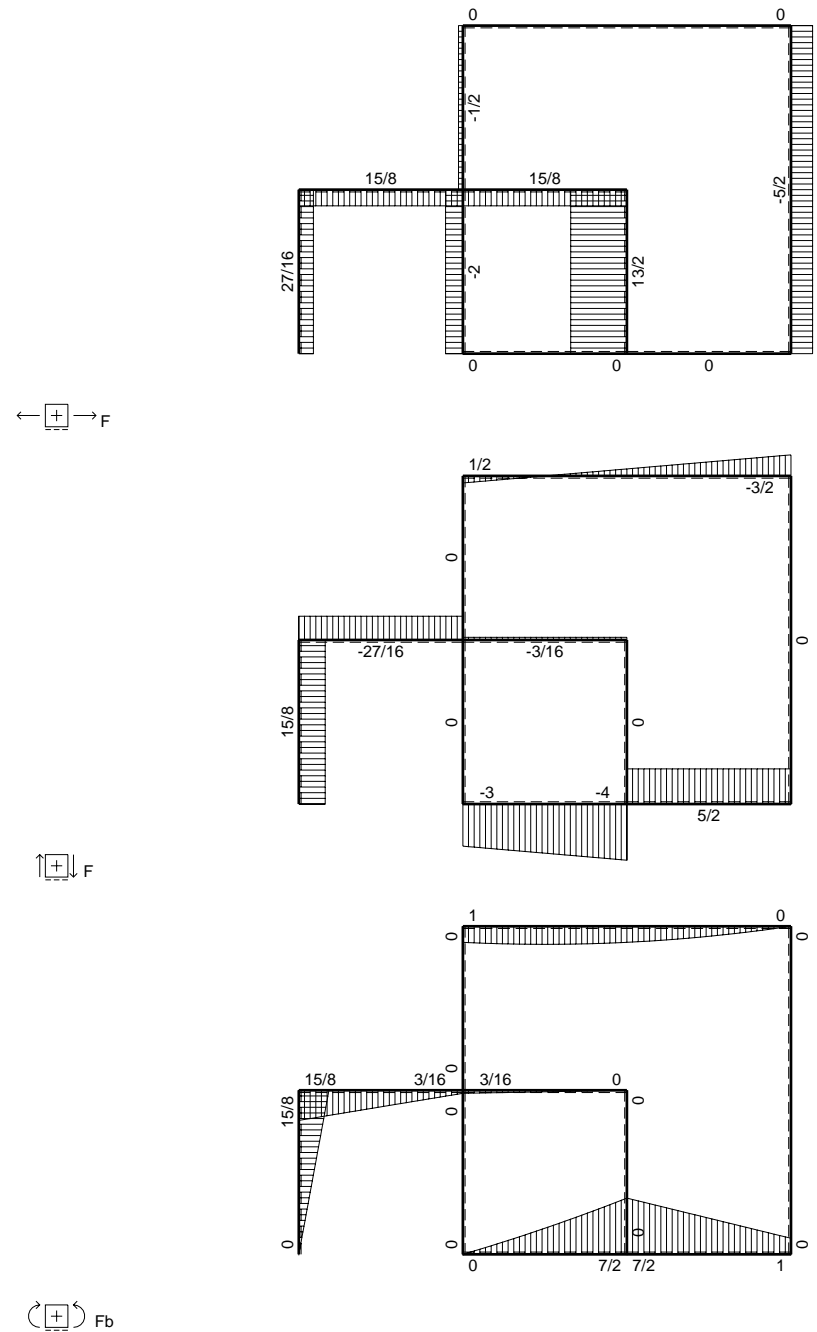
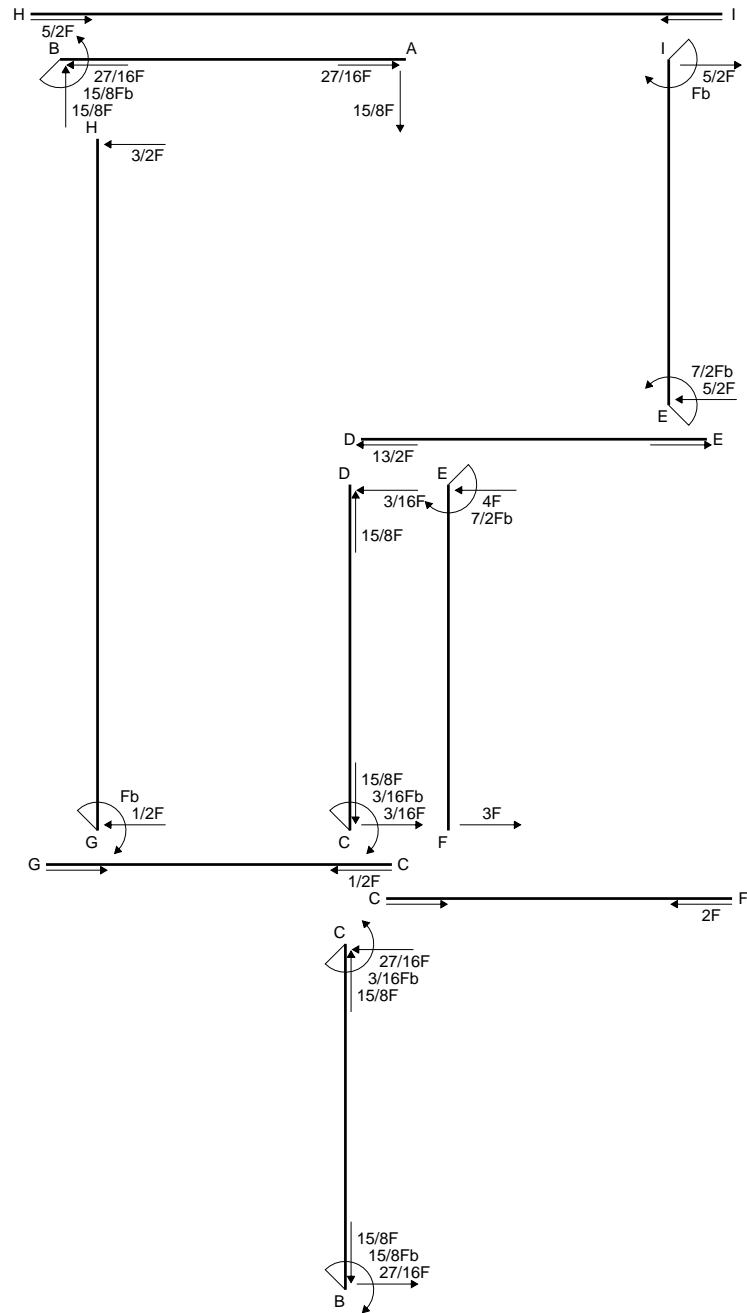
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

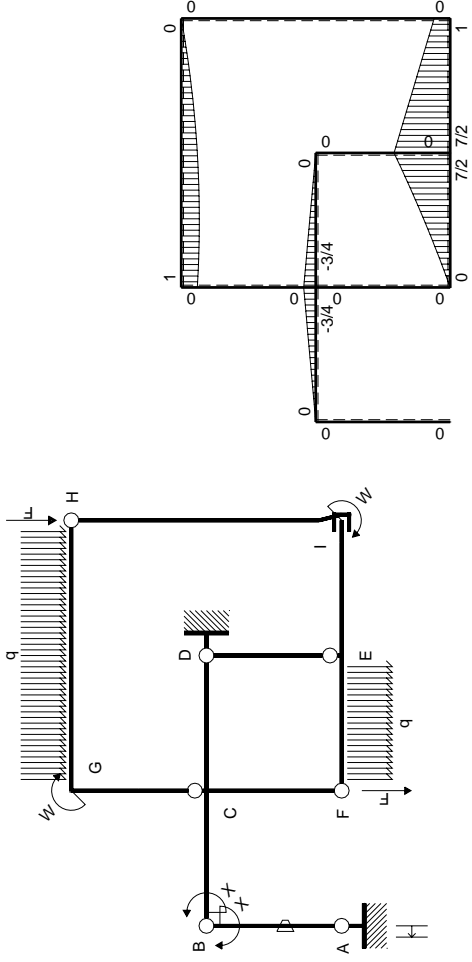
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

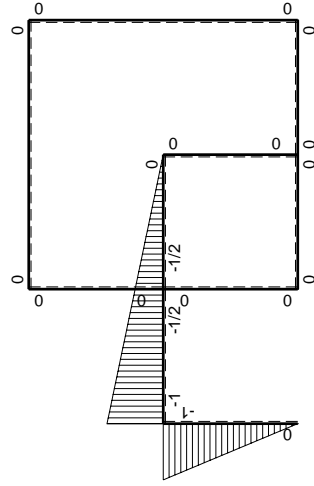






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                          |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                          |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                          |
| CG b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                          |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                       | 0             | 0                       |                             |                          |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                          |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                          |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                          |
|       | totali                      |                     |          |                         |               |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-15/8Fb$                   |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

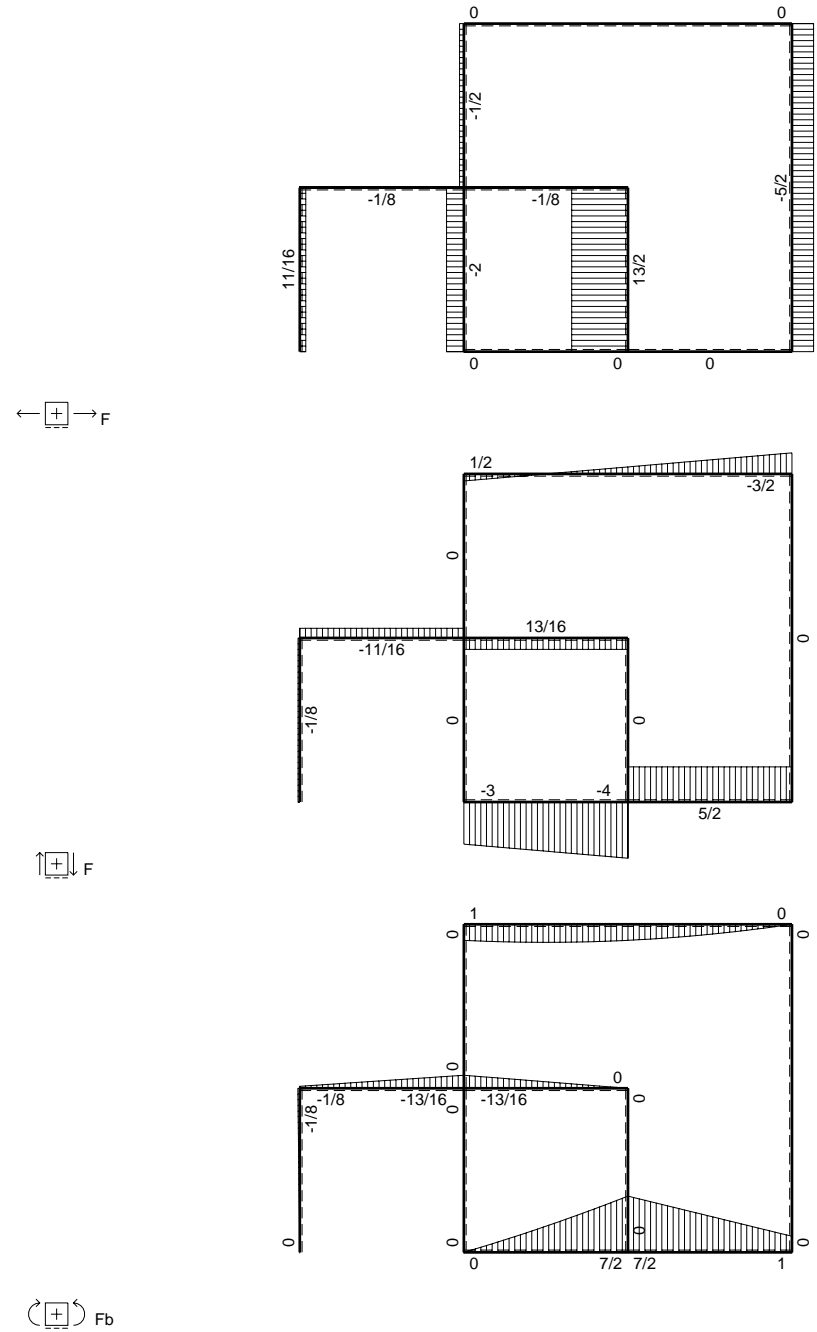
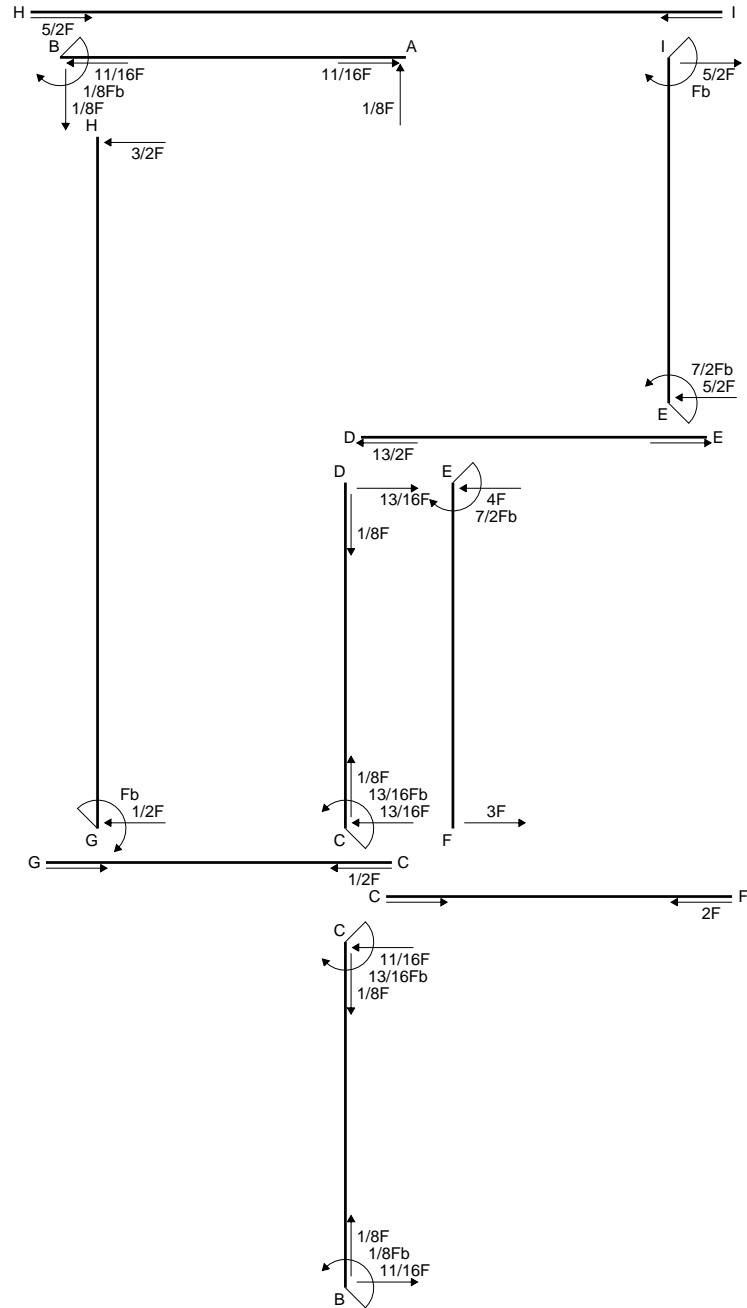
$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

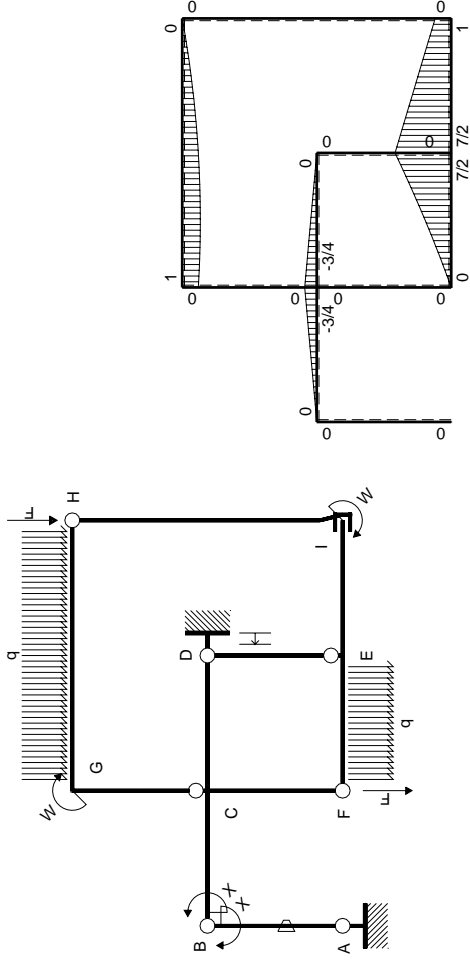
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | 0        | $3/8Fb-3/4Fx+3/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | 0        | $3/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |               |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

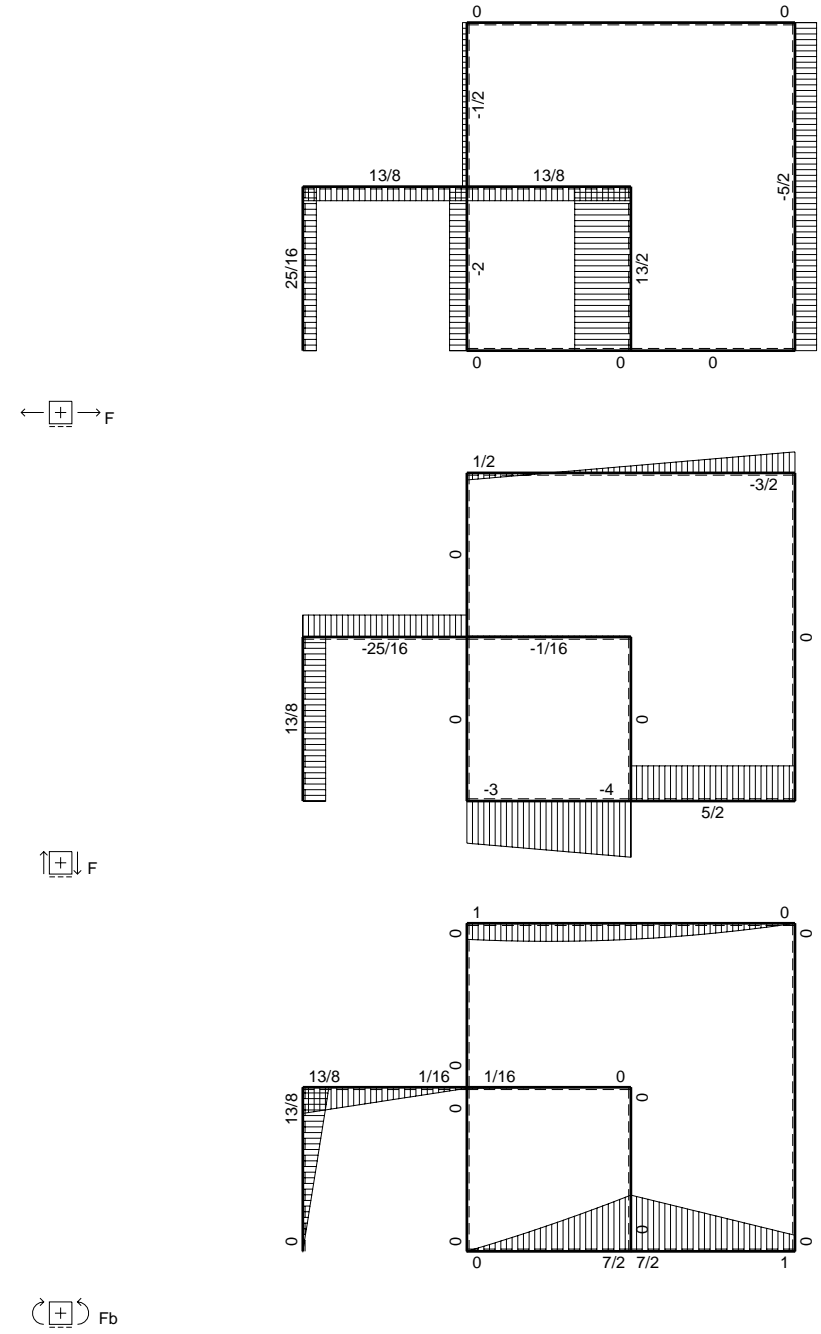
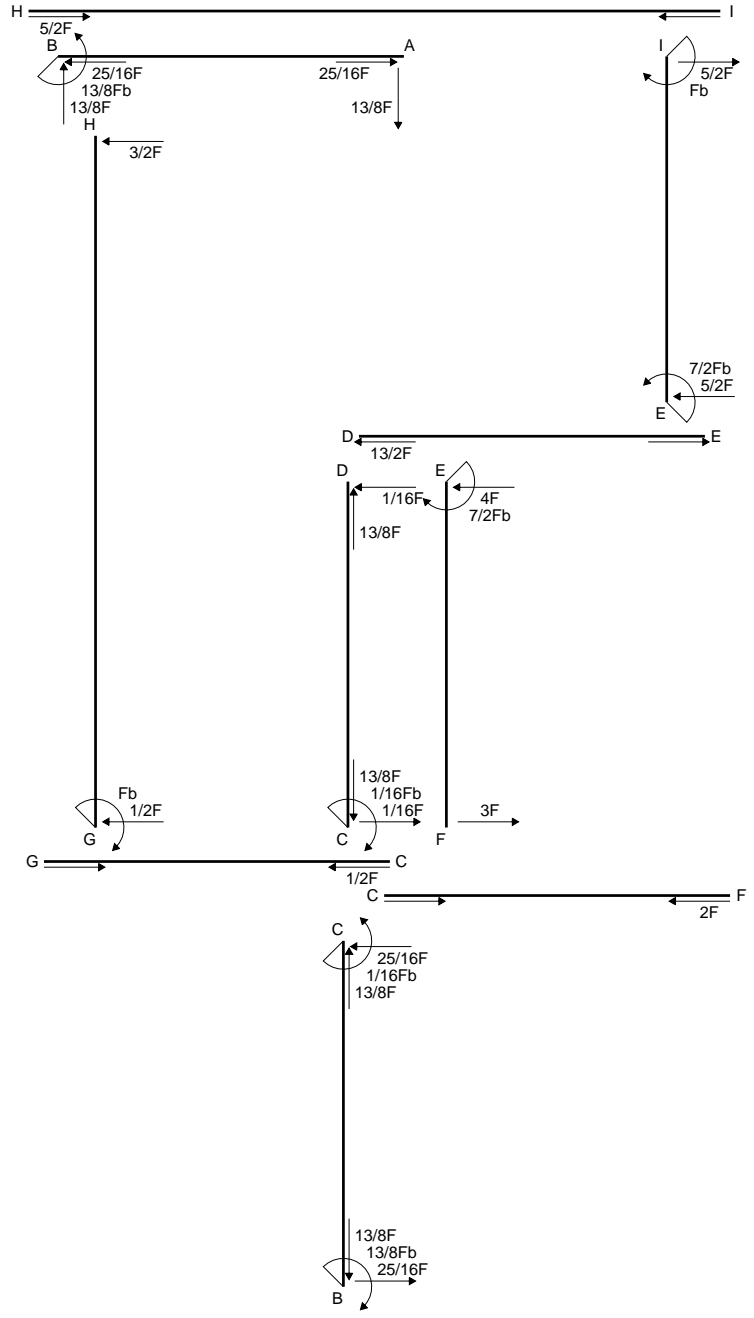
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ$$

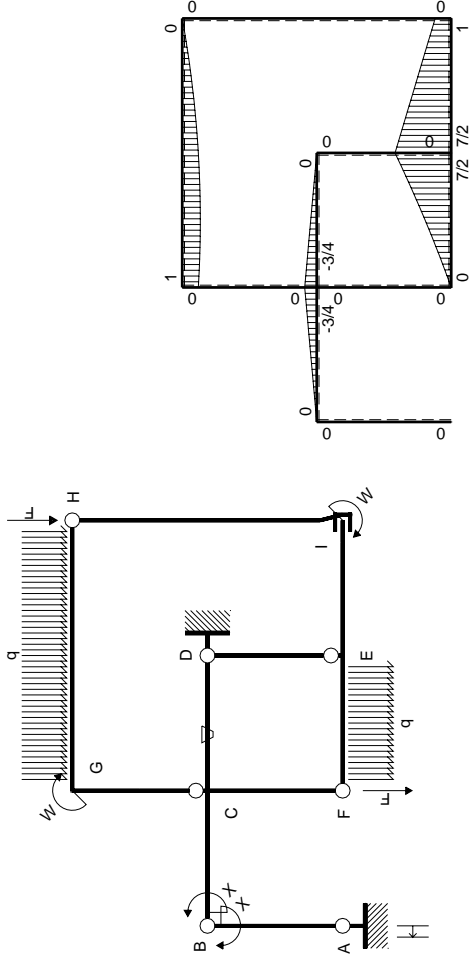
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx = [1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/8 b) Fb 1/EJ = 1/8 Fb^2/EJ$$

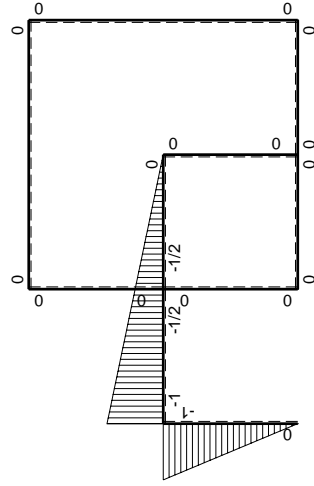






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $13/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-13/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

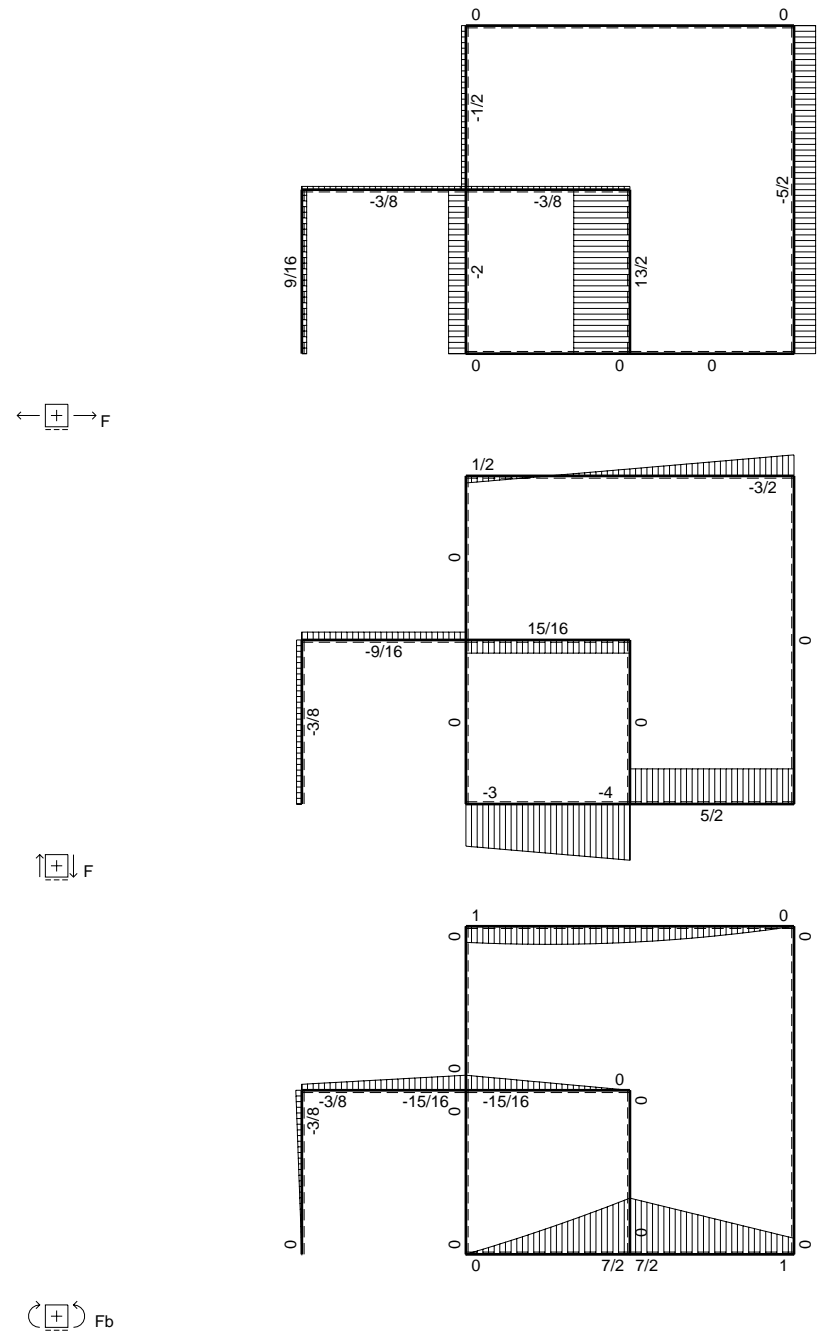
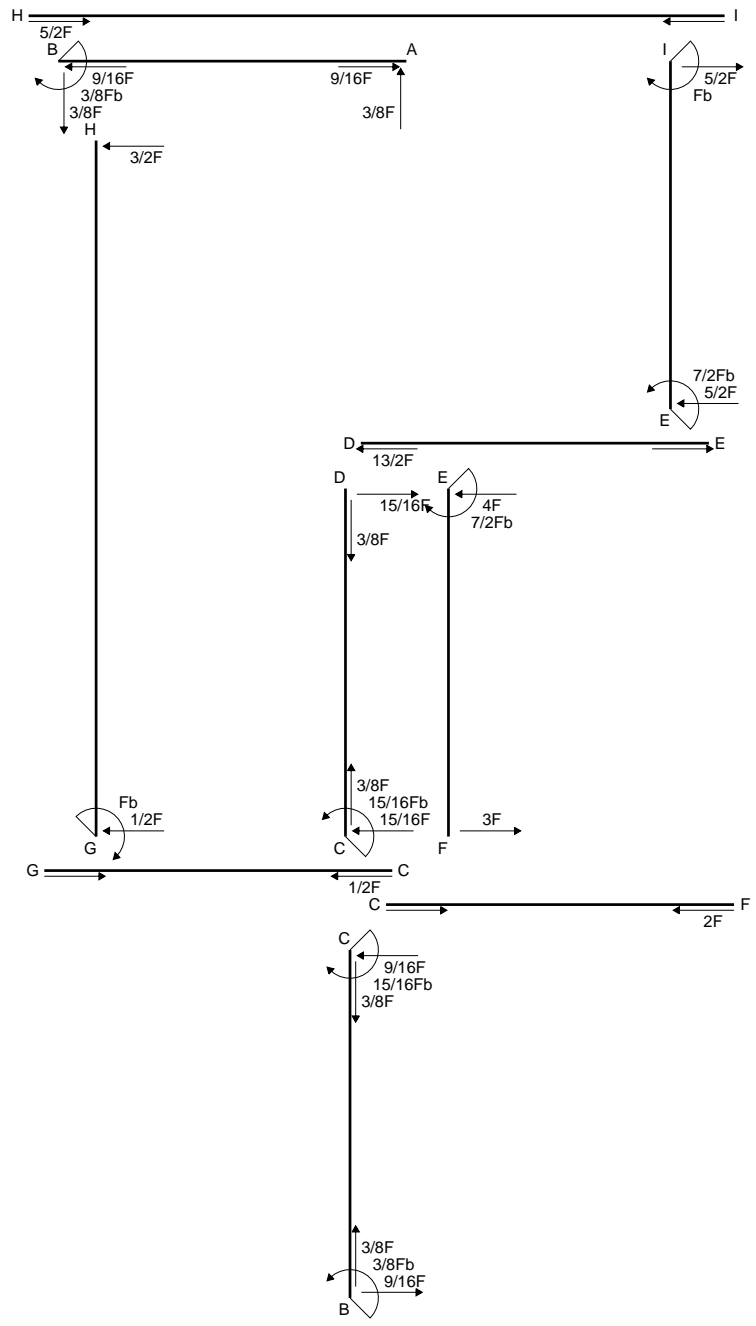
$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

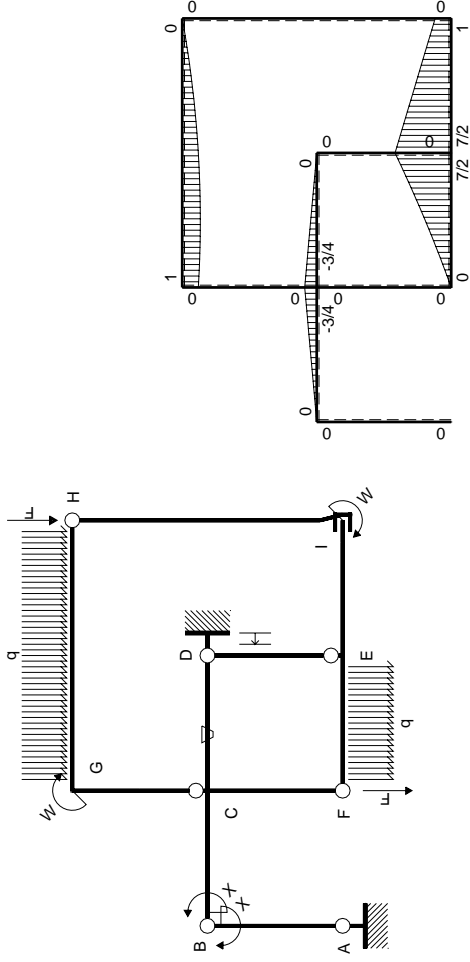
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/4Fx$            | 0        | $3/4Fx-3/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/4+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/4Fb-3/4Fx$       | 0        | $3/8Fb-3/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-3/4Fb+3/4Fx$      | $-Fb/EJ$ | $3/8Fb-3/4Fx+3/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/8+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $3/4Fx$             | $Fb/EJ$  | $3/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-4Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-3/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $3/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/4 x/b - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x^2/b - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/8 - 3/8 x^2/b^2) Fb 1/EJ dx = [3/8 x - 1/8 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/8 b - 1/8 b) Fb 1/EJ = 1/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (3/8 - 3/4 x/b + 3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

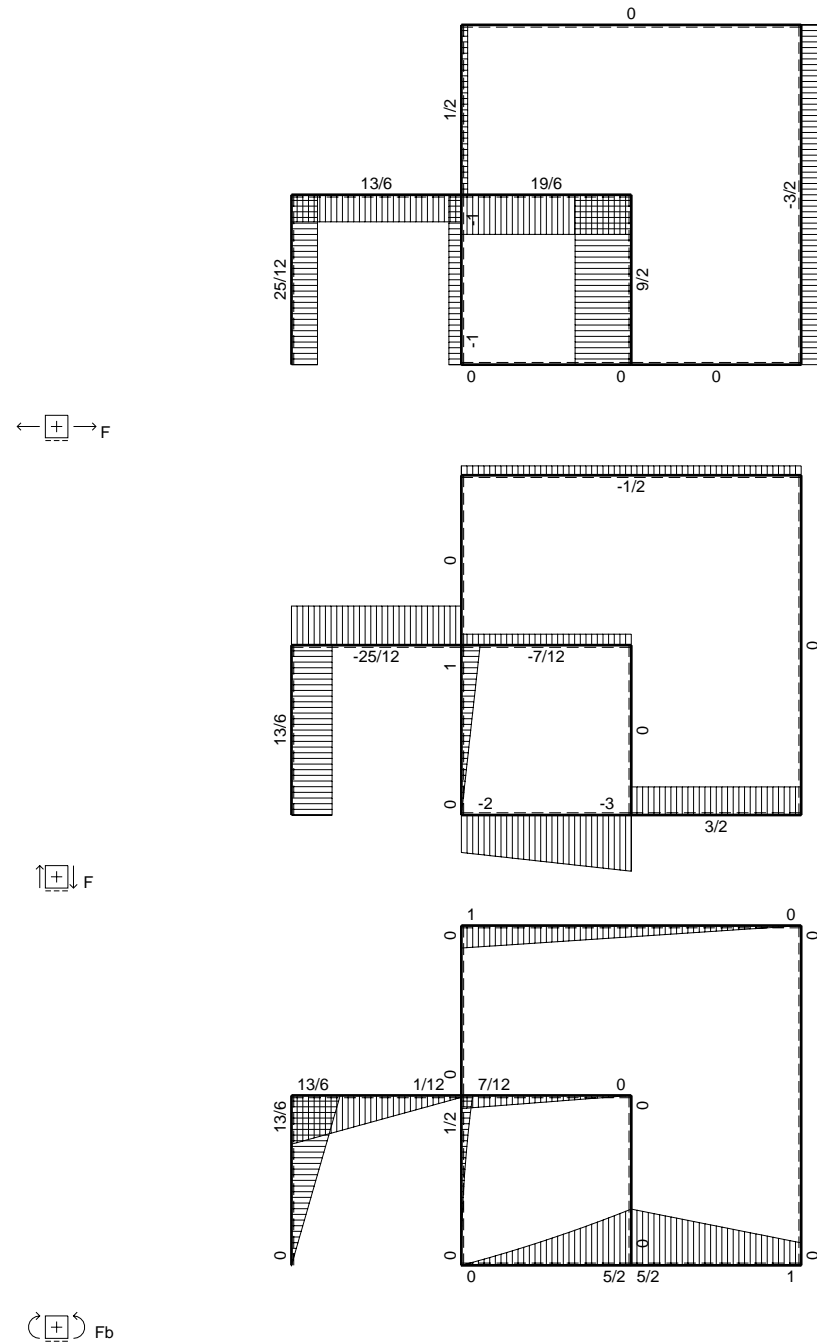
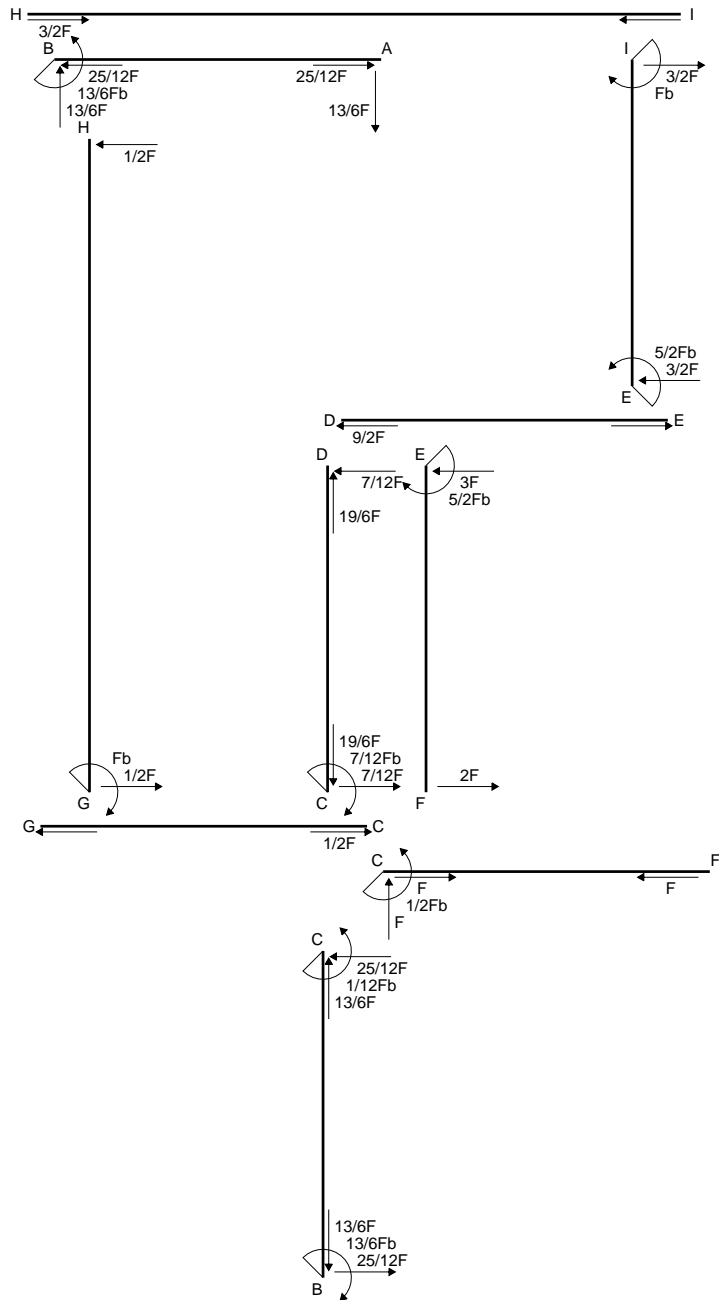
$$= [3/8 x - 3/8 x^2/b + 1/8 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

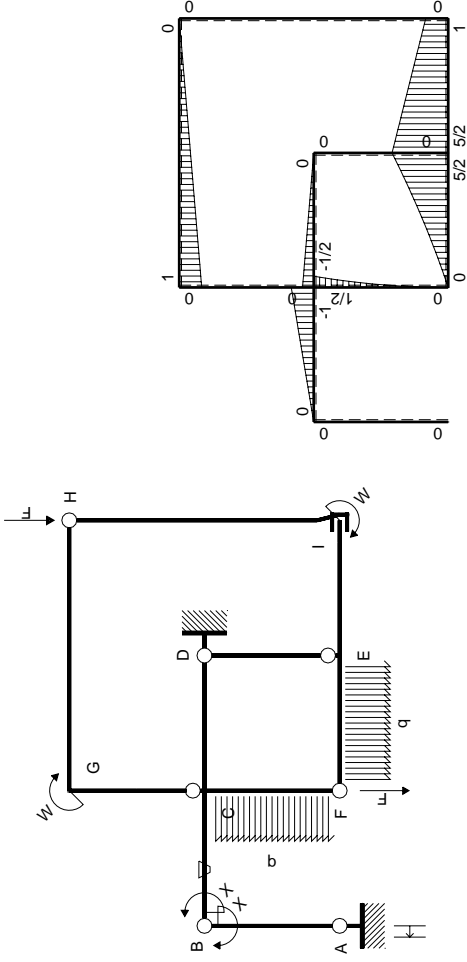
$$= (3/8 b - 3/8 b + 1/8 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 3/8 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (3/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/8 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/8 b) Fb 1/EJ + (-1/4 b) \theta = 3/8 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

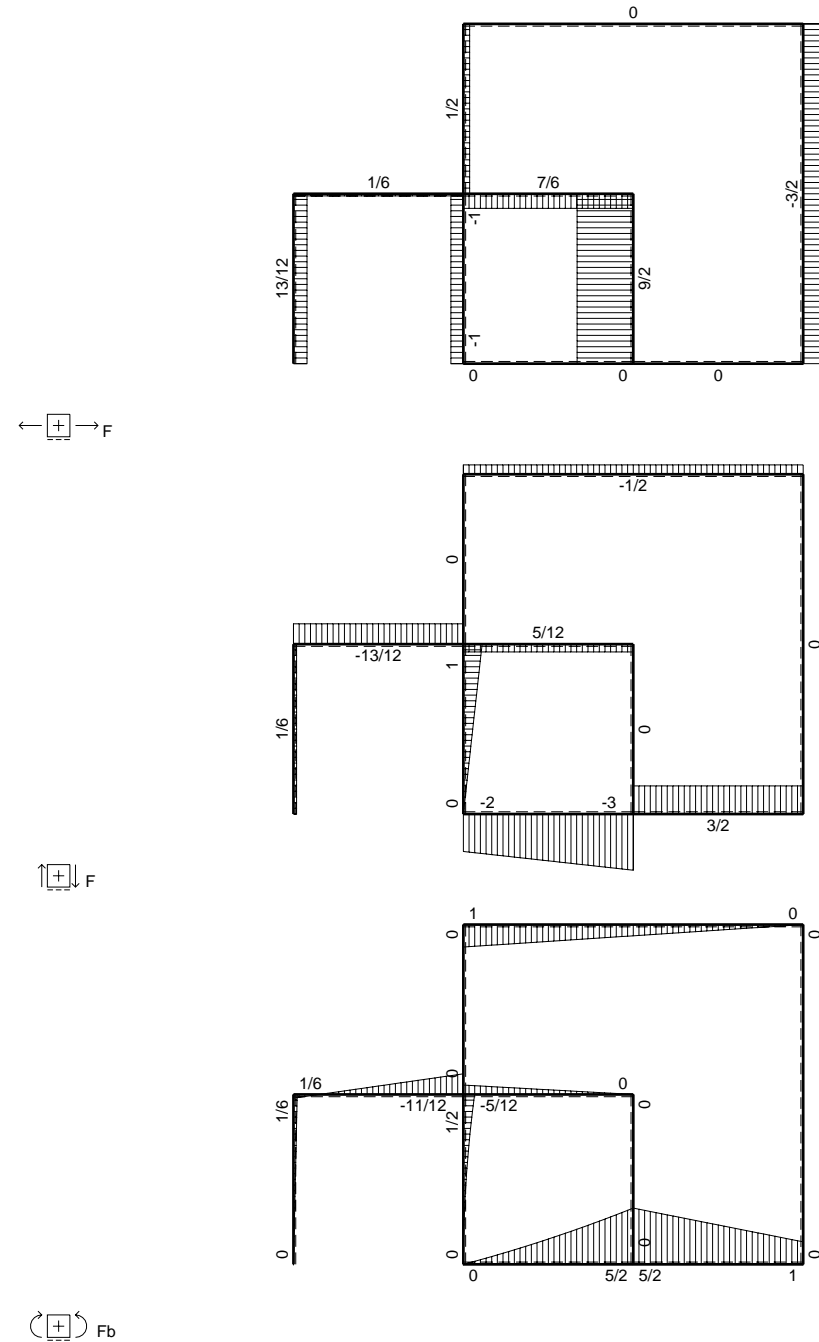
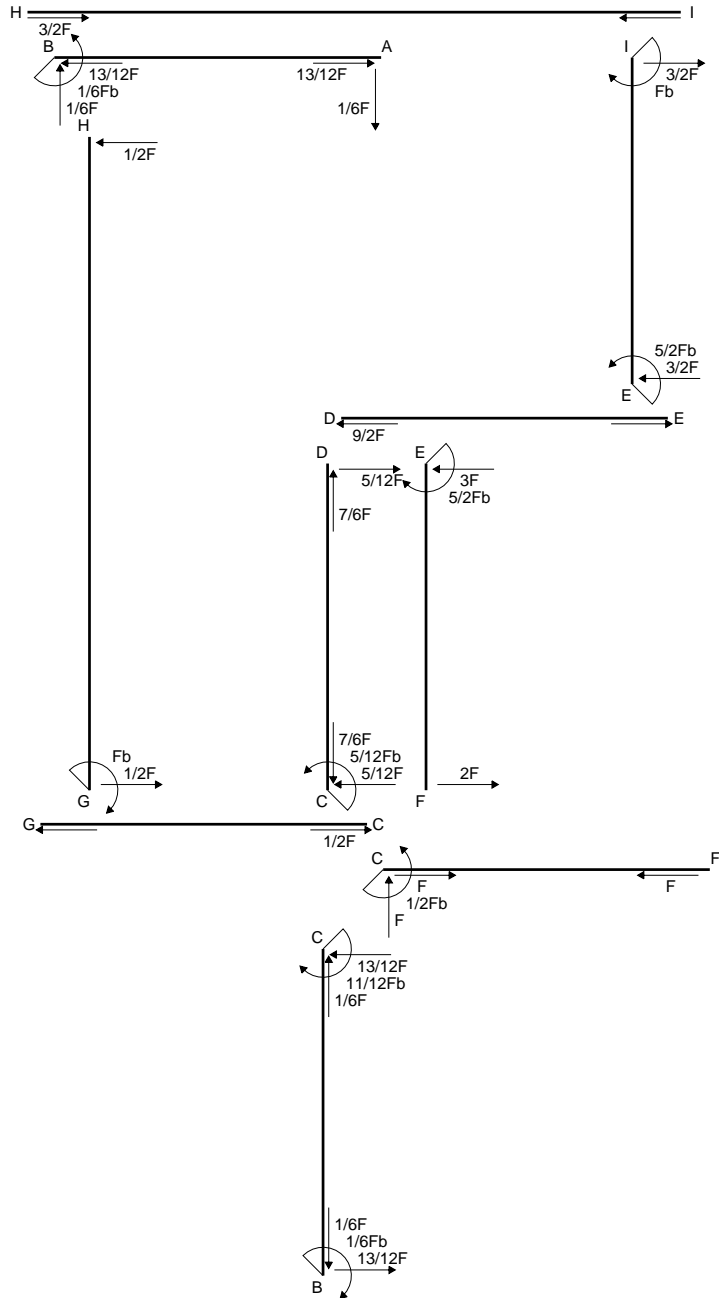
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

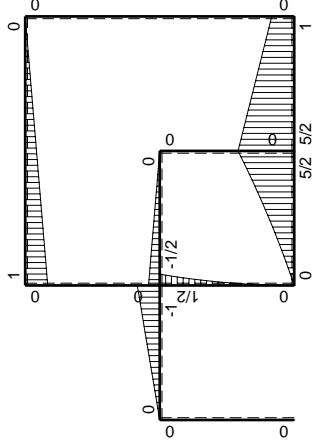
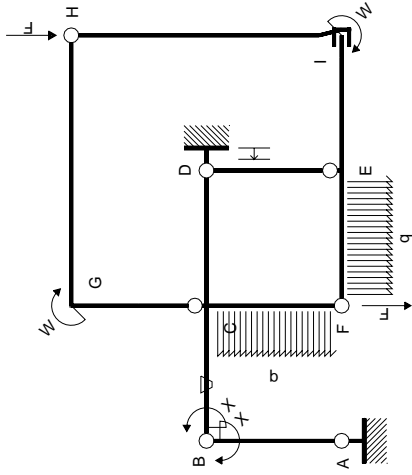
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

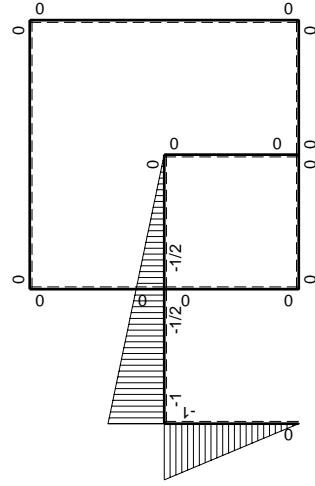
$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

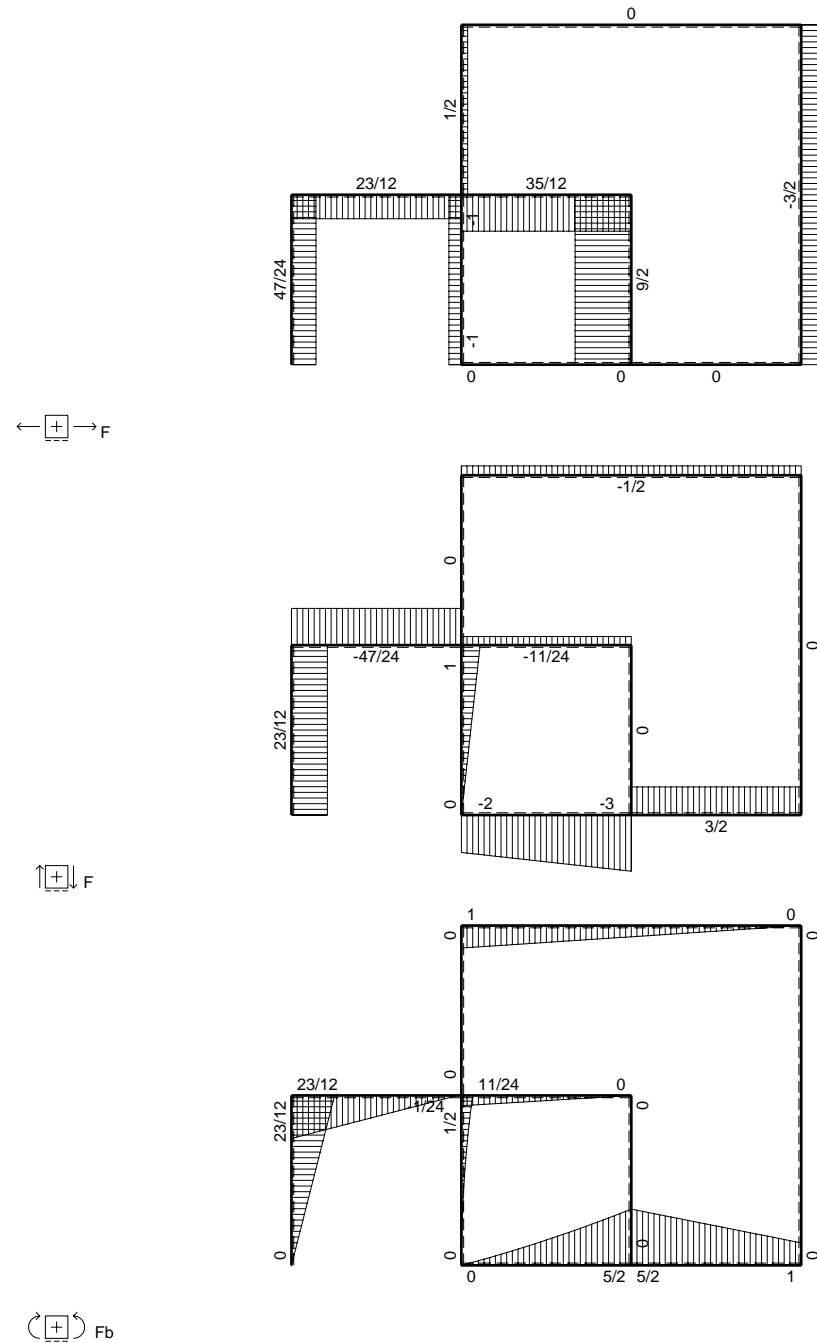
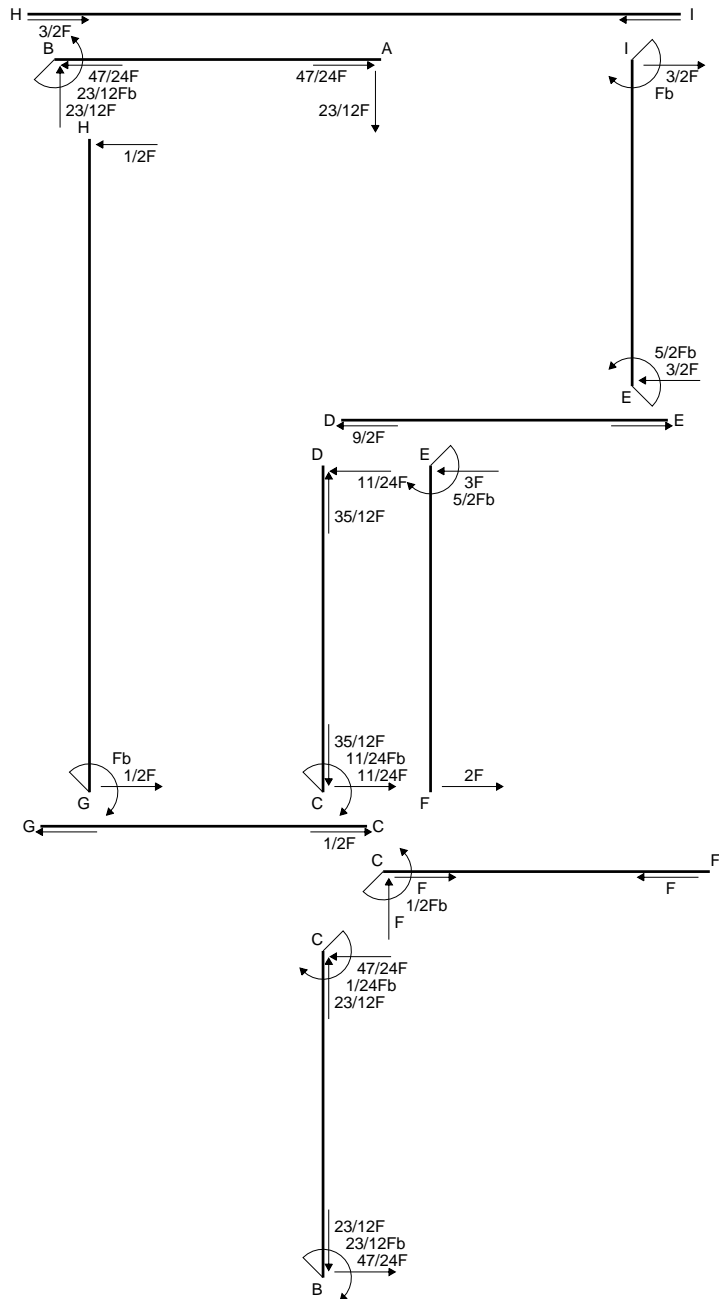
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

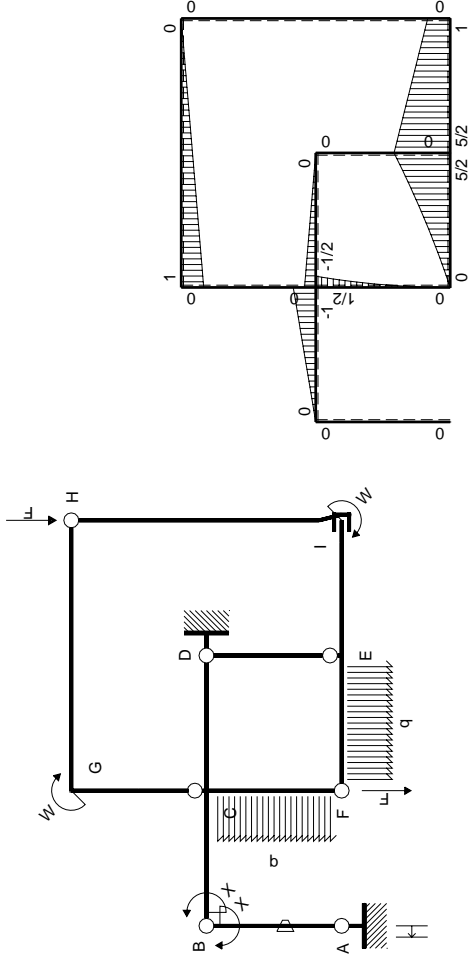
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

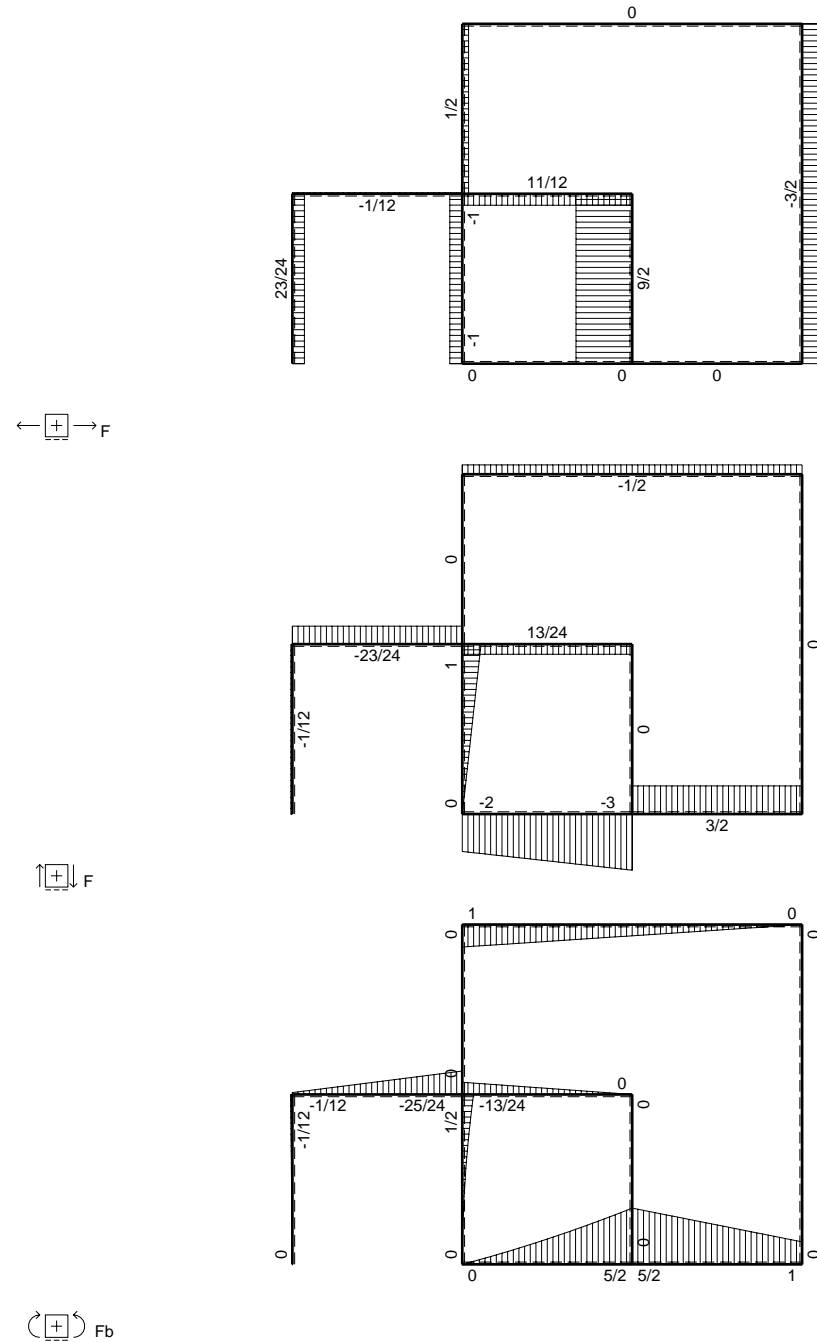
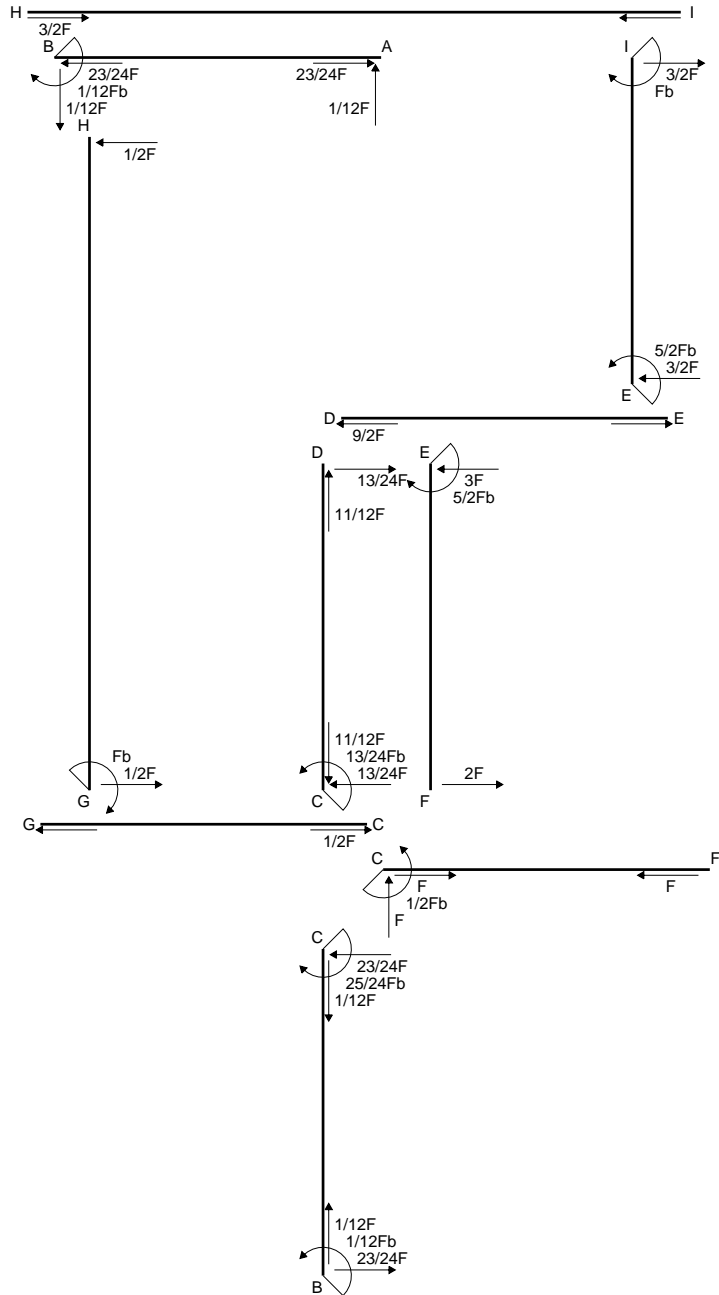
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

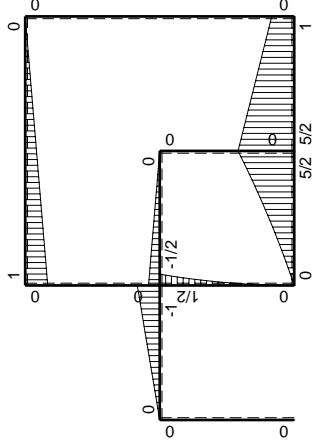
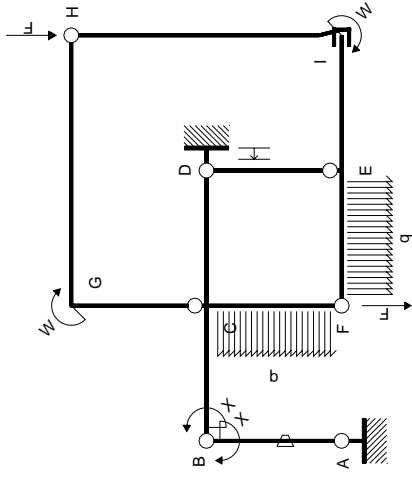
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

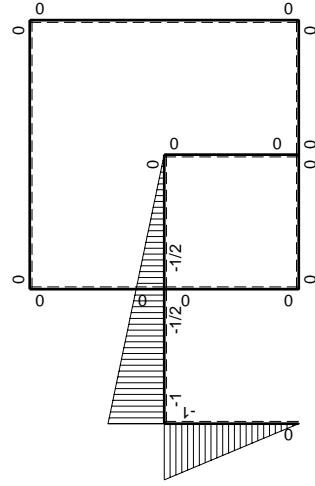


⊕ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |               |                         | $-1/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $1/12Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

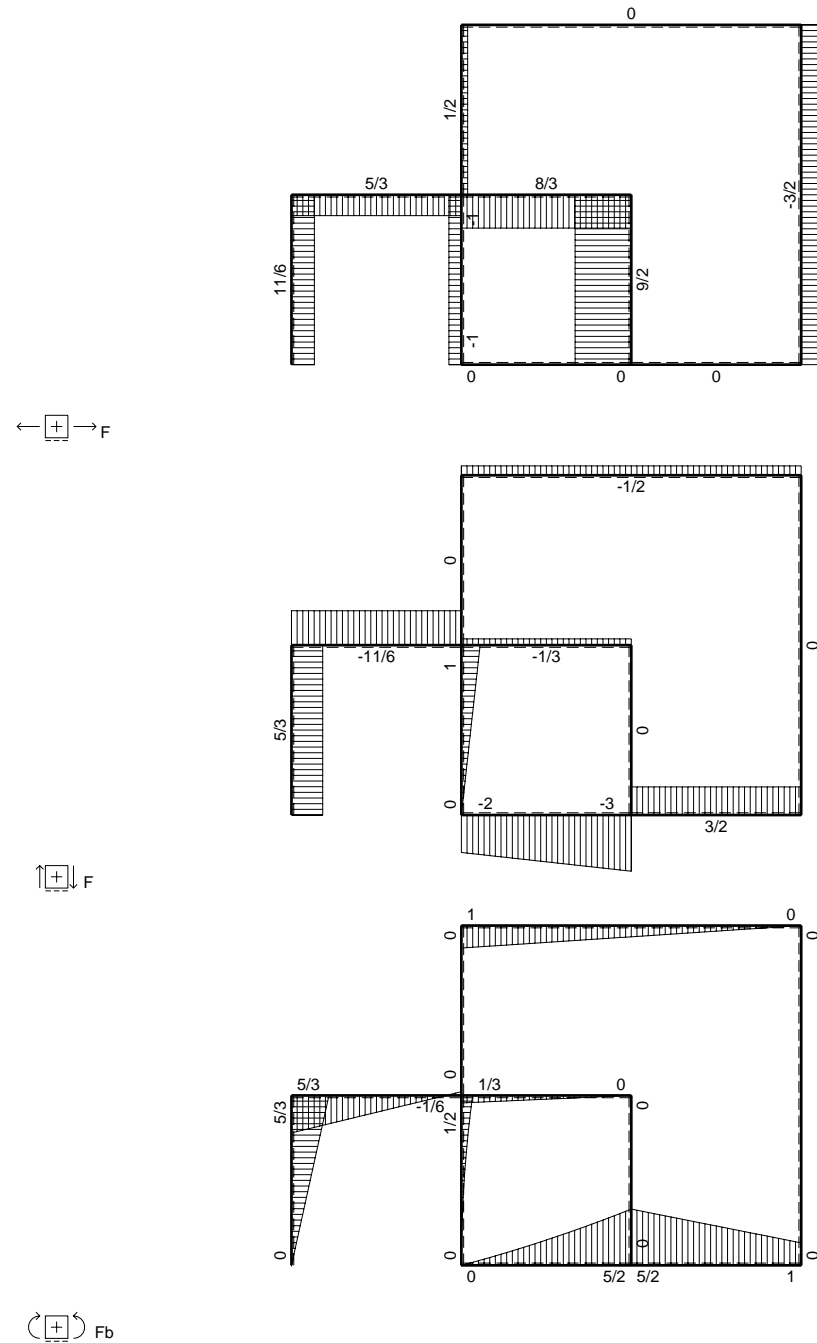
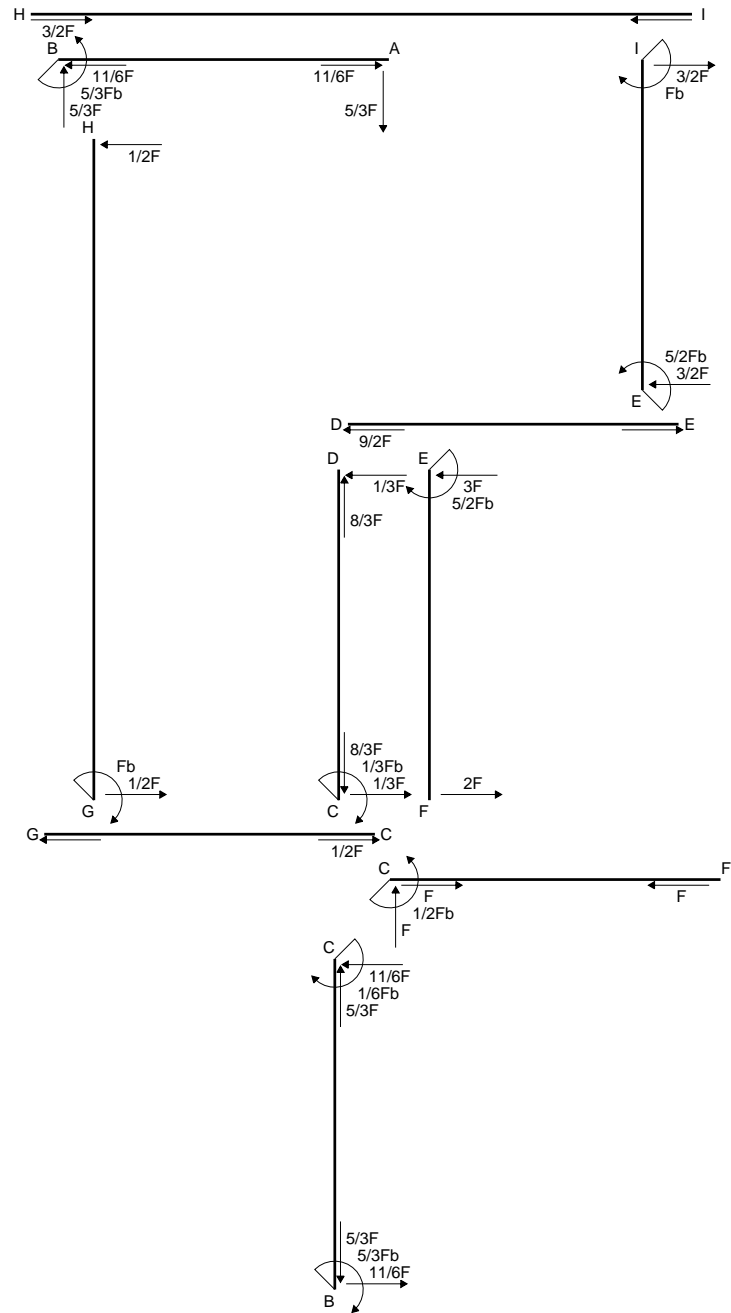
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

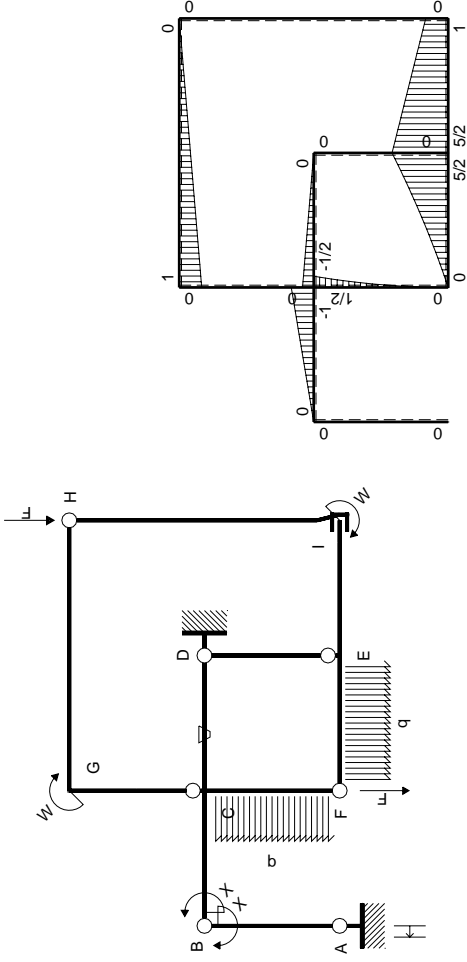
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $5/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-5/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

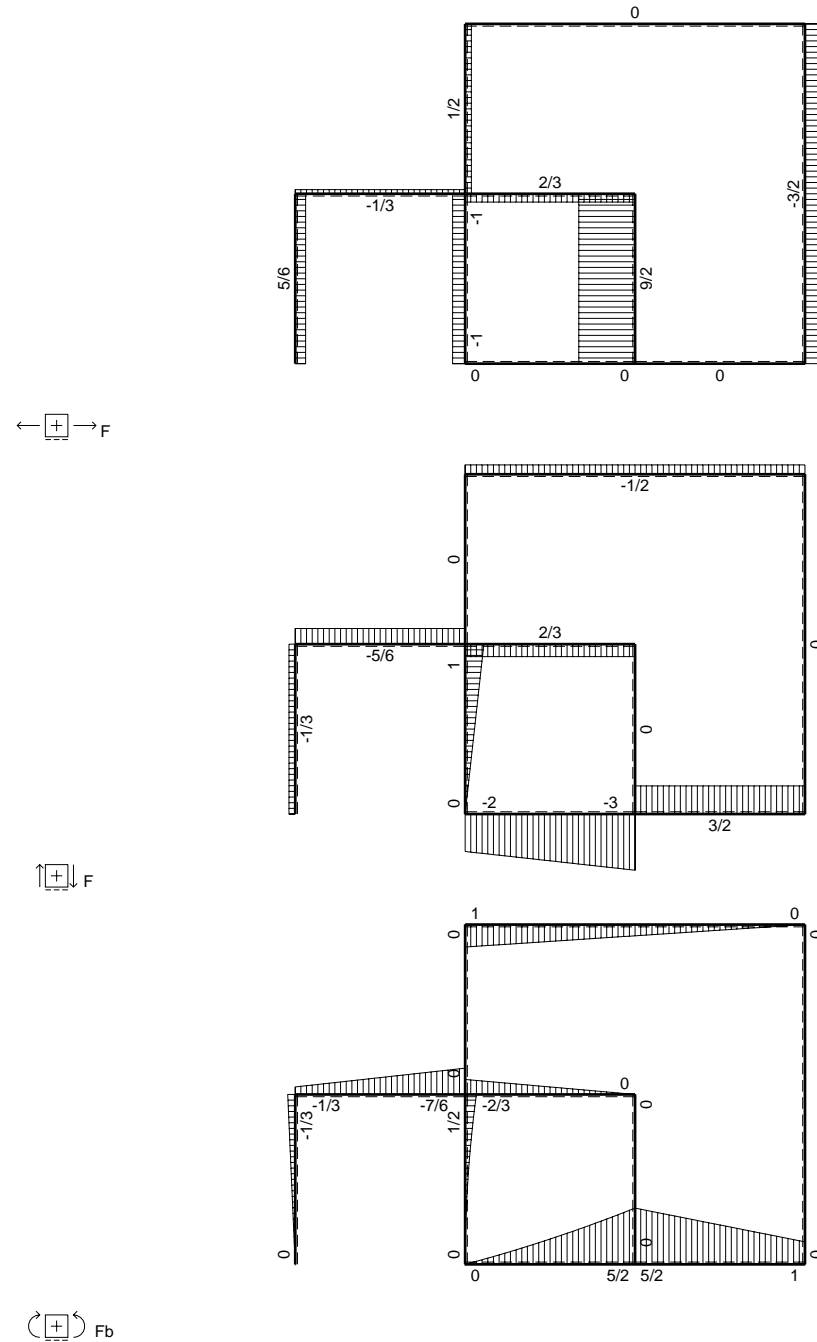
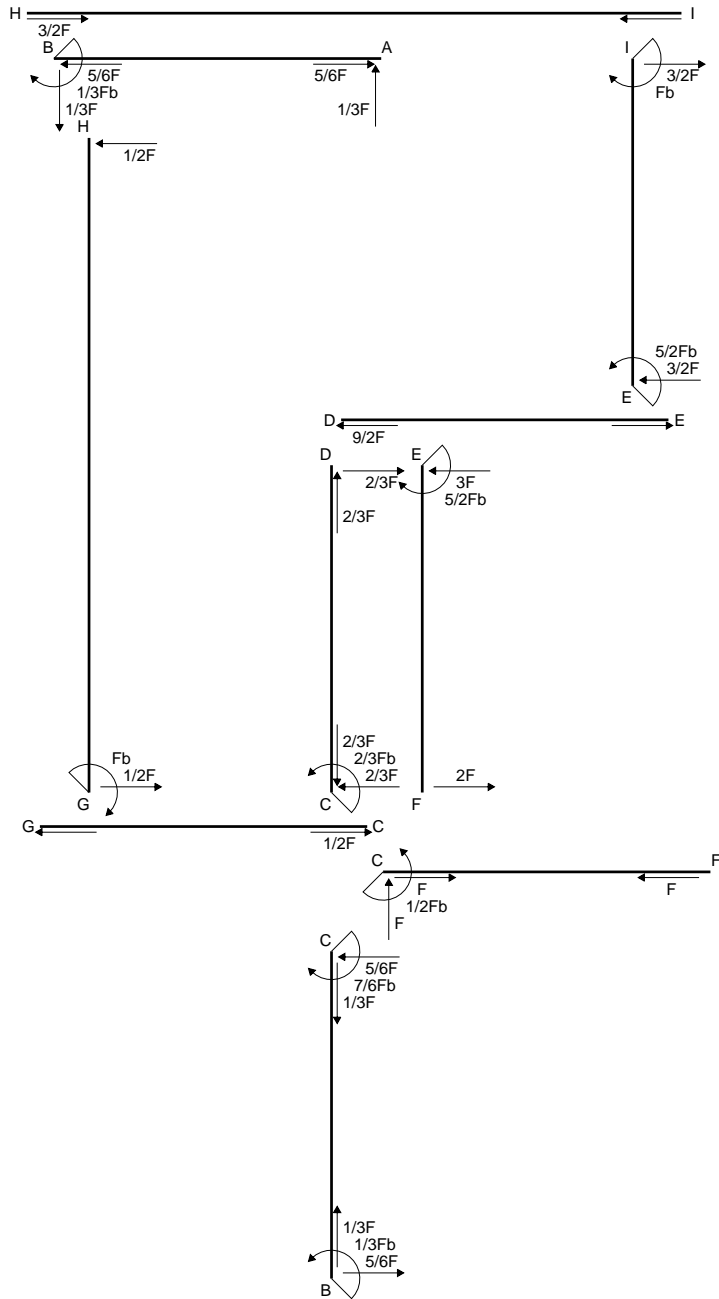
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

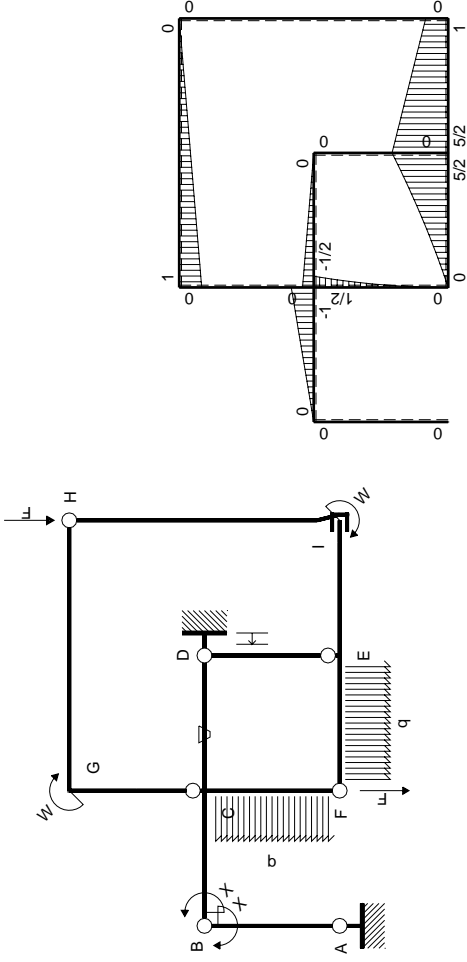
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$



$\left(\oplus\right) F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$           | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-1/3Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $1/3Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

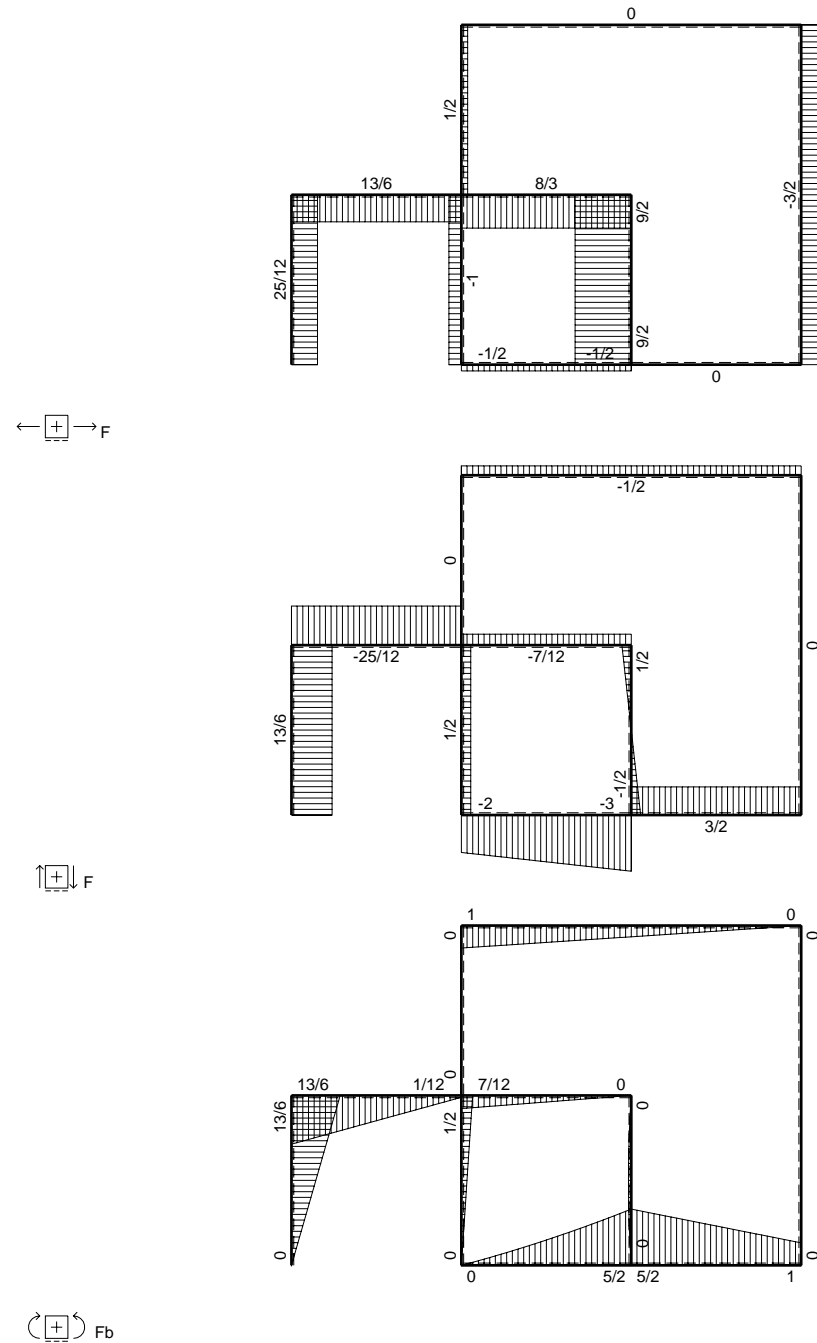
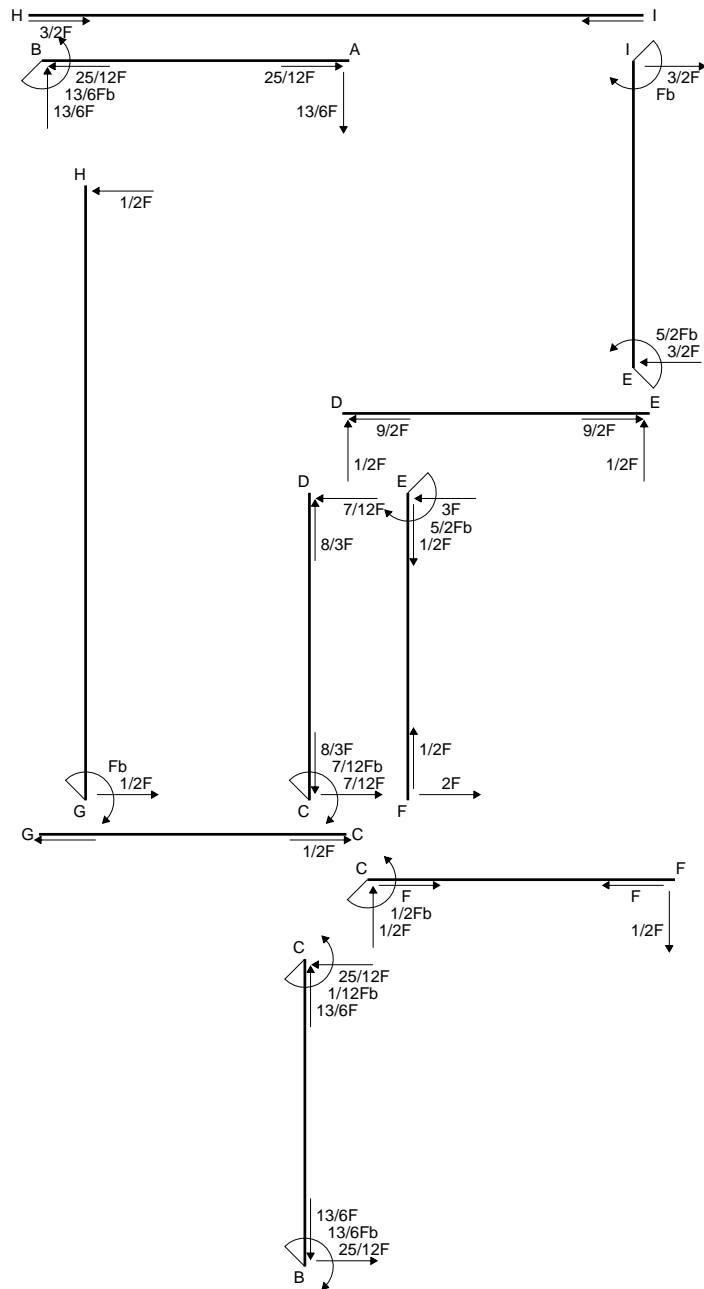
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

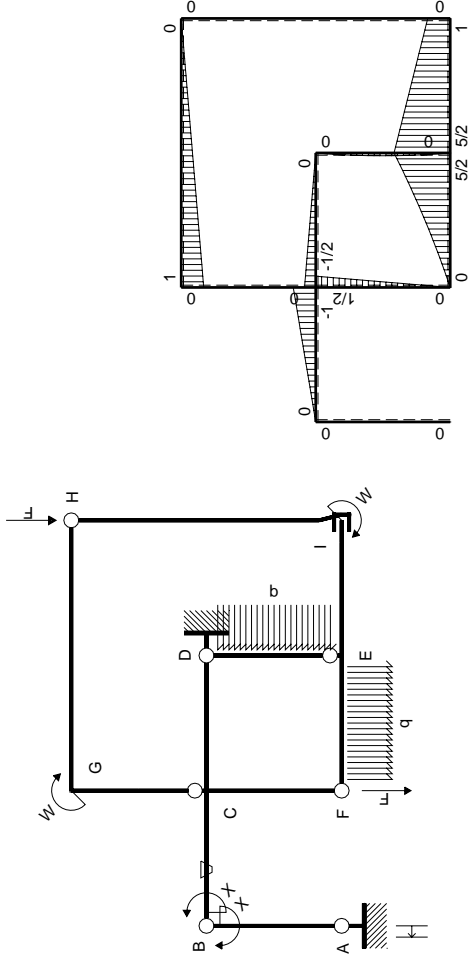
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

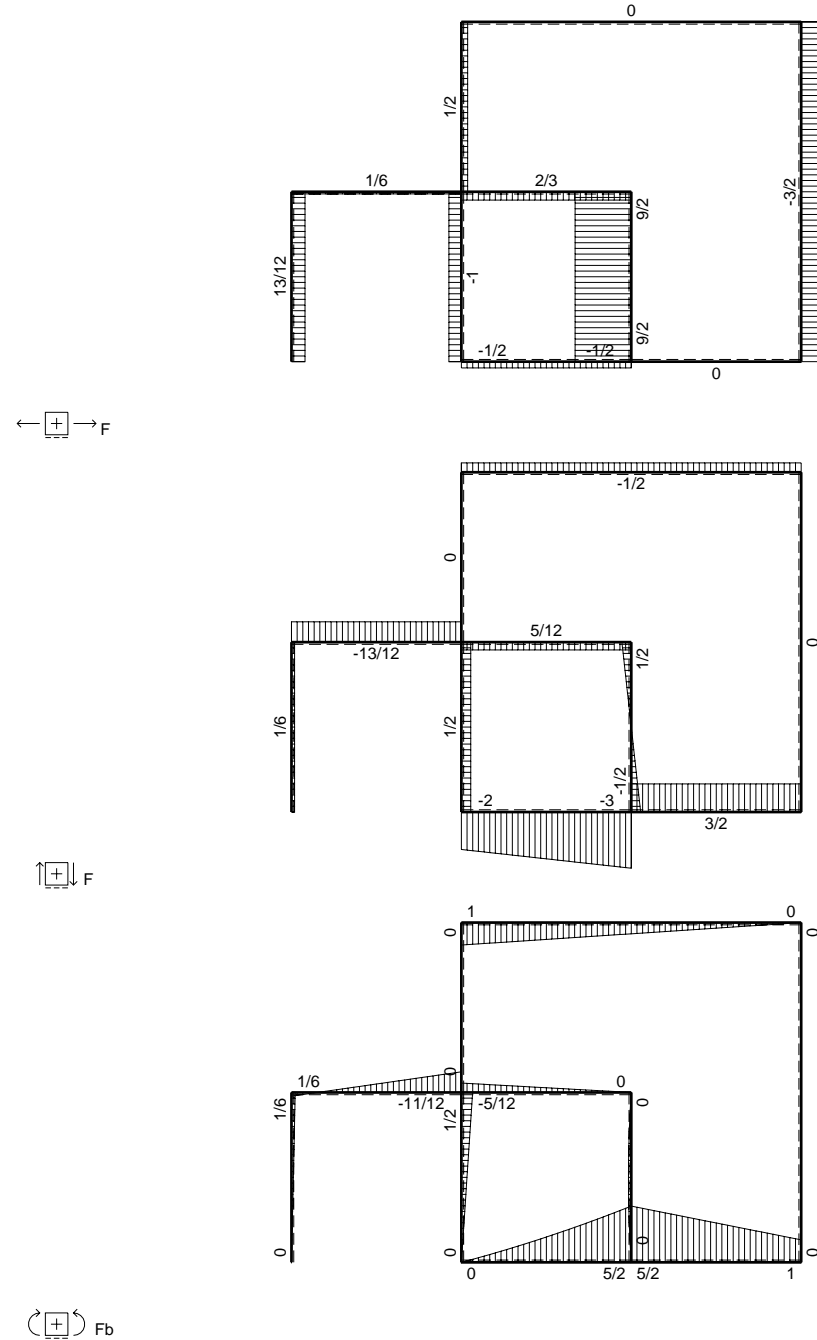
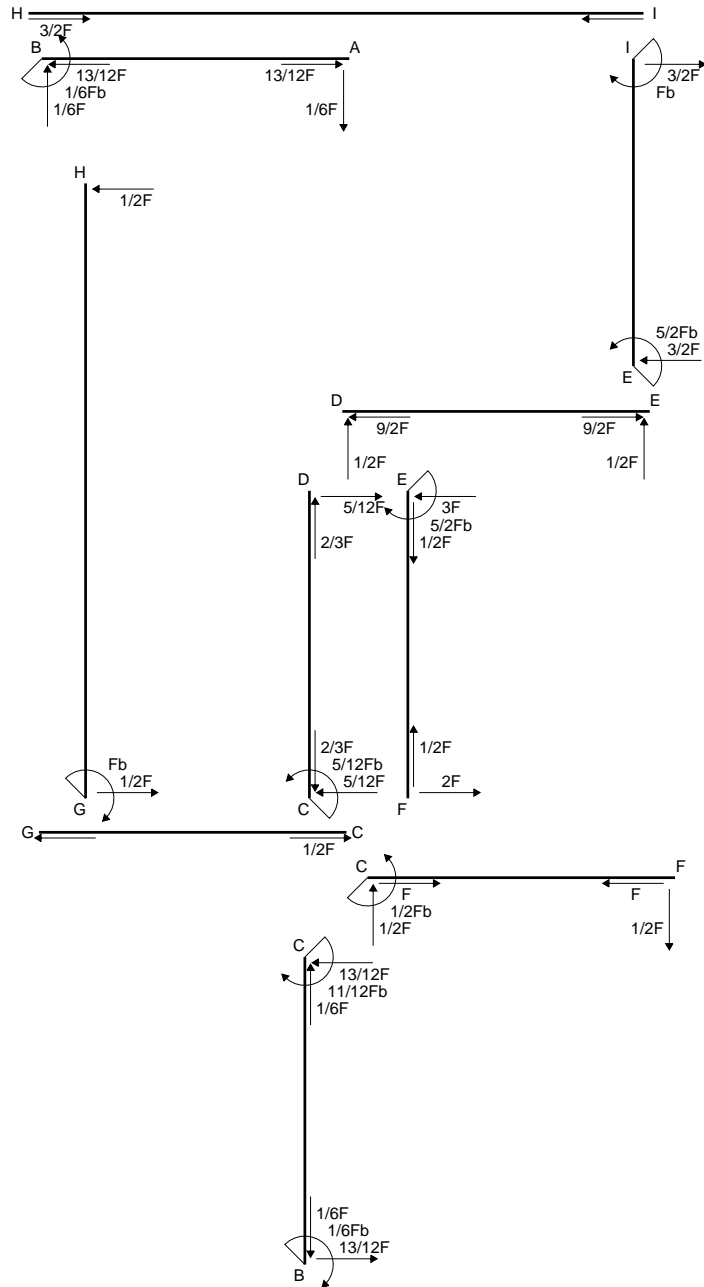
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

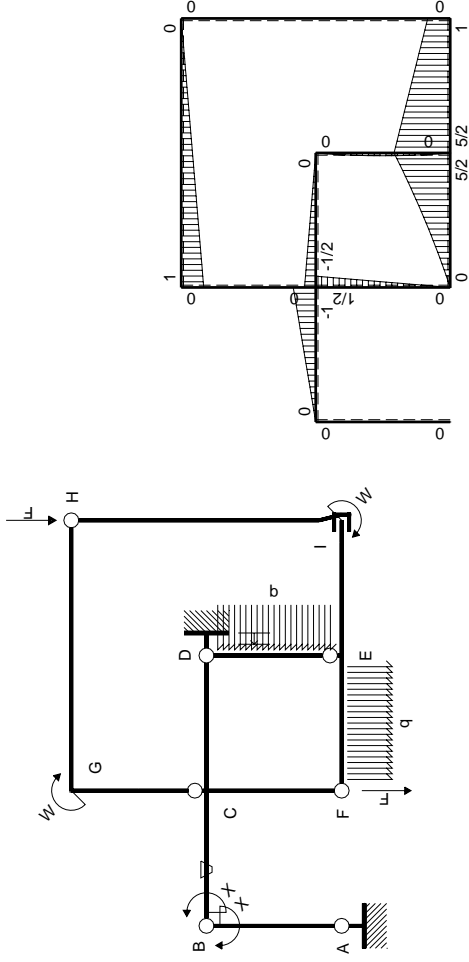
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | $-Fb/EJ$ | $Fx-1/2Fx^2/b$          | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $1/6Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $-1/6Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

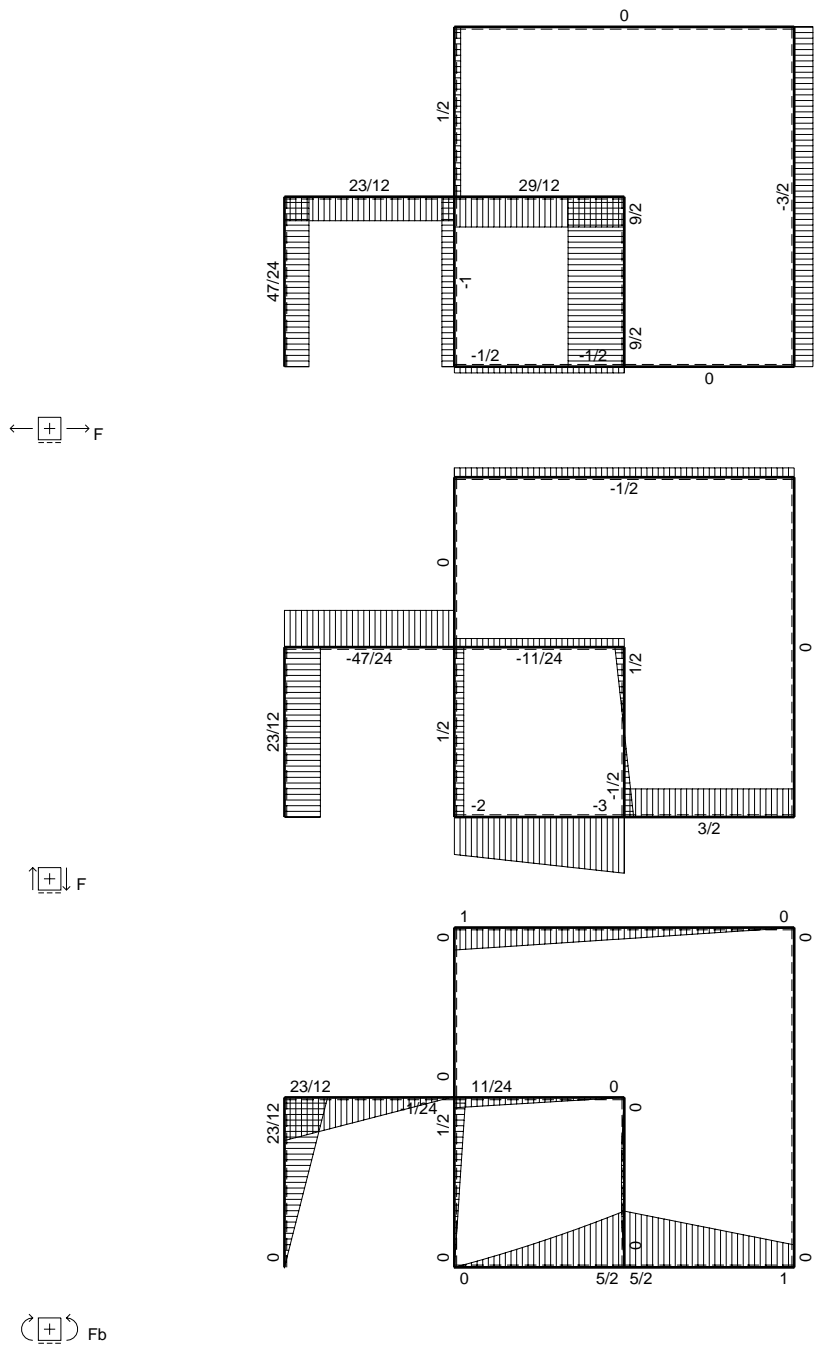
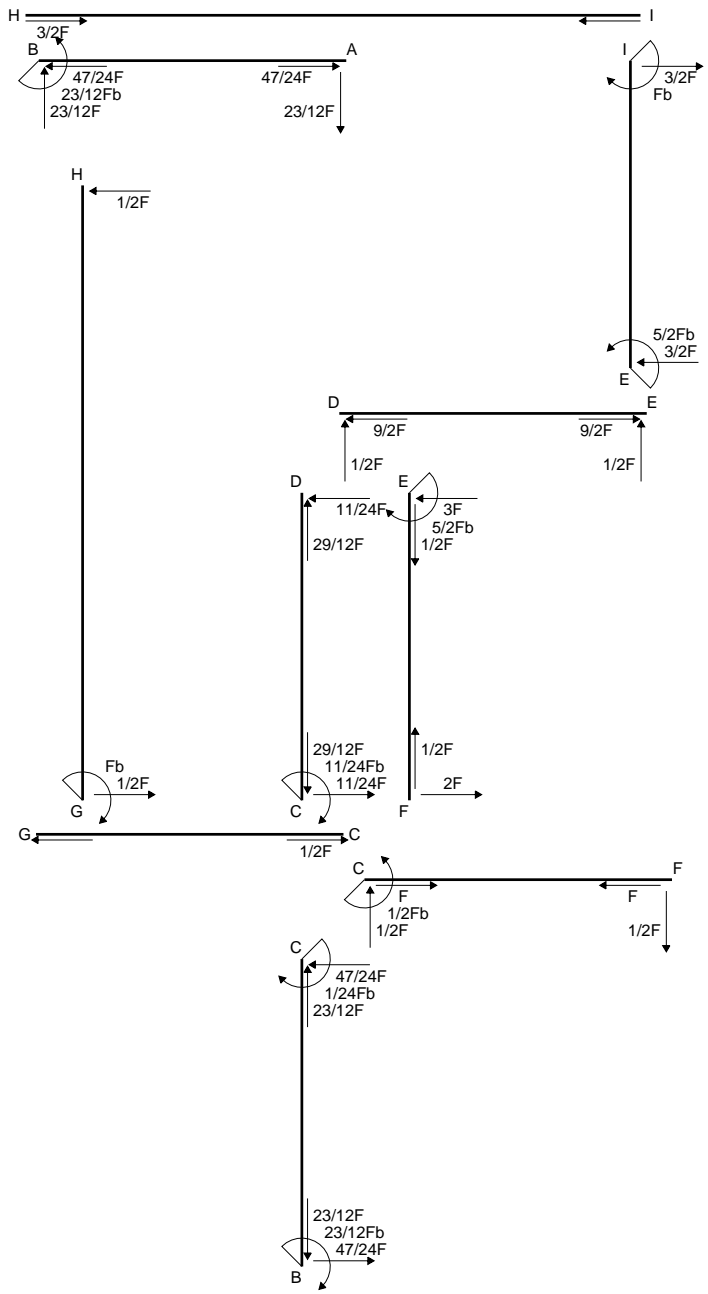
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

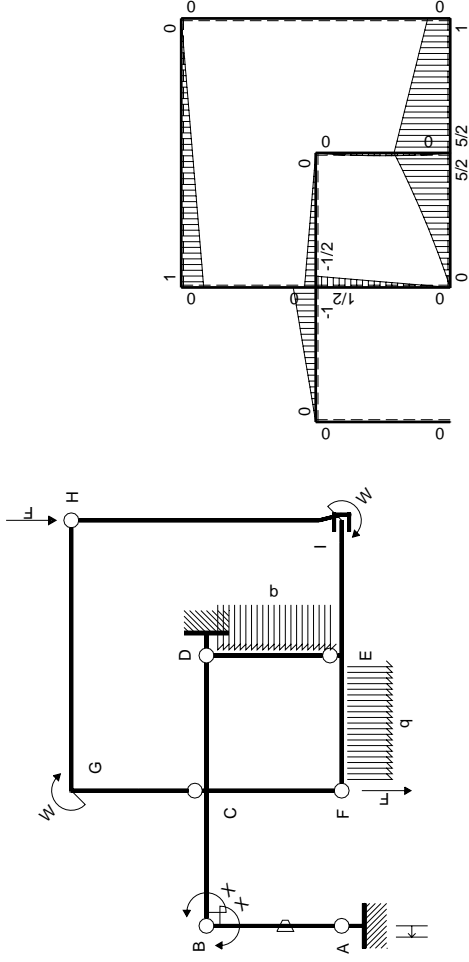
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |               |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

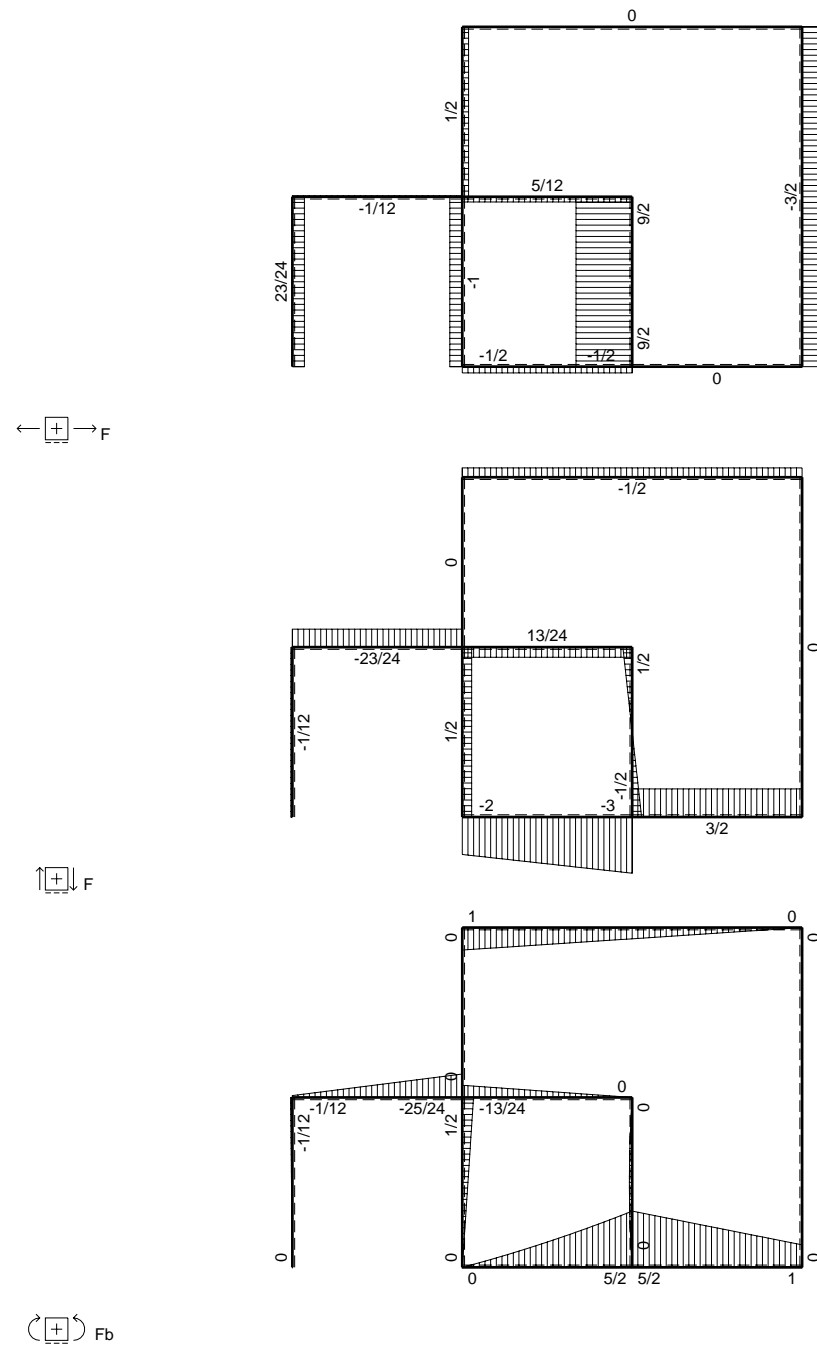
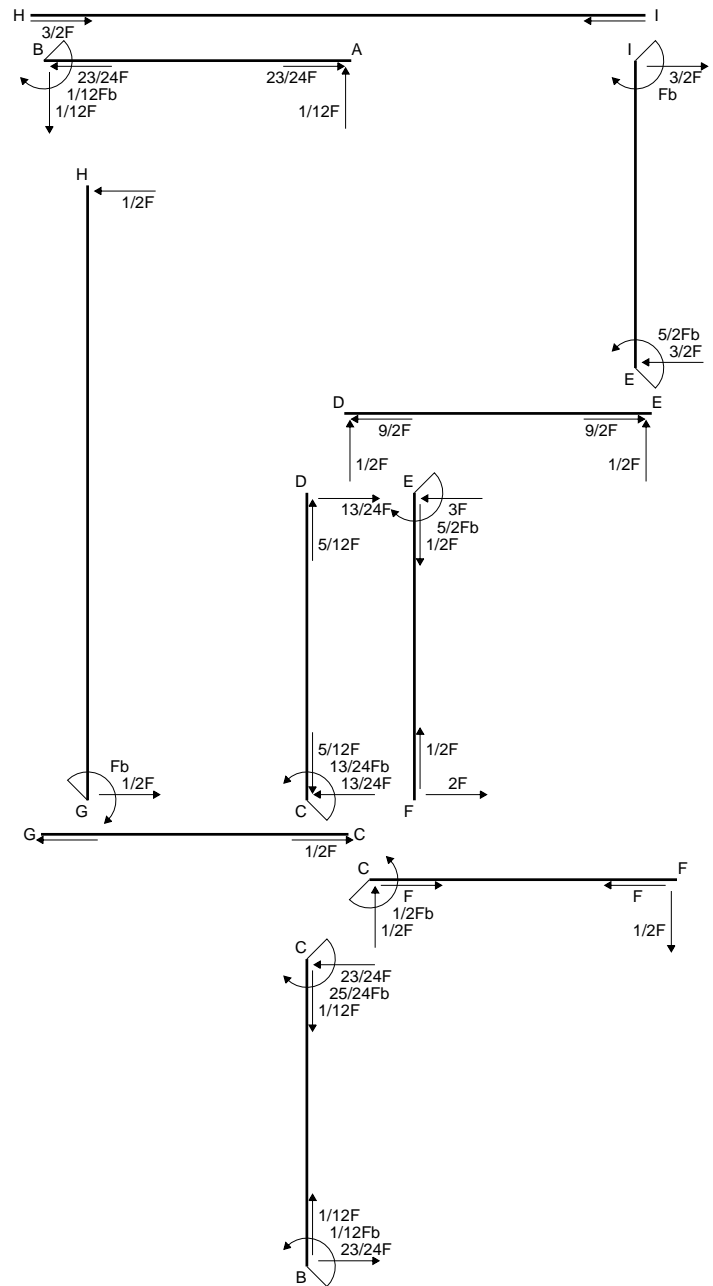
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

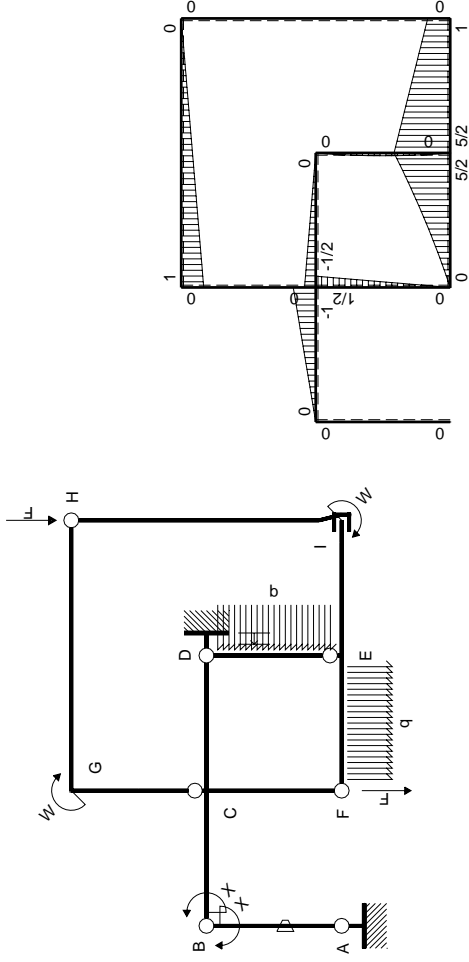
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | 0        | $1/4Fb-1/2Fx+1/4Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | 0        | $1/4Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                         |               |                         | $-1/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |               |                         | $1/12Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

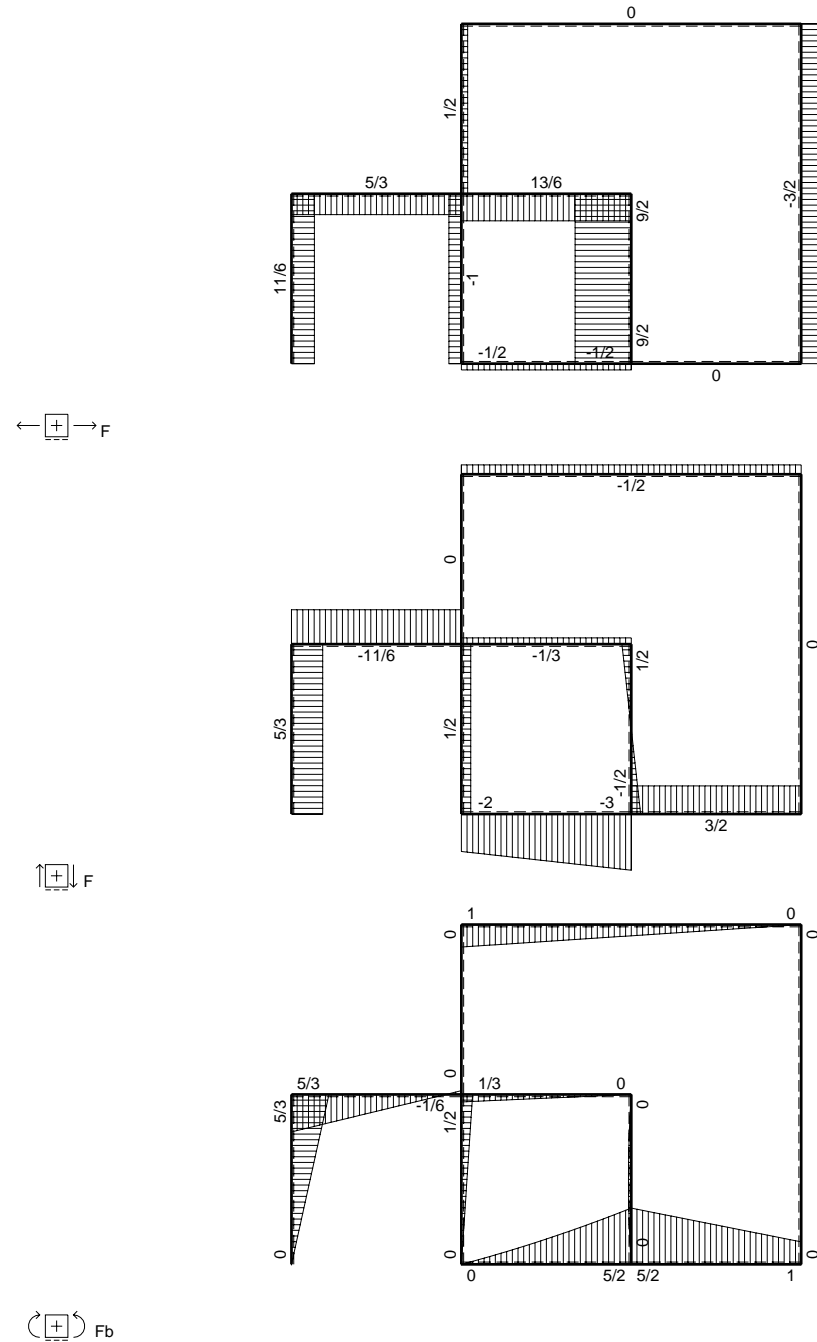
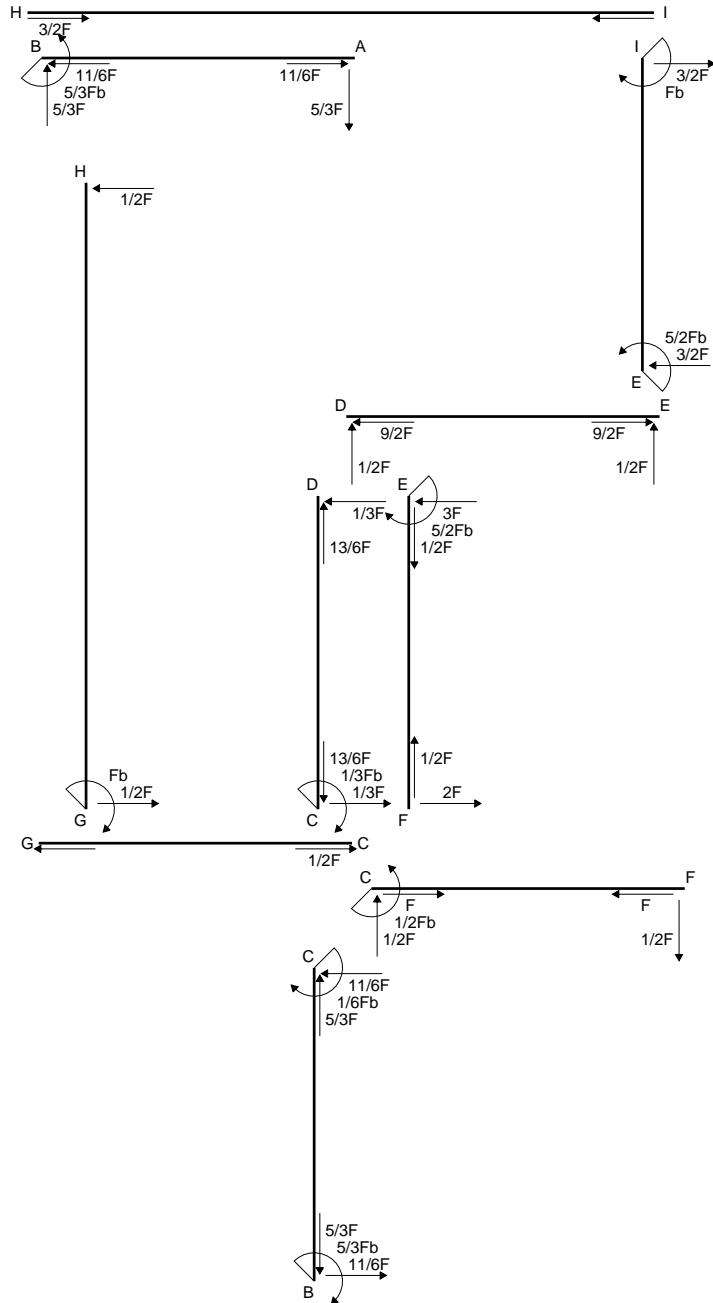
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ$$

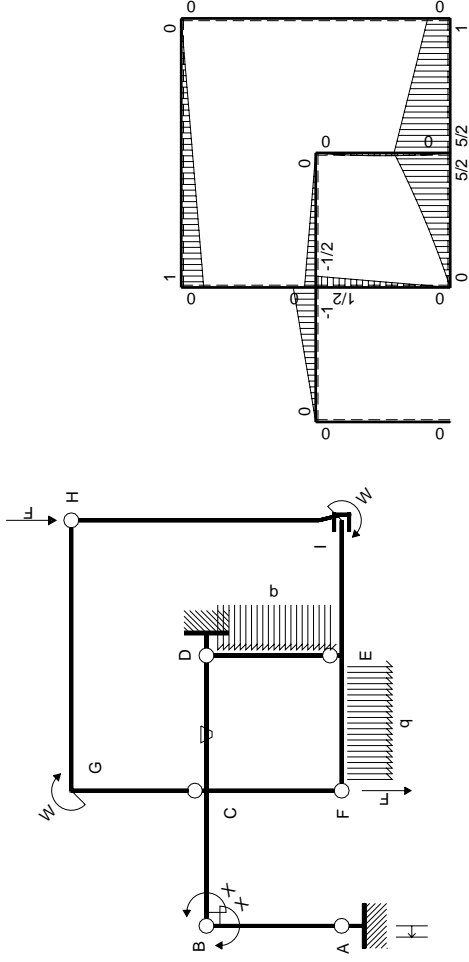
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx = [1/12 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/12 b) Fb 1/EJ = 1/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                     |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                     |          |                         |                     |                         | $5/3Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         | $-5/3Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

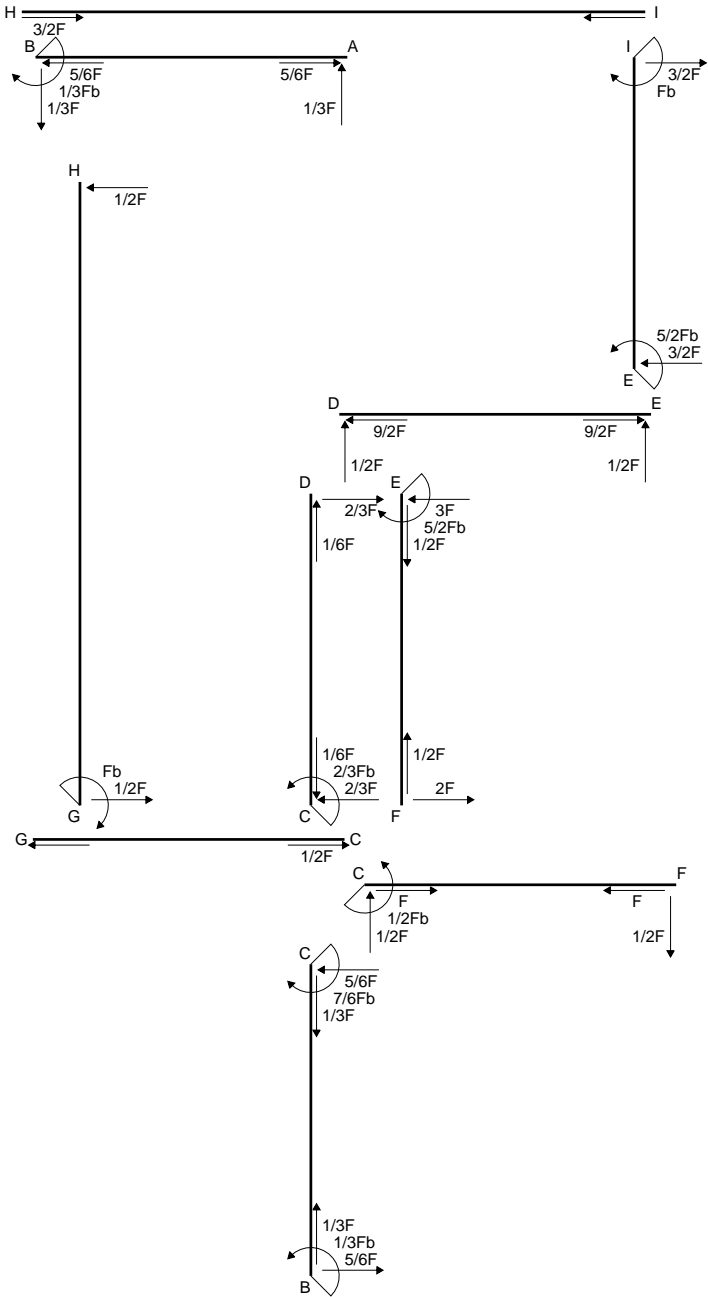
$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

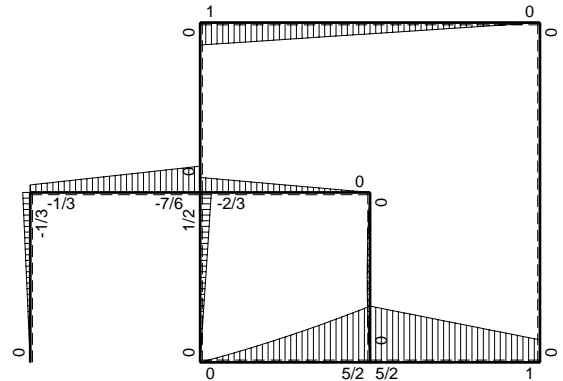
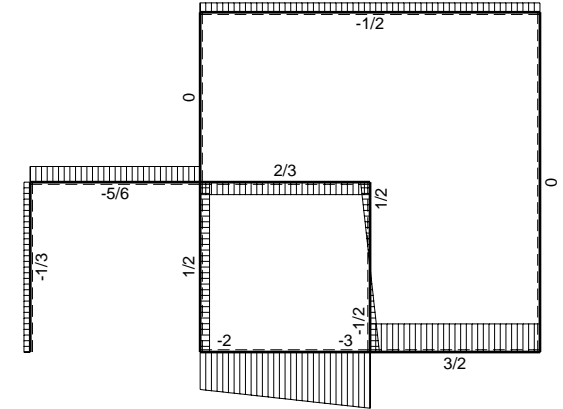
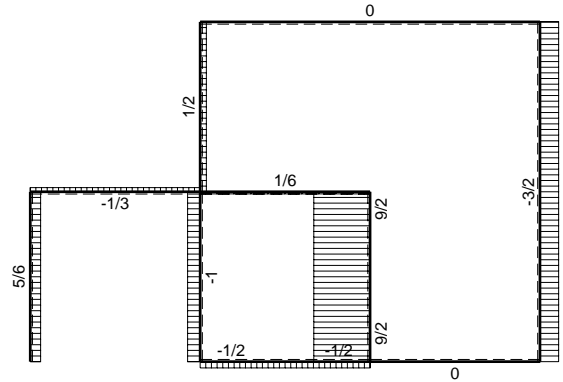
$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$

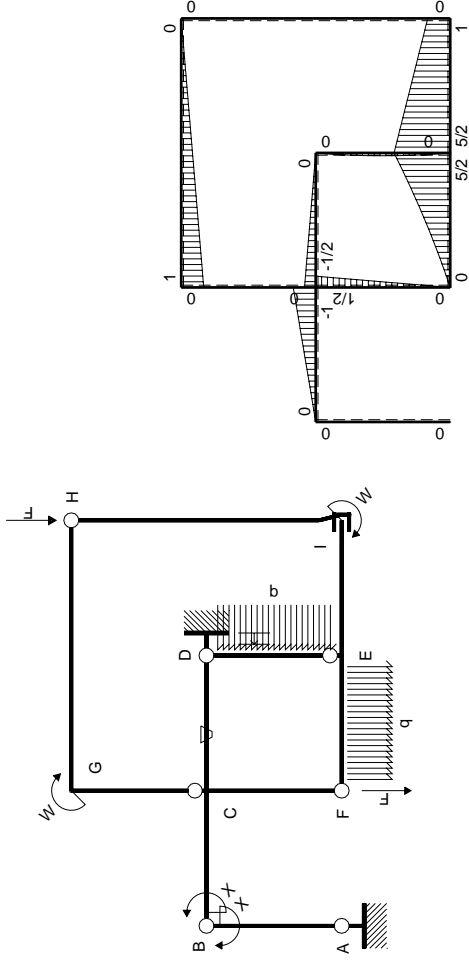


← ⊕ → F

↑ ⊕ ↓ F

⊕ ⊖ F<sub>b</sub>





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$               | 0        | $Fx-1/2Fx^2/b$          | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$             | 0        | $1/2Fb-1/2Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+1/2Fx$      | $-Fb/EJ$ | $1/4Fb-1/2Fx+1/4Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/12+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2Fx$             | $Fb/EJ$  | $1/4Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$     | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$             | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                         |                     |                         |                             | $-1/3Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                         |                     |                         |                             | $1/3Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

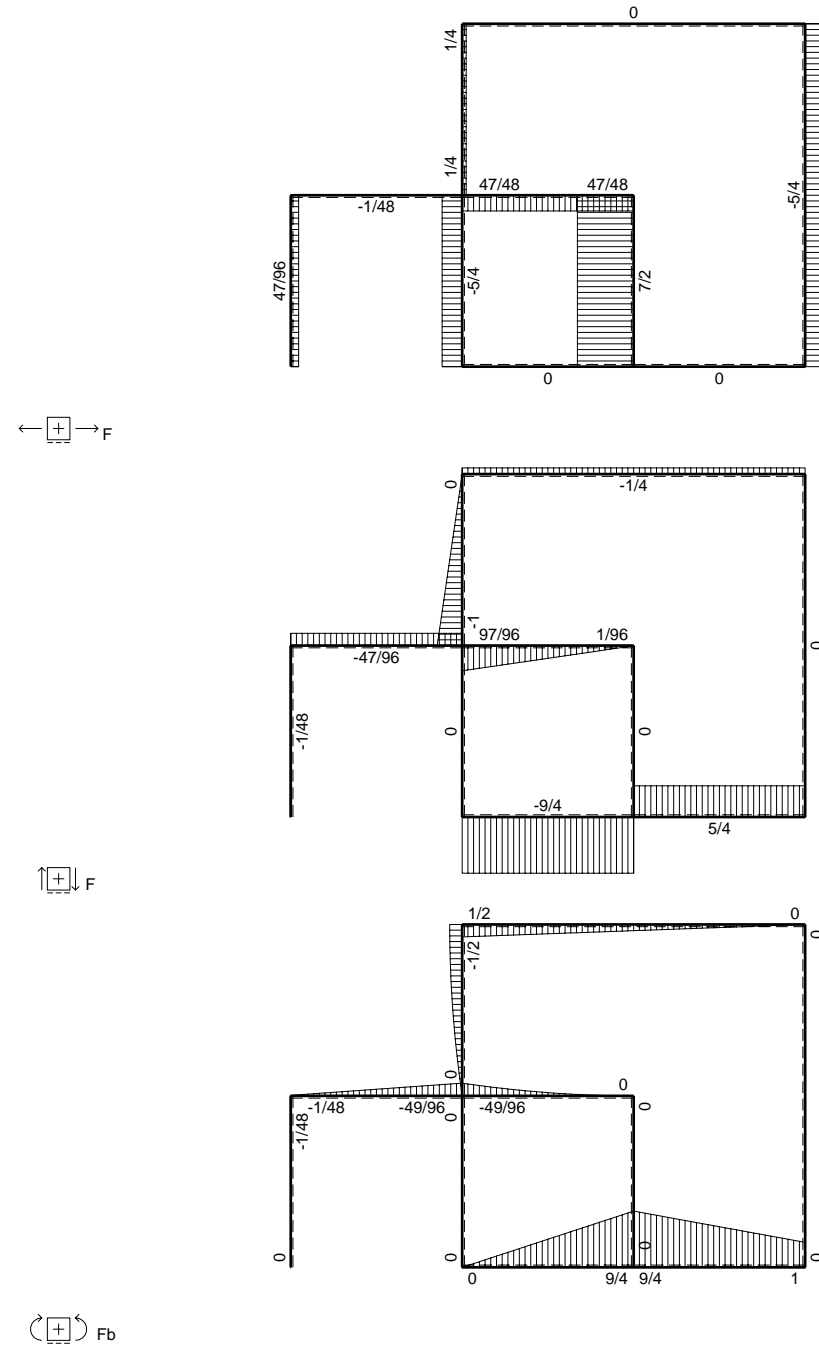
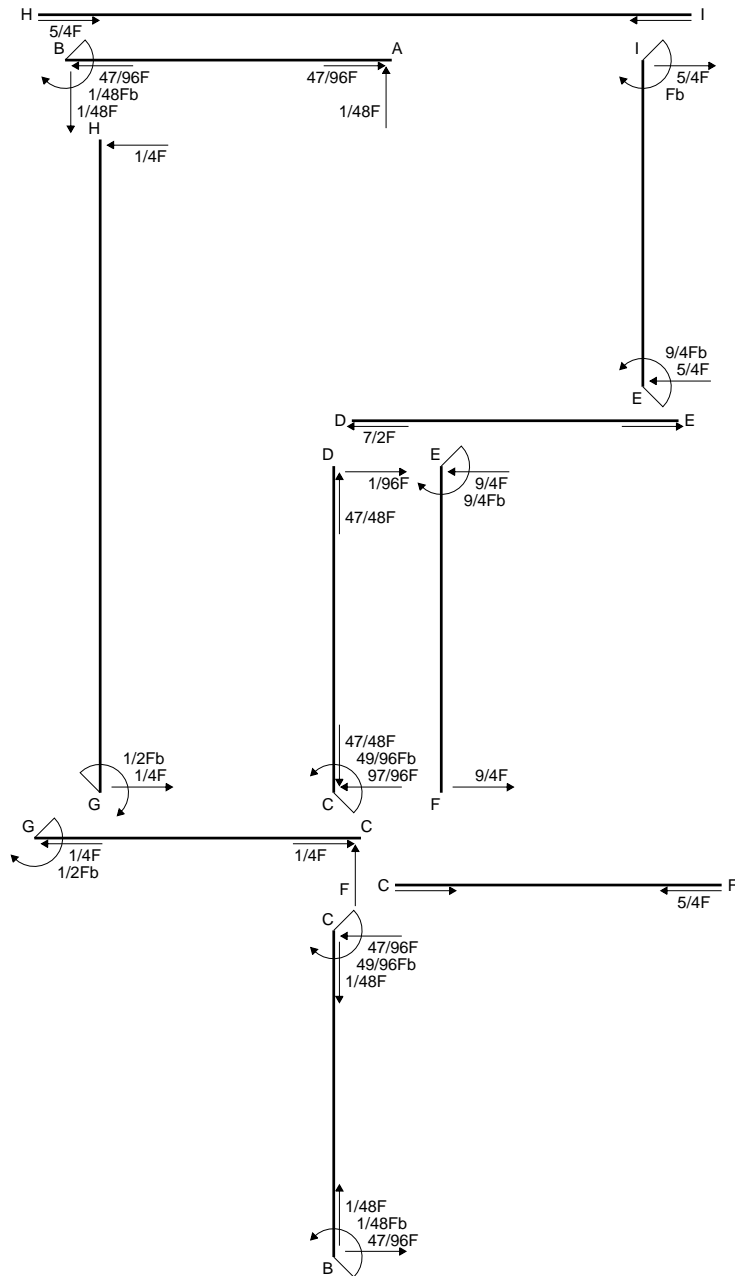
$$= [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

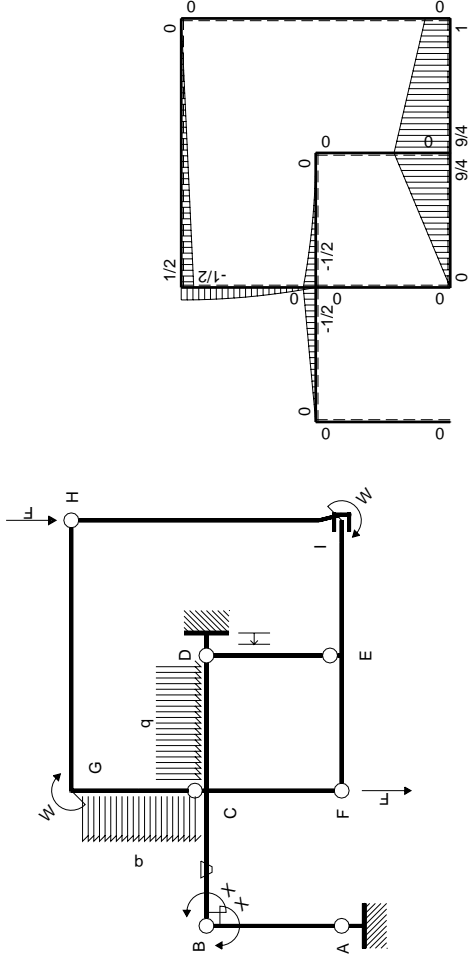
$$= (1/4 b - 1/4 b + 1/12 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 1/3 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/12 b) Fb 1/EJ + (-1/4 b) \theta = 1/3 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|---------------------|----------|-----------------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                                 | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                                 | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$            | $-Fb/EJ$ | $1/2Fx-1/4Fx^2/b$                 | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/6+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$       | $Fb/EJ$  | $1/4Fb-1/4Fx^2/b$                 | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+Fx-1/2qx^2$ | 0        | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $1/2qx^2$           | 0        | $1/4qx^3/b$                       | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $9/4Fb-9/4Fx$       | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-9/4Fx$            | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$       | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$     | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$       | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$            | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx$          | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-9/4Fb+5/4Fx$      | 0        | 0                                 | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                     |          |                                   |                     |                         |                             | $-1/48Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                     |          |                                   |                     |                         |                             | $1/48Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

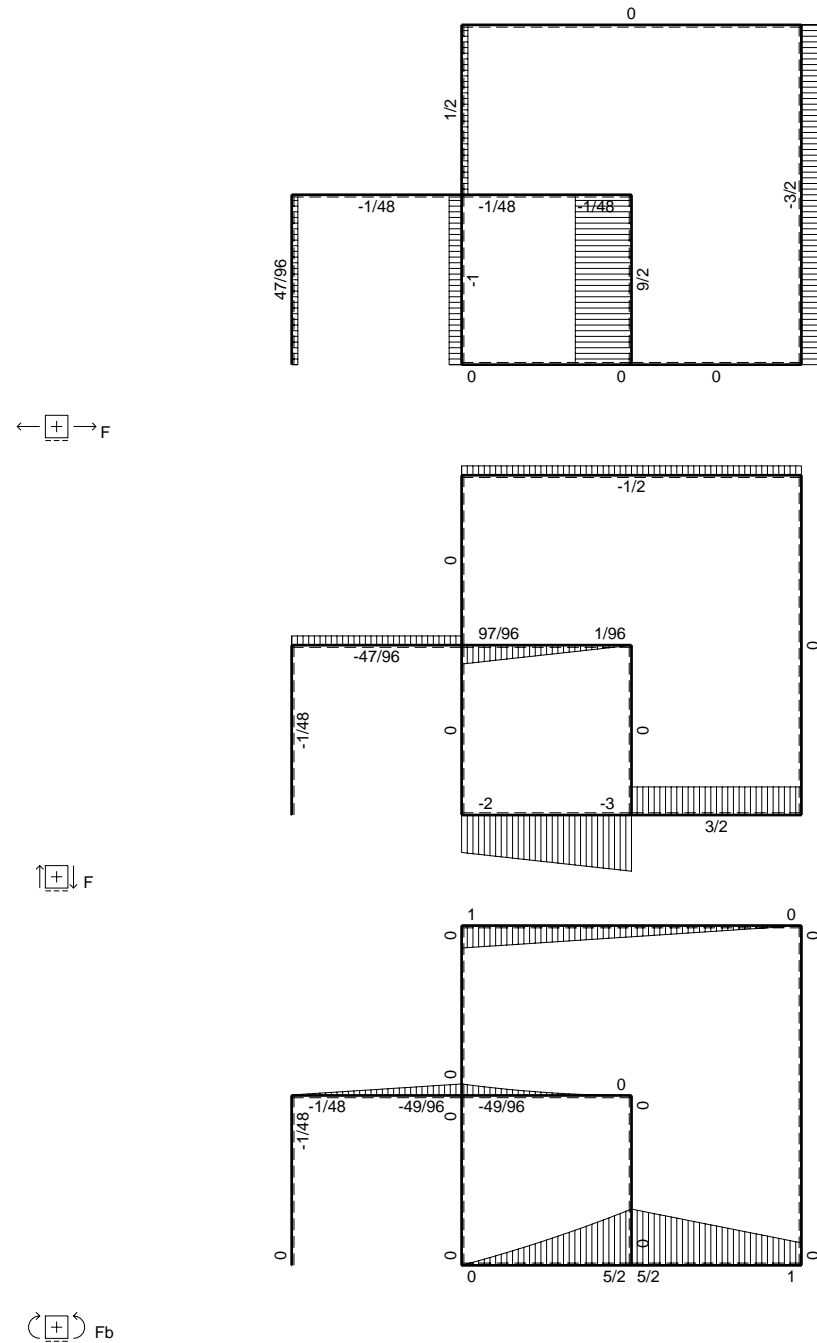
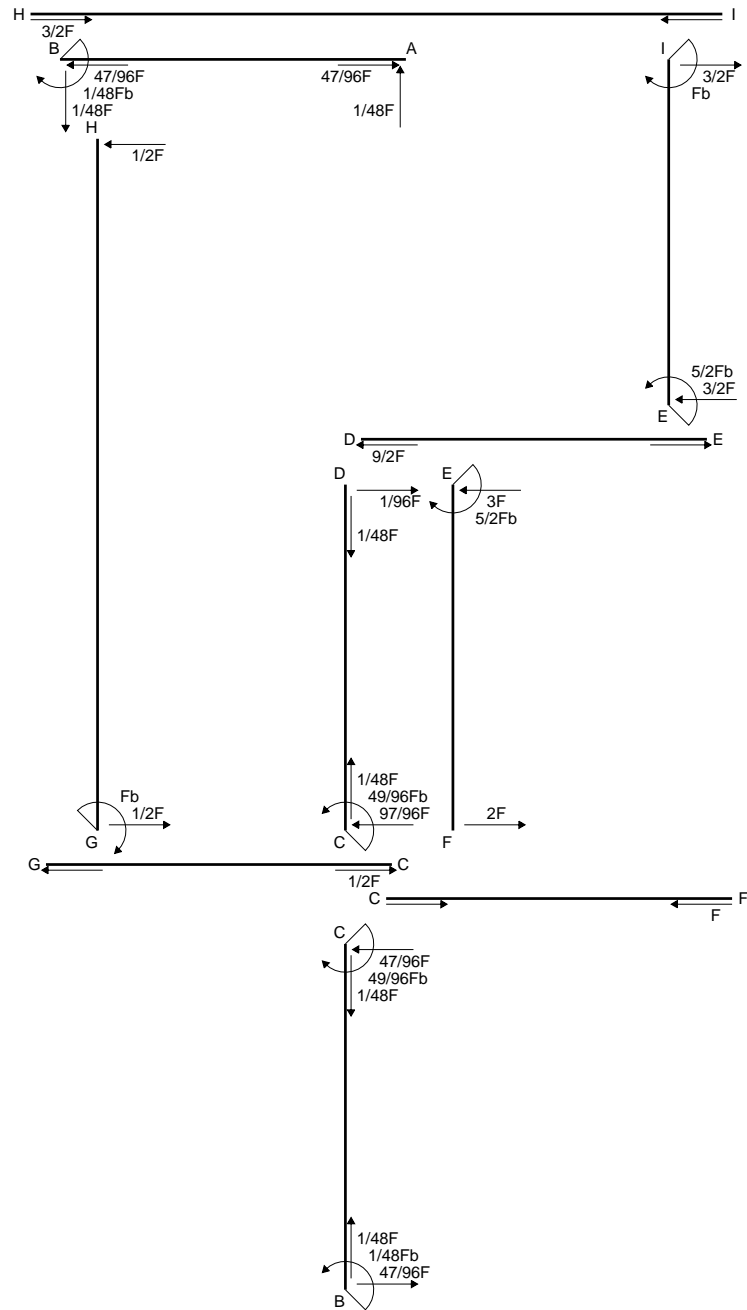
$$L_{CD}^{xo} = \int_0^b (1/4 - 3/4 x/b + 3/4 x^2/b^2 - 1/4 x^3/b^3) Fb 1/EJ dx$$

$$= [1/4 x - 3/8 x^2/b + 1/4 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb 1/EJ$$

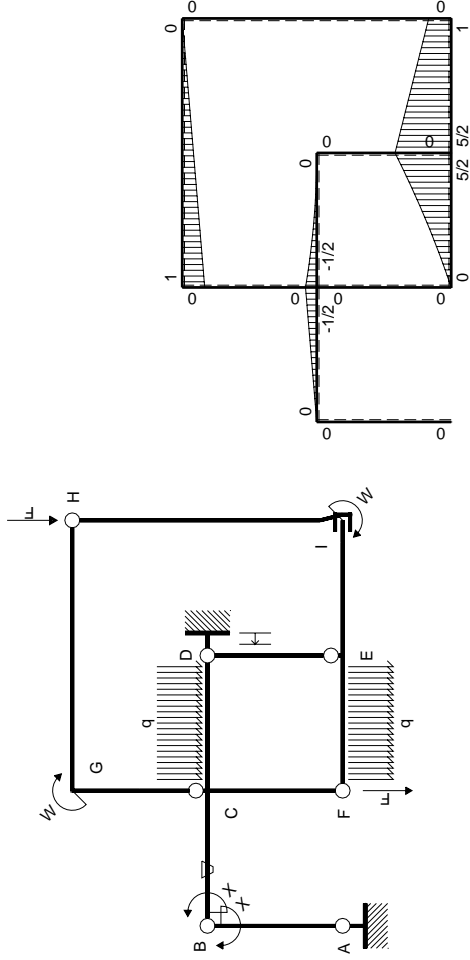
$$= (1/4 b - 3/8 b + 1/4 b - 1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^3/b^3) Fb 1/EJ dx = [1/16 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$



⊕ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$            | $\theta$ | $M_x M_o$                         | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|---------------------|----------|-----------------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                   | 0        | 0                                 | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                   | 0        | 0                                 | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-1/2Fx$            | $-Fb/EJ$ | $1/2Fx-1/4Fx^2/b$                 | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/6+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $1/2Fb-1/2Fx$       | $Fb/EJ$  | $1/4Fb-1/4Fx^2/b$                 | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-1/2Fb+Fx-1/2qx^2$ | 0        | $1/4Fb-3/4Fx+3/4Fx^2/b-1/4qx^3/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/16+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $1/2qx^2$           | 0        | $1/4qx^3/b$                       | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $5/2Fb-3Fx+1/2qx^2$ | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-2Fx-1/2qx^2$      | 0        | 0                                 | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$          | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$            | 0        | 0                                 | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                   | 0        | 0                                 | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx$          | 0        | 0                                 | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-5/2Fb+3/2Fx$      | 0        | 0                                 | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                     |          |                                   |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                     |          |                                   |                     |                         | $-1/48Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                     |          |                                   |                     |                         | $1/48Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (1/2 x/b - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/4 x^2/b - 1/12 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/4 - 1/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/4 x - 1/12 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/12 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 11/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/4 - 3/4 x/b + 3/4 x^2/b^2 - 1/4 x^3/b^3) Fb 1/EJ dx$$

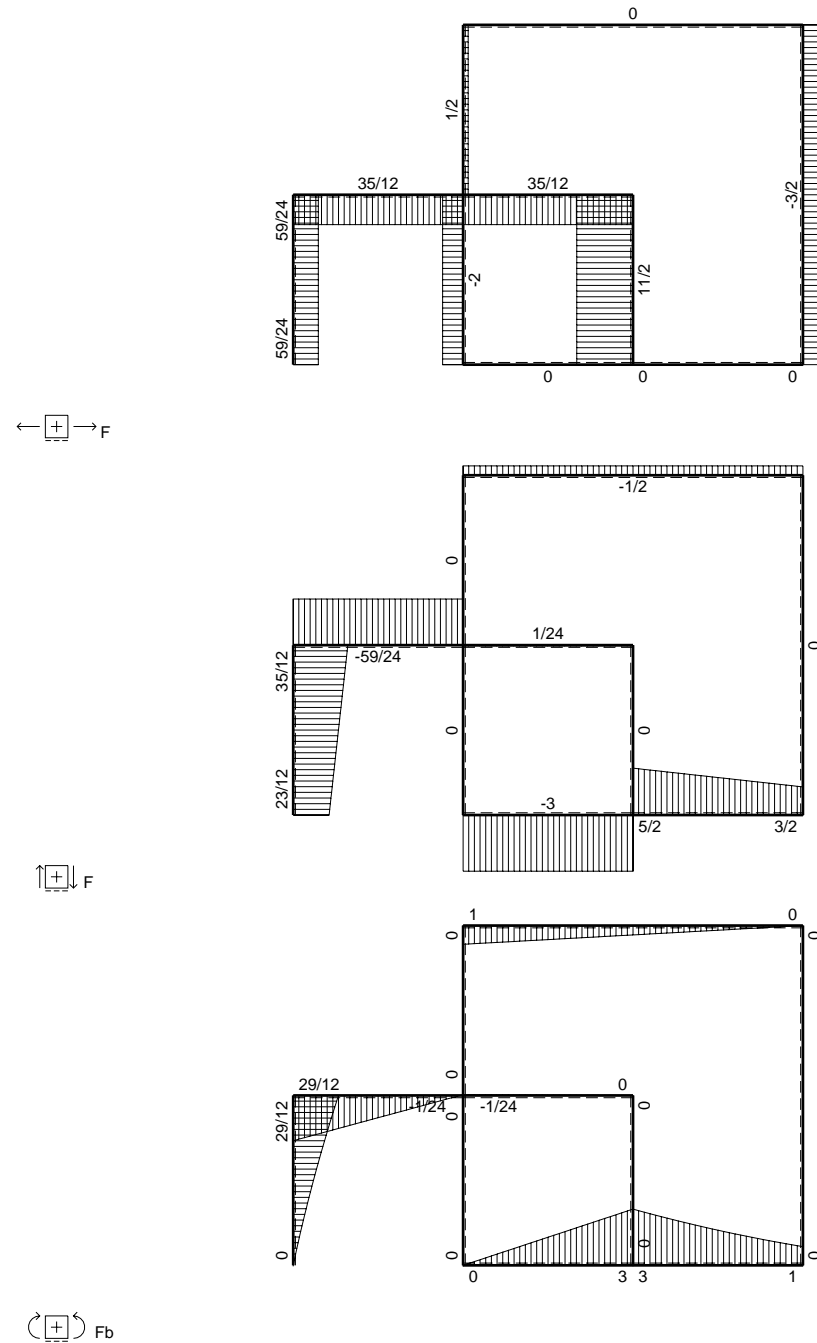
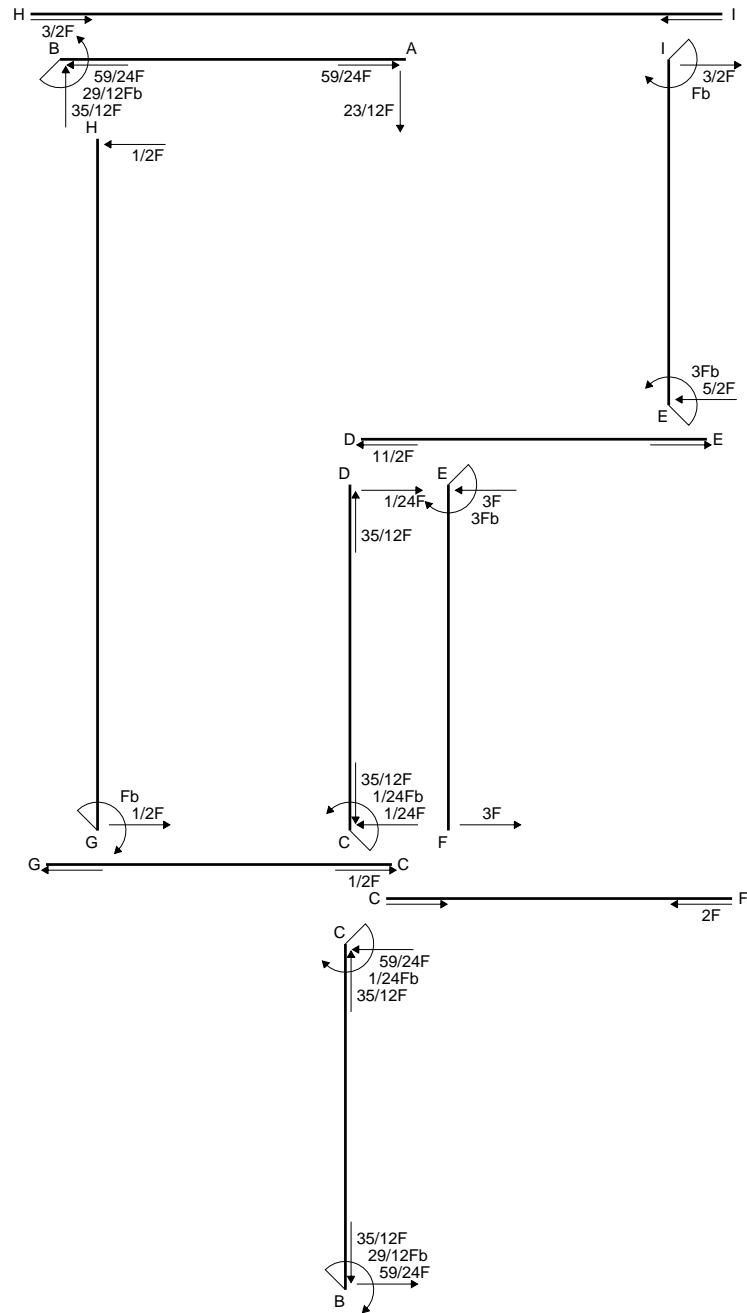
$$= [1/4 x - 3/8 x^2/b + 1/4 x^3/b^2 - 1/16 x^4/b^3]_0^b Fb 1/EJ$$

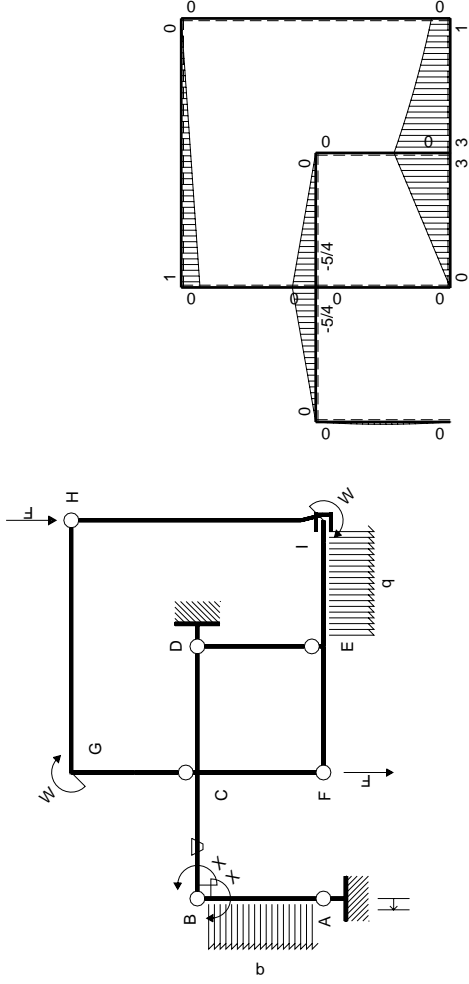
$$= (1/4 b - 3/8 b + 1/4 b - 1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/4 x^3/b^3) Fb 1/EJ dx = [1/16 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/16 b) Fb 1/EJ = 1/16 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|-----------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$            |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                       |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$           |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$           |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                       |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                       |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                       |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                       |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                       |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                       |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                       |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                     |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                       |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                     |                         | $Fb^2/EJ$                   |                       |
|       | totali                      |                      |          |                          |                     |                         | $29/12Fb^2/EJ$              | $Xb/EJ$               |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         | $-29/12Fb$                  |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

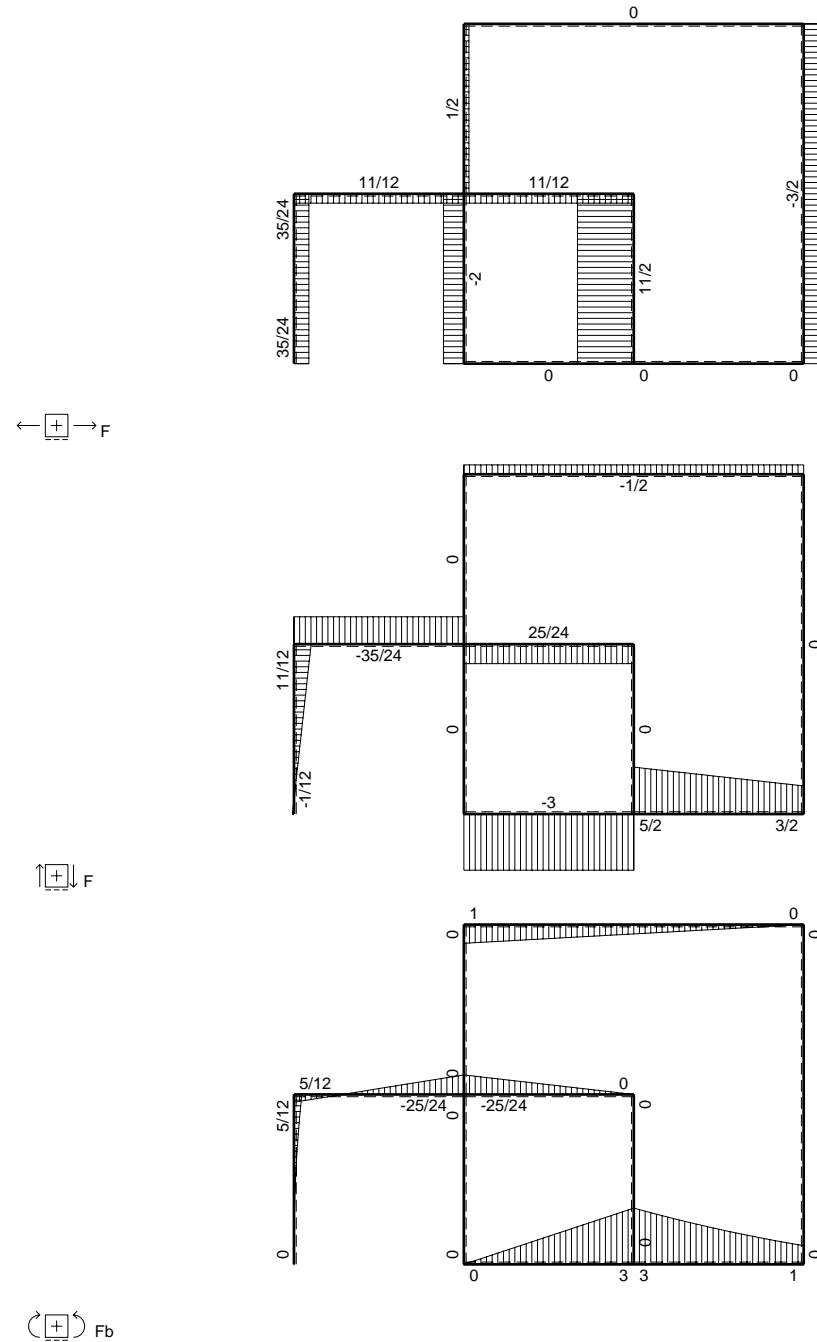
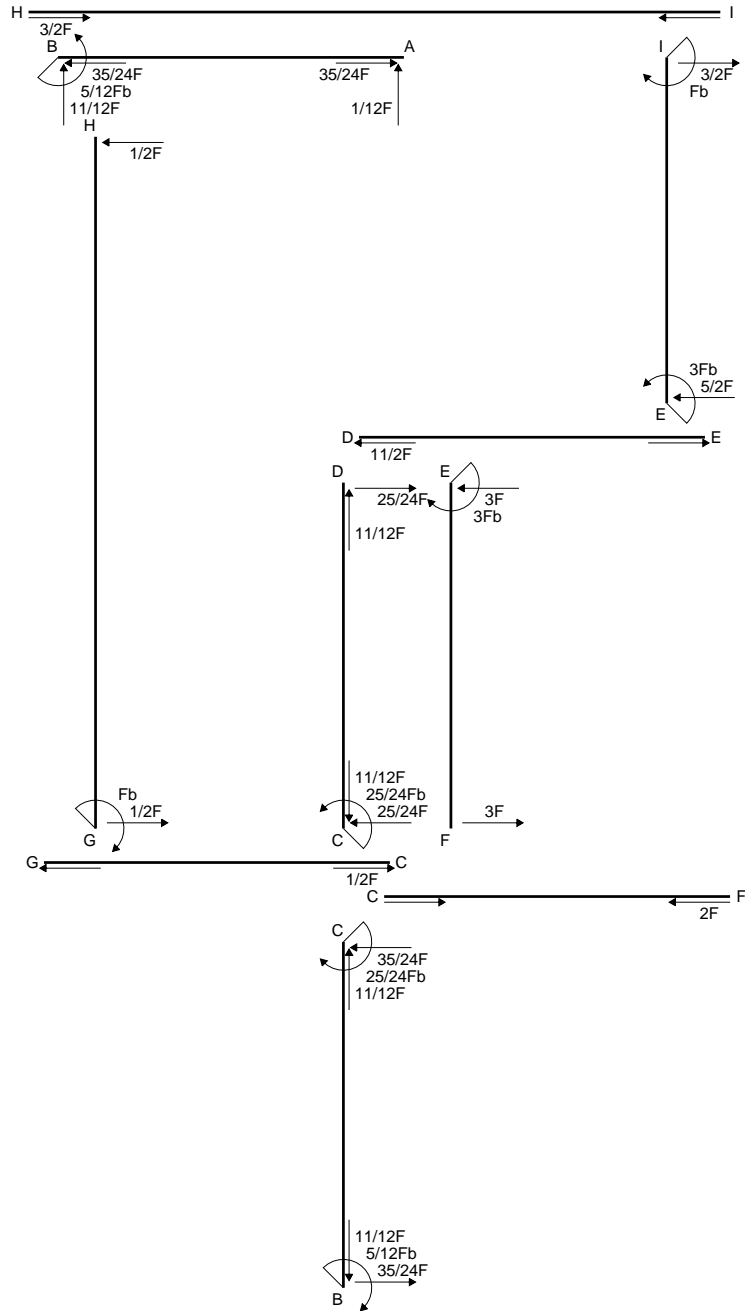
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

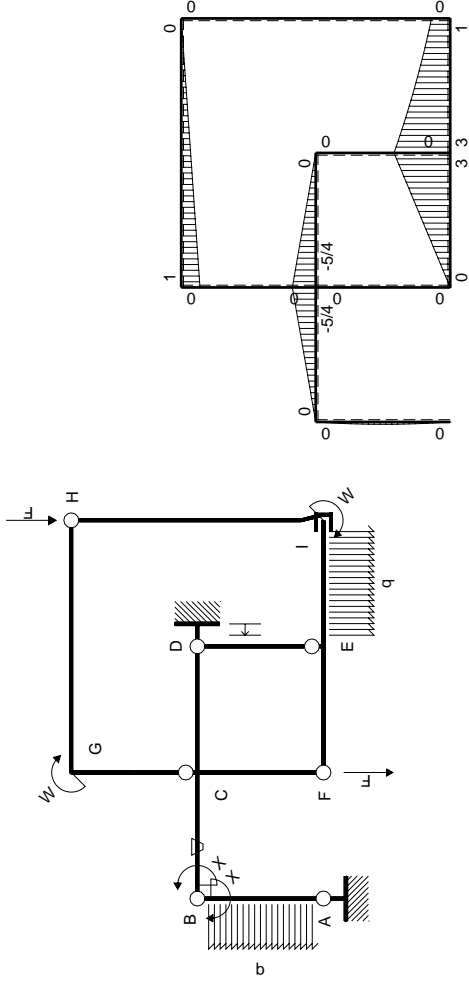
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$        | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                      |          |                          |                     |                         | $5/12Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         | $-5/12Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

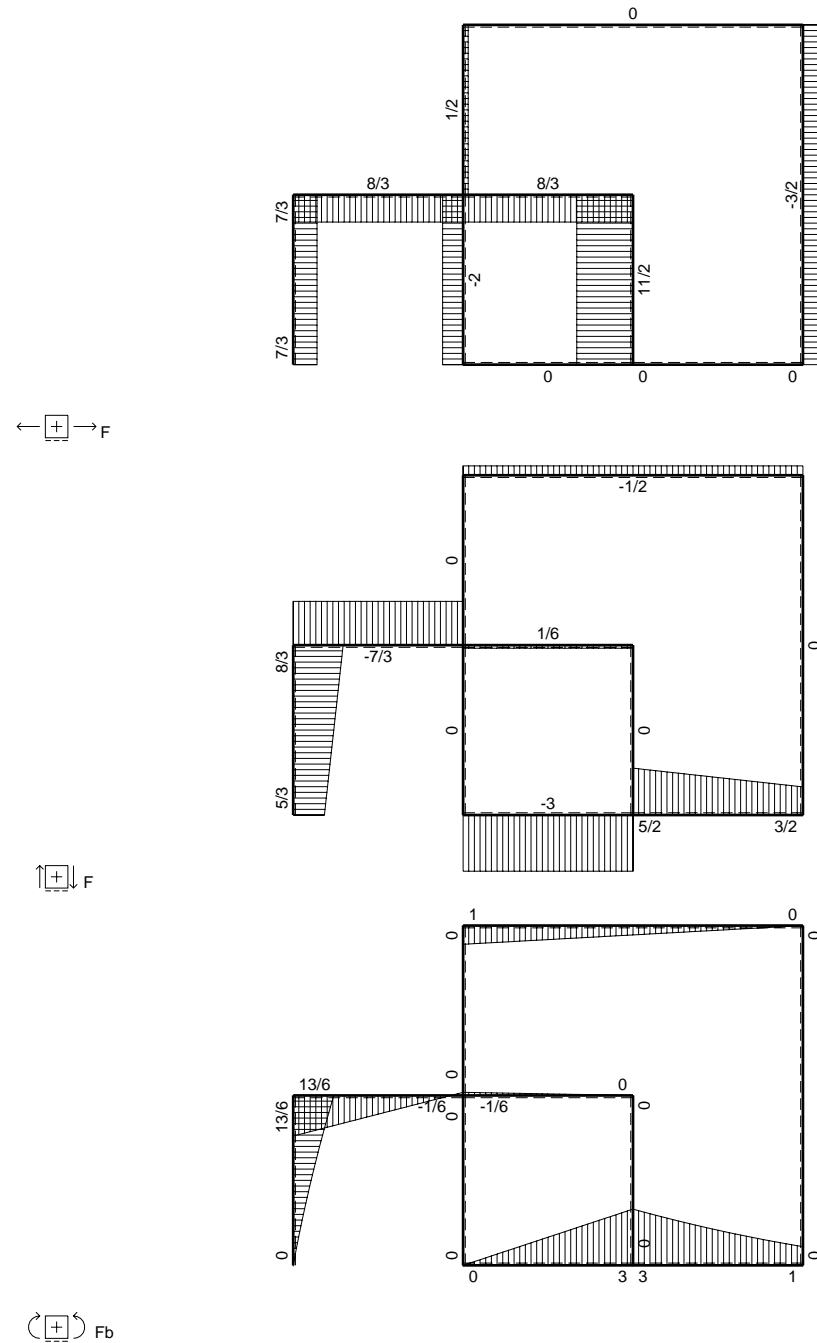
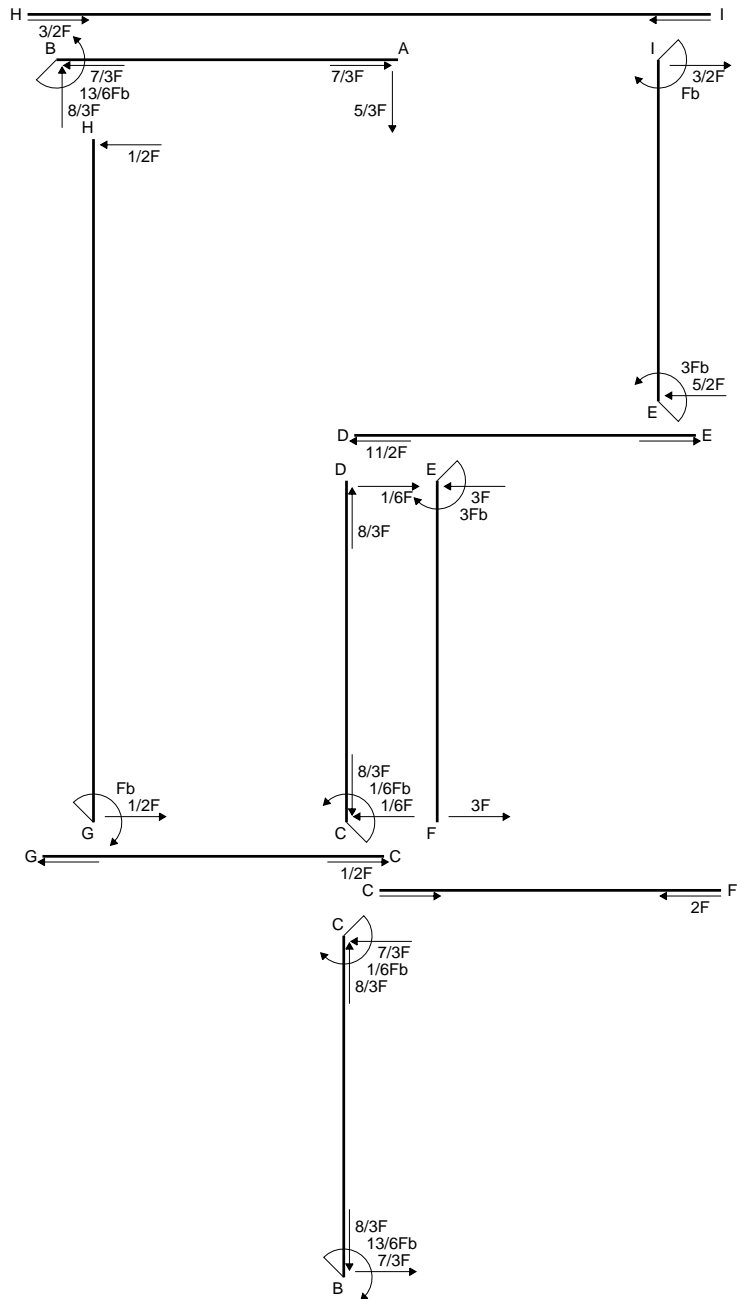
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

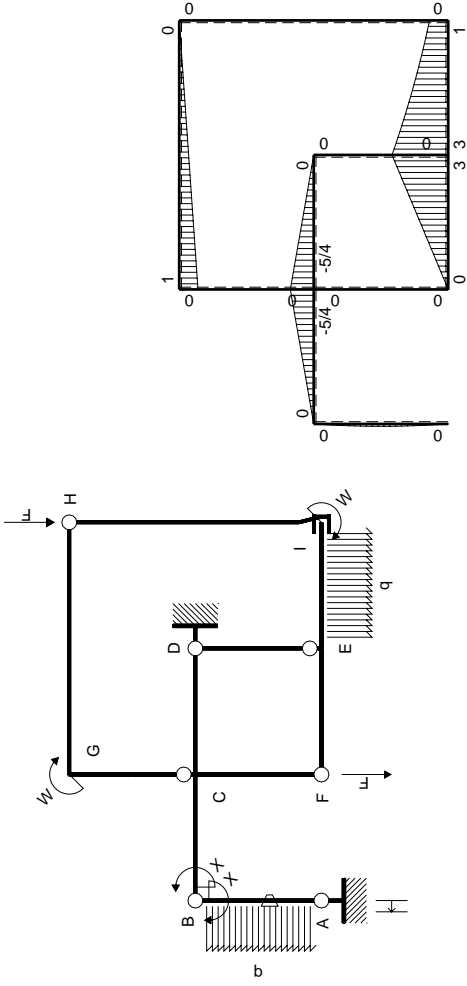
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

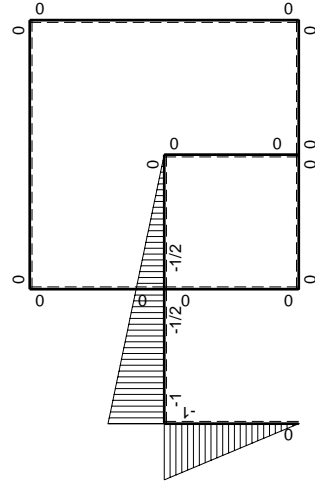






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                          |               |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |               |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

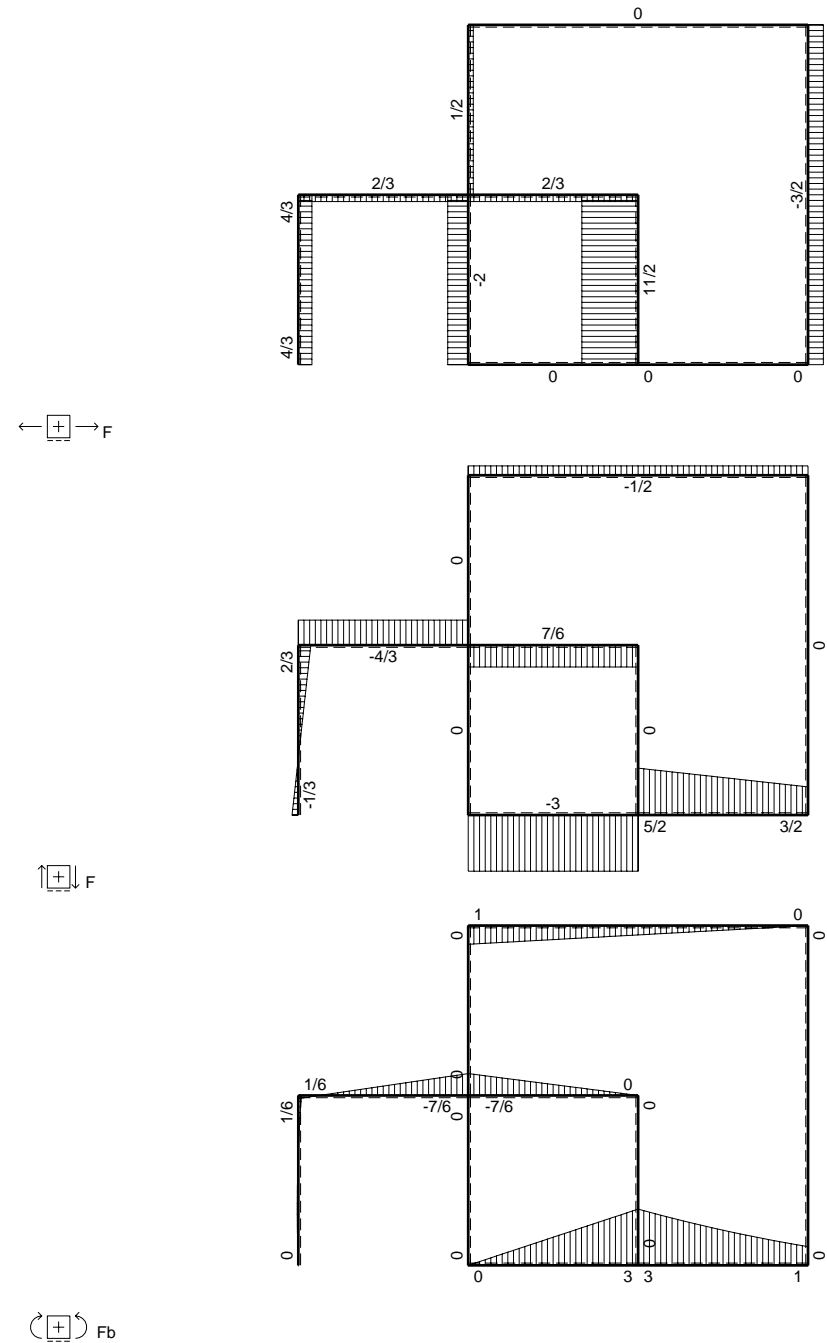
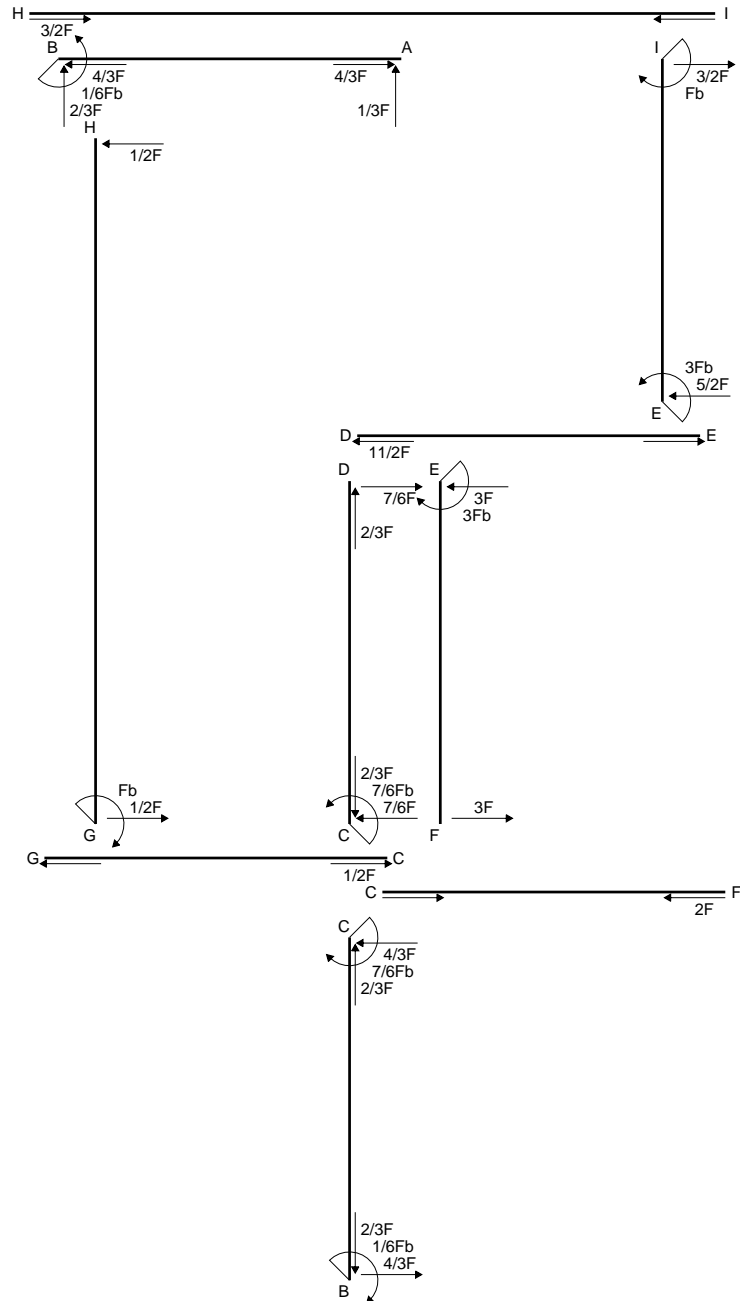
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

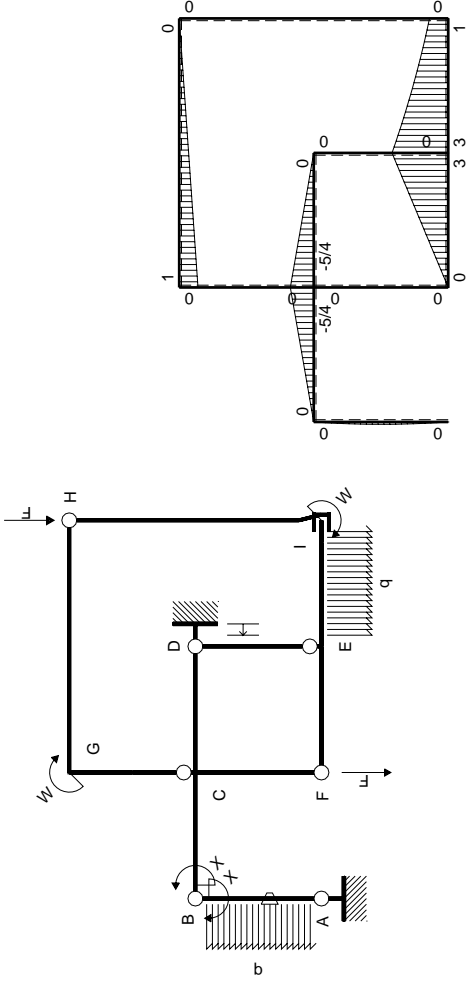
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$



⊕ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |  |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |  |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$           |  |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |  |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$  | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$           |  |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |  |
| DE b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| ED b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                       |  |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0             | 0                       |                             |                       |  |
| FC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| CF b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                       |  |
| CG b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| GC b  | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                       |  |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0             | 0                       |                             |                       |  |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0             | 0                       |                             |                       |  |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       |                             |                       |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |               |                         | $-Fb^2/EJ$                  |                       |  |
|       | totali                      |                      |          |                          |               |                         | $1/6Fb^2/EJ$                | $Xb/EJ$               |  |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |               |                         | $-1/6Fb$                    |                       |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

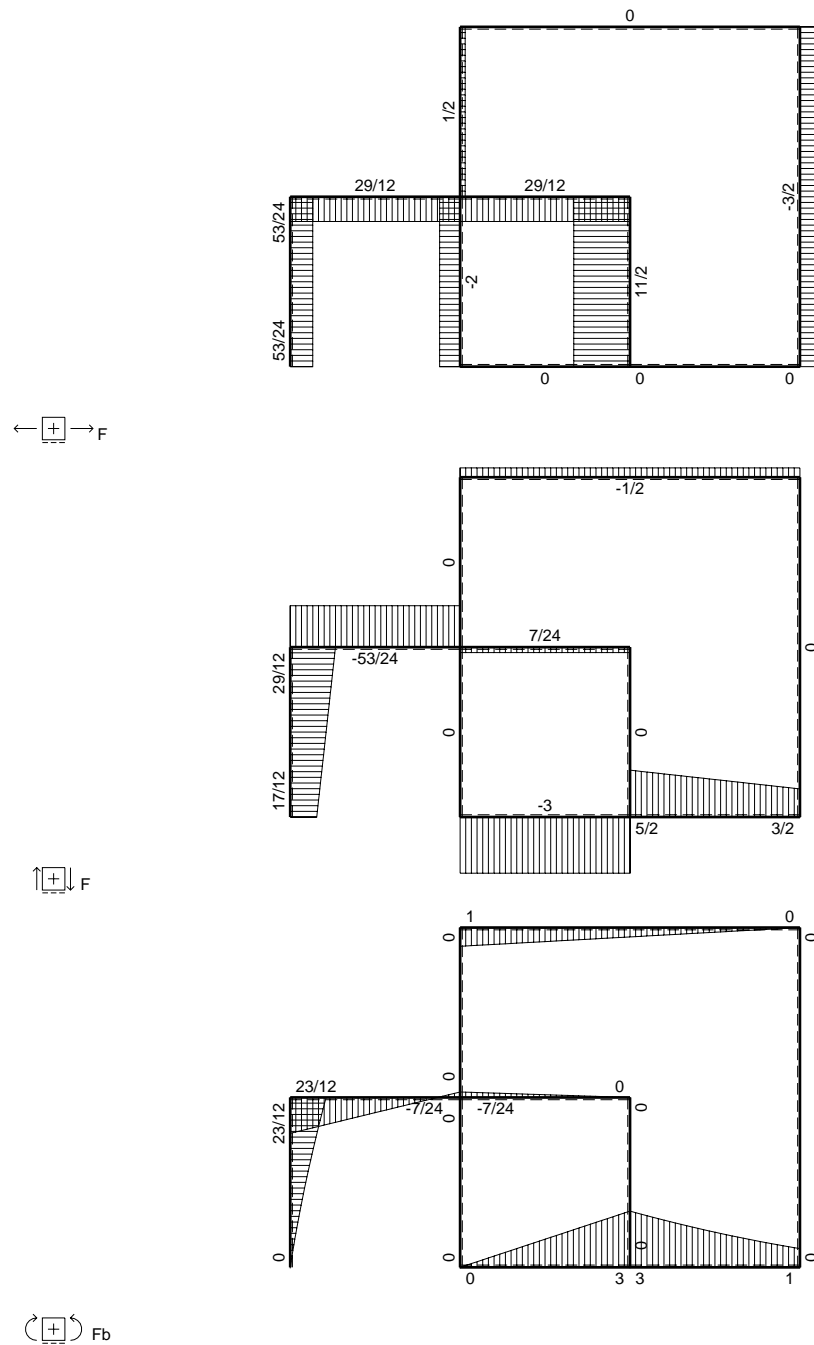
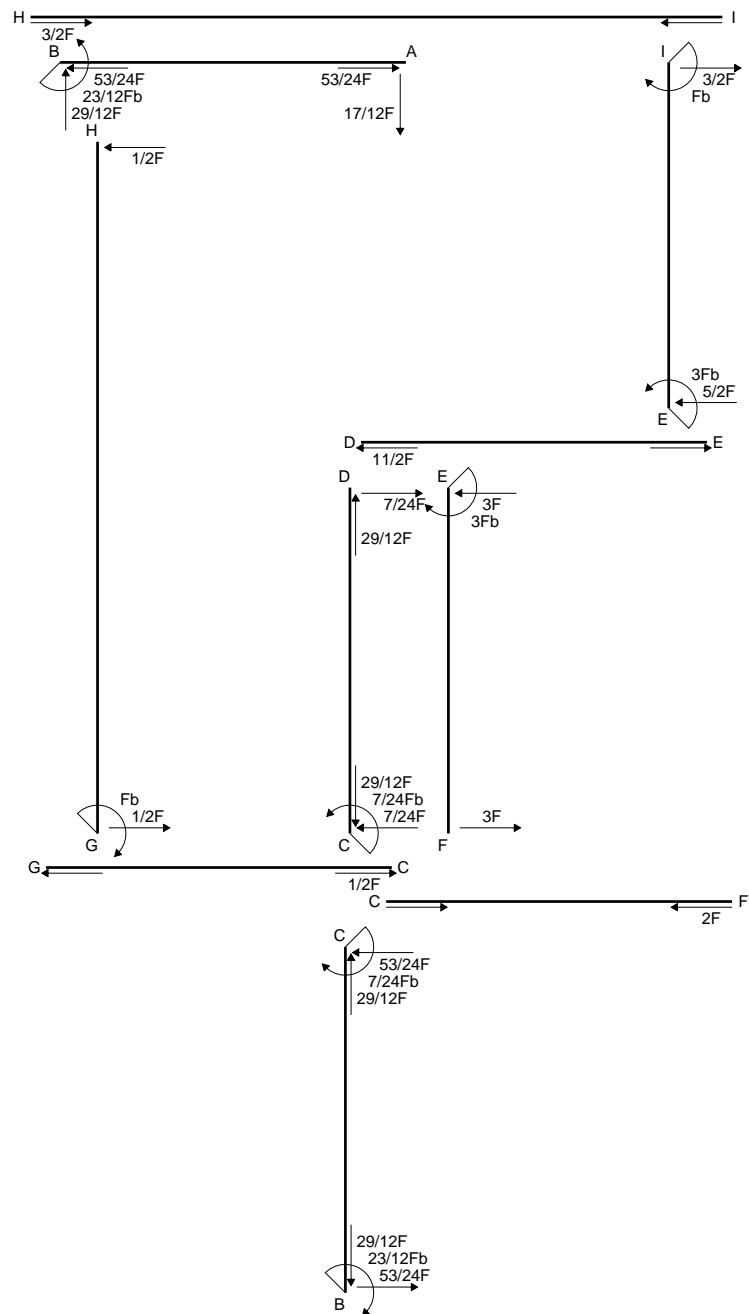
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

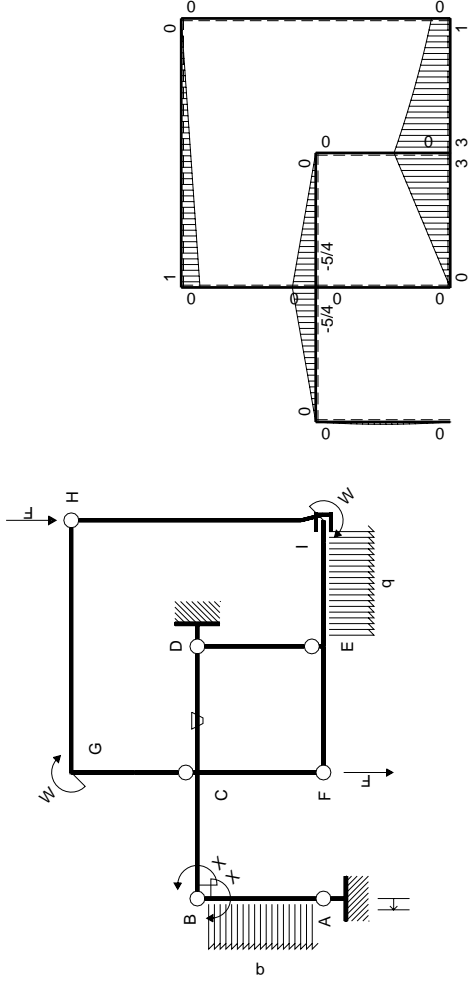
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                          |                     |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

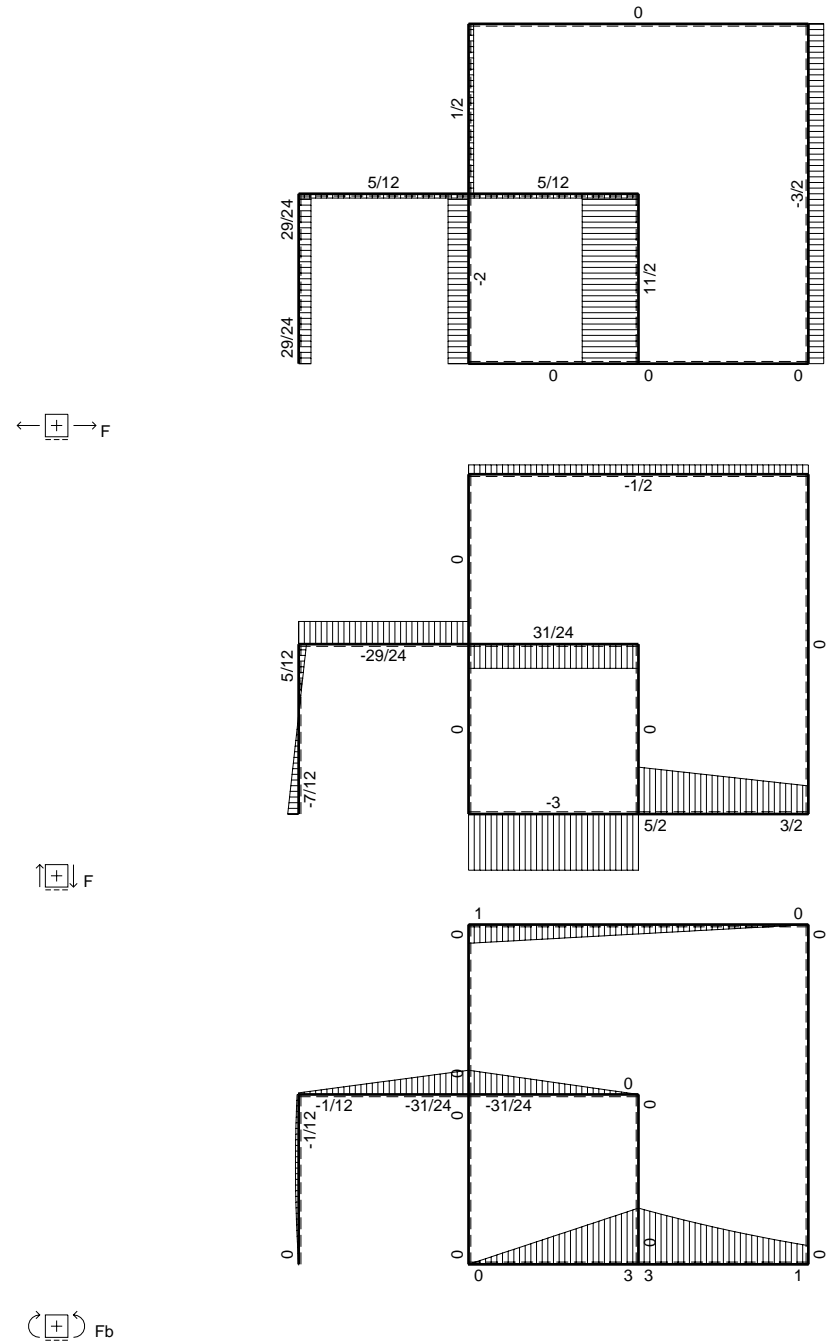
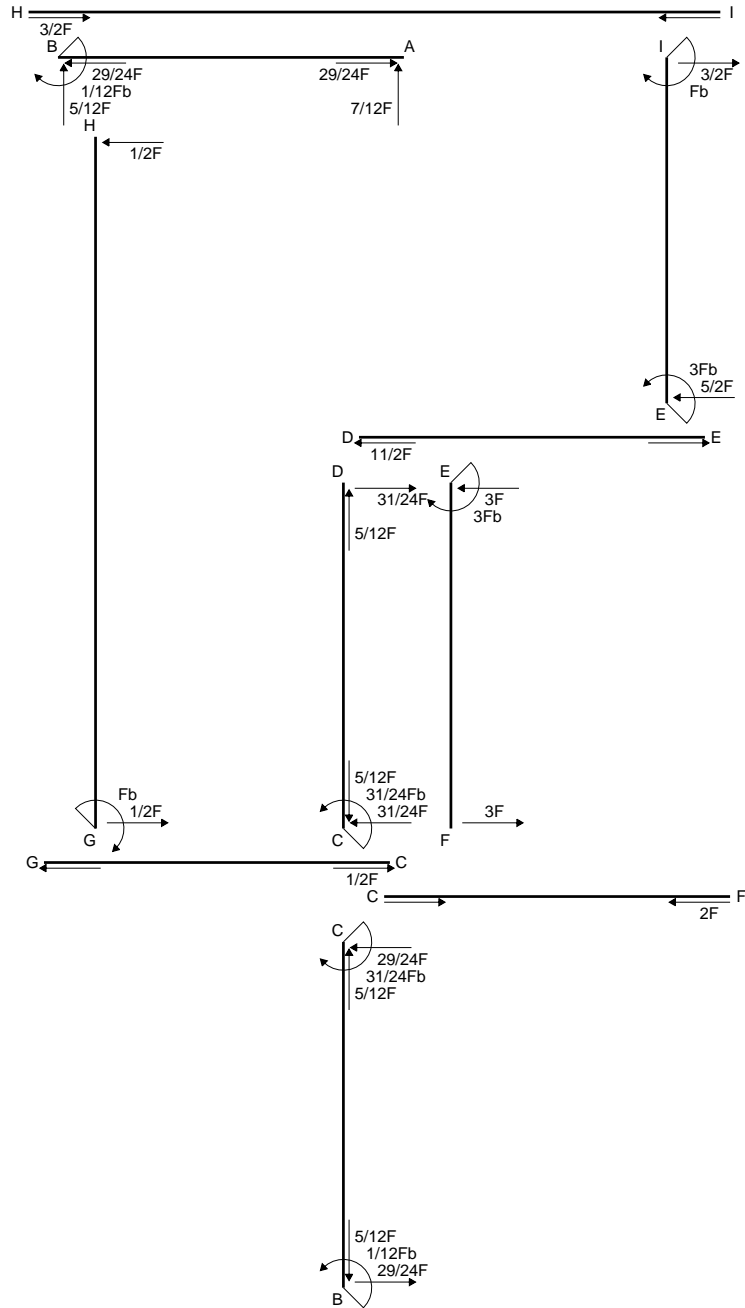
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

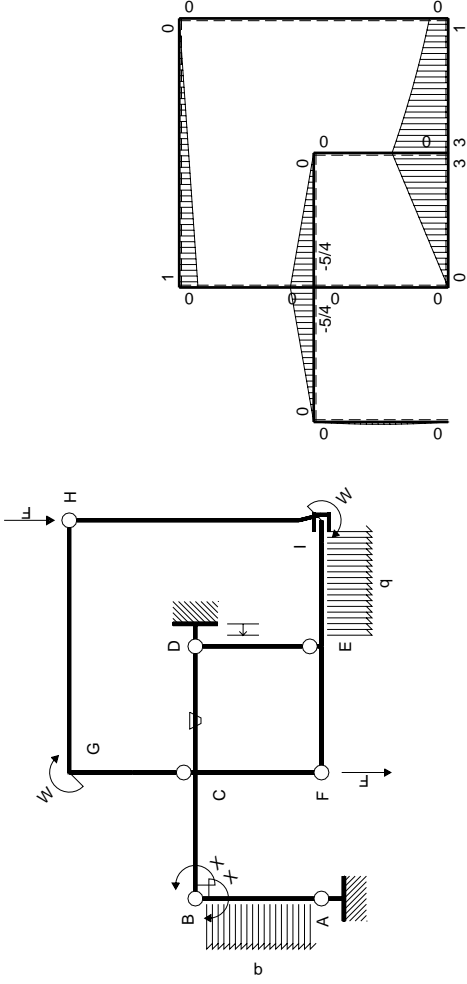
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$     | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$      | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$        | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$  | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                          |                     |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                          |                     |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

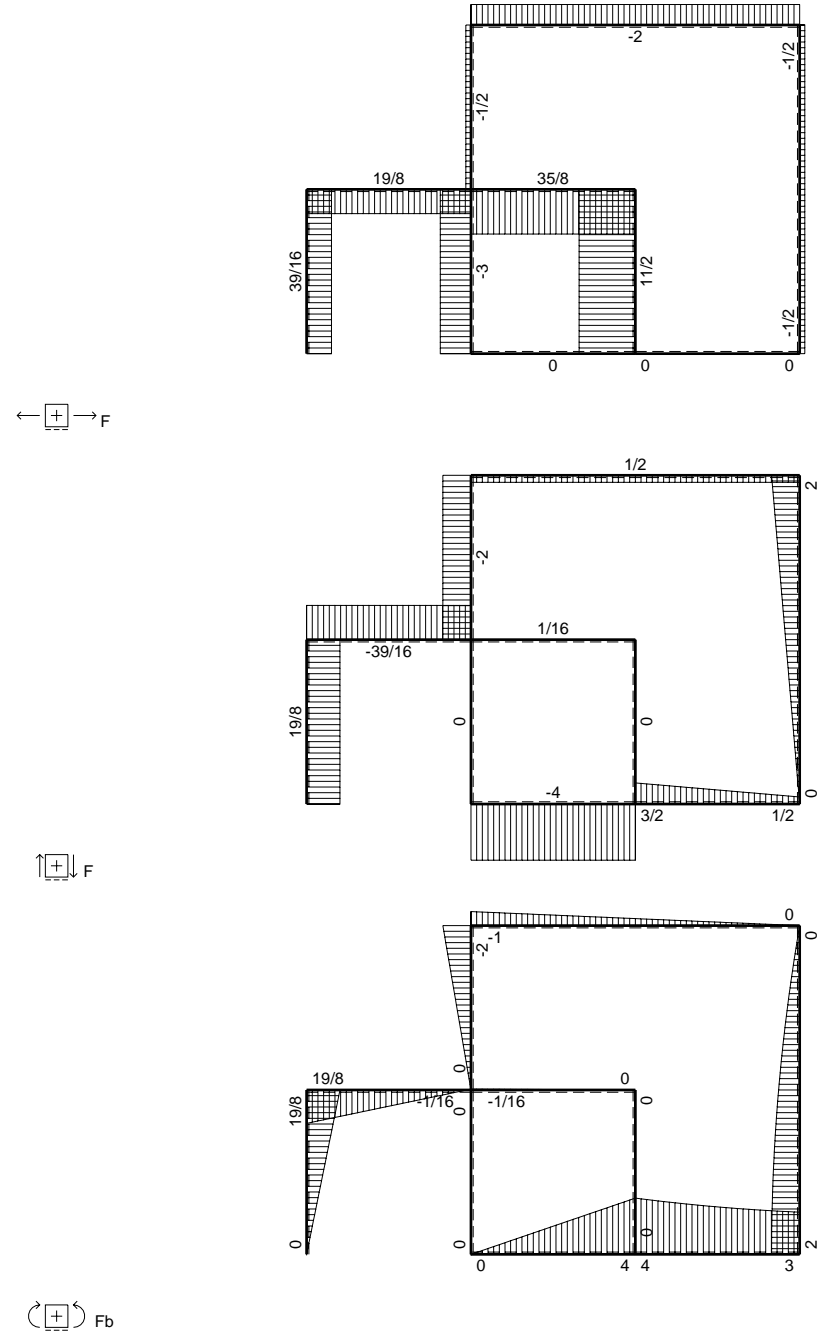
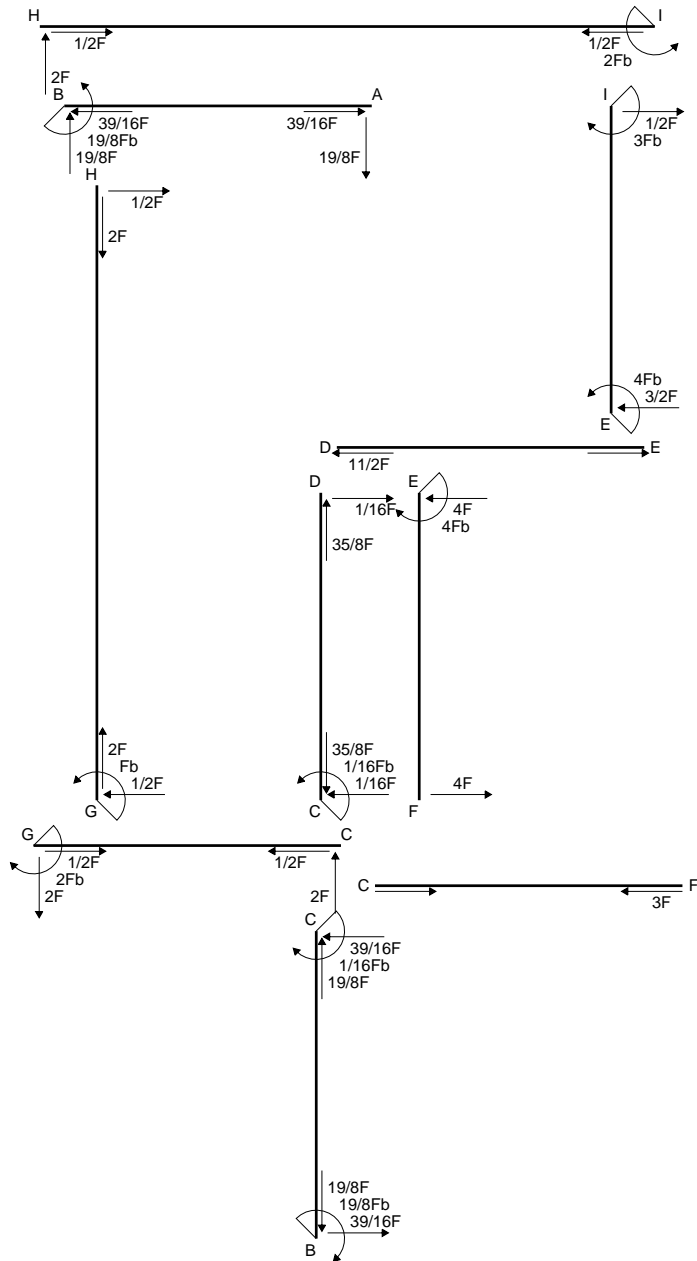
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

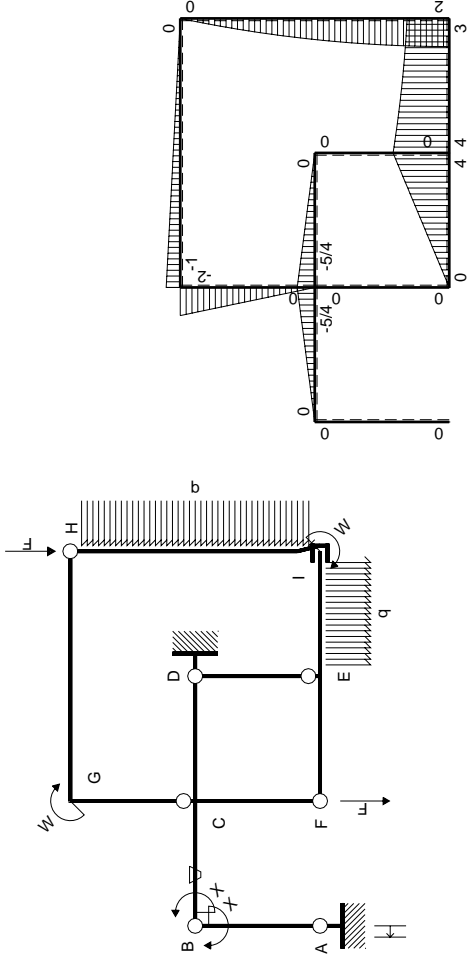
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$

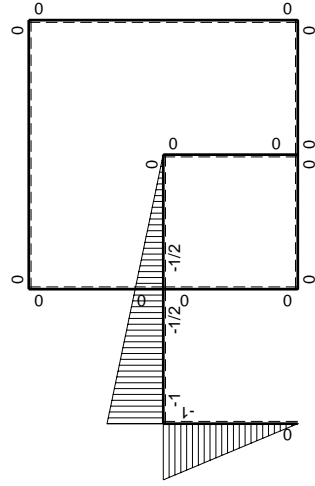






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |                     |                         | $19/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-19/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

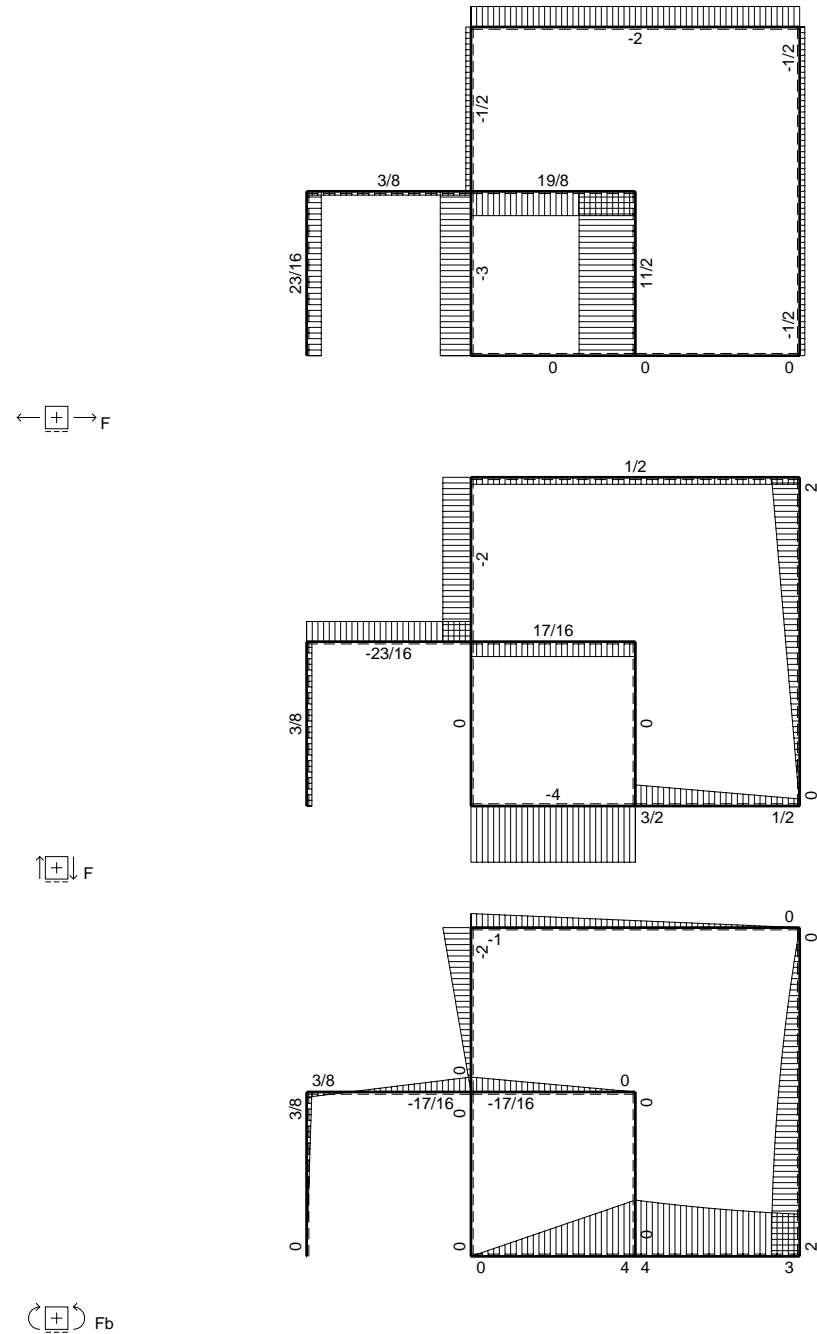
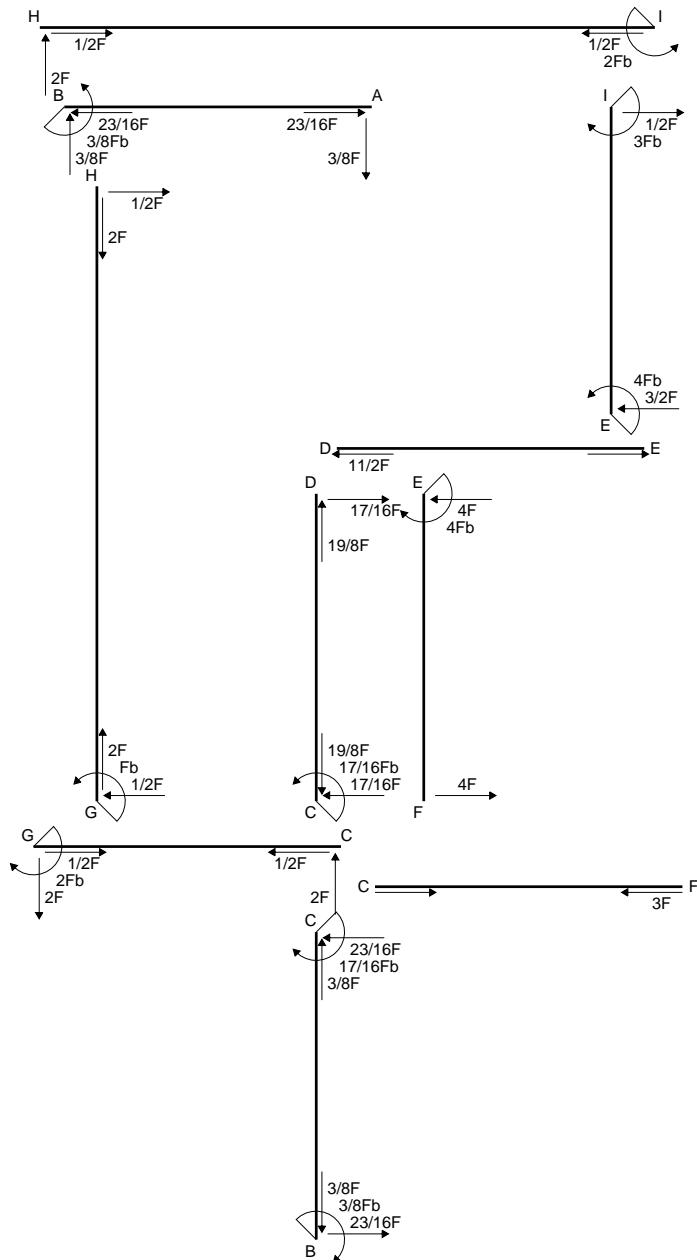
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

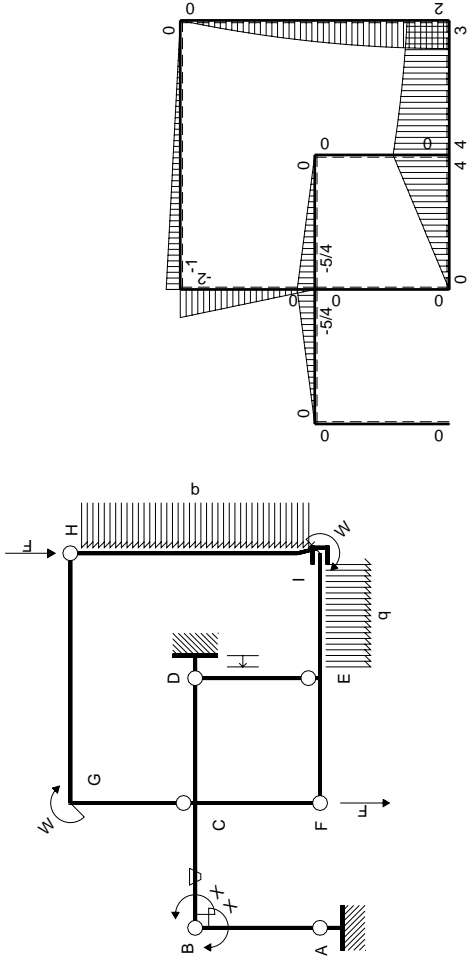
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

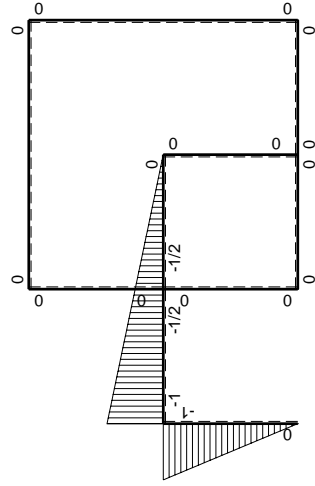


⊕ F<sub>b</sub>



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x / EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|--------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$               |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                          |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$              |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                          |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$              |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                          |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                          |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                          |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                          |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                          |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                          |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                          |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                        |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                          |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |                     |                         | $-Fb^2/EJ$                  |                          |
|       | totali                      |                      |          |                         |                     |                         | $3/8Fb^2/EJ$                | $Xb/EJ$                  |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-3/8Fb$                    |                          |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

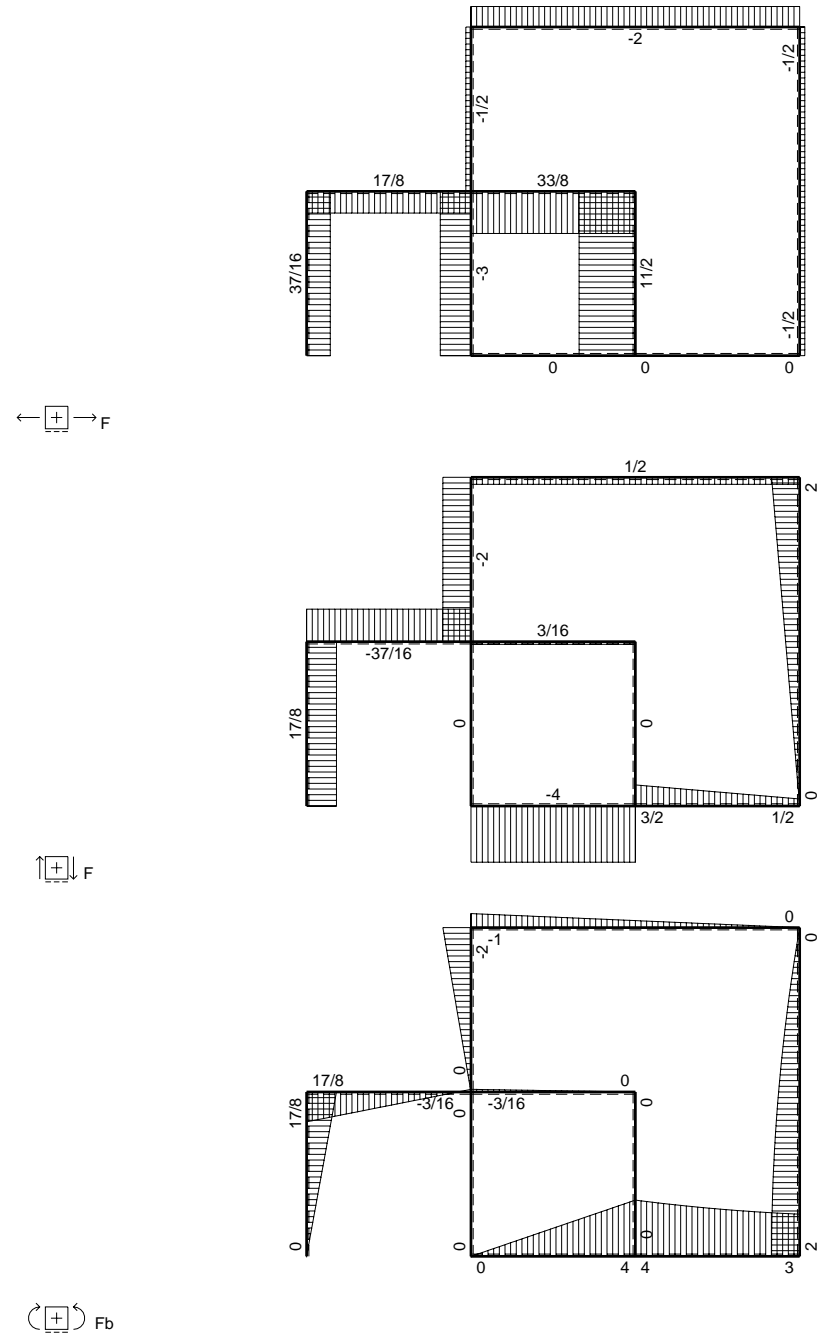
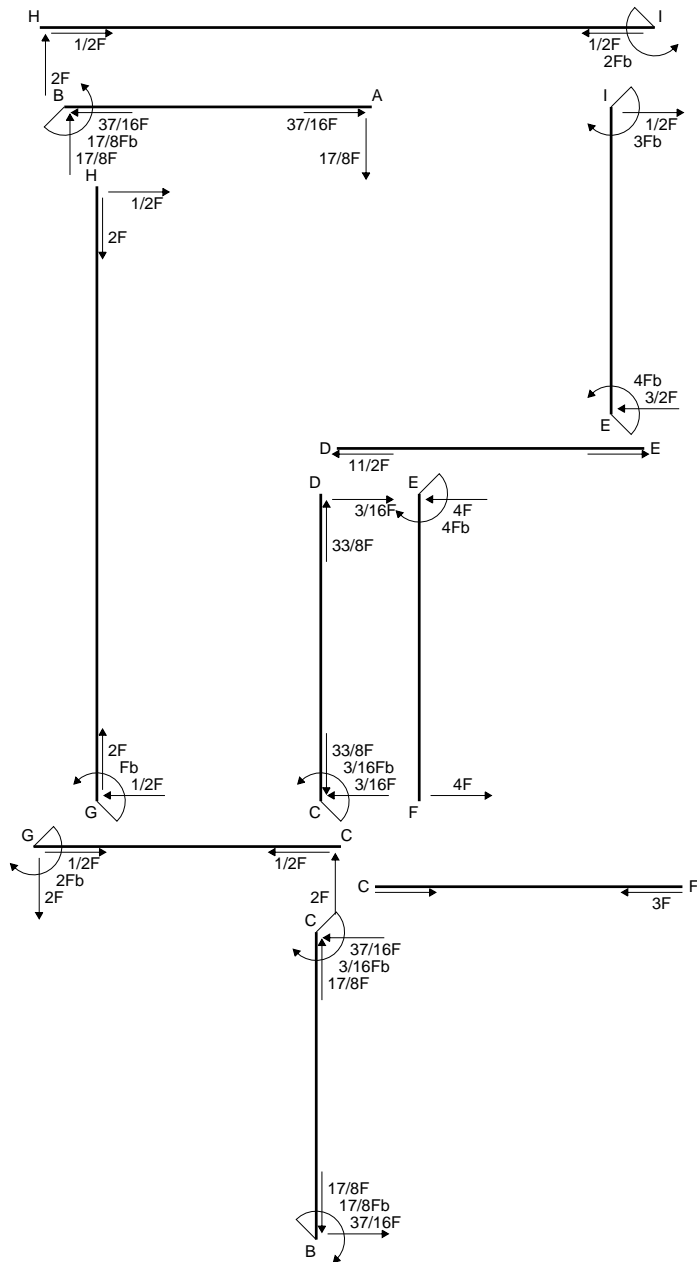
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

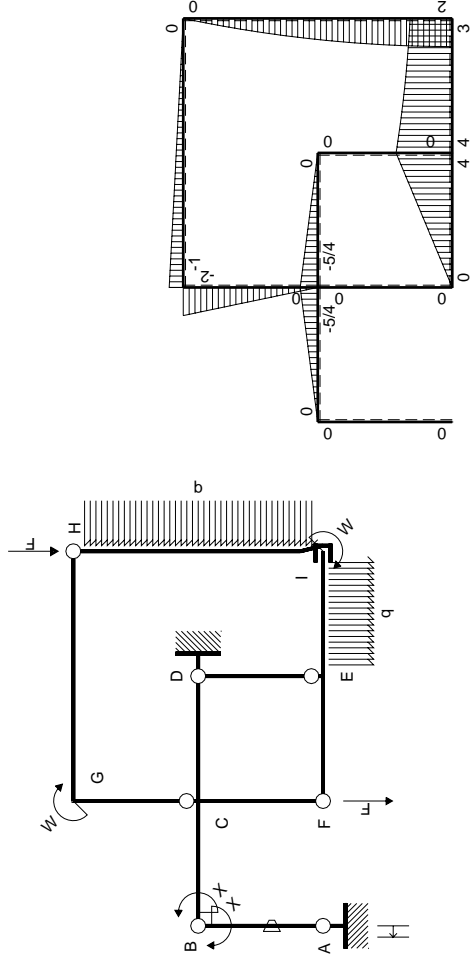
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

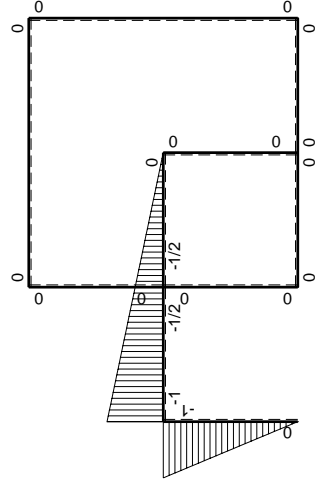






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |               |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |               |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

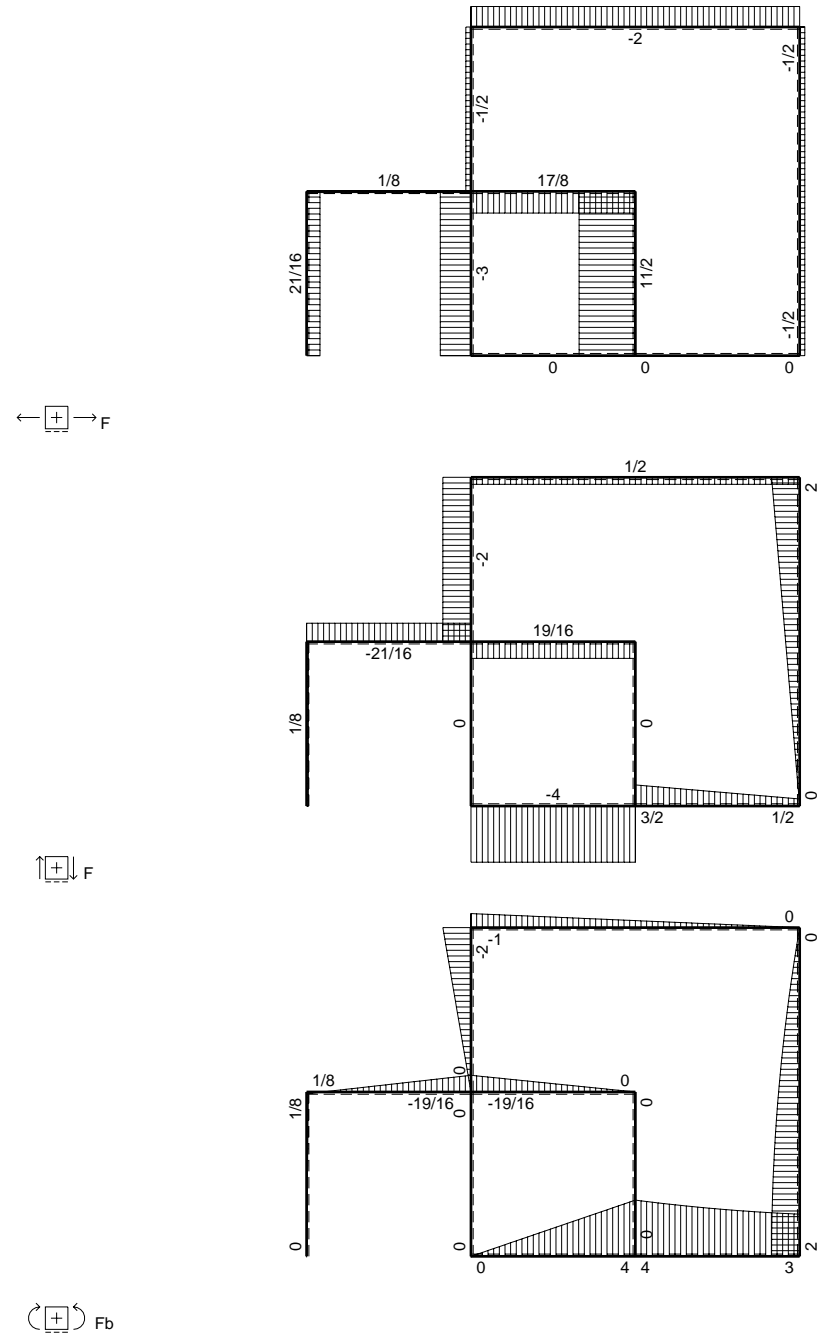
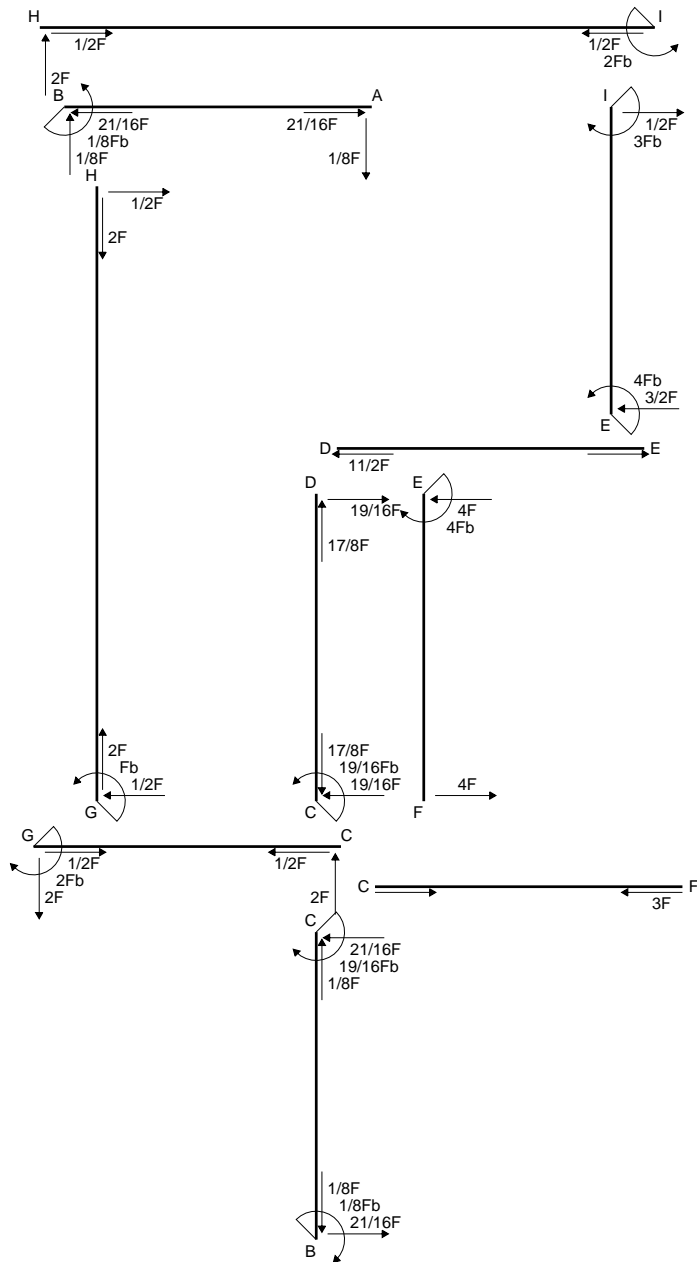
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

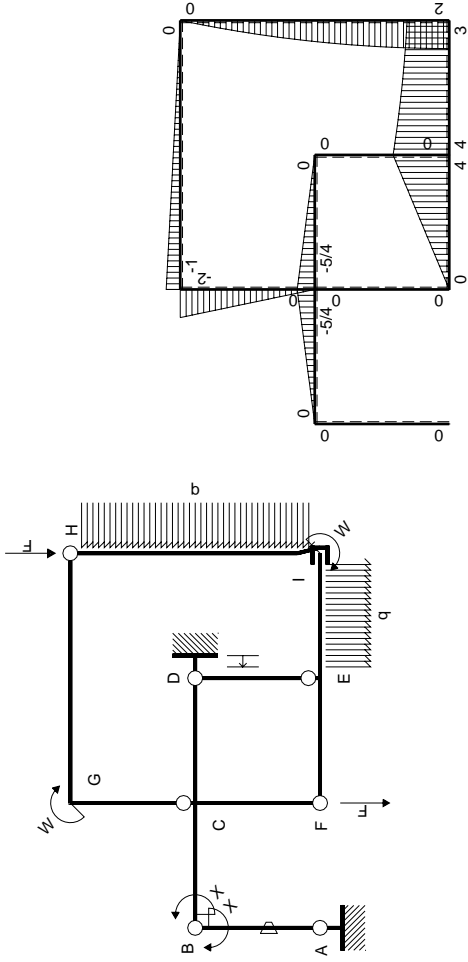
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

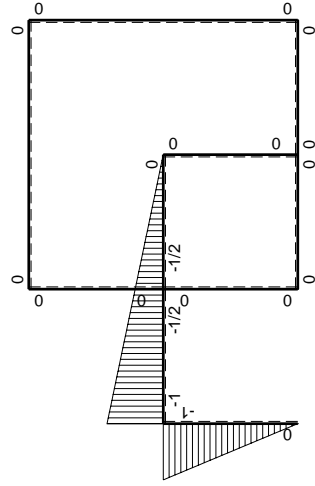
$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |               |                         |                             | $1/8Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |               |                         |                             | $-1/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

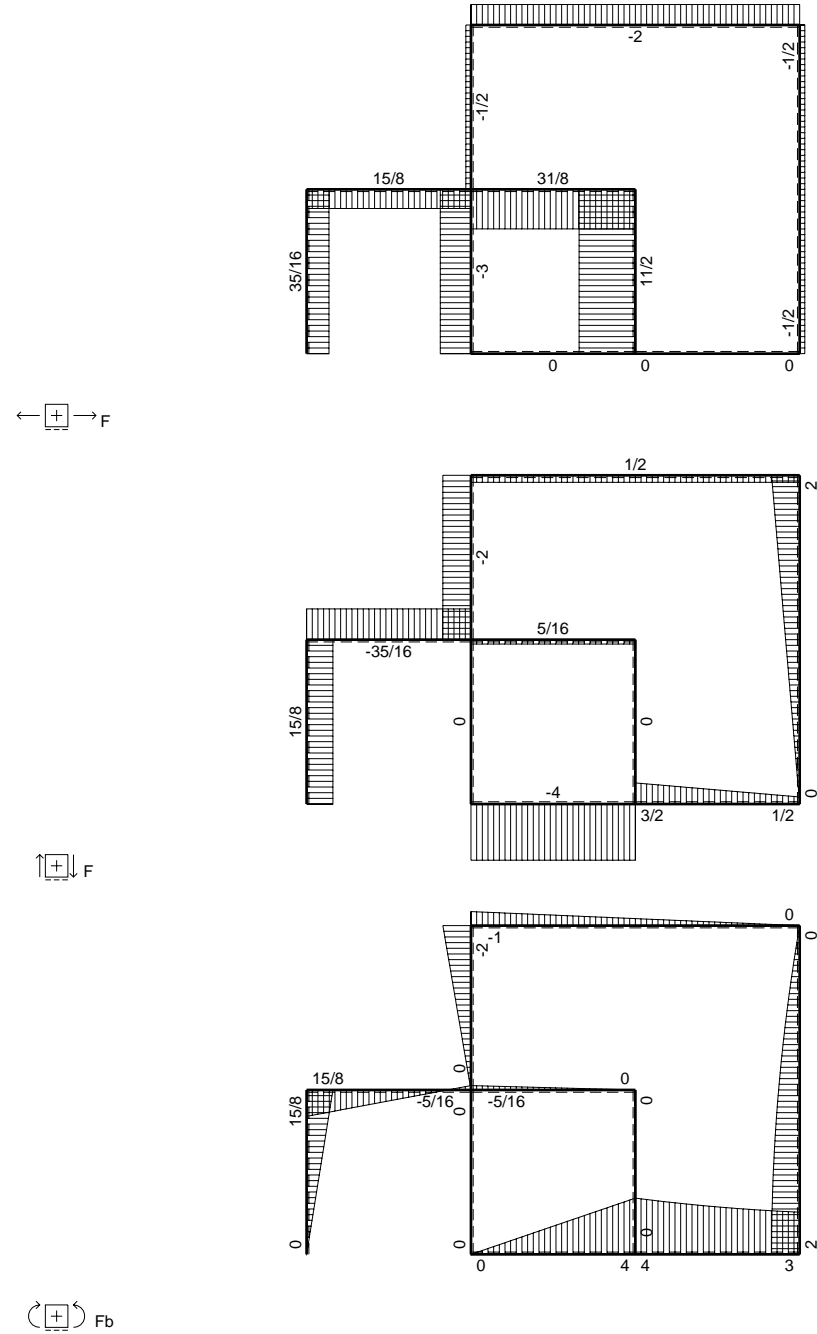
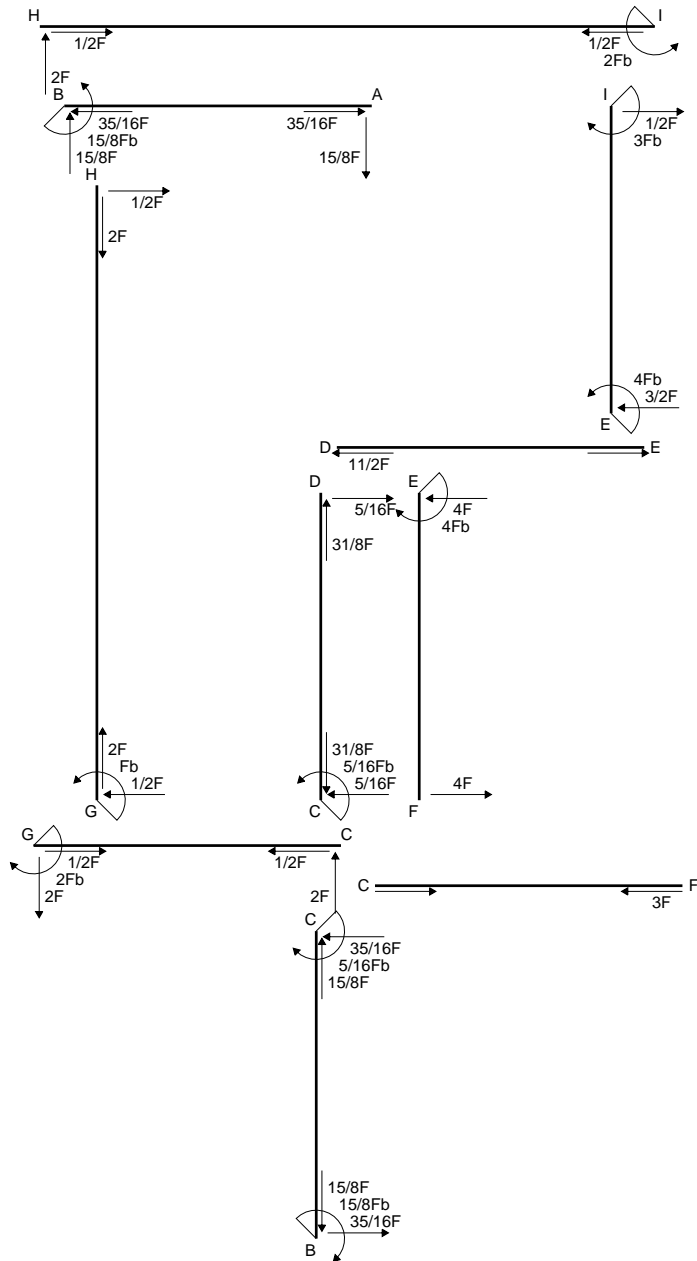
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

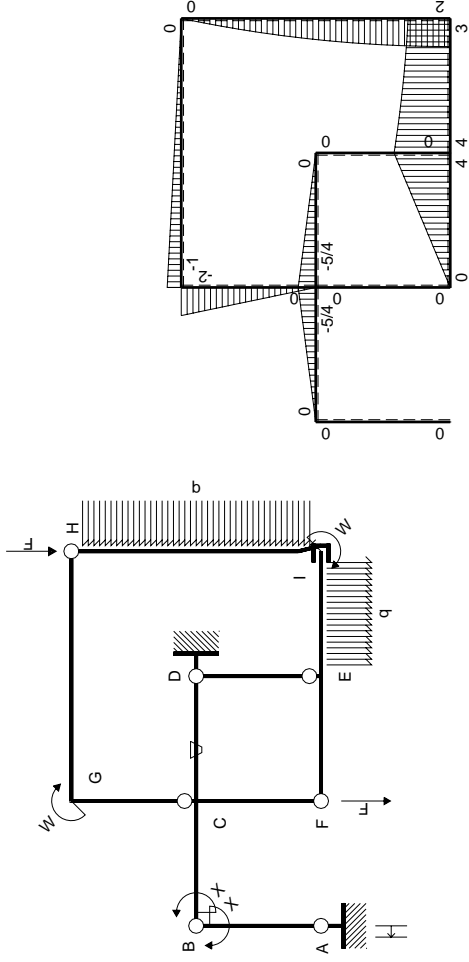
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

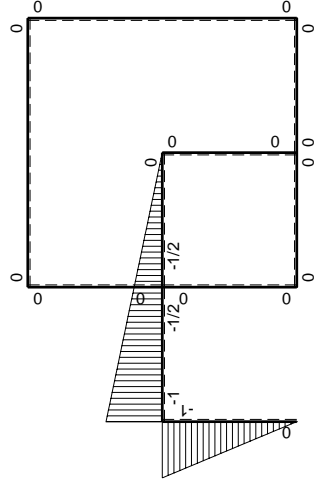






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |                     |                         |                             | $Fb^2/EJ$              |         |
|       | totali                      |                      |          |                         |                     |                         |                             | $15/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         |                             | $-15/8Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

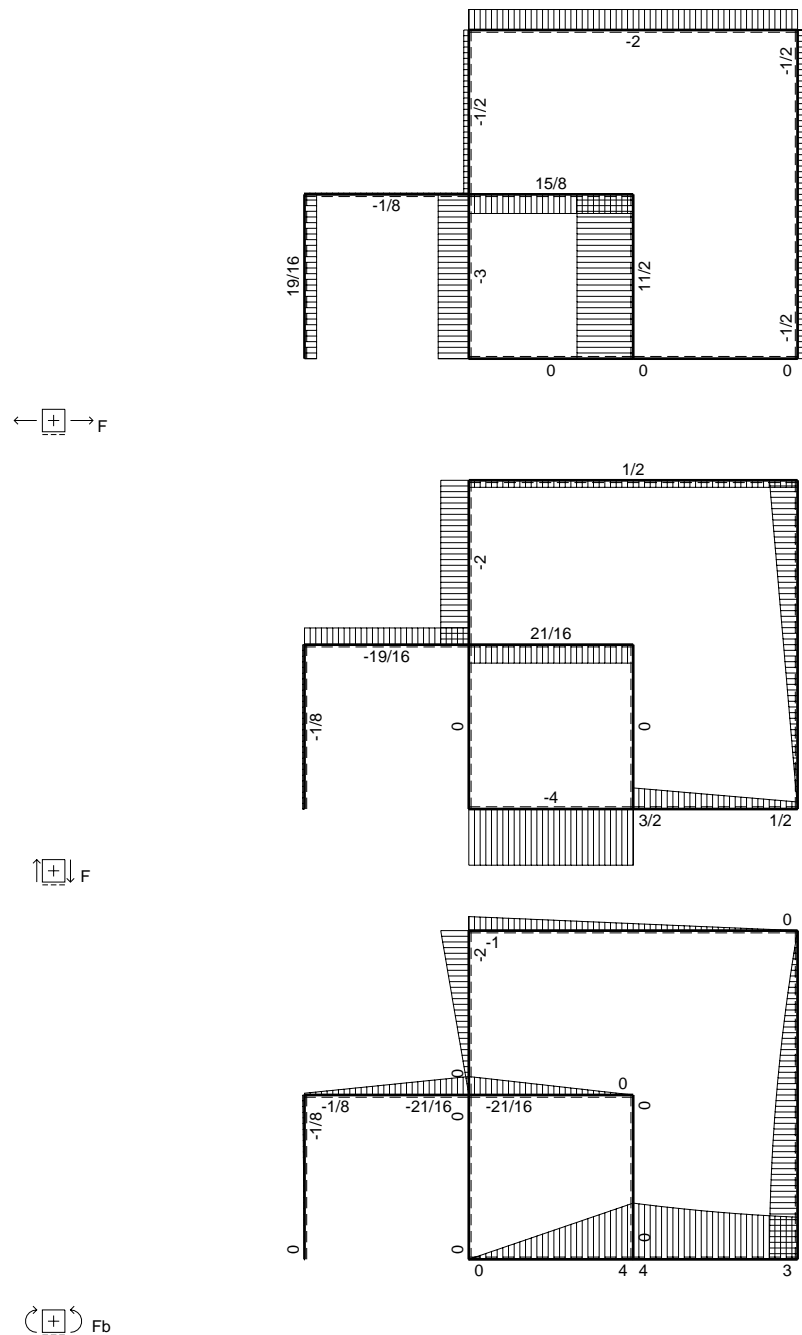
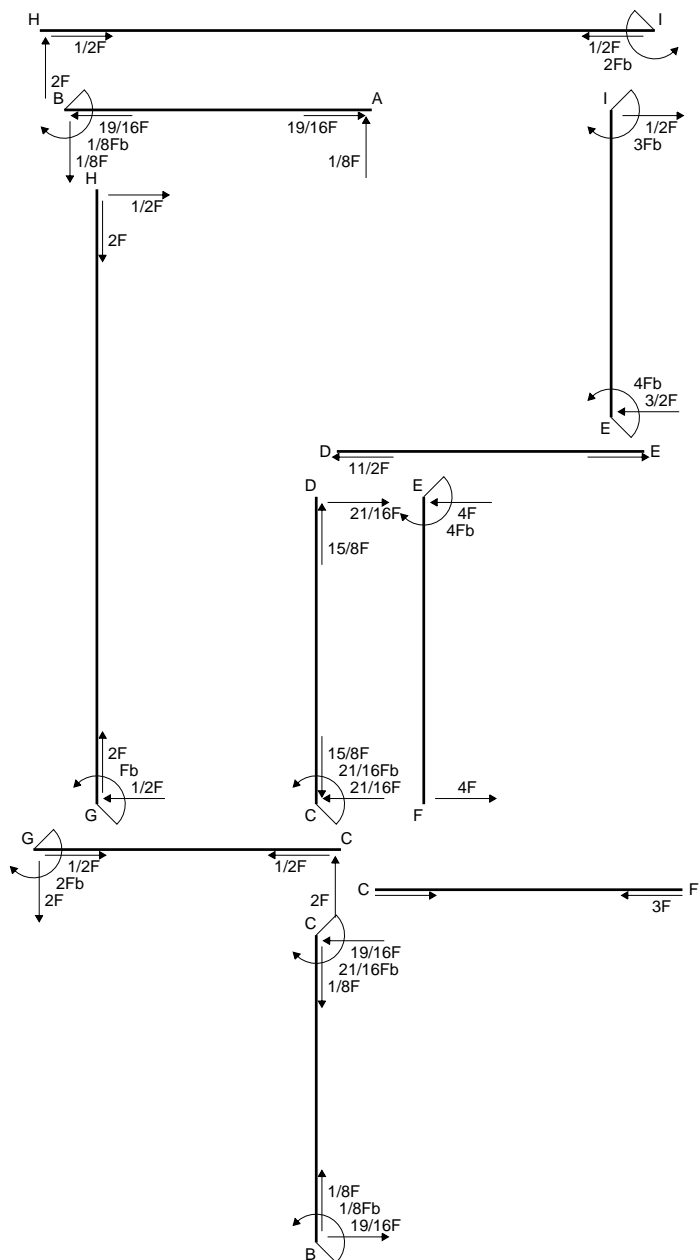
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

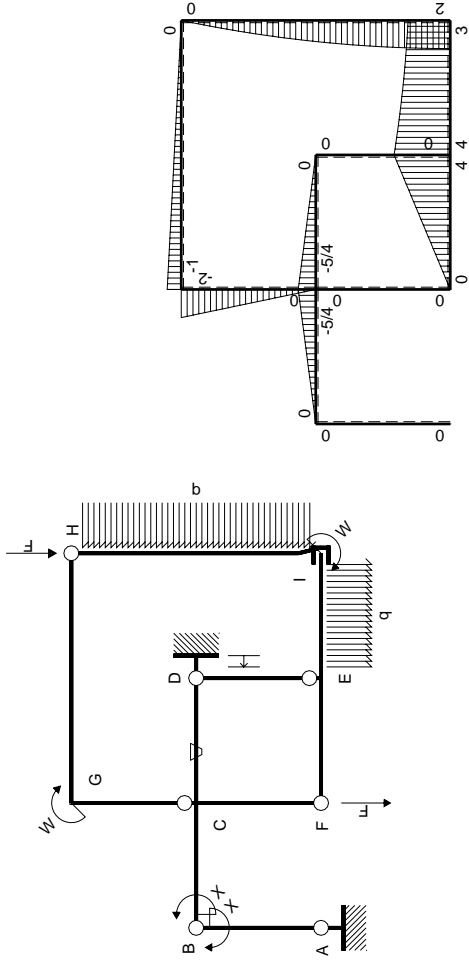
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

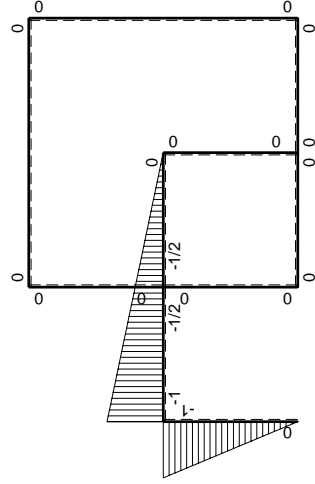
$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-2Fx$               | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $2Fb-2Fx$            | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $-Fb+1/2Fx$          | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $1/2Fx$              | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | $2Fx-1/2qx^2$        | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | $-2Fb+1/2qx^2$       | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $3Fb+1/2Fx+1/2qx^2$  | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+3/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |                     |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

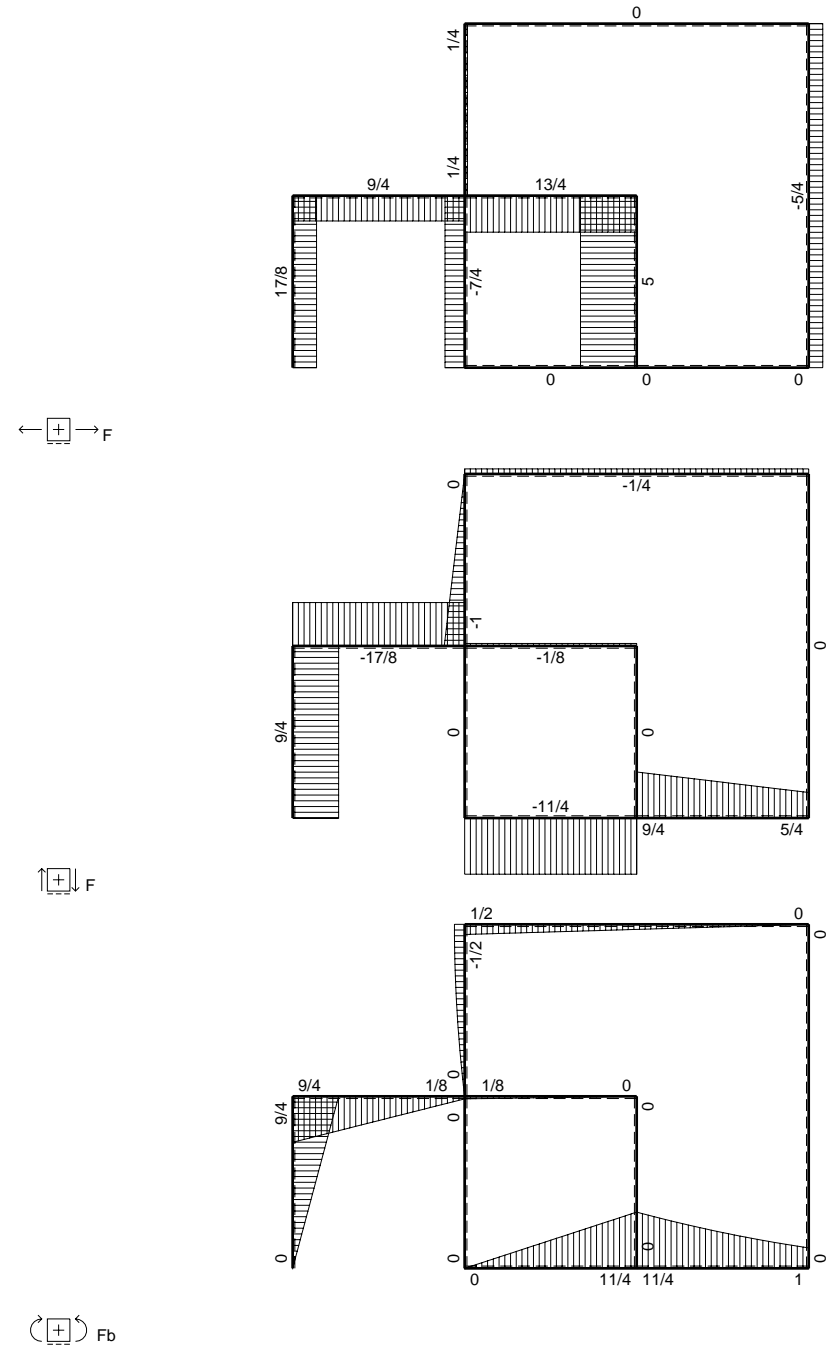
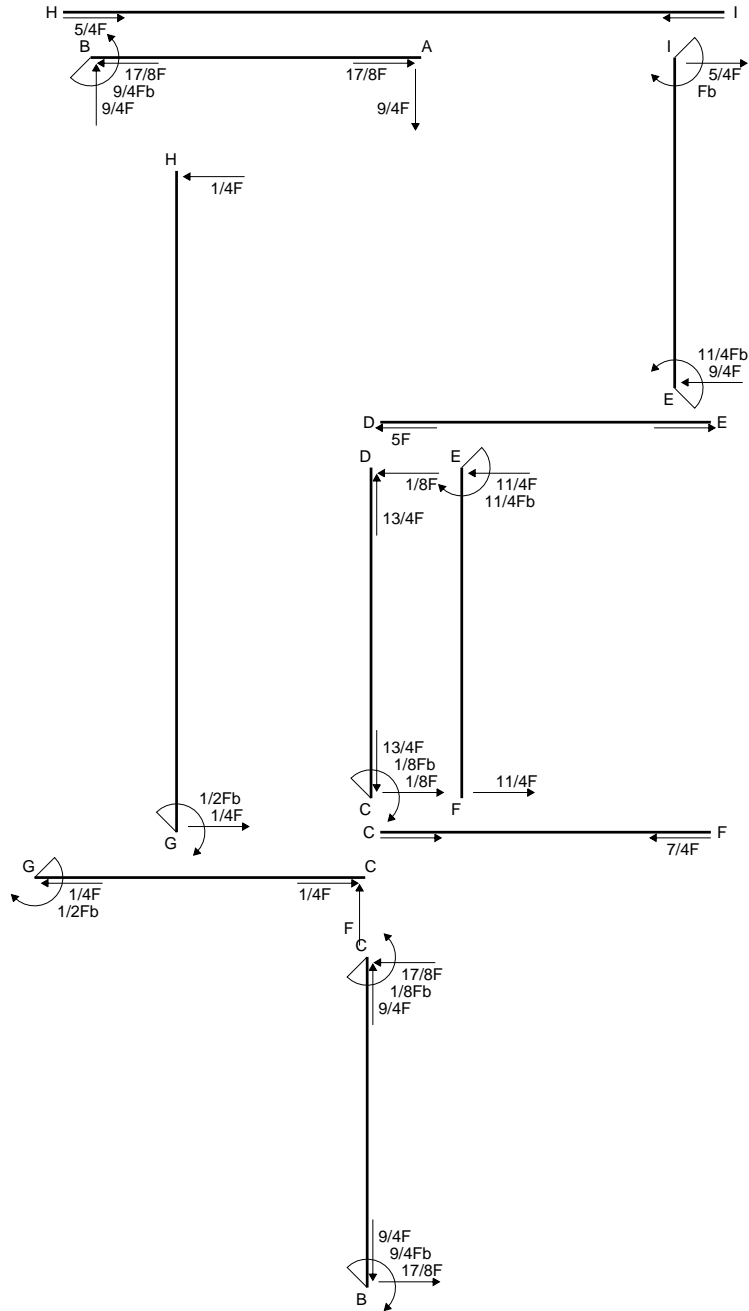
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

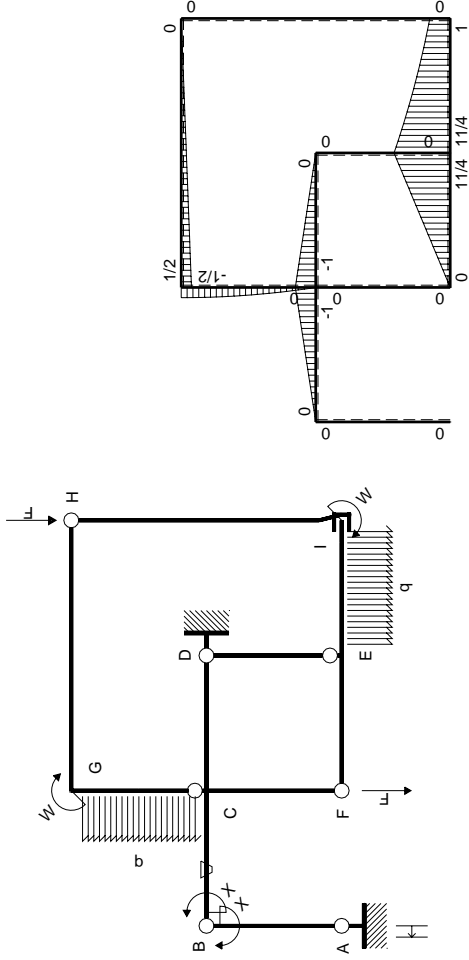
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                    | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                         |          |                      |                     |                         | $9/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         | $-9/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

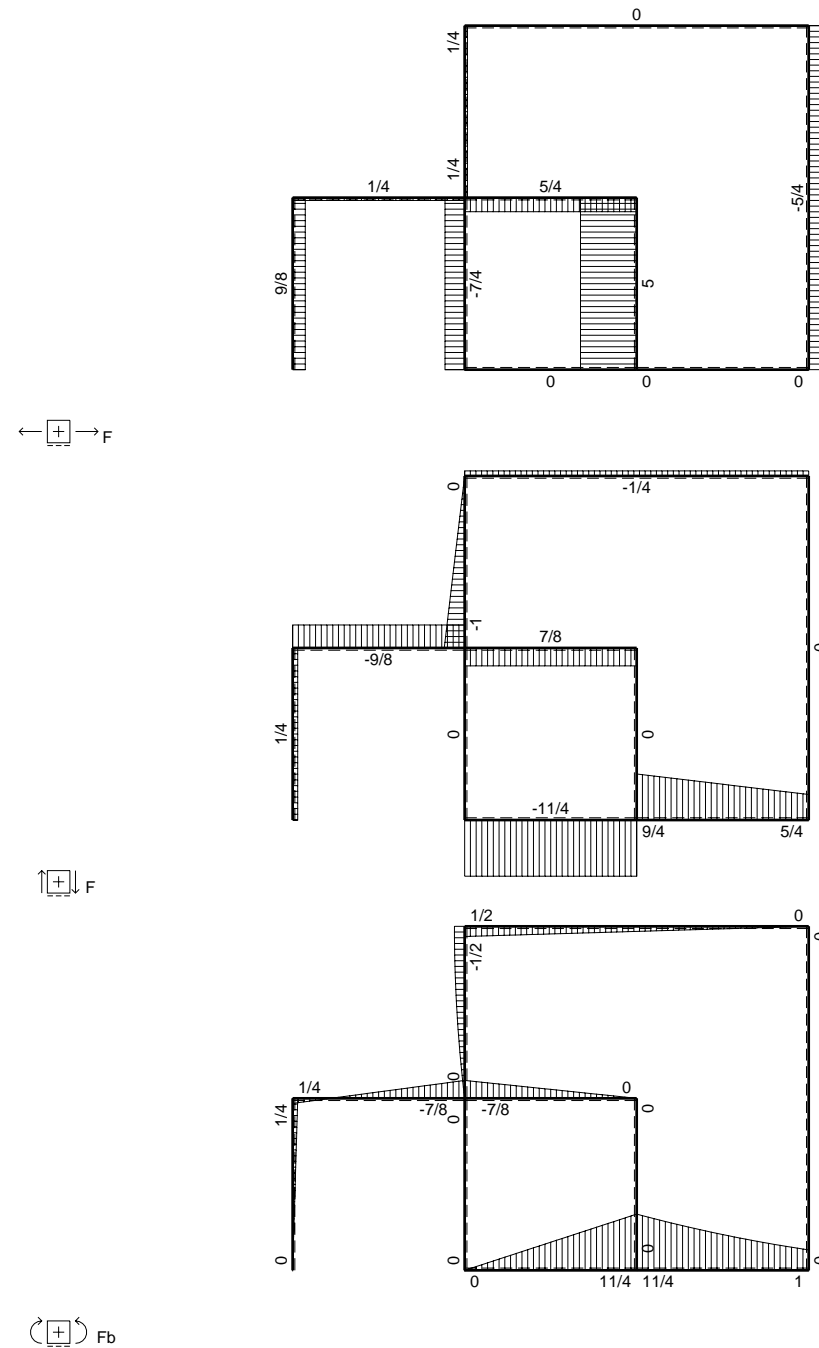
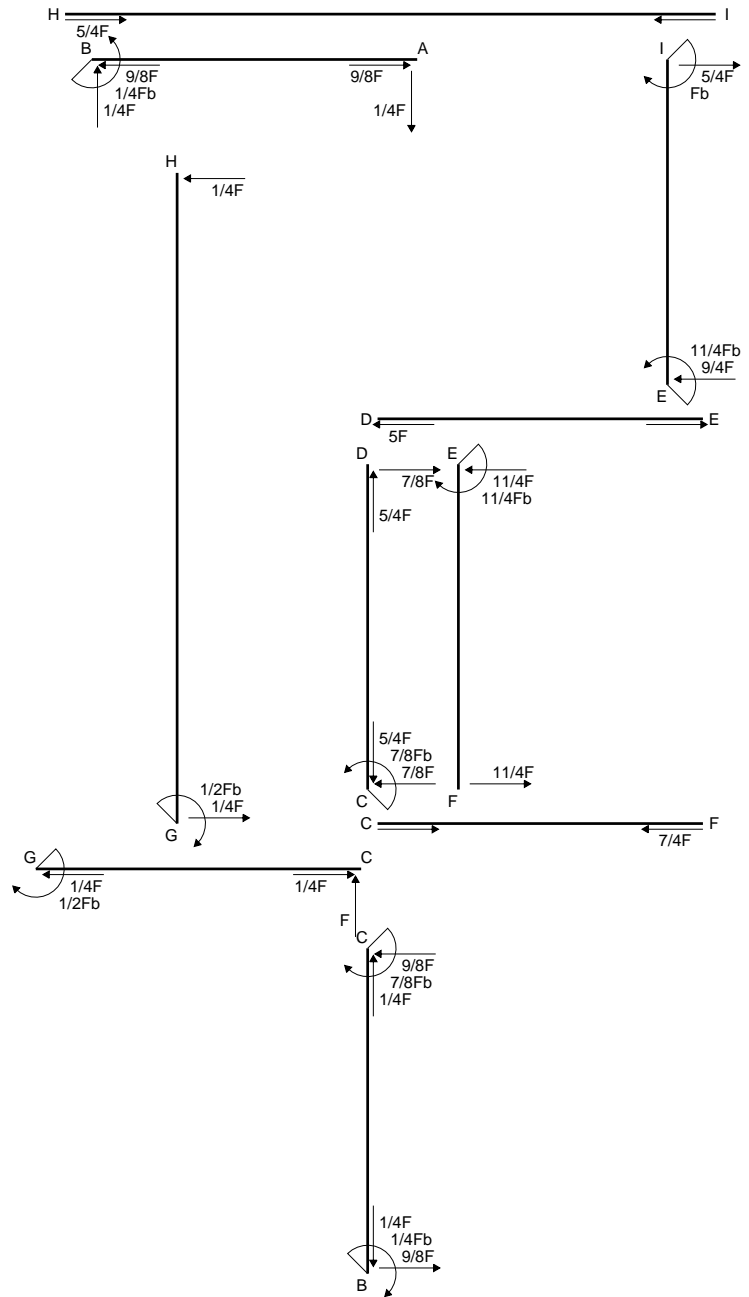
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

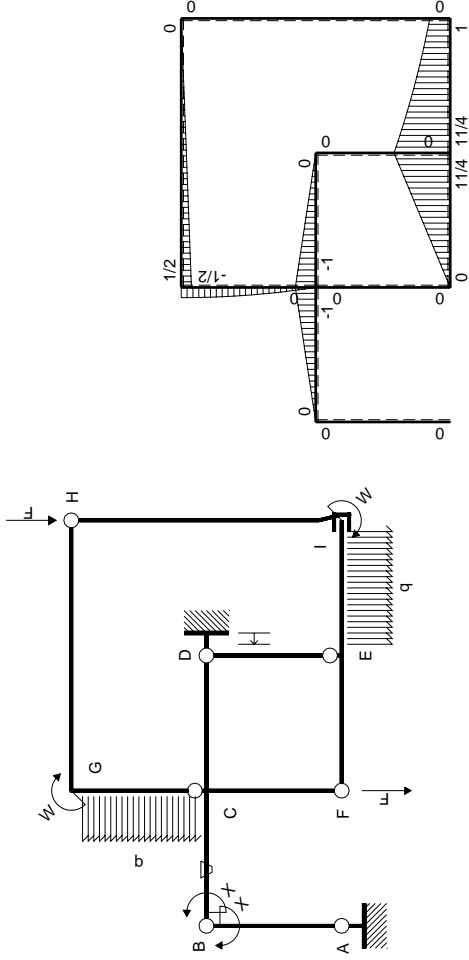
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                    | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                         |          |                      |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                         |          |                      |                     |                         | $1/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         | $-1/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

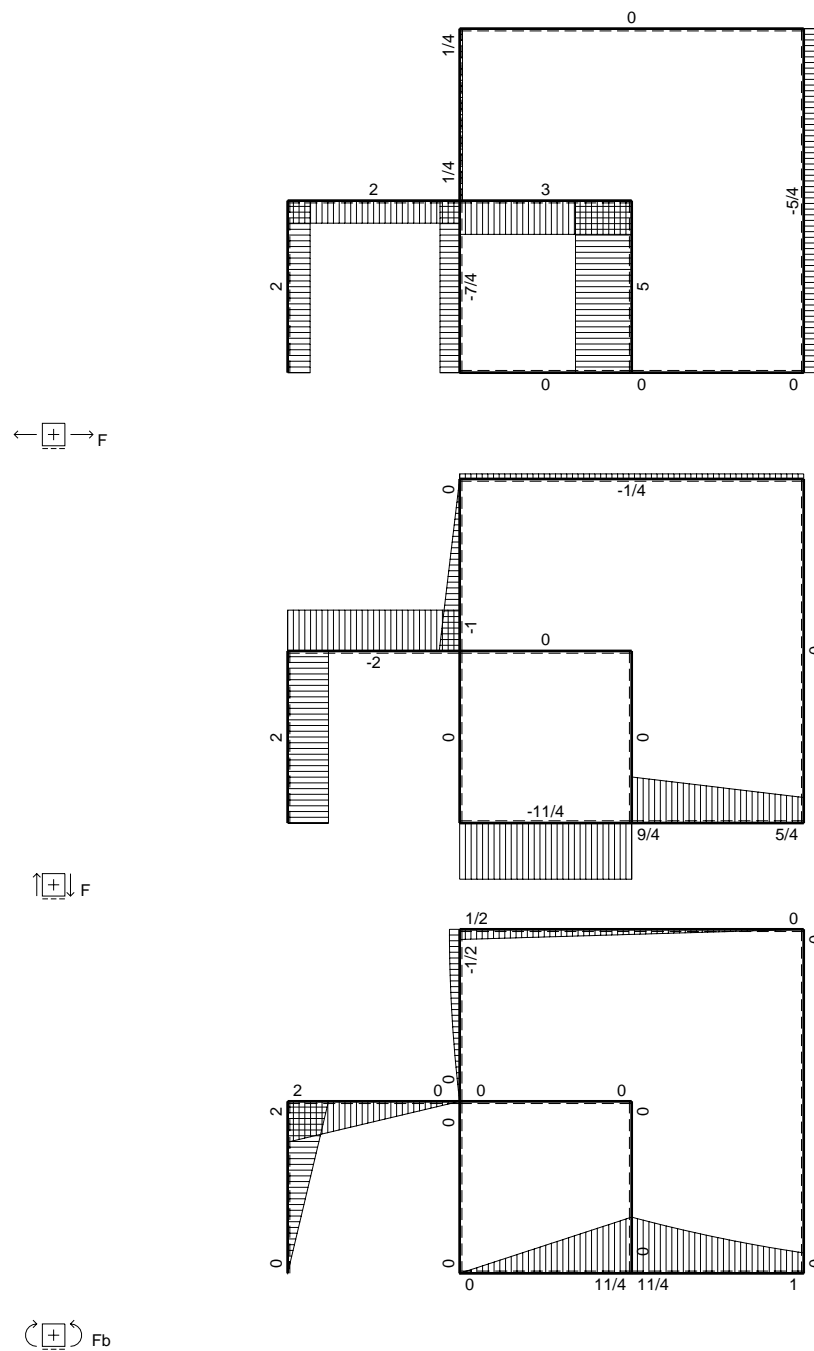
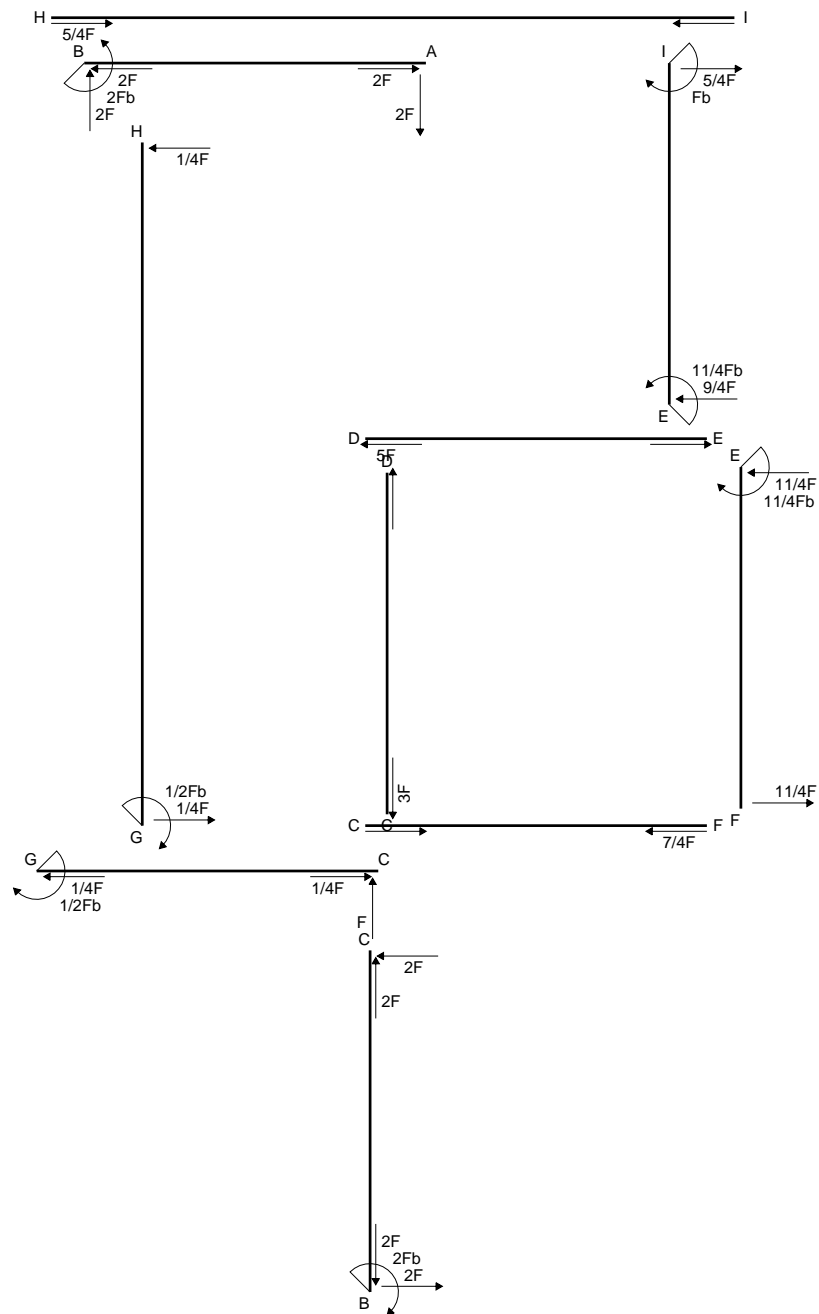
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

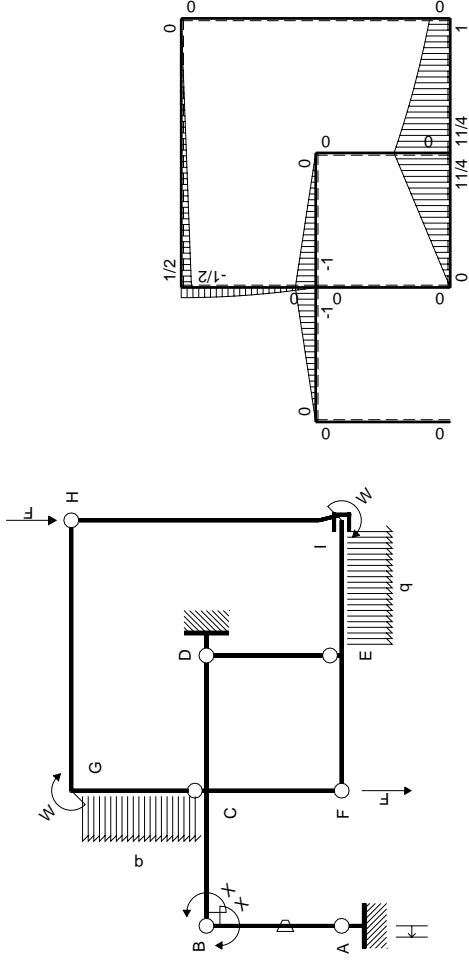
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | 0        | $Fx-1/2Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | 0        | $1/2Fb-1/2Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                    | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                         |          |                      |               |                         | $2Fb^2/EJ$                  | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |               |                         | $-2Fb$                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

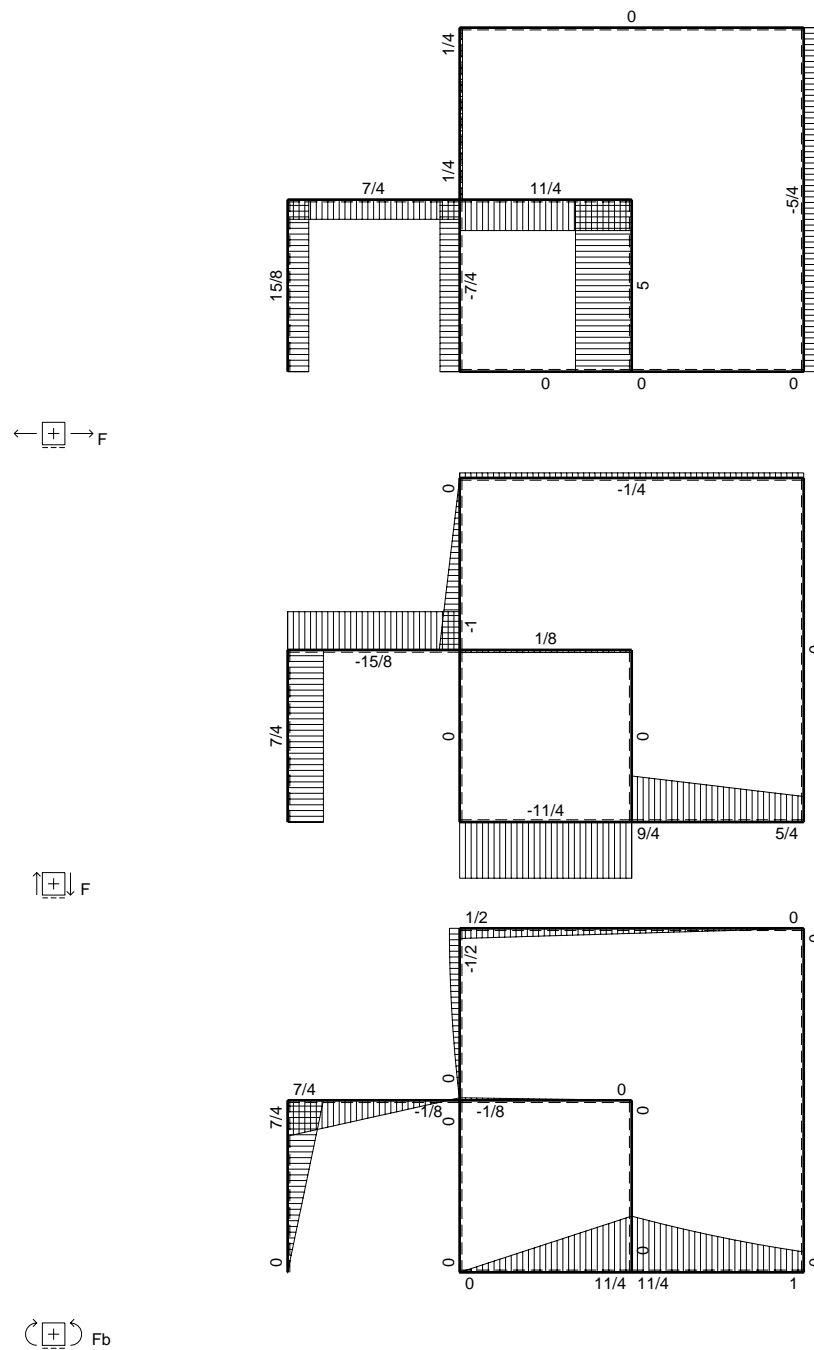
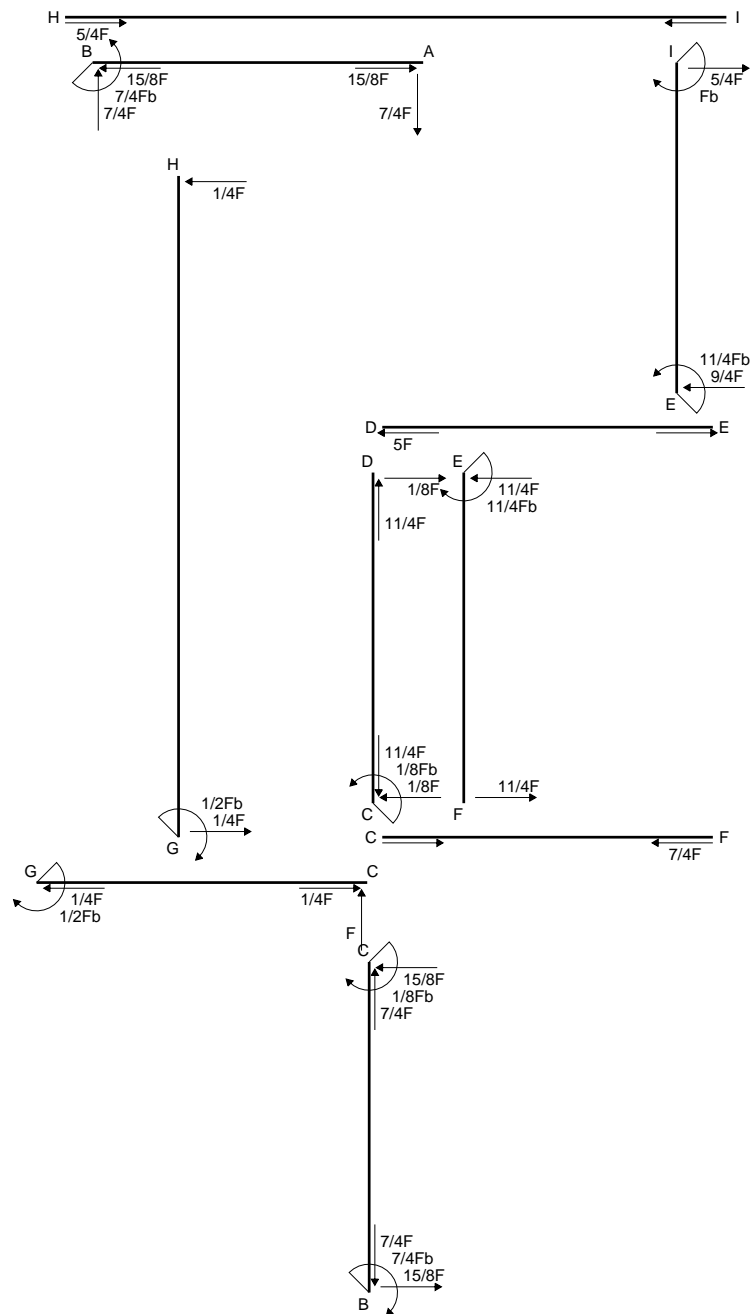
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

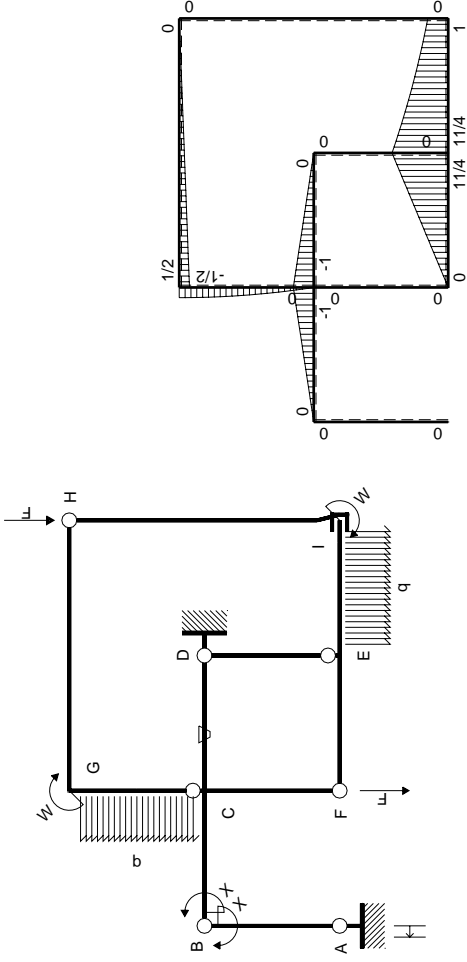
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                    | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                         |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                         |          |                      |                     |                         | $7/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         | $-7/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

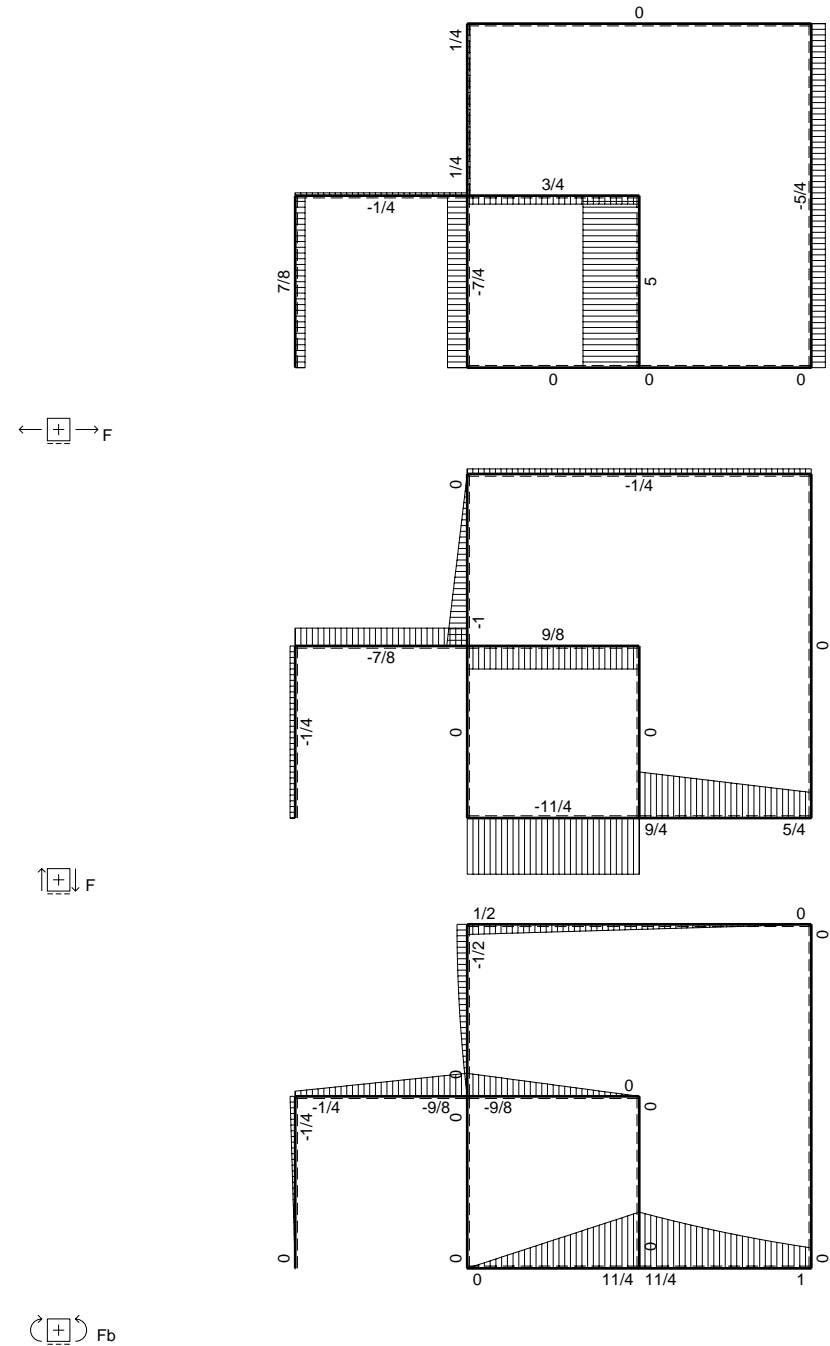
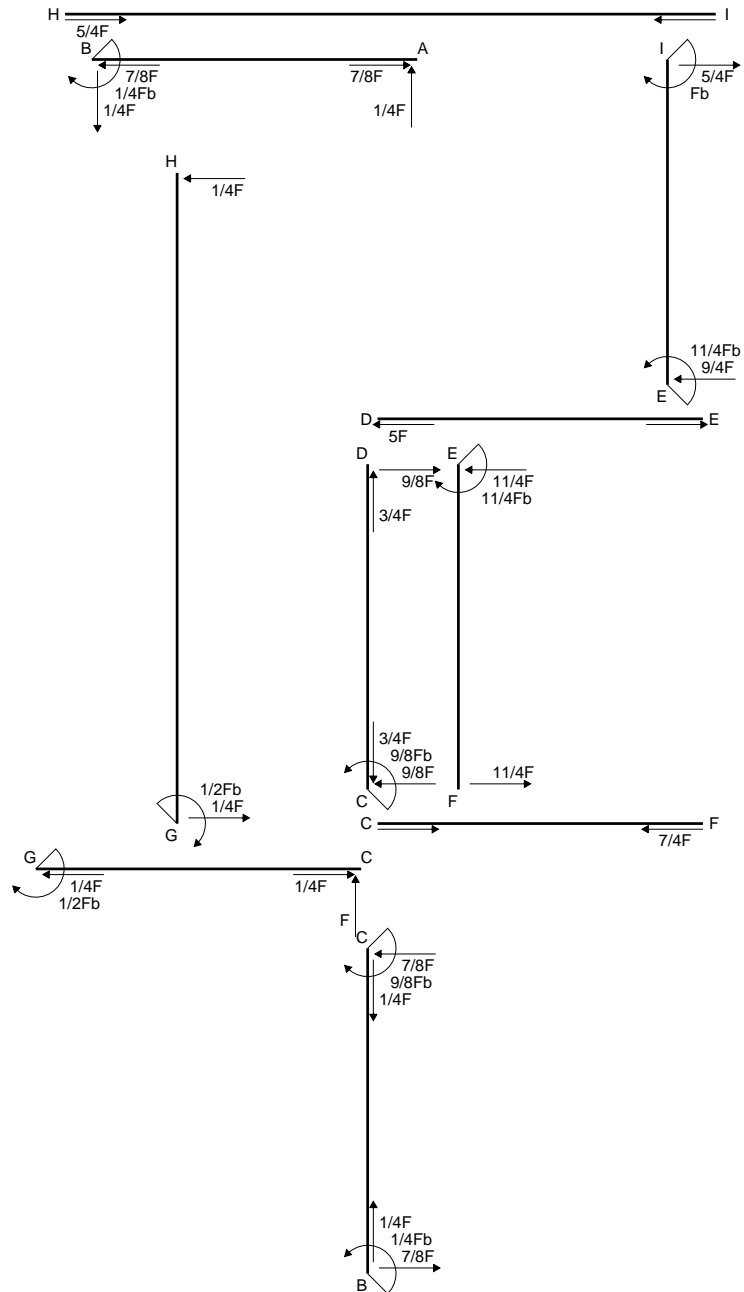
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

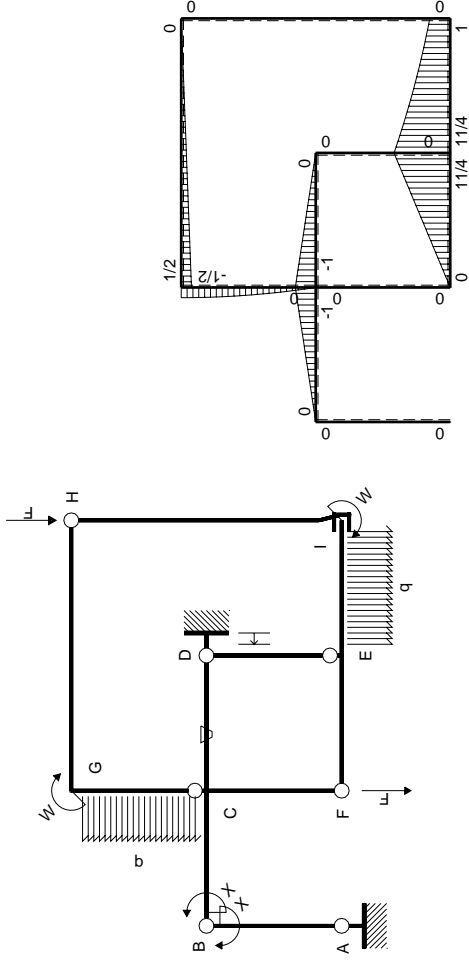
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$                | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|-------------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                       | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                       | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$                   | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$                 | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$                | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                    | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $11/4Fb-11/4Fx$         | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-11/4Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | $-Fx+1/2qx^2$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | $1/2Fb-1/2qx^2$         | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $1/2Fb-1/4Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/4Fx$                | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/4Fx+1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-11/4Fb+9/4Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                         |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                         |          |                      |                     |                         |                             | $-1/4Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                         |          |                      |                     |                         |                             | $1/4Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

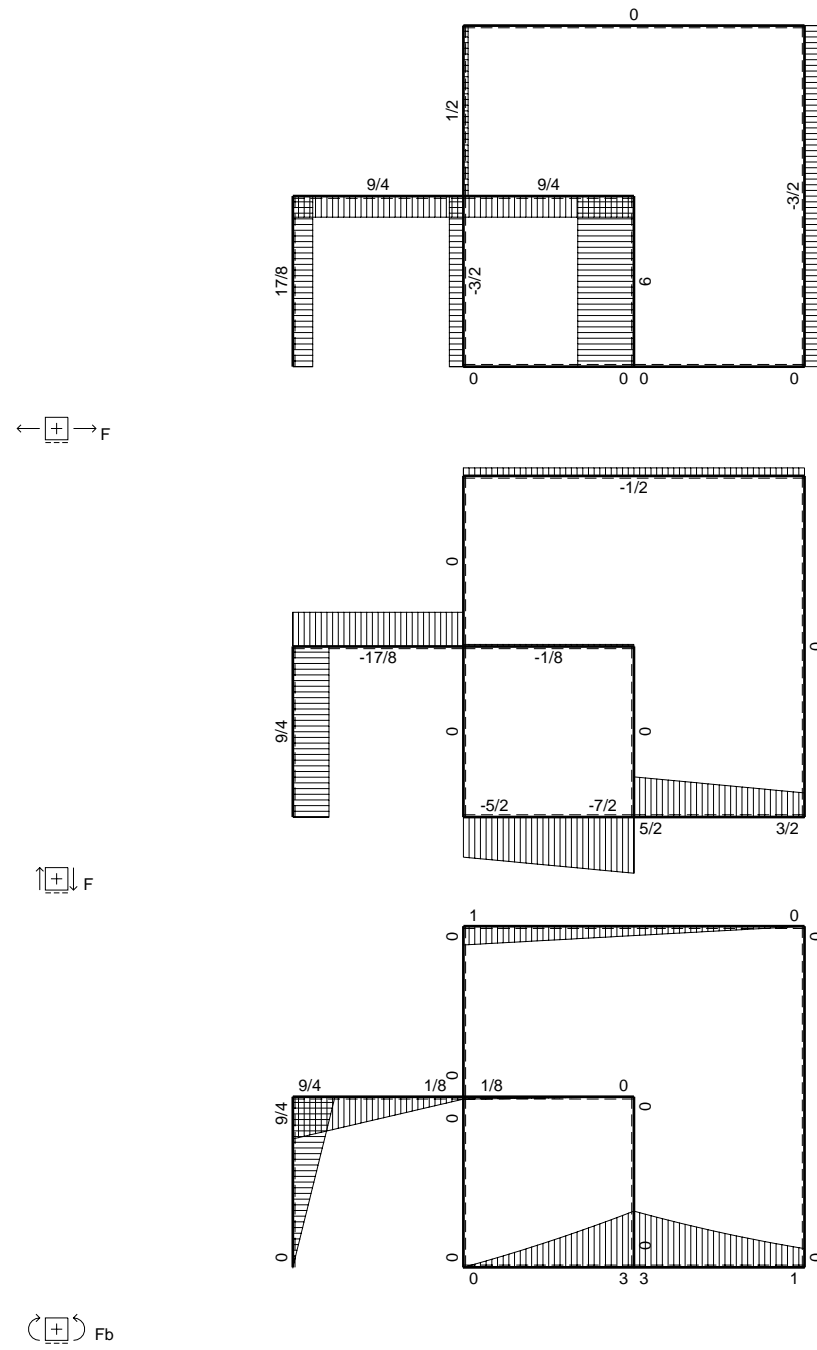
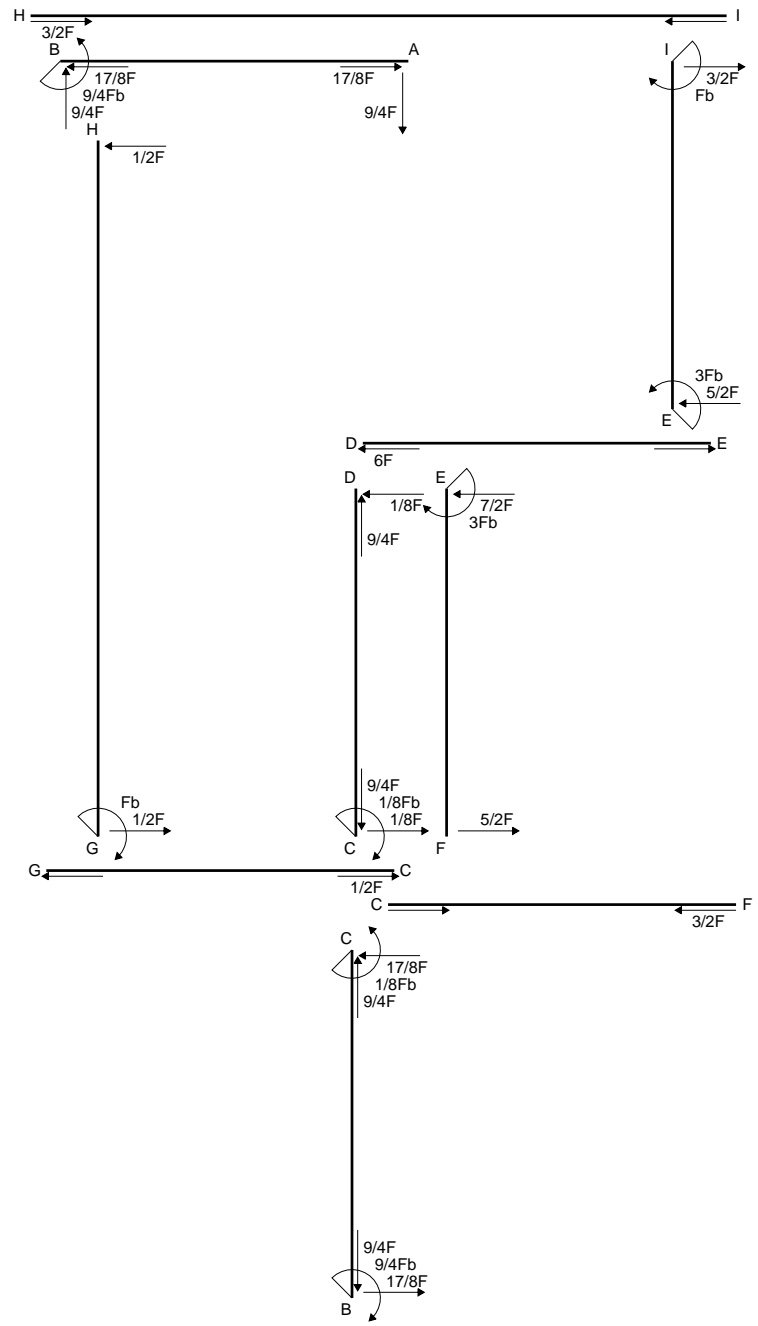
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

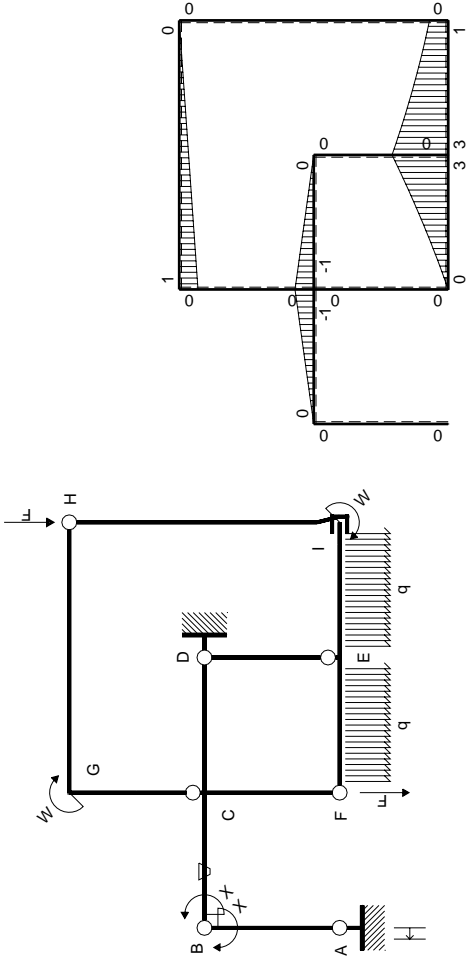
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$

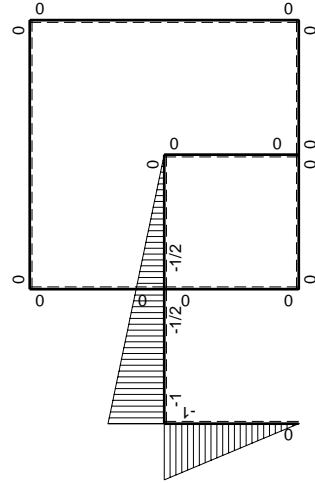


$\left(\oplus\right) F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |                     |                         | $9/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-9/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

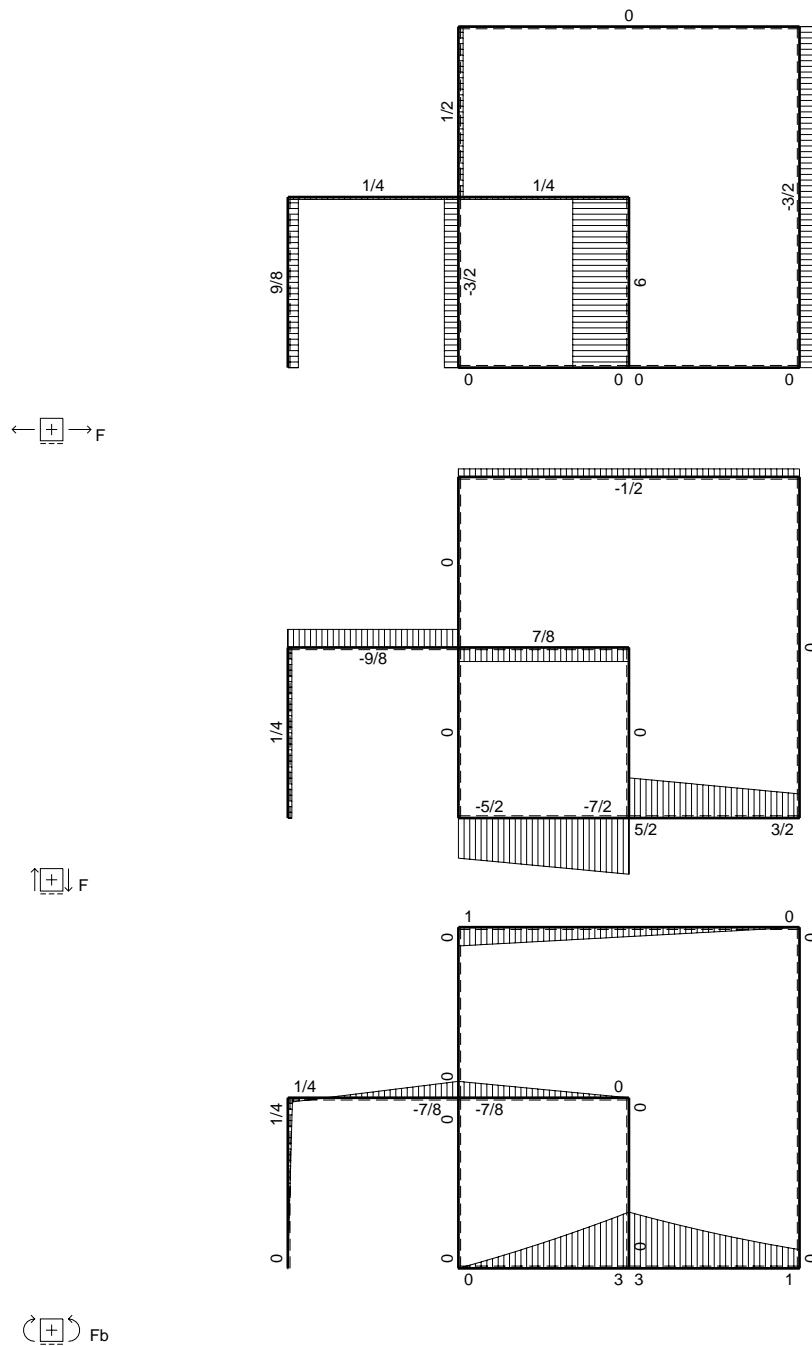
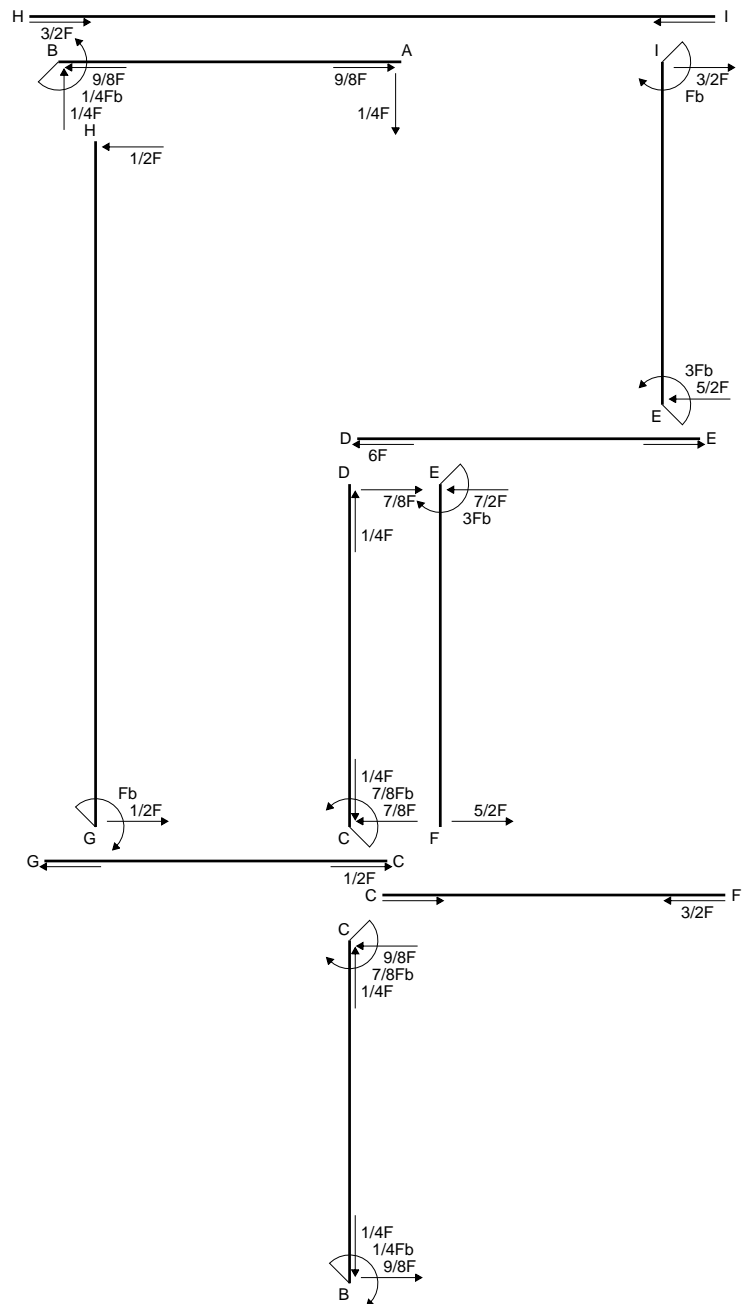
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

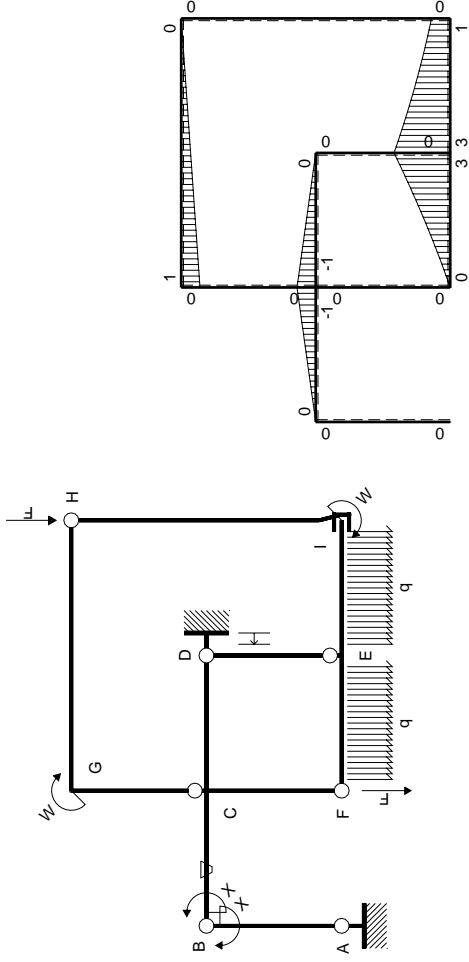
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | $-Fb/EJ$ | $Fx-1/2Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $1/4Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $-1/4Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/6 b) Fb 1/EJ + (b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

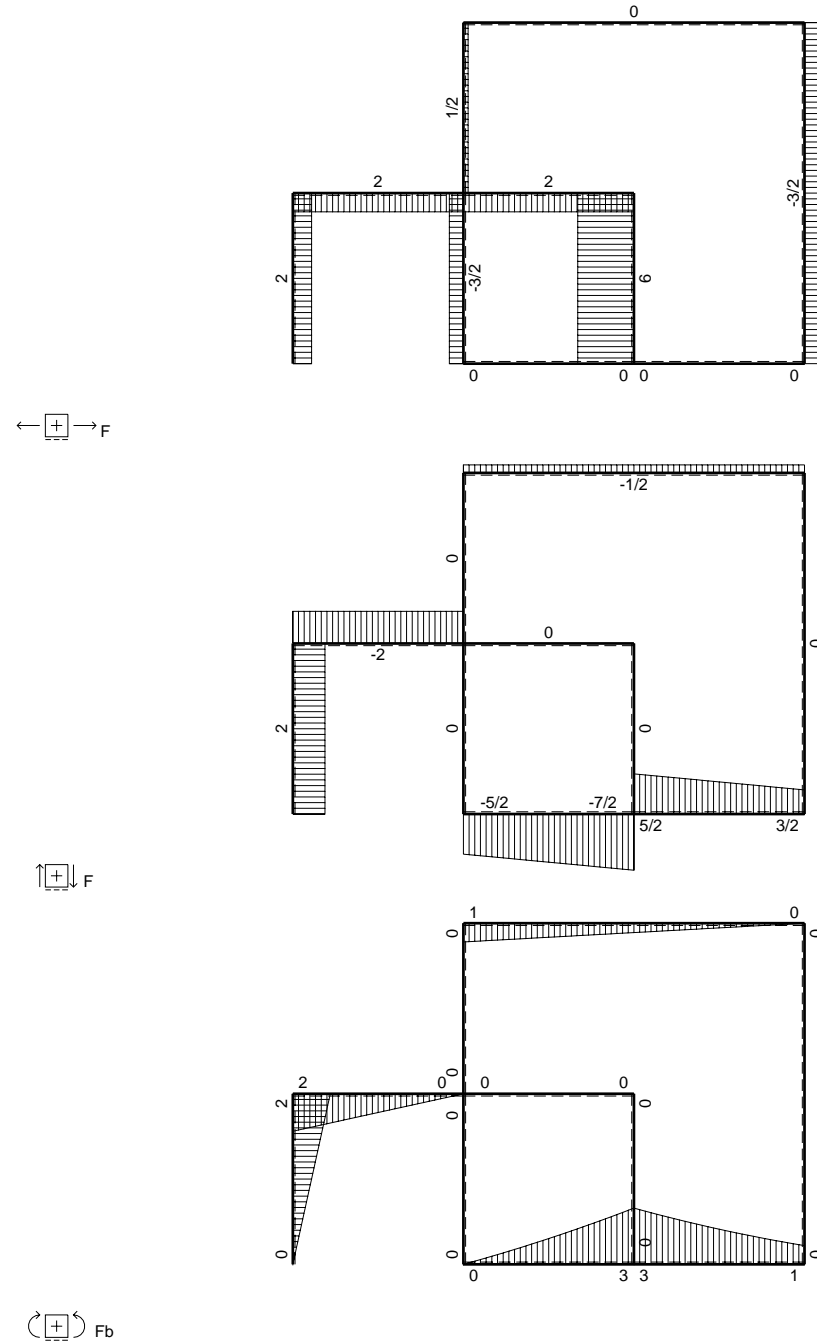
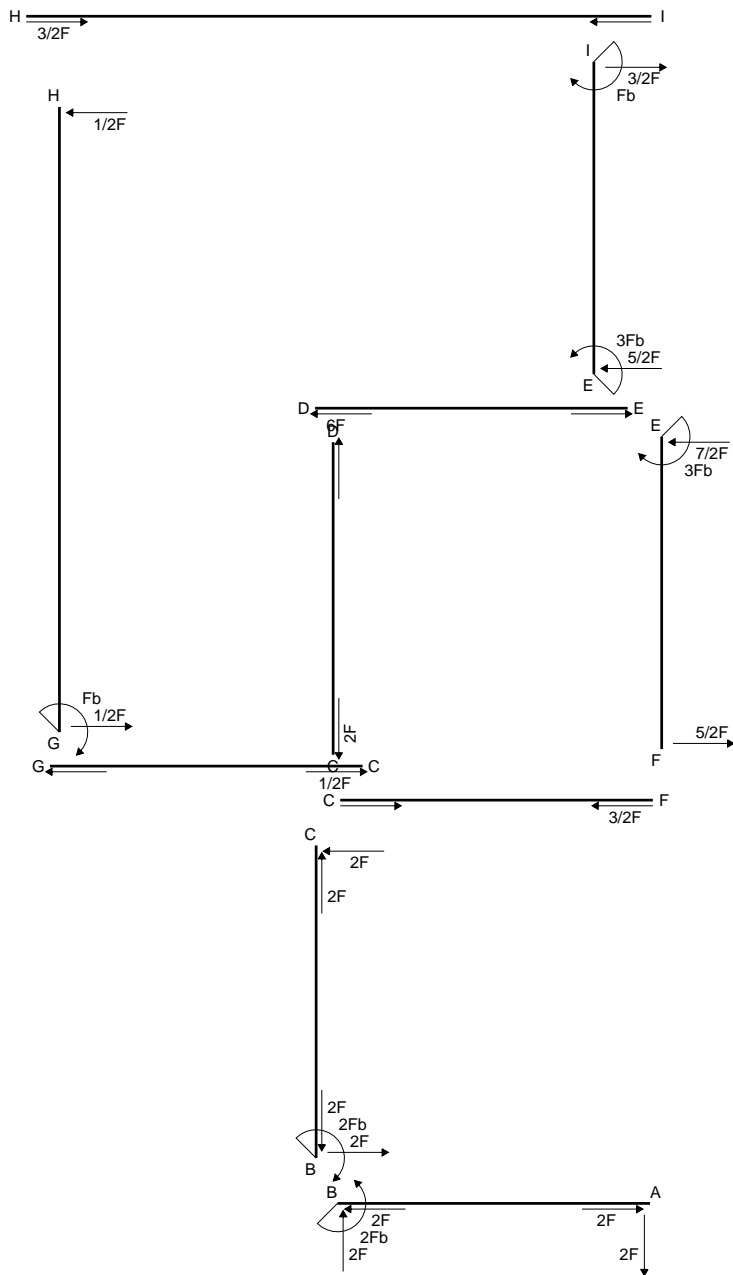
$$= (1/2 b - 1/6 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 13/12 Fb^2/EJ$$

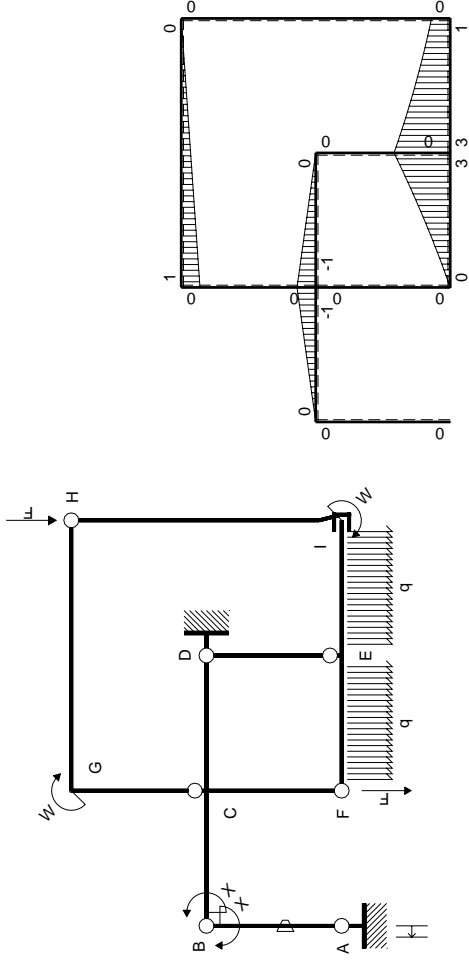
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | 0        | $Fx-1/2Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | 0        | $1/2Fb-1/2Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |               |                         | $2Fb^2/EJ$                  | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |               |                         | $-2Fb$                      |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

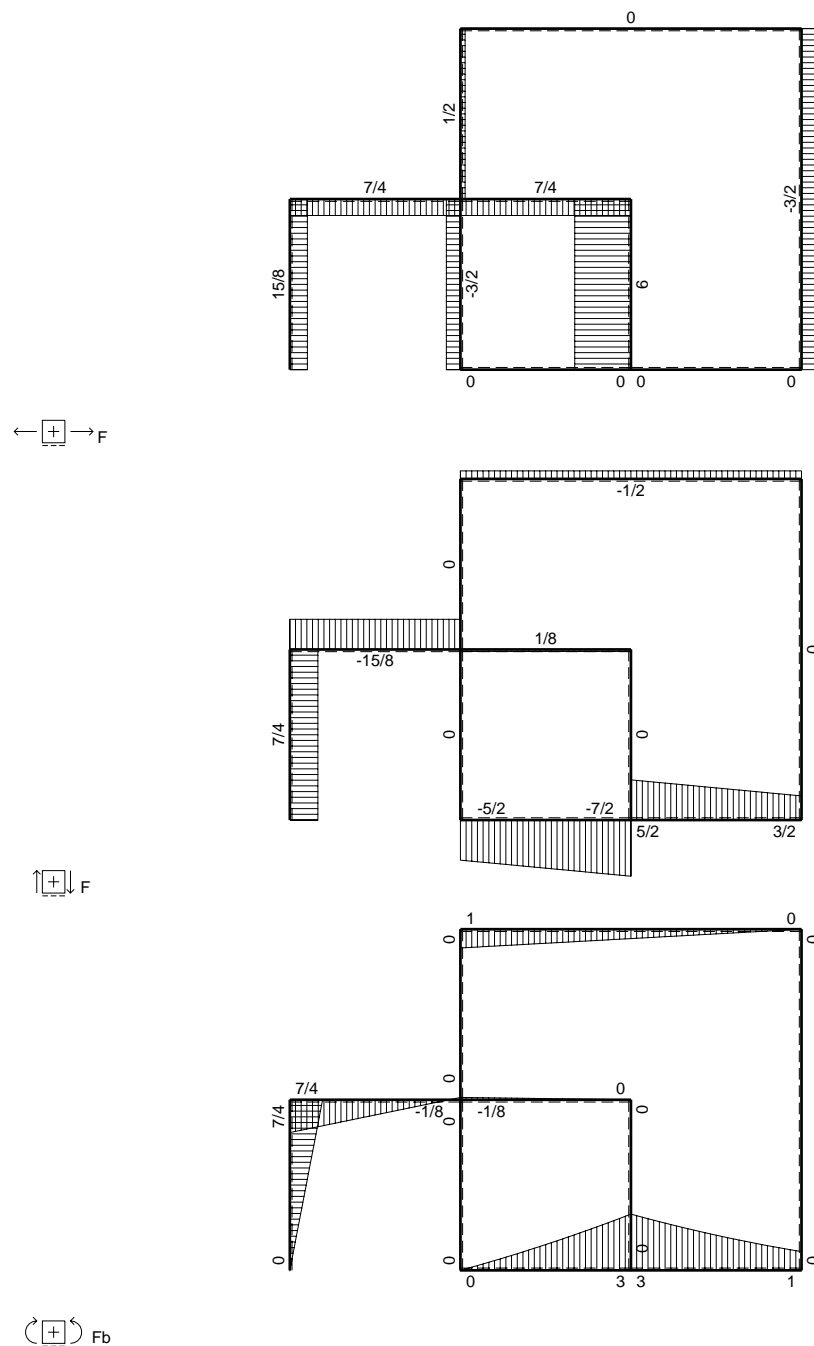
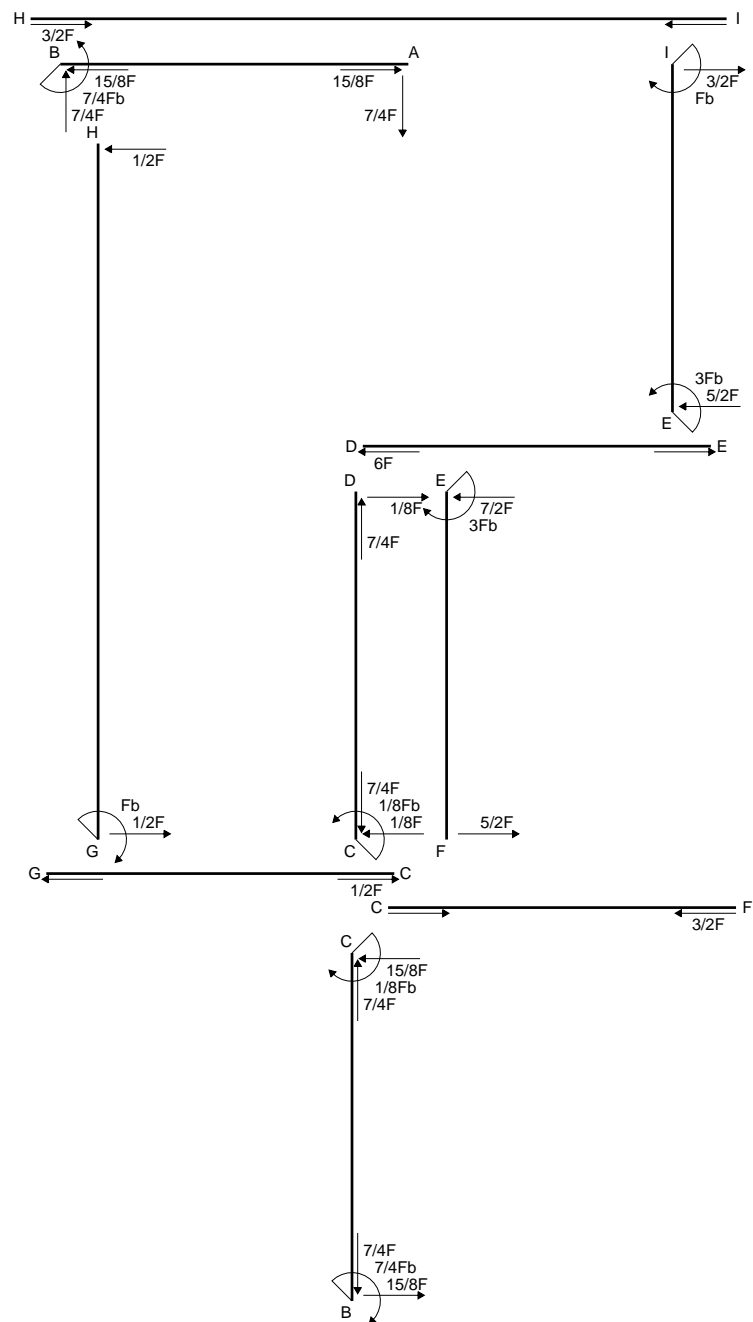
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

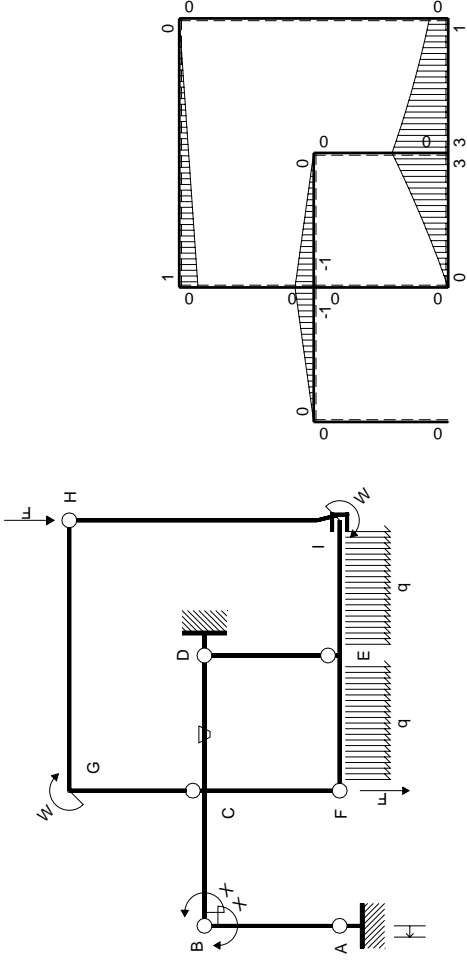
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |                     |                         | $7/4Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-7/4Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

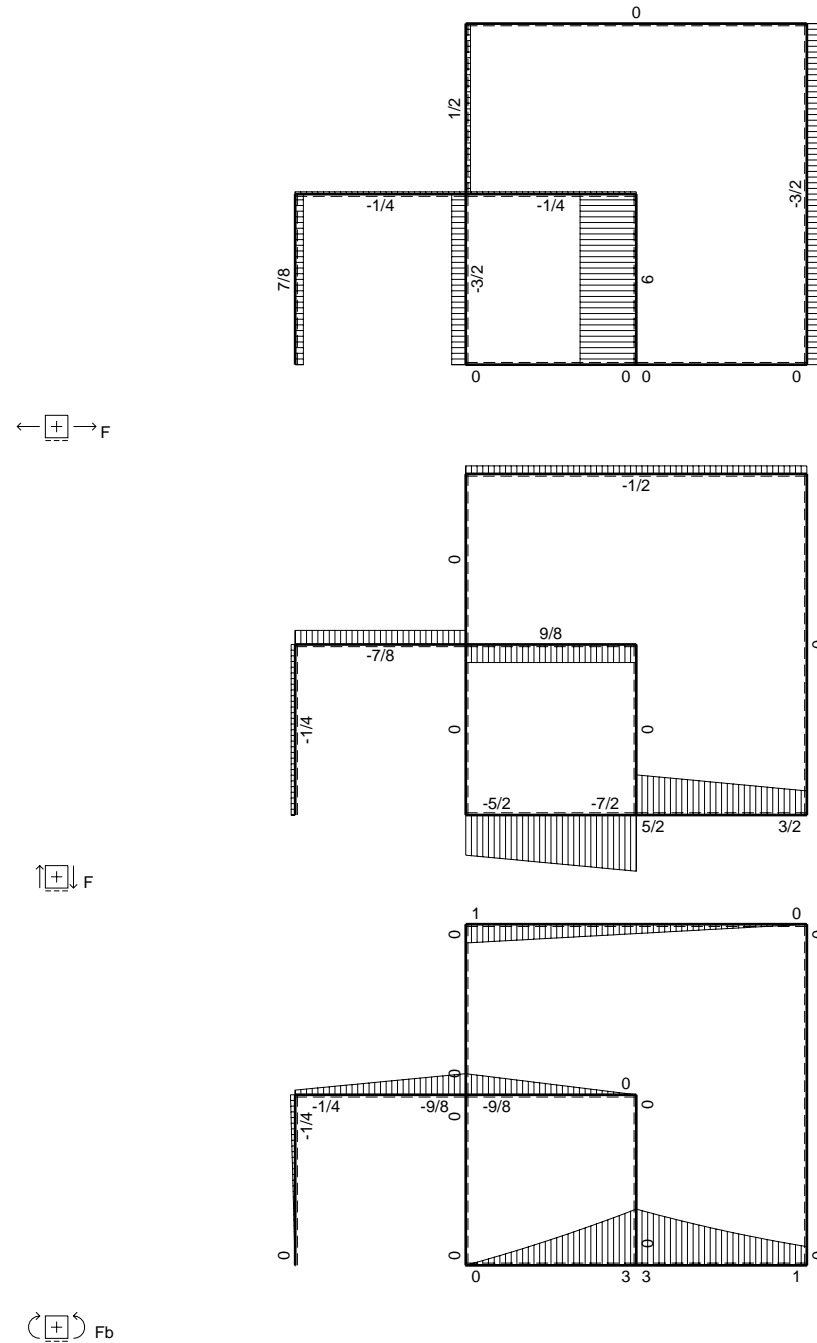
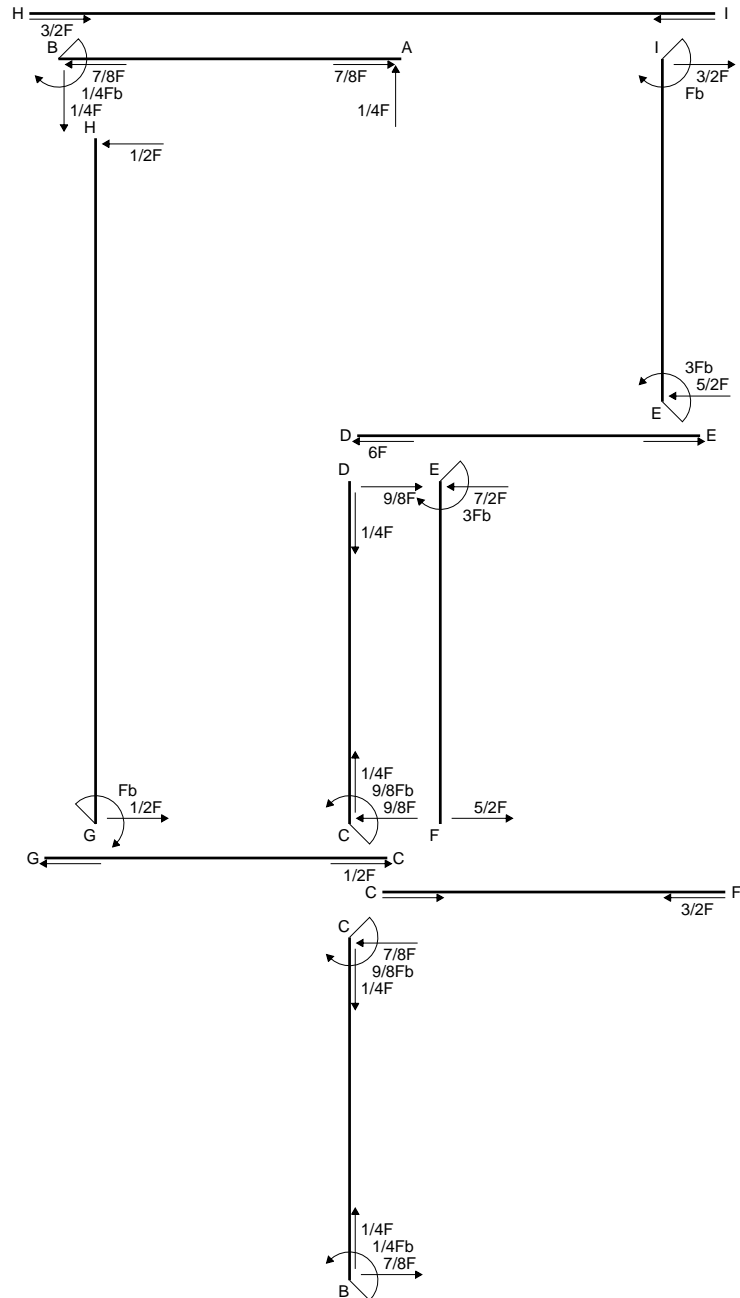
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

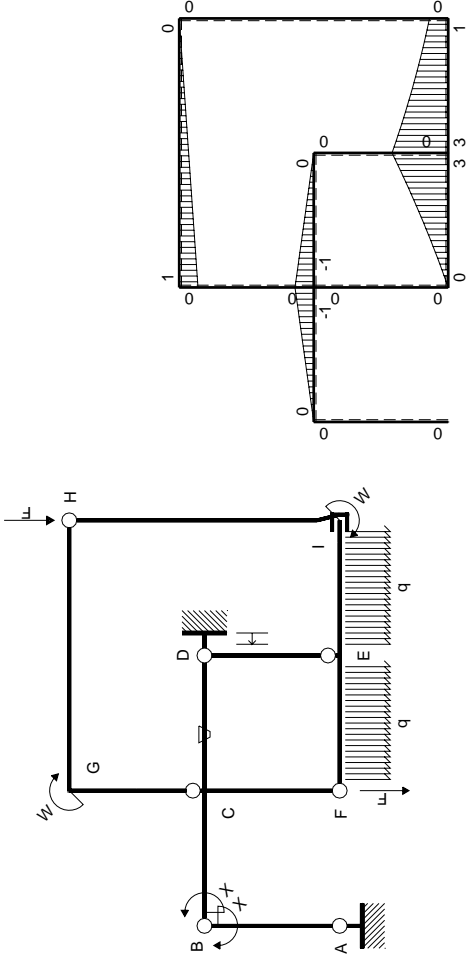
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$



⊕ Fb



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$                | 0        | $Fx-1/2Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$              | 0        | $1/2Fb-1/2Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-7/2Fx+1/2qx^2$  | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-5/2Fx-1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $-1/4Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $1/4Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

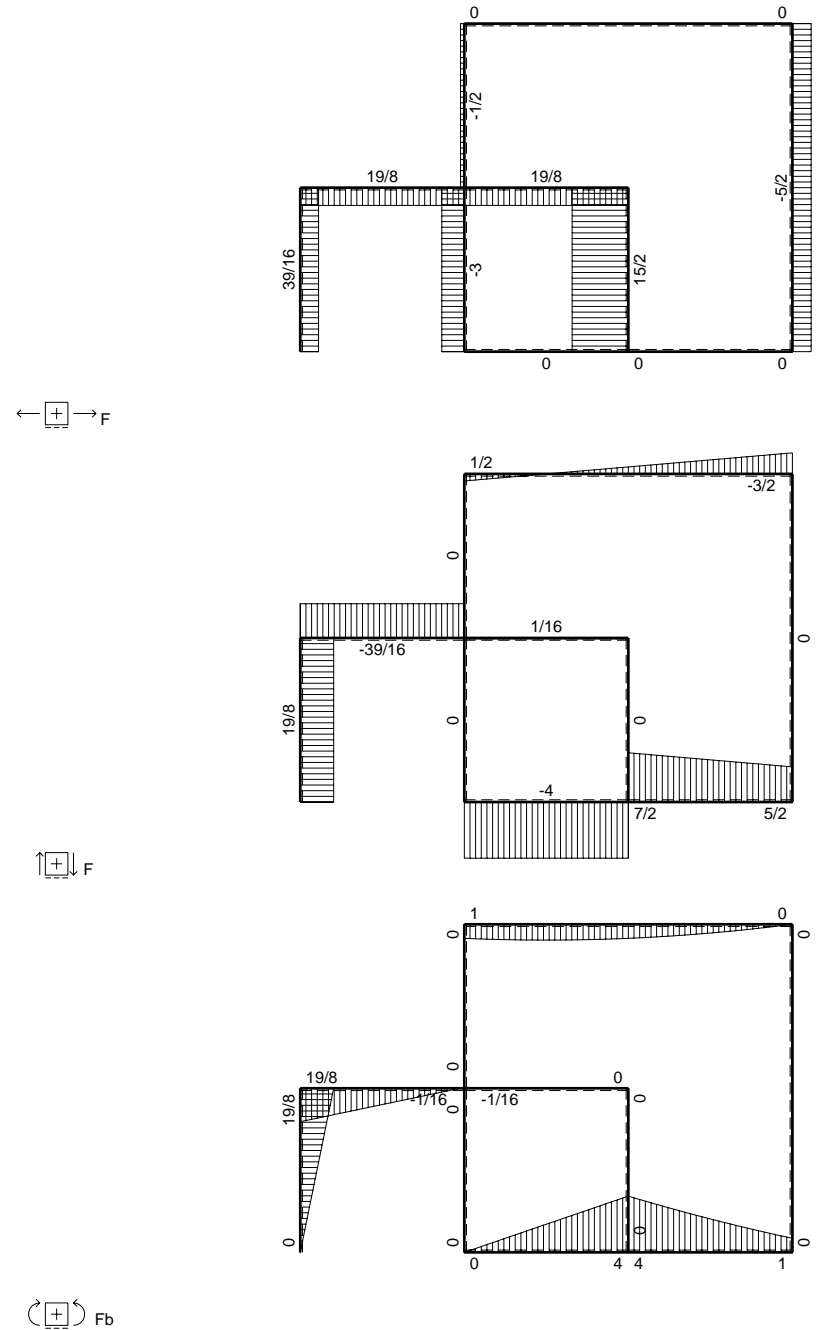
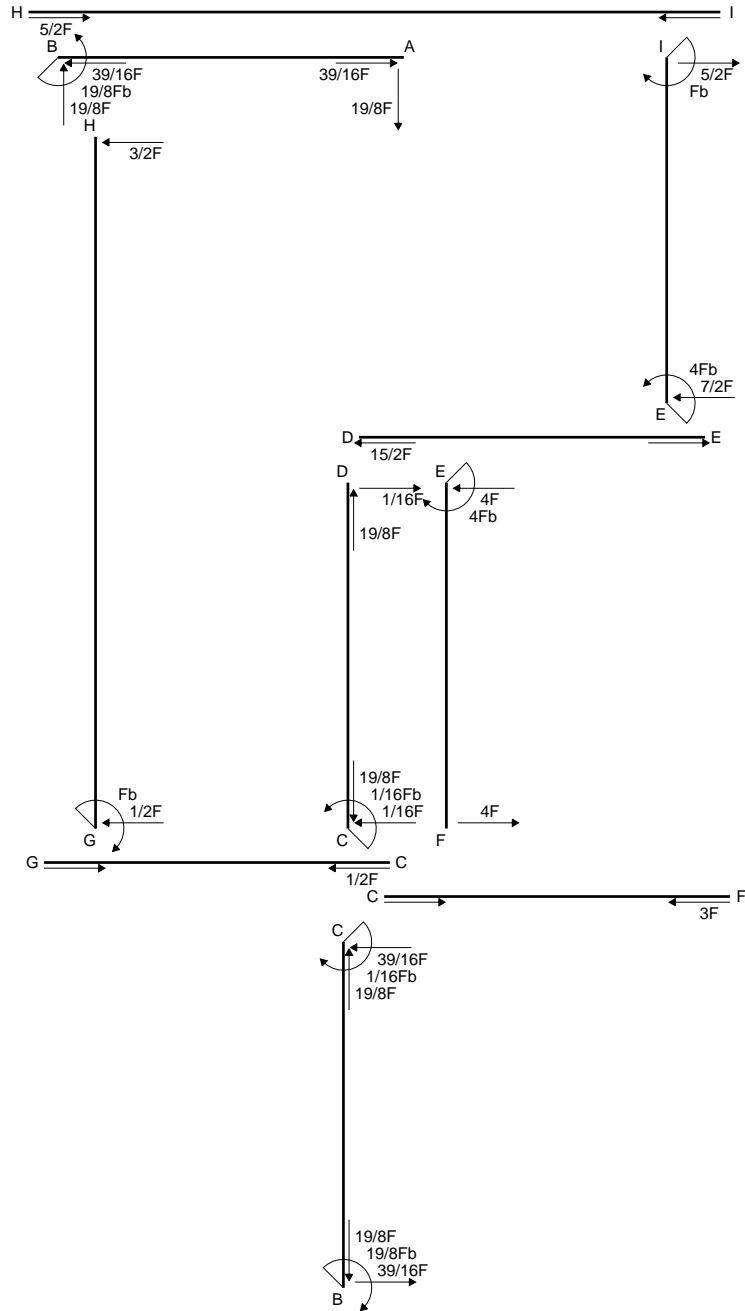
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

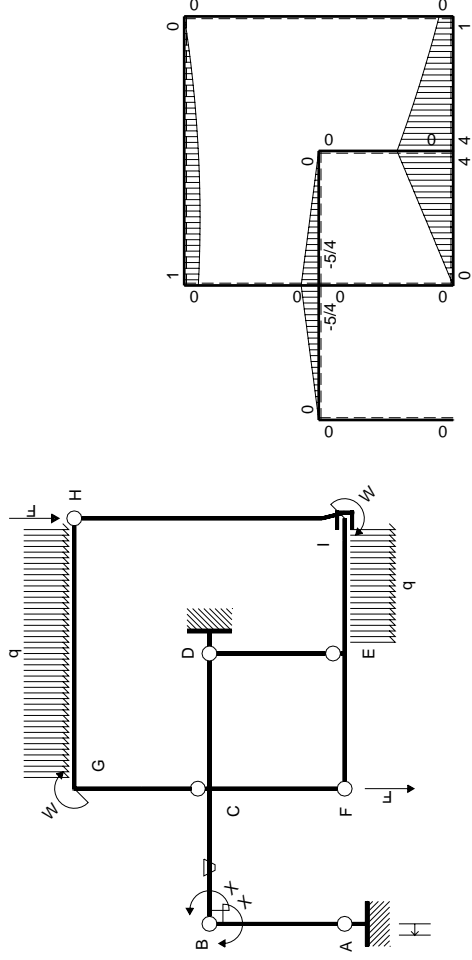
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$

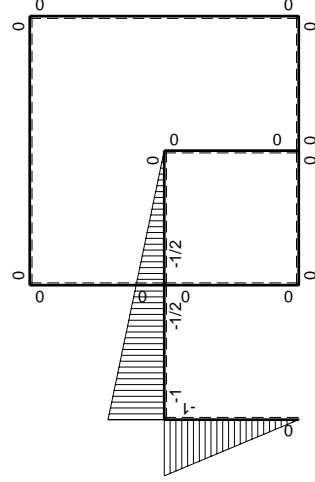






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| CG b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| GC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| HI 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| IE b  | 0                           | $Fb+5/2Fx+1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |  |
| EI b  | 0                           | $-4Fb+7/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |  |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |                     |                         | $Fb^2/EJ$                   |                        |  |
|       | totali                      |                      |          |                         |                     |                         | $19/8Fb^2/EJ$               | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-19/8Fb$                   |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

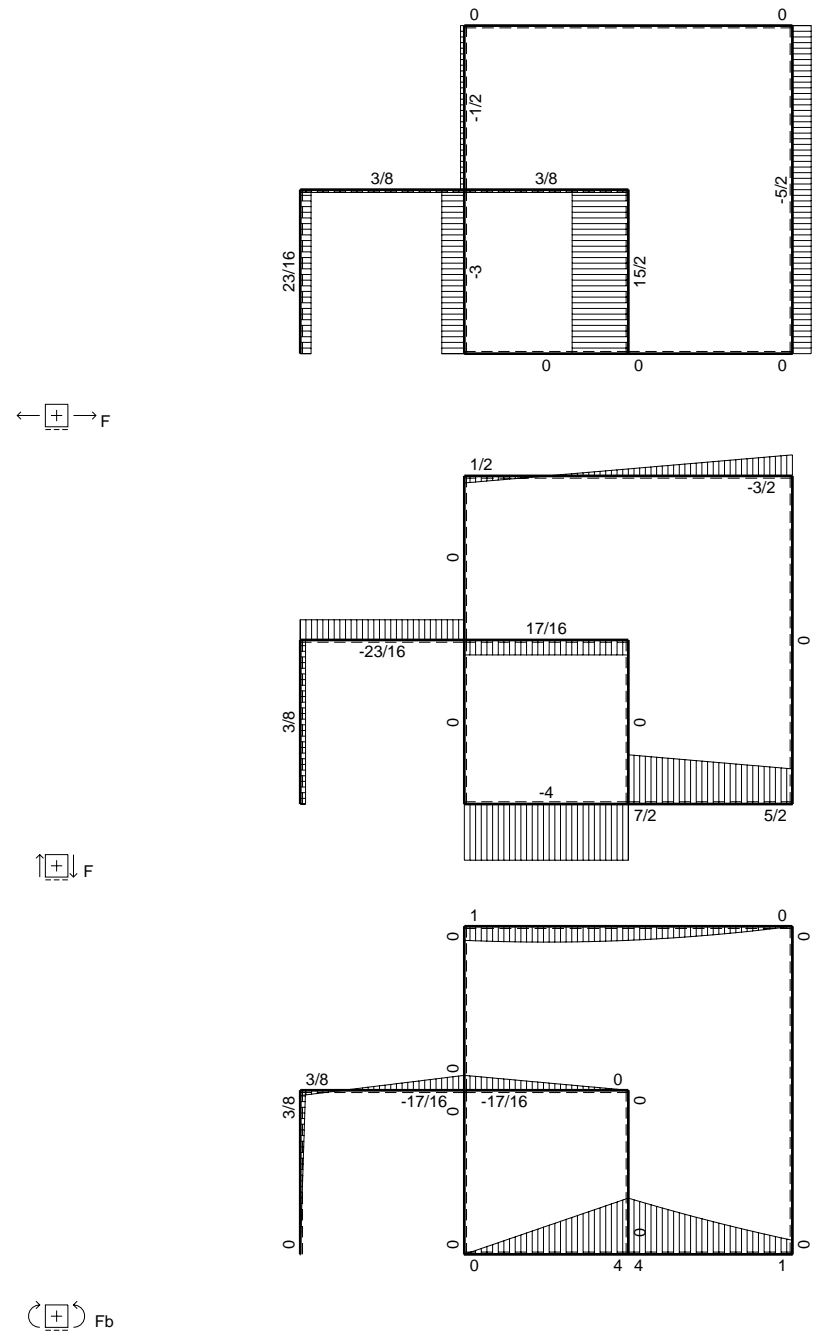
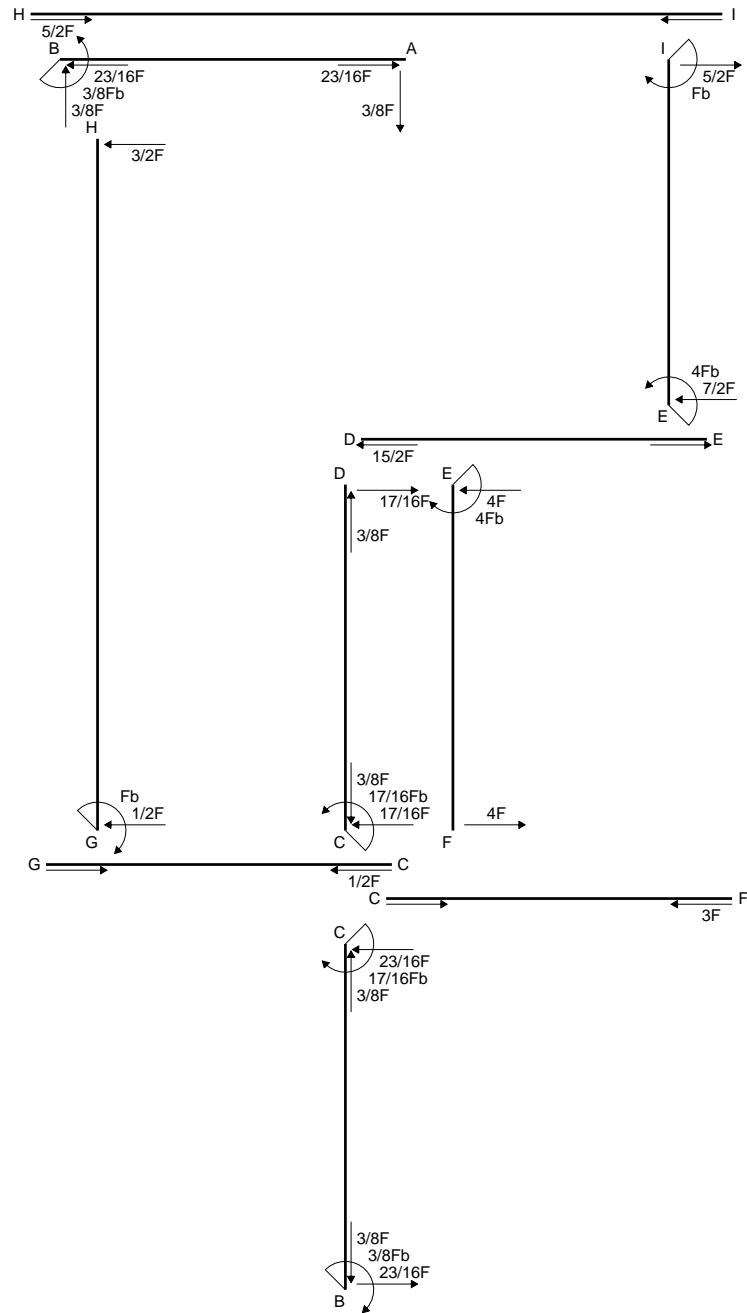
$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

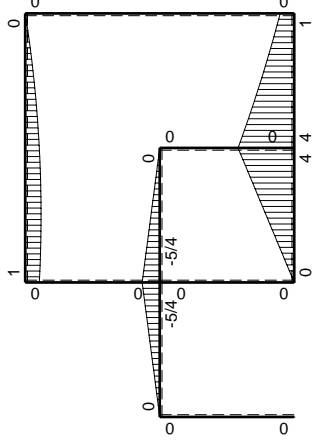
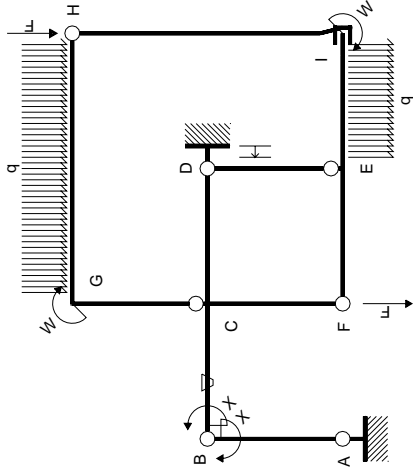
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

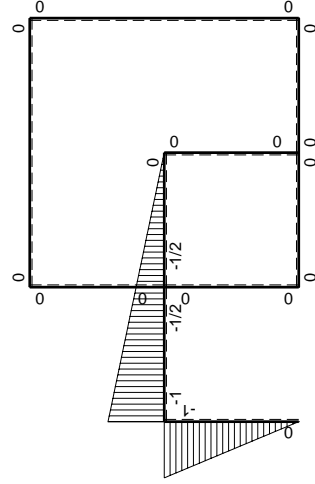
$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | $-Fb/EJ$ | $5/4Fx-5/8Fx^2/b$       | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(5/12+3/4)Fb^2/EJ$         | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | $Fb/EJ$  | $5/8Fb-5/8Fx^2/b$       | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0                   | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx+1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+7/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |                     |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                      |          |                         |                     |                         | $3/8Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-3/8Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/24 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 7/6 Fb^2/EJ$$

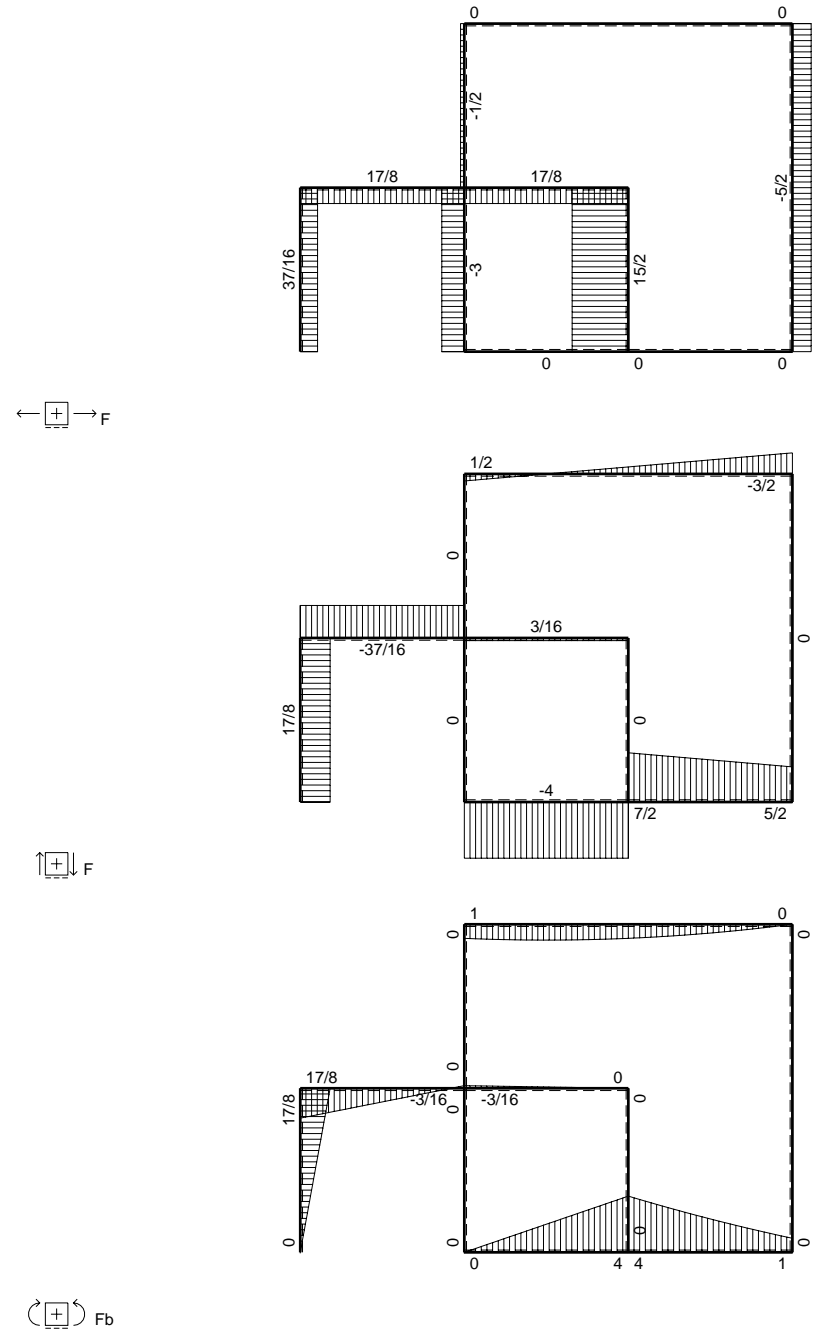
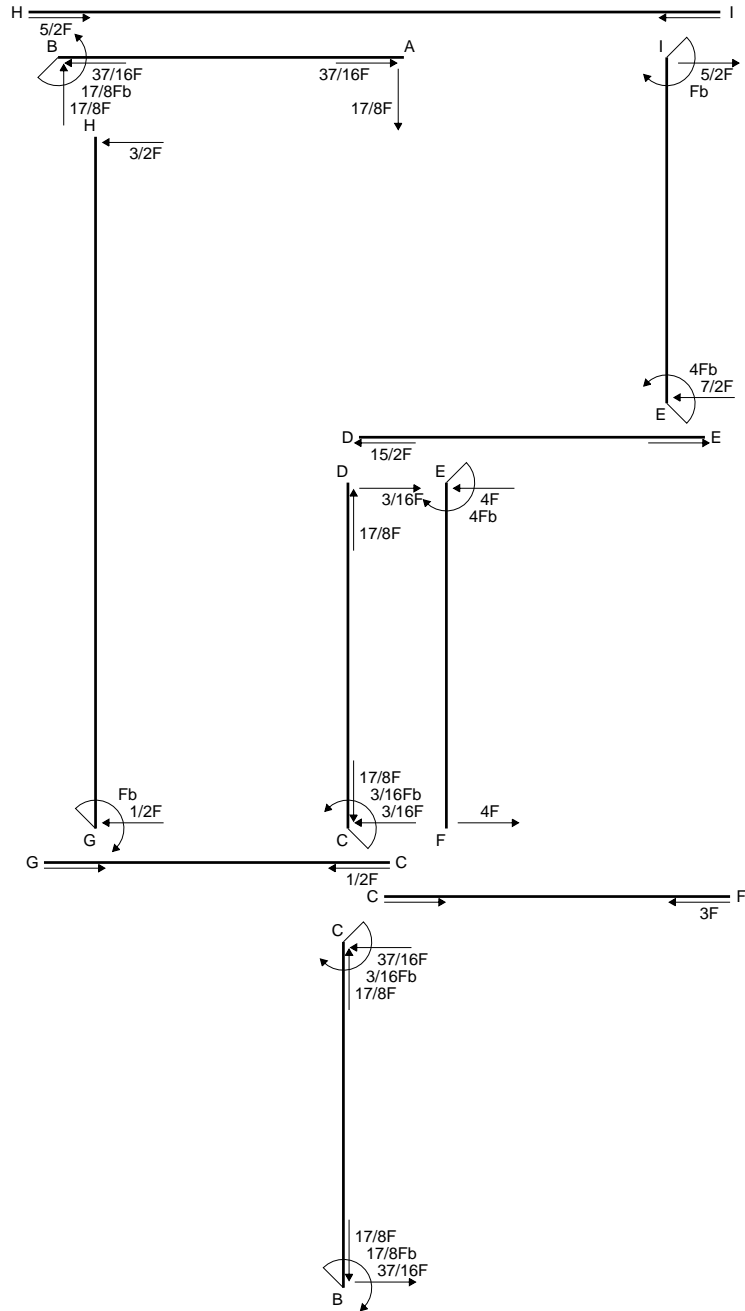
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$







Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$     | 0        | 0                       | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx+1/2qx^2$   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+7/2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |               |                         | $17/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |               |                         | $-17/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

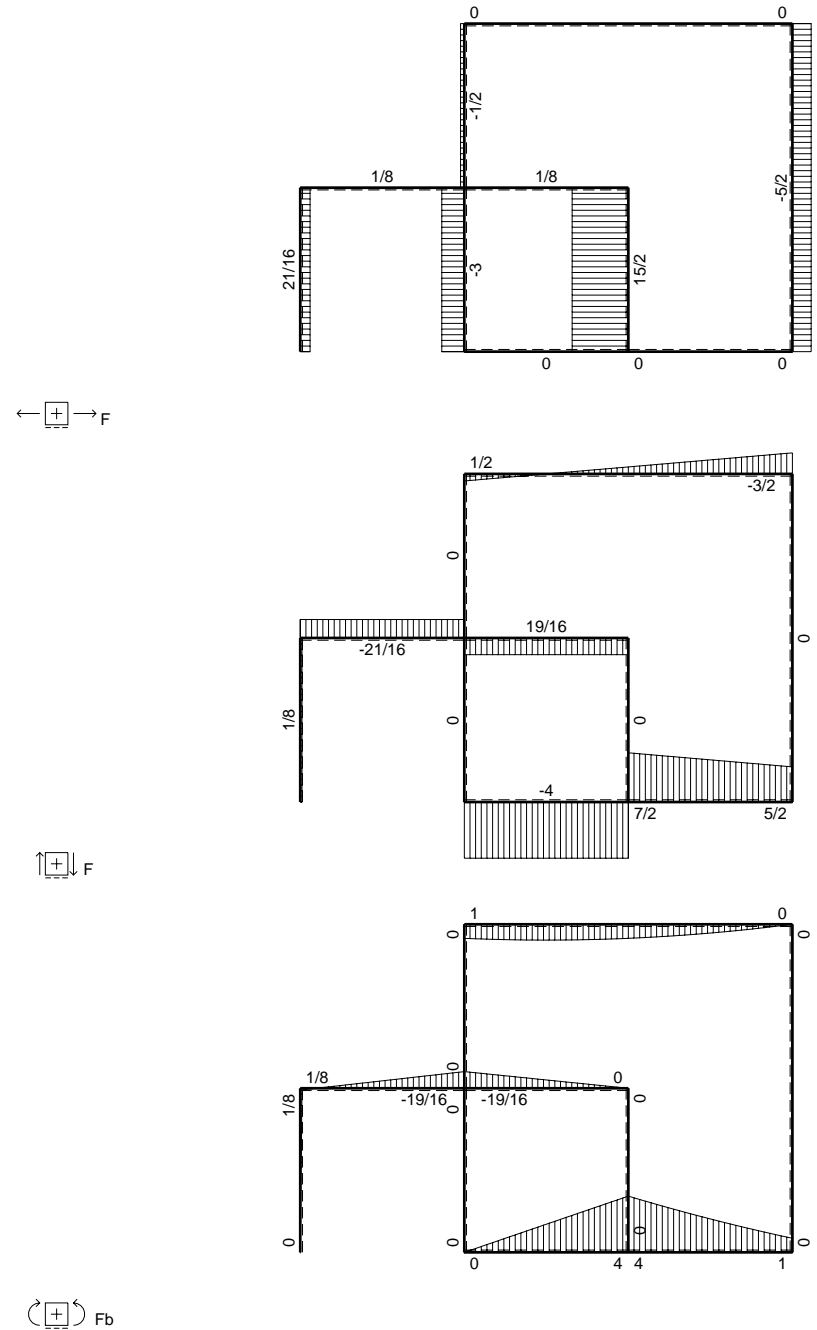
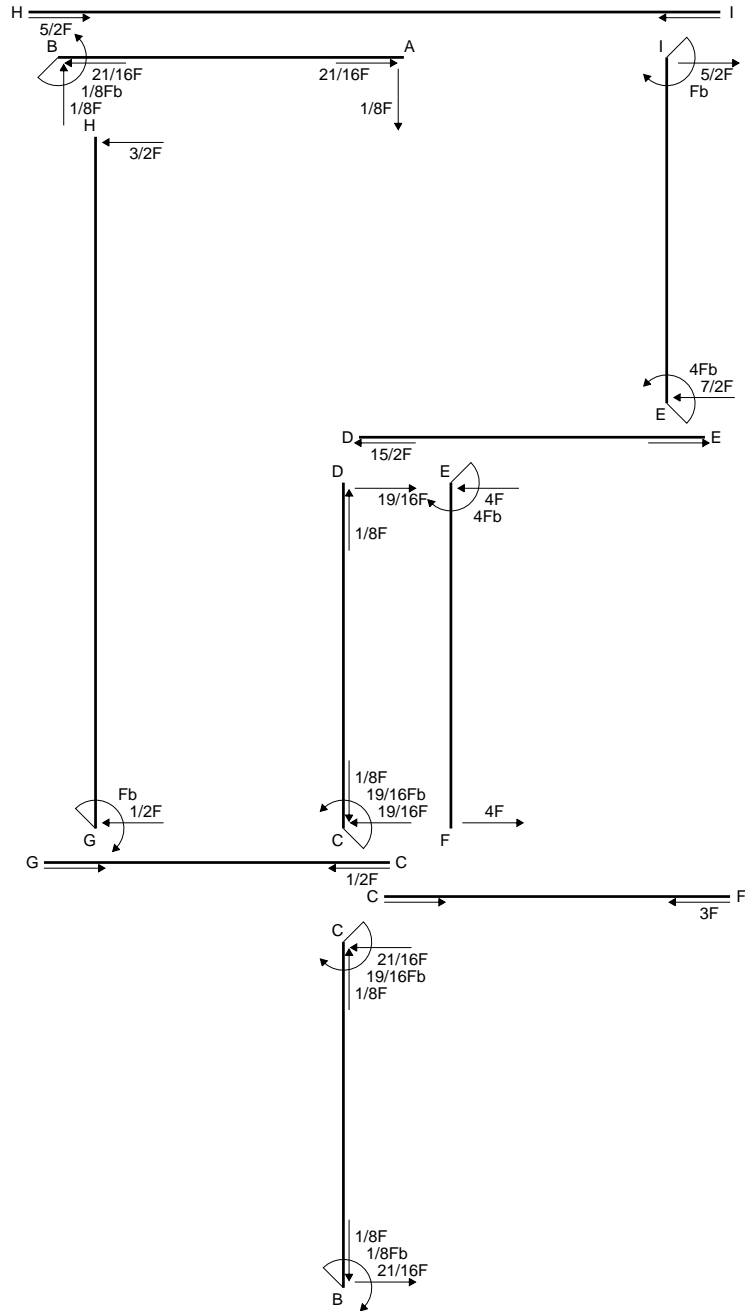
$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

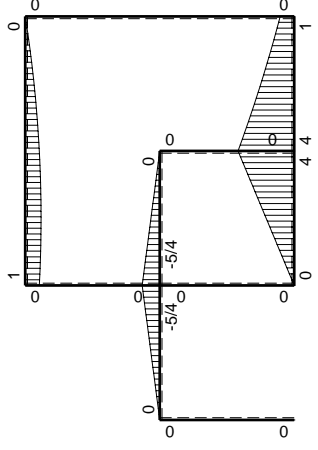
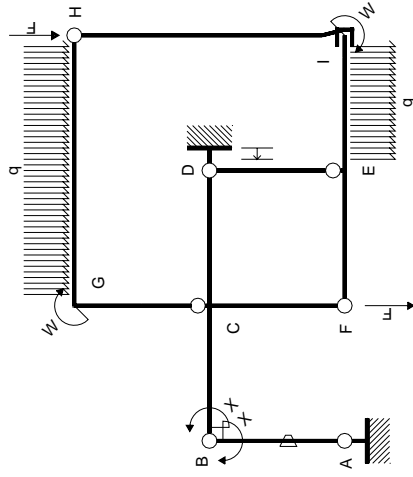
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

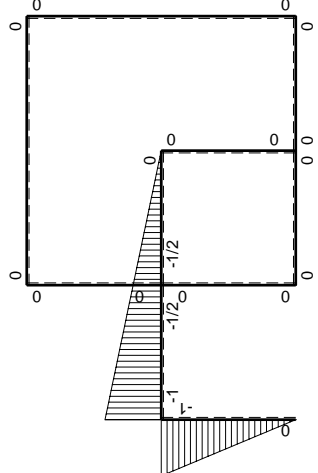
$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                       | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                       | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0             | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | 0        | $5/8Fb-5/4Fx+5/8Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+0)Fb^2/EJ$           | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | 0        | $5/8Fx^2/b$             | 0             | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0             | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$     | 0        | 0                       | 0             | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                       | 0             | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx+1/2qx^2$   | 0        | 0                       | 0             | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+7/2Fx-1/2qx^2$ | 0        | 0                       | 0             | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |               |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |               |                         |                             | $1/8Fb^2/EJ$           | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |               |                         |                             | $-1/8Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

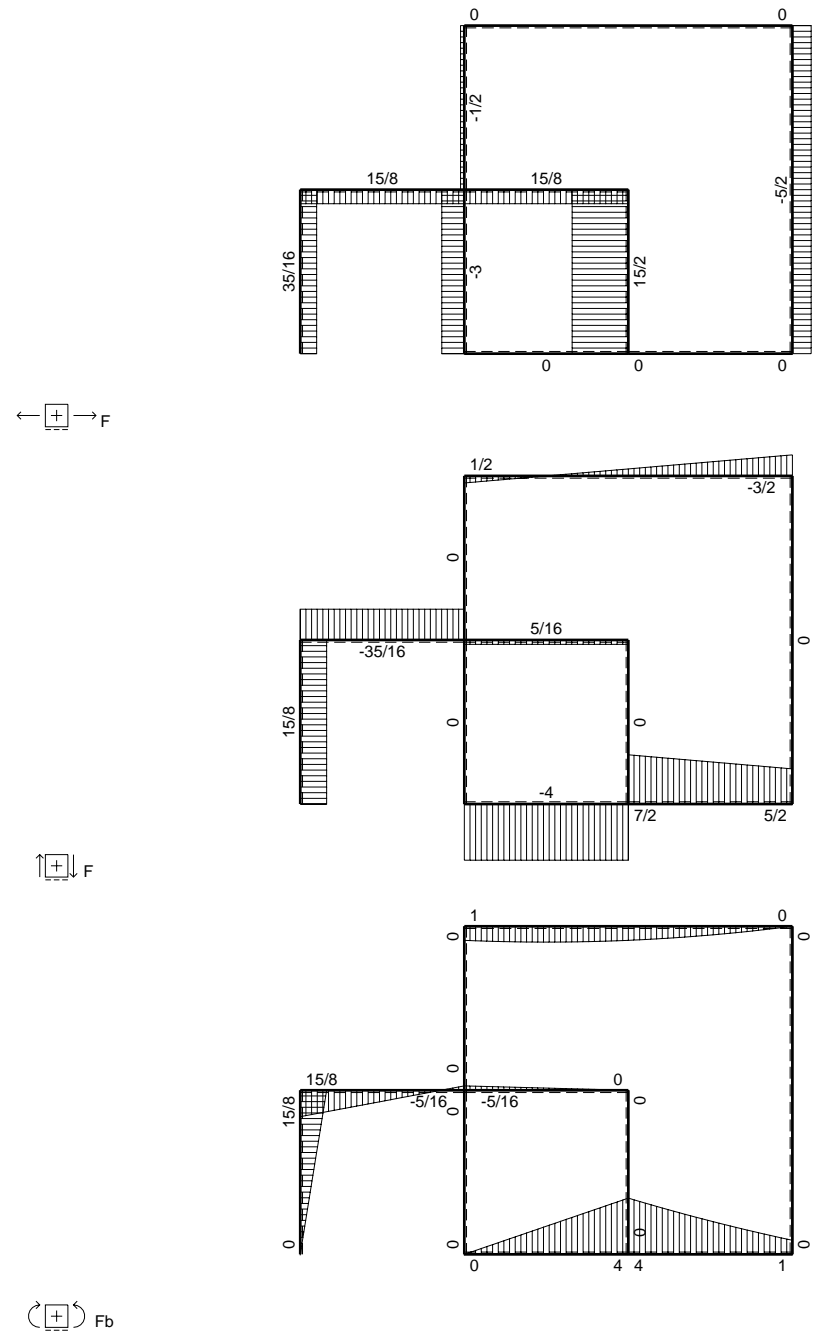
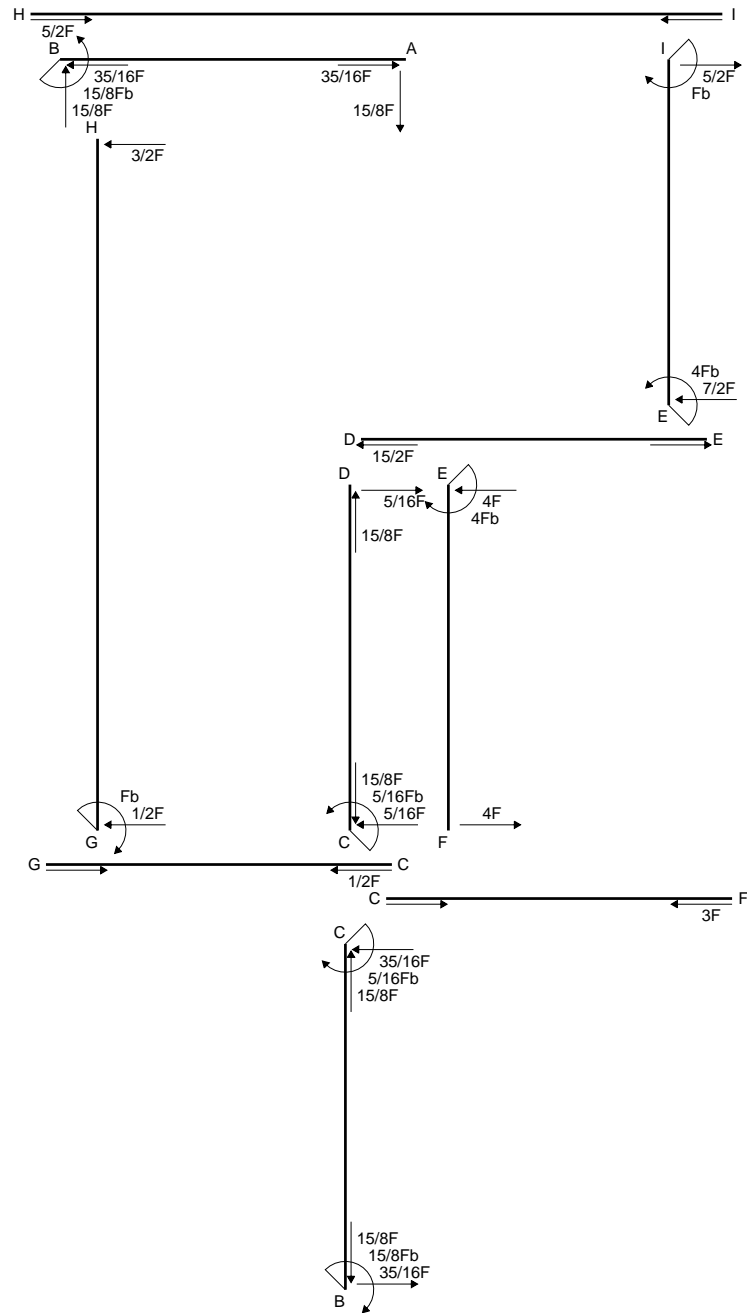
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ$$

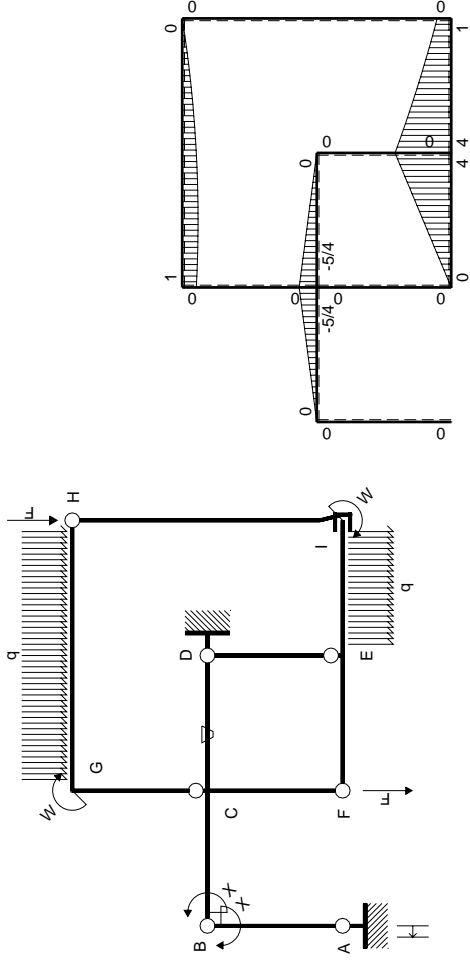
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx = [5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/24 b) Fb 1/EJ = 5/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx+1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-4Fb+7/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                         |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                         |                     |                         | $15/8Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         | $-15/8Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

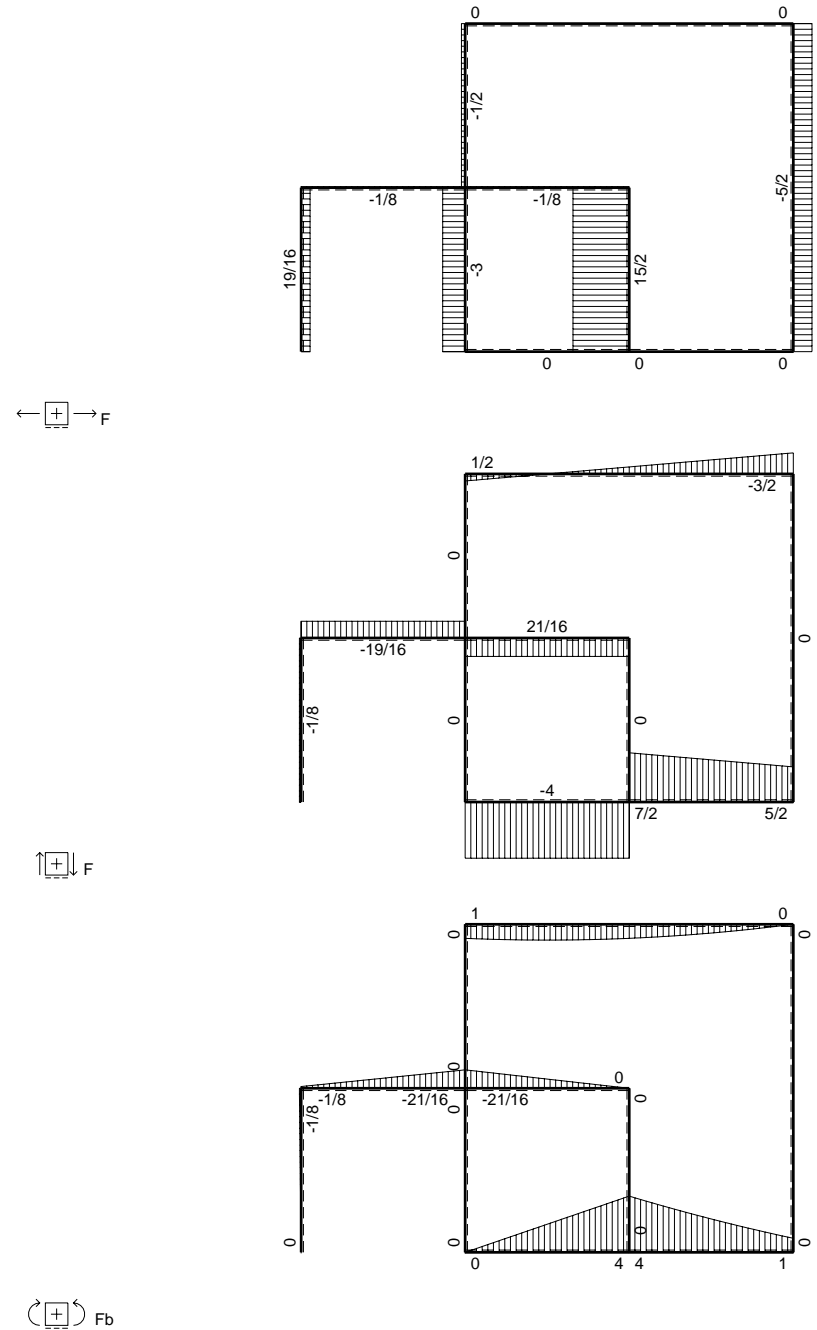
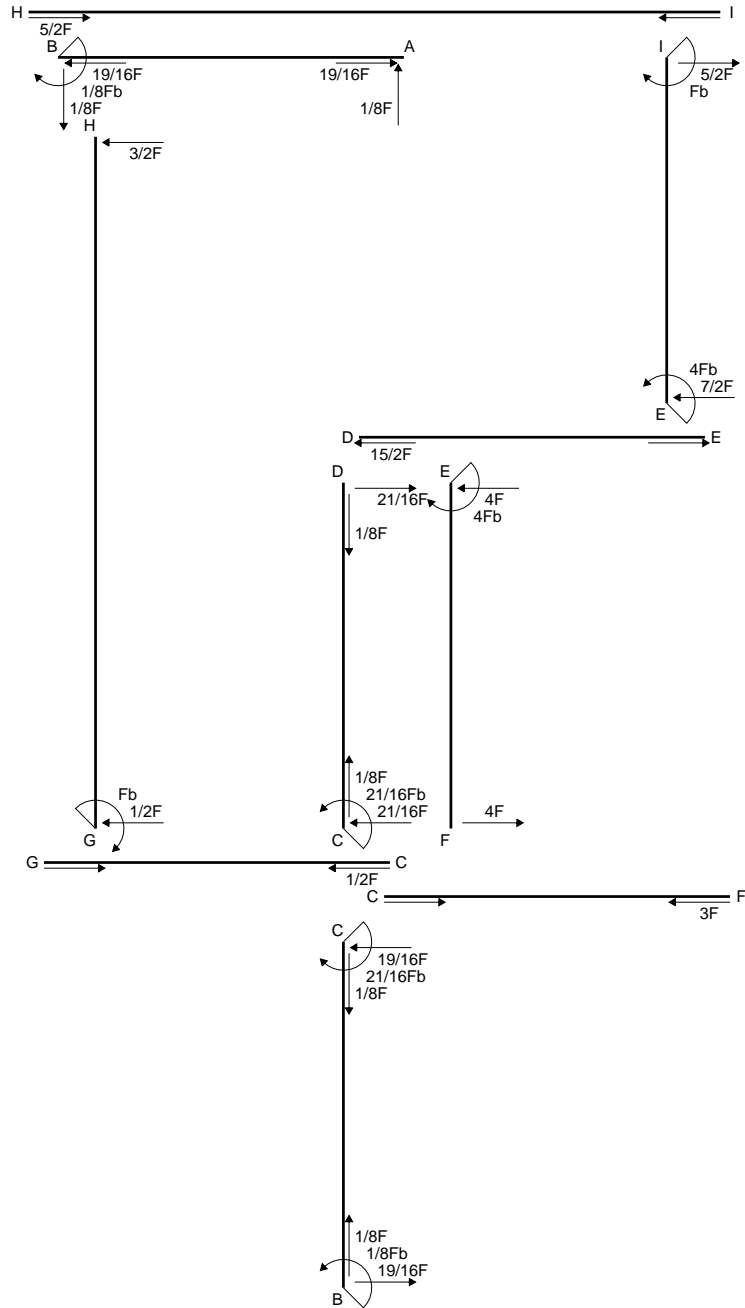
$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

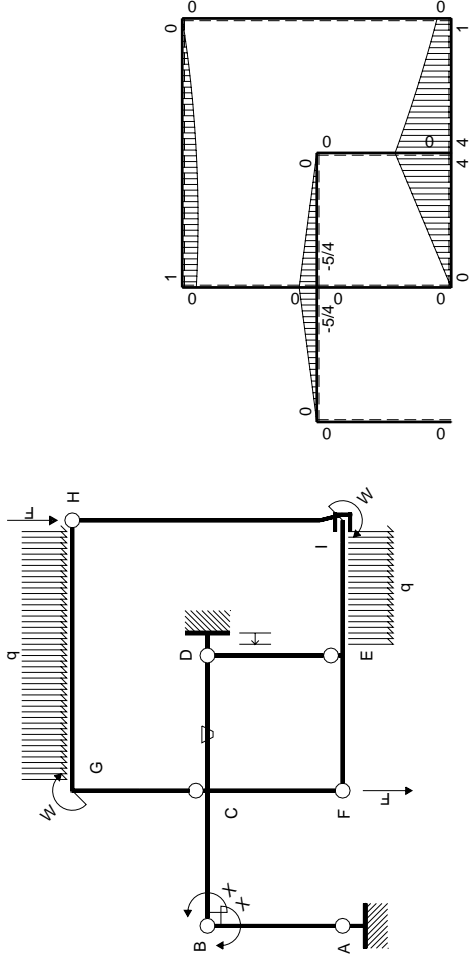
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$               | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|-------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                       | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                       | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-5/4Fx$             | 0        | $5/4Fx-5/8Fx^2/b$       | 0                   | $1-x/b+1/4x^2/b^2$      | $(5/12+0)Fb^2/EJ$           | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $5/4Fb-5/4Fx$        | 0        | $5/8Fb-5/8Fx^2/b$       | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-5/4Fb+5/4Fx$       | $-Fb/EJ$ | $5/8Fb-5/4Fx+5/8Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(5/24+1/4)Fb^2/EJ$         | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $5/4Fx$              | $Fb/EJ$  | $5/8Fx^2/b$             | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $4Fb-4Fx$            | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-4Fx$               | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$     | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx+1/2qx^2$   | 0        | 0                       | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-4Fb+7/2Fx-1/2qx^2$ | 0        | 0                       | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                         |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                         |                     |                         |                             | $-1/8Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                         |                     |                         |                             | $1/8Fb$                |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (5/4 x/b - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x^2/b - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (5/8 - 5/8 x^2/b^2) Fb 1/EJ dx = [5/8 x - 5/24 x^3/b^2]_0^b Fb 1/EJ$$

$$= (5/8 b - 5/24 b) Fb 1/EJ = 5/12 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (5/8 - 5/4 x/b + 5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

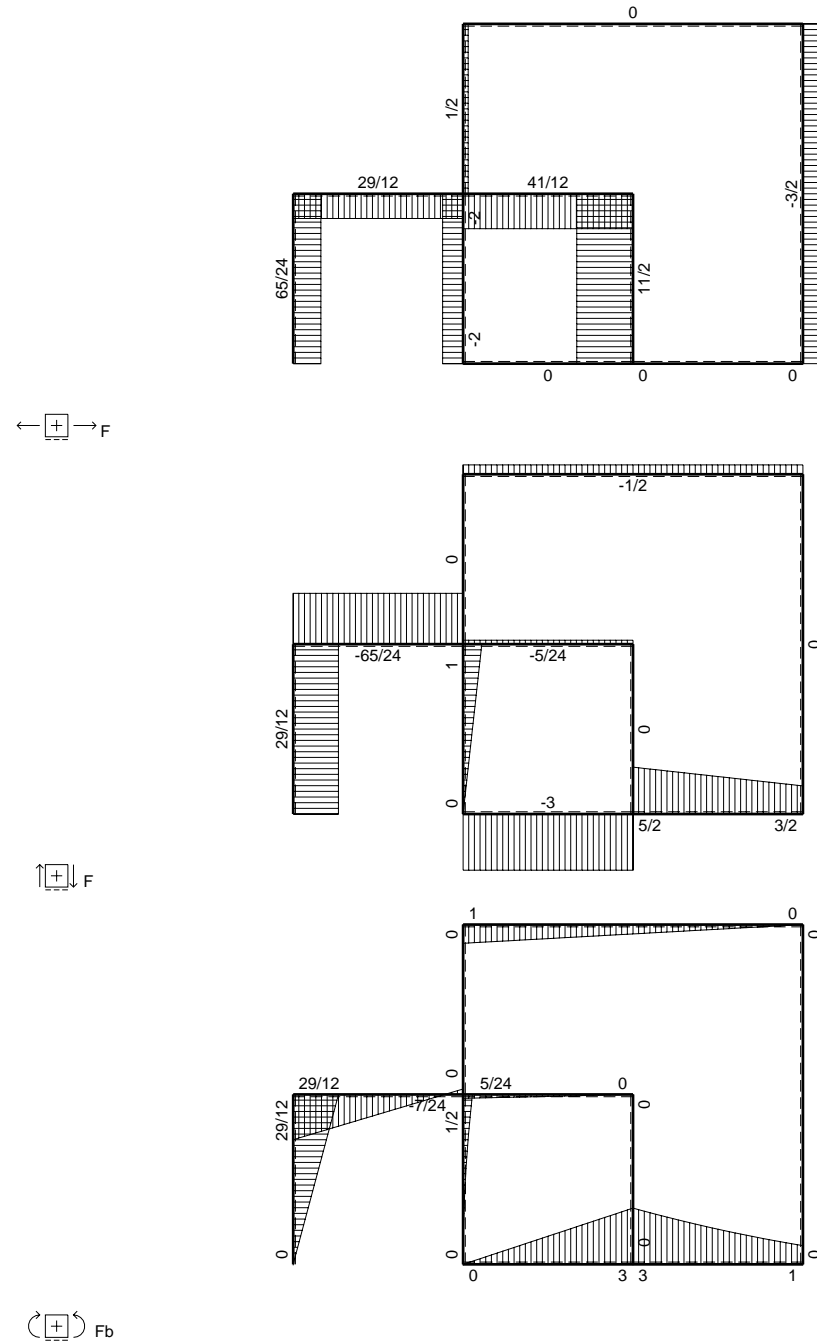
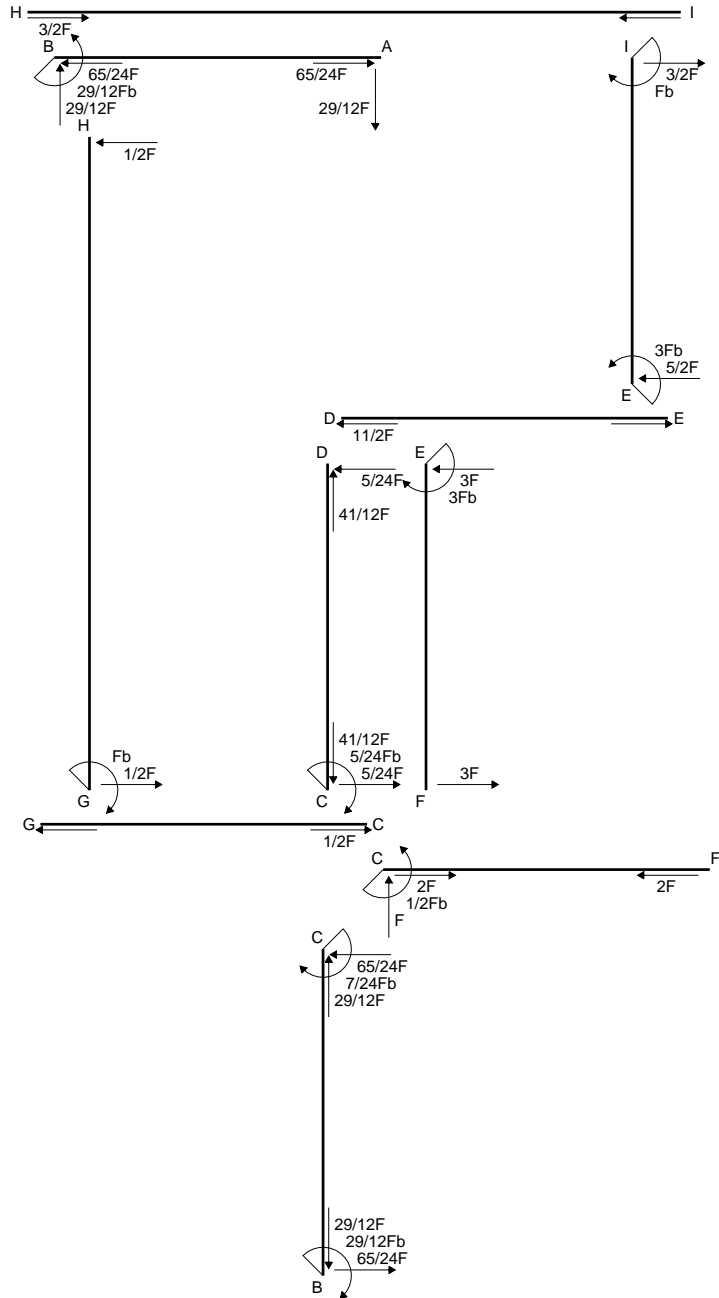
$$= [5/8 x - 5/8 x^2/b + 5/24 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

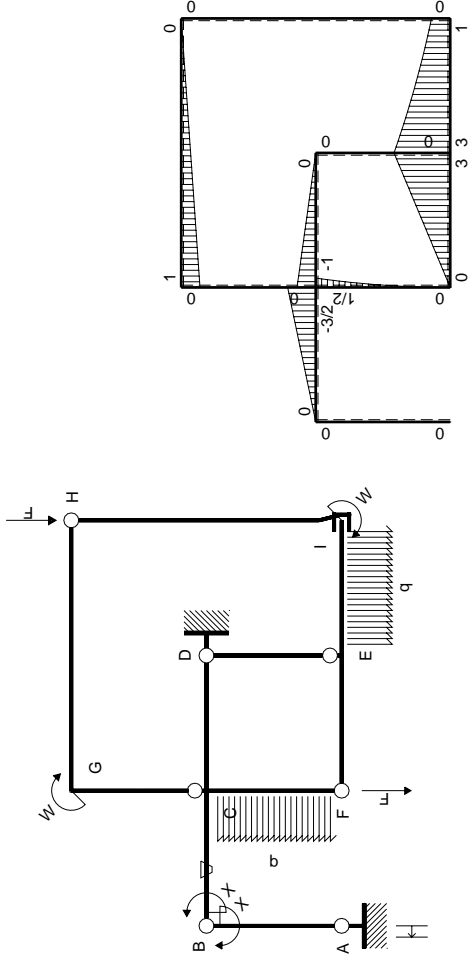
$$= (5/8 b - 5/8 b + 5/24 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 11/24 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (5/8 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [5/24 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (5/24 b) Fb 1/EJ + (-1/4 b) \theta = 11/24 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |  |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|--|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |  |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |  |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | $-Fb/EJ$ | $3/2Fx-3/4Fx^2/b$    | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/2+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |  |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | $Fb/EJ$  | $3/4Fb-3/4Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |  |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |  |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |  |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |  |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |  |
|       | totali                      |                      |          |                      |                     |                         | $29/12Fb^2/EJ$              | $Xb/EJ$                |  |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-29/12Fb$                  |                        |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb 1/EJ + (b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

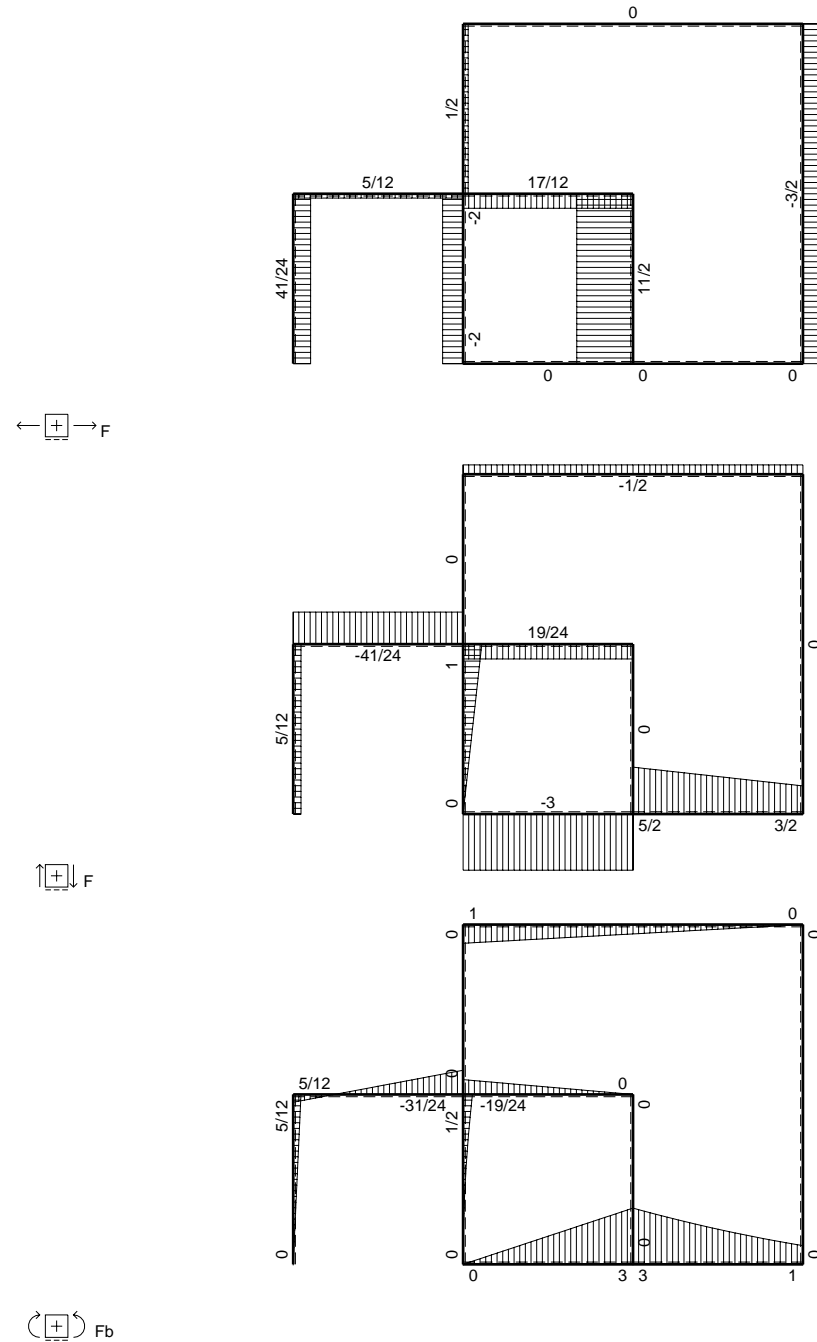
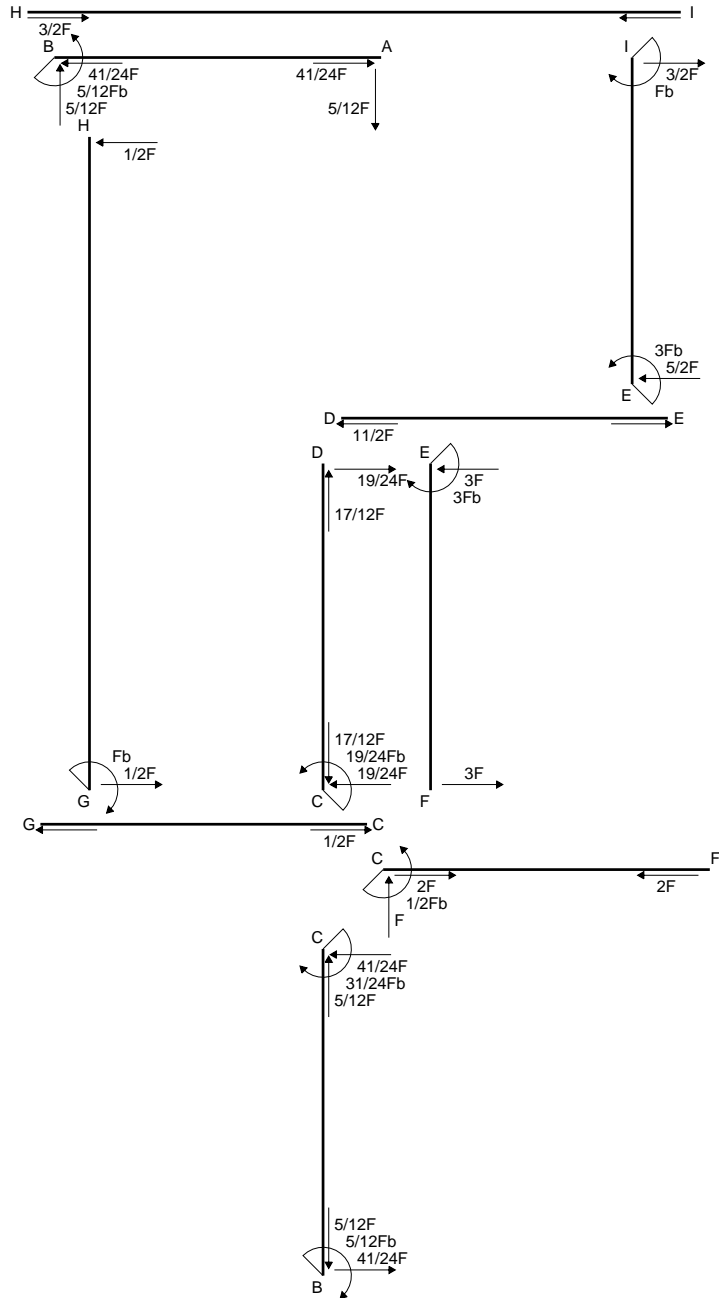
$$= (3/4 b - 1/4 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

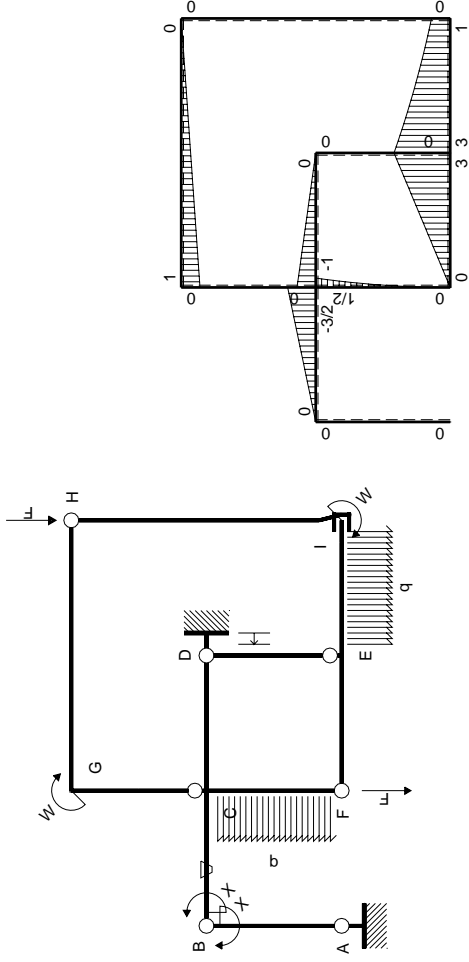
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | $-Fb/EJ$ | $3/2Fx-3/4Fx^2/b$    | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/2+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | $Fb/EJ$  | $3/4Fb-3/4Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $5/12Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $-5/12Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb 1/EJ + (b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

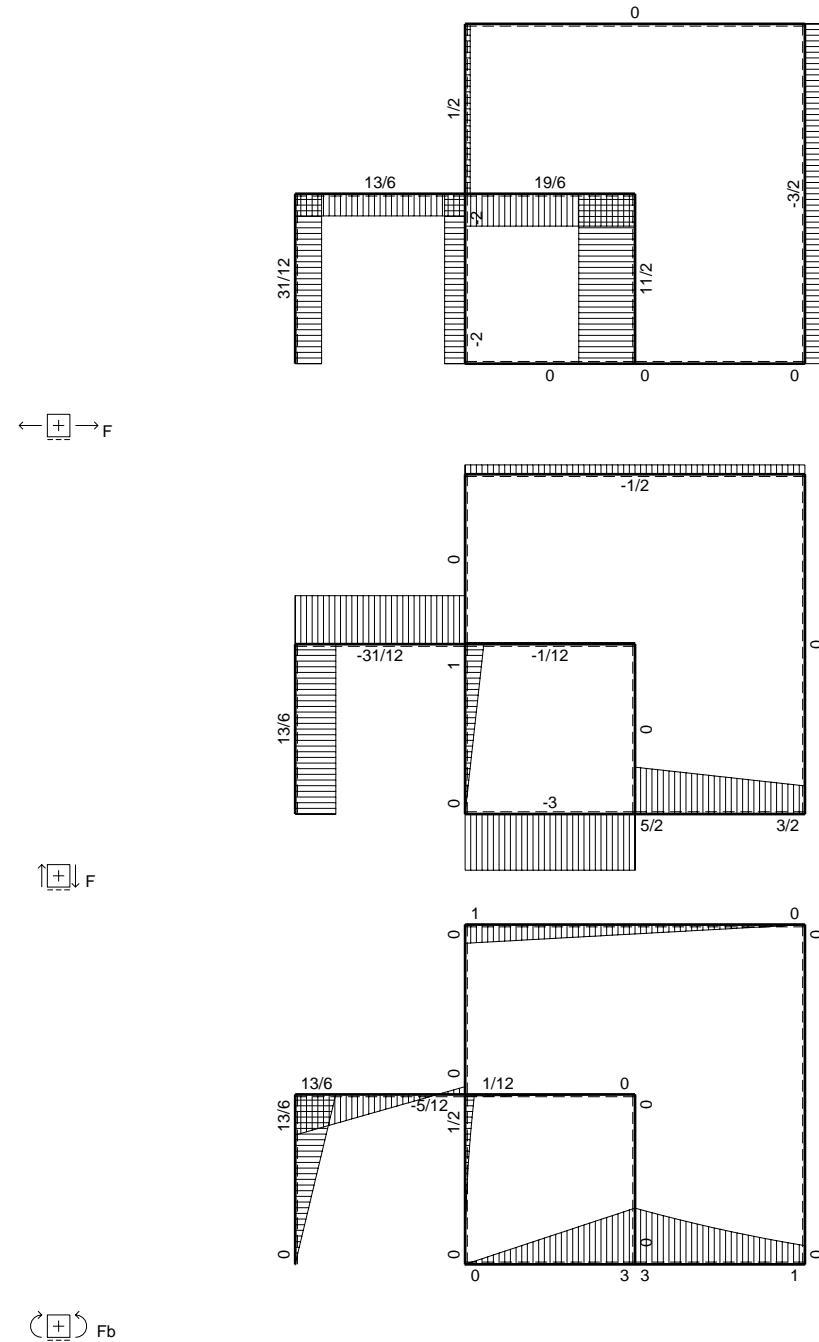
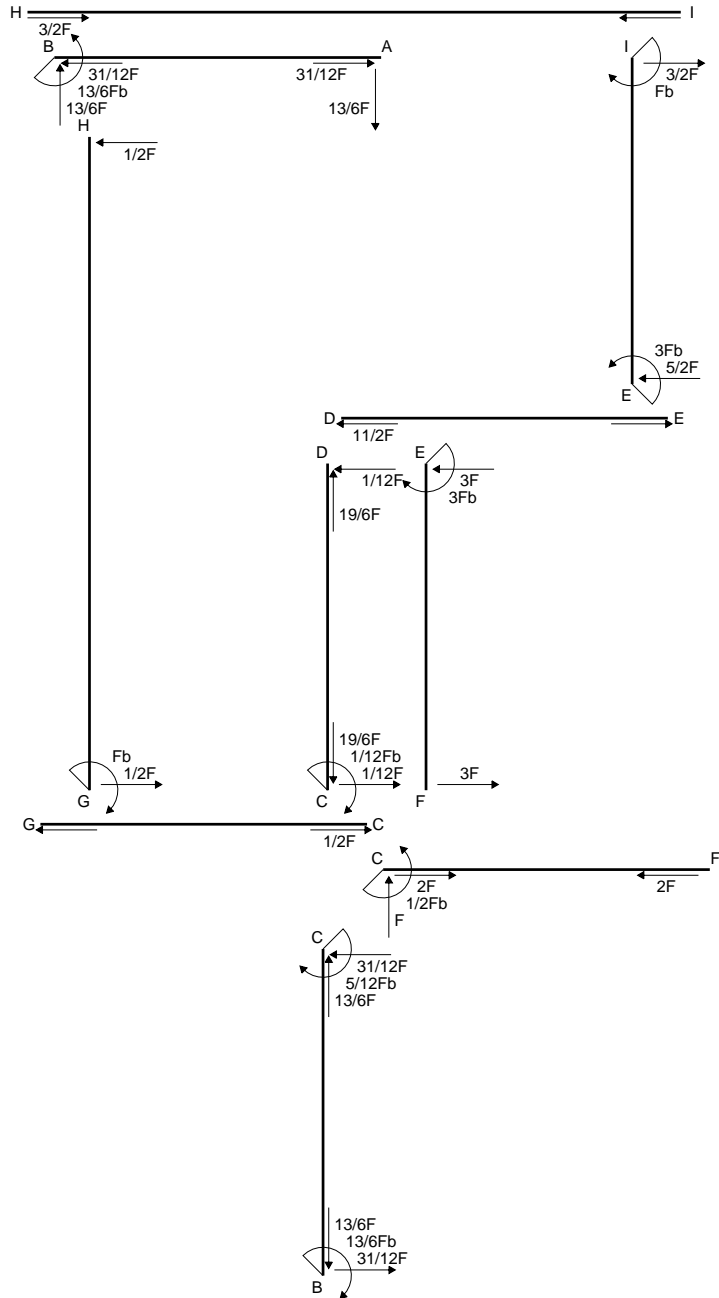
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

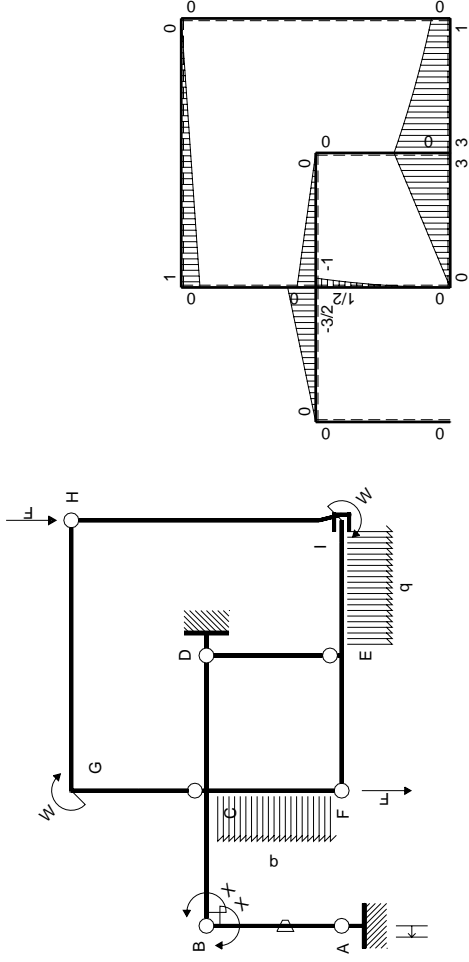
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0             | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |               |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |               |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

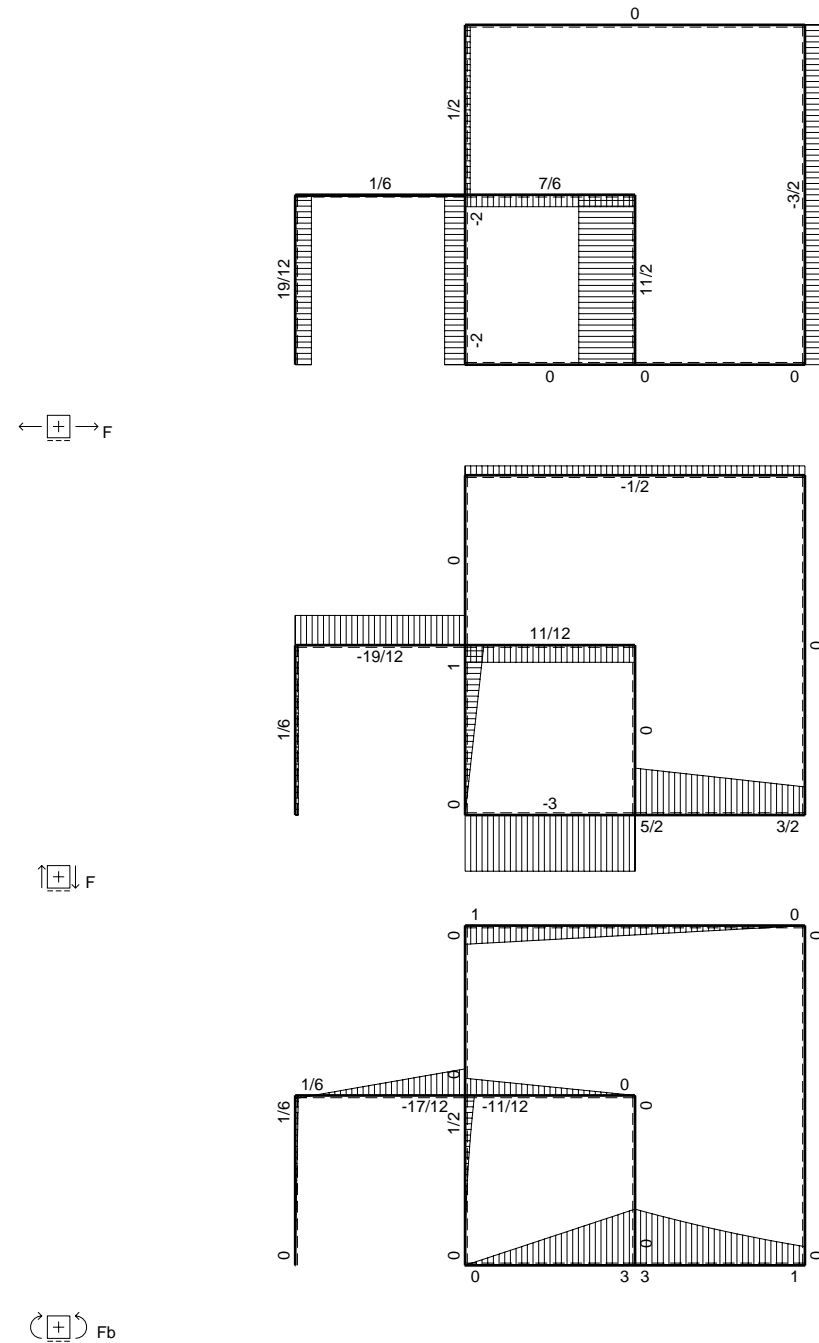
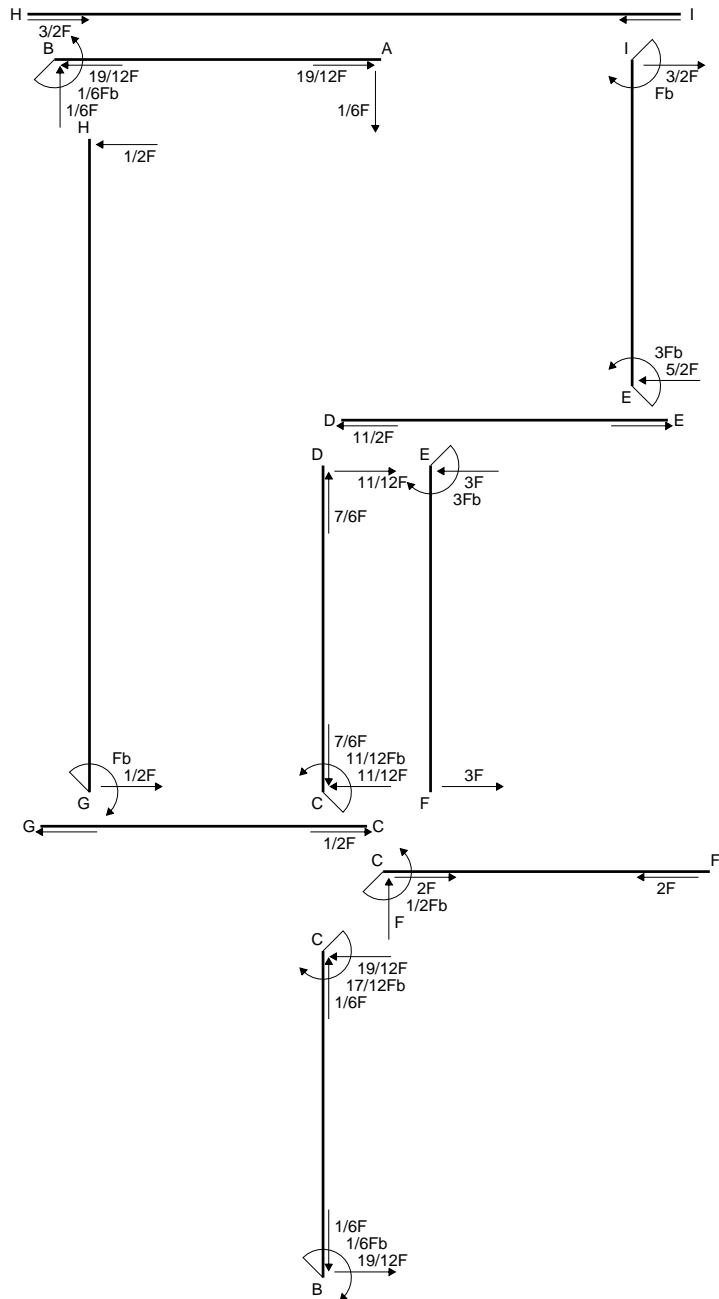
$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

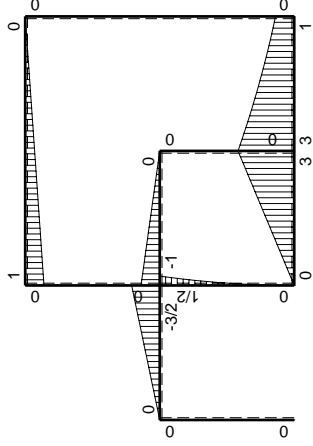
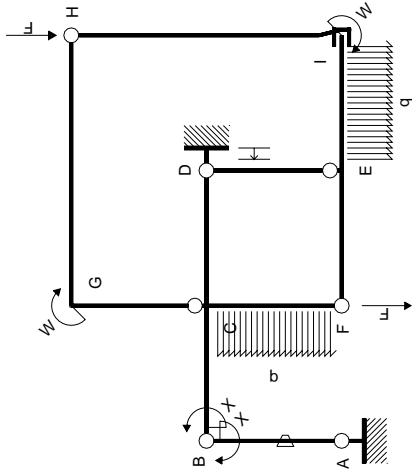
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

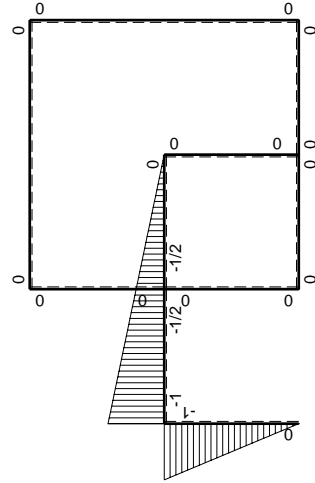
$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica X=1

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0             | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                      |          |                      |               |                         | $1/6Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |               |                         | $-1/6Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

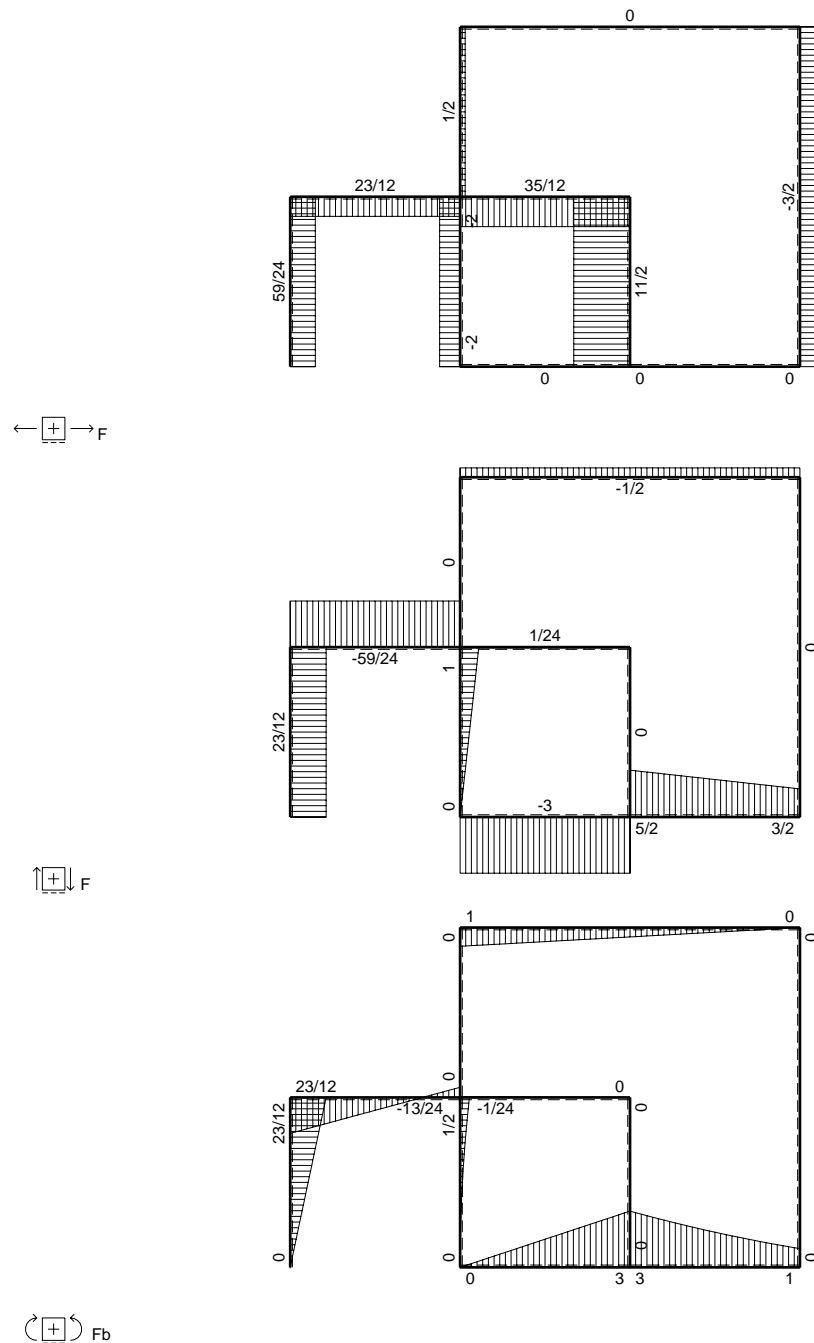
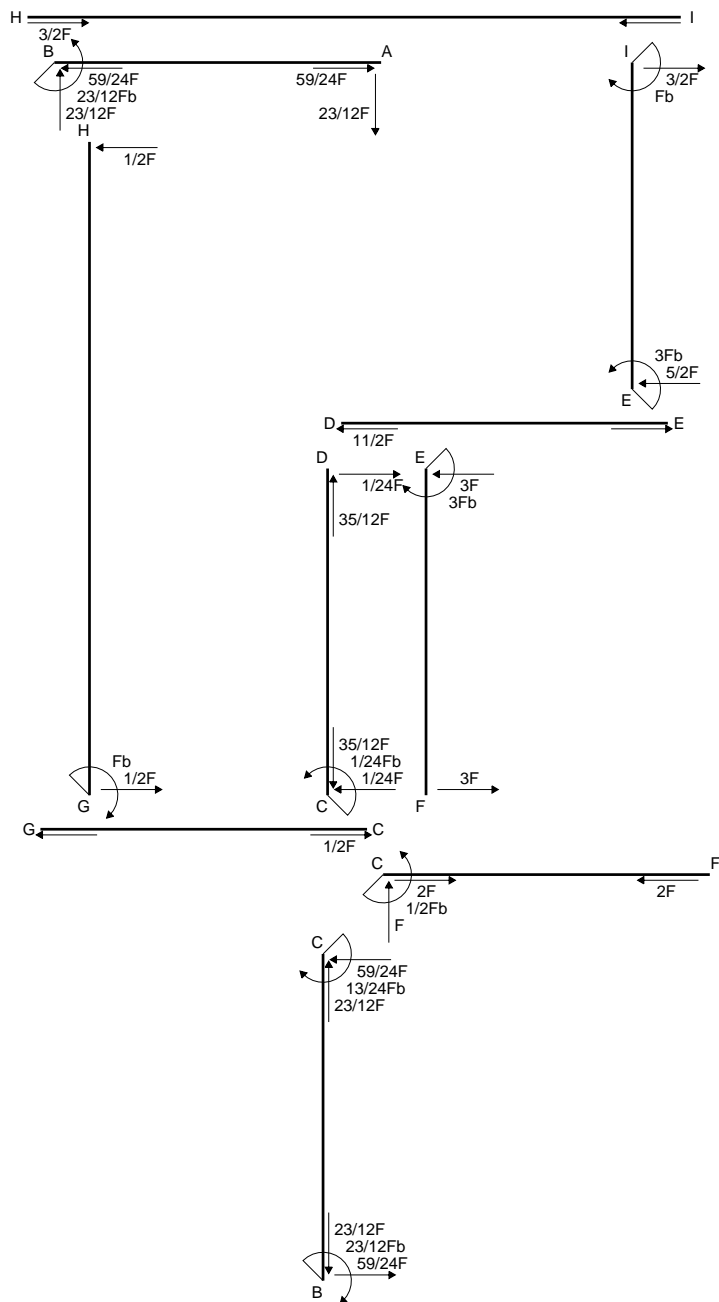
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

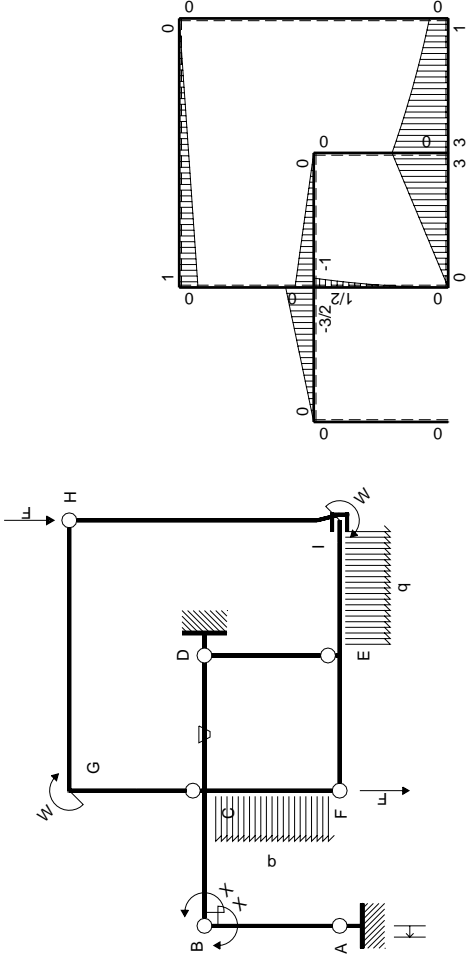
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |                     |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

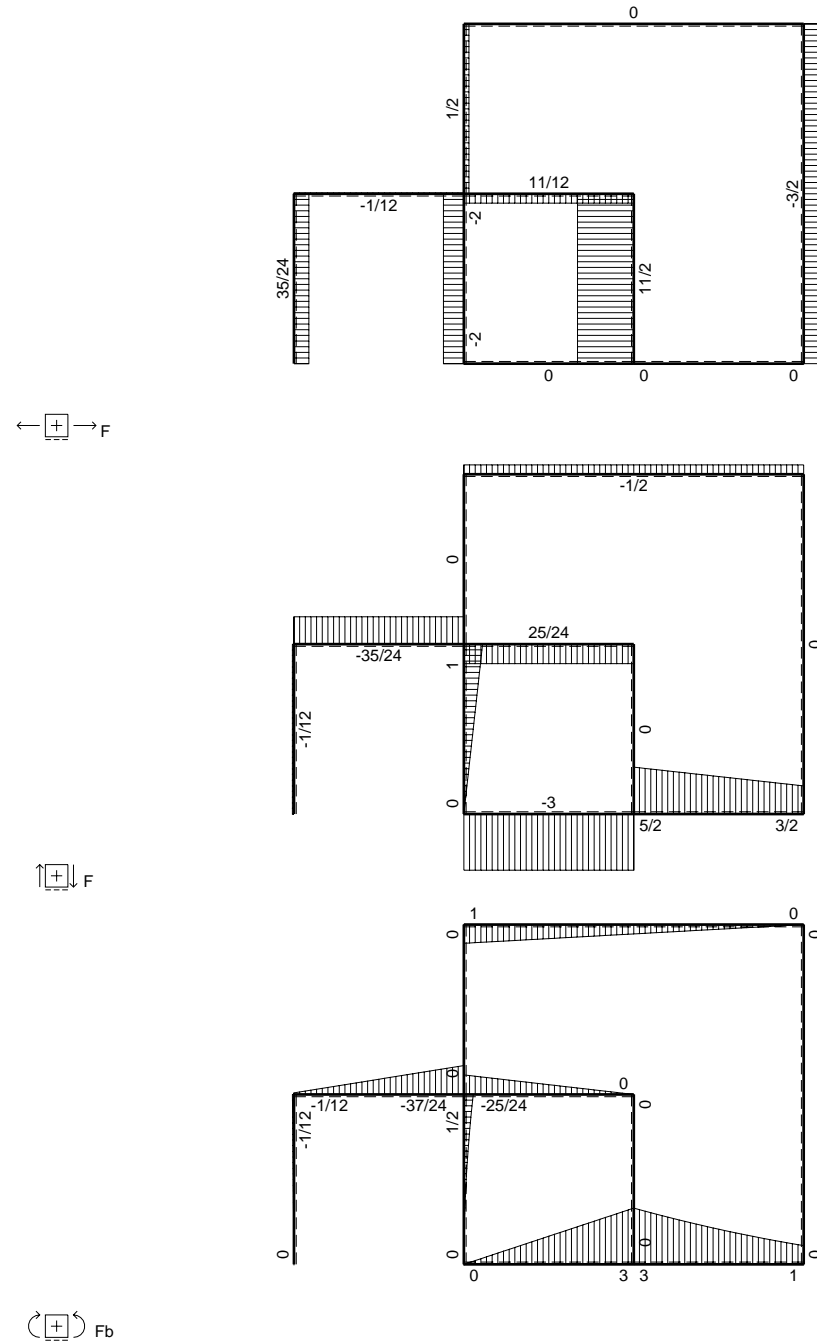
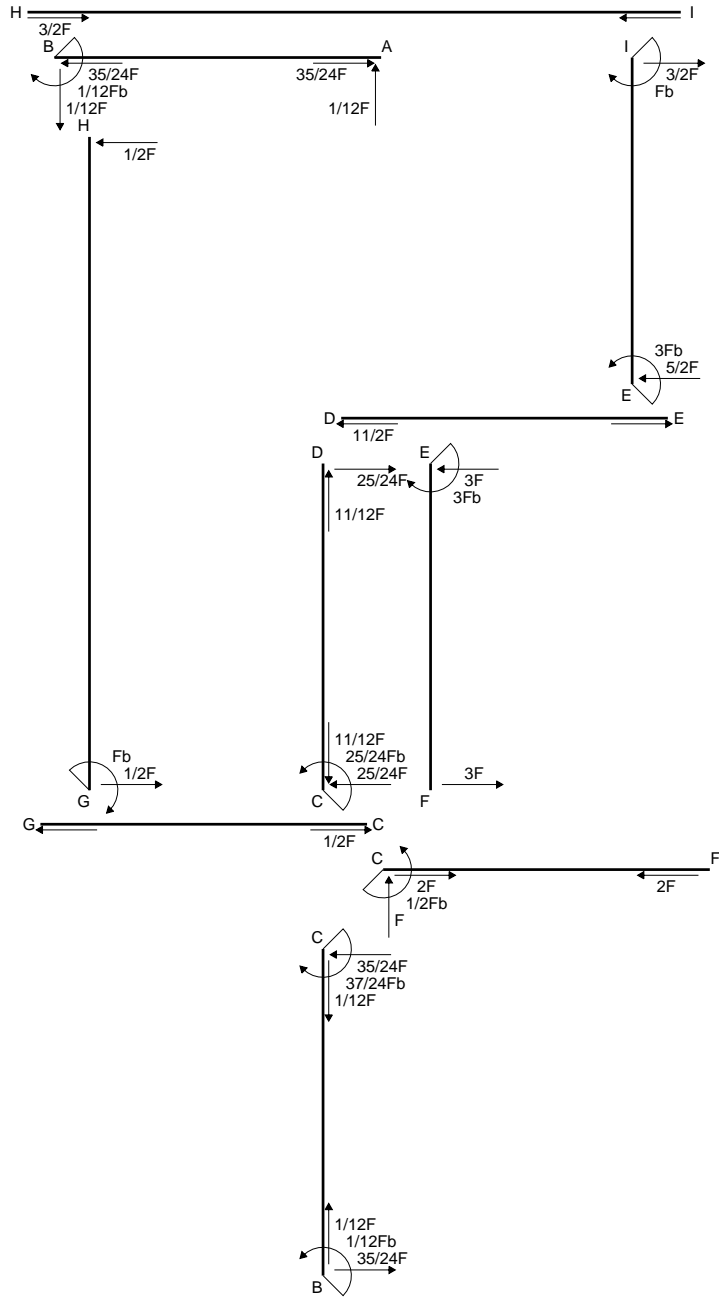
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

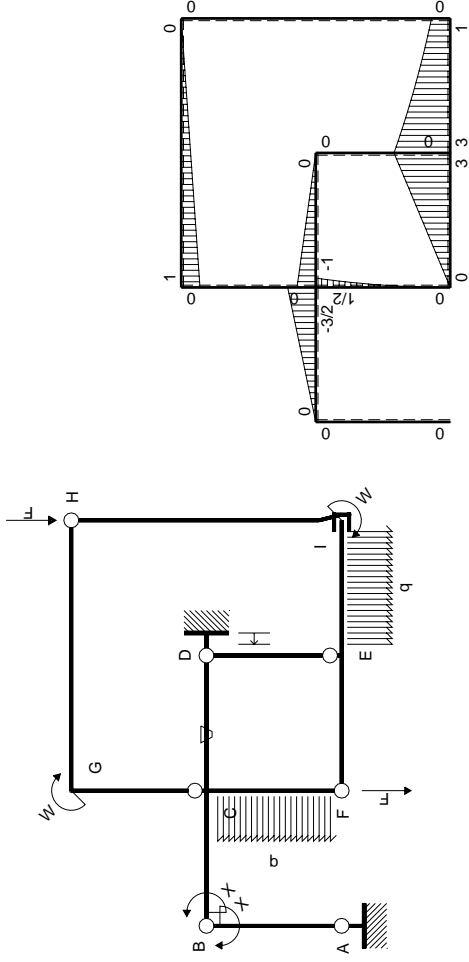
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2qx^2$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+Fx-1/2qx^2$  | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

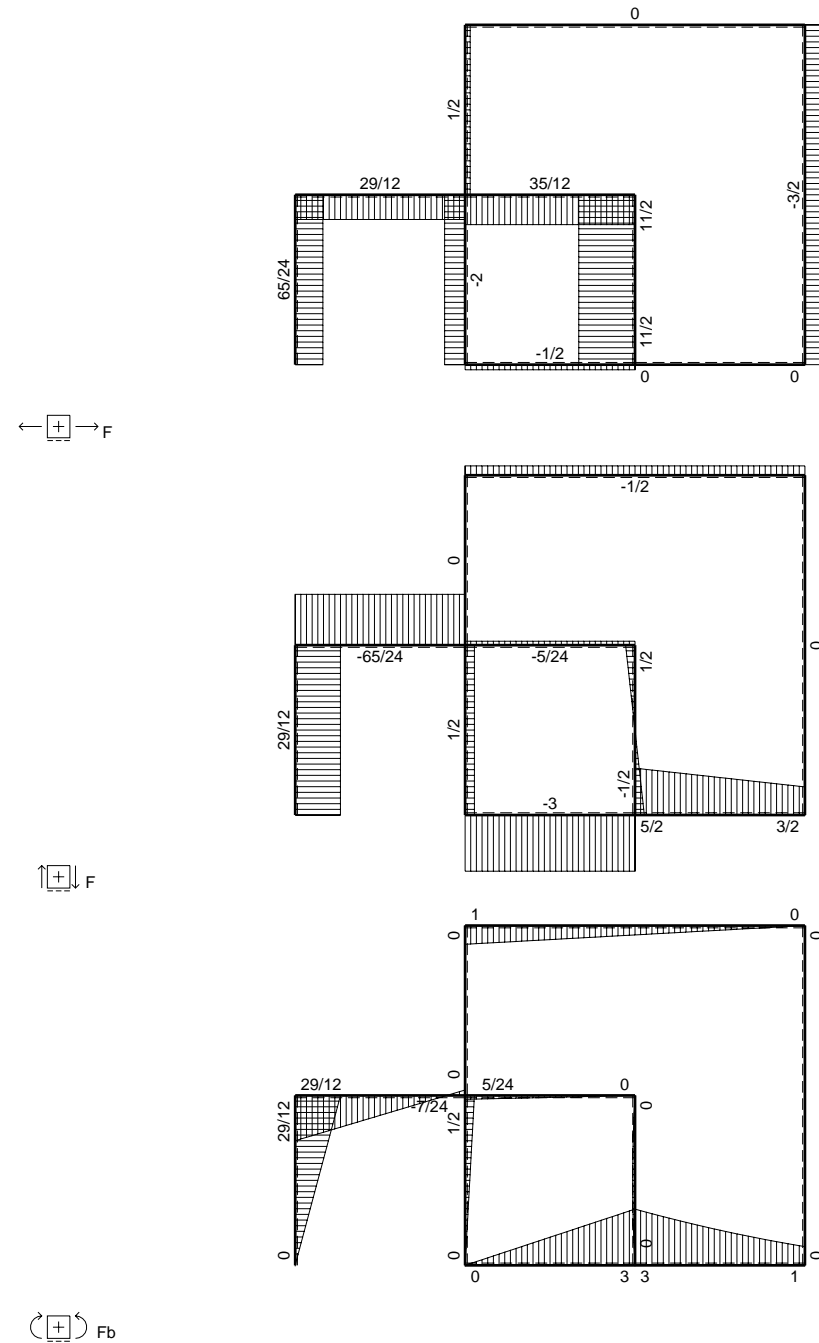
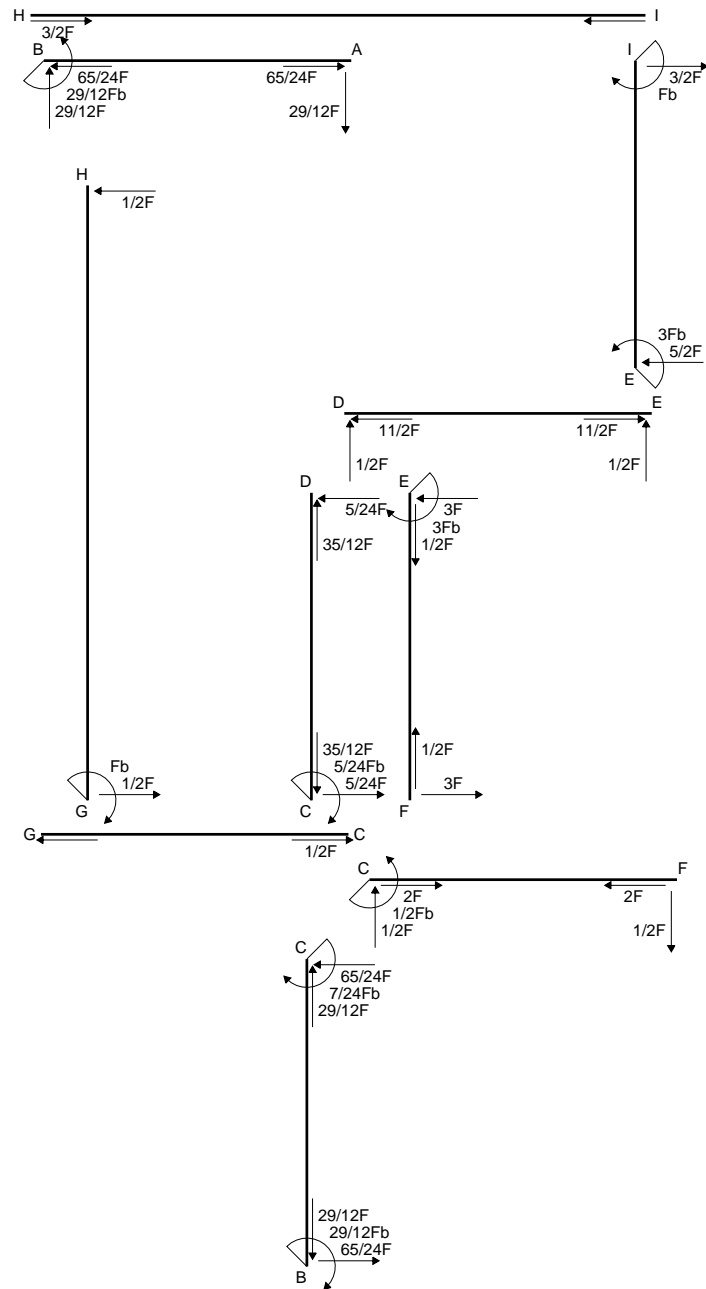
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

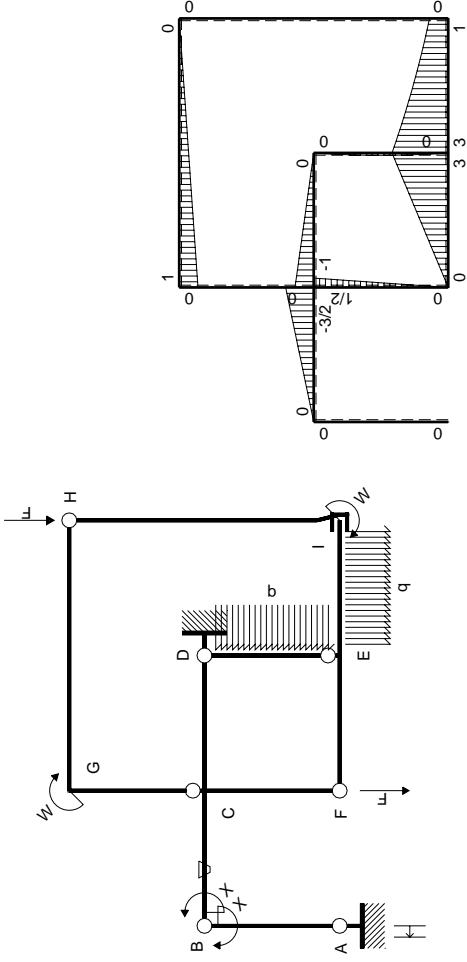
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|-----------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$            |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                       |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | $-Fb/EJ$ | $3/2Fx-3/4Fx^2/b$    | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/2+3/4)Fb^2/EJ$          | $7/12Xb/EJ$           |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | $Fb/EJ$  | $3/4Fb-3/4Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$           |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                       |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                       |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                       |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0                   | 0                       |                             |                       |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                       |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                       |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                       |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                     |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                       |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                       |
|       | totali                      |                      |          |                      |                     |                         | $29/12Fb^2/EJ$              | $Xb/EJ$               |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-29/12Fb$                  |                       |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb 1/EJ + (b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

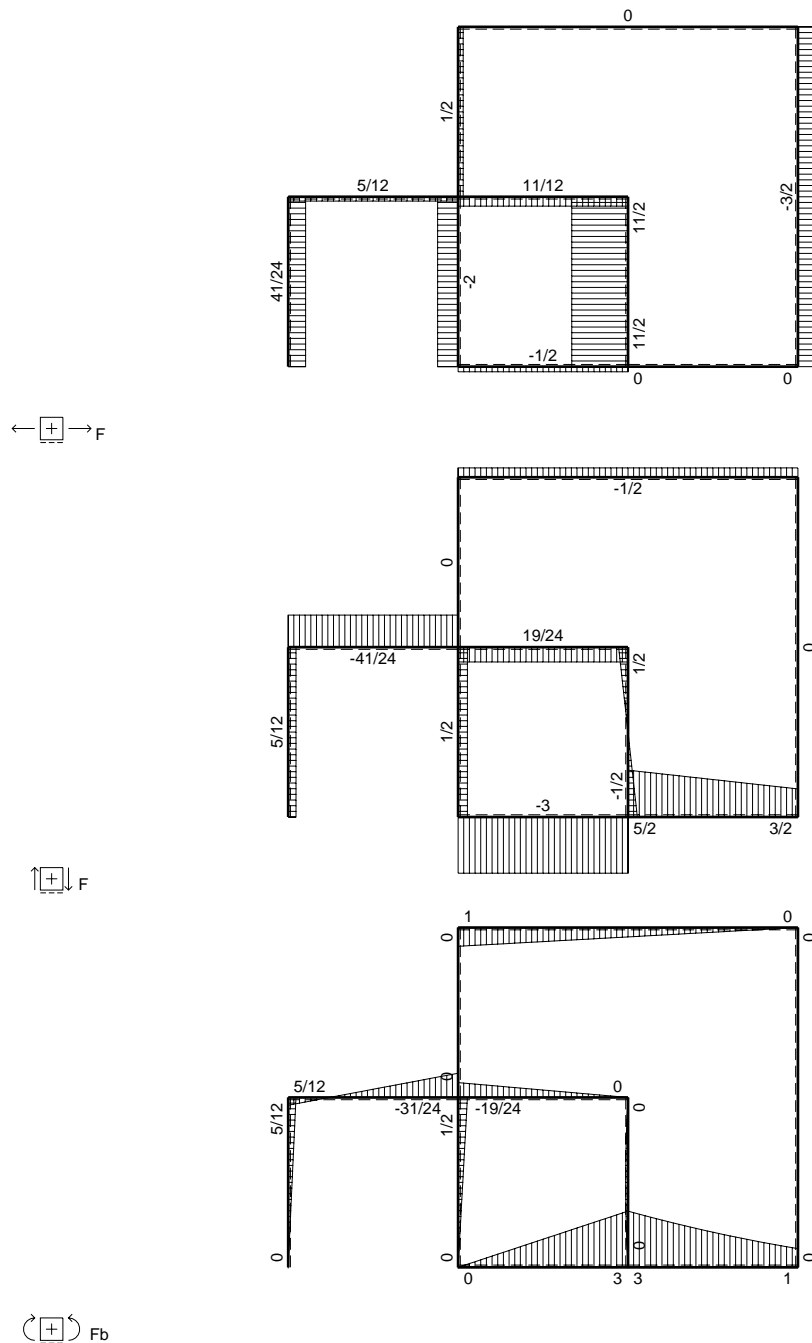
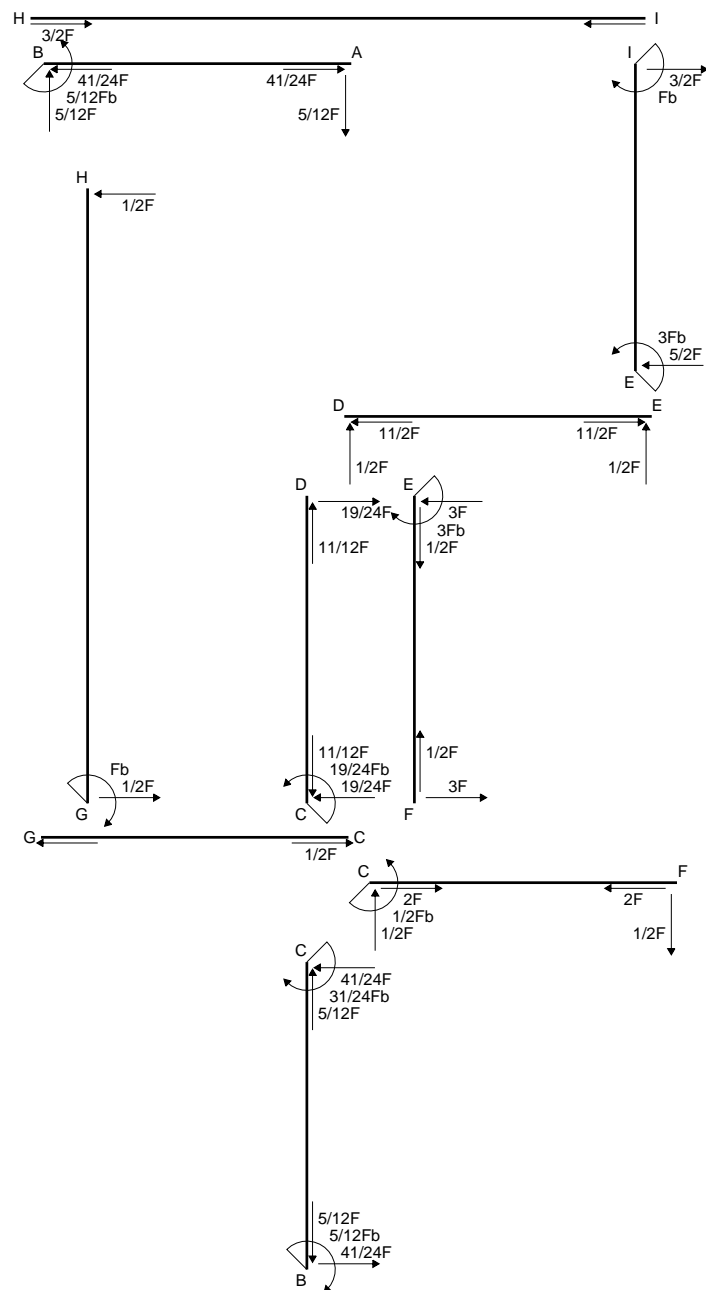
$$= (3/4 b - 1/4 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

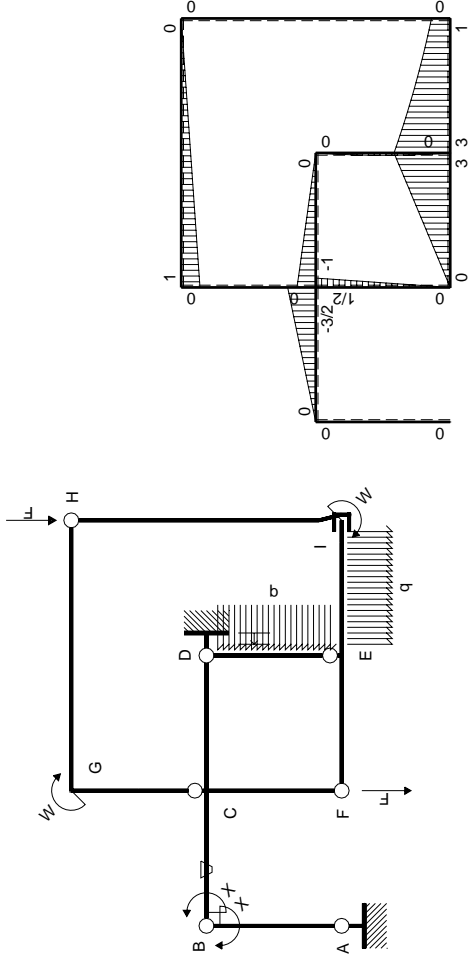
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$



$\left[ \begin{matrix} + \\ \curvearrowright \end{matrix} \right] F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | $-Fb/EJ$ | $3/2Fx-3/4Fx^2/b$    | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/2+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | $Fb/EJ$  | $3/4Fb-3/4Fx^2/b$    | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $5/12Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $-5/12Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (1 - 1/2 x/b) \theta dx$$

$$= [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ + [x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb 1/EJ + (b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 - 1/2 x/b) \theta dx$$

$$= [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ + [-1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (3/4 b - 1/4 b) Fb 1/EJ + (-1/2 b - 1/4 b) \theta = 5/4 Fb^2/EJ$$

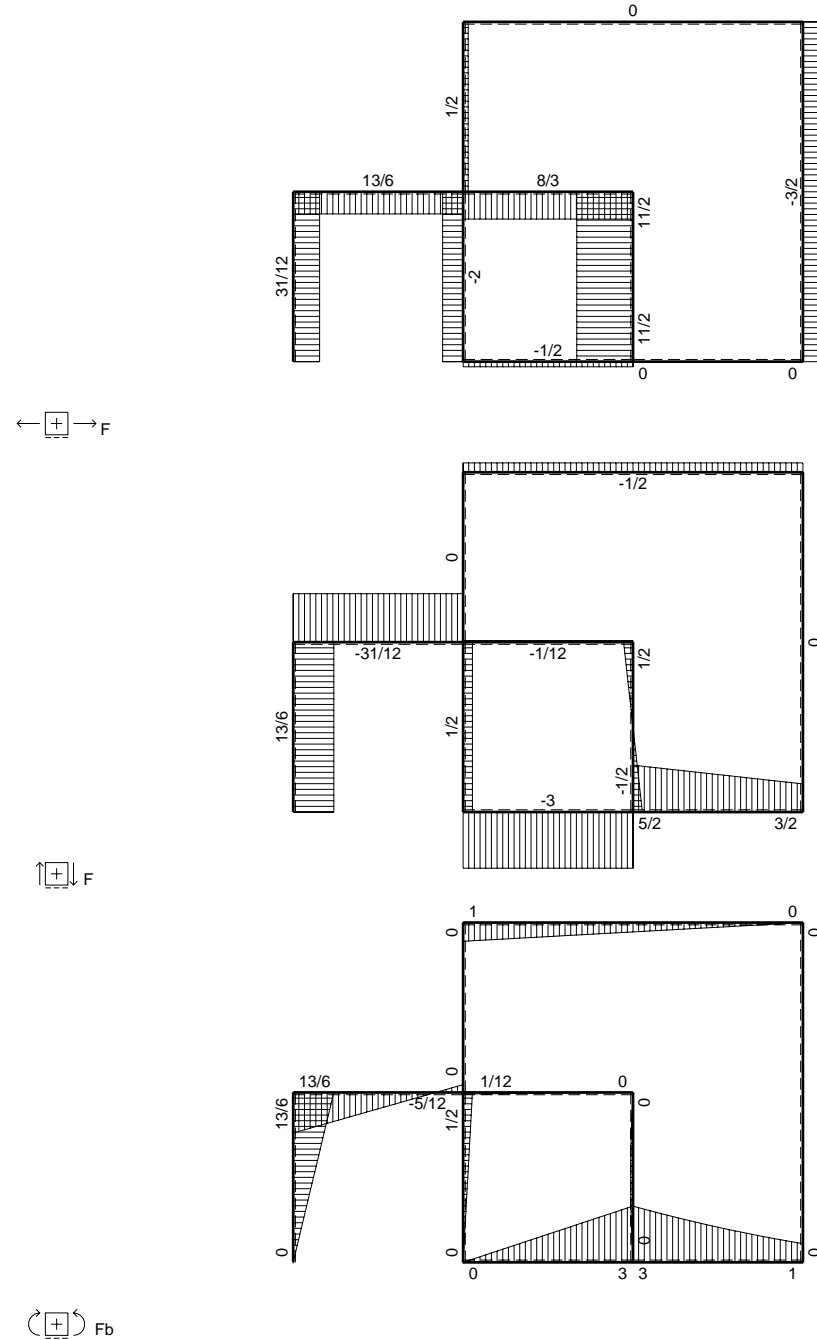
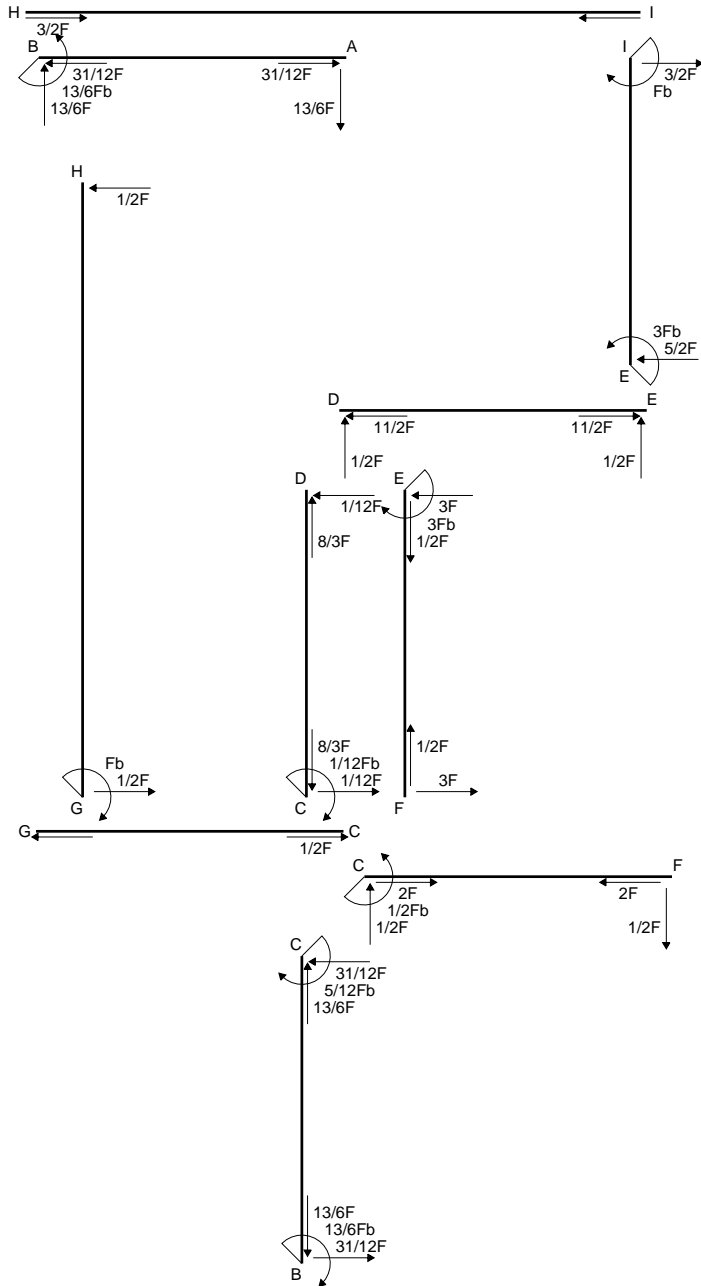
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

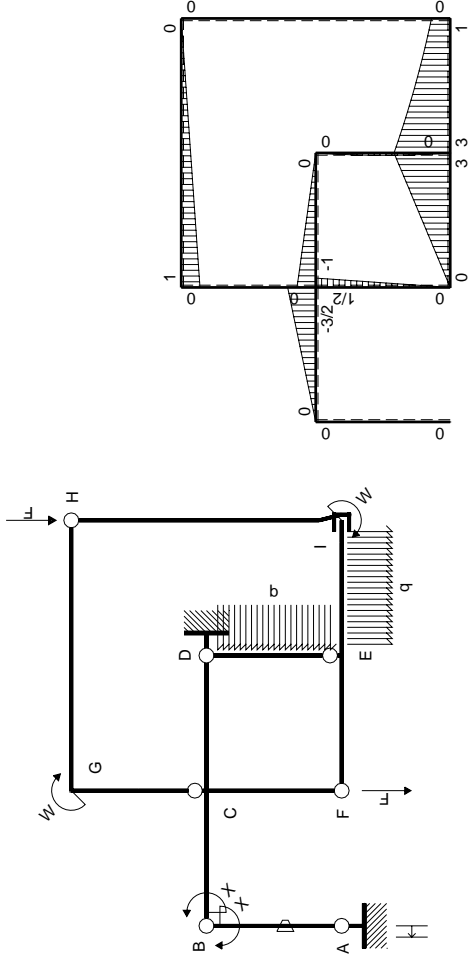
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0             | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |               |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |               |                         | $13/6Fb^2/EJ$               | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |               |                         | $-13/6Fb$                   |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

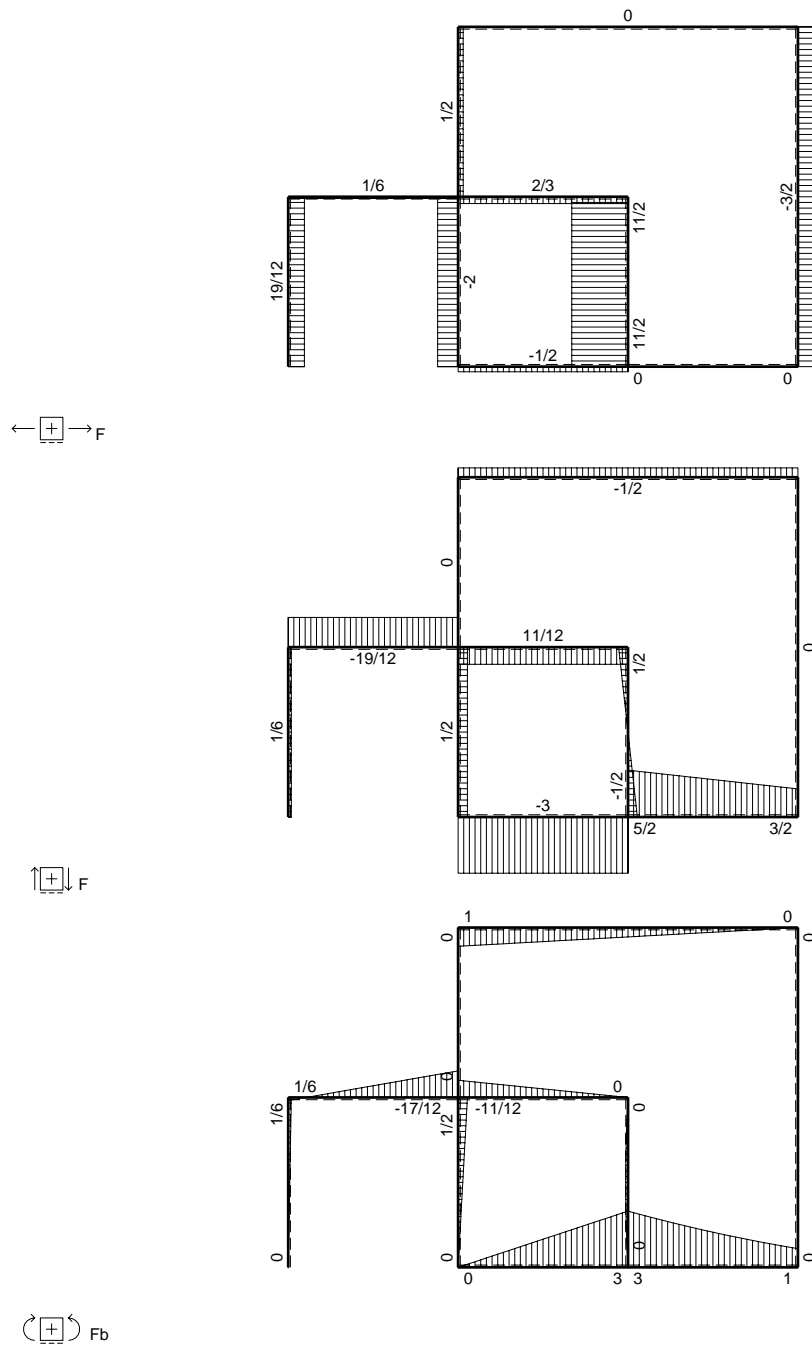
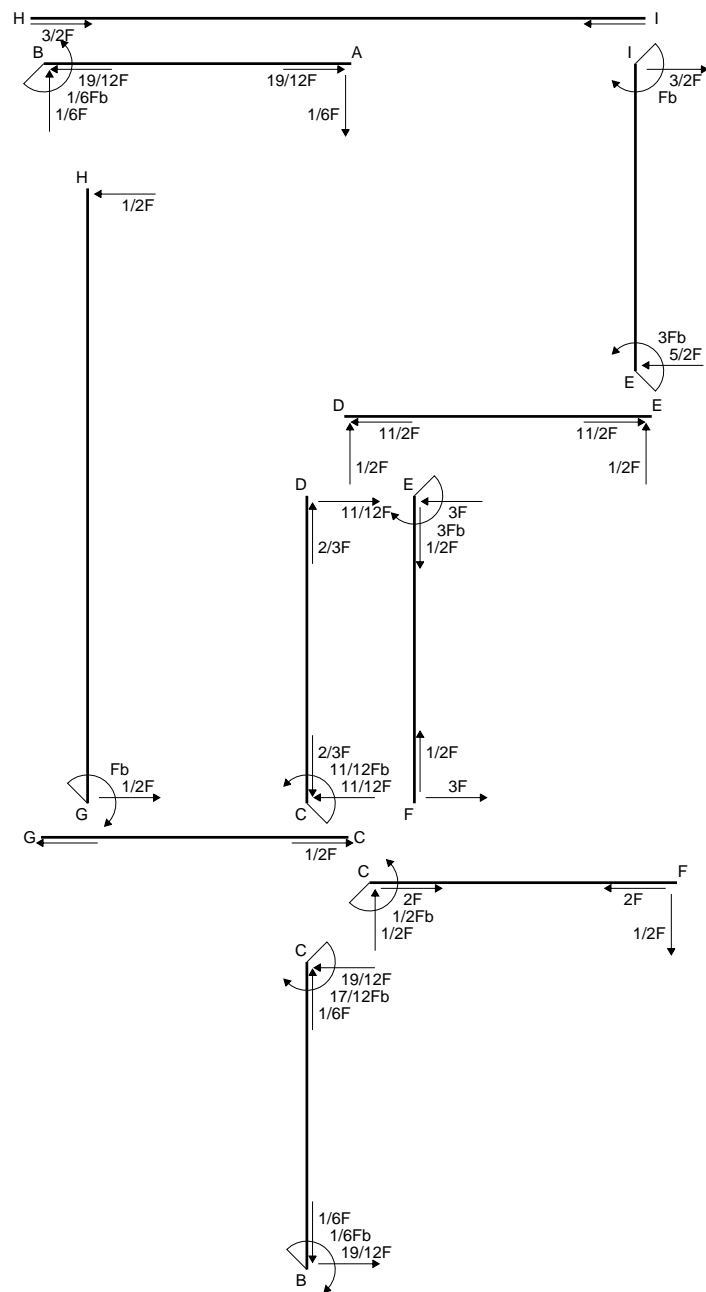
$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

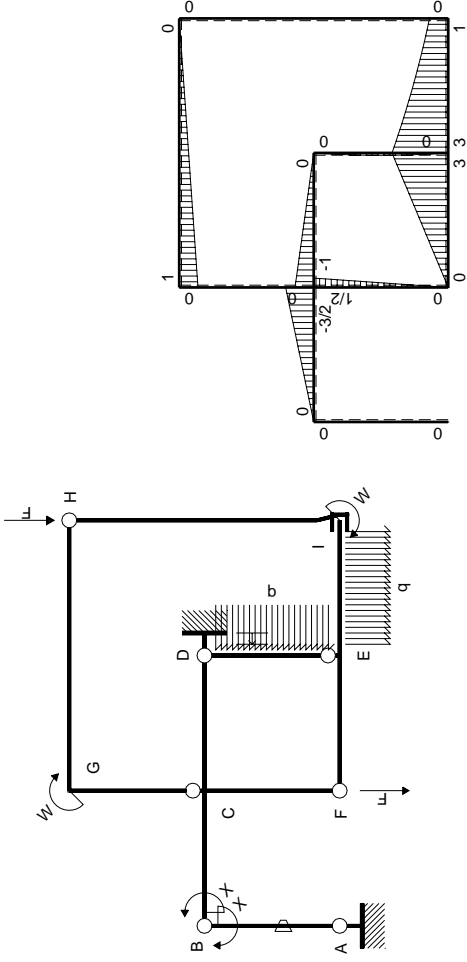
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$



$\left[ \begin{matrix} + \\ - \end{matrix} \right] F_b$



Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | $-Fb/EJ$ | 0                    | $Fx/EJ$       | $x^2/b^2$               | $(0+1/2)Fb^2/EJ$            | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | $Fb/EJ$  | 0                    | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0             | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | 0        | $1/2Fb-Fx+1/2Fx^2/b$ | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | 0        | $1/2Fx^2/b$          | 0             | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0             | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0             | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0             | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0             | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0             | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0             | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0             | 0                       |                             |                        |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |               |                         | $-Fb^2/EJ$                  |                        |
|       | totali                      |                      |          |                      |               |                         | $1/6Fb^2/EJ$                | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |               |                         | $-1/6Fb$                    |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (x/b) \theta dx = [1/2 x^2/b]_0^b \theta$$

$$= (1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (-1 + x/b) \theta dx = [-x + 1/2 x^2/b]_0^b \theta$$

$$= (-b + 1/2 b) \theta = 1/2 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

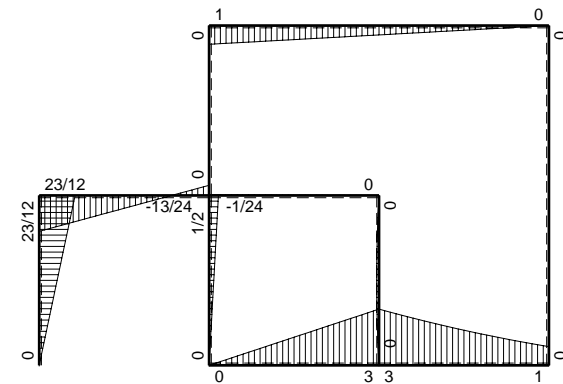
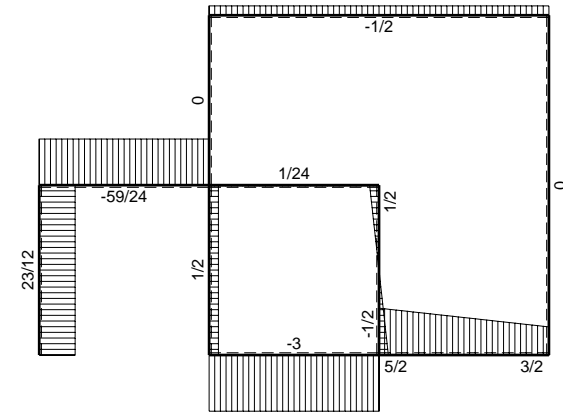
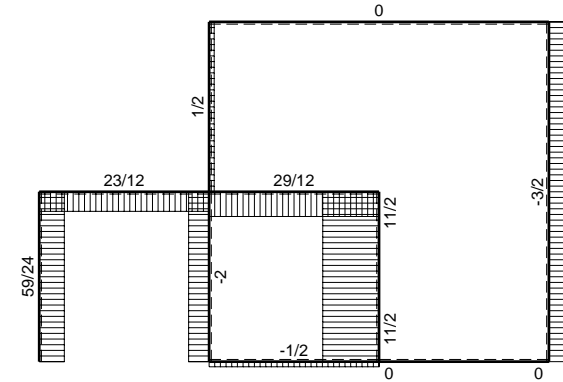
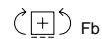
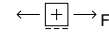
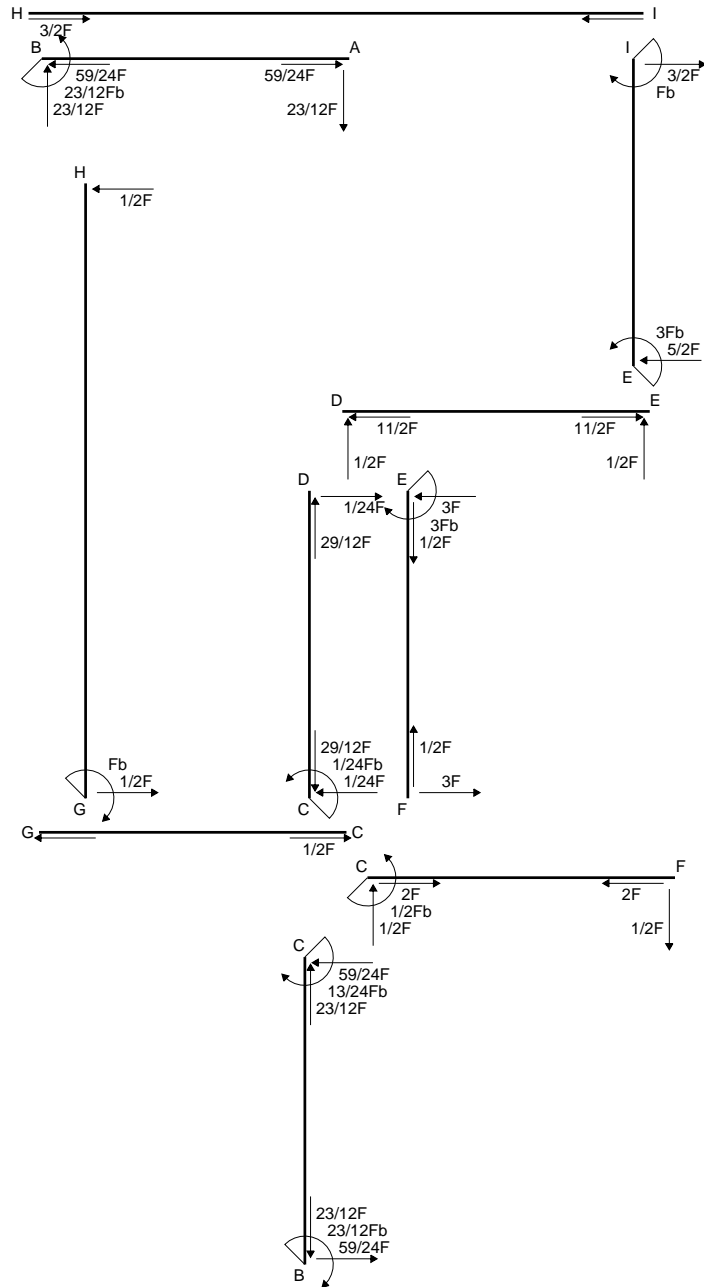
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

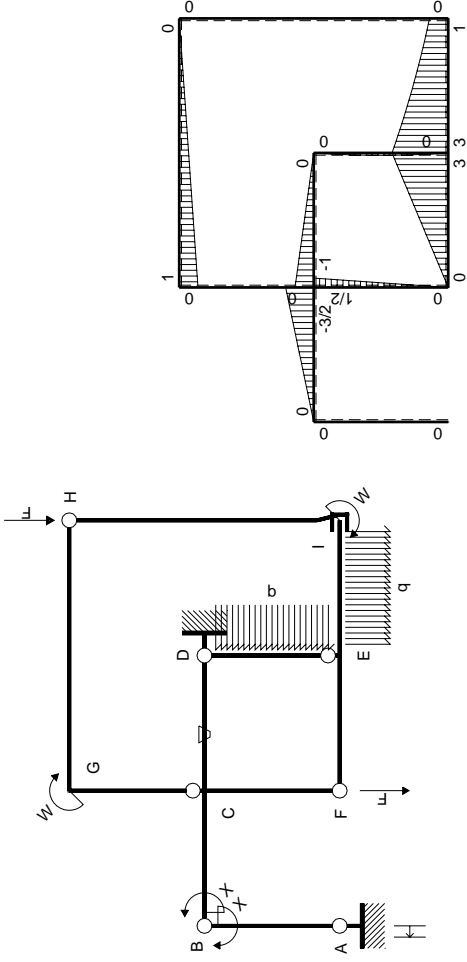
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$







Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                      |          |                      |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                      |          |                      |                     |                         | $23/12Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         | $-23/12Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

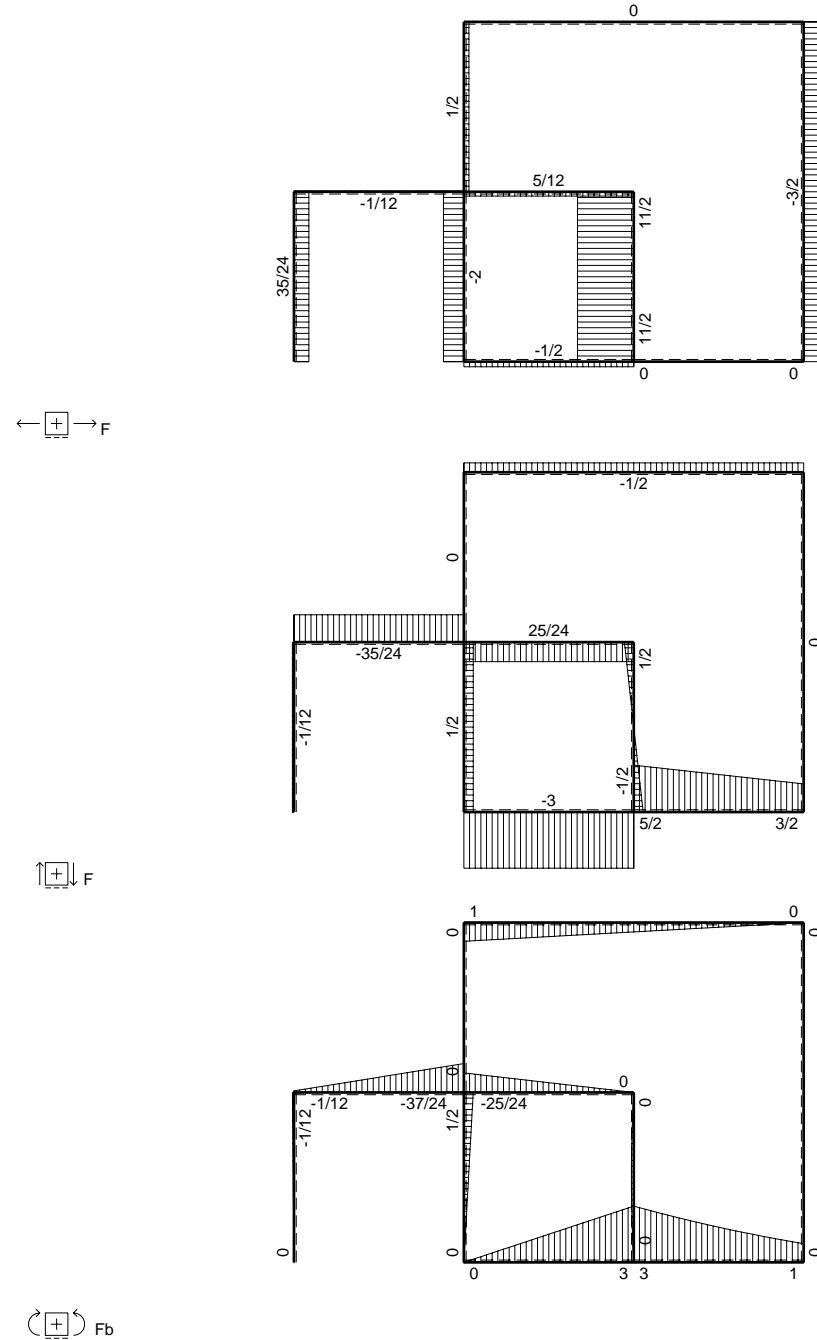
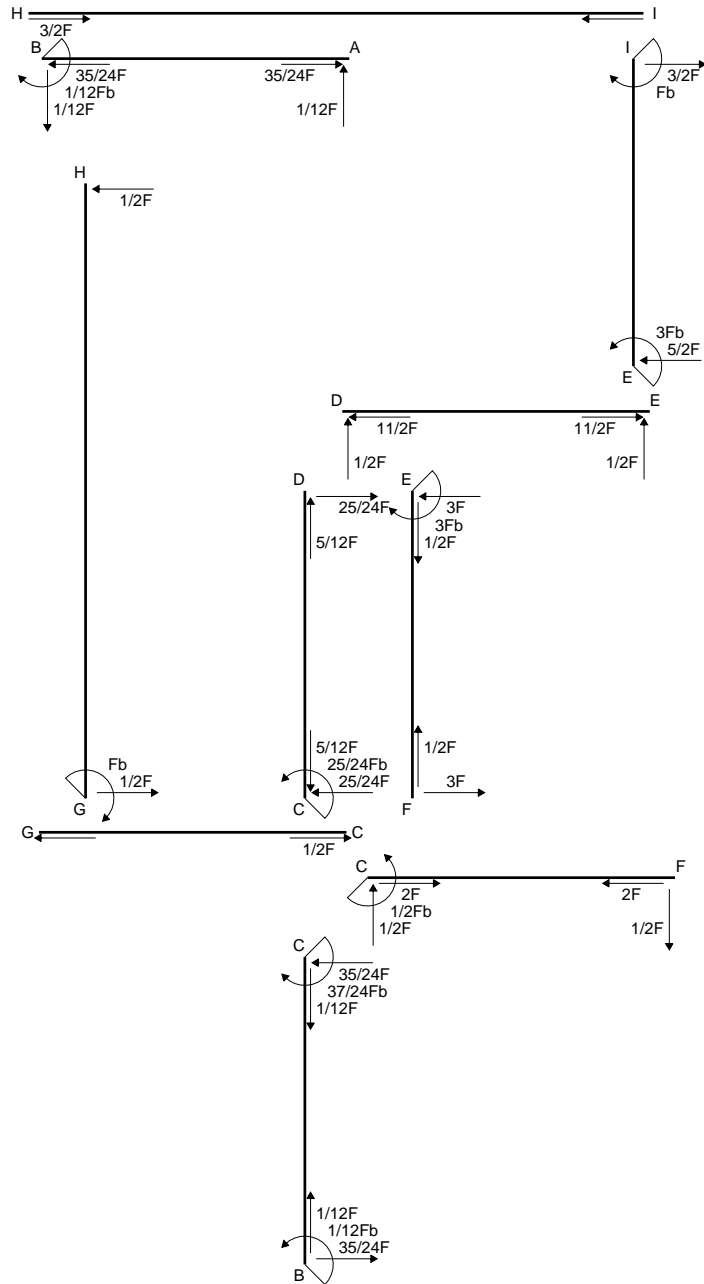
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

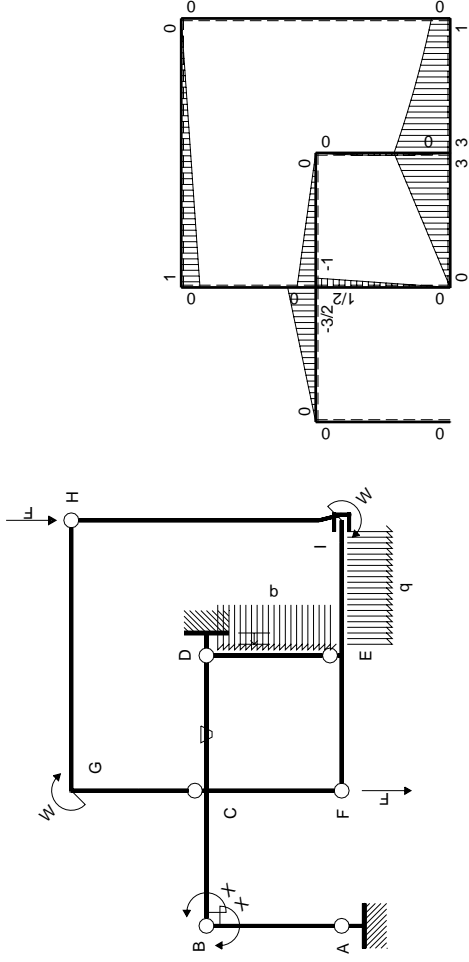
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$





Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$             | $\theta$ | $M_x M_o$            | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|----------------------|----------|----------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | 0                    | 0        | 0                    | 0                   | $x^2/b^2$               | 0+0                         | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | 0                    | 0        | 0                    | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-3/2Fx$             | 0        | $3/2Fx-3/4Fx^2/b$    | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/2+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $3/2Fb-3/2Fx$        | 0        | $3/4Fb-3/4Fx^2/b$    | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$             | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$ | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$                 | $Fb/EJ$  | $1/2Fx^2/b$          | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | $1/2Fx-1/2qx^2$      | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | $-1/2Fx+1/2qx^2$     | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $3Fb-3Fx$            | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-3Fx$               | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | $1/2Fx$              | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | $-1/2Fb+1/2Fx$       | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb-1/2Fx$           | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-1/2Fx$             | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                    | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+3/2Fx+1/2qx^2$   | 0        | 0                    | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-3Fb+5/2Fx-1/2qx^2$ | 0        | 0                    | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                      |          |                      |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                      |          |                      |                     |                         |                             | $-1/12Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                      |          |                      |                     |                         |                             | $1/12Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{BC}^{xo} = \int_0^b (3/2 x/b - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x^2/b - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (3/4 - 3/4 x^2/b^2) Fb 1/EJ dx = [3/4 x - 1/4 x^3/b^2]_0^b Fb 1/EJ$$

$$= (3/4 b - 1/4 b) Fb 1/EJ = 1/2 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

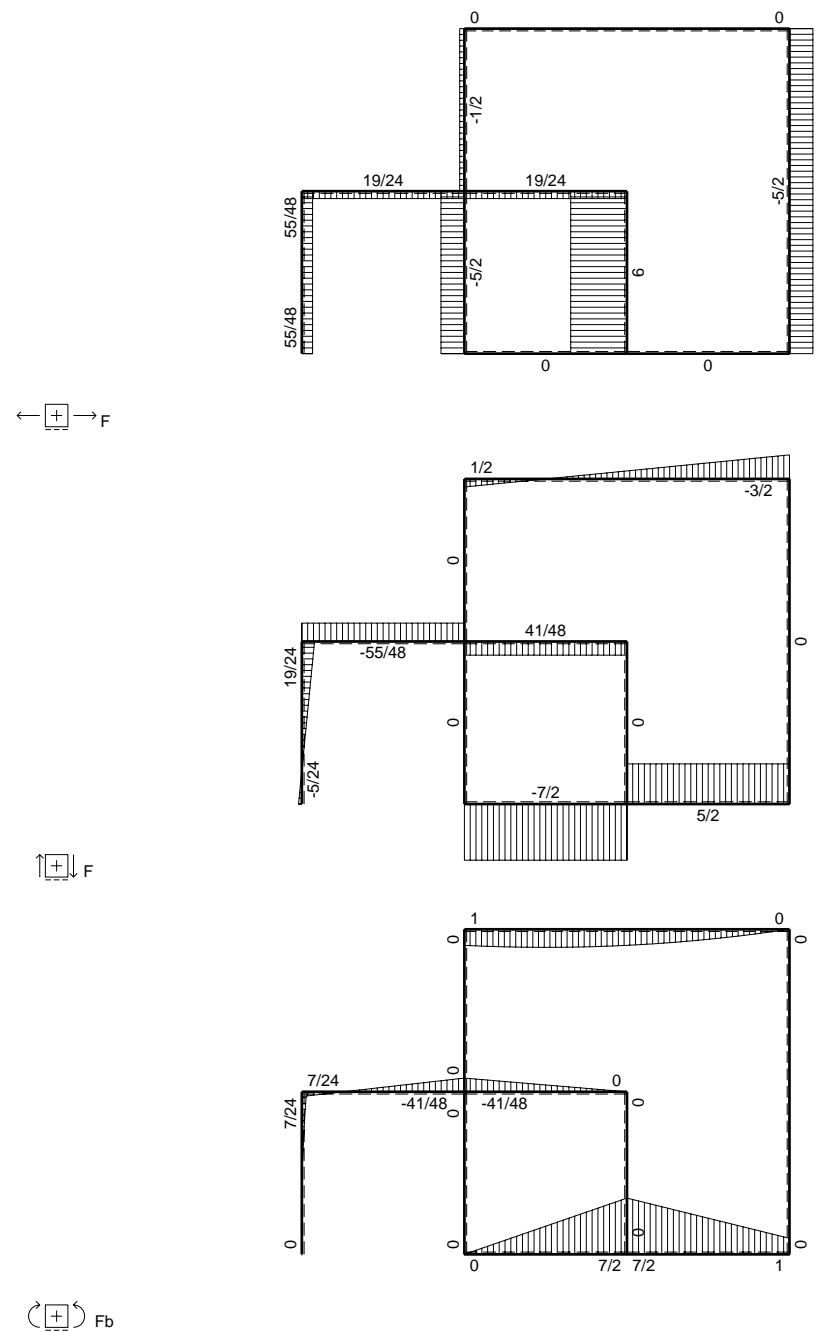
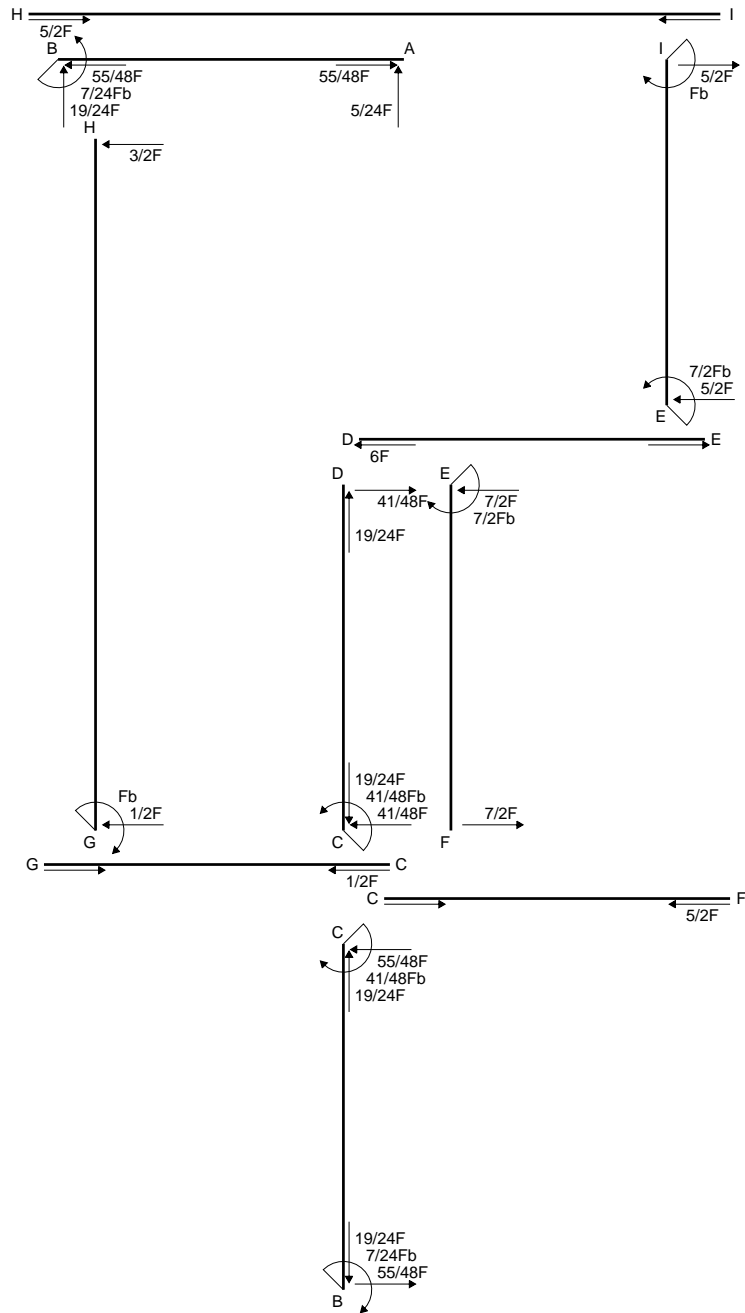
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

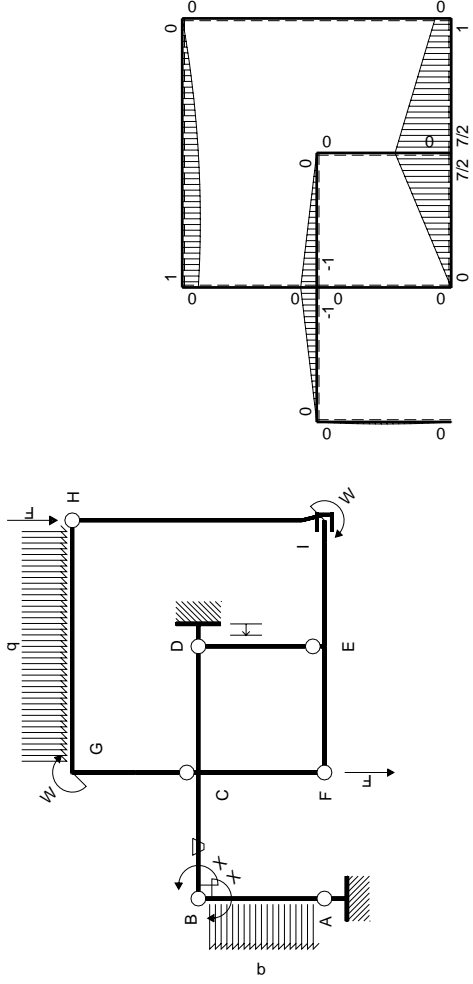
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$

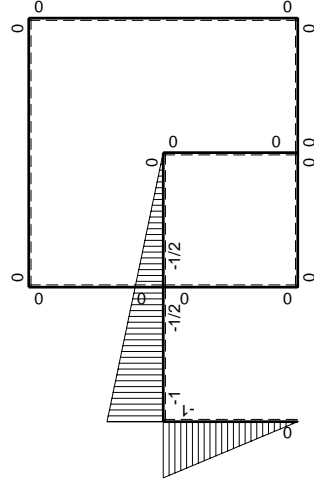






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | $-Fb/EJ$ | $Fx-1/2Fx^2/b$           | $Fb/EJ-1/2Fx/EJ$    | $1-x/b+1/4x^2/b^2$      | $(1/3+3/4)Fb^2/EJ$          | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | $Fb/EJ$  | $1/2Fb-1/2Fx^2/b$        | $1/2Fb/EJ+1/2Fx/EJ$ | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0                   | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0                   | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| ED b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CF b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CG b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                    |          |                          |                     |                         |                             | $7/24Fb^2/EJ$          | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |                     |                         |                             | $-7/24Fb$              |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b \left( \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{3} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{3} b \right) \frac{1}{EJ} = \frac{1}{3} \frac{b}{EJ}$$

$$L_{BA}^{xx} = \int_0^b \left( 1 - 2 \frac{x}{b} + \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ x - \frac{x^2}{b} + \frac{1}{3} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( b - b + \frac{1}{3} b \right) \frac{1}{EJ} = \frac{1}{3} \frac{b}{EJ}$$

$$L_{BC}^{xx} = \int_0^b \left( 1 - \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( b - \frac{1}{2} b + \frac{1}{12} b \right) \frac{1}{EJ} = \frac{7}{12} \frac{b}{EJ}$$

$$L_{CB}^{xx} = \int_0^b \left( \frac{1}{4} + \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{4} x + \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b + \frac{1}{4} b + \frac{1}{12} b \right) \frac{1}{EJ} = \frac{7}{12} \frac{b}{EJ}$$

$$L_{CD}^{xx} = \int_0^b \left( \frac{1}{4} - \frac{1}{2} \frac{x}{b} + \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{4} x - \frac{1}{4} \frac{x^2}{b} + \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{4} b + \frac{1}{12} b \right) \frac{1}{EJ} = \frac{1}{12} \frac{b}{EJ}$$

$$L_{DC}^{xx} = \int_0^b \left( \frac{1}{4} \frac{x^2}{b^2} \right) \frac{1}{EJ} dx = \left[ \frac{1}{12} \frac{x^3}{b^2} \right]_0^b \frac{1}{EJ}$$

$$= \left( \frac{1}{12} b \right) \frac{1}{EJ} = \frac{1}{12} \frac{b}{EJ}$$

$$L_{AB}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x^2}{b^2} - \frac{1}{2} \frac{x^3}{b^3} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{6} \frac{x^3}{b^2} - \frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{6} b - \frac{1}{8} b \right) Fb \frac{1}{EJ} = \frac{1}{24} \frac{Fb^2}{EJ}$$

$$L_{BA}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x}{b} - \frac{x^2}{b^2} + \frac{1}{2} \frac{x^3}{b^3} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{4} \frac{x^2}{b} - \frac{1}{3} \frac{x^3}{b^2} + \frac{1}{8} \frac{x^4}{b^3} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{4} b - \frac{1}{3} b + \frac{1}{8} b \right) Fb \frac{1}{EJ} = \frac{1}{24} \frac{Fb^2}{EJ}$$

$$L_{BC}^{xo} = \int_0^b \left( \frac{x}{b} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx + \int_0^b \left( 1 - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} \frac{x^2}{b} - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ} + \left[ x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb \frac{1}{EJ} + \left( b - \frac{1}{4} b \right) \theta = \frac{13}{12} \frac{Fb^2}{EJ}$$

$$L_{CB}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{1}{2} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx + \int_0^b \left( -\frac{1}{2} - \frac{1}{2} \frac{x}{b} \right) \theta dx$$

$$= \left[ \frac{1}{2} x - \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ} + \left[ -\frac{1}{2} x - \frac{1}{4} \frac{x^2}{b} \right]_0^b \theta$$

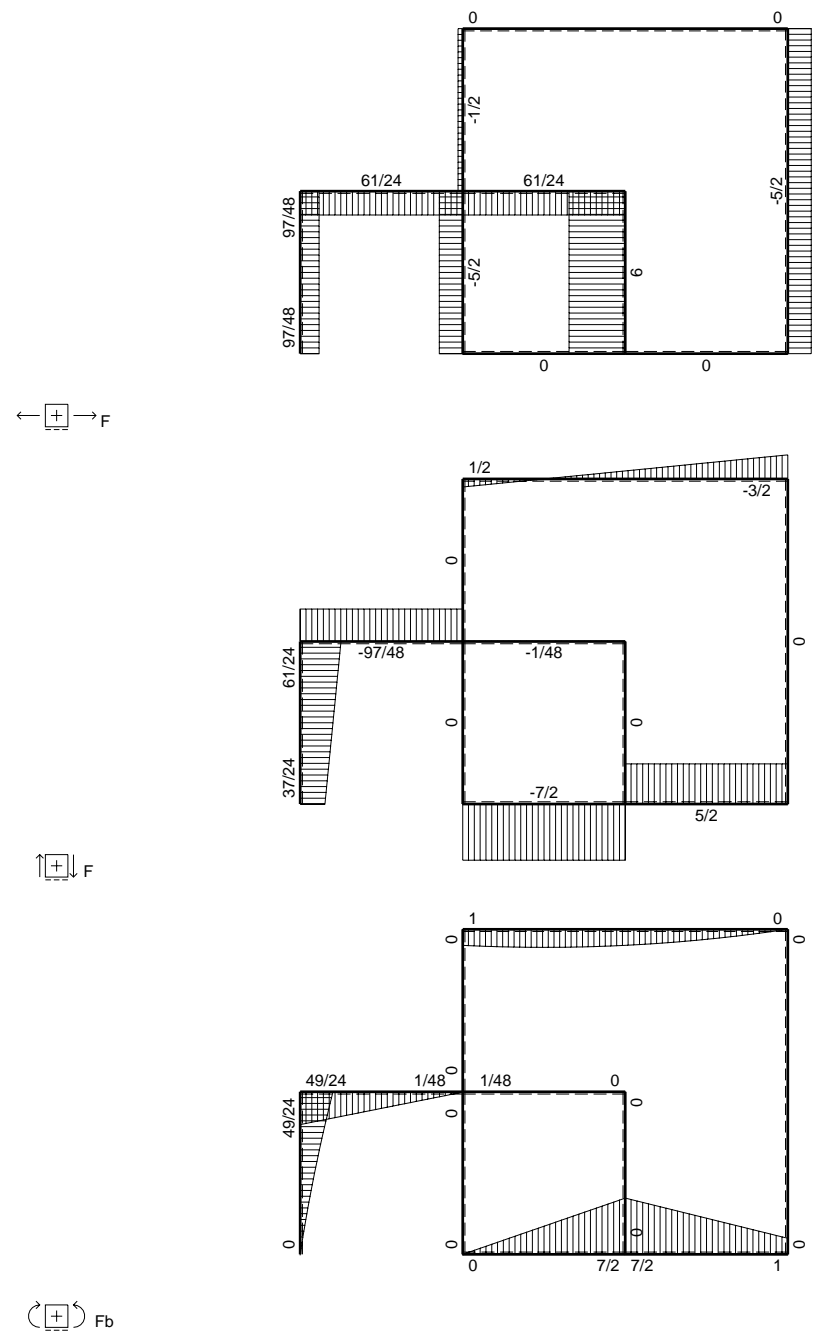
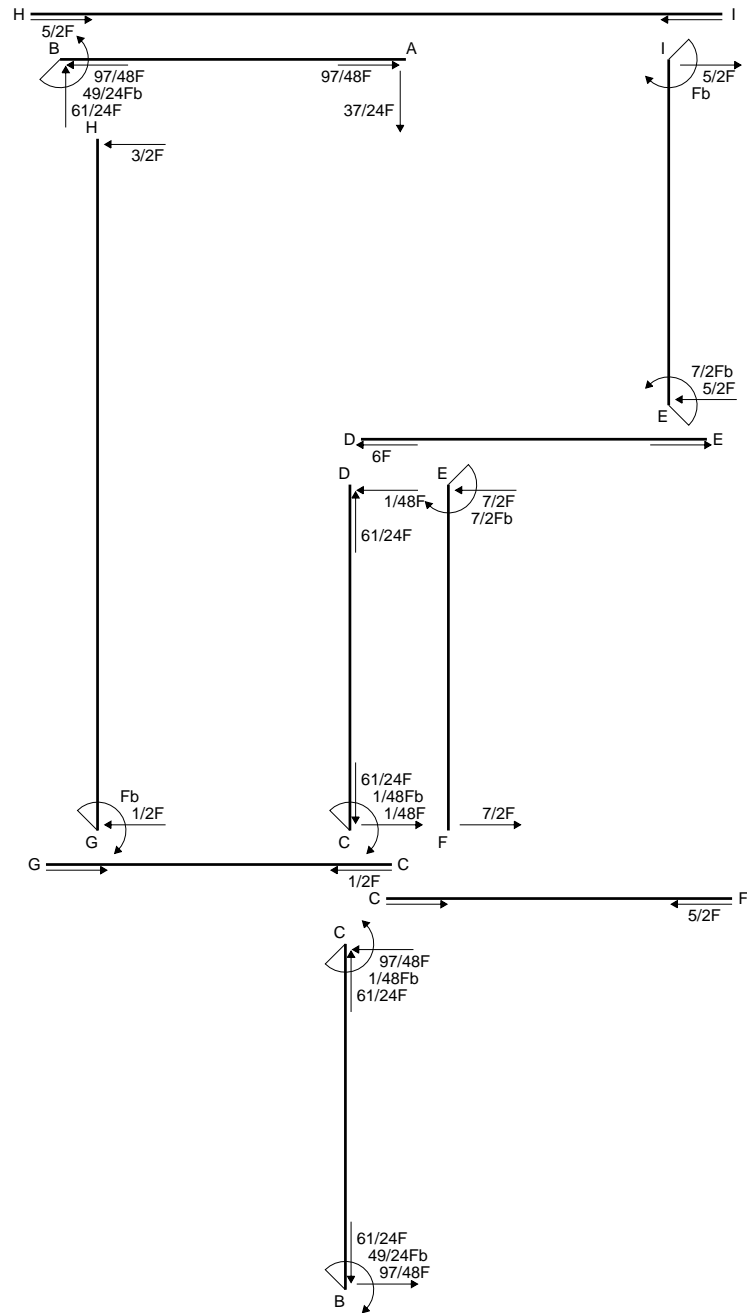
$$= \left( \frac{1}{2} b - \frac{1}{6} b \right) Fb \frac{1}{EJ} + \left( -\frac{1}{2} b - \frac{1}{4} b \right) \theta = \frac{13}{12} \frac{Fb^2}{EJ}$$

$$L_{CD}^{xo} = \int_0^b \left( \frac{1}{2} - \frac{x}{b} + \frac{1}{2} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{2} x - \frac{1}{2} \frac{x^2}{b} + \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{2} b - \frac{1}{2} b + \frac{1}{6} b \right) Fb \frac{1}{EJ} = \frac{1}{6} \frac{Fb^2}{EJ}$$

$$L_{DC}^{xo} = \int_0^b \left( \frac{1}{2} \frac{x^2}{b^2} \right) Fb \frac{1}{EJ} dx = \left[ \frac{1}{6} \frac{x^3}{b^2} \right]_0^b Fb \frac{1}{EJ}$$

$$= \left( \frac{1}{6} b \right) Fb \frac{1}{EJ} = \frac{1}{6} \frac{Fb^2}{EJ}$$





Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |  |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$           |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$           |  |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |  |
| DE b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| ED b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0             | 0                       |                             |                       |  |
| FC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| CF b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| CG b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| GC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                       |  |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0             | 0                       |                             |                       |  |
| A     | cedimento nodo $-H_{1A}u_A$ |                    |          |                          |               |                         | $Fb^2/EJ$                   |                       |  |
|       | totali                      |                    |          |                          |               |                         | $49/24Fb^2/EJ$              | $Xb/EJ$               |  |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |               |                         | $-49/24Fb$                  |                       |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

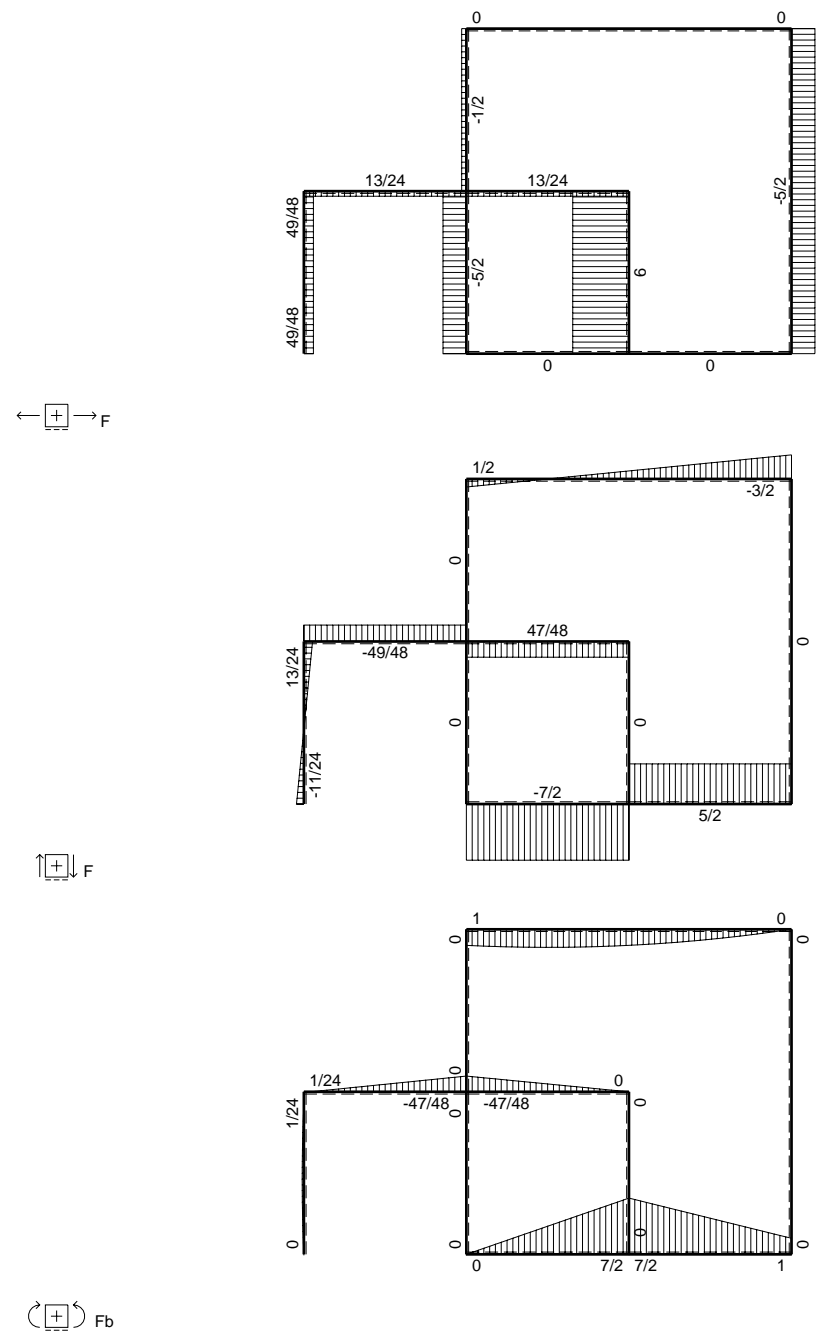
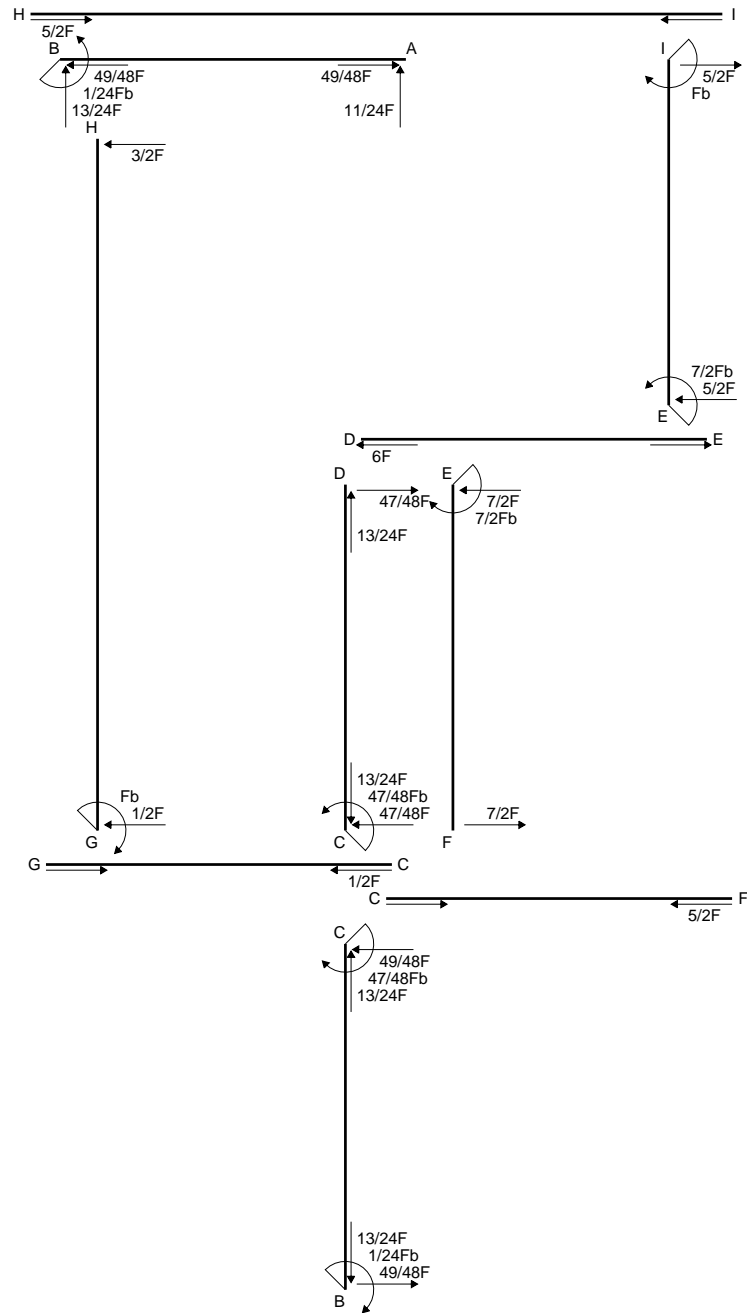
$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

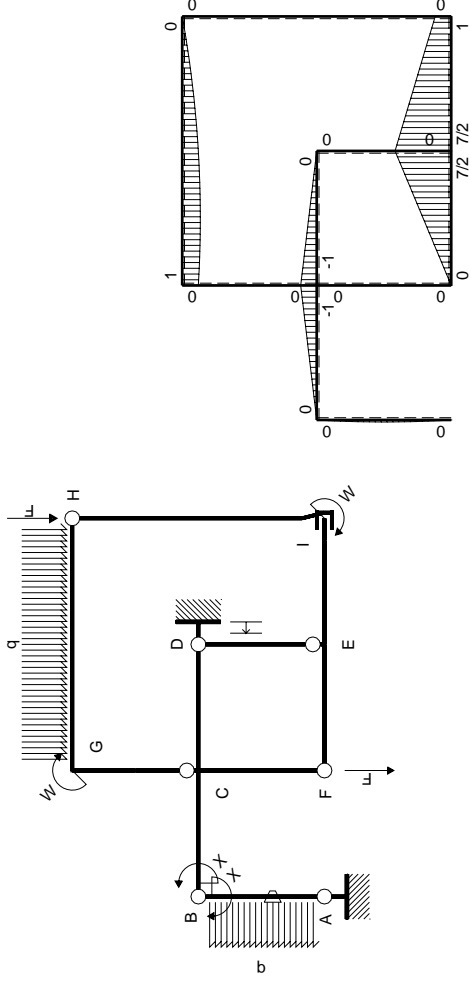
$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

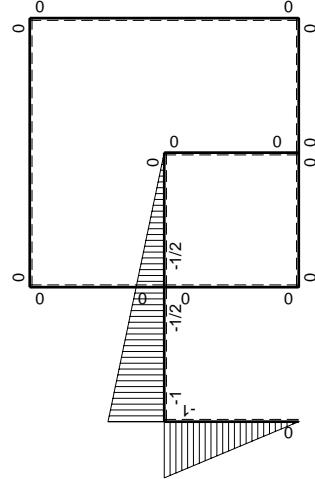






Schema di calcolo iperstatico

$M_0$  flessione da carichi assegnati



$M_x$  flessione da iperstatica  $X=1$

Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$  | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJdx$ |  |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------|-------------------------|-----------------------------|-----------------------|--|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | $-Fb/EJ$ | $1/2Fx^2/b-1/2qx^3/b$    | $Fx/EJ$       | $x^2/b^2$               | $(1/24+1/2)Fb^2/EJ$         | $1/3Xb/EJ$            |  |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | $Fb/EJ$  | $1/2Fx-Fx^2/b+1/2qx^3/b$ | $Fb/EJ-Fx/EJ$ | $1-2x/b+x^2/b^2$        |                             |                       |  |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0             | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$           |  |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0             | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                       |  |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | 0        | $1/2Fb-Fx+1/2Fx^2/b$     | 0             | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+0)Fb^2/EJ$            | $1/12Xb/EJ$           |  |
| DC b  | $1/2x/b$                    | $Fx$               | 0        | $1/2Fx^2/b$              | 0             | $1/4x^2/b^2$            |                             |                       |  |
| DE b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| ED b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0             | 0                       |                             |                       |  |
| FC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| CF b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| CG b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| GC b  | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0             | 0                       |                             |                       |  |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0             | 0                       |                             |                       |  |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0             | 0                       | 0+0                         | 0                     |  |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0             | 0                       |                             |                       |  |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |               |                         | $-Fb^2/EJ$                  |                       |  |
|       | totali                      |                    |          |                          |               |                         | $1/24Fb^2/EJ$               | $Xb/EJ$               |  |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |               |                         | $-1/24Fb$                   |                       |  |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (x/b) \theta dx$$

$$= [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ + [1/2 x^2/b]_0^b \theta$$

$$= (1/6 b - 1/8 b) Fb 1/EJ + (1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx + \int_0^b (-1 + x/b) \theta dx$$

$$= [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ + [-x + 1/2 x^2/b]_0^b \theta$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ + (-b + 1/2 b) \theta = 13/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

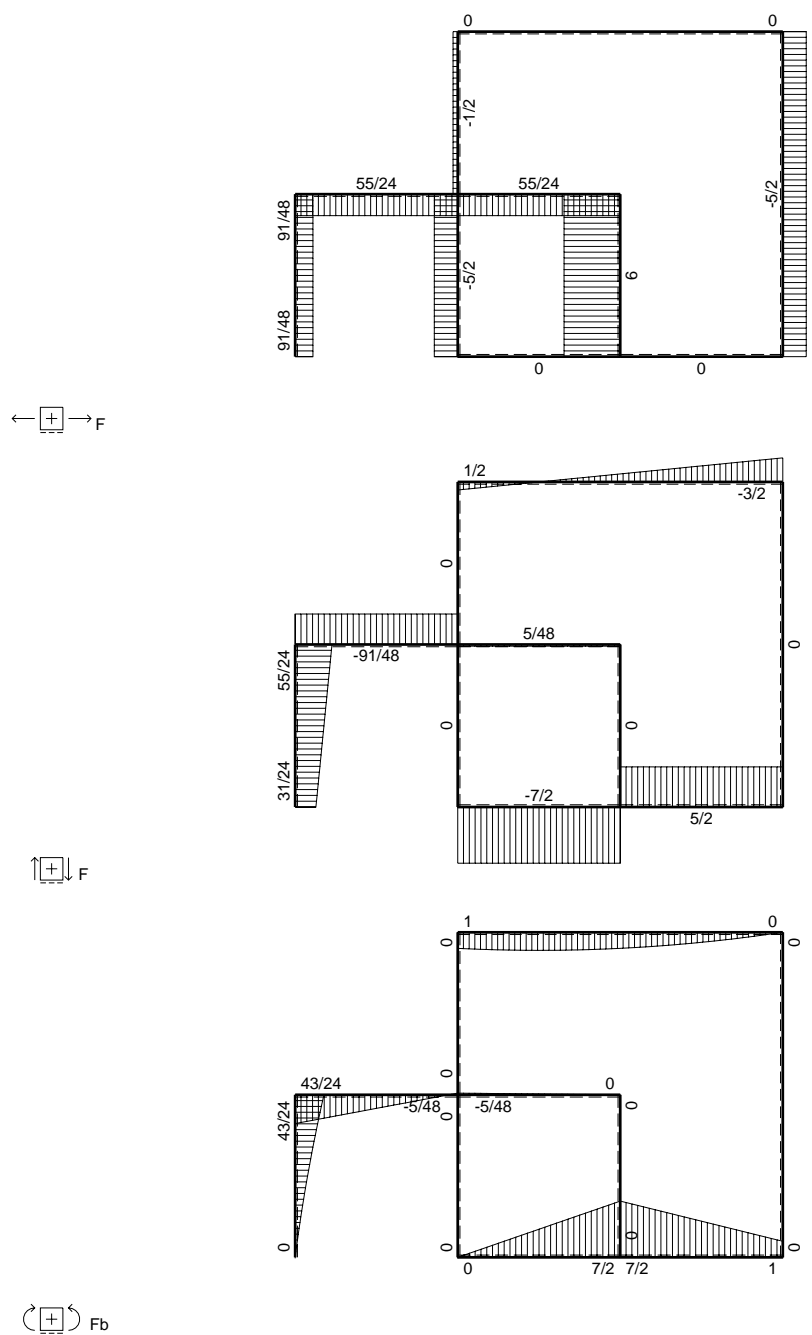
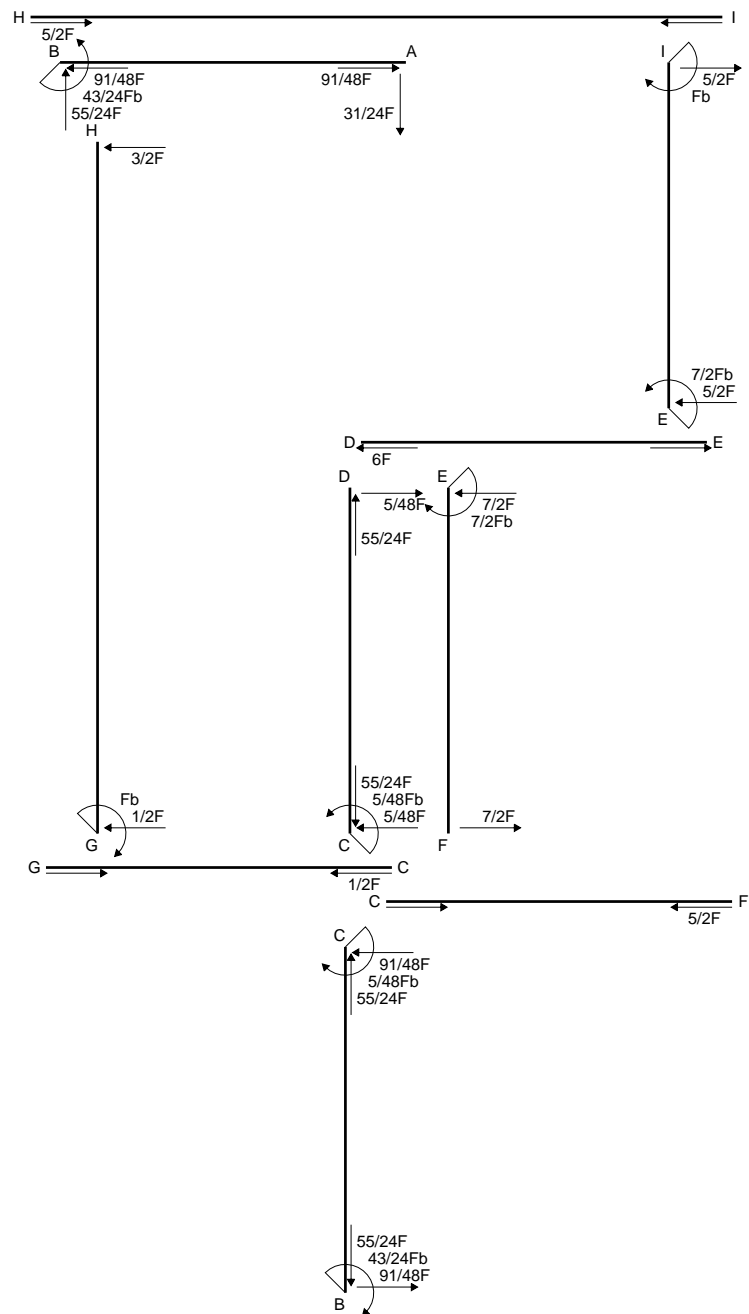
$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx = [1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/6 b) Fb 1/EJ = 1/6 Fb^2/EJ$$





Quadro contributi PLV per iperstatica  $X=W_{BC}$

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$     | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |
| DC b  | $1/2x/b$                    | $Fx$               | $Fb/EJ$  | $1/2Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |
| DE b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| ED b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0                   | 0                       |                             |                        |
| FC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| CF b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| CG b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| GC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                        |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0                   | 0                       |                             |                        |
| A     | cedimento nodo $-H_{1A}u_A$ |                    |          |                          |                     |                         | $Fb^2/EJ$                   |                        |
|       | totali                      |                    |          |                          |                     |                         | $43/24Fb^2/EJ$              | $Xb/EJ$                |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |                     |                         | $-43/24Fb$                  |                        |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

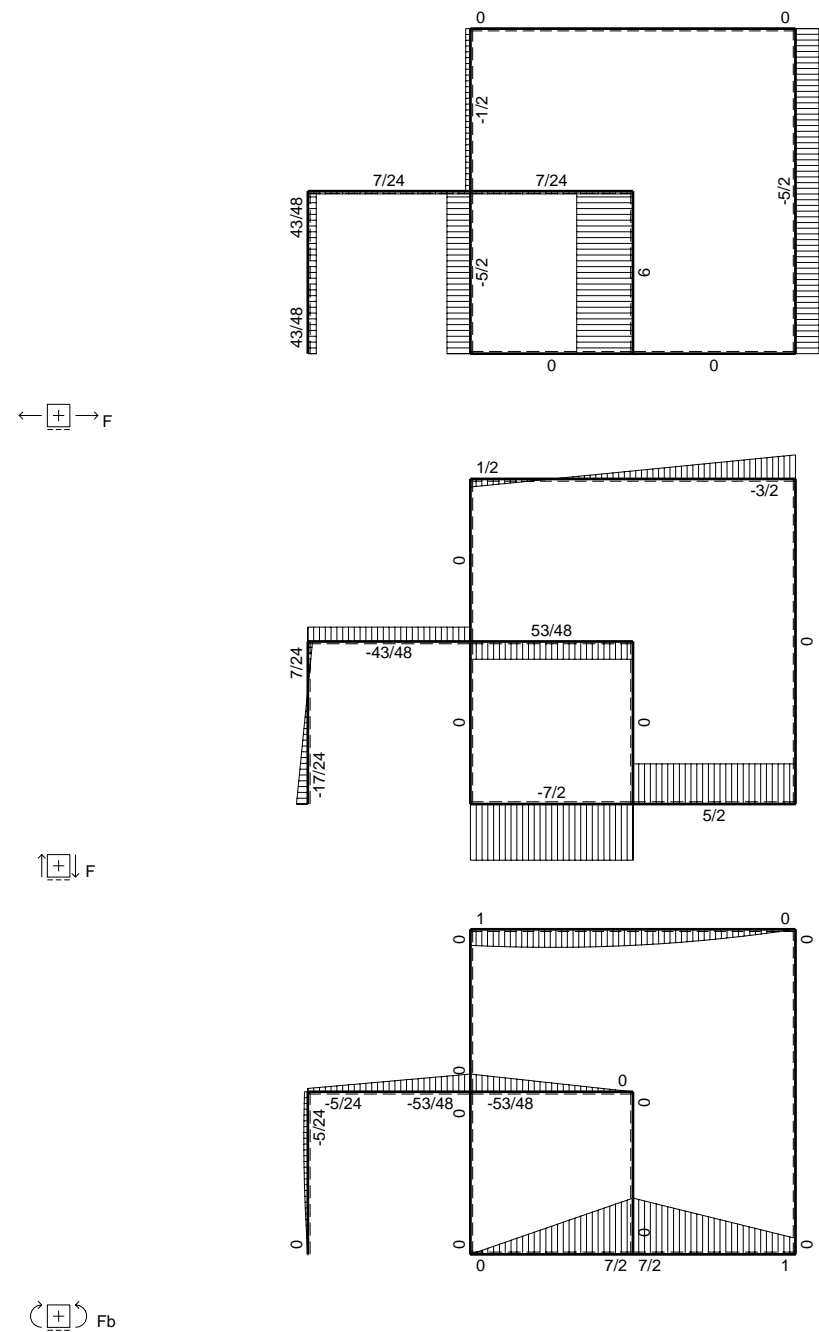
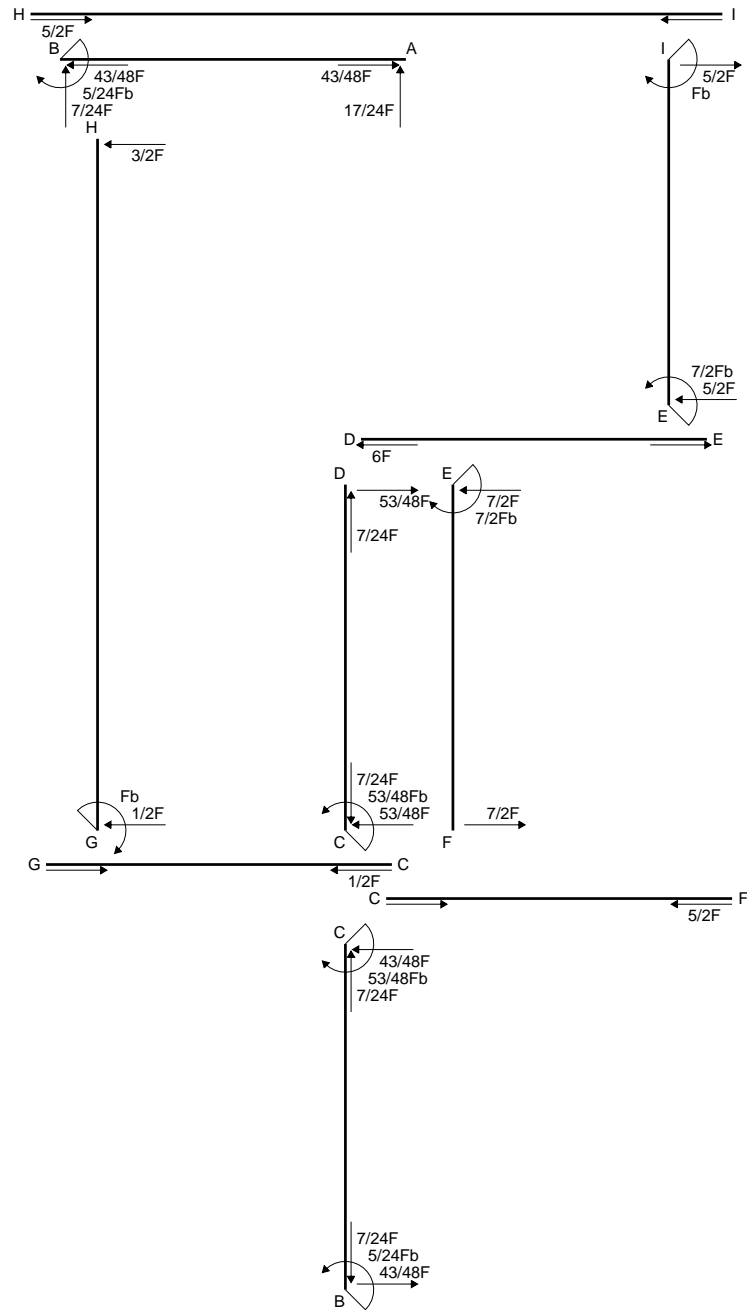
$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$







Quadro contributi PLV per iperstatica  $X=W_{BC}$ 

| →     | $M_x(x)$                    | $M_o(x)$           | $\theta$ | $M_x M_o$                | $M_x \theta$        | $M_x M_x$               | $\int M_x(M_o/EJ+\theta)dx$ | $\int X M_x M_x/EJ dx$ |         |
|-------|-----------------------------|--------------------|----------|--------------------------|---------------------|-------------------------|-----------------------------|------------------------|---------|
| AB b  | $-x/b$                      | $-1/2Fx+1/2qx^2$   | 0        | $1/2Fx^2/b-1/2qx^3/b$    | 0                   | $x^2/b^2$               | $(1/24+0)Fb^2/EJ$           | $1/3Xb/EJ$             |         |
| BA b  | $1-x/b$                     | $1/2Fx-1/2qx^2$    | 0        | $1/2Fx-Fx^2/b+1/2qx^3/b$ | 0                   | $1-2x/b+x^2/b^2$        |                             |                        |         |
| BC b  | $-1+1/2x/b$                 | $-Fx$              | 0        | $Fx-1/2Fx^2/b$           | 0                   | $1-x/b+1/4x^2/b^2$      | $(1/3+0)Fb^2/EJ$            | $7/12Xb/EJ$            |         |
| CB b  | $1/2+1/2x/b$                | $Fb-Fx$            | 0        | $1/2Fb-1/2Fx^2/b$        | 0                   | $1/4+1/2x/b+1/4x^2/b^2$ |                             |                        |         |
| CD b  | $-1/2+1/2x/b$               | $-Fb+Fx$           | $-Fb/EJ$ | $1/2Fb-Fx+1/2Fx^2/b$     | $1/2Fb/EJ-1/2Fx/EJ$ | $1/4-1/2x/b+1/4x^2/b^2$ | $(1/6+1/4)Fb^2/EJ$          | $1/12Xb/EJ$            |         |
| DC b  | $1/2x/b$                    | $Fx$               | $Fb/EJ$  | $1/2Fx^2/b$              | $1/2Fx/EJ$          | $1/4x^2/b^2$            |                             |                        |         |
| DE b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| ED b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| EF b  | 0                           | $7/2Fb-7/2Fx$      | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| FE b  | 0                           | $-7/2Fx$           | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| FC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| CF b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| CG b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| GC b  | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| GH 2b | 0                           | $Fb+1/2Fx-1/2qx^2$ | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| HG 2b | 0                           | $-3/2Fx+1/2qx^2$   | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| HI 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| IH 2b | 0                           | 0                  | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| IE b  | 0                           | $Fb+5/2Fx$         | 0        | 0                        | 0                   | 0                       | 0+0                         | 0                      |         |
| EI b  | 0                           | $-7/2Fb+5/2Fx$     | 0        | 0                        | 0                   | 0                       |                             |                        |         |
| D     | cedimento nodo $-H_{1D}u_D$ |                    |          |                          |                     |                         |                             | $-Fb^2/EJ$             |         |
|       | totali                      |                    |          |                          |                     |                         |                             | $-5/24Fb^2/EJ$         | $Xb/EJ$ |
|       | iperstatica $X=W_{BC}$      |                    |          |                          |                     |                         |                             | $5/24Fb$               |         |

Sviluppi di calcolo iperstatica

$$L_{AB}^{xx} = \int_0^b (x^2/b^2) 1/EJ dx = [1/3 x^3/b^2]_0^b 1/EJ$$

$$= (1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BA}^{xx} = \int_0^b (1 - 2x/b + x^2/b^2) 1/EJ dx = [x - x^2/b + 1/3 x^3/b^2]_0^b 1/EJ$$

$$= (b - b + 1/3 b) 1/EJ = 1/3 b/EJ$$

$$L_{BC}^{xx} = \int_0^b (1 - x/b + 1/4 x^2/b^2) 1/EJ dx = [x - 1/2 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (b - 1/2 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CB}^{xx} = \int_0^b (1/4 + 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x + 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b + 1/4 b + 1/12 b) 1/EJ = 7/12 b/EJ$$

$$L_{CD}^{xx} = \int_0^b (1/4 - 1/2 x/b + 1/4 x^2/b^2) 1/EJ dx = [1/4 x - 1/4 x^2/b + 1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/4 b - 1/4 b + 1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{DC}^{xx} = \int_0^b (1/4 x^2/b^2) 1/EJ dx = [1/12 x^3/b^2]_0^b 1/EJ$$

$$= (1/12 b) 1/EJ = 1/12 b/EJ$$

$$L_{AB}^{xo} = \int_0^b (1/2 x^2/b^2 - 1/2 x^3/b^3) Fb 1/EJ dx = [1/6 x^3/b^2 - 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/6 b - 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BA}^{xo} = \int_0^b (1/2 x/b - x^2/b^2 + 1/2 x^3/b^3) Fb 1/EJ dx = [1/4 x^2/b - 1/3 x^3/b^2 + 1/8 x^4/b^3]_0^b Fb 1/EJ$$

$$= (1/4 b - 1/3 b + 1/8 b) Fb 1/EJ = 1/24 Fb^2/EJ$$

$$L_{BC}^{xo} = \int_0^b (x/b - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x^2/b - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CB}^{xo} = \int_0^b (1/2 - 1/2 x^2/b^2) Fb 1/EJ dx = [1/2 x - 1/6 x^3/b^2]_0^b Fb 1/EJ$$

$$= (1/2 b - 1/6 b) Fb 1/EJ = 1/3 Fb^2/EJ$$

$$L_{CD}^{xo} = \int_0^b (1/2 - x/b + 1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (1/2 - 1/2 x/b) \theta dx$$

$$= [1/2 x - 1/2 x^2/b + 1/6 x^3/b^2]_0^b Fb 1/EJ + [1/2 x - 1/4 x^2/b]_0^b \theta$$

$$= (1/2 b - 1/2 b + 1/6 b) Fb 1/EJ + (1/2 b - 1/4 b) \theta = 5/12 Fb^2/EJ$$

$$L_{DC}^{xo} = \int_0^b (1/2 x^2/b^2) Fb 1/EJ dx + \int_0^b (-1/2 x/b) \theta dx = [1/6 x^3/b^2]_0^b Fb 1/EJ + [-1/4 x^2/b]_0^b \theta$$

$$= (1/6 b) Fb 1/EJ + (-1/4 b) \theta = 5/12 Fb^2/EJ$$